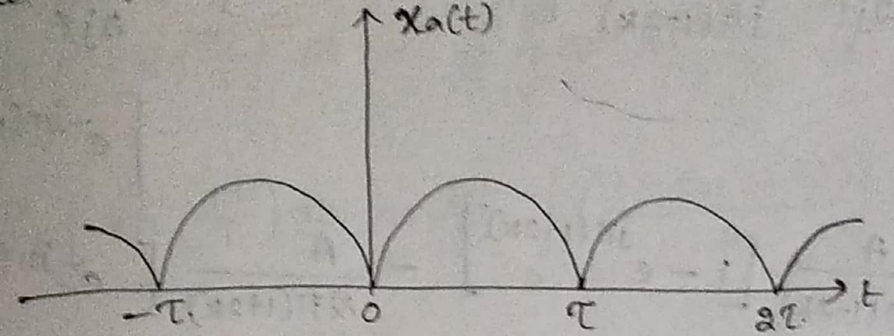
Problems :-

1.) Consider the full wave rectified sinusoid in fig.

(a) Determine its spectrum $X_a(F)$.

(b) Compute the power of the sig.



(c) Plot the power spectral density.

(d) Check the validity of parseval's relation for this sig.

Sol: (a) $x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k f t}$

Time period $T = \tau$.

$\Rightarrow f = \frac{1}{\tau}$.

$\Rightarrow x(t) = \sum_{k=-\infty}^{\infty} c_k e^{\frac{j2\pi k t}{\tau}}$ $\Rightarrow x_a(t) = A \sin\left(\frac{\pi t}{\tau}\right)$

$$c_k = \frac{1}{\tau}\int_0^{\tau} x_a(t)\, e^{-\frac{j2\pi k t}{\tau}}\, dt$$

$$= \frac{1}{\tau}\int_0^{\tau} A\sin\left(\frac{\pi t}{\tau}\right) \cdot e^{-\frac{j2\pi k t}{\tau}}\, dt$$

$$= \frac{A}{\tau}\int_0^{\tau}\left(\frac{e^{\frac{j\pi t}{\tau}} - e^{-\frac{j\pi t}{\tau}}}{2j}\right) \cdot e^{-\frac{j2\pi k t}{\tau}}\, dt$$

$$= \frac{A}{\tau 2j}\int_0^{\tau}\left(e^{\frac{j\pi t}{\tau}} \cdot e^{-\frac{j2\pi k t}{\tau}} - e^{-\frac{j\pi t}{\tau}} \cdot e^{-\frac{j2\pi k t}{\tau}}\right) dt$$

$$= \frac{A}{2j\tau}\left[\int_0^{\tau} e^{j\pi(1-2k)\frac{t}{\tau}}\, dt - \int_0^{\tau} e^{-j\pi(1+2k)\frac{t}{\tau}}\, dt\right]$$

$$\Rightarrow \frac{A}{2j\tau}\left\{\left[\frac{e^{j\pi(1-2k)\frac{t}{\tau}}}{\frac{j\pi(1-2k)}{\tau}}\right]_0^\tau - \left[\frac{e^{-j\pi(1+2k)\frac{t}{\tau}}}{\frac{-j\pi(1+2k)}{\tau}}\right]_0^\tau\right\}$$

$$\Rightarrow \frac{A}{2j\cancel{\tau}}\times\frac{\cancel{\tau}}{j\pi(1-2k)}\left[e^{j\pi(1-2k)}-1\right] + \frac{A}{2j\cancel{\tau}}\times\frac{\cancel{\tau}}{j\pi(1+2k)}\left[e^{-j\pi(1+2k)}-1\right]$$

$$\Rightarrow \frac{A}{2\pi(1-2k)}\left[1-e^{j\pi(1-2k)}\right] - \frac{A}{2\pi(1+2k)}\left[e^{-j\pi(1+2k)}-1\right]$$

$$\Rightarrow \frac{A}{2\pi(1-2k)}\left[1-e^{j\pi}\cdot e^{-j2\pi k}\right] - \frac{A}{2\pi(1+2k)}\left[e^{-j\pi}\cdot e^{-j2\pi k}-1\right]$$

$$e^{j\pi} = \cos\pi + j\sin\pi \; ; \quad e^{-j2\pi k} = \cos(2\pi k) + j.\sin(2\pi k)$$

$$= -1 \qquad\qquad = 1$$

$$\Rightarrow \frac{A}{2\pi(1-2k)}\left[1-(-1)\right] - \frac{A}{2\pi(1+2k)}\left[(-1)-1\right]$$

$$= \frac{2A}{2\pi(1-2k)} + \frac{2A}{2\pi(1+2k)}$$

$$C_k = \frac{2A}{\pi(1-4k^2)}$$

$$\Rightarrow X_a(F) = \int_{-\infty}^{\infty} x_a(t)\cdot e^{-j2\pi f t}\,dt.$$

$$= \int_{-\infty}^{\infty}\sum_{K=-\infty}^{\infty} C_k e^{\frac{j2\pi K t}{\tau}}\cdot e^{-j2\pi f t}\,dt$$

$$= \sum_{K=-\infty}^{\infty}\int_{-\infty}^{\infty} C_k\, e^{-j2\pi\left(f-\frac{K}{\tau}\right)t}\,dt$$

$$= \sum_{K=-\infty}^{\infty} C_k\int_{-\infty}^{\infty} e^{-j2\pi\left(f-\frac{K}{\tau}\right)t}\,dt$$

$$\Rightarrow X_a(f) = \sum_{k=-\infty}^{\infty} C_k \,\delta\left(f - \frac{k}{\tau}\right)$$

(b) $$P = \frac{1}{\tau}\int_0^{\tau} |x_a(t)|^2\, dt$$

$$= \frac{1}{\tau}\int_0^{\tau}\left(A \sin\frac{\pi t}{\tau}\right)^2 dt = \frac{1}{\tau}\int_0^{\tau} A^2 \sin^2\left(\frac{\pi t}{\tau}\right) \cdot dt$$

$$= \frac{A^2}{\tau}\int_0^{\tau} \sin^2\left(\frac{\pi t}{\tau}\right) dt = \frac{A^2}{\tau}\int_0^{\tau}\left(\frac{1 - \cos 2\left(\frac{\pi t}{\tau}\right)}{2}\right) dt$$

$$= \frac{A^2}{\tau}\int_0^{\tau}\left(\frac{1}{2} - \frac{\cos 2\left(\frac{\pi t}{\tau}\right)}{2}\right) dt$$

$$= \frac{A^2}{\tau}\int_0^{\tau} \frac{1}{2} \cdot dt - \frac{A^2}{2\tau}\int_0^{\tau} \cos 2\left(\frac{\pi t}{\tau}\right) \cdot dt$$

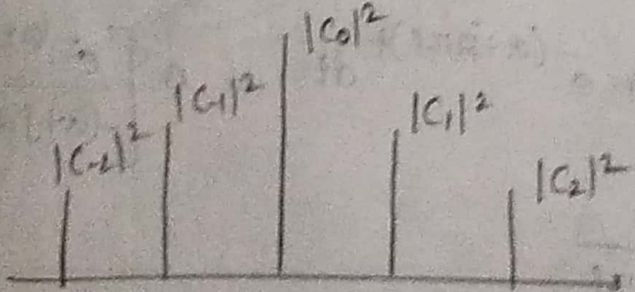
$$= \frac{A^2}{\cancel{\tau}}\left(\frac{\cancel{\tau}}{2}\right) - \frac{A^2}{2\tau}\left(\frac{\sin 2\left(\frac{\pi t}{\tau}\right)}{\frac{2\pi}{\tau}}\right)_0^{\tau}$$

$$= \frac{A^2}{2} - \frac{A^2 \cdot \cancel{\tau}}{2\cancel{\tau} \times 2\pi}\left[\sin 2(\pi) - \sin(0)\right]$$

$$= \frac{A^2}{2} - \frac{A^2}{2\pi}(0)$$

$$\therefore P = \frac{A^2}{2}$$

(c)



(d) $$P = \sum_{k=-\infty}^{\infty} |C_k|^2$$

$$= \sum_{k=-\infty}^{\infty}\left(\frac{2A}{\pi(1-4k^2)}\right)^2$$

$$= \frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(1-4k^2)}$$

Scanned with CamScanner

$$\Rightarrow \frac{4A^2}{\pi^2}\left[\frac{1}{(4k^2-1)^2}\Big|_{K=0} + 2\sum_{K=1}^{\infty}\frac{1}{(4k^2-1)^2}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1 + 2\left(\frac{1}{3^2}+\frac{1}{15^2}+\cdots\right)\right]$$

$$= \frac{4A^2}{\pi^2}(1.231)$$

$$= 0.498A^2$$

$$\simeq 0.5A^2$$

$$\therefore P = \frac{A^2}{2}.$$

4.2 Compute and sketch the magnitude and phase spectra for the following slg's $(a>0)$.

(a) $x_a(t) = \begin{cases} Ae^{-at} & ; t \geq 0 \\ 0 & ; t < 0. \end{cases}$

(b) $x_a(t) = Ae^{-a|t|}$

Sol:- (a) $X_a(F) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi ft}\, dt$

$$= \int_{0}^{\infty} Ae^{-at} e^{-j2\pi ft}\, dt$$

$$= \int_{0}^{\infty} A \cdot e^{-(a+j2\pi f)t}\, dt. = A\left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)}\right]_0^{\infty}$$

$$= \frac{A}{a+j2\pi f}$$

$\Rightarrow$ Magnitude. $|X_a(F)| = \dfrac{A}{\sqrt{a^2+(2\pi f)^2}} = \dfrac{A}{\sqrt{a^2+4\pi^2 f^2}}$

$\Rightarrow$ phase $\angle X_a(F) = -\tan^{-1}\left(\dfrac{2\pi f}{a}\right).$

(b) $X_a(F) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j2\pi f t} \, dt$

$$= \int_{-\infty}^{0} A \cdot e^{+at} \cdot e^{-j2\pi f t} \, dt + \int_{0}^{\infty} A \, e^{-at} \cdot e^{-j2\pi f t} \, dt$$

$$= \int_{0}^{\infty} A \cdot e^{-at} \cdot e^{j2\pi f t} \, dt + \int_{0}^{\infty} A \cdot e^{-at} \cdot e^{-j2\pi f t} \, dt$$

$$= A \int_{0}^{\infty} e^{-(a-j2\pi f)t} \, dt + A \int_{0}^{\infty} e^{-(a+j2\pi f)t} \, dt$$

$$= A \cdot \left[\frac{e^{-(a-j2\pi f)t}}{-(a-j2\pi f)} \right]_{0}^{\infty} + A \cdot \left[\frac{e^{-(a+j2\pi f)}}{-(a+j2\pi f)} \right]_{0}^{\infty}$$

$$= \frac{A}{(a-j2\pi f)} + \frac{A}{(a+j2\pi f)}$$

$$= \frac{Aa + Aj2\pi f + Aa - Aj2\pi f}{a^2 - (j2\pi f)^2}$$

$$= \frac{2Aa}{a^2 + (2\pi f)^2}$$

$\Rightarrow$ Magnitude $|X_a(F)| = \dfrac{2Aa}{\sqrt{a^2 + 4\pi^2 f^2}}$

$\Rightarrow$ phase $\angle X_a(F) = -\tan^{-1}(0)$

$= 0$

Scanned with CamScanner

4.3 Consider the slg $x(t) = \begin{cases} 1-|t|/\tau ; & |t| \le \tau \\ 0 ; & \text{otherwise} \end{cases}$

(a) Determine and sketch its magnitude & phase spectra, $|X_a(F)|$ and $\angle X_a(F)$.

(b) Create a periodic slg $x_p(t)$ with fundamental period. $T_p \ge 2\tau$. so that $x(t) = x_p(t)$ for $|t| < T_p/2$. What are the fourier coeff's C_k for the slg $x_p(t)$?

(c) Using the results in parts (a) & (b). show that

$$C_k = \frac{1}{T_p} X_a(k/T_p)$$

Sol:- (a) $X_a(F) = \int_{-\tau}^{0} \left(1+\frac{t}{\tau}\right) e^{-j2\pi f t} dt + \int_{0}^{\tau} \left(1-\frac{t}{\tau}\right) e^{-j2\pi f t} dt$

Let $y(t) = x'(t) = \frac{d}{dt} x(t) = \begin{cases} \left(1+\frac{t}{\tau}\right) ; & -\tau < t < 0. \\ \left(1-\frac{t}{\tau}\right) ; & 0 < t < \tau. \end{cases}$

$$= \begin{cases} \frac{1}{\tau} ; & -\tau < t < 0. \\ -\frac{1}{\tau} ; & 0 < t < \tau. \end{cases}$$

$$\Rightarrow Y(F) = \int_{-\tau}^{0} \frac{1}{\tau} e^{-j2\pi f t} dt - \int_{0}^{\tau} \frac{1}{\tau} e^{-j2\pi f t} dt$$

$$= \frac{1}{\tau}\left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{-\tau}^{0} - \frac{1}{\tau}\left[\frac{e^{-j2\pi f t}}{-j2\pi f}\right]_{0}^{\tau}$$

$$= \frac{1}{\tau}\left[\frac{1}{-j2\pi f} + \frac{e^{j2\pi f \tau}}{j2\pi f}\right] - \frac{1}{\tau}\left[\frac{e^{-j2\pi f \tau}}{-j2\pi f} + \frac{1}{j2\pi f}\right]$$

$$= \frac{1}{j2\pi f \tau}\left[e^{j2\pi f \tau} - 1\right] + \frac{1}{j2\pi f \tau}\left[e^{-j2\pi f \tau} - 1\right]$$

$$= \frac{1}{j2\pi f \tau}\left[e^{j2\pi f \tau} + e^{-j2\pi f \tau} - 2\right]$$

$$\Rightarrow Y(F) = \frac{1}{j\pi f \tau}\cos(2\pi f \tau) - \frac{\cancel{2}}{\cancel{2}j\pi f \tau}$$

