Assignment Problems

2.1) A Discrete time Signal $x[n]$ is defined as

$$x[n] = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le 1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

a) Determine it's Values and sketch signal $x[n]$.

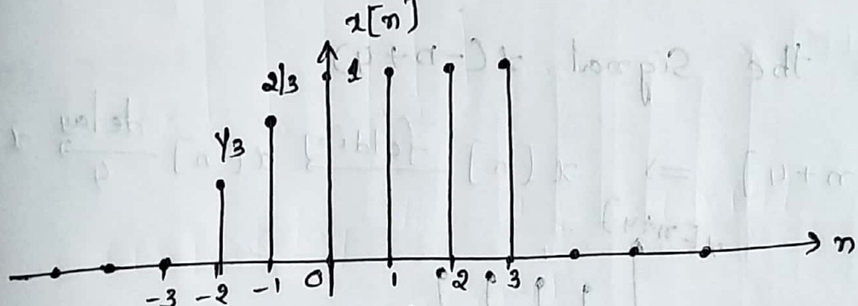
b) sketch the signals that results if we

1) first fold $x[n]$ and then delay the resulting signal by four samples.

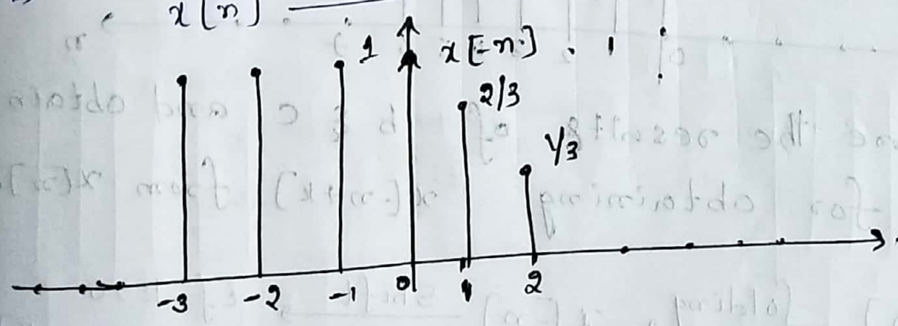
2) first delay $x[n]$ and then fold the resulting signal.

A) a) $$x[n] = \begin{cases} 1 + \frac{n}{3} & -3 \le n \le 1 \\ 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

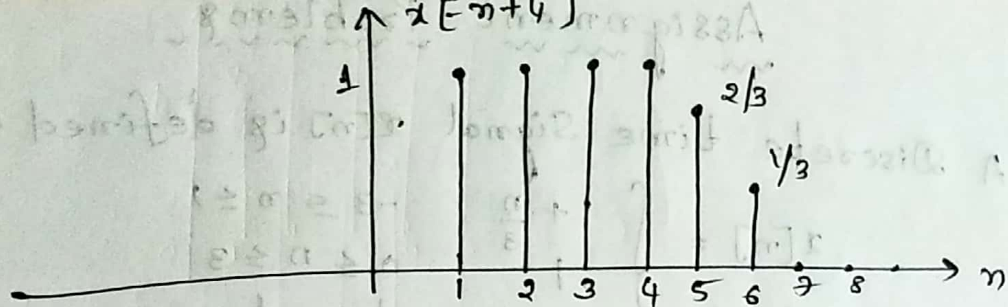
$$= \{\ldots, 0, \tfrac{1}{3}, \tfrac{2}{3}, \underset{\uparrow}{1}, 1, 1, 1, 0, \ldots\}$$



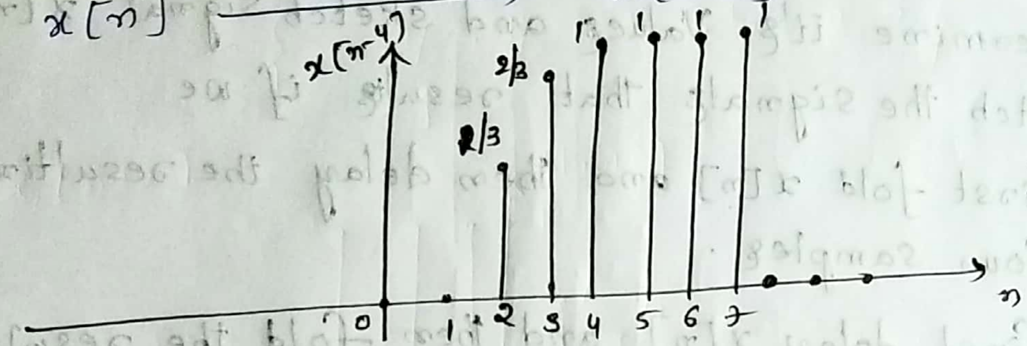
b) $x[n] \xrightarrow{\text{fold}} x[-n] \xrightarrow{\text{delay by 4}} x[-n+4]$



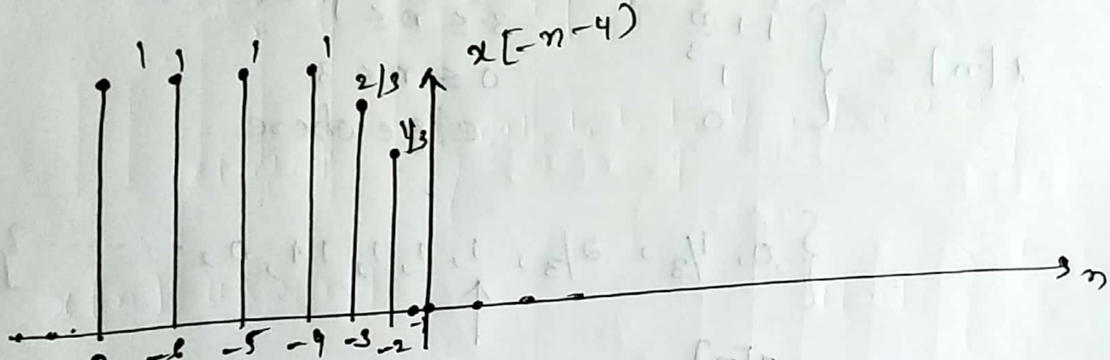
$$x[-n] = \{\ldots, 0, 0, 1, 1, 1, \underset{\uparrow}{1}, \tfrac{2}{3}, \tfrac{1}{3}, 0, \ldots\}$$



b) $x[n] \xrightarrow{\text{delay by } 4} x[n-4]$

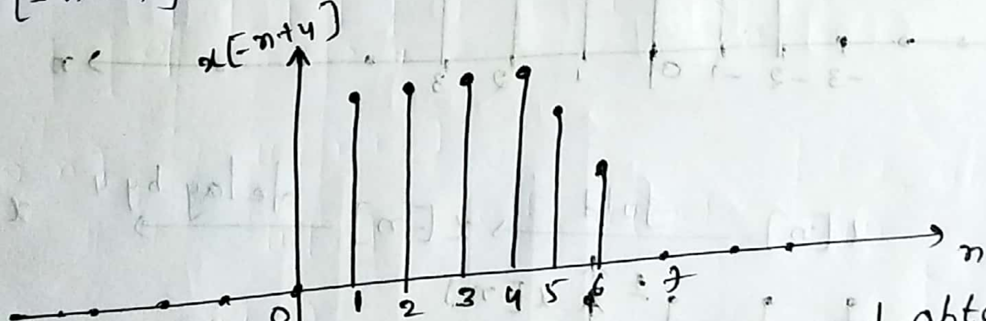


$x[n-4] \xrightarrow{\text{folding}} x[-n-4]$



c) sketch the Signal $x(-n+4)$.

A) $x[-n+4] \Rightarrow x[n] \xrightarrow{\text{folding}} x[-n] \xrightarrow[4]{\text{delay}} x[-n+4]$



d) Compare the results of b & c and obtain a rule for obtaining $x[-n+k]$ from $x[n]$.

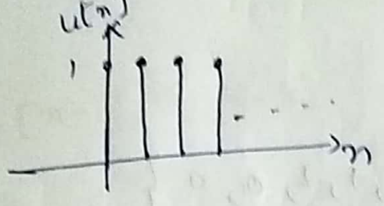
A) $x[n] \xrightarrow{\text{folding}} x[-n] \xrightarrow[\text{by } k]{\text{shift}}$ if $k>0$ $x[-n+k]$

if $k<0$ $x[-n-k]$

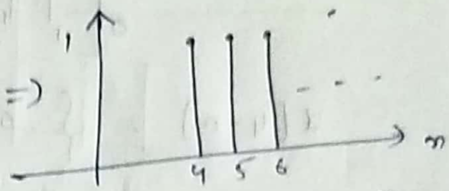
c) Can you express the signal in terms of $\delta(n)$ and $u(n)$.

A) yes.

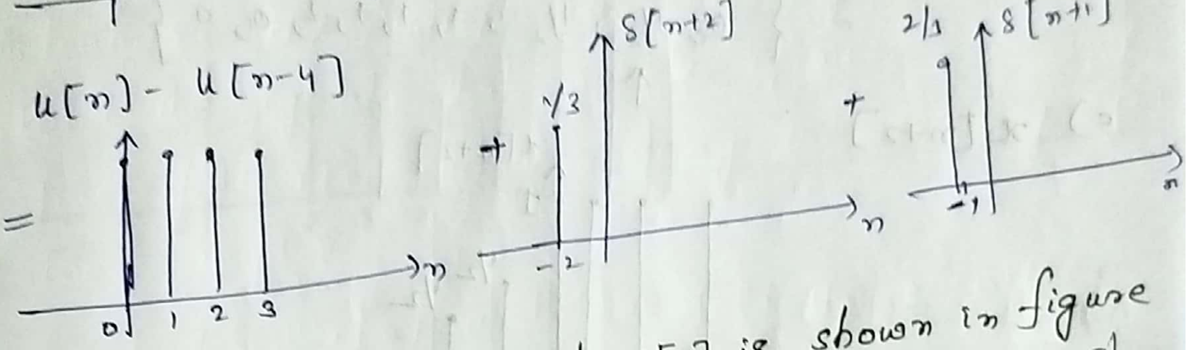
$$x[n] = 0\cdot\delta[n+3] + \frac{1}{3}\delta[n+2] + \frac{2}{3}\delta[n+1] + u[n] - u[n-4].$$



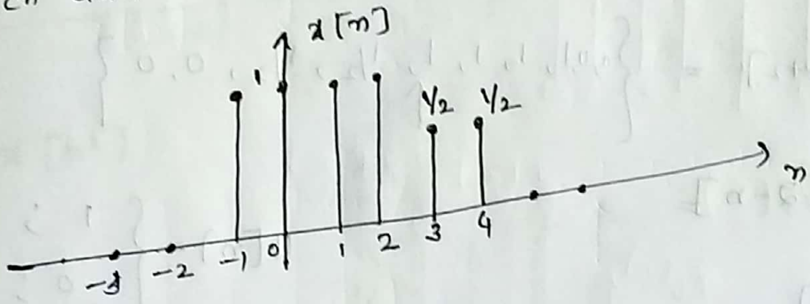
$u[n-4] \Rightarrow$



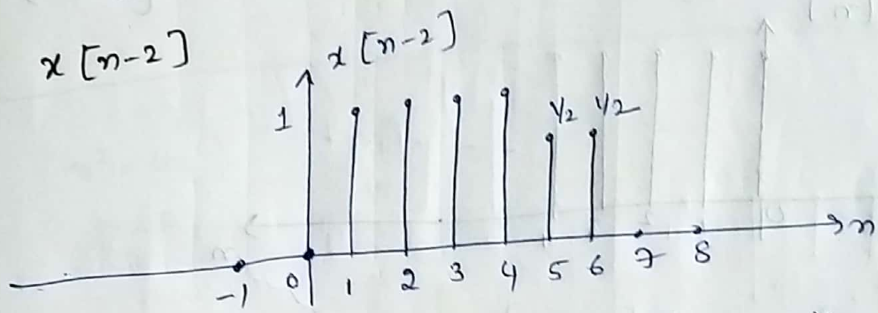
$x[n] =$



2.2) A discrete time signal $x[n]$ is shown in figure sketch and label carefully each of the following sig.



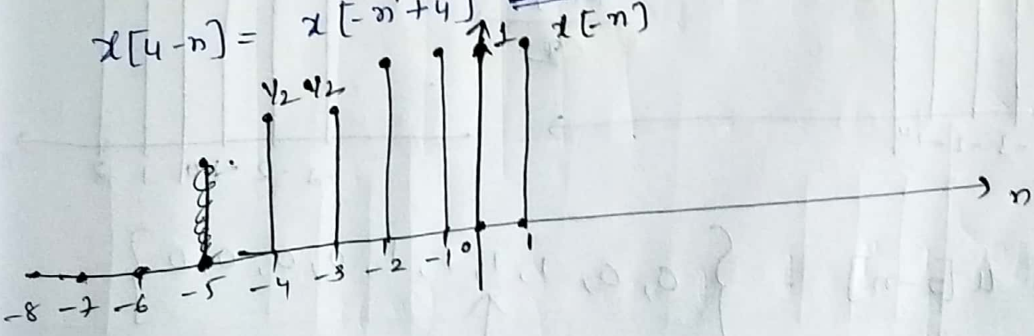
A) a) $x[n-2]$



$$x[n-2] = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1, 1/2, 1/2, 0, 0\}$$

b) $x[4-n] \longrightarrow x(n) \xrightarrow{\text{fold}} x[-n] \xrightarrow[\text{by } 4]{\text{delay}} x[-n+4]$.

$x[4-n] = x[-n+4] \Longleftarrow x[-n]$



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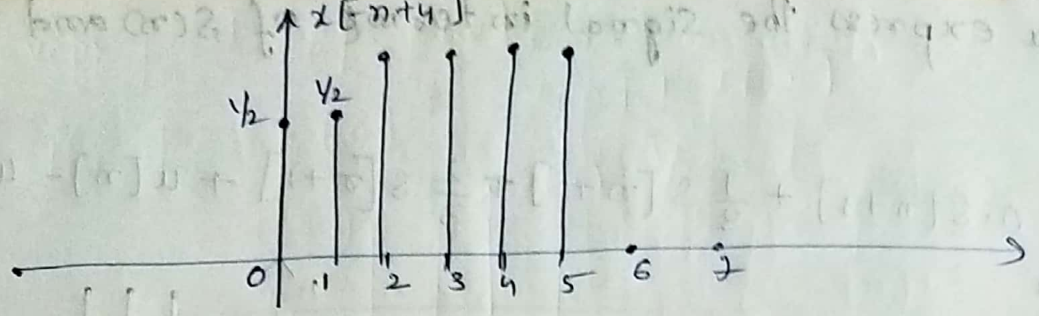
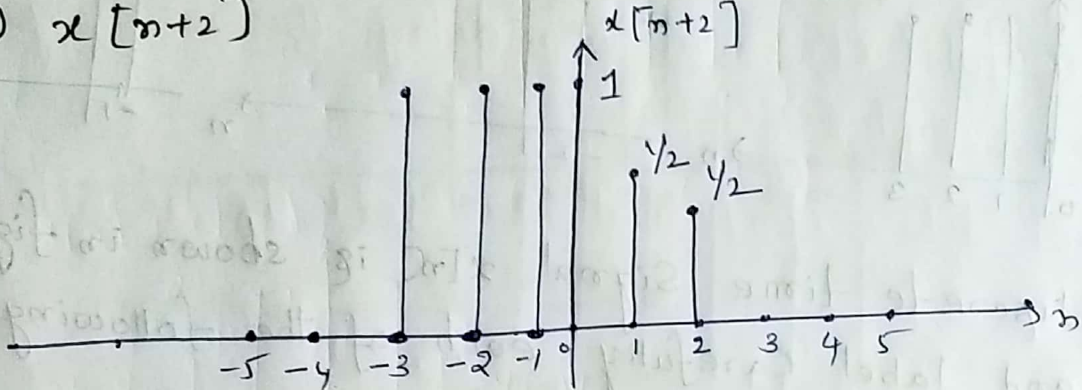


$$x[4-n] = \{ \cancel{0,1,2,3,} 4, 5 \}$$

$$= \{ \tfrac{1}{2}, \tfrac{1}{2}, 1, 1, 1, 1, 0, 0 \}$$
$\uparrow$

c) $x[n+2]$



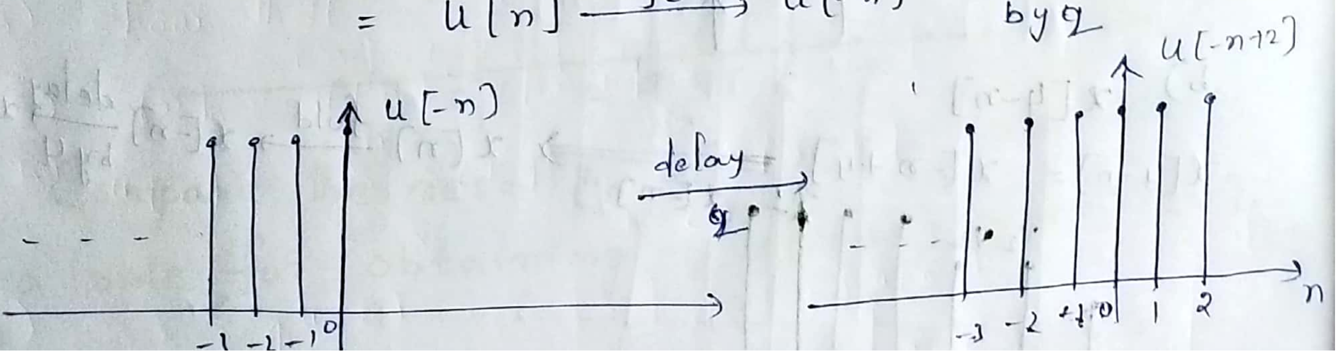
$$x[n+2] = \{0, 0, 1, 1, 1, 1, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0\}$$
$\uparrow$

