problems

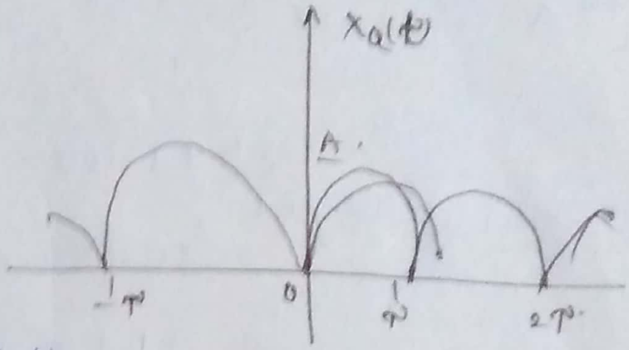
1) consider the full-wave rectified sinusoid in fig

a) Determine its spectrum $x_a(F)$

b) compute power of signal

c) plot the power spectral density

d) check the validity of parseval's relation for this signal.



Given $x_a(t)$ is periodic, so we go for Fourier series.

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k t/T}$$

$$c_k = \frac{1}{T}\int_0^T A \sin(\pi t/T) \cdot e^{-j2\pi k t/T} \, dt$$

$$= \frac{A}{T}\int_0^T \left(\frac{e^{j\pi t/T} - e^{-j\pi t/T}}{2j}\right) e^{-j2\pi k t/T} \, dt$$

$$\cancel{= \frac{A}{2Tj}\int_0^T e^{j\pi t(2k+1)}}$$

$$= \frac{A}{2jT}\int_0^T \left(e^{j\pi t/T(1-2k)} - e^{-j\pi t/T(1+2k)}\right) dt$$

$\frac{j\pi t}{T} - \frac{2j\pi k t}{T}$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi t/T(1-2k)}}{(1-2k)\frac{j\pi}{T}} - \frac{e^{-j\pi t/T(1+2k)}}{-(1+2k)\, j\pi\frac{1}{T}}\right]_0^T$$

$\frac{j\pi}{T} - \frac{2j\pi k}{T}$

$\frac{j\pi - 2j\pi k}{T}$

$\frac{j\pi(1-2k)}{T}$

$$= \frac{A}{2jT}\left\{\left[\frac{e^{j\pi(1-2k)}}{(1-2k)\frac{j\pi}{T}} + \frac{e^{-j\pi(1+2k)}}{(1+2k)\frac{j\pi}{T}}\right] - \left[\frac{1}{(1-2k)\frac{j\pi}{T}} + \frac{1}{(1+2k)\frac{j\pi}{T}}\right]\right\}$$

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1) consider the full-wave rectified sinusoid in fig

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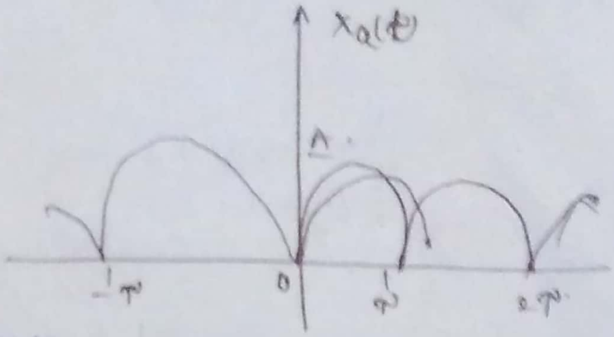

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$$= \frac{A}{T}\int_0^T \left(\frac{e^{j\pi t/T} - e^{-j\pi t/T}}{2j}\right) e^{-j2\pi k t/T}\,dt$$

$$= \cancel{\frac{A}{2Tj}\int_0^T e^{j\pi t(2k+1)}}$$

$$= \frac{A}{2jT}\int_0^T \left(e^{j\pi t/T(1-2k)} - e^{-j\pi t/T(1+2k)}\right)dt$$

$$\frac{j\pi t}{T} - \frac{2j\pi k t}{T} \qquad = \frac{A}{2jT}\left[\frac{e^{j\pi t/T(1-2k)}}{(1-2k)\frac{j\pi}{T}} - \frac{e^{-j\pi t/T(1+2k)}}{-(1+2k)\frac{j\pi}{T}}\right]_0^T$$

$$\frac{j\pi}{T} - \frac{2j\pi k}{T}$$

$$\frac{j\pi - 2j\pi k}{T}$$

$$\frac{j\pi(1-2k)}{T}$$

$$= \frac{A}{2jT}\left\{\left[\frac{e^{j\pi(1-2k)}}{(1-2k)\frac{j\pi}{T}} + \frac{e^{-j\pi(1+2k)}}{(1+2k)\frac{j\pi}{T}}\right] - \left[\frac{1}{(1-2k)\frac{j\pi}{T}} + \frac{1}{(1+2k)\frac{j\pi}{T}}\right]\right\}$$

$$= \frac{A}{2j T}\left\{\left[\frac{e^{j\pi(1-2k)}-1}{(1-2k)\frac{j\pi}{T}}\right]+\left[\frac{e^{-j\pi(1+2k)}-1}{(1+2k)(\frac{j\pi}{T})}\right]\right\}$$

$$= \frac{A}{2j\cancel{T}}\frac{\cancel{T}}{j\pi}\left\{\frac{e^{j\pi(1-2k)}-1}{(1-2k)}+\frac{e^{-j\pi(1+2k)}-1}{(1+2k)}\right]$$

$$= -\frac{A}{2\pi}\left\{\frac{e^{j\pi(1-2k)}-1}{1-2k}+\frac{e^{-j\pi(1+2k)}-1}{1+2k}\right\}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\pi(1-2k)} = \cos\{\pi(1-2k)\} + j\sin\{\pi(1-2k)\}$$

$$= -1 + 0 \Rightarrow \underline{\underline{-1}}$$

$$e^{-j\pi(1+2k)} = \cos\{\pi(1-2k)\} - j\sin\{\pi(1+2k)\}$$

$$= -1 - 0 = \underline{\underline{-1}}$$

$$= \frac{-A}{2\pi}\left\{\left(\frac{-1-1}{1-2k}\right)+\left(\frac{-1-1}{1+2k}\right)\right\}$$

$$= \frac{-A}{2\pi}\left\{\frac{-2}{1-2k}-\frac{2}{1+2k}\right\}$$

$$= \frac{-A}{\cancel{2}\pi}\left\{-\cancel{2}\left(\frac{1}{1-2k}+\frac{1}{1+2k}\right)\right\}$$

$$= \frac{A}{\pi}\left\{\frac{1+\cancel{2k}+1-\cancel{2k}}{(1-2k)(1+2k)}\right\}$$

$$= \frac{A}{\pi}\frac{2}{(1-4k^2)} = \frac{2A}{\pi(1-4k^2)}$$

$$x_a(t) = \sum_{k=-\infty}^{\infty}\frac{2A}{\pi(1-4k^2)}e^{j2\pi kt/T}$$

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$$X_P(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi F t} \cdot dt$$

$$= \int_{-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} \frac{2A}{\pi(1-4k^2)} \cdot e^{j2\pi k t/T} \cdot e^{-j2\pi F t} \right) dt$$

$$= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_n \cdot e^{-j2\pi t(F - k/T)} \cdot dt .$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_n \cdot e^{-j2\pi(F-k/T)t} \cdot dt .$$

$$= \sum_{k=-\infty}^{\infty} c_n \cdot \int_{-\infty}^{\infty} e^{-j2\pi(F-k/T)t} \cdot dt$$

