Problems

2.1 A discrete time slg $x[n]$ is defined as.

$$x(n) = \begin{cases} 1+\frac{n}{3} & ; -3 \le n \le -1 \\ 1 & ; 0 \le n \le 3 \\ 0 & ; \text{otherwise} \end{cases}$$

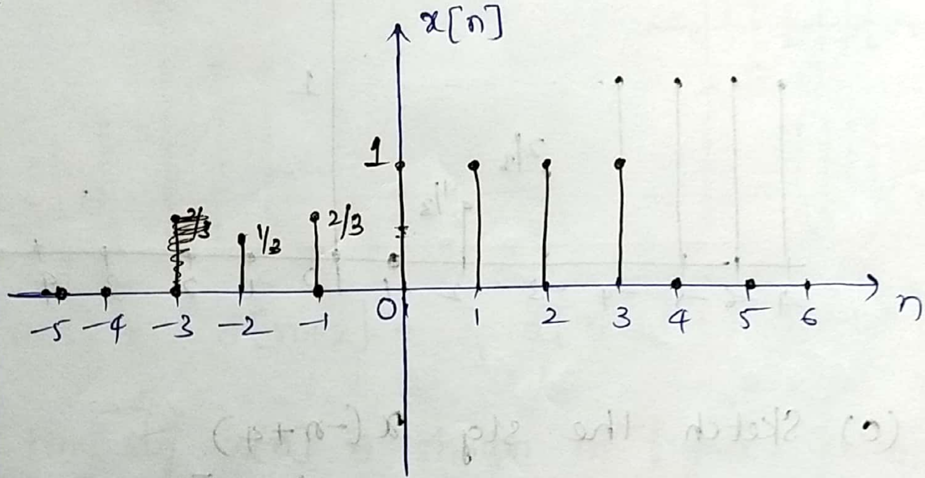
a) Determine its values and sketch the slg $x[n]$.

Sol:- $x[n] = \{ \ldots 0, \frac{1}{3}, \frac{2}{3}, 1, 1, 1, 1, 0, \ldots \}$

$-3: 1 + \frac{-3}{3} = 1-1 = 0$

$-2: 1 + \frac{-2}{3} = 1 - \frac{2}{3} = \frac{1}{3}$

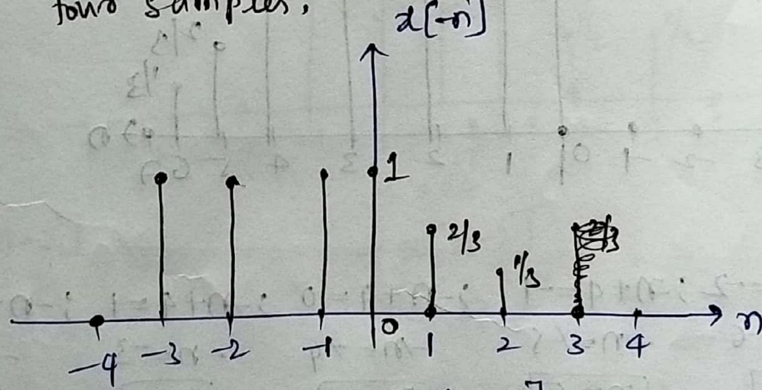
$-1: 1 + \frac{-1}{3} = \frac{3-1}{3} = \frac{2}{3}$

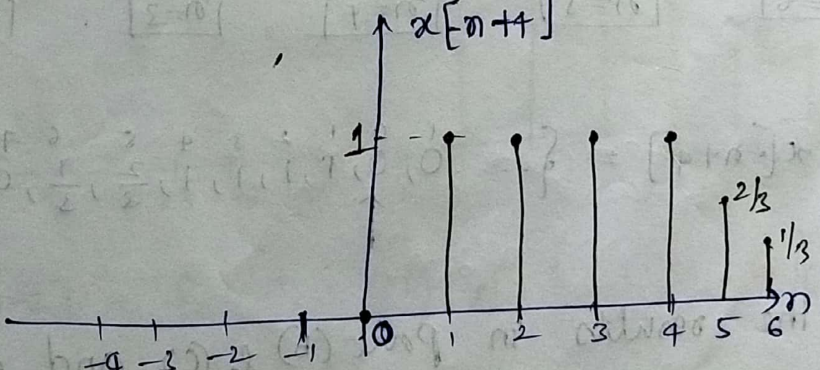


b) Sketch the slg's that result if we:

(1) First fold $x[n]$ & then delay the resulting slg. by four samples.

Sol:-



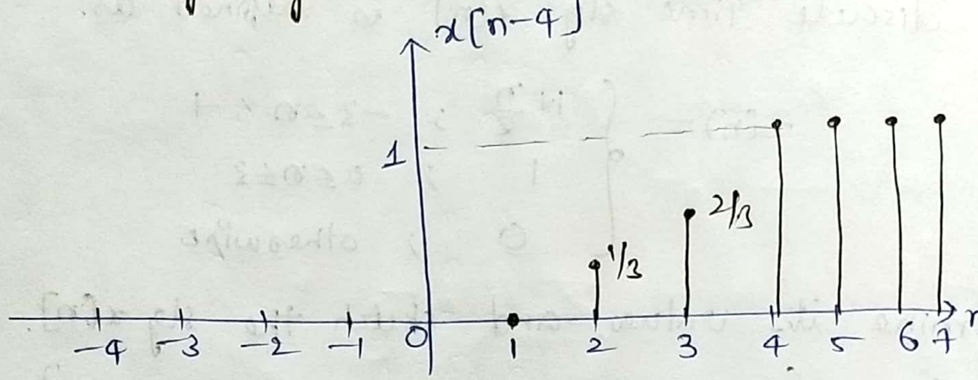


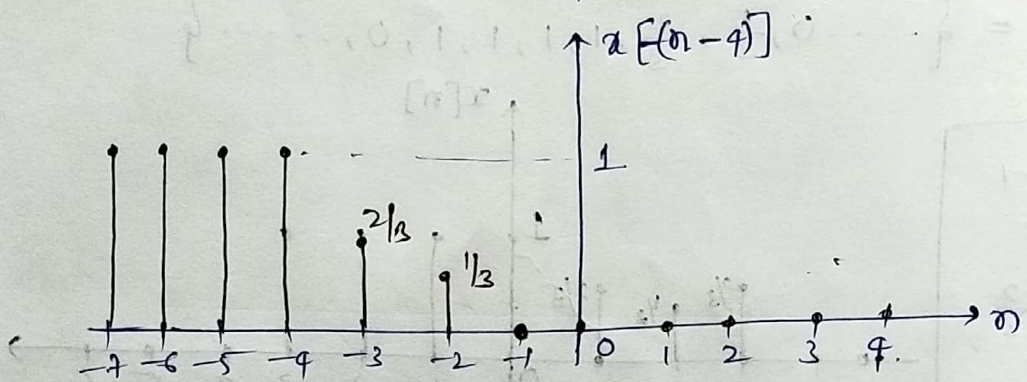
$-n-4=0$
$n=4$

$x[-n+4] = \{ \ldots 0, 1, 1, 1, 1, \frac{2}{3}, \frac{1}{3}, 0, \ldots \}$

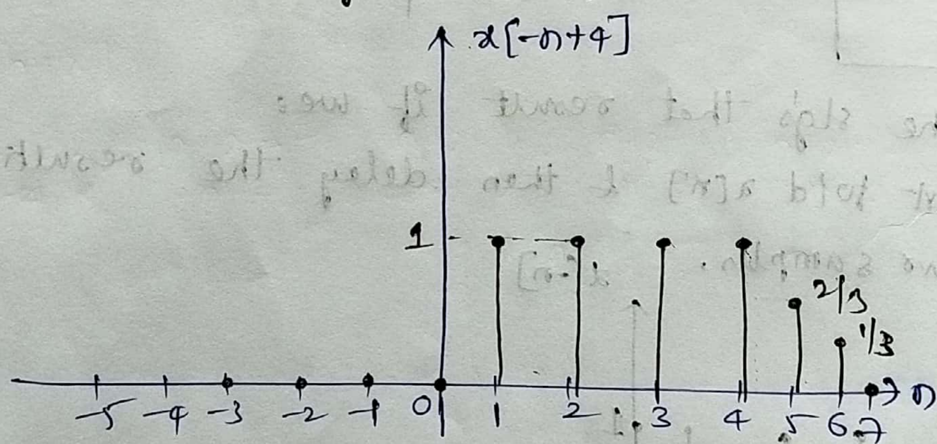
Scanned by CamScanner

b(2) First delay $x[n]$ by 4 samples and then fold the resulting slg.





(c) Sketch the slg $x(-n+4)$.



$-n+4=-3$; $-n+4=-2$; $-n+4=-1$; $-n+4=0$; $-n+4=1$; $-n+4=2$

$-n=-7$; $-n=-6$; $-n=-5$; $-n=-4$; $-n=-3$; $-n=-2$

$\boxed{n=7}$; $\boxed{n=6}$; $\boxed{n=5}$; $\boxed{n=4}$; $\boxed{n=3}$; $\boxed{n=2}$

$-n+4=3$

$-n=-1$

$\boxed{n=1}$

$$x[-n+4] = \{\ldots \overset{-1}{0}, \underset{\uparrow}{\overset{0}{0}}, \overset{1}{1}, \overset{2}{1}, \overset{3}{1}, \overset{4}{1}, \overset{5}{\tfrac{2}{3}}, \overset{6}{\tfrac{1}{3}}, \overset{7}{0}, \ldots\}$$

(d) Compare the results in part (b) & (c) and derive a rule for obtaining the slg $x[-n+k]$ from $x[n]$.

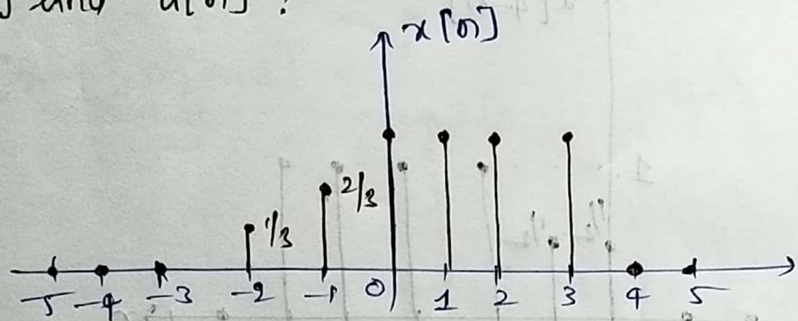
By folding $x[n]$ and then shifting by 'k' samples we obtain $x[n+k]$.

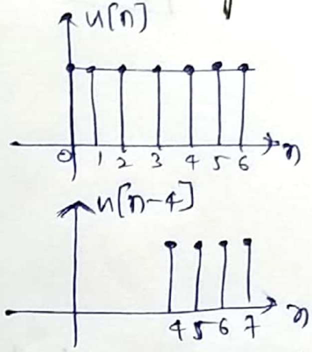
$\Rightarrow$ shift $x[-n]$ by "k" samples to the right if $k>0$

$\Rightarrow$ shift $x[-n]$ by 'k' samples to the left if $k<0$.

(e) Can you express the slg $x[n]$ in terms of slg's $\delta[n]$ and $u[n]$?

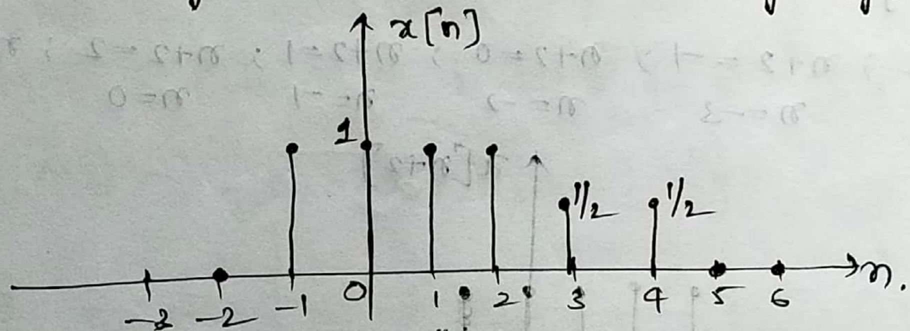
Sol:-



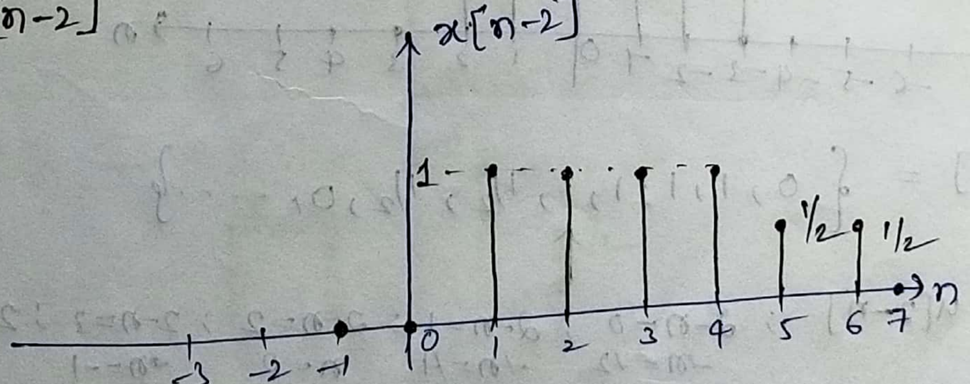


$$x[n] = 0\cdot\delta[n+3] + \frac{1}{3}\delta[n+2] + \frac{2}{3}\delta[n+1] + u[n] - u[n-4]$$

Q.2. A discrete-time slg $x[n]$ is shown in figure. Sketch & label carefully each of the following slg's.



(a) $x[n-2]$



$$x[n-2] = \{\ldots 0, \underset{\uparrow}{0}, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0 \ldots\}$$

$\Rightarrow x[-(n-4)]$

(b) $x[4-n]$

$4-n=-2$; $4-n=-1$; $4-n=0$; $4-n=1$; $4-n=2$

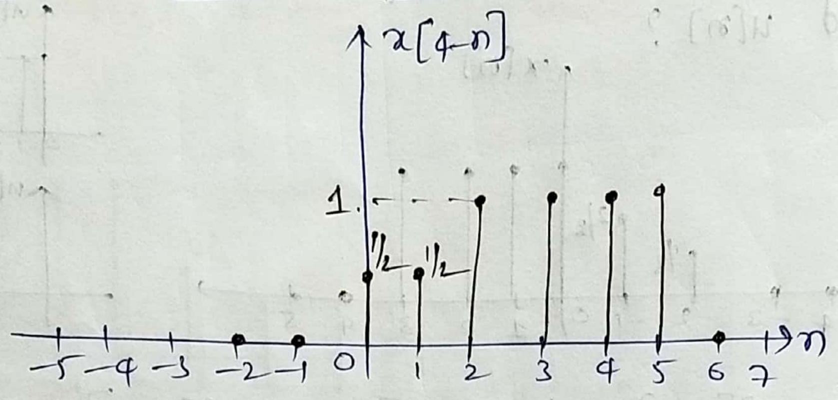
$-n=-6$ $\quad$ $-n=-5$ $\quad$ $-n=-4$ $\quad$ $-n=-3$ $\quad$ $-n=-2$

$\boxed{n=6}$ $\quad$ $\boxed{n=5}$ $\quad$ $\boxed{n=4}$ $\quad$ $\boxed{n=3}$ $\quad$ $\boxed{n=2}$

$4-n=3$; $4-n=4$; $4-n=5$

$-n=-1$ $\quad$ $-n=0$ $\quad$ $-n=1$

$\boxed{n=1}$ $\quad$ $\boxed{n=0}$ $\quad$ $\boxed{n=-1}$

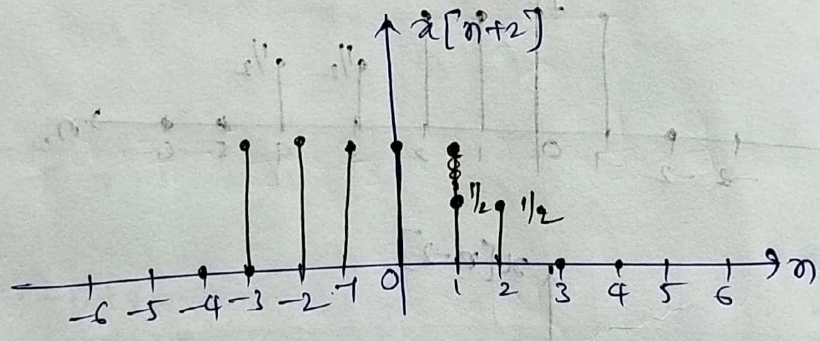


$x[4-n] = \{0, 1/2, 1/2, 1, 1, 1, 1, 0, \ldots\}$
$\uparrow$

(c) $x[n+2]$

$n+2=-2$; $n+2=-1$; $n+2=0$; $n+2=1$; $n+2=2$; $n+2=3$; $n+2=$

$n=-4$ $\quad$ $n=-3$ $\quad$ $n=-2$ $\quad$ $n=-1$ $\quad$ $n=0$ $\quad$ $n=1$ $\quad$ $n=$

