DSP Assignment

2.1) A discrete time signal $x(n)$ is defined as

$$x[n] = \begin{cases} 1+\frac{n}{3} & -3 \le n \le -1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

a) Determine its value and sketch the signal $x[n]$

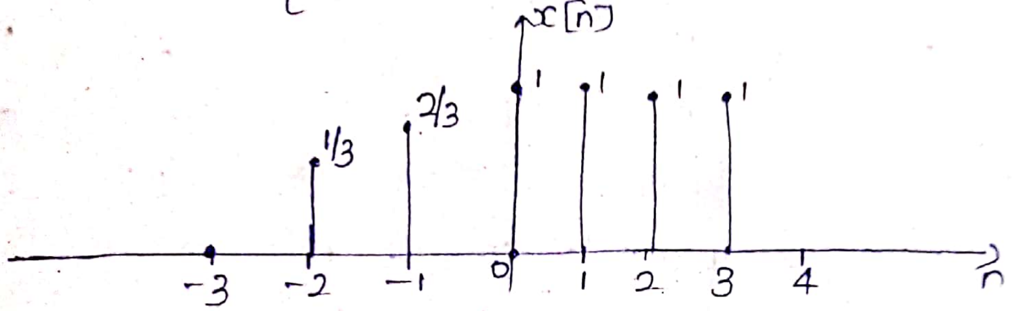
$-3 \le n \le -1$ $\quad x[n] = 1+\frac{n}{3}$, $n=-3$ $\quad x[n] = 1+\frac{(-3)}{3} = \frac{3-3}{3} = 0$

$n=-2$ $\quad x[n] = 1-\frac{2}{3} = 1/3$

$n=-1$ $\quad x[n] = 1-\frac{1}{3} = 2/3$.

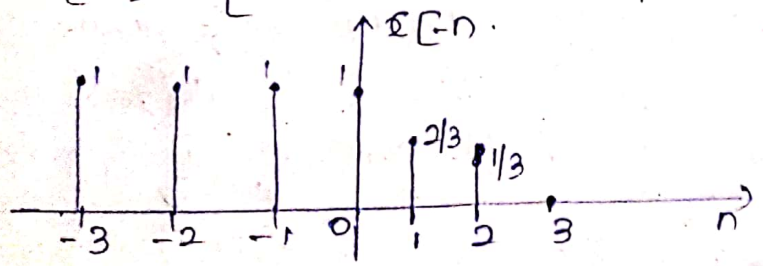
$0 \le n \le 3$ $\quad x[n] = 1$.

$\therefore x[n] = \{ \ldots 0, 0, 1/3, 2/3, 1, 1, 1, 1, 0, 0, \ldots \}$.



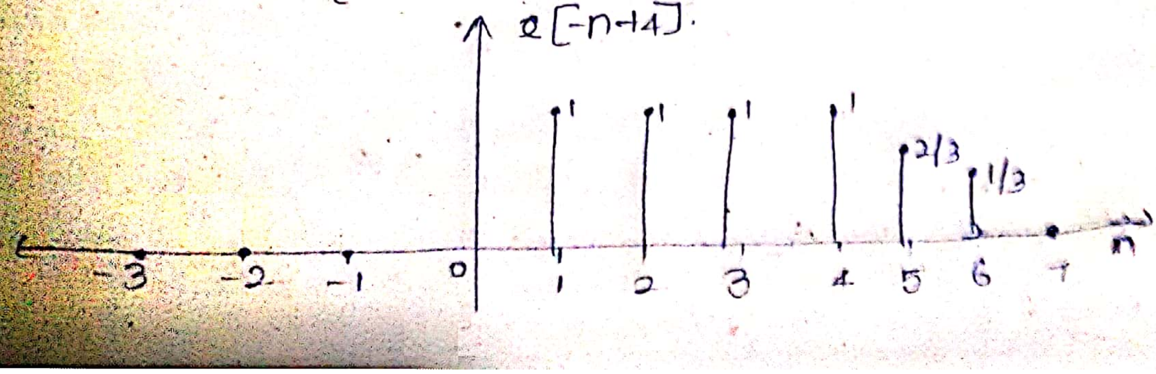
b) i) first fold the $x[n]$ and delay the resulting signal by four samples.

$x[-n] = \{ \ldots 0, 1, 1, 1, 1, 2/3, 1/3, 0, \ldots \}$.



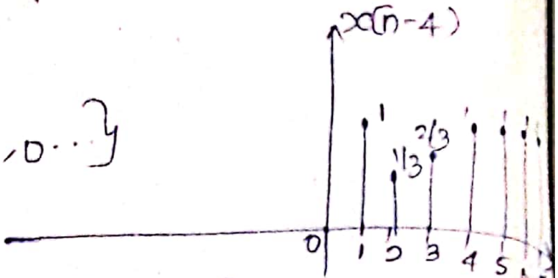
After delaying the folded signal by 4 samples we have

$x[-n+4] = \{ \ldots 0, 0, 1, 1, 1, 1, 2/3, 1/3, 0 \ldots \}$.

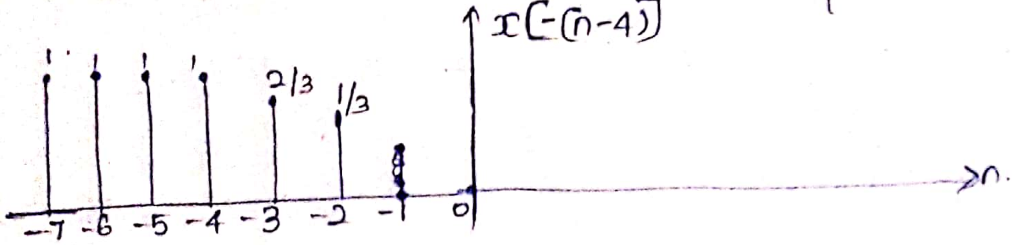


ii] First delay $x[n]$ by four samples and then fold the resulting signal.

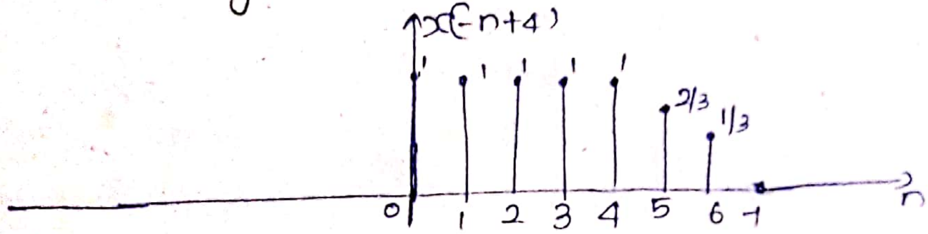
$x(n-4) = \{\ldots 0, 0, 1/3, 2/3, 1, 1, 1, 1, 0 \ldots\}$



By folding $x(n-4)$ we have $x(-n-4) = \{\ldots 0, 1, 1, 1, 1, 2/3, 1/3, 0 \ldots\}$



c] Sketch the signal $x(-n+4)$



d] Compare results in part (b) and (c), derive a rule for obtaining the signal $x[-n+5]$ from $x[n]$.

To obtain $x[-n+k]$ from $x[n]$, first we have to fold the signal then delay the signal by some L units.

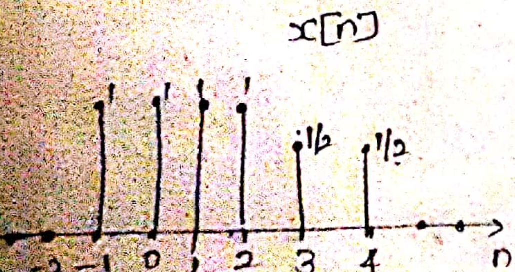
e] Can you express the signal in terms of signals $\delta[n]$ & $u[n]$.

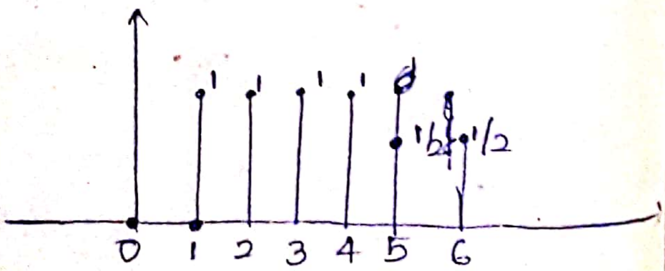
i] $x[n] = \frac{1}{3}\delta[n+2] + \frac{2}{3}\delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3]$

ii] $x[n] = \frac{1}{3}\delta[n+2] + \frac{2}{3}\delta[n+1] - u[n-4] + u[n]$.

2.2] A discrete time signal $x[n]$ is shown. Sketch and label carefully each of the following signal.

a] $x(n-2)$





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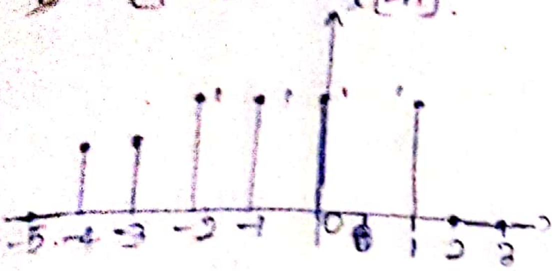
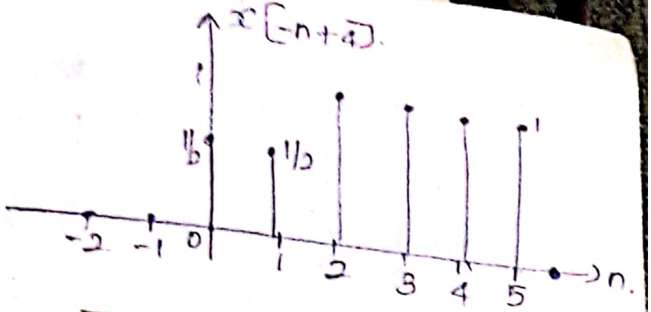
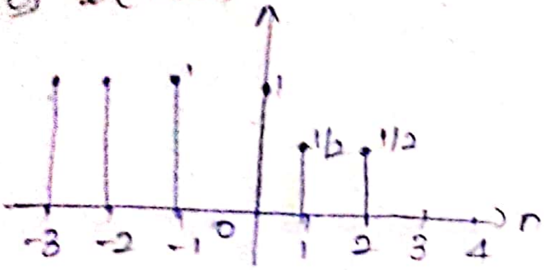
b) $x[4-n]$.

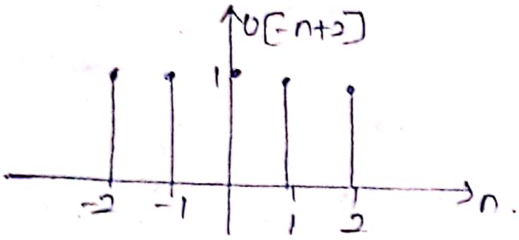


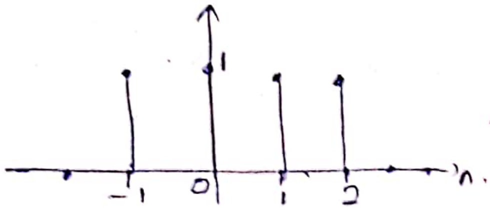


c) $x(n+2)$

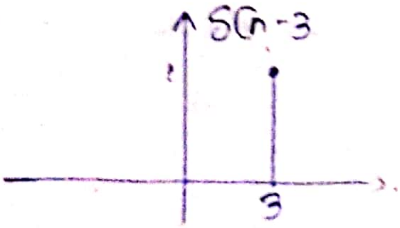


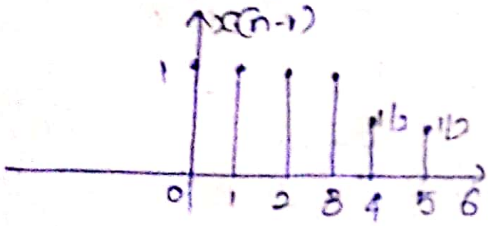
d) $x[n]u[2-n]$.

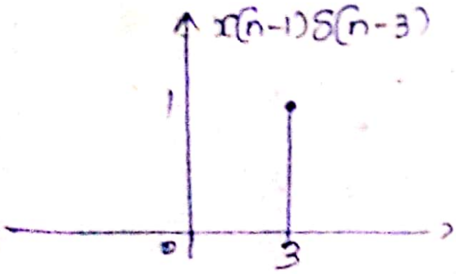




e) $x(n-1)\,\delta(n-3)$







$x[n]\,x[n-n_0] = x[n_0]\,\delta[n-n_0]$

$x[n-1]\,\delta[n-3] = x[2]\,\delta[n-3]$.

f) $x[n^2]$.

$x[n^2] = \{\ldots 0, x(4), x(1), x(0), x(1), x(4), 0\}$

$= \{0, 1/2, 1, 1, 1, 1/2, 0 \ldots\}$

$y(0) = x(0)$. $\quad y(3) = x(9)$

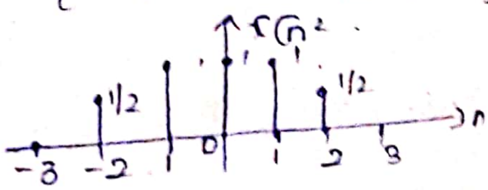
$y(1) = x(1) \quad y(-1) = x(1)$

$y(2) = x(4) \quad y(-2) = x(4)$

$y(-3) = x(9)$

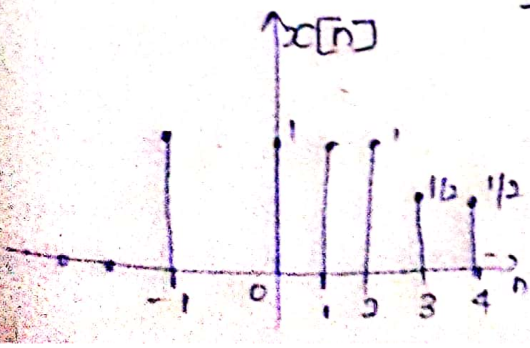
$x(n^2) = \{\ldots x(9), x(4)\, x(1), x(0), x(1), x(4), x(9) \ldots\}$.

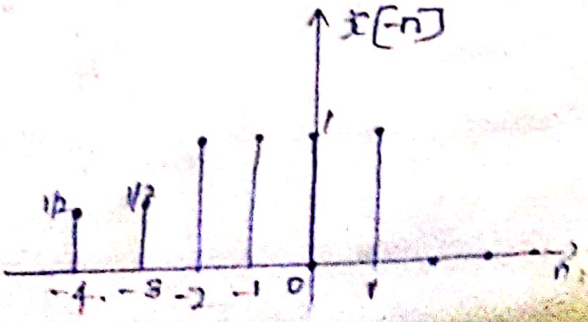
$= \{0, 1/2, 1, 1, 1, 1/2, 0 \ldots\}$.

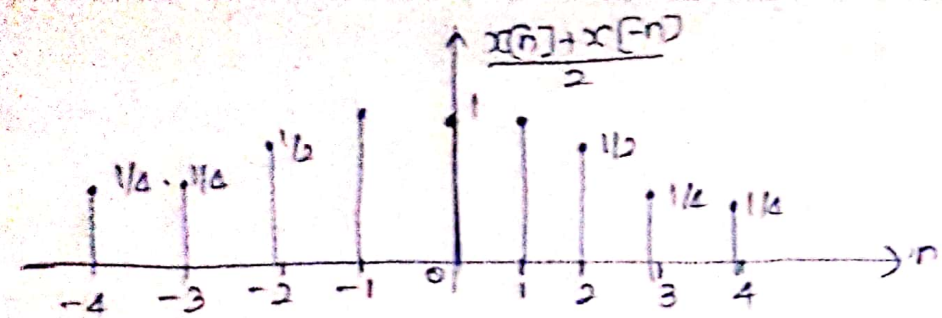


g) even part of $x[n]$.

$x[n] = \dfrac{x[n] + x[-n]}{2}$

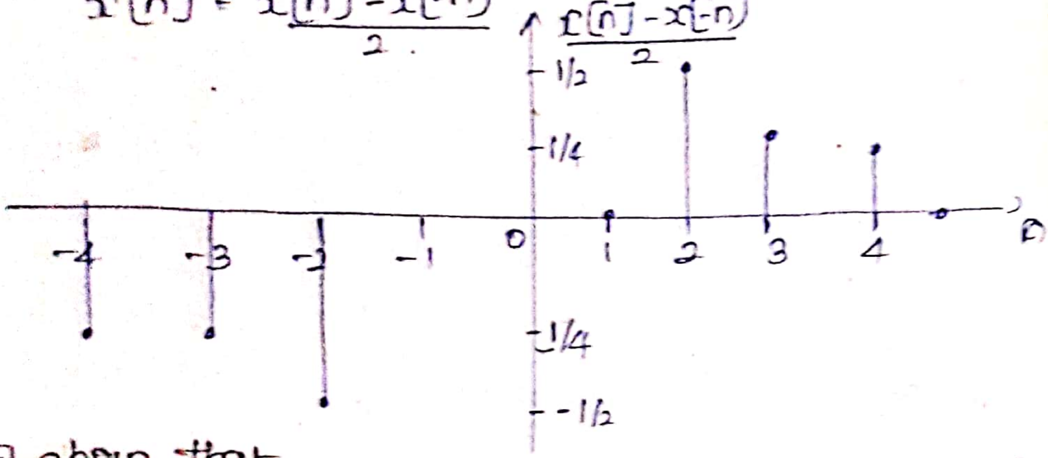






h] odd part of $x[n]$.

$$x[n] = \frac{x[n]-x[-n]}{2}.$$

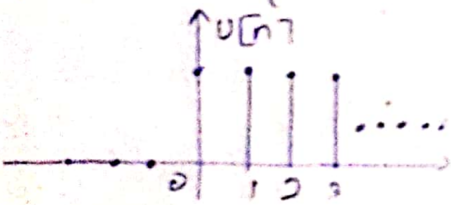


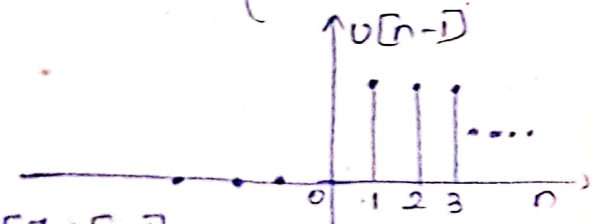
2.3] show that

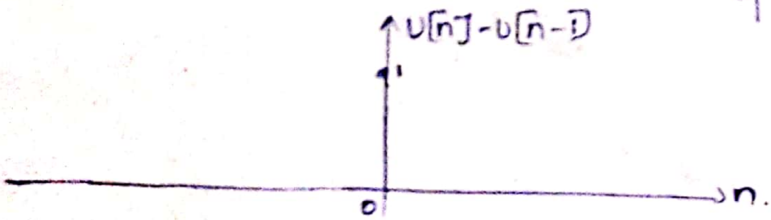
a] $\delta[n] = u[n] - u[n-1]$.

we know that $\delta[n]$ is defined as $\delta[n] = \begin{cases} 1 & n=0 \\ 0 & n \neq 0 \end{cases}$

and $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$ $\quad u[n-1] = \begin{cases} 1 & n \geq 1 \\ 0. & n < 1. \end{cases}$







b] $u[n] = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$.

$$\sum_{k=-\infty}^{n} \delta(k) = \delta(-\infty) + \cdots + \delta[n]$$

$$① \longrightarrow u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0. \end{cases}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \delta(n) + \delta(n-1) + \cdots + \delta(n-\infty)$$

$$② \longrightarrow u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{if } n < 0 \end{cases}$$

Because 0, ∞ are non negative numbers

if $n-k=0 \Rightarrow \delta(0)=1$. hence [n] must be a positive integer. Hence, $\sum_{n=\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k) = u[n]$.