$$= \sum_{k=-\infty}^{\infty} c_n \cdot \delta\left(F - \frac{k}{T}\right)$$

b) $\text{power} = \frac{1}{T} \int_0^T x_a^2(t)\, dt .$

$$= \frac{1}{T} \int_0^T \left(A \sin \frac{\pi t}{T}\right)^2 \cdot dt$$

$$= \frac{A^2}{T} \int_0^T \sin^2\left(\frac{\pi}{T} \cdot t\right) dt .$$

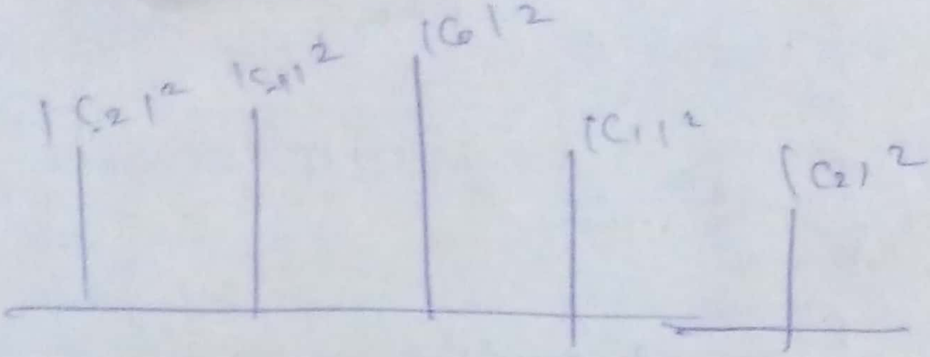
$$= \frac{A^2}{T} \int_0^T \left(\frac{1 - \cos 2\left(\frac{\pi}{T} t\right)}{2} \right) \cdot dt$$

$$= \frac{A^2}{T} \left[\frac{t}{2} - \frac{\sin 2\left(\frac{\pi}{T} t\right)}{2\left(\frac{\pi}{T}\right)} \right]_0^T$$

$$= \frac{A^2}{T} \left[\frac{T}{2} - \frac{\sin 2\left(\frac{\pi}{T} T\right)}{2\left(\frac{\pi}{T}\right)} \right] - 0$$

$$= \frac{A^2}{T} \left(\frac{T}{2} - 0 \right) = \frac{A^2}{2}$$

c)



d)

$$P_x = \frac{1}{T}\int_0^T x_a^2(t)\cdot dt$$

$$= |c_k|^2.$$

$$\sum_{k=-\infty}^{\infty} |c_k|^2 = \sum_{k=-\infty}^{\infty}\left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(1-4k^2)^2}$$

$$= \cancel{\frac{4A^2}{4\pi^2}}\ \cancel{1} + \frac{2}{3^2}\cancel{+}$$

$$= \frac{4A^2}{\pi^2}\left\{\frac{1}{(1-4k^2)^2}\bigg|_{k=0} + 2\sum_{k=1}^{\infty}\frac{1}{(1-4k^2)^2}\right\}$$

$$= \frac{4A^2}{\pi^2}\left\{1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots\right\}$$

$$= \cancel{\frac{4A^2}{\pi^2}}\left(\frac{\pi^2}{8}\right) \quad \text{infinite series sum to } \frac{\pi^2}{8}$$

$$= \frac{A^2}{2}.$$

$$= \frac{2}{15^2} \times \frac{3^2}{2}$$

$$= 0.04$$

$$\frac{2}{15^2} - \frac{2}{3^2}$$

$$\frac{1}{1-}$$

compute & sketch the magnitude & phase spectra for the following signals (a>0)

a) $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

b) $x_a(t) = Ae^{-a|t|}$

a) $x_a(t) = \begin{cases} Ae^{-at}, & t \geq 0 \\ 0, & t < 0 \end{cases}$

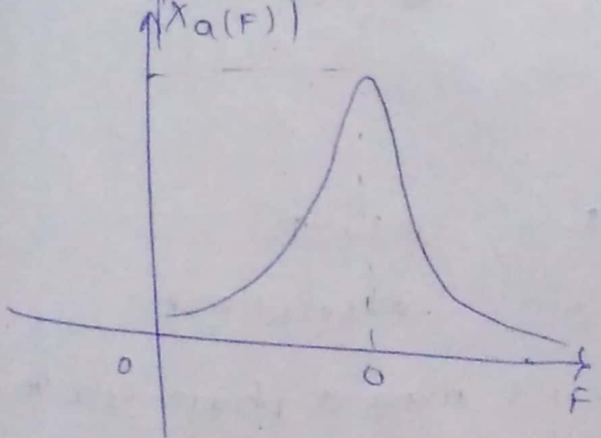
$$X_a(F) = \int_0^{\infty} Ae^{-at} \cdot e^{-j2\pi f t} \cdot dt$$

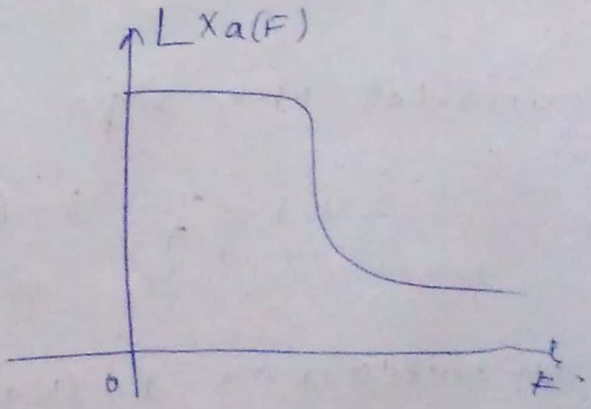
$$= A \int_0^{\infty} e^{-t(a+j2\pi f)} \cdot dt$$

$$= A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \Big|_0^{\infty} \right]$$

$$= A \left(\frac{1}{a+j2\pi f} \right) \therefore = \frac{A}{a+j2\pi f}.$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2 + (2\pi f)^2}} \quad ; \quad \angle X_a(F) = -\tan^{-1}\left(\frac{2\pi f}{a}\right)$$





b) $x_a(t) = Ae^{-a|t|}$

$$X_a(F) = \int_{-\infty}^{\infty} Ae^{-a|t|} \cdot e^{-j2\pi Ft} \cdot dt$$

$$= \int_{-\infty}^{0} Ae^{at} \cdot e^{-j2\pi Ft} \cdot dt + \int_{0}^{\infty} Ae^{-at} \cdot e^{-j2\pi Ft} \cdot dt$$

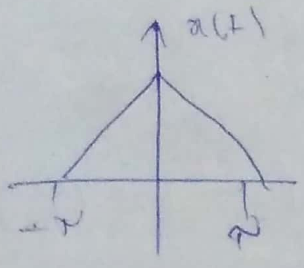
$$= \left[\frac{Ae^{t(a-j2\pi f)}}{a-j2\pi f} \Big|_{-\infty}^{0} \right] + \left[\frac{Ae^{-t(a+j2\pi f)}}{-(a+j2\pi f)} \Big|_{0}^{\infty} \right]$$

