Problems

1) A discrete-time sig $x(n)$ is defined as

$$x(n) = \begin{cases} 1+\frac{n}{3} & , -3 \le n \le -1 \\ 1 & , 0 \le n \le 3 \\ 0 & , \text{elsewhere.} \end{cases}$$

a) Determine its value and sketch the sig $x(n)$.

Sol

$1+\frac{n}{3}$, $-3 \le n \le -1$ $\Rightarrow$ at $n=-3$ 0

at $n=-2$ $\frac{1}{3}$

at $n=-1$ $\frac{2}{3}$

from $0 \le n \le 3$ 1

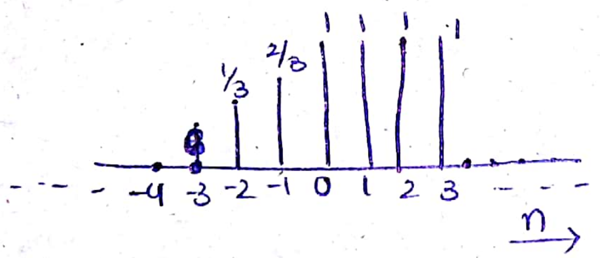
elsewhere 0.

so $x(n) = \{ \ldots -0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots \}$

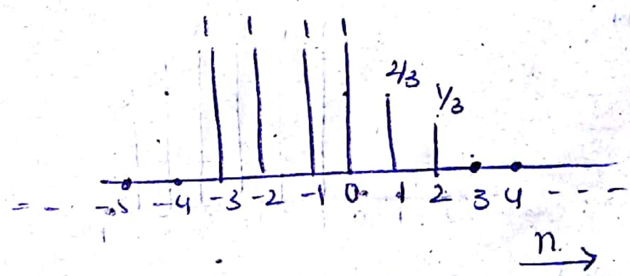
b) Sketch its value and the signals that results if we

i) First fold $x(n)$ and then delay the resulting by 4 samples

Sol:- $x(n)$



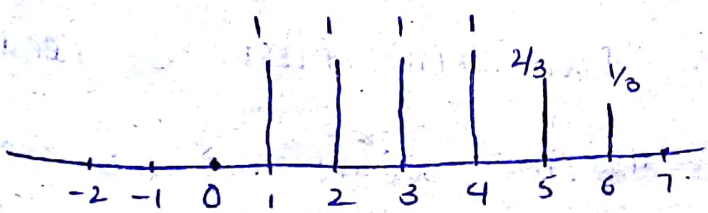
folding $x(n) = x(-n)$



• $x(-n+4)$ → after delaying $(-3 \le n+4 \le 2)$

$-7 \le -n \le -2$

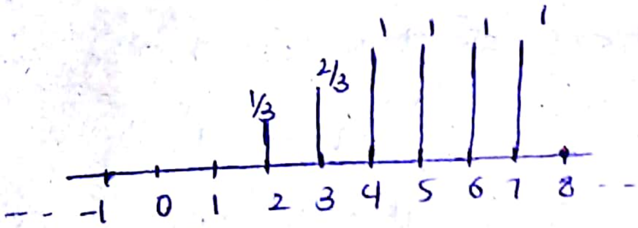
$7 \ge n \ge -2$



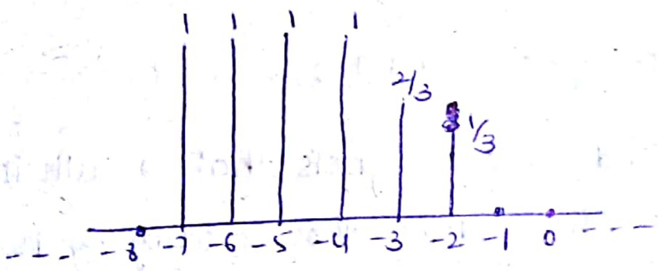
ii) First delay $x(n)$ by four samples & then find the resulting s/g.

Sol $x(n) \rightarrow$ delay by 4 samples

$x(n-4)$

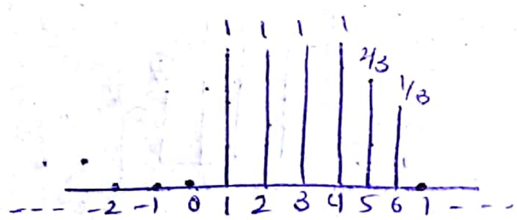


folding $x(n-4) \rightarrow \cancel{x(-(n-4))}\ x(-n-4)$



c) Sketch the s/g $x(-n+4)$

Sol :-



d) Compare the results in parts (b) & (c) and derive a rule for obtaining the s/g $x(-n+k)$ from $x(n)$

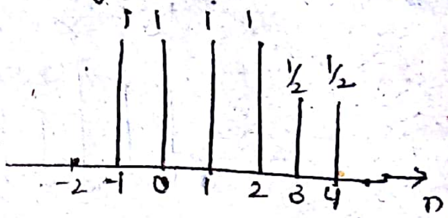
Sol:- By comparing results in parts (b) & (c) we can say that to get $x(-n+k)$ from $x(n)$ first we need to fold $x(n)$ which results in $x(-n)$ and then we need to shift by k samples to right if $k>0$ (or) to left if $k<0$ results in $x(-n+k)$.

e) Can you express the s/g $x(n)$ in terms of s/g's $\delta(n)$ & $u(n)$.

Sol Yes.

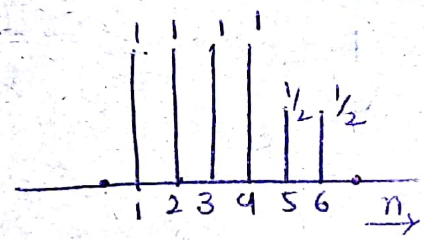
$$x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$$

2) A discrete-time s/g $x(n)$ is shown in figure. Sketch & label carefully each of the following s/g's.



a) $x(n-2)$

Sol:-

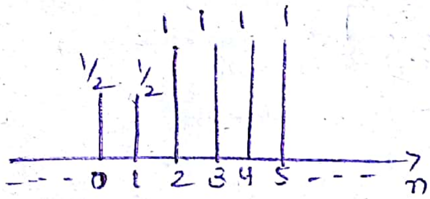


b) $x(4-n)$

$-1 \le 4-n \le 4$

$-5 \le -n \le 0$

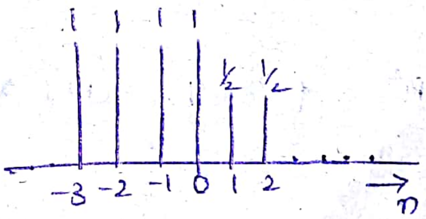
$5 \ge n \ge 0$



c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n \le 2$



d) $x(n)\, u(2-n)$

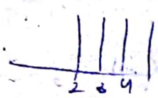
Sol

~~$0 \le 2\; n \le \infty$~~

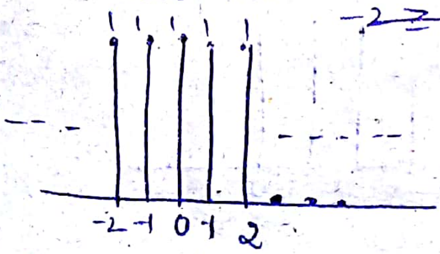
~~$-2 \le n \le \infty$~~

~~$-2 \ge n \ge \infty$~~

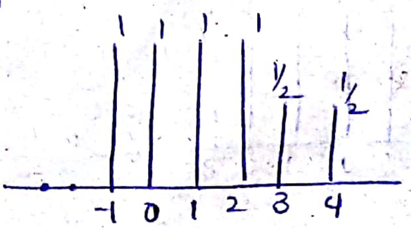
$u[-(n-2)]$



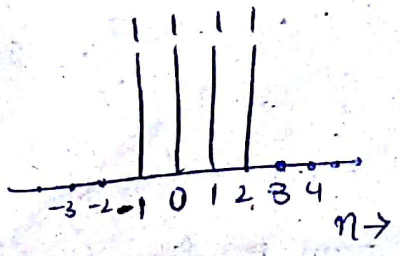
$u(2-n)$



$x(n)$



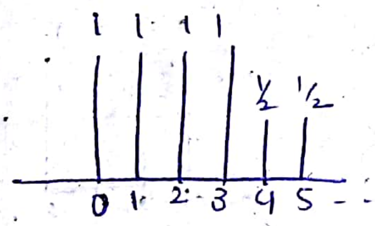
$x(n)\,u(n-2)$



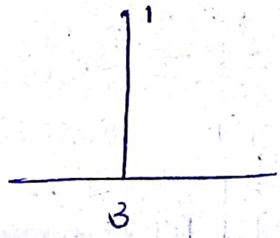
e) $x(n-1)\,\delta(n-3)$

Sol:-

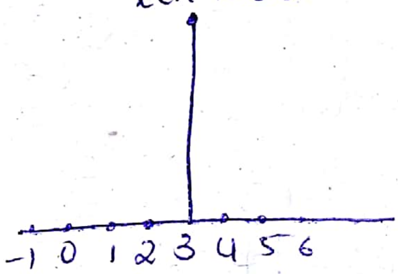
$x(n-1)$



$\delta(n-3)$



$x(n-1)\,\delta(n-3)$



f) $x(n^2)$

Sol:- $x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4) \dots\}$

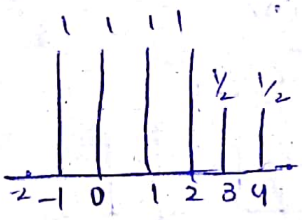
$x(n^2) = \{x(4), x(1), x(0), x(1), x(4), x(9), x(16) \dots\}$

