Chapter-4
Problems

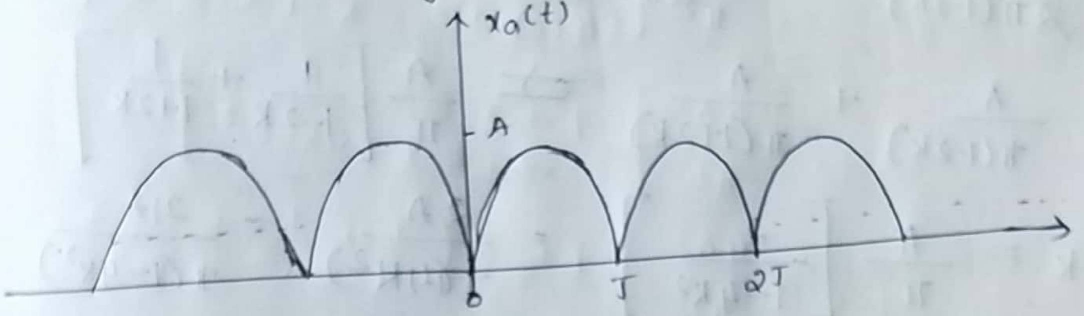
1) Consider the full-wave rectified sinusoid in fig P4.1

a) Determine it's spectrum $X_a[F]$

b) Compute the power of the signal.

c) plot the power spectral density

d) check the validity of parseval's Relation.



A) $x_a(t) = A\sin\left(\frac{\pi t}{T}\right)$

Since $x_a(t)$ is a periodic signal $X_a(F)$ can be determined from the Fourier series expansion of $x_a(t)$.

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi k t}{T}}$$

$$c_k = \frac{1}{T}\int_{-T/2}^{T/2} x_a(t)\, e^{-j\frac{2\pi k t}{T}}\, dt$$

$$= \frac{1}{T}\int_{0}^{T} A\sin\left(\frac{\pi t}{T}\right) e^{-j\frac{2\pi k t}{T}}\, dt$$

$$= \frac{A}{T}\int_{0}^{T}\left[\frac{e^{\frac{j\pi t}{T}} - e^{-\frac{j\pi t}{T}}}{2j}\right] e^{-j\frac{2\pi k t}{T}}\, dt$$

$$= \frac{A}{2jT}\int_{0}^{T}\left(e^{\frac{j\pi t}{T}} \cdot e^{-j\frac{2\pi k t}{T}} - e^{-\frac{j\pi t}{T}} \cdot e^{-\frac{j2\pi k t}{T}}\right)dt$$

$$= \frac{A}{2jT}\int_{0}^{T} e^{\frac{j\pi t}{T}(1-2k)}\, dt - \frac{A}{2jT}\int_{0}^{T} e^{-\frac{j\pi t}{T}(1+2k)}\, dt$$

$$= \frac{A}{2jT} \times \left[\frac{e^{\frac{j\pi t}{T}(1-2k)}}{\frac{j\pi}{T}(1-2k)} \right]_0^T - \frac{A}{2jT} \left[\frac{e^{-\frac{j\pi t}{T}(1+2k)}}{-\frac{j\pi t}{T}(1+2k)} \right]_0^T$$

$$= \frac{A}{2jT} \times \frac{T}{\pi j(1-2k)} \left[e^{j\pi(1-2k)} - 1 \right] + \frac{A}{2jT} \times \frac{T}{j\pi t} \left[e^{-j\pi(1-2k)} - 1 \right]$$

$$= \frac{-A}{2\pi(1-2k)} \left[-1 - 1 \right] + \left(\frac{-A}{2\pi(1+2k)} \right) \left[-1 - 1 \right]$$

$$= \frac{A}{\cancel{2}\pi(1-2k)} (\cancel{2}) + \frac{A}{\cancel{2}\pi(1+2k)} (\cancel{2})$$

$$= \frac{A}{\pi(1-2k)} + \frac{A}{\pi(1+2k)} \Rightarrow \frac{A}{\pi} \left[\frac{1}{1-2k} + \frac{1}{1+2k} \right]$$

$$c_k = \frac{A}{\pi} \left[\frac{2}{1-4k^2} \right] = \frac{2A}{\pi(1-4k^2)} = \frac{2A}{\pi(1-4k^2)}$$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi Ft}\, dt$$

$$x_a(t) = \sum_{k=-\infty}^{\infty} c_k\, e^{+j\frac{2\pi kt}{T}}$$

$$= \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_k\, e^{j\frac{2\pi kt}{T}} \cdot e^{-j2\pi Ft}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k \int_{-\infty}^{\infty} e^{-j\left(F-\frac{k}{T}\right)2\pi t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k \left[\underbrace{\int_{-\infty}^{\infty} e^{j\frac{2\pi k}{T}t} \cdot e^{-j2\pi Ft}\, dt}_{\text{F.T of } e^{j\frac{2\pi k}{T}t}} \right]$$

$$\boxed{X_a[F] = \sum_{k=-\infty}^{\infty} c_k\, \delta\left(F - \frac{k}{T}\right)} \qquad k = 0, \pm 1, \pm 2, \ldots$$

$$e^{j2\pi \times \frac{k}{T} t} \xrightarrow{F.T} \delta\left[F - \frac{k}{T}\right]$$

b) Power $(P) = \frac{1}{T} \int_0^T x_a^2(t)\, dt$

$$= \frac{1}{T} \int_0^T A^2 \sin^2\left(\frac{\pi t}{T}\right) dt$$

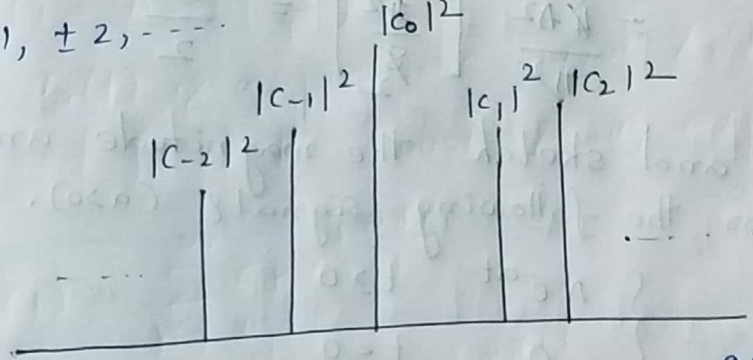
$$= \frac{A^2}{T} \int_0^T \frac{1-\cos\left(\frac{2\pi t}{T}\right)}{2}\, dt$$

$$= \frac{A^2}{2T} \int_0^T 1\, dt - \frac{A^2}{2T} \int_0^T \cancel{\cos\left(\frac{2\pi t}{T}\right) dt}^{\,0}$$

$$= \frac{A^2}{2T} \left[t\right]_0^T = \frac{A^2}{2T}\left[T\right] = \frac{A^2}{2}.$$

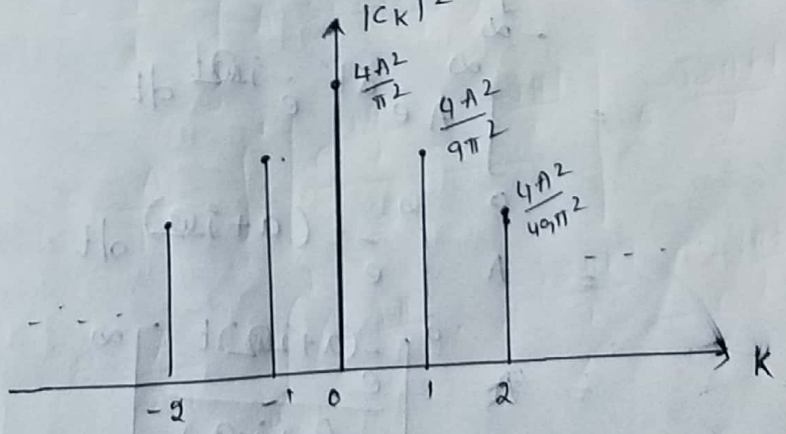
Hence Power $P = \frac{A^2}{2}$

c) power spectral density is a plot of $|c_k|^2$ versus $k = 0, \pm 1, \pm 2, \ldots$



$c_0 = \frac{2A}{\pi}$ $\quad c_1 = \frac{-2A}{3\pi}$ $\quad c_2 = \frac{-2A}{7\pi} \ldots$

$|c_0|^2 = \frac{4A^2}{\pi^2}$ $\quad |c_1|^2 = \frac{4A^2}{9\pi^2}$ $\quad |c_2|^2 = \frac{4A^2}{49\pi^2}, \ldots$



d) parsevel's Relation gives

$$\frac{1}{T}\int_0^T x_a^2(t)\,dt = \sum_{K=-\infty}^{\infty} |c_K|^2$$

$$= \sum_{K=-\infty}^{\infty} \left|\frac{2A}{\pi(1-4K^2)}\right|^2 = \sum_{K=-\infty}^{\infty} \left|\frac{2A}{\pi(1-4K^2)}\right|^2$$

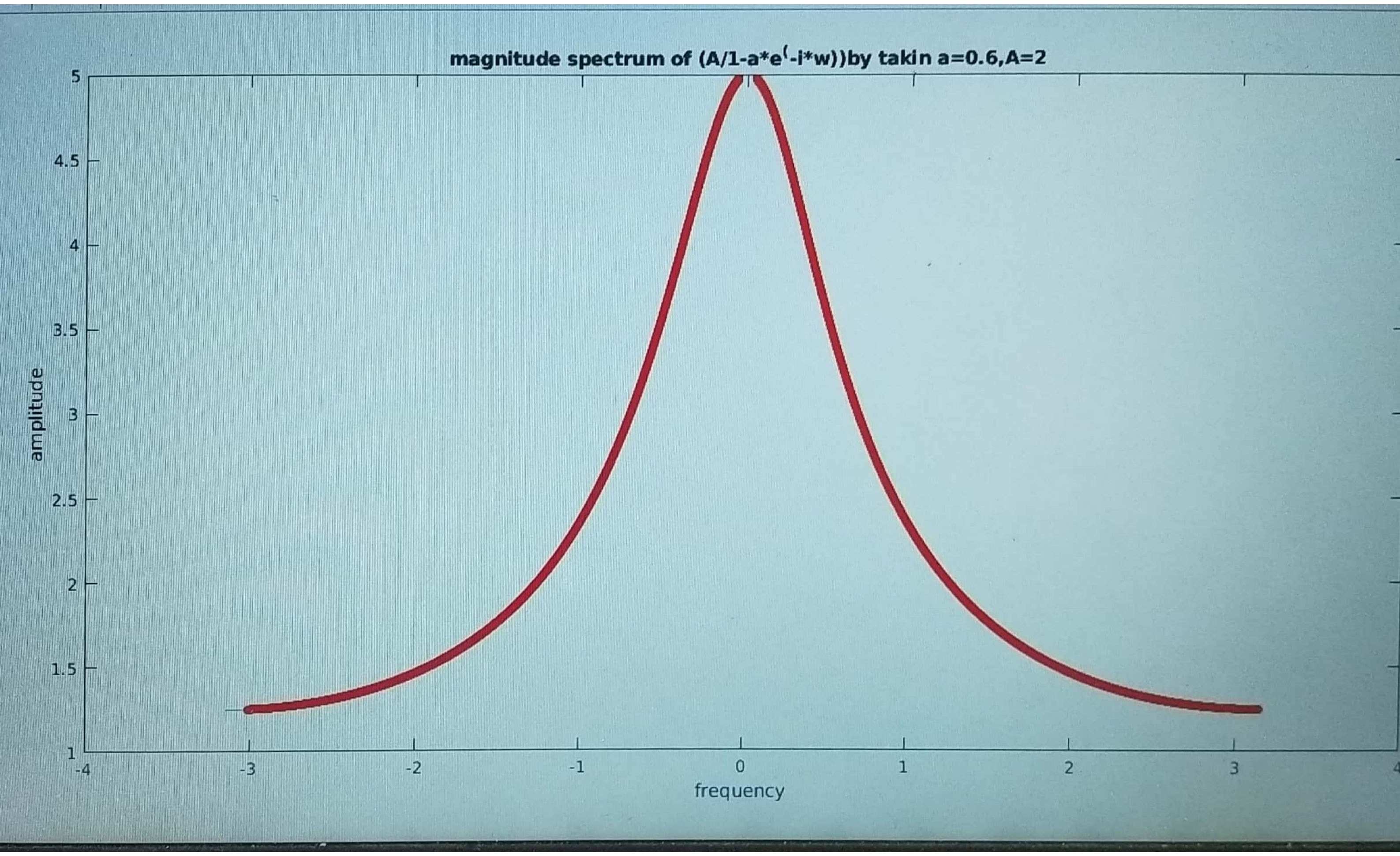
$$= \frac{4A^2}{\pi^2} \sum_{K=-\infty}^{\infty} \frac{1}{(4K^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[\frac{1}{(4(0)-1)^2} + 2\sum_{K=1}^{\infty}\frac{1}{(4K^2-1)^2}\right]$$

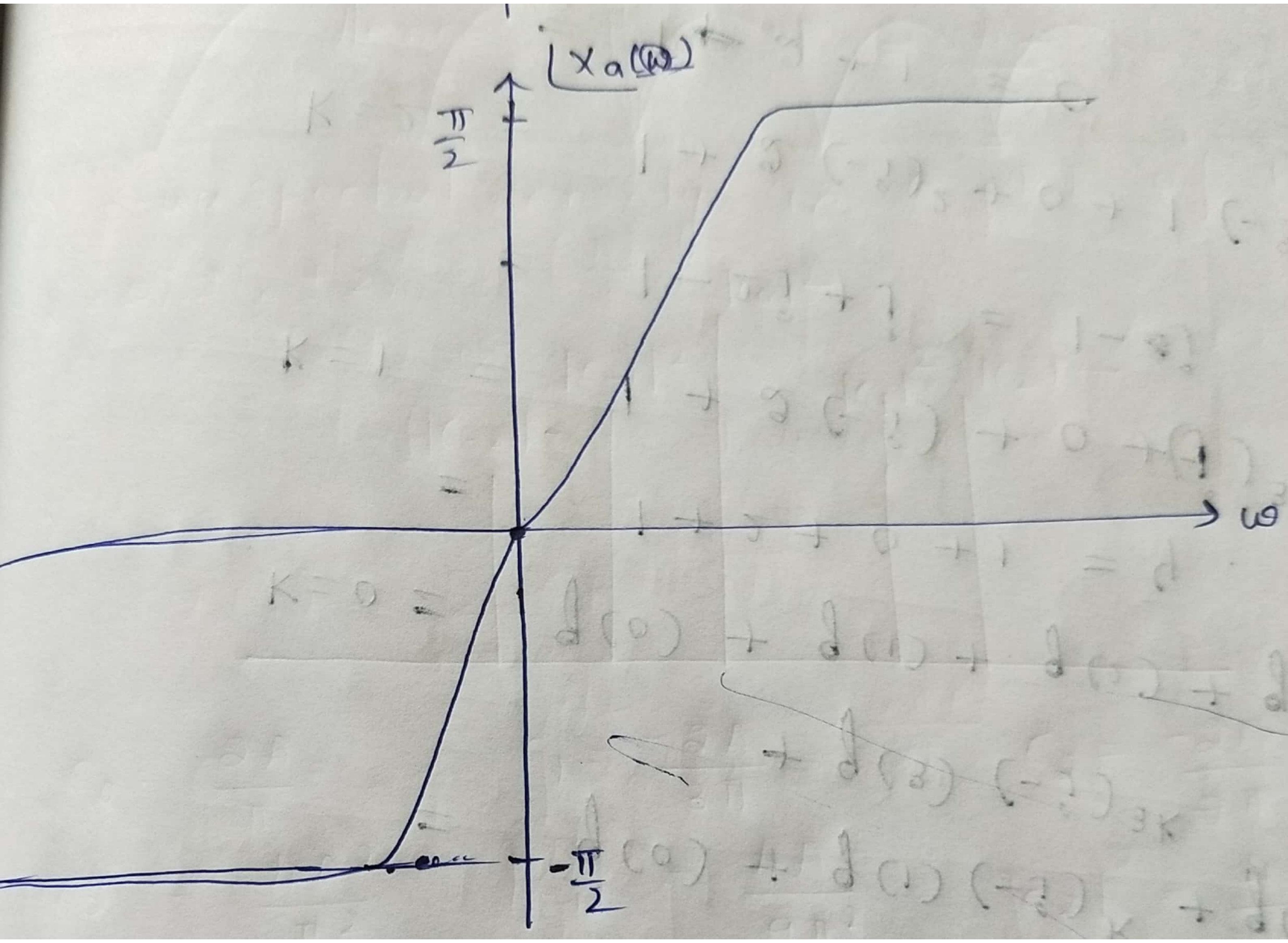
$$= \frac{4A^2}{\pi^2}\left[1 + 2\sum_{K=1}^{\infty}\frac{1}{(4K^2-1)^2}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1 + 2\left(\frac{1}{3^2} + \frac{1}{15^2} + \frac{1}{35^2} + \cdots\right)\right]$$

$$= \frac{\cancel{4}A^2}{\cancel{\pi^2}}\left[\frac{\cancel{\pi^2}}{\cancel{8}_2}\right] = \frac{A^2}{2}.$$



magnitude spectrum of (A/1-a*e^(-i*w))by takin a=0.6,A=2
amplitude
frequency
5
4.5
4
3.5
3
2.5
2
1.5
1
-4
-3
-2
-1
0
1
2
3
4



