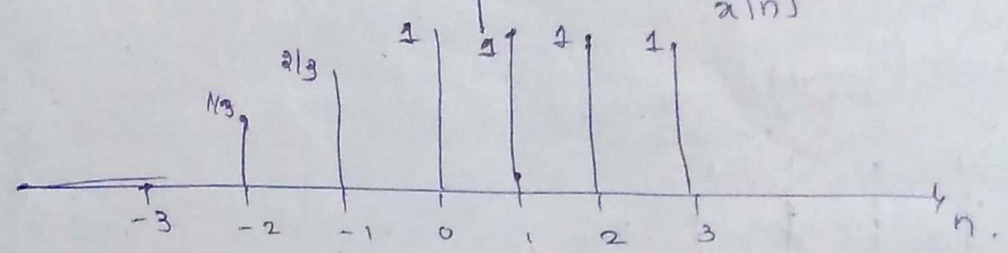
(1)

$$x(n) = \begin{cases} 1 + \frac{n}{3} ; & -3 \le n \le -1 \\ 1 ; & 0 \le n \le 3 \\ 0 ; & \text{elsewhere} \end{cases}$$

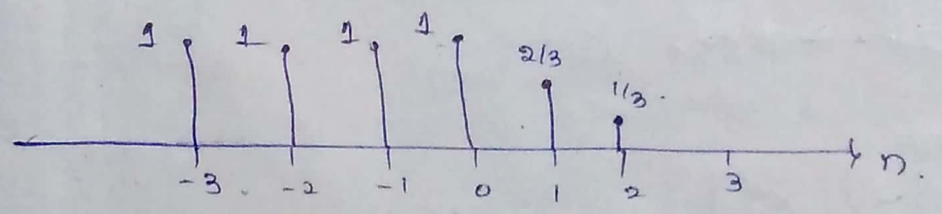
a) Determine its values and sketch signal

$$x(n) = \{ \ldots 0, 1/3, 2/3, \underset{\uparrow}{1}, 1, 1, 1, 0 \ldots \}$$

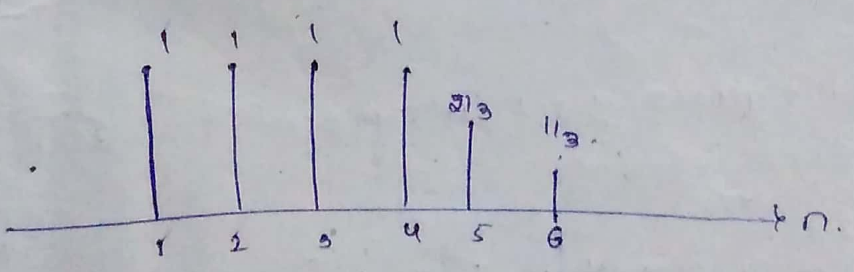


b)

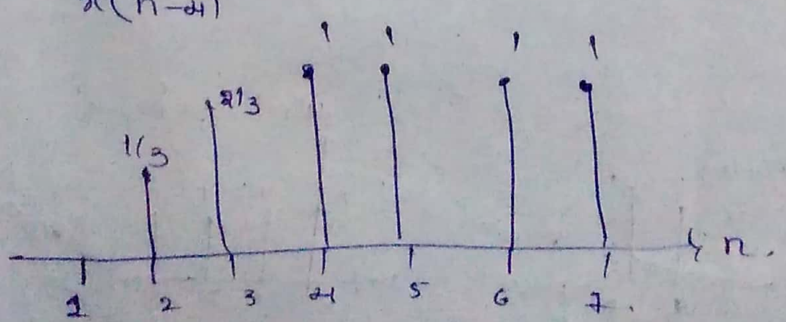
1) $x(-n)$



$x(-n-4) = x[-(n+4)]$.



2) $x(n-4)$



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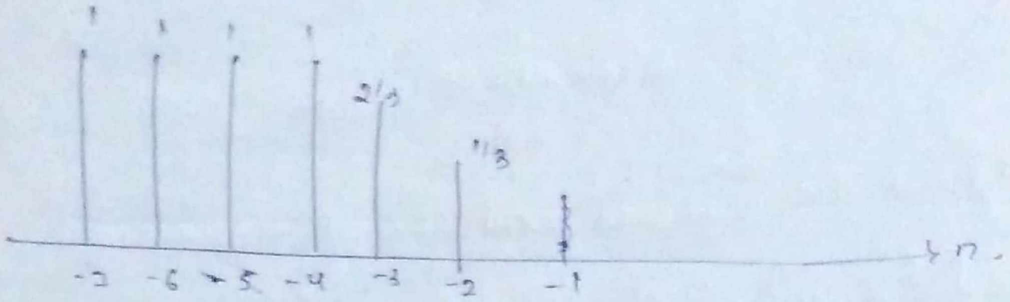
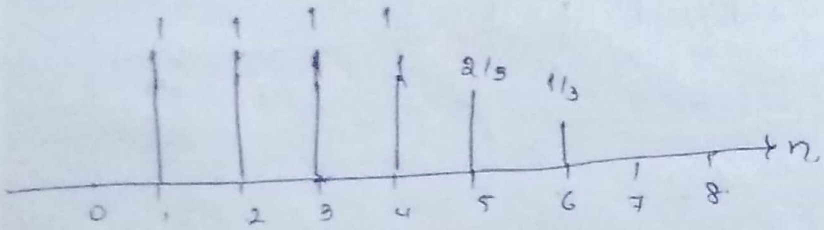
$x(-(n-4))$



c)

$x(-n+4)$.



d)

Reverse $x(n) \rightarrow x(-n)$

$x(-n+k)$

$x(-n+k) = x[-(n-k)]$

shift signal by four samples right and then reverse the signal to obtain $x(-n+k)$.

e)

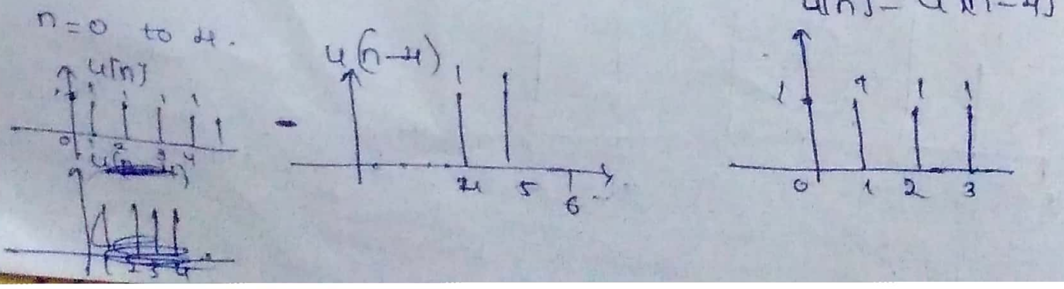
Yes, we can express $x(n)$ interms of $\delta(n)$ and $u(n)$.

$$x(n) = \tfrac{1}{3}\,\delta(n+2) + \tfrac{2}{3}\,\delta(n+1) + u[n] - u[n-4]$$

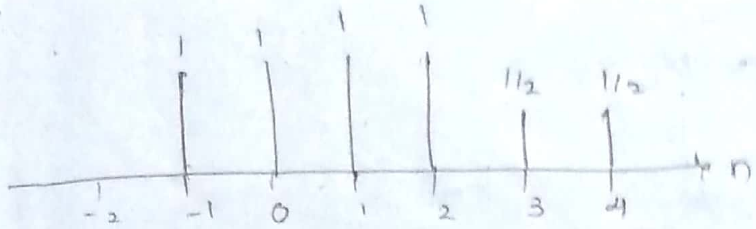
$n=-2 \rightarrow 1/3$

$n=-1 \rightarrow 2/3$

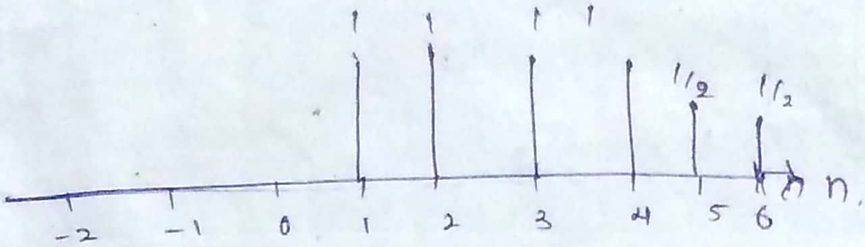
$n=0$ to 4.



(2)

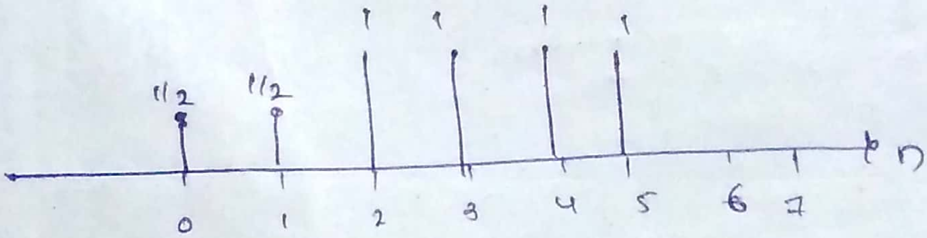


a) $x(n-2)$



b) $x(4-n)$

$x(4-n) = x(-n+4) \Rightarrow$

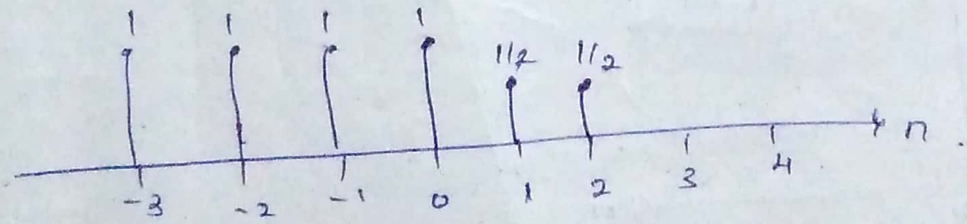


c) $x(n+2)$

$n+2=4$ $\quad$ $n+2=-2$

$\boxed{n=2}$ $\quad$ $\boxed{n=-4.}$

$x(n+2)$



d) $x(n)\, u(2-n)$

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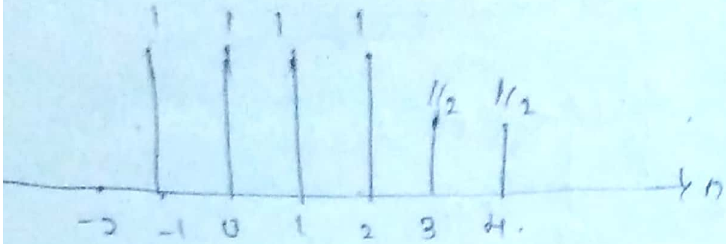
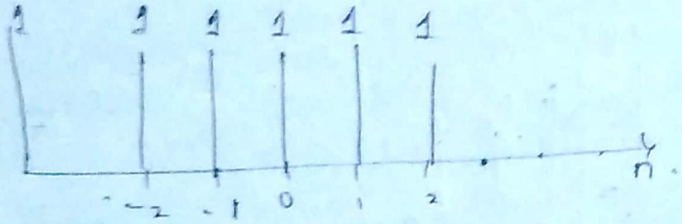
$x(n)\,u(2-n)$