$= \{\dots \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0\}$

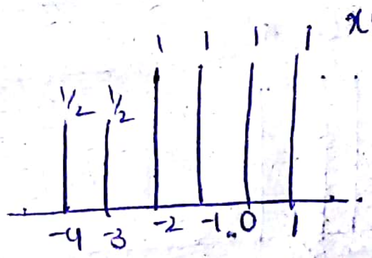
g) even part of $x_e(n) = \dfrac{x(n) + x(-n)}{2}$

Sol:

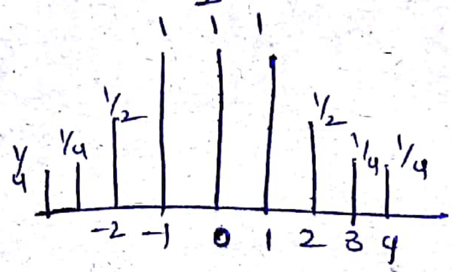
$x(n)$



$x(-n)$

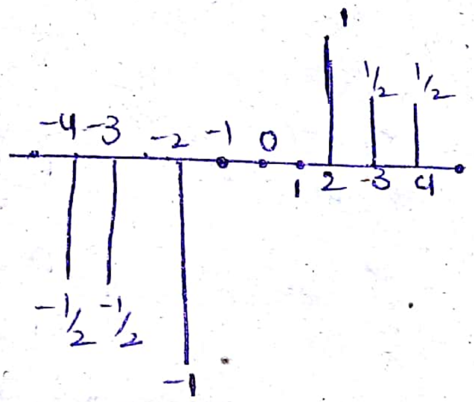


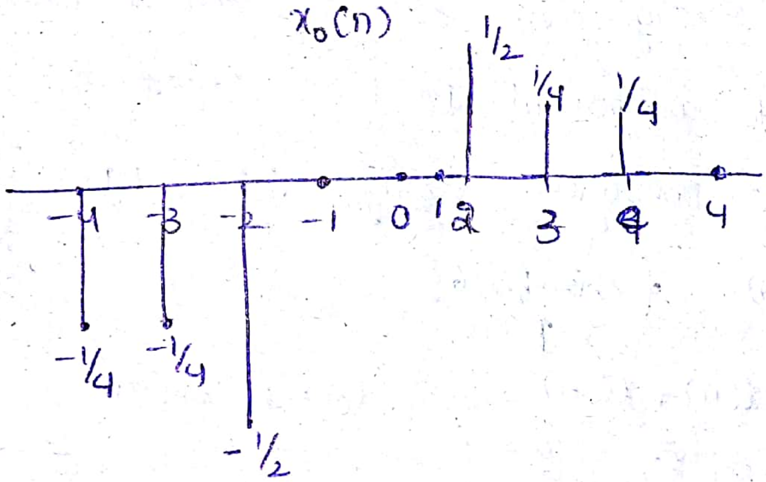
$\frac{x(n)+x(-n)}{2}$



h) oddpart of $x_o(n) = \frac{x(n) - x(-n)}{2}$

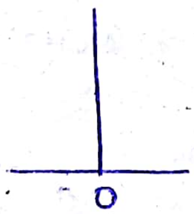
$x(n) - x(-n)$



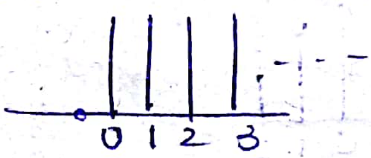


3) a) show that $\delta(n) = u(n) - u(n-1)$

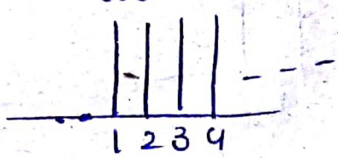
S.I We know $\delta(n)$



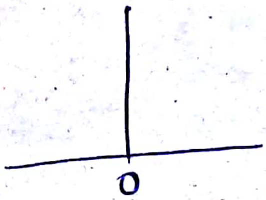
u(n)



u(n-1)



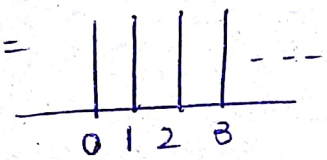
$u(n) - u(n-1)$



$\therefore \delta(n) = u(n) - u(n-1)$

b) $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$.

Sol: $u(n) =$



$\Rightarrow \sum_{k=-\infty}^{n} \delta(k) = u(n) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{k=0}^{\infty} \delta(n-k) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

4) Show that any s/g can be decomposed into an even & an odd component. Is the decomposition unique? Illustrate your component using the s/g.

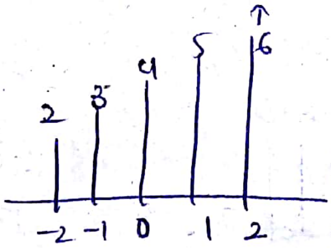
$$x(n) = \{2, 3, 4, 5, \underset{\uparrow}{6}\}$$

Sol:- $x_e(n) = \frac{x(n) + x(-n)}{2}$ $\qquad x_e(n) = x_e(-n)$

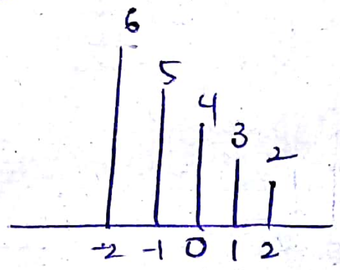
$x_o(n) = \frac{x(n) - x(-n)}{2}$ $\qquad x_o(n) = -x_o(n)$

$\Rightarrow x(n) = x_e(n) + x_o(n)$

$x(n) = \{2, 3, 4, 5, \underset{\uparrow}{6}\}$

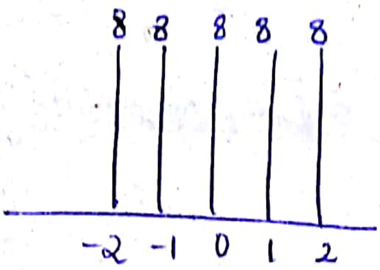


$x(n)$

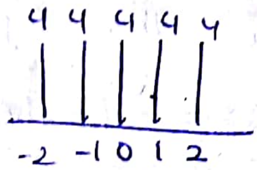


$x(-n)$

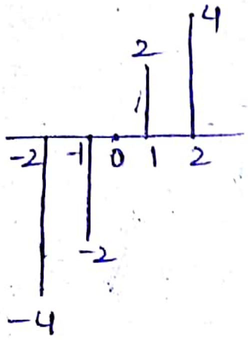
$x(n)+x(-n)$



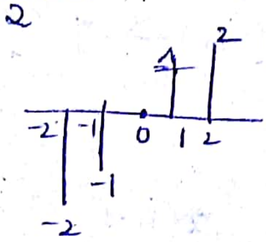
$$x_e(n) = \frac{x(n)+x(-n)}{2} \Rightarrow$$



$x(n)-x(-n)$



$$\frac{x(n)-x(-n)}{2}$$



5) Show that the energy (power) of a real-valued energy (power) signal is equal to the sum of the energy (power) of its even and odd components.

Sol

$$\sum_{n=-\infty}^{\infty} x_e(n)+x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n)+x_o(n) = \sum_{m=-\infty}^{\infty} x_e(-m)\, x_o(m)$$

$$= -\sum_{m=-\infty}^{\infty} x_e(m)\, x_o(m)$$

$$= -\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= 0$$

Energy (power)

$$\Rightarrow \sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n)+x_o(n)]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2\, x_e(n)\, x_o(n)$$

$$= \sum_{n=-\infty}^{\infty} x_e^2(n) + \sum_{n=-\infty}^{\infty} x_o^2(n) + 2\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$$

$$= E_e + E_o + 0$$

$$E = E_e + E_o$$

6) Consider the system $y(n) = T[x(n)] = x(n^2)$

a) Determine if the system is time invariant.

Sol Given $y(n) = T[x(n)] = x(n^2)$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$$

$$= x[n^2 + k^2 - 2nk]$$

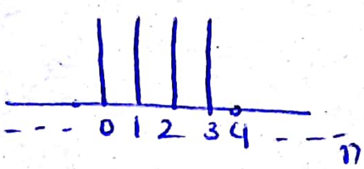
$$x(n-k) \neq y(n-k)$$

so the s/m is time variant

b) Clarify the result in part (a) assume that the s/g $x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$ is applied into the s/m

1) sketch the s/g $x(n)$.

Sol) $x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, 0, \ldots\}$



2) Determine and sketch the s/g $y(n) = T[x(n)]$

Sol $y(n) = T[x(n)] = x(n^2) \Rightarrow \{x(0), x(1), x(2^2), x(3^2), \ldots\}$

$$\Rightarrow \{x(0), x(1), x(4), x(9), \ldots\}$$

$y(n) = x(n^2) = \{\ldots 1, \underset{\uparrow 0}{1}, 0, 0, 0, \ldots\}$

3) Sketch the s/g $y_2'(n) = y(n-2)$

Sol $y(n-2) = \{\ldots 0, 0, 1, \underset{\uparrow}{1}, 0, 0, 0 \ldots\}$

4) Sketch the s/g $x_2(n) = x(n-2)$

Sol $x(n-2) = \{\ldots 0, 0, 1, \underset{\uparrow 0}{1}, 1, 1, 0, \ldots\}$

5) Determine & sketch the s/g $y_2(n) = T[x_2(n-2)]$

Sol $y_2(n) = T[x_2(n-2)] = \{\ldots x(0), x(1), x(2), x(3), \ldots\}$

$= \{\ldots 0, 1, 0, \underset{\uparrow}{0}, 0, 1, 0 \ldots\}$

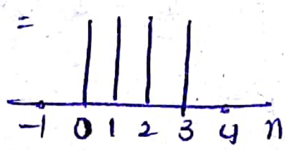
6) Compare the s/g's $y_2(n)$ & $y(n-2)$. what is your conclusion.

