12

Application 10: Testing for Change Over Time: The Latent Growth Curve Model

Behavioral scientists have long been intrigued with the investigation of change. From a general perspective, questions of interest in such inquiry might be: "Do the rates at which children learn differ in accordance with their interest in the subject matter?" From a more specific perspective, such questions might include: "To what extent do perceptions of ability in particular school subjects change over time?" and "Does the rate at which self-perceived ability in math and/or science change differ for adolescent boys and girls?" Answers to questions of change such as these necessarily demand repeated measurements on a sample of individuals at multiple points in time. This chapter addresses these types of change-related questions.

The application demonstrated herein is based on a study by Byrne and Crombie (2003) in which self-ratings of perceived ability in math, language, and science were measured for 601 adolescents in a three-year period that targeted Grades 8, 9, and 10. In this chapter, however, we focus on subscale scores related only to the subject areas of math and science. Consistent with most longitudinal research, some subject attrition occurred during the three-year period: 101 cases were lost, thereby leaving 500 complete-data cases. In the original study, we addressed this issue of missingness by employing a multiple-sample missing-data model that involved three time-specific groups.[1] However, because the primary focus of this

[1]Group 1 ($n = 500$) represented subjects for whom complete data were available across the three-year time span; Group 2 ($n = 543$) represented subjects for whom data were available only for Years 2

chapter is to walk you through a basic understanding and application of a simple latent growth curve (LGC) model, as well as in the interest of space and expediency, the current example is based only on the group with complete data across all three time points.[2] Nonetheless, readers are urged to become familiar with the pitfalls that might be encountered when working with incomplete data in the analysis of LGC models (Duncan & Duncan, 1994, 1995; and Muthén, Kaplan, & Hollis, 1987) and to study the procedures involved in working with a missing-data model (Byrne & Crombie, 2003; Duncan & Duncan, 1994, 1995; and Duncan, Duncan, Strycker, Li, & Alpert, 1999). In EQS, this can be as simple as adding MISSING=ML (see chap. 9, for example). For an elaboration of missing-data issues in general, see Byrne (2001), Little & Rubin (1987), and Muthén et al. (1987).

Historically, researchers typically based analyses of change on two-wave panel data, a strategy that Willett and Sayer (1994) deemed inadequate because of limited information. As they noted (p. 363), "When true development follows an interesting trajectory over time, 'snapshots' of status taken before and after are unlikely to reveal the intricacies of individual change." Addressing this weakness in longitudinal research, Willett (1988) and others (Bryk & Raudenbush, 1987; Rogosa, Brandt, & Zimowski, 1982; and Rogosa & Willett, 1985) outlined methods of individual growth modeling that in contrast capitalized on the richness of multiwave data, thereby allowing for more effective testing of systematic interindividual differences in change. (For a comparative review of the many advantages of LGC modeling over the former approach to the study of longitudinal data, see Tomarken & Waller, 2005.)

In a novel extension of this earlier work, researchers (e.g., McArdle & Epstein, 1987; Meredith & Tisak, 1990; and Muthén, 1991) showed how individual growth models can be tested using the analysis of mean and covariance structures within the framework of SEM. Considered within this context, it has become customary to refer to such models as latent growth curve (LGC) models. Given its many appealing features (for elaboration, see Willett & Sayer, 1994) together with the ease in which researchers can tailor its basic structure for use in innovative applications (see, e.g., Cheong, MacKinnon, & Khoo, 2003; Duncan, Duncan, Okut, Strycker, & Li, 2002; Hancock, Kuo, & Lawrence, 2001; and Li, Duncan, Duncan, McAuley, Chaumeton, & Harmer, 2001), it seems evident that LGC modeling has the potential to revolutionize analyses of longitudinal research.

In this chapter, the topic of LGC modeling is introduced via three gradations of conceptual understanding. First, I present a general overview of measuring change in individual growth over time. Next, I illustrate the testing of an LGC model that measures change in perceptions of math and science ability for adolescents during

and 3; and Group 3 ($n = 601$) represented subjects for whom data were available only for Year 1 of the study.

[2]In this case, however, the same pattern of results replicates those based on the multigroup missing-data model.

a three-year period from Grades 8 through 10. Finally, I demonstrate the addition of gender to the LGC model as a possible time-invariant predictor of change that may account for any heterogeneity in the individual growth trajectories (i.e., intercept and slope) of perceived ability in math and science.