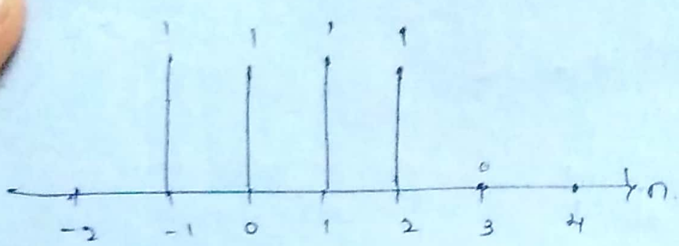
$x(n)$



$u(2-n)$

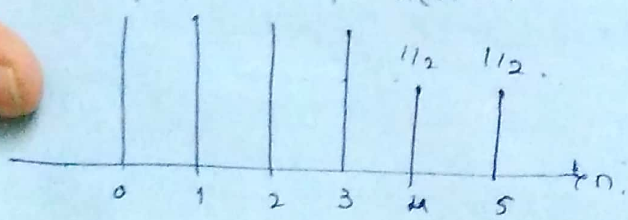


$x(n)\,u(2-n)$

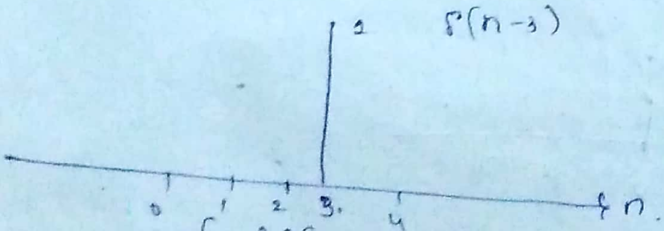


e) $x(n-1)\,\delta(n-3)$

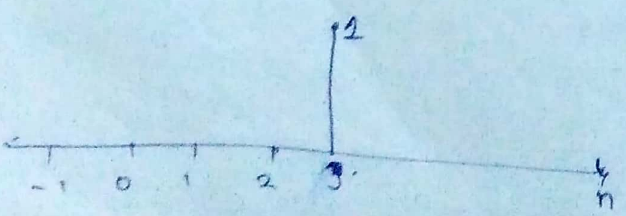
$x(n-1)$



$\delta(n-3)$



$x(n-1)\,\delta(n-3)$



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#) $x(n^2)$

$x(n^2) = \{ \ldots x((-2)^2), x((-1)^2), x(0), x(1), x(4), x(9), x(16) \}$

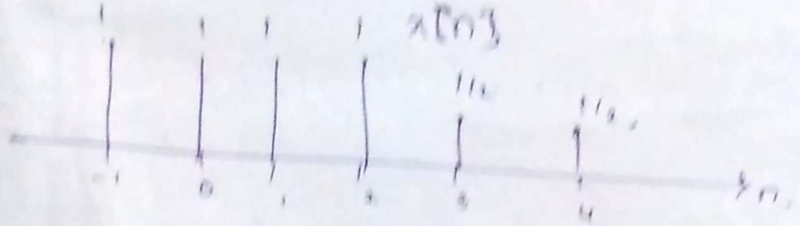
$= \{ \ldots x(4), x(1), x(0), x(1), x(4), x(9), x(16) \ldots \}$

$x(n^2)$

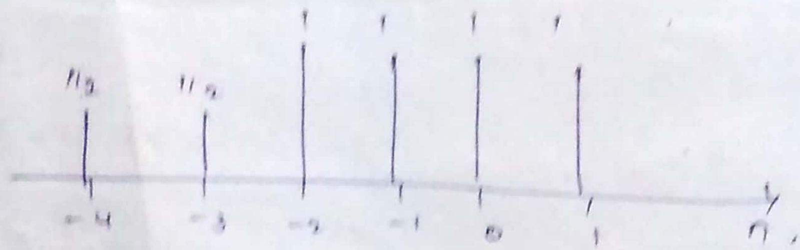
#) even part of $x(n)$

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

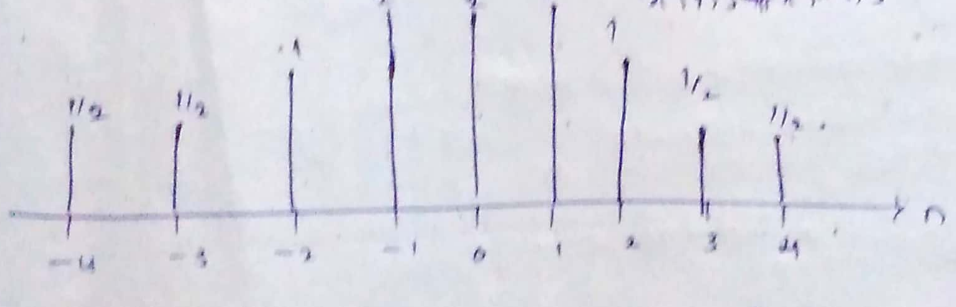
$x[n]$



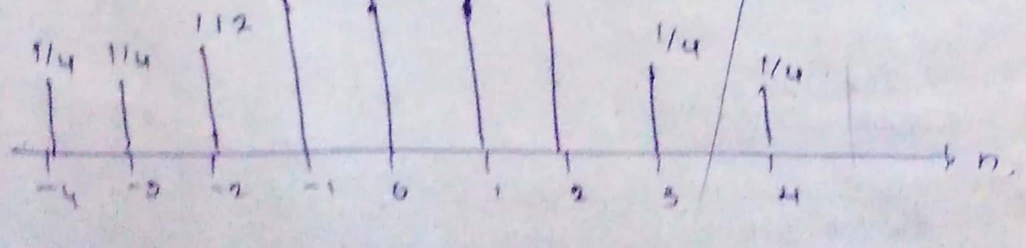
$x[-n]$



$x[n] + x[-n]$



$\frac{x[n] + x[-n]}{2}$



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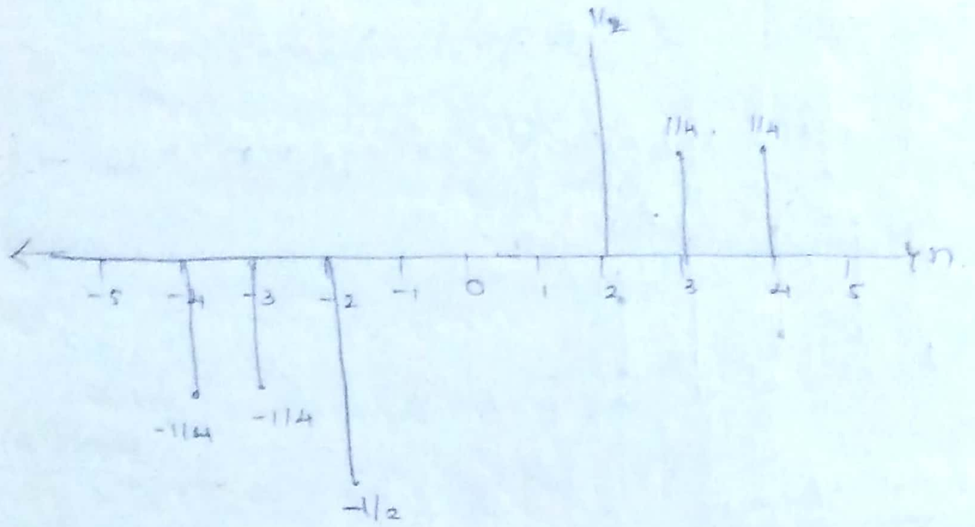
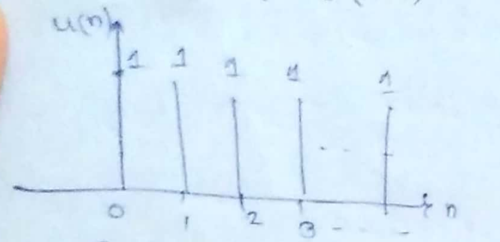
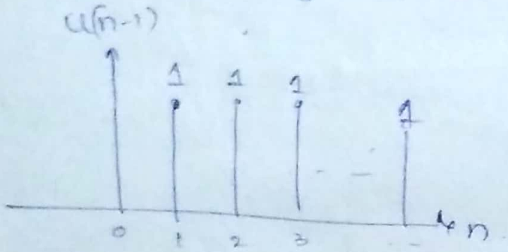
h) odd part of $x(n)$

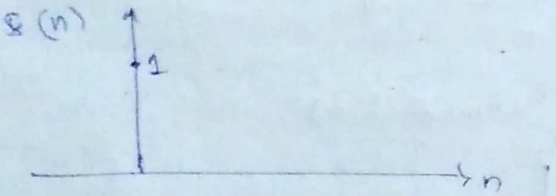


③ show that

a) $\delta(n) = u(n) - u(n-1)$







$\therefore \quad \delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$

$$u(n) = \begin{cases} 1 \; ; & n \geq 0 \\ 0 \; ; & n < 0 \end{cases}$$

$$\sum_{k=-\infty}^{n} \delta(k) = \sum_{k=-\infty}^{0} \delta(k) + \sum_{k=0}^{n} \delta(k)$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \underbrace{\delta(n) + \delta(n-1) + \dots + \delta(n-\infty)}$$

$\therefore u(n) = \sum_{k=0}^{\infty} \delta(n-k)$ all are positive integers $\therefore n \geq 0$

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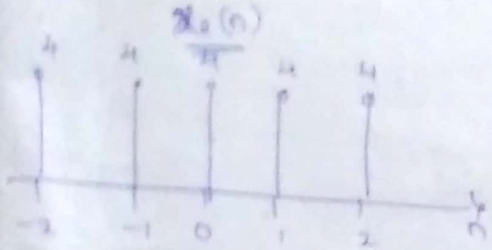
④ $x(n) = \{2, 3, 4, 5, 6\}$
$\uparrow$

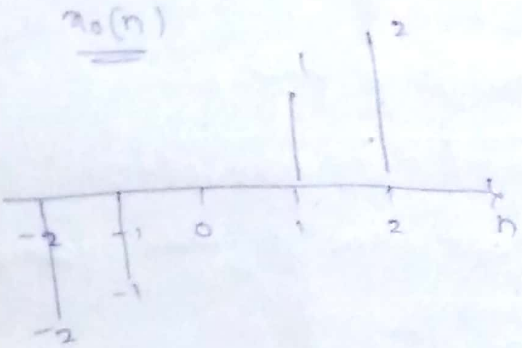
$x_e(n) = \frac{x(n) + x(-n)}{2}$ $\quad x_o(n) = \frac{x(n) - x(-n)}{2}$

$x(-n) = \{6, 5, 4, 3, 2\}$
$\uparrow$

$x_e(n) = \frac{x(n) + x(-n)}{2} = \left\{\frac{8}{2}, \frac{8}{2}, \frac{8}{2}, \frac{8}{2}, \frac{8}{2}\right\} = \{4, 4, 4, 4, 4\}$

$x_o(n) = \left\{\frac{-4}{2}, \frac{-2}{2}, \frac{0}{2}, \frac{2}{2}, \frac{4}{2}\right\} = \{-2, -1, 0, 1, 2\}$
$\uparrow$





Hence the decomposition is unique.
we get the original sig $x(n)$ by adding $x_e(n)$ and $x_o(n)$

$$x(n) = x_e(n) + x_o(n).$$

⑤

$x_e(n) = \frac{x(n) + x(-n)}{2}$; $\quad x_o(n) = \frac{x(n) - x(-n)}{2}$

$$\sum_{n=-\infty}^{\infty} |x_e(n)|^2 = \sum_{n=-\infty}^{\infty}\left(\frac{x(n) + x(-n)}{2}\right)^2 = \frac{1}{4}\left\{\sum_{n=-\infty}^{\infty} x(n)^2 + \sum_{n=-\infty}^{\infty} x(-n)^2 + \sum_{n=-\infty}^{\infty} 2x(n)\,x(-n)\right\}$$

$$\sum_{n=-\infty}^{\infty} |x_o(n)|^2 = \sum_{n=-\infty}^{\infty} \frac{1}{4}\left\{x^2(n) + x^2(-n) - 2x(n)\,x(-n)\right\}$$

$$\sum_{n=-\infty}^{\infty} |x_e(n)|^2 + \sum_{n=-\infty}^{\infty} |x_o(n)|^2 = \frac{1}{4}\sum_{n=-\infty}^{\infty} x(n)^2 + \frac{1}{4}\sum_{n=-\infty}^{\infty} x(-n)^2 + \cancel{\frac{1}{4}\sum_{n=-\infty}^{\infty} 2x(n)\,x(-n)}$$
$$+ \frac{1}{4}\sum_{n=-\infty}^{\infty} x^2(n) + \frac{1}{4}\sum_{n=-\infty}^{\infty} x^2(-n) - \cancel{\frac{1}{4}\sum_{n=-\infty}^{\infty} 2x(n)\,x(-n)}$$

$$= \sum_{n=-\infty}^{\infty} \frac{1}{2} x^2(n) + \frac{1}{2} x^2(-n) \qquad \boxed{x^2(n) = x^2(-n)}$$

$$= \sum_{n=-\infty}^{\infty} x^2(n)$$

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$$\boxed{\therefore \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) = \sum_{n=-\infty}^{\infty} x^2(n)}$$

$\therefore$ The energy of sig is equal to sum of energy of its even and odd components.