Sol $y_2(n) \neq y(n-2) \rightarrow$ s/m is time variant

c) Repeat part (b) for the s/m

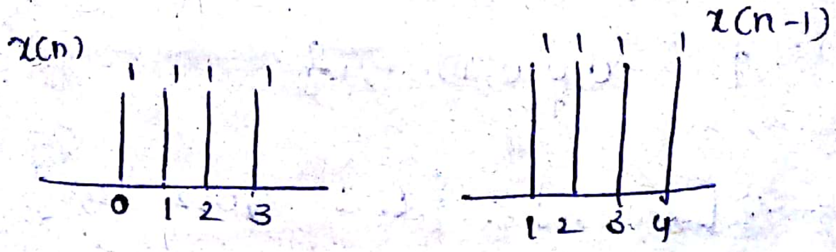
$y(n) = x(n) - x(n-1)$. Can you use this statement about the time invariance of the s/m.

Sol 1) $x(n) =$

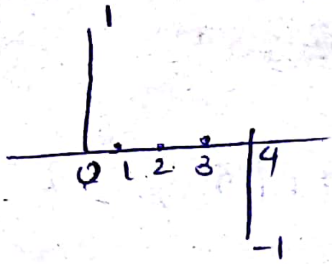


$= \{\underset{\uparrow}{1}, 1, 1, 1\}$

2) $y(n) = x(n) - x(n-1)$

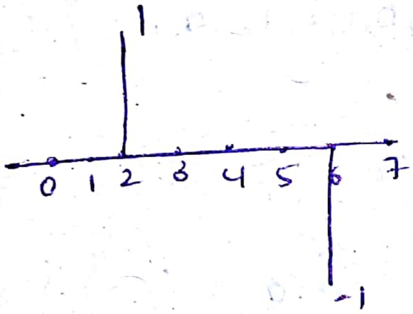


$y(n) = x(n) - x(n-1)$



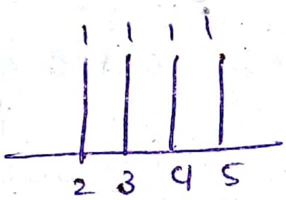
$\{0, \underset{\uparrow}{1}, 0, 0; 0, -1\}$

3) $y(n-2)$



$\{0, 0, 1, \underset{\uparrow}{0}, 0, 0, -1\}$

4) $x(n-2) \Rightarrow$



$\Rightarrow \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$

5) $y_2(n) \Rightarrow \{0, 0, 1; 0, 0, 0, -1\}$

6) $y_2(n) = y(n-2) \Rightarrow$ s/m is time invariant

d) Repeat parts (b) & (c) for the s/m $y(n) = T[x(n)] = nx(n)$

So 1) $y(n) = n\,x(n)$

$x(n) = \{\ldots 0, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$ $\quad n \to 0, 1, 2, \ldots$

2) $y(n) = \{\ldots 0, \underset{\uparrow}{1}, 2, 3, 4, \ldots\}$

3) $y(n-2) = \{\ldots 0, \underset{\uparrow}{0}, 0; 1, 2, 3, 4, \ldots\}$

4) $x(n-2) = \{\ldots 0, 0; \underset{\uparrow}{0}, 1, 1, 1, 1 \ldots\}$

5) $y_2(n) = T[x(n-2)] = \{\ldots\ldots 0, 0, 2, 3, 4, 5\}$

6) $y_2(n) \neq y(n-2) \Rightarrow$ s/m is time variant

7) (1) Static or dynamic (2) Linear or non-linear

(3) causal or non-causal (4) Time invariant or varying.

5) Stable or unstable

Examine the following systems w.r.t the properties above

a) $y(n) = \cos[x(n)]$

i) Static (only present i/p)

ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos[x_2(n)]$

$y(n) = \cos[x_1(n)] + \cos[x_2(n)]$

$y'(n) = \cos[x_1(n) + x_2(n)]$

Non-linear

iii) Only present i/p → causal

iv) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos[x(n-n_0)]$

Time variant

v) stable

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Sol: Dynamic (depends on future values)

Linear, Time invariant, non causal, unstable.

c) $y(n) = x(n) \cos(\omega_0 (n))$

Sol static, linear, time variant, causal, stable

$$y(n) = x(n-n_0) \cos(\omega_0 n - n_0)$$

$$y'(n) = x(n-n_0) \cos \omega_0 n$$

d) $y(n) = x(-n+2)$

Sol Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

$\downarrow$

future value

$$y_1(n) = x_1(-n+2) + x_2(-n+2)$$

$$y_2(n) = x_1(-n+2) + x_2(-n+2)$$

Linear

Non causal, stable, time invariant

e) $y(n) = \text{Tran}[x(n)]$

Sol static, non linear, time invariant, causal, stable

f) $y(n) = \text{Round}[x(n)]$

Sol Static, non-linear, time invariant, causal, stable

g) $y(n) = |x(n)|$

Sol static, non linear, time invariant, causal; stable

h) $y(n) = x(n)\, u(n)$

Sol static, linear, time variant, ~~Non~~ causal; ~~unstab~~ stable

i) $y(n) = x(n) + n\, x(n+1)$

Sol dynamic, linear, time variant, non-causal, unstable

(j) $y(n) = x(2n)$

S/ dynamic, linear, time variant, non-causal, stable

k) $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0; & \text{if } x(n) < 0 \end{cases}$

S/ static, linear, time invariant, non-causal, stable

l) $y(n) = x(-n)$

S/ dynamic, linear, time variant, causal, stable

m) $y(n) = \text{sign}[x(n)]$

S/ static, non-linear, time invariant, causal, stable

n) The ideal sampling s/m with i/p $x_a(t)$ & o/p

$x(n) = x_a(nT)$, $-\infty < n < \infty$

S/ $x(n) = x_a(nT)$

static, linear, time variant, non causal, stable

Q) Let T be an LTI, relaxed and BIBO stable s/m with i/p $x(n)$ & o/p $y(n)$. show that.

a) If $x(n)$ is periodic with period N [i.e., $x(n) = x(n+N)$ for all $n \geq 0$], the o/p $y(n)$ tends to a periodic s/g with the same period.

S/

$$x(n) = x(n+N) \quad \forall n \geq 0$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(k)x(n-k) + \sum_{k=\infty}^{n} h(k)x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)x(n-k)$$

For BIBO system $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$$\therefore y(N) = y(n+N)$$

b) If $x(n)$ is bounded & tends to a constant, the o/p will also tend to a constant

Sol $x(n) = x_0(n) + a\,u(n)$ $\quad x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\Rightarrow y(n) = a\sum_{k=0}^{\infty} h(k)u(n-k) + \sum_{k=0}^{\infty} h(k)x_0(n-k) = a\sum_{k=0}^{n} h(k) + y_0(n)$$

$$\Rightarrow \sum_n x_0^2(n) < \infty = \sum_n y_0^2(n) < \infty$$

Hence $\lim_{n\to\infty} |y_0(n)| = 0 = a\sum_{k=0}^{n} h(k) =$ constant

c) If $x(n)$ is energy s/g, the o/p $y(n)$ will also be an energy s/g.

Sol $y(n) = \sum_k h(k)\, x(n-k)$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k)\, x(n-k)\right]^2$$

$$= \sum_k \sum_l h(k)h(l) \sum_n x(n-k)\, x(n-l)$$

but $\sum_n x(n-k)x(n-l) \le \sum_n x^2(n) |h(l)|$

for BIBO stable s/m $\sum_k |h(k)| < M$

Hence $E_y \le m^2 E_x$, so that $E_y < 0$ if $E_x < 0$

10) The following input-output pairs have been observed during the operation of a time invariant s/m.

$$x_1(n) = \{\underset{\uparrow}{1},0,2\} \xleftrightarrow{T} y_1(n) = \{\underset{\uparrow}{0},1,2\}$$

$$x_2(n) = \{0,\underset{\uparrow}{0},3\} \xleftrightarrow{T} y_2(n) = \{0,\underset{\uparrow}{1},0,2\}$$

$$x_3(n) = \{0,\underset{\uparrow}{0},0,1\} \xleftrightarrow{T} y_3(n) = \{1,2,\underset{\uparrow}{1}\}$$

Can you draw any conclusion regarding the linearity of the s/m what is the impulse response of the s/m

Sol As this is a time-invariant s/m

$y_2(n)$ should have only 3 elements &

$y_3(n)$ should have 4 elements.

So, it is non-linear

11) The following i/p - o/p pairs have been observed during the operation of a linear s/m:

$$x_1(n) = \{-1,\underset{\uparrow}{2},1\} \xrightarrow{T} y_1(n) = \{1,\underset{\uparrow}{2},-1,0,1\}$$

$$x_2(n) = \{1,-1,-1\} \xrightarrow{T} y_2(n) = \{-1,\underset{\uparrow}{1},0,2\}$$

$$x_3(n) = \{0,1\} \xrightarrow{T} y_3(n) = \{\underset{\uparrow}{1},2,1\}$$

Can you draw any conclusions about the time invariance of this system.

S.I since $x_1(n) + x_2(n) = \delta(n)$

& s/m is linear, the impulse response of the s/m is

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

if s/m were time invariant. the response of $x_3(n)$

would be $\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$

12) The only available information about the s/m consists of N input-output pairs of signals $y_i(n) = T[x_i(n)]$; $i = 1, 2, 3, \ldots N$.

a) what is the class of input s/g's for which we can determine the o/p, using the information above, if the s/m is known to be linear.

S.I Any linear combination of s/g in the form of

$x_i(n)$; $i = 1, 2, \ldots N$

because if we take $i = 1, 3$.

$y_1(n) = x_1(n)$

$\Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

$y_3(n) = x_3(n)$

$y(n) = x_1(n) + x_3(n)$

Linear.

b) Same repeat, for the s/m is invariant.