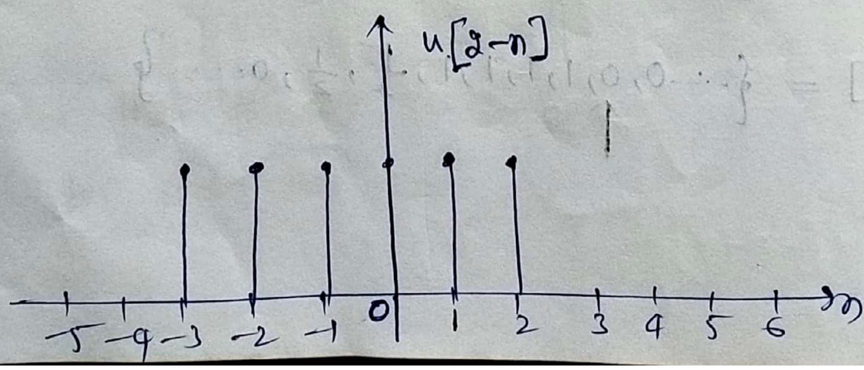


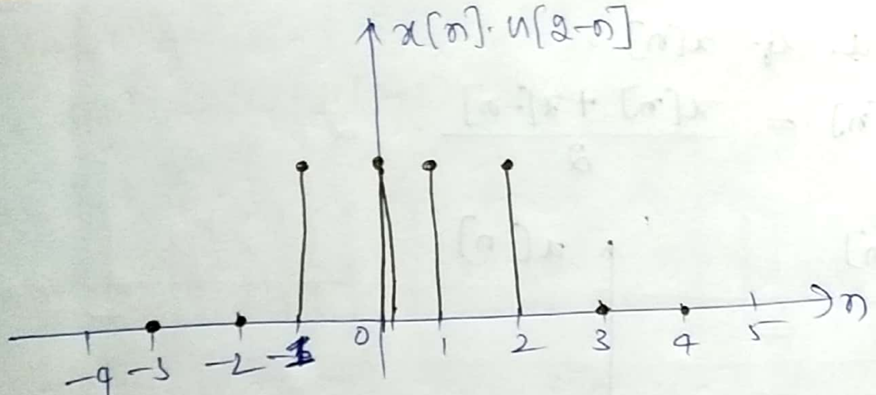
$x[n+2] = \{0, 1, 1, 1, 1, 1/2, 1/2, 0, \ldots\}$
$\uparrow$

(d) $x[n]\, u[2-n]$; $2-n=0$; $2-n=1$; $2-n=2$; $2-n=3$; $2-n=4$;

$-n=+2$ $\quad$ $-n=+1$ $\quad$ $n=0$ $\quad$ $-n=-1$ $\quad$ $n=-2$

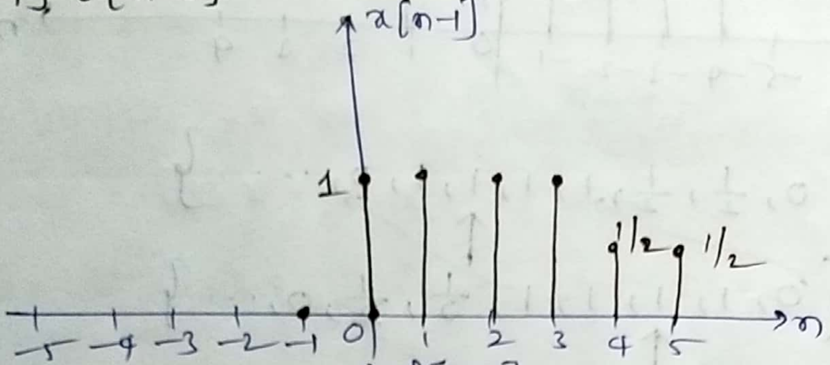
$2-n=-2$
$-n=-4$
$\boxed{n=4}$

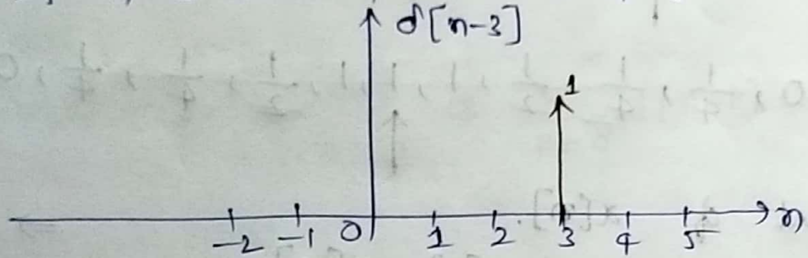




$x[n]\cdot u[2-n] = \{0, 1, \underset{\uparrow}{1}, 1, 1, 0, 0, \dots\}$

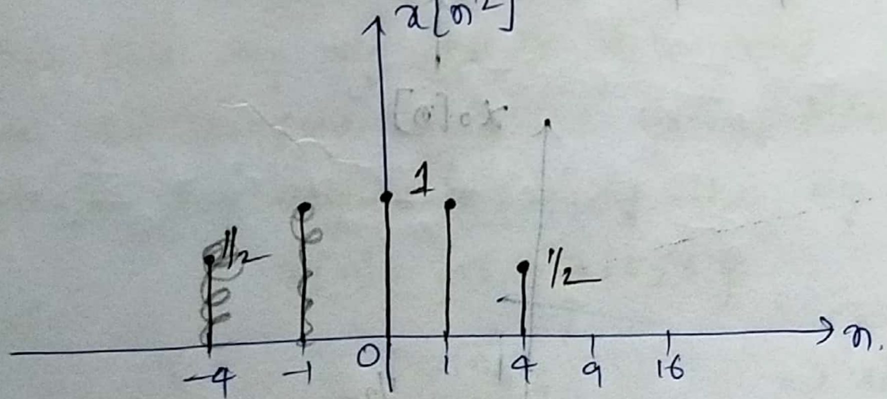
(e) $x[n-1]\,\delta[n-3]$





$x[n-1]\cdot\delta[n-3] = \{\dots, \underset{\uparrow}{0}, 0, 0, 1, 0, 0, \dots\}$

(f) $x[n^2]$



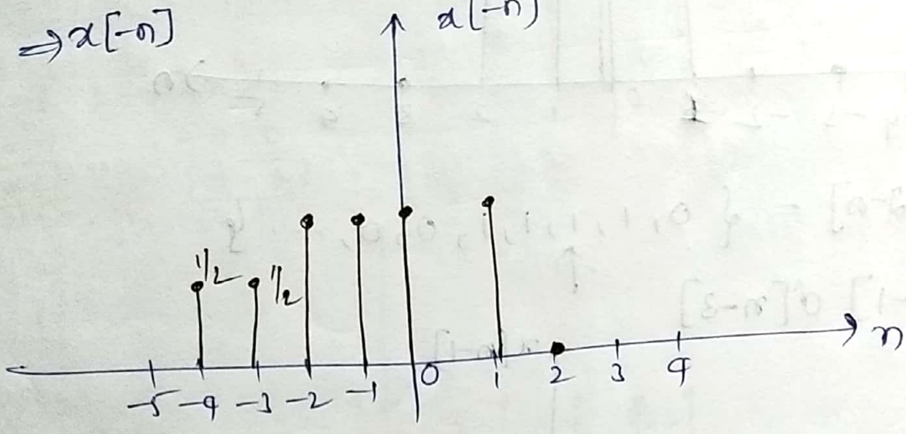
$x[n^2] = \{1/2, 1, \underset{\uparrow}{1}, 1, 1/2, \dots\}$

$= \{x(4), x(1), x(0), x(1), x(4), 0, \dots\}$

(g) even part of $x[n]$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$\Rightarrow x[-n]$



$$x[-n] = \{0, \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, 1, 1, 0, \ldots\}$$

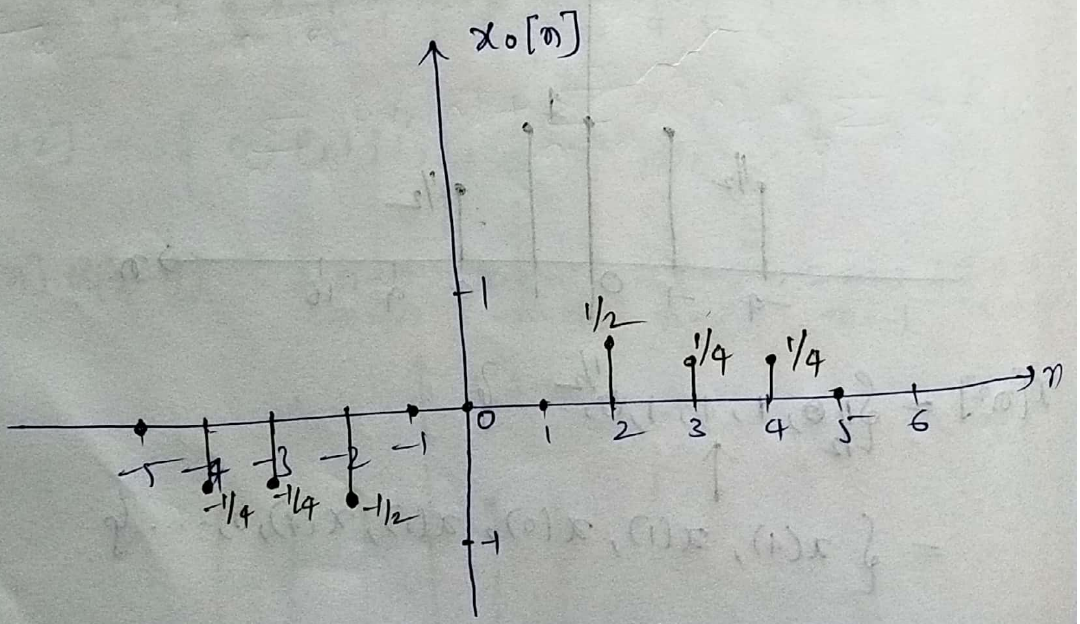
$$x[n] = \{0, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, \ldots\}$$

$$x_e[n] = \{0, \tfrac{1}{4}, \tfrac{1}{4}, \tfrac{1}{2}, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \ldots\}$$

(h) Odd part of $x[n]$. $\Rightarrow \frac{0 - (\frac{1}{2})}{2}$

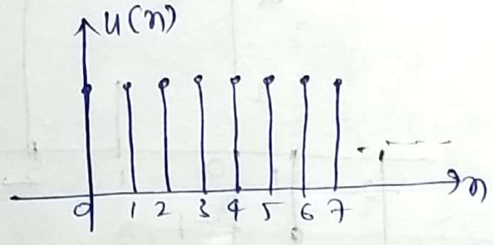
$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$$x_o[n] = \{0, \tfrac{-1}{4}, \tfrac{-1}{4}, \tfrac{-1}{2}, 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{4}, 0, \ldots\}$$

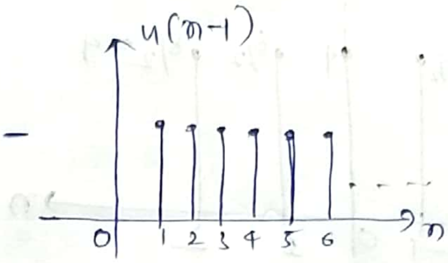


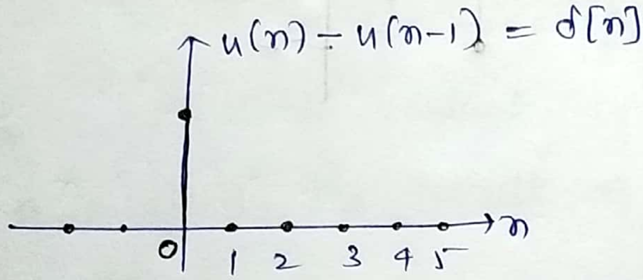
2.3 Show that

(a) $\delta[n] = u(n) - u(n-1)$



−



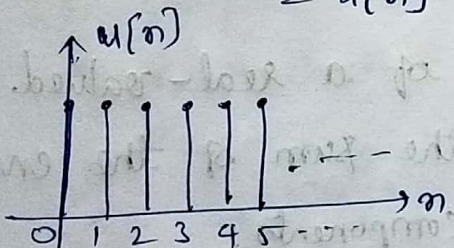


$$\Rightarrow u[n] - u[n-1] = \delta[n] = \begin{cases} 0 ; & n<0 \\ 1 ; & n=0 \\ 0 ; & n>0 \end{cases}$$

(b) $u[n] = \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$

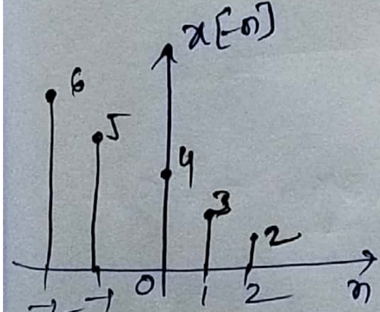
$$\sum_{k=-\infty}^{n} \delta[k] = \begin{cases} 0 ; & n<0 \\ 1 ; & n\geq 0 \end{cases} \quad ; \quad \sum_{k=0}^{\infty} \delta[n-k] = \begin{cases} 0 ; & n<0 \\ 1 ; & n\geq 0 \end{cases}$$

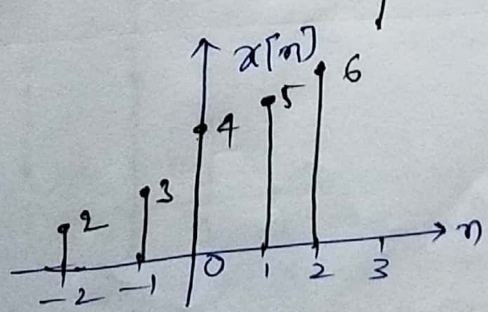
$$= u[n] \qquad\qquad = u[n]$$



2.4 Show that any sig can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the sig.

$$x[n] = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$



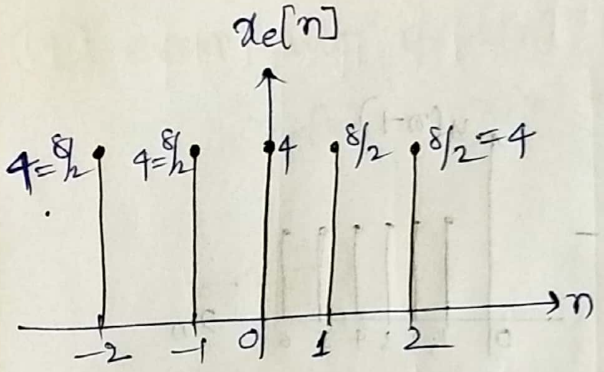


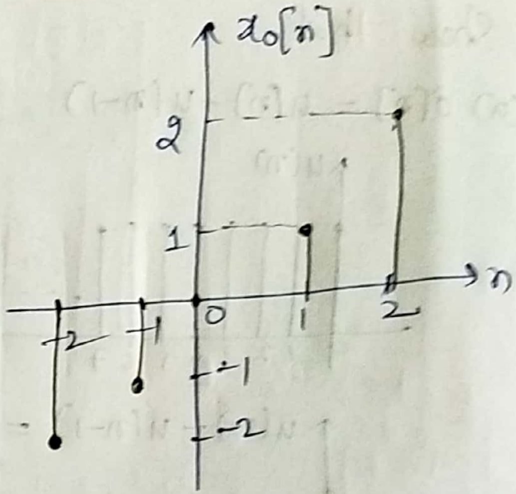
$\Rightarrow x_e[n] = \dfrac{x[n]+x[-n]}{2}$

$\Rightarrow x_o[n] = \dfrac{x[n]-x[-n]}{2}$

$x_e[n] = x_e[-n]$

$x_o[n] = -x_o[-n]$

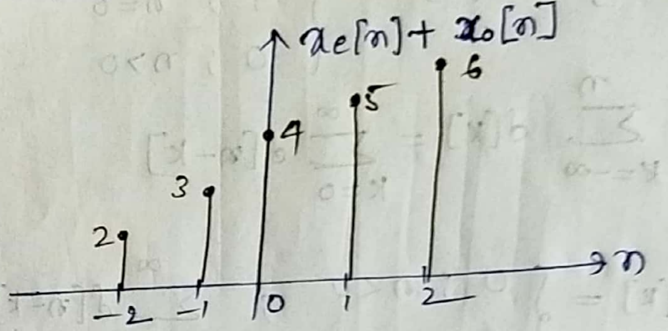




$x_e[n] = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$

$x_o[n] = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

Since $x[n] = x_e[n] + x_o[n]$. The decomposition is unique.



2.5 Show that the energy (power) of a real-valued energy (power) slg is equal to the sum of the energies (power) of its even and odd components.

Sol:- $\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} [x_e[n] + x_o[n]]^2$

$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + \sum_{n=-\infty}^{\infty} 2\, x_e[n] \cdot x_o[n] \xrightarrow{} 0$

$\Rightarrow \sum_{n=-\infty}^{\infty} x_o[n]\, x_e[n] = \sum_{m=-\infty}^{\infty} x_e[-m]\, x_o[-m] = -\sum_{m=-\infty}^{\infty} x_e[m]\, x_o[m]$

$= -\sum_{n=-\infty}^{\infty} x_e[n]\, x_o[n].$

$= 0.$

$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n]$

$\boxed{E_x = E_e + E_o}$

Scanned by CamScanner

Q6 Consider the system $y[n] = T[x[n]] = x(n^2)$.