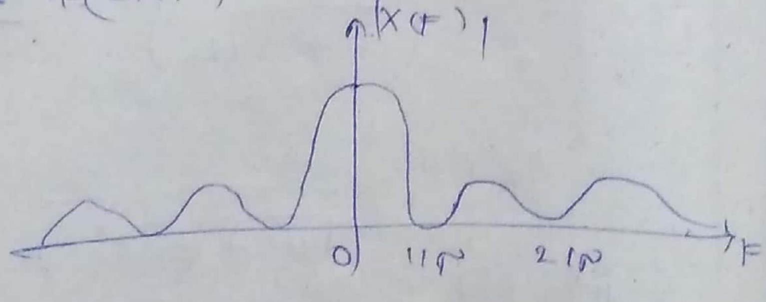
$$= \left[\frac{A}{a-j2\pi f} - 0 \right] + \left(0 + \frac{A}{a+j2\pi f} \right)$$

$$= \frac{A}{a-j2\pi f} + \frac{A}{a+j2\pi f}$$

$$= \frac{2aA}{a^2+(2\pi f)^2}$$

$$|X_a(F)| = \frac{2aA}{a^2+(2\pi F)^2} \quad ; \quad \angle X_a(F) = 0.$$





consider the sig

$$x(t) = \begin{cases} 1 - |t|/\tau, & |t| \le \tau \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine & sketch its mag & phase spectra, $|X_a(F)|$ and $\angle X_a(F)$, respectively

b) create a periodic sig $x_p(t)$ with fundamental period $T_p \ge 2T$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$

what are Fourier coeff c_k ...

c) using results in parts (a) and (b), show that $c_k = (1/T_p) \cdot X_a(k/T_p)$.

a) $$X_a(F) = \int_{-\tau}^{0} \left(1+\frac{t}{\tau}\right) e^{-j2\pi Ft}\, dt + \int_{0}^{\tau} \left(1-\frac{t}{\tau}\right) e^{-j2\pi Ft}\, dt$$

Let $$y(t) = \begin{cases} 1/\tau, & -\tau < t \le 0 \\ 1/\tau, & 0 < t \le \tau \end{cases}$$

$$Y(F) = \int_{-\tau}^{\tau} y(t)\, e^{-j2\pi Ft}\, dt$$

$$= \int_{-\tau}^{0} \frac{1}{\tau} e^{-j2\pi Ft}\, dt + \int_{0}^{\tau} \left(-\frac{1}{\tau}\right) e^{-j2\pi Ft}\, dt.$$

$$= -\frac{2\sin^2 \pi F\tau}{j\pi F\tau}.$$

$$X(F) = \frac{1}{j2\pi F} Y(F)$$

$$= \tau \left(\frac{\sin \pi F\tau}{\pi F\tau}\right)^2$$

$$|X(F)| = \tau \left(\frac{\sin \pi F\tau}{\pi F\tau}\right)^2$$

$$\angle X_a(F) = 0$$

b) $$c_k = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi kt/T_p}\, dt$$

$$= \frac{1}{T_p}\left[\int_{-\tau}^{0} \left(1+\frac{t}{\tau}\right) e^{-j2\pi kt/T_p}\, dt + \int_{0}^{\tau} \left(1-\frac{t}{\tau}\right) e^{-j2\pi kt/T_p}\, dt\right]$$

$$= \frac{\tau}{T_p}\left[\frac{\sin \pi k\tau/T_p}{\pi k\tau/T_p}\right]^2$$

c) $c_k = \frac{1}{T_p} \cdot X_a\left(\frac{k}{T_p}\right)$ Hence proved from (a) & (b)

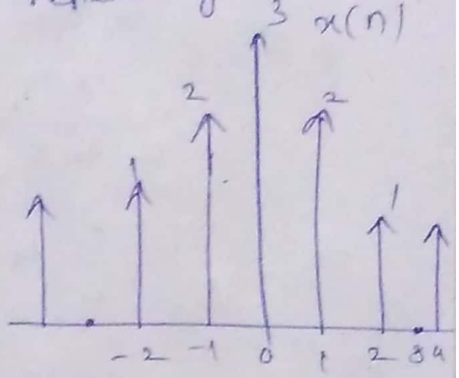
A) Consider the following periodic signal:

$x(n) = \{ \ldots 1, 1, 0, 1, 1, 2, \underset{\uparrow}{3}, 2, 1, 1, 0, 1, 1, \ldots \}$

a) sketch signal $x(n)$ and its mag and phase spectrum.

b) using results in part (a), verify parseval's relation by computing power in time and frequency domains.

$x(n) = \underbrace{1, 0, 1, 1, 2, \underset{\uparrow}{3}}_{N=6}$ $\overset{n=0}{}$ $, 2, 1, 1, 0, 1, 1, \ldots$ repeating



$C_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi kn}{N}}$

$= \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi kn}{6}}$

$n=0;$ $\cancel{\frac{1}{6}(3) e^{\frac{-j2\pi k(0)}{6}}} = \cancel{3/6}$

$n=0;$ $x(0) \cdot e^{\frac{-j2\pi k(0)}{6}} = 3 \times 1 = 3$

$n=1;$ $x(1) \cdot e^{\frac{-j2\pi k(1)}{6}} = 2 e^{\frac{-j2\pi k}{6}}$

$n=2;$ $x(2) \cdot e^{\frac{-j2\pi k 2}{6_3}} = e^{\frac{-j2\pi k}{3}}$

$n=3;$ $x(3) \cdot e^{\frac{-j2\pi k 3}{2}} = 0$

$n=4;$ $x(4) \cdot e^{\frac{-j2\pi k 4}{6_3}} = e^{\frac{-j4\pi k}{3}}$

$n=5;$ $x(5) \cdot e^{\frac{-j2\pi k 5}{6_3}} = 2 e^{\frac{-j5\pi k}{3}}$

$= \frac{1}{6} \left[3 + 2 e^{\frac{-j2\pi k}{6}} + e^{\frac{-j2\pi k}{3}} + 0 + e^{\frac{-j4\pi k}{3}} + 2 e^{\frac{-j5\pi k}{3}} \right]$

For $k=0$

$= \frac{1}{6} [3 + 2 + 1 + 0 + 1 + 2]$

$= 9/6$

$\underline{\underline{k=1}}$

$$= \frac{1}{6}\left[3 + 2e^{-j\frac{2\pi}{6}} + e^{-j\frac{2\pi}{3}} + e^{-j\frac{4\pi}{3}} + 2e^{-j\frac{5\pi}{3}}\right]$$

$$= \frac{1}{6}\left[3 + 2\left[\cos(\pi/3) - j\sin(\pi/3)\right] + \left[\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right]\right.$$

$$\left. + \left[\cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right)\right] + \left(2\left(\cos\left(\frac{5\pi}{3}\right) - j\sin\left(\frac{5\pi}{3}\right)\right)\right]$$

$$\sqrt{(0.5)^2 + (0.87)^2} = 1$$

$$= \frac{1}{6}\left[3 + 2[0.5 - j\cancel{0.87}] + [-0.5 - j\cancel{0.87}] + [-0.5 + j\cancel{0.870}]\right.$$
$$2(0.5 + j\cancel{0}.87)$$

0.25 1.760

$$= \frac{1}{6}\left[3 + 2 + 1 + 1 + 2\right]$$

$$= \frac{1}{6}\left[3 + 1 - \cancel{0.5} - \cancel{0.5} + 1\right]$$

$$= \frac{1}{6}(4) = \frac{4}{6}$$

$\underline{\underline{k=2}}$

$$= \frac{1}{6}\left[3 + 2e^{-j\frac{4\pi}{6}} + e^{-j\frac{4\pi}{3}} + 0 + e^{-j\frac{8\pi}{3}} + 2e^{-j\frac{10\pi}{3}}\right]$$

$= \underline{0}$

Ily $k=3$; $C_3 = 1/6$; $k=4$, $C_4 = 0$

$k=5$; $C_5 = 4/6$.

b) $P_t = \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$

$= \frac{1}{6}\left[1^2+0^2+1^2+2^2+3^2+2^2\right]$

$= \frac{1}{6}\left[1+1+4+9+4\right] = \frac{19}{6}$

$P_f = \sum_{n=0}^{5} |c(n)|^2 = \left(\frac{9}{6}\right)^2+\left(\frac{4}{6}\right)^2+0+\left(\frac{1}{6}\right)^2+\left(\frac{4}{6}\right)^2$

$= \frac{114}{36} = \frac{19}{6}$

$$\boxed{\therefore\ P_t = P_f = 19/6}$$

Hence, parsvel's relation is verified.