MEASURING CHANGE IN INDIVIDUAL GROWTH OVER TIME: THE GENERAL NOTION

In answering questions of individual change related to one or more domains of interest, a representative sample of individuals must be observed systematically over time and their status in each domain measured on several temporally spaced occasions (Willett & Sayer, 1994). However, several conditions may also need to be met. First, the outcome variable representing the domain of interest must be of a continuous scale. Second, although the time lag between occasions can be either evenly or unevenly spaced, both the number and the spacing of these assessments must be the same for all individuals. Third, when the focus of individual change is structured as an LGC model with analyses conducted using an SEM approach, data must be obtained for each individual on three or more occasions. Finally, the sample size must be large enough to allow for the detection of person-level effects (Willett & Sayer, 1994). Moreover, when analyses entail SEM and researchers want to use robust statistics to correct for non-normal distributions, sample-size requirements become even more critical. Accordingly, one would expect minimum sample sizes of not less than two hundred at each time point (Boomsma, 1985; and Boomsma & Hoogland, 2001).

THE HYPOTHESIZED MODEL

Willett and Sayer (1994) noted that the basic building blocks of the LGC model comprise two underpinning submodels that they termed *Level 1* and *Level 2* models. The Level 1 model can be viewed as a "within-person" regression model that represents individual change over time with respect to (in the present example) two single outcome variables: Perceived Ability in Math and Perceived Ability in Science. The Level 2 model can be viewed as a "between-person" model that focuses on interindividual differences in change with respect to these outcome variables. We turn now to the first of these two submodels which addresses the issue of intra-individual change.

Modeling Intra-Individual Change

The first step in building an LGC model is to examine the within-person growth trajectory. In the present case, this task translates into determining for each individual

the direction and extent to which his or her score in Self-Perceived Ability in Math and Science changes from Grades 8 through 10. Of critical importance in most appropriately specifying and testing the LGC model, however, is that the shape of the growth trajectory be known a priori. If the trajectory of hypothesized change is considered linear (a typical assumption underlying LGC modeling in practice), then the specified model includes two growth parameters: (a) an intercept parameter representing an individual's score on the outcome variable at Time 1, and (b) a slope parameter representing the individual's rate of change over the time period of interest. Within the context of our work here, the intercept represents an adolescent's Perceived Ability in Math and Science at the end of Grade 8; the slope represents the rate of change in this value during the three-year transition from Grades 8 through 10. As reported in Byrne and Crombie (2003), this assumption of linearity was tested and found to be tenable.[3] (For an elaboration of tests of underlying assumptions, see Byrne & Crombie, 2003; and Willett & Sayer, 1994.)

Of the many advantages in testing for individual change within the framework of a SEM over other longitudinal strategies, two are of primary importance. First, this approach is based on the analysis of mean and covariance structures and, as such, can distinguish group effects observed in means from individual effects observed in covariances. Second, a distinction can be made between observed and unobserved (or latent) variables in the specification of models. This allows for both the modeling and estimation of measurement error. With these basic concepts, we turn to Fig. 12.1, in which the hypothesized multiple-domain model to be tested in this first application is presented schematically.

Focus first on the six observed variables enclosed in rectangles at the bottom of the path diagram. Each variable constitutes a subscale score at one of three time points, with the first three (V16, V31, and V44) representing Perceived Math Ability and the latter three (V18, V33, and V46) representing Perceived Science Ability. Of course, associated with each is their matching random measurement error term (E). Moving up to the middle portion of the diagram, we find two latent factors associated with each of these constructs: Factors 1 and 2 represent the intercept and slope, respectively, for Perceived Math Ability; and likewise Factors 3 and 4 for Perceived Science Ability. In contrast to previous models presented thus far, the disturbance terms (D1–D4) take on a somewhat different connotation in LGC models. Interpreted within this context, they represent individual differences in the intercept and linear growth trajectories, respectively. Finally, at the top of the diagram we see the Constant variable with its typical EQS labeling of V999. As discussed in chapters 9 and 10, in keeping with the Bentler–Weeks representation system of EQS, this variable provides the mechanism by which a covariance structure is transformed into a mean and covariance (or moment) structure.