(a) Determine if the system is time invariant

Sol:-

$$x[n] \longrightarrow y[n] = x[n^2]$$

$$x[n-k] \longrightarrow y_1[n] = x[(n-k)^2]$$

$$= x[n^2 + k^2 - 2nk]$$

$$\neq y[n-k]$$

$\therefore$ It is a time variant system.

(b) To clarify the result of part (a) assume that the sig $x[n] = \begin{cases} 1 \ ; & 0 \le n \le 3 \\ 0 \ ; & \text{otherwise} \end{cases}$ is applied into the system.

(1) Sketch the sig $x[n]$.

(2) Determine & sketch the sig $y[n] = T[x[n]]$.

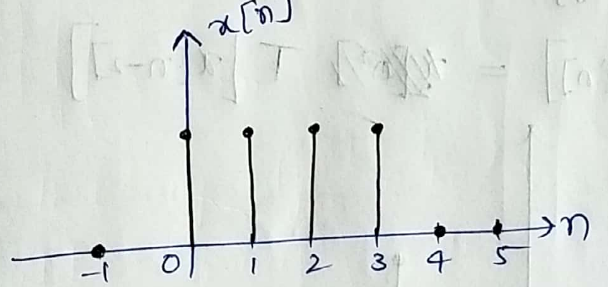
(3) Sketch the sig $y_2[n] = y[n-2]$.

(4) Determine & sketch the sig $x_2[n] = x[n-2]$.

(5) Determine and sketch the sig $y_2[n] = T[x_2[n]]$.

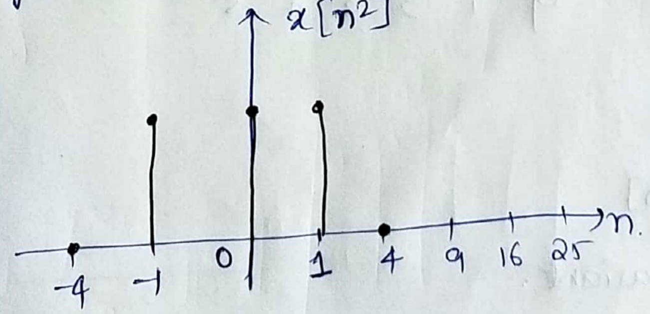
(6) Compare the sig's $y_2[n]$ and $y[n-2]$. what is your conclusion?

Sol:- (1)



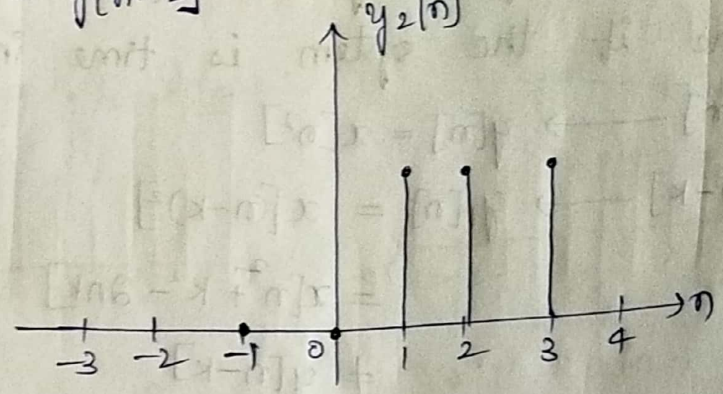
$$x[n] = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$$

(2) $y[n] = T[x(n)] = x[n^2]$



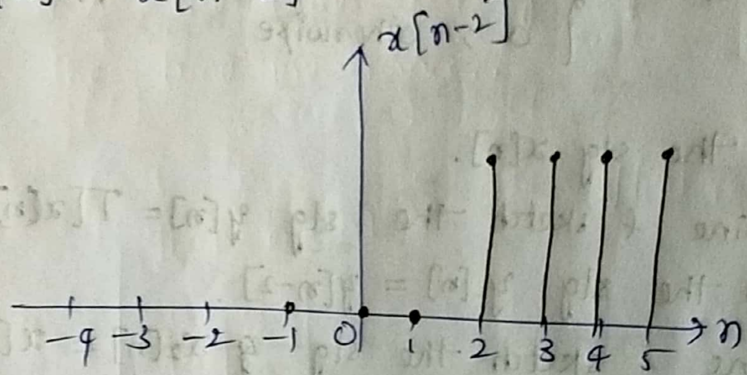
$$x[n^2] = \{\ldots, 0, 1, \underset{\uparrow}{1}, 1, 0, \ldots\}$$

(3) $y_2[n] = y[n-2]$



$$y_2[n] = \{0, \underset{\uparrow}{0}, 1, 1, 1, 0, 0, \ldots\}$$

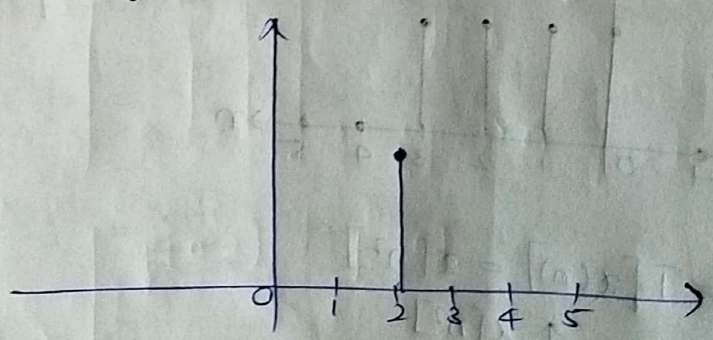
(4) $x_2[n] = x[n-2]$



$$x[n-2] = \{0, \underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0, 0\}$$

(5) $y_2[n] = T[x_2[n]]$

$y_2[n] = T[x_2[n]] = T[x[n-2]]$



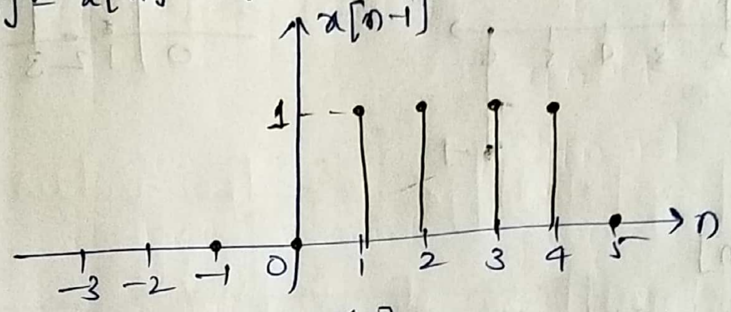
(6) $y[n-2] \neq y_2[n]$

So it's time variant.

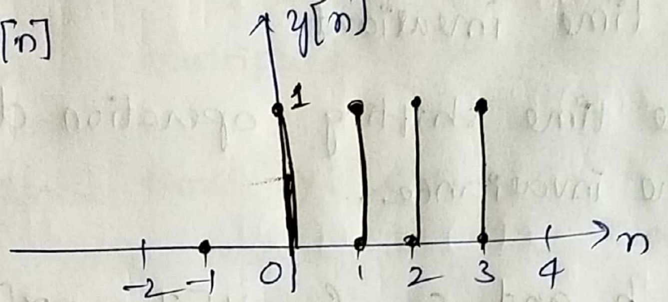
c) Repeat part (b) for the sys. $y[n] = x[n] - x[n-1]$

Can you use this result to make any statement about time invariance of this sys? why?

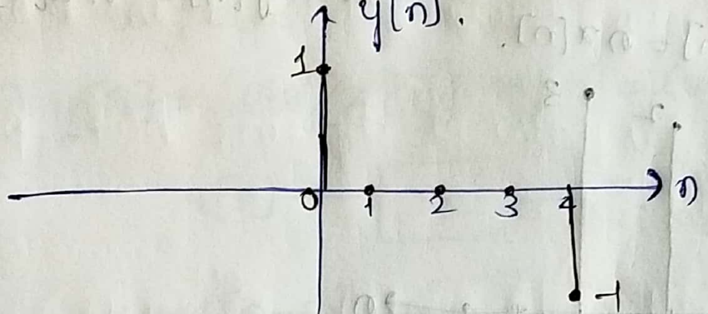
Sol:- $y[n] = x[n] - x[n-1]$



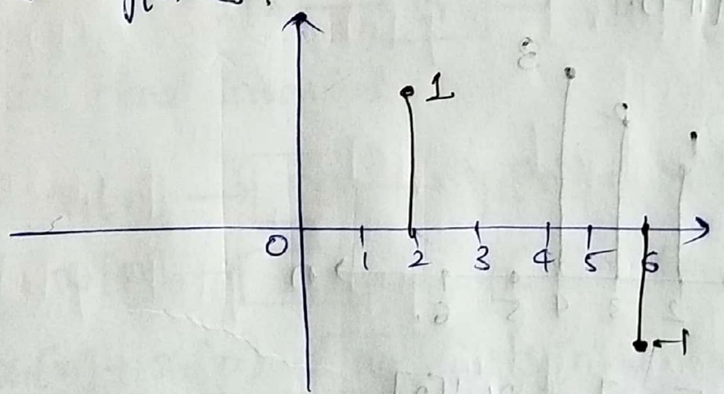
(1) $y[n] = x[n]$



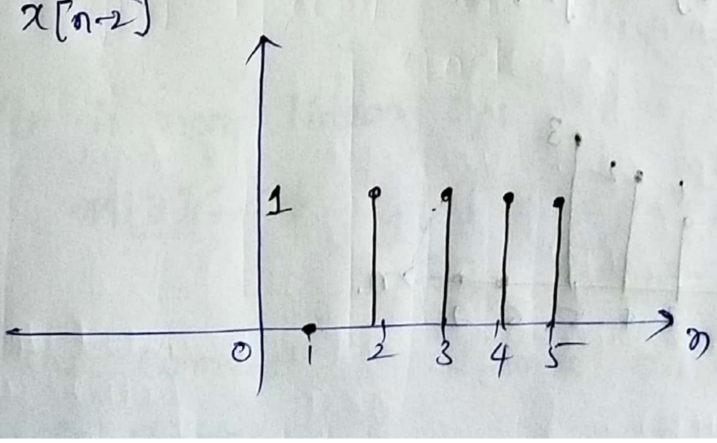
(2)



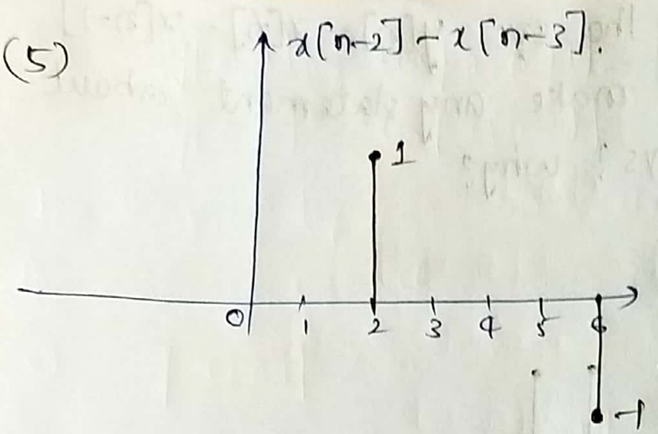
(3) $y_2'[n] = y[n-2]$.

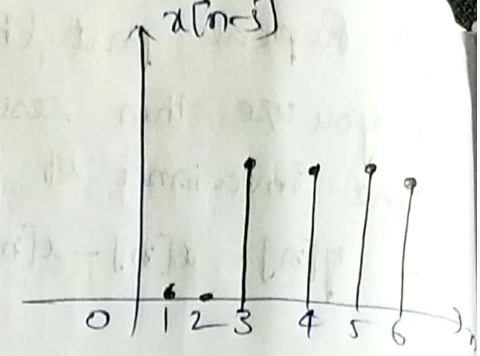


(4) $x[n-2]$



(5)





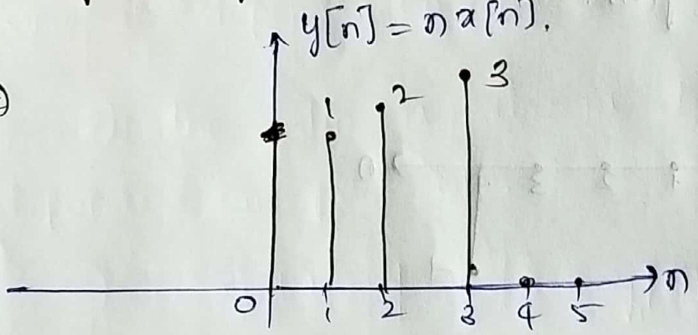
(6) $y_2[n] = y_2'[n]$

The system is time invariant

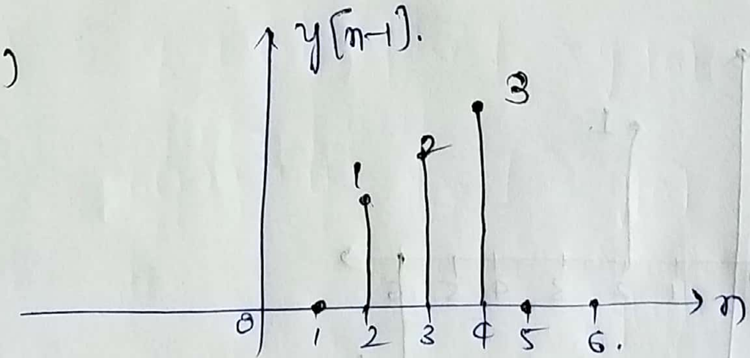
⇒ from this the time shifting operation doesn't affect the time invariance.

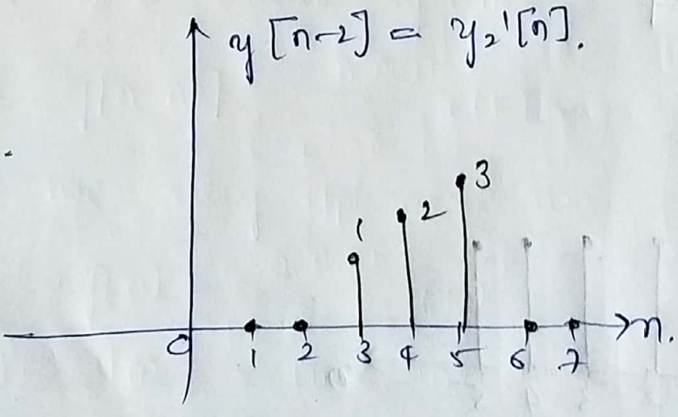
d) Repeat part b and c for $y[n] = n x[n]$.

(2)



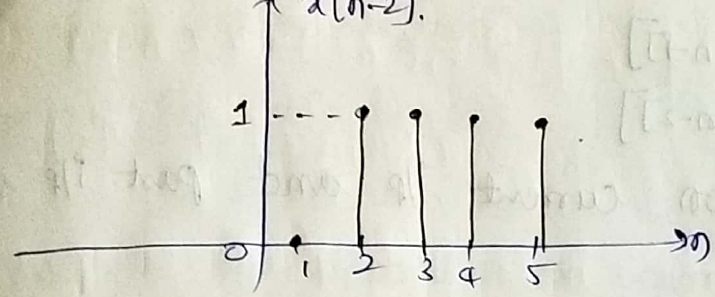
(3)



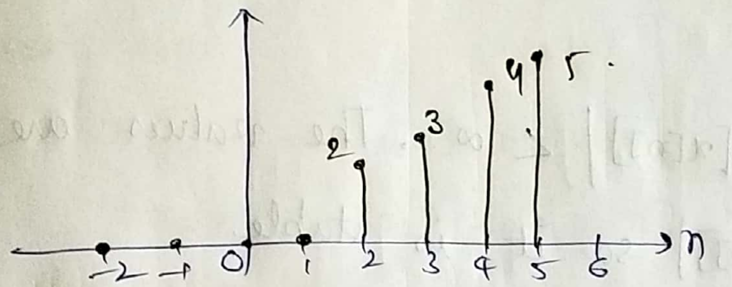


Scanned by CamScanner

(4)



(5)



$\Rightarrow y_2[n] \neq y_2'[n]$.
Time Variant

$\Rightarrow$ So if $x[n]$ is multiplied by 'n' then it will be time variant system.

Q.7 A discrete time sys can be. Examine the follow. systems with respect to the above properties:

(a) $y[n] = \cos[x[n]]$.

Sol:- 1) ~~y[0] ≠ cos[x[0]]~~ $y[n] = \cos[x[n]]$.

$x[n] \rightarrow \boxed{\text{sys}} \rightarrow y[n] = x[n-n_0]$

$x[n-n_0] \rightarrow \boxed{\text{Delay}} \rightarrow y[n] = \cos[x[n-n_0]]$

$x[n] \rightarrow \boxed{\text{sys}} \rightarrow y[n-n_0] = \cos[x[n-n_0]]$

$\therefore$ It is time invariant.

