THE FINAL FORM OF BALANCE EQUATIONS

The balance equations are

$$\begin{aligned}\frac{\partial n_k}{\partial t} &= \left\langle \frac{\partial n_k}{\partial t} \right\rangle_{\mathrm{coll}}, \\ \frac{3}{2} n_k \frac{\partial T_k}{\partial t} &= \frac{9}{2}\Omega^2 \eta_k + \left\langle \frac{\partial}{\partial t}\left(\frac{3}{2} n_k T_k\right)\right\rangle_{\mathrm{coll}} + \left(\frac{9}{8} m_k \Omega^2 x^2 - \frac{3T_k}{2}\right)\left\langle \frac{\partial n_k}{\partial t} \right\rangle_{\mathrm{coll}},\end{aligned} \tag{1}$$

where

$$\begin{aligned}\left\langle \frac{\partial n_1}{\partial t} \right\rangle_{\mathrm{coll}} &= \frac{1}{2}\sum_{i+j>2} K_{ij}^{\mathrm{frag}} \cdot n_i n_j - n_1 \sum_j K_{1j}^{\mathrm{agg}} \cdot n_j, \\ \left\langle \frac{\partial n_k}{\partial t} \right\rangle_{\mathrm{coll}} &= \frac{1}{2}\sum_{i+j=k} K_{ij}^{\mathrm{agg}} \cdot n_i n_j - n_k \sum_j \left(K_{kj}^{\mathrm{agg}} + K_{kj}^{\mathrm{frag}}\right) \cdot n_j, \quad k>1,\end{aligned} \tag{2}$$

and

$$\begin{aligned}\left\langle \frac{\partial}{\partial t}\left(\frac{3}{2} n_1 T_1\right)\right\rangle_{\mathrm{coll}} &= \frac{1}{2}\sum_{i+j>2} K_{ij}^{\mathrm{frag}} \cdot n_i n_j \cdot \mathcal{G}_{ij}^{\mathrm{frag}} - n_1 \sum_j \left(K_{1j}^{\mathrm{agg}} \cdot \mathcal{L}_{1j}^{\mathrm{agg}} + K_{1j}^{\mathrm{res}} \cdot \mathcal{L}_{1j}^{\mathrm{res}}\right) \cdot n_j \\ \left\langle \frac{\partial}{\partial t}\left(\frac{3}{2} n_k T_k\right)\right\rangle_{\mathrm{coll}} &= \frac{1}{2}\sum_{i+j=k} K_{ij}^{\mathrm{agg}} \cdot n_i n_j \cdot \mathcal{G}_{ij}^{\mathrm{agg}} - n_k \sum_j \left(K_{kj}^{\mathrm{agg}} \cdot \mathcal{L}_{kj}^{\mathrm{agg}} + K_{kj}^{\mathrm{res}} \cdot \mathcal{L}_{kj}^{\mathrm{res}} + K_{kj}^{\mathrm{frag}} \cdot \mathcal{L}_{kj}^{\mathrm{frag}}\right) \cdot n_j, \quad k>1.\end{aligned} \tag{3}$$

The kernels for each specific type of collision are

$$\begin{aligned}K_{ij}^{\mathrm{agg}} &= K_{ij} \cdot \left[1 - \left(1 + C_{ij} g_{\mathrm{agg}}^2\right)\exp\left(-C_{ij} g_{\mathrm{agg}}^2\right)\right], \\ K_{ij}^{\mathrm{res}} &= K_{ij} \cdot \left[\left(1 + C_{ij} g_{\mathrm{agg}}^2\right)\exp\left(-C_{ij} g_{\mathrm{agg}}^2\right) - \exp\left(-C_{ij} g_{\mathrm{frag}}^2\right)\right], \\ K_{ij}^{\mathrm{frag}} &= K_{ij} \cdot \exp\left(-C_{ij} g_{\mathrm{frag}}^2\right), \\ K_{ij} &= \frac{2\pi^2 \sigma_{ij}^2 \alpha_{ij}}{n_i n_j C_{ij}^2}\left(\frac{\pi}{A_{ij}}\right)^{3/2}.\end{aligned} \tag{4}$$

The amounts of energy lost and gained on average per single collision, for each specific type of collision are

$$\begin{aligned}\mathcal{G}_{ij}^{\mathrm{agg}} &= \frac{3m_k}{4A_{ij}}\left[1 + \frac{R_{ij}^2}{3A_{ij}C_{ij}}\left(1 - f_{ij}^{\mathrm{agg}}\right)\right], \\ \mathcal{L}_{ij}^{\mathrm{agg}} &= \frac{3m_k}{4A_{ij}}\left[1 + \frac{\left(R_{ij} + 2\mu_j A_{ij}\right)^2}{3A_{ij}C_{ij}}\left(1 - f_{ij}^{\mathrm{agg}}\right)\right], \\ \mathcal{L}_{ij}^{\mathrm{res}} &= \left[\frac{1-\varepsilon^2}{2}\frac{\mu_{ij}^2}{C_{ij} m_k} + \frac{1+\varepsilon}{2}\frac{\mu_{ij} R_{ij}}{A_{ij} C_{ij}}\right]\left(1 + f_{ij}^{\mathrm{res}}\right), \\ \mathcal{G}_{ij}^{\mathrm{frag}} &= \frac{i+j}{4\pi} \cdot \frac{3m_k}{2A_{ij}}\left(1 + f_{ij}^{\mathrm{frag},1}\right), \\ \mathcal{L}_{ij}^{\mathrm{frag}} &= \frac{3m_k}{4\pi A_{ij}}\left(1 + f_{ij}^{\mathrm{frag}}\right),\end{aligned} \tag{5}$$