5) consider signal

$x(n) = 2+2\cos\left(\frac{\pi n}{4}\right)+\cos\left(\frac{\pi n}{2}\right)+\frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$

a) Determine and sketch its power spectral density spectrum.

~~b) Determine and sketch the mag & phase spectrum of following periodic sig.~~

b) evaluate power of the signal.

$x(n) = 2+2\cos\left(\frac{\pi n}{4}\right)+\cos\left(\frac{\pi n}{2}\right)+\frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$

$= 2+\cancel{2}\left[\frac{e^{\frac{j\pi n}{4}}+e^{-\frac{j\pi n}{4}}}{\cancel{2}}\right]+\left[\frac{e^{\frac{j\pi n}{2}}+e^{-\frac{j\pi n}{2}}}{2}\right]+$

$\frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}}+e^{-\frac{j3\pi n}{4}}}{2}\right]$

$= 2+e^{\frac{j\pi n}{4}}+e^{-\frac{j\pi n}{4}}+\frac{1}{2}e^{\frac{j\pi n}{2}}+\frac{1}{2}e^{-\frac{j\pi n}{2}}+\frac{1}{4}e^{\frac{j3\pi n}{4}}+\frac{1}{4}e^{-\frac{j3\pi n}{4}}$

From these $N=8$

$$c_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-j\frac{\pi k n}{4}}$$

$$c_0 = \frac{1}{8}\sum_{n=0}^{7} x(n)$$

$$= \frac{1}{8}\left[x(0) + x(1) + x(2) + \dots x(7)\right]$$

$$= \frac{1}{8}\left[\frac{11}{2} + \left[2 + \cancel{\tfrac{3}{4}}\sqrt{2}\right] + [1] + \left[2 - \cancel{\tfrac{3}{4}}\sqrt{2}\right] + \left[\tfrac{1}{2}\right] + \left[2 - \cancel{\tfrac{3}{4}}\sqrt{2}\right] + [1] + \left[2 + \cancel{\tfrac{3}{4}}\sqrt{2}\right]\right]$$

$$= \frac{1}{8}\left[\frac{11}{2} + 10 + \frac{1}{2}\right] \qquad = \cancel{\frac{1}{8}}\left[\cancel{\frac{11+2+1}{2}}\right]$$

$$= \frac{1}{8}\left[\frac{11 + 20 + 1}{2}\right] \qquad = \cancel{\frac{8}{8}} = 1$$

$$= \frac{1}{\cancel{8}}\left(\frac{\cancel{32}^{16}}{\cancel{2}}\right)^{2} = 2$$

$\underline{\text{11y}}$

$c_1 = c_7 = 1,$

$c_2 = 1/2\;,\; c_3 = c_5 = 1/4\;,\; c_4 = 0.$

b) $$\text{power} = \sum_{k=0}^{7} |c_k|^2$$

$$= \left[2^2 + 1^2 + 1^2 + (1/2)^2 + (1/2)^2 + \left(\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{4}\right)^2\right]$$

$$= \left(4 + 2 + \frac{1}{2} + \frac{1}{8}\right)$$

$$= 53/8.$$

6) Determine and sketch the mag & phase spectra of following periodic signals.

a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

$x(n) = 4 \sin\left[\frac{\pi(n-2)}{3}\right]$

$= 4 \sin\left(\frac{\pi n - 2\pi}{3}\right)$

$\sin(\omega n)$

$\omega = \frac{\pi}{3}$

$2\pi f = \frac{\pi}{3}$

$f = \frac{\cancel{\pi}}{2\cancel{\pi}3}$

$\boxed{f = \frac{1}{6}}$

$N = 6$

$\frac{\pi n - 2\pi}{3}$

$\pi \quad \omega = \frac{\pi}{3}$

$2\pi f = \frac{\pi}{3}$

$f = \frac{\pi}{2\pi 3}$

$f =$

$$c_k = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi kn}{\cancel{6}3}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} \left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right] \cdot e^{\frac{-j\pi kn}{3}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} \left[\frac{e^{\frac{j\pi n}{3}} \cdot e^{\frac{-j\pi 2}{3}} - e^{\frac{-j\pi n}{3}} \cdot e^{\frac{2\pi j}{3}}}{2j}\right] e^{\frac{-j\pi kn}{3}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} e^{\frac{j\pi n}{3}(\cancel{k-k})} \quad e^{\frac{j\pi}{3}(n-2-kn)} \quad -1-k \quad e^{-\frac{j\pi}{3}(n-2+kn)}$$

$$e^{\frac{2j\pi}{3}} - e^{\frac{-2j\pi}{3}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} e^{\frac{j\pi n}{3}} \left(e^{\frac{-j\pi 2}{3}} - e^{\frac{-2\pi j}{3}}\right)$$

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b) $x[n] = \cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{5}\right)n$

$C_k = C_{1k} + C_{2k}$

$N = \text{LCM}(3, 5) = 15$

$\therefore N = 15$

$$\cos\left(\frac{2\pi}{3}n\right) = \frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right]$$

$$C_{1k} = \begin{cases} 1/2, & k = 5, 10 \\ 0, & \text{otherwise} \end{cases}$$

$$\sin\frac{2\pi n}{5} = \frac{1}{2j}\left(e^{j\frac{2\pi n}{5}} - e^{-j\frac{2\pi n}{5}}\right)$$

$$C_{2k} = \begin{cases} 1/2j & ; k = 3 \\ -1/2j & ; k = 12 \\ 0 & ; \text{otherwise} \end{cases}$$

$$C_k = \begin{cases} 1/2j, & k = 3 \\ -1/2j, & k = 5 \\ 1/2, & k = 10 \\ -1/2j, & k = 12 \\ 0, & \text{otherwise}, \end{cases}$$

c) $x(n) = \cos \frac{2\pi}{3} n \sin \frac{2\pi}{5} n$.

$x(n) = \cos \frac{2\pi n}{3} \cdot \sin \frac{2\pi n}{5}$

$= \frac{1}{2} \sin \underbrace{\frac{16\pi n}{15}}_{C_{1k}} - \frac{1}{2} \sin \underbrace{\left(\frac{4\pi n}{15}\right)}_{C_{2k}}$

$N = 15$.

$\sin \frac{16\pi n}{15} = \frac{1}{2j}\left(e^{j\frac{16\pi n}{15}} - e^{-j\frac{16\pi n}{15}}\right)$

$$C_{1k} = \begin{cases} 1/2j & ; \\ -1/2j & ; \\ 0 & ; \end{cases}$$

$\cos \frac{2\pi n}{3} = \frac{1}{2}\left[e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}}\right]$

$$C_{2k} = \begin{cases} 1/2 & ; \ k = \\ 0 & \end{cases}$$

d) $x(n) = \{ \ldots, -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2 \}$

$N = 5$

$C_k = \frac{1}{5} \sum_{n=0}^{4} x(n) e^{-\frac{j2\pi nk}{5}}$

$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2 \cdot e^{-\frac{j4\pi k}{5}} - 2e^{-\frac{j6\pi k}{5}} - e^{-\frac{j8\pi k}{5}}\right]$

$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$

$C_0 = 0,$

$C_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) + 2\sin\left(\frac{4\pi}{5}\right)\right]$

$C_2 = \frac{2j}{5}\left[\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{2\pi}{5}\right)\right]$

As from the given seq.