2.4 Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique? Illustrate your arguments using the signal.

Any signal can be written or formed by the combination of even and odd components of the signal.

So decomposition is unique

$$x[n] = x_e[n] + x_o[n].$$

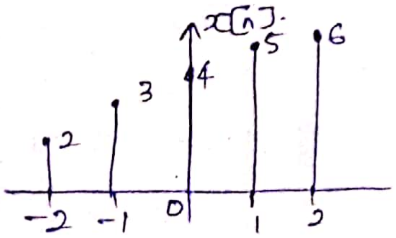
We know that $x_e[n] = \frac{x[n]+x[-n]}{2}$ & $x_o[n] = \frac{x[n]-x[-n]}{2}$

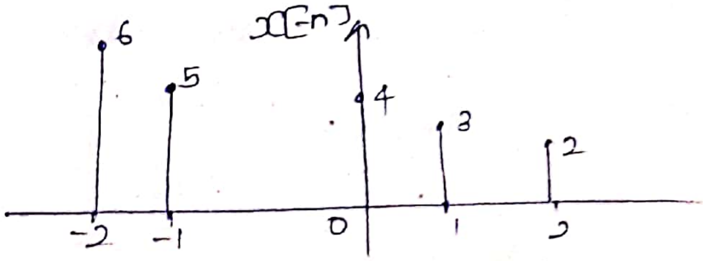
$$\therefore x[n] = \frac{x[n]+x[-n]}{2} + \frac{x[n]-x[-n]}{2} = \frac{x[n]}{2} + \frac{x[n]}{2}.$$

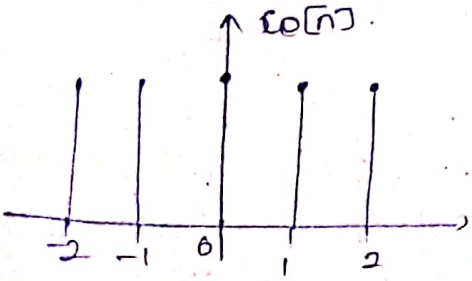
$x[n] = x[n]$. ∴ Hence proved.

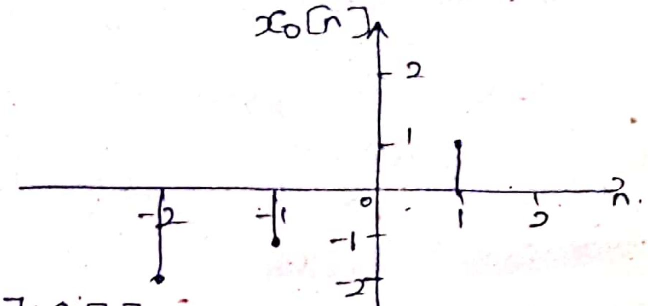
Given $x[n] = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$.

The odd & even components of signal is given as.

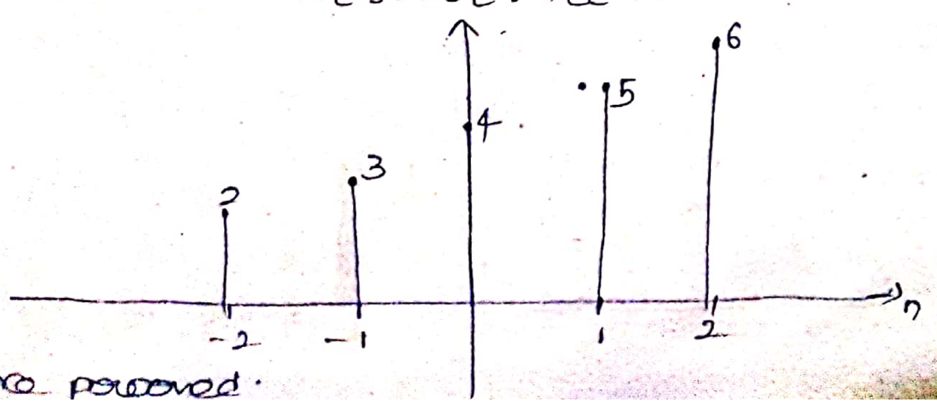








$x[n] = x_o[n] + x_e[n]$



Hence proved.

2.5 Show that the energy (power) of a real valued energy (power) signal is equal to the sum of the energy (power) of its even and odd components

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} \left[x_e[n] + x_o[n]\right]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n] + 2\sum_{n=-\infty}^{\infty} x_e[n]\,x_o[n].$$

$$= \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n].$$

$$\sum_{n=-\infty}^{\infty} x_e[n]x_o[n] = \sum_{n=-\infty}^{\infty} x_e[-m]x_o[-m]$$

$$= -\sum_{m=-\infty}^{\infty} x_e[m]\,x_o[m]$$

$$= -\sum_{n=-\infty}^{\infty} x_e[n]\,x_o[n] = \sum_{m=-\infty}^{\infty} x_e[n]\,x_o[n].$$

$$= 0$$

2.6 Consider the system

$$y[n] = T[x[n]] = x[n^2].$$

a) Determine if the system is time invariant.

$$y[n] = x(n^2).$$

Time invariance checking.

$$x[n] \rightarrow y[n] = x(n^2)$$

$$x(n-k) \rightarrow y_1(n) = x[(n-k)^2] = y(n-k)$$

$$= x(n^2+k^2-2nk)$$

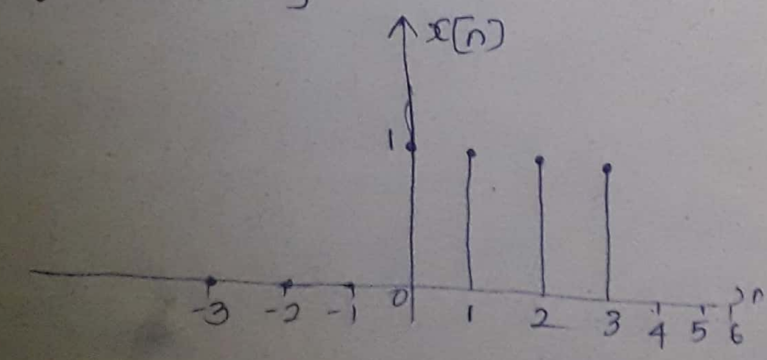
thus $x(n-k) \neq y(n-k)$.

thus $x(n^2) = y(n)$ is Time variant system.

b) prove (a) part by considering signal $x(n) = \begin{cases} 1 & 0<n\leq 3 \text{ (approx)} \\ 0 & \text{elsewhere} \end{cases}$

Also find & draw $T\,x(n)$ signal

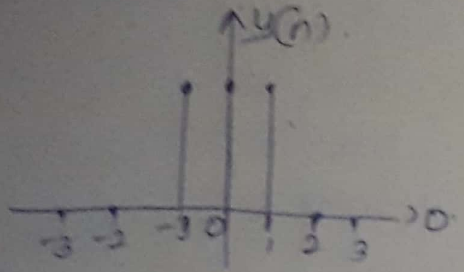
$x(n) = \{0, 1, 1, 1, 1, 0 \ldots\}$



2] $y(n) = T[x(n)]$ signal

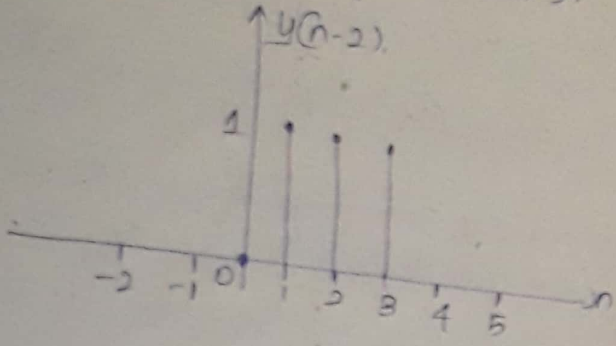
$= \{x(9), x(4), x(1), x(0), x(1), x(4), x(9)\}$

$= \{0, 0, 1, 1, 1, 0, 0\}$



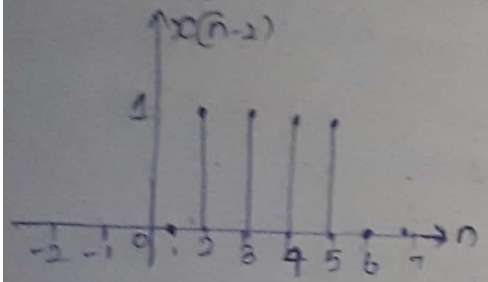
3] $y_2(n) = y(n-2)$

$y(n-2) = \{0, 0, 1, 1, 1, 0, 0\}$



4] $x_2(n) = x(n-2)$

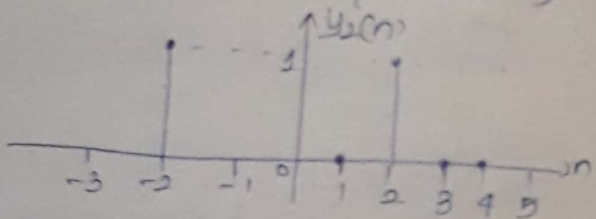
$= \{0, 0, 0, 1, 1, 1, 1, 0 ..\}$



5] $y_2(n) = T[x_2(n)]$

$y_2(n) = \{x(9), x(4), x(1), x(0), x(1), x(4), x(9)\}$

$= \{.. 0, 1, 0, 0, 0, 1, 0 ...\}$



6] Conclude about $y_2(n)$ & $y(n-2)$

$y_2(n)$ & $y(n-2)$ both having different values thus.

$y_2(n) \neq y(n-2) \Rightarrow$ says that system is time variant.

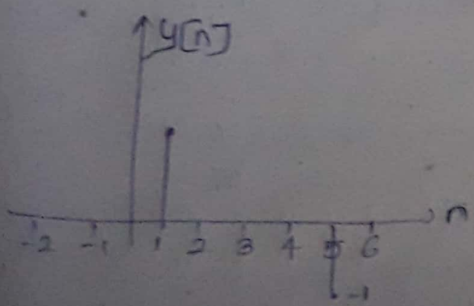
c] Repeat part (b) for $y[n] = x[n] - x[n-1]$ & conclude over time invariance of system.

1] $x[n] = \{0, 1, 1, 1, 1\}$

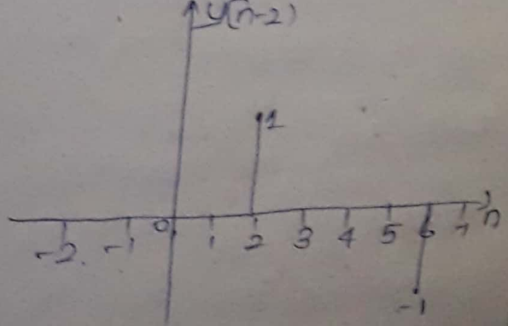
$x[n-1] = \{0, 0, 1, 1, 1, 1\}$

2] $y(n) = x(n) - x(n-1)$

$= \{0, 1, 0, 0, 0, -1, 0 ..\}$



3] $y(n-2) = \{.. 0, 0, 0, 1, 0, 0, 0, -1, 0 ..\}$



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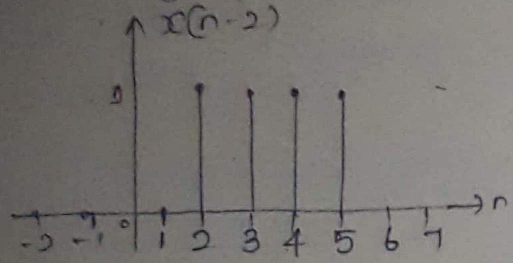
4] $x(n-2) = x_2(n)$

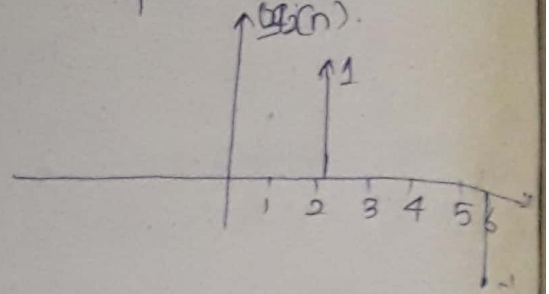
$= \{0, 0, 0, 1, 1, 1, 1, 0 \cdots\}$



5] $y_2(n) = x_2(n) - x_2(n-1)$

$= \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 0\} - \{\underset{\uparrow}{0}, 0, 0, 1 \ldots$

$y_2(n) = \{\underset{\uparrow}{0}, 0, 1, 0, 0, 0, 0, -1\}$

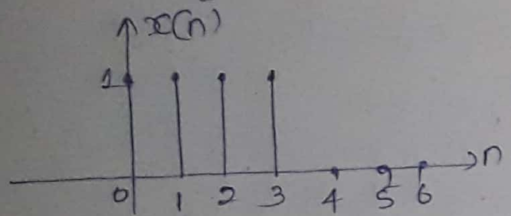


6] here $y_2(n) = y(n-2)$

$\Rightarrow$ system is time invariant.

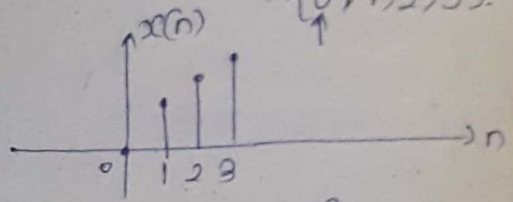
d] Repeat part c & b) for $y[n] = T[x(n)] = n x(n)$.

1] $x[n] = \{\cdots 0, 1, 1, 1, 1, 0 \cdots\}$

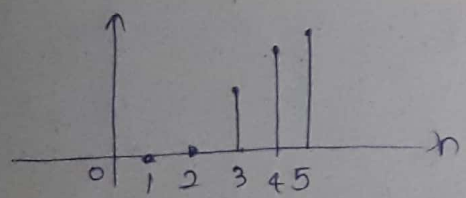


2] $y[n] = n x(n) = \{\underset{\uparrow}{1}, 1, 1, 1\}$

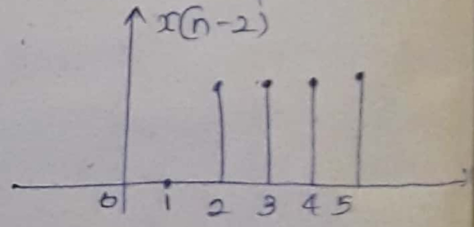
$= \{\underset{\uparrow}{0}, 1, 2, 3\}$.



3] $y(n-2) = \{\underset{\uparrow}{0}, 0, 0, 1, 2, 3\}$



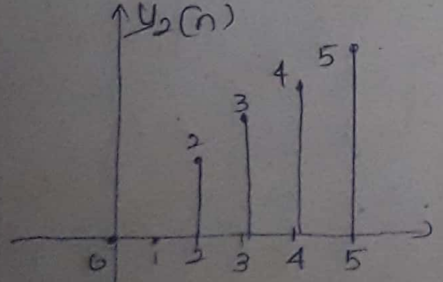
4] $x(n-2) = x_2(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$



5] $y_2(n) = T(x_2(n))$

0	0	1	1	1	1
×	×	×	×	×	×
0	1	2	3	4	5.

$y_2(n) = \{\underset{\uparrow}{0}, 0, 2, 3, 4, 5\}$



6] $y_2(n)$ & $y(n-2)$ both are have different values.

thus

$y_2(n) \neq y(n-2)$

$\Rightarrow$ Says that system is time invariant.

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2.7] A DTS System can be 1] static or dynamic
2] linear (or) non-linear
3) Time invariant (or) time varying
4) causal (or) non-causal
5) stable (or) unstable., Examine the following systems with respect to the properties above.

a] $y(n) = \cos[x(n)]$ → static, Non-linear, Time invariant, causal, stable

b] $y(n) = \sum_{k=-\infty}^{k=n+1} x(k)$ → Dynamic, linear, TI, noncausal, unstable

c] $y(n) = x(n)\cos(\omega_0 n)$ static, linear, time variant, causal, stable

d] $y(n) = x(-n+2)$. Dynamic, linear, time invariant, noncausal, stable.

e] $y(n) = Tr[x(n)]$, where $Tr[x(n)]$ denotes the integer part of $x[n]$ obtained by truncation. static, non-linear, time-invariant, causal, stable.

f] $y[n] = Round[x(n)]$, where $Round[x(n)]$ denotes the integer part of $x(n)$ obtained by Rounding. static, non-linear, time-invariant, causal, stable.

g] $y[n] = |x(n)|$ static, non-linear, time invariant, causal, stable

h] $y[n] = x(n)u(n)$ static, linear, time invariant, causal, stable

i] $y[n] = x[n] + n\,x(n+1)$. Dynamic, linear, time variant, non-causal, unstable. note:- Bounded i/p $x(n)$ gives $y(n)$ which is unbounded o/p

j] $y[n] = x[2n]$ Dynamic, linear, time variant, non-causal, stable.

k] $y[n] = \begin{cases} x(n) & \text{if } x(n) \geq 0 \\ 0 & \text{if } x(n) < 0 \end{cases}$ static, non-linear, time invariant, causal, stable

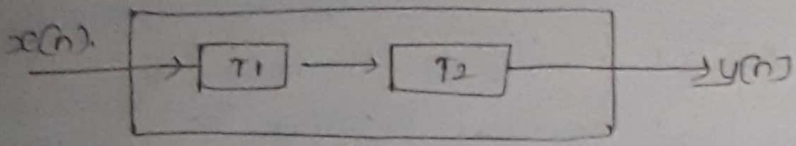
l] $y[n] = x(-n)$. Dynamic, linear, time invariant, non-causal, stable.

m] $y[n] = sign[x(n)]$ static, non linear, time invariant, causal, stable.

n] The ideal sampling system with i/p $x_a(t)$ & o/p $x_n = x_a(nT)$, $-\infty < n < \infty$.

stable, linear, time invariance, causal, static

2.8 Q DTS T_1 & T_2 are connected in cascade to form a new system as shown in fig below. prove (or) disprove following statement.



$T = T_1 T_2$

a) If T_1 & T_2 are linear then T is linear [i.e cascade connection of two linear systems is linear].

=> True.