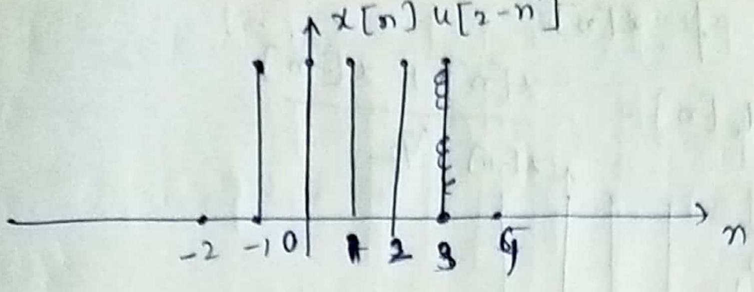
d) $x[n]\, u[2-n]$

$$u[n] = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & \text{elsewhere} \end{cases}$$

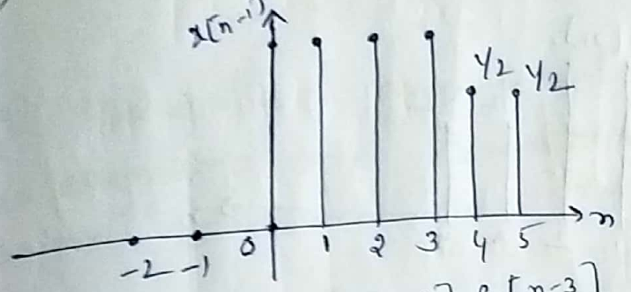
$$u[2-n] = u[-n+2]$$

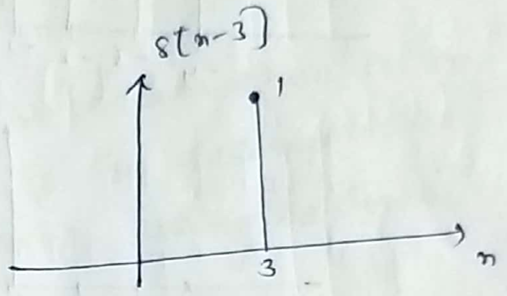
$$= u[n] \xrightarrow{\text{fold}} u[-n] \xrightarrow[\text{by } 2]{\text{delay}} u[-n+2]$$

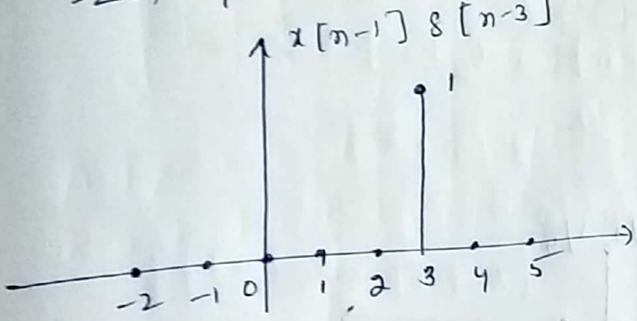




e) $x[n-1]\ \delta[n-3]$



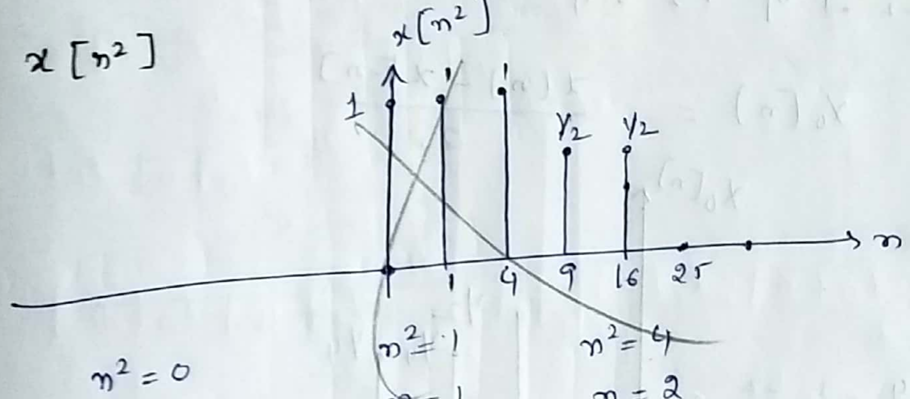




$x[n-1]\ \delta[n-3]$

$= \{0, 0, \underset{\uparrow}{0}, 0, 0, 1, 0, 0\}$

f) $x[n^2]$



$n^2 = 0$

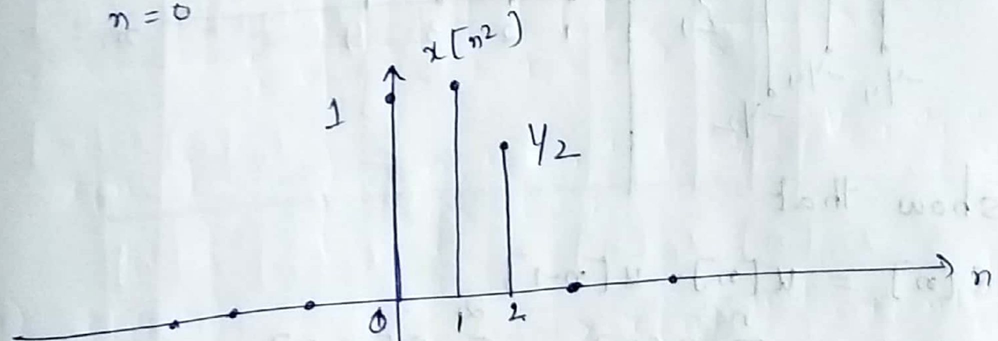
$n = 0$

$n^2 = 1$

$n = 1$

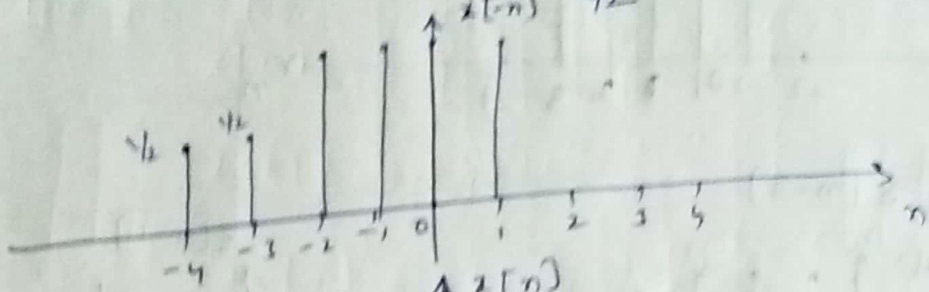
$n^2 = 4$

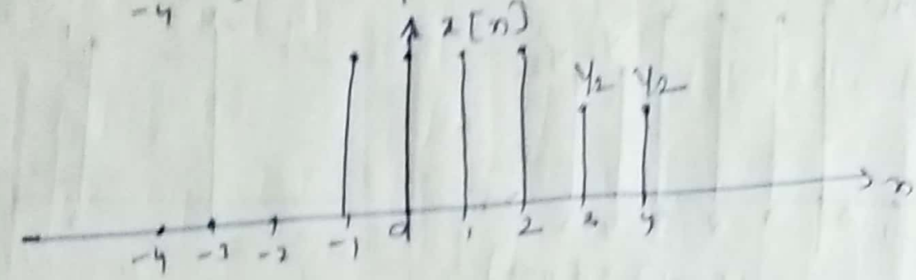
$n = 2$

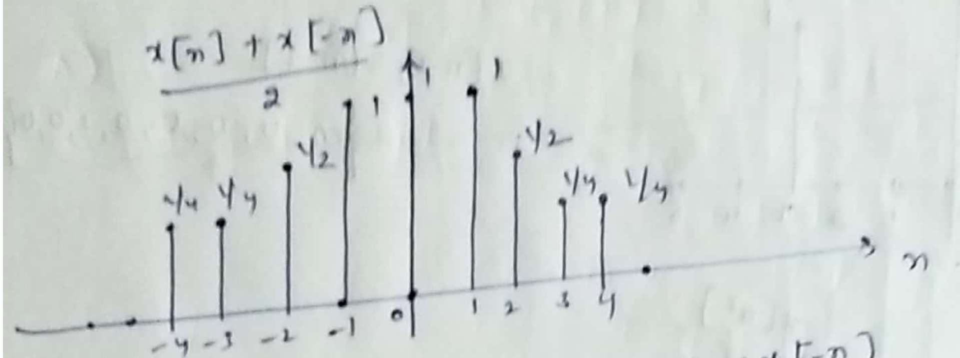


g) even part of $x[n]$

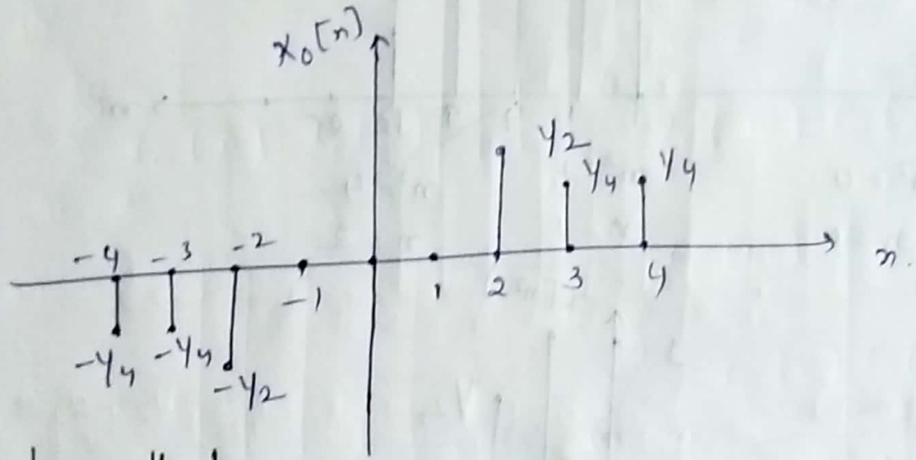
$$x_e[n] = \frac{x[n] + x[-n]}{2}$$







h) $$x_o[n] = \frac{x[n] - x[-n]}{2}.$$

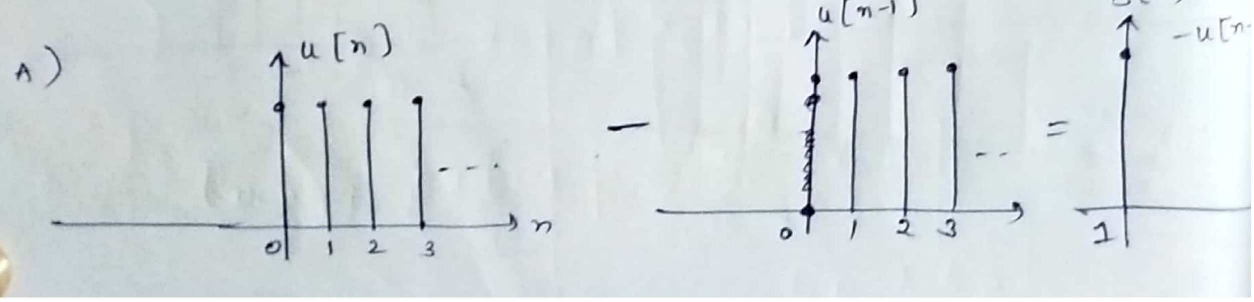


2.3) show that

a) $\delta[n] = u[n] - u[n-1]$

b) $u[n] = \sum_{k=-\infty}^{n} \delta(k] = \sum_{k=0}^{\infty} \delta[n-k]$

A)



b) $\sum_{k=-\infty}^{n} \delta(k) = u[n] = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n < 0 \end{cases}$

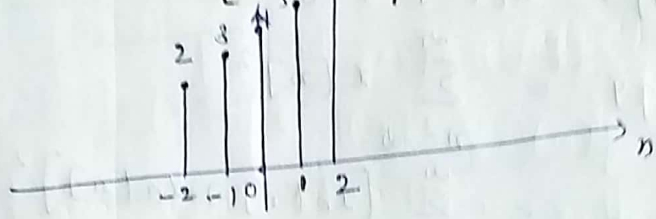
$\sum_{k=0}^{\infty} \delta[n-k] = u[n] = \begin{cases} 1 ; & n \geq 0 \\ 0 ; & n \leq 0 \end{cases}$

$\therefore \sum_{k=-\infty}^{n} \delta[k] = \sum_{k=0}^{\infty} \delta[n-k]$

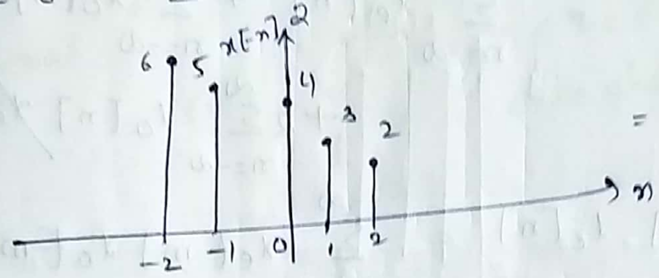
2.4) Show that any signal can be decomposed into an even and odd component. Is the decomposition unique? Illustrate your arguements using the signal.

$x[n] = \{ 2, 3, \underset{\uparrow}{4}, 5, 6 \}$

1)



$x_e[n] = \frac{x[n] + x[-n]}{2}$



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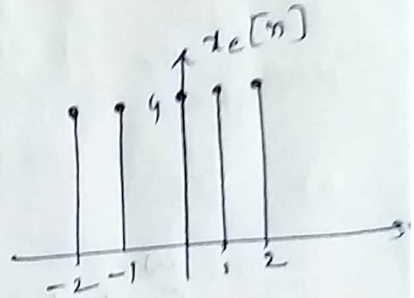
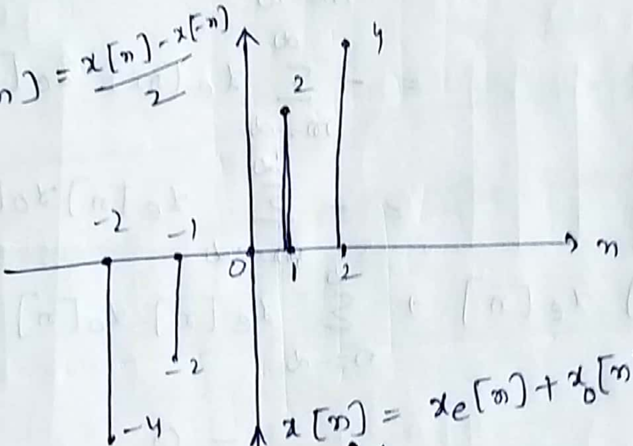
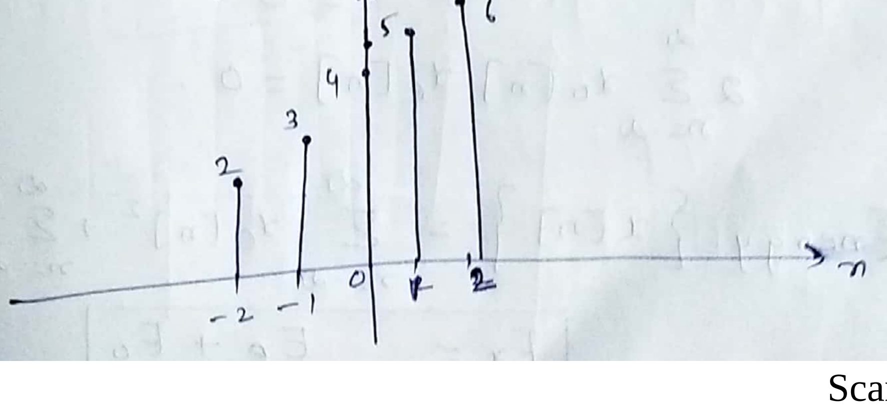


$x_o[n] = \frac{x[n] - x[-n]}{2}$



$x[n] = x_e[n] + x_o[n]$



Hence $x[n] = x_e[n] + x_o[n]$

The decomposition is unique.

$$x[n] = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

$$x_e[n] = \{4, 4, \underset{\uparrow}{4}, 4, 4\}$$

$$x_o[n] = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$$

2.5) Show that the energy of a real valued energy signal is equal to the sum of the energy of it's even and odd components.

A) $$x[n] = x_e[n] + x_o[n]$$

$$\text{Energy} = \sum_{n=-\infty}^{\infty} x[n]^2$$

$$= \sum_{n=-\infty}^{\infty} [x_e[n] + x_o[n]]^2$$

$$= \sum_{n=-\infty}^{\infty} x_e[n]^2 + \sum_{n=-\infty}^{\infty} x_o[n]^2$$

$$+ 2\sum_{n=-\infty}^{\infty} x_o[n]\, x_e[n] \xrightarrow{} 0$$

$$\sum_{n=-\infty}^{\infty} x_o[n]\, x_e[n] = \sum_{m=-\infty}^{\infty} x_e[-m]\, x_o[-m]$$

$$= -\sum_{m=-\infty}^{\infty} x_e[m]\, x_o[m]$$

$$= -\sum_{n=-\infty}^{\infty} x_e[n]\, x_o[n]$$

$$\sum_{n=-\infty}^{\infty} x_o[n]\, x_e[n] + \sum_{n=-\infty}^{\infty} x_e[n]\, x_o[n]$$

$$= 0$$

$$2\sum_{n=-\infty}^{\infty} x_o[n]\, x_e[n] = 0.$$

$$\text{Energy}\{x[n]\} = \sum_{n=-\infty}^{\infty} x_e[n]^2 + \sum_{n=-\infty}^{\infty} x_o[n]^2$$

$$\boxed{E_x = E_e + E_o}$$

2.6) Consider the system

$$y[n] = T[x[n]] = x[n^2]$$

a) Determine the system is time invariant (or) not.