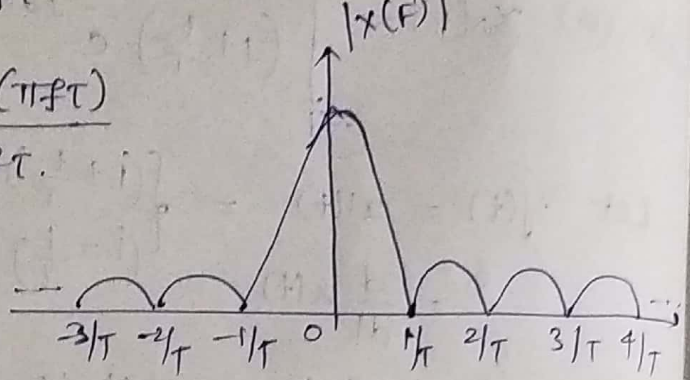
$$= \frac{\cos(2\pi f \tau)}{j\pi f \tau} - \frac{1}{j\pi f \tau}.$$

$(\cos 2\theta = 1 - 2\sin^2\theta)$.

$$= \frac{1 - 2\sin^2(\pi f \tau)}{j\pi f \tau} - \frac{1}{j\pi f \tau}$$

$$= \frac{1 - 2\sin^2(\pi f \tau) - 1}{j\pi f \tau}.$$

$$Y(F) = \frac{-2 \cdot \sin^2(\pi f \tau)}{j\pi f \tau}.$$



$$\Rightarrow y(t) = \frac{d}{dt}x(t).$$

$$Y(F) = j\omega \cdot X(F).$$

$$Y(F) = j2\pi f\, X(F).$$

$$X(F) = \frac{1}{j2\pi f} Y(F) = \frac{1}{j\cancel{2}\pi f}\left(\frac{-\cancel{2}\cdot\sin^2(\pi f \tau)}{j\pi f \tau}\right);$$

$$= \frac{\sin^2(\pi f \tau)}{\pi^2 f^2 \tau \cdot \frac{\tau}{\tau}}$$

$$= \frac{\sin^2(\pi f \tau)\cdot \tau}{\pi^2 f^2 \cdot \tau^2}$$

$$\therefore X(F) = \tau \cdot \mathrm{sinc}^2(f\tau)$$

$\Rightarrow |Xa(F)| = Xa(F).$

$\Rightarrow \underline{|Xa(F)} = 0$

Scanned with CamScanner

(b) $C_K = \frac{1}{T_P} \int_{-T_P/2} x(t)\, e^{-\frac{j2\pi kt}{T_P}}\, dt.$

$$= \frac{1}{T_P} \int_{-\tau}^{0} \left(1+\frac{t}{\tau}\right) e^{-\frac{j2\pi kt}{T_P}}\, dt + \frac{1}{T_P} \int_{0}^{\tau} \left(1-\frac{t}{\tau}\right) \cdot e^{-\frac{j2\pi kt}{T_P}}\, dt.$$

Let $y(t) = x'(t) = \frac{1}{\tau}; \; -\tau < t < 0.$

$\qquad\qquad\qquad\quad \frac{-1}{\tau}; \; 0 < t < \tau.$

$$= \frac{1}{T_P} \int_{-\tau}^{0} \frac{1}{\tau} \cdot e^{-\frac{j2\pi kt}{T_P}} \cdot dt + \frac{1}{T_P} \int_{0}^{\tau} e^{-\frac{j2\pi kt}{T_P}} \cdot dt.$$

$$\Rightarrow C_K = \frac{\tau}{T_P} \left[\frac{\sin \frac{\pi k \tau}{T_P}}{\left(\frac{\pi k \tau}{T_P}\right)} \right]^2$$

(c) $C_K = \frac{1}{T_P} \cdot X_a\left(\frac{K}{T_P}\right)$

$$= \frac{1}{T_P} \cdot \tau \left(\frac{\sin \pi \frac{K}{T_P} \cdot \tau}{\pi \frac{K}{T_P} \cdot \tau} \right)^2$$

$$= \frac{\tau}{T_P} \left(\frac{\sin \pi K \tau / T_P}{\pi K \tau / T_P} \right)^2$$

$C_K = \frac{1}{T_P} \cdot X_a\left(\frac{K}{\tau}\right)$

4.4 Consider the following periodic slg.

$$x[n] = \{ \ldots -1, 0, 1, 2, 3, 2, 1, 0, 1, \ldots \}.$$

(a) Sketch the slg $x[n]$ & its mag & phase spectra.

(b) Using the results in part (a). Verify Parseval's theorem relation by Computing Power in time & freq. domain.

Sol:- (a) $x[n] = \{ \ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots \}$

$\Rightarrow x[n] = \{ \ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, \ldots \}$

$\therefore N = 6.$

$$\Rightarrow C_K = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \, e^{-\frac{j2\pi k n}{N}}$$

$$= \frac{1}{6} \sum_{n=0}^{5} x[n] \, e^{-\frac{j2\pi k n}{6}}$$

$$= \frac{1}{6} \left[3 \cdot e^{-\frac{j2\pi k(0)}{6}} + 2 \cdot e^{-\frac{j2\pi k(1)}{6}} + e^{-\frac{j2\pi k}{3}} + 0 + e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j10\pi k}{6}} \right]$$

$$= \frac{1}{6} \left[3 + 2e^{-\frac{j\pi k}{3}} + e^{-\frac{j2\pi k}{3}} + e^{-\frac{j4\pi k}{3}} + 2e^{-\frac{j5\pi k}{3}} \right]$$

$$= \frac{1}{6} \left[3 + 2 \cdot e^{-\frac{j\pi k}{3}} + e^{-\frac{j2\pi k}{3}} + e^{-j\left(2\pi k - \frac{2\pi k}{3}\right)} + 2 \cdot e^{-j\left(2\pi k - \frac{\pi k}{3}\right)} \right]$$

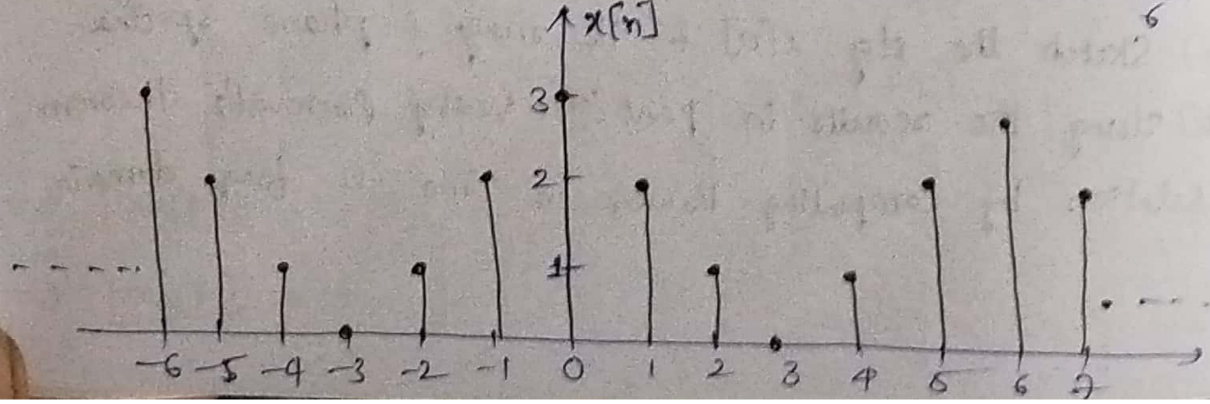
$$= \frac{1}{6} \left[3 + 2\left(e^{\frac{j\pi k}{3}} + e^{-\frac{j\pi k}{3}} \right) + 1 \cdot \left(e^{-\frac{j2\pi k}{3}} + e^{\frac{j2\pi k}{3}} \right) \right]$$

$$\therefore C_k = \frac{1}{6} \left[3 + 4\cos\left(\frac{\pi k}{3}\right) + 2\cos\left(\frac{2\pi k}{3}\right) \right]$$

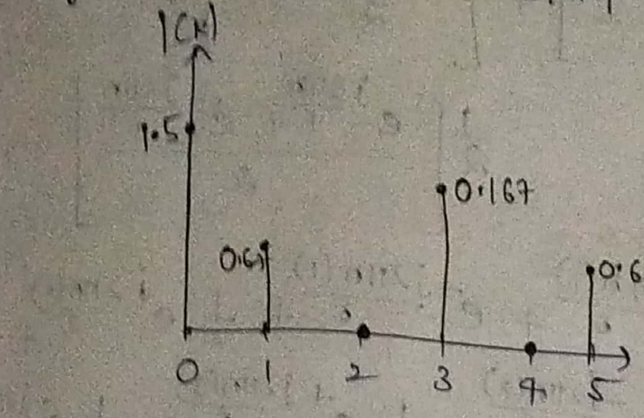
If $K = 0$.

$$C_0 = \frac{1}{6}\left[3 + 4 + 2\right] = \frac{9}{6} = \frac{3}{2}.$$

$C_1 = \frac{4}{6}$; $C_2 = 0$; $C_3 = \frac{1}{6}$; $C_4 = 0$; $C_5 = \frac{4}{6}$



Magnitude spectrum $\longrightarrow$ $|C_K|$ Vs K.



phase spectrum $\angle C_K = 0$.

(b) $P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$

$= \frac{1}{6} \sum_{n=0}^{5} \left[1^2 + 0^2 + 1^2 + 2^2 + 3^2 + 2^2\right]$

$= \frac{1}{6}\left[1+1+4+9+4\right] = \frac{19}{6}$.

$P = \sum_{K=0}^{N-1} |C_K|^2$

$= \left(\frac{9}{6}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{4}{6}\right)^2$

$= \frac{114}{36} = \frac{19}{6}$.

4.5 Consider the slg. $x[n] = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$.

(a) Determine & sketch its P.S.D.

(b) Evaluate the power of the slg

Sol:- (a) $P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$

$x[n] = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$.

$\omega_1 = \frac{\pi}{4}$; $\omega_2 = \frac{\pi}{2}$; $\omega_3 = \frac{3\pi}{4}$.

$f_1 = \frac{1}{8}$; $f_2 = \frac{1}{4}$; $f_3 = \frac{3}{8}$.

$\Rightarrow N_1 = 8$; $N_2 = 4$; $N_3 = 8$.

$\boxed{f = \frac{k}{N}}$

$\therefore N = L.C.M(8,4,8)$

$\boxed{N = 8}$

$$x[n] = 2 + 2\left[\frac{e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}}{2}\right] + \left[\frac{e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}}{2}\right] + \frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}}{2}\right]$$

$$= 2\cdot e^{\frac{j2\pi(0)}{8}} + 1\cdot e^{\frac{j2\pi n(1)}{8}} + e^{-\frac{j2\pi n(1)}{8}} + \frac{1}{2} e^{\frac{j2\pi n(2)}{8}} + \frac{1}{2} e^{-\frac{j2\pi n(2)}{8}} + \frac{1}{4} e^{+\frac{j2\pi n(3)}{8}} + \frac{1}{4} e^{-\frac{j2\pi n(3)}{8}}$$

$\Rightarrow C_0 = 2,\ C_1 = 1,\ C_2 = \frac{1}{2},\ C_3 = \frac{1}{4}$

$$\boxed{C_K = C_{K+N}}$$

$\Rightarrow C_7 = C_{-1+8} = C_{-1} = 1$

$\Rightarrow C_6 = C_{-2+8} = C_{-2} = \frac{1}{2}$

$\Rightarrow C_5 = C_{-3+8} = C_{-3} = \frac{1}{4}$

$\Rightarrow C_4 = C_{4+8} = C_{-4} = 0$

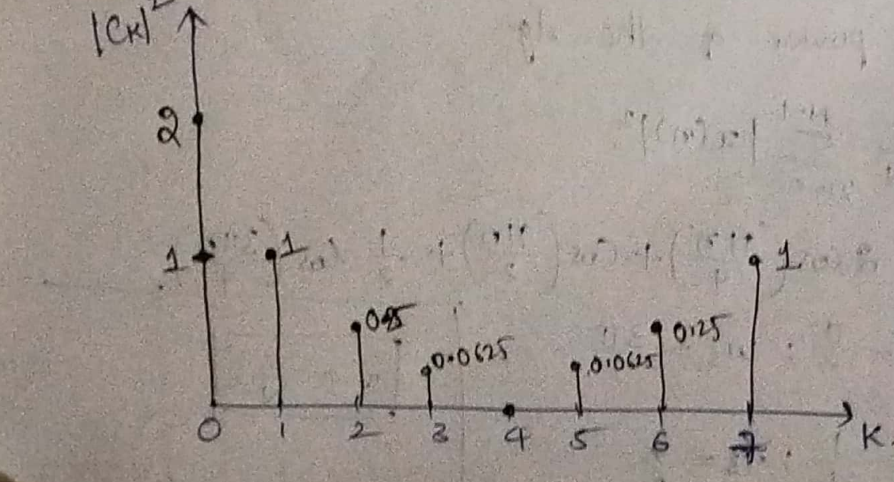
(b)

$$\Rightarrow P = \frac{1}{8}\sum_{n=0}^{7} |x(n)|^2 = \sum_{K=0}^{7} |C_K|^2$$

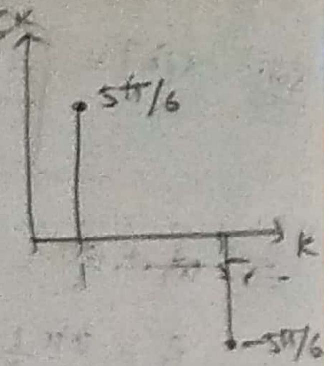
$$= \left[(2)^2 + (1)^2 + \left(\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{4}\right)^2 + \left(\tfrac{1}{2}\right)^2 + (1)^2\right]$$

$$\therefore P = \frac{53}{8}$$

$\Rightarrow$ P.S.D.