If $v_1(n) = T_1(x_1(n))$ & $v_2(n) = T_1(x_2(n))$ then $a_1x_1(n)+a_2x_2(n)$ gives

$a_1v_1(n) + a_2v_2(n)$ -> from linearity property of T_1

similarly, if $y_1(n) = T_2[v_1(n)]$ & $y_2(n) = T_2[v_2(n)]$. then

$\beta_1 v_1(n) + \beta_2 v_2(n) \rightarrow y(n) = \beta_1 y_1(n) + \beta_2 y_2(n)$] from linearity property of T_2

since $v_1(n) = T_1(x_1(n))$ & $v_2(n) = T_2[x_2(n)]$, we've -

$A_1 x_1(n) + A_2 x_2(n) \rightarrow A_1 T(x_1(n)) + A_2 T(x_2(n))$

where $T = T_1 T_2$ & T is linear.

b) If T_1 & T_2 are time invariant, then T is time invariant

=> True.

for T_1, if $x(n) \rightarrow v(n)$ & $x(n-k) \rightarrow v(n-k)$.

For T_2, if $v(n) \rightarrow y(n)$ & $v(n-k) \rightarrow y(n-k)$.

For T_1T_2, if $x(n) \rightarrow y(n)$ & $x(n-k) \rightarrow y(n-k)$

therefore $T = T_1 T_2$ is time invariant.

c) If T_1 & T_2 are causal, then T is causal.

=> True, T_1 is causal => $v(n)$ depends only on $x(k)$ for $k \le n$

T_2 is causal => $y(n)$ depends only on $v(k)$ for $k \le n$

there $y(n)$ depends only on $x(k)$ for $k \le n$ thus T is causal

d) If T_1 & T_2 are linear & time invariant, same holds for T

True proof from both a & b.

e) If T_1 & T_2 are linear & time invariant, then interchang their order doesn't change the system

⇒ True.
From the reason as $h_1(n) * h_2(n) = h_2(n) * h_1(n)$

f] As in part (e) except that T_1, T_2 are now Time varying.

⇒ False.

For Ex: $T_1: y(n) = nx(n)$

$T_2: y(n) = x(n+1)$

then $T_2[T_1(\delta(n))] = T_2(0) = 0$.

$T_1[T_2(\delta(n))] = T_1[\delta(n+1)]$

$= -\delta(n+1)$

$T_1[T_2(\delta(n))] \neq 0$.

Thus both are different.

g] If T_1 & T_2 are non-linear, then T is non-linear.

⇒ False.

$T_1: y_1(n) = x(n) + b$ & $T_2: y_2(n) = x(n) - b$ where $b \neq 0$.

then $T[x(n)] = T_2[T_1[x(n)]]$.

$= T_2[x(n) + b]$

$T[x(n)] = x(n)$ ⇒ says that T is linear.

h] If T_1 & T_2 are stable, then T is stable.

⇒ True.

T_1 is stable when ⇒ $v(n)$ is bounded possible if $x(n)$ is bounded.

T_2 is stable when ⇒ $y(n)$ is bounded - possible if $v(n)$ is bounded.

Hence $y(n)$ is bounded if $x(n)$ is bounded ⇒ $T = T_1 T_2$ is stable

i] show by an example that the invers of part c & h) don't hold in general.

Inverse of (c)

If T_1 & T_2 are non causal. then T is non causal.

Ex:- $T_1: y(n) = x(n+1)$

$T_2: y(n) = x(n-2)$

⇒ $T: y(n) = x(n-1)$

⇒ causal.

⇒ Inverse of (c) is false

Inverse of (h)

If T_1 &/or T_2 is unstable. then T is unstable.

Ex:- $T_1: y(n) = e^{x(n)}$ ⇒ stable

$T_2 \cdot y(n) = \ln[x(n)]$ ⇒ unstable.

⇒ $T: y(n) = x(n)$ ⇒ stable.

Inverse of (h) is false.

2.9 Let T be an LTI, relaxed & BIBO stable system with i/p $x(n)$ & o/p Then show that,

a) If $x(n)$ is periodic with period N [i.e $x(n) = x(n+N)$ for all $n \geq 0$]

then the o/p $y(n)$ tends to a periodic signal with the same period.

$x[n] \rightarrow$ LTI $\rightarrow y[n]$

$y[n] = \sum_{k=-\infty}^{n} h[k]x[n-k] = \sum_{k=-\infty}^{n} x[k]h[n-k].$

Let $y[n] = \sum_{k=-\infty}^{n} h[k]x[n-k].$

replacing 'n' by n+N' we get

$y[n+N] = \sum_{k=-\infty}^{n+N} h[k]\cdot x[n+N-k] = \sum_{k=-\infty}^{n+N} h[k]$ ..

where $x[n]$ is a periodic signal

$y[n+N] = \sum_{k=-\infty}^{n+N} h[k]x[n-k]$

$= \sum_{k=-\infty}^{n} h(k)x(n-k) + \sum_{k=n+1}^{N+n} h(k)x(n-k).$

$= y[n] + \sum_{k=n+1}^{n+N} h[k]x[n-k].$

Applying limit on both sides,

$\lim_{n\to\infty} y[n+N] = y[n] + \lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)x(n-k].$

for a BIBO system, $\lim_{n\to\infty} |h(n)| = 0$

$\therefore \sum_{k=n+1, n\to\infty}^{n+N} \lim h(k)x(n-k) = 0.$

$\Rightarrow \lim_{n\to\infty} y(n+N) = y(n)$ is a periodic signal.

b] If $x(n)$ is bounded and tends to a constant, the o/p will also tend to a constant.

Let $y(n) = x_0(n) + au(n)$ where a is a constant & $x_0(n)$ is a bounded signal with $\lim_{n\to\infty} x_0(n) = 0.$

i.e $y(n) = x_0(n) + au(n)$

$y[n] = \sum_{k=0}^{\infty} h(k)x_0(n-k) + a\sum_{k=0}^{\infty} h(k)u(n-k)$

$$y(n) = a y_0(n) + a \sum_{k=0}^{n} h(k)$$

thus

$$y(n) = y_0(n) + a \sum_{k=0}^{n} h(k) \quad \text{--- (1)}$$

Clearly easy that when $\sum_n x_0^2(n) < \infty$

$$\Rightarrow \sum_n y_0^2(n) < \infty$$

Hence $\lim_{n \to \infty} |y_0(n)| = 0$.

thus (1) eqn becomes.

$$y(n) = 0 + a \sum_{k=0}^{n} h(k)$$

$$\Rightarrow y(n) = a \sum_{k=0}^{n} h(k) \Rightarrow \text{Constant}$$

[c] If $x(n)$ is an energy signal the o/p $y(n)$ will also be an energy signal.

we know that

$$y(n) = \sum_k h(k) x(n-k)$$

Summing from $-\infty$ to ∞ & squaring on both sides gives us.

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty} \left[\sum_k h(k) x(n-k) \right]^2$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_k \sum_l h(k) h(l) \sum_n x(n-k) x(n-l)$$

$$\text{But } \sum_n x(n-k) x(n-l) \le \sum_n x^2(n) = E_x.$$

then

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)| \quad \text{--- (1)}$$

for BIBO system $\sum_k |h(k)| < M$.

then we have.

$\sum_n y^2(n) \rightarrow$ has energy: E_y

$\sum_k h(k) \rightarrow$ has energy M.

$\sum_l h(l) \rightarrow$ has energy M

(1) becomes

$$\sum_n y^2(n) \le E_x \sum_k h(k) \sum_l h(l)$$

$$E_y \le M \cdot M \cdot E_x$$

$$\boxed{E_y \le M^2 E_x}$$

such that $E_y < \infty$ if $E_x < \infty$

2.10] The following i/p - o/p pairs have been observed during the operation of a time invariant system.

$(n, x_1[n]) = \{\underset{\uparrow}{1}, 0, 2\} \longrightarrow y_1[n] = \{0, 1, 2\}$.

$x_2[n] = \{\underset{\uparrow}{0}, 0, 3\} \longrightarrow y_2[n] = \{0, 1, 0, 2\}$.

$x_3[n] = \{\underset{\uparrow}{0}, 0, 0, 1\} \longrightarrow y_3[n] = \{1, 2, 1\}$.

can you draw any conclusion regarding the linearity of system what is the impulse response of the system

The given system is non-linear

Coming to $x_2[n]$ & $x_3[n]$

$x_2[n] \cdot \{0, 0, 0, 3\} \xrightarrow{\{T\}} \{0, 1, 0, 2\}$.

$x_3(n-1) = \{0, 0, \underset{\uparrow}{0}, 1\} \xrightarrow{T} \{1, \underset{\uparrow}{2}, 1\}$.

$x_3(n+1) = \{0, 0, 1\} \xrightarrow{(T.I)} \{1, 2, 1\}$.

Now if the system is linear then.

$3x_3(n+1) \longrightarrow \{3, 6, 3\}$

But $\{3, 6, 3\} \neq \{0, 1, 0, 2\}$ Hence the system is non-linear.

2.11] The following i/p - o/p pairs have been observed during the operation of a linear s/m.

$x_1[n] = \{-1, 2, 1\} \xrightarrow{T} y_1[n] = \{1, 2, -1, 0, 1\}$

$x_2[n] = \{1, -1, -1\} \longrightarrow y_2[n] = \{-1, 1, 0, 2\}$

$x_3[n] = \{0, 1, 1\} \longrightarrow y_3[n] = \{1, 2, 1\}$

can you draw any conclusions about time variance of this s/m.

$\because x_1[n] + x_2[n] = \delta(n)$ and the s/m is linear

The impulse response of the system is given by

$y_1[n] + y_2[n] = \{0, 3, -1, 2, 1\}$

If the s/m is further time - invariant then it would be an LTI s/m.

Then o/p $x_3[n] * h[n] = \{0, 1, 1\} * \{0, 3, -1, 2, 1\}$

$y[n] = \{3, 2, 1, 3, 1\}$

But the given o/p $y_3[n] : \{1, 2, 1\}$

2.12) The only available information about a s/m consists of N i/p-o/p pairs of signals $y_i(n) = T[x_i(n)]$ $i = 1, 2 \ldots N$. Then find.

a] what is the class of i/p signals for which we can determine the o/p, using the info above, if the system is known to be linear?

system is linear, then our i/p should be any weighted linear combination of the signals

$x_i(n)$, $i = 1, 2 \ldots N$.

b] The same as above, if the s/m is known to be time invariant.

system is time invariant, then our i/p should be any as of the form $x_i(n-k)$ where k = any integer $i = 1, 2 \ldots N$.

2.13] Show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is:-

$\sum_{n=-\infty}^{\infty} |h(n)| \leq M_h < \infty$ for some constant M_h.

Deriving BIBO stability from convolution formula we know that.

Convolution formula:-

$$y[n] = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$$

Taking absolute value on both sides, for above eq

$$\Rightarrow |y(n)| = \left| \sum_{k=-\infty}^{\infty} h(k)x(n-k) \right|$$

$$\Rightarrow |y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)||x(n-k)|$$

If i/p is bounded with M_x then, we have

$$\Rightarrow |y(n)| \leq M_x \sum_{k=-\infty}^{\infty} |h(k)| \quad \left(\because |x(n)| < M_x \quad |x(n-k)| \leq M_x \right)$$

thus we have

$$|y(n)| \leq M_x \cdot \sum_{k=-\infty}^{\infty} |h(k)|$$

here o/p $y(n)$ is bounded if the impulse response satisfies the condition $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

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2.14] show that

a] A relaxed LTI is causal if and only if for any i/p $x(n)$ such that $x(n)=0$ for $n<n_0 \Rightarrow y(n)=0$ for $n<n_0$

from convolution formula we've.

$$y(n) = \sum_{k=0}^{n} x(k)h(n-k) = \sum_{k=0}^{n} h(k)x(n-k)$$

* If i/p $x(n)$ is causal.

i.e $x(n)=0$ for $n<n_0$.

thus $y(n)=0$ for $n<n_0$.

b] A relaxed LTI s/m is causal if & only if $h(n)=0$, for $n<0$

from convolution formula we have.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n_0-k)$$

$$= \sum_{k=0}^{\infty} h(k)\, x(n_0+k) + \sum_{k=-\infty}^{-1} h(k)\, x(n_0-k)$$

$$= \underbrace{[h(0)x(n_0)+h(1)x(n_0-1)+h(2)x(n_0-2)+\dots]}_{\text{past \& present values}} +$$

$$\underbrace{[h(-1)x(n_0+1)+h(-2)x(n_0+2)+\dots]}_{\text{future values}}.$$

$\Rightarrow$ To make the system causal, o/p should depend on only past & present values, this is possible if $h(n)=0$ for $n<0$.

2.15 a] show that for any real (or) complex constant a & any finite integer no's M & N we have -

$$\sum_{n=M}^{N} a^n = \begin{cases} \frac{a^M - a^{N+1}}{1-a} & \text{if } a \neq 1 \\ N-M+1 & \text{if } a=1. \end{cases}$$

we know that sum of no's in a range from L to P

$$P-(L-1) = P-L+1$$

similarly $\sum_{n=M}^{N} a^n = N-M+1$ for $a=1$

for $a \neq 1$ $\sum_{n=M}^{N} a^n$ is.

$$\sum_{n=M}^{N} a^n = a^M + a^{M+1} + \dots + a^N.$$

$$(1-a)\sum_{n=M}^{N} a^n = (a^M + a^{M+1} + \dots + a^N)(1-a)$$

$$= a^M + a^{M+1} + \dots + a^N)(1-a).$$

$$= a^M + a^{M+1} - a^{M+1} + \dots + a^N - a^N + a^{N+1}$$

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$$(1-a)\sum_{n=M}^{N} a^n = a^M - a^{N+1}$$

$$\sum_{n=M}^{N} a^n = \frac{a^M - a^{N+1}}{1-a}$$

thus we have
$$\sum_{n=M}^{N} a^n = \begin{cases} \frac{a^M - a^{N+1}}{1-a} & ; a \neq 1 \\ N - M + 1 & ; a = 1 \end{cases}$$

b) show that if $|a| < 1$, then $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$.

we know that,

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}, \quad |r| < 1$$

similarly $\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad |a| < 1$

this can be obtained if we substitute $M=0$, $N=\infty$.

if $\sum_{n=M}^{N} a^n = \frac{a^n - a^{N+1}}{1-a}$, $a \neq 1$

2.16 a) If $y(n) = x(n) * h(n)$, show that $\mathcal{E}_y = \mathcal{E}_x \mathcal{E}_h$ where

$\mathcal{E}_x = \sum_{n=-\infty}^{\infty} x(n)$.

from convolution formula, we w.

$$y(n) = \sum_{k} h(k) x(n-k)$$

summation on both sides w.r.t n.

$$\sum_n y(n) = \sum_n \sum_k h(k) x(n-k)$$

$$= \sum_k h(k) \cdot \sum_{n=-\infty}^{\infty} x(n-k) \quad \text{from give } \mathcal{E}_x = \sum_{n=-\infty}^{\infty} x(n)$$

$$\sum_n y(n) = \sum_k h(k) \cdot \sum_n x(n)$$

thus,
$$\sum_n y(n) = \left[\sum_k h(k)\right] \cdot \left[\sum_n x(n)\right]$$

$$\boxed{\mathcal{E}_y = \mathcal{E}_h \cdot \mathcal{E}_x} \Rightarrow \text{is proved.}$$

b) Compare the convolution $y(n) = x(n) * h(n)$ of the following signals & check the correction of the results by using the test in (a)

1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$x(n) * h(n)$

$x(n) \backslash h(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

$\Rightarrow x(n) * h(n) = y(n)$

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$y(n) = 35$ — ①

proof

$x(n) = [1, 2, 4] = 7$

$h[n] = [1, 1, 1, 1, 1] = 5$

$x(n) * h(n) = 35$ — ②

Hence both ① & ② are true

2] $x(n] = \{1, 2, -1\}$, $h[n] = x(n)$

$x(n) * h(n)$

$x(n) \backslash h(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$y(n) = x(n) * h(n)$

$= \{1, 4, 2, -4, 1\}$, $y(n) = 4$ — ①

proof $x(n) = [1, 2, -1] = 2$

$h(n) = x(n) = [1, 2, -1] = 2$ — ②

$x(n) * h(n) = y(n) = 4$

① & ② are equal.

3] $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

$x(n) \backslash h(n)$	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$x(n) * h(n) = y(n)$

$y(n) = \{0, 1/2, -1/2, 3/2, -2, 0, -5/2 ...$

$y(n) = -5$ — ①

proof $x(n) = \{0, 1, -2, 3, -4\} = -2$

$h(n) = \{1/2, 1/2, 1, 1/2\} = 2.5$

$x(n) * h(n) = -5$ — ②

Hence ① & ② are same.

4] $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$.

$h(n) \backslash x(n)$	1	2	3	4	5
1	1	2	3	4	5

$x(n) * h(n) = y(n) \Rightarrow \{1, 2, 3, 4, 5\} = 15$ — ①

$x[n] = \{1, 2, 3, 4, 5\} = 15$ — ①

$h(n) = \{1\} = 1$

$x(n) * h(n) = 15\ y(n)$ — ②

thus ① & ② are equal.

5] $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$.

$h(n) \backslash x(n)$	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$x(n) * h(n) = y(n)$

$y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$y(n) = 8$ — ①

$x(n) = \{1, -2, 3\} = 2$

$h(n) = \{0, 0, 1, 1, 1, 1\} = 4$.

$x(n) * h(n) = 8$ — ②. Hence ① & ② are true.

6] $x(n) = \{0,0,1,1,1,1\}$, $h(n) = \{1,-2,3\}$.

$h(n)$ \ $x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
→ -2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$x(n) * h(n) = y(n)$

$y(n) = \{0,0,1,-1,2,2,1,3\}$

$y(n) = 8$ — ①

$x(n) = \{0,0,1,1,1,1\} = 4$

$h(n) = \{1,-2,+3\} = 2$

$x(n) * h(n) = 8$ — ②

Hence ① & ② are true

7] $x(n) = \{0,1,4,-3\}$, $h(n) = \{1,0,-1,-1\}$

$h(n)$ \ $x(n)$	0	1	4	-3
→ 1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

$x(n) * h(n) = y(n)$

$y(n) = \{0,1,4,-4,-5,-1,3\}$

$y(n) = -2$ — ①

$x(n) = \{0,1,4,-3\}$, $h(n) = \{1,0,-1,-1\}$

$x(n) * h(n) = -2$

Hence ① & ② are true

8] $x(n) = (1,1,2)$, $h(n) = u(n)$, $h(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

$x(n) * h(n) = y(n)$

$(1,1,2) * u(n) = y(n)$

$y(n) = u(n) + u(n-1) + 2u(n-2)$.

$y(n) = \infty$ — ①

$x(n) = \{1,1,2\} = 3$

$h(n) = u(n) = \infty$

$x(n) * h(n) = y(n) = \infty$ — ②

Hence ① & ② are true.

9] $x(n) = \{1,1,0,1,1\}$, $h(n) = \{1,-2,-3,4\}$.