For a system to be time invariant

delayed i/p Response = delayed Response.

delayed i/p Response:

$x[n] \to$ delay $\xrightarrow{x[n-n_0]}$ system $\xrightarrow{x[(n-n_0)^2]} y[n]$

delayed Response:

$x[n] \to$ system $\xrightarrow{y[n] = x[n^2]}$ delay $\to y[n] = x[n^2 - n_0]$

delayed i/p Response $\neq$ delayed Response

So the system is time invariant

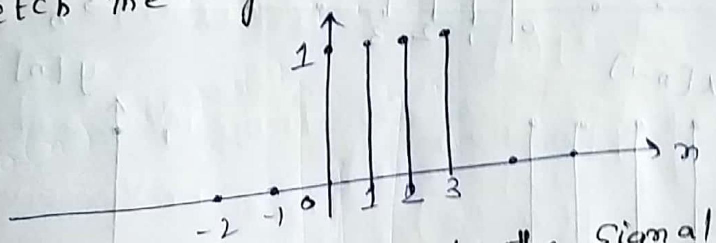
b) To clarify the result in part (a) assume that the signal

$$x[n] = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$$

is applied

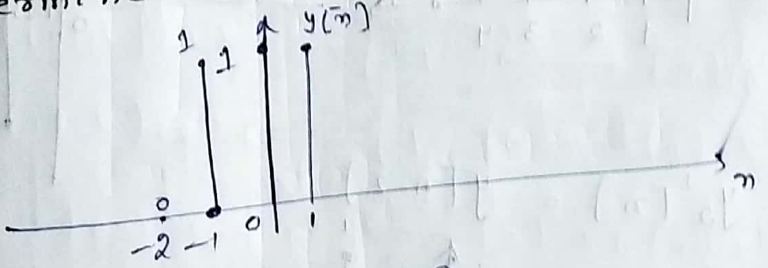
1) sketch the signal $x[n]$.

A)

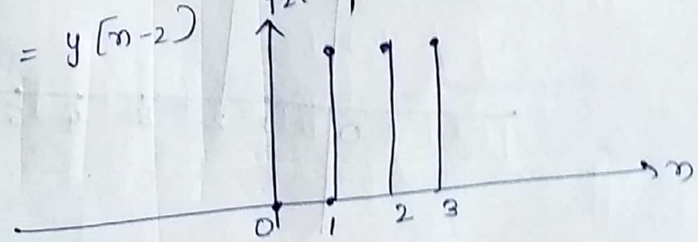


2) Determine and sketch the signal $y[n] = T[x[n]]$

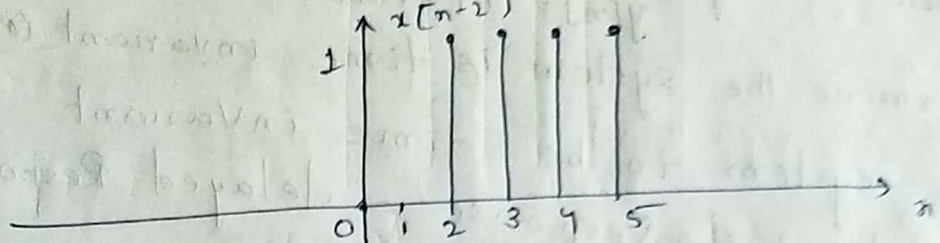
A)



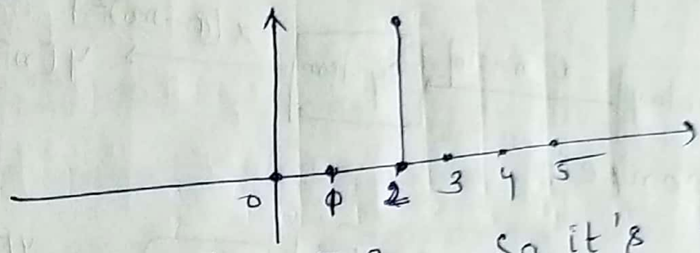
3) $y_2[n] = y[n-2]$



4) $x_2[n] = x[n-2]$



5) $y_2[n] = T[x_2[n]]$.

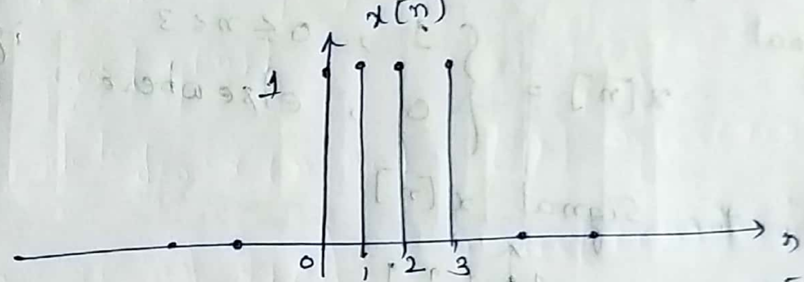


So it's time invariant

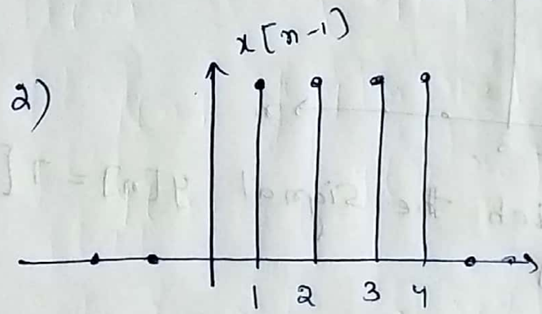
6) $y[n-2] \neq y_2[n]$.

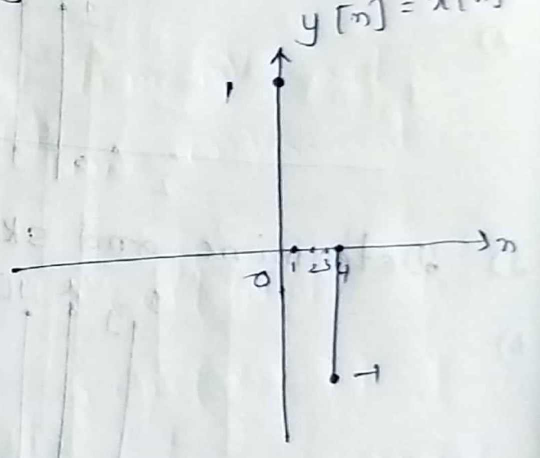
c) Repeat part b for the system
$y(n) = x[n] - x[n-1]$
Can you use this result to make any statement about the time invariance of the system? why?

A) 1)

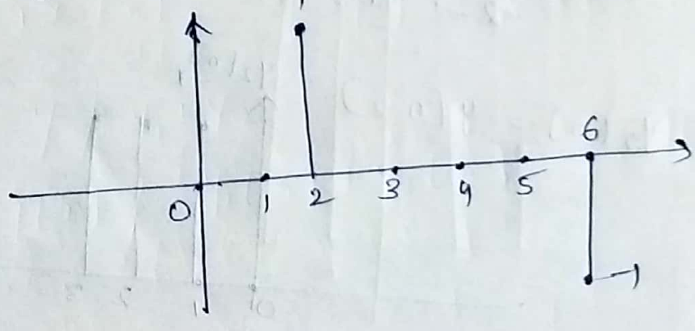


2)

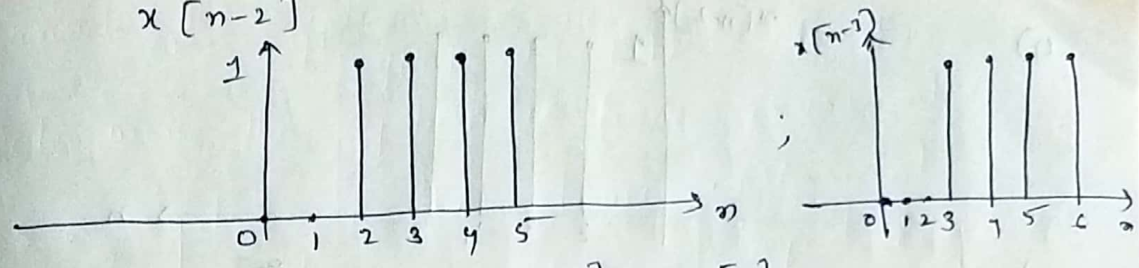




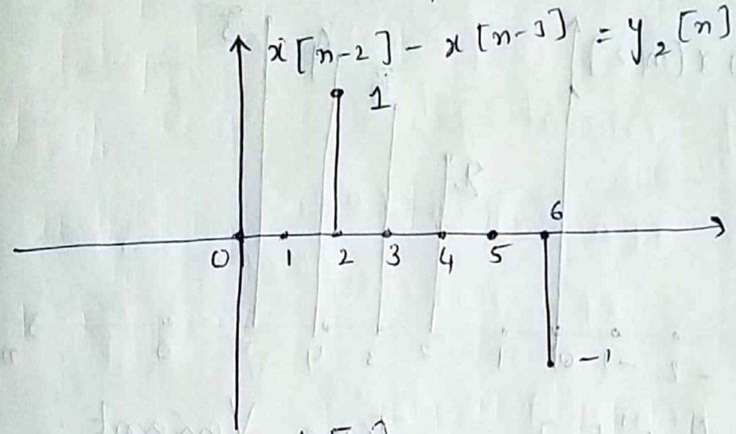
3) $y_2'[n] = y[n-2]$



4)



5)



6) $y_2[n] = y_2'[n]$

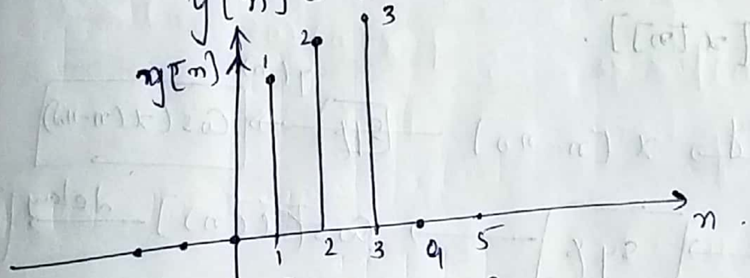
The system is time invariant.

→ from this the time shifting operation doesn't affect the time invariance.

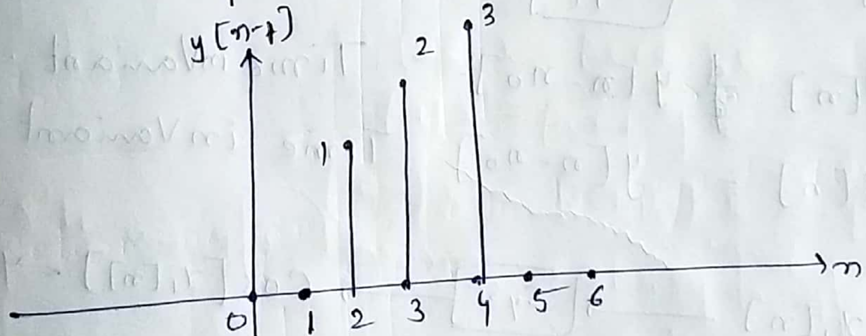
d) Repeat part b and c for $y[n] = n\,x[n]$

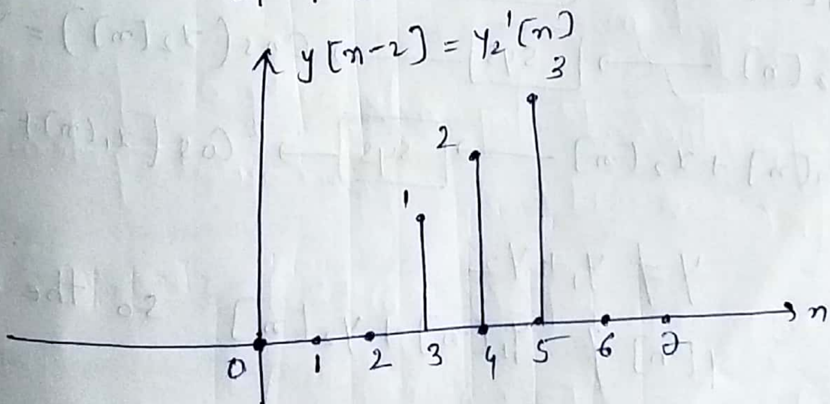
$y[n] = n\,x[n]$.

2)

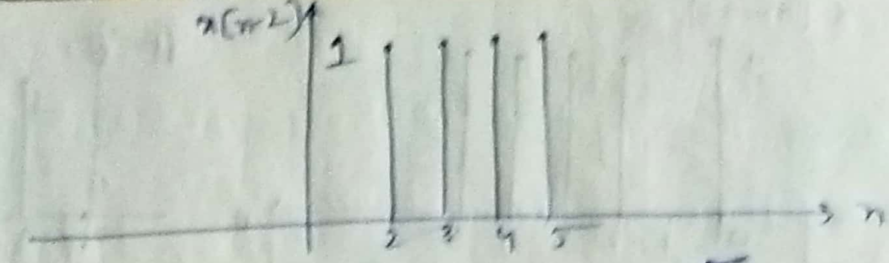


3)

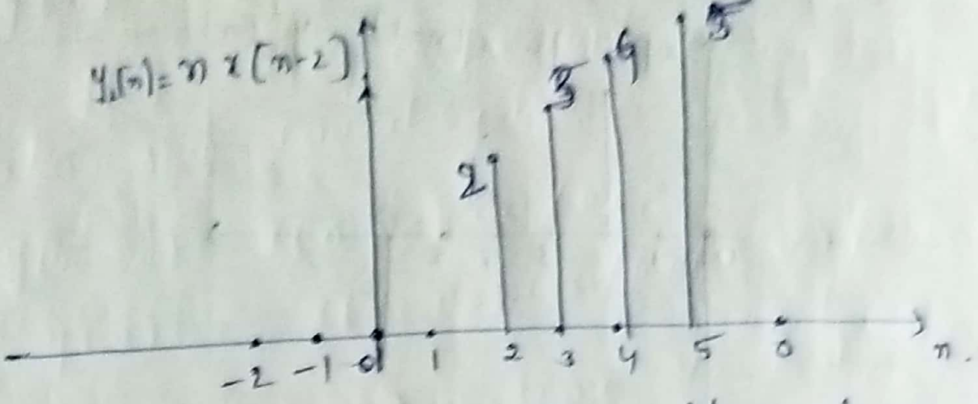




4)



5)



$y_2[n] \neq y_2'[n]$. Time Variant.

So if the $x[n]$ is multiplied by n then it will be a time variant system.

2.7) Examine the properties of the system with respect to their properties.

a) $y[n] = \cos[x[n]]$.

1) $x[n] \xrightarrow{\text{delay}} x[n-n_0] \longrightarrow \boxed{sys} \xrightarrow{y(n)'} \boxed{\cos(x(n-n_0))}$

$x[n] \longrightarrow \boxed{sys} \longrightarrow \cos[x(n)] \xrightarrow{\text{delay}} \boxed{\cos(x(n-}$

$y[n] \neq y[n-n_0]$ Time invariant

$y[n] = y[n-n_0]$ Time invariant.

2) $x_1[n] \longrightarrow \boxed{sys} \longrightarrow \cos[x_1[n]] = y_1$

$x_2[n] \longrightarrow \boxed{sys} \longrightarrow \cos(x_2[n]) = y_2$

$x_1[n] + x_2[n] \longrightarrow \boxed{sys} \longrightarrow \cos[x_1(n) + x_2(n)]$

$y \neq y_1 + y_2$

$y[n] \neq y_1[n] + y_2[n]$ so the system is non linear.

3) $y[n] = \cos[x[n]]$

$y[0] = \cos[x[0]]$

$y[1] = \cos[x[1]]$

The system depends on present i/p's so it's a static system.

4) Since it is static it is also causal because all static systems are causal. Vice Versa is not possible.

5) $|x[n]| < \infty$ & $|\cos(x(n))| < \infty$ so it's a stable system.

6) $y[n] = \sum_{K=-\infty}^{n+1} x(K)$ (IIR system).

1) $y[2] = \sum_{K=-\infty}^{3} x(K) = \cdots + x(-1) + x(0) + x(1) + x(2) + x(3)$.

since the present o/p depends upon past i/p's the present o/p depends on future i/p's so it's a non causal sys.

it's a Dynamic system.

→ unstable because it's sum of infinite terms

→ $x_1[n] \longrightarrow \sum_{K=-\infty}^{n+1} x_1[K] = y_1[n]$

$x_2[n] \longrightarrow \sum_{K=-\infty}^{n+1} x_2[K] = y_2[n]$

$x_1[n] + x_2[n] \longrightarrow \sum_{K=-\infty}^{n+1} x_1[K] + x_2[K] = y[n]$

$y[n] = y_1[n] + y_2[n]$.

So it's a linear System.

→ time Variancy :-

$y[n-n_0] = \sum_{K=-\infty}^{n-n_0+1} x(K)$; $y[n] = \sum_{n=-\infty}^{n-n_0+1} x(K)$.

∴ Time Invariant s/m.

$\Rightarrow$ If $x(K) = u(K)$ (for bounded i/p).

$$y[n] = \sum_{K=-\infty}^{n+1} u(K) = \begin{cases} 0 & ; n < -1 \\ n+2 & ; n \geq -1 \end{cases}$$

Since $y[n] \to \infty$ as $n \to \infty$
The s/m is unstable

c) $y[n] = x[n] \cos[\omega_0 n]$.

A) $y[0] = x[0] \cos[\omega_0 (0)]$

$\rightarrow$ static and causal system.

$\rightarrow$ $y[n-n_0] = x[n-n_0] \cos(\omega_0 n)$

$y[n] = x[n-n_0] \cos(\omega_0 (n-n_0))$

$y[n] \neq y(n-n_0]$ time Variant.

$\rightarrow$ $x_1[n] \longrightarrow y_1[n] = x_1[n] \cos[\omega_0 n]$

$x_2[n] \longrightarrow y_2[n] = x_2[n] \cos[\omega_0 n]$

$x_1[n] + x_2[n] \longrightarrow y_3[n] = (x_1[n] + x_2[n]) \cos(\omega_0 n)$

$y[n] = y_1[n] + y_2[n]$ so it's a linear System

$\rightarrow$ as $|x(n)| < \infty$; $|x(n) \cos \omega_0 n)| < \infty$ so it's a stable system.