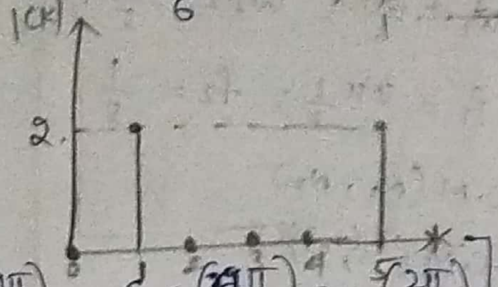


4.6 Determine & sketch the mag & phase spectra of the following periodic sig's.



(a) $x[n] = 4\sin\left(\frac{\pi(n-2)}{3}\right)$.

Sol:- $x[n] = 4\sin\frac{\pi(n-2)}{3} = 4\sin\frac{2\pi(n-2)}{6}$



$= 4\sin\left(\frac{n\pi}{3} - \frac{2\pi}{3}\right)$

$\neq \cancel{4\sin\left(\frac{\pi}{3}(n-2)\right)}$

$= 4\left[\sin\left(\frac{n\pi}{3}\right)\cdot\cos\left(\frac{2\pi}{3}\right) - \cos\left(\frac{n\pi}{3}\right)\sin\left(\frac{2\pi}{3}\right)\right]$

$= 4\left[\left(-\frac{1}{2}\right)\sin\left(\frac{n\pi}{3}\right) - \frac{\sqrt{3}}{2}\cos\left(\frac{n\pi}{3}\right)\right]$

$= -2\sin\left(\frac{n\pi}{3}\right) - 2\sqrt{3}\cos\left(\frac{n\pi}{3}\right)$

$= -2\left[\frac{e^{\frac{jn\pi}{3}} - e^{-\frac{jn\pi}{3}}}{2j}\right] - 2\sqrt{3}\left[\frac{e^{\frac{jn\pi}{3}} + e^{-\frac{jn\pi}{3}}}{2}\right]$

$= e^{\frac{jn\pi}{3}}\left(-\frac{1}{j} - \sqrt{3}\right) + e^{-\frac{jn\pi}{3}}\left(-\sqrt{3} + \frac{1}{j}\right)$

$= e^{\frac{j2\pi n(1)}{6}}\left(-\frac{1}{j} - \sqrt{3}\right) + e^{-\frac{j2\pi n(-1)}{6}}\left(\frac{1}{j} - \sqrt{3}\right)$

$= e^{\frac{j2\pi n}{6}}(-\sqrt{3}+j) + e^{-\frac{2\pi n}{6}}(-\sqrt{3}-j)$.

$\Rightarrow |C_1| = \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{3+1}$

$= \sqrt{4}$

$\Rightarrow |C_1| = 2$; $|C_5| = 2$; $C_0 = 0$; $C_2 = C_3 = C_4 = 0$.

$|C_0| = |C_2| = |C_3| = |C_4| = 0$; $|C_1| = |C_5| = 2$.

$\Rightarrow \angle C_1 = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right)$; $\angle C_5 = \tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right)$

$= 180 - \theta$ $\qquad = 180 + 30$

$= 180° - 30°$ $\qquad = 210°$

$= 150° = \frac{5\pi}{6}$ $\qquad = -150° = \frac{-5\pi}{6}$

$\angle C_0 = \angle C_2 = \angle C_3 = \angle C_4 = 0$.

(b) $x[n] = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n$.

Sol:- $x[n] = \left[\frac{e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}}{2}\right] + \left[\frac{e^{j\frac{2\pi}{5}n} + e^{-j\frac{2\pi}{5}n}}{2j}\right]$

$\Rightarrow \omega_1 = \frac{2\pi}{3}$; $\omega_2 = \frac{2\pi}{5}$.

$f_1 = 2\pi\frac{1}{3}$; $f_2 = \frac{1}{5}$.

$N = L.C.M(N_1, N_2)$

$N = L.C.M(3, 5)$

$\therefore \boxed{N = 15}$

$$x[n] = \frac{1}{2}e^{j\frac{2\pi}{3}n} + \frac{1}{2}e^{-j\frac{2\pi}{3}n} + \frac{1}{2j}e^{j\frac{2\pi}{5}n} + \frac{1}{2j}e^{-j\frac{2\pi}{5}n}$$

$$= \frac{1}{2}e^{\frac{j2\pi(5)(1)}{15}} + \frac{1}{2}e^{\frac{-j2\pi(5)(1)}{15}} + \frac{1}{2j}e^{\frac{j2\pi(3)}{15}} - \frac{1}{2j}e^{\frac{-j2\pi(3)}{15}}$$

$\Rightarrow \frac{1}{2}e^{j\frac{2\pi}{3}n} = e^{\frac{j2\pi kn}{N}}$

$N-K = 15-5 = 10$

$K = \frac{N}{3} = \frac{15}{3} = 5$

$\Rightarrow 15-5 = 10$.

$\Rightarrow \frac{1}{2j}e^{j\frac{2\pi}{5}n} = e^{\frac{j2\pi kn}{N}}$

$N-K = 15-3 = 12$

$K = \frac{N}{5} = \frac{15}{5} = 3$.

$\Rightarrow 15-3 = 12$

$$C_{1K} = \begin{cases} \frac{1}{2} & ; k = 5, 10. \\ 0 & ; \text{otherwise} \end{cases}$$

$$C_{2K} = \begin{cases} \frac{1}{2j} & ; k = 3, 12 \\ \frac{-1}{2j} & ; k = 12 \\ 0 & ; \text{otherwise.} \end{cases}$$

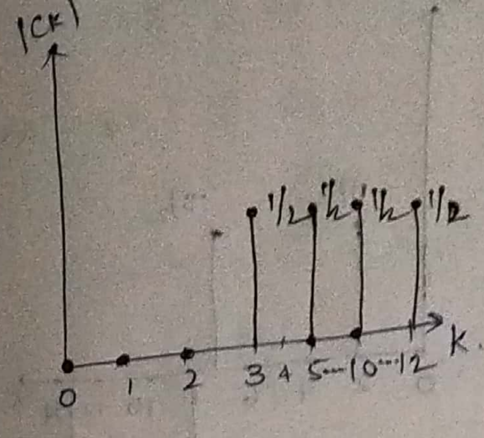
$$\Rightarrow C_K = C_{1K} + C_{2K} = \begin{cases} \frac{1}{2j} & ; k = 3. \\ \frac{1}{2} & ; k = 5 \\ \frac{1}{2} & ; k = 10 \\ \frac{-1}{2j} & ; k = 12 \\ 0 & ; \text{otherwise} \end{cases}$$

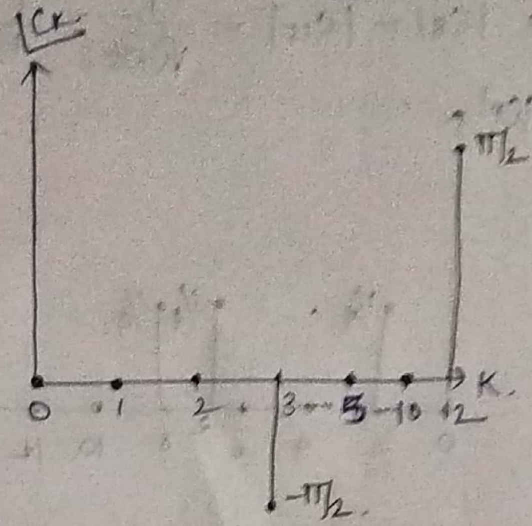
$\Rightarrow |C_3| = \frac{1}{2}$; $|C_5| = |C_{10}| = \frac{1}{2}$; $|C_{12}| = \frac{1}{2}$.

Scanned with CamScanner

$\angle C_3 = \tan^{-1}\left(\frac{2}{1}\right)$; $\angle C_5 = \angle C_{10} = 0$; $\angle C_{12} = -\tan^{-1}\left(\frac{-2}{1}\right)$

$= -\frac{\pi}{2}$ $\qquad\qquad = \frac{\pi}{2}$





(c) $x[n] = \cos\left(\frac{2\pi}{3}n\right) \cdot \sin\left(\frac{2\pi}{5}n\right).$

Sol:- $x[n] = \frac{1}{2}\sin\left(\frac{16\pi n}{3}\right) - \frac{1}{2}\sin\left(\frac{4\pi n}{15}\right)$

$\Rightarrow \omega_1 = \frac{16\pi}{3}$; $\omega_2 = \frac{4\pi}{15}$

$\Rightarrow f_1 = \frac{16\pi}{3 \times 2\pi}$; $f_2 = \frac{4\pi}{15 \times 2\pi}$

$\Rightarrow f_1 = \frac{8}{3}$; $f_2 = \frac{2}{15}$

$N_1 = 3$; $N_2 = 15 \Rightarrow N = \text{L.C.M}(2, 15)$

$\boxed{N = 15}$

$$x[n] = \frac{1}{4j}\left[e^{\frac{j16\pi n}{3}} - e^{-\frac{j16\pi n}{3}}\right] - \frac{1}{4j}\left[e^{\frac{j4\pi n}{15}} - e^{-\frac{j4\pi n}{15}}\right]$$

$\Rightarrow \frac{K}{N} = \frac{8}{15}$ $\qquad \Rightarrow \frac{K}{N} = \frac{2}{15}$

$\frac{K}{\cancel{15}} = \frac{8}{\cancel{15}}$ $\qquad \frac{K}{\cancel{15}} = \frac{2}{\cancel{15}}$.

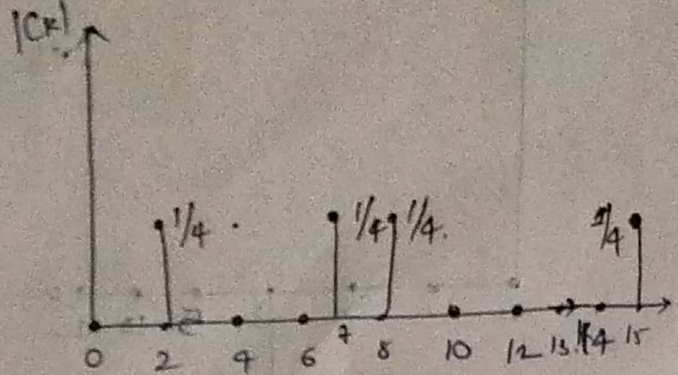
$K = 8 \rightarrow \frac{1}{4j}$ $\qquad K = 2 \rightarrow \frac{-1}{4j}$

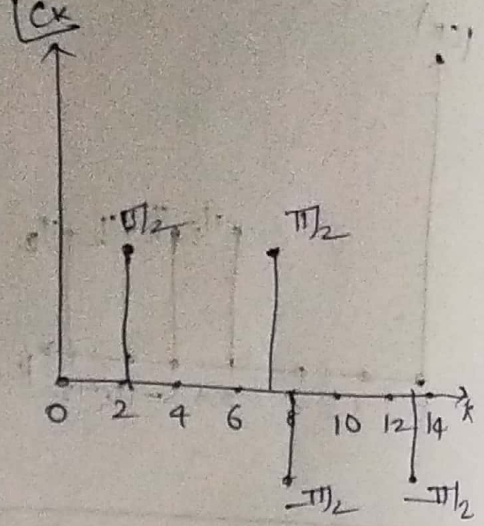
$15 - 8 = 7 \rightarrow \frac{-1}{4j}$ $\qquad 15 - 2 = 13 \rightarrow \frac{+1}{4j}$

$$\therefore C_K = \begin{cases} \frac{-1}{4j} ; & K = 2, 7 \\ \frac{1}{4j} ; & K = 8, 13 \\ 0 ; & \text{otherwise} \end{cases}$$

$\Rightarrow |C_2| = \frac{1}{\sqrt{(4)^2}} = \frac{1}{4} = |C_7|.$

$\Rightarrow |C_8| = |C_{15}| = \frac{1}{\sqrt{(4)^2}} = \frac{1}{4}.$





(d) $x[n] = \{\ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2, \ldots\}$

$N = 5$

Sol:-

$$c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\cdot e^{-\frac{j2\pi kn}{N}}$$

$$= \frac{1}{5}\sum_{n=0}^{4} x[n]\cdot e^{-\frac{j2\pi kn}{5}}$$

$$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j4\pi k}{5}} - 2e^{-\frac{j6\pi k}{5}} - e^{-\frac{j8\pi k}{5}}\right]$$

$$= \frac{1}{5}\left[0 + e^{-\frac{j2\pi k}{5}} + 2\cdot e^{-\frac{j2\pi(2)k}{5}} - 2e^{-\frac{j2\pi(3)k}{5}} - e^{-\frac{j2\pi(4)k}{5}}\right]$$

$$= \frac{1}{5}\left[\left(e^{-\frac{j2\pi k}{5}} - 2e^{-\frac{j2\pi k(3)}{5}}\right) + \left(2\cdot e^{-j\frac{4\pi k}{5}} - e^{-j\frac{4\pi k(}{5}}\right.\right.$$

$$= \frac{1}{5}\left[e^{-\frac{j2\pi k}{5}} + 2e^{-j\frac{4\pi k}{5}} - 2e^{\frac{j4\pi k}{5}} - e^{\frac{j2\pi k}{5}}\right]$$

$$= \frac{1}{2j}\frac{1}{5}\left[\sin\left(\frac{2\pi k}{5}\right) - 4j\sin\left(\frac{4\pi k}{5}\right)\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right].$$

$\Rightarrow$ $k=0$; $C_0=0$

$k=1$; $C_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) - 2\sin\left(\frac{4\pi}{5}\right)\right]$ $\boxed{C_{k+N}=C_k.}$

$k=2$; $C_2 = \frac{2j}{5}\left[-\sin\left(\frac{4\pi}{5}\right) - 2\sin\left(\frac{8\pi}{5}\right)\right]$

$\Rightarrow$ $k=3$; $C_3 = C_{5-2}$.

$C_3 = -C_2$

$k=4$; $C_4 = C_{5-1} = -C_1$

(e) $x[n] = \{\ldots\ldots -1, 2, \overset{\downarrow}{1}, 2, -1, 0, -1, 2, 1, 2, \ldots\ldots\}$

$N=6$.