Sol Any $x_i(n-k)$ where k is any integer, $i = 1, 2, \ldots N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n-n_0) \Rightarrow x_i(n-k-n_0)$

[Time invariant]

(3) Show that the necessary and sufficient condition for a relaxed LTI s/m to be BIBO stable is

$$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_n < \infty \quad \text{for constant } M_n.$$

Sol A s/m to be BIBO stable only when bounded O/P produce bounded input

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$|y(n)| = \sum_k |h(k)|\, |x(n-k)|$$

$$= \sum_k |x(n-k)| \leq M_n \quad \text{[some constant]}$$

So $|y(n)| = M_n \sum_k |h(k)|$

$|y(n)| < \infty$ for all n, if & only if $\sum_k |h(k)| < \infty$

So $\sum_{n=-\infty}^{\infty} |y(n)|$

$\Rightarrow$ A s/m to be BIBO stable only when bounded i/p produces bounded O/p.

$$y(n) = \sum_{k=\infty}^{\infty} h(k)\, x(n-k) \;;\; n \leq n-k$$

$$k \geq 0$$

$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)|.$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \leq M_n$ for some constant

$$|y(n)| = M_n \sum_{k=-\infty}^{\infty} |h(k)| \; ; \; n \le n-k$$
$$k \ge 0$$

$|y(n)|$ is $< \infty$ if and only if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

$$\text{so} \quad \sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

14) Show that

a) A relaxed line s/m is causal if & only if for any input $x(n)$ such that $x(n) = 0$ for $n < n_0 \Rightarrow y(n) = 0$ for $n < n_0$

Sol If a system is causal output depends only on the present and past inputs as $x(n) = 0$ for $n < n_0$ then $y(n)$ also because zero for $n < n_0$

b) A relaxed LTI s/m is causal if and only if $h(n) = 0$ for $n < 0$.

Sol :- $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

For finite impulse response

$$h(n) = 0 \; , \; n < 0 \text{ and } n \ge M$$

So $y(n)$ reduces to $y(n) = \sum_{k=0}^{n-1} h(k)\, x(n-k)$

$$\therefore y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

15) Show that for any real or complex constant a, and any finite integer numbers m and N, we have

a) $$\sum_{n=M}^{N} n = a^n = \begin{cases} \dfrac{a^m - a^{N+1}}{1-a} \; , & \text{if } a \ne 1 \\ N - m + 1 \; ; & \text{if } a = 1 \end{cases}$$

Sol for $a=1$, $\sum_{n=M}^{N} a^n = N-m+1$

for $a \neq 1$, $\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a}$

$$(1-a)^N \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \dots a^N - a^N - a^{N+1}$$

$$= a^m - a^{N+1}$$

6) For $M=0$, $|a|<1$ & $N \to \infty$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \ |a|<1.$$

16) a) If $y(n) = x(n) * h(n)$, show that $\mathcal{E}_y = \mathcal{E}_x \mathcal{E}_n$ where

$$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} x(n)$$

Sol

$$y(n) = \sum_{k} h(k)\, x(n-k)$$

$$\sum_{n} y(n) = \sum_{n}\sum_{k} h(k)\, x(n-k)$$

$$\sum_{n} y(n) = \sum_{k} h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_{n} y(n) = \left(\sum_{k} h(k)\right)\left(\sum_{n} x(n)\right)$$

b) Compute the convolution $x(n)*h(n)$ of the following slg's and check the corrections of the results by using the test in (a)

1) $x(n) = \{1,2,4\}$, $h(n) = \{1,1,1,1,1\}$

Sol

$$y(n) = \{1,3,7,7,7,6,4\}$$

$$\sum_{n} y(n) = 35; \quad \sum_{n} x(n) = 7. \quad \sum_{n} h(n) = 5$$

By Tabular method

$$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$$

$35 = 7 \times 5$

$35 = 35$

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

Sol. $x(n) = \{1, 2, -1\}$, $h(n) = \{1, 2, -1\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) \sum_n h(n)$

$4 = 4$

h(n) \ x(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

Sol $y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, 2\}$

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$, $\sum_n h(n) = 5/2$

$\sum_n y(n) = \sum_n x(n)\, h(n)$

$-5 = -5$

h(n) \ x(n)	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

4) $x(n) = \{1,2,3,4,5\}$; $h(n) = \{1\}$

Sol $y(n) = \{1,2,3,4,5\}$

$\sum_n y(n) = \sum_n x(n) \cdot \sum_n h(n)$

$15 = 15(1)$

$15 = 15$

5) $x(n) = \{1,2,3\}$, $h(n) = \{0,0,1,1,1,1\}$

Sol $y(n) = \{0,0,1,-1,2,2,1,3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

h(n) \ x(n)	1	-2	3
→ 0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

6) $x(n) = \{0,0,1,1,1,1\}$, $h(n) = \{1,-2,3\}$

Sol $y(n) = \{\underset{\uparrow}{0},0,1,-1,2,2,1,3\}$

$\sum_n y(n) = 8$, $\sum_n x(n) = 4$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$8 = 8$

h(n) \ x(n)	0	0	1	1	1
1	0	0	1	1	1
-2	0	0	-2	-2	-2
3	0	0	3	3	3

7) $x(n) = \{\underset{\uparrow}{0},1,4,-3\}$, $h(n) = \{1,0,-1,-1\}$

Sol $y(n) = \{0,1,4,-4,-5,-1,3\}$

$\sum_n y(n) = -2$, $\sum_n x(n) = -2$, $\sum_n h(n) = 1$

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$$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	+3
-1	0	-1	-4	3

8) $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$.

Sol $y(n) = \{1, 2, 4, 3, 2\}$

$\sum_n y(n) = 12$; $\sum_n x(n) = 4$; $\sum_n h(n) = 3$

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$12 = 4 \times 3$

$12 = 12$

h(n) \ x(n)	1	1	2
1	1	1	2
1	1	1	2
1	1	1	2

9) $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$

Sol $y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$

$\sum_n y(n) = 0$, $\sum_n x(n) = 4$, $\sum_n h(n) = 0$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	-4	-4	0	-4	-4

10) $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$, $h(n) = x(n)$

Sol $y(n) = \{1, 4, 4, 4, 10, 4, 4, 4, 4, 1\}$

$\sum_n y(n) = 36$, $\sum_n x(n) = 6$ $\sum_n h(n) = 6$

y(n) \ x(n)	1	2	0 ↓	2	1
1	1	2	0	2	1
2	2	4	0	4	2
→ 0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 36$

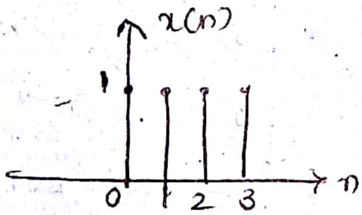
11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

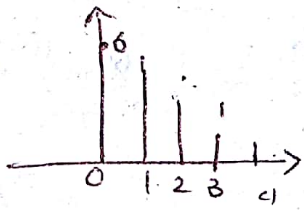
Sol $y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$ $\sum_n x(n) = 2$

17) Compute & plot convolutions $x(n) * h(n)$ and $h(n) * x(n)$ for the pairs of sig's shown below.

a)





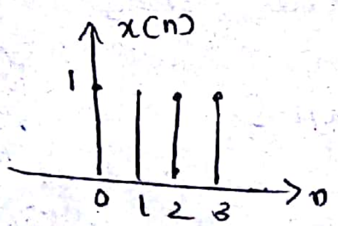
h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1
0			

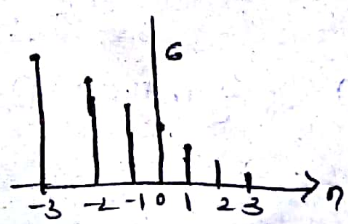
Sol $x(n) = \{1, 1, 1, 1\}$, $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

b)





6)

S.I $x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $h(n) = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1, 0\}$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$

h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

18) Determine and sketch the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

a) graphically.

Sol:- $x(n) = \{0, 1/3, 2/3, 1, 4/3, 5/3, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, \underset{\uparrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$

h(n) \ x(n)	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2

b) analytically.

S.I $x(n) = \frac{1}{3}n\,[u(n) - u(n-7)]$

$h(n) = u(n+2) - u(n-3)$

$y(n) = \frac{1}{3}n\,[u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3}\,n u(n) * u(n+2) - \frac{1}{3}\,n u(n) * u(n-3) - \frac{1}{3}\,n\,u(n-7) *$

$u(n+2) + \frac{1}{3}\,n\,u(n-7) * u(n-3)$

20) Consider the following three operations.

a) Multiply the integer numbers : 131 & 122

Sol $131 \times 122 = 15982$

b) Compute the convolution of sig's : $\{1,3,1\} * \{1,2,2\}$

Sol $y(n) = \{15, 9, 8, 2\}$

$u(n) \backslash x(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) Multiply the polynomials.

$1+3z+z^2$ & $1+2z+2z^2$

Sol. $(z^2+3z+1)\,(2z^2+2z+1)$

$= 2z^4 + 5z^3 + 9z^2 + 5z + 1$

d) Repeat part (a) for the numbers 1.31 and 12.2

Sol: $1.31 \times 12.2 = 15.982$

e) Comment on your result.

Sol These are different ways to perform convolution.