(2) $x_1[n] \rightarrow \boxed{\text{sys}} \rightarrow y_1[n] = \cos[x_1[n]]$

$x_2[n] \rightarrow \boxed{\text{sys}} \rightarrow y_2[n] = \cos[x_2[n]]$

$x_1[n] + x_2[n] \rightarrow \boxed{\text{sys}} \rightarrow y[n] = \cos[x_1[n] + x_2[n]]$

$\neq \cos[x_1[n]] + \cos[x_2[n]]$

$\therefore$ It is non-linear sys.

(3) ~~x₁[0]~~ $y[0] = \cos[x[0]]$

$y[2] = \cos[x[2]]$.

So the current i/p depends on current o/p so the sys is static.

④ $y[n-1] = \cos[x[n-1]]$.

$y[n-2] = \cos[x[n-2]]$ . . .

The sys o/p depends on current i/p and past i/p so the sys is causal.

⑤ $|x[n]| < \infty$

$|y[n]| = |\cos[x[n]]| < \infty$ ∴ . The values are finite for $|\cos[x[n]]|$ so it is stable.

(b) $y[n] = \sum_{k=-\infty}^{n+1} x(k)$

Sol:- 1) $y[1] = \sum_{K=-\infty}^{2} x(K) = x(-\infty) + \dots \dots x(0) + x(1) + x(2)$

Since it depends on future values. it is non-causal

② The s/m is also a dynamic bcoz present i/p o/p depends on past o/p.

③ $x_1(K) \longrightarrow \boxed{\text{sys}} \longrightarrow y_1[n] = \sum_{K=-\infty}^{n+1} x_1(K)$

$x_2(K) \longrightarrow \boxed{\text{sys}} \longrightarrow y_2[n] = \sum_{K=-\infty}^{n+1} x_2(K)$

$x_1(K) + x_2(K) \longrightarrow \boxed{\text{sys}} \longrightarrow y[n] = \sum_{K=-\infty}^{n+1} [x_1(K) + x_2(K)]$

$y[n] = y_1[n] + y_2[n]$

∴ It is a linear system.

④ $y[n-n_0] = \sum_{K=-\infty}^{n-n_0+1} x(K)$

→ If we delay i/p also we get the same

↳ $y[n] = \sum_{K=-\infty}^{n-n_0+1} x(K)$

∴ The s/m is time invariant. → For bounded i/p. $x(K) = u(K)$

⑤ $y[n] = \sum_{K=-\infty}^{n+1} u(K) = \begin{cases} 0 ; & n < -1 \\ n+2 ; & n \geq -1 \end{cases}$

Since $y[n] \rightarrow \infty$ as $n \rightarrow \infty$, the s/m is unstable

Scanned by CamScanner

c) $y[n] = x[n] \cos[\omega_0 n]$

Sol:- 1) $y[0] = x[0] \cdot \cos[(0)]$

$y[1] = x[1] \cdot \cos[(1)]$

The s/m o/p depends on current i/p so the sys is static s/m.

2) $x_1[n] = x_1[n] \cos[\omega_0 n]$

$x_2[n] = x_2[n] \cos[\omega_0 n]$

$x_1[n] + x_2[n] = \cos(\omega_0 n) \left[x_1[n] + x_2[n]\right] = y[n]$.

$y[n] = y_1[n] + y_2[n]$.

∴ The s/m is linear.

3) $y[n-1] = x[n-1] \cos[(n-1)\omega_0]$

$y[n-2] = x[n-2] \cos[(n-2)\omega_0]$

The s/m is causal bcoz the o/p depends on past & current i/p.

4) The s/m is static as it depends on current i/p's

5) $|x[n]| < \infty$.

$|y[n]| = |x[n] \cos[\omega_0 n]| < \infty$.

The s/m is stable for the $(|x[n]| < \infty)$ finite values of 'n'.

(d) $y[n] = x[-n+2]$.

Sol:- 1) $y[0] = x[2]$

$y[1] = x[-1+2] = x[1]$

$y[2] = x[-2+2] = x[0]$

The s/m is non-causal it depends on future values.

2) $x_1[n] \longrightarrow \boxed{\text{sys}} \longrightarrow x_1[-n+2]$

$x_2[n] \longrightarrow \boxed{\text{sys}} \longrightarrow x_2[-n+2]$.

Scanned by CamScanner

$$x_1[n] + x_2[n] \longrightarrow \boxed{sys} \longrightarrow x_1(-n+2) + x_2(-n+2)$$

$$x_1[n] + x_2[n] = x[n]$$

$\therefore$ The s/m is linear.

3) The s/m is dynamic bcoz it depends on past values and current values.

4) $y[n-n_0] = x[-(n-n_0)+2]$

Shift $x[n]$ by 'n_0' $\longrightarrow x[n-n_0]$

$$y[n] = x[-n+n_0+2]$$

$\therefore$ It is time invariant.

5) $|x[n]| < \infty$.

$y[n] = |x[-n+2]| < \infty$.

For finite values of 'n' the s/m is also finite so. it is stable

(g) $y[n] = |x[n]|$

Sol:- 1) $y[0] = |x[0]|$

$y[1] = |x[1]|$

$y[-1] = |x[-1]|$

The s/m is causal as well as static, it depends on current i/p and past i/p.

3) $x_1[n] \longrightarrow \boxed{s/m} \longrightarrow |x_1[n]| = y_1[n]$

$x_2[n] \longrightarrow \boxed{s/m} \longrightarrow |x_2[n]| = y_2[n]$.

$x_1[n] + x_2[n] \longrightarrow \boxed{s/m} \longrightarrow y[n] = |x_1[n] + x_2[n]|$

$= |x_1[n]| + |x_2[n]|$

$\therefore$ The s/m is linear. s/m.

4) $y[n-n_0] = |x[n-n_0]|$

$x[n-n_0] = |x[n-n_0]|$

$\therefore$ The s/m is time invariant.

5) $|x[n]| < \infty$.

$|y[n]| < \infty$

The s/m is finite for finite 'n' values.

(h) $y[n] = x[n]\, u[n]$.

Sol:- 1) $y[0] = x[0]\, u[0]$; $y[-1] = x[-1]\, u[-1]$

$y[1] = x[0]\, u[1]$.

The s/m is static and causal ~~the~~ it depends on current and past values.

3) $x_1[n] \longrightarrow \boxed{s/m} \longrightarrow x_1[n]\, u[n] = y_1[n]$

$x_2[n] \longrightarrow \boxed{s/m} \longrightarrow x_2[n] \cdot u[n] = y_2[n]$

$x_1[n] + x_2[n] \longrightarrow \boxed{s/m} \longrightarrow u[n]\,(x_1[n] + x_2[n]) = y_3[n]$

$\therefore$ The s/m is linear

4) $y[n-n_0] = x[n-n_0]\, u[n-n_0]$.

$y[n] = x[n-n_0] = x[n-n_0] \cdot u[n]$.

$\therefore$ It is time variant.

5) $|x[n]| < \infty$.

For bounded i/p of $x[n]$. we will get bounded o/p. of $x[n]$ so the amp is finite. The s/m is stable

(i) $y[n] = x[n] + n\,x[n+1]$

Sol:- 1) $y[0] = x[0] + 0\, x[0+1]$; $y[-1] = x[-1] + (-1)\, x[0]$

$y[1] = x[1] + x[2]$; $= x[-1] - x[0]$.

Since the o/p. depends on present and future values, it is Dynamic s/m and causal system it depends on past values.

③ eg. $x_1[n] \longrightarrow \boxed{s/m} \longrightarrow x_1[n] + n\, x_1[n+1]$.

$x_2[n] \longrightarrow \boxed{s/m} \longrightarrow x_2[n] + n\, x_2[n+1]$.

$x_1[n] + x_2[n] \longrightarrow \boxed{s/m} \longrightarrow x_1[n] + n(x_1[n+1] + x_2[n+1]) + x_2[n]$.

∴ The s/m is linear.

④ $y[n-n_0] = x[n-n_0] + (n-n_0)\, x[n-n_0+1]$.

$y[n] = x[n-n_0] = x[n-n_0] + n\, x[n-n_0+1]$

∴ The s/m is time variant

⑤ $|x[n]| < \infty$

Since for bounded i/p the s/m will be get bounded o/p. The s/m is stable

(j) $y[n] = x[2n]$

Sol:- 1) $y[0] = x[0]$

$y[1] = x[2]$

$y[-1] = x[-2]$

The s/m is Dynamic it depends on future values

② $x_1[n] \longrightarrow \boxed{s/m} \longrightarrow x_1[2n]$

$x_2[n] \longrightarrow \boxed{s/m} \longrightarrow x_2[2n]$.

$x_1[n] + x_2[n] \longrightarrow \boxed{s/m} \longrightarrow x_1[2n] + x_2[2n]$.

∴ The s/m is linear.

③ $y[n-n_0] = x[2(n-n_0)]$

$y[n] = x[n-n_0] = x[2n-n_0]$.

∴ The s/m is time variant.

④ $y[0] = x[0]$.

$y[-2] = x[-4]$.

The s/m is non-causal. bcoz it doesn't depends on past values.

5) $|y[n]| < \infty$

For bounded i/p the s/m. gets bounded o/p the s/m is stable. s/m.

(k) $y[n] = \begin{cases} x[n], & \text{if } x[n] \geq 0. \\ 0; & \text{if } x[n] < 0 \end{cases}$

Sol:- 1) $y[n-n_0] = x[n-n_0]$.

$y[n] = x[n-n_0] = x[n-n_0]$

∴ The s/m is time invariant.

2) $x_1[n] \rightarrow \boxed{s/m} \rightarrow x_1[n] = y_1[n]$

$x_2[n] \rightarrow \boxed{s/m} \rightarrow x_2[n] = y_2[n]$.

$x_1[n] + x_2[n] \rightarrow \boxed{s/m} \rightarrow$

∴ The s/m is non-linear.

3) $y[0] = x[0]$

$y[1] = x[1]$

∴ The system is causal and static bcoz it depends on present values.

5) $|y[n]| < \infty$.

For bounded i/p of $x[n]$. there will be bounded. o/p. so the s/m is stable

(l) $y[n] = x[-n]$

Sol:- 1) $y[1] = x[-1]$

$y[2] = x[-2]$

$y[-1] = x[1]$

∴ The s/m is Dynamic, causal.

③ $x_1[n] \rightarrow \boxed{} \rightarrow x_1[-n]$

$x_2[n] \rightarrow \boxed{} \rightarrow x_2[-n]$.

$x_1[n] + x_2[n] \rightarrow \boxed{} \rightarrow x_1[-n] + x_2[-n] = y[n]$.

$y[n] = y_1[n] + y_2[n] \Rightarrow$ linear s/m.

④ $y[n-n_0] = x[-n+n_0]$.

$y(n) = x[n-n_0] = x[-n-n_0]$.

∴ The s/m is time variant s/m.

⑤ $|x[n]| < \infty$.

For bounded values of 'n' the value of o/p is bounded o/p. So the s/m is stable

(m) $y[n] = sign[x[n]]$.

Sol:- 1) $y[n-n_0] = sign[x(n-n_0)]$.

$y[n] = x[n-n_0] = sign[x[n-n_0]]$.

∴ The s/m is time invariant

2) $y[0] = sign[x[0]]$

$y[1] = sign[x[1]]$.

The s/m is static s/m. and as well as causal s/m.

④ The amp is finite for 'n' finite values. Therefore the s/m is stable

$|y[n]| < \infty$.

⑤ $x_1[n] \rightarrow \boxed{} \rightarrow sign[x_1[n]]$

$x_2[n] \rightarrow \boxed{} \rightarrow sign[x_2[n]]$.

$x_1[n] + x_2[n] \rightarrow \boxed{} \rightarrow sign[x_1[n] + x_2[n]]$

∴ The s/m is linear s/m.

Q.9. Let T be an LTI, relaxed and BIBO stable s/m. with i/p $x[n]$ and o/p $y[n]$. show that.

(a) If $x[n]$ is periodic with period N {i.e., $x[n] = x[n+N]$ ∀ $n \geq 0$}, the o/p $y[n]$ tends to a periodic slg with the same period.

(b) If $x[n]$ is bounded and tends to a const, the o/p will also tend to a const.

(c) If $x[n]$ is an energy slg, the o/p $y[n]$ will also be an energy slg?

(a) $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$, $x[n] = 0$, $n<0$.

$$y[n+N] = \sum_{k=-\infty}^{n+N} h[k]\, x[n+N-k] = \sum_{k=-\infty}^{n+N} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{n} h[k]\, x[n-k] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k].$$

$$= y[n] + \sum_{k=n+1}^{n+N} h[k]\, x[n-k]$$

For a BIBO s/m,

$$\lim_{n\to\infty} |h[n]| = 0.$$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h[k]\, x[n-k] = 0.$$

$$\lim_{n\to\infty} y[n+N] = \lim_{n\to\infty} y[n].$$

(b) Let $y[n] = x_0[n] + a u[n]$

$x_0[n]$ is a bounded sig with $\lim_{n\to\infty} x_0[n] = 0$

$$y[n] = a\sum_{k=0}^{\infty} h[k]u[n-k] + \sum_{k=0}^{\infty} h[k]x_0[n-k]$$

$$= a\sum_{k=0}^{\infty} h[k] + y_0[n]$$

$$\Rightarrow \sum_n x_0^2[n] < \infty \Rightarrow \sum_n y_0^2[n] < \infty.$$

$$\lim_{n\to\infty} |y_0[n]| = 0.$$

$$\Rightarrow \lim_{n\to\infty} y[n] = a\sum_{k=0}^{n} h[k] = \text{Const}.$$

(c) $y[n] = \sum_k h[k]x[n-k]$.

$$\sum_{-\infty}^{\infty} y^2[n] = \sum_{-\infty}^{\infty}\left[\sum_k h[k]x[n-k]\right]^2$$

$$= \sum_k \sum_l h[k]h[l] \cdot \sum_n x[n-k]\,x[n-l]$$

But

$$\sum_n x[n-k]x[n-l] \le \sum_n x^2[n] = E_x.$$

$$\sum_n y^2[n] \le E_x \cdot \sum_k |h[k]| \cdot \sum_l |h(l)|$$

For BIBO stable,

$$\sum_k |h[k]| < M$$

$$E_y \le M^2 E_x.$$

$$E_y < 0 \text{ if } E_x < 0.$$

2.13 Show that the necessary and sufficient for a relaxed LTI s/m to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h[n]| \le M_h < \infty \quad \text{for some Const } M_h.$$

Sol:- If Bounded i/p produces Bounded o/p the s/m is BIBO stablity s/m.

$$y[n] = \sum_{K} h[K]\, x[n-K]$$

$$|y[n]| \le \sum_{K} |h[K]|\, |x[n-K]| \qquad |x[n-0]| \le M_x.$$

$$\le M_x \sum_{K} |h[K]|$$

$\to |x[n-K]|$

$$|y[n]| < \infty \;\; \forall\, n, \text{ if and only if } \sum_{K} |h[K]| < \infty.$$

2.14 Show that ~~the necessary & sufficient~~ condition ~~for a relaxed LTI s/m to be BIBO stable~~ is

$$\cancel{\sum_{n=-\infty}^{\infty} |h[n]|}$$

(a) A relaxed linear s/m is causal if only only if for any i/p $x[n]$ such that $x[n]=0$ for $n<n_0 \Rightarrow y[n]=0$ for $n<n_0$.

(b) A relaxed linear s/m is Causal. if and only if $h[n]=0$, for $n<0$.

(a) Causal. s/m $\to$ non-zero i/p. is given the o/p also should be non-zero.

$$x[n]=0,\; n<n_0 \Rightarrow y[n]=0 \;;\; n<n_0.$$

(b) $$y[n] = \sum_{-\infty}^{0} h[K]\, x[n-K] + \sum_{0}^{\infty} h[K]\, x[n-K]$$

The s/m should be causal only if $\sum_{-\infty}^{0} h[K]\, x[n-K] = 0$ it tends to zero only if $h[K]=0$; for $K<0$.

Q.15 a) Show that for any real (or) complex constant, a, and any finite integer numbers M and N, we have.

$$\sum_{n=M}^{N} a^n = \begin{cases} \dfrac{a^M - a^{N+1}}{1-a} ; & \text{if } a \neq 1 \\ N-M+1 ; & \text{if } a = 1. \end{cases}$$

b) Show that if $|a| < 1$, then.

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

Sol:- (a) If $a = 1$; $\sum_{n=M}^{N} a^n = \sum_{n=M}^{N} (1)^n = \sum_{n=M}^{N} (1)$

$$= N - M + 1$$

If $a=1$; $\boxed{\sum_{n=M}^{a^n} = N-M+1}$

If $a \neq 1$; $\sum_{n=M}^{N} a^n = a^M + a^{M+1} + a^{M+2} \cdots\cdots + a^N.$

$$\Rightarrow (1-a) \sum_{n=M}^{N} a^n = a^M - \cancel{a^{M+1}} + \cancel{a^{M+1}} - \cancel{a^{M+2}} + \cancel{a^{M+2}} \cdots + a^N - a^N - a^{N+1}.$$

$$\boxed{\sum_{n=M}^{n} a^n = \frac{a^M - a^{N+1}}{1-a}.}$$

(b) $\Rightarrow -1 < a < 1$

For $|a| < 1$ it forms a G.P (progression series).