$\angle X_a(\omega)$
$\frac{\pi}{2}$
ω
$-\frac{\pi}{2}$

2) Compute and sketch the magnitude and phase spectra for the following signals $(a>0)$.

a) $x_a(t) = \begin{cases} A e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$

b) $x_a(t) = A e^{-a|t|}$

A) a) $X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j\Omega t}\, dt$, $a>0$

$$= \int_{0}^{\infty} A e^{-at} e^{-j\Omega t} dt$$

$$= A \int_{0}^{\infty} e^{-t(a+j\Omega)} dt$$

$$= A \left[\frac{e^{-(a+j\Omega)t}}{-(a+j\Omega)} \right]_0^{\infty}$$

$$X_a(\omega) = A \left[\frac{1}{a+j\Omega} - 0 \right] = \frac{A}{a+j\Omega}$$

b) $x_a(t) = A e^{-a|t|}$

$= A e^{at} u(-t) + A e^{-at} u(t)$

$X_a(\Omega) = A \int_{-\infty}^{0} e^{at} e^{-j\Omega t} dt + A \int_{0}^{\infty} e^{-at} e^{-j\Omega t} d$

$= A \int_{-\infty}^{0} e^{t(a-j\Omega)} dt + A \int_{0}^{\infty} e^{-t(a+j\Omega)}$

$$X_a(\Omega) = A\left[\frac{e^{t(a-j\Omega)}}{a-j\Omega}\right]_{-\infty}^{0} + A\left[\frac{e^{-t(a+j\Omega)}}{-(a+j\Omega)}\right]_{0}^{\infty}.$$

$$= \frac{A}{a-j\Omega} + \frac{A}{a+j\Omega}.$$

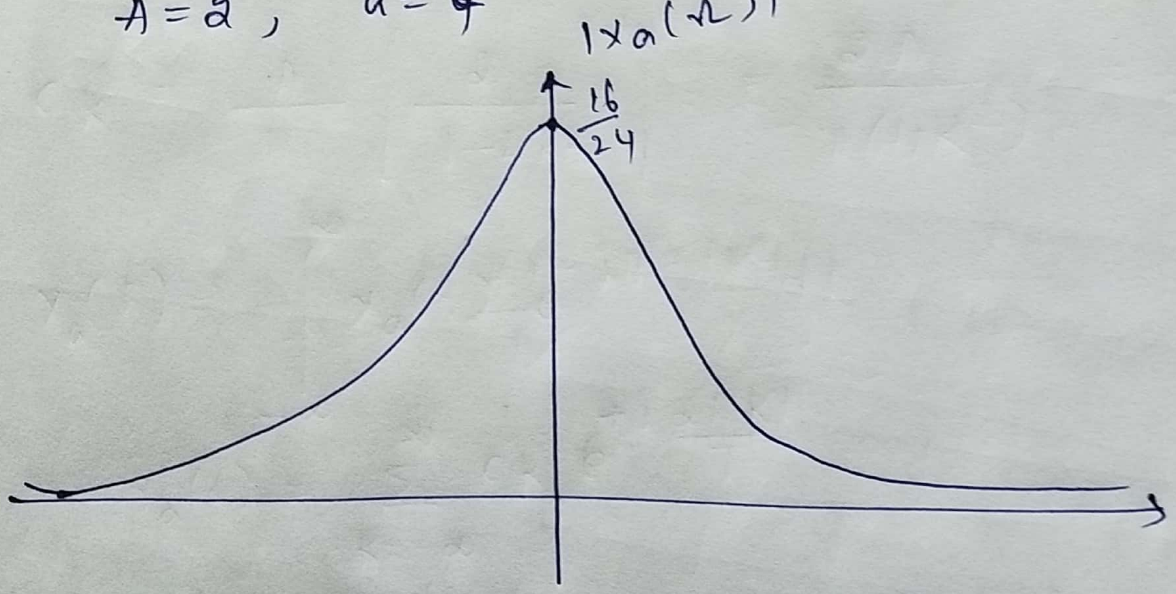
$$X_a(\Omega) = \frac{2aA}{a^2+\Omega^2}$$

$$X_a(F) = \frac{2aA}{a^2+4\pi^2F^2}$$

$$|X_a(\Omega)| = \frac{2aA}{a^2+4\pi^2F^2}$$

$\angle X_a(\Omega) = 0$ because there is no imaginary part.

$A = 2, \quad a = 4$



3) Consider the signal

$$x(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| < T \\ 0, & \text{elsewhere.} \end{cases}$$

a) Determine and sketch it's magnitude and phase spectra.

b) Create a periodic signal $x_p(t)$ with fundamental time period $T_p \geq 2T$. so that $x(t) = x_p(t)$ for $|t| < \frac{T_p}{2}$. what are Fourier coefficients c_k for the signal $x_p(t)$?

c) using results in part (a) and part (b) show that

$$c_k = \frac{1}{T_p} X_a\left(\frac{k}{T_p}\right)$$

A)

$$x(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \quad ; \quad -T \leq t \leq T \\ 0, & \text{otherwise} \end{cases}$$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi F t}\, dt.$$

Then it ...

Let $y(t) = x'(t) = \begin{cases} \frac{1}{T} & -T < t \leq 0 \\ -\frac{1}{T} & 0 < t \leq T \end{cases}$

$$Y(F) = \int_{-T}^{0} \frac{1}{T} e^{-j2\pi F t} dt + \int_{0}^{T} e^{-j2\pi F t} \left(\frac{-1}{T}\right) dt$$

$$= \frac{1}{T} \left[\frac{e^{-j2\pi F t}}{-j2\pi F} \right]_{-T}^{0} - \frac{1}{T} \left[\frac{e^{-j2\pi F t}}{-j2\pi F} \right]_{0}^{T}$$

$$= \frac{1}{T} \frac{\left[e^{j2\pi F T} - 1 \right]}{+j2\pi F} + \frac{1}{T} \frac{\left[e^{-j2\pi F T} - 1 \right]}{j2\pi F}$$

$$= \frac{1}{j2\pi F T} \left[e^{j2\pi F T} + e^{-j2\pi F T} \right] - \frac{\cancel{2}}{\cancel{2}j2\pi F T}$$

$$= \cancel{\frac{\sin(2\pi F T)}{\pi F T}}$$

$$= \cancel{\frac{\sin(2\pi F T)}{\pi F T}}$$

$$= \frac{1}{j\pi F T} \left(\cos(2\pi F T) \right) - \frac{1}{j\pi F T}$$

$$= \frac{1}{j\pi F T} \left(1 - 2\sin^2(\pi F T) \right) - \frac{1}{j\pi F T}$$

$$Y(F) = \frac{-2\sin^2(\pi F T)}{j\pi F T}$$

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$$y(t) = x'(t) = \frac{d}{dt}x(t)$$

$$Y(F) = j2\pi F\, X(F)$$

$$X(F) = \frac{1}{j2\pi F}\, Y(F)$$

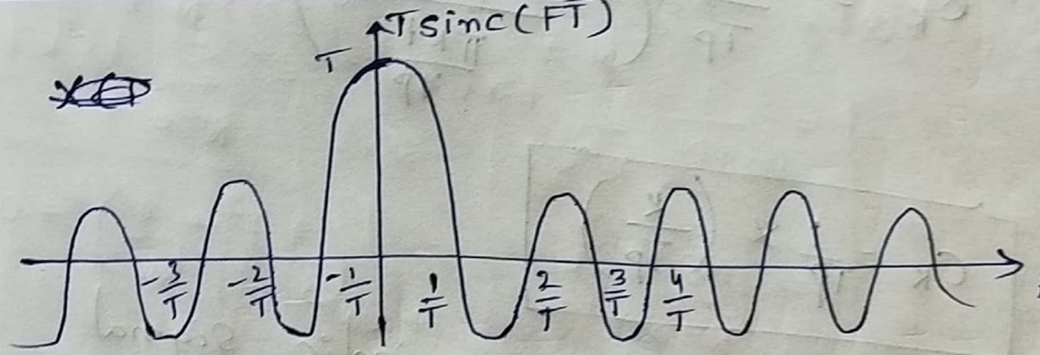
$$= \frac{1}{j\cancel{2}\pi F} \times \left(-\frac{\cancel{2}\sin^2(\pi F T)}{j\pi F T}\right)$$

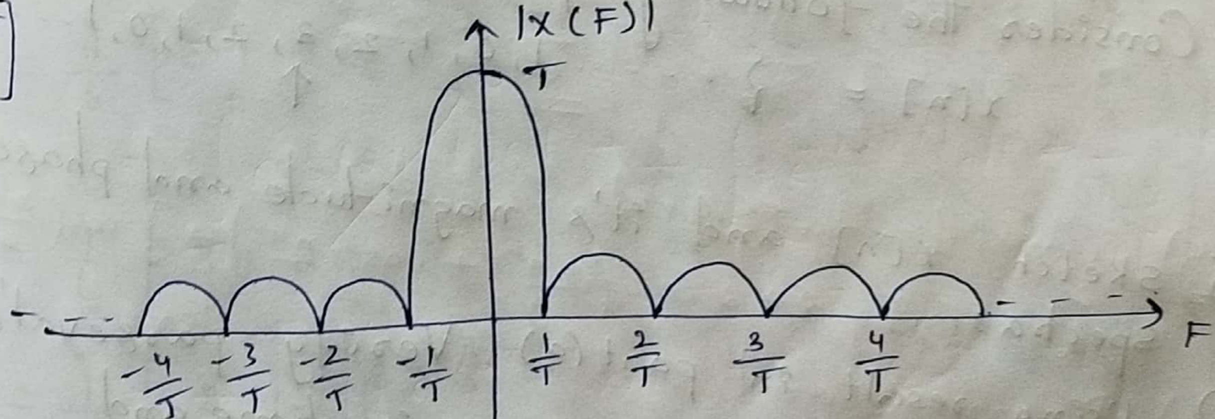
$$= \frac{\sin^2(\pi F T)}{(\pi F)^2 T}$$

$$= \frac{\sin^2(\pi F T)}{(\pi F)^2 \times T \times \frac{T}{T}} = T\left(\frac{\sin(\pi F T)}{\pi F T}\right)^2$$

$$\boxed{X(F) = T\,\mathrm{sinc}^2(FT)}$$

$$\boxed{\begin{aligned} \angle X(F) &= 0 \\ |X(F)| &= T\,\mathrm{sinc}^2(FT) \end{aligned}}$$





b) $$C_k = \frac{1}{T_p}\int_{-T_p/2}^{T_p/2} x(t)\, e^{\frac{-j2\pi kt}{T_p}}\, dt$$

$$= \frac{1}{T_p}\left[\int_{-T}^{0}\left(1+\frac{t}{T}\right) e^{\frac{-j2\pi kt}{T_p}}\, dt + \int_{0}^{T}\left(1-\frac{t}{T}\right) e^{\frac{-j2\pi kt}{T_p}}\, dt\right]$$

$$= \frac{1}{T_p}\left[\frac{1}{T}\int_{-T}^{0} t\, e^{\frac{-j2\pi kt}{T_p}}\, dt - \frac{1}{T}\int t\, e^{\frac{-j2\pi kt}{T_p}}\, dt\right]$$

$$= \frac{1}{T_p}\left[T\left(\frac{\sin\left(\frac{\pi kt}{T_p}\right)}{\frac{\pi kt}{T_p}}\right)^2\right]$$

$$= \frac{T}{T_p}\left(\frac{\sin\left(\frac{\pi kt}{T_p}\right)}{(\pi kt/T_p)}\right)^2$$

c) from a and b

$$X(F) = T\left(\frac{\sin(\pi FT)}{\pi FT}\right)^2$$

$$C_k = \frac{T}{T_p}\left(\frac{\sin\left(\frac{\pi kt}{T_p}\right)}{\frac{\pi kt}{T_p}}\right)^2$$

$$\boxed{C_k = \frac{1}{T_p} X\left(\frac{k}{T_p}\right)}$$

4) Consider the following periodic signal

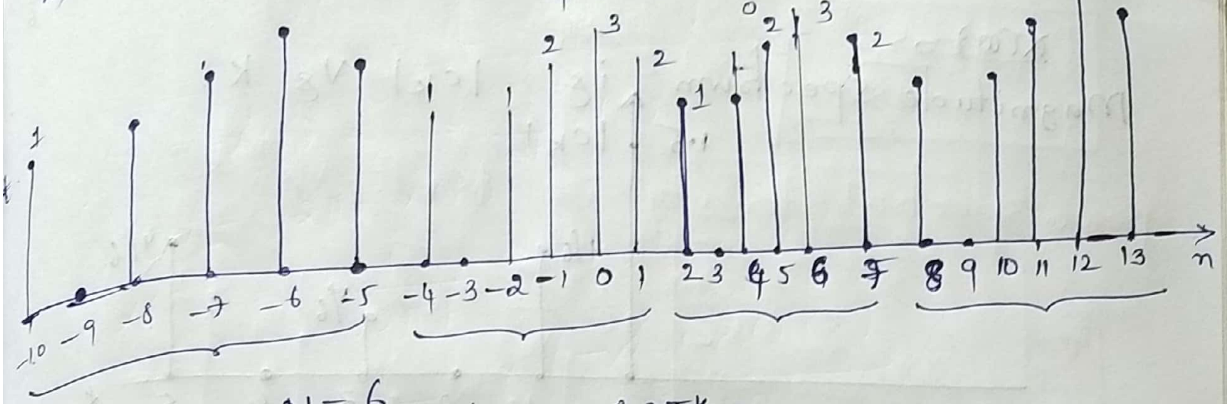
$$x[n] = \{\ldots 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1 \ldots\}$$

a) Sketch $x[n]$ and it's magnitude and phase spectra.

b) Using result in part (a) Verify parseval's Relation by Computing power in time and frequency domain.