$h(n)$ \ $x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
4	4	4	0	4	4

$x(n) * h(n) \Rightarrow y(n) = \{1,-1,-5,3,-5,-1,4\}$

$y(n) = 0$ — ①

$x(n) = \{1,1,0,1,1\} * h(n) = \{1,-2,-3,4\}$

$= 4 * 0$

$= 0$ — ②

Hence ① & ② are true

10] $x(n) = \{1,2,0,2,1\}$, $h(n) = x(n)$

$h(n)$ \ $x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

$x(n) * h(n) \Rightarrow y(n) = \{1,3,4,4,10,4,4,4,1\}$

$y(n) = 36$ — ①

$x(n) = \{1,2,0,2,1\} = 6$

$h(n) = \{1,2,0,2,1\} = 6$

$x(n) * h(n) = 36$ — ②

① = ②

11] a] $x(n) = (1/2)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

$\Rightarrow$ $x(n) * h(n) = y(n)$

$$y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u(n)$$

$$\sum y(n) = 8/3 \quad \text{--- } ①$$

$\Rightarrow$

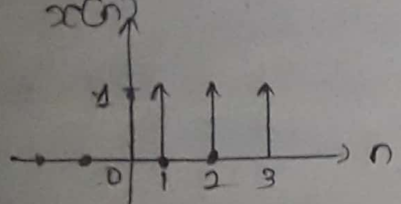
$\sum x(n) = \left(\frac{1}{2}\right)^n u(n) = 2$ Hence ① & ② are true.

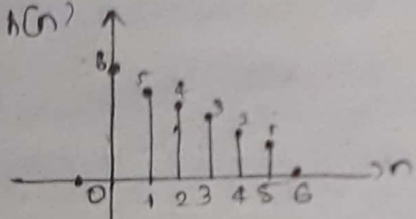
$\sum h(n) = \left(\frac{1}{4}\right)^n u(n) = 4/3$

$$x(n) * h(n) = 2 \times 4/3 = 8/3 = y(n) \text{ --- } ②$$

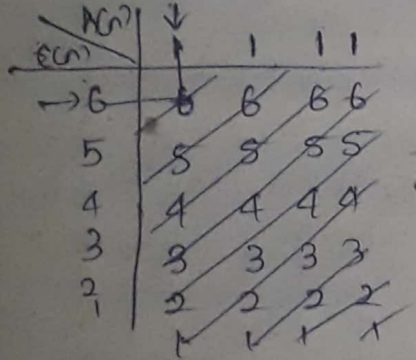
2.17 Compute and plot the convolutions $x(n) * h(n)$ & $h(n) * x(n)$ for the following pairs

a]

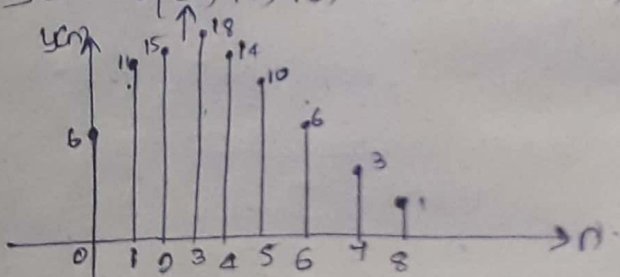




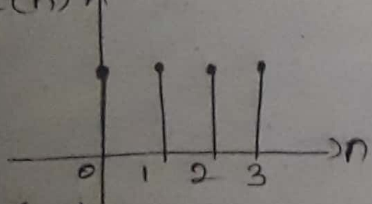
$$x(n) * h(n) = \sum x(k)\, h(n-k) = y(n)$$

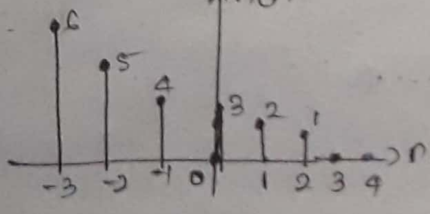


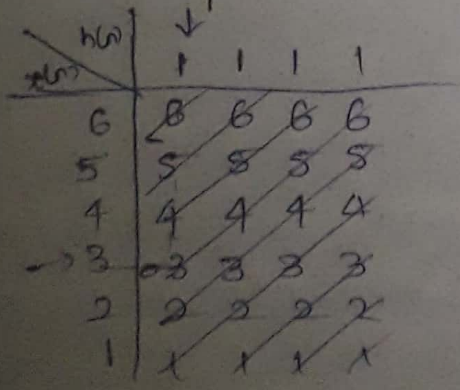
$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$



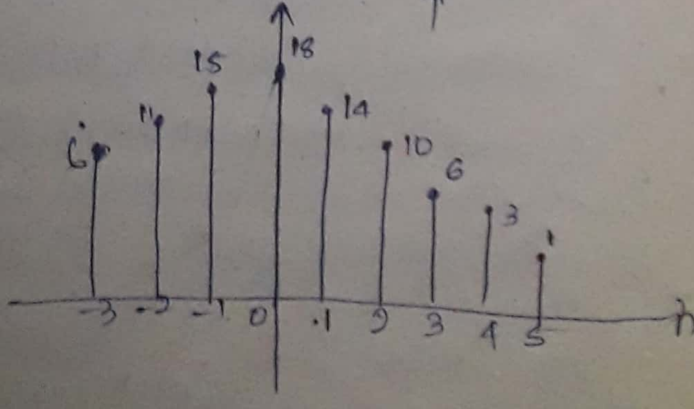
b]



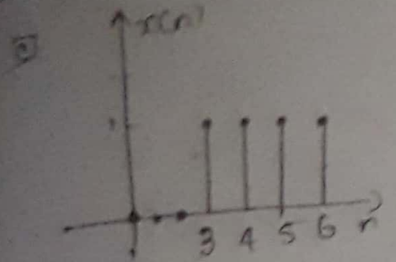


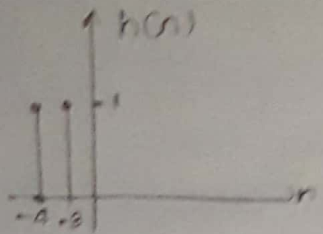


$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$



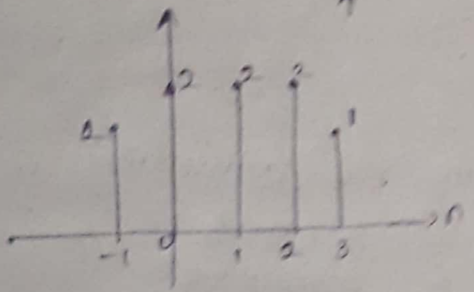
c)



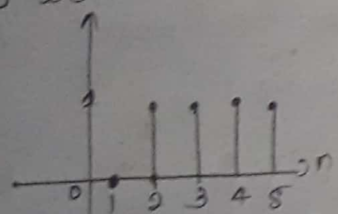


x(n) \ h(n)	1	1	0	0	0
→ 0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

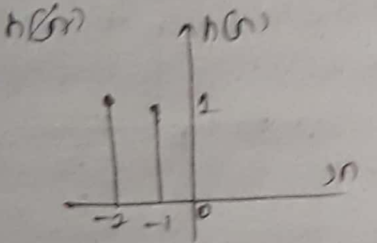
$y(n) = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0\}$



d) x(n)

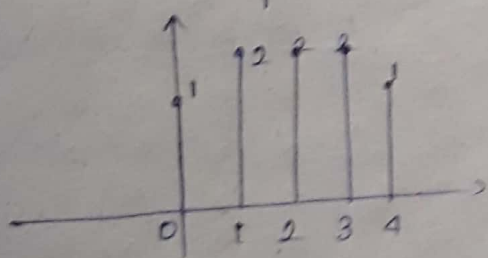


h(n)



x(n) \ h(n)	1	1	0
→ 0	0	0	0
0	0	0	0
1	1	1	0
1	1	1	0
1	1	1	0
1	1	1	0

$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0\}$



Q.18 Determine & sketch the following y[n] of the signal

$$x[n] = \begin{cases} n/3 & 0 \le n \le 6 \\ 0 & \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1 & -2 \le n \le 2 \\ 0, & \text{elsewhere} \end{cases}$$

both Graphically & Analytically

i) $x[n] = \{\underset{\uparrow}{0}, 1/3, 2/3, 1, 4/3, 5/3, 2\}$.

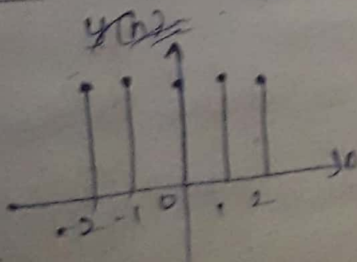
$h[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

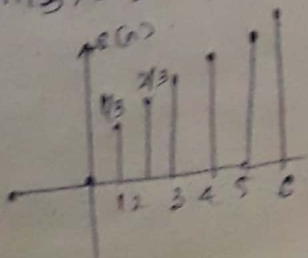
$y[n] = x[n] * h[n]$

$y[n] = \{0, 1/3, 1, \underset{\uparrow}{2}, 10/3, 4, 17/3, 5, 4, 11/3, 2\}$.

x \ h	1	1	1	1	1
→ 0	0	0	0	0	0
1/3	1/3	1/3	1/3	1/3	1/3
2/3	2/3	2/3	2/3	2/3	2/3
1	1	1	1	1	1
4/3	4/3	4/3	4/3	4/3	4/3
5/3	5/3	5/3	5/3	5/3	5/3
2	2	2	2	2	2

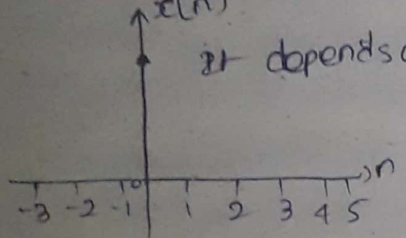
y(n)

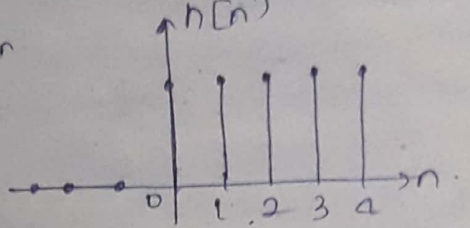




2.19] compute the convolution $y(n)$ of the signals:-

$$x(n) = \begin{cases} \alpha^n & -3 \le n \le 5 \\ 0 & , \text{elsewhere} \end{cases} \qquad h(n) = \begin{cases} 1 & , 0 \le n \le 4 \\ 0 & , \text{elsewhere} \end{cases}$$





$$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5\}.$$

$$h[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}.$$

	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5
→ 1	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5
1	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5
1	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5
1	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5
1	α^{-3}	α^{-2}	α^{-1}	1	α	α^2	α^3	α^4	α^5

$$y(n) = \{\alpha^{-3}, \alpha^{-3}+\alpha^{-2}, \alpha^{-1}+\alpha^{-2}+\alpha^{-3}, 1+\alpha^{-1}+\alpha^{-2}+\alpha^{-3}, \alpha^{-3}+\alpha^{-2}+\alpha^{-1}+1+\alpha, \alpha^{-2}+\alpha^{-1}+1+\alpha+\alpha^2, \alpha^{-1}+1+\alpha+\alpha^2+\alpha^3, \alpha+1+\alpha^2+\alpha^3+, \alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5, \alpha^2+\alpha^3+\alpha^4+\alpha^5, \alpha^3+\alpha^4+\alpha^5, \alpha^5\}$$

2.20] consider the following operations

a] multiple the integer number 131 & 122.

131×122 = 15982.

b] Compute the convolution of signal: $\{1,3,1\} * \{1,2,2\}$.

	$\underset{\downarrow}{1}$	2	2
→ 1	1	2	2
3	3	6	6
1	1	2	2

$$y(n) = \{1, 5, 9, 8, 2\}.$$

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c] multiply the polynomials $1+3z+z^2$ & $1+2z+2z^2$

$(1+3z+z^2)(1+2z+2z^2) = 1+3z+z^2+2z+6z^2+2z^3+2z^2+6z^3+2z^4$

$= 2z^4+8z^3+9z^2+5z+1$

thus,

$(1+3z+z^2)(1+2z+2z^2) = 2z^4+8z^3+9z^2+5z+1$

d] multiply 1.31 & 12.2

$1.31 \times 12.2 = 15.982$

e) comment on your results.

Those are several ways of perform convolution.

2.21] Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

a] $x(n) = \alpha^n u(n)$, $h(n) = \beta^n u(n)$ where when $\alpha = \beta$

$y(n) = \sum_k h(k) x(n-k) \approx \sum_k x(k) h(n-k)$

$y(n) = \sum_{k=0}^{n} a^k u(k) \cdot b^{n-k} u(n-k)$

$= \sum_{k=0}^{n} a^k . b^n . b^{-k} (n)$

$y(n) = b^n \sum_{k=0}^{n} (ab^{-1})^k \rightarrow y(n) = \begin{cases} \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n), & a \neq b \\ b^n (n+1) u(n), & a = b. \end{cases}$

b] $x(n) = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{elsewhere.} \end{cases}$ $\quad h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$

$x(n) = \{0, 1, 2, \underset{\uparrow}{1}, 1, 0\}$, $h(n) = \{\underset{\downarrow}{1}, -1, 0, 0, 1, 1\}$.

x(n) \ h(n)	1	-1	0	0	1	1
0	0	0	0	0	0	0
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
→ 1	1	-1	0	0	1	1
1	1	-1	0	0	1	1
0	0	0	0	0	0	0

$y(n) = \{0, 1, +1, \underset{\uparrow}{-1}, 0, 0, 3, 3, 2, 1, 0\}$.

c] $x(n) = u(n+1) - u(n-4) - \delta(n-5)$, $h(n) = [u(n+2) - u(n-3)](3 - |n|)$

$x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$. $h[n] = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$.

	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
→ 3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

$y[n] = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, -1\}$

d] $x[n] = u[n] - u[n-5]$, $h[n] = u[n-2] - u[n-8] + u(n-11) - u(n-17)$

$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$.

$h[n] = \{0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1\}$

	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1
1	0	0	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1

$y[n] = \{0, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 2, 3, 4, 5, 5, 4, 3, 2, 1\}$

2.22 Let $x[n]$ be be i/p signal to a discrete time filter with impulse response $h_i[n]$ & $y_i[n]$ be corresponding o/p.

a] Compute & sketch $x[n]$ & $y[n]$ using the same scale in all figures:

$x[n] = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$

$h_1[n] = \{1, 1\}$, $h_2[n] = \{1, 2, 1\}$, $h_3[n] = \{1/2, 1/2\}$.

$h_4[n] = \{1/4, 1/2, 1/4\}$, $h_5[n] = \{\frac{1}{4}, -1/2, 1/4\}$.

Sketch $x(n), y_1(n), y_2(n)$ on one graph & $x(n), y_3(n), y_4(n), y_5(n)$ on another graph.

i]

x(n) \ h(n)	1	1
1	1	1
4	4	4
2	2	2
3	3	3
5	5	5
3	3	3
3	3	3
4	4	4
5	5	5
7	7	7
6	6	6
9	9	9

$y[n] = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 12, 15, 9\}$

⇓

similar to $x(n) + x(n-1)$

ii]

	1	2	1
1	1	2	1
4	4	8	4
2	2	4	2
3	3	6	3
5	5	10	5
3	3	6	3
3	3	6	3
4	4	8	4
5	5	10	5
7	7	14	7
6	6	12	6
9	9	18	9

$y[n] = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$

iii]

	1/2	1/2
1	1/2	1/2
4	2	2
2	1	1
3	3/2	3/2
5	5/2	5/2
3	3/2	3/2
3	3/2	3/2
4	2	2
5	5/2	5/2
7	7/2	7/2
6	3	3
9	9/2	9/2

$y_3[n] = \{0.5, 2.5, 3, 2.5, 4, 4, 3, 3.5, 4.5, 6, 6, 7.5, 4.5\}$

iv]

	1/4	1/2	1/4
1	1/4	1/2	1/4
4	1	2	1
2	1/2	1	1/2
3	3/4	3/2	3/4
5	5/4	5/2	5/4
3	3/4	3/2	3/4
3	3/4	3/2	3/4
4	1	2	1
5	5/4	5/2	5/4
7	7/4	7/2	7/4
6	3/2	3	3/2
9	9/4	9/2	9/4

$y[n] = \{0.25, 1.5, 2.75, 2.75, 3.25, 4, 3.5, 3.25, 3.75, 5.25, 6.25, 7, 6, 2.25\}$

v]

	1/4	-1/2	1/4
1	1/4	-1/2	1/4
4	1	-2	1
2	1/2	-1	1/2
3	3/4	-3/2	3/4
5	5/4	-5/2	5/4
3	3/4	-3/2	3/4
3	3/4	-3/2	3/4
4	1	-2	1
5	5/4	-5/2	5/4
7	7/4	-7/2	7/4
6	3/2	-3	3/2
9	9/4	9/2	9/4

$y[n] = \{0.25, 0.5, -1.25, 0.75, 0.25, -1, 0.5, 0.25, 0, 0.25, -0.75, 1, -3, 2.25\}$

b] what is the difference b/w $y_1(n)$ & $y_2(n)$ & b/w $y_3(n)$ & $y_4(n)$

$y_3(n) = 1/2\, y_1(n)$ because $h_3(n) = 1/2\, h_1(n)$

$y_4(n) = 1/4\, y_2(n)$; $h_4(n) = 1/4\, h_2(n)$

c] Comment on smoothness of $y_2(n)$ & $y_4(n)$, which factors affect the smoothness.

$y_2(n)$ & $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scalar factor.

d) Compare $y_4(n)$ with $y_5(n)$ what is the differences can you explain it?

system A results in a smoother o/p. The negative value of $h_5(0)$ is responsible for non-smooth characteristics of $y_5(n)$

e) Let $h_6(n) = \{1/2, 1/2\}$. compute $y_6(n)$ + sketch $x(n)$, $y_2(n)$ & $y_6(n)$ on the same figure & comment on the results

$y_6(n) = \{1/2, 3/2, -1, -1/2, 1, -1, 0, 1/2, 1/2, 1, -1/2, 3/2, -9/2\}$

$y_2[n]$ is smoother than $y_6(n)$

2.23] The DTS having $y(n) = ny(n-1) + x(n)$; $n \geq 0$ is at rest $y(-1) = 0$ check if the s/m is linear time invariant & BIBO stable.

=) If $y_1(n) = ny_1(n-1) + x_1(n)$ & $y_2(n) = ny_2(n-1) + x_2(n)$

then $x(n) = ax_1(n) + bx_2(n)$ similarly $x(n) = ax_1(n) + bx_2(n)$

↓ produces o/p ↓ gives

$y(n) = ay_1(n) + by_2(n)$ $y(n) = ny(n-1) + x(n)$

↓

thus s/m is linear

=) If i/p is $x(n-1)$ then $y(n-1) = (n-1)y(n-2) + x(n-1)$ — ⓐ

If s/m is delay $\longrightarrow y(n-1) = ny(n-2) + x(n-1)$ — ⓑ

from ⓐ & ⓑ s/m is time invariant

=) If $x(n) = u(n)$, then $|x(n)| \leq 1$

But for this bounded i/p,

the o/p is $y(0) = 1$
$y(1) = 1+1 = 2$
$y(2) = 2\times2+1 = 5$

which is unbounded

thus, s/m is unstable.