Sol:-

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x[n]\, e^{-\frac{j2\pi kn}{6}}$$

$$= \frac{1}{6}\left[1 + 2e^{-\frac{j2\pi k}{6}} - 1\cdot e^{-\frac{j4\pi k}{6}} + 0 - 1\cdot e^{-\frac{j8\pi k}{6}} + 2\cdot e^{-\frac{j10\pi k}{6}}\right]$$

$$= \frac{1}{6}\left[1 + 2\cdot e^{-\frac{j\pi k}{3}} - 1\cdot e^{-\frac{j2\pi k}{3}} - e^{-\frac{j4\pi k}{3}} + 2\cdot e^{-\frac{j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 2\cdot e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{-j\left(2\pi k - \frac{2\pi k}{3}\right)} + 2\cdot e^{-j\left(2\pi k - \frac{\pi k}{3}\right)}\right]$$

$$= \frac{1}{6}\left[1 + 2\cdot e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{+\frac{j2\pi k}{3}} + 2\cdot e^{j\frac{\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi k}{3}\right) - 2\cdot\cos\left(\frac{2\pi k}{3}\right)\right]$$

$\Rightarrow$ $k=0$; $C_0 = \frac{1}{6}(1+4-2) = \frac{1}{2}$.

$k=1$; $C_1 = \frac{1}{6}(1+2-2(-1)) = \frac{1}{6}(1+2+1) = 4/6 = 2/3$

$k=2$; $C_2 = \frac{1}{6}[1+(-2)+1] = \frac{1}{6}(2-2) = 0$.

$k=3$; $C_3 = \frac{1}{6}[1+(-4)-2] = \frac{1}{6}[1-6] = \frac{-5}{6}$

$k=4$; $C_4 = \frac{1}{6}(1-2+1) = \frac{1}{6}(2-2) = 0$.

$k=5$; $C_5 = \frac{1}{6}(1+2+1) = \frac{1}{6}(4) = \frac{4}{6} = 2/3$.

$\therefore$ $C_0 = 0$, $C_1 = 2/3$, $C_2 = 0$, $C_3 = -5/6$, $C_4 = 0$, $C_5 = 2/3$.

(f) $x[n] = \{ \cdots, 0, 0, \overset{\downarrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0, \cdots \}$.

$\underbrace{\qquad\qquad}_{N=5}$

Sol:-

$$C_K = \frac{1}{5} \sum_{n=0}^{4} x[n] \cdot e^{-\frac{j2\pi k n}{5}}$$

$$= \frac{1}{5} \left[1 + 1 \cdot e^{-\frac{j2\pi k}{5}} + 0 + 0 + 0 \right]$$

$$= \frac{1}{5} \left[1 + e^{-\frac{j2\pi k}{5}} \right] = \frac{1}{5} \left[1 \cdot e^{-\frac{j2\pi k(0)}{5}} + e^{-\frac{j2\pi k(1)}{5}} \right]$$

$$= \frac{1}{5} \left[1 + \cos\left(\frac{2\pi k}{5}\right) - j \sin\left(\frac{2\pi k}{5}\right) \right]$$

$\cancel{C_K = \frac{1}{5}\{1+e^{-\frac{j2\pi k}{5}}\}}$

$C_0 = \frac{2}{5}$; $C_1 = \frac{1}{5}\left[1 + e^{-j\frac{2\pi}{5}}\right]$; $C_2 = \frac{1}{5}\left[1 + e^{-j\frac{4\pi}{5}}\right]$.

$C_3 = \frac{1}{5}\left[1 + e^{-j\frac{6\pi}{5}}\right]$, $C_4 = \frac{1}{5}\left[1 - e^{-\frac{j8\pi}{5}}\right]$.

(g) $x[n] = 1$, $-\infty < n < \infty$.

Sol:- $N = 1$

$$C_K = \frac{1}{1} \sum_{n=0}^{0} x[0] = 1$$

$C_0 = 1$

(h) $x[n] = (-1)^n$; $-\infty < n < \infty$.

Sol:- $N = 2$.

$$C_K = \frac{1}{2} \sum_{n=0}^{1} x[n] \cdot e^{-j\pi n k}$$

$$= \frac{1}{2}\left(1 - e^{-j\pi n k}\right)$$

$k=0 \Rightarrow C_0 = \frac{1}{2}(1-1) = 0$.

$k=1 \Rightarrow C_1 = \frac{1}{2}\left(1 - e^{-j\pi k}\right)$

$= \frac{1}{2}(2)$

$C_1 = 1$

4.7 Determine the periodic slg's $x[n]$ with fundamental period $N=8$. if their Fourier coeff's are given by:

(a) $c_K = \cos\left(\frac{k\pi}{4}\right) + \sin\left(\frac{3k\pi}{4}\right)$.

Sol:- $N=8$.

$$x[n] = \sum_{K=0}^{7} c_K e^{+\frac{j2\pi nk}{8}}$$

If $c_K = e^{\frac{j2\pi pk}{N}}$; then

$$\Rightarrow \sum_{K=0}^{7} e^{\frac{j2\pi pk}{8}} \cdot e^{+j\frac{2\pi nk}{8}} = \sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{8}}$$

$$= 8 \; ; \; p=-n.$$

$$= 0 \; ; \; p \neq -n.$$

$$\Rightarrow c_K = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{-\frac{j2\pi k}{8}}\right] + \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{-\frac{j6\pi k}{8}}\right]$$

$$= \frac{1}{2}\left[e^{\frac{j2\pi k(1)}{8}} + e^{-\frac{j2\pi k(1)}{8}}\right] + \frac{1}{2j}\left[e^{\frac{j2\pi k(3)}{8}} - e^{-\frac{j2\pi k(3)}{8}}\right]$$

$$\Rightarrow x[n] = 4\delta(n+1) + 4\delta(n-1) + 4j\,\delta(n+3) - 4j\,\delta(n-3)$$

$$; \; -3 \leq n \leq 5.$$

(b) $c_K = \begin{cases} \sin\frac{k\pi}{3} & ; \; 0 \leq K \leq 6 \\ 0 & ; \; K=7 \end{cases}$

Sol:- $\boxed{N=8}$

$c_0 = 0$; $c_1 = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$; $c_2 = \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$

$c_3 = \sin(\pi) = 0$, $c_4 = \sin\left(\frac{4\pi}{3}\right) = \frac{-\sqrt{3}}{2}$; $c_5 = \sin\left(\frac{5\pi}{3}\right) = \frac{-\sqrt{3}}{2}$.

$c_6 = \sin\left(\frac{6\pi}{3}\right) = \sin(2\pi) = 0$. ; $c_7 = 0$.

$$\Rightarrow x[n] = \sum_{K=0}^{7} c_K \cdot e^{+\frac{j2\pi kn}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n(2)}{4}} - e^{+\frac{j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n}{2}} - e^{+\frac{j\pi n}{2}} - e^{+\frac{j\pi n}{4}}\right]$$

$$= \sqrt{3}\cdot\left[\sin\left(\frac{\pi n}{4}\right)+\sin\left(\frac{\pi n}{2}\right)\right]\cdot e^{\frac{j(3n-2)}{4}}$$

(c) $\{c_k\} = \{\cdots 0, \tfrac{1}{4}, \tfrac{1}{2}, 1, \underset{\uparrow}{2}, 1, \tfrac{1}{2}, \tfrac{1}{4}, 0, \cdots\}$

Sol:- $\boxed{N=8}$

$$x[n] = \sum_{k=-3}^{4} c_k\, e^{\frac{j2\pi nk}{8}}$$

$$= 2 + 1\cdot e^{\frac{j\pi n}{4}} + \frac{1}{2}\cdot e^{\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}} + \frac{1}{4} e^{-\frac{j3\pi n}{4}} + e^{-\frac{j\pi n}{4}} - \frac{1}{2}\cdot e^{-\frac{j\pi n}{2}}$$

$$= 2 + \left(e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}\right) + \frac{1}{2}\left(e^{\frac{j\pi n}{2}} + e^{-\frac{j\pi n}{2}}\right) + \frac{1}{4}\left(e^{\frac{j3\pi n}{4}} + e^{-\frac{j3\pi n}{4}}\right)$$

$$\therefore\ x[n] = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi n}{2}\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right).$$

4.9 Compute the Fourier Transform of the following

(a) $x[n] = u[n] - u[n-6]$

Sol:- $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n}$

$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

$= \sum_{n=0}^{5} e^{-j\omega n}$

$= 1 + e^{-j\omega(1)} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega}$

$= \frac{1(1 - e^{-j\omega 6})}{(1 - e^{-j\omega(1)})}$

$= \frac{1 - e^{-j\omega 6}}{1 - e^{-j\omega}}$

(b) $x[n] = 2^n u[-n]$

Sol:- $X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$ $\boxed{m = -n}$

$\frac{e^{2j\omega}}{\cancel{\ }_2} \times \frac{2}{e^{j\omega}}$

$r = \frac{e^{j\omega}}{2}$

$= \sum_{m=\infty}^{0} 2^m e^{+j\omega m}$

$= \sum_{m=0}^{\infty} \frac{e^{j\omega m}}{2^m}$

$\boxed{S_\infty = \frac{a}{1-r}}$

$= 1 + \frac{e^{j\omega}}{2} + \frac{e^{j2\omega}}{2^2} + \frac{e^{j3\omega}}{2^3} + \cdots$

$= 1 \cdot \frac{\cancel{e}}{\cancel{2}} + \frac{e^{j\omega}}{2^1} + \frac{e^{j2\omega}}{2^2} + \frac{e^{j3\omega}}{2^3} + \cdots$

$= \cancel{\frac{2}{2 - (2e^{+j\omega})}} = \frac{1}{1 - \frac{e^{j\omega}}{2}} = \frac{1}{\frac{2 - e^{j\omega}}{2}}$

(c) $x[n] = \left(\frac{1}{4}\right)^n u(n+4)$

Sol:- $X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$

Let $m = n+4$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m-4} \cdot e^{-j\omega(m-4)}$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m (4)^4 e^{-j\omega m} \cdot e^{j4\omega}$$

$$= \left(\sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m}\right)(4)^4 \cdot e^{j4\omega}$$

$$= \cancel{4^4}\left(1 + \frac{1}{4} e^{-j\omega} + \frac{1}{4^2} e^{-j2\omega} + \frac{1}{4^3} e^{-j3\omega} + \dots\right)(4)^4 e^{j4\omega}$$

$$\frac{e^{-2j\omega}}{\cancel{16}\,4} \times \frac{4}{e^{-j\omega}} \Rightarrow \frac{e^{-2j\omega} \cdot e^{j\omega}}{4} \quad \frac{e^{-2j\omega+j\omega}}{4}$$

$$= \left(\frac{1}{1 - \frac{e^{-j\omega}}{4}}\right)(4)^4 e^{j4\omega} = \left(\frac{4}{4 - e^{-j\omega}}\right)(4)^4 e^{j4\omega}$$

$$\therefore X(\omega) = \frac{(4)^5 \cdot e^{j4\omega}}{4 - e^{-j\omega}}$$

(d) $x[n] = (\alpha^n \sin n\omega_0 n) u(n)$, $|\alpha| < 1$

(e) $x[n] = |\alpha^n| \sin \omega_0 n$; $|\alpha| < 1$

Sol:- (d)
$$= \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{+j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] \cdot e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{2j}\left(\alpha^n e^{-j(\omega_0+\omega)n}\right) - \frac{1}{2j}\sum_{n=0}^{\infty}\left(\cancel{\alpha}\, \alpha^n e^{-j(\omega+\omega_0)n}\right)$$

$$= \sum_{n=0}^{\infty} \frac{1}{2j}\left[\left(\alpha e^{-j(\omega-\omega_0)}\right)^n\right] - \sum_{n=0}^{\infty} \frac{1}{2j}\left[\left(\alpha e^{-j(\omega+\omega_0)}\right)^n\right]$$

$$= \frac{1}{2j}\left[1 + \alpha e^{-j(\omega-\omega_0)} + \alpha e^{-j2(\omega-\omega_0)} + \dots\right] - \cancel{\sum_{n=0}} \frac{1}{2j}$$

$$\left[1 + \alpha e^{-j(\omega+\omega_0)} + \alpha e^{-j2(\omega+\omega_0)} + \dots\right]$$

$$= \frac{1}{2j}\left[\frac{1}{1 - \alpha e^{-j(\omega-\omega_0)}} - \frac{1}{1 - \alpha e^{-j(\omega+\omega_0)}}\right]$$

$$= \frac{1}{2j}\left[\frac{\cancel{1} - \alpha e^{-j(\omega+\omega_0)} - \cancel{1} + \alpha e^{-j(\omega-\omega_0)}}{(1-\alpha e^{-j(\omega-\omega_0)})(1+\alpha e^{-j(\omega+\omega_0)})}\right]$$

$$X(\omega) = \frac{\alpha \sin\omega_0 e^{-j\omega}}{1 - 2\alpha\cos\omega_0 e^{-j\omega} + \alpha^2 e^{-j2\omega}}$$

(f) $x[n] = \begin{cases} 2-\left(\frac{1}{2}\right)n & ; |n| \leq 4. \\ 0 & ; \text{otherwise} \end{cases}$

Sol: $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$= \sum_{n=-4}^{4} \left(2-\left(\frac{1}{2}\right)n\right) e^{-j\omega n}.$$

let $m = n+4$

$$= \sum_{m=0}^{8} \left(2 \cdot e^{-j\omega(m-4)} - \frac{(m-4)}{2} \cdot e^{-j\omega(m-4)}\right)$$

$$\Rightarrow \sum_{m=0}^{8} 2 \cdot e^{-j\omega m} \cdot e^{j\omega 4} \Rightarrow 2 + 2\cdot e^{-j\omega} + 2e^{-j2\omega} + 2e^{-j3\omega} + \cdots - - - + 2e^{-j8\omega}$$

$$\boxed{\Rightarrow \frac{2e^{-j2\omega}}{2e^{-j\omega}} \quad e^{-2j\omega+j\omega} \quad r = e^{-j\omega}}$$

$$= \left(\frac{2\, e^{j\omega 4}}{1-e^{-j\omega}}\right)$$

$$\Rightarrow \sum_{m=0}^{8} \frac{(m-4)}{2} e^{-j\omega m} \cdot e^{j\omega 4} = \left(\sum_{m=0}^{\infty} \frac{m}{2} e^{-j\omega m}\right) e^{j\omega 4} - \left(2 \cdot \sum_{m=0}^{\infty} e^{-j\omega m}\right) e^{j\omega 4}$$

$$= \frac{1}{2}\left[e^{-j\omega} + 2\cdot e^{-j2\omega} + 3\cdot e^{-j\omega} + \cdots\right] e^{j\omega 4} -$$

$$2\left[1 + e^{-j\omega} + e^{-j2\omega} + \cdots\right] e^{j\omega 4}$$

$=$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \frac{1}{2}\Big[-4e^{j4\omega} + 4e^{-j\omega 4} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{-j2\omega} - e^{j\omega} + e^{-j\omega}\Big].$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\Big[4\sin 4\omega + 3\sin 3\omega + 2\sin 2\omega + \sin\omega\Big].$$

(e) $x[n] = |\alpha^n| \sin\omega_0 n, \quad |\alpha| < 1$

Note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin\omega_0 n|$

If $\omega_0 = \frac{\pi}{2}$; $|\sin\omega_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \longrightarrow \infty.$$

$\therefore$ F·T doesn't exist.