(6)

(a) $y(n) = T[x(n)] = x(n^2)$

$x(n) \longrightarrow y(n) = x(n^2)$

ip delay

$x(n-k) \longrightarrow y(n-k) = x[(n-k)^2]$

op delay $\therefore$

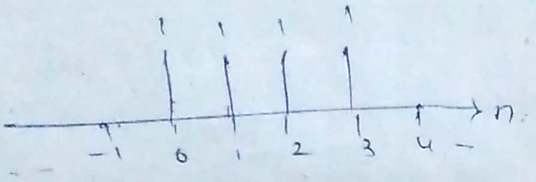
$= x(n^2-k)$

$\therefore y(n-k) \neq x(n^2-k)$

The sim is time variant

(b) $x(n) = \begin{cases} 1 ; & 0 \le n \le 3 \\ 0 ; & \text{elsewhere} \end{cases}$

1] $x[n]$

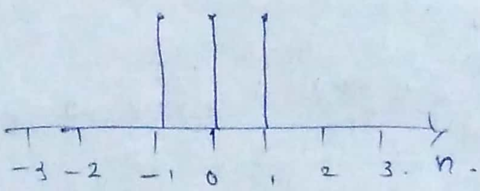


2) $x[n^2] \Rightarrow y(n) = T[x(n)] = x(n^2)$

$n^2 = 3 \Rightarrow n = \pm\sqrt{3}$.

$n^2 = 1 \Rightarrow n = \pm 1$

$n^2 = 0 \Rightarrow n = 0$

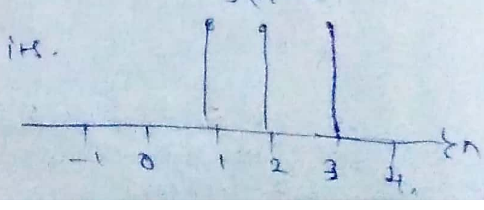


3) $y_2'(n) = y(n-2)$

$y(n-2) = x[(n-2)^2]$.

shift $y(n)$ right side by 2 units.

$y(n-2)$



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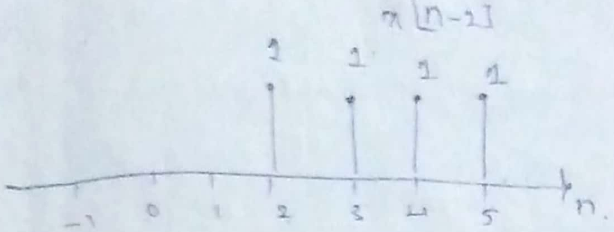
4) $x_2(n) = x[n-2]$

$x[n-2]$



5) $y_2(n) = T[x_2(n)]$

$y_2(n) = T[x_2(n)] = x[(n-2)^2]$

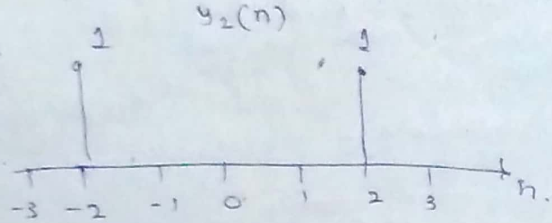
$= x[(n-2)^2]$

$n^2 = 2 \Rightarrow n = \pm\sqrt{2}$

$n^2 = 3 \quad n = \pm\sqrt{3}$

$n^2 = 4 \quad n = \pm 2$

$y_2(n)$



6)

$y_2(n) = T[x_2(n)] = T[x(n-2)]$.

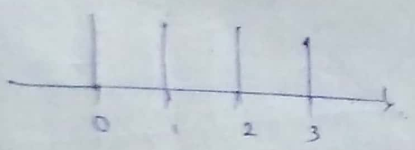
$\Rightarrow y(n-2)$

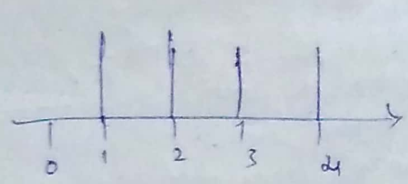
Here $y_2(n) \neq y(n-2)$

$\therefore$ The s/m is time variant s/m.

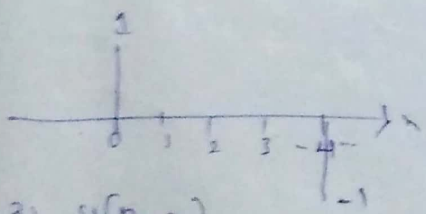
c) $y(n) = x(n) - x(n-1)$

1) $x(n)$. $\quad x(n-1)$

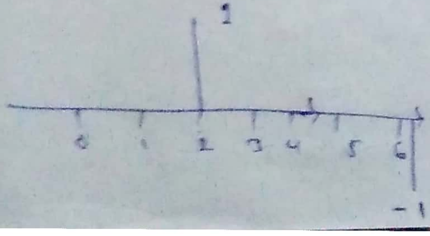


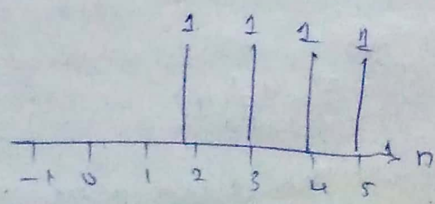


2) $y(n)$



3) $y(n-2)$ $\quad$ 4) $x(n-2)$





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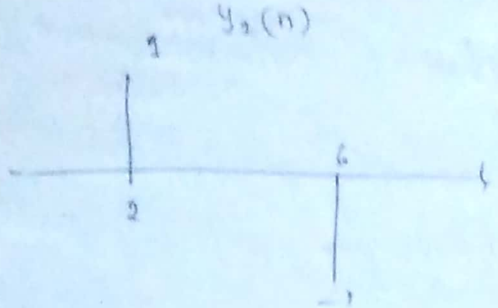
5) $y_2(n) = T[x_2(n)]$

$x(n-2) - x(n-3)$

6) $y_2(n) = y'(n-2)$

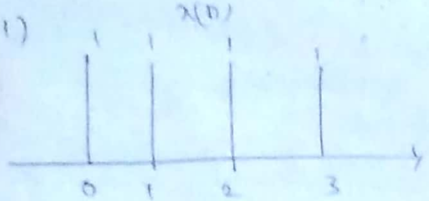
Hence the sim is time invariant

$y_2(n)$

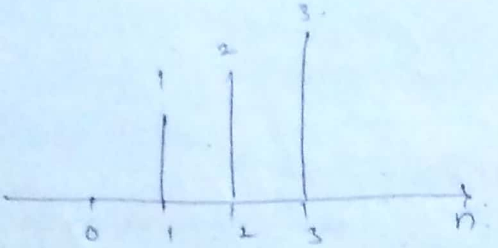


d) $y(n) = T[x(n)] = n\,x(n)$

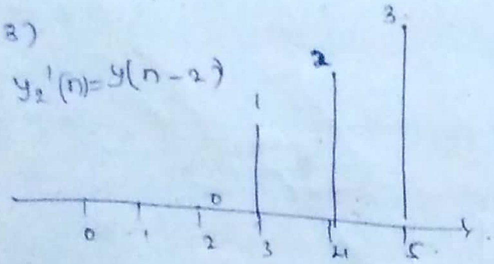
1) $x(n)$



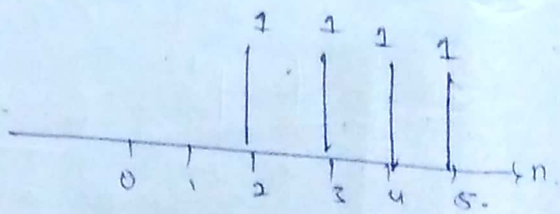
2) $y(n)$



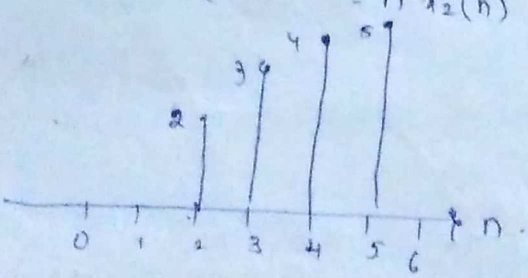
3) $y_2'(n) = y(n-2)$



4) $x_2(n) = x(n-2)$



5) $y_2(n) = T(x_2(n)) = n\,x_2(n)$



6) $y_2(n) \neq y_2'(n)$

$\therefore$ The given sim is time variant.

(7)

a) $y[n] = \cos[x(n)]$

1) static

⇒ Because ~~static~~ s/m i/p and o/p depends only on present samples.

2) Non-linear.

$y_1(n) = \cos(x_1(n))$

$y_2(n) = \cos(x_2(n))$

$y_1(n) + y_2(n) = \cos(x_1(n)) + \cos(x_2(n))$

$\cos[x_1(n) + x_2(n)] \neq y_1(n) + y_2(n)$.

∴ It is a non-linear s/m.

3) $y(n) = \cos[x(n)] \Rightarrow y(n,k) = \cos[x(n-k)]$

$y[n-k] = \cos[x(n-k)]$.

∴ s/m is time invariant.

4) causal

⇒ since it is static only depends on present

∴ This is causal.

5) Stable.

⇒ As cos is bounded b/w -1 to 1.

$|x(n)| < \infty$

∴ It is stable

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

1) $y(n) = x(-\infty) + \dots x(n) + x(n+1)$

↓

future value.

∴ This is dynamic.

2) ~~y(n)~~ = $a x_1(k) + b x_2(k)$. $a y_1(n) = a\sum_{k=-\infty}^{n+1} x_1(k)$

$b y_2(n) \dots$

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$y_1(n) + y_2(n) = \sum_{k=-\infty}^{n+1} x_1(n) + x_2(n) \neq y(n).$

$\therefore$ This is linear.

3) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$; $y(n,k) = \sum_{k=-\infty}^{n+1-k} x(k).$

$y(n-k) = \sum_{k=-\infty}^{n+1-k} x(k)$

Hence time invariant

4) As sim is dynamic

$\therefore$ it is non-causal

5)

sim is unstable.

6) $y(n) = x(n) \cos(\omega_0 n)$

1) static

2) $x(n) = a x_1(n) + b x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$

$a y_1(n) = a x(n) \cos(\omega_0 n)$

$y_2(n) = b x_2(n) \cos(\omega_0 n)$

$y'(n) = a x(n) \cos(\omega_0 n) + b x_2(n) \cos \omega_0(n).$

$y(n) = a x_1(n) \cos(\omega_0 n) + b x_2(n) \cos(\omega_0 n)$

$\Rightarrow$ it is a linear sim.