2.24] Consider the signal $\gamma(n) = a^n u(n)$, $0 < a < 1$

a) show that any sequence $x(n)$ can be decomposed as $x(n) = \sum_{n=-\infty}^{\infty} C_k \gamma(n-k)$ & express C_k in terms of $x(n)$

from given $\gamma(n)$, $\delta(n)$ is expressed as.

$\delta(n) = \gamma(n) - a\gamma(n-1)$

$\delta(n-k) = \delta(n-k) - a\delta(n-k-1)$

Then $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

$= \sum_{k=-\infty}^{\infty} x(k)\left[\delta(n-k) - a\delta(n-k-1)\right]$

$= \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - a\sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k-1)$

$= \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k) - a\sum_{k=-\infty}^{\infty} x(k-1)\,\delta(n-k)$

$x(n) = \sum_{k=-\infty}^{\infty}\left[x(k) - ax(k-1)\right]\delta(n-k)$

$x(n) = \sum_{k=-\infty}^{\infty} C_k\,\delta(n-k)$

where $C_k = x(k) - ax(k-1)$.

b] use the properties of linearity & time invariance to express the o/p $y(n) = T(x(n))$ in term of i/p $x(n)$ & the signal $g(n) = T[u(n)]$, where $T(\cdot)$ is an LTI s/m.

$y[n] = T[x(n)]$.

$= T\left[\sum_{k=-\infty}^{\infty} C_k\,\delta(n-k)\right]$

$= \sum_{k=-\infty}^{\infty} C_k\left(T\left[\delta(n-k)\right]\right)$ i.e the o/p is expressed interms of $g(n)$.

$y(n) = \sum_{k=-\infty}^{\infty} C_k\, g(n-k)$

$= T(g(n))$

c] Express the impulse response $h(n) = T[\delta(n)]$ in terms of $g(n)$.

$h(n) = T[\delta(n)]$

$= T[\delta[n] - a\delta(n-1)]$

$= T[\delta(n)] - T[a\delta(n-1)]$

$h(n) = g(n) - ag(n-1)$.

2.25] Determine the zero-i/p response of the s/m described by second-order difference equation

$x(n) - 3y(n-1) - 4y(n-2) = 0$.

Given $x(n) - 3y(n-1) - 4y(n-2) = 0$.

zero-i/p response $\Rightarrow x(n) = 0$

$\Rightarrow x(n) - 3y(n-1) - 4y(n-2) = 0$.

$0 - 3y(n-1) - 4y(n-2) = 0$.

$3y(n-1) + 4y(n-2) = 0$

$y(n-1) + \frac{4}{3}y(n-2) = 0$

$y(n-1) = -4/3\, y(n-2)$

when $n = 0$.

$y(-1) = \frac{-4}{3} y(-2)$

similarly

$y(0) = \left(\frac{-4}{3}\right)^2 y(-2)$

$y(1) = \left(\frac{-4}{3}\right)^3 y(-2)$

$\vdots$

zero i/p response. $y(k) = \left(\frac{-4}{3}\right)^{k+2} y(-2)$

2.2.5 Determine the particular solution of the difference equation $y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$ when the forcing function is $x(n) = 2^n u(n)$

=

Given $y(n) = \frac{5}{6}y(n-1) - 1/6\, y(n-2) + x(n)$

finding homogenous solution by considering $x(n) = 0$

$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 0$.

$\lambda^n - \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2} = 0$

$\lambda^{n-2}\left[\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6}\right] = 0$.

$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0$

$\lambda = 1/2, 1/3$.

$\Rightarrow y_h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n$.

finding particular solution to $x(n)$

$x(n) = 2^n u(n)$

for a unit step sequence, the particular solution is of o/p constant k multiplied with i/p $u(n)$

i.e $y_p(n) = k\, 2^n u(n)$

Substituting this $y_p(n)$ into D.E equation.

$$y[n] = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = x(n)$$

$$k2^n u(n) - \frac{5}{6}k2^{n-1}u(n-1) + \frac{1}{6}k2^{n-2}u(n-2) = 2^n u(n)$$

for $n=2$ $\quad 2^2 k - \frac{5}{6}2k + \frac{1}{6}k2^0 = 2^2$.

$$4k - \frac{5}{3}k + \frac{k}{6} = 4$$

$$\boxed{k = 8/5}$$

thus, $y_p(n) = \frac{8}{5}2^n u(n)$.

then, the total solution is.

$$y[n] = y_p(n) + y_h(n)$$

$$y(n) = \frac{8}{5}2^n u(n) + C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n u(n)$$

Determining C_1, C_2 :-

* Assuming initial conditions as zero

i.e $y(-1) = y(-2) = 0$ — ⓐ

Substitute $n=0,1$, in D.E equation.

$$y(n) = \frac{5}{6}y(n-1) - \frac{1}{6}y(n-2) + x(n)$$

$n=0$ $\quad y(0) = \frac{5}{6}y(-1) - \frac{1}{6}y(0-2) + 2u(0)$

$= \frac{5}{6}(0) - \frac{1}{6}(0) + 1$ $\quad$ from ⓐ

$y(0) = 1$ — ⓑ

$n=1$ $\quad y(1) = \frac{5}{6}y(1-1) - \frac{1}{6}y(1-2) + 2^1 u(1)$

$= \frac{5}{6}y(0) - \frac{1}{6}y(-1) + 2$

$= \frac{5}{6}(1) - \frac{1}{6}(0) + 2$

$y(1) = \frac{17}{6}$ — ⓒ

* Substitute $n=0,1$ in total solution, then, we no

$$y(n) = \frac{8}{5}2^n u(n) + C_1\left(\frac{1}{2}\right)^n u(n) + C_2\left(\frac{1}{3}\right)^n u(n)$$

$n=0$ $\quad y(0) = \frac{8}{5}2^0 u(0) + C_1\left(\frac{1}{2}\right)^0 u(0) + C_2\left(\frac{1}{3}\right)^0 u(0)$

From ⓑ

$$1 = \frac{8}{5} + C_1 + C_2$$

$$\boxed{c_1 + c_2 = \frac{-3}{5}} \quad \text{--- (1)}$$

$n = -1$

$$y(1) = \frac{8}{5} 2^1 u(1) + c_1\left(\frac{1}{2}\right)^1 u(1) + c_2\left(\frac{1}{3}\right)^1 u(1)$$

from (c)

$$\frac{17}{6} = \frac{16}{5} + \frac{c_1}{2} + \frac{c_2}{3}$$

$$\boxed{3c_1 + 2c_2 = \frac{-11}{5}} \quad \text{--- (2)}$$

Solving (1) & (2) gives us

$$c_1 = -1, \ c_2 = \frac{2}{5}$$

=> thus total solution is

$$y[n] = \left[\frac{8}{5}(2^n) - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right] u[n]$$

2.27] Determine the response $y(n), n \geq 0$ of the sm described by the 2nd order difference equation.

$y[n] - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$ to the i/p $x(n) = 4^n u$

Given $y[n] - 3y(n-1) - 4y(n-2) = x[n] + 2x(n-1)$

-> C.E equation

$$y(n) - 3y(n-1) - 4y(n-2) = 0$$

$$\Rightarrow \lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\Rightarrow \lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$$

$$\Rightarrow \lambda^2 - 3\lambda - 4 = 0$$

$$\Rightarrow \lambda = 4, -1$$

$$\Rightarrow y(n) = c_1 4^n + c_2(-1)^n$$

-> particular solution

we have, $x(n) = 4^n u(n)$

since 4 is a characteristic root & excitation value

we have $y_p(n) = kn4^n u(n)$

Substitute this $y_p(n)$ in D.E equation

$$kn4^n u(n) - 3k(n-1)4^{n-1}u(n-1) - 4k(n-2)4^{n-2}u(n-2) = 4^n u(n)$$

$$2(4)^{n-1} u(n-1)$$

for $n=2$

$\rightarrow k(2)4^2 - 3k(2-1)4^1 - 4k(2-2)4^0 = 4^2 + 2(4)^1$

$\rightarrow 32k - 12k - 0 = 16+8$

$20k = 24$

$k = \frac{24}{20} = \frac{6}{5}$

$k = 6/5$

$$\boxed{y_p[n] = \frac{6}{5} n 4^n u(n)}$$

» Total solution is

$y(n) = y_p(n) + y_h(n)$

$y(n) = \left[\frac{6}{5} n 4^n + c_1 4^n + c_2(-1)^n\right] u(n)$

» Determining c_1 & c_2

Assuming initial conditions as zero

$\Rightarrow y(-1) = y(-2) = 0$ — ⓐ

substitute $n=0, 1$ in D.E equation

$n=0$

$\rightarrow y(0) - 3y(0-1) - 4y(0-2) = x(0) + 2x(0-1)$

$y(0) - 3y(-1) - 4y(-2) = 4^0 u(0) + 2 4^{-1} u(-1)$

$y(0) - 0 - 0 = 1+0$

$y(0) = 1$

$n=1$

$\Rightarrow y(1) - 3y(1-1) - 4y(1-2) = x(1) + 2x(1-1)$

$y(1) - 3y(0) - 4y(-1) = 4u(1) + 2 4^0 u(0)$

$y(1) - 3 - 0 = 4+2$

$y(1) = 6+3$

$y(1) = 9$

* substitute $n=0, 1$ in total equation

$n=0$ $y[0] = \left[\frac{6}{5}(0)4^0 + c_1 4^0 + c_2(-1)^0\right] u(n)$

$1 = c_1 + c_2$ — ⓑ

$n=1$ $y[1] = \left[\frac{6}{5}(1)4^1 + c_1 4^1 + c_2(-1)^1\right] u(1)$

$9 = \frac{24}{5} + 4c_1 - c_2$

$4c_1 - c_2 = \frac{21}{5}$ — ⓒ

solving ⓑ & ⓒ we get $C_1 = \frac{26}{25}$, $C_2 = \frac{-1}{25}$

then the total eqn is

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25}(-1)^n\right] u(n)$$

2.28 Determine impulse response of the following causal s/m. $y[n] - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$y[n] - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$

$\hookrightarrow \lambda = 4, -1$, $y_h(n) = C_1 4^n + C_2(-1)^n$

$y(n) = C_1 4^n + C_2(-1)^n$

when $x(n) = \delta(N) \Rightarrow y_p(n) = 0$

then C_1 & C_2 are

*) Assuming initial conditions as zero

$y(-1) = y(-2) = 0$

*) Substitute $n = 0, 1$ in DE equation

$n = 0$

$\Rightarrow y(0) - 3y(0-1) - 4y(0-2) = x(0) + 2x(0-1)$

$\Rightarrow y(0) - 3y(-1) - 4y(-2) = \delta(0) + 2\delta(-1)$

$\Rightarrow y(0) - 0 - 0 = 1 + 0$

$\Rightarrow y(0) = 1$

$n = 1$

$\Rightarrow y(1) - 3y(1-1) - 4y(1-2) = x(1) + 2x(1-1)$

$\Rightarrow y(1) - 3y(0) - 4(-1) = \delta(1) + 2\delta(0)$

$\Rightarrow y(1) - 3(1) - 0 = 0 + 2$

$\Rightarrow y(1) = 2 + 3$

$\Rightarrow y(1) = 5$

* Substitute $n = 0, 1$ in total solution.

then we get

$y(0) = C_1 + C_2 \Rightarrow C_1 + C_2 = 1$ — ⓑ

$y(1) = 4C_1 - C_2 \Rightarrow 4C_1 - C_2 = 5$ — ⓒ

solving ⓑ & ⓒ we get

$C_1 = \frac{6}{6}$, $C_2 = -1/5$

then the total solution is

impulse response $h(n) = y(n) = \left[\frac{6}{5}4^n - \frac{1}{5}(-1)^n\right]u(n)$

2.29] let $x(n) \rightarrow N_1 \le n \le N_2$ & $h(n) \rightarrow M_1 \le n \le M_2$ be 2 finite duration signals.

a] Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1 & M_2.

b] Determine.

Convolution of $x(n)$ & $h(n)$ gives the range $L_1 \le n \le L_2$ where $L_1 = N_1 + M_1$ & $L_2 = N_2 + M_2$

b] Determine the limits of the cases of partial overlap from left full overlap & partial overlap from the right. For convenience assume that $h(n)$ has shorter duration than $x(n)$

L_1 & L_2 for different cases:-

Partial overlap from left :- $L_1 = N_1 + M_1$, $L_2 = N_1 + M_2 - 1$

Full overlap :- $L_1 = N_1 + M_2$, $L_2 = N_2 + M_1$

partial overlap from right :- $L_1 = N_2 + M_1 + 1$, $L_2 = N_2 + M_2$

c] Illustrate the validity of your results by computing the convolution of the signal $x(n) = \begin{cases} 1 & -2 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$ & $h(n) = \begin{cases} 2 & -1 \le n \le 2 \\ 0 & \text{else} \end{cases}$

findinding the above values of L_1 & L_2 for the different mentional above cases for $x(n) = \{1, 1, 1, \underset{\uparrow}{1}, 1, 1, 1\}$

$h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$

here $N_1 = -2$, $N_2 = 4$ $\Rightarrow N_1 \le n \le N_2$

$M_1 = -1$, $M_2 = 2$ $\Rightarrow M_1 \le n \le M_2$

then L_1, L_2 for below cases are:-

partial overlap from left :- $L_1 = -3$, $L_2 = -1$

full overlap :- $L_1 = 0$, $L_2 = 3$

partial overlap from right :- $L_1 = 4$, $L_2 = 6$.

2.30] Determine the impulse response & the unit step response of the systems described by the difference equation.

a] $y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

Impulse response = total solution.

a. I] Impulse response

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$\Rightarrow y_h(n)$:-

$x(n) = 0$

$\Rightarrow y(n) = 0.6y(n-1) - 0.08y(n-2)$

$\lambda^n = 0.6\lambda^{n-1} - 0.08\lambda^{n-2}$.

$\Rightarrow \lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$.

$\Rightarrow \lambda^{n-2}[\lambda^2 - 0.6\lambda + 0.08] = 0$

$\Rightarrow \lambda^2 - 0.6\lambda + 0.08 = 0$

$\Rightarrow \lambda = 0.2, 0.4$

then.

$y_h(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

$\rightarrow y_p(n)$:-

$x(n) = \delta(n)$

$\Rightarrow y_p(n) = 0$

$\rightarrow y(n) = y_p(n) + y_h(n)$

$\Rightarrow y(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

$\rightarrow$ Determining C_1, C_2:-

$y(-1) = y(-2) = 0$

* $n = 0, 1$ in DE equation.

$n = 0$

$\Rightarrow y(0) = 0.6y(-1) - 0.08y(-2) + \delta(0)$

$\Rightarrow y(0) = 0.6(0) - 0.08(0) + 1$

$\Rightarrow y(0) = 1$

$n = 1$ $y(1) = 0.6y(0) - 0.8y(-1) + \delta(1)$

$= 0.6(1) - 0 + 0$

$y(1) = 0.6$

$\Rightarrow n=0,1$ in total equation gives us.

$$c_1+c_2=1 \text{ --- (b)}$$

$$\frac{1}{5}c_1+\frac{2}{5}c_2=0.6 \text{ --- (c)}$$

solving (b) & (c) gives us -

$$c_1=-1,\ c_2=3.$$

then $h(n)=y(n)$ - Impulse response is

$$h(n)=\left[-\left(\frac{1}{5}\right)^n+2\left(\frac{2}{5}\right)^n\right]u(n)$$

ii) step response

$$s(n)=\sum_{k=0} h(n-k),\ n\geq 0$$

$$=\sum_{k=0}\left[2\left(\frac{2}{5}\right)^{n-k}-\left(\frac{1}{5}\right)^{n-k}\right)$$

$$s(n)=\left\{\frac{1}{0.12}\left[\left(\frac{2}{5}\right)^{n+1}-1\right]-\frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1}-1\right]\right\}u(n).$$

b) $y(n)=0.7y(n-1)-0.1y(n-2)+2x(n)-x(n-2)$

i) Impulse response

$$y(n)=0.7y(n-1)-0.1y(n-2)+2x(n)-x(n-2)$$

$\Rightarrow y_h(n)$:-

$$x(n)=0 \ \& \ x(n-2)=0$$

$$\Rightarrow y(n)=0.7y(n-1)-0.1y(n-2)$$

$$\lambda^n=0.7\lambda^{n-1}-0.1\lambda^{n-2}$$

$$\lambda^n-0.7\lambda^{n-1}+0.1\lambda^{n-2}=0$$

$$\lambda^{n-2}\left[\lambda^2-0.7\lambda+0.1\right]=0$$

$$\Rightarrow \lambda^2-0.7\lambda+0.1=0.$$

$$\lambda=1/2,\ 1/5$$

then $y_h(n)=c_1\left(\frac{1}{2}\right)^n+c_2\left(\frac{1}{5}\right)^n.$

$\Rightarrow y_p(n)$:- $\quad x(M)=\delta(N)$

$$\Rightarrow y_p(n)=0.$$

$$\Rightarrow y(n)=y_p(n)+y_h(n)$$

$$\Rightarrow y(n)=c_1\left(\frac{1}{2}\right)^n+c_2\left(\frac{1}{5}\right)^n$$

Determining c_1, c_2:-

$\Rightarrow y(-1) = y(-2) = 0$

$\Rightarrow n = 0, 1$ in D.E equation

$n=0$, $y(0) = 0.7y(-1) - 0.1y(-2) + 2\delta(0) - \delta(-2)$

$y(0) = 0.7(0) - 0.1(0) + 2 - 0$

$y(0) = 2$

$n=1$

$y(1) = 0.7y(0) - 0.1y(-1) + 2\delta(1) - \delta(-1)$

$= 0.7y(0) - 0.1(0) + 2(0) - 0$

$y(1) = 0.7 \times 2$

$y(1) = 1.4$

$\Rightarrow n = 0, 1$ in total equation of solution gives us

$\rightarrow c_1 + c_2 = 2$ — (b)

$\rightarrow \frac{1}{2}c_1 + \frac{1}{5}c_2 = 1.4$ — (c)

Solving (b) & (c) gives us

$c_1 = \frac{10}{3}$, $c_2 = -4/3$.

then

$h(n) = y(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right]u(n)$.

ii) Step response -

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$s(n) = \left[\frac{10}{3}\left[\frac{1}{2}^n\left(2^{n+1}-1\right)\right] - \left[\frac{1}{3}\left(\frac{1}{5}^n\left(5^{n+1}-1\right)\right)\right]\right]$$

2.31] Consider the sm with impulse response

$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$ Determine the i/p $x(n)$ for

$0 \le n \le 8$ that will generate the o/p sequence

$y(n) = \{\underset{\uparrow}{1}, 2, 2, 5, 3, 3, 3, 2, 1, 0 \ldots\}$.