[3] If, on the other hand, the growth trajectory were considered nonlinear, the hypothesized model would include a third parameter representing curvature (for elaboration on this parameterization, see Byrne & Crombie, 2003; and Duncan et al., 1999). Fitting a nonlinear model such as a polynomial model requires more time points of measurement (Bentler, 2005).

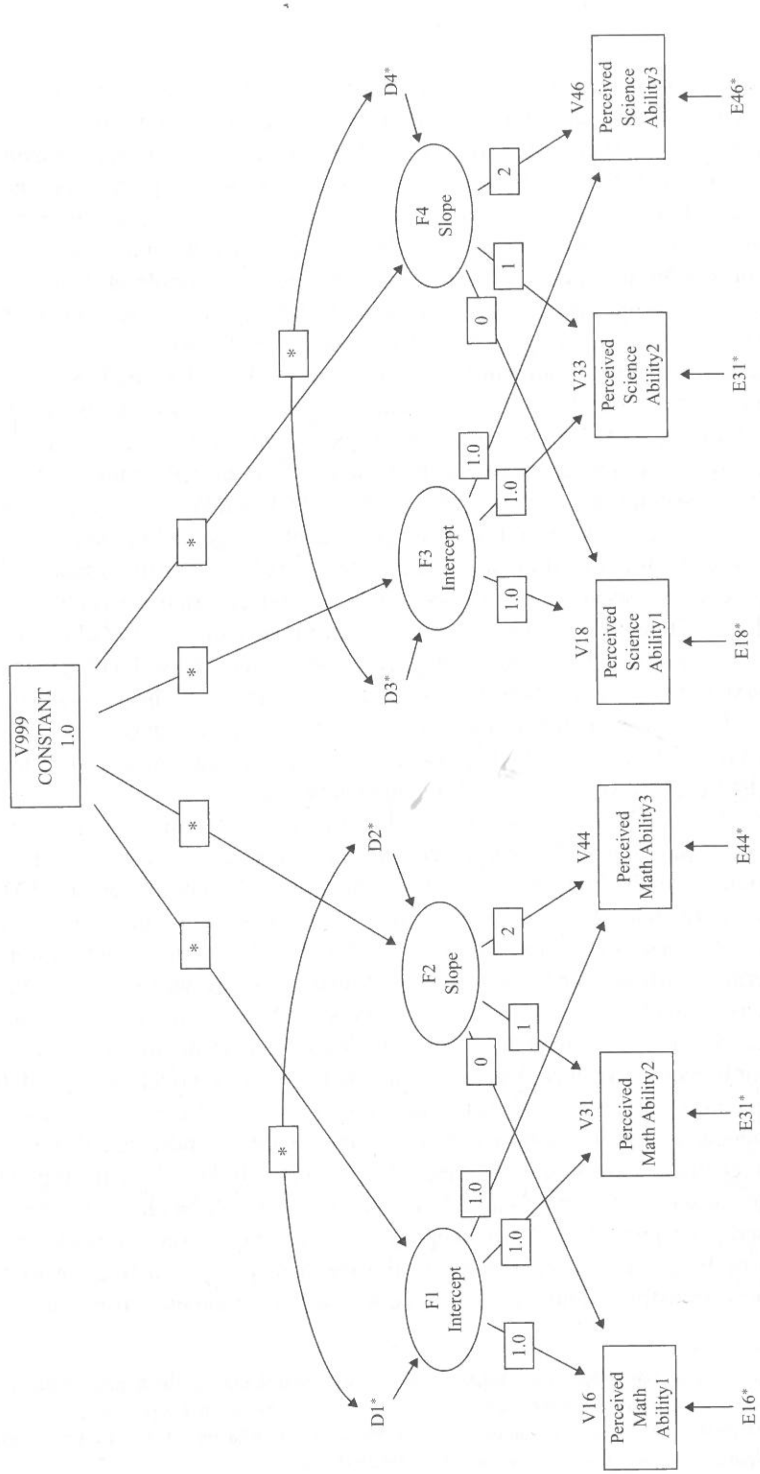

FIG. 12.1. Hypothesized multiple-domain latent growth curve model.