Sum of series $= \dfrac{a}{1-r}$ $\Rightarrow \boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}}$

2.16 (a) If $y[n] = x[n] * h[n]$, show that $\sum_y = \sum_x \sum_h$

where $\sum_x = \sum_{n=-\infty}^{\infty} x[n]$.

(b) Compute the conv. $y[n] = x[n] * h[n]$ of the following sig's and check the correctness of the results by using the test in (a).

(a) $$y[n] = \sum_k h[k]\, x[n-k]$$

$$\sum_n y[n] = \sum_n \sum_k h[k]\, x[n-k] \quad \text{[linearity]}.$$

$$\sum_n y[n] = \sum_k h[k] \cdot \sum_{n=-\infty}^{\infty} x[n-k]$$

$$\sum_n y[n] = \left(\sum_k h[k]\right)\left(\sum_n x[k]\right)$$

(b) (1) $x[n] = \{1, 2, 4\}$, $h[n] = \{1, 1, 1, 1, 1\}$.

Sol:- $y[n] = x[n] * h[n]$.

$x[n]$ \ $h[n]$	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$y[n] = \{1, 3, 7, 7, 7, 6, 4\}$.

$\sum_n y[n] = 35$; $\sum_k h[k] = 5$.

$\sum_k x[n] = 7$.

$$\Rightarrow \sum_n y[n] = \sum_k h[k] \sum_k x[k]$$

$$35 = 5 \times 7 \Rightarrow 35 = 35.$$

(2) $x[n] = \{1, 2, -1\}$; $h[n] = x[n]$.

$x[n]$ \ $h[n]$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y[n] = \{1, 4, 2, -4, 1\}$

$\sum_n y[n] = 4$; $\sum_k h[k] = 2$; $\sum_k x[k] = 2$

$\therefore 4 = 2 \times 2 \Rightarrow 4 = 4$.

Scanned by CamScanner

(3) $x[n] = \{0, 1, -2, 3, -4\}$; $h[n] = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$

Sol:-

$x[n]$ \ $h[n]$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
0	0	0	0	0
1	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$
-2	-1	-1	-2	-1
3	$\frac{3}{2}$	3/2	3	3/2
-4	-2	-2	-4	-2

$y[n] = \{0, \frac{1}{2}, -1/2, \frac{3}{2}, -2, 0, -5/2, -2\}$

$\sum_n y[n] = -5$; $\sum_k h[k] = 2.5$; $\sum_k x[k] = -2$

$\therefore \sum_n y[n] = \sum_k h[k] \cdot \sum_k x[k]$

$-5 = 2.5(-2) \Rightarrow -5 = -5.$

(4) $x[n] = \{1, 2, 3, 4, 5\}$; $h[n] = \{1\}$.

Sol:-

$x[n]$ \ $h[n]$	1
1	1
2	2
3	3
4	4
5	5

$y[n] = \{1, 2, 3, 4, 5\} \Rightarrow \sum_n y[n] = 15$; $\sum_k x[k] = 15.$

$\sum_k h[k] = 1.$

$\therefore \sum_n y[n] = \sum_k x[k] \cdot \sum_k h[k]$

$15 = 15(1) \Rightarrow 15 = 15$

(5) $x[n] = \{\underset{\uparrow}{1}, -2, 3\}$; $h[n] = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$

$y[n] = \{0, 0, 1, -1, 2, 2, +1, +3\}$

$\sum_n y[n] = 8$; $\sum_k x[k] = 2$; $\sum_k h[k] = 4$

$x[n]$ \ $h[n]$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
+3	0	0	+3	+3	+3	+3

$$\therefore \sum_n y[n] = \sum_k x[k] \sum_k h[k]$$

$$8 = 4 \times 2 \Rightarrow 8 = 8.$$

6) $x[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$; $h[n] = \{1, \underset{\uparrow}{-2}, 3\}$

$h[n]$ \ $x[n]$	0	0	1	1	1	1
1	0	0	1	1	1	1
$\rightarrow$ -2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y[n] = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y[n] = 8$; $\sum_k x[k] = 4$. ; $\sum_k h[k] = 2$.

$$\sum_n y[n] = \sum_k x[k] \times \sum_k h[k]$$

$$8 = 4 \times 2 \Rightarrow 8 \times 8.$$

7) $x[n] = \{\underset{\uparrow}{0}, 1, 4, -3\}$; $h[n] = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$x[n]$ \ $h[n]$	1	0	-1	-1
$\rightarrow$ 0	0	0	0	0
1	1	0	-1	-1
4	4	0	-4	-4
-3	-3	0	+3	+3

$y[n] = \{0, 1, 4, -4, -5, -1, 3\}$.

$\sum_n y[n] = -2$

$\sum_k x[k] = 2$

$\sum_k h[k] = -1$

$\Rightarrow -2 = 2(-1)$

$-2 = -2$

8) $x[n] = \{\underset{\uparrow}{1}, 1, 2\}$ and $h[n] = u[n]$.

$x[n]$ \ $h[n]$	$u[n]$	$u[n-1]$	$u[n-2]$ ...
1	$u[n]$	$u[n-1]$	$u[n-2]$
1	$u[n]$	$u[n-1]$	$u[n-2]$.
2	$2u[n]$	$2u[n-1]$	$2u[n-2]$

$\sum_n y[n] = u[n] + u[n-1] + 2u[n-2] + \ldots = \infty$

$\sum_k x[n] = 4$.

$\sum_k h[n] = \infty$.

$x[n]$ \ $h[n]$	1	1	1	1
1	1	1	1	1
1	1	1	1	1
2	2	2	2	2

$\therefore \sum_n y[n] = \sum_k x[k] \times \sum_k h[k]$

$\infty = \infty \times 4$.

$\infty = \infty$

$y[n] = \{1, 2, 4, 4, 3, 2\}$

9) $x[n] = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$ and $h[n] = \{1, -2, -3, \underset{\uparrow}{-4}\}$

$x[n]$ \ $h[n]$	1	-2	-3	$\overset{\downarrow}{-4}$
1	1	-2	-3	-4
1	1	-2	-3	-4
$\rightarrow$ 0	0	0	0	0
1	1	-2	-3	-4
1	1	-2	-3	-4

$y[n] = \{1, -1, -5, -6, -5, -5, -7, -4\}$

$\sum_n y[n] = -32$

$\sum_k x[k] = 4$; $\sum_k h(k) = -8$.

$\therefore \sum_n y[n] = \sum_k x[k] \times \sum_k h[k]$.

$-32 = -8 \times 4 \Rightarrow -32 = -32$

10) $x[n] = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$; $h[n] = x[n]$.

11) $x[n] = \left(\frac{1}{2}\right)^n u[n]$. ; $h[n] = \left(\frac{1}{4}\right)^n u[n]$.

(10)

$x[n]$ \ $h[n]$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$y[n] = \{1, 4, 4, 4, 10, 4, 4, 4, 1\}$

$\sum_n y[n] = 36$

$\sum_k x[k] = \sum_k h[k] = 6$

$\therefore \sum_n y[n] = \sum_k x[k] \cdot \sum_k h[k] \Rightarrow 36 = 36$

---

(11) $x[n] = \left(\frac{1}{2}\right)^n u[n] \; \cancel{\left(\frac{1}{4}\right)^n u[n]}$; $h[n] = \left(\frac{1}{4}\right)^n u[n]$

$x[n]$ \ $h[n]$	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	 0
1	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{128}$	
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$	
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{32}$	$\frac{1}{128}$	$\frac{1}{512}$	
⋮					
0					

$y[n] = \left\{1, \frac{3}{4}, \frac{7}{16}, \frac{15}{64}, \frac{7}{128}, \frac{3}{256}, \frac{1}{512}\right\}$

$\cancel{\sum} \, y[n] = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u[n]$

$\sum_n y[n] = 2\left(\frac{1}{2}\right)^n u[n] - \left(\frac{1}{4}\right)^n u[n]$

$= \sum_{n=0}^{\infty} 2\left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n$

$\sum_n y[n] = 2\left(\frac{1}{1-\frac{1}{2}}\right) - \left(\frac{1}{1-1/4}\right) = 2\left(\frac{1}{1/2}\right) - \left(\frac{1}{3/4}\right)$

$= 4(1) - \frac{4}{3} = 4 - \frac{4}{3}$

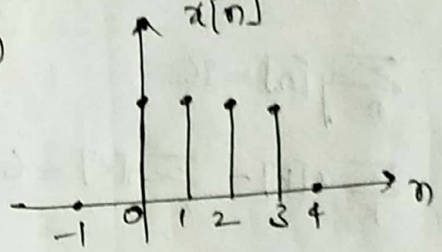
$= \frac{8}{3}$

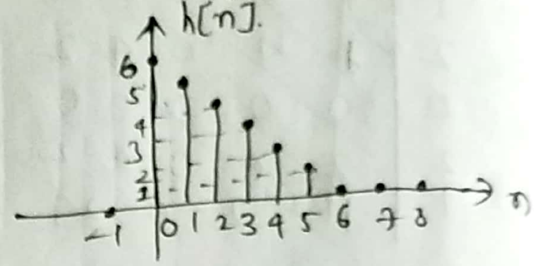
$\sum_k x[k] = 2$

$\sum_k h[k] = \frac{4}{3}$

Q.17 Compute and plot the convolutions $x[n] * h[n]$ and $h[n] * x[n]$ for the pairs of sig's.

(a)

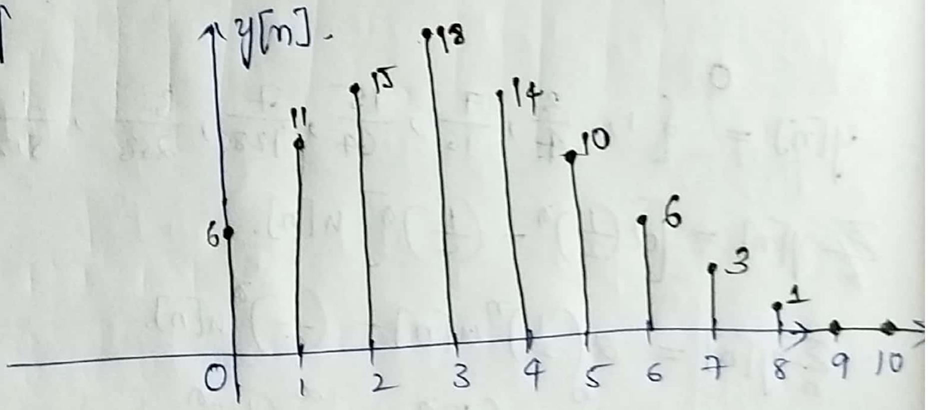




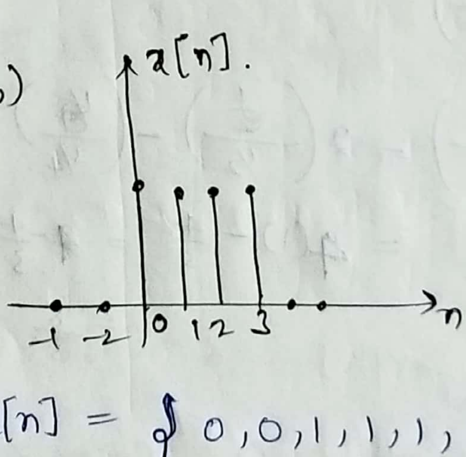
$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$; $h[n] = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1\}$

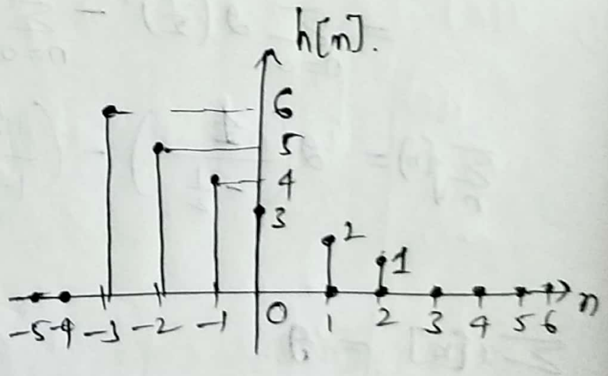
h[n] / x[n]	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0

$y[n] = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1, 0\}$



(b)

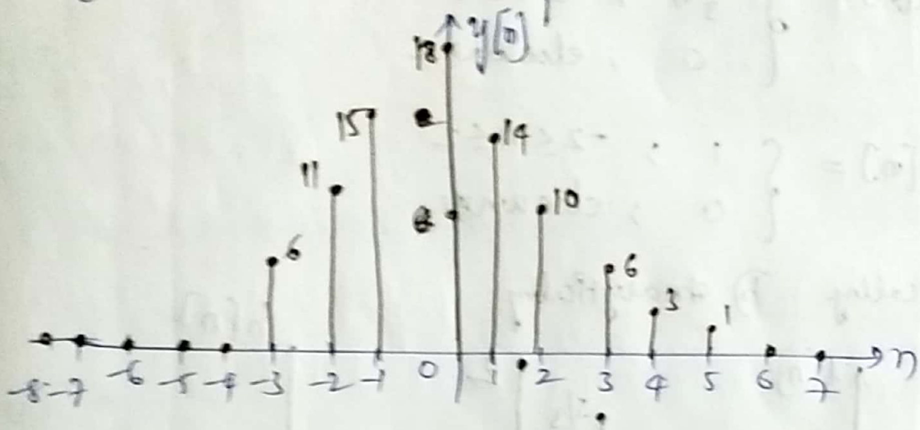




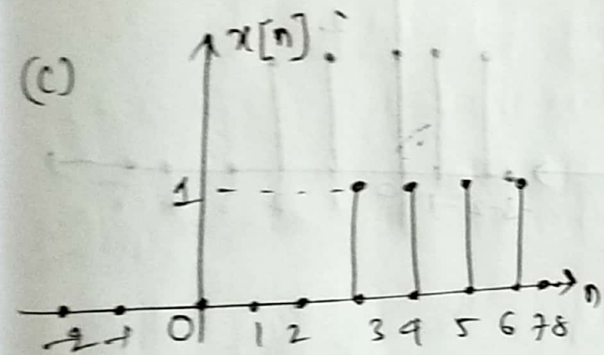
$x[n] = \{0, 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0\}$; $h[n] = \{0, 0, 6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0, 0, 0, 0$

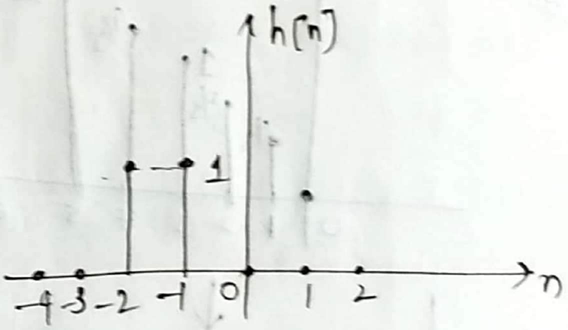
x[n] \ h[n]	0	0	6	5	4	3	2	1	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
→ 1	0	0	6	5	4	3	2	1	0	0
1	0	0	6	5	4	3	2	1	0	0
1	0	0	6	5	4	3	2	1	0	0
1	0	0	6	5	4	3	2	1	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$$y[n] = \{0,0,0,0,6,11,15,\underset{\uparrow}{18},14,10,6,3,1,0,0,0,0\}$$



(c)

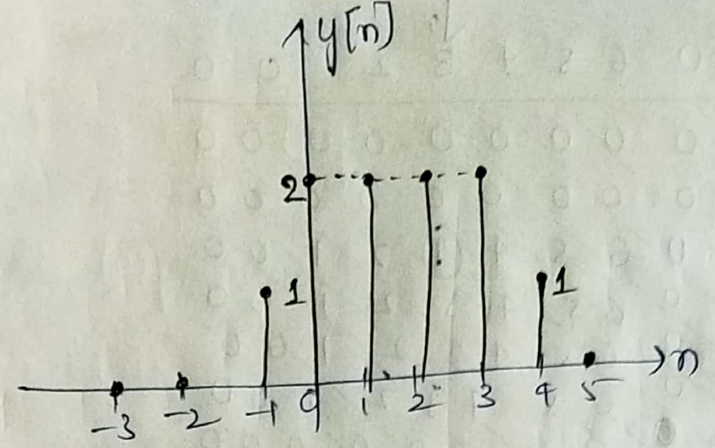




$$x[n] = \{0,0,\underset{\uparrow}{0},0,0,1,1,1,1,0,0\}; \quad h[n] = \{0,0,1,1,\underset{\uparrow}{0},0,0\}$$

x[n] \ h[n]	0	1	1	0	0
→ 0	0	0	0	0	0
1	0	1	1	0	0
1	0	1	1	0	0
1	0	1	1	0	0
1	0	1	1	0	0
0	0	1	1	0	0

$$y[n] = \{0,0,1,\underset{\uparrow}{2},2,2,2,1,0\}$$



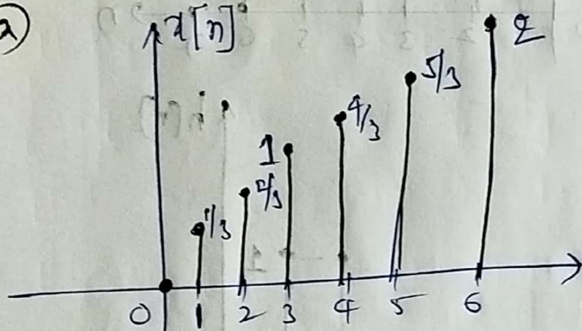
2.18 Determine & sketch the conv $y[n]$ of the sig's.