2.3
in
re
b)
c]
a
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$h(n) = \{1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}\}$

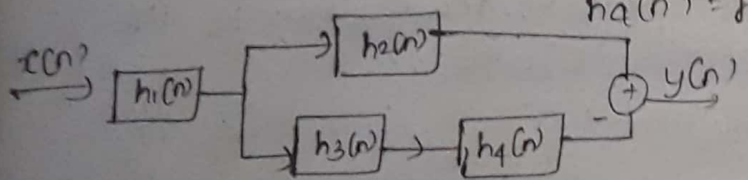
$y[n] = \sum_{k=-\infty}^{\infty} x(k) h[n-k]$

2.32) Consider the interconnection of LTI s/m as shown in fig below. a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$ & $h_4(n)$

b) Determine the $h(n)$ when $h_1(n) = \{1/2, 1/4, 1/2\}$

$h_2(n) = h_3(n) = (n+1)u(n)$

$h_4(n) = \delta(n-2)$



c) Determine the response of the s/m in part b) if

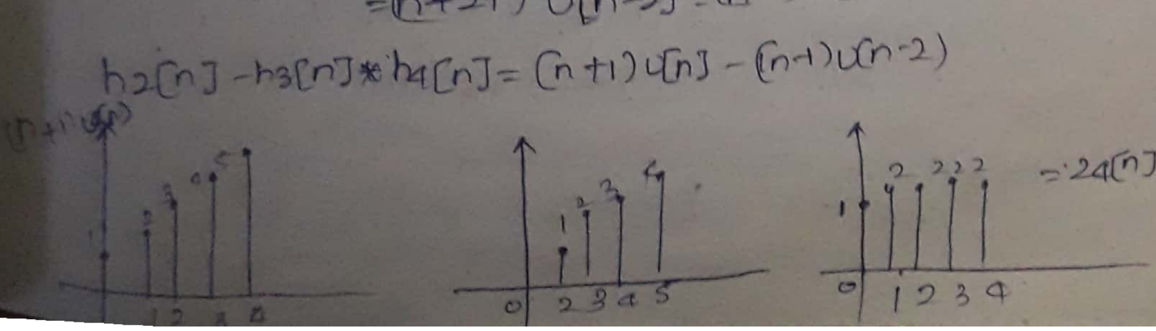
$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3)$

a) $h[n] = h_1[n] * [h_2[n] - h_3[n] * h_4[n]]$.

b) $h_3[n] * h_4[n] = (n+1) u[n] * \delta(n-2)$

$= (n-2+1) u[n-2] = (n-1)u(n-2)$

$h_2[n] - h_3[n] * h_4[n] = (n+1)u[n] - (n-1)u(n-2)$



$= 2u[n] - \delta[n]$

$$h[n] = \left(\tfrac{1}{2}\delta[n] + \tfrac{1}{4}\delta[n-1] + \tfrac{1}{2}\delta[n-2]\right) * [2u[n] - \delta[n]]$$

$$= u[n] + \tfrac{1}{2}u[n-1] + u[n-2] - \tfrac{1}{2}\delta[n] - \tfrac{1}{4}\delta[n-1] - \tfrac{1}{2}\delta[n]$$

$$= 1/2\,\delta[n] + \tfrac{5}{4}\delta[n-1] + 2\delta[n-2] + \tfrac{5}{2}u[n-3]$$

c) $y[n] = x[n] * h[n]$

$$= (\delta[n+2] + 3\delta[n-1] - 4\delta[n-3]) * h[n]$$

$$= h[n+2] + 3h[n-1] - 4h[n-3]$$

$$= \tfrac{1}{2}\delta[n+2] + \tfrac{5}{4}\delta[n+1] + 2\delta[n] + \tfrac{5}{4}u[n-1] + \tfrac{3}{2}\delta[n-1]$$
$$+ \tfrac{15}{4}\delta[n-2] + 6\delta[n-3] + \tfrac{15}{2}u[n-4] - 2\delta[n-3] - 5\delta[n-4]$$
$$- 8\delta[n-5] - 10\delta[n-6]$$

$n=-2$; $y[n] = 1/2$, $n=-1 \to 5/4$, $n=0 \to 2$; $n=1 \to \frac{5}{2} + \frac{3}{2} = 4$

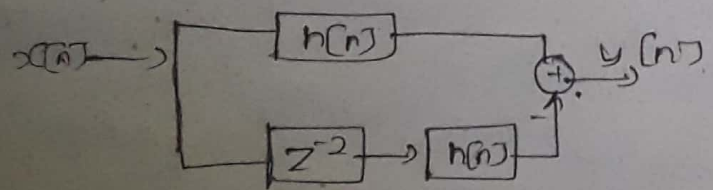
$n=2 \to \frac{15}{4} + \frac{5}{2} = \frac{25}{2}$; $n=3 \to \frac{5}{2} + 6 - 2 = \frac{13}{2}$

$n=3 \to \frac{5}{2} + \frac{15}{2} - 5 = 5$

$n=5 \to \frac{5}{2} + \frac{15}{2} - 8 = 2$; $n=6 \to -10 + \frac{15}{2} + \frac{5}{2} = 0$

$$\therefore y[n] = \left\{\tfrac{1}{2}, \tfrac{5}{4}, \underset{\uparrow}{2}, \tfrac{25}{4}, \tfrac{13}{2}, 5, 2, 0, 0\right\}$$

2.33) Consider the system with $h[n] = a^n u[n]$, $-1 < a < 1$. Determine the response $y[n]$ of the system to the excitation $x[n] = u[n+5] - u[n-10]$.



$h'[n] = h[n] - h[n-2] = a^n\delta[n] - a^{n-2}u[n-2]$

$y[n] = x[n] * h[n]$

$$[u[n+5] - u[n-10]] * [a^n u[n] - a^{n-2}u[n-2]]$$

$$\Rightarrow a^n u[n] * u[n+5] - a^{n-2}u[n-2] * u[n+5] - a^n u[n] * u[n]$$
$$+ a^{n-2}u[n-2] * u[n-10]$$

$$\Rightarrow a^n u[n] * u[n+5] = \sum_{k=-\infty}^{\infty} u(k+5)\,a^{n-k}\,u(n-k) = \sum_{k=-5}^{n} a^{n-k}$$

$$= 1 + a + \dots + a^{n+5} = \frac{a^{n+6} - 1}{a-1}\, u[n+5]$$

$$a^{n-2}u[n-2]*u[n+5] = \sum_{k=-\infty}^{\infty} u[k+5]\, a^{n-k-2}\, u(n-k-2)$$

$$= \sum_{k=-5}^{n-2} a^{n-k-2} \quad = \sum_{k=3}^{\infty} a^{n-k} \text{ [replacing k by k-1]}.$$

$$= 1+a+\cdots+a^{n+3} = \frac{a^{n+4}-1}{a-1} u[n+3]$$

$$a^{n}u[n]*u[n-10] = \sum_{k=-\infty}^{\infty} u(k-10)\, a^{n-k}\, u(n-k) = \sum_{k=10}^{n} a^{n-k}$$

$$= 1+a+\cdots+a^{n-10} = \frac{a^{n-10+1}-1}{a-1} u[n-10]$$

$$= \frac{a^{n-9}-1}{a-1} u[n-10]$$

$$a^{n-2}u[n-2]*u[n-10] = \sum_{k=-\infty}^{\infty} u(k-10)\, a^{n-k-2}\, u(n-k-2)$$

$$\Rightarrow \sum_{k=10}^{n-2} a^{n-k-2} = \sum_{k=12}^{n} a^{n-k} = 1+a+\cdots+a^{k-12}.$$

$$= \frac{a^{n-11}-1}{a-1} u(n-12)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u[n+3] - \frac{a^{n+4}-1}{a-1} u[n+3] - \frac{a^{n-9}-1}{a-1} u[n-10]$$

$$+ \frac{a^{n-11}-1}{a-1} u[n+12]$$

2.34 Compute ~~and sketch~~ the step response of the s/m.

$$y(n) = \frac{1}{M}\sum_{k=0}^{M-1} x(n-k)$$

$$h[n] = \frac{1}{M}\left[u[n] - u[n-M]\right]$$

$$y[n] = u[n]*h[n] = u[n]*\frac{1}{M}\cdot\left[u[n]-u[n-M]\right]$$

$$= \frac{1}{M}\left[u[n]*u[n]\right] - \frac{1}{M}\left[u[n]*u[n-M]\right]$$

$$= \frac{1}{M}\sum_{k=-\infty}^{\infty} u(n-k)\,u(k) - \frac{1}{M}\sum_{k=-\infty}^{\infty} u(k)\,u[n-k-M].$$

$$= \frac{1}{M}\sum_{k=0}^{n}(1) - \frac{1}{M}\sum_{k=0}^{n-M}(1)$$

$$= \frac{1}{M}\sum_{k=0}^{n}(1) - \frac{1}{M}\sum_{k=M}^{n}(1)$$

If $n \geq M = \frac{1}{M}\left[n+1-(n-M+1)\right] = 1$.

$n < M = \frac{1}{M}\left[n+1-0\right] = \frac{n+1}{M}$.

$\therefore S[n] = \begin{cases} \frac{n+1}{M} ; & n < M \\ 1 ; & n \geq M \end{cases}$

$2 \cdot \boxed{35}$ Determine the range of values of the parameters from which the LTI s/m with impulse response

$h[n] = \begin{cases} a^n, & n \geq 0, \text{ reven} \\ 0, & \text{otherwise} \end{cases}$ is stable

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0}^{\infty} |a^n| = \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

stable if $|a| < 1$

$2 \cdot 3\boxed{B}$ Determine the response of the system with impulse response $h(n) = a^n u(n)$ to the i/p signal $x(n) = u(n) - u(n-10)$

$\underline{y(n)} = x(n) * h(n)$

$= \sum_{k=0}^{\infty} x(k) h(n-k)$

$= \sum_{k=0}^{\infty} u(k) h(n-k)$

$= \sum_{k=0}^{n} h(n-k)$

$= \sum_{k=0}^{n} a^{n-k}$

$= a^n \sum_{k=0}^{n} a^{-k}$

$= a^n \cdot \frac{1-a}{1-a} u(n)$

$y_1(n) = \frac{1-a^{n+1}}{1-a} u(n)$

Similarly

$y_1(n-10)$ is the o/p when $x(n) = u(n-10)$

$\underline{y_1(n-10)} = \frac{1-a^{n-9}}{1-a} u(n-10)$

then $\underline{y(n) = y_1(n) - y_1(n-10)}$

$$\underline{y(n)} = \frac{1}{1-a}\left[(1-a^{n+1}) u(n) - (1-a^{n-9}) u(n-10)\right]$$

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Q.37] Determine the response of the sim characterized by the impulse $h(n) = (\frac{1}{2})^n u(n)$ to i/p signal

$x(n) = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$

we know that

$$y(n) = \frac{1}{1-a}\left[(1-a^{n+1})u(n) - (1-a^{n-9})u(n-10)\right]$$

now when $h(n) = (\frac{1}{2})^n u(n)$

$y(n) = ?$ It is clear that $a = \frac{1}{2}$

then $$y[n] = \frac{1}{1-\frac{1}{2}}\left[\left(1-(\tfrac{1}{2})^{n+1}\right)u(n) - \left(1-(\tfrac{1}{2})^{n-9}\right)u(n-10)\right]$$

$$y[n] = 2\left[\left(1-(\tfrac{1}{2})^{n+1}\right)u(n) - \left(1-(\tfrac{1}{2})^{n-9}\right)u(n-10)\right]$$

Q.38] Determine the response of the (relaxed) sim characterized by impulse response $h(n) = (\frac{1}{2})^n u(n)$ to i/p signals

a] $x(n) = 2^n u(n)$ b] $x(n) = u(n)$.

a] $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (\tfrac{1}{2})^k u(k) \cdot 2^{n-k} u(n-k)$$

$$= \sum_{k=-\infty}^{n} (\tfrac{1}{2})^k \cdot 2^{n-k}$$

$$= \sum_{k=-\infty}^{n} (\tfrac{1}{2})^k \cdot 2^n (\tfrac{1}{2})^k$$

$$= 2^n \sum_{k=-\infty}^{\infty} (\tfrac{1}{4})^k$$

$$= 2^n \left[1 - (\tfrac{1}{4})^{n+1}\right] \cdot \frac{4}{3}$$

$$= 2^{n+1} \left[1 - (\tfrac{1}{4})^{n+1}\right] \cdot \frac{2}{3}$$

$$= \left[2^{n+1} - (\tfrac{2}{4})^{n+1}\right] \cdot \frac{2}{3}$$

$$y[n] = \frac{2}{3}\left[2^{n+1} - (\tfrac{1}{2})^{n+1}\right] u(n)$$

b] $y(n) = x[n] * h(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^k u(k) u(n-k)$

$$y(n) = \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

a] $y(n) = 0, \ n < 0$.

b] $y[n] = \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1} \left(\frac{1}{2}\right)^k$$

$$= 2 - \left(\frac{1 - \left(\frac{1}{2}\right)^n}{1/2}\right)$$

$$y(n) = 2\left(\frac{1}{2}\right)^n, \ n \geq 0.$$

thus $y(n) = \begin{cases} 2 & , n < 0 \\ 2\left(\frac{1}{2}\right)^n & , n \geq 0. \end{cases}$

2.39] Three s/m with impulse response $h_1(n) = \delta(n) - \delta(n-1)$; $h_2(n) = h(n)$ & $h_3(n) = u(n)$ are connected in cascade.

a] what is the impulse response of overall s/m $h_e(n)$

b] does the order of the interconnection affect the overall s/m?

a] $h_e[n] = h_1[n] * h_2(n) * h_3[n]$.

$$= [\delta[n] - \delta[n-1]] * h[n] * u[n]$$

$$= [\delta[n] - \delta[n-1]] * u[n] * h[n]$$

$$= [u(n) - u[n-1]] * h[n].$$

$$= u[n] * h[n]$$

$$h_e[n] = h[n]$$

b] No, it doesn't affect, because convolutions satisfies commutative law

$$\boxed{a * b = b * a}$$

2.40] a] prove and explain graphically the difference b/w the relations $x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$ & $x(n) * \delta(n-n_0) = x(n-n_0)$

i] $x(n)\delta(n-n_0) = x(n_0)\delta(n-n_0)$

This true. Because only at $n = n_0$ the value of $x[n]$ is existed with an impulse.

$x[n] * \delta(n-n_0) = x(n-n_0)$

This can be obtained from the shifted version of $x(n)$

b] Show that a discrete-time slm, which is described by a convolution summation is LTI & relaxed.

$$y[n] = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = h(n) * x(n)$$

Linearity $x_1(n) \longrightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \longrightarrow y_2(n) = h(n) * x_2(n)$

$x(n) = ax_1(n) + bx_2(n) \longrightarrow y(n) = h(n) * x(n)$

$y(n) = h(n) * [ax_1(n) + bx_2(n)]$

$= ah(n) * x_1(n) + bh(n) * x_2(n).$

thus, $\boxed{y(n) = ay_1(n) + by_2(n)}$

Time invariance

$x(n) \longrightarrow y(n) = h(n) * x(n) \longrightarrow$ delay

$= h(n) * x(n-n_0)$

$= y(n-n_0)$

$x(n-n_0) \longrightarrow y_1(n) = h(n) * x(n-n_0$.

$= \sum_{k=0}^{\infty} h(k) x(n-n_0-k)$

$y_1(n) ? y(n-n_0)$

c] what is the impulse response of the slm described by $y[n] = x(n-n_0)$

$y[n] = h[n] * x(n)$

$= x(n-n_0)$

To get o/p $x(n-n_0)$, let $h(n)$ be

$\Rightarrow \delta(n-n_0) * x(n) = x(n-n_0)$

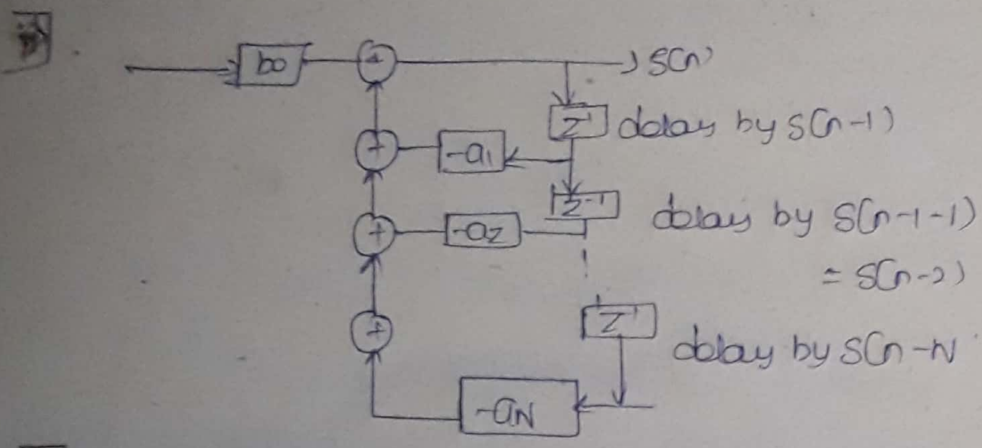
therefore, impulse response, $h(n) = \delta(n-n_0)$.

2.41] Two signals $s(n)$ & $v(n)$ are related through the following difference equation.

$s(n) + a_1 s(n-1) + \cdots + a_N s(n-N) = b_0 v(n).$

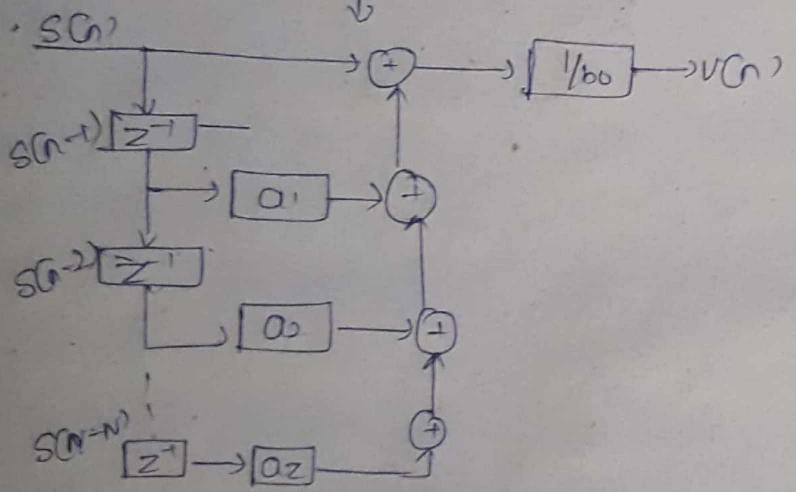
i] Design the s/m that generates s(n) when excited by v(n)

$s(n) = -a_1 s(n-1) - a_2 s(n-2) + \dots + a_N s(n-N) + b_0 v(n)$



ii] The s/m that generates v(n) when excited by s(n)

$v(n) = \frac{1}{b_0}[s(n) + a_1 s(n-1) + \dots + a_N s(n-N)]$



iii] what is the impulse response of the cascade interconnection of the s/m in parts of (i) & (ii)

In (i) → impulse response is $-a_1, -a_2, -a_3, \dots -a_N = -a_n$

In ii) → Impulse response is $a_1, a_2, a_3 \dots a_N = +a_n$

2.42] Compute the zero state response of the s/m described by the difference equation

$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$ to the i/p

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$ by solving the difference equation recursively.