$\Downarrow$

$|x(n)| < \infty$

d) $y[n] = x[-n+2]$

$y[0] = x[-0+2] = x[2]$

$y[1] = x[-1+2] = x[1]$

$y[4] = x[-4+2] = x[-2]$

$\rightarrow$ present o/p doesn't depend on present i/p so it's a Dynamic system.

$\rightarrow$ present o/p doesn't depends only on present and past i/ps so it's a non Causal system.

$\rightarrow$ $y[n-n_0] = x[-(n-n_0)+2]$

$y[n] = x[-(n-n_0)+2]$

Time invariant system.

$\rightarrow$ $x_1[n] \longrightarrow x_1[-n+2] = y_1[n]$

$x_2[n] \longrightarrow x_2[-n+2] = y_2[n]$

$x_1[n] + x_2[n] \longrightarrow x_1[-n+2] + x_2[-n+2] = y[n]$

$y[n] = y_1[n] + y_2[n]$ linear system.

$|x(n)| < \infty$ then $|x(-n+2)| < \infty$. stable.

$\longrightarrow$

g) $y[n] = |x[n]|$

$\rightarrow$ $y[0] = |x(0)|$ static system, Causal system.

$y[1] = |x(1)|$

$\longrightarrow$ $y[n-n_0] = |x(n-n_0)|$

$y[n] = |x(n-n_0)|$

$y[n] = y[n-n_0] \Rightarrow$ Time invariant system.

$\longrightarrow$ $x_1[n] \xrightarrow{sys} |x_1[n]| = y_1[n]$

$x_2[n] \xrightarrow{sys} |x_2[n]| = y_2[n]$.

$x_1[n] + x_2[n] \longrightarrow |x_1[n] + x_2[n]| = y[n]$

$y[n] = y_1[n] + y_2[n]$ Non linear system.

$\longrightarrow$ $|x(n)| < \infty$, $||x(n)|| < \infty$. stable.

$\Downarrow$

$|x(n)| < \infty$

h) $y[n] = x[n]u[n]$

A) $\rightarrow$ $y[0] = x(0)\,u(0)$

$y(1) = x(1)\,u(1)$ } static & Causal system.

$\longrightarrow$ $y[n-n_0] = x[n-n_0]\,u[n-n_0]$

$y[n] = x[n-n_0]\,u[n]$

$y[n] \neq y(n-n_0)$ Time Variant.

$\longrightarrow$ $x_1[n] \xrightarrow{sys} x_1[n]\,u[n] = y_1[n]$

$x_2[n] \xrightarrow{sys} x_2[n]\,u[n] = y_2[n]$

$x_1[n] + x_2[n] \longrightarrow [x_1[n] + x_2[n]]\,u[n] = y[n]$

$y_1[n] + y_2[n] = y[n] \Rightarrow$ linear.

$\longrightarrow$ $|x(n)| < \infty$ then $|x(n)u(n)| < \infty$. Stable

i) $y[n] = x[n] + nx[n+1]$

A) $\rightarrow$ It's a Dynamic and non causal system as it depends on ~~past~~ future i/p's.

$\longrightarrow$ $y[n-n_0] = x[n-n_0] + (n-n_0)\, x[n-n_0+1]$

$y[n] = x[n-n_0] + n\, x[n-n_0+1]$

$y[n] \neq y[n-n_0] \Rightarrow$ Time Variant system.

$\longrightarrow$ $x_1[n] \xrightarrow{sys} x_1[n] + n\, x_1[n+1] = y_1[n]$

$x_2[n] \xrightarrow{sys} x_2[n] + n\, x_2[n+1] = y_2[n]$

$x_1[n] + x_2[n] \xrightarrow{sys} x_1[n] + x_2[n] + n\,(x_1(n+1) + x_2(n+1)) = y(n)$

$y[n] = y_1[n] + y_2[n]$ so it's a linear sy

$\longrightarrow$ stability :- for Bounded i/p we are getting Bounded o/p so it's stable.

ii) $y[n] = x(2n)$ $\qquad y[1] = x[2]$.

A) $\rightarrow$ It's a Dynamic and non Causal system as it depends on past i/p's.

$\longrightarrow$ $y[n-n_0] = x[2(n-n_0)]$

$y[n] = x[2n-n_0]$

$y[n-n_0] \neq y[n]$ So time Variant system.

$\longrightarrow$ $x_1[n] \xrightarrow{sys} x_1[2n] = y_1[n]$

$x_2[n] \xrightarrow{sys} x_2[2n] = y_2[n]$

$x_1[n] + x_2[n] \longrightarrow x_1[2n] + x_2[2n] = y[n]$

$y[n] = y_1[n] + y_2[n]$ linear System.

$\rightarrow$ $|x(n)| < \infty$ then $|x(2n)| < \infty$ so Stable

k) $y[n] = \begin{cases} x[n] & ; \text{ if } x[n] \geq 0 \\ 0 & ; \quad x[n] \leq 0 . \end{cases}$

$y[1] = x[1]$ So static system & also

A) Causal for $x[n] \geq 0$.

$$\left. \begin{aligned} y[n-n_0] &= x[n-n_0] \\ y[n] &= x[n-n_0] \end{aligned} \right\} \text{time invariant.}$$

as $|x(n)| < \infty$ then $y[n] = x[n]$ so it's a stable system.

$y[n] \neq y_1[n] + y_2[n]$ nonlinear.

l) $y[n] = x[-n]$

A) $\rightarrow$ $\left. \begin{aligned} y[0] &= x[0] \\ y[1] &= x[-1] \end{aligned} \right\}$ It's a Dynamic & Causal system.

$\rightarrow$ $\left. \begin{aligned} y[n-n_0] &= x[-(n)+n_0)] \\ &= x[-n+n_0] \end{aligned} \right\}$ timeallaviant sys.

$y[n] =$

$\rightarrow$ $x_1[n] \xrightarrow{sys} x_1[-n] = y_1[n]$

$x_2[n] \xrightarrow{sys} x_2[-n] = y_2[n]$

$x_1[n] + x_2[n] \xrightarrow{sys} x_1[-n] + x_2[-n] = y[n]$

$= y_1[n] + y_2[n]$ $\Rightarrow$ linear system.

$y[n] = y_1[n] + y_2[n]$

$\rightarrow$ as $|x(n)| < \infty$ then $|x(-n)| < \infty$. so it's a Stable system.

m) $y[n] = \text{Sign}[x[n]]$

$\left. \begin{aligned} y[0] &= \text{Sign}[x[0]] \\ y[1] &= \text{Sign}[x[1]] \end{aligned} \right\}$ Static & Causal system.

$\longrightarrow$ $y[n-n_0] = \text{Sign}[x(n-n_0)]$
$y[n] = \text{Sign}[x[n-n_0]]$ } time invariant

$\longrightarrow$ $x_1[n] \xrightarrow{sys} \text{Sign}[x_1[n]] = y_1[n]$
$x_2[n] \xrightarrow{sys} \text{Sign}[x_2[n]] = y_2[n]$
$x_1[n] + x_2[n] \xrightarrow{sys} \text{Sign}[x_1(n) + x_2[n]] = y[n]$
$y_1[n] + y_2[n] = y[n]$ linear.

$\longrightarrow$ as $|x(n)| < \infty$ then $|\text{Sign}(x(n))| < \infty$
So it's a stable system.

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2.9) Let T be an LTI, relaxed, and BIBO stable sys with input $x[n]$ and output $y[n]$. show that

a) if $x[n]$ is periodic with period N [i.e. $x[n] = x[n+N]$ for all $n \geq 0$], the output $y[n]$ tends to a periodic signal with the same period.

b) if $x[n]$ is bounded and tends to constant, the o/p will also tend to constant.

c) if $x[n]$ is an energy signal the output $y[n]$ will also be an energy signal.

A) a) $y[n] = \sum_{k=-\infty}^{\infty} x[n-k] h[k]$, $x[n]=0$; $n<0$.

$$y[n+N] = \sum_{k=-\infty}^{n+N} x[n+N-k] h[k]$$

$$= \sum_{k=-\infty}^{n+N} x[n-k] h[k]$$

$$= \sum_{k=-\infty}^{n} x[n-k] h[k] + \sum_{k=n+1}^{n+N} x[n-k] h[k]$$

$$= y[n] + \sum_{k=n+1}^{n+N} x[n-k] h[k]$$

for a system to be bibo system

$$\lim_{n\to\infty} |h(n)| = 0$$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} x[n-k] h[k] = 0.$$

$$\lim_{n\to\infty} y[n+N] = \lim_{n\to\infty} y[n]$$

So $y[n]$ is also periodic with period N.

b) let $x[n] = x_0[n] + a u[n]$, where a is constant and $x_0[n]$ is a bounded signal with $\lim_{n \to \infty} x_0[n]$.

$$y[n] = a \sum_{k=0}^{\infty} h(k) u(n-k) + \sum_{k=0}^{\infty} h(k) x_0(n-k)$$

$$= a \sum_{k=0}^{n} h(k) + y_0[n].$$

clearly $\sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$ Hence

$$\lim_{n \to \infty} |y_0(n)| = 0.$$

thus $\lim_{n \to \infty} y(n) = a \sum_{k=0}^{n} h(k) = \text{Constant}$

c)

$$y[n] = \sum_k h(k) x(n-k)$$

$$\sum_{n=-\infty}^{\infty} y^2(n) = \sum_{n=-\infty}^{\infty} \left[\sum_k h(k) x(n-k) \right]^2$$

$$= \sum_k \sum_l h(k) h(l) \sum_n x(n-l) x(n-k)$$

$$\sum_n x(n-k) x(n-l) \le \sum_n x^2(n) = E_x.$$

$$\sum_n y^2(n) \le E_x \sum_k |h(k)| \sum_l |h(l)|$$

for a BIBO stable system

$$\sum_k |h(k)| < M.$$

$$E_y \le M^2 E_x \quad \text{So that}$$

$$E_y < \infty \text{ if } E_x < \infty.$$

2.13) show that the necessary and sufficient condition for a relaxed LTI system to be BIBO stable is $\sum_{n=-\infty}^{\infty} |h(n)| \le M_h < \infty$ for some constant M_h.

A) A system is BIBO stable if and only if a bounded input produces a bounded output.

$$y[n] = \sum_k h(k)\, x(n-k) \quad \forall n \quad \begin{cases} |x(n)| < \infty \\ |y(n)| < \infty \end{cases}$$

$$|y(n)| = \sum_k |h(k)\, x(n-k)|$$

$$|y(n)| \le \sum_k |h(k)|\, |x(n-k)|$$

$$\le m_x \sum_k |h(k)|$$

m_x is the maximum amplitude $x(t)$ and it is less than ∞

$$|x(n-k)| \le m_x \quad \text{so} \quad |y(n)| < \infty \text{ for all } n$$

if and only if

$$\boxed{\sum_k |h(k)| < \infty}$$

2.14) show that

a) A relaxed linear system is causal if and only if for any i/p $x(n)$ such that $x(n) = 0$ for $n < n_0 \Rightarrow$ $y[n] = 0$ for $n < n_0$.

b) A relaxed LTI system is causal if and only if $h(n) = 0$, for $n < 0$.

A) a) A system to be causal the o/p becomes nonzero after the i/p becomes non zero. Hence

$x(n) = 0$ for $n < n_0 \Rightarrow y[n] = 0$ for $n < n_0$.

b) $$y[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y[n] = \sum_{-\infty}^{0} h(k)\, x(n-k) + \sum_{k=0}^{\infty} h(k)\, x(n-k).$$

tends to zero then the system is causal

$$\boxed{h(k) = 0 \text{ for } k < 0}$$

2.15) a) show that for any real ~~complex~~ Constant a, and any finite integer numbers m and N, we have

$$\sum_{n=m}^{N} a^n = \begin{cases} \frac{a^m - a^{N+1}}{1-a} & \text{if } a \neq 1 \\ N-m+1 & \text{if } a = 1. \end{cases}$$

b) show that if $|a| < 1$ then

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}.$$

$a \cdot a^{N-1}$

a^N

A) $$\sum_{n=0}^{N} (1)^n = \sum_{n=0}^{N} (1) = N+1$$

$$a = 1 \Rightarrow \sum_{n=m}^{N} a^n = (N-m)+1 = N-m+1.$$

if $a \neq 1$. $$\sum_{n=m}^{N} a^n = a^m + a^{m+1} + \cdots + a^N.$$

$$\cancel{\sum_{n=m}^{N} a^n a = a^m + a^{m+1} + \cdots +}$$

$$(1-a)\sum_{n=m}^{N} a^n = a^m - \cancel{a^{m+1}} + \cancel{a^{m+1}} - \cancel{a^{m+2}} + \cancel{a^{m+2}} - \cancel{a^{m+3}} + \cdots + \cancel{a^N} - \cancel{a^N} \cancel{+a^{N+1}} - a^{N+1}$$

$$\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a} \quad \text{if } a \neq 1.$$

b) if $|a| < 1$
$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}.$$

as a lies between -1 and 1 each and every value of a Cambe written as a Rational number (i.e. $1/x$) which forms a GP so sum of infinite terms in G.P is $\frac{a}{1-r}$ $\Rightarrow$ $$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}.$$ Here initial term is 1.

(a.)

$$\sum_{n=m}^{N} a^n = \frac{a^m - a^{N+1}}{1-a} \quad \text{as } m \to 0 \text{ and } N \to \infty$$

$$= \frac{1}{1-a} \quad \text{for } |a| < 1$$

2.16) a) if $y[n] = x[n] * h[n]$ show that $\Sigma_y = \Sigma_x \Sigma_h$

where $\sum_x = \sum_{n=-\infty}^{\infty} x(n)$.

$$y[n] = \sum_k h(k)\, x(n-k)$$

$$\sum_n y[n] = \sum_k \sum_n h(k)\, x(n-k)$$

$$= \sum_k h(k) \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_n (y(n)) = \sum_k h(k) \sum_n x(n)$$

Since $\sum_{n=-\infty}^{\infty} x(n-k) = \sum_{n=-\infty}^{\infty} x(n)$

$$\boxed{\sum_n y(n) = \sum_k h(k) \sum_n x(n)}$$

b) Compute the Convolution $y[n] = x[n] * h[n]$ of the following signals and check the correctness of the results by using the test in (a).

1) $x[n] = \{1, 2, 4\}$, $h[n] = \{1, 1, 1, 1, 1\}$.

A)

$x[n]$ \ $h[n]$	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$y[n] = \{1, 3, 7, 7, 7, 6, 4\}$.

$\sum_n y[n] = 35$; $\sum_n x[n] = 4 + 2 + 1 = 7$

$\sum_k h[k] = 1+1+1+1+1 = 5$

$$\sum_n y[n] = \sum_n x[n] \sum_k h[k]$$

$$= 7 \times 5$$

$$= 35$$

2) $x[n] = \{1, 2, -1\}$, $h[n] = x[n]$

A)

$x[n]$ \ $h[n]$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$$y[n] = x[n] * h[n] = \{1, 4, 2, -4, 1\}$$

$$\sum_n y[n] = 1+4+2-4+1 = 4$$

$$\sum_n x[n] = 1+2-1 = 2$$

$$\sum_k h[k] = 1+2-1 = 2$$

$$\sum_n y[n] = \sum_n x[n] \sum_k h[k] = 2 \cdot 2 = 4.$$

3) $x[n] = \{0, 1, -2, +3, -4\}$, $h[n] = \{\frac{1}{2}, \frac{1}{2}, 1, 1/2\}$

$x[n]$ \ $h[n]$	1/2	1/2	1	1/2
0	0	0	0	0
1	1/2	1/2	1	1/2
-2	-1	-1	-2	-1
3	3/2	3/2	3	3/2
-4	-2	-2	-4	-2

$$y[n] = x[n] * h[n]$$

$$= \{0, 1/2, -1/2, 3/2, -2, 0, -5/2, -2\}$$

$$\sum_n y[n] = -5$$

$\sum_n x[n] = 0+1-2+3-4$ $\qquad$ $\sum_k h(k) = 2+\frac{1}{2}$

$\sum_n x[n] = -2$ $\qquad$ $= 5/2$

$\sum_n y[n] = \sum_n x[n] \sum_k h[k] = -5$

4) $x[n] = \{1,2,3,4,5\}$; $h[n] = \{1\}$.

A)

$x[n]$ \ $h[n]$	1
1	1
2	2
3	3
4	4
5	5

$x[n] * h[n] = y[n] = \{1,2,3,4,5\}$.

$\sum_n y[n] = 15$ $\qquad$ $\sum_n x[n] = 15$ $\qquad$ $\sum_k h[k] = 1$

$\sum_n y[n] = \sum_n x[n] \sum_k h(k) = 15$.

5) $x[n] = \{\underset{\uparrow}{1}, -2, 3\}$, $h[n] = \{0, \underset{\uparrow}{0}, 1, 1, 1, 1\}$.

A).

$x[n]$ \ $h[n]$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y[n] = x[n] * h[n] = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$.

$\sum_n y[n] = 8$; $\sum_n x[n] = 2$; $\sum_k y[k] = 4$.

$\sum_n y[n] = \sum_n x[n] \sum_k y[k] = 8$.

6) $x[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$, $h[n] = \{1, \underset{\uparrow}{-2}, 3\}$

A) $\sum_n x[n] = 4$; $\sum_k h[k] = 2$.

$x[n]$ \ $h[n]$	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$$y[n] = x[n] * h[n] = \{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$$

$$\sum_n y[n] = 8 = \sum_n x[n] \sum_k h[k]$$

7) $x[n] = \{\underset{\uparrow}{0}, 1, 4, -3\}$; $h[n] = \{\underset{\uparrow}{1}, 0, -1, -1\}$

$$\sum_n x[n] = 2 \quad ; \quad \sum_k h[k] = -1$$

$x[n]$ \ $h[n]$	1	0	−1	−1
0	0	0	0	0
1	1	0	−1	−1
4	4	0	−4	−4
−3	−3	0	3	3

$y[n] = \{0, 1, 4, -4, -5, -1, 3\}$ (↑ at 0)

$\sum_n y[n] = -2 = \sum_n x[n] = \sum_k h[k]$

8) $x[n] = \{1, 1, 2\}$ (↑ at first 1), $h[n] = u[n]$.