3) $y'(n) = y(n-n_0) = x(n-n_0) \cos(\omega_0(n-n_0))$

$y(n) = x(n-n_0) \cos(\omega_0 n)$

$y'(n) \neq y(n)$

Hence, time variant.

4) causal, since it is static

5) stable as cos is a bounded sim.

$\therefore$ stable

d) $y(n) = x(-n+2)$

1) Dynamic

2) $y_1(n) = x_1(-n+2)$

$y_2(n) = x_2(-n+2)$

$\Rightarrow y(n) = y_1(n) + y_2(n)$

$\therefore$ Linear

3) $y(n) = x(-n+2)$

$y(n-n_0) = x(-(n-n_0)+2) = x(-n+n_0+2)$

$y'(n,n_0) = x(-n-n_0+2)$

$y(n,n_0) \neq y(n-n_0)$

Time variant.

4) Non causal as it is dynamic

5) Stable; as $|y(n)| < \infty$.

e) $y(n) = \text{Trun}[x(n)]$

1) Static

$\Rightarrow$ Depends on present i/p

2) $x_1(n), x_2(n)$

$y_1(n) = \text{Trun}[x_1(n)]$; $y_2(n) = \text{Trun}[x_2(n)]$

$y[n] \neq y_1(n) + y_2(n)$

$$\underset{1}{\underset{\downarrow}{2.3}} + \underset{1}{\underset{\downarrow}{1.3}} = 2.$$

$= 2.3 + 1.3$

$= 3.6$

$\neq 2.$

$\therefore$ Non-linear.

3) $y(n,n_0) = y(n-n_0) = \text{Trun}[x(n-n_0)]$

$\therefore$ Hence time invariant.

4) Causal as it is static

5) Stable as it is bounded

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(f) $y(n) = \text{Round}[x(n)]$;

1) Static
2) Non-linear
3) Time variant.
4) Causal.
5) Stable.

g) $y(n) = |x(n)|$

1) Static

2) $a_0 y_1(n) + b y_2(n) = |a_0 x_1(n) + b x_2(n)|$

$a|x_1(n)| + b|x_2(n)| \neq a|x_1(n)| + b|x_2(n)|$

$\therefore$ Non-linear.

3) Time invariant

4) Causal

5) Stable

$\Rightarrow$ Bounded i/p give bounded o/p.

h) $y(n) = x(n)\, u(n)$.

1) Static

2) $a x_1(n) u(n) + b x_2 u(n) = a x_1(n) u(n) + b x_2(n) u(n)$

Linear.

3) $y(n) = x(n)\, u(n)$

$y(n-n_0) = x(n-n_0)\, u(n-n_0)$

$x(n-n_0) u(n-n_0) = y(n)$, $y(n, n_0) = x(n-n_0) u(n)$

$\Rightarrow$ Time variant

4) Causal

5) Stable

i) $y(n) = x(n) + n\, x(n+1)$

1) Dynamic

$\Rightarrow$ s/m depends on future values.

2) $y_1(n) = x_1(n) + n\, x_1(n+1)$

$y_2(n) = x_2(n) + n\, x_2(n+1)$

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$y = y_1(n) + y_2(n)$

$= [x_1(n) + x_2(n)] + n[x_1(n+1) + x_2(n+1)]$

$\therefore$ Linear

3) $x(n-n_0) + n\,x(n+1-n_0) = y(n)$

$y'(n) = y(n-n_0) = x(n-n_0) + (n-n_0)\cdot x(n+1-n_0)$.

$y'(n) \neq y(n)$

$\therefore$ Time variant

4) Non-causal s/m

5) stable

$\Rightarrow$ bounded i/p give bounded o/p

3) $y(n) = x(2n)$

1) $y(1) = x(2)$

Dynamic s/m

2) $y_1(n) = x_1(2n)$

$y_2(n) = x_2(2n)$

$y = [y_1(n) + y_2(n)]$

$= x_1(2n) + x_2(2n)$

~~Time-vari~~ $\therefore$ Linear

3) $y(n,k) = x(2n-k)$

$y(n-k) = x(2n-2k)$

$\therefore$ $y(n,k) \neq y(n-k)$

Time variant.

4) Non-causal (as it is Dynamic)

5) stable

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k. $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0 \end{cases}$

1) $y(0) = x(0)$

Static sim

2) Non linear $x(n) = 0$

3) $y(n-n_0) = x(n-n_0)$

→ Time invariant sim

4) Causal sim

5) stable

l) $y(n) = x[-n]$

1) Dynamic

2) Linear

$y_1[n] = x_1(-n)$

$y_2(n) = x_2(-n)$

$y = y_1(n) + y_2(n) = x_1(-n) + x_2(-n)$

∴ Linear

3) Time variant

$y(n) = x(-n)$

$y(n-n_0) = x(-n-n_0)$

$y(n+n_0) = \cancel{x-n}$

4) Non causal

5) Stable

m) $y(n) = \text{sig}[x(n)]$

1) Static sim

2) Non-linear

3) Time invariant.

$y(n,k) = \text{sig}[x(n-k)]$

$y(n-k) = \text{sig}(x(n-k)]$

4) Causal

5) Stable.

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n) i/p → $x_a(t)$

o/p → $x(n) = x_a(nT)$ $\quad -\infty < n < \infty$

1) static

2) Linear, $\quad x_1 a(nT) + x_2 a(nT) = y(n)$

3) $x(n-k) = x_a(nT-k)$

$x(n-k) = x_a(nT-k)$

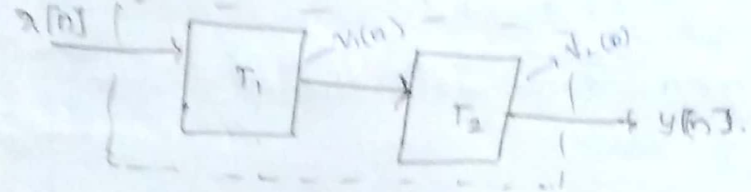
∴ time invariant

4) causal

5) stable

(5)

a)



$T = T_1 T_2$.

$v_1(n) = T_1[x_1(n)]$

$v_2(n) = T_1[x_2(n)]$

$\alpha_1 x_1(n) + \alpha_2 x_2(n) \Rightarrow \alpha_1 v_1(n) + \alpha_2 v_2(n)$

$y_1(n) = T_2[v_1(n)]$

$y_2(n) = T_2[v_2(n)]$.

$y(n) = a y_1(n) + b y_2(n)$

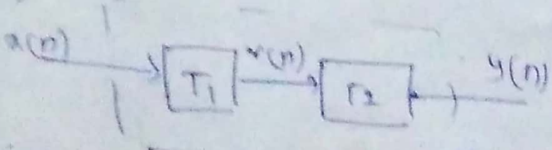
$= a v_1(n) + b v_2(n)$

$a T[x_1(n)] + b T[x_2(n)]$

$\boxed{T = T_1 T_2}$

so the overall sm is linear.

b)



$x(n) \rightarrow v(n)$ $\qquad$ $v(n) \rightarrow y(n)$

$x(n-k) \rightarrow v(n-k)$ } T_1 $\qquad$ $v(n-k) \rightarrow y(n-k)$ } T_2.

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$x(n) \longrightarrow y(n)$

$x(n-k) \longrightarrow y(n-k)$

$\therefore$ $T = T_1 T_2$ is time invariant.

c)

Proof:

T_1 is causal $\Rightarrow$ $w(n)$ dependens $x(k)$ for $k \le n$

T_2 is causal $\Rightarrow$ $y(n)$ depends only on $x(k)$ for $k \le n$.

Hence, T is causal.

d) We can prove from b and c

e) Yes, interchanging their order doesn't change the slm

$h_1(n) * h_2(n) = h_2(n) * h_1(n)$

$x(n) \longrightarrow \boxed{T_1} \longrightarrow \boxed{T_2} \longrightarrow y(n)$

$x(n). \longrightarrow \boxed{T_2} \longrightarrow \boxed{T_1} \longrightarrow y(n)$

(f)

T_1 : $y(n) = n\, x(n)$ and

T_2 : $y(n) = n \cdot x(n+1)$.

here two slm are not in time varying

then,

if $\delta(n)$ is i/p

$T_2[T_1[\delta(n)]] = T_2[0] = 0$

$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$

$= -\delta(n+1)$

$\ne 0$

h) $x(n)$ bounded i/p, $y(n)$ is bounded o/p

so [illegible]

i) Inverse of (e)

T_1 and T_2 are non causal

$\Rightarrow T$ is non causal

$T_1: y(n) = x(n+1)$

$T_2: y(n) = x(n-3)$

$\Rightarrow T: y(n) = x(n-2)$;

(10)

$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} \{\underset{\uparrow}{0}, 1, 0, 2\}$

$x_3(n) = \{0, 0, 0, 1\} \xleftrightarrow{T} \{1, \underset{\uparrow}{2}, 1\}$

If slm is linear then

$2 * x_3(n) \longrightarrow \{2, 4, 2\}$.

$\{0, 0, 0, 2\}$.

$\{2, 4, 2\} \neq \{\underset{\uparrow}{0}, 1, 0, 2\}$

$\therefore$ The slm is non-linear.

(12)

a) A linear combination of signals $x_i(n) \Rightarrow i = 1, 2, .., N$

b) Any $x_i(n-k)$, where k is any integer and $i = 1, 2, - N$

(13)

slm $y(n) =$

(16)

(6)

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

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$y(n)$ \ $x(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$y[n] = \{1, 3, 7, 7, 7, 6, 4\}$.

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$.

$h(n)$ \ $x(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y(n) = \{1, 4, 2, -4, 1\}$

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

$h(n)$ \ $x(n)$	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1	-1	3/2	-2

$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$

4) $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

$y(n) = \{1, 2, 3, 4, 5\}$,

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5) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

$h(n) \backslash x(n)$	1	-2	3
0	0	0	0
0	0	0	0
0	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

6) $x(n) = \{\underset{\substack{\uparrow \\ n=0}}{0}, 0, 1, 1, 1, 1\}$, $h(n) = \{1, \underset{\substack{\uparrow \\ n=0}}{-2}, 3\}$

$h(n) \backslash x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y(n) = \{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$

7) $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$h(n) \backslash x(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
1	0	1	4	-3

$y(n) = \{0, 1, 4, -4, -3, 7, -3$

(8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$

$h(n) \backslash x(n)$	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

$y(n) = \{1, 2, 4, 3, 2\}$

(9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

$h(n) \backslash x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	4	4	0	4	4

$y(n) = \{1, -1, -5, 1, 2, \underset{\uparrow}{-5}, 1, 4.\}$

(10) $x(n) = \{1, 2, 0, 2, 1\}$. $h(n) = x(n)$.