(g) $x[n] = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

Sol:- $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$= -2\cdot e^{+j2\omega} - 1\cdot e^{j\omega(1)} + 0 + 1\cdot e^{-j\omega n} + 2\cdot e^{-j\omega(2)}$$

$$= -1\cdot(e^{j\omega} - e^{-j\omega}) - (2\cdot e^{j2\omega} - 2e^{-j2\omega})$$

$$= -2j\Big[2\sin(2\omega) + \sin(\omega)\Big].$$

(h) $x[n] = \begin{cases} A(2M+1-|n|) & ; |n| \leq M \\ 0 & ; |n| > M \end{cases}$

Sol:- $X(\omega) = \sum_{n=-M}^{M} x[n]\cdot e^{-j\omega n}$.

$$= \sum_{n=-M}^{M} A(2M+1-|n|)\cdot e^{-j\omega n} \qquad \boxed{K = n}$$

$$= A(2M+1) + A\sum_{K=1}^{M}(2M+1-K)(e^{-j\omega k} + e^{j\omega k})$$

$$= A(2M+1) + 2A\sum_{K=1}^{M}(2M+1-K)\cos(\omega K)$$

Scanned with CamScanner

4.10 Determine the slg's having the following F-Transform

(a) $X(\omega) = \begin{cases} 0, & 0 \le |\omega| < \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

Sol:- $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega + \frac{1}{2\pi} \int_{\omega_0}^{\pi} e^{j\omega n} d\omega \longrightarrow ①$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi}^{-\omega_0} + \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{\omega_0}^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} \right] + \frac{1}{2\pi} \left[\frac{e^{j\pi n} - e^{j\omega_0 n}}{jn} \right]$$

$$= \frac{1}{2\pi} \left[2\left(\frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} \right) + 2\left(\frac{e^{j\pi n} - e^{-j\pi n}}{2jn} \right) \right]$$

$$= \frac{1}{2\pi} \left[\frac{2}{n} \sin(\pi n) - \frac{2}{n} \sin(\omega_0 n) \right]$$

$$= \frac{1}{\pi n} \left[\cancel{\sin(\pi n)}^{0} - \sin(\omega_0 n) \right]$$

$$= \frac{-\sin(\omega_0 n)}{\pi n} \quad ; n \neq 0$$

For $n=0$; from ①

$$= \frac{1}{2\pi} (\pi - \omega_0) + \frac{1}{2\pi} (\pi + \omega_0)$$

$$= \frac{\cancel{2}(\pi - \omega_0)}{\cancel{2}\pi}$$

$$= \frac{\pi - \omega_0}{\pi} \quad ; n = 0$$

(b) $X(\omega) = \cos^2(\omega)$

$$= \left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left(e^{j2\omega} + 2e^{j\omega}\times\frac{1}{e^{j\omega}} + e^{-j2\omega}\right)$$

$$= \frac{1}{4}\left(e^{j2\omega}+e^{-j2\omega}+2\right)$$

$$X(\omega) = \frac{1}{4}e^{j2\omega} + \frac{1}{4}e^{-j2\omega} + \frac{1}{2}$$

$$\cancel{x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cdot e^{j\omega n}\,d\omega}$$

$$\cancel{= \frac{1}{8\pi}\int_{-\pi}^{\pi}\left(e^{j2\omega}+e^{-j2\omega}+2\right)e^{j\omega n}\,d\omega}$$

$$\cancel{= \frac{1}{8\pi}\int_{-\pi}^{\pi} e^{j(2\omega}}$$

$$x[n] = \frac{1}{4}\delta(n+2) + \frac{1}{4}\delta(n-2) + \frac{1}{2}\delta(n)$$

(c) $$X(\omega) = \begin{cases} 1 ; & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2} \\ 0 ; & \text{else where} \end{cases}$$

Sol:- $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cdot e^{+j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_0+\frac{\delta\omega}{2}} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{\omega_0+\frac{\delta\omega}{2}}^{\pi} e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}} e^{j\omega n}\cdot d\omega = \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}}$$

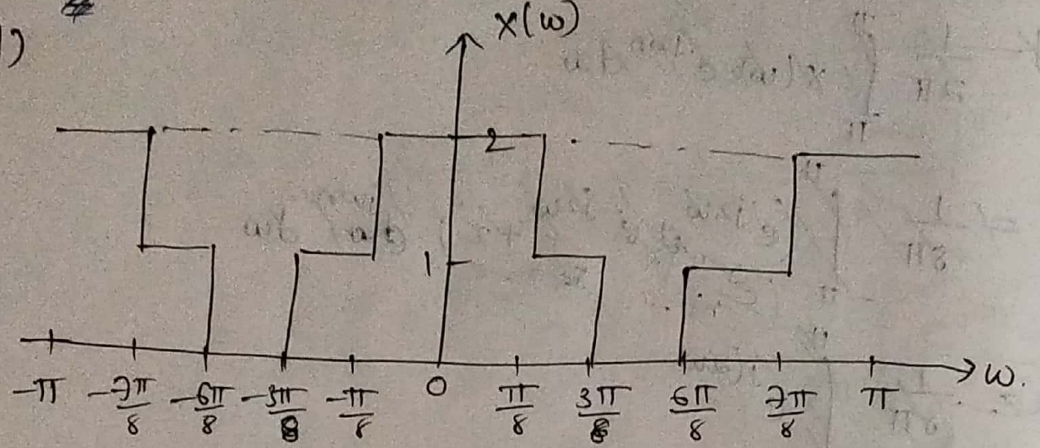
$$= \frac{1}{2\pi jn}\left[e^{j\omega_0 n + j\frac{\delta\omega n}{2}} - e^{j\omega_0 n - j\frac{\delta\omega n}{2}}\right]$$

Scanned with CamScanner

$$= \frac{1}{2\pi j n}\left[e^{j(\omega_0+\frac{d\omega}{2})n} - e^{-j(\omega_0-\frac{d\omega}{2})n}\right]$$

$$= \frac{2}{\pi}\cdot d\omega\left[\frac{\sin\left(\frac{d\omega n}{2}\right)}{\frac{n d\omega}{2}}\right]e^{j\omega_0 n}$$

(d)



Sol:- $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi}\left[\int_0^{\pi/8} 2e^{j\omega n}d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n}d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}d\omega + \int_{7\pi/8}^{\pi} 2e^{j\omega n}d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{j\omega n}}{jn}\right)_0^{\pi/8} + \left(\frac{e^{j\omega n}}{jn}\right)_{\pi/8}^{3\pi/8} + \left(\frac{e^{j\omega n}}{jn}\right)_{6\pi/8}^{7\pi/8} + 2\left(\frac{e^{j\omega n}}{jn}\right)_{7\pi/8}^{\pi}\right.$$

$$= \frac{1}{jn\,2\pi}\left[2\left(e^{\frac{j\pi n}{8}}-1\right) + \left(e^{\frac{j3\pi n}{8}} - e^{\frac{j\pi n}{8}}\right) + \left(e^{\frac{j7\pi n}{8}} - e^{\frac{j6\pi n}{8}}\right)\right.$$
$$\left. + 2\left(e^{j\pi n} - e^{\frac{j7\pi n}{8}}\right)\right]$$

Scanned with CamScanner

(d) $x[n] = \frac{1}{2\pi} Re \left\{ \int_0^{\pi/8} 2e^{j\omega n} d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n} d\omega + \int_{\pi}^{7\pi/8} e^{j\omega n} d\omega + \int_{7\pi/8}^{6\pi/8} 2e^{j\omega n} d\omega \right\}$

$= \frac{1}{2\pi} \left[\int_0^{\pi/8} 2Cos(\omega n) d\omega + \int_{\pi/8}^{3\pi/8} Cos(\omega n) d\omega + \int_{6\pi/8}^{7\pi/8} Cos(\omega n) d\omega + \int_{7\pi/8}^{\pi} 2Cos(\omega n) d\omega \right]$

$= \frac{1}{2\pi} \left[-Sin\left(\frac{7\pi n}{8}\right) + Sin\left(\frac{6\pi n}{8}\right) - Sin\left(\frac{3\pi n}{8}\right) - Sin\left(\frac{\pi n}{8}\right) \right]$

4.11 Consider the slg $x[n] = \{\overset{-3}{1}, \overset{-2}{0}, \overset{-1}{-1}, \underset{\uparrow}{\overset{0}{2}}, \overset{1}{3}\}$ with F.T

$X(\omega) = X_R(\omega) + jX_I(\omega)$. Determine & sketch the slg $y[n]$ with F.T.

$Y(\omega) = X_I(\omega) + X_R(\omega) \cdot e^{j2\omega}$.

Sol: $x_e[n] = \frac{x[n] + x[-n]}{2}$ $\Rightarrow x[-n] = \{\overset{-1}{3}, \underset{\uparrow}{\overset{0}{2}}, \overset{1}{-1}, \overset{2}{0}, \overset{3}{1}\}$

$\Rightarrow x_e[n] = \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\}$

$\Rightarrow x_o[n] = \{\frac{1}{2}, 0, -2, \underset{\uparrow}{0}, \frac{2}{2}, \frac{0}{2}, \frac{+1}{2}\}$

$X_R(\omega) = \sum_{n=-3}^{3} x_e[n] e^{-j\omega n}$; $jX_I(\omega) = \sum_{n=-3}^{5} x_o[n] e^{-j\omega n}$.

$\Rightarrow Y(\omega) = X_I(\omega) + X_R(\omega) e^{+j2\omega}$

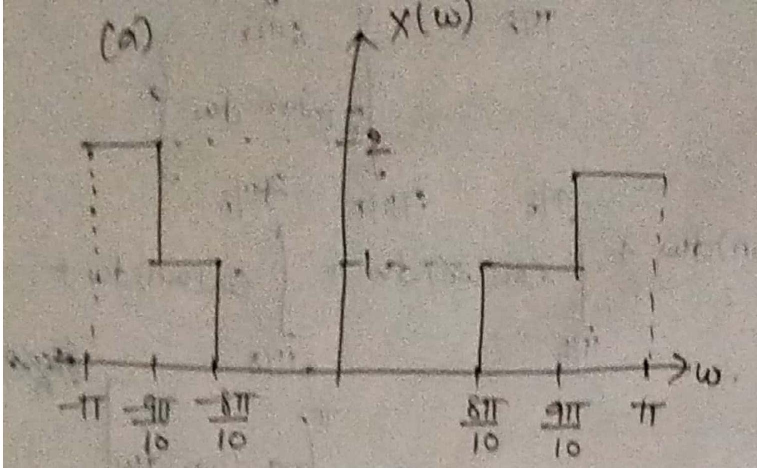
$y[n] = F^{-1}\{X_I(\omega)\} + F^{-1}\{X_R(\omega) e^{j2\omega}\}$

$= -jx_o[n] + x_e(n+2)$

$= \{\frac{1}{2}, 0, 1 - \frac{j}{2}, 2, 1 + \frac{j}{2}, \underset{\uparrow}{0}, \frac{1}{2} - j2, 0, \frac{j}{2}\}$.