$x[n]$ \ $h[n]$	1	1	1	1	1	1	1	- - -
1	1	1	1	1	1	1	1	- -
1	1	1	1	1	1	1	1	- -
2	2	2	2	2	2	2	2	- - -

$y[n] = \{1, 2, 4, 4, \dots\}$ (↑ at 1)

$x[n]$ \ $h[n]$	$u[n]$	$u[n-1]$	$u[n-2]$	$u[n-3]$ - -
1	$u[n]$	$u[n-1]$	$u[n-2]$	$u[n-3]$ - -
1	$u[n]$	$u[n-1]$	$u[n-2]$	$u[n-3]$
2	$2u[n]$	$2u[n-1]$	$2u[n-2]$	$2u[n-3]$

$y[n] = \{u(n),\ u(n)+u(n-1),\ 2u(n)+u(n-1)+u(n-2) \dots\}$

$+ 2u[n-2]$.

$y[n] = u[n] + u[n-1]$

$\sum_n y[n] = \infty \qquad \sum_n x[n] = 4, \qquad \sum_k h[k] = \infty.$

$\sum_n y[n] = \sum_n x[n] \sum_k h[k] = \infty.$

11) $x[n] = \left(\frac{1}{2}\right)^n u[n]$, $h[n] = \left(\frac{1}{4}\right)^n u[n]$.

$x[n]$ \ $h[n]$	1	1/4	1/16	1/64	...	0
1	1	1/4	1/16	1/64	...	0
1/2	1/2	1/8	1/32	1/128	...	0
1/4	1/4	1/16	1/64	1/256	...	0
⋮	⋮	⋮	⋮	⋮		
0	0	0	0	0		0

$$y[n] = \left\{1, \frac{3}{4}, \frac{7}{16}, \frac{15}{64}, \ldots, 0\right\}$$

$$y[n] = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n\right] u[n].$$

$$\sum_n y[n] = \sum_{n=0}^{\infty} 2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u[n]$$

$$= \frac{2}{1-1/2} - \frac{1}{1-1/4}$$

$$= 4 - \frac{4}{3} = \frac{8}{3}.$$

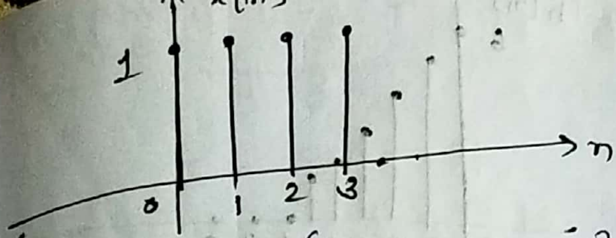
$$\sum_n x[n] = \sum_n \left(\frac{1}{2}\right)^n u(n) = 2.$$

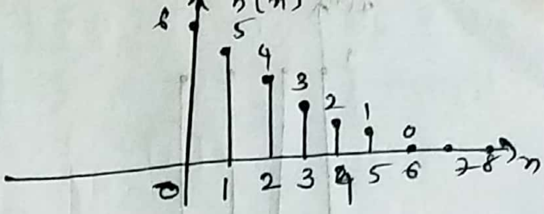
$$= \frac{1}{1-1/2} = 2.$$

$$\sum_k h[k] = \sum_{k=0}^{n} \left(\frac{1}{4}\right)^n = \frac{1}{1-1/4} = \frac{4}{3}.$$

$$\sum_n y[n] = \sum_n x[n] \sum_k h[k] = \frac{8}{3}.$$

2.19 Compute and plot the convolutions $x[n] * h[n]$ and $h[n] * x[n]$ for the pairs of signals shown in fig P2.19.





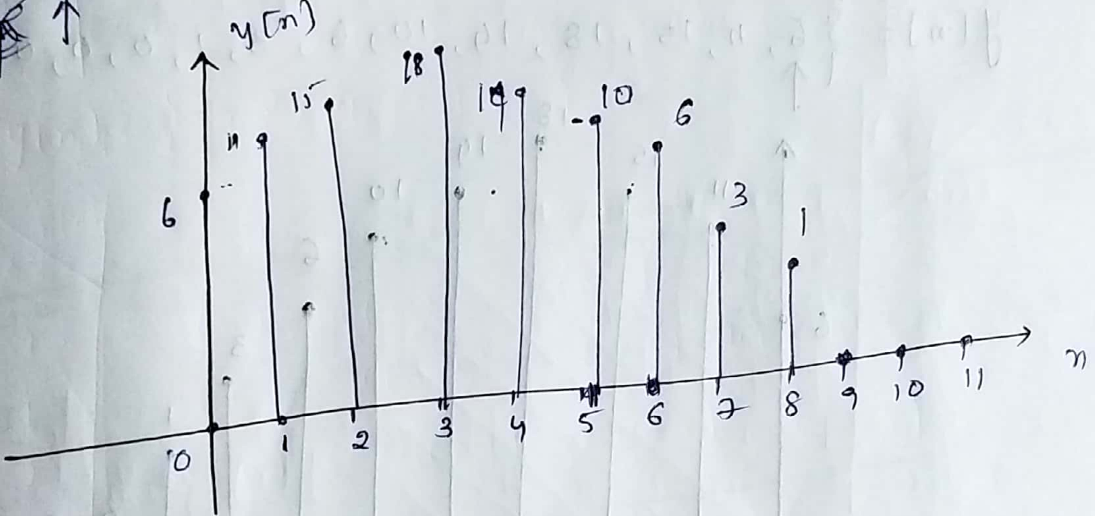
$x[n] = \{ 0, \underset{\uparrow}{1}, 1, 1, 1 \}$

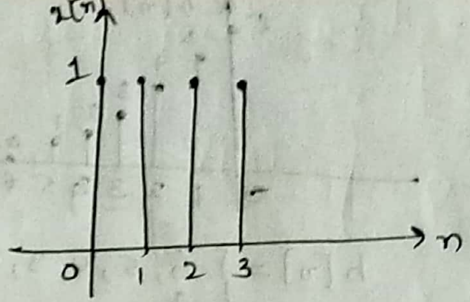
$h[n] = \{ \underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0, 0, 0 \}$

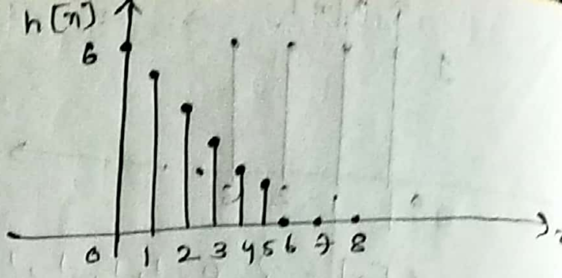
A)

$x[n]$ \ $h[n]$	6	5	4	3	2	1	0	0	0
0	0	0	0	0	0	0	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0

$y[n] = \{ 0, \underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1, 0, 0, 0 \}$.



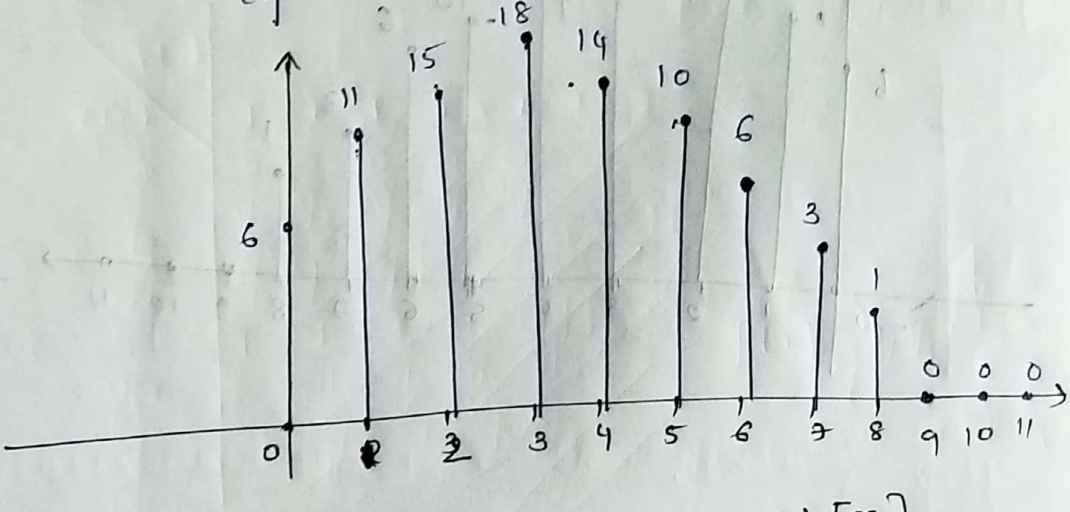




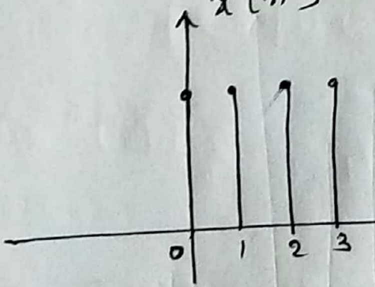
$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $\qquad$ $h[n] = \{\underset{\uparrow}{6}, 5, 4, 3, 2, 1, 0, 0, 0\}$

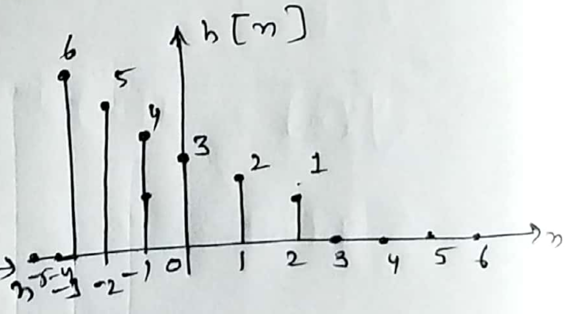
x[n] \ h[n]	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0
1	6	5	4	3	2	1	0	0	0

$y[n] = \{\underset{\uparrow}{6}, 11, 15, 18, 14, 10, 6, 3, 1, 0, 0, 0\}$.



b)



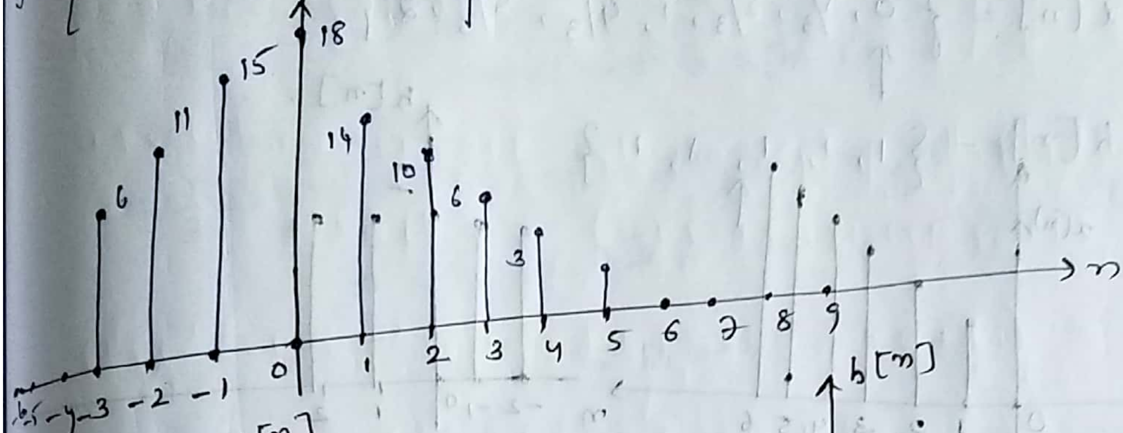


A) $x[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$ $\qquad$ $h[n] = \{6, 5, 4, \underset{\uparrow}{3}, 2, 1\}$

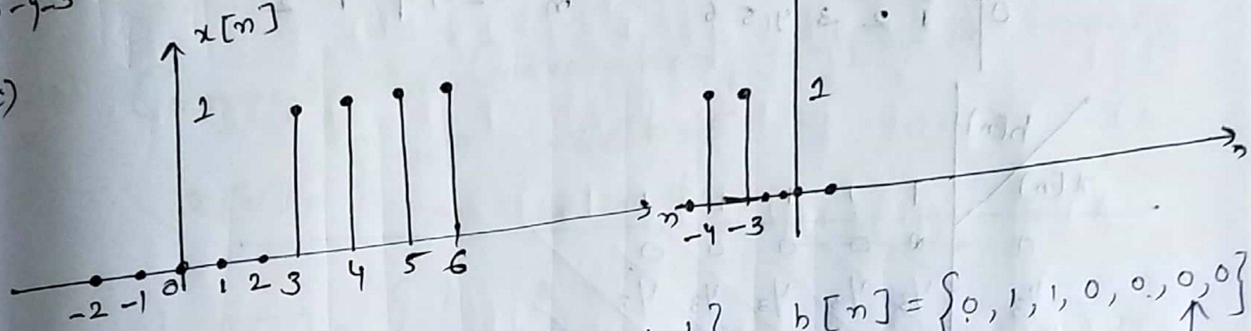
$x[n]$ \ $h[n]$	0	0	6	5	4	3	2	1	0	0	0	0
	6	5	4	3	2	1						
1	0	0	6	5	4	3	2	1	0	0	0	0
1	0	0	6	5	4	3	2	1	0	0	0	0
1	0	0	6	5	4	3	2	1	0	0	0	0
1	0	0	6	5	4	3	2	1	0	0	0	0

$$g[n] = \{0, 0, 6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1, 0, 0, 0, 0\}$$

-3 -2 -1



c)

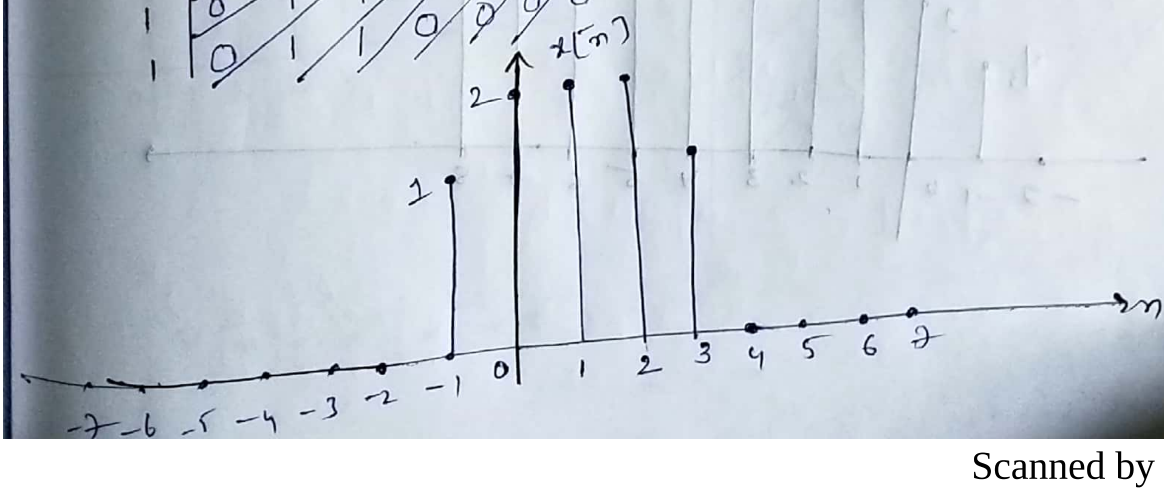


$$h[n] = \{0, 1, 1, 0, 0, \underset{\uparrow}{0}, 0\}$$

A) $x[n] = \{0, 0, \underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

$x[n]$ \ $h[n]$	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
1	0	1	1	0	0	0	0
1	0	1	1	0	0	0	0
1	0	1	1	0	0	0	0
1	0	1	1	0	0	0	0

$$y[n] = \{0, 0, 0, 0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0, 0\}$$



2.18) Determine and sketch the Convolution $y[n]$ of the Signals:

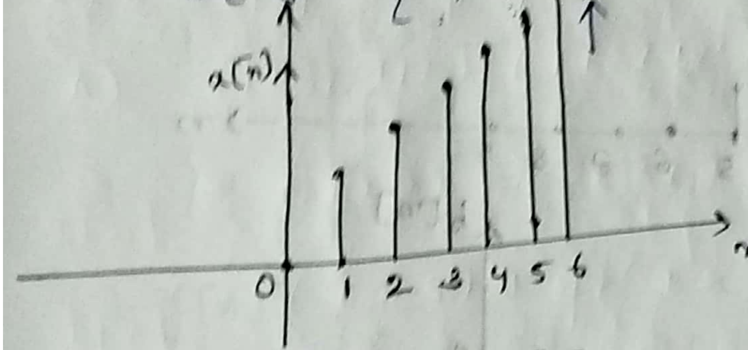
$$x[n] = \begin{cases} \frac{1}{3} n & 0 \le n \le 6 \\ 0 & \text{elsewhere} \end{cases}$$

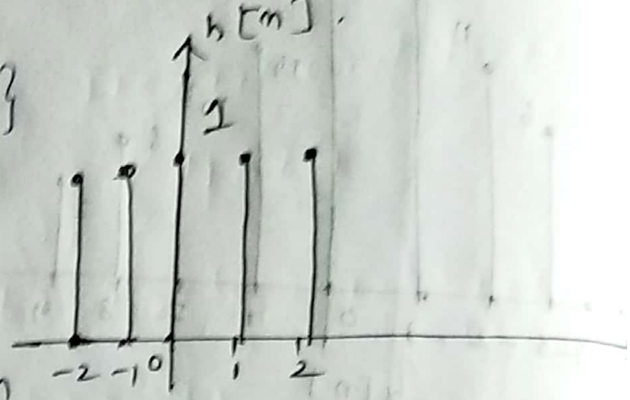
$$h[n] = \begin{cases} 1 & -2 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

a) Graphically b) analytically.