A) $x[n] = \{ \ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots \}$

$\quad -4\ -3\ -2\ -1\ \ 0\ \ 1\ \ 2$



$$N = 6$$

$$x[n] = \sum_{k=0}^{N-1} c_k\, e^{+j\frac{2\pi k}{N} n}$$

$$c_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi k}{N} n}$$

$$c_k = \frac{1}{6} \sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi k}{6} n}$$

$$= \frac{1}{6} \sum_{n=0}^{5} x[n]\, e^{-j\frac{\pi k}{3} n}$$

$$= \frac{1}{6} \left[3\, e^{-j\frac{\pi}{3}(0)k} + 2\, e^{-j\frac{\pi}{3}k} + 1\cdot e^{-j\frac{\pi}{3}(2)k} + 0 + 1\cdot e^{-j\frac{\pi k}{3}(4)} + 2\cdot e^{-j\frac{\pi k}{3}(5)} \right]$$

$$= \frac{1}{6} \left[3 + 2\, e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}} + 2\cdot e^{-j\frac{10\pi k}{6}} \right]$$

$$= \frac{1}{6} \left[3 + 2\, e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi k}{3}} + e^{-j\frac{4\pi k}{3}} + 2\, e^{-j\frac{5\pi k}{3}} \right]$$

$$c_k = \frac{1}{6} \left[3 + 2\, e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi k}{3}} + e^{-j\left(2\pi k - \frac{2\pi k}{3}\right)} + 2\, e^{-j\left(2\pi k - \frac{\pi k}{3}\right)} \right]$$

$$= \frac{1}{6} \left[3 + 2\, e^{-j\frac{\pi k}{3}} + e^{-j\frac{2\pi k}{3}} + e^{j\frac{2\pi k}{3}} + 2\, e^{j\frac{\pi k}{3}} \right]$$

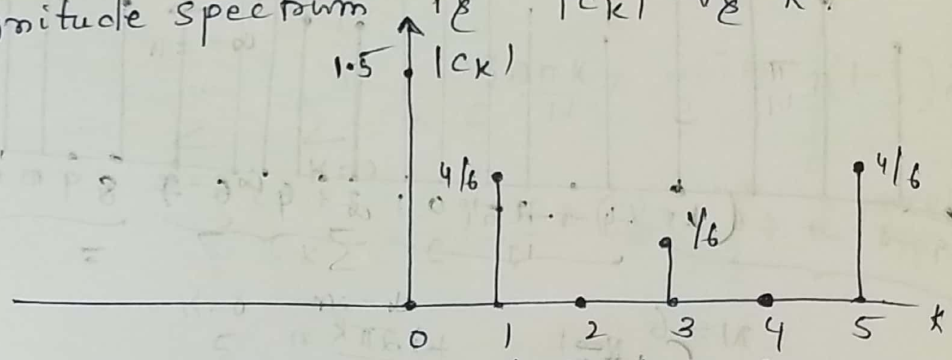
$$c_k = \frac{1}{6} \left[3 + 2\left(e^{-j\frac{\pi k}{3}} + e^{j\frac{\pi k}{3}} \right) + e^{j\frac{2\pi k}{3}} + e^{-j\frac{2\pi k}{3}} \right]$$

$$c_k = \frac{1}{6} \left[3 + 4\cos\left(\frac{\pi k}{3}\right) + 2\cos\left(\frac{2\pi k}{3}\right) \right]$$

$C_0 = \frac{9}{6}$, $C_1 = \frac{4}{6}$, $C_2 = 0$, $C_3 = \frac{1}{6}$, $C_4 = 0$, $C_5 = \frac{4}{6}$

~~X(ω)~~

Magnitude spectrum is $|C_K|$ vs K:



$\angle C_K = 0$ since we don't have imaginary value.

b) $$\text{Power } P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2$$

$$= \frac{1}{6}\sum_{n=0}^{5} |x(n)|^2$$

$$= \frac{1}{6}\left[3^2 + 2^2 + 1^2 + 0^2 + 1^2 + 2^2\right]$$

$$= \frac{1}{6}\left[9 + 4 + 1 + 1 + 4\right] = \underline{\underline{\frac{19}{6}}}.$$

Parsevals Relation says that

$$P = \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2 = \sum_{k=0}^{N-1} |C_K|^2$$

$$= \sum_{K=0}^{5} |C_K|^2 = \left[\left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0^2 + \left(\frac{1}{6}\right)^2 + \left(\frac{4}{6}\right)^2\right]$$

$$= \frac{114}{36} = \underline{\underline{\frac{19}{6}}}.$$

4.5) Consider the Signal

$$x[n] = 2 + 2\cos\left(\frac{n\pi}{4}\right) + \cos\left(\frac{n\pi}{2}\right) + \frac{1}{2}\cos\left(\frac{n\pi 3}{4}\right)$$

a) Determine and sketch it's power density spectrum.

b) Evaluate the power of the signal.

A) first we need to find out timeperiod.

$\omega_1 = \frac{\pi}{4}$, $\omega_2 = \frac{\pi}{2}$, $\omega_3 = \frac{3\pi}{4}$

$f_1 = \frac{1}{8}$, $f_2 = \frac{1}{4}$, $f_3 = \frac{3}{8}$.

$N_1 = 8$ $N_2 = 4$ $N_3 = 8$.

$N = LCM(8, 4, 8)$

$\boxed{N = 8}$.

$$x[n] = 2 + 2\left[\frac{e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}}{2}\right] + \left[\frac{e^{j\frac{\pi n}{2}} + e^{-j\frac{\pi n}{2}}}{2}\right]$$

$$+ \frac{1}{2}\left[\frac{e^{j\frac{3\pi}{4}n} + e^{-j\frac{3\pi}{4}n}}{2}\right]$$

$$= 2e^{\frac{j2\pi n(0)}{8}} + 1 \cdot e^{\frac{j2\pi n(1)}{8}} + 1 \cdot e^{\frac{-j2\pi n(1)}{8}}$$

$$+ \frac{1}{2}e^{\frac{j2\pi n(2)}{8}} + \frac{1}{2}e^{\frac{-j2\pi n(2)}{8}}$$

$$+ \frac{1}{4}e^{\frac{j2\pi n(3)}{8}} + \frac{1}{4}e^{\frac{-j2\pi n(3)}{8}}.$$

$\Rightarrow C_0 = 2$, $C_1 = 1$, $C_2 = \frac{1}{2}$, $C_3 = \frac{1}{4}$

$C_7 = C_{-1} = 1$

$C_{K+N} = C_{K+8}$

$C_6 = C_{-2+8} = C_{-2} = \frac{1}{2}$.

$C_4 = C_{-4+8} = 0$

$C_5 = C_{-3+8} = C_{-3} = \frac{1}{4}$.

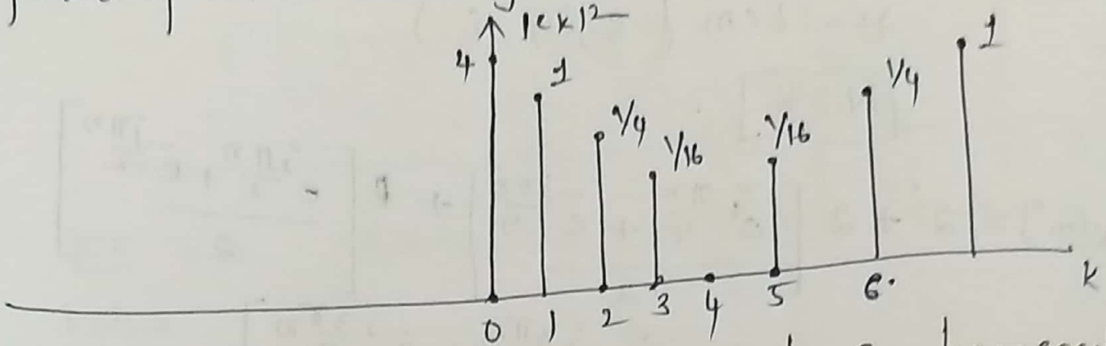
b) Power $P = \sum_{K=0}^{N-1} |C_K|^2$

$$= \sum_{K=0}^{7} |C_K|^2 = 2^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + 0^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + 1^2$$

$P = 4 + 1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + \frac{1}{4} + 1$

$$\boxed{P = \frac{53}{8}}$$

power spectral density is $|C_K|^2$ vs K.



6) Determine and sketch magnitude & phase spectra of the following periodic signals.

a) $x[n] = 4 \sin\left(\frac{\pi}{3}(n-2)\right)$

A) $x[n] = 4 \sin\left[\frac{n\pi}{3} - \frac{2\pi}{3}\right]$

$x[n] = 4 \sin\left(\frac{n\pi}{3}\right) \cos\left(\frac{2\pi}{3}\right) - 4 \cos\left(\frac{n\pi}{3}\right) \sin\left(\frac{2\pi}{3}\right)$

$= 4\left(\frac{-1}{2}\right) \sin\left(\frac{n\pi}{3}\right) - 4\left(+\frac{\sqrt{3}}{2}\right) \cos\left(\frac{n\pi}{3}\right)$

$= -2 \sin\left(\frac{n\pi}{3}\right) - 2\sqrt{3} \cos\left(\frac{n\pi}{3}\right)$

$= -\cancel{2}\left[\frac{e^{\frac{jn\pi}{3}} - e^{\frac{-jn\pi}{3}}}{\cancel{2}j}\right] - \cancel{2}\sqrt{3}\left[\frac{e^{\frac{jn\pi}{3}} + e^{\frac{-jn\pi}{3}}}{\cancel{2}}\right]$

$= e^{\frac{jn\pi}{3}}\left[\frac{-1}{j} - \sqrt{3}\right] + e^{\frac{-jn\pi}{3}}\left[\frac{1}{j} - \sqrt{3}\right]$

$= e^{\frac{j2\pi n}{6}}\left[-\sqrt{3} + j\right] + e^{\frac{-j2\pi n}{6}}\left[-\sqrt{3} - j\right]$

$C_0 = 0$, $C_1 = -\sqrt{3} + j$ $\quad |C_1| = 2$;

$C_2 = C_3 = C_4 = 0 \quad C_5 = C_{-1+6} = C_{-1} = -\sqrt{3} - j$

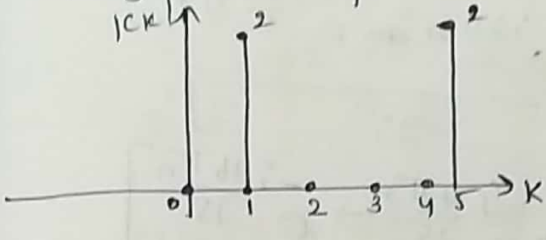
$|C_5| = 2$.

$\angle C_1 = \tan^{-1}\left(\frac{1}{-\sqrt{3}}\right) = 180 - 30 = 150^\circ = \frac{5\pi}{6}$.

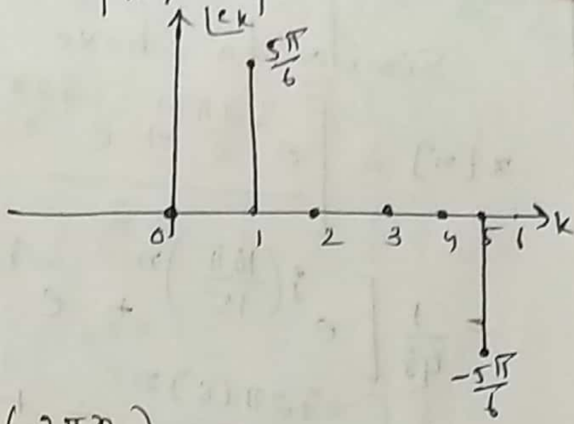
$\angle C_5 = \tan^{-1}\left(\frac{-1}{-\sqrt{3}}\right) = -\frac{5\pi}{6}$

$\angle c_2 = \angle c_3 = \angle c_4 = 0$.

Magnitude spectrum



phase spectrum



b) $x[n] = \cos\left(\frac{2\pi n}{3}\right) + \sin\left(\frac{2\pi n}{5}\right)$

A) $\omega_1 = \frac{2\pi}{3}, \quad \omega_2 = \frac{2\pi}{5}$

$f_1 = \frac{1}{3}, \quad f_2 = \frac{1}{5}$

$N_1 = 3, \quad N_2 = 5 \quad \Rightarrow \quad N = \mathrm{Lcm}(N_1, N_2)$

$= \mathrm{Lcm}(3,5)$

$\boxed{N = 15}$

$$x[n] = \frac{e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi n}{3}}}{2} + \frac{e^{j\frac{2\pi n}{5}} - e^{-j\frac{2\pi n}{5}}}{2j}$$

$$= \frac{1}{2} e^{j\frac{2\pi(5)n}{15}} + \frac{1}{2} e^{-j\frac{2\pi(5)n}{15}} + \frac{1}{2j} e^{j\frac{2\pi(3)n}{15}} - \frac{1}{2j} e^{-j\frac{2\pi(3)n}{15}}$$

$c_5 = \frac{1}{2} \qquad c_{-5} = \frac{1}{2}$

$c_{10} = c_{-5+15} = \frac{1}{2}$

$|c_5| = |c_{10}| = \frac{1}{2}$

$\angle c_5 = \angle c_{10} = 0$

$c_3 = \frac{1}{2j} \qquad c_{-3} = \frac{-1}{2j}$

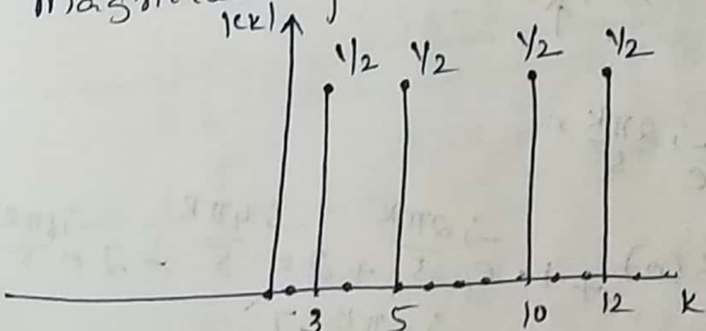
$c_{-3+15} = c_{12} = \frac{-1}{2j}$

$|c_3| = |c_{12}| = \frac{1}{2}$

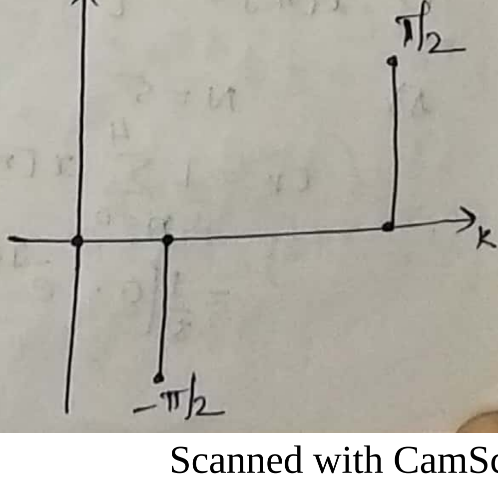
$\angle c_3 = \tan^{-1}\left(\frac{-\frac{1}{2}}{0}\right) = -\frac{\pi}{2}$

