ALGT - The manual

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Contents

Chapter 1

Overview

This document gives an overview of the *ALGT*-tool. First, a general overview of what the tool does is given and why it was developped. Second, a hands-on tutorial develops a Simply Typed Functional Language (STFL). Thirdly, the reference manual gives an in-depth overview of the possibilities and command line flags. Fourth, some concepts and algorithms are explained more thoroughly, together with properties they use. And at last, the `dynamize` and `gradualize` options are explained in depth, as these are what the master dissertation is about.

This documentation is about version **0.1.24** (`Totally dissapearing builtins`), generated on 2017-3-9 .

Chapter 2

What is ALGT?

ALGT (*Automated Language Generation Tool* [1]) is a tool to formally specify any programming language. This is done by first declaring a syntax, using BNF, and then declaring semantics by introducing logical deduction rules, such as evaluation or typing rules. Eventually, properties can be introduced which can be tested. Of course, these can be run.

The tool is kept as general as possible, so any language can be modelled.

[1] Note the similarity of *ALGT* and *AGT* . The tool started its life as *AGT*, based on the paper *Abstracting Gradual Typing*. When it became more general, another name and acronym was chosen.

Chapter 3

Tutorial: developing a simple programming language

We will develop a programming language which can work with booleans and integers. Apart from doing basic arithmetic, we can also use anonymous functions.

Example programs are:

```
True
False
If True Then False Else True
If If True Then False Else True Then True Else False
42
20 + 22
1 + 2 + 3
(\x : Int . x + 1) 41
```

The expression `(\x : Int . x + 1)` is a *lambda expression.* This is an anonymous function, taking one argument - named x- of type `Int`. When applied (e.g. `(\x : Int . x + 1) 41`, the expression right of the `.` is returned, with the variable `x` substituted by the argument, becoming `41 + 1`.

3.1 Setting up a language

A language is declared inside a `.language` file [1]. Create `STFL.language`, and put a title in it:

```
 STFL
******
```

[1]Actually, the extension doesn't matter at all.

```
# A Simply Typed Functional Language
```

You can put comments wherever you want after a `#`, e.g. to give some explanation about the language

3.2 Declaring the syntax

For a full reference on syntax, see the reference manual on syntax.

3.2.1 Simple booleans

A program is nothing more then a string of a specific form. To describe strings of an arbitrary structure, *BNF* [2] can be used.

The syntax of our programming language is defined in the **Syntax**section of *STFL.language*:

```

 STFL
******

# A Simply Typed Functional Language

 Syntax
========
```

What do we write here? Let's start with declaring the boolean values `True` and `False`. We express how these can be parsed by writing `bool    ::= "True" | "False"`. This tells the tool that a syntactic form named `bool` exists and it is either `True` of `False`. Note the double quotes, these are important to indicate that we want this string literally. The `|` epxresses that it can choose between those forms.

STFL.language now looks like:

```

 STFL
******

# A Simply Typed Functional Language

 Syntax
========

bool    ::= "True" | "False"
```

Lets try running this! Create `examples.stfl`, which contains:

```
True
False
```

We can parse these by running (in your terminal) `./ALGT STFL.language examples.stfl bool -l`. The first argument is the language file, the second the examples, the `bool` tells ALGT what syntactic rule to parse. The `-l` flag expresses that each line should be treated individually.

[2]Backus-Naur-form, as introduced by John Backus in the ALGOL60-report.

If all went well, you should get the following output:

```
# "True" was parsed as:
True     bool.0
# "False" was parsed as:
False     bool.1
```

The most interesting part here is that `True` has been parsed with `bool.0`, thus the first choice of the `bool`-form, while `False` has been parsed with the second form.

3.2.2 If-statements

Now, let's add expressions of the form `If True Then False Else True`. We define a new syntactic form: `expr    ::= "If" bool "Then" bool "Else" bool`.[3] This tell *ALGT* that an expression starts with a literal `If`, is followed by a `bool` (so either `True` or `False`), is followed by a literal `Then`, ... The tool uses the double quotes `"` to distinguish between a literal string and another syntactic form.

STFL.language now looks like:

```

 STFL
******

# A Simply Typed Functional Language

 Syntax
========

bool    ::= "True" | "False"
expr    ::= "If" bool "Then" bool "Else" bool
```

This captures already some example expressions. Let's add `If True Then False Else True` to *examples.stfl*:

```
True
False
If True Then False Else True
```

Let's run our tool, this time with `./ALGT STFL.language examples.stfl` **expr** `-l`

```
"examples.stfl (line 0)" (line 1, column 1):
unexpected "T"
expecting "If"
Could not parse expression of the form expr
```

Oops! Seems like our parser now always wants to see a `If` in the beginning, and can't handle `True` anymore. Perhaps we should tell that a `bool` is a valid expression to:

```

 Syntax
========
```

[3]Don't worry about spaces and tabs, we deal with them. If you want need to parse stuff like "duizendeneen" or whitespace sensitive languages, please refer to the reference manual

```
bool     ::= "True" | "False"
expr     ::= "If" bool "Then" bool "Else" bool
           | bool
```

Lets see what this gives:

```
+  If        expr.0
|  True      bool.0
|  Then      expr.0
|  False     bool.1
|  Else      expr.0
|  True      bool.0
```

Looks a lot better! The third example shows clearly how the expression falls apart in smaller pieces.

What with a nested `If`?

`If If True Then False Else True Then True Else False` clearly can't be parsed, as the condition should be a `bool`, according to our current syntax.

Well, we can just write `expr` instead of `bool` in our syntax:

```
expr     ::= "If" expr "Then" expr "Else" expr
           | bool
```

Running this gives

```
# "If If True Then False Else True Then True Else False" was parsed
+  If           expr.0
|  +  If        expr.0
|  |  True      bool.0
|  |  Then      expr.0
|  |  False     bool.1
|  |  Else      expr.0
|  |  True      bool.0
|  Then         expr.0
|  True         bool.0
|  Else         expr.0
|  False        bool.1
```

This clearly shows how the parse trees are nested. This can be rendered too:[4]

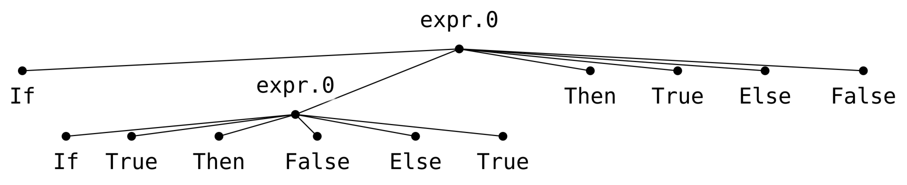


Figure 3.1: ParseTree of a nested condition[5]

[4]These images can be created with `ALGT STLF.language examples.stfl -l --ptsvg Outputname`
[5]These images can be created with `ALGT STLF.language examples.stfl -l --ptsvg Outputname`

3.2.3 Adding numbers, subtyping and forbidden left recursion

Time to spice things up with numbers. To make things easier, integers are built in as `Number`. It's good practice to introduce a new syntactic rule for them:

```
int      ::= Number
```