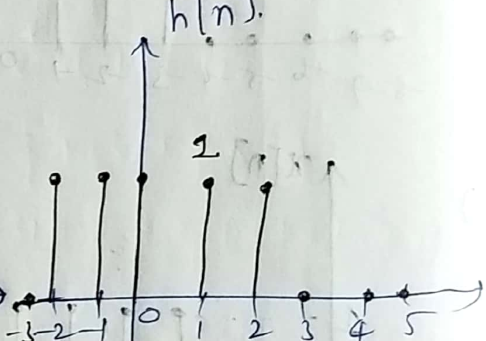
$$x[n] = \begin{cases} \frac{1}{3}n & ; 0 \le n \le 6. \\ 0 & ; \text{elsewhere.} \end{cases}$$

$$h[n] = \begin{cases} 1 & ; -2 \le n \le 2 \\ 0 & ; \text{elsewhere.} \end{cases}$$

a) Graphically b) Analytically

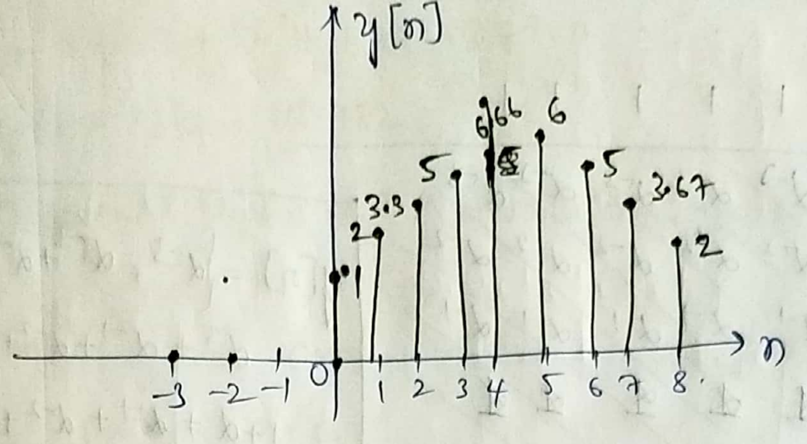
Sol:- a)





$x[n]$ \ $h[n]$	0	1	1	1	1	1	0
0	0	0	0	0	0	0	0
$\frac{1}{3}$	0	$\frac{1}{3}$	1/3	1/3	1/3	1/3	0
$\frac{2}{3}$	0	2/3	2/3	2/3	2/3	2/3	0
1	0	1	1	1	1	1	0
$\frac{4}{3}$	0	4/3	4/3	4/3	4/3	4/3	0
5/3	0	5/3	5/3	5/3	5/3	5/3	0
2	0	2	2	2	2	2	0

$$y[n] = \{0, 0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$$



b) $x[n] = \frac{1}{3} n [u[n] - u[n-7]]$.

$h[n] = u[n+2] - u[n-3]$.

$y[n] = x[n] * h[n]$.

$= \frac{1}{3} n [u(n) - u(n-7)] * [u[n+2) - u[n+3)]$

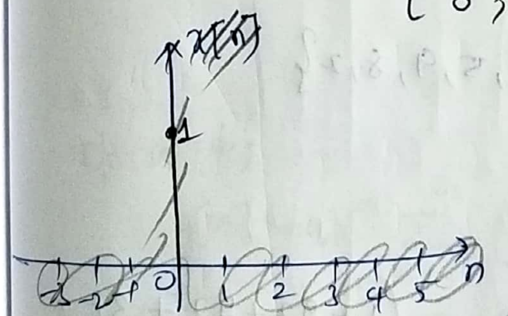
$= \frac{1}{3} n \Big[u(n) * u(n+2) - u(n) * u(n-3) - u(n-7) * u(n+2)$
$+ u(n-7) * u(n-3)\Big]$.

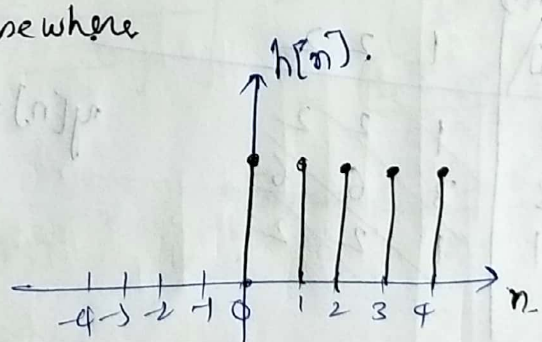
$y[n] = \frac{1}{3}\delta[n+1] + \delta[n] + 2\delta[n-1] + \frac{10}{3}\delta[n-2] + 5\delta[n-3] +$
$\frac{20}{3}\delta[n-4] + 6\delta[n-5] + 5\delta(n-6) + 5\delta[n-6] +$
$\frac{11}{3}\delta[n-7] + 2\delta[n-8]$.

2.19 Compute the Conv. $y[n]$ of the $s/g's$.

$$x[n] = \begin{cases} a^n ; & -3 \le n \le 5 \\ 0 ; & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 1 ; & 0 \le n \le 4 \\ 0 ; & \text{elsewhere} \end{cases}$$





$x[n] = \{\frac{1}{\alpha^3}, \frac{1}{\alpha^2}, \frac{1}{\alpha}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5\}$.
$\uparrow$

$h[n] = \{0, 0, 1, 1, 1, 1, 1, 0\}$
$\uparrow$

$x[n]$	1	1	1	1	1
α^{-3}	α^{-3}	α^{-3}	α^{-3}	α^{-3}	α^{-3}
α^{-2}	α^{-2}	α^{-2}	α^{-2}	α^{-2}	α^{-2}
α^{-1}	α^{-1}	α^{-1}	α^{-1}	α^{-1}	α^{-1}
1	1	1	1	1	1
α^{1}	α^{1}	α	α	α	α
α^{2}	α^{2}	α^{2}	α^{2}	α^{2}	α^{2}
α^{3}	α^{3}	α^{3}	α^{3}	α^{3}	α^{3}
α^{4}	α^{4}	α^{4}	α^{4}	α^{4}	α^{4}
α^{5}	α^{5}	α^{5}	α^{5}	α^{5}	α^{5}

$$y[n] = \{\alpha^{-3},\ \alpha^{-3}+\alpha^{-2},\ \alpha^{-1}+\alpha^{-3}+\alpha^{-1}+\alpha^{-2}+\alpha^{-3},\ 1+\alpha+\alpha^{-1}+\alpha^{-2}+\alpha^{-3},\ \alpha^{2}+\alpha+\alpha^{-1}+\alpha^{-2}+1,\ \alpha^{3}+\alpha^{2}+\alpha+1,+\alpha^{-1},\ \alpha^{4}+\alpha^{3}+\alpha^{2}+\alpha+1,\ \alpha^{5}+\alpha^{4}+\alpha^{3}+\alpha^{2}+\alpha,\ \alpha^{5}+\alpha^{4}+\alpha^{3}+\alpha^{2},\ \alpha^{5}+\alpha^{4},\ \alpha^{5}\}$$

Q.20 Consider the following '3' operations.

(a) Multiply the integer no.'s 131 & 122.

(b) Compare the conv. of slg $\{1,3,1\} * \{1,2,2\}$.

(c) Mul. the polynomials $(1+3z+z^2)$ & $(1+2z+2z^2)$.

(d) Repeat part (a) for 1.31 & 12.2.

(e) Comment

Sol:- (a) $131 \times 122 = 15982$

(b) $x[n] = \{1,3,1\}$ & $h[n] = \{1,2,2\}$

$x[n]$ \ $h[n]$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

$y[n] = \{1, 5, 9, 8, 2\}$

(c) $(1+3z+z^2)(1+2z+2z^2) = (1+2z+2z^2+3z+6z^2+6z^3+2z^2+2z^3+2z^4)$

$= 8z^3 + 9z^2 + 2z^4 + 5z + 1$

$= 1 + 5z + 9z^2 + 8z^3 + 2z^4$

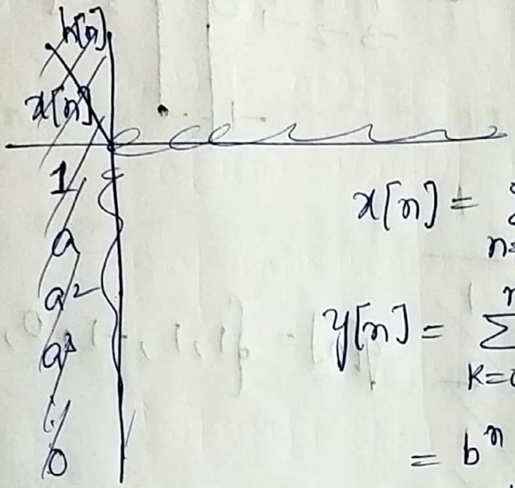
(d) $1.31 \times 12.2 = 15.982$

(e) Different types of convolutions performed.

2.21 Compute the conv. $y[n] = x[n] * h[n]$ of the following pairs of sig's.

a) $x[n] = a^n u[n]$, $h[n] = b^n u[n]$ when $a \neq b$ and. when $a = b$.

Sol:-



$$x[n] = \sum_{n=0}^{\infty} a^n \quad ; \quad h[n] = \sum_{n=0}^{\infty} b^n$$

$$y[n] = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k \cdot b^{-k}$$

$$= b^n \sum_{k=0}^{n} (ab^{-1})^k$$

If $a \neq b$.

$$\Rightarrow S_n = \frac{1-r^n}{1-r} = b^n \left[\frac{\left(1 - \left(\frac{a}{b}\right)^n\right)}{1 - \left(\frac{a}{b}\right)} \right]$$

$$= b^n \left[\frac{\left(\frac{b^n - a^n}{b^n}\right)}{\left(\frac{b-a}{b}\right)} \right] = \cancel{b^n} \left[\frac{b^n - a^n}{\cancel{b^n}} \times \frac{b}{b-a} \right]$$

$$= \frac{(b^n - a^n)\, b}{b-a}$$

$$y[n] = \frac{b^{n+1} - a^n}{b-a}.$$

If $a = b$;

$$y[n] = a^n \sum_{k=0}^{n} (a a^{-1})^k$$

$$= a^n \sum_{k=0}^{n} \left(\frac{a}{a}\right)^k$$

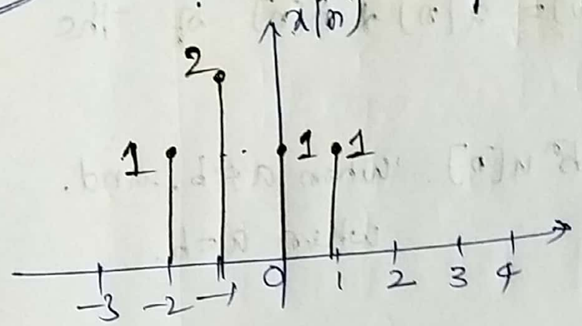
$$= a^n \sum_{k=0}^{\infty} (1)^k = a^n + N + 1$$

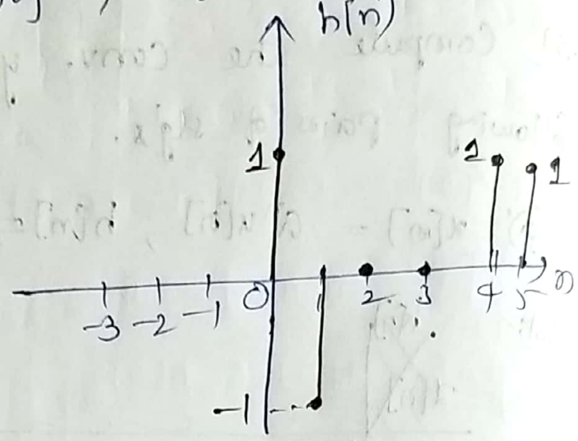
Scanned by CamScanner

b) $x[n] = \begin{cases} 1 & ; n = -2, 0, 1 \\ 2 & ; n = -1 \\ 0 & ; \text{elsewhere} \end{cases}$

$h[n] = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

Sol:- $x[n] = \{1, 2, \underset{\uparrow}{1}, 1, 0, 0\}$; $h[n] = \{\overset{\downarrow}{1}, -1, 0, 0, 1, 1\}$





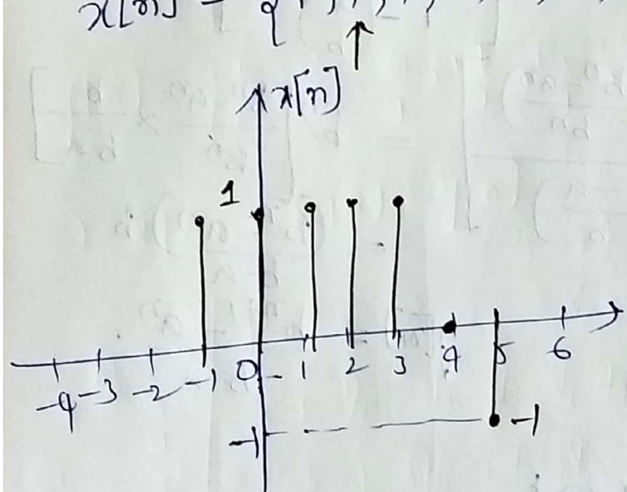
x[n] \ h[n]	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

$y[n] = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

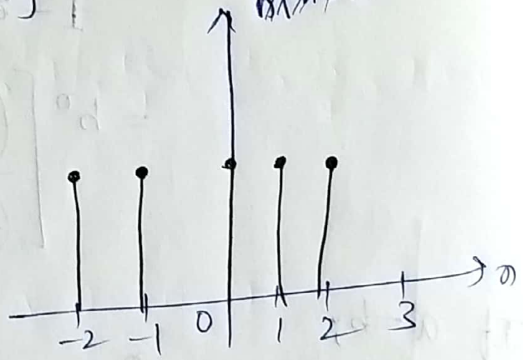
(c) $x[n] = u[n+1] - u(n-4) - \delta[n-5]$.

$h[n] = [u(n+2] - u[n-3]] \cdot (3 - |n|)$

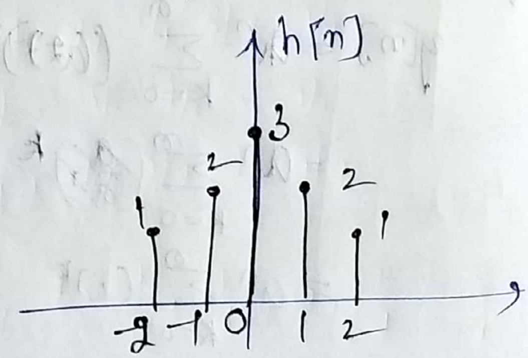
$x[n] = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$



$u(n+2) - u[n-3]$



$h[n] = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$



h[n] \ x[n]	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

$$y[n] = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, -1\}.$$

d) $x[n] = u[n] - u[n-5]$; $h[n] = u[n-2] - u(n-8) + u(n-11) - u(n-17)$

Sol:- $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

x[n] \ h[n]	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1

$$y[n] = \{0, 1, 3, 4, 5, 5, 5, 5, 4, 3, 2, 1\}.$$

2.2.2 Let $x[n]$ be the i/p slg to a discrete time filter with impulse response $h_1[n]$ and let $y_1[n]$ be the corresponding o/p.

(a) Compute and sketch $x[n]$ and $y_1[n]$ in the following cases, using the same scale in all figures.

$x[n] = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$.

$h_1[n] = \{1, 1\}$; $h_2[n] = \{1, 2, 1\}$; $h_3[n] = \{1/2, 1/2\}$

$h_4[n] = \{1/4, \frac{1}{2}, 1/4\}$; $h_5[n] = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

Scanned by CamScanner

Sol: (a) $x[n] * h_3[n] = y[n]$.