$h(n) \backslash x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	1
0	0	0	0	0	0
2	2	4	0	4	1
1	1	2	0	2	1

$y(n) = \{1, 4, 4, 4, \underset{\uparrow}{10}, 3, 4, 3, 1\}$

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11) $x(n) = (1/2)^n u(n)$; $h(n) = (1/4)^n u(n)$

$$y(n) = \sum_{k=-\infty}^{\infty} (1/2)^k u(k) (1/4)^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} (1/2)^k (1/4)^{n-k} = \sum_{k=0}^{n} (1/2)^{2n-k}$$

$$y(n) = (1/2)^{2n} \cdot \sum_{k=0}^{n} 2^k$$

$$= (1/2)^{2n} \frac{1-2^{n+1}}{1-2}$$

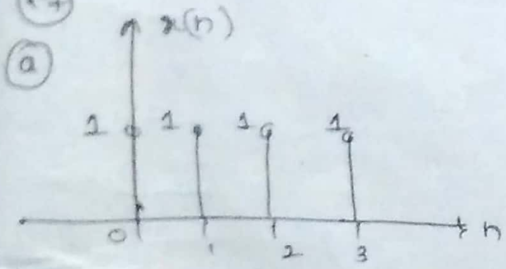
$$= (1/2)^{2n} (2^{n+1} - 1)$$

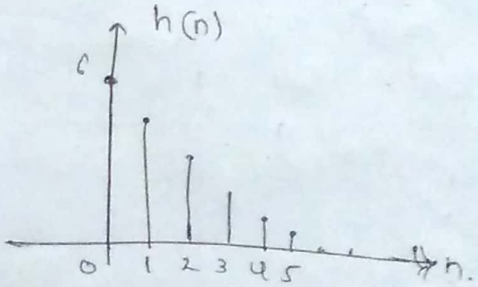
$$\Rightarrow 2^{-n+1} - 2^{-2n}.$$

$$= 2 (1/2)^n - (1/4)$$

(17)

(a)

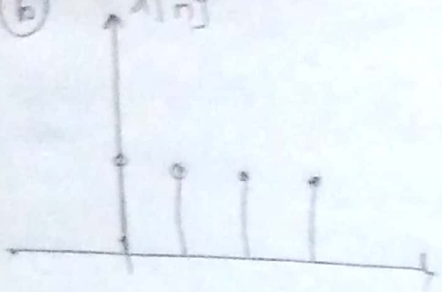


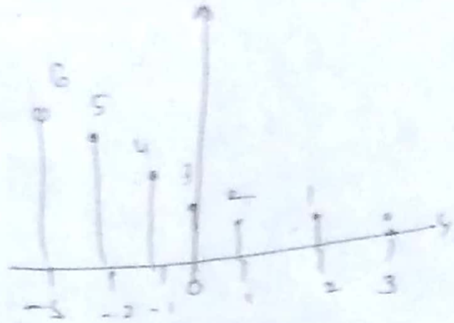


x(n) \ h(n)	1	1	1	1
6	6	6	6	6
5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1
0				

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

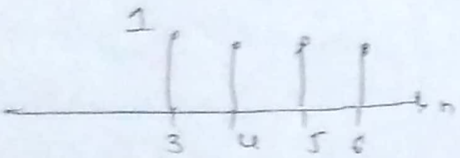
(b)

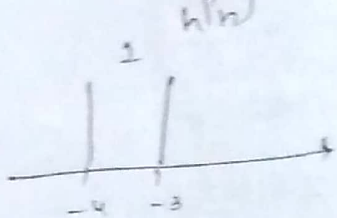




$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$

c) x(n)

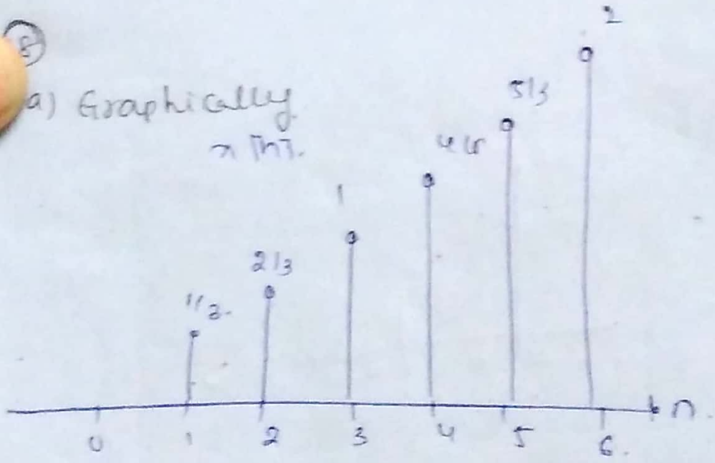


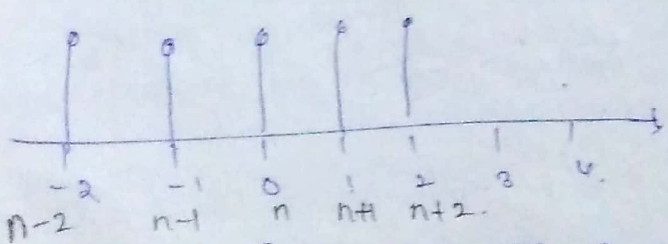


{

(5)

a) Graphically





$y[n] = \{1/3, \underset{\uparrow}{1}, 2, 10/3, 5, 20/3, 6, 5, 11/3, 2\}$

$n+2 < 0$, $n < -2 \Rightarrow o/p = 0$

$n+2 > 0$ and $n-2 < 0$

$n > -2$ and $n < 2$

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$$\sum_{0}^{n+2} (1/3)^n = \cancel{(1)^{1}}$$

$n = -2$; 0

$n = -1$; $\sum_{0}^{1} (1/3) \Rightarrow 1/3.$

$n = 0$; $\sum_{0}^{2} (1/3)^0 \Rightarrow 1/3 (2+1) = 1$

$n = 1$; $\sum_{0}^{3} (1/3) \Rightarrow 1/3 (1+2+3) = 2$

$n = 2$; $\sum_{0}^{4} (1/3)^2 \Rightarrow 1/3 (1+2+3+4) = 10/3$

$\Rightarrow$ $n-2 \geq 0 \Rightarrow$ $\quad n+2 < 6$

$n \geq 2$; $\quad n < 4.$

$n=2$; $10/3$

$n=3$; 5

$n=4$; $20/3$

$\cancel{n=6}$; $\cancel{2}$

$\Rightarrow$ $n+2 > 6 \Rightarrow n > 4.$

$\cancel{\sum_{n-2}^{6} (1/3)^n = 1+3+1}$

$n = 5 \Rightarrow \sum_{3}^{6} (1/3)^n \Rightarrow 1/3 [6+5+4+3]$

$= \underline{\underline{6}}$

$y[n] = \{ 1/3, \underset{\uparrow}{1}, 2, 10/3, 5, 20/3, 6, 5, 1/3, 2 \}$

b) <u>Analytically!</u>

h(n) \ x(n)	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2

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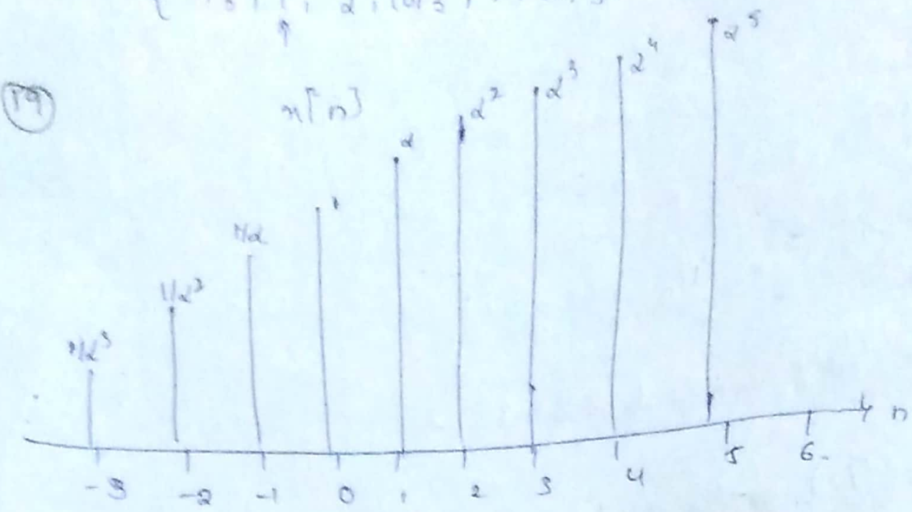
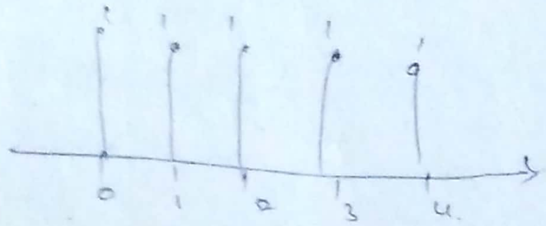
$y(n) = \{ 1/3, \underset{\uparrow}{1}, 2, 10/3, 5, 20/3, 6, 5, 11/3, 2 \}$

(19)



$h(n) = h(k)$



~~Analytic~~

$h(n) \backslash x(n)$	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5

$y[n] = \{ 1/\alpha^3, 1/\alpha^2 + 1/\alpha^3, 1/\alpha^3 + 1/\alpha^2 + 1/\alpha, 1/\alpha^3 + 1/\alpha^2 + 1/\alpha + 1,$

$\underset{\uparrow}{\alpha + 1 + 1/\alpha + 1/\alpha^2 + 1/\alpha^3}, 1/\alpha^3 + 1/\alpha^2 + 1/\alpha + 1 + \alpha, \alpha^2 + \alpha + 1 + 1/\alpha +$

$1/\alpha^2, 1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5, \alpha^2 + \alpha^3 +$

$\alpha^3 + \alpha^4 + \alpha^5, \alpha^4 + \alpha^5, \alpha^5 \}$

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(20)

a) 131×122

$= 15982$

b)

$h(n) \backslash x(n)$	1	3	1
1	1	3	1
2	2	6	2
2	2	6	2

$y[n] = \{1, 5, 9, 8, 2\}$

c) $(1+3z+z^2)(1+2z+2z^2)$

$1+3z+z^2+2z+6z^2+2z^3+2z^2+6z^3+2z^4$

$9z^2+8z^3+2z^4+5z+1$

$\Rightarrow 2z^4+8z^3+9z^2+5z+1$

d) $1.31 \times 12.2 = 15.982$

e) These are ways to perform convolution operatio

(21) $x(n) = a^n u(n)$; $h(n) = b^n u(n)$

$y(n) = x(n) * h(n)$

$= \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$

$= \sum_{0}^{n} a^k b^{n-k} = b^n \sum_{0}^{n} (a/b)^k$

$= b^n \left[\dfrac{1-(a/b)^{n+1}}{1-a/b}\right]$

$= \dfrac{b^{n+1}\left[1-(a/b)^{n+1}\right]}{b-a}$

$= \dfrac{b^{n+1} - a^{n+1}}{b-a}\, u(n) \quad a \neq b.$

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$$y(n) = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a}\, u(n) & a \neq b \\ b^n (n+1)\, u(n); & a = b \end{cases}$$

(b)

$$x(n) = \begin{cases} 1 ; & n = -2, 0, 1 \\ 2 ; & n = -1 \\ 0 ; & \text{elsewhere.} \end{cases}$$

$$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$$

$$x(n) = \{1, 2, \underset{\uparrow}{1}, 1\};$$

$$h(n) = \{\underset{\uparrow}{1}, -1, 0, 0, 1, 1\}$$

$$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$$

	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

c)

$$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$$

$$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$$

	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

$$y(n) = \{1, 3, 6, \ldots$$

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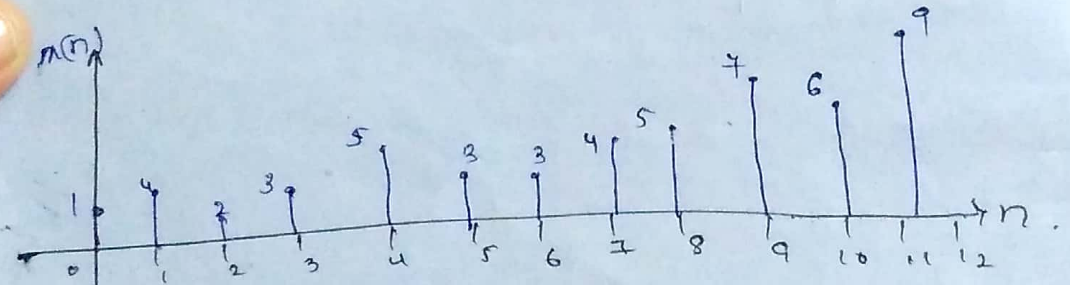
22