$C_3 = -C_2$ and $C_4 = -C_1$

e) $x(n) = \{ \cdots \cancel{1}, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2, \cdots \}$

$N = 6.$

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-\frac{j2\pi nk}{6}}$$

$$= \frac{1}{6}\left[1 + 2e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j5\pi k}{3}}\right]$$

$\cos\left(\frac{\pi k}{3}\right)$

$\cancel{= \frac{1}{6}\{1}$

$\cos\left(\frac{2\pi k}{3}\right)$

120
$\frac{360}{3}$

$$= \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi k}{3}\right) - 2\cos\left(\frac{2\pi k}{3}\right)\right]$$

$C_0 = \frac{1}{6}[1 + 4 - 2] = \frac{3}{6} = 1/2$

$C_1 = \frac{1}{6}[1 + 2 - (-1)] = \frac{4}{6} = 2/3.$

$C_2 = 0$

$C_3 = -5/6$; $C_4 = 0$; $C_5 = 2/3$

f) $x(n) = \{ \cdots, 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \cdots \}$

$N = 5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi nk}{5}}$$

$$= \frac{2}{2} \times \frac{1}{5}\underbrace{\left[1 + e^{-\frac{j2\pi k}{5}}\right]}_{2} \times \frac{e^{\frac{j2\pi k}{5}}}{e^{j\pi k/5}}$$

$$= \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{-j\pi k/5}$$

$C_0 = 2/5$

$C_1 = \frac{2}{5}\cos(\frac{\pi}{5}) e^{-j\pi/5}$

$C_2 = \frac{2}{5}\cos(\frac{2\pi}{5}) e^{-j2\pi/5}$

$C_3 = \frac{2}{5}\cos(\frac{3\pi}{5}) e^{-j3\pi/5}$

$C_4 = \frac{2}{5}\cos(\frac{4\pi}{5}) e^{-j4\pi/5}$

g) $x(n) = 1, \quad -\infty < n < \infty$

$N = 1$

$C_k = x(0) = 1 = C_0$

h) $x(n) = (-1)^n, \quad -\infty < n < \infty$

$N = 2$

$C_k = \frac{1}{2} \sum_{n=0}^{1} x(n) e^{-j\pi nk}$

$= \frac{1}{2}(1 - e^{-j\pi k})$

$\therefore C_0 = 0 \; ; \; C_1 = 1$

7) Determine the periodic signals $x(n)$, with fundamental period $N = 8$, if their Fourier coefficients are given by:

a) $C_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

$x(n) = \sum_{k=0}^{7} c_k e^{\frac{j2\pi nk}{8}}$

if $C_k = e^{\frac{j2\pi pk}{8}}$, then

$\sum_{k=0}^{7} e^{\frac{j2\pi pk}{8}} \cdot e^{\frac{j2\pi nk}{8}} = \sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{8}}$

$= 8 \; ; \; p = -n$

$0 \; ; \; p \neq -n$

from given problem

$$c_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}\right] + \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j6\pi k}{8}}\right]$$

$$x(n) = \sum_{k=0}^{7} c_k \cdot e^{\frac{j2\pi nk}{8}}.$$

$$x(n) = 4\,\delta(n+1) + 4\,\delta(n-1) - 4j\,\delta(n+3) + 4j\,\delta(n-3),$$

$$-3 \le n \le 5.$$

b) $$c_k = \begin{cases} \sin \frac{k\pi}{3}, & 0 \le k \le 6 \\ 0, & k = 7 \end{cases}$$

$c_0 = 0$; $\quad c_1 = \sqrt{3}/2$

$c_3 = 0$; $\quad c_2 = \sqrt{3}/2$

$c_6 = c_7 = 0$; $\quad c_4 = -\sqrt{3}/2$

$c_5 = -\sqrt{3}/2$.

$$x(n) = \sum_{k=0}^{7} c_k\, e^{\frac{j2\pi nk}{8}}$$

$$= \sqrt{3}/2\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\cancel{2}\pi n}{\cancel{4}\,2}} - e^{\frac{j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\sin\frac{\pi n}{4} + \sin\frac{\pi n}{2}\right] e^{\frac{j\pi(3n-2)}{4}}$$

~~c) $x(n) = \cos\frac{2\pi}{3}n\,\sin\frac{2\pi}{5}n$~~

~~$x(n) = \sum_{k=-3}^{4} c_k\, e^{\frac{j2\pi nk}{8}}$~~

$= 2 + .$

c) $\{c_k\} = \{\ldots, 0, 1/4, 1/2, 1, \underset{\uparrow}{2}, 1, 1/2, 1/4, 0, \ldots\}$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{\frac{-j\pi n}{2}} + \frac{1}{4}e^{j3\pi n/4} + \frac{1}{4}e^{-j3\pi n/4}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

8) Two DT signals, $S_k(n)$ & $S_l(n)$, are said to be orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} S_k(n) S_l^*(n) = \begin{cases} A_k, & k = l \\ 0, & k \neq l. \end{cases}$$

if $A_k = 1$, the signals are called orthonormal.

a) prove the relation

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \begin{cases} N, & k = 0, \pm N, \pm 2N, \ldots \\ 0, & \text{otherwise}. \end{cases}$$

if $k = 0, \pm N, \pm 2N \ldots$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \sum_{n=0}^{N-1} 1 + 0 + \ldots$$

$$= \sum_{n=0}^{N-1} 1 = N.$$

If $k \neq 0, \pm N, \pm 2N, \ldots$

$$\sum_{n=0}^{N-1} e^{j2\pi kn/N} = \frac{1 - e^{j2\pi k}}{1 - e^{j2\pi k/N}}$$

$\frac{1-a^r}{1-a}$ $\quad$ $\frac{1-r}{1-r^n}$

$$= 0$$

b)

c)

9) Compute F.T of following

a) $x(n) = u(n) - u(n-6)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$= \frac{1 - e^{-j\omega n}}{1 - e^{-j\omega}}$$

b) $x(n) = 2^n u(-n)$

$$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$$

$$= \cancel{\sum_{m=0}^{\infty}} \cancel{\left(\frac{e^{j\omega}}{2}\right)^n}$$

$$= \sum_{n=0}^{-\infty} 2^n e^{-j\omega n}$$

$n = -n$.

$$= \sum_{n=0}^{\infty} (2 e^{-j\omega})^{-n}$$

$$= \sum_{n=0}^{\infty} (2^{-1} e^{j\omega})^n \qquad \sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha}$$

$$= \frac{1}{1 - 2^{-1} e^{j\omega}} = \frac{1}{1 - \frac{1}{2} e^{j\omega}}.$$

c) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

Let $n+4 = m$.

