Sin & Cos Error @ $x = 1/N$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

Approx $\sin(x) \approx x$

Error $O()$ = Sin series − Approx at $\frac{1}{N}$

$$\left(\frac{1}{N} - \frac{\left(\frac{1}{N}\right)^3}{3!} + \frac{\left(\frac{1}{N}\right)^5}{5!} - \dots\right) - \frac{1}{N} = \frac{-1}{3!\cdot N^3} + \frac{1}{5!\cdot N^5} = \text{Error}$$

Error $O() = \frac{1}{6N^3} \rightarrow \boxed{O\left(\frac{1}{N^3}\right)}$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

Approx $\cos(x) \approx 1 - \frac{x^2}{2}$

Error $O()$ at $x = 1/N$ = (series − approx.)

$$\left(1 - \frac{\left(\frac{1}{N}\right)^2}{2!} + \frac{\left(\frac{1}{N}\right)^4}{4!} - \dots\right) - \left(1 - \frac{\left(\frac{1}{N}\right)^2}{2}\right) = \frac{1}{4!\cdot N^4} \dots = \text{Error}$$

Error $O()$ @ $x = \frac{1}{N} = \frac{1}{4!} \cdot \frac{1}{N^4}$

upper bound

$$= \boxed{O\left(\frac{1}{N^4}\right)}$$