$x[n]$ \ $h_3[n]$	$1/2$	$1/2$
1	$1/2$	$1/2$
4	2	2
2	1	1
3	$3/2$	$3/2$
5	$5/2$	$5/2$
3	$3/2$	$3/2$
3	$3/2$	$3/2$
4	2	2
5	$5/2$	$5/2$
7	$7/2$	$7/2$
6	3	3
9	$9/2$	$9/2$

$$y_3[n] = \left\{ \frac{1}{2}, \frac{5}{2}, 3, \frac{5}{2}, 4, 4, 3, \frac{7}{2}, \frac{9}{2}, 6, \frac{13}{2}, \frac{15}{2}, \frac{9}{2} \right\}$$

$\Rightarrow x[n] * h_4[n] = y[n]$

$h_4[n]$ \ $x[n]$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$1/4$	1	$1/2$	$3/4$	$5/3$	$3/4$	$3/4$	1	$5/4$	$7/4$	$3/2$	$9/4$
$\frac{1}{2}$	$1/2$	2	1	$3/2$	$5/2$	$3/2$	$3/2$	2	$5/2$	$7/2$	3	$9/2$
$1/4$	$1/4$	1	$1/2$	$3/4$	$5/3$	$3/4$	$3/4$	1	$5/4$	$7/4$	$3/2$	$9/4$

$$y[n] = \left\{ \frac{1}{4}, \frac{1}{2}, \frac{11}{4}, \frac{17}{4}, \frac{13}{4}, 4, \frac{7}{2}, \frac{13}{4}, \frac{21}{4}, \frac{25}{4}, 7, 6, \frac{9}{4} \right\}$$

$\Rightarrow x[n] * h_5[n] = y_4[n]$

$h_5[n]$ \ $x[n]$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$1/4$	1	$1/2$	$3/4$	$5/3$	$3/4$	$3/4$	1	$5/4$	$7/4$	$3/2$	$9/4$
$-\frac{1}{2}$	$-1/2$	-2	-1	$-3/2$	$-5/2$	$-3/2$	$-3/2$	2	$-5/2$	$-7/2$	-3	$-9/2$
$+1/4$	$1/4$	1	$1/2$	$3/4$	$5/3$	$3/4$	$3/4$	1	$5/4$	$7/4$	$3/2$	$9/4$

$$y_5[n] = \left\{ \frac{1}{4}, \frac{1}{2}, \frac{-5}{4}, \frac{3}{4}, \frac{1}{4}, -1, \frac{-1}{2}, \frac{1}{4}, 0, \frac{1}{4}, \frac{-3}{4}, 1, -3, -9/4 \right\}$$

b) what is the difference b/w $y_1[n]$ & $y_3[n]$ & b/w $y_2[n]$ and $y_4[n]$?

Sol:- $y_3[n] = \frac{1}{2} y_1[n]$; bcoz $h_3[n] = \frac{1}{2} h_1[n]$

$y_4[n] = \frac{1}{4} y_2[n]$; bcoz $h_4[n] = \frac{1}{4} h_2[n]$

c) Comment on the smoothness of $y_2[n]$ & $y_4[n]$. which factors affect the smoothness.

Sol:- $y_2[n]$ & $y_4[n]$ are smoother than $y_1[n]$, But $y_4[n]$ will appear even smoother bcoz of the smaller scale factor.

d) Compare $x_4[n]$ with $y_5[n]$. what is the diff? Can you explain it?

Sol:- $x_4[n] \longrightarrow$ Smoother o/p.

⇒ Negative value in $h_5[n]$ results for the non-smooth char's of $y_5[n]$.

2.23 The Discrete-time sys $y(n) = n y[n-1] + x[n]$; $n \geq 0$ is at rest [i.e, $y(-1)=0$]. Check if the s/m is linear, T.IM & BIBO s/m.

Sol:- $x_1[n] \longrightarrow \boxed{s/m} \longrightarrow n y_1[n-1] + x_1[n]$

$x_2[n] \longrightarrow \boxed{s/m} \longrightarrow n y_2[n-1] + x_2[n]$

$n y[n-1] + [x_1(n) + x_2(n)]$

$x_1[n] + x_2[n] \longrightarrow \boxed{s/m} \longrightarrow n [y_1[n-1] + x_1[n] + y_2[n-1] + x_2[n]]$

∴ The s/m is linear. s/m.

$y[n-n_0] = (n-n_0)\, y[n-n_0+1] + x[n-n_0]$

$y[n] = x[n-n_0] = n y[n-1] + x[n-n_0]$

∴ It is time variant s/m.

$|x[n]| < \infty \Rightarrow$ For Bounded i/p the o/p is unbounded

$y[0] = 1$, $y(1) = 1+1 = 2$; $y(2) = 4+1 = 5$

$\therefore$ The sys is unstable

2.24 Consider the sys slg $y[n] = a^n u[n]$, $0<a<1$

(a) show that any sequence $x[n]$ can be decomp as $x[n] = \sum_{n=-\infty}^{\infty} C_k y[n-k]$

Sol:-

$$\cancel{\delta[n] = x[k]\, a\, T[n-k]}$$

$$\cancel{\delta[n-k] = T[n-k]\, \delta[n-k-1]}$$

$$\delta[n] = \tau[n] - a\tau[n-k]$$

$$\delta[n-k] = \tau[n-k] \overset{a.\tau}{\cancel{a}}[n-k-1]$$

$$\Rightarrow x[n] = \sum_{k=-\infty}^{\infty} x(k)\, \overset{\tau}{\cancel{g}}[n-k]$$

$$= \sum_{k=-\infty}^{\infty} \{x(k)\left[\tau(n-k) - a\tau(n-k-1)\right]$$

$$= \sum_{k=-\infty}^{\infty} x(k)\tau(n-k) - a\sum_{n=-\infty}^{\infty} x(k-1)\,\tau(n-k)$$

$$\cancel{= \sum_{k=-\infty}^{\infty} (x(k) - a x(k-1))\tau(n-k)}$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - a x(k-1)\right] \tau(n-k)$$

$$\therefore \boxed{C_k = x(k) - a x(k-1)}$$

(b) Use the prop's of linearity, T-I & V to ~~expto~~ expr the o/p $y[n] = T[x[n]]$ in terms of the i/p $x[n]$ and the slg $g[n] = T[x[n]]$, where $T[\cdot]$ is an d.T-I sys.

Sol:-

$$y[n] = T[x[n]]$$

$$= T\left[\sum_{k=-\infty}^{\infty} C_k \tau[n-k]\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k T\left[\tau(n-k)\right] = \sum_{n=-\infty}^{\infty} C_k\, g(n-k)$$

c) Express the impulse response $h[n] = T[\delta[n]]$ in terms of $g(n)$.

Sol:- $h[n] = T[\delta[n]]$.

$= T[r[n] - r(n-1)]$

$= g(n) - a\,g(n-1)$

2.25 Determine the zero-i/p response of the s/m described by the 2nd order difference eq.

$$x[n] - 3y[n-1] - 4y[n-2] = 0$$

Sol:- $x[n] = 0 \rightarrow$ for '0'-i/p response

$$-3y[n-1] = 4y[n-2]$$

$$y[n-1] = -\frac{4}{3}y[n-2]$$

$n = 0$ $\quad y[-1] = -\frac{4}{3}y[-2]$

$n = 1$ $\quad y[0] = \left(-\frac{4}{3}\right)^2 y[-1]$

$n = 1$ $\quad y[1] = \left(-\frac{4}{3}\right)^3 y[1]$.

$\vdots$

$$y[k] = \left(-\frac{4}{3}\right)^{k+2} y[\ ]$$

2.26 Determine the particular solution of the difference equation. $y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + x[n]$ when the forcing function is $x[n] = 2^n u[n]$.

Sol:- $y[n] = \frac{5}{6}y[n-1] - \frac{1}{6}y[n-2] + 2^n u[n]$.

$\Rightarrow x[n] = 0$

$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 0$. [Homogenous eq.]

$\Rightarrow \lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0 \Rightarrow \lambda = \frac{1}{2}, \frac{1}{3}$.

$$y_h[n] = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n.$$

Particular soln:- $x[n] = 2^n u[n]$.

$y_p[n] = k \cdot 2^n u[n] \longrightarrow$ sub in difference eq

$$k(2^n)u[n] - k\left(\frac{5}{6}\right)(2^{n-1})u[n-1] + k\left(\frac{1}{6}\right)2^{n-2}u[n-2] = 2^n u[n]$$

For $n=2$

$$4k - \frac{5k}{3} + \frac{k}{4} = 4.$$

$$k\left(4 - \frac{5}{3} + \frac{1}{4}\right) = 4.$$

$$\boxed{k = \frac{8}{5}}$$

Total solution is.

$$y[n] = y_p[n] + y_h[n].$$

$$= \frac{8}{5}[2^n]u[n] + c_1\left(\frac{1}{2}\right)^n u[n] + c_2\left(\frac{1}{3}\right)^n u[n]$$

Let $y(-2) = y(-1) = 0$ (to find c_1 & c_2).

$y(0) = 1$.

$y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$.

$\Rightarrow \frac{8}{5} + c_1 + c_2 = 1 \Rightarrow c_1 + c_2 = \frac{-3}{5}$

$\Rightarrow \frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6} \Rightarrow 3c_1 + 2c_2 = \frac{-11}{5}$

$\Rightarrow c_1 + c_2 = \frac{-3}{5}$ $\qquad c_1 = -1.$

$3c_1 + 2c_2 = \frac{-11}{5}$ $\qquad c_2 = \frac{2}{5}.$

$4c_1 + 3c_2 = \frac{-14}{5}$

Total solution is $y[n] = \left[\frac{8}{5}2^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$.

2.27 Determine the response $y[n]$, $n \geq 0$ of the s/m described by the 2nd order difference eq

$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$ to the i/p $x[n] = 4^n u[n]$.

Sol: $y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$. → ①

Homogenous sol^n → $x[n] = 0$.

$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$.

$4\lambda^2$ $\quad \lambda^2 - 4\lambda + \lambda - 4 = 0$.

$4\lambda \quad 1\lambda$ $\quad \lambda(\lambda-4) + 1(\lambda-4) = 0$

$(\lambda-4)(\lambda+1) = 0$.

$\lambda = +4, -1$.

$\Rightarrow y_h[n] = C_1(\lambda_1)^n + C_2(\lambda_2)^n$.

$y_h[n] = c_1(4)^n + c_2(-1)^n$.

To find particular sol^n, it is ~~same~~ assumed to be an exp. sequence of the same form as $x[n]$.

$y_p[n] = K(4)^n u[n]$

Since $y_p[n]$ is already contained in homogenous sol^n, So this particular sol^n is ~~redun~~ redundant.

⇒ We select particular sol^n to be linearly independent of the terms in homo... sol^n.

$y_p[n] = Kn(4)^n u[n]$ → ②

Sub ② in ①

$\Rightarrow Kn(4)^n u[n] - 3K(n-1)(4)^{n-1}u(n-1) - 4K(n-2)(4)^{n-2}u[n-2]$
$= (4)^n u[n] + 2(4)^{n-1}u[n-1]$.

Let $n=2$.

~~$K(32)u[n]4$~~ $\Rightarrow 32K - 12K - 4(0) = 16 + 8$.

$20K = 24$

$K = \frac{24^6}{20_5} = \frac{6}{5}$.

$\Rightarrow y_p[n] = \frac{6}{5} n (4)^n u[n]$.

$y[n] = y_p[n] + y_h[n]$

$= C_1(4)^n + C_2(-1)^n + \frac{6}{5} n (4)^n$; $n \geq 0$

$n=0$ $\quad y[0] = 3y(-1) + 4y(-2) + 1 \rightarrow$ ①

$y[1] = 3y(0) + 4y(-1) + 6$

$= 13y(-1) + 12y(-2) + 9 \rightarrow$ ②

$\Rightarrow$ $y_p[n]$ If $n=0$;

$y[0] = c_1 + c_2 \rightarrow$ ①

$y[1] = -c_1 + 4c_2 + \frac{24}{5} \rightarrow$ ②

$\Rightarrow$ To get c_1 & c_2 we are equating the two sets of eq's.

$y[-1] = y[-2] = 0$.

$\Rightarrow c_1 + c_2 = 1 \rightarrow$ ①.

$\Rightarrow -c_1 + 4c_2 + \frac{24}{5} = 9 \rightarrow$ ②.

$\Rightarrow c_1 = \frac{-1}{25}$; $c_2 = \frac{26}{25}$.

$\Rightarrow y_p[n] = \frac{-1}{25}(4)^n + \frac{26}{25}(-1)^n + \frac{6}{5}n(4)^n$; $n \geq 0$.

2.28. Determine the impulse response of the following causal s/m.

$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$.

Sol:- $\Rightarrow \lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1$

$y_h[n] = c_1(4)^n + c_2(-1)^n$

when $x[n] = \delta[n]$; $y[0] = 1$

$y[1] - 3y[0] = 2$

$\Rightarrow y[1] - 3 = 2$.

$y[1] = 2+3$

$y[1] = 5$.

$\Rightarrow c_1 + c_2 = 1$ and $4c_1 - c_2 = 5$

$\Rightarrow c_1 = \frac{6}{5}$; $c_2 = \frac{-1}{5}$.

$\therefore h[n] = \left[\frac{6}{5}4^n - \frac{1}{5}(-1)^n\right]u[n]$

2.29 Let $x[n]$, $N_1 \le n \le N_2$ & $h[n]$, $M_1 \le n \le M_2$ be two finite duration slg's.

(b) Determine the limits of the cases of partial overlap from the left, full overlap & partical overlap from the right. For convenience, assume that $h[n]$ has shorter duration than $x[n]$.

(a) Determine the range $L_1 \le n \le L_2$ of their conv. in terms of N_1, N_2, M_1 & M_2.

(c) Illustrate the validity of your results by computing the conv. of the slg's.

$$x[n] = \begin{cases} 0 & \text{otherwise} \\ 1 & -2 \le n \le 4. \end{cases} \quad ; \quad h[n] = \begin{cases} 2; & -1 \le n \le 2 \\ 0; & \text{otherwise.} \end{cases}$$

Sol:- (a) $L_1 = N_1 + M_1$; $L_2 = N_2 + M_2$

(b) partial overlap from left :

low $N_1 + M_1$ high $N_1 + M_2 - 1$

Full overlap :-

Low $N_1 + M_2$ high $N_2 + M_1$

Partial overlap from right :

low $N_2 + M_1 + 1$ high $N_2 + M_2$.

(c) $x[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$; $h[n] = \{2, \underset{\uparrow}{2}, 2, 2\}$

$N_1 = -2$; $N_2 = 4$

$M_1 = -1$; $M_2 = 2$

partial overlap from left : $n = -3$ $n = -1$ $L_1 = -3$.

Full overlap : $n = 0$ $n = 3$.

partial overlap from right : $n = 4$ $n = 6$ $L_1 = 6$.

2.30 Determine the impulse response & unit step response of the slm's described by the difference equation

(a) $y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + x[n]$.

(b) $y[n] = 0.7\,y[n-1] - 0.1\,y[n-2] + 2x[n] - x[n-2]$.

(a) $y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + x[n]$

$\Rightarrow \lambda^2 - 0.6\lambda + 0.08 = 0.$

$\lambda = 0.2, 0.4.$

$y_h[n] = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{2}{5}\right)^n.$

If $x[n] = \delta[n]$

$\Rightarrow y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + \delta[n].$

$\underline{n=0}$

$y[0] = 0.6\,y[-1] - 0.08\,y[-2] + \delta[0].$

$y[0] = 1$ (for causal s/m. $y[n] = 0$; $n \le 0$)

$\underline{n=1}$

$y[1] = 0.6\,y[0] - 0.08\,y[-1] + \delta[1].$

$= 0.6(1)$

$y[1] = 0.6.$

$\Rightarrow c_1 + c_2 = 1$

$\left(\frac{1}{5}\right)c_1 + \frac{2}{5}c_2 = 0.6 \Rightarrow c_1 = -1\ ;\ c_2 = 2.$

$\therefore h[n] = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u[n].$

<u>step response is</u>

$$s[n] = \sum_{k=0}^{n} h[n-k]\ ,\ n \ge 0.$$

$$= \sum_{k=0}^{n}\left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= \left[\frac{1}{0.12}\left[\left(\frac{2}{5}\right)^{n+1} - 1\right) - \frac{1}{0.16}\left(\left(\frac{1}{5}\right)^{n+1} - 1\right)\right] u[n]$$

(b) $\lambda^2 - 0.7\lambda + 0.1 = 0.$

$\lambda = \frac{1}{2}, \frac{1}{5}.$

If $x[n] = \delta[n].$

Scanned by CamScanner

$y[n] = 0.7\,y[n-1] - 0.1\,y[n-2] + 2\delta[n] - 2\delta[n-2]$.

If $n=0$

$y[0] = 2$;

If $n=1$

$y[1] = 0.7\,y[0]$

$= 0.7 \times 2$

$y[1] = 1.4$.

$\Rightarrow c_1 + c_2 = 2 \rightarrow \frac{1}{2}c_1 + \frac{1}{5}c_2 = 1.4$

$c_1 + \frac{2}{5}c_2 = \frac{14}{5}$

$c_1 = \frac{10}{3}$; $c_2 = -\frac{4}{3}$

$h[n] = \left[\cancel{\left(\frac{10}{3}\right)^n +} \frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]$.

Step response

$s[n] = \sum_{k=0}^{n} h[n-k]$

$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$.

$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n}\left(\frac{1}{2}\right)^{-k} - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n}\left(\frac{1}{5}\right)^{-k}$.

$\boxed{\frac{a(r^{n+1}-1)}{r-1} ; r>1}$

$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n}(2)^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n}(5)^k$

$= \frac{10}{3}\left\{\left(\frac{1}{2}\right)^n (2^{n+1}-1)u(n) - \frac{1}{3}\left(\left(\frac{1}{5}\right)^n \cdot (5^{n+1}-1)\right)u(n)\right)$

2.31 Consider a s/m with impulse response.

$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n; & 0 \le n \le 4. \\ 0, & \text{elsewhere} \end{cases}$$

Determine the input $x[n]$ for $0 \le n \le 8$ that will generate the o/p sequence.