$\angle c_{12} = \tan^{-1}\left(\frac{\frac{1}{2}}{0}\right) = \frac{\pi}{2}$

Magnitude spectrum



phase spectrum.



c) $x[n] = \cos\left[\frac{2\pi}{3} n\right] \sin\left[\frac{2\pi}{5} n\right]$

Similar to above $N = 15$

$$x[n] = \left[\frac{e^{j\frac{2\pi n}{3}} + e^{-j\frac{2\pi}{3} n}}{2}\right]\left[\frac{e^{j\frac{2\pi}{5} n} - e^{-j\frac{2\pi}{5} n}}{2j}\right]$$

$$= \frac{1}{4j}\left[e^{j\left(\frac{16\pi}{15}\right)n} + e^{-j\frac{4\pi}{15} n} - e^{j\frac{4\pi}{15} n} - e^{-j\frac{16\pi}{15} n}\right]$$

$$= \frac{1}{4j} e^{\frac{j2\pi(8)n}{15}} - \frac{1}{4j} e^{-j\frac{2\pi(8)n}{15}} + \frac{1}{4j} e^{-j\frac{2\pi(2)n}{15}} - \frac{1}{4j} e^{j\frac{2\pi(2)n}{15}}$$

$C_8 = \frac{1}{4j}$ $\quad C_{-8} = \frac{-1}{4j}$

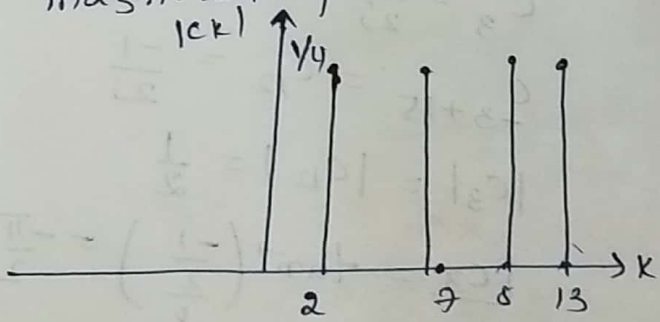
$C_7 = C_{-8+15} = C_{-8} = \frac{-1}{4j}$

$C_2 = \frac{-1}{4j}$ $\quad C_{13} = C_{-2+15} = C_{-2} = \frac{1}{4j}$.

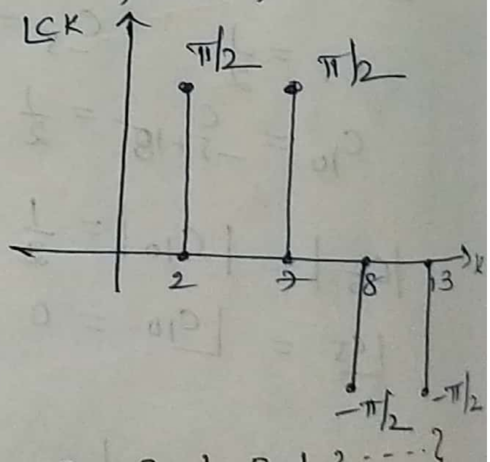
$|C_8| = |C_7| = \frac{1}{4}$ $\quad |C_2| = |C_{13}| = \frac{1}{4}$.

$\angle C_8 = -\frac{\pi}{2}$, $\angle C_7 = \frac{\pi}{2}$, $\angle C_2 = \frac{\pi}{2}$, $\angle C_{13} = -\frac{\pi}{2}$

Magnitude spectrum



phase spectrum



d) $x[n] = \{ \ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2 \ldots \}$

A) $N = 5$

$$C_K = \frac{1}{5} \sum_{n=0}^{4} x[n]\, e^{-j\frac{2\pi k}{5} n}.$$

$$= \frac{1}{5}\left[0 \cdot e^{-j\frac{2\pi k}{5}(0)} + 1 \cdot e^{-j\frac{2\pi k}{5}} + 2 e^{-j\frac{4\pi k}{5}} - 2 e^{-j\frac{6\pi k}{5}} - e^{-j\frac{8\pi k}{5}}\right]$$

$$c_k = \frac{1}{5}\left[e^{-j\frac{2\pi k}{5}} + 2e^{-j\frac{4\pi k}{5}} - 2e^{-j\frac{6\pi k}{5}} - e^{-j\frac{8\pi k}{5}}\right]$$

$$= \frac{1}{5}\left[e^{-j\frac{2\pi k}{5}} + 2e^{-j\frac{4\pi k}{5}} - 2e^{-j\left(2\pi k - \frac{4\pi k}{5}\right)} - e^{-j\left(2\pi k - \frac{8\pi k}{5}\right)}\right]$$

$$= \frac{1}{5}\left[e^{-j\frac{2\pi k}{5}} + 2e^{-j\frac{4\pi k}{5}} - 2e^{+j\frac{4\pi k}{5}} - e^{+j\frac{2\pi k}{5}}\right]$$

$$= \frac{1}{5}\left[-2j\sin\left(\frac{2\pi k}{5}\right) + 4j\sin\left(\frac{4\pi k}{5}\right)\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) + 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

$$c_0 = 0 \qquad c_1 = \frac{2j}{5}\left[-\sin\left(\frac{2\pi}{5}\right) + 2\sin\left(\frac{4\pi}{5}\right)\right]$$

$$c_2 = \frac{2j}{5}\left[-\sin\left(\frac{4\pi}{5}\right) + 2\sin\left(\frac{8\pi}{5}\right)\right]$$

$$c_3 = \frac{2j}{5}\left[-\sin\left(\frac{6\pi}{5}\right) + 2\sin\left(\frac{12\pi}{5}\right)\right]$$

$$c_4 = \frac{2j}{5}\left[-\sin\left(\frac{8\pi}{5}\right) + 2\sin\left(\frac{16\pi}{5}\right)\right]$$

e) $x[n] = \{\cdots\cdots, \underbrace{-1, 2, \underset{\uparrow}{1}, 2, -1, 0}, -1, 2, 1, 2, \cdots\cdots\}$

$N = 6$.

$2\pi - \frac{2\pi k}{6}$

$$c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-j\frac{2\pi k n}{6}}$$

$$= \frac{1}{6}\sum_{n=0}^{5} x[n]\, e^{-j\frac{2\pi k n}{6}}$$

$$= \frac{1}{6}\left[1\times e^{-j\frac{2\pi k(0)}{6}} + 2\times e^{-j\frac{2\pi k(1)}{6}} - 1\times e^{-j\frac{2\pi k(2)}{6}} + 0 - 1\times e^{-j\frac{2\pi k(4)}{6}} + 2\times e^{-j\frac{2\pi k(5)}{6}}\right]$$

$$= \frac{1}{6}\left[1 + 2e^{-j\frac{2\pi k}{6}} - e^{-j\frac{4\pi k}{6}} - e^{-j\frac{8\pi k}{6}} + 2e^{-j\frac{10\pi k}{6}}\right]$$

$$\cancel{= \frac{1}{6}\left[1 + 2e^{-j\frac{2\pi k}{3}} - e^{-j\frac{4\pi k}{6}} - e^{-j\frac{5\pi k}{3}} + 2e^{-j2\pi k}\right]}$$

$$= \frac{1}{6}\left[1 + 2e^{-j\frac{\pi k}{3}} - e^{-j\frac{2\pi k}{3}} - e^{-j\frac{4\pi k}{3}} + 2e^{-j\frac{5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 2e^{-j\frac{\pi}{3}k} - e^{-j\frac{2\pi k}{3}} - e^{-j\left(2\pi k - \frac{2\pi k}{3}\right)} + 2e^{-j\left(2\pi k - \frac{\pi}{3}k\right)}\right]$$

$$C_K = \frac{1}{6}\left[1 + 2e^{-j\frac{\pi K}{3}} - e^{-j\frac{2\pi K}{3}} - e^{j\frac{2\pi K}{3}} + 2e^{j\frac{\pi K}{3}}\right]$$

$$C_K = \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi K}{3}\right) - 2\cos\left(\frac{2\pi K}{3}\right)\right]$$

$C_0 = \frac{1}{2}$, $C_1 = \frac{2}{3}$, $C_2 = 0$, $C_3 = \frac{-5}{6}$, $C_4 = 0$, $C_5 = \frac{2}{3}$

f) $x[n] = \{\cdots 0, 0, \underset{\uparrow}{1}, \underbrace{1, 0, 0, 0,} 1, 1, 0, 0, \cdots\}$

$N = 5$

$$C_K = \frac{1}{5}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi K}{N}n}$$

$$= \frac{1}{5}\sum_{n=0}^{4} x[n]\, e^{-j\frac{2\pi K n}{5}}$$

$$= \frac{1}{5}\left[1 \times e^{-j\frac{2\pi K}{5}(0)} + 1 \times e^{-j\frac{2\pi K}{5}(1)} + 0 + 0 + 0\right]$$

$$C_K = \frac{1}{5}\left[1 + e^{-j\frac{2\pi K}{5}}\right]$$

$$C_K = \frac{1}{5}\left[1 + \cos\left(\frac{2\pi K}{5}\right) - j\sin\left(\frac{2\pi K}{5}\right)\right]$$

$C_0 = 0$, $C_1 = \frac{1}{5}\left[1 + e^{-j\frac{2\pi}{5}}\right]$, $C_2 = \frac{1}{5}\left[1 + e^{-j\frac{4\pi}{5}}\right]$

$C_3 = \frac{1}{5}\left[1 + e^{-j\frac{6\pi}{5}}\right]$, $C_4 = \frac{1}{5}\left[1 + e^{-j\frac{8\pi}{5}}\right]$

g) $x[n] = 1 \quad -\infty < n < \infty$.

$N = 1$

$$C_K = \frac{1}{N}\sum_{n=0}^{0} x[n]\, e^{-j\frac{2\pi K}{N}n}$$

$$= \sum_{n=0}^{0} x[n]\, e^{-j2\pi K n}$$

$$\boxed{C_K = x(0) = 1} \quad ; \quad C_0 = 1$$

h) $x[n] = (-1)^n \quad -\infty < n < \infty$.

$N = 2$

$$C_K = \frac{1}{2}\sum_{n=0}^{1} x[n]\, e^{-j\pi K n}$$

$$= \frac{1}{2}x(0) + \frac{1}{2}x(1)\, e^{-j\pi K}$$

$$c_K = \frac{1}{2} - \frac{1}{2} e^{-j\pi K}.$$

$$\boxed{C_k = \frac{1}{2}\left(1 - e^{-j\pi k}\right)}$$

$$\boxed{C_0 = 0, \quad c_1 = 1}$$

7) Determine the periodic signals $x[n]$, with fundamental time period $N=8$, if their FC coefficient are given by.

(a) $C_K = \cos\left(\frac{K\pi}{4}\right) + \sin\left(\frac{3\pi K}{4}\right)$

A) $x[n] = \sum_{K=0}^{N-1} C_K e^{\frac{j2\pi K n}{8}}$

$$x[n] = \sum_{K=0}^{7} C_K e^{\frac{j2\pi K n}{8}}$$

$$C_K = \frac{e^{\frac{j\pi K}{4}} + e^{-\frac{j\pi K}{4}}}{2} + \frac{e^{\frac{j3\pi K}{4}} - e^{-\frac{j3\pi K}{4}}}{2j}$$

$$C_K = \frac{1}{2}e^{\frac{j2\pi K}{8}} + \frac{1}{2}e^{-\frac{j2\pi K}{8}} + \frac{1}{2j}e^{\frac{j2\pi K(3)}{8}} - \frac{1}{2j}e^{\frac{-j2\pi K(3)}{8}}$$

$$x[n] = 4\delta[n+1] + 4\,\delta(n-1) + 4j\,\delta(n+3) - 4j\,\delta(n-3)$$

$$-3 \le n \le 5.$$

b) $C_K = \begin{cases} \sin\left(\frac{\pi K}{3}\right) & 0 \le K \le 6 \\ 0, & K = 7 \end{cases}$

$C_0 = 0,\ C_1 = \frac{\sqrt{3}}{2},\ C_2 = \frac{\sqrt{3}}{2},\ C_3 = 0,\ C_4 = -\frac{\sqrt{3}}{2},$

$C_5 = -\frac{\sqrt{3}}{2},\ C_6 = C_7 = 0.$

$$x[n] = \sum_{K=0}^{7} C_K e^{\frac{j2\pi K n}{N}}.$$

$$= \frac{\sqrt{3}}{2} \cdot e^{\frac{j2\pi n}{8}} + \frac{\sqrt{3}}{2} e^{\frac{j4\pi n}{8}} - \frac{\sqrt{3}}{2} e^{\frac{j8\pi n}{8}} - \frac{\sqrt{3}}{2} e^{\frac{j10\pi n}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j2\pi n}{8}} + e^{\frac{j4\pi n}{8}} - e^{\frac{j8\pi n}{8}} - e^{\frac{j10\pi n}{8}}\right]$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j2\pi n}{4}} - e^{\frac{j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\sin\frac{\pi n}{2} + \sin\left(\frac{\pi n}{4}\right)\right] e^{\frac{j\pi(3n-2)}{4}}$$

c) $\{c_K\} = \{ \cdots 0, \frac{1}{4}, \frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}, \frac{1}{4}, 0, \cdots \}$

A) $x[n] = \sum_{K=0}^{7} c_K e^{\frac{j2\pi K}{8} n}$

$= 2 + e^{\frac{j2\pi n}{8}} + \frac{1}{2} e^{\frac{j4\pi n}{8}} + \frac{1}{4} e^{\frac{j6\pi n}{8}} + \cancel{0}$

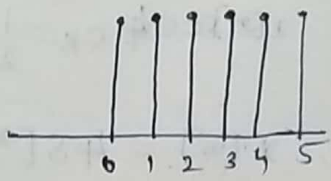
$+ e^{-\frac{j2\pi n}{8}} + \frac{1}{2} e^{-\frac{j4\pi n}{8}} + \frac{1}{4} e^{-\frac{j6\pi n}{8}}.$

$x[n] = 2 + 2\cos\left(\frac{n\pi}{4}\right) + \cos\left(\frac{n\pi}{2}\right) + \frac{1}{2}\cos\left(\frac{3n\pi}{4}\right)$

4.9) Compute the fourier transform of the following signals

a) $x[n] = u[n] - u[n-6]$

$x[n] = u[n] - u[n-6]$



$N = 6.$

$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$= \sum_{n=0}^{6} x[n] e^{-j\omega n}. \quad = \sum_{n=0}^{6} (1) e^{-j\omega n}$

$= [1 + e^{-j\omega} + e^{(j\omega)2} + \cdots + e^{-j\omega 6}]$

$= \frac{1 - r^N}{1 - r}$

$$\boxed{X(\omega) = \frac{1 - e^{-j\omega 6}}{1 - e^{-j\omega}}}$$

b) $x[n] = 2^n u[-n]$

$X(\omega) = \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$

let $-m = n$

$= \sum_{m=0}^{\infty} 2^{-m} e^{j\omega m} = \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^m.$

$n = m-4$

c) $x[n] = \left(\frac{1}{4}\right)^n u(n+4)$

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}.$$

let $m = n+4 \Rightarrow n = m-4$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m-4} e^{-j\omega(m-4)}$$

$$= \left(\sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^m e^{-j\omega m}\right) \cdot e^{j\omega 4} \cdot 4^4$$

$$\boxed{X(\omega) = \frac{4^4 e^{j\omega 4}}{1 - \frac{1}{4} e^{-j\omega}}}$$

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d) $x[n] = \alpha^n \sin(\omega_0 n)\, u(n) \qquad |\alpha|<1$