As an `int` is a valid expression, we add it to the `expr` form:

```
expr     ::= "If" expr "Then" expr "Else" expr
           | bool
           | int
```

Note that every `int` now also is an `expr`, just as every `bool` is an `expr`. This typing relationship can be visualized with `ALGT STFL.language -lsvg Subtyping.svg`[6] :

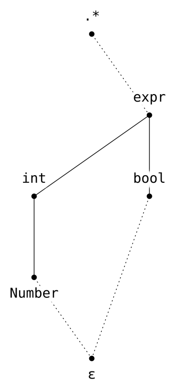


Figure 3.2: Subtyping relation of STFL.language

Now that numbers have been added, let's run this with a number as example:

```
42
```

should give

```
# "42" was parsed as:
42    Number.0
```

So far, so good! Time to add addition:

```
expr     ::= "If" expr "Then" expr "Else" expr
           | bool
           | int
```

expr "+" expr
We add some example:

[6]Creating this svg might take a long time for complicated syntaxes, as ALGT calculates the ordering of labels resulting in the least intersecting lines.

```
20 + 22
1 + 2 + 3
```

And run it:

```
Error:
  While checking the syntax:
    Potential infinite left recursion detected in the syntax.
    Left cycles are:
        expr -> expr
```

Oops! Looks like we did something wrong. What is this left recursion? Whenever the parser wants to parse an expression, it tries every choice from left to right. This means that whenever it tries to parse `expr`, it should first try to parse `expr`. That's not really helpfull, so the parser might get in an infinite loop then.

Not allowing left recursion also means that no loops in the subtypings occur. In other words, the subtyping relationship is a lattice.

The solution to this problem is splitting `expr` in two parts: a `term` with simple elements in front, and `expr` with advanced forms:

```
expr     ::= term
          | term "+" expr
term     ::= "If" expr "Then" expr "Else" expr
          | bool
          | int
```

Let's retry this:

```
Error:
  While checking the syntax:
    While checking for dead choices in expr:
      The choice 'term "+" expr' will never be parsed.
      The previous choice 'term' will already consume a part of it.
      Swap them and you'll be fine.
```

What went wrong this time? The parser tries choice after choice. When parsing `20 + 22` against `expr ::= term | term "+" term`, it'll first try `term` (and not `term "+" term`). It succesfully parses `20` against the lonely `term`, thus the input string `+ 22` is left. The parser doesn't know what to do with this leftover part, so we get an error.

To fix this, we change the order:

```
expr     ::= term "+" expr
          | term
```

When we try again, we get:

```
# "20 + 22" was parsed as:
+  20    Number.0
|  +     expr.0
|  22    Number.0
# "1 + 2 + 3" was parsed as:
+  1        Number.0
|  +        expr.0
|  +  2     Number.0
|   |   +     expr.0
|   |   3     Number.0
```

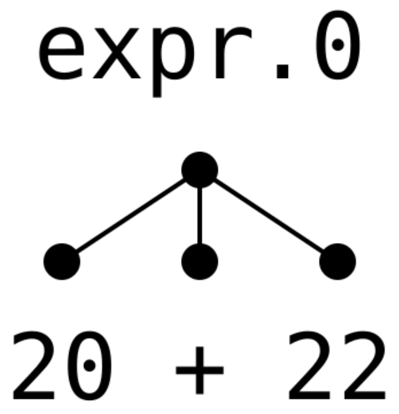


Figure 3.3: Parsetree of `20+22`

3.2.4 Lambda expressions

The lambda expression is the last syntactic form we'd like to add. Recall that these look like `(\x : Int . x + 1)`.

Variables

The first thing we should deal with, are variables. A builtin is provided for those, namely `Identifiers`, matching all words starting with a lowercase (matching `[a-z][a-zA-Z0-9]*`). Let's introduce them in our syntax:

```
var      ::= Identifier
```

A `var` is valid in expressions too, e.g. in the expression `x + 1`, so we want to add it to our `term`:

```
term     ::= "If" expr "Then" expr "Else" expr
         | bool
         | int
         | var
```

Types

The second ingredient we still need, are types, to annotated the input types. Valid types, for starters, are `Bool` and `Int`.

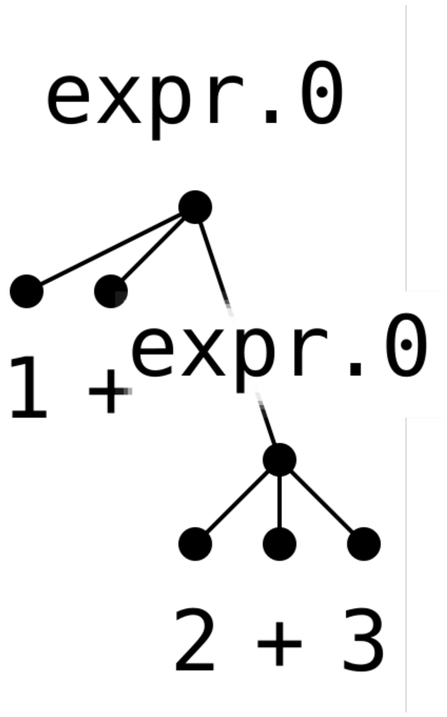


Figure 3.4: Parsetree of `1 + 2 + 3`

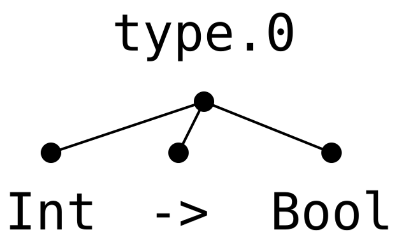
But what is the type of `(\x : Int . x + 1)`? It's something that takes an `Int` and gives back an `Int`. We type this as `Int -> Int`.

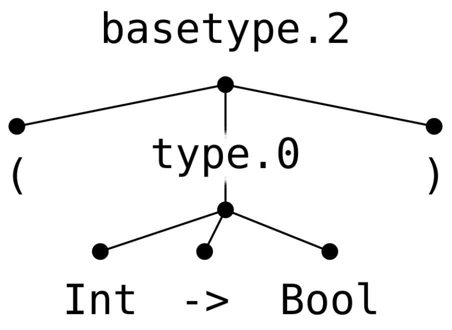
And what is the type of a function, taking another function as an argument? That would be, for example, `(Int -> Int) -> Int`, meaning we need to add a form with parentheses.