A) $x[n] = \{ \underset{\uparrow}{0}, 1/3, 2/3, 1, 4/3, 5/3, 2 \}$

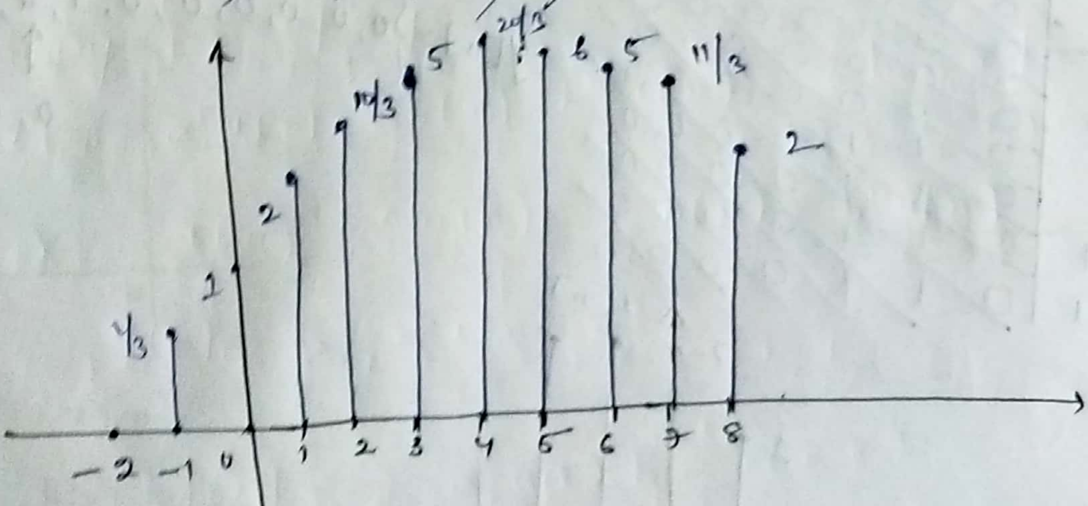
a) $h[n] = \{ 1, 1, \underset{\uparrow}{1}, 1, 1 \}$





x[n] \ h[n]	1	1	1	1	1
0	0	0	0	0	0
1/3	1/3	1/3	1/3	1/3	1/3
2/3	2/3	2/3	2/3	2/3	2/3
1	1	1	1	1	1
4/3	4/3	4/3	4/3	4/3	4/3
5/3	5/3	5/3	5/3	5/3	5/3
2	2	2	2	2	2

$y[n] = \{ 0, 1/3, \underset{\downarrow}{1}, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2 \}$



b) analytically

$h[n] = u[n+2] - u[n-3]$

$x[n] = \frac{1}{3}n[u[n] - u[n-7]]$

$y[n] = x[n] * h[n]$

$= \left(\frac{1}{3}[nu[n] - u[n-7]]\right) * \left(u[n+2] - u[n-3]\right)$

$= \frac{1}{3}nu[n] * u[n+2] - \frac{1}{3}nu[n] * u[n-3] - nu[n-7] * u[n+2]$

$+ nu[n-7] * u[n-3]$.

$= \frac{1}{3}\delta[n+1] + 1\,\delta[n] + 2\,\delta[n-1] + \frac{10}{3}\delta[n-2] + 5\,\delta[n-3]$

$+ \frac{20}{3}\delta[n-4] + 6\,\delta[n-5] + 5\,\delta[n-6] + \frac{11}{3}\delta(n-7)$

$+ 2\,\delta(n-8)$.

2.19) Compute the Convolution $y[n]$ of the signals

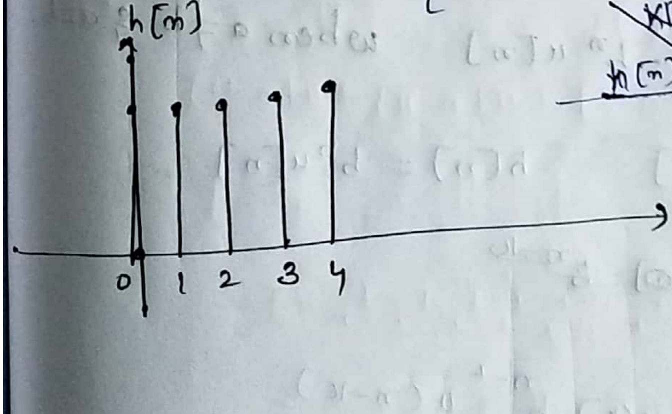
$$x[n] = \begin{cases} \alpha^n & -3 \le n \le 5 \\ 0 & \text{elsewhere.} \end{cases}$$

$$h[n] = \begin{cases} 1 & 0 \le n \le 4 \\ 0, & \text{elsewhere.} \end{cases}$$

A) $x[n] = \left\{ \frac{1}{\alpha^3}, \frac{1}{\alpha^2}, \frac{1}{\alpha}, \underset{\uparrow}{1}, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5 \right\}$

$h[n] = \{1, 1, 1, 1, 1\}$

h[n]



h[n] \ x[n]	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5

$y[n] = \{\alpha^{-3}, \alpha^{-3} + \alpha^{-2}, \alpha^{-3} + \alpha^{-2} + \alpha^{-1}, \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1, \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha, \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2}, \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}, 1 + \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4}, \alpha + \alpha^{+2} + \alpha^{+3} + \alpha^{+4}, \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}, \alpha^{3} + \alpha^{4} + \alpha^{5}, \alpha^{5}\}$

Q.20) Consider the following operations.

a) Multiply the integer numbers 131 and 122

A) 15982

b) Compute the Convolution of signals. $\{1,3,1\} * \{1,2,2\}$

$y[n]$ \ $h[n]$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

$y[n] = \{1, 5, 9, 8, 2\}$

c) Multiply the polynomials $1+3z+z^2$ and $1+2z+2z^2$

A) $(1+3z+z^2)(1+2z+2z^2)$ = ~~(1+82+~~

$= (1+2z+2z^2+3z+6z^2+6z^3+z^2+2z^3+2z^4)$

$= (1+5z+9z^2+8z^3+2z^4)$

d) Repeat part a for 1.31 and 12.2

A) $(1.31)(12.2) = 15.982$

e) Comment on your result.

A) These are the different types of Convolution methods.

Q.21) Compute the Convolution $y[n] = x[n] * h[n]$ of the following pair of signals. when $a \neq b$ & $a = b$.

a) $x[n] = a^n u[n]$ $h[n] = b^n u[n]$

A) a) $x[n] = a^n u[n]$ $h[n] = b^n u[n]$

$y[n] = \sum_{n=0}^{\infty}$ ~~$a^n u[n]$~~ ~~b^{n-k}~~

$$y[n] = \sum_{k=-\infty}^{\infty} a^k u(k)\, b^{n-k} u(n-k)$$

$$= \sum_{k=0}^{n} a^k b^{n-k} = b^n \sum_{k=0}^{n} (ab^{-1})^k$$

if $a \neq b$
It forms a G.P. with $r = \frac{a}{ab}$.

$$S_n = \frac{1-r^N}{1-r}$$

$$y[n] = b^n \left[\frac{1-\left(\frac{a}{ab}\right)^n}{1-\frac{a}{ab}} \right] = \frac{b^n - a^n}{b^n} \times \frac{b}{b-a}$$

$$= b^n \left[\frac{(ab)^n - 1}{(ab)^n} \cdot \frac{ab}{ab - b} \right] = \frac{(b^n - a^n)b}{b-a}$$

$$y[n] = \frac{b^{n+1} - a^n b}{b-a}$$

if $a = b$

$$y[n] = b^n \sum_{k=0}^{n} (1)^k = b^n (n+1) u(n)$$

$$x[n] = \begin{cases} 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$$

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$$

$$x[n] * h[n] = y(n)$$

$$x[n] = 2\delta[n+2]$$

$$y[n] = 2\delta[n+2] * \delta[n] - 2\delta[n+2]\delta[n-1] + 2\delta[n+2] * \delta[n-4] + 2\delta[n+2] * \delta[n-5]$$

$$y[n] = 2\delta[n+2] - 2\delta[n+1] + 2\delta[n-2] + 2\delta[n-3]$$

$$y[n] = \{2, -2, \underset{\uparrow}{0}, 0, 2, 2\}$$

$$x[n] = \begin{cases} 1; & n = -2, 0, 1 \\ 2; & n = -1 \\ 0; & \text{elsewhere} \end{cases}$$

$$h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$$

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$x[n] = \delta[n+2] + 2\delta[n+1] + \delta[n] + \delta[n-1]$

$h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$

$x[n]$ \ $h[n]$	1	-1	0	0	1	1
1	1	-1	0	0	1	1
2	2	-2	0	0	2	2
1	1	-1	0	0	1	1
1	1	-1	0	0	1	1

$y[n] = \{1, 1, -1, 0, 0, 1, 3, \ldots$
$\uparrow$

c) $x[n] = u[n+1] - u[n-4] - \delta[n-5]$

$h[n] = [u[n+2] - u[n-3]]\,(3 - |n|)$

$3 - |-2| = 3 - 2 = 1$

$3 - |-1| = 3 - 1 = 2$

$3 - |0| = 3 - 0 = 3$

$3 - |1| = 3 - 1 = 2$

$x[n] = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h[n] = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$x[n]$ \ $h[n]$	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
1	1	2	3	2	1
0	0	0	0	0	0
-1	-1	-2	-3	-2	-1

$y[n] = \{1, 3, 6, \underset{\uparrow}{8}, 9, 8, 5, 1, -2, -2, \ldots$

d) $x[n] = u[n] - u[n-5]$

$h[n] = u[n-2] - u[n-8] + u[n-11] - u[n-17]$

$x[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$h[n] = \{$ [illegible]

$h'[n] = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1, 1, 1\}$

$h[n] = h'[n] + h'[n-9]$

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$y[n] = y'[n] + y'[n-9]$

$y'[n] = \{ \underset{\uparrow}{0}, 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1 \}$

2.22) Let $x[n]$ be the i/p Signal to a discrete-time filter with impulse response $h_i(n)$ and $y_i(n)$ be the Corresponding output.

a) Compute and sketch and $y_i[n]$ in the following cases, using the same scale in all figures.

$x[n] = \{1, 4, 2, 8, 5, 3, 3, 4, 5, 7, 6, 9\}.$

$h_1[n] = \{1, 1\}$; $h_2[n] = \{1, 2, 1\}$, $h_3[n] = \{1/2, 1/2\}$

$h_4[n] = \{1/4, 1/2, 1/4\}$, $h_5[n] = \{1/4, -1/2, 1/4\}$.

A) $x[n] * h_1[n]$

h[n] \ x[n]	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_1[n] = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}.$

$x[n] * h_2[n]$

h[n] \ x[n]	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_2[n] = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}.$

$y_3[n] = x[n] * h_3[n]$

$y_3[n] = \{ 1/2, 5/2, 3, 5/2, 4, 4, 3, 7/2, 9/2, 6, \frac{11}{2}, \frac{15}{2}, \frac{3}{2} \}$

$y_4[n] = x[n] * h_4[n]$

$= \{ 1/4, 1/2, 11/4, 11/4, 13/4, 4, \frac{7}{2}, 13/4, \frac{21}{4}, \frac{25}{4}, 7, 1 \}$

$y_5[n] = x[n] * h_5[n]$

$= \{ \frac{1}{4}, \frac{1}{2}, -\frac{5}{4}, \frac{-3}{4}, \frac{1}{4}, -1, \frac{1}{2}, \frac{1}{4}, 0, \frac{1}{4}, -\frac{3}{4}, 1, -3, \frac{-9}{4} \}$

5) what is the difference between $y_1[n]$ and $y_3[n]$ and b/w $y_2[n]$ & $y_4[n]$?

A)

$$h_3[n] = \frac{1}{2} h_1[n]$$

$$y_3[n] = \frac{1}{2} y_1[n]$$

$$h_4[n] = \frac{1}{4} h_2[n]$$

$$y_4[n] = \frac{1}{4} y_2[n].$$

c) Comment on smoothness of $y_2[n]$ and $y_4[n]$.

A) $y_2[n]$ & $y_4[n]$ are smoother than $y_1[n]$ and $y_4[n]$ is even smoother than $y_2[n]$ since it's scaled by a small factor.

d) Compare $y_4[n]$ and $y_5[n]$. what is the difference Can you explain it?

A) $y_4[n]$ has smoother o/p than $x_5[n]$ because $x_5[n]$ has some -ve value.

2.23) The Discrete time system

$$y[n] = ny[n-1] + x[n] \quad n>0$$

is at rest [i.e. $y(-1)=0$]. check if the system is linear time invariant and BIBO system.

A) $\rightarrow$ $x_1[n] \xrightarrow{sys} y_1[n] = n\, y_1[n-1] + x_1[n]$

$x_2[n] \xrightarrow{\;\;"\;\;} y_2[n] = n\, y_2[n-1] + x_2[n]$.

$x_1[n] + x_2[n] \longrightarrow y[n] = n\, y[n-1] + x_1[n] + x_2[n]$

$y[n] = y_1[n] + y_2[n]$. So linear system.

$\rightarrow$ $y[n-n_0] = y[n-n_0-1]\,(n-n_0) + x[n-n_0]$

$y[n] = n\, y[n-1] + x[n-n_0]$

Time Veriant system.

$\rightarrow$ let. $x[n] = u[n]$ $\Rightarrow$ $|u[n]| \le 1$ it's a bonded on

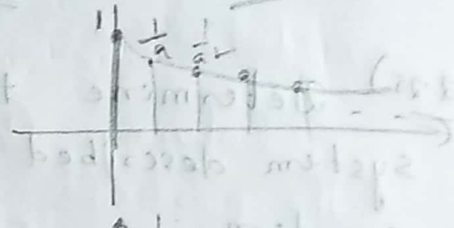
$y[0] = 1$, $y[1] = 1+1 = 2$, $y[2] = 2\times2+1 = 5, \ldots$

$\rightarrow$ So it's a unbounded. Hence system is unstable.

2.24) Consider the signal $x[n] = a^n u[n]$ $0 < a < 1$

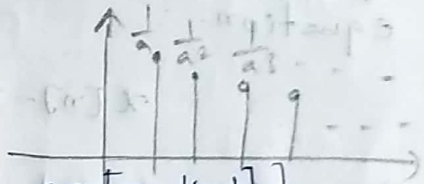
a) Show that any sequence of $x[n]$ can be decompos as $x[n] = \sum_{n=-\infty}^{\infty} c_k\, \delta(n-k)$ and express c_k interms of $x[n]$.

A) $\delta[n] = x[n] - a\, x(n-1)$



$x[n] = x[n] * \delta[n]$ $\because$ Convolution identity.

$a \cdot a^{n-1}$



$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\, \delta[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\left[\gamma(n-k) - a\gamma[n-k-1]\right]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, \gamma[n-k] - a \sum_{k=-\infty}^{\infty} x[k]\, \gamma[n-k-1]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, \gamma[n-k] - a \sum_{k=-\infty}^{\infty} x[k-1]\, \gamma[n-k]$$

$$x[n] = \sum_{k=-\infty}^{\infty} \left(x[k] - a\, x[k-1]\right) \gamma[n-k].$$

$c_k = x[k] - a\, x[k-1]$.

b) Use the properties of linearity and time invariance to express the o/p $y[n] = T[x[n]]$ in terms of the i/p $x[n]$ and the signal $g[n] = T[r[n]]$, where $T[\cdot]$ is an LTI system.

A)
$$y[n] = T[x[n]]$$
$$= T\left[\sum_{K=-\infty}^{\infty} c_K\, r(n-k)\right]$$
$$= \sum_{K=-\infty}^{\infty} c_K T[r(n-k)]$$
$$= \sum_{K=-\infty}^{\infty} c_K\, g(n-k) \quad \because g[n] = T[r[n]]$$

c) Express the impulse Response $h[n] = T[\delta[n]]$ in terms of $g[n]$.

A)
$$h[n] = T[\delta[n]]$$
$$h[n] = T[r[n] - a\,r[n-1]] \quad \because \text{it's a LTI system we can split } T.$$
$$= T[r[n]] - a\,T[r[n-1]]$$
$$= g[n] - a\,g[n-1]$$

2.25) Determine the zero-input response of the system described by the second order difference equation.

$$x[n] - 3y[n-1] - 4y[n-2] = 0$$

A) Zero input Response (i.e. $x[n] = 0$)
$$-3y[n-1] - 4y[n-2] = 0$$
$$4y[n-2] = -3y[n-1]$$
$$-\frac{4}{3}y[n-2] = y[n-1]$$
$$n=0 \qquad y[-1] = \frac{-4}{3}y[-2]$$

2.25) Determine the particular Solution of the difference equation.

$$y[n] = \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2] + x[n].$$ when the forcing function is $x[n] = 2^n u[n]$.