21) Compute the convolution $y(n) * h(n)$ of the following pairs of sig's.

a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ when $a \neq b$ and when $a = b$

Sol:

$y(n) = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$= [a^n * b^n]\, u(n)$

$$y(n) = \sum_{k=0}^{n} a^k u(k)\, b^{n-k} u(n-k)$$

$$= b^n \sum_{k=0}^{n} a^k u(k)\, b^{-k}$$

$$= b^n \cdot \sum_{k=0}^{n} (ab)^{-k}$$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1)\, u(n)$

b) $x(n) = \begin{cases} 1 & ; n = -2, 0, 1 \\ 2 & ; n = -1 \\ 3 & ; \text{elsewhere} \end{cases}$

$h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

Sol $x(n) = \{1, 2, 1, 1\}$ $\quad h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1\}$

h(n) \ x(n)	1	2	1	1
1	1	2	1	1
-1	-1	-2	-1	-1
0	0	0	0	0
0	0	0	0	0
1	1	2	1	1
1	1	2	1	1

22) Let $x(n)$ be the input s/g to a discrete time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding o/p.

a) Compute & sketch $x(n)$ & $y_i(n)$ in the the following cases using the small scale in all the figures

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

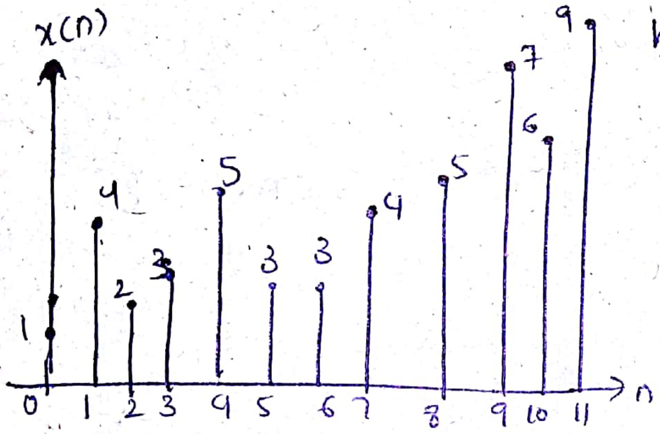
$h_1(n) = \{1,1\}$, $h_2(2) = \{1,2,1\}$, $h_3(n) = \{\frac{1}{2},\frac{1}{2}\}$, $h_4(n) = \{\frac{1}{4},\frac{1}{2},\frac{1}{4}\}$

$h_5(n) = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$

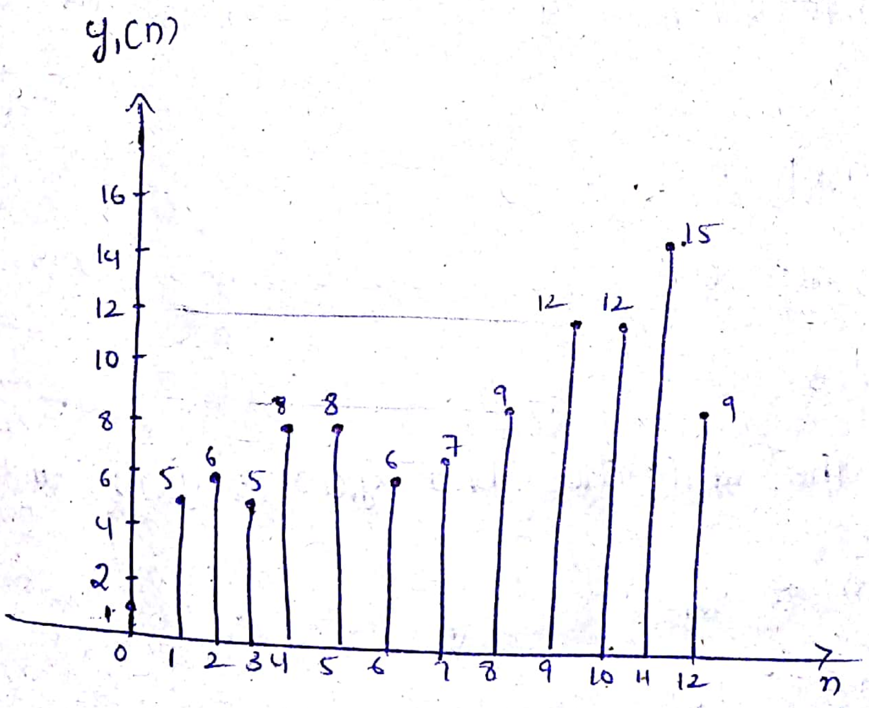
Sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph and $x(n)$, $y_3(n)$, $y_4(n)$, $y_5(n)$ on another graph.

Sol $y_1(n) = x(n) * h_1(n)$

$$y_1(n) = \{\underset{\uparrow}{1},5,6,5,8,8,6,7,9,12,12,15,9\}$$

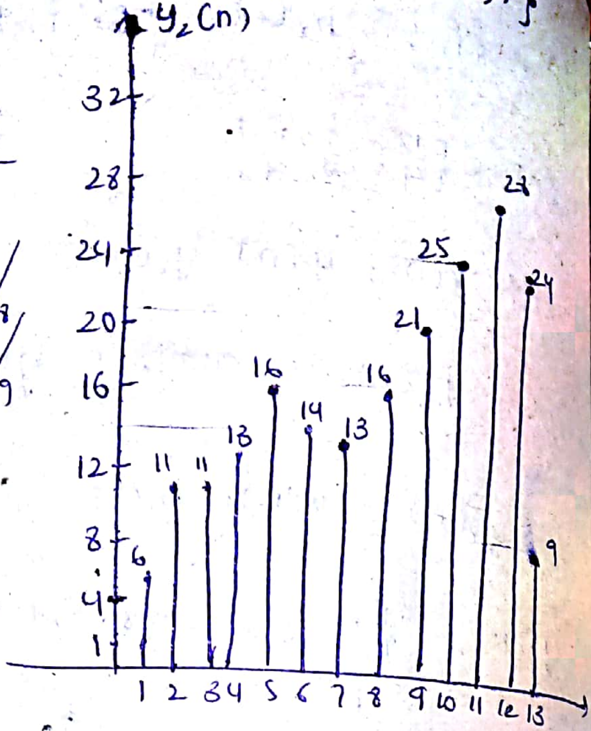


$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9



$y_2(n) = x(n) * h_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

$h_2(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9



$\rightarrow y_3(n) = x(n) * h_3(n)$

$= \{\frac{1}{2}, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

$\Rightarrow y_4(n) = x(n) * h_4(n)$

$= \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 5.25, 6.25, 6.25, 7, 6, 2.25\}$

$\Rightarrow y_5(n) = x(n) * h_5(n)$

$= \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0.25, 0, 0.25, -0.75, 1, -3, -2.25\}$

b) What is the difference b/w $y_1(n)$ & $y_2(n)$ and b/w $y_3(n)$ & $y_4(n)$

Sol

$y_3(n) = \frac{1}{2} y_1(n)$, $h_3(n) = \frac{1}{2} h_1(n)$

$y_4(n) = \frac{1}{4} y_2(n)$, $h_4(n) = \frac{1}{4} h_2(n)$

c) Comment on the smoothness of $y_2(n)$ and $y_4(n)$. which factors affect the smoothness?

S) $y_2(n)$ & $y_4(n)$ are smoother than $y_1(n)$ because of smaller scale factor.

d) Compare $y_4(n)$ with $y_5(n)$. What is the difference Can you explain it?

S) $y_4(n)$ results in smoother output than $y_5(n)$.

The negative value of $h_5(0)$ is responsible for the non-smooth characteristics of $y_5(n)$

e) Let $h_6(n) = \{\frac{1}{2}, -\frac{1}{2}\}$ compute $y_6(n)$. sketch $x(n)$, $y_2(n)$ & $y_6(n)$ on the same figure and comment on the result?

S) $y_6(n) = x(n) * h_6(n)$

$$y_6(n) = \{\frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2}\}.$$

$y_2(n)$ is more smaller than $y_6(n)$.

Q3) Express the output $y(n)$ of a linear time-invariant s/m with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ & the input $x(n)$.

Sol:- We can express $\delta(n) = u(n) - u(n-1)$

$$h(n) = h(n) * \delta(n)$$

$$= h(n) * [u(n) - u(n-1)]$$

$= h(n) * u(n) - h(n) * u(n-1)$

$= \delta(n) - \delta(n-1)$

then $y(n) = h(n) * x(n)$

$= [\delta(n) - \delta(n-1)] * x(n)$

$= \delta(n) * x(n) - \delta(n-1) * x(n)$

24). The discrete time system $y(n) = n\,y(n-1) + x(n), n \geq 0$ is [i.e, $y(-1) = 0$]. Check if the s/m is linear time invariant & BIBO stable.

Sol $y(n) = n\,y(n-1) + x(n)$, $n \geq 0$

$y_1(n) = n y_1(n-1) + x_1(n)$ — ①

$y_2(n) = n y_2(n-1) + x_2(n)$

① + ② ⇒ $y(n) = n y_1(n-1) + x_1(n) + n y_2(n-1) + x_2(n)$

$y(n) = n y(n-1) + x(n)$

$x(n) = a x_1(n) + b x_2(n)$

$y(n) = a y_1(n) + b y_2(n)$

Hence the s/m is linear.

⇒ $y(n-1) = (n-1)\,y(n-2) + x(n-1)$

delayed ⇒ $y(n-1) = n\,y(n-2) + x(n-1)$

So the s/m is time variant.

If $x(n) = u(n)$, then $|x(n)| \leq 1$, for this bounded i/p, output is $y(0) = 0$, $y(1) = 2$, $y(2) = 5$, ... unbounded

so s/m is unstable.

25) Consider the s/g $\gamma(n) = a^n u(n)$, $0<a<1$.

a) show that any sequence $x(n)$ can be decomposed as $x(n) = \sum_{n=-\infty}^{\infty} C_k \delta(n-k)$ and express C_k in terms of $x(n)$.

Sol

$$\delta(n) = \gamma(n) - a\gamma(n-1)$$

$$\delta(n-k) = \gamma(n-k) + a\gamma(n-k-1)$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$= \sum_{k=-\infty}^{\infty} x(k)\left[\gamma(n-k) - a\gamma(n-k-1)\right]$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\gamma(n-k-1).$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\gamma(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\gamma(n-k)$$

$$= \sum_{k=-\infty}^{\infty} \left[x(k) - a(x(k-1))\right)\gamma(n-k)$$

Thus $C_k = x(k) - a\,x(k-1)$

b) Use the property of linearity and time invariance to express the o/p $y(n) = T[x(n)]$ in terms of the input $x(n)$ and the s/g $g(n) = T[\gamma(n)]$, where $T[\,]$ is an LTI s/m.