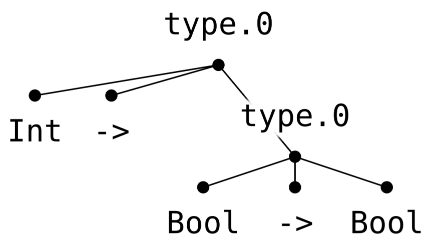
Recalling the trouble we had with left recursion and ordering, we write `type` as following:

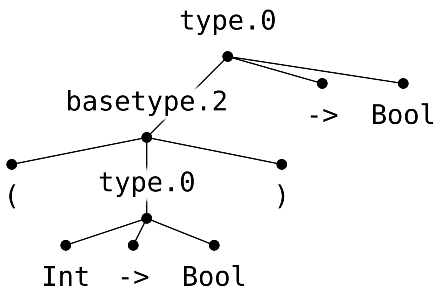
```
basetype::= "Bool" | "Int" | "(" type ")"
type    ::= basetype "->" type | basetype
```

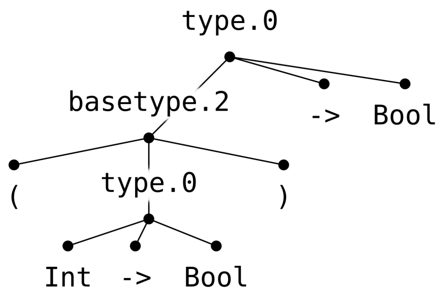
Some examples of types are:











3.2.5 Lambda expressions

Now we have what we need to define lambda expressions. As they act as a term, we add it there:

```
term     ::= "If" expr "Then" expr "Else" expr
         | "(" "\\" var ":" type "." expr ")"
         | bool
         | int
         | var
```

Backslash is the escape character, so use two of them to represent a single backslash.

We can also apply arguments to a lambda expression. We expand `expr`:

```
expr     ::= term "+" expr
         | term expr
         | term
```

3.2.6 What about nonsensical input?

With the current syntax, expresions as `If 5 Then True else False`, `True + 5`, `True 5` or `(\x : Int : x + 1) True` can be written. We allow these forms to be parsed, as the next stage of the compiler (the typechecker) will catch these errors. How to construct this, will be explained in a following section.

3.2.7 Recap

Our *STFL.language* contains

```

 STFL
******

# A Simply Typed Functional Language

 Syntax
========

basetype::= "Bool" | "Int" | "(" type ")"
type     ::= basetype "->" type | basetype

bool     ::= "True" | "False"
int      ::= Number
var      ::= Identifier

expr     ::= term "+" expr
         | term expr
         | term


term     ::= "If" expr "Then" expr "Else" expr
         | "(" "\\" var ":" type "." expr ")"
         | bool
         | int
         | var
```

Our *examples.stfl* contains

```
True
False
If True Then False Else True
If If True Then False Else True Then True Else False
42
20 + 22
1 + 2 + 3
(\x : Int . x + 1) 41
```

We run this with

- `ALGT STFL.language examples.stfl expr -l` to show the parsetrees
- `ALGT STFL.language examples.stfl expr -l --ptsvg SVGnames` to render the parsetrees as SVG
- `ALGT STFL.language --lsvg SVGname.svg` to visualize the subtyping relationship.

3.3 Functions

For a full reference on syntax, see the reference manual on functions.

3.3.1 Domain and codomain

It'll come in handy later on to be able to calculate the *domain* and *codomain* of a function type. The *domain* of a function is the type it can handle as input. The *codomain* of a function is the type it gives as output.

Table 3.1: Examples of domain and codomain

Function type	dom	cod
`Int -> Bool`	`Int`	`Bool`
`(Int -> Bool)`	`Int`	`Bool`
`Int -> Bool -> Bool`	`Int`	`Bool -> Bool`
`Int -> (Bool -> Bool)`	`Int`	`Bool -> Bool`
`(Int -> Bool) -> Bool`	`Int -> Bool`	`Bool -> Bool`
`Int`	*Undefined*	*Undefined*
`Bool`	*Undefined*	*Undefined*

Now, let's define these functions!

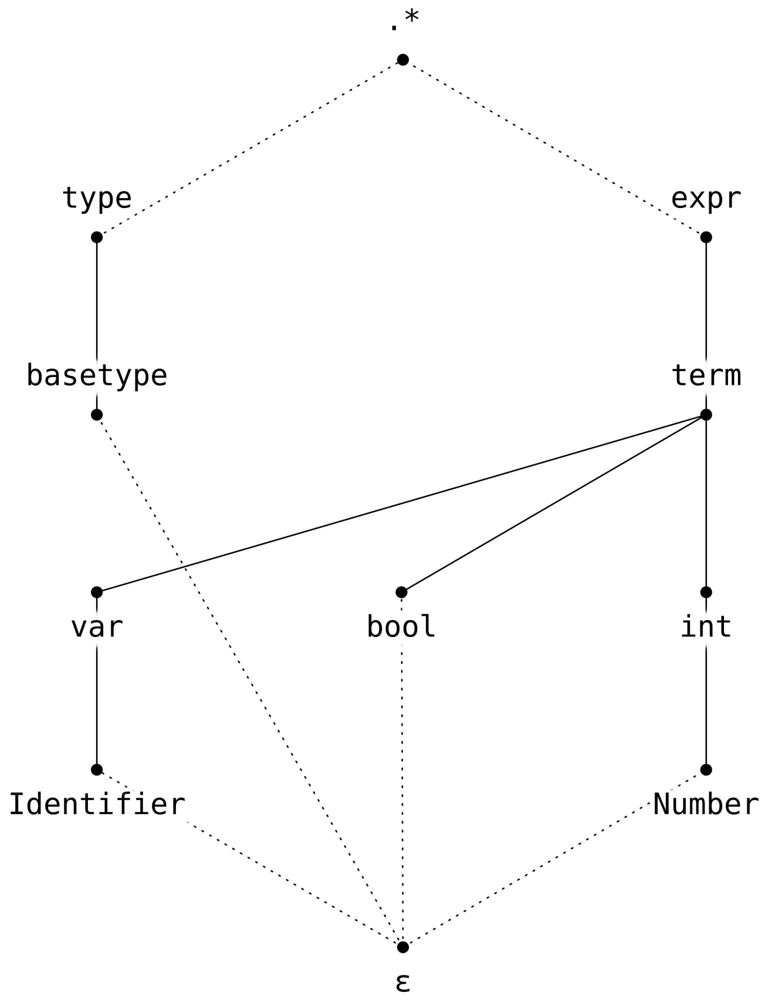


Figure 3.5: The final subtyping relationship of STFL.language

3.3.2 The function section

We add a new header to *STFL.language*:

```
 Functions
===========
```

In this function section, we can define the function *dom* in the following way:

```
domain                             : type -> type
domain(T1 "->" T2)                 = T1
```

So, what is going on here? Let's first take a look to the first line:

```
domain                             : type -> type
```

The `domain` is the name of the function. The `type -> type` indicates what syntactic form is taken as input (a `type`) before the `->` and what is given as output (again a `type`).

WHAT'S THIS?
DOUGLAS HOFSTADTER'S SIX-WORD AUTOBIOGRAPHY. AFTER ALL THOSE 700-PAGE TOMES, I GUESS HE WANTED TO TRY FOR BREVITY.
HUH. LET'S SEE...
I'M SO META, EVEN THIS ACRONYM
...WHOA.
I THINK HE NAILED IT.