A) $$y[n] = \frac{5}{6} y[n-1] - \frac{1}{6} y[n-2] + x[n]$$

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n]$$

for finding homogeneous solution

$$y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = 0$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0.$$

$$\left(\lambda - \frac{1}{2}\right)\left(\lambda - \frac{1}{3}\right) = 0 \Rightarrow \lambda = 1/2, 1/3.$$

$$y_h(n) = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{3}\right)^n$$

particular solution

$$x[n] = 2^n u[n]$$

$$y_p[n] = K(2)^n u[n].$$

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 2^n u[n]$$

$$y_p[n] = \frac{2^n u[n]}{\left(\lambda - \frac{1}{2}\right)\left(\lambda - \frac{1}{3}\right)}$$

$$\lambda = 0$$

$$y_p[n] = \frac{2^n u[n]}{1/6}$$

$$y_p[n] = K\, 2^n u[n]$$

Substitute those equations in difference equation

$$K(2^n)u(n) - \frac{K5}{6}(2^{n-1})u(n-1) + K\left(\frac{1}{6}\right)2^{n-2}u(n-2) = 2^n u$$

for $n = 2$,

$$4K - \frac{5K}{3} + \frac{K}{6} = 4 \Rightarrow K = \frac{8}{5}$$

So Solution is

$$y[n] = y_p[n] + y_h[n]$$

$$y[n] = \frac{8}{5}(2^n)u[n] + c_1\left(\frac{1}{2}\right)^n u[n] + c_2\left(\frac{1}{3}\right)^n u[n]$$

to determine c_1 and c_2 assume $y(-2) = y(-1)$:

then $y(0) = 1$ and

$$y(1) = \frac{5}{6}y(0) + 2 = \frac{17}{6}$$

$$\frac{8}{5} + c_1 + c_2 = 1$$

$$c_1 + c_2 = 1 - \frac{8}{5}$$

$$c_1 + c_2 = \frac{-3}{5} \longrightarrow (1)$$

$$y[1] = \frac{16}{5} + \frac{1}{2}c_1 + \frac{1}{3}c_2 = \frac{17}{6}$$

$$3c_1 + 2c_2 = -\frac{11}{5} \longrightarrow (2)$$

Solving (1) & (2)

$$c_1 + c_2 = -3/5$$

$$3c_1 + 2c_2 = -11/5$$

$$c_1 = -1 \; ; \; c_2 = \frac{2}{5}$$

Therefore total solution is

$$y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \left(\frac{2}{5}\right)\left(\frac{1}{3}\right)^n\right]u[n]$$

2.25) Determine Zero input Response of the System described by the second order differential eqn

$$x[n] - 3y[n-1] - 4y[n-2] = 0.$$

Zero input Response $(x[n] = 0)$

$$-3y[n-1] = 4y[n-2]$$

$$4y[n-2] + 3y[n-1] = 0$$

$$4\lambda^{n-2} + 3\lambda^{n-1} = 0$$

$$\lambda^{n-2}(4 + 3\lambda) = 0.$$

$$4 + 3\lambda = 0.$$

$$\boxed{\lambda = \frac{-4}{3}}$$

$$y_h(n) = c_1(\lambda)^n$$

$$= c_1\left(\frac{-4}{3}\right)^n$$

$$-3y[n-1] = 4y[n-2]$$

$$y[n-1] = \frac{-4}{3}y[n-2]$$

$n=0$ $\quad y[-1] = \frac{-4}{3}.y[-2]$

$n=1$ $\quad y[0] = \frac{-4}{3}y[-1]$

$$= \left(\frac{-4}{3}\right)\left(\frac{-4}{3}\right)y[-2]$$

$$= \left(\frac{-4}{3}\right)^2 y[-2]$$

$n=2$ $\quad y[1] = \left(\frac{-4}{3}\right)^3.y[-2]$

$\vdots$

$n = k+1$ $\quad \boxed{y[k] = \left(\frac{-4}{3}\right)^{k+2} y[-2]}$

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2.27) Determine Response $y[n]$, $n \geq 0$ of the system described by the second-order difference equation.

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$$

to the input $x[n] = 4^n u[n]$.

$$y[n] = y_h[n] + y_p[n]$$

to find homogeneous solution $x[n] = 0$

$$y[n] - 3y[n-1] - 4y[n-2] = 0$$

$$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - 3\lambda - 4] = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

$$\lambda_1 = 4, \lambda_2 = -1$$

$$y_h[n] = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$= c_1 (4)^n + c_2 (-1)^n.$$

Since 4 is a characteristic root and the excitation is

$$x[n] = 4^n u[n]$$

The particular solution is assumed to be of the same form as an exponential sequence $x[n]$.

$$y_p[n] = k 4^n u[n]$$

Since this term is already present in $y_h[n]$ we will take another linearly independent of the terms contained in $y_h[n]$.

$$y_p[n] = kn 4^n u[n] \longrightarrow (1)$$

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1] \rightarrow (2)$$

Substituting (1) in (2).

$$Kn\,4^n u[n] - 3K(n-1)4^{n-1} u[n-1] - 4K(n-2)4^{n-2} u(n-2)$$
$$= 4^n u[n] + 2(4^{n-1} u(n-1))$$

To determine K, we evaluate equation for any $n \geq 2$, where none of the unit step terms vanish.

let $n = 2$.

$$32K - 12K - 4(0) = 16 + 8$$

$$\boxed{\begin{aligned} 20K &= 24 \\ K &= \frac{24}{20} = \frac{6}{5} \end{aligned}}$$

$$y_p[n] = \frac{6}{5} n (4)^n u(n)$$

$$\begin{aligned} y[n] &= y_h[n] + y_p[n] \\ &= c_1(4)^n + c_2(-1)^n + \frac{6}{5} n (4)^n u(n). \\ &= c_1(4)^n + c_2(-1)^n + \frac{6}{5} n (4)^n \quad n \geq 0. \\ &= c_1(4)^n + c_2(-1)^n + \frac{6}{5} n (4)^n \end{aligned}$$

$$\begin{aligned} y[n] &= 3y[n-1] + y[n-2] + x[n] + 2x[n-1] \\ y[0] &= 3y[-1] + 4y[-2] + 4^n u[n] + 2 \times 4^{n-1} u[n-1]. \\ &= 3y[-1] + 4y[-2] + 1 \\ y[1] &= 3y[0] + 4y[-1] + 4 + 2 \times (1) \\ &= 3y[0] + 4y[-1] + 6. \\ &= 3(3y(-1) + 4y(-2) + 1) + 4y(-1) + 6. \\ &= 13y[-1] + 12y[-2] + 9 \end{aligned}$$

On the other hand

$$y[n] = c_1(4)^n + c_2(-1)^n + \frac{6}{5} n (4)^n$$
$$y[0] = c_1 + c_2$$

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$y[1] = -c_1 + 4c_2 + \frac{24}{5}$

let $y(-1)$ & $y(-2) = 0$.

$y[0] = c_1 + c_2 = 1$

$y[1] = -c_1 + 4c_2 + \frac{24}{5} = 9$

$c_1 + c_2 = 1$

$-c_1 + 4c_2 = \frac{21}{5}$

By solving $c_1 = \frac{-1}{25}$, $c_2 = \frac{26}{25}$

$y[n] = \frac{-1}{25} 4^n + \frac{26}{25} (-1)^n + \frac{6}{5} n (4^n) u(n)$.

$$\boxed{y[n] = \frac{-1}{25} 4^n + \frac{26}{25} (-1)^n + \frac{6}{5} n 4^n} \quad n \geq 0$$

2.28) Determine the impulse response of the following Causal system.

$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1]$

$y[n] - 3y[n-1] - 4y[n-2] = 0$.

$\lambda^n - 3\lambda^{n-1} - 4\lambda^{n-2} = 0$.

$\lambda^{n-2} [\lambda^2 - 3\lambda - 4] = 0$

$\lambda^2 - 3\lambda - 4 = 0$.

$\lambda_1 = 4$, $\lambda_2 = -1$

$y_h[n] = c_1(4^n) + c_2(-1)^n$.

~~let~~ for an impulse Response $x[n] = \delta[n]$

$y[n] = 3y[n-1] + 4y[n-2] + x[n] + 2x[n-1]$

$= 3y[n-1] + 4y[n-2] + \delta[n] + 2\delta[n-1]$

Causal system means $y[n] = 0; n < 0$.

$y[0] = 0 + 0 + 1 + 0 = 1$

$y[1] = 3y[0] + 2.$

$y[1] = 3(1) + 2 = 5$

for an impulse Response particular sol = 0.

$y(n) = y_h(n)$

$y(n) = c_1 4^n + c_2 (-1)^n.$

$y[0] = c_1 + c_2 = 1$

$y[1] = 4c_1 - c_2 = 5$

By solving $c_1 = \frac{6}{5}$, $c_2 = \frac{-1}{5}$

$h(n) = y(n) = \frac{6}{5}(4)^n + (-\frac{1}{5})(-1)^n \quad n \geq 0.$

$y(n) = \frac{6}{5}(4)^n - \frac{1}{5}(-1)^n \quad n \geq 0$

2.29) let $x[n]$, $N_1 \leq n \leq N_2$ and $h(n)$, $M_1 \leq n \leq M_2$ be two finite duration signals.

a) Determine the range $L_1 \leq n \leq L_2$ of their Convolution interms of N_1, N_2, M_1 and M_2

b) Determine the limits of the cases of partial overlap from the left, full overlap, and partial overlap from right. for Convenience assume that $h[n]$ has shorter duration than $x[n]$.

c) Illustrate the Validity of your results by Computing following

$$x[n] = \begin{cases} 1, & -2 \leq n \leq 4 \\ 0, & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 2, & -1 \leq n \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

A) a) $L_1 = N_1 + M_1$ and $L_2 = N_2 + M_2$

b) partial overlap from left

low N_1+M_1 high N_1+M_2-1

Full overlap

low N_1+M_2 high N_2+M_1

partial overlap from right

low N_2+M_1+1 high N_2+M_2

c) $x[n] = \{1,1,\underset{\uparrow}{1},1,1,1,1\}$

$h[n] = \{2,\underset{\uparrow}{2},2,2\}$

$N_1=-2$, $N_4=2$, $M_1=-1$, $M_2=2$

partial overlap from right : $n=4$ $n=6$ $L_2=6$

partial " " left : $n=-3$ $n=-1$ $L_1=-3$

full overlap : $n=0$ $n=3$

2.30) Determine the impulse Response and unit step Response of the systems described below.

a) $y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + x[n]$

b) $y[n] = 0.7\,y[n-1] - 0.1\,y[n-2] + 2x[n] - x[n-2]$

A) a) $y[n] - 0.6\,y[n-1] + 0.08\,y[n-2] = x[n]$

$y[n] - 0.06\,y[n-1] + 0.08\,y[n-2] = 0$

$\lambda^n - 0.06\,\lambda^{n-1} + 0.08\,\lambda^{n-2} = 0$

$\lambda^2 - 0.06\lambda + 0.08 = 0$

$\lambda_1 = 0.2$, $\lambda_2 = 0.4$

$y_h[n] = C_1\lambda_1^n + C_2\lambda_2^n$

$= C_1(0.2)^n + C_2(0.4)^n$

for an impulse Response particular solution is '0'.

$y[n] = 0.6\,y[n-1] - 0.08\,y[n-2] + \delta[n]$

$y[0] = 1$ if the system is Caus

$y[1] = 0.6\, y[0]$

$y[1] = 0.6\,(1) = 0.6$

$y[n] = c_1 (0.2)^n + c_2 (0.4)^n$

$y(0) = c_1 + c_2 = 1$

$y[1] = 0.2 c_1 + 0.4 c_2 = 0.6$

By solving $\quad c_1 = -1, \; c_2 = +2$

Therefore $\quad h[n] = \left[-(0.2)^n + 2\,(0.4)^n\right] u[n]$

The step Response is

$$s[n] = \sum_{k=0}^{n} h[n-k] \qquad n \geq 0$$

$$= \sum_{k=0}^{n} \left(2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k} \right)$$

$$= 2\left(\frac{2}{5}\right)^{n} \sum_{k=0}^{n} \left(\frac{2}{5}\right)^{-k} - \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{-k}$$

$$= 2\left(\frac{2}{5}\right)^{n} \sum_{k=0}^{n} \left(\frac{5}{2}\right)^{+k} - \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= 2\left(\frac{2}{5}\right)^{n} \frac{\left(\frac{5}{2}\right)^{n+1} - 1}{\frac{5}{2} - 1} - \left(\frac{1}{5}\right)^{n} \frac{\left(5^{n+1} - 1\right)}{5}$$

$$= \frac{4}{3}\left(\left(\frac{2}{5}\right)^{n}\left(\left(\frac{5}{2}\right)^{n+1} - 1\right)\right) - \left(\frac{1}{5}\right)^{n+1} \left(5^{n+1} - 1\right)$$

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b) $y[n] + 0.7\, y[n-1] + 0.1\, y[n-2] = 2x[n] - 2x[n-1]$

$$\lambda^2 - 0.7\lambda + 0.1 = 0.$$

$$\lambda_1 = \frac{1}{2}, \quad \lambda_2 = 1/5$$

$$y_h[n] = c_1 \left(\frac{1}{2}\right)^n + c_2 \left(\frac{1}{5}\right)^n.$$

impulse Response $x[n] = \delta[n]$.

$$y[0] = 2$$

$$y[1] = 0.7\, y[0]$$

$$= 1.4$$

$y[0] = c_1 + c_2 = 2$; $y[1] = 0.5c_1 + 0.2c_2$

$= 1.4$

By solving $c_1 = \frac{10}{3}$, $c_2 = \frac{-4}{3}$

$$h[n] = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n \right] u[n].$$

Step Response

$$s[n] = \sum_{k=0}^{n} h[n-k]$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^{+k} - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$\frac{a(1-r^{n+1})}{1-r} \quad r>1$$

$$\frac{a(r^{N+1}-1)}{r-1} = \frac{1(2^{n+1}-1)}{1}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1}-1) - \frac{4}{3}\left(\frac{1}{5}\right)^n (5^{n+1}-1)$$

$$s[n] = \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1}-1) - \frac{4}{3}\left(\frac{1}{5}\right)^n (5^{n+1}-1) \quad n \geq 0.$$

2.31 Consider a system with impulse Response $h[n] = \begin{cases} (1/2)^n; & 0 \leq n \leq 4 \\ 0, & \text{elsewhere}. \end{cases}$

Determine input $x[n]$ for $0 \leq n \leq 8$ that will generate the output sequence

$$y[n] = \{ \underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 \}.$$

A) $h[n] = \{ \underset{\uparrow}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16} \}$

$$y[n] = \{ \underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 \}$$

$$y[n] = \sum_{k=0}^{n} x[k]\, h[n-k]$$

$$y[0] = x(0) \times h(0) \quad \Rightarrow \quad x(0) = 1$$

$$1 = x(0) \times 1$$

$$y[1] = x(0)\, h(1) + x(1)\, h(0)$$

$$2 = \frac{1}{2} x(0) + x(1)$$

$$2 = \frac{1}{2} + x(1) \quad \Rightarrow \quad x(1) = 3/2.$$

$$y[2] = x[0]\, h[2] + x[1]\, h[1] + x[2]\, h[0]$$

$$2.5 = 1 \cdot (0.25) + 3/2\,(0.5) + x[2](1)$$

$$x[2] = 3/2.$$

$$x[n] = \left\{ 1,\ 3/2,\ 3/2,\ \frac{7}{4},\ \ldots \right\}.$$

2.34) Compute and sketch the step response of the system

$$y[n] = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k)$$

$$h[n] = \frac{u[n] - u[n-M]}{M}.$$

$$s[n] = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k) = \begin{cases} \frac{n+1}{N} & n < M \\ 1, & n \geq M. \end{cases}$$

2.35) Determine the range of values of the parameter a for which the linear time invariant system with impulse response

$$h[n] = \begin{cases} a^n & n \geq 0,\ n \text{ even} \\ 0 & \text{otherwise.} \end{cases}$$ is stable.

A) $$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\ n \text{ even}}^{\infty} |a|^n$$

$$= \sum_{n=0}^{\infty} |a|^{2n} = \frac{1}{1-|a|^2}$$

stable if $|a| < 1$

2.36) Determine the response of the system with impulse response $h(n) = a^n u[n]$ to the input signal $x[n] = u[n] - u[n-10]$.

Response to $u[n]$ is

$$y_1[n] = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k} = a^n \sum_{k=0}^{\infty} a^{-k}$$

$$= a^n \left[1 + \frac{1}{a} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots \right]$$

$$= a^n \left[\frac{1-a^{n+1}}{1-a} \right] u[n].$$

$$y[n] = y_1[n] - y_1(n-10)$$

$$= \frac{1}{1-a}\left[a^n (1-a^{n+1}) u(n) - a^{n-10}(1-a^{n-9}) u(n-10) \right]$$

$$y[n] = \frac{1}{1-a}\left[a^n (1-a^{n+1})\, u(n) - a^{n-10}(1-a^{n-9})\, u(n-10) \right]$$

2.37 Determine the response of the relaxed system characterized by the impulse response $h[n] = \left(\frac{1}{2}\right)^n u[n]$ to the input signal

$$x[n] = \begin{cases} 1, & 0 \le n < 10 \\ 0, & \text{otherwise}. \end{cases}$$

A) $h[n] = \left(\frac{1}{2}\right)^n u[n]$

$$y[n] = \sum_{k=0}^{\infty} x[k]\, h(n-k)$$

we can use result in 2.36 by substituting $a = \frac{1}{2}$.

$$y[n] = \frac{2}{2^n}\left[\left(1 - \left(\frac{1}{2}\right)^{n+1}\right) u(n) \right] - \left(\frac{1}{2}\right)^{n-10} \times \frac{2}{2^{n-10}} \left(1 - \left(\frac{1}{2}\right)^{n-9}\right) u(n-10)].$$

2.38) Determine the response of the system characterized by the impulse Response $h[n] = \left(\frac{1}{2}\right)^n u[n]$.
to the i/p signals

a) $x[n] = 2^n u[n]$

b) $x[n] = u[-n]$

a) a)
$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$
$$= \sum_{k=0}^{n} \left(\frac{1}{2}\right)^k 2^{n-k}.$$
$$= 2^n \sum_{k=0}^{n} 2^{-k} 2^{-k}$$
$$= 2^n \sum_{k=0}^{n} \left(\frac{1}{4}\right)^k.$$
$$= 2^n \sum_{k=0}^{n} \left[\frac{1-\left(\frac{1}{4}\right)^{n+1}}{\frac{3}{4}}\right]$$
$$= 2^n \left[1-\left(\frac{1}{4}\right)^{n+1}\right]\frac{4}{3}$$
$$= \frac{2}{3}\left[2^{n+1} - \frac{1}{2^{n+1}}\right] u[n].$$

b) $x[n] = u[-n]$
$$y[n] = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$
$$= \sum_{k=0}^{\infty} h(k)$$
$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{1-\frac{1}{2}} = 2;\ n<0.$$
$$y[n] = \sum_{k=n}^{\infty} h(k)$$
$$= \sum_{k=n}^{\infty} \left(\frac{1}{2}\right)^k = \sum_{k=0}^{\infty}\left(\frac{1}{2}\right)^k - \sum_{k=0}^{n-1}\left(\frac{1}{2}\right)^k$$
$$2 - \left(\frac{1-\left(\frac{1}{2}\right)^n}{\ldots}\right)$$

$$y[n] = 2\left(\frac{1}{2}\right)^n ; \quad n \geq 0.$$

2.39) Three systems with impulse Responses $h_1[n] = \delta[n] - \delta[n-1]$, $h_2[n] = h[n]$ and $h_3[n] = u[n]$ are connected in Cascade.

a) what is the impulse responce $h_e(n)$ of the the overall system.

b) Does the order of interconnection affect the overall system?