4.12 Determine the sig. $x[n]$ if its F.T is as given a

(a)



Soln

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(w)\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\left[\int_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} 1\cdot e^{jwn}\, dw + \int_{\frac{9\pi}{10}}^{\pi} 2\cdot e^{jwn}\, dw\right]$$

$$= \frac{1}{2\pi}\left[\int_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} \cos(wn)\, dw + \int_{\frac{9\pi}{10}}^{\pi} 2\cos(wn)\, dw\right]$$

$$= \frac{1}{2\pi}\left[\left[\frac{\sin(wn)}{n}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} + 2\left[\frac{\sin(wn)}{n}\right]_{9\pi/10}^{\pi}\right]$$

$$= \frac{1}{2\pi n}\left[\cancel{\sin\left(\frac{9\pi n}{10}\right)} - 2\sin\left(\frac{8\pi n}{10}\right) + \cancel{\sin\left(\frac{\pi n}{1}\right)} - 2\sin\left(\frac{9\pi n}{10}\right)\right]$$

$$= \frac{-2}{2\pi n}\left[\sin\left(\frac{8\pi n}{10}\right) + \sin\left(\frac{9\pi n}{10}\right)\right]$$

$$= \frac{-1}{\pi n}\left[\sin\left(\frac{8\pi n}{10}\right) + \sin\left(\frac{9\pi n}{10}\right)\right]$$

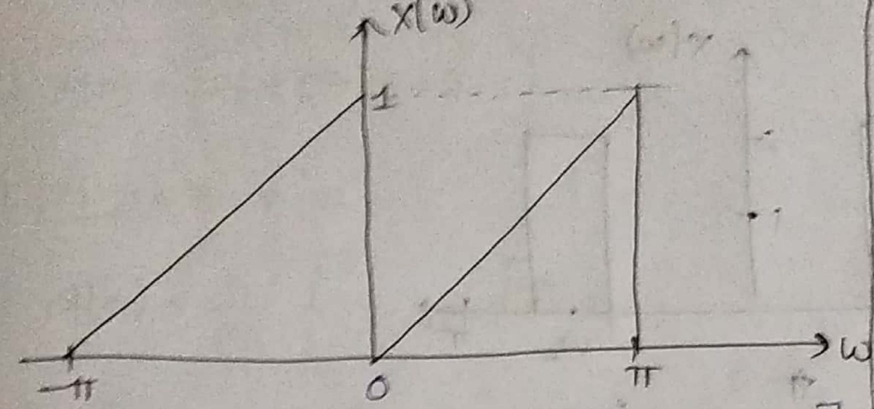
(or)

(a) $x[n] = \frac{1}{2\pi}\left[\frac{1}{jn}\left(e^{\frac{j9\pi n}{10}} - e^{-\frac{j9\pi n}{10}} - e^{-\frac{j8\pi n}{10}} + e^{\frac{j8\pi n}{10}}\right) + \frac{2}{jn}\left(-e^{-\frac{j9\pi n}{10}} + e^{\frac{j9\pi n}{10}} + e^{j\pi n} - e^{-j\pi n}\right)\right]$

$= \frac{1}{n\pi}\left[\sin(\pi n) - \sin\left(\frac{8\pi n}{10}\right) - \sin\left(\frac{9\pi n}{10}\right)\right]$

$= \frac{-1}{n\pi}\left[\sin\left(\frac{8\pi n}{10}\right) + \sin\left(\frac{9\pi n}{10}\right)\right]$.

(b)



$x_1, y_1 \quad x_2, y_2$

$(-\pi, 0) \quad (0, 1)$

$(y-0) = \frac{1}{\pi}(x+\pi)$

$y = \frac{1}{\pi}(x+\pi)$

$X(\omega) = \frac{(\omega+\pi)}{\pi}$

$X(\omega) = \frac{\omega}{\pi} + 1$

$(0,0) \quad (\pi, 1)$

$y - 0 = \frac{1}{\pi}(x)$

$X(\omega) = \frac{\omega}{\pi}$

Ans. $x[n] = \frac{1}{2\pi}\left[\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}\, d\omega + \int_{0}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega\right]$

$= \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} \frac{\omega}{\pi}\cdot e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{0} \frac{\omega}{\pi}\cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\pi}^{0} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{0}^{\pi} \frac{\omega}{\pi}\cdot e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{\omega}{\pi} e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\pi}^{0} e^{j\omega n}\, d\omega$

$= \frac{1}{2\pi^2}\left[\frac{\omega e^{j\omega n}}{jn} + \frac{e^{j\omega n}}{n^2}\right]_{-\pi}^{\pi} + \frac{1}{2\pi}\int_{0}^{\pi} e^{j\omega n}\, d\omega$.

$= \frac{1}{2\pi^2}\left[e^{j\omega n}\left[\frac{\omega}{jn} + \frac{1}{n^2}\right]\right]_{-\pi}^{\pi}$

$= \frac{1}{2\pi^2}\left[e^{j\pi n}\left[\frac{\pi}{jn} + \frac{1}{n^2}\right] - e^{-j\pi n}\left[\frac{-\pi}{jn} + \frac{1}{n^2}\right]\right]$

$$= \frac{1}{2\pi^2}\left[\frac{\pi}{jn}e^{j\pi n} + \frac{1}{n^2}e^{j\pi n} + \frac{\pi}{jn}e^{-j\pi n} - \frac{1}{n^2}e^{-j\pi n}\right]$$

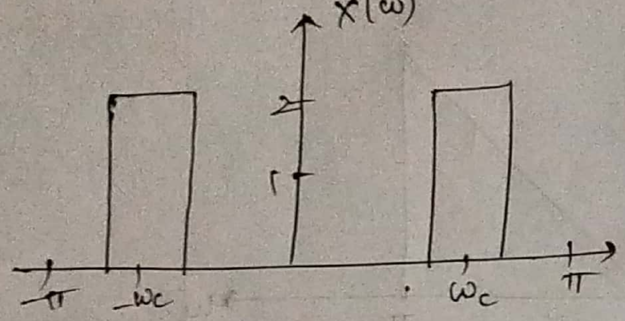
$$= \frac{1}{n\pi}\left[\cancel{\sin(n\pi)}^{\,0}\right] + \frac{1}{2\pi^2 n^2}\left[\cancel{e^{-j\pi n} - e^{-j\pi n}}^{\,0}\right]$$

$I_1 = 0$; $I_2 = \frac{1}{2\pi j n}\left[1 - e^{-j\pi n}\right]$

$$= \frac{1}{2\pi j n}\left[e^{\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}\right]\cdot e^{-\frac{j\pi n}{2}}$$

$$\therefore x[n] = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right)e^{-\frac{jn\pi}{2}}$$

(c)



Sol:- $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi}\int_{-\omega_c-\omega/2}^{-\omega_c+\omega/2} 2\cdot e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c-\omega/2}^{\omega_c+\omega/2} 2\cdot e^{j\omega n}\, d\omega$$

$$= \frac{1}{\pi}\left[\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c-\omega/2}^{-\omega_c+\omega/2} + \left[\frac{e^{j\omega n}}{jn}\right]_{\omega_c-\omega/2}^{\omega_c+\omega/2}\right]$$

$$= \frac{1}{j\pi n}\left[e^{j(-\omega_c+\omega/2)n} - e^{-j(\omega_c+\omega/2)n} + e^{j(\omega_c+\omega/2)n} - e^{j(\omega_c-\omega/2)n}\right]$$

$$= \frac{1}{j\pi n}\left[e^{-j(\omega_c-\omega/2)n} - e^{-j(\omega_c-\omega/2)n}\right] + \frac{1}{\pi j n}\left[e^{j(\omega_c+\omega/2)n} - e^{-j(\omega_c+\omega/2)n}\right]$$

$$x[n] = \frac{2}{n\pi}\left[\sin(\omega_c+\omega/2)n + \sin(\omega_c-\omega/2)n\right]$$

4.14 Consider the sig. $x[n] = [\overset{-2}{-1}, \overset{-1}{-2}, \underset{\uparrow}{\overset{0}{-3}}, \overset{1}{2}, \overset{2}{-1}]$ with F.T $X(\omega)$.
Compute the following quantities, without explicitly computing $X(\omega)$:

(a) $X(0)$ (b) $\angle X(\omega)$ (c) $\int_{-\pi}^{\pi} X(\omega)\,d\omega$ (d) $X(\pi)$ (e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

Sol:- (a) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$

$$X(0) = \sum_{n=-2}^{2} x[n]\, e^{-j(0)n}$$

$$\Rightarrow X(0) = [-1+\cancel{2}-3+2-1] = -1.$$

(b) $\angle X(\omega) = \pi \;\; \forall \; '\omega'$.

(c) $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$2\pi\, x[n] = \int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega.$$

Sub. $\boxed{n=0}$

$$2\pi\, x[0] = \int_{-\pi}^{\pi} X(\omega).\, e^{j\omega(0)}\, d\omega.$$

$$\Rightarrow \int_{-\pi}^{\pi} X(\omega)\, d\omega = 2\pi\, x[0] = 2\pi \times (-3) = -6\pi.$$

(d) $X(\pi) = \sum_{n=-\infty}^{\infty} x[n]\, .e^{-jn\pi}$

$\boxed{e^{-jn\pi} = \cos(\pi n) - j\sin(\pi n) = -1}$

$$= \sum_{n=-2}^{2} x[n].e^{-jn\pi}$$

$$= \sum_{n=-2}^{2} (-1)^n\, x[n] = (-1)^{-2}(-1) + (-1)^{-1}(2) + (-1)^0(-3) + (-1)^1 \times 2 + (-1)^2(-1)$$

$$= -1-2-3-2-1$$

$$= -3-4-2 = -9.$$

$$= -3-\cancel{4}-2 = -9$$

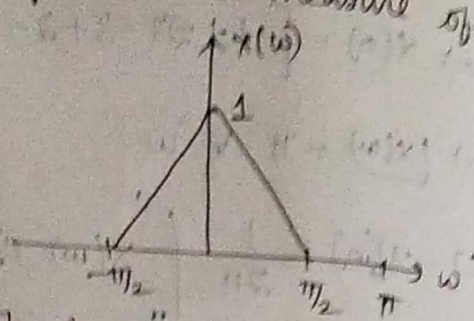
(e) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = 2\pi \sum_{n=-2}^{2} |x(n)|^2$

$= 2\pi \left[(1)^2 + (2)^2 + (3)^2 + (2)^2 + (1)^2 \right]$

$= 2\pi \left[1+4+9+4+1 \right] = 2\pi(19)$

$= 38\pi$

4.15 The center of gravity of a sig $x[n]$ is defined as $c = \frac{\sum_{n=-\infty}^{\infty} n x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$ and provides a measure of the "time delay" of the sig



(a) Express 'c' in terms of $X(\omega)$.

(b) Compute 'c' for the sig $x[n]$ whose FT is shown in fig

Sol:- (a) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

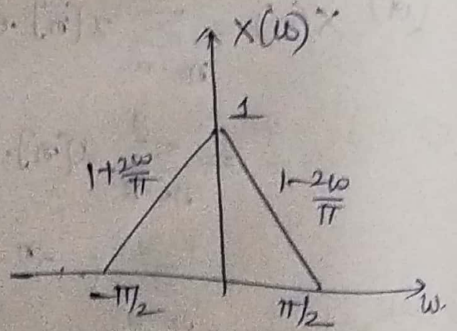
$\boxed{n x[n] \longleftrightarrow j \frac{d}{d\omega} X(\omega)}$

$\Rightarrow X(0) = \sum_{n=-\infty}^{\infty} x[n]$

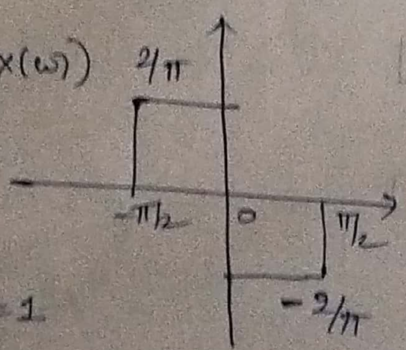
$\Rightarrow \left. \frac{dX(\omega)}{d\omega} \right|_{\omega=0} = -j \sum_{n=-\infty}^{\infty} n \cdot x[n] \cdot e^{-j\omega n} \Big|_{\omega=0}$

$= -j \sum_{n=-\infty}^{\infty} n x[n]$

$\therefore c = \frac{+j \left. \frac{dX(\omega)}{d\omega} \right|_{\omega=0}}{X(0)}$



(b) $\frac{d}{d\omega}(X(\omega))$



$X(0) = 1$

$\Rightarrow c = \left. \frac{\frac{d}{d\omega} X(\omega)}{X(\omega)} \right|_{\omega=0}$

$= \frac{0}{1} = 0$

4.16 Consider the F.T pair.

$$a^n u(n) \longleftrightarrow \frac{1}{1-ae^{-j\omega}}, \ |a|<1$$

Use the differentiation in freq ~~domain~~ theorem and induction to show that

$$x[n] = \frac{(n+l-1)!}{n!(l-1)!} a^n u[n] \longleftrightarrow X(\omega) = \frac{1}{(1-ae^{-j\omega})^l}$$

Sol:- $x_1[n] = a^n u[n]$

$$\Big\downarrow F$$

$$\frac{1}{1-ae^{-j\omega}}$$

$\Rightarrow$ Suppose $x_K[n] = \frac{(n+K-1)!}{n!(K-1)!} a^n u[n].$

$$\xleftrightarrow{F.T} \frac{1}{(1-ae^{-j\omega})^K}$$

$$x_{K+1}[n] = \frac{(n+K+\cancel{1}-\cancel{1})!}{n!(K+\cancel{1}-\cancel{1})!} a^n u[n]$$

$$= \frac{n+K.}{n!K!} \left(\frac{1}{(1-ae^{-j\omega})^K}\right)$$

$$= \frac{n+K.}{n!K!} . x_K[n].$$

$$\Rightarrow X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{(n+K)!}{n!K!}\right) x_K[n] . e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \frac{\cancel{n!}(n+K)}{n!\,K} x_K[n] e^{-j\omega n}$$

~~$X(K+1)$~~

$$\Rightarrow X_{K+1}(\omega) = \frac{1}{K} \sum_{n=-\infty}^{\infty} n x_K[n] e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_K[n] e^{-j\omega n}.$$

$$= \frac{1}{k} j \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$n x[n] \xleftrightarrow{FT} +j \frac{dX(\omega)}{d\omega}$

$$\Rightarrow \frac{d}{d\omega} X_k(\omega) = \frac{d}{d\omega}\left(\sum_{n=-\infty}^{\infty} x_k[n] e^{-j\omega n}\right)$$

$$= \sum_{n=-\infty}^{\infty} \frac{d}{d\omega}\left(e^{-j\omega n}\right) \cdot x_k[n]$$

$$= \frac{ae^{-j\omega}}{(1-ae^{-j\omega})^{K+1}} + \frac{1}{(1-ae^{-j\omega})^{K}}$$

4.17 Let $x[n]$ be an arbitrary slg, not necessarily real valued, with F.T. $X(\omega)$. Express the F.T. of the following slg's in terms of $X(\omega)$.

(a) $x^*[n]$.

Sol:- $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-j\omega n}$ $= \sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n}$

$$= \sum_{n=-\infty}^{\infty} x^*[n] \cdot \left(e^{-j(-\omega)n}\right)^*$$

$$= \sum_{n=-\infty}^{\infty} \left(x[n] \cdot e^{j\omega n}\right)^* = X^*(-\omega)$$

(b) $x^*[-n]$.