$$\sum_{m=0}^{\infty} \left(\frac{1}{4} e^{-j\omega}\right)^n = \sum_{m=0}^{\infty} \left(\frac{1}{4} e^{-j\omega}\right)^{m-4}.$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4} e^{-j\omega}\right)^m \left(\frac{1}{4} e^{-j\omega}\right)^{-4}.$$

$$= \left(\frac{1}{4}e^{-j\omega}\right)^{-4} \sum_{m=0}^{\infty} \left(\frac{1}{4}e^{-j\omega}\right)^{m}$$

$$= 4^{4} e^{j4\omega} \cdot \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$= \frac{4^{4} e^{j4\omega}}{1-\frac{1}{4}e^{-j\omega}}$$

d) $x(n) = \alpha^{n} \sin\omega_0 n\, u(n), \quad |\alpha| < 1$

$$X(\omega) = \sum_{n=0}^{\infty} \alpha^{n} \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{-j(\omega-\omega_0)}\right]^{n} - \frac{1}{2j} \sum_{n=0}^{\infty} \left[\alpha e^{-j(\omega+\omega_0)}\right]^{n}$$

$$= \frac{1}{2j}\left[\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}} - \frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}\right]$$

$$= \frac{1}{2j}\left(\frac{\cancel{1} - \alpha e^{-j(\omega+\omega_0)} - \cancel{1} + \alpha e^{-j(\omega-\omega_0)}}{\left(1-\alpha e^{-j(\omega-\omega_0)}\right)\left(1-\alpha e^{-j(\omega+\omega_0)}\right)}\right)$$

$$= \frac{1}{2j} \frac{\alpha e^{-j\omega} \cdot e^{j\omega_0} - \alpha e^{-j\omega} \cdot e^{-j\omega_0}}{\cancel{1-\alpha e^{-j(\omega+\omega_0)} - \alpha e^{-j\omega}} \quad 1 - \alpha e^{-j(\omega+\omega_0)} - \alpha e^{-j(\omega-\omega_0)} + \alpha^{2} e^{-j(\omega-\omega_0)} e^{-j(\omega+\omega_0)}}$$

$$= \frac{\alpha \sin\omega_0\, e^{-j\omega}}{1 - 2\alpha\cos\omega_0\, e^{-j\omega} + \alpha^{2} e^{-j2\omega}}$$

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e) $x(n) = |\alpha|^n \sin \omega_0 n, |\alpha| < 1$

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin \omega_0 n|$$

If $\omega_0 = \pi/2$.

$$|\sin(\omega_0 n)| = 1.$$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty$$

F.T doesn't exist.

f) $x(n) = \begin{cases} 2-(1/2)^n, & |n| \le 4 \\ 0, & \text{otherwise} \end{cases}$

$$X(\omega) = \sum_{n=-4}^{4} x(n) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left[2 - (1/2)^n\right] e^{-j\omega n}$$

$$= \cancel{2e^{j4\omega}}$$

$$= \sum_{n=-4}^{4} 2e^{-j\omega n} - \sum_{n=-4}^{4} \left(1/2\, e^{-j\omega}\right)^n$$

$n+4=m$.

$$= 2\sum_{m=0}^{8} e^{-j\omega n} - \cancel{\frac{1}{1-1/2\, e^{-j\omega}}} \sum_{m=0}^{8} \left(1/2\, e^{-j\omega}\right)^n$$

$$= 2\,\frac{1-e^{-j\omega 9}}{1-e^{-j\omega}} - \frac{1-\left(1/2\, e^{-j\omega}\right)^9}{1-\left(1/2\, e^{-j\omega}\right)}$$

$=$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\left[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\right]$$

9) $x(n) = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$= -2e^{j2\omega} - \cancel{e^{j\omega}} + e^{j\omega} + 2e^{-j2\omega}$$

$$= -2j[2\sin 2\omega + \sin\omega]$$

10) Determine sig's having following F.T

a) $X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\omega_0}^{\pi} e^{j\omega n} d\omega$$

$$x(0) = \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0)$$

$$= \frac{\pi - \omega_0}{\pi}$$

$n \neq 0$

$$\int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega = \frac{1}{jn} e^{j\omega n} \Big|_{-\pi}^{-\omega_0}$$

$$= \frac{1}{jn}\left(e^{-j\omega_0 n} - e^{-j\pi n}\right)$$

$$\int_{\omega_0}^{\pi} e^{j\omega n} d\omega = \frac{1}{jn} e^{j\omega n} \Big|_{\omega_0}^{\pi}$$

$$= \frac{1}{jn}\left(e^{j\pi n} - e^{j\omega_0 n}\right)$$

$$x(n) = -\frac{\sin n\omega_0}{n\pi}, \quad n \neq 0.$$

b) $X(\omega) = \cos^2\omega$.

$$= \left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right)^2$$

$$= \frac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right)$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{8\pi}\left[2\pi\,\delta(n+2) + 4\pi\,\delta(n) + 2\pi\,\delta(n-2)\right]$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

c) $X(\omega) = \begin{cases} 1, & \omega_0 - \delta\omega/2 \le |\omega| \le \omega_0 + \delta\omega/2 \\ 0, & \text{elsewhere.} \end{cases}$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0-\delta\omega/2}^{\omega_0+\frac{\delta\omega}{2}} e^{j\omega n}\, d\omega$$

$$= \frac{2}{\pi}\,\delta\omega\left(\frac{\sin(n\delta\omega/2)}{n\delta\omega/2}\right) e^{jn\omega_0}$$

d) ii) consider the signal

$$x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$$

with F.T $X(\omega) = X_R(\omega) + j(X_I(\omega))$. Determine & sketch signal of (n) with F.T

$$Y(\omega) = X_I(\omega) + X_R(\omega)\, e^{j2\omega}$$

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$X_R(\omega)$ = even of $x(n)$

$X_I(\omega)$ = img of $x(n)$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$= \left\{ 1/2, 0, 1, \underset{\uparrow}{2}, 1, 0, 1/2 \right\}$$

$$x_o(n) = \frac{x(n) - x(-n)}{2}$$

$$= \left\{ 1/2, 0, -2, \underset{\uparrow}{0}, 2, 0, 1/2 \right\}$$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n) \cdot e^{-j\omega n}$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n)\, e^{-j\omega n}$$

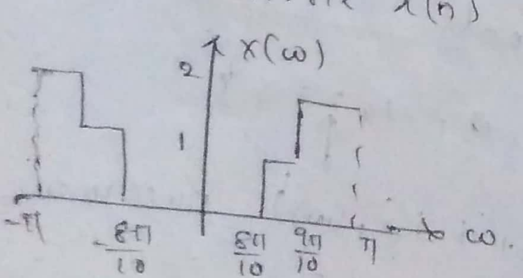
Given $Y(\omega) = X_I(\omega) + X_R(\omega)\, e^{j2\omega}$

$$y(n) = F^{-1}[Y(\omega)]$$

$$= -j x_o n + x_e(n+2)$$

$$= \left\{ 1/2, 0, 1, -j/2, 2, 1 + j/2, \underset{\uparrow}{0}, 1/2 - j2, 0, j/2 \right\}$$

2) Determine $x(n)$ if $X(\omega)$ is given



$$x(n) = \frac{1}{2\pi}\left\{\int_{8\pi/10}^{9\pi/10} e^{j\omega n}\,d\omega + \int_{-9\pi/10}^{8\pi/10} e^{j\omega n}\,d\omega + 2\int_{9\pi/10}^{\pi} e^{j\omega n}\,d\omega + 2\int_{-\pi}^{-9\pi/10} e^{j\omega n}\,d\omega\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{1}{jn}\left(e^{j9\pi n/10} - e^{-9\pi n j/10} - e^{-j8\pi n/10} + e^{-j8\pi n/10}\right) + \frac{2}{jn}\left(-e^{j9\pi n/10} + e^{-j9\pi n/10} + e^{j\pi n} - e^{-j\pi n}\right)\right\}$$

$$= \frac{1}{n\pi}\left[\sin \pi n - \sin 8\pi n/10 - \sin 9\pi n/10\right]$$

$$= -\frac{1}{n\pi}\left[\sin 4\pi n/5 + \sin 9\pi n/10\right]$$

b) $$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi} X(\omega)\, e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{0}^{\pi}\frac{\omega}{\pi}\, e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\frac{\omega}{jn\pi}\, e^{j\omega n}\Big|_{-\pi}^{\pi} + \frac{e^{j\omega n}}{jn}\Big|_{-\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}\, e^{-jn\pi/2}$$