A) $h_e(n) = h_1(n) * h_2(n) * h_3(n)$ $\because$ Cascaded

$$= (\delta[n] - \delta[n-1]) * u[n] * h[n]$$

$$= (\delta[n] * u[n] - \delta[n-1] * u[n-1]) * h[n]$$

$$= [u(n) - u(n-1)] * h[n]$$

$$= \delta[n] * h[n]$$

$$= \delta[n].$$

The order of interconnection doesn't alter the overall system.

2.40) a) Prove and explain graphically the difference between the relations

$$x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0] \text{ and}$$

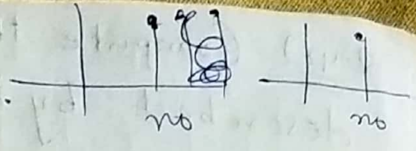
$$x[n] * \delta[n-n_0] = x[n-n_0].$$

b) show that a discrete time system which is described by a Convolution summation formula is LTI and relaxed.

c) what is the impulse response of the system described by $y[n] = x[n-n_0]$?

a) $x[n]\,\delta[n-n_0] = x[n_0]\,\delta[n-n_0]$

only $x[n]$ value is at $n=n_0$.



$x[n] * \delta[n-n_0] = x[n-n_0]$

it is the shift version of the sequence $x[n]$.

b) $$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

$$= h[n] * x[n]$$

linearity

$$x_1[n] \longrightarrow y_1[n]$$

$$y_1[n] = h[n] * x_1[n]$$

$$x_2[n] \longrightarrow y_2[n]$$

$$y_2[n] = h[n] * x_2[n]$$

$$x_b[n] = \alpha x_1[n] + \beta x_2[n] \longrightarrow y[n]$$

$$= h[n] * (\alpha x_1[n] + \beta x_2[n])$$

$$= \alpha h[n] * x_1[n] + \beta h[n] * x_2[n]$$

$$= \alpha y_1[n] + \beta y_2[n]$$

$\alpha y_1[n] + \beta y_2[n] =$

so it's linear

Time invariance:

$$x[n] \longrightarrow y[n] = h[n] * x[n]$$

$$x[n-n_0] \longrightarrow y_1[n] = h[n] * x[n-n_0]$$

$$= \sum_k h[k]\,x[n-n_0-k]$$

$$x[n-n_0] = y[n-n_0]$$

Time invariant so it's an LTI system.

c) Impulse Response is

$$y[n] = x[n-n_0]$$

$$h[n] = \delta[n-n_0].$$

2.42) Compute the zerostate Response of the system described by the difference equation

$$y[n] + \frac{1}{2} y[n-1] = x[n] + 2x[n-2]$$ to the

$$x[n] = \{1, 2, 3, 4, 2, 1\}$$ By solving equation recursively.

$\uparrow$ -2 -1

A)

$$y[n] = -\frac{1}{2} y[n-1] + x[n] + 2x[n-2]$$

$$y[-2] = -\frac{1}{2} y[-3] + x[-2] + 2x[-4]$$

$$= 1$$

$$y[-1] = -\frac{1}{2} y[-2] + x[-1] + 2x[-3]$$

$$= -\frac{1}{2}(1) + 2 + 2(0) = \frac{3}{2}$$

$$y[0] = -\frac{1}{2} y(-1) + x(0) + 2x(-1)$$

$$= -\frac{1}{2}\left(\frac{3}{2}\right) + 3 + 2(2) = \frac{1\cancel{3}}{4}$$

$$y[1] = -\frac{1}{2} y(0) + x(1) + 2x(-1) = \frac{47}{8}$$

|

|

2.45) Consider the system described by the difference equation

$$y[n] = a\, y[n-1] + b\, x[n]$$

a) Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

b) Compute the zerostate Response s[n] of the system and choose b so that s(∞) = 1

c) Compare the values of b obtained in parts (a) and (b) what did you notice.

A) $y[n] = a y[n-1] + b x[n]$

$h[n] = b a^n u[n]$.

$$\sum_{n=0}^{\infty} h(n) = \frac{b}{1-a} = 1$$

$$\underline{\underline{b = 1-a}}$$

b) $$S[n] = \sum_{k=0}^{n} h[n-k]$$

$$= b \sum_{k=0}^{n} a^{n-k} u[n-k].$$

$$= b a^n \sum_{k=0}^{n} \left(\frac{1}{a}\right)^k$$

$$= b a^n \frac{1-\left(\frac{1}{a}\right)^{n+1}}{1-\frac{1}{a}}$$

$$= b a^{n+1} \left[\frac{1-a^{-n-1}}{a-1}\right] u[n]$$

$$= b \left[\frac{a^{n+1}-1}{a-1}\right] u[n]$$

$$S(\infty) = \frac{-b}{a-1}$$

$$-b = a-1$$

$$\boxed{b = 1-a}$$

c) $b = 1-a$ in both Cases.

2.51) Compute ~~the~~ and sketch the Convolution $y_i[n]$ and Correlation ~~r~~$_i(n)$ sequences for the following pair of Signals and Comment on the results obtained.

a) $x_1[n] = \{1, 2, 4\}$ $h_1[n] = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

b) $x_2[n] = \{0, \underset{\uparrow}{1}, -2, 3, -4\}$ $h_2[n] = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, 1/2\}$

c) $x_3[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3[n] = \{\underset{\uparrow}{4}, 3, 2, 1\}$

d) $x_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_4[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$

A)

a) $y_1[n] = x[n] * h[n]$

$x[n] \backslash h[n]$	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

$y_1[n] = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

$y_1[n] = x_1[n] * h_1[-n]$

$h_1[-n] = \{1, 1, 1, 1, \underset{\uparrow}{1}\}$

$x[n] \backslash h[-n]$	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

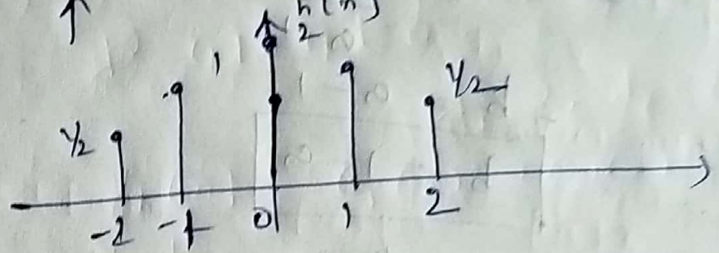
$y_2[n] = \{1, 3, 7, 7, \underset{\uparrow}{7}, \ldots$

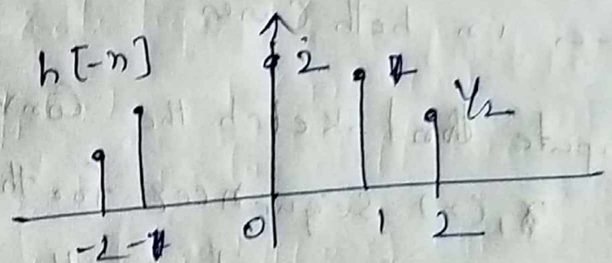
b) $y_2[n] = x_2[n] * h_2[n]$

$x[n] \backslash h[n]$	1/2	1	2	1	1/2
0	0	0	0	0	0
1	1/2	1	2	1	1/2
-2	-1	-2	-4	-2	-1
3	3/2	3	6	3	3/2
-4	-2	-4	-8	-4	-2

$y_2[n] = \{0, \frac{1}{2}, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -\frac{5}{2}, -2\}$

$h[n] =$





$h[n] = h[-n]$ so $y_2[n] = x[n] * h[-n]$

$= x[n] * h[n]$

$y_2[n] = \{0, 1/2, \underset{\uparrow}{0}, 3/2, -2, 1/2, -6, -5/2, -2\}$

2.52) The zero state Response of a Causal LTI system to the input $x[n] = \{\underset{\uparrow}{1}, 3, 3, 1\}$ is $y[n] = \{\underset{\uparrow}{1}, 4, 6, 4, 1\}$

Determine it's impulse Response.

obviously the length of $h[n]$ is 2.

$$h[n] = \{h_0, h_1\}.$$

$$h_0 = 1$$

$$y[0] = x[0]h[0] \qquad y[1] = x[0]h[1] + x[1]h[0]$$

$$1 = 1 \cdot h[0] \qquad 4 = h[1] + 3h[0].$$

$$h[0] = 1 \qquad h[1] = 1$$

$$h[n] = \{1, 1\}.$$

2.54) Determine the Response $y[n], n \geq 0$ of the system described by the second-order difference eqn

$$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1] \text{ when}$$

input is $x[n] = (-1)^n u[n]$ with initial Conditions

$y(-1) = y(-2) = 0$

A)
$$y[n] - 4y[n-1] + 4y[n-2] = 0$$

$$\lambda^2 - 4\lambda + 4 = 0.$$

$$\lambda_1 = 2, \lambda_2 = 2.$$

$$y_h[n] = C_1 (2)^n + C_2 n (2)^n$$

particular solution is

$$y_p[n] = k(-1)^n u[n]$$

Substituting it into difference equation

$$k(-1)^n u[n] - 4k(-1)^{n-1} u[n-1] + 4k(-1)^{n-2} u[n-2]$$

$$= (-1)^n u[n] - (-1)^{n-1} u[n-1]$$

$n \geq 2$ to avoid vanishing of unit steps.

$n = 2.$

$$k(1+4+4) = 2 \Rightarrow k = \frac{2}{9}$$

$y[n] = y_h[n] + y_p[n]$.

$y[n] = \left[C_1 (2)^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u[n]$

from the initial conditions $y[0]$ ~~$y[-2]$~~

$y[0] = 4y[-1] - 4y[-2] + (-1)^n u[n] - (-1)^{n-1} u[n-1]$

$= 4y[-1] - 4y[-2] + 1 - 0$.

$y[0] = 1$

$y[1] = 4y[0] - 4y[-1] + (-1) - 1$

$= 4 - 2 = 2$.

$y[0] = C_1 + C_2(0) + \frac{2}{9} = 1$

$= C_1 + \frac{2}{9} = 1 \Rightarrow C_1 = \frac{7}{9}$.

$y[1] = 2C_1 + 2C_2 - \frac{2}{9} = 2$

$C_2 = 1/3$.

$y[n] = \left[\frac{7}{9} 2^n + \frac{1}{3} n 2^n + \frac{2}{9}(-1)^n\right] u[n]$.

2.55) Determine the impulse Response $h[n]$ for the system described by the second order dif equation.

$y[n] - 4y[n-1] + 4y[n-2] = x[n] - x[n-1]$.

A) $y_h[n] = C_1(2)^n + C_2 n (2)^n$.

for an impulse Response there is no particular solution.

$y[n] = 4y[n-1] - 4y[n-2] + x[n] - x[n-1]$

$= 4y[n-1] - 4y[n-2] + \delta[n] - \delta[n-1]$

$y[0] = 4y[-1] - 4y[-2] + 1$

$y[0] = 1$ by taking $y[-1]$ & $y[-2] = 0$

$y[1] = 4y[0] - 4y[-1] + 0 - 1$
$= 4(1) - 0 - 1$

$y(1) = 3$

$y[n] = y_h[n] = C_1(2)^n + C_2 n(2)^n$

$y(0) = C_1 = 1$

$C_1 = 1$

$y[1] = 2C_1 + 2C_2 = 3$

$2 + 2C_2 = 3$

$C_2 = 1/2$

$$y[n] = h[n] = \left[1 \times (2)^n + \frac{1}{2} n(2)^n \right) u[n]$$

$$= 2^n \left[1 + \left(\frac{1}{2}\right)^n \right] u[n]$$

$$h[n] = 2^n \left[1 + \frac{n}{2} \right] u[n]$$

2.56) Show that any discrete time signal $x[n]$ can be expressed as $x[n] = \sum_{k=-\infty}^{\infty} [x[k] - x[k-1]] u[n-k]$ where $u[n-k]$ is a unit step delayed by k units in time that is

$$u[n-k] = \begin{cases} 1, & n \geq k \\ 0, & \text{elsewhere} \end{cases}$$

$x[n] = x[n] * \delta[n]$ By Convolution identity.

$= x[n] * [u[n] - u[n-1]]$

$= (x[n] - x[n-1]) * u[n]$

$$x[n] = \sum_{k=-\infty}^{\infty} (x[k] - x[k-1]) u[n-k]$$

2.57) Show that the output of an LTI System can be expressed in terms of it's unit step response $s[n]$ as follows

$$y[n] = \sum_{k=-\infty} (s(k) - s(k-1)) \; x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} (x[k] - x[k-1]) s(n-k)$$

A) let $h[n]$ be the impulse response of system.

$$S[k] = \sum_{m=-\infty}^{k} h[m].$$

$$h[k] = S[k] - S[k-1]$$

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \; x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] \; x[n-k].$$

2.58) Compute the Correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for the following signal sequences.

$$x[n] = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

A) $$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x[n] x[n-l]$$

The range of Non zero Values of $r_{xx}(l)$ is defined by

$$n_0 - N \le n \le n_0 + N$$

$$n_0 - N \le n - l \le n_0 + N.$$

$$-2N \le l \le 2N$$

implies for a given shift l, the number of terms in the summation for which both $x[n]$ and $x[n-l]$ are non zero is $2N+1-|l|$ and the value of each term is 1.

$$r_{xx}(l) = \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise.} \end{cases}$$

for $r_{xy}(l)$ we have

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0-2N \le l \le n_0+2N \\ 0, & \text{otherwise.} \end{cases}$$

2.59) Determine the autocorrelation sequences of the following signals.

a) $x[n] = \{1, \underset{\uparrow}{2}, 1, 1\}$ b) $y[n] = \{1, 1, \underset{\uparrow}{2}, 1\}$

what is your Conclusion.

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\,x(n-l)$$

$$r_{xx}(-3) = x(0)\,x(3) = 1$$

$$r_{xx}(-2) = x(0)\,x(2) + x(1)\,x(3) = 3$$

$$r_{xx}(-1) = x(0)\,x(1) + x(1)\,x(2) + x(2)\,x(3) = 5$$

$$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

Also $r_{xx}(-l) = r_{xx}(l)$

$$r_{xx}(l) = \{1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1\}$$

b) $$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\,y(n-l)$$

$$r_{yy}(-3) = y(0)\,y(3) = 1.$$

$$r_{yy}(-2) = y(0)\,y(2) + y(1)\,y(3) = 2+1=3$$

$$r_{yy}(-1) = y(0)\,y(1) + y(1)\,y(2) + y(2)\,y(3) = 5$$

$$r_{yy}(0) = \sum_{n=0}^{3} y^2(n) = 7$$

$$r_{yy}(-l) = r_{yy}(l)$$

$$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}.$$

We observe that $y[n] = x[-n+3]$ which is equivalent to reversing sequence $x[n]$ this not changed the autocorrelation sequence.

2.60) what is the normalized auto correlation sequence of signal $x(n)$ given by

$$x[n] = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

A) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$

$$= \begin{cases} 2N+1-|l| & -2N \le l \le 2N \\ 0 & \text{otherwise.} \end{cases}$$

$r_{xx}(0) = 2N+1$

Therefore the normalized auto correlation is

$$P_{xx}(l) = \begin{cases} \dfrac{r_{xx}(l)}{r_{xx}(0)} = \dfrac{2N+1-|l|}{2N+1} & ; -2N \le l \le 2N \\ 0 & \text{otherwise.} \end{cases}$$

2.61 An audio sig $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coeff's. r_1 & r_2. The sig $x(t)$ recorded by a microphone close to the loudspeaker, after sampling, is -

$$x[n] = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)$$

where k_1 & k_2 are the delays of the two echos.

(a) Determine the auto correlation $r_{xx}(l)$ of the sig $x[n]$.

(b) Can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}(l)$?

(c) What happens if $r_2 = 0$

A) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$$= \sum_{n=-\infty}^{\infty} \left[s(n) + r_1 s(n-k_1) + r_2 s(n-k_2)\right] \times$$

$$\left[s(n-l) + r_1 s(n-k_1-l) + r_2 s(n-k_2-l)\right]$$

$$= \left(1 + r_1^2 + r_2^2\right) r_{ss}(l) + r_1\left(r_{ss}(l+k_1) + r_{ss}(l-k_1)\right) +$$

$$r_2\left(r_{ss}(l+k_2) + r_{ss}(l-k_2)\right)$$

$$+ r_1 r_2\left[r_{ss}(l+k_1-k_2) + r_{ss}(l+k_2-k_1)\right]$$

b) $r_{yy}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1 + k_2)$. Suppose that $k_1 < k_2$ then we can determine r_1 and k_1. The problem is to determine r_2 and k_2 from the other peaks.

c) if $r_2 = 0$, the peak occur at $l = 0$ and $l = \pm k_1$. Then it is easy to obtain r_1 and k_1.