Figure 3.6: Relevant XKCD (by Randall Munroe, #917)

3.3.3 Pattern matching

Let's have look at the body of the function:

```
domain(T1 "->" T2)                 = T1
```

What happens if we throw in `Int -> Bool`? Remember that this is parsed as a tree, with three leafs. Notice that there are three elements in the pattern match to: a variable `T1`, a literal `->` and a variable `T2`. These leafs are matched respectively:

The same principle applies with more advanced inputs:

We thus always bind `T1` to the part before the top-level `->`, in other words: we always bind the input type (or domain type) to `T1`. As that is exactly what we need, we return it!

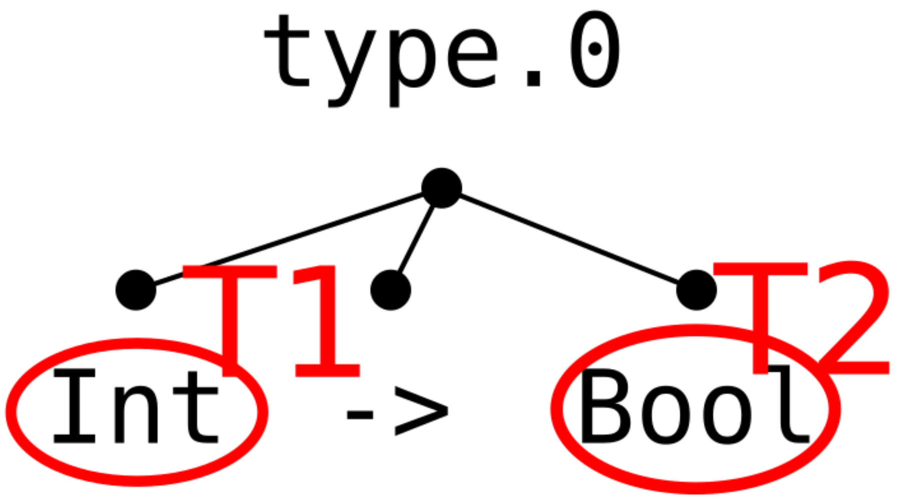


Figure 3.7: Pattern matching in action

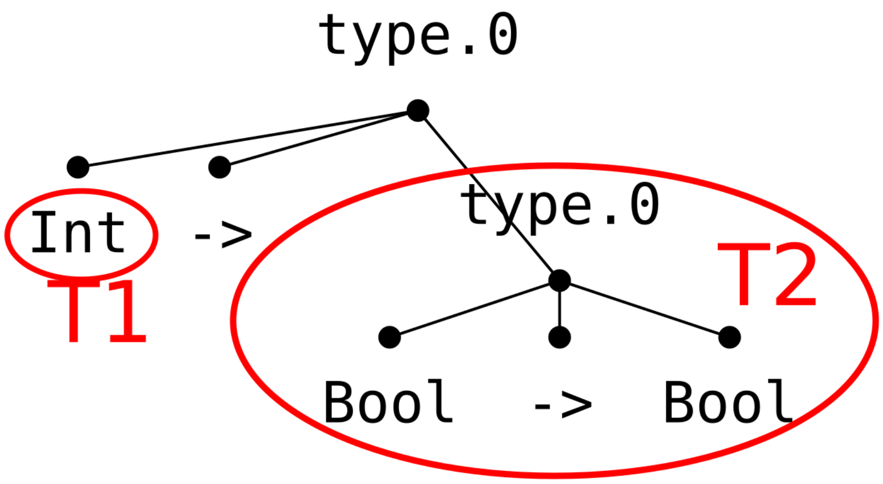


Figure 3.8: Pattern matching in action (more advanced)

3.3.4 Missing cases

This already gives us the most important part. However, what should we do if the first argument type is a function? e.g. `(Int -> Bool) -> Bool`:

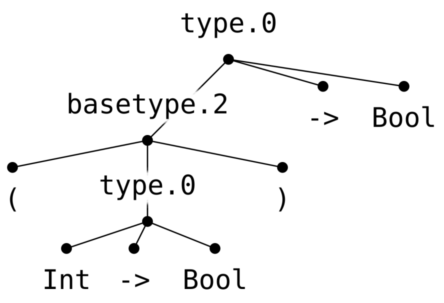


Figure 3.9: Typetree of function argument

This will match the pattern `T1 "->" T2` as following:

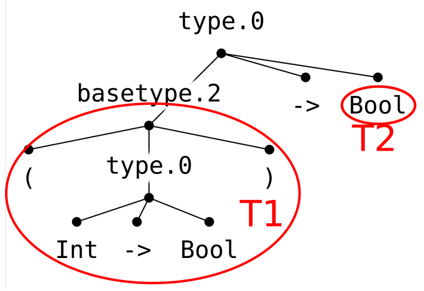


Figure 3.10: Typetree of function argument, matched

We see that `T1` includes the `(` and `)`, which we don't want. We simply solve this by adding a extra clause:

```
domain                              : type -> type
domain(("(" T1 ")") "->" T2)        = T1
domain(T1 "->" T2)                  = T1
```

We expect that the part *before* the arrow now is surrounded by parens. Note that we put parens once without double quotes and once with. These parens capture this part of the parsetree and match it against the patterns inside the parens, as visible in the green ellipse:

3.3.5 Recursion

There is a last missing case, namely if the entire type is wrapped in parens.

We match a type between parens, and then calculate its domain simply by calling the domain function again.

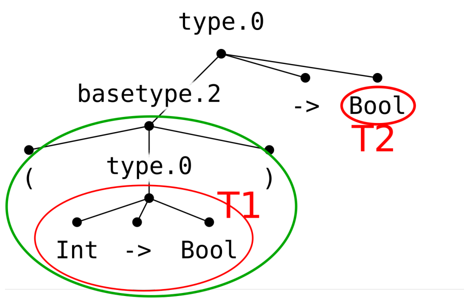


Figure 3.11: Recursive pattern matching

```
domain                              : type -> type
domain("(" T ")")                   = domain(T)
domain(("(" T1 ")") "->" T2)        = T1
domain(T1 "->" T2)                  = T1
```

3.3.6 Clause determination

As you can see, there are two clauses now. How do we now what clause is executed?

Simply put, when the function is evaluated, the arguments are pattern matched against the first clause. If this pattern match succeeds, the expression on the right hand side is returned. Do the arguments not match the patterns? Then the next clause is considered.

In other words, clauses are tried **from top to bottom** and tested. The first clause of which the patterns match, is executed.

When no clauses match, an error message is given.

3.3.7 Executing functions

Let's put this to a test. We can calculate the domain of our examples.

