Assignment 2 - Computational Mathematics
Week 3

Instructions

For each problem, please include:
- Source code with comments explaining all important steps.
- A screenshot of the program output and explanation.
- Include graphs or tables if the task requires them.

Save your answers as a PDF report and submit it to the Moodle.

Task 1: Jacobi Method and Convergence Analysis.

Problem:

1. Implement the Jacobi method to solve the system of linear equations:

$$10x_1 - x_2 - 2x_3 = 5$$
$$-2x_1 + 10x_2 - x_3 = -6$$
$$-x_1 - 2x_2 + 10x_3 = 15$$

2. Start with an initial guess x_0=[0,0,0].
3. Check the convergence of the system using the diagonal dominance criterion.

Required:

1. Provide a detailed calculation of the iterations.
2. Check whether the diagonal dominance criterion is satisfied.
3. Explain how convergence depends on the structure of the system.

Task 2: Gaussian method with choice of leading element.

Problem:

1. Using Gauss's method, solve the system:

$$2x_1 + 3x_2 + x_3 = 10$$
$$4x_1 + 11x_2 + -1x_3 = 33$$
$$-2x_1 + x_2 + 7x_3 = 15$$

2. Enable leading element selection to minimize numerical errors.

Required:

1. Print the upper triangular matrix and the result of the inverse substitution.
2. Explain why the choice of the pivot is important for numerical stability.

Task 3: Gauss-Jordan method.

Problem:

1. Solve the following system using the Gauss-Jordan method:

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

2. Transform the augmented matrix of the system into diagonal form.

Required:

1. Provide the final diagonal matrix and the values of the variables.
2. Explain the advantages of the Gauss-Jordan method over the Gauss method.

Task 4: Gauss-Seidel method and stopping criterion.

Problem:

1. Implement the Gauss-Seidel method for the system:

$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$6x_1 + 3x_2 + 12x_3 = 36$$

2. Start with an initial guess x_0=[0,0,0] and use a precision of 10^{-5}.

Required:

1. Provide a table of iterations with current values of variables.
2. Explain how the stopping criterion (accuracy) affects execution time.

Task 5: Relaxation method.

Problem:

1. Solve the following system using the relaxation method with parameter ω=1.1:

$$5x_1 + x_2 + x_3 = 10$$
$$x_1 + 4x_2 + 2x_3 = 12$$
$$x_1 + x_2 + 5x_3 = 15$$

2. Compare the execution time and number of iterations for ω=1.1 and ω=1.5.

Required:

1. Provide solutions for both values of ω.
2. Explain how the relaxation parameter affects the convergence of the method.

Task 6: Ill-conditioned systems.

Problem:

1. Solve the following ill-conditioned system:

$$1.001x_1 + 0.999x_2 = 2$$
$$1.002x_1 + 1.000x_2 = 2.001$$

2. Find the solution analytically and numerically.
3. Show how a small change in the coefficients affects the result.

Required:

1. Derive an analytical and numerical solution.
2. Explain why ill-conditioned systems are sensitive to change.

P.S. Please be prepared to explain your code/solution/answers.