A) $X(\omega) = \sum_{n=0}^{\infty} \alpha^n \sin(\omega_0 n)\, e^{-j\omega n}$

$$= \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] e^{-j\omega n}$$

$$= \frac{1}{2j}\sum_{n=0}^{\infty} \left[\alpha e^{-j(\omega-\omega_0)}\right]^n - \frac{1}{2j}\sum_{n=0}^{\infty}\left[\alpha e^{-j(\omega+\omega_0)}\right]^n$$

$$= \frac{1}{2j}\,\frac{1}{1-\alpha e^{-j(\omega-\omega_0)}} - \frac{1}{2j}\,\frac{1}{1-\alpha e^{-j(\omega+\omega_0)}}$$

$$= \frac{1}{2j}\left[\frac{2-\alpha e^{-j(\omega-\omega_0)} - \alpha e^{-j(\omega+\omega_0)}}{(1-\alpha e^{-j(\omega-\omega_0)})(1-\alpha e^{-j(\omega+\omega_0)})}\right]$$

$$\boxed{X(\omega) = \frac{\alpha \sin\omega_0\, e^{-j\omega}}{1-2\alpha\cos\omega_0\, e^{-j\omega} + \alpha^2 e^{-j2\omega}}}$$

e) $x[n] = |\alpha|^n \sin\omega_0 n \qquad |\alpha|<1$

$x[n] = |\alpha|^n \sin(\omega_0 n)$

Note that $\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin(\omega_0 n)|$

Suppose $\omega_0 = \frac{\pi}{2}$ $\qquad |\sin(\omega_0 n)| = 1$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty.$$

So FT Doesn't exist.

f) $x[n] = \begin{cases} 2-\left(\frac{1}{2}\right)^n & |n| \le 4 \\ 0 & \text{elsewhere} \end{cases}$

$$X(\omega) = \sum_{n=-4}^{4} \left(2-\left(\tfrac{1}{2}\right)^n\right) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} 2 e^{-j\omega n} - \sum_{n=-4}^{4} \left(\tfrac{1}{2}\right)^n e^{-j\omega n}$$

$$= 2\frac{e^{j4\omega}}{1-e^{-j\omega}} - \Big[16 e^{+j\omega 4} + 8 e^{j3\omega} + 4 e^{j2\omega} + 2 e^{j\omega} + 1 + \tfrac{1}{2} e^{-j\omega} + \tfrac{1}{4} e^{-j2\omega} + \tfrac{1}{8} e^{-j3\omega} + \tfrac{1}{16} e^{-j4\omega}\Big]$$

(or)

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \sum_{m=0}^{8} \left(\tfrac{1}{2}\right)^{m-4} e^{-j\omega(m-4)}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \sum_{m=0}^{8} \left(\tfrac{1}{2}\right)^{m} e^{-j\omega m} \cdot 2^4 \cdot e^{j\omega 4}$$

$$X(\omega) = \frac{2e^{j4\omega}}{1-e^{-j\omega}} - \frac{2^4 e^{j\omega 4}}{1-\frac{1}{2}e^{-j\omega}} \left((1-e^{-j\omega 8}\right)$$

g) $x[n] = \{-2, -1, \underset{\uparrow}{0}, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-2}^{2} x[n] e^{-j\omega n}$$

$$= -2e^{j\omega 2} - 1 e^{j\omega} + 1 e^{-j\omega} + 2 e^{-j2\omega}$$

$$= -2\left(e^{j2\omega} - e^{-j2\omega}\right) - \left(e^{j\omega} - e^{-j\omega}\right)$$

$$= -4j\sin(2\omega) - 2j\sin(\omega)$$

$$X(\omega) = -2j\left(2\sin(2\omega) + \sin(\omega)\right)$$

h) $x[n] = \begin{cases} A(2M+1-|n|) & |n| \le M \\ 0 & |n| > M \end{cases}$

$$X(\omega) = A\sum_{n=-M}^{M}(2M+1-|n|)\,e^{-j\omega n}$$

$$= (2M+1)A + A\sum_{k=1}^{M}(2M+1-k)\left(e^{-j\omega k} + e^{j\omega k}\right)$$

$$X(\omega) = (2M+1)A + 2A\sum_{k=1}^{M}(2M+1-k)\cos\omega k$$

$$=$$

4.10) Determine the signals having following FT

a) $X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-\omega_0} 1\, e^{+j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{-\omega_0} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0}^{\pi}$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j(\omega_0)n} - e^{-j\pi n}}{jn}\right] + \frac{1}{2\pi}\left[\frac{e^{j\pi n} - e^{j\omega_0 n}}{jn}\right]$$

$$= \frac{1}{2\pi jn}\left[e^{-j\omega_0 n} - e^{+j\omega_0 n} - e^{-j\pi n} + e^{j\pi n}\right]$$

$$= \frac{1}{2\pi jn}\left[-\left(e^{j\omega_0 n} - e^{-j\omega_0 n}\right) + e^{j\pi n} - e^{-j\pi n}\right]$$

$$= \frac{1}{\pi n}\left(\sin(n\pi) - \sin(n\omega_0)\right)$$

$$\boxed{x[n] = -\frac{\sin(n\omega_0)}{n\pi}} \quad n \neq 0$$

$$\boxed{x[n] = \frac{1}{2\pi}\left[\pi - \omega_0\right] + \frac{1}{2\pi}\left[\pi - \omega_0\right]}; \quad n = 0$$

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b) $X(\omega) = \cos^2\omega$

$$= \left[\frac{e^{j\omega} + e^{-j\omega}}{2}\right]^2$$

$$= \frac{1}{4} e^{j2\omega} + \frac{1}{4} e^{-j2\omega} + \frac{1}{2}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{4} e^{j2\omega} e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{4} e^{-j2\omega} e^{j\omega n} d\omega$$

$$+ \frac{1}{2\pi}\int \frac{1}{2} e^{j\omega n} d\omega$$

$$= \frac{1}{4}\left[\underbrace{\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j2\omega} \cdot e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j2\omega} e^{j\omega n} d\omega}_{} + \frac{1}{2\pi}\int \frac{1}{2} \cdot e^{j\omega n} d\omega\right]$$

$$\boxed{x[n] = \frac{1}{4}\delta(n+2) + \frac{1}{4}\delta[n-2] + \frac{1}{2}\delta[n] \; ; \; n \neq 0}$$

$$x[n] = \frac{1}{\pi}\int_{-\pi}^{\pi} \cancel{\cos(2\omega)}^{\,0} + \frac{1}{4\pi}\int_{-\pi}^{\pi} d\omega \qquad n=0$$

$$= \frac{1}{4\cancel{\pi}}[2\cancel{\pi}] = \frac{1}{2} \; ; \quad n=0$$

$$\boxed{x[n] = \frac{1}{2} \; ; \; n=0}$$

c) $$X(\omega) = \begin{cases} 1 & \omega_0 - \frac{\delta\omega}{2} \leq |\omega| \leq \omega_0 + \frac{\delta\omega}{2} \\ 0 & \text{else where} \end{cases}$$

$$x[n] = \frac{1}{2\pi}\int_{-\omega_0-\frac{\delta\omega}{2}}^{-\omega_0+\frac{\delta\omega}{2}} 1 \cdot e^{j\omega n} d\omega + \frac{1}{2\pi}\int_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}} 1 \cdot e^{j\omega n} d\omega$$

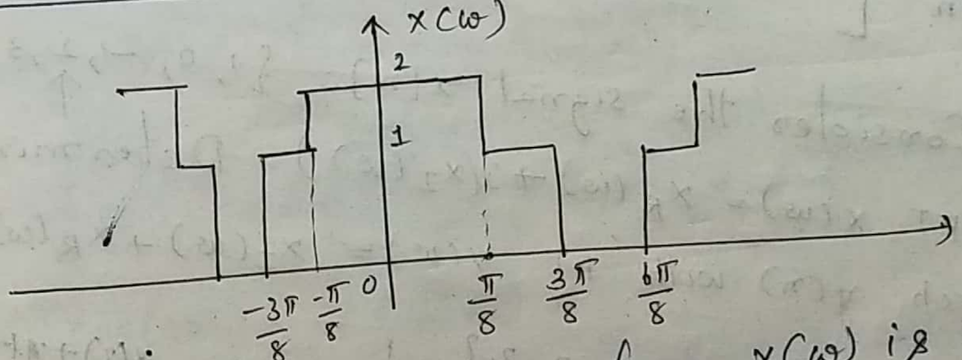
$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_0-\frac{\delta\omega}{2}}^{-\omega_0+\frac{\delta\omega}{2}} + \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0-\frac{\delta\omega}{2}}^{\omega_0+\frac{\delta\omega}{2}}$$

$$= \frac{1}{2\pi j n}\left[e^{j(-\omega_0+\cancel{\frac{\delta\omega}{2}})n} - e^{-j(\omega_0+\frac{\delta\omega}{2})n} + e^{j(\omega_0+\frac{\delta\omega}{2})} - e^{+j(\cancel{\omega_0-\frac{\delta\omega}{2}})}\right]$$

$$= \frac{\sin\left(\omega_0+\frac{\delta\omega}{2}\right)n}{\pi n} + \frac{1}{2\pi j n}\left[\;\right] - \frac{1}{n\pi}\sin\left[\omega_0-\frac{\delta\omega}{2}\right]$$

$$\boxed{x[n] = \frac{\sin\left(\omega_0+\frac{\delta\omega}{2}\right)n}{\pi n} - \frac{1}{n\pi}\sin\left[\left(\omega_0-\frac{\delta\omega}{2}\right)n\right]}$$

d)



A) Since the Fourier Transform $X(\omega)$ is even and Real signal $x[n]$ also be Real and even.

$$\text{so } x[n] = \mathrm{Re}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega\right\}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cos\omega n\, d\omega.$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi/8} 2\cos\omega n\, d\omega + \int_{\pi/8}^{3\pi/8} \cos\omega n\, d\omega + \int_{\frac{6\pi}{8}}^{\frac{7\pi}{8}} 1\cdot\cos\omega n\, d\omega + \int_{\frac{7\pi}{8}}^{\pi} 2\cos\omega n\, d\omega\right.$$

$$= \frac{1}{2\pi}\left[2\left[\frac{\sin \omega n}{n}\right]_0^{\pi/8} + \left[\frac{\sin \omega n}{n}\right]_{\pi/8}^{3\pi/8} + \left[\frac{-\sin \omega n}{n}\right]_{6\pi/8}^{7\pi/8} + 2\left[\frac{-\sin \omega n}{n}\right]_{7\pi/8}^{\pi} \right]$$

$$= \frac{1}{2\pi n}\left[-2\sin\left(\frac{\pi n}{8}\right) + \sin\left(\frac{\pi n}{8}\right) - \sin\left(\frac{3\pi n}{8}\right) + \sin\left(\frac{6\pi n}{8}\right) - \sin\left(\frac{7\pi n}{8}\right) + 2\sin\left(\frac{7\pi n}{8}\right) - 2\sin\left(\cancel{\pi n}\right)^{0} \right]$$

$$x[n] = \frac{1}{2\pi n}\left[\sin\left(\frac{7\pi n}{8}\right) + \sin\left(\frac{6\pi n}{8}\right) - \sin\left(\frac{3\pi n}{8}\right) - \sin(\ldots$$

4011) Consider the signal $x[n] = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$ with FT $X(\omega) = X_R(\omega) + j(X_I(\omega))$. Determine & sketch $y(n)$ with FT $Y(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}$

A) $x[n] = \{1, 0, -1, 2, 3\}$
$\quad\quad\quad\quad -3\ -2\ -1\ \underset{\uparrow}{0}\ 1$

$$x_e[n] = \frac{x[n] + x[-n]}{2}$$

$$x_e(3) = \frac{x(3) + x(-3)}{2} = \frac{1}{2}$$

$$x_e(2) = \frac{x(2) + x(-2)}{2} = 0$$

$$x_e(0) = \frac{x[0] + x[0]}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

$$x_e(1) = \frac{x(1) + x(-1)}{2} = \frac{3-1}{2} = 1$$

$$x_e(-3) = \frac{x(-3) + x(3)}{2} = \frac{1}{2}$$

$$x_e(-2) = \frac{x(-2) + x(2)}{2} = 0$$

$$x_e(-1) = \frac{x(-1) + x(1)}{2} = 1$$

$$x_e[n] = \{\tfrac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \tfrac{1}{2}\}$$

$$x_o[n] = \frac{x[n] - x[-n]}{2}$$

$x_o(0) = 0$, $x_o(-1) = -2$, $x_o(-2) = 0$, $x_o(1) = 2$, $x_o(2) = 0$,

$x_o(3) = \frac{1}{2}$

$$x_o[n] = \left\{ \frac{1}{2}, 0, -2, \underset{\uparrow}{0}, 2, 0, \frac{1}{2} \right\}$$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n)\, e^{-j\omega n}$$

$$= \frac{1}{2} e^{+j3\omega} + 1 \times e^{+j\omega} + 2 + 1 \times e^{-j\omega} + \frac{1}{2} e^{-j3\omega}$$

$$= \frac{1}{2}\left[e^{j3\omega} + e^{-j3\omega}\right] + 2 + e^{j\omega} + e^{-j\omega}$$

$$X_R(\omega) = \cos(3\omega) + 2 + 2\cos(\omega)$$

$$jX_I(\omega) = \sum_{n=-3}^{3} x_o(n)\, e^{-j\omega n}$$

$$= \frac{1}{2} e^{j3\omega} - 2e^{+j\omega} + 0 + 2e^{-j\omega} + \frac{1}{2} e^{-j3\omega}$$

$$= \cos(3\omega) - 4\cos(\omega)$$

$$Y(\omega) = X_I(\omega) + X_R(\omega)\, e^{j2\omega}.$$

$$y[n] = -j\, x_o[n] + x_e(n+2)$$

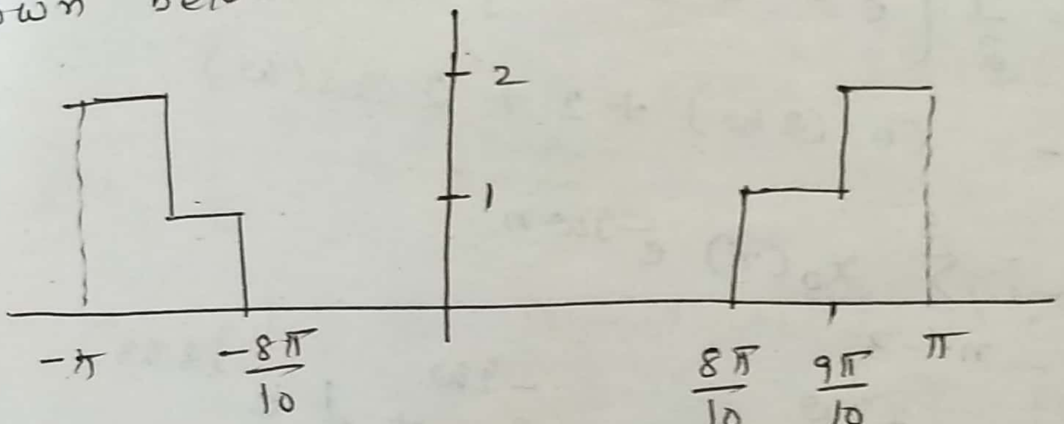
$$= -j \left\{ \frac{1}{2}, 0, -2, \underset{\uparrow}{0}, 2, 0, \frac{1}{2} \right\} + \left\{ \frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2} \right\}$$

$$= \left\{ \frac{-j}{2}, 0, +2j, \underset{\uparrow}{0}, -2j, 0, -\frac{j}{2} \right\} + \left\{ \frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2} \right\}$$

$$y[n] = \left\{ \frac{-j}{2}, 0, \frac{1}{2} - 2j, \underset{\uparrow}{0}, 1-2j, 2, 1-\frac{j}{2}, 0, \frac{1}{2} \right\}$$

4.12) Determine the signal $x[n]$ if its FT is as shown below.

a)



A) $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$.