Sol

$$y(n) = T[x(n)]$$

$$= T\left[\sum_{k=-\infty}^{\infty} C_k\,\gamma(n-k)\right]$$

$$= \sum_{k=-\infty}^{\infty} C_k\,T[\gamma(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} C_k\, g(n-k)$$

c) Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

Sol

$$h(n) = T[\delta(n)]$$

$$h(n) = T[\gamma(n) - a\gamma(n-1)]$$

$$= g(n) - a\, g(n-1)$$

26) Determine the zero-input response of the s/m described by the 2nd order difference equation

$$x(n) - 3y(n-1) - 4y(n-2) = 0$$

Sol With $x(n) = 0$

$$-3y(n-1) - 4y(n-2) = 0 \quad (\div(-3))$$

$$y(n-1) + \frac{4}{3} y(n-2) = 0$$

at $n=0$

$$y(-1) = -\frac{4}{3} y(-2)$$

at $n=1$

$$y(0) = -\frac{4}{3} y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

⋮

$$y(k) = \left(-\frac{4}{3}\right)^{k+2} y(-2)$$

↳ zero i/p response

27) Determine the particular solution of the difference equation $y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$ when the forcing function is $x(n) = 2^n u(n)$

Sol $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$

$x(n) = y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2)$

characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{3}$$

So $y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2\left(\frac{1}{3}\right)^n$

$x(n) = 2^n u(n)$

$y_p(n) = k(2^n)u(n)$

So $k(2^n)u(n) - k\left(\frac{5}{6}\right)(2^{n-1})u(n-1) + k\left(\frac{1}{6}\right)(2^{n-2})u(n-2)$

$= 2^n u(n)$

for $n=2$

$$4k - \frac{5k}{3} + \frac{k}{6} = 4$$

$$k = 8/5$$

Total solution is

$$y_p(n) + y_n(n) = y(n)$$

$$y(n) = \frac{8}{5}(2^n)u(n) + c_1\left(\frac{1}{2}\right)^n u(n) + c_2\left(\frac{1}{3}\right)^n u(n)$$

Assume $y(-2) = y(-1) = 0$, so $y(0) = 1$

then $y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$

so $\frac{8}{5} + c_1 + c_2 = 1$

$$c_1 + c_2 = \frac{3}{5} \rightarrow (1)$$

$$\frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = \frac{-11}{5} \rightarrow (2)$$

By solving (1) & (2)

$c_1 = -1, \quad c_2 = 2/5$

So the total solution is

$$y(n) = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right] u(n)$$

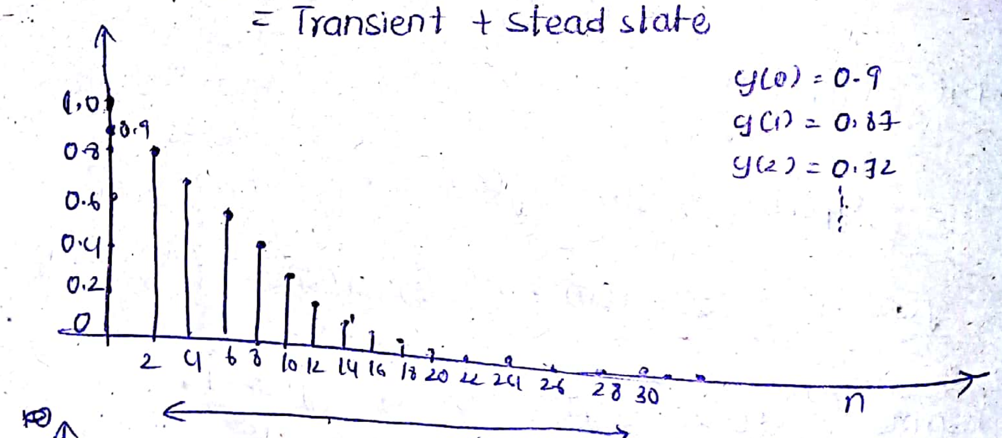
Q) In the given equation $y(n) = (-a_1)^{n+1} \cdot y(-1) + \frac{(1-(-a_1)^{n+1})}{1+a_1}$ for $n \geq 0$, seperate o/p sequence $y(n)$ into the transient response & the steady-state response. plot the response for $a_1 = -0.9$.

Sol at $y(-1) = 1$

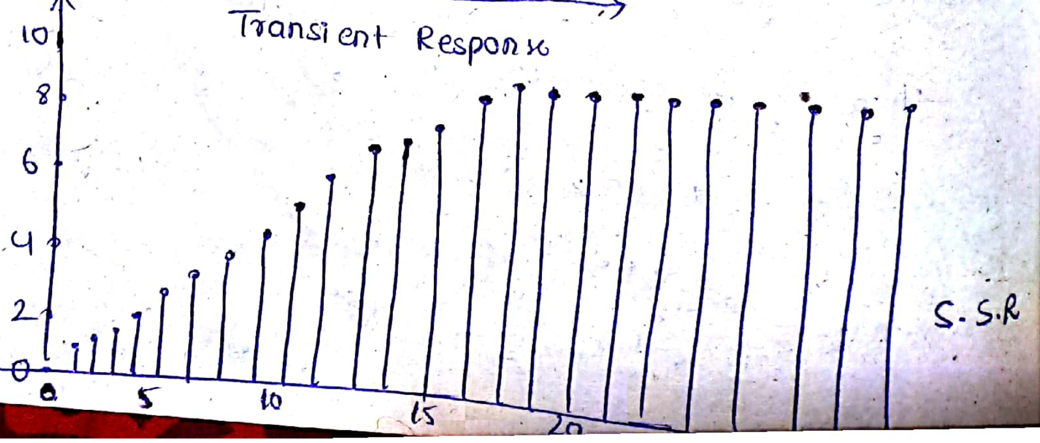
The given eqn $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

$$y(n) = y_{zi}(n) + y_{zs}(n)$$

$$= \text{Transient} + \text{stead slate}$$



$y(0) = 0.9$
$y(1) = 0.87$
$y(2) = 0.72$
$\vdots$



29) Determine the impulse response for the cascade of two linear time invariant s/m's having impulse responses:

$h_1(n) = a^n [u(n) - u(n-N)]$ and $h_2(n) = [u(n) - u(n-M)]$

Sol $h(n) = h_1(n) * h_2(n)$

$$= \sum_{k=-\infty}^{\infty} a^k [u(k) u(k-N)] [u(n-k) - u(n-k-m)]$$

$$= \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k) - \sum_{k=-\infty}^{\infty} a^k u(k) u(n-k-m) -$$

$$\sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k) + \sum_{k=-\infty}^{\infty} a^k u(k-N) u(n-k-m)$$

$$= \left(\sum_{k=0}^{n} a^k - \sum_{k=0}^{n-M} a^k \right) - \left[\sum_{k=N}^{m} a^k - \sum_{k=N}^{n-m} a^k \right]$$

$h(n) = 0$

30. Determine the response $y(n)$, $n \geq 0$, of the s/m decribed by the 2^{nd} order difference equations.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the input

$x(n) = 4^n u(n)$.

Sol $y(n) = 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

characteristic eqn is

$\lambda^2 - 3\lambda - 4 = 0$

$\lambda = 4, -1$

So $y_h(n) = c_1 4^n + c_2 (-1)^2$

$x(n) = 4^n u(n)$

$y_p(n) = kn 4^n u(n)$

$$kn\,4^n u(n) - 3k(n-1)4^{n-1}u(n-1) - 4k(n-2)4^{n-2}u(n-2) =$$

$$4^n u(n) + 2(4)^{n-1}u(n-1)$$

for $n=2$ $\quad k(32-12) = 4^2+8 = 24 \rightarrow k = 6/5.$

Total solution is

$$y(n) = y_p(n) + y_h(n)$$

$$= \left[\frac{6}{5}n4^n + c_1 4^n + c_2(-1)^n\right]u(n)$$

to find c_1 & c_2, Let $y(-2)$ & $y(-1)=0$ then $y(0)=1$

$$y(1) = 3y(0) + 4 + 2 = 9$$

$$c_2 + c_1 = 1 \rightarrow ①$$

$$\frac{24}{5} + 4c_1 - c_2 = 9$$

$$4c_1 - c_2 = \frac{21}{5} \rightarrow ②$$

$$c_1 = \frac{26}{25} \;\&\; c_2 = \frac{-1}{25}$$

So $y(n) = \left[\frac{6}{5}n4^n + \frac{26}{25}4^n - \frac{1}{25}(-1)^n\right]u(n)$

31) Determine the impulse response of the following causal s/m $y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

Sol Characteristic eqn $\lambda^2 - 3\lambda - 4 = 0$

$$\lambda = -4, 1$$

$$y_h(n) = c_1 4^n + c_2(-1)^n$$

$$x(n) = \delta(n)$$

$$y(0) = 1 \;\&\; y(1) - 3y(0) = 2$$

$$y(1) = 5$$

So $c_1 + c_2 = 1 \rightarrow ①$

$4c_1 - c_2 = 5 \rightarrow ②$

from ① & ② $c_1 = \frac{6}{5}$ & $c_2 = -1/5$

$\therefore h(n) = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)$

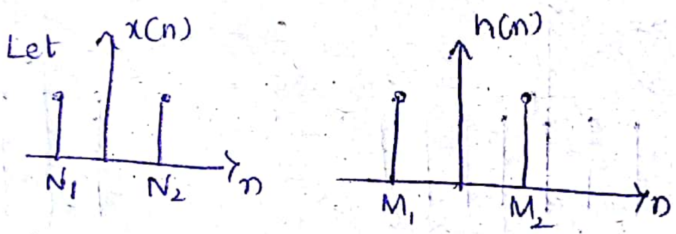
32) Let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $m_1 \le n \le m_2$ be two finite duration sig's.

a) Determine the range $L_1 \le n \le L_2$, of their convolution in terms of N_1, N_2, M_1 & M_2

Sol $L_1 = N_1 + M_1$

$L_2 = N_2 + M_2$

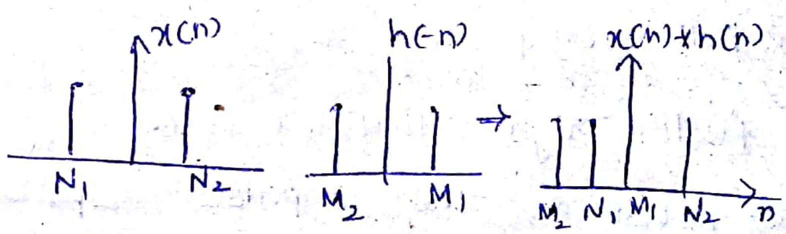
Let



b) Determine the limits of the cases of partial overlap from the left, full overlap and partial overlap from right. For convinience, assume that $h(n)$ has shortest duration than $x(n)$.