c) $$x(n) = \frac{1}{2\pi}\int_{\omega_c-\omega/2}^{\omega_c+\omega/2} 2e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{-\omega_c-\omega/2}^{-\omega_c+\omega/2} 2e^{j\omega n}\,d\omega$$

$$= \frac{1}{\pi}\left\{\frac{1}{jn\pi}\, e^{j\omega n}\Big|_{\omega_c-\omega/2}^{\omega_c+\omega/2} + \frac{e^{j\omega n}}{jn}\Big|_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\omega/2}\right\}$$

$$= \frac{2}{\pi n}\left[\frac{e^{j(\omega_c+\omega/2)n} - e^{j(\omega_c-\omega/2)n} + e^{-j(\omega_c-\omega/2)n} - e^{-j(\omega_c+\omega/2)n}}{2j}\right]$$

$$= \frac{2}{\pi n}\left[\sin\left(\omega_c+\frac{\omega}{2}\right)n - \sin\left(\omega_c-\frac{\omega}{2}\right)n\right]$$

14)

$x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$

a) $X(0)$

$X(0) = \sum_n x(n) = -1$

b) $\angle X(\omega)$

$\angle X(\omega) = \pi$ for all ω

c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega = \int_{-\pi}^{\pi} X(\omega) \cdot d\omega$

$= 2\pi\, x(0) = -6\pi.$

d) $X(\pi)$

$X(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\pi}$

$= \sum_n (-1)^n x(n)$

$= -3 + (-4) - 2 = -9$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

$= 2\pi \sum_{n} |x(n)|^2$

$= 2\pi (19)$

$= 38\pi$

15) $x(n) \Rightarrow c = \dfrac{\sum_{n=-\infty}^{\infty} n\, x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$

a) express c in terms of $X(\omega)$

$X(\omega) = \sum_n x(n) e^{-j\omega n}$

$X(0) = \sum_n x(n)$

$\frac{d}{d\omega} X(\omega)\Big|_{\omega=0} = -j \sum_n n\, x(n) e^{-j\omega n}\Big|_{\omega=0}$

$= -j \sum_n n\, x(n).$

$C = \dfrac{j \frac{dX(\omega)}{d\omega}}{X(0)}$

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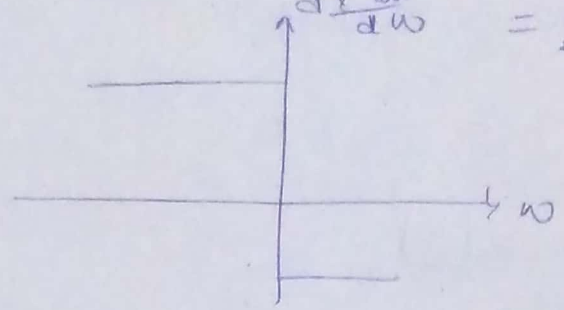
b) From given fig, $X(0) = 1 \therefore c = \frac{0}{1}$

$\frac{dX(\omega)}{d\omega} = 0$



17) F.T interms of $X(\omega)$

a) $x^*(n)$

$$\sum_n x^*(n) e^{-j\omega n} = \left(\sum_n x(n) e^{-j(-\omega)n}\right)^*$$

$$= X^*(-\omega)$$

b) $x^*(-n)$

$$\sum_n x^*(-n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n} = X^*(\omega)$$

c) $y(n) = x(n) - x(n-1)$

$$\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega n}$$

$$Y(\omega) = X(\omega) + X(\omega) e^{-j\omega}$$

$$= (1 - e^{-j\omega}) X(\omega)$$

d) $y(n) = \sum_{k=-\infty}^{n} x(k)$

$\cancel{\sum_n y(n) e^{-j\omega n} = \sum_n x(n) e^{-j\omega n} - \sum_n x(n-1) e^{-j\omega}}$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)$$

$$= y(n) - y(n-1) \Rightarrow x(n)$$

$$X(\omega) = (1 - e^{-j\omega}) Y(\omega)$$

$$\Rightarrow Y(\omega) = \frac{X(\omega)}{\ldots}$$

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e) $y(n) = x(2n)$

$$Y(\omega) = \sum_n x(2n) e^{-j\omega n}$$

$$= \sum_n x(n) e^{-j\frac{\omega}{2} n}$$

$$= X\left(\frac{\omega}{2}\right)$$

f)
$$y(n) = \begin{cases} x(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$$

$$Y(\omega) = \sum_n x(n/2) e^{-j\omega n}$$

$$= \sum_n x(n) e^{-j2\omega n}$$

$$= X(2\omega)$$

18) <u>F.T</u>

a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

$$X_1(\omega) = \sum_n x(n) e^{-j\omega n}$$

$$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$$

$$= 1 + 2\cos\omega + 2\cos 2\omega$$

b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1\}$

$$X_2(\omega) = \sum_n x_2(n) e^{-j\omega n}$$

$$= e^{j4\omega} + e^{j2\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}$$

$$= 1 + 2\cos 2\omega + 2\cos 4\omega.$$

c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

$$X_3(\omega) = \sum_n x_3(n) e^{-j\omega n}$$

$$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$$

$$= 1 + 2\cos 3\omega + 2\cos 6\omega$$

d) From above we get to know that

$X_2(\omega) = X_1(2\omega)$ and $X_3(\omega) = X_1(3\omega)$

e)

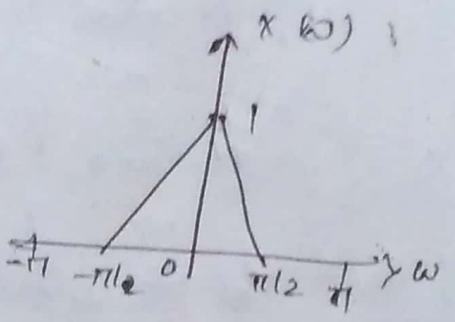
Given $x_k(n) = \begin{cases} x(n/k), & n/k \text{ an integer} \\ 0, & \text{otherwise.} \end{cases}$

$$X_k(\omega) = \sum_{n,\ \frac{n}{k}\ \text{integer}} x_k(n) e^{-j\omega n}$$

$$= \sum_n x(n) e^{-jk\omega n}$$

$$= X(k\omega).$$

19) F.T



a) $x_1(n) = x(n)\cos(\pi n/4)$

$$x_1(n) = \frac{1}{2}\left(e^{j\pi n/4} + e^{-j\pi n/4}\right) x(n)$$

$$X_1(\omega) = \frac{1}{2}\left\{X(\omega - \pi/4) + X(\omega + \pi/4)\right\}$$

b) $x_2(n) = x(n)\sin(\pi n/2)$

~~$X_2(\omega) = \frac{1}{2j}\left[X(\omega - \pi/2) + e^{j\pi/2}\right.$~~

$$x_2(n) = \frac{1}{2j}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right] x(n)$$

$$X_2(\omega) = \frac{1}{2j}\left[X(\omega - \pi/2) + X(\omega + \pi/2)\right]$$

c) $x_3(n) = x(n)\cos(\pi n/2)$

$$x_3(n) = \frac{1}{2}\left(e^{j\pi n/2} + e^{-j\pi n/2}\right) x(n)$$

$$= \frac{1}{2}\left\{X(\omega - \pi/2) + X(\omega + \pi/2)\right]$$

d) $x_H(n) = x(n)\cos \pi n$

$x_H(n) = \frac{1}{2}\left(e^{j\pi n} + e^{-j\pi n}\right) x(n)$

$X_H(\omega) = \frac{1}{2}\left[X(\omega - \pi) + X(\omega + \pi)\right]$

$= X(\omega - \pi)$

20) $x(n) \rightarrow$ Aperiodic $F.T = X(\omega)$

C_k^y of periodic sig $y(n) = \sum_{l=-\infty}^{\infty} x(n - lN)$

$c_k^y = \frac{1}{N} X\left(\frac{2\pi}{N} k\right), \quad k = 0, 1, \ldots N-1.$