Create a file `typeExamples.stfl`, with contents

```
Int -> Bool
(Int -> Bool)
Int -> Bool -> Bool
Int -> (Bool -> Bool)
(Int -> Bool) -> Bool
Int
Bool
```

Run this with `ALGT STFL.language typeExamples.stfl type -l -f domain`:

```
Warning:
  While checking the totality of function "domain":
    Following calls will fall through:
      domain("Bool")
      domain("Int")
```

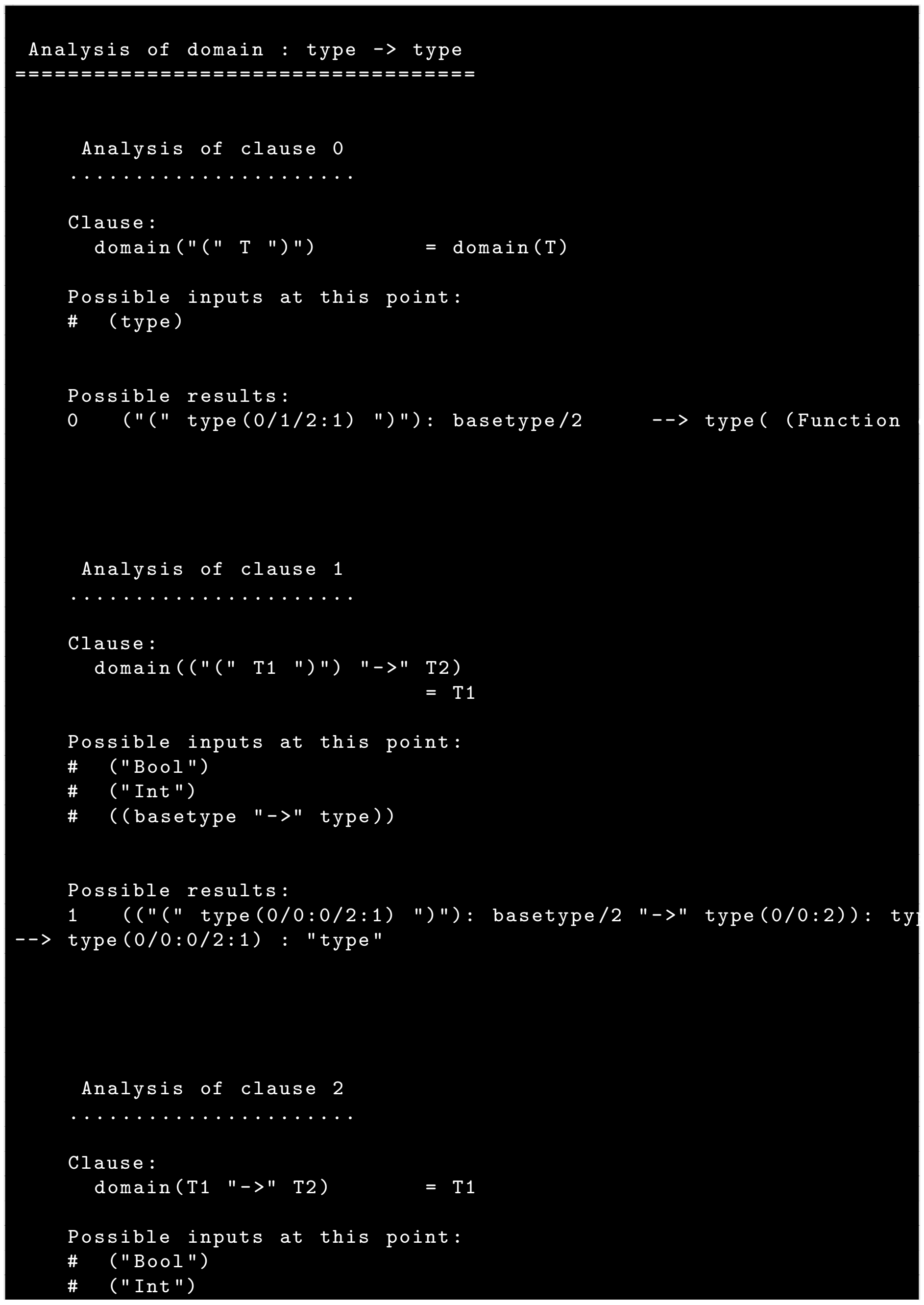
```
# "Int -> Bool" applied to domain
Int

# "(Int -> Bool)" applied to domain
Int

# "Int -> Bool -> Bool" applied to domain
Int

# "Int -> (Bool -> Bool)" applied to domain
Int

# "(Int -> Bool) -> Bool" applied to domain
Int -> Bool

# "Int" applied to domain
Not a single clause of domain matched:
  While pattern matching clause 0:
       In Pattern matching clause 0 with arguments: (Int)
       In domain(Int)
    :
      FT: Could not pattern match ’"Int"’ over ’"(" T ")"’
  While pattern matching clause 1:
       In Pattern matching clause 1 with arguments: (Int)
       In domain(Int)
    :
      FT: Could not pattern match ’"Int"’ over ’("(" T1 ")") "->" T2
  While pattern matching clause 2:
       In Pattern matching clause 2 with arguments: (Int)
       In domain(Int)
    :
      FT: Could not pattern match ’"Int"’ over ’T1 "->" T2’

# "Bool" applied to domain
Not a single clause of domain matched:
  While pattern matching clause 0:
       In Pattern matching clause 0 with arguments: (Bool)
       In domain(Bool)
    :
      FT: Could not pattern match ’"Bool"’ over ’"(" T ")"’
  While pattern matching clause 1:
       In Pattern matching clause 1 with arguments: (Bool)
       In domain(Bool)
    :
      FT: Could not pattern match ’"Bool"’ over ’("(" T1 ")") "->" T
  While pattern matching clause 2:
       In Pattern matching clause 2 with arguments: (Bool)
       In domain(Bool)
    :
      FT: Could not pattern match ’"Bool"’ over ’T1 "->" T2’
```

What does this output tell?

For starters, we get a warning that we forgot two cases, namely `Bool` and `Int`. For some functions this is a problem, but domain is not defined for those values.

Then, we see an overview for each function what result it gives (or an error message if the pattern matches failed).

If you want more information about the behaviour of a function, specify `--ia` or `--ifa FUNCTION-TO-ANALYZE` to get a clause-per-clause overview:



```
 Analysis of domain : type -> type
===================================



     Analysis of clause 0
    .....................

    Clause:
      domain("(" T ")")         = domain(T)

    Possible inputs at this point:
    #  (type)


    Possible results:
    0   ("(" type(0/1/2:1) ")"): basetype/2     --> type( (Function




     Analysis of clause 1
    .....................

    Clause:
      domain(("(" T1 ")") "->" T2)
                                = T1

    Possible inputs at this point:
    #  ("Bool")
    #  ("Int")
    #  ((basetype "->" type))


    Possible results:
    1   (("(" type(0/0:0/2:1) ")"): basetype/2 "->" type(0/0:2)): ty
--> type(0/0:0/2:1) : "type"




     Analysis of clause 2
    .....................

    Clause:
      domain(T1 "->" T2)        = T1

    Possible inputs at this point:
    #  ("Bool")
    #  ("Int")
    #  (("Bool" "->" type))
    #  (("Int" "->" type))


    Possible results:
    2   ("Bool" "->" type(0/0:2)): type/0        --> "Bool"
: "basetype"
    2   ("Int" "->" type(0/0:2)): type/0         --> "Int"
: "basetype"





   Falthrough
  ------------

  ("Bool")
  ("Int")
```

3.3.8 Codomain

`codomain` can be implemented totally analogously:

```
codomain                        : type -> type
codomain("(" T ")")             = codomain(T)
codomain(T1"->" ("(" T2 ")"))   = T2
codomain(T1 "->" T2)            = T2
```

3.4 Relations and Rules: building the evaluator

While we could build an function which evaluates our programming languages, language designers love *natural deduction* more. Don't worry if you never heard about that before, we'll explain it right away!

