Assignment 1 - Computational Mathematics
Week 1 & 2

Instructions

For each problem, please include:
- Source code with comments explaining all important steps.
- A screenshot of the program output and explanation.
- Include graphs or tables if the task requires them.

Save your answers as a PDF report and submit it to the Moodle.

Task 1: Graphical method and absolute error.

(You can replace the given equations with more complex ones if you prefer)

Problem:

1. Plot the graph of the function $f(x)=x^3-2x^2-5$ in the range $x\in[1,4]$.
2. Using the graph, find the approximate value of the root.
3. Calculate the value of $f(x)$, where x is the found approximate value of the root.
4. Find the absolute error between the approximate value of the root and the true value (using any other method for the exact solution).

Required:

1. Plot a graph.
2. Give a detailed calculation of the absolute error.
3. Explain why the graphical root search method is only approximate.

Task 2: Comparison of root finding methods.

Problem:

1. Find the root of the equation $f(x)=e^x-2x-3$ in the interval [0,2] using the following methods:
    - Bisection method.
    - Secant method.
2. For each method, measure the number of iterations required to achieve an accuracy of 10^{-6}.
3. Calculate the relative error for both methods with respect to the exact value of the root.

Required:

1. Write functions for each method.
2. Print the number of iterations and the exact value of the root.
3. Explain which method is more efficient and why.

Task 3: Newton-Raphson Method and Error Estimation.

Problem:

1. Using the Newton-Raphson method, find the root of the equation $f(x)=x^2-3x+2$, starting with the initial guess $x_0=2.5$.
2. Calculate the absolute and relative errors at each iteration step.
3. Plot a convergence graph, where the iteration number is plotted on the x-axis and the absolute error is plotted on the y-axis.

Required:

1. Provide a table of iterations with the current guess, absolute and relative errors.
2. Explain how the choice of the initial guess affects the convergence of the method.

Task 4: Muller's Method and Complex Roots.

Problem:

1. Find one of the roots (including complex roots) for the function $f(x)=x^3+x^2+x+1$ using Muller's method, starting with the initial approximations $x_0=-1$, $x_1=0$, $x_2=1$.
2. Check the result by substituting the found root value into the function.
3. Calculate the absolute error between the calculated value of f(x) and 0.

Required:

1. Explain how Muller's method handles complex roots.
2. Provide an interpretation of the result.

Task 5: Error estimation when using False position method.

Problem:

1. Find the root of the equation $f(x)=x^2-2^x$ using the False position method.
2. Calculate the absolute and relative errors after each iteration.
3. Plot a graph of the absolute error as a function of the iteration number.

Required:

1. Explain why the false position method converges more slowly than the Newton-Raphson method.

Task 6: Iteration method and errors.

Problem:

1. Solve the equation $f(x)=x^2-6x+5$ using the iteration method. Transform the equation into the form $x=g(x)$.
2. Perform 10 iterations, starting with the initial value $x_0=0.5$.
3. Find the absolute error at each iteration, comparing the result with the true root.

Required:

1. Explain how the choice of function g(x) affects convergence.

P.S. Please be prepared to explain your code/solution/answers.