$C_k^y = \frac{1}{N} \sum_{n=0}^{N-1} y(n) e^{-j2\pi k n / N}$

$= \frac{1}{N} \sum_{n=0}^{N-1} \left[\sum_{l=-\infty}^{\infty} x(n - lN)\right] e^{-j2\pi k n / N}.$

$= \frac{1}{N} \underbrace{\sum_{l=-\infty}^{\infty} \cdot \sum_{m=-lN}^{N-1-lN} x(m) e^{-j2\pi k (m + lN) / N}}_{X(\omega)}.$

$= \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$

$\therefore$ Hence proved.

21) P.T $X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n}$

as $X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin(2N+1)(\omega - \theta)/2}{\sin[(\omega - \theta)/2]} \cdot d\theta$

Let $x_N(n) = \frac{\sin \omega_c n}{\pi n}$, $-N \leq n \leq N$

$= x(n) \cdot w(n)$

$x(n) = \frac{\sin \omega_c n}{\pi n}$, $-\infty \leq n \leq \infty$

$w(n) = 1$, $-N \leq n \leq N$

0, otherwise.

Then $\frac{\sin \omega_c n}{\pi n} \longleftrightarrow X(\omega)$

$= 1$, $|\omega| \leq \omega_c$

0, otherwise

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\theta)\, W(\omega - \theta)\, d\theta$$

$$= \int_{-\omega}^{\omega_c} \frac{\sin (2N+1)(\omega-\theta)/2}{\sin(\omega-\theta)/2} \cdot d\theta$$

22)

$$x(n) \longleftrightarrow X(\omega) = \frac{1}{1 - a e^{-j\omega}}.$$

$\hookrightarrow$ $x(n) = a^n$

Determine F.T

a) $x(2n+1)$

$$X_1(\omega) = \sum_n x(2n+1) e^{-j\omega n}$$

$$= \sum_k x(k) e^{-j\omega k/2} \cdot e^{j\omega/2}.$$

$$= X\left(\frac{\omega}{2}\right) e^{j\omega/2}$$

$$= \frac{e^{j\omega/2}}{1 - a e^{j\omega/2}}.$$

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b) $x_2(\omega) = x(n+2)\, e^{\pi n/2}$

$$X_2(\omega) = \sum_n x(n+2)\, e^{\pi n/2}\, e^{-j\omega n}$$

$$= -\sum_k x(k)\, e^{-j(k\omega + j\pi/2)} - e^{j2\omega}$$

$$= -X(\omega + j\pi/2)\, e^{j2\omega}$$

c) $x(-2n)$

$$X_3(n) = \sum_n x(-2n)\, e^{-j\omega n}$$

$$= -\sum_k x(k)\, e^{-jk\omega/2}$$

$$= X(-\omega/2)$$

d) $x_4(n) = x(n)\cos(0.3\pi n)$

$$X_4(\omega) = \sum_n \frac{1}{2}\left(e^{j0.3\pi n} + e^{-j0.3\pi n}\right) x(n)\, e^{-j\omega n}$$

$$= \frac{1}{2}\sum_n x(n)\left[e^{-j(\omega-0.3\pi)n} + e^{-j(\omega+0.3\pi)n}\right]$$

$$= \frac{1}{2}\left[X(\omega-0.3\pi) + X(\omega+0.3\pi)\right]$$

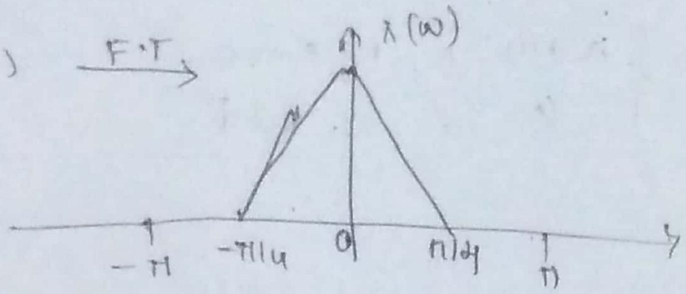
e) $x(n) * x(-n)$

$$X(\omega) = X(\omega)\cdot X(-\omega)$$

$$= \frac{1}{(1-ae^{-j\omega})(1-ae^{j\omega})}$$

$$= \frac{1}{1-2a\cos\omega + a^2}$$

23) $x(n) \xrightarrow{F.T}$



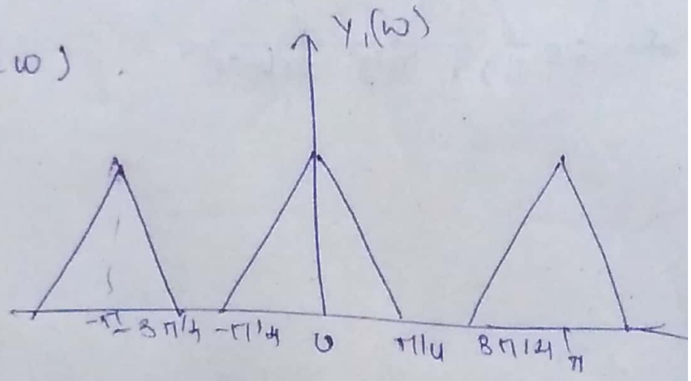
a) $Y_1(n) = \begin{cases} x(n), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

$y_1(n) = x(n)\, s(n)$; $s(n) = \{ \ldots 0,1,1,0,1,1,0,1,1,0$

$Y_1(\omega) = \sum_n y_1(n) e^{-j\omega n}$

$y_1(n) = \begin{cases} y_2(n/2), & n \text{ even} \\ 0, & n \text{ odd.} \end{cases}$

$Y_1(\omega) = Y_2(2\omega)$.



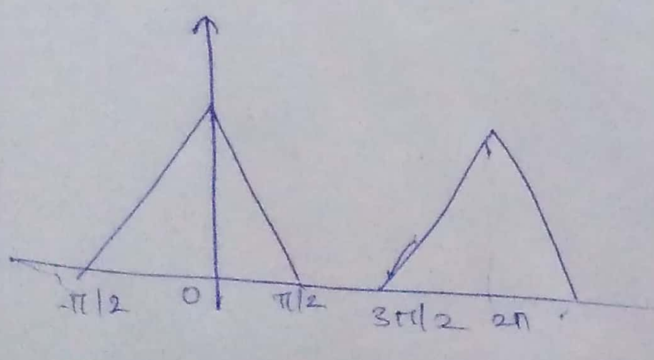
b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$Y_2(\omega) = \sum_n y_2(n) e^{-j\omega n}$

$= \sum_n x(2n)\, e^{-j\omega n} = \sum_m x(m)\, e^{-j\omega m/2}$

$= X(\omega/2)$



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c) $Y_3(n) = \begin{cases} x(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

$y_3(n) = ?$

$$Y_3(n) = \sum_n y_3(n) e^{-jwn}$$

$$= \sum_{n\,even} x(n/2) e^{-jwn}$$

$$= \sum_m x(m) e^{-j2wm}$$

$$= X(2w).$$