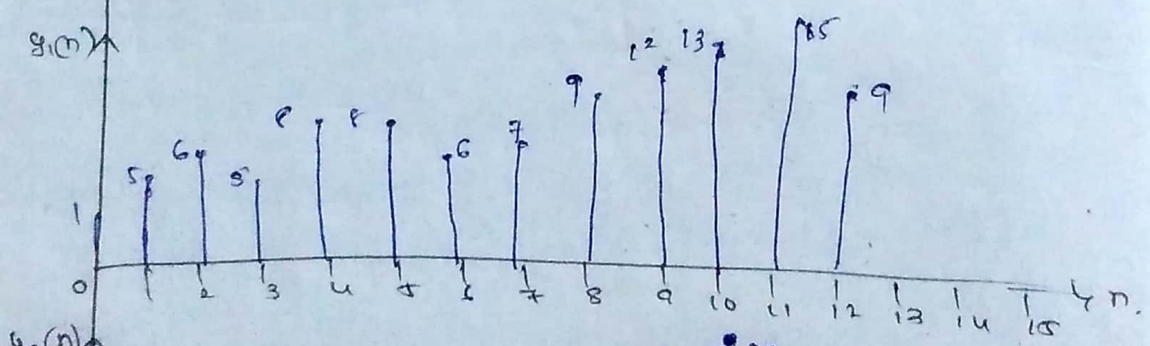
$x[n]$ / $h_1[n]$	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9

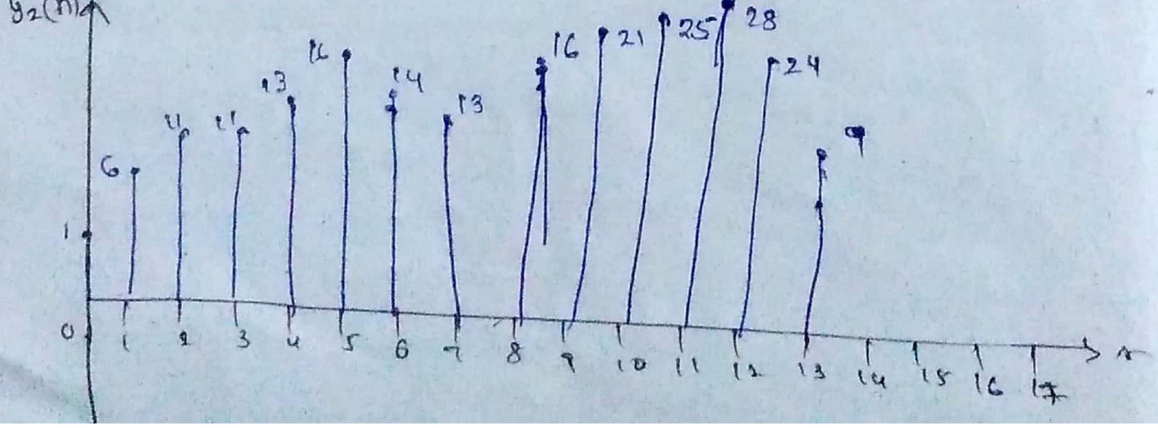
$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$.

$x[n]$ / $h_2[n]$	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$.

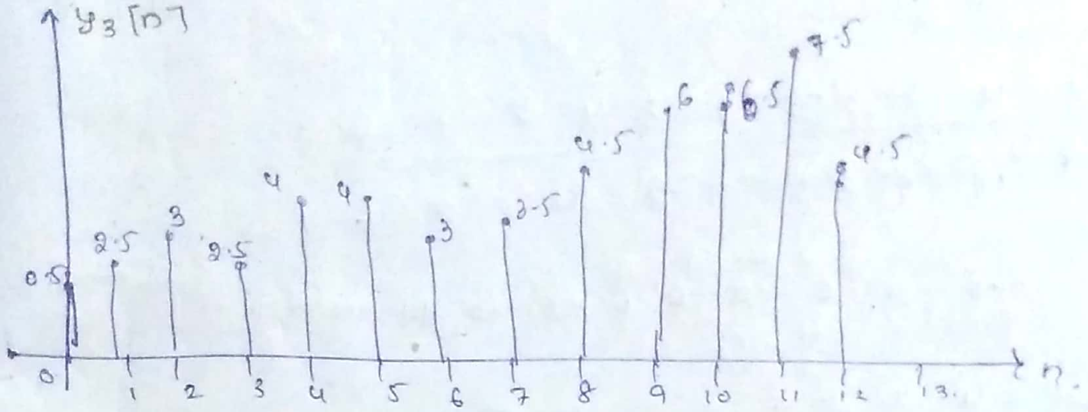






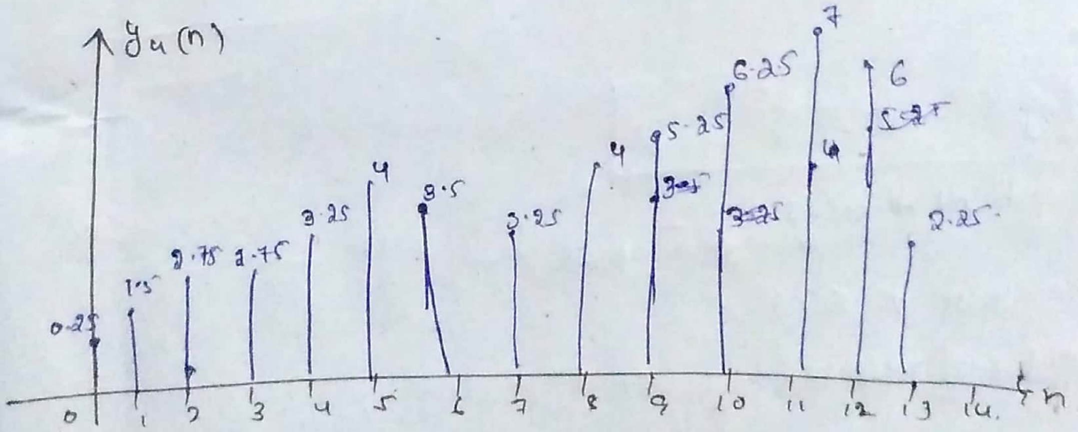
$y_3[n] = 1/2 \; y_1[n]$

$= \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6.5, 7.5, 4.5\}$



$h_4(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	3/2	9/4
1/2	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2
1/4	1/4	1	1/2	3/4	5/4	3/4	3/4	1	5/4	7/4	3/2	9/4

$y_4(n) = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 4, 5.25, 6.25, 7, 6, 2.25\}.$



(b) $\Rightarrow y_3[n] = 1/2 \; y_1[n]$

as $h_3[n] = 1/2 \; h_1(n)$

$\Rightarrow y_4(n) = 1/4 \; y_2(n)$

as $h_4(n) = 1/4 \; h_2(n)$

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(26)

$x(n) - 3y(n-1) - 4y(n-2) = 0$

$x(n) = 0$

$-3y(n-1) - 4y(n-2) = 0$

$3y(n-1) + 4y(n-2) = 0$

$y(n-1) + \frac{4}{3}y(n-2) = 0$

$y(n-1) = -\frac{4}{3}y(n-2)$

$n = 0$

$y(-1) = \frac{-4}{3}y(-2) \quad — (1)$

$n = 1$

$y(0) = -\frac{4}{3}y(1-2)$

$y(0) = -\frac{4}{3}y(-1)$

$y(0) = -\frac{4}{3} \times -\frac{4}{3}y(-2) \quad — (2)$

$n = 2$

$y(1) = -\frac{4}{3}y(0)$

$= \left(-\frac{4}{3}\right)^3 \cdot y(-2)$

$n = K$

$\boxed{y(K) = \left(-\frac{4}{3}\right)^{K+2} y(-2)}$

zero i/p response.

(27)

$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

when $x(n) = 0$

$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 0$

$y(n) = c\lambda^n$

$c\left[\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2}\right] = 0$

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$$C\lambda^{n-2}\left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right] = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$$

$$\lambda_1 = 1/2, \quad \lambda_2 = 1/3$$

$$y_h(n) = C_1 (1/2)^n + C_2 (1/3)^n$$

$$y_p(n) = K\, 2^n u(n)$$

$$K 2^n u(n) - \frac{5}{6} 2^{n-1} u(n-1) + K \frac{1}{6} 2^{n-2} u(n-2) = 2^n u(n)$$

for $n = 2$

$$4K - \frac{5}{6_3} \times 2 + K\frac{1}{6} = 2^2 \qquad \frac{5}{2}K = 4$$

$$\boxed{K = 8/5}$$

$$4K - \frac{5}{3}K + \frac{K}{6} - 4 = 0$$

$$y(n) = y_p(n) + y_h(n)$$

$$= \frac{8}{5} 2^n u(n) + C_1 (1/2)^n u(n) + C_2 (1/3)^n u(n).$$

<u>initial conditions</u>

$$y(0) = 1; \quad y(1) = \frac{5}{6} + 2 = \frac{17}{6}$$

$$\frac{17}{6} = \frac{8}{5} + \frac{C_1}{2} + \frac{C_2}{3}$$

$$\frac{C_1}{2} + \frac{C_2}{3} = \frac{-77}{30} \quad — ①$$

$$1/2\, C_1 + 1/2\, C_2 = -3/5 \times 1/2 \longrightarrow ②$$

$$(-) \quad (-) \quad +$$

$$-1/6\, C_2 = -\frac{14}{5}$$

$$C_2 = -\frac{12}{3} = -4.$$

Thus. $\frac{8}{5} + C_1 + C_2 = 1 \Rightarrow C_1 + C_2 = -3/5$ — ①

$$\frac{16}{5} + \frac{1}{2}C_1 + \frac{1}{3}C_2 = \frac{17}{6} \Rightarrow 3C_1 + 2C_2 = -2/5 \quad — ②$$

solving ① and ②

$$C_1 = -1; \quad C_2 = 2/5$$

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$$y(n) = \left[\frac{8}{5}(2)^n - (1/2)^n - \frac{2}{5}(1/3)^n\right]u(n)$$

(3c)

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

$$x(n) = 4^n u(n)$$

let $y(n) = c\lambda^n$

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = \lambda^n + 2\lambda^{n-1}$$

characteristic eq is

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda = 4, -1$$

$$y_h(n) = c_1 4^n + c_2(-1)^n$$

$$y_p(n) = Kn\, 4^n u(n)$$

for n=2

$$K[n4^2u(n) - 3(n-1)\cdot 4u(n-1) - 4(n-2)u(n-2)]$$

$$= 4^2u(n) + 2(n-1)4^{n-1}u(n-1)$$

$$K[32-12] = 16+8 \Rightarrow K = 6/5$$

$$\therefore\ y(n) = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5}n4^n + c_1 4^n + c_2(-1)^n\right]u(n)$$

$y(-1) = y(-2) = 0,$

$y(0) = 1$ and $c_1 + c_2 = 1$

$y(1) = 9$ $\quad \frac{6}{5}\times 4 + 4c_1 - c_2 = 9$

$$\Rightarrow c_1 = \frac{26}{25} \quad c_2 = -1/25$$

$$y(n) = \left[\frac{6}{5}n4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right]u(n)$$

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(31) $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$x(n) = \delta(n)$

$y_c(n) = c_1 4^n + c_2 (-1)^n$

$y_p(n) = K n \delta(n)\ K(n-1)\ 3\delta(n-1) - K(n-2)4\ \delta(n-2) = \delta[n] + 2\delta[n-1]$

for $n=2$

$\frac{-K(2-2)}{8} = \delta(n) + 2\delta(n-2)$

$y[n] = c_1 4^n + c_2 (-1)^n$.