$y[n] = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$.

Sol:- $h[n] = \{\underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$.

$y[n] = \{\underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$.

$$\sum_{k=0}^{n} y[n] = \sum_{k=0}^{n} x[n] \cdot \sum_{k=0}^{n} h[n].$$

$$y[0] = x[0] \cdot h[0] \Rightarrow 1 = x[0] \cdot 1 \Rightarrow x[0] = 1$$

$$\Rightarrow y[1] = x[0]h(1) + x(1)h(0)$$

$$2 = \frac{1}{2}x(0) + x(1) \Rightarrow x(1) = 2 - 1/2 = 3/2$$

$$\Rightarrow y(2) = x(0)h(2) + x(2)h(0) + x(1)h(1)$$

$$2.5 = 1 \times 1/4 + x(2) \times (1) + \frac{3}{2}\left(\frac{1}{2}\right)$$

$$2.5 = \frac{1}{4} + x(2) + \frac{3}{4} \Rightarrow x(2) = 2.5 - 1 = 3/2$$

$$x[n] = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \ldots \right\}.$$

2.39 Compute and sketch the step response of the s/m.

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x[n-k]$$

Sol:- $h[n] = (u[n] - u[n-M]) / M.$

$$s(n) = \sum_{k=-\infty}^{\infty} u[k]\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M}, & n < M \\ 1, & n \geq M \end{cases}$$

Q.35 Determine the range of values of the parameter 'a' for which the linear time-invariant s/m with impulse response $h[n] = \begin{cases} a^n, & n\geq 0, n \text{ even} \\ 0, & \text{otherwise} \end{cases}$ is stable.

Sol:-
$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0, n \text{ even}}^{\infty} |a|^n$$
$$= \sum_{n=0}^{\infty} |a|^{2n}$$
$$= \frac{1}{1-|a|^2}$$

Stable if $|a|<1$.

Q.36 Determine the response of the s/m with impulse response $h[n] = a^n u[n]$ to the i/p slg. $x[n] = u[n] - u[n-10]$.

Sol:- $h[n] = a^n u[n]$,
$$y_1[n] = \sum_{k=0}^{\infty} u[k]\, h[n-k] = \sum_{k=0}^{n} a^{n-k} = a^n \sum_{k=0}^{n} a^{-k}$$
$$= \frac{1-a^{n+1}}{1-a} u[k].$$
$$y[n] = y_1[n] - y_1[n-10]$$
$$= \frac{1}{1-a}\left[(1-a^{n+1})u[n] - (1-a^{n-9})u[n-10]\right].$$

Q.37 Determine the response of the s/m characterized by the impulse response to the i/p slg. $h[n] = (\frac{1}{2})^n u[n]$.
$$x[n] = \begin{cases} 1; & 0\leq n \leq 10 \\ 0; & \text{otherwise} \end{cases}$$

Sol:- $h[n] = (\frac{1}{2})^n u[n]$, $y[n] = \sum_{k=0}^{\infty} u[k]\, h[n-k]$.
$$\Rightarrow y[n] = 2\left[1-\left(\frac{1}{2}\right)^{n+1}\right]u[n] - 2\left[1-\left(\frac{1}{2}\right)^{n-9}\right]u[n-10]$$

Q.38 Determine the response of the s/m char by the impulse response $h[n] = (\frac{1}{2})^n u[n]$ to the i/p slg's.

(a) $x[n] = 2^n u[n]$,

(b) $x[n] = u[-n]$

Sol:- $h[n] = \left(\frac{1}{2}\right)^n u[n]$; $x[n] = 2^n u[n]$.

(a) $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$.

$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$

$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$.

$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right) = \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u[n]$

(b) $y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$

$= \sum_{k=0}^{\infty} h[k]$

$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \; n<0$.

$\Rightarrow y[n] = \sum_{k=n}^{\infty} h[k] = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$.

$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$

$= 2 - \frac{\left(1 - \left(\frac{1}{2}\right)^n\right)}{1/2}$

$= 2\left(\frac{1}{2}\right)^n ; \; n \geq 0$.

Q.39:- Three s/m's with impulse responses $h_1[n] = \delta[n] - \delta[n-1]$, $h_2[n] = h[n]$ and $h_3[n] = u[n]$, are connected in cascade.

(a) what is the impulse response, $h_e[n]$ of the overall s/m ?.

(b) Does the order of the interconnection after the overall s/m ?

Sol:- (a) $h_e[n] = h_1[n] * h_2[n] * h_3[n]$

$= [\delta[n] - \delta[n-1]] * u[n] * h[n]$.

$= [u[n] - u[n-1]] * h[n] = \delta[n] * h[n] = h[n]$.

Scanned by CamScanner

(k) No.

Q.40 (a) Prove and explain graphically the difference b/w the relation $x[n]\delta[n-n_0] = x[n_0]\delta[n-n_0]$ and $x[n] * \delta[n-n_0] = x[n-n_0]$.

(b) Show that a discrete-time s/m. which is described by a conv. summation is LTI and relaxed

(c) What is the impulse response of the s/m described by $y[n] = x[n-n_0]$?

Sol:- (a) $x[n]\cdot\delta[n-n_0] = x[n_0]$. Only the value of $x[n]$ at $n = n_0$.

$x[n] * \delta[n-n_0] = x[n-n_0]$, we obtain the shifted the sequence $x[n]$.

(b) $$y[n] = \sum_{K=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= h[n] * x[n].$$

Linearity:- $x_1[n] \rightarrow y_1[n] = h[n] * x_1[n]$.

$x_2[n] \rightarrow y_2[n] = h[n] * x_2[n]$

$x[n] = \alpha x_1[n] + \beta x_2[n] \longrightarrow y[n] = h[n] * x[n]$.

$y[n] = h[n] * [\alpha x_1[n] + \beta x_2[n]]$.

$= \alpha h[n] * x_1[n] + \beta h[n] * x_2[n]$.

$= \alpha y_1[n] + \beta y_2[n]$.

Time invariance :-

$x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$.

$$= \sum_k h(k)\, x(n-n_0-k)$$

$= y(n-n_0)$.

(c) $h[n] = \delta[n-n_0]$.

Q.42 . Compute the 0-state response of the s/m described by the difference eq.

$$y[n] + \frac{1}{2} y[n-1] = x[n] + 2x[n-2] \text{ to the i/p.}$$

$x[n] = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$ by solving the difference eq recursively

Sol: $y[n] = -\frac{1}{2}y[n-1] + x[n] + 2x[n-2]$.

$n=-2$
$y[-2] = -\frac{1}{2}y[-3] + x(-2) + 2x(-4) = 1$.

$n=-1$ $y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2}$.

$n=0$
$y(0) = -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4}$.

$n=1$
$y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1)$
$= -\frac{1}{2}\left(\frac{17}{4}\right) + 4 + 2(2) = \frac{47}{8}$.

2.95 Consider the s/m described by the diff. eq.
$y[n] = a\,y[n-1] + b\,x[n]$.

(a) Determine 'b' in terms of 'a' so that $\sum_{n=-\infty}^{\infty} h[n] = 1$.

(b) Compute the 0-state step response $s[n]$ of the s/m and choose b so that $s(\infty) = 1$.

(c) Compare the values of "b" obtained in parts (a) & (b). What did you notice?

Sol: (a) $y[n] = a\,y[n-1] + b\,x[n]$.

$\Rightarrow h[n] = b\,a^n u[n]$.

$$\sum_{n=0}^{\infty} h[n] = \frac{b}{1-a} = 1$$

$\Rightarrow b = 1-a$.

(b) $s(n) = \sum_{k=0}^{n} h[n-k]$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right]u[n]$$

$$s(\infty) = \frac{b}{1-a} = 1$$

$\Rightarrow b = 1-a$.

(c) $b = 1-a$ in both cases.

Scanned by CamScanner

Q5) Compute the sketch the conv. $y_1[n]$ & correlation $r[n]$. sequences for the following pair of signals and comment on the results obtained.

a) $x_1[n] = \{\underset{\uparrow}{1}, 2, 4\}$; $h_1[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

b) $x_2[n] = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$; $h_2[n] = \{1/2, 1, \underset{\uparrow}{2}, 1, 1/2\}$.

c) $x_3[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$; $h_3[n] = \{\underset{\uparrow}{4}, 3, 2, 1\}$.

d) $x_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$. ; $h_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

Sol: (a) $y_1[n] = x_1[n] * h_1[n]$. → conv.

$h[n]$ \ $x_1[n]$	1	2	4
→ 1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$y_1[n] = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$.

$y_1[n] = x_1[n] * h_1[-n]$. → correlation

$x_1[n]$ \ $h_1[-n]$	1	1	1	1	1
→ 1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$y_1[n] = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$.

(b) $y_2[n] = x_2[n] * h_2[n]$

$x_2[n]$ \ $h_2[n]$	1/2	1	2	1	1/2
→ 0	0	0	0	0	0
1	1/2	1	2	1	1/2
-2	-1	-2	-4	-2	-1
-3	-3/2	-3	-6	-3	-3/2
-4	-2	-4	-8	-4	-2

$y_2[n] = \{1/2, \underset{\uparrow}{0}, 3/2, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$.

, $y_2[n] = x_2[n] * h_2[-n]$.

$x_2[n]$ \ $h_2[n]$	1/2	1	2	1	1/2
→ 0	0	0	0	0	0
1	1/2	1	2	1	1/2
-2	-1	-2	-4	-2	-1
-3	-3/2	-3	-6	-3	-3/2
-4	-2	-4	-8	-4	-2

$y_2[n] = \{1/2, \underset{\uparrow}{0}, 5/2, -2, 1/2, -6, -5/2, -2\}$

(e) $y_3[n] = \{1, 2, 3, 4\}$ $\qquad$ $y_3[n] = x_3[n] * h_3[-n]$.

$x_3[n] * h_3[n]$.

$x_3[n]$ \ $h_3[n]$	$\underset{\downarrow}{4}$	3	2	1
$\rightarrow 1$	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$$y_3[n] = \{\underset{\uparrow}{4}, 11, 20, 30, 11, 4\}$$

$x_3[n]$ \ $h_3[-n]$	$\underset{\downarrow}{4}$	3	2	1
$\rightarrow 1$	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

$$y_3[n] = \{\cancel{4}, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}.$$

Q52 The zero-state response of a causal L.T.I s/m to the i/p $x[n] = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y[n] = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$. Determine its impulse response.

Sol:- length of $h[n] = 2$.

$$h[n] = \{h_0, h_1\}.$$

$$h_0 = 1.$$

$$3h_0 + h_1 = 4.$$

$$\Rightarrow 3 + h_1 = 4.$$

$$\boxed{h_1 = 1} \; ; \; \boxed{h_0 = 1}$$

(or)

$\Rightarrow y[0] = x[0] \cdot h[0]$ $\qquad$ $\Rightarrow y[1] = x[0]\,h[1] + x(1)\,h[0]$.

$1 = 1 \cdot h[0]$. $\qquad$ $4 = h[1] + 3\,h[0]$.

$\boxed{h[0] = 1}$. $\qquad$ $4 = h[1] + 3$

$$\boxed{h[1] = 1}$$

Q54 Determine the response $y[n]$, $n \geq 0$ of the s/m described by the 2nd order difference eqn

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$, when the i/p is $x[n] = (-1)^n u[n]$, and the initial conditions are $y(-1) = y(-2) = 0$.

Sol: Characteristic eq. is:

$$\lambda^2 - 4\lambda + 4 = 0.$$

$$\lambda = 2, 2.$$

$$y_h[n] = c_1 2^n + c_2 n 2^n$$

particular sol^n. is. $y_p[n] = k(-1)^n u[n]$.

$$\Rightarrow k(-1)^n u[n] - 4k(-1)^{n-1} u[n-1] + 4k(-1)^{n-2} u[n-2] = (-1)^n u(n) - (-1)^{n-1} u(n-1).$$

$n=2$

$$k(1+4+4) = 2 \Rightarrow k = 2/9.$$

total sol^n is $y[n] = \left[c_1 2^n + c_2 n \cdot 2^n + \frac{2}{9}(-1)^n\right] u[n]$.

$n=0$

$y(0) = 1$.

$n=1$

$y(1) = 2$

$$\Rightarrow c_1 + \frac{2}{9} = 1.$$

$$c_1 = 7/9.$$

$$\Rightarrow 2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\Rightarrow c_2 = 1/3.$$

$$\Rightarrow y(n) = y_p[n] + y_h[n]$$

$$y[n] = \frac{2}{9}(-1)^n u[n] + \frac{7}{9}(2)^n + 1/3\, n (2)^n.$$

2.55 Determine the imp. response $h[n]$ for the sys described by the 2nd order difference eq.

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$$

Sol:

$$y[n] = 4y[n-1] - 4y[n-2] - x[n] + x[n-1].$$

$$h[n] = [c_1 2^n + c_2 n 2^n] u[n].$$

$y[0] = 1$; $y[1] = 3$.

$c_1 = 1$; $2c_1 + 2c_2 = 3$.

$$\Rightarrow c_2 = 1/2.$$

$$h[n] = \left[2^n + \frac{1}{2} 2^n \times n\right] u[n].$$

Scanned by CamScanner

Q.56 Show that any discrete time sig $x[n]$ can be expressed as. $x[n] = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, u[n-k]$ where $u[n-k]$ is a unit step delayed by k units in time, that is $u[n-k] = \begin{cases} 1 ; & n \geq k \\ 0 ; & \text{otherwise.} \end{cases}$

Sol:- $x[n] = x[n] * \delta[n]$.

$= x[n] * [u[n] - u[n-1]]$.

$= x[n] * u[n] - x[n] * u[n-1]$

$= [x(n) - x(n-1)] * u[n]$.

$= \sum_{k=-\infty}^{\infty} (x(k) - x(k-1))\, u(n-k)$

Q.57 Show that ~~any~~ the o/p of an LTI s/m can be expressed in terms of its unit step response s[n] as follows.

$y[n] = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x[n-k]$.

$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k)$.

Sol:- Let $h[n]$ be the impulse response of the s/m.

$s(k) = \sum_{m=-\infty}^{k} h[m]$

$\Rightarrow h[k] = s(k) - s(k-1)$

$y[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$.

$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$.

Q.58 Compute the correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following sig seq.

$x[n] = \begin{cases} 1 ; & n_0 - N \leq n \leq n_0 + N. \\ 0 ; & \text{otherwise.} \end{cases}$

Scanned by CamScanner

$$y[n] = \begin{cases} 1; & -N \le n \le N. \\ 0; & \text{otherwise} \end{cases}$$

Sol: $r_{xx}(l)$ (range of non-zero values).

$$n_0 - N \le n \le n_0 + N.$$

$$n_0 - N \le n - l \le n_0 + N.$$

$$-2N \le l \le 2N.$$

⇒ For a given shift l, the no. of terms in the summation for which both $x[n]$ & $x[n-l]$ are non-zero is $2N+1-|l|$

$$\Rightarrow r_{xx}(l) = \begin{cases} 2N+1-|l|; & -2N \le l \le 2N. \\ 0; & \text{otherwise} \end{cases}$$

$$\Rightarrow r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|; & n_0-2N \le l \le n_0+2N. \\ 0; & \text{otherwise} \end{cases}$$

2.59 Determine the autocorrelation sequences of the following sig's. (a) $x[n] = \{\underset{\uparrow}{1}, 2, 1, 1\}$. (b) $y[n] = \{1, 1, 2, \underset{\uparrow}{1}\}$.

What is your conclusion?

Sol: (a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

$l=-3$: $r_{xx}(-3) = x(0)\,x(3) = 1$

$l=-2$: $r_{xx}(-2) = x(0)x(2) + x(1)\cdot x(3) = 3$

$l=-1$: $r_{xx}(-1) = x(0)x(1) + x(1)x(2) + x(2)x(3) = 5$.

$$\Rightarrow r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

$$\boxed{r_{xx}(-l) = r_{xx}(l)}$$

$$\therefore r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}.$$

(b) $r_{yy}(l) = \sum_{n=-\infty}^{\infty} y[n]\cdot y[n-l]$.

$$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}.$$

We observe that $y[n] = x[-n+3]$ is equivalent to reversing the seq. $x[n]$. Not changed the autocorrelation sequence.

Scanned by CamScanner

260 What is the normalized auto correlation seq of the sig $x[n]$ given by $x[n] = \begin{cases} 1 & ; -N \leq n \leq N, \\ 0 & ; \text{otherwise} \end{cases}$

Sol:- $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n]\, x[n-l]$

$$= \begin{cases} 2N+1-|l| & ; -2N \leq l \leq 2N. \\ 0 & ; \text{otherwise}. \end{cases}$$

$r_{xx}(0) = 2N+1$

The normalized auto correlation is

$$\rho_{xx}(l) = \frac{1}{2N+1}\left(2N+1-|l|\right); -2N \leq l \leq 2N$$

$$= 0 \;;\; \text{otherwise}$$

Scanned by CamScanner