Sol. Partial overlap from left

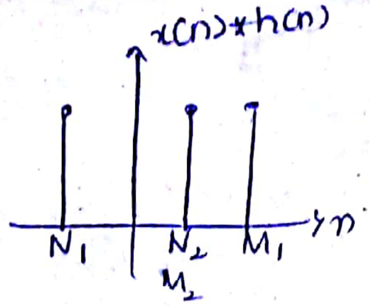
→ $x(n) * h(n)$ ⇒



Low, $N_1 + M_1$ & high $M_2 + N_1 - 1$

If fully overlap then $N_1 + M_2$ (low) & high $N_2 + M_1$

partial overlap from right



Low → $N_2 + M_1 + 1$

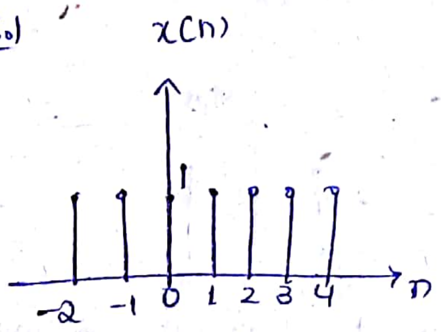
high → $N_2 + M_2$

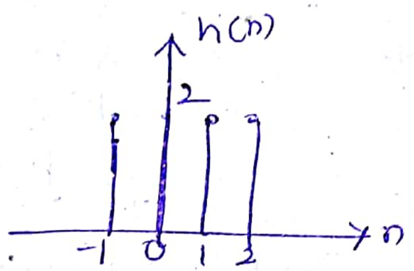
if fully overlapped high $N_2 + M_1$; Low = $N_1 + M_2$

c) Illustrate the validity of your results by computing the convolution of the sig.

$$x(n) = \begin{cases} 1 & , -2 \le n \le 4 \\ 0 & , \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} +2 & , -1 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Sol



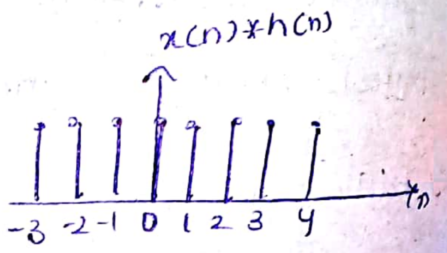


$N_1 = -2$, $N_2 = 4$ $\qquad$ $m_1 = -1$, $m_2 = 2$

Partial overlap from left

Low $N_1 + M_1 = 3$

high $m_2 + N_1 - 1 = 2 - 2 - 1 = -1$



full overlap $n = 0$, $n = 3$

partial right ; $n = 4$, $n = 6$, $L_2 = 6$

33) Determine the impulse response and the unit step response of the s/m's described by the difference equation.

a) $y(n) = 0.6\, y(n-1) - 0.08\, y(n-2) + x(n)$

Sol

$$x(n) = y(n) - 0.6\, y(n-1) + 0.08\, y(n-2)$$

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characteristic eqn

$$\lambda^2 - 0.6\lambda + 0.08 = 0$$

$$\lambda = 1/2 , 2/5$$

$$y_h(n) = c_1\left(\frac{1}{5}\right)^n + c_2\left(\frac{2}{5}\right)^n$$

Impulse response $x(n) = \delta(n)$ with $y(0)=1$

$$y(1) - 0.6\, y(0) = 0 \Rightarrow y(1) = 0.6$$

so $c_1 + c_2 = 1 \rightarrow ①$

$$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \rightarrow ②$$

from ① & ② $c_1 = -1, c_2 = 3$

$$h(n) = \left[-(1/5)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$$

step response $x(n) = u(n)$

$$s(n) = \sum_{k=0}^{n} h(n-k), \; n \geq 0$$

$$= \sum_{k=0}^{n}\left[2(2/5)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$$

$$= 2\left(\frac{2}{5}\right)^n \sum_{k=0}^{n}\left(\frac{5}{2}\right)^k - \left(\frac{1}{5}\right)^n \sum_{k=0}^{n}(5)^k$$

$$= \left[2\left(\frac{2}{5}\right)^n\left(\left(5/2\right)^{n+1} - 1\right)\right] - \left[\left(\frac{1}{5}\right)^k\left[5^{n+1} - 1\right]\right] u(n)$$

b) $y(n) = 0.7y(n-1) - 0.1\, y(n-2) + 2x(n) - x(n-2)$.

Sol

$$2x(n) - x(n-2) = y(n) - 0.7y(n-1) + 0.1\, y(n-2)$$

characteristic eqn

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$$\lambda = 1/2, 1/5$$

$$y_h(n) = C_1(1/2)^n + C_2(1/5)^n$$

Impulse Response $x(n) = \delta(n)$ $y(0) = 2$

$$y(1) - 0.7\,y(0) = 0$$

$$y(1) = 1.4$$

$$C_1 + C_2 = 2$$

$$\frac{1}{2}C_1 + \frac{1}{5}C_2 = \frac{7}{5} \rightarrow ①$$

$$C_1 + \frac{2}{5}C_2 = \frac{14}{5} \rightarrow ②$$

solving ① & ②

$$C_1 = \frac{10}{3} \quad \& \quad C_2 = -\frac{4}{3}$$

So $h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$

Step Response $s(n) = \sum_{k=0}^{n} h(n-k)$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n\sum_{k=0}^{n}2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n\sum_{k=0}^{n}5^k$$

$$= \frac{10}{3}\left[\left(\frac{1}{2}\right)^n(2^{n+1}-1)\,u(n)\right] - \frac{4}{3}\left[\left(\frac{1}{5}\right)^n(5^{n+1}-1)\right]u(n)$$

34) Consider a s/m with impulse response

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

Determine the i/p $x(n)$ for $0 \le n \le \infty$ that will generate the o/p sequence $y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$
$\uparrow$

Sol $h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots\}$
$\uparrow$

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0\}$
$\uparrow$

$y(0) = x(0)\, h(0)$

$y(0) = x(0) \cdot 1$

$\Rightarrow x(0) = 1$

$y(1) = x(1) + h(1) \cdot x(0)$

$2 = x(1) + \frac{1}{2}(1) \Rightarrow x(1) = \frac{3}{2}$

$y(2) = x(2) + h(2)\, x(1) + h_1\, x(0)$

$2.5 = x(2) + \frac{1}{4}(3/2) + 1/2\,(1)$

So $x(n) = \{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2}, \dots\}$

37) Compute & sketch the step response of the s/m

$$y(n) = \frac{1}{M} \sum_{k=0}^{m-1} x(n-k)$$

Sol:

$$h(n) = \left[\frac{u(n) - u(n-m)}{m}\right]$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{M} & , n < M \\ 1 & , n \geq M \end{cases}$$

38) Determine the range of values of the parameter a for which the linear time-invariant s/m with impulse response

$$h(n) = \begin{cases} a^n, & n \geq 0 \quad n \text{ even is stable} \\ 0, & \text{otherwise} \end{cases}$$

Sol

$$\sum_{k=-\infty}^{\infty} |h(n)| = \sum_{\substack{n=0 \\ n \to even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

stable if $|a| < 1$.

39) Determine the response of the s/m with impulse response $h(n) = a^n u(n)$ to i/p s/g $x(n) = u(n) - u(n-10)$

Sol $h(n) = a^n u(n)$

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k}$$

$$= \frac{1-a^{n+1}}{1-a} u(n)$$

$$y(n) = y_1(n) - y_1(n-10)$$

$$= \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]$$

41) Determine the response of system characterized by the impulse response $h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the i/p s/g's

a) $x(n) = 2^n u(n)$

Sol $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}$$

$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k$$

$$= 2^n \left[1 - \left(\frac{1}{4}\right)^{n+1}\right]\left(\frac{4}{3}\right)$$

$$= \frac{2}{3}\left[2^{n+1} - \left(\frac{1}{2}\right)^{n+1}\right] u(n)$$

b) $x(n) = u(-n)$

Sol $$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=0}^{\infty} h(k)$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = 2, \ n<0$$

$$y(n) = \sum_{k=n}^{\infty} h(k)$$

$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left[\frac{1-\left(\frac{1}{2}\right)^n}{1/2}\right]$$

$$= 2\left(\frac{1}{2}\right)^n, \ n \geq 0$$

42) Three s/m's with impulse responses $h(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$ and $h_3(n) = u(n)$ are connected in cascade.

a) What is the impulse response, $h_c(n)$ of overall s/m

Sol $$h_c(n) = h_1(n) * h_2(n) * h_3(n)$$

$$= [\delta(n) - \delta(n-1)] * u(n) * h(n)$$

$$= [u(n) - u(n-1)] * h(n)$$

$$= \delta(n) * h(n) = h(n)$$

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b) Does the order of interconnection affect the overall s/m?