$c_1 + c_2 = 1$ — (1)

$y(1) = 3y(0) - 0 + \delta(n) + 2\delta(1-1)$

$= 3(1) + 0 + 2(1)$

$= 5$

$c_1 4 - c_2 = 5$ — (2)

solving (1) & (2)

$c_1 = 6/5 \qquad c_2 = -1/5$

$y[n] = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$

(32)

(a) $y(n) = x(n) * h(n)$ (o/p of convolution)

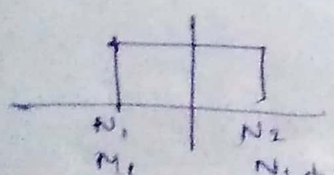
range will be $N_1 + M_1 \le n \le N_2 + M_2$

$L_1 = N_1 + M_1 \quad L_2 = N_2 + M_2$.

(b) $h(n)$ has shorter duration than $x(n)$

→ partial overlap from left

$N_1 + M_1 \qquad N_2 + M_2 - 1$



overlap from $N_1 + M_2 \qquad N_2 + M_1$

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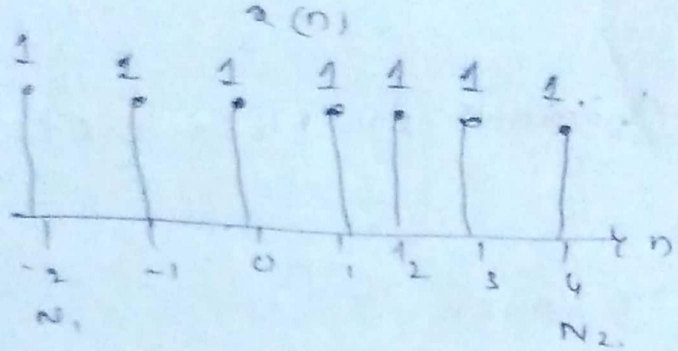
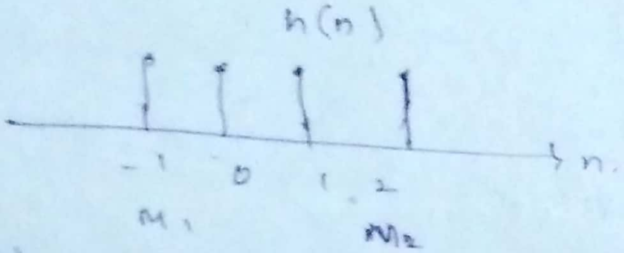
partial overlap from right

N_1+M_1+1 — N_2+M_2.

e)





h(n) \ x(n)	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

$y(n) = \{1, 2, 3, \underset{\uparrow}{4}, 4, 4, 4, 3, 2, 1\}$

$L_1 = -3$
↓
Lower

$L_2 = 6$
↓
higher.

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(8)

(a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$x(n) = \delta(n)$

$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$

$\lambda^2 - 0.6\lambda + 0.08 = 0$ $\quad \lambda_1 = 2/5,\ \lambda_2 = 1/5$

$y_c(n) = c_1\left(\frac{2}{5}\right)^n + c_2(1/5)^n$

$y_p(n) = ?$

$n\,k\,\delta(n) - (n-1)k\,0.6\,\delta(n-1) + 0.08\,k\,(n-2)\,\delta(n-2) = \delta$

$n = 2$

$y_p(n) = 0$

$y(n) = y_c(n) = c_1(2/5)^n + c_2(1/5)^n$

$y(0) = x(0) = \delta(0) = 1$

$c_1 + c_2 = 1$ ——— (1)

$y(1) = 0.6\,y(0) + x(1) = 0.6\,(1)$

$0.6 = \frac{2c_1}{5} + \frac{c_2}{5}$

$2c_1 + c_2 = 3$ — (2)

Solving (1) & (2)

$c_1 = 2 \quad c_2 = -1$

$x(n) = u(n)$ $\quad \boxed{y(n) = 2(2/5)^n - (1/5)^n}$ → impulse re

$n\,k\,u(n) - k(n-1)\,0.6\,u(n-1) + 0.8\,k(n-2)\,u(n-2) = u(n)$

$y(0) = 1$

$y(1) = 0.6 + 1 = 1.6$

$n = 2$

$2k - k\,0.6 = 1$ $\quad 1.4k = 1$

$2k - 0.6k = 1$ $\quad \boxed{k = 1/1.4}$

$\boxed{k = 5/7}$

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$y(n) = \frac{5}{7} n\, u(n) +$

$\boxed{\cancel{y(n) = 2(2/5)^n - (1/6)^n}}$ → ~~impulse response of stm.~~

$y(n) = \frac{5}{7} n\, u(n) + \left[c_1 (2/5)^n + c_2 (1/6)^n \right] u(n)$

$1 = c_1 + c_2$ — (1)

$1.6 = 5/7 + c_1 \times 2/5 + c_2\, 1/5$

$\frac{31}{7} = 2c_1 + c_2$.

$2c_1 + c_2 = 31/7$ — (2)

solving (1) & (2)

$c_1 = 24/7 \qquad c_2 = 1 - \frac{24}{7} = \frac{-17}{7}$.

$y(n) = \left[\frac{5}{7} n + \frac{24}{7} (2/5)^n - \frac{17}{7} (1/6)^n \right] u(n)$

(34)

$h(n) = \{1, 1/2, 1/4, 1/8, 1/16\}$

$y(n) = x(n) * h(n)$

$y(n) = \sum_{n=-\infty}^{\infty} x(n) \cdot h(n-k)$

$x(0) \cdot h(0) = y(0) \Rightarrow x(0) = 1$

$1/2\, x(0) + x(1) = y(1)$

$x(1) = 3/2$.

$x(0)\, h(0) = y(0)$

$x(0) = 1$

$x(0)\, h(1) + h(0)\, x(1) = y(1)$

$\therefore x(n) = \{ 1, 3/2, 3/2, 7/4, 3/2 \ldots \}$

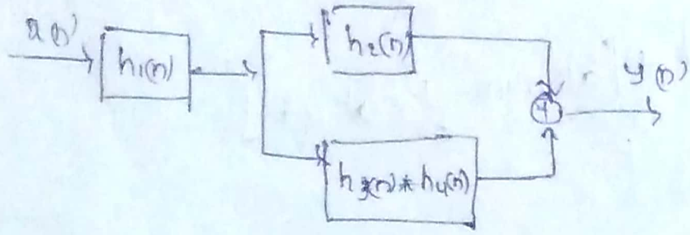
$h(n) \backslash x(n)$	$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$	$x(8)$
$h(0)$	$x(0)h(0)$	$h(0)x(1)$	$h(0)x(2)$	$h(0)x(3)$	$h(0)x(4)$	$h(0)x(5)$	$h(0)x(6)$	$h(0)x(7)$	$h(0)x(8)$
$h(1)$	$x(0)h(1)$	$x(1)h(1)$	$x(2)h(1)$	$x(3)h(1)$	$x(4)h(1)$	$x(5)h(1)$	$x(6)h(1)$	- -	$h(8)h(1)$
$h(2)$	$h(2)x(0)$	$x(1)h(2)$	$x(2)h(2)$	-	-				$x(8)h(2)$
$h(3)$	.								$x(8)h(3)$
$h(4)$	$h(4)x(0)$	$h(4)x(1)$							$x(8)h(4)$

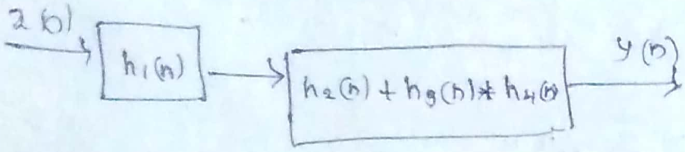
(33)

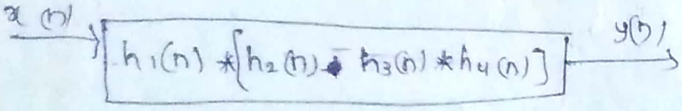
(a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n).$

(35)

(a)







impulse response

$y(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)] * x(n)$

(b)

$h(n) = h_3(n) * h_4(n)$

$= (n+1)\, u(n) * \delta(n-2)$

$= \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-1)\right] * \left[(n+1)\, u(n) - (n+1)\, u(n) * \delta(n-2)\right]$

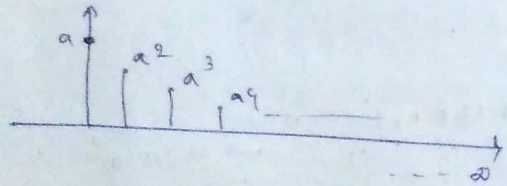
(c)

$x(n) = \{1, 0, 0, 3, 0, -4\}$

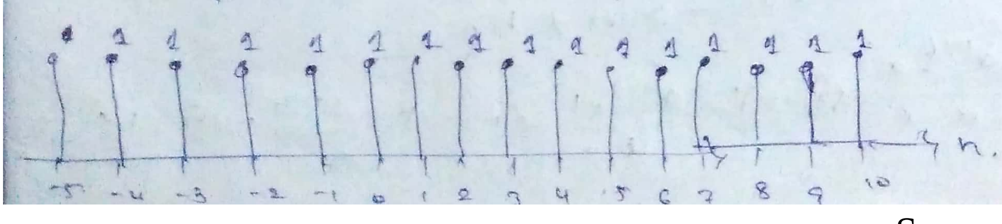
$y(n) = x(n) * h(n).$

(36)

$h(n) = a^n u(n)$



x(n)



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Pa:

from block diagram.

$$y(n) = x(n) * \left[h(n) - \frac{1}{3^2} * h(n) \right]$$

$$= \left[u(n+5) - u(n-10)\right] * \left[a^n u(n) - \frac{1}{3^2} * a^n u(n)\right]$$

$$s(n) = u(n) * h(n)$$

$$= \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} h(n-k) \rightarrow \sum_{k=0}^{\infty} a^{n-k}.$$

$$= a^n + a^{n-1} + a^{n-2} + a^{n-3} + \cdots$$

$$\frac{a^{n-1}}{a^n} = \frac{a^n/a}{a^n} = 1/a \implies \text{Geometric series}$$

$$= \frac{a(1-r^n)}{1-r}$$

$$= \frac{a^n\left(1-(1/a)\right)}{1 - 1/a} = \frac{a^n\left(1 - \frac{1}{a^n}\right)}{\frac{a-1}{a}}$$

$$= \frac{\frac{a^n(a^n-1)}{a^n}}{\frac{a-1}{a}}$$

$$= \frac{a(a^n-1)}{a-1} = \frac{a^{n+1}-a}{a-1}$$

$$x(n) = u(n+5) - u(n-10)$$

$$s(n+5) - s(n-10) = \frac{a^{n+6} - a}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6}-a}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}}{a-1} - u(n+3) + \frac{a}{a}$$

(37)