3.4.1 Declaring relation "evaluation"

First, we declare a new section inside our *STFL.language*, with a relation declaration inside:

```
 Relations
===========

(→)     : expr (in), expr (out)         Pronounced as "small step"
```

Let us break this line down.

The first part, `(→)`, says that we declare a relation with name →. Except from some builtin symbols, you can use whatever string you want, including unicode[7]. If you don't want to use the unicode-arrow for this tutorial, you can replace → by `->`.

[7]to enter an unicode character on a linux machine, type `Left-Ctrl + Shift + U`, release, and type the hexcode of the desired character, e.g. `2192` to get the right-arrow

The second part, `: expr (in), expr (out)` states that this is a relation between two `expr`. As example, `2 + 3 , 5` will be in `(→)` or written more conventionally `2 + 3 → 5`.

What about the `(in)` and `(out)` parts? These are to help the computer. Given `2 + 3`, it's pretty easy for the computer to calculate `5`. Given `5`, the computer can't magically deduce that this was computed by calculating `2 + 3`, especially because an infinite amount of possible calculations might lead to the result `5`. We call this the *mode* of the argument.

The last part, `Pronounced as "evaluation"` defines a name for the relation. It's just an extra, to help users of your language to know what a relation is supposed to do or to help them searching it on a search engine.

Note that the goal of → is to make a small, fundamental step - just one addition or simplification, e.g. `1 + 2 + 3 → 1 + 5`. We'll design another relation later on which will give us the end result immediatly, giving us `6`.

3.4.2 Defining relation "evaluation"

Simple deduction rules: If

Defining relations works with one or more rules.

We start with a simple one:

```

----------------------------------------         [EvalIfTrue]
 "If" "True" "Then" e1 "Else" e2 → e1
```

How should you read this rule? The part under the line says that this is part of the relation; in other words; `"If" "True" "Then" e1 "Else" e2` will evaluate to `e1`. This is equivalent to writing the function `eval("If" "True" "Then" e1 "Else" e2) = e1`.

The part on the right of the line `[EvalIfTrue]` gives the name of the rule. You can put there pretty much everything you want.

Analogously, you can add a rule for `If False`:

```

-----------------------------------------        [EvalIfFalse]
 "If" "False" "Then" e1 "Else" e2 → e2
```

This is already enough to run our third example. To run a relation, specify `-r <name-of-relation>`, thus `./ALGT STFL.language examples.stfl expr -l -r →`:

```

# If True Then False Else True applied to →
# Proof weight: 1, proof depth: 1


----------------------------------- [EvalIfTrue]
If True Then False Else True → False
```

Deduction rules with predicates: plus

How do we add numbers? We can add a deduction rule just as well for this:

```
 n1:Number               n2:Number
-----------------------------------    [EvalPlus]
 n1 "+" n2 → !plus(n1, n2)
```

First, take a look at the bottom line. The left part is straightforward; we match a parsetree with form `n1 + n2`. But what is this `!plus(n1, n2)`. It's a function call with arguments `n1` and `n2`. The exclamation mark `!` indicates that this is a builtin function.

In other words, this rule indicates that `1 + 2` should be evaluated with `!plus(1,2)`, giving three.

There is a catch, though. `!plus` has type `Number -> Number -> Number` (recall, this means that plus takes two `Numbers` and gives us a `Number` in return). We can't pass in other types, or it would fail. We thus have to check that we get correct input for this rule. To do this, we have those predicates on top: `n1:Number` (read this as *`n1` is of syntactic form `Number`*) and `n2:Number`.

Let give this a run!

```

# 20 + 22 applied to →
# Proof weight: 3, proof depth: 2


20 : Number    22 : Number
-------------------------- [EvalPlus]
20 + 22 → 42




# Could not apply relation → to the input "1 + 2 + 3", because:
While trying to proof that (→) is applicable to "1 + 2 + 3":
  Not a single rule matched:
  While trying to intepret the rule EvalPlus with 1 + 2 + 3:
    n2 = 2 "+" 3 is not a "Number" but a "expr"
  While trying to intepret the rule EvalIfTrue with 1 + 2 + 3:
    Sequence lengths are not the same: "If" "True" "Then" e1 "Else"
  While trying to intepret the rule EvalIfFalse with 1 + 2 + 3:
    Sequence lengths are not the same: "If" "False" "Then" e1 "Else"
```

We can see that our simple example, `20 + 22` neatly gives us the answer[8]. The other example, `1 + 2 + 3` however, fails. Note that it gives a detailed overview of what rules it attempted to apply and why those rules failed.

If with complicated conditions

But what with our fourth example, `If If True Then False Else True Then True Else False`? The condition itself as an `If`-expression as well.

Herefore we introduce a more complicated rule:

```

 cond0 → cond1
```

[8]Luckily, your computer didn't have to run for 10 million years. And it conveniently gave the question too, so that we wouldn't forget it.

```
--------------------------------------------------- [EvalIfCond]
 "If" cond0 "Then" e1 "Else" e2 → "If" cond1 "Then" e1 "Else" e2
```

The part above the line should be read as a condition. It states that, whenever `cond0` evaluates to `cond1`, then we can evaluate the bigger expression.

Evaluation contexts for congruence rules

However, there is a way to write this even shorter:

```

 expr0 → expr1
----------------------------            [EvalCtx]
 expr[expr0] → expr[expr1]
```

The part `expr[expr0]` will search, within the expression we want to evaluate, a nested expression that satisfies the conditions. In other words, it will search in the parsetree (`If (If True Then False Else False) Then True Else False`) a part that can be evaluated (e.g. `If True Then False Else False`).

The evaluated expression will then be put back in the original, bigger parsetree at the same location.

Make sure to name the nested expr `expr0`, thus *syntactic-form-name* followed by a number. That's how the tool figures out what kind of parsetree to search for.

```

# If If True Then False Else True Then True Else False applied to →
# Proof weight: 2, proof depth: 2


----------------------------------- [EvalIfTrue]
If True Then False Else True → False
-------------------------------------------------------------------
If If True Then False Else True Then True Else False → If False Then
```

As a bonus, it'll also solve our `1 + 2 + 3`!

```

# 1 + 2 + 3 applied to →
# Proof weight: 4, proof depth: 3


2 : Number    3 : Number
------------------------ [EvalPlus]
2 + 3 → 5
------------------------- [EvalCtx]
1 + 2 + 3 → 1 + 5
```

These proofs are getting a bit harder to read, but always start from the bottom if you get lost.