Sol NO.

43) a. Prove & explain graphically the difference betw the relations $x(n)\delta(n-n_0) = x(n_0)\,\delta(n-n_0)$ and

$$x(n) * \delta(n-n_0) = x(n-n_0).$$

Sol $x(n)\,\delta(n-n_0) = x(n_0)$. Thus only the value of $x(n)$ at $n=n_0$ is of interest.

$x(n) * \delta(n-n_0) = x(n-n_0)$. Thuse, we obtain shifted version of $x(n)$ sequence.

b. Show that a discrete-time s/m, which is described by convolution summation, is LTI and relazed.

Sol
$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\,x(n-k)$$

$$= h(n) * x(n)$$

Linearity:- $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$$= \alpha\, h(n) * x_1(n) + \beta\, h(n) * x_2(n)$$

$$= \alpha\, y_1(n) + \beta y_2(n)$$

Time invariance:-

$x(n) \longrightarrow y_1(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$$= \sum_k h(k)\,x(n-n_0-k)$$

$$= y(n-n_0)$$

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c) What is the impulse response of the s/m described by $y(n) = x(n-n_0)$.

Sl $h(n) = \delta(n-n_0)$.

45) Compute the zero state response of the s/m described by the difference equation $y(n)+\frac{1}{2}y(n-1) = x(n) + 2x(n-2)$ to the input $x(n) = \{1,2,3,4,2,1\}$ by solving the difference equation recursively.

Sl $y(n) = -\frac{1}{2}y(n-1) + x(n) + 2x(n-2)$

at $y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1$

$y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = \frac{3}{2}$

$y(0) = -\frac{1}{2}y(-1) + 2x(-2) + x(0) = \frac{17}{4}$

$y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1) = \frac{47}{8}$

48) Consider the s/m described by the difference equations $y(n) = ay(n-1) + bx(n)$

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

b) Compute the zero-state step response $s(n)$ of the s/m & choose b so that $s(\infty) = 1$

c) Compare the values of b obtained in parts (a) and (b). What did you notice.

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a) $y(n) = a y(n-1) + b x(n)$

$h(n) = b a^n u(n)$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$b = 1-a$

b) $$S(n) = \sum_{k=0}^{n} h(n-k)$$

$$= b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

$$S(\infty) = \frac{b}{1-a} = 1$$

$b = 1-a$

c) $b = 1-a$ in both the cases.

54) Compute & sketch the convolution $y_i(n)$ and corelation $r_i(n)$ sequences for the following pair of s/g's and comment of the result obtained

a) $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$, $h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3(n) = \{4, \underset{\uparrow}{3}, 2, 1\}$

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$, $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Sol

a) Convolution : $y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

Correlation : $r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b) convolution: $y_2(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

correlation: $r_1(n) = \{\frac{1}{2}, \underset{\uparrow}{0}, \frac{3}{2}, -2, \frac{1}{2}, -6, -\frac{5}{2}, -2\}$

Note: $y_2(n) = r_2(n)$ $\because$ $h_2(-n) = h_2(n)$ (C)

convolution : $y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$

correlation : $r_1(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

c) convolution : $y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$

correlation : $r_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

Note that $h_3(-n) = h_4(n+3)$

hence $r_3(n) = y_4(n+3)$

$h_4(-n) = h_3(n+3)$

$r_4(n) = y_3(n+3)$

55) The zero-state response of a causal LTI s/m to the input $x(n) = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y(n) = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$. Determine its impulse response.

Sol

$x(n) * y(n) = h(n)$

Length of $h(n) = 2$

$h(n) = \{h_0, h_1\}$

$h_0 = 1$

$3h_0 + h_1 = 4$

$\Rightarrow h_0 = 1, h_1 = 1$

57) Determine the response $y(n)$, $n \geq 0$ of the s/m described by the second-order differential equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$, when the input is $x(n) = (-1)^n u(n)$ and initial conditions are

$y(-1) = y(-2) = 0$

Sol
$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The characteristic equation is

$$\lambda^2 - 4\lambda + 4 = 0$$

$\lambda = 2, 2$. Hence

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is

$$y_p(n) = K(-1)^n u(n)$$

Substituting this solution into the difference equation, we obtain $K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n)$

For $n=2$, $K(1+4+4) = 2 \Rightarrow K = \frac{2}{9}$. The total solution is

$$y(n) = \left[C_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

from the initial conditions, we obtain $y(0)=1$; $y(1)=2$

$$C_1 + \frac{2}{9} = 1$$

$$C_1 = \frac{7}{9}$$

$$2C_1 + 2C_2 - \frac{2}{9} = 2$$

$$C_2 = \frac{1}{3}$$

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58) Determine the impulse response $h(n)$ for the s/m described by the second-order difference equation.

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

Sol From problem (57)

$$h(n) = [C_1 2^n + C_2 n 2^n] u(n)$$

with $y(0) = 1$, $y(1) = 3$, we have

$$C_1 = 1, \quad 2C_1 + 3C_2$$

$$C_2 = \frac{1}{2}$$

Thus $h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$.

59) Show that any discrete-time s/g $x(n)$ can be expressed as $x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$

where $u(n-k)$ is a unit step delayed by k units in time that is,

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise.} \end{cases}$$

Sol

$$x(n) = x(n) * \delta(n)$$

$$= x(n) * [u(n) - u(n-1)]$$

$$= [x(n) - x(n-1)] * u(n)$$

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k)$$

60) Show that the o/p of an LTI s/m can be expressed in terms of its unit step response $s(n)$ as follows

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)]\, s(n-k)$$

Sol Let $h(n)$ be the impulse response of s/m

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$$h(k) = s(k) - s(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)]\, x(n-k)$$

61) Compute the correlation sequence $r_{xx}(l)$ and $r_{xy}(l)$ for the following sequences.

$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{elsewhere} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{elsewhere} \end{cases}$$

Sol

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

The range of non-zero values of $r_{xx}(l)$ is determined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N$$

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which implies

$$-2N \le l \le 2N$$

for a given shift l, the number of terms in the summation for which both $x(n)$ & non-zero is $2N+1-|l|$ and the value of each term is 1. Hence

$$\gamma_{xx}(l) = \begin{cases} 2N+1-|l| & , -2N \le l \le 2N \\ 0 & , \text{elsewhere} \end{cases}$$

for $\gamma_{xy}(l)$ we have

$$\gamma_{xy}(l) = \begin{cases} 2N+1-|l-n_0| & , n_0-2N \le l \le n_0+2N \\ 0 & , \text{elsewhere} \end{cases}$$

62) Determine the autocorrelation sequence of the following sig's a) $x(n) = \{1,\underset{\uparrow}{2},1,1\}$ b) $y(n) = \{1,1,\underset{\uparrow}{2},1\}$. What is your conclusion.

Sol a) $\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

$\gamma_{xx}(-3) = x(0)\,x(3) = 1$

$\gamma_{xx}(-2) = x(0)\,x(2) + x(1)\,x(3) = 3$

$\gamma_{xx}(-1) = x(0)\,x(1) + x(1)\,x(2) + x(2)\,x(3) = 5$

$\gamma_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$

also $\gamma_{xx}(-l) = \gamma_{xx}(l)$

$\therefore \gamma_{xx}(l) = \{1,3,5,\underset{\uparrow}{7},5,3,1\}$

by $\gamma_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$

we obtain $\gamma_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

we obtain $y(n) = x(-n+3)$ which is equivalent to reversing the sequence $x(n)$.

63) Determine normalized auto corelation sequence of the s/g $x(n)$ given by

$$x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{elsewhere} \end{cases}$$

Sol:

$$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \begin{cases} 2N+1 - |l|, & -2N \le l \le 2N \\ 0, & \text{elsewhere} \end{cases}$$

$$\gamma_{xx}(0) = 2N+1$$

the normalized auto correlation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}[2N+1-|l|], & -2N \le l \le 2N \\ 0, & \text{elsewhere} \end{cases}$$

64) An audio s/g $s(t)$ generated by a loud speaker is reflected at two different walls with reflection coefficients r_1 and r_2. The s/g $x(t)$ recorded by a microphone close to the loudspeaker, after sampling is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 and k_2 are the delays of the two echoes.

a) Determine the auto correlation $r_{xx}(l)$ of the sig $x(n)$

b) can we obtain r_1, r_2, k_1 & k_2 by observing $r_{xx}(l)$?

c) what happens if $r_2 = 0$?

a) $$\gamma_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) + \gamma_1 s(n-k_1) + \gamma_2 s(n-k_2)] *$$

$$[s(n-l) + \gamma_1 s(n-l-k_1) + \gamma_2 s(n-l-k_2)]$$

$$= (1+\gamma_1^2+\gamma_2^2)\, \gamma_{ss}(l) + \gamma_1 [\gamma_{ss}(l+k_1) + \gamma_{ss}(l-k_1)] +$$

$$\gamma_2 [\gamma_{ss}[l+k_2) + \gamma_{ss}(l-k_2)] + \gamma_1\gamma_2 [\gamma_{ss}(l+k_1-k_2) +$$

$$\gamma_{ss}[l+k_2-k_1]$$

b) $\gamma_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$ suppose that $k_1 < k_2$. Then we can determine γ_1 & k_1. The problem is to determine γ_2 & k_2 from the other peaks.

c) If $\gamma_2 = 0$, the peaks occur at $l = 0$ and $l = \pm k_1$. Then is easy to obtain γ_1 & k_1