$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$

$y(n) = \frac{-1}{2}y(n-1) + x(n) + 2x(n-2)$

at $n=-2$ $x(n)=1$ then $y(-2) = -\frac{1}{2}y(-3) + x(-2) + 2x(-4) = 1$

$n=-1$ $x(n)=2$ then $y(-1) = -\frac{1}{2}y(-2) + x(-1) + 2x(-3) = 3/2$

$n=0$ $x(n)=3$ then $y(0) = -1/2\, y(-1) + x(0) + 2x(-2)$

$= -3/4 + 3 + 2 = 17/4$

$n=1$, $x(n)=4$ then $y(1) = -\frac{1}{2}y(0) + x(1) + 2x(-1)$.

$= -\frac{17}{2} + 4 + 4 = \frac{47}{8}$

$n=2$, $x(n)=2$ then $y(2) = -\frac{1}{2}y(1) + x(2) + 2x(0)$

$= \frac{-47}{16} + 2 + 6 = \frac{81}{16}$.

$n=3$, $x(n)=1$ then $y(3) = -\frac{1}{2}y(2) + x(3) + 2x(1)$

$= \frac{-81}{32} + 1 + 8 = \frac{207}{32}$

thus,
$y[n] = \{1, 3/2, \underset{\uparrow}{17/4}, \frac{47}{8}, \frac{81}{16}, \frac{207}{32}\}$

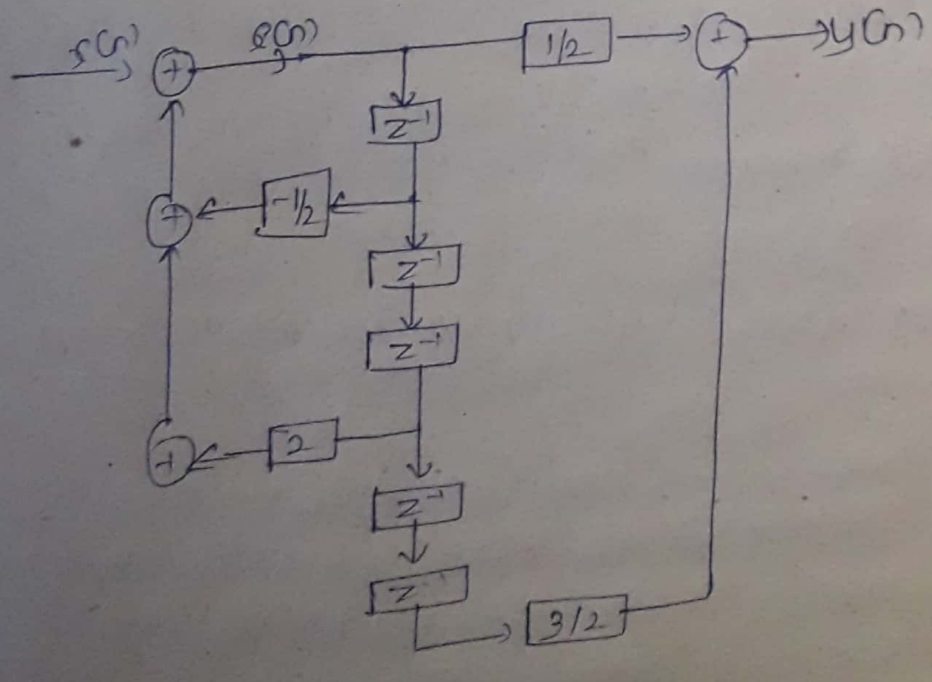
2.43 Determine the direct form of realization for each of the following LTI s/m.

a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

$2y(n) + y(n-1) + 4y(n-3) = x(n) + 3x(n-5)$

$y(n) = \frac{1}{2}[-y(n-1) + 4y(n-3) + x(n) + 3x(n-5)]$

$= \frac{-1}{2}[y(n-1) + 2y(n-3) + \frac{1}{2}x(n) + \frac{3}{2}x(n-5)]$.



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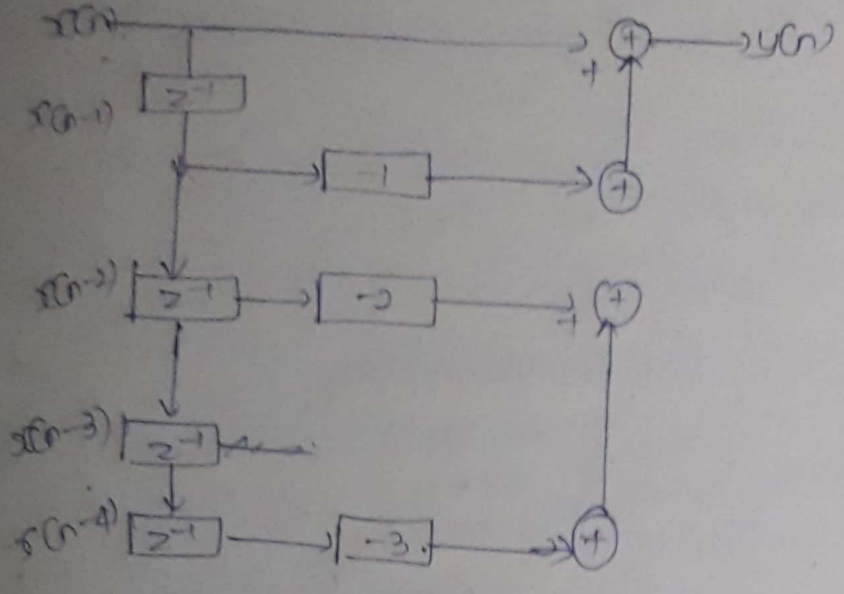
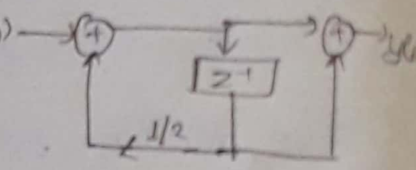
b) $y(n) = x[n] - x(n-1) + 2x(n-2) - 3x(n-4)$

$y(n) = x(n) - x(n-1) + 2x(n-2) - 3x(n-4)$



2.44 Consider the discrete s/m



a) Compute the 1st 10 signals of its impulse response.

$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

At let $x(n) = \{0, 1, 0, 0, 0, \ldots\} = \delta(n)$

$n=0 \to x(n)=1 \to$ then $y(0) = \frac{1}{2}y(-1) + x(0) + x(-1) = 1$

$n=1 \to x(n)=0 \to$ then $y(1) = \frac{1}{2}y(0) + x(1) + x(0) = \frac{1}{2} + 0 + 1 = 3/2$

$n=2 \to x(n)=0 \to$ then $y(2) = \frac{1}{2}y(1) + x(2) + x(1) = \frac{1}{2}\left(\frac{3}{2}\right) + 0 + 0 = 3/4$

$n=3 \to x(n)=0 \to$ then $y(3) = \frac{1}{2}y(2) + x(3) + x(2) = \frac{1}{2}\left(\frac{3}{4}\right) + 0 + 0 = \frac{3}{8}$

$n=n \to x(n)=0 \to$ then $y(n) = \{1, 3/2, 3/4, 3/8, 3/16, 3/32 \ldots$

b) Find i/p & o/p relation

$y[n] = \frac{1}{2}y[n-1] + x(n) + x(n-1)$

c) Apply i/p $x[n] = \{1, 1, \ldots\}$ & Compute the 1st 10 samples of o/p.

$x(n) = \{1, 1, 1 \ldots\}$

at $n=0 \to x(n)=1 \to$ then $y(0) = \frac{1}{2}y(-1) + x(0) + x(-1) = 1$

$n=1 \to x(n)=1 \to$ then $y(1) = \frac{1}{2}y(0) + x(1) + x(0) = \frac{1}{2} + 1 + 1 = \frac{5}{2}$

$n=2 \to x(n)=1 \to$ then $y(2) = \frac{1}{2}y(n) + x(2) + x(1) = \frac{13}{4}$

$n=3 \to x(n)=1 \to$ then $y(3) = \frac{1}{2}y(n) + x(3) + x(2) = \frac{29}{8}$

$n=n \to x(n)=1 \to$ then $y(n) = \{1, 5/2, 13/4, \frac{29}{8} \ldots\}$

d) Compute the 1st 10 samples of o/p for the i/p given in part (c) by using Convolution.

$$y(n) = u(n) * h(n) = \sum_{k=0}^{\infty} u(k) h(n-k)$$

$$y(n) = \sum_{k=0}^{n} h(n-k)$$

At $n=0$ $y(0) = h(0) = 1$

At $n=1$, $y(1) = h(0) + h(1) = 5/2$.

At $n=2$, $y(2) = h(0) + h(1) + h(2) = 13/4$.

e) Is the system causal & stable -

Yes the system is causal & stable

from the ~~zero~~state response equation

from $\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4} \cdots\right)$

stable o/p

2.45 Consider the s/m described by the eqn to

$y[n] = ay(n-1) + bx(n)$ then.

a) Determine b in terms of a, so that $\sum_{n=-\infty}^{\infty} h(n) = 1$.

$$y[n] = ay(n-1) + bx(n)$$

$$h(n) = ba^n u(n)$$

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

b) Compute zero-state step response s(n) of the system & Choose b so that $s(\infty) = 1$

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$$

at $n = \infty$

$$\Rightarrow s(\infty) = \frac{b}{1-a} = 1$$

$$\boxed{b = 1-a}$$

c) Compare the values of b obtained in part (a) & (b).

In both cases b is same & equal.

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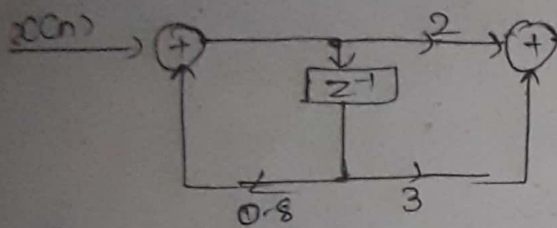
2.46 A discrete time system is realized by the struc..



then

(a) determine impulse response.

from the figure

$$y[n] = 2x(n) + 3x(n-1) + 0.8y(n-1)$$

$$y[n] - 0.8y[n-1] = 2x(n) + 3x(n-1)$$

$y_h(n)$:-

$$y(n) - 0.8y(n-1) = 0$$

$$\lambda^n - 0.8\lambda^{n-1} = 0$$

$$\lambda^{n-1}[\lambda - 0.8] = 0$$

$$\lambda - 0.8 = 0$$

$$\lambda = 0.8$$

$$y_h(n) = C(0.8)^n$$

$y_p(n)$. let $x(n) = \delta(n) \Rightarrow y_p(n) = 0$

$$y(n) = y_p(n) + y_h(n)$$

$$\Rightarrow y(n) = C(0.8)^n$$

Substitute $n=0$ in d-E equation & initial condition

$$y(-1) = 0$$

$$\Rightarrow y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$$

At $n=0$ we have

$$y(0) - 0.8y(-1) = 2x(0) + 3x(-1)$$

$$y(0) - 0.8(0) = 2\delta(0) + 3\delta(-1)$$

$$y(0) = 2$$

Let us consider the response of the system is

$$y(n) - 0.8y(n-1) = x(n)$$

→ substitute $n=0$ in above equation at initial conditions $y(-1) = 0$

then, we have

$$y(0) - 0.8y(-1) = x(0)$$

$$\Rightarrow y(0) - 0.8(0) = \delta(0)$$

$$y(0) = 1 \quad \text{—} (1)$$

Similarly substitute n=0 in y(n)

then we've $y(0) = c(0.5)^0$

$\Rightarrow y(0) = c$ — ⓑ

Equating ⓐ & ⓑ gives us

$\boxed{c=1}$

then impulse response h(n) is

$h(n) = y(n) = 2(0.5)^n u(n) + 3(0.5)^{n-1} u(n-1)$

$\Rightarrow h(n) = 2\delta(n) + 4.6(0.5)^{n-1} u(n-1)$

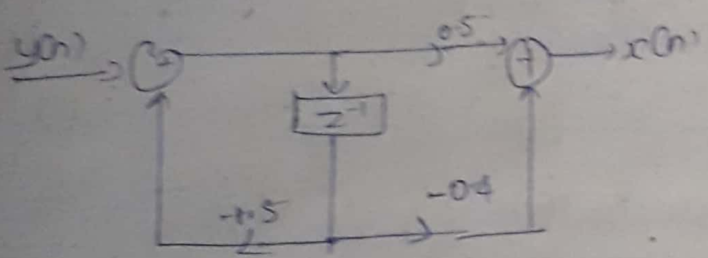
b) Determine a realization for its inverse of the s/m.

Inverse of the s/m is characterized by the DE

$y(n) = 2x(n) + 3x(n-1) + 0.5y(n-1)$

$y(n) - 0.5y(n-1) + 3x(n-1) = x(n)$

$\Rightarrow x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$

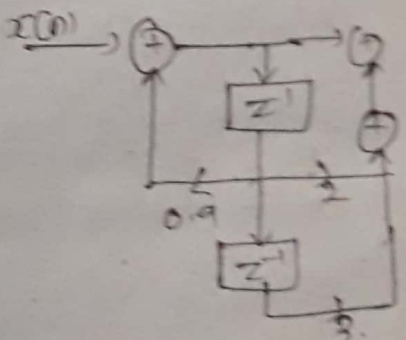


2.41 Consider the discrete time s/m below.

a) Compute the 1st six value of impulse response of s/m.

b) Compute the 1st six values of the zero-state step response of the s/m.

c) Determine an analytical expression for impulse response of the s/m.



a) From the figure $y(n) = x(n) + 0.9y(n-1) + 2x(n-1) + 3x(n-2)$

$x(n) = \{\underset{\uparrow}{1}, 0, 0, \ldots\}$

At $n=0$ $x(n)=1$: then $y(0) = 0.9y(-1) + x(0) + 2x(-1) + 3x(-2) = 1$

At $n=1 \rightarrow x(n)=0$: then $y(1) = 0.9y(0) + x(1) + 2x(0) + 3x(-1) = 2.9$

At $n=2 \rightarrow x(n)=0$; then $y(2) = 0.9y(1) + x(2) + 2x(1) + 3x(0) = 5.61$

thus $y(n) = \{1, 2.9, 5.61, 5.049, 4.544, 4.090 \ldots\}$.

b) $x(n) = \{1, 1, 1, 1, 1 \ldots\}$
$\uparrow$

at $n=0 \rightarrow x(n) = 1$; then $y(0) = 0.9y(-1) + x(0) + 2x(-1) + 3x(-2)$

at $n=1 \rightarrow x(n) = 1$; then $y(1) = 0.9y(0) + x(1) + 2x(0) + 3x(-1)$
$= 3.9$

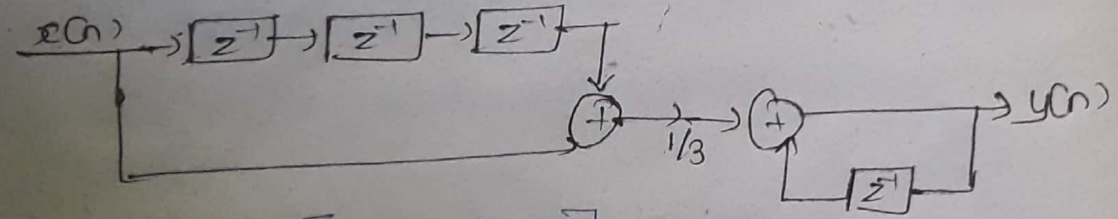
at $n=2 \rightarrow x(n) = 1$; then $y(2) = 0.9y(1) + x(2) + 2x(1) + 3x(0)$
$= 9.51$

then
$y(n) = \{1, 3.9, 9.51, 14.56, 19.10, 23.19 \ldots\}$.

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$h(n) = \delta(n) + 2.9\delta(n-1) + 5.61(0.9)^{n-2} u(n-2)$

2.48 Determine & sketch the impulse response of the following s/m for $n=0$.

a) 

$y(n) = \frac{1}{3}[x(n) + x(n-3)] + y(n-1)$

at $x(n) = \delta(n) = \{1, 0, 0 \ldots\}$

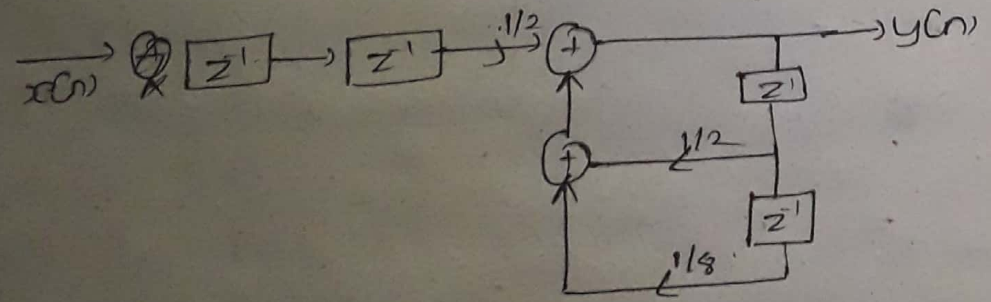
at $n=0 \rightarrow x(n) = 1 \Rightarrow y(0) = \frac{1}{3}x(0) + \frac{1}{3}x(-3) + y(-1) = 1/3$

at $n=1$, $x(n) = 0 \Rightarrow y(1) = \frac{1}{3}x(1) + \frac{1}{3}x(-2) + y(0) = 1/3$.

at $n=2$, $x(n) = 0 \Rightarrow y(2) = \frac{1}{3}x(2) + \frac{1}{3}x(-1) + y(1) = 1/3$.

at $n=3$, $x(n) = 0 \Rightarrow y(3) = \frac{1}{3}x(3) + \frac{1}{3}x(0) + y(2) = 2/3$.

$h(n) = y(n) = \{\frac{1}{3}, 1/3, 1/3, 1/3, 2/3, 2/3, 2/3 \ldots\}$.
$\uparrow$

b) 

$y(n) = \frac{1}{2}y(n-1) + \frac{1}{8}y(n-2) + 1/2\, x(n-2)$

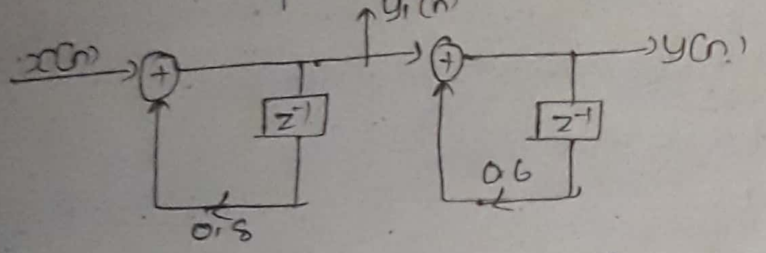
$x(n) = \delta(n) = \{1, 0, 0, \ldots\}$
$\uparrow$

At $n=0$, $x(n)=1$; then $y(0)=\frac{1}{2}y(-1)+\frac{1}{8}y(-2)+\frac{1}{2}x(-2)=0$.

