(1.1) $\psi(n,p) = \dfrac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p} + k(n,p)}$

if $p > p_0$, $\exists p_0$

$\dfrac{\varphi(n)}{p} < \dfrac{\varphi(n)}{p_0}$; $k(n,p) = C\log p > k(n,p_0) = C\log p_0$

s.t. $\dfrac{\varphi(n)}{p_0}$ 下降速度較 $k(n,p_0)$ 上升速度快.

$$\Rightarrow \psi(n,p) \le \frac{\sigma(n)+\varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p} + k(n,p)} < \frac{\sigma(n)+\varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p_0} + k(n,p_0)} = \psi(n,p_0)$$ #.

(1.2)

$$\varepsilon(n,p') \le \frac{\sigma(n)+\varphi(n)}{p'\left(\sigma(n) + \frac{\varphi(n)}{p'} + k(n,p')\right)} = \frac{\sigma(n)+\varphi(n)}{p'\sigma(n) + \varphi(n) + p'k(n,p')}$$

if $p' > p$, $\sigma(n) \ge 0$, $k(n,p') \ge 0$

$$p'\sigma(n) + \varphi(n) + p'k(n,p') \ge p\sigma(n) + \varphi(n) + pk(n,p)$$

$$\Rightarrow \varepsilon(n,p') \le \frac{\sigma(n)+\varphi(n)}{p'\sigma(n)+\varphi(n)+p'k(n,p')} \le \frac{\sigma(n)+\varphi(n)}{p\sigma(n)+\varphi(n)+pk(n,p)} = \varepsilon(n,p)$$ #

(1.3) $n = 1{,}000{,}000$

$\chi = 10e^{-9}$ s

$\lambda = 0.0001$ s

execution time in finding maximum is

$$(\lceil \tfrac{n}{p} \rceil - 1)\chi + \lceil \log p \rceil (\lambda + \chi)$$

```
T-seq = n * χ
def parallel_time (p):
    levels = math.ceil ( math.log2 (p))
    T-comp = (math.ceil ( n/p ) -1) * χ
    T-comm = levels * ( λ + χ)
    return T-comp + T-comm
dep speedup (p):
    return T-seq / parallel-time (p)

p = list (range (2, 17))

speedup = { p: speedup(p) for p in P }.
```

P	speedup($\psi(n,p)$)
1	x
2	1.96
3	2.83
4	3.70
5	4.35
6	5.08
7	5.79
8	6.45
9	6.62
10	7.14
11	7.64
12	8.11
13	8.55
14	8.97
15	9.37
16	9.76

(1.4) $\frac{\sigma(n)}{\sigma(n)+\varphi(n)} = 5\%$, $p = 10$

$$\psi \le \frac{\sigma(n)+\varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p} + k(n,p)}$$

By Amdahl's Law.

$$f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)} = 0.05$$

$$\psi \le \frac{1}{f+(1-f)/p} = \frac{1}{0.05+0.95/10} = \frac{1}{0.145} \approx 6.9$$

(1.5) $f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)} = 0.06$

$$\psi \le \frac{1}{0.06+0.94/p} \le 10$$

$$1 \le 0.6 + 9.4/p$$

$$0.4p \le 9.4$$

$$p \le 23.5$$

The minimize $p = 24$.

(1.6)

假設理想情況 $p \to \infty$.

$$f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)}$$

$$\psi \le \lim_{p\to\infty} \frac{\sigma(n)+\varphi(n)}{\sigma(n)+\frac{\varphi(n)}{p}} = \frac{\sigma(n)+\varphi(n)}{\sigma(n)} = \frac{1}{f} = 50$$

$$f = \frac{1}{50} = 0.02$$

The maximum fraction of executive time that can be spend performing inherently sequential operations is 2%.

(1.7) $f = \frac{\sigma(n)}{\sigma(n)+\varphi(n)}$

By Amdahl's Law

$$\psi \le \frac{1}{f+(1-f)/p} = \frac{1}{f+(1-f)/10} \le 9.$$

$$1 \le 9(f+(1-f)/10)$$

$$1 \le \cancel{89}\,81 f$$

$$\Rightarrow f = \cancel{\frac{1}{89}}\ \frac{1}{81}$$

(7.8) By Barsis's Law

$\sigma(n) = 9$
$\varphi(n) = 233$
$p = 16$

$$s = \frac{\sigma(n)}{\sigma(n) + \varphi(n)/p} = \frac{9}{9 + 233/16} = 0.38196$$

$$\psi = \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p}} = s + p(1-s) = s + 16(1-s) = 10.27$$

(7.9) $p = 40$

假設 $\sigma(n) = 1s$
$\varphi(n) = 99s$

$$s = \frac{\sigma(n)}{\sigma(n) + \frac{\varphi(n)}{p}} = \frac{1s}{1s + \frac{99s}{40}} = \frac{\frac{40}{40}s}{\frac{40+99}{40}s} = \frac{40}{139}$$

$$\psi = s + p(1-s) = \frac{40}{139} + 40\left(1 - \frac{40}{139}\right)$$
$$= \frac{40}{139} + 40 \cdot \frac{99}{139} \doteq 28.78$$

(7.10) Karp-Flatt matrix

	I	II	III	IV	V	VI
2	0.198	0.038	0.058	0.02	0.149	0.031
3	0.201	0.070	0.060	0.02	0.152	0.032
4	0.200	0.079	0.060	0.03	0.153	0.033
5	0.200	0.090	0.060	0.03	0.155	0.033
6	0.200	0.100	0.060	0.03	0.155	0.033
7	0.200	0.110	0.060	0.03	0.156	0.033
8	0.200	0.121	0.060	0.04	0.157	0.033

program I : (B)
program II : (C)
program III : (B)
program IV : (C)
program V : (C)
program VI : (B)

(7.11) In Amdahl's Law, the maximum speedup converges to $1/f$ because $f = \frac{\sigma(n)}{\sigma(n) + \varphi(n)}$ is constrained by the inherently sequential portion of the workload. In contrast, Gustafson-Barsis's Law accounts for the number of processor p, showing that $s = \frac{\sigma(n)}{\sigma(n) + \frac{\varphi(n)}{p}}$ increase with p without limit.

(7.12) No. In Amdahl's Law, even with many processors, the speedup is limited by the sequential portion of the problem. If the significant part of the communication cannot be parallelized, adding more processor won't help reduce the runtime below a certain threshold.

(7.13) Iso. relationship $n \geq f(p)$; Scalability function $M(f(p))/p$

(a) $n \geq Cp$; $M(n) = n^2$
$M(f(p))/p = C^2p^2/p = C^2p$

(b) $n \geq C\sqrt{p}\log p$; $M(n) = n^2$
$M(f(p))/p = C^2 p \log^2 p / p = C^2 \log^2 p$

(c) $n \geq C\sqrt{p}$; $M(n) = n^2$
$M(f(p))/p = C^2 p/p = C^2$

(d) $n \geq Cp\log p$; $M(n) = n^2$
$M(f(p))/p = C^2p^2\log^2 p/p = C^2 p\log^2 p$

(e) $n \geq Cp$; $M(n) = n$
$M(f(p))/p = Cp/p = C$

(f) $n \geq p^C$; $M(n) = n$
$M(f(p))/p = p^C/p = p^{C-1}$, $1 < C < 2$

(g) $n \geq p^C$; $M(n) = n$
$M(f(p))/p = p^C/p = p^{C-1}$, $C > 2$

Ans: e, c, b, f, a, d, g.