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{9\pi}{10}} 2e^{j\omega n}\, d\omega + \int_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} e^{j\omega n}\, d\omega + \int_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} e^{j\omega n}\, d\omega + \int_{\frac{9\pi}{10}}^{\pi} 2e^{j\omega n} d\omega\right.$$

$$= \frac{1}{2\pi}\left[2\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{j\omega n}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{j\omega n}}{jn}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} + 2\left[\frac{e^{j\omega n}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right.$$

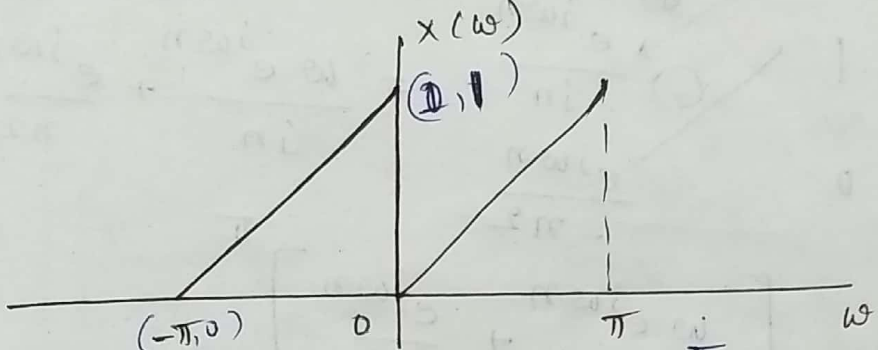
$$= \frac{1}{jn2\pi}\left[2e^{-j\frac{9\pi n}{10}} - 2e^{-j\pi n} + e^{-j\frac{8\pi n}{10}} - e^{-j\frac{9\pi n}{10}} + e^{j\frac{9\pi n}{10}} - e^{j\frac{8\pi n}{10}} + 2e^{j\pi n} - 2e^{j\frac{9\pi n}{10}}\right]$$

$$= \frac{1}{n\pi}\left[\frac{e^{-j\frac{9\pi n}{10}} - e^{j\frac{9\pi n}{10}}}{2j} + \frac{e^{-j\frac{8\pi n}{10}} - e^{j\frac{8\pi n}{10}}}{2j} + \frac{2e^{j\pi n} - 2e^{-j\pi n}}{2j}\right.$$

$$= \frac{1}{n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) - \sin\left(\frac{8\pi n}{10}\right) + 2\sin\cancel{(n\pi)}^{\,0}\right]$$

$$= \frac{-1}{n\pi}\left[\sin\left(\frac{9\pi n}{10}\right) + \sin\left(\frac{4\pi n}{5}\right)\right].$$

b)



A) $$x[n] = \frac{1}{2\pi}\left[\int_{-\pi}^{0} X(\omega)\, e^{j\omega n}\, d\omega + \int_{0}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega\right]$$

$$X(\omega) - 0 = \frac{y_2 - y_1}{x_2 - x_1}\,(\omega + \pi)$$

$$X(\omega) = \frac{1-0}{0+\pi}\,(\omega+\pi) = \frac{\omega+\pi}{\pi} = 1 + \frac{\omega}{\pi}$$

$(0,0)$ $(\pi,1)$

$$X(\omega) = \frac{1-0}{\pi-0}\,(\omega - 0)$$

$$X(\omega) = \frac{\omega}{\pi}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\, d\omega + \int_{0}^{\pi}\left(\frac{\omega}{\pi}\right) e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\omega}{\pi}\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{-\pi}^{0} 1\, e^{j\omega n}\, d\omega.$$

$$= \underbrace{\frac{1}{2\pi^2}\int_{-\pi}^{\pi}\omega\, e^{j\omega n}\, d\omega}_{I_1} + \underbrace{\frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{0}}_{I_2}$$

$$I_2 = \frac{1}{2\pi jn}\left[1 - e^{-j\pi n}\right]$$

$$I_1 = \frac{1}{2\pi^2}\int_{-\pi}^{\pi} \omega e^{j\omega n}\, d\omega .$$

$$\begin{array}{ccc} \omega & & e^{j\omega n} \\ & \oplus & \\ 1 & & \dfrac{e^{j\omega n}}{jn} \\ & \ominus & \\ 0 & & \dfrac{e^{j\omega n}}{-n^2} \end{array} \quad = \frac{\omega e^{j\omega n}}{jn} + \frac{e^{j\omega n}}{n^2}$$

$$= \frac{1}{2\pi^2}\left[\frac{\omega e^{j\omega n}}{jn} + \frac{e^{j\omega n}}{n^2}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi^2}\left[e^{j\omega n}\left[\frac{\omega}{jn} + \frac{1}{n^2}\right]\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi^2}\left[e^{j\pi n}\left[\frac{\pi}{jn} + \frac{1}{n^2}\right] - e^{-j\pi n}\left[\frac{-\pi}{jn} + \frac{1}{n^2}\right]\right]$$

$$= \frac{1}{2\pi^2}\left[\frac{\pi}{jn} e^{j\pi n} + \frac{1}{n^2} e^{j\pi n} + \frac{\pi}{jn} e^{-j\pi n} - \frac{1}{n^2} e^{-j\pi n}\right]$$

$$= \frac{1}{n\pi}\left[\cancelto{0}{\sin(n\pi)}\right] + \frac{1}{2\pi^2 n^2}\left[\cancelto{0}{e^{j\pi n} - e^{-j\pi n}}\right]$$

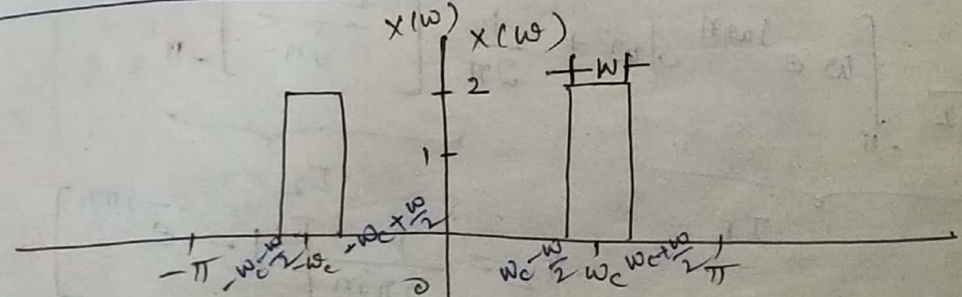
$$I_1 = 0$$

$$I_2 = \frac{1}{2\pi j n}\left[1 - e^{-j\pi n}\right]$$

$$= \frac{1}{2\pi j n}\left[e^{\frac{j\pi n}{2}} - e^{-\frac{j\pi n}{2}}\right] e^{-\frac{j\pi n}{2}}$$

$$\boxed{x[n] = \frac{1}{n\pi}\sin\left(\frac{n\pi}{2}\right) e^{-\frac{jn\pi}{2}}}$$

c)



A) $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(\omega)\, e^{j\omega n}\, d\omega.$

$= \frac{1}{2\pi}\int_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}} 2\, e^{j\omega n}\, d\omega + \frac{1}{2\pi}\int_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}} 2\, e^{j\omega n}\, d\omega.$

$= \frac{1}{\pi}\left[\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c-\frac{\omega}{2}}^{-\omega_c+\frac{\omega}{2}} + \left[\frac{e^{j\omega n}}{jn}\right]_{\omega_c-\frac{\omega}{2}}^{\omega_c+\frac{\omega}{2}}\right]$

$= \frac{1}{\pi j n}\left[e^{j(-\omega_c+\frac{\omega}{2})n} - \cancel{e^{j(\omega_c+\frac{\omega}{2})n}} + \cancel{e^{j(\omega_c+\frac{\omega}{2})n}} - e^{j(\omega_c-\frac{\omega}{2})n}\right]$

$= \frac{1}{\pi j n}\left[e^{-j(\omega_c-\frac{\omega}{2})n} - e^{j(\omega_c-\frac{\omega}{2})n}\right] + \frac{1}{\pi j n}\left[e^{j(\omega_c+\frac{\omega}{2})} - e^{-j(\omega_c+\frac{\omega}{2})}\right]$

$$\boxed{x[n] = \frac{-2}{n\pi}\left[\sin\left(\left(\omega_c-\frac{\omega}{2}\right)n\right)\right] + \frac{2}{n\pi}\sin\left(\left(\omega_c+\frac{\omega}{2}\right)n\right)}$$

$$\boxed{x[n] = \frac{2}{n\pi}\left[\sin\left(\omega_c+\frac{\omega}{2}\right)n + \sin\left(\omega_c-\frac{\omega}{2}\right)n\right]}$$

4.13) $x[n] = \begin{cases} 1, & -m \le n \le m \\ 0, & \text{otherwise} \end{cases}$ The FT of $x[n]$

is shown to be $X(\omega) = 1 + 2\sum_{n=1}^{m} \cos\omega n.$

show that th FT of

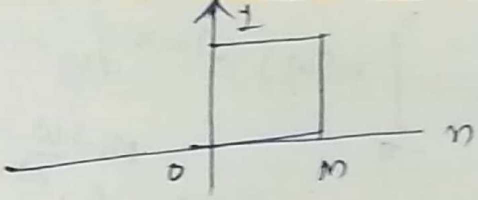
$x_1[n] = \begin{cases} 1, & 0 \le n \le m \\ 0, & \text{otherwise.} \end{cases}$

$x_2[n] = \begin{cases} 1, & -m \le n \le -1 \\ 0, & \text{otherwise.} \end{cases}$

are Respectively

$X_1(\omega) = \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}, \quad X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(m+1)}}{1-e^{j\omega}}$

A) $x_1[n] =$

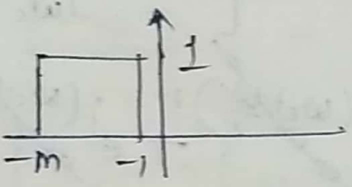


$$X_1(\omega) = \sum_{n=0}^{M} 1 \cdot e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j2\omega} + \cdots + e^{-j\omega M}.$$

$$= \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}$$

$x_2[n] =$



$$X_2(\omega) = \sum_{n=-M}^{-1} e^{-j\omega n}$$

$\Rightarrow$ let $m = -n$.

$$= \sum_{m=1}^{M} e^{j\omega m}.$$

$$= e^{j\omega} + (e^{j\omega})^2 + \cdots + e^{j\omega M}$$

$$X_2(\omega) = \frac{e^{j\omega}(1 - e^{+j\omega M})}{1 - e^{j\omega}}.$$

$$X_1(\omega) + X_2(\omega) = X(\omega)$$

$$\frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}} + \frac{e^{j\omega}(1 - e^{j\omega M})}{1 - e^{j\omega}}.$$

$$\frac{1 - e^{-j\omega M} \cdot e^{-j\omega}}{1 - e^{-j\omega}} + \frac{e^{j\omega} - e^{j\omega(M+1)}}{1 - e^{j\omega}}.$$

$$\frac{\cancel{e^{-j\omega}}(e^{j\omega} - e^{-j\omega M})}{\cancel{e^{-j\omega}}(e^{j\omega} - 1)} + \frac{\cancel{e^{j\omega}}(1 - e^{j\omega M})}{\cancel{e^{j\omega}}(e^{-j\omega} - 1)}$$

$$\frac{e^{j\omega} - e^{-j\omega m}}{e^{j\omega} - 1} + \frac{1 - e^{j\omega m}}{e^{-j\omega} - 1}$$

$$= \frac{(e^{-j\omega} - 1)(e^{j\omega} - e^{-j\omega m}) + (1 - e^{j\omega m})(e^{j\omega} - 1)}{(e^{j\omega} - 1)(e^{-j\omega} - 1)}$$

$$= \frac{\cancel{1} - e^{-j\omega(m+1)} - \cancel{e^{j\omega}} + e^{-j\omega m} + \cancel{e^{j\omega}} - \cancel{1} - e^{j\omega(m+1)} + e^{j\omega m}}{1 - e^{j\omega} - e^{-j\omega} + 1}$$

$$= \frac{-e^{-j\omega(m+1)} - e^{+j\omega(m+1)} + e^{-j\omega m} + e^{j\omega m}}{2 - e^{j\omega} - e^{-j\omega}}$$

$$X(\omega) = \frac{2\cos\omega m - 2\cos(\omega(m+1))}{2 - 2\cos\omega}$$

$$= \frac{2\cos\omega m - 2\cos\omega m\cos\omega + 2\sin(\omega m)\sin\omega}{2(1 - \cos\omega)}$$

$$= \frac{2\cos\omega m(1 - \cos\omega) + 2\sin\omega m\sin\omega}{2\left(\sin^2\frac{\omega}{2}\right)}$$

$$\cos 2\cdot\frac{\theta}{2} = 1 - 2\sin^2\frac{\theta}{2}$$

$$2\sin^2\frac{\theta}{2} = 1 - \cos 2\theta$$

$$= \frac{2\sin^2\omega/2\,\cos\omega m + 2\sin\omega m\sin\omega}{2\sin^2\frac{\omega}{2}}$$

$$= \frac{\cancel{2}\sin\left(\omega m + \frac{\omega}{2}\right)\cos\frac{\omega}{2}}{\cancel{2}\sin^2\frac{\omega}{2}}$$

$$\boxed{X(\omega) = \frac{\sin\left(m + \frac{1}{2}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}}$$

4.14) Consider the signal

$x[n] = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$ with

FT $X(\omega)$: Compute the following without explicitly Computing $X(\omega)$

a) $X(0)$ b) $\angle X(\omega)$ c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$ d) $X(\pi)$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2\, d\omega$.

A) a) $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$X(0) = \sum_{n=-2}^{2} x[n] \quad = -1+2-3+2-1$

$= -1$

b) $X(\omega) = \sum x[n] \cos\omega n + j \sum x[n] \sin\omega n$.

$\angle X(\omega) = \pi$

c) $\int_{-\pi}^{\pi} X(\omega) = ?$

$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$.

$2\pi x[0] = \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$.

$2\pi x[0] \cancel{e^{j\omega n}} = \int_{-\pi}^{\pi} X(\omega)\, d\omega$.

$= 2\pi(-3) = -6\pi$

d) $X(\pi) = \sum_{n=-2}^{2} x(n) e^{-j\pi n}$.