Sol:- $X(\omega) = \sum_{n=-\infty}^{\infty} x^*[-n] e^{-j\omega n}$ $= \sum_{n=-\infty}^{\infty} x^*[n] e^{j\omega n} = X^*(\omega)$

(c) $y[n] = x[n] - x[n-1]$

Sol:- $\sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$

$$Y(\omega) = X(\omega) - X(\omega) e^{-j\omega}$$

$$\Rightarrow Y(\omega) = (1-e^{-j\omega}) X(\omega)$$

(d) $y[n] = \sum_{k=-\infty}^{n} x[k]$

Sol:- $y[n] = \sum_{k=-\infty}^{n} x[k]$

$= y[n] - y[n-1]$

$= x[n].$

$\Rightarrow X(\omega) = (1-e^{-j\omega})Y(\omega)$

$Y(\omega) = \dfrac{X(\omega)}{(1-e^{-j\omega})}$

(e) $y[n] = x[2n]$.

Sol:- $Y(\omega) = \sum_{n=-\infty}^{\infty} x[2n] e^{-j\omega n}$

$= \sum_{n} x[n] \cdot e^{-j\frac{\omega}{2}n}$

$= X\left(\frac{\omega}{2}\right)$

(f) $y[n] = \begin{cases} x(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

Sol:- $Y(\omega) = \sum_{n} x\left[\frac{n}{2}\right] e^{-j\omega n}$

$= \sum_{n} x[n] e^{-2j\omega n}$

$= X(2\omega).$

4.18 Determine and sketch the F.T's $X_1(\omega)$, $X_2(\omega)$ & $X_3(\omega)$ of the following sig's.

(a) $x_1[n] = \{\overset{-2}{1}, \overset{-1}{1}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{1}, \overset{2}{1}\}$

Sol:- $X_1(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$= \sum_{n=-2}^{2} x[n] e^{-j\omega n} = 1 \cdot e^{+2j\omega} + 1 \cdot e^{+j\omega} + 1 \cdot e^{-j\omega(0)} +$

$1 e^{-j\omega(1)} + e^{-j2\omega}$

$= e^{j2\omega} + e^{j\omega} + e^{-j\omega} + 1 + e^{-j2\omega}$

$= (e^{2j\omega} + e^{-2j\omega}) + (e^{j\omega} + e^{-j\omega}) + 1$

$= 1 + 2\cos(2\omega) + 2\cos(\omega).$

(b) $x_2[n] = \{\overset{-4}{1}, \overset{-3}{0}, \overset{-2}{1}, \overset{-1}{0}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{0}, \overset{2}{1}, \overset{3}{0}, \overset{4}{1}\}$.

Sol:- $X_2(\omega) = \sum_{n=-4}^{4} x[n] e^{-j\omega n}$

$= 1 \cdot e^{j4\omega} + 1 \cdot e^{2j\omega} + 1 \cdot e^{-j\omega(0)} + e^{-j2\omega} + e^{-j4\omega}$

$= 1 + \left(e^{4j\omega} + e^{-4j\omega}\right) + \left(e^{2j\omega} + e^{-2j\omega}\right)$

$= 1 + 2\cos(4\omega) + 2\cos(2\omega)$

(c) $x_3[n] = \{\overset{-6}{1}, \overset{-5}{0}, \overset{-4}{0}, \overset{-3}{1}, \overset{-2}{0}, \overset{-1}{0}, \underset{\uparrow}{\overset{0}{1}}, \overset{1}{0}, \overset{2}{0}, \overset{3}{1}, \overset{4}{0}, \overset{5}{0}, \overset{6}{1}\}$.

$= \sum_{n=-6}^{6} x[n] e^{-j\omega n}$

$= 1 \cdot e^{j6\omega} + 1 \cdot e^{j3\omega} + 1 + e^{-j3\omega} + e^{-6j\omega}$.

$= 1 + 2\cos(6\omega) + 2\cos(3\omega)$.

Scanned with CamScanner

(d) Is there any relation b/w $X_1(\omega)$, $X_2(\omega)$ & $X_3(3)$?

Sol:- $X_2(\omega) = X_1(2\omega)$

$X_3(\omega) = X_1(3\omega)$

(e) Show that if $x_k(n) = \begin{cases} x\left(\frac{n}{k}\right) & ; \text{ if } \frac{n}{k} \text{ integer} \\ 0 & ; \text{ otherwise} \end{cases}$

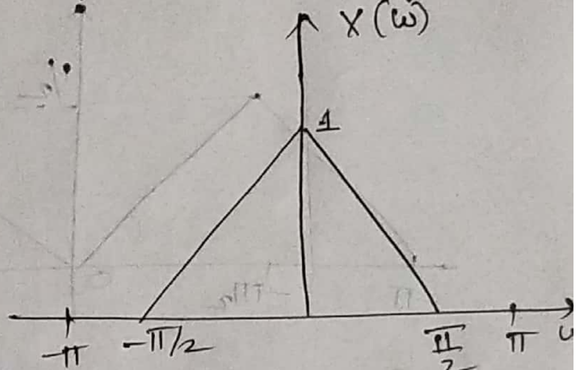
then $X_k(\omega) = X(k\omega)$

Sol:- $X_k(\omega) = \sum_n x_k[n].e^{-j\omega n}$

$= \sum_n x[n]\, e^{-jk\omega n}$

$= X(k\omega)$.

4.19 Let $x[n]$ be a sig with F.T as shown in fig. Determine and sketch the F.T of the following sig's.



(a) $x_1[n] = x[n]\cos\left(\frac{\pi n}{4}\right)$.

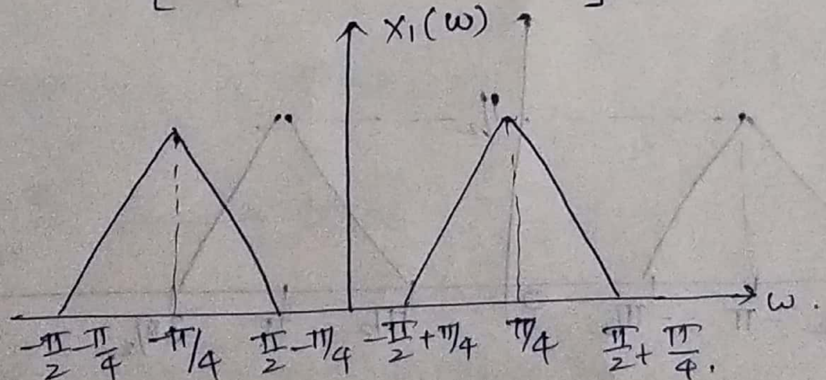
(b) $x_2[n] = x[n]\sin\left(\frac{n\pi}{2}\right)$

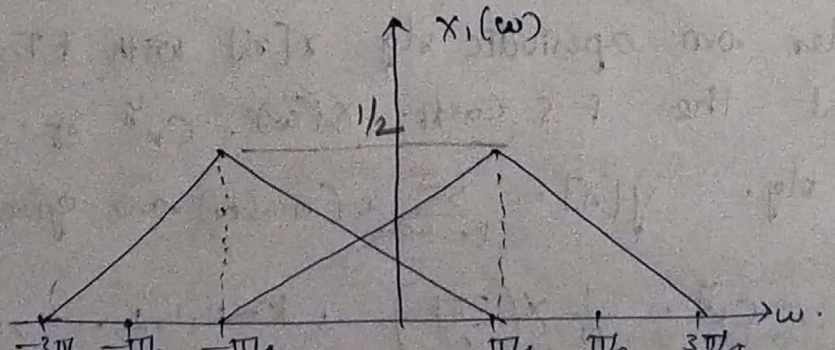
(c) $x_3[n] = x[n]\cos\left(\frac{n\pi}{2}\right)$

(d) $x_4[n] = x[n]\cos(\pi n)$.

Sol:- (a) $x_1[n] = \frac{1}{2}\left(e^{\frac{j\pi n}{4}} + e^{-\frac{j\pi n}{4}}\right) x[n]$

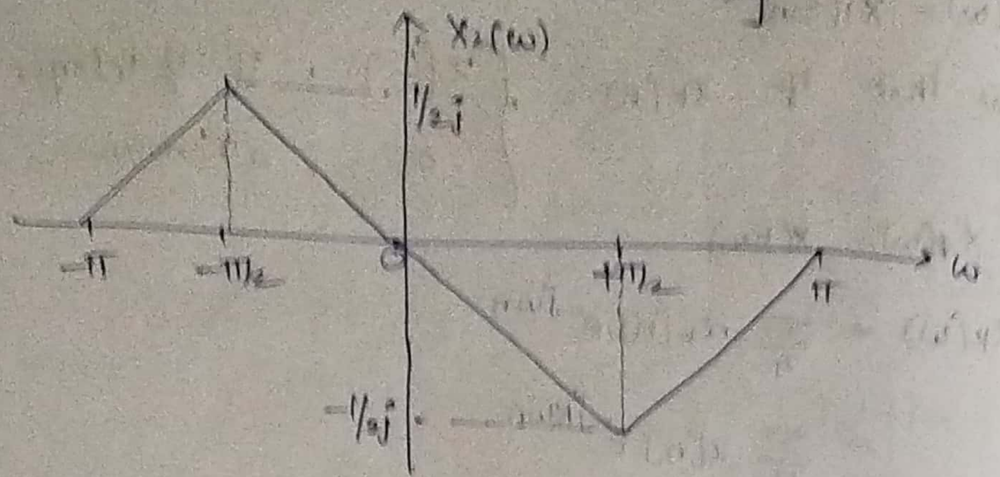
$\Rightarrow X_1(\omega) = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{4}\right) + X\left(\omega + \frac{\pi}{4}\right)\right]$





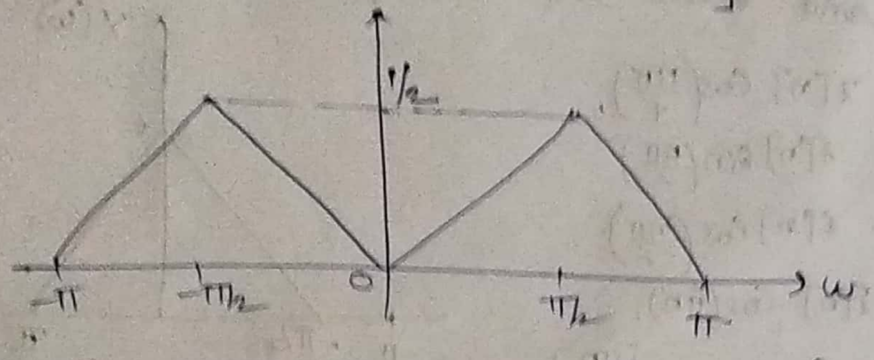
(b) $x_3[n] = x[n]\cdot\sin\left(\frac{\pi n}{2}\right) = x[n]\left(\frac{e^{j\pi n/2} + e^{-j\pi n/2}}{2j}\right)$

$$= \frac{1}{2j}\left[X\left(\omega+\frac{\pi}{2}\right) - X\left(\omega-\frac{\pi}{2}\right)\right]$$



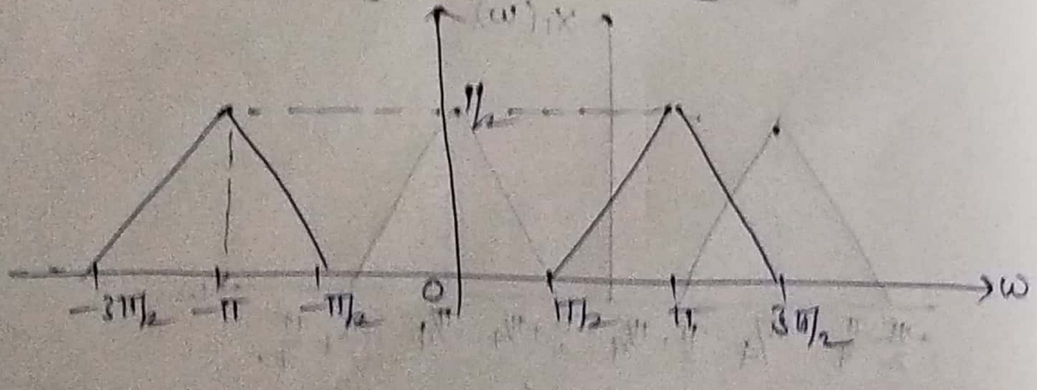
(c) $x_5[n] = x[n]\cos\left(\frac{\pi n}{2}\right)$

$$= \frac{1}{2}\left[X\left(\omega-\frac{\pi}{2}\right) + X\left(\omega+\frac{\pi}{2}\right)\right]$$



(d) $x_4[n] = x[n]\cos(\pi n)$

$$= \frac{1}{2}\left[X(\omega-\pi) + X(\omega+\pi)\right]$$



4.20. Consider an aperiodic sig $x[n]$ with FT $X(\omega)$. Show that the F.S coeff. c_k^y of the periodic sig. $y[n] = \sum_{l=-\infty}^{\infty} x(n-lN)$ are given by

$$c_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right) \quad ; \; k = 0, 1, 2, \ldots, N-1$$

Sol:- $C_k^y = \frac{1}{N}\sum_{n=0}^{N-1} y[n]\, e^{-\frac{j2\pi k n}{N}}$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right]\cdot e^{-\frac{j2\pi k n}{N}}$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=lN}^{N-1-lN} x(m)\, e^{-\frac{j2\pi k(m+lN)}{N}}$$

Let $m = n - lN$; $n = m + lN$

$m = -lN$ to $N-1-lN$.

$$X(\omega) = \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x[m]\, e^{-\frac{j2\pi k m}{N}} \cdot e^{-j2\pi k l}$$

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right).$$

4.21 prove that $X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_0 N}{\pi n} e^{-j\omega n}$ may be expressed as $X_N(\omega) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{\sin\left((2N+1)(\omega - \frac{\theta}{2})\right)}{\sin\left((\omega-\theta)/2\right)}\, d\theta$

Sol:- $x_N[n] = \frac{\sin \omega_c n}{\pi n}$; $-N \le n \le N$

$$= x[n]\, w[n]$$

$x[n] = \frac{\sin \omega_c n}{\pi n}$; $-\infty \le n \le \infty$.