where the correction functions are

$$
\begin{aligned}
f_{ij}^{\text{agg}} &= \frac{C_{ij} g_{\text{agg}}^2}{2} \frac{C_{ij} g_{\text{agg}}^2 \exp\left(-C_{ij} g_{\text{agg}}^2\right)}{1 - \left(1 + C_{ij} g_{\text{agg}}^2\right) \exp\left(-C_{ij} g_{\text{agg}}^2\right)}, \\
f_{ij}^{\text{res}} &= \frac{C_{ij} g_{\text{agg}}^2}{2} \frac{C_{ij} g_{\text{agg}}^2 \exp\left(-C_{ij} g_{\text{agg}}^2\right) - 2(g_{\text{frag}}/g_{\text{agg}})^2 \exp\left(-C_{ij} g_{\text{frag}}^2\right)}{\left(1 + C_{ij} g_{\text{agg}}^2\right) \exp\left(-C_{ij} g_{\text{agg}}^2\right) - \exp\left(-C_{ij} g_{\text{frag}}^2\right)}, \\
f_{1j}^{\text{frag},1} &= \frac{R_{1j}^2}{6A_{1j}C_{1j}} \left(2 + C_{1j} g_{\text{frag}}^2\right) - \frac{2\gamma_{1j} A_{1j}}{3C_{1j}} \left(1 + C_{1j} g_{\text{frag}}^2 + C_{1j}^2 g_{\text{frag}}^4\right), \\
f_{ij}^{\text{frag}} &= \frac{\left(R_{ij} + 2\mu_j A_{ij}\right)^2}{6A_{ij}C_{ij}} \left(2 + C_{ij} g_{\text{frag}}^2\right).
\end{aligned}
\tag{6}
$$

The A_{ij}, R_{ij} and C_{ij} terms are the functions of temperatures, given by

$$
\begin{aligned}
A_{ij} &= \frac{m_i}{2T_i} + \frac{m_j}{2T_j}, \\
R_{ij} &= \frac{\mu}{T_j} - \frac{\mu}{T_i}, \\
C_{ij} &= \frac{m_i m_j}{2T_i m_j + 2T_j m_i}, \\
\frac{R_{ij}^2}{A_{ij}C_{ij}} &= \frac{4\mu^2}{m_i m_j} \frac{\left(T_j - T_i\right)^2}{T_i T_j}, \\
\frac{\left(R_{ij} + 2\mu_j A_{ij}\right)^2}{A_{ij}C_{ij}} &= \frac{4T_i}{T_j} \frac{m_j}{m_i}.
\end{aligned}
\tag{7}
$$

We can also introduce the functions

$$
\begin{aligned}
Q_{ij} &= C_{ij} g_{\text{agg}}^2, \\
P_{ij} &= C_{ij} g_{\text{frag}}^2.
\end{aligned}
\tag{8}
$$

BINARY SYSTEM

Let us consider a system of only monomers and dimers, e.g a system where the aggregates cannot exceed the size of $k = 2$. The collisional terms for such a system are written as

$$
\begin{aligned}
\left\langle \frac{\partial n_1}{\partial t} \right\rangle_{\text{coll}} &= \frac{1}{2} K_{22}^{\text{frag}} \cdot n_2^2 + K_{12}^{\text{frag}} \cdot n_1 n_2 - K_{11}^{\text{agg}} \cdot n_1^2, \\
\left\langle \frac{\partial n_2}{\partial t} \right\rangle_{\text{coll}} &= \frac{1}{2} K_{11}^{\text{agg}} \cdot n_1^2 - K_{12}^{\text{frag}} \cdot n_1 n_2 - K_{22}^{\text{frag}} \cdot n_2^2, \\
\left\langle \frac{\partial}{\partial t} \left(\frac{3}{2} n_1 T_1\right) \right\rangle_{\text{coll}} &= \frac{1}{2} K_{22}^{\text{frag}} \cdot n_2^2 \cdot \mathcal{G}_{22}^{\text{frag}} + K_{12}^{\text{frag}} \cdot n_1 n_2 \cdot \mathcal{G}_{12}^{\text{frag}} - (K_{11}^{\text{agg}} \cdot \mathcal{L}_{11}^{\text{agg}} + K_{11}^{\text{res}} \cdot \mathcal{L}_{11}^{\text{res}}) \cdot n_1^2 - K_{12}^{\text{res}} \cdot \mathcal{L}_{12}^{\text{res}} \cdot n_1 n_2, \\
\left\langle \frac{\partial}{\partial t} \left(\frac{3}{2} n_2 T_2\right) \right\rangle_{\text{coll}} &= \frac{1}{2} K_{11}^{\text{agg}} \cdot n_1^2 \cdot \mathcal{G}_{11}^{\text{agg}} - \left(K_{12}^{\text{res}} \cdot \mathcal{L}_{12}^{\text{res}} + K_{12}^{\text{frag}} \cdot \mathcal{L}_{12}^{\text{frag}}\right) \cdot n_1 n_2 - \left(K_{22}^{\text{res}} \cdot \mathcal{L}_{22}^{\text{res}} + K_{22}^{\text{frag}} \cdot \mathcal{L}_{22}^{\text{frag}}\right) \cdot n_2^2,
\end{aligned}
\tag{9}
$$

where we have removed the terms containing K_{12}^{agg} and K_{22}^{agg}, since they correspond to creation of particles of size larger than $k = 2$. Needs recalculation!