$= \sum_{n=-2}^{2} (-1)^n x(n)$

$= -1 - 2 - 3 - 2 - 1 \quad = -9$

e) By parseval's Relation

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega .$$

$$\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega = 2\pi \sum_{n=-\infty}^{\infty} |x(n)|^2$$

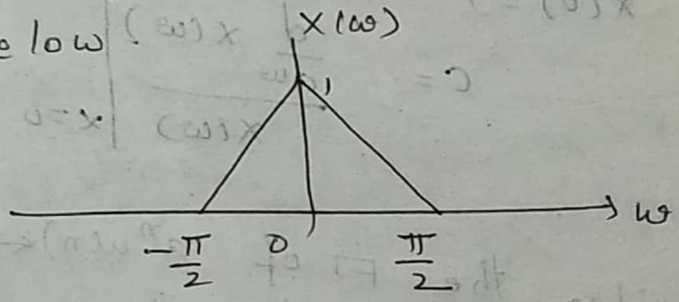
$$= 2\pi [1+4+9+4+1]$$

$$= 38\pi$$

4.15) The centre of gravity of a signal $x[n]$ is defined as $c = \dfrac{\sum_{n=-\infty}^{\infty} n\, x[n]}{\sum_{n=-\infty}^{\infty} x[n]}$ and provides a measure of timedelay of the signal.

a) Express c in terms $X(\omega)$.

b) Compute c for signal $x[n]$ whose FT is shown below



A) $$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$X(0) = \sum_{n=-\infty}^{\infty} x[n]$$

$$\frac{d X(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} x[n]\, (-jn)\, e^{-j\omega n} .$$

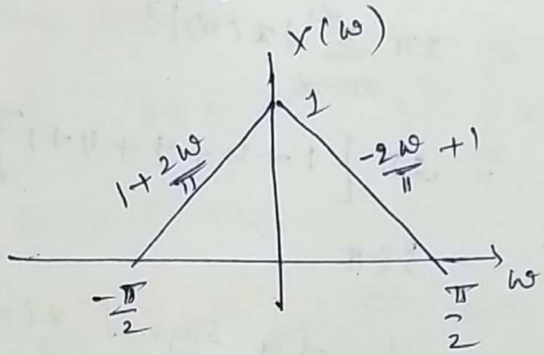
$$= -j \sum_{n=-\infty}^{\infty} n\, x[n]\, e^{-j\omega n} .$$

$\omega = 0$

$$\left.\frac{d X(\omega)}{d\omega}\right|_{\omega=0} = -j \sum_{n=-\infty}^{\infty} n\, x[n]$$

so $c = \dfrac{\sum_{n=-\infty}^{\infty} n x[n]}{\sum_{n=-\infty}^{\infty} x[n]} = \dfrac{j \frac{d}{d\omega} X(\omega)}{X(\omega)} \Bigg|_{\omega=0}$

b)



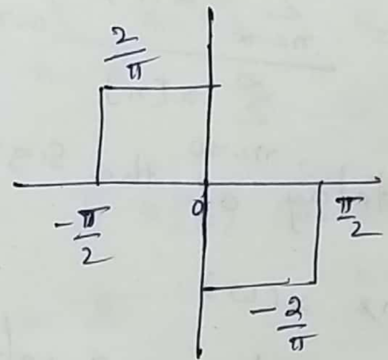
$X(\omega) = \dfrac{\omega + \frac{\pi}{2}}{\frac{\pi}{2}} = 1 + \dfrac{2\omega}{\pi}$

$(0, 1) \qquad (\frac{\pi}{2}, 0)$

$X(\omega) - 1 = \dfrac{1-0}{0-\frac{\pi}{2}} (\omega)$

$X(\omega) = -\dfrac{2\omega}{\pi} + 1$

$\dfrac{d}{d\omega}(X(\omega)) =$



$X(0) = 1$

$c = \dfrac{\frac{d}{d\omega} X(\omega)}{X(\omega)} \Bigg|_{\omega=0} = \dfrac{0}{1} = 0$

4.16) Consider the FT of $a^n u(n) \xleftrightarrow{F} \dfrac{1}{1-ae^{-j\omega}}$ $|a|<1$

use the differential in frequency theorem and induction to show that

$x[n] = \dfrac{(n+l-1)!}{n!(l-1)!} a^n u(n) \longleftrightarrow X(\omega) = \dfrac{1}{(1-ae^{-j\omega})^l}$

~~$x_{k+1}[n] =$~~ $x_k[n] = \dfrac{(n+k-1)!}{n!(k-1)!} a^n u(n) \xleftrightarrow{FT} \dfrac{1}{(1-ae^{-j\omega})^k}$

holds, then $x_{k+1}[n] = \dfrac{(n+k)!}{n!\,k!} a^n u(n)$

$$= \frac{(n+K)(n+K-1)!}{n!\, K\cancel{}\,(K-1)!} a^n u(n)$$

$$= \frac{n+K}{K} x_K(n)$$

$$X_{K+1}(\omega) = \frac{1}{K} \sum_n (n+K)\, x_K(n) e^{-j\omega n}$$

$$= \frac{1}{K} \sum_n n\, x_K(n) e^{-j\omega n} + \sum_n x_K(n) e^{-j\omega n}.$$

$$= \frac{1}{k} j \frac{d}{d\omega} X_k(\omega) + X_K(\omega)$$

$$X_{K+1}(\omega) = \frac{a e^{-j\omega}}{(1-ae^{-j\omega})^{K+1}} + \frac{1}{(1-ae^{-j\omega})^K}$$

$$X_K(\omega) = \frac{1}{(1-ae^{-j\omega})^K}.$$

4.17) Let $x[n]$ be an arbitary signal, not necessarily realvalued, with FT $X(\omega)$. express the FT of following signals in terms of $X(\omega)$.

a) $x^*(n)$ b) $x^*(-n)$ c) $y(n) = x(n) - x(n-1)$ f) $y(n) = \begin{cases} x(\frac{n}{2}) & n\,\text{even} \\ 0 & n\,\text{odd} \end{cases}$

d) $y(n) = \sum_{k=-\infty}^{n} x(k)$ e) $y(n) = x(2n)$

A)

a) $x^*(n)$

$$\sum_n x^*(n) e^{-j\omega n} = \sum_n x^*(n) \left(e^{-j(-\omega)n}\right)^*$$

$$= \left(\sum_n x(n) e^{-j(-\omega)n}\right)^*$$

$$= X^*(-\omega)$$

b) $x^*(-n)$

$$\sum_n x^*(-n) e^{-j\omega n} = \sum_n x^*(+n) e^{+j\omega n}$$

$$= X^*(\omega)$$

c) $y[n] = x[n] - x[n-1]$

$$Y(\omega) = \sum_n x(n) e^{-j\omega n} - \sum_m x(n-1) e^{-j\omega n}.$$

$$Y(\omega) = X(\omega) - \sum_n x(n-1) e^{-j\omega n}$$

Let $m = n-1$

$n = m+1$

$$= X(\omega) - \sum_m x(m) e^{-j\omega(m+1)}$$

$$= X(\omega) - X(\omega) e^{-j\omega}.$$

d) $$y(n) = \sum_{k=-\infty}^{n} x(k)$$

$$= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{n} x(k) e^{-j\omega n}.$$

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

$$= y[n] - y[n-1] = x[n]$$

$$= \left(1 - e^{-j\omega}\right) Y(\omega) = X(\omega)$$

$$\boxed{Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}}$$

e) $$Y(\omega) = \sum_n x(2n) e^{-j\omega n}$$

$m = 2n$

$$= \sum_m x(m) e^{-j\omega \frac{m}{2}}$$

$$= \sum_m x(m) e^{-j\frac{\omega}{2} m}.$$

$$= X\left(\frac{\omega}{2}\right)$$

f) $y(\omega) = \sum_{n} x\left(\frac{n}{2}\right) e^{-j\omega n}$ let $m = \frac{n}{2}$

$= \sum_{m} x(m)\, e^{-j2\omega m}$

$y(\omega) = X(2\omega)$

$=$ -

4.18) Determine and sketch FTs of $X_1(\omega), X_2(\omega)$ & $X_3(\omega)$ of the following signals.

a) $x_1[n] = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

A) $X_1(\omega) = \sum_{n=-\infty}^{\infty} x_1[n]\, e^{-j\omega n}$.

$= \sum_{n=-2}^{2} x_1[n]\, e^{-j\omega n}$

$= e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}$.

$= 1 + 2\cos(\omega) + 2\cos(2\omega)$.

b) $x_2[n] = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

A) $X_2(\omega) = \sum_{n=-\infty}^{\infty} x_2[n]\, e^{-j\omega n}$

$X_2(\omega) = \sum_{n=-4}^{4} x_2[n]\, e^{-j\omega n}$.

$= 1 e^{j4\omega} + 1 \cdot e^{j2\omega} + 1 + 1 \cdot e^{-j2\omega} + 1 \cdot e^{-j4\omega}$.

$= 1 + 2\cos(4\omega) + 2\cos(2\omega)$.

c) $x_3[n] = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

$X_3(\omega) = e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$.

$= 1 + 2\cos(6\omega) + 2\cos(3\omega)$.

d) find Relation between $X_1(\omega), X_2(\omega), X_3(\omega)$?

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d) $x_1[n] = x_2\left[\frac{2n}{2}\right]$ $\qquad x_3[n] = x_1\left[\frac{n}{3}\right]$

$x_2[n] = x_1\left[\frac{n}{2}\right]$ $\qquad X_3(\omega) = X_1(3\omega)$

$X_2(\omega) = X_1(2\omega)$

e) show that if

$$x_k[n] = \begin{cases} x\left(\frac{n}{k}\right) & \text{if } \frac{n}{k} \text{ integer} \\ 0 & \text{otherwise.} \end{cases}$$

then $X_k(\omega) = X(k\omega)$

A) $$X_k(\omega) = \sum_{n,\ \frac{n}{k}\text{ are integer}} x_k(n)\, e^{-j\omega n}.$$

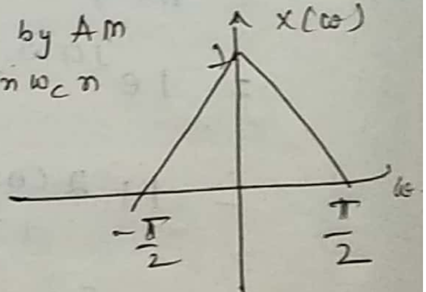
$$= \sum x\left(\frac{n}{k}\right) e^{-j\omega n}.$$

let $m = \frac{n}{k}$.

$$= \sum_m x(m)\, e^{-j(\omega k)m}.$$

$$\boxed{X_k(\omega) = X(k\omega)}$$

4.19) Let $x[n]$ be a signal with FT as shown in fig. Determine and sketch the FT of following signals. Note these are obtained by AM of carrier $\cos\omega_c n$, or $\sin\omega_c n$



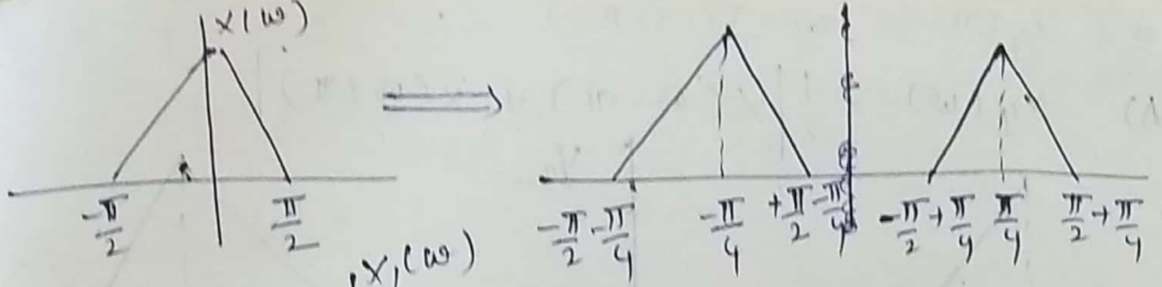
a) $x_1[n] = x(n)\cos\left(\frac{\pi n}{4}\right)$

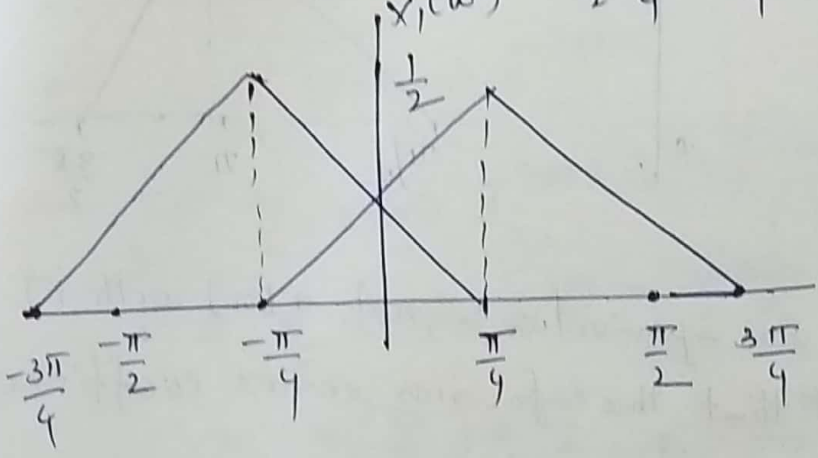
A) $x_1[n] = x[n]\cos\left[\frac{\pi n}{4}\right]$

$$= x[n]\left[\frac{e^{j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}}{2}\right]$$

$$= \frac{1}{2}x[n]\, e^{j\frac{\pi n}{4}} + \frac{1}{2}x(n)\, e^{-j\frac{\pi n}{4}}$$

$$X_1(\omega) = \frac{1}{2}X\left(\omega + \frac{\pi}{4}\right) + \frac{1}{2}X\left(\omega - \frac{\pi}{4}\right).$$





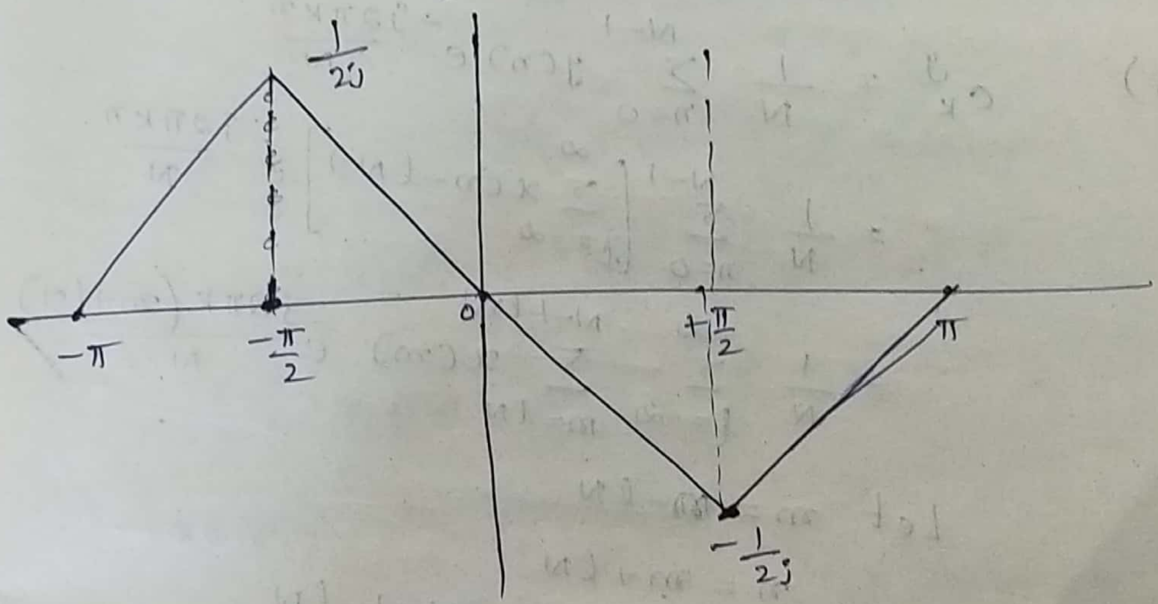
$\frac{\pi}{2} + \frac{\pi}{4}$

$\frac{2\pi + \pi}{4} = \frac{3\pi}{4}$.