Given $y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$

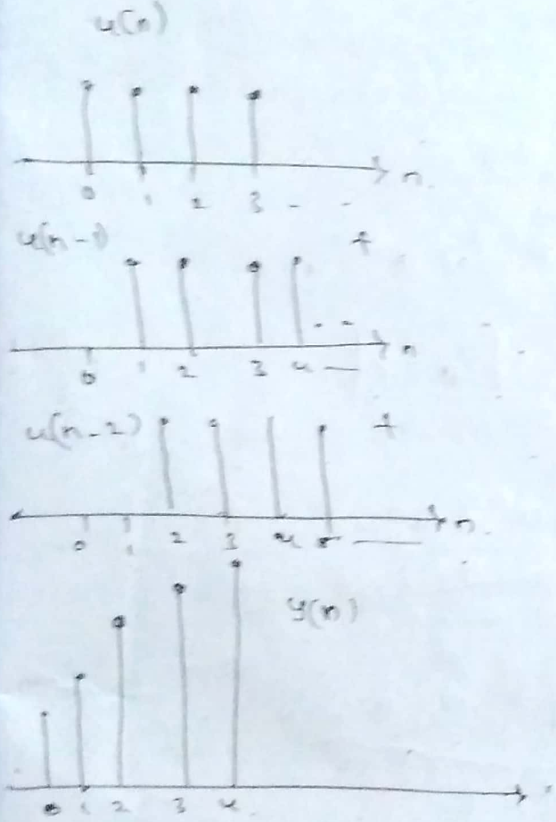
$x(n) = u(n)$

$y(n) = h(n) * x(n)$

$y(n) = \frac{1}{M} \sum_{k=0}^{m-1} x(n-k)$

$= \frac{1}{M} \sum_{k=0}^{m-1} u(n-k)$

$= \frac{1}{M} \left[u(n) + u(n-1) + u(n-2) + u(n-3) + \cdots + u(n-m) \right]$



(38)

As s/m is stable $|x(n)| < \infty$

$|h(n)| < \infty$

$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0, \text{even}}^{\infty} |a|^n$

$= \sum_{n=0, \text{even}}^{\infty} |a|^n$

$= 1 + |a|^2 + |a|^4 + |a|^6 + \cdots$

As n is even.

$$= \frac{1}{1-|a|^2}.$$

$\Rightarrow$ To be stable $|a| < 1$

(39)

$h(n) = a^n u(n)$

$x(n) = u(n) - u(n-10)$

$y(n) = x(n) * h(n)$

$= [u(n) - u(n-10)] * a^n u(n)$

$$y(n) = \sum_{n=-\infty}^{\infty} u(k) \cdot h(n-k)$$

$$= \sum_{k=0}^{\infty} h(n-k)$$

$$= \sum a^{n-k} = a^n + a^{n-1} + a^{n-2} + \cdots =$$

$$\frac{a^{n-1}}{a^n} = \frac{a^n/a}{a^n} = \boxed{1/a = r}$$

$$S_n = \frac{a^n(1-(1/a)^n)}{1-1/a} = \frac{\frac{a^n(a^n-1)}{a^n}}{\frac{a-1}{a}}$$

$$= \frac{a(a^n-1)}{a-1}$$

$$S_n = \frac{a^{n+1}-a}{a-1}$$

$$S(n-10) = \frac{a^{n-10+1}-a}{a-1} = \frac{a^{n-9}-a}{a-1}$$

$$y(n) = \frac{a^{n+1}-a}{a-1} u(n) - \frac{a^{n-9}-a}{a-1} u(n-10).$$

(40)

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(40).

$$x(n) = u(n) - u(n-10)$$

$$y(n) = \frac{(1/2)^{n+1} - (1/2)}{1/2 - 1} u(n) - \frac{\left(\frac{1}{2}\right)^{n-9} - 1/2}{1/2 - 1} u(n-10)$$

$$= 2\left[\frac{2(1/2)^{n-9} - 1}{2} u(n-10) - \frac{2(1/2)^{n+1} - 1}{2} u(n)\right]$$

$$y(n) = \left(2(1/2)^{n-9} - 1\right) u(n-10) - \left(2(1/2)^{n+1} - 1\right) u(n))$$

(41)

(a) $x(n) = 2^n u(n)$

$$h(n) = (1/2)^n u(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} 2^k (1/2)^{n-k} = \sum_{k=0}^{\infty} 2^k\, 2^{-n+k}$$

$$= \sum_{k=0}^{\infty} 2^{k-n+k} = \sum_{k=0}^{\infty} 2^{2k-n}$$

$$= 2^{-n} + 2^{2-n} + 2^{4-n} + 2^{6-n} + \cdots$$

$$\cancel{S_n} = \cancel{\frac{2^{-n}(1-4^n)}{1-4}} \qquad \frac{2^{2-n}}{2^{-n}} = \frac{2^2}{2^{\cancel{n}}} \times \frac{\cancel{2}^n}{1}$$

$$= 4.$$

$$\therefore S_n = \frac{2^{-n}(1-4^n)}{1-4}.$$

(b) $y(n) = \sum_{k=}$

Q2

(a) $h_e(n) = h_1(n) * h_2(n) * h_3(n)$

$= [\delta(n) - \delta(n-1)] * h(n) * u(n)$

$= [u(n) - u(n-1)] * h(n)$

$= \delta(n) * h(n)$

$= h(n-0)$

$= h(n)$

(b) No, the order of interconnection doesn't affect the overall sim.

Q3

(a) $x(n) \cdot \delta(n-n_0)$

→ Gives the value of $x(n)$ at $n = n_0$

$x(n) * \delta(n-n_0)$

→ Gives the shifted version of seq..

$x(n) * \delta(n-n_0) = x(n-n_0)$.

(b) $y(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot h(n-k) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$

$\cancel{= h(k) *}$

$= h(n) * x(n)$

Linearity

$x_1(n)$, $x_2(n)$

$y_1(n) = h(n) * x_1(n)$

$y_2(n) = h(n) * x_2(n)$

$x(n) = a x_1(n) + b x_2(n)$

$y(n) = h(n) * x(n)$

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$y(n) = h(n) * [a x_1(n) + b x_2(n)]$

$= a h(n) x_1(n) + b \cdot h(n) x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$

There given sim is Linear ~~time invariant~~.

$x(n) \longrightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \Sigma h(k) \cdot x(n-n_0-k)$

$= y(n-n_0)$

c)

$y(n) = x(n-n_0)$

Impulse response $h(n) = \delta(n-n_0)$

45

$x(n) = \{1, 2, \underset{\uparrow 0}{3}, 4, 2, 1\}$

$y(-2) = x(-2) + 2x(-4) - \frac{1}{2} y(-3)$

$= 1 + 2(0) - \frac{1}{2}(0)$

$= \underline{1}$

$y(-1) = x(-1) + 2x(-3) - 1/2\, y(-2)$

$= 2 + 0 + 1/2 = 3/2$

$y(0) = x(0) + 2x(-2) - 1/2\, y(-1)$

$= 3 + 2(1) - 1/2(3/2)$

$= 5 - 3/4$

$= \underline{17/4}$

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(48)

(a) $h(n) = b a^n u(n)$

$$\sum_{n=-\infty}^{\infty} b a^n u(n) = \sum_{n=0}^{\infty} b a^n$$

$$= \frac{b}{1-a}$$

$$= 1$$

$$\frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

(54)

a) convolution

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

correlation

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution

$y(n) = \{ \underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4 \}$

correlation

$r(n) = \{ 1, 3, 7, 7, 7, 6, 4 \}$

c) $x_3(n) = \{1, 2, 3, 4\}$ $h_3(n) = \{4, 3, 2, 1\}$

convolution

$h_3(n)$ \ $x_3(n)$	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

correlation

$h_3(n)$ \ $x_3(n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

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convolution

$y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$
$\uparrow$ (under the 4)

correlation

$r_3(n) = \{1, 4, 10, 20, 25, 24, 16\}$
$\uparrow$ (under the 20)

d) $x_4(n) = \{1, 2, 3, 4\}$ $\uparrow$ (under the 1) $h_4(n) = \{1, 2, 3, 4\}$ $\uparrow$ (under the 1)

Convolution

$h_4(n)$ \ $x_4(n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

correlation

$h_4(n)$ \ $x_4(n)$	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

convolution

$y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$
$\uparrow$ (under the 1)

correlation

$r_4(n) = \{4, 11, 20, 30, 20, 11, 4\}$
$\uparrow$ (under the 30)

59

$h(n)$ \ $x(n)$	1	3	3	1
$h(0)$	$h(0)x(0)$	$h(0)x(1)$	$h(0)x(2)$	$h(0)x(3)$
$h(1)$	$h(1)x(1)$	$h(1)x(1)$	$h(1)x(2)$	$h(1)x(3)$

$h(0)\,x(0) = 1$

$\boxed{h(0) = 1}$

$h(1)\,x(0) + h(0)\,x(1) = 4$

$h(1)\,1 + 1(3) = 4$

$h(1) = 4 - 3$

$\boxed{h(1) = 1}$

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47

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

Complimentary solution:

$c\lambda^n - 4c\lambda^{n-1} + 4c\lambda^{n-2} = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda_1 = 2 \quad \lambda_2 = 2$

$y_h(n) = c_1 2^n + c_2 n 2^n$

$= (c_1 + c_2 n) 2^n$

particular solution:

$y_p(n) = k(-1)^n u(n)$

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) =$
$x(n) - x(n-1)$

let $n = 2$

$k + 4k + 4k = x(2) - x(1)$

$k(1+8) = x(2) - x(1)$

$9k = x(2) - x(1)$

$x(2) = (-1)^n u(n) = 1$

$x(1) = -1$

$9k = 2$

$\boxed{k = 2/9}$

$y(n) = y_h(n) + y_p(n)$

$= \frac{2}{9}(-1)^n u(n) + c_1 2^n + c_2 n 2^n$

$y(0) = \underbrace{4y(0)}_{0} + 0 + x(0) - 0$

$y(0) = x(0)$

$y(0) = 1$

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~~$c_1 \neq c_2$~~

$\frac{2}{9} + c_1 + c_2 = 1$

$c_1 + c_2 = 1 - 2/9$

$c_1 + c_2 = 7/9$ — ①

$y(1) = 4y(0) + x(1) - x(0)$

$= 4(1) - 1 - 1$

$= 4 - 2$

$= 2$

$\boxed{y(1) = 2}$

$-\frac{2}{9} + 2c_1 + 2c_2 = 2$

$2(c_1 + c_2) = 2 + \frac{2}{9} = \frac{20}{9}$

$c_1 + c_2 = \frac{\cancel{20}\,10}{9 \times \cancel{2}}$

$c_1 + c_2 = 10/9$ —— ②

solving ① & ②.

$c_1 = 7/9$

$c_2 = 1/3$.

$\therefore y(n) = \frac{2}{9}(-1)^n u(n) + 7/9\, 2^n + 1/3\, n\, 2^n$.

58

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$h(n) = (c_1 2^n + c_2 n 2^n) u(n)$

$y(0) = 1$; $y(1) = 3$

$c_1 = 1$

$2c_1 + 2c_2 = 3$

$c_2 = 1/2$.

$\therefore h(n) = (2^n + \frac{1}{2} n 2^n) u(n)$

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(59)

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u(n-k).$$

(60)

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]$$

(62)

a)

$x(n)$ \ $x(n)$	1	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1

$$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$$

b)

$y(n)$ \ $y(n)$	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
1	1	1	2	1

$$r_{yy}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$$

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(5c)

a) $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) \cdot x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [\delta(n) + \gamma_1(n-k_1) + \gamma_2 \delta(n-k_2)] \cdot$$

$$[\delta(n-l) + \gamma_1(n-l-k_1) + \gamma_2 \delta(n-l-k_2)]$$

b) $r_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$, γ_2 and k_2 can be determined from other peaks.

c) If $\gamma_2 = 0$,

peaks occur at $l=0$ and $l = \pm k$, then it is easy to obtain γ_1 and k_1

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