At $n=1$, $x(n)=0$; then $y(1)=\frac{1}{2}y(0)+\frac{1}{8}y(-1)+\frac{1}{2}x(-1)=0$.

At $n=2$, $x(n)=0$, then $y(2)=\frac{1}{2}y(1)+\frac{1}{8}y(0)+\frac{1}{2}x(0)=1/2$.

$$y[n]=\{\underset{\uparrow}{0}, 0, 1/2, 1/4, 3/16 \ldots\}.$$

c) 

$$y(n)=1.4y(n-1)-0.48y(n-2)+x(n)$$

$$x(n)=\delta(n)=\{1, 0, 0 \ldots\}.$$

At $n=0$, $\delta(n)=1$; then $y(0)=1.4y(-1)-0.48y(-2)+x(0)=1$.

At $n=1$, $\delta(n)=0$; then $y(1)=1.4y(0)-0.48y(-1)+x(1)=1.4$

At $n=2$, $\delta(n)=0$; then $y(2)=1.4y(1)-0.48y(0)+x(2)=1.48$.

$$y[n]=\{1, 1.4, 1.48, 1.4, 1.2496 \ldots\}.$$

d) classify the s/m above as FIR or IIR.

all the 3 s/m's are IIR.

e) find an explicit expression for impulse response of the s/m in part c.

$$y(n)=1.4y(n-1)-0.48y(n-2)+x(n)$$

then $y_h(n) = x(n)=0$.

$$\Rightarrow y(n)=1.4y(n-1)-0.48y(n-2)=0.$$

$$\Rightarrow y(n)-1.4y(n-1)+0.48y(n-2)=0.$$

$$\Rightarrow \lambda^n-1.4\lambda^{n-1}+0.48\lambda^{n-2}=0.$$

$$\lambda^{n-2}[\lambda^2-1.4\lambda+0.48]=0.$$

$$\lambda^2-1.4\lambda+0.48=0.$$

$$\lambda=0.8, 0.6.$$

then $y_h(n)=C_1(0.8)^n+C_2(0.6)^n$.

$y_p(n)$:

For $x(n)=\delta(n)$, $y_p(n)=0$.

$$y[n]=y_h(n)+y_p(n)=C_1(0.8)^n+C_2(0.6)^n.$$

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Assuming conditions $y(-1) = y(-2) = 0$.

Substitute $n = 0, 1$ in D.E e

$\rightarrow y(0) = 1.4y(-1) - 0.48y(-2) + \delta(0)$

$y(0) = 1.4y(0) - 0.48y(0) + \delta(0)$

$y(0) = 1$

$\rightarrow y(1) = 1.4y(0) - 0.48y(-1) + x(1)$

$y(1) = 1.4 - 0 + 0$

$y(1) = 1.4$.

Substitute $n = 0, 1$ in $y(n)$ we've

$\Rightarrow y(0) = C_1 + C_2$

$\Rightarrow C_1 + C_2 = 1$ — (b)

$\rightarrow y(1) = C_1(0.8) + C_2(0.6)$

$\Rightarrow 0.8C_1 + 0.6C_2 = 1.4$ — (c)

Substitute (b) & (c) we get

$C_1 = 4,\ C_2 = -3$.

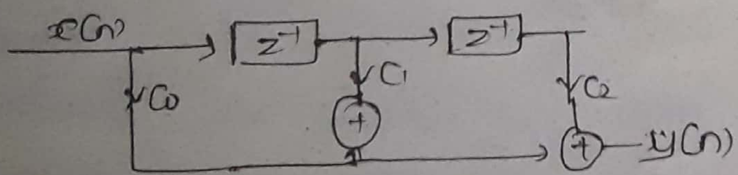
then $h(n) = y(n) = [4(0.8)^n - 3(0.6)^n]u(n)$

2.49 Consider the s/m below

a) Determine & sketch their impulse responses, $h_1(n), h_2(n)$ & $h_3(n)$

b) Is it possible to choose the coefficients of these systems in such a way that $h_1(n) = h_2(n) = h_3(n)$

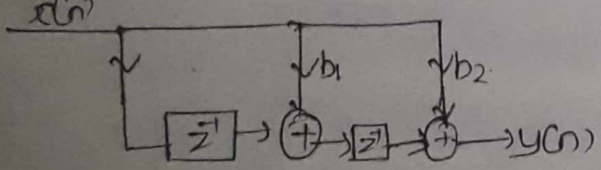
a) i)



$h_1(n) = C_0\delta(n) + C_1\delta(n-1) + C_2\delta(n-2)$

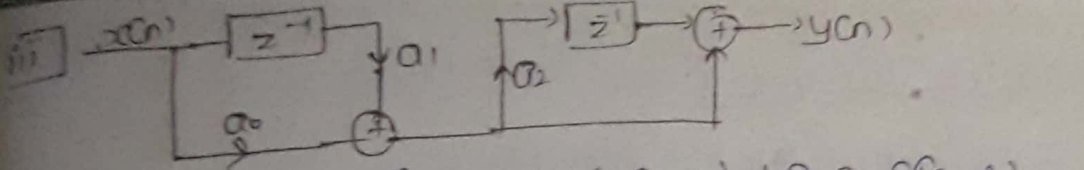
$y_1(n) = C_0x(n) + C_1x(n-1) + C_2x(n-2)$

ii)



$h_2 = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$y_2(n) = b_2x(n) + b_1x(n-1) + b_0x(n-2)$

iii 

$h_3(n) = a_0\delta(n) + (a_1 + a_0a_2)\delta(n-1) + a_1a_2\delta(n-2)$

$y_3(n) = a_0x(n) + a_1x(n-1) + a_0a_2x(n-1) + a_1a_2x(n-2)$

b. To make $h_3(n) = h_2(n) = h_1(n)$

Let $a_0 = c_0$, $a_1 + a_2c_0 = c_1$, $a_2a_1 = c_2$.

Thus we've

$$\frac{c_2}{a_2} + a_2c_0 - c_1 = 0$$

$$\Rightarrow c_0a_2^2 - c_1a_2 + c_2 = 0$$

for $c_0 \neq 0$, the Q.E has 0 real solutions if & only if

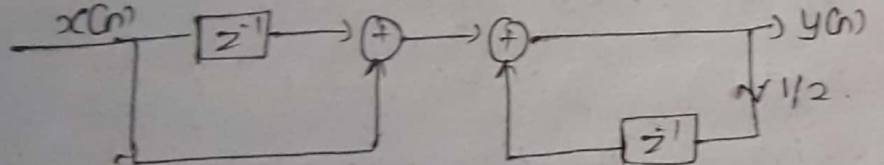
$$c_1^2 - 4c_0c_2 \geq 0.$$

2.50 Consider the s/m shown below

a. Determine it's impulse response $h(n)$

b. Show that $h(n)$ is equal to the convolution of the following signal. $h_1(n) = \delta(n) + \delta(n-1)$

$h_2(n) = (\frac{1}{2})^n u(n)$



a.

$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

$y(n) - \frac{1}{2}y(n-1) = x(n) + x(n-1)$

$y_h(n)$:- $x(n)$ & $x(n-1) = 0$.

$\Rightarrow y(n) - \frac{1}{2}y(n-1) = 0$.

$\lambda - 1/2 = 0$

$\lambda = 1/2$

$\Rightarrow y_h(n) = C(\frac{1}{2})^n u(n)$

$y_p(n)$:- $x(n) = \delta(n)$

$\Rightarrow y_p(n) = 0$

$\Rightarrow y(n) = y_p(n) + y_h(n)$

$y(n) = C(\frac{1}{2})^n u(n)$

Assuming $y(-1) = 0$

substitute $n=0$ in D.E

$n=0 \rightarrow y(0) - \frac{1}{2}y(-1) = x(0) + x(-1)$

$y(0) - \frac{1}{2}(0) = \delta(0) + \delta(-1)$

$y(0) = 1$

substitute $n=0$ in $y(n)$

$\Rightarrow y(0) = c\left(\frac{1}{2}\right)^0 u(0)$

$\Rightarrow y(0) = c$

$\Rightarrow c = 1$

then impulse response

$h(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

$\boxed{b}$

$h_1(n) * h_2(n) = [\delta(n) + \delta(n-1)] * \left(\frac{1}{2}\right)^n u(n)$

$h_1(n) * h_2(n) = \left[\left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)\right]$

2.$\boxed{5}$ Compute the sketch the Convolution $y_1(n)$ & Correlation $r_1(n)$ sequence for the following pair of signals & Comment on the result obtained.

$\boxed{a}$ $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$\boxed{b}$ $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$, $h_2(n) = \{1/2, 1, \underset{\uparrow}{2}, 1, 1/2\}$

$\boxed{c}$ $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$

$\boxed{d}$ $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

Note

$\Rightarrow$ Convolution is done as $h(n) * x(n)$

$\Rightarrow$ Correlation means writing $h(n)$ in reverse order.

$\boxed{a}$ $x_1(n) = \{\underset{\uparrow}{1}, 2, 4\}$, $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h_1(n)$ \ $x(n)$	1	2	1
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution : $y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 1\}$

Correlation $y_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b]

$h_2(n) \backslash x(n)$	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
→ 2	0	2	-4	6	-8
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

Convolution $y_2(n) = \{0, 1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$.

Correlation $y_2(n) = \{0, 1/2, \underset{\uparrow}{0}, 3/2, -2, \frac{1}{2}, -6, -5/2, -2\}$

c]

$h_3(n) \backslash x(n)$	1	2	3	4
→ 4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

Convolution :- $\{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$.

Correlation :- $\{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 6\}$.

d]

$h_4(n) \backslash x(n)$	1	2	3	4
→ 1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Convolution :- $y_4(n) = \{1, \underset{\uparrow}{4}, 10, 20, 25, 24, 16\}$

Correlation : $y_3(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

2.52] The zero state response of a causal LTI s/m to the i/p $x(n) = \{1, 3, 3, 1\}$ is $y(n) = \{1, 4, 6, 4, 1\}$. Determine it's impulse response.

$h(n) \backslash x(n)$	1	3	3	1
h_0	$h_0 x(0)$	$h_0 x(1)$	$h_0 x(2)$	$h_0 x(3)$
h_1	$h_1 x(0)$	$h_1 x(1)$	$h_1 x(2)$	$h_1 x(3)$

$h(0) x(0) = 1$

$h(0) = 1$

$\Rightarrow h(1) \cdot x(0) + h(0) x(1) = 4$.

$h(1) \cdot 1 + 1 \cdot (3) = 4$.

$h(1) = 4 - 3$

$h(1) = 1$

thus $h(0) = 1, h(1) = 1$

$\Rightarrow h(n) = \{h_0, h_1\} = \{1, 1\}$

2.53] prove by direct substitution the equivalence of equations (2.5.9) & (2.5.10), which describe the direct form II structures, to the relation (2.5.6), which describes the direct form I structure.

$$y[n] = -\sum_{k=1}^{N} a_k y(n-k) + \sum_{k=0}^{M} b_k x(n-k) \quad - 2.5.6.$$

$w[n] = -\sum_{k=1}^{N} a_k w[n-k] + x(n)$ — 2.5.9

$y[n] = \sum_{k=0}^{M} b_k w(n-k)$ — 2.5.10

from 2.5.9 obtain.

$x[n] = w[n] + \sum_{k=1}^{N} a_k w(n-k)$ — (a)

similarly 2.5.10 ;

writing 2.5.10 for $y(n)$ and (a) in 2.5.9

$\sum_{k=0}^{M} b_k w(n-k)$

2.5(a) Determine the response $y(n), n \geq 0$ of the system describes by the 2nd order difference equation.

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the i/p is $x(n) = (-1)^n u(n)$

$y[n] - 4y(n-1] + 4y(n-2) = x(n) - x(n-1)$

$y_h(n) = [C_1 2^n + C_2 n 2^n] u[n]$.

$y_p(n)$;-

$x[n] = (-1)^n u(n) \quad n \geq 0$

$u(n) \to$ has a constant k.

thus $x(n)$ has $y_p(n) = k \cdot (-1)^n u(n)$

Substituting this $y_p(n)$ into D.E

$y(n) = 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2)$

$= k(-1)^n u(n) - (-1)^{n-1} u(n-1)$

for $n=2$

$k(-1)^2 u(2) - 4k(-1)^1 u(1) + 4k(-1)^0 u(0) = (-1)^2 u(2) - (-1)^1 u(1)$

$k + 4 + 4k - 4 = 1 + 1$

$9k = 2$

$k = \frac{2}{9}$

then $y_p(n) = \frac{2}{9} (-1)^n u(n)$

then $y(n) = y_h(n) + y_p(n)$

$= \left[C_1 2^n + C_2 n 2^n + \frac{2}{9} (-1)^n\right] u[n]$.

Assumining initial condition

$y(-1) = y(-2) = 0$.

substitute $n=0, n=1$ in D.E

$\Rightarrow y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1) \Rightarrow y(0) = 1$.

$\Rightarrow y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4 + 0 = -1 - 1$

$y(1) = 4 - 2$

$y(1) = 2$

substitute $n=0, n=1$ in $y(n)$

$\Rightarrow y(0) = (C_1 + \frac{2}{9}) u(n)$

$\Rightarrow C_1 + \frac{2}{9} = 1 \Rightarrow \boxed{C_1 = 7/9}$

$\Rightarrow y(1) = 2C_1 + 2C_2 - \frac{2}{9}$

$\Rightarrow 2C_1 + 2C_2 - \frac{2}{9} = 2$

$\Rightarrow C_1 + C_2 = \frac{10}{9}$

$\Rightarrow C_2 = \frac{10}{9} - \frac{7}{9} = \frac{3}{9} = 1/3$

$\boxed{C_2 = 1/3}$

then $h(n) = y(n) = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9} (-1)^n\right] u(n)$

55. Determine the impulse response $h(n)$ for the s/m described by the 2nd order difference equation.

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$y_h(n)$:- $x(n) = x(n-1) = 0$.

$\Rightarrow y(n) - 4y(n-1) + 4y(n-2) = 0$.

$\Rightarrow \lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$.

$\Rightarrow \lambda^{n-2}[\lambda^2 - 4\lambda + 4] = 0$.

$\Rightarrow \lambda^2 - 4\lambda + 4 = 0$

$\Rightarrow \lambda = 2, 2$.

$y_h(n) = [c_1 2^n + c_2 n 2^n] u(n)$

$y_p(n)$: $x(n) = \delta(n)$

$\Rightarrow y_p(n) = 0$.

$y(n)$:

$y(n) = y_p(n) + y_h(n)$

$\Rightarrow y(n) = (c_1 2^n + c_2 n 2^n) u(n)$

Assuming $y(-1) = y(-2) = 0$.

Substituting $n=0, n=1$, in D.E

$\rightarrow y(0) - 4y(-1) + 4y(-2) = x(0) - x(-1)$

$y(0) - 4(0) + 4(0) = \delta(0) - \delta(-1)$

$\Rightarrow y(0) = 1$.

$\rightarrow y(1) - 4y(0) + 4y(-1) = x(1) - x(0)$

$y(1) - 4(1) + 4(0) = \delta(1) - \delta(0)$

$y(1) - 4 + 0 = 0 - 1$

$y(1) = 4 - 1$

$y(1) = 3$

Substituting $n=0, n=1$ in $y(n)$

$\rightarrow y(0) = c_1 + c_2(0)$

$\Rightarrow \boxed{c_1 = 1}$

$\rightarrow y(1) = c_1(2) + c_2(2)$

$\Rightarrow y(1) = 2 + 2c_2$

$\Rightarrow 3 - 2 = 2c_2$

$\Rightarrow \boxed{c_2 = 1/2}$

Q.56] Show that any DT signal $x(n)$ can be expressed as $x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u[n-k]$ where $u[n-k]$ is a unit step delayed by k units in time, that is $u(n-k) = \begin{cases} 1 & n \geq k \\ 0 & \text{otherwise.} \end{cases}$

$$x(n) = x(n) * \delta(n)$$
$$= x(n) * [u(n) - u(n-1)]$$
$$= [x(n) - x(n-1)] * u[n]$$
$$= x[n] * u(n) - x(n-1) * u(n)$$
$$= \sum_k x(k) u(n-k) - \sum_k x(k-1) u(n-k)$$
$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u[n-k]$$

2.57] Show that the o/p of an LTI s/m can be expressed in terms of the unit-step response s(n) as follows

$$\underline{y[n]} = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$
$$\underline{y[n]} = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k).$$

Let $h(k) = s(k) - s(k-1)$

where $s(k)$ = unit step response

$$s(k) = \sum_{m=-\infty}^{\infty} h(m)$$

then $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k)$$
$$\underline{y(n)} = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k).$$

2.58] Compute the correlation sequences $r_{xx}(l)$ & $r_{xy}(l)$ for the following signal sequences.

$$x(n) = \begin{cases} 1 & n_0 - N \le n \le n_0 + N \\ 0 & \text{otherwise.} \end{cases}$$

$$\underline{y[n]} = \begin{cases} 1 & -N \le n \le N. \\ 0 & \text{otherwise.} \end{cases}$$

i] . we know that
$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n) x(n-l)$$

Range of $r_{xx}(l)$ is i] $n_0 - N \le n \le n_0 + N$: $x(n)$

ii] $n_0 - N \le n - l \le n_0 + N$

$\Rightarrow n_0 - N + l \le n \le n_0 + N + l$; $x(n-l)$

we know that $x(n)x(n-l)$ summation values gives us $r_{xx}(l)$

Similarly summing those range values give us $r_{xy}(l)$

i.e adding / summing (a) & (b) gives us $-2N+1-|l|$

$r_{xx}(l) \rightarrow$ range : $-2N \le l \le 2N$.

ii) thus we've

$$r_{xx}(l) = \begin{cases} 2N+1-|l| \;; & -2N \le l \le 2N \\ 0 \;; & \text{otherwise.} \end{cases}$$

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0| \;; & n_0-2N \le r \le n_0+2N \\ 0 \;; & \text{otherwise.} \end{cases}$$

2.59 Determine the autocorrelation sequences of the following signals & give the conclusion.

a] $x(n) = \{\underset{\uparrow}{1}, 2, 1, 1\}$ b] $y(n) = \{1, \underset{\uparrow}{1}, 2, 1\}$.

x(n) \ x(n)	1	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
→ 1	1	2	1	1

y(n) \ y(n)	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
→ 1	1	1	2	1

$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$.

$r_{yy}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$.

we observe that $y(n) = x(-n+3)$ which is equivalent to reversing the sequence $x(n)$. this has not changed the autocorrelation sequence

2.60 what is the normalized autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise} \end{cases}$$

we know that,

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

range: $= \begin{cases} 2N+1-|l| & ; -2N \le l \le 2N \\ 0 & ; \text{otherwise} \end{cases}$

here $r_{xx}(0) = 2N+1$

then normalized autocorrelation is -

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) & ; -2N \le l \le 2N. \\ 0. & ; \text{otherwise} \end{cases}$$