b) $x_2[n] = x[n] \sin\left(\frac{\pi n}{2}\right)$

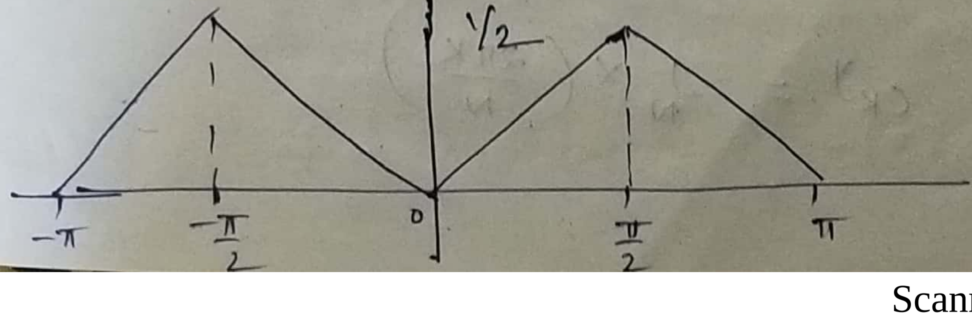
A) $X_2(\omega) = \frac{1}{2j}\left[X\left(\omega + \frac{\pi}{2}\right) - X\left(\omega - \frac{\pi}{2}\right)\right]$

$= \frac{1}{2j}\left[X\left(\omega + \frac{\pi}{2}\right)\right] - \frac{1}{2j} X\left[\omega - \frac{\pi}{2}\right]$



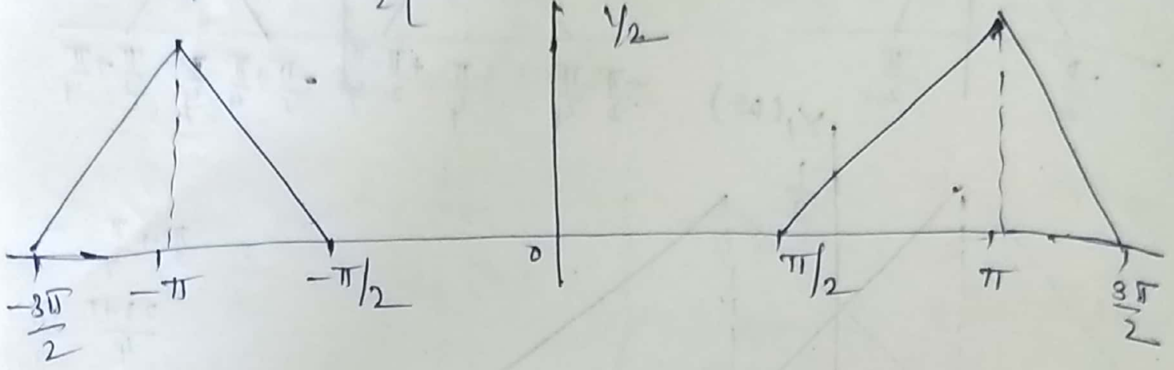
c) $x_3[n] = x[n] \cos\left(\frac{\pi n}{2}\right)$

A) $X_3[\omega] = \frac{1}{2}\left[X\left(\omega - \frac{\pi}{2}\right) + X\left(\omega + \frac{\pi}{2}\right)\right]$



d) $x_4(n) = x[n].\cos(\pi n)$

A) $X_4(\omega) = \frac{1}{2}\left[X(\omega-\pi) + X(\omega+\pi)\right]$



4.20) Consider an aperiodic signal $x[n]$ with FT $X(\omega)$. show that the fourier series coefficien C_K^y of the periodic signal

$$y[n] = \sum_{l=-\infty}^{\infty} x(n-lN) \quad \text{are given b}$$

$$C_K^y = \frac{1}{N} X\left(\frac{2\pi K}{N}\right) \quad K=0,1,---N-1$$

A) $$C_K^y = \frac{1}{N}\sum_{n=0}^{N-1} y(n) e^{-\frac{j2\pi kn}{N}}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right] e^{-\frac{j2\pi kn}{N}}.$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=lN}^{N-l+lN} x(m)\, e^{-\frac{j2\pi k(m+lN)}{N}}.$$

Let $m = n - lN$

$n = m + lN$

$m = -lN$ to $N-1-lN$.

$$X(\omega) = \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m).\, e^{-\frac{j2\pi km}{N}}.\, e^{-j2\pi kl}$$

$$C_K^y = \frac{1}{N} X\left(\frac{2\pi K}{N}\right)$$

4.21) ~~Consi~~ prove that

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c N}{\pi n} e^{-j\omega n}$$

may be expressed as

$$X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin\left((2N+1)(\omega-\theta)/2\right)}{\sin\left((\omega-\theta)/2\right)}$$

A)

$$x_N(n) = \frac{\sin \omega_c n}{\pi n} \quad ; \quad -N \le n \le N$$

$$= x(n)\, w(n)$$

$$x(n) = \frac{\sin \omega_c n}{\pi n} \quad ; \quad -\infty \le n \le \infty$$

$$w(n) = 1 \quad ; \quad -N \le n \le N$$

$$\frac{\sin \omega_c n}{\pi n} \xrightarrow{F} X(\omega)$$

$$= 1, \quad |\omega| \le \omega_c$$

$$= 0, \quad \text{otherwise.}$$

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\theta)\, W(\omega-\theta)\, d\theta$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin(2N+1)(\omega-\theta)/2}{\sin(\omega-\theta)/2}$$

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4·22) prove that $x[n]$ has the following FT

$$X(\omega) = \frac{1}{1-ae^{-j\omega}}$$ then determin the follo

a) $x(2n+1)$

A) $= \sum_n x(2n+1)\, e^{-j\omega n}$ Let $m = 2n+1$

$n = \frac{m-1}{2}$

$= \sum_n x(m)\, e^{-j\omega\left(\frac{m-1}{2}\right)}$

$= \sum_n x(m)\, e^{-j\frac{\omega}{2}m} \cdot e^{+j\frac{\omega}{2}}$

$= X\left(\frac{\omega}{2}\right) e^{\frac{j\omega}{2}} = \frac{e^{j\omega/2}}{1-ae^{-j\frac{\omega}{2}}}$

b) $e^{\frac{\pi n}{2}} x(n+2)$

$= \sum_n x(n+2)\, e^{\frac{\pi n}{2}} \cdot e^{-j\omega n}$ $K = n+2$

$= -\sum_K x(K)\, e^{\frac{n\pi}{2}} \cdot e^{-j\omega(K-2)}$

$= -\sum_K x(K)\, e^{\frac{-jn\pi}{j2}} \cdot e^{-j\omega K} \cdot e^{j2\omega}$

$= -\sum_K x(K)\, e^{-j\left(\omega K + \frac{2\pi}{2}\right)} \cdot e^{j2\omega}$

$= -X\left(\omega + \frac{j\pi}{2}\right) e^{j2\omega}$

c) $x(-2n)$

$= \sum_n x(-2n)\, e^{-j\omega n}$ let $m = -2n$

$= \sum_n x(m)\, e^{-j\omega\left(-\frac{2m}{2}\right)}$

$= \sum_n x(m)\, e^{-j\left(-\frac{\omega}{2}\right)m}$

$= X\left(-\frac{\omega}{2}\right)$

$=$

d) $x[n] \cos(0.3\pi n)$

$$= \sum_n \frac{1}{2}\left(e^{j(0.3\pi n)} + e^{-j(0.3\pi n)}\right) x(n)\, e^{-j\omega n}$$

$$= \frac{1}{2}\sum_n e^{j(0.3\pi n)} \cdot x[n]\, e^{-j\omega n} + \frac{1}{2}\sum_n e^{-j(0.3\pi n + \omega n)}\, x[n]$$

$$= \frac{1}{2}\sum_n e^{-j(\omega - 0.3\pi)n}\, x[n] + \frac{1}{2}\sum_n e^{-j[\omega + 0.3\pi]n}\, x[n]$$

$$= \frac{1}{2}\left[X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right]$$

e) $x(n) * x(n-1)$

A) Convolution in one domain $\longleftrightarrow$ Multiplication in another domain

$$x(n) * x(n-1) \xleftrightarrow{FT} X(\omega) \cdot X(\omega)\, e^{-j\omega}.$$

$$\longrightarrow X^2(\omega)\, e^{-j\omega}.$$

f) $x(n) * x(-n)$

$$x(n) * x(-n) \xrightarrow{FT} X(\omega)\, X(-\omega)$$

$$\longrightarrow \frac{1}{1 - a e^{-j\omega}} \cdot \frac{1}{1 - a e^{j\omega}}$$

$$\longrightarrow \frac{1}{1 - a e^{j\omega} - a e^{-j\omega} + a^2}$$

$$\longrightarrow \frac{1}{1 - 2a\cos\omega + a^2}$$

4.23) from a discrete-time signal $x[n]$ with FT of $X(\omega)$ as shown in fig. determine and sketch the following signals.

a) $y_1[n] = \begin{cases} x(n) & , n \text{ even} \\ 0 & \quad n \text{ odd} \end{cases}$

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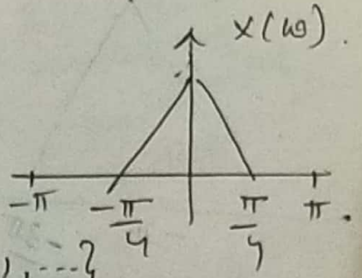
b) $y_2[n] = x(2n)$

c) $y_3(n) = \begin{cases} x(n/2), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

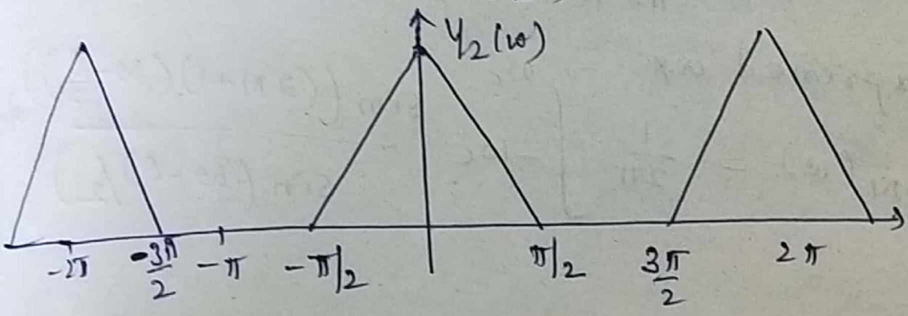


Note that $y_1[n] = x[n]\,s[n]$

where $s[n] = \{\cdots 0,1,0,1,0,1,0,1,\cdots\}$

b) $y_2[n] = x(2n)$

$$Y_2[\omega] = X\left(\frac{\omega}{2}\right)$$

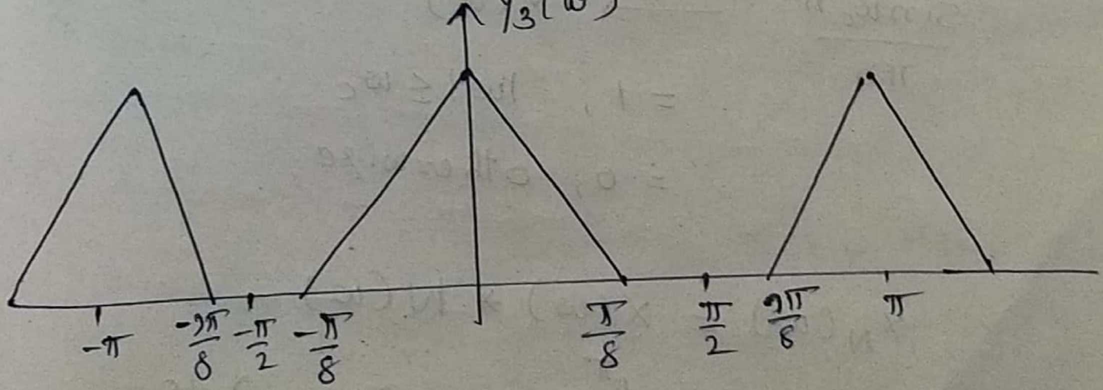


c) $y_3[n] = \begin{cases} x\left(\frac{n}{2}\right), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

$$Y_3[\omega] = \sum_n y_3(n)\, e^{-j\omega n}$$

$$= \sum_{n \text{ even}} x\left(\frac{n}{2}\right) e^{-j\omega n}$$

$$= X(2\omega)$$



a) $y_1[n] = \begin{cases} y_2\left(\frac{n}{2}\right), & n \text{ even} \\ 0, & n \text{ odd} \end{cases}$

$$Y_1(\omega) = \sum_n y_2\left(\frac{n}{2}\right) e^{-j\omega n}$$

$$= Y_2(2\omega)$$

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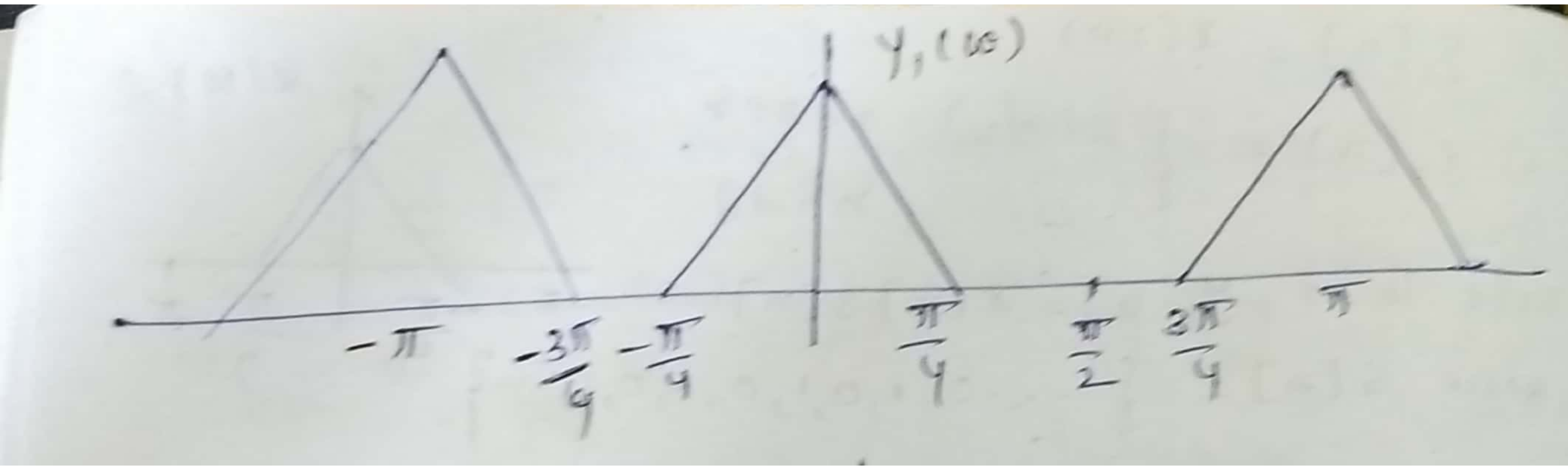
Y1(ω)
−π
−3π/4
−π/4
π/4
π/2
3π/4
π