It basically states that `1 + (2 + 3)` makes a step to `1 + 5`, because of rule `EvalCtx`; this rule could be used because `2 + 3` evaluates to `5`.

The proof for that part of the relation is given more on top, by rule `EvalPlus`, which could be invoked because both `2` and `3` are `Numbers`.

Application

As last, we'd like to apply functions, such as `(\\x : Int . x + 1) 41`.

Our intution is that, given something as `(\\x : someType . someExpr) someArg`, we want to evaluate this to `someExpr`, where we replace every `x` in `someExpr`. However, a builtin function does exactly that: `!subs`. This gives us the following rule:

```
------------------------------------------------------  [EvalLamApp]
 ("(" "\\" var ":" type "." e ")") arg → !subs:expr(var, arg, e)
```

```
# (\x : Int . x + 1) 41 applied to →
# Proof weight: 1, proof depth: 1


--------------------------------- [EvalLamApp]
( \ x : Int . x + 1 ) 41 → 41 + 1
```

3.4.3 Evaluation-relation: recap

Our evaluation rule is defined as:

```

 Relations
===========

(→)      : expr (in), expr (out)         Pronounced as "small step"


 Rules
=======


 expr0 → expr1
----------------------------            [EvalCtx]
 expr[expr0] → expr[expr1]



 n1:Number              n2:Number
------------------------------------    [EvalPlus]
 n1 "+" n2 → !plus(n1, n2)




---------------------------------------         [EvalIfTrue]
 "If" "True" "Then" e1 "Else" e2 → e1


----------------------------------------        [EvalIfFalse]
 "If" "False" "Then" e1 "Else" e2 → e2

```

```
-------------------------------------------------------  [EvalLamApp]
 ("(" "\\" var ":" type "." e ")") arg → !subs:expr(var, arg, e)
```

3.4.4 Is canonical

It is usefull to known when an expression is *canonical*, thus is fully evaluated. This can be simply stated by a relation taking just one argument and *giving no output*[9]. It would contain exactly the `Numbers`, `True` and `False`.

First, the declaration in the `Relations`-section:

```
(✓)      : expr (in)                        Pronounced as "is canonical"
```

And the implementation:

```
 (i:int)
-----------                          [CanonInt]
 (✓) i


 (b:bool)
-----------                          [CanonBool]
 (✓) b
```

3.4.5 Bigstep

Of course, when we input `1 + 2 + 3`, we would like te get `6`, and not `1 + 5`. For this, we can define a second relation,

First, let us declare this:

```
(→*)     : expr (in), expr (out)            Pronounced as "big step"
```

First, the basecase. If something is canonical, we are done and just return that unchanged. We express this as following:

```
 (✓) e
---------                            [BigStepBase]
 e →* e
```

What if we are not done? That means that we can make a single step, `e0 →e1` and that we calculate this `e1` to its canonical from `e2` with the relation itself!

```
 e0 → e1         e1 →* e2
-------------------------            [BigStepRec]
 e0 →* e2
```

So, we finally did it! Time to see our examples in all their glory!

```
# True applied to →*
# Proof weight: 3, proof depth: 3
```

[9]Mathematicians would call this a *set*.

```
True : bool
----------- [CanonBool]
(✓) True
----------------------- [BigStepBase]
True ⟶* True
```

```
# False applied to ⟶*
# Proof weight: 3, proof depth: 3


False : bool
------------ [CanonBool]
(✓) False
------------------------ [BigStepBase]
False ⟶* False
```

```
# If True Then False Else True applied to ⟶*
# Proof weight: 5, proof depth: 4


                                        False : bool
                                        ------------
                                        (✓) False
------------------------------------    --------------
If True Then False Else True → False    False ⟶* False
------------------------------------------------------
If True Then False Else True ⟶* False
```

```
# 42 applied to ⟶*
# Proof weight: 3, proof depth: 3


42 : int
-------- [CanonInt]
(✓) 42
------------------- [BigStepBase]
42 ⟶* 42
```

```
# 20 + 22 applied to ⟶*
# Proof weight: 7, proof depth: 4


                              42 : int
                              --------
20 : Number    22 : Number    (✓) 42
--------------------------    --------
20 + 22 → 42                  42 ⟶* 42
--------------------------------------
20 + 22 ⟶* 42
```

```
# 1 + 2 + 3 applied to →*
# Proof weight: 12, proof depth: 5


                                                        6 : int
                                                        -------
2 : Number    3 : Number    1 : Number    5 : Number    (✓) 6
------------------------    ------------------------    -------
2 + 3 → 5                   1 + 5 → 6                   6 →* 6
------------------------    -----------------------------------
1 + 2 + 3 → 1 + 5           1 + 5 →* 6
---------------------------------------------------------------
1 + 2 + 3 →* 6
```

```
# (\x : Int . x + 1) 41 applied to →*
# Proof weight: 9, proof depth: 5


                                                         42 : int
                                                         --------
                                  41 : Number 1 : Number (✓) 42
                                  ---------------------- --------
                                  41 + 1 → 42            42 →* 42
--------------------------------- -------------------------------
( \ x : Int . x + 1 ) 41 → 41 + 1 41 + 1 →* 42
-----------------------------------------------------------------
( \ x : Int . x + 1 ) 41 →* 42
```

3.5 Building the typechecker

3.6 Properties

3.7 Recap: used command line arguments

Chapter 4

Reference manual

4.1 General

A language is defined in a `.language`-file. It starts (optionally) with a title:

```
 Language Name
***************
```

Comments start with a `#` and can appear quasi everywhere.

```
# This is a comment
```

Syntax, functions, relations, ... are all defined in their own sections:

```
 Syntax
========
```

A section header starts with an upper case, is underlined with `=` and followed by a blank line.

4.2 Syntax

All syntax is defined in the `Syntax` section. It consists out of `BNF`-rules, of the form

```
name    ::= "literal" | choice | seq1 seq2
```

Choices might be written on multiple lines, as long as at least one tab precedes them:

```
name    ::= choice1 | choice2
        | choice3
```

4.2.1 Literals

A string that should be matched exactly, is enclosed in " (double quotes). Some characters can be escaped with a backslash, namely:

Table 4.1: Escape sequences

Sequence	Result
`\n`	newline
`\t`	tab
`\"`	double quote
`\\`	backslash

4.2.2 Parsing order

Rules are parsed **left to right**, in other words, choices are tried in order. No backtracking happens when a choice is made; the parser is a *recursive descent parser*. This has two drawbacks: left recursion results in an infinite loop and the ordering of choices does matter.

```
# Left recursion, error message.
expr    ::= expr "+" int

# ‘int‘ already consumes a part of ‘int + int‘, error message
expr    ::= int | int "+" int

# Common part extraction, error message

# Correct
expr    ::= int "+" int | int
```

4.2.3 Builtin syntactic forms

Some syntactic forms are already provided for your convenience, namely:

Table 4.2: Builtin syntax

Builtin	Meaning	Regex
`Identifier`	Matches an identifier	`[a-z][a-zA-Z0-9]*`
`Number`	Matches an (negative) integer. Integers parsed by this might be passed into the builtin arithmetic functions.	`-?[0-9]*`
`Any`	Matches a single character, whatever it is, including newline characters	`.`
`Lower`	Matches a lowercase letter	`[a-z]`
`Upper`	Matches an uppercase letter	`[A-Z]`

Builtin	Meaning	Regex
`Digit`	Matches an digit	`[0-9]`
`Hex`	Matches a hexadecimal digit	`[0-9a-fA-F]`
`String`	Matches a double quote delimted string, returns the value including the double quotes	`"([^"\]|\"|\\)*"`
`StringUnesc`	Matches a double quote delimeted string, returns the value without the double quotes	`"([^"\]|\"|\\)*"`
`LineChar`	Matches a single character that is not a newline	`[^\n]`
`ParO`	Matches a '(', which will dissapear in the parsetree	`(`
`ParC`	Matches a ')', which will dissapear in the parsetree	`)`

4.2.4 Subtyping relationship

A syntactic form equals a (possibly infinite) set of strings. By using a syntactic form `a` as choice in other syntactic form `b`, `a` will be a subset of be, giving the natural result that `a` is a subtype of `b`.

In the following examle, `bool` and `int` are both subsets of `expr`. This can be visualised with the `--lsvg Output.svg`-flag.

```
bool     ::= "True" | "False"
int      ::= Number
expr     ::= ... | bool | int
```

4.2.5 Whitespace in sequences

Whitespace (the characters `" "`,`"\t"`), is parsed by default (and ignored completely). If you want to parse a whitespace sensitive language, use other symbols to declare the rule:

Table 4.3: Whitespace modes

Operator	Meaning
`::=`	Totally ignore whitespace
`~~=`	Parse whitespace for this rule only
`//=`	Parse whitespace for this rule and all recursively called rules

This gives rise to the following behaviour:

Table 4.4: Whitespace mode examples

Syntax	Matching String
`a ::= "b" "c" x`	`b c x y`
`x ::= "x" "y"`	`bcxy`
	`b\tc\tx\ty`
	`b c\txy`
	`...`
`a ~~= "b" "c" x`	`bcx y`
`x ::= "x" "y"`	`bcxy`
	`bcx\ty`
`a //= "b" "c" x`	`bcxy`
`x ::= "x" "y"`	

4.2.6 Grouping sequences

Sometimes, you'll want to group an entire rule as a token (e.g. comments, an identifier, ...)

Add a `$` after the assignment to group it.

```
text                        ::= LineChar line | LineChar
commentLine                 ::= $ "#" text "\n"

customIdentifier            ::= $ Upper Number
```

When such a token is used in a pattern or expression, the contents of this token are parsed against this rule:

```
f               : customIdentifier -> statement
f("X10")        = "X9" "# Some comment"
```

4.3 Functions

4.3.1 Patterns and expressions

Functions transform their input. A function is declared by first giving its type, followed by one or more clauses:

```
f               : a -> b
f(a, "b")       = "c"
f(a, b)         = "d"
```

When an input is given, arguments are pattern matched against the patterns on between parentheses. If the match succeeds, the expression on the right is given. If not, the next clause is given.

Note that using the same variable multiple times is allowed, this will only work if these arguments are the same:

```
f(a, a)          = ...
```

Recursion can be used just as well:

```
f("a" a)         = f(a)
```

This is purely functional, heavily inspired on Haskell.

Possible expressions

Expr	Name	As expression
`x`	Variable	Recalls the parsetree associated with this variable
`_`	Wildcard	*Not defined*
`42`	Number	This number
`"Token"`	Literal	This string
`func(arg0, arg1, ...)`	Function call	Evaluate this function
`!func:type(arg0, ...)`	Builtin function call	Evaluate this builtin function, let it return a `type`
`(expr or pattern:type)`	Ascription	Checks that an expression is of a type. Bit useless
`e[expr or pattern]`	Evaluation context	Replugs `expr` at the same place in `e`. Only works if `e` was created with an evaluation context
`a "b" (nested)`	Sequence	Builds the parse tree

Possible patterns

Expr	As pattern
`x`	Captures the argument as the name. If multiple are used in the same pattern, the captured arguments should be the same or the match fails.
`_`	Captures the argument and ignores it
`42`	Argument should be exactly this number
`"Token"`	Argument should be exactly this string
`func(arg0, arg1, ...)`	Evaluates the function, matches if the argument equals the result. Can only use variables which are declared left of this pattern
`!func:type(arg0, ...)`	Evaluates the builtin function, matches if the argument equals the result. Can only use variables which are declared left of this pattern

Expr	As pattern
`(expr or pattern:type)`	Check that the argument is an element of `type`
`e[expr or pattern]`	Matches the parsetree with `e`, searches a subpart in it matching `pattern`
`a "b" (nested)`	Splits the parse tree in the appropriate parts, pattern matches the subparts

4.3.2 Typechecking

Equality

```
 Syntax
========

a        ::= ...
b        ::= ...
c        ::= a | b | d

 Functions
===========

f        : a -> b -> c
f(x, x) = x
```

When equality checks are used in the pattern matching, the variable will be typed as the smallest common supertype of both types. If such a supertype does not exist, an error message is given.

Note that using a supertype might be a little *too* loose, but won't normally happen in real-world examples.

In the given example, `x` will be typed as `c`, the common super type. In this example, `x` might also be a `d`, while this is not possible for the input. This can be solved by splitting of `a | b` as a new rule.

4.3.3 Totality- and liveabilitychecks

Can be disabled with `--no-check`, when they take to long.

4.3.4 Higher order functions and currying?

Are not possible for now (v 0.1.24). Perhaps in a future version or when someone really needs it and begs for it.

4.3.5 Builtin functions

name	Descr	Arguments
`plus`	Gives a sum of all arguments (0 if none given)	`Number* -> Number`

name	Descr	Arguments
`min`	Gives the first argument, minus all the other arguments	`Number -> Number* -> Number`
`mul`	Multiplies all the arguments. (1 if none given)	`Number* -> Number`
`div`	Gives the first argument, divided by the product of the other arguments. (Integer division, rounded down))	`Number -> Number* -> Number`
`mod`	Gives the first argument, module the product of the other arguments.	`Number -> Number* -> Number`
`neg`	Gives the negation of the argument	`Number -> Number`
`equal`	Checks that all the arguments are equal. Gives 1 if so, 0 if not.	`.* -> .* -> Number`
`error`	Stops the function, gives a stack trace. When used in a rule, this won't match a predicate	`.** ->` ε
`subs`	(expression to replace, to replace with, in this expression) Replaces each occurence of the first expression by the second, in the third argument. You'll want to explictly type this one, by using `subs:returnType("x", "41", "x + 1")`	`.* -> .* -> .* -> .*`
`group`	Given a parsetree, flattens the contents of the parsetree to a single string	`.* -> StringUnesc`

4.4 Relations and Rules

4.5 Properties

4.6 Command line flags

Chapter 5

Used concepts and algorithms

Chapter 6

Dynamization and gradualization

Chapter 7

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