$w(n) = 1$; $-N \le n \le N$.

$$\frac{\sin \omega_c n}{\pi n} \longleftrightarrow X(\omega)$$

$= 1$, $|\omega| \le \omega_c$

$= 0$, otherwise

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\theta)\, W(\omega-\theta)\, d\theta.$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{\sin\{(2N+1)(\omega-\theta)\}/2}{\sin(\omega-\theta)/2}\, d\theta$$

4.22 prove that $x[n]$ has the following F.T :

$X(\omega) = \frac{1}{1-ae^{-j\omega}}$ then determine the following.

(a) $x[2n+1]$

Sol:- $X(\omega) = \sum_n x(2n+1)\, e^{-j\omega n}$

Let $m = 2n+1$. $n = \frac{m-1}{2}$

$= \sum_m x(m)\, e^{-j\omega\left(\frac{m-1}{2}\right)}$

$= \sum_n x[m]\, e^{-\frac{j\omega m}{2}} \cdot e^{\frac{j\omega}{2}}$

$= X\left(\frac{\omega}{2}\right) \cdot e^{\frac{j\omega}{2}} = \frac{e^{j\omega/2}}{1-ae^{-\frac{j\omega}{2}}}$

(b) $e^{\frac{\pi n}{2}}\, x[n+2]$

Sol:- $= \sum_n e^{\frac{\pi n}{2}}\, x[n+2] \cdot e^{-j\omega n}$ let $k = n+2$.

$= -\sum_K x[K] \cdot e^{\frac{n\pi}{2}} \cdot e^{-j\omega(k-2)}$

$= -\sum_K x[K] \cdot e^{\frac{-jn\pi}{-j2}} \cdot e^{-j\omega k} \cdot e^{j\omega 2}$

$= -\sum_K x[K] \cdot e^{-j\left(\omega k + \frac{j\pi}{2}\right)} \cdot e^{j2\omega}$

$= -X\left(\omega + \frac{j\pi}{2}\right) e^{j2\omega}$

(c) $x[-2n]$.

Sol:- $= \sum_n x[-2n]\, e^{-j\omega n}$ Let $m = -2n$.

$= \sum_n x[m]\, e^{-j\omega\left(\frac{-m}{2}\right)}$

$= \sum_n x[m]\, e^{-j\left(-\frac{\omega}{2}\right)m}$

$= X\left(-\frac{\omega}{2}\right)$.

(d) $x[n] \cos(0.3\pi n)$

Sol:- $= \sum_n \frac{1}{2}\left(e^{j(0.3\pi n)} + e^{-j(0.3\pi n)}\right) x[n] \cdot e^{-j\omega n}$

Scanned with CamScanner

$$= \frac{1}{2} \sum_n e^{j(0.3\pi n)} \cdot x[n] \cdot e^{-j\omega n} + \frac{1}{2} \sum_n e^{-j(0.3\pi n + \omega n)} \cdot x[n]$$

$$= \frac{1}{2} \sum_n e^{-j(\omega - 0.3\pi)n} x[n] + \frac{1}{2} \sum_n e^{-j(\omega + 0.3\pi)n} \cdot x[n].$$

$$= \frac{1}{2}\left[X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right]$$

(e) $x[n] * x[n-1]$

Sol:- $x[n] * x[n-1] \xleftrightarrow{FT} X(\omega) \cdot X(\omega) \cdot e^{-j\omega}$

$$X^2(\omega)\, e^{-j\omega}$$

(f) $x[n] * x[-n]$

Sol:- $x[n] * x[-n] \xleftrightarrow{FT} X(\omega) \cdot X(-\omega)$

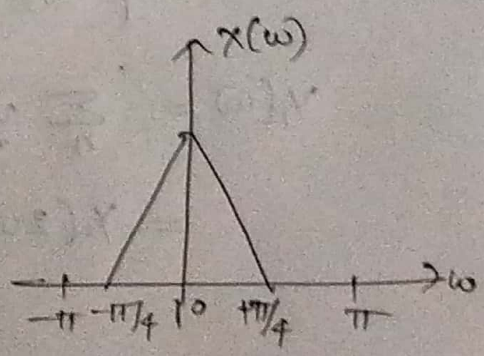
$$= \frac{1}{(1-ae^{-j\omega})} \cdot \frac{1}{(1-ae^{j\omega})}$$

$$= \frac{1}{1-ae^{j\omega}-ae^{-j\omega}+a^2}$$

$$= \frac{1}{1-2a\cos\omega+a^2}$$

4.23 From a discrete-time sig $x[n]$ with FT of $X(\omega)$ as shown in fig. determine and sketch the following sig's.

(a) $y_1[n] = \begin{cases} x[n] & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases}$



Sol:- (b) $y_2[n] = x[2n]$

(c) $y_3[n] = \begin{cases} x(n/2) & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases}$

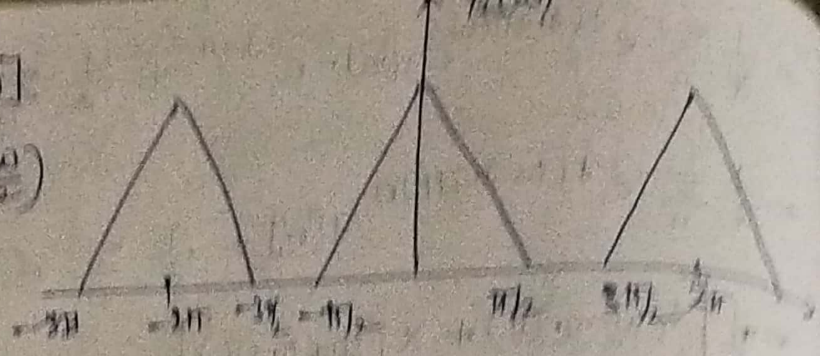
Note that

$$y_1[n] = x[n] \cdot s[n]. \text{ where}$$

$$s[n] = \{\ldots\ 0,1,0,1,0,1,0,1,\ldots\}$$

Sol: (b) $y_2[n] = x[2n]$

$Y_2(\omega) = X\left(\frac{\omega}{2}\right)$

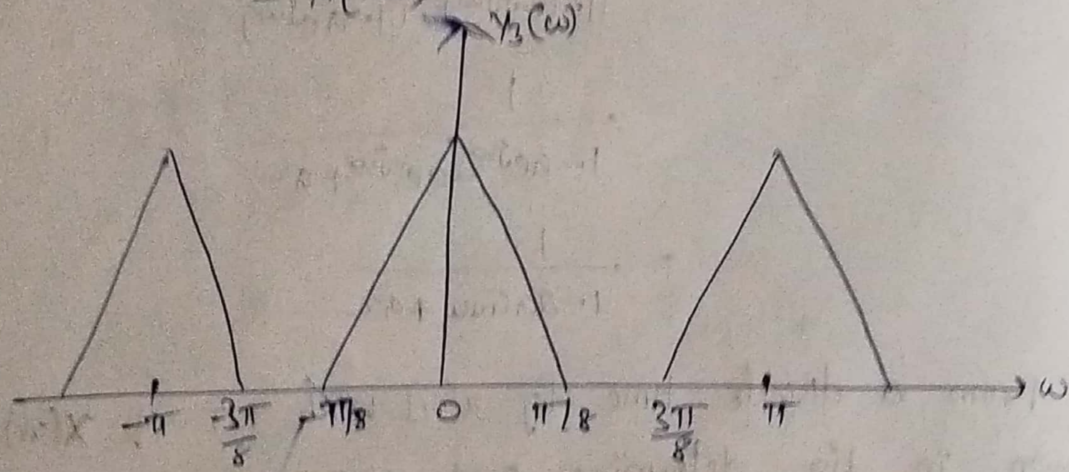


(c) $y_3[n] = \begin{cases} x\left(\frac{n}{2}\right), & n \text{ even.} \\ 0, & n \text{ odd.} \end{cases}$

$Y_3(\omega) = \sum_n y_3[n] e^{-j\omega n}$

$= \sum_{n \text{ even}} x\left(\frac{n}{2}\right) e^{-j\omega n}$

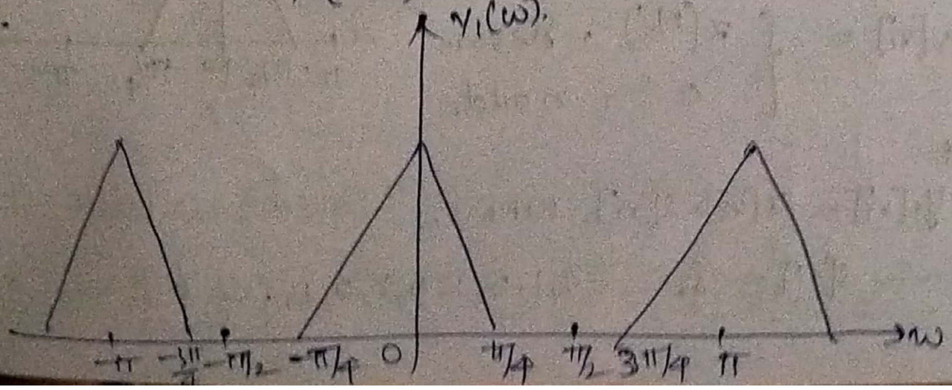
$= X(2\omega)$



(a) $y_1[n] = \begin{cases} y_2\left(\frac{n}{2}\right), & n \text{ even.} \\ 0, & n \text{ odd.} \end{cases}$

$Y_1(\omega) = \sum_n y_2\left[\frac{n}{2}\right] e^{-j\omega n}$

$= Y_2(2\omega)$

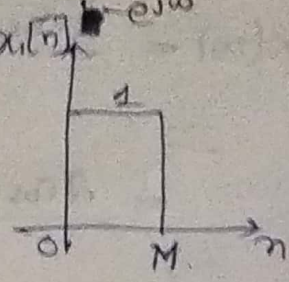


4.13 $x[n] = \begin{cases} 1; & -m \le n \le m \\ 0; & \text{otherwise} \end{cases}$ The F.T of $x[n]$ is shown

to be $X(\omega) = 1 + 2\sum_{n=1}^{M} \cos \omega n$. Show that the F.T of

$x_1[n] = \begin{cases} 1; & 0 \le n \le m. \\ 0; & \text{otherwise} \end{cases}$ $x_2[n] = \begin{cases} 1; & -m \le n \le -1. \\ 0; & \text{otherwise} \end{cases}$ are

respectively $X_1(\omega) = \dfrac{1 - e^{-j\omega(m+1)}}{1 - e^{-j\omega}}$, $X_2(\omega) = \dfrac{e^{j\omega} - e^{j\omega(m+1)}}{1 - e^{j\omega}}$

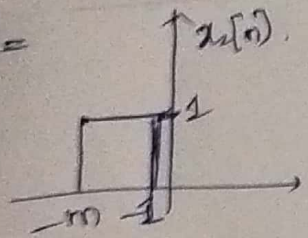


Sol: ~~$x[n]$~~ $X_1(\omega) = \sum_{n=0}^{M} 1 \cdot e^{-j\omega n}$

$= 1 + e^{-j\omega} + e^{-j2\omega} + \cdots + e^{-j\omega m}$

$= \dfrac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$

$x_2[n] =$



$X_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n} = \sum_{m=1}^{m} e^{j\omega m}$.

let $\boxed{m = -n}$

$= e^{j\omega} + (e^{j\omega})^2 + \cdots + e^{j\omega m}$.

$X_2(\omega) = \dfrac{e^{j\omega}(1 - e^{j\omega m})}{1 - e^{j\omega}}$

$\Rightarrow X(\omega) = X_1(\omega) + X_2(\omega)$

$= \dfrac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}} + \dfrac{e^{j\omega}(1 - e^{j\omega m})}{1 - e^{j\omega}}$.

$= \dfrac{e^{j\omega} - e^{-j\omega m}}{e^{j\omega} - 1} + \dfrac{1 - e^{j\omega m}}{e^{-j\omega} - 1}$.

$= \dfrac{(e^{-j\omega} - 1)(e^{j\omega} - e^{-j\omega m}) + (1 - e^{j\omega m})(e^{j\omega} - 1)}{(e^{j\omega} - 1)(e^{-j\omega} - 1)}$

$$= \frac{1 - e^{-j\omega(m+1)} - e^{j\omega} + e^{-j\omega m} + e^{j\omega} - 1 - e^{j\omega(m+1)} + e^{j\omega m}}{1 - e^{j\omega} - e^{-j\omega} + 1}$$

$$= \frac{-e^{-j\omega(m+1)} - e^{j\omega(m+1)} + e^{-j\omega m} + e^{j\omega m}}{2 - e^{j\omega} - e^{-j\omega}}$$

$$X(\omega) = \frac{2\cos\omega m - 2\cos\omega(m+1)}{2 - 2\cos\omega}$$

$$= \frac{2\cos\omega m - 2\cos\omega m \cdot \cos\omega + 2\sin\omega m \cdot \sin\omega}{2(1-\cos\omega)}$$

$$= \frac{2\cos\omega m(1-\cos\omega) + 2\sin\omega m \cdot \sin\omega}{2\sin^2\left(\frac{\omega}{2}\right)}$$

$$\Rightarrow \left(\cos 2\cdot\frac{\theta}{2} = 1 - 2\sin^2\theta/2\right).$$

$$\left(2\sin^2\theta/2 = 1 - \cos 2\theta\right)$$

$$= \frac{2\sin^2\omega/2 \cdot \cos\omega m + 2\sin\omega m \cdot \sin\omega}{2\sin^2\omega/2}$$

$$= \frac{\cancel{2}\sin(\omega m + \omega/2)\cos\omega/2}{\cancel{2}\sin^2\omega/2}$$

$$X(\omega) = \frac{\sin(m+1/2)\omega}{\sin(\omega/2)}$$