Let's turn now to the modeled paths in the diagram. As usual, the arrows leading from F1 and F2 to each of the related observed variables represent the regression of Perceived Math Ability scores onto both the Intercept and Slope at each of three time points; likewise for the regression paths related to Perceived Science Ability. In contrast, the arrows leading from the Constant to each of the Intercept and Slope factors represent the average intercept (or starting point of the growth curve) and average linear growth coefficient, respectively. As usual, the arrows leading from the E's to the observed variables represent the influence of random measurement error and, as stated previously, those from the D's represent the impact of individual differences in the intercept and linear growth trajectories, respectively. Finally, the modeled covariance between D1 and D2 and between D3 with D4 is assumed in the specification of an LGC model. The numerical values assigned to the regression paths leading from the Intercept and Slope factors to the observed variables are explained later in this chapter.

The primary focus in this subsection is to model intra-individual change. Within the framework of SEM, this focus is captured by the measurement model, the portion of a model that incorporates only linkages between the observed variables and their underlying unobserved factors. As you are well aware by now, of primary interest in any measurement model is the strength of the factor loadings or regression paths linking the observed and unobserved variables. As such, the only parts of the model in Fig. 12.1 that are relevant in the modeling of intraindividual change are the four factors (i.e., two intercepts and two slopes), the regression paths linking the six observed variables to these factors, the variances and covariances among the factors as given by the D's, and the related measurement errors associated with these observed variables.

We now consider this measurement model within a statistical framework. However, for simplicity, I present this summary pertinent to only one of the constructs: Perceived Math Ability (the same basic principles apply to the Perceived Science Ability construct). Accordingly, the measurement model as shown in Fig. 12.1 and expressed in matrix format is represented by the following regression equation:

$$\begin{bmatrix} \text{Perceived Math Abil1} \\ \text{Perceived Math Abil2} \\ \text{Perceived Math Abil3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} \text{Intercept} \\ \text{Slope} \end{bmatrix} + \begin{bmatrix} \text{E16} \\ \text{E31} \\ \text{E44} \end{bmatrix} \qquad (1)$$

Essentially, this is an ordinary factor analysis model with two special features. First, all the loadings are fixed—there are no unknown factor loadings. Second, the particular pattern of fixed loadings plus the mean structure allows for interpretation of the factors as intercept and slope factors. As in all factor models, this equation states that each adolescent's Perceived Math Ability score at each of three time points (i.e., Time 1 = 0, Time 2 = 1 and Time 3 = 2) is a function of three distinct components: (a) a factor-loading matrix of constants (1, 1, 1) and known time values (0, 1, 2) that remain invariant across all individuals, multiplied by (b) an

LGC vector containing individual-specific and unknown factors, herein called individual growth parameters (Intercept and Slope), plus (c) a vector of individual-specific and unknown errors of measurement. Whereas the LGC vector represents the within-person true change in Perceived Math Ability over time, the error vector represents the within-person noise that erodes these true change values (Willett & Sayer, 1994).

In preparing for a transition from the modeling of intra-individual change to the modeling of interindividual change, it is important to briefly review the basic concepts underlying the analyses of mean and covariance structures in SEM. When population means are of no interest in a model, analysis is based only on covariance structure parameters. As such, all scores are considered deviations from their means; thus, the constant term (represented as α in a regression equation) equals zero. Given that mean values played no part in the specification of the Level 1 (or within-person) portion of the LGC model, only the analysis of covariance structures is involved. However, in the Level 2 (or between-person) portion of the model, interest focuses on mean values associated with the Intercept and Slope factors; these values, in turn, influence the means of the observed variables. Because both levels are involved in the modeling of interindividual differences in change, analyses are now based on both mean and covariance structures. Again, this explanation is limited to the Perceived Math Ability construct.

Modeling Interindividual Differences in Change

Level 2 argues that over and above hypothesized linear change in Perceived Math Ability over time, trajectories necessarily vary across adolescents as a consequence of different intercepts and slopes. Within the SEM framework, this portion of the model reflects the "structural model" component that, in general, portrays relations among unobserved factors and postulated relations among their associated residuals. Within the more specific LGC model, however, this structure is limited to the regression paths linking the Constant to the Intercept and Slope factors (F1,V999; F2,V999) along with their related residuals, as reflected in the upper tier of the model shown in Fig. 12.1. Expressed in simple matrix terms, this portion of the model is summarized as follows:

$$\begin{bmatrix} \text{Intercept} \\ \text{Slope} \end{bmatrix} = \text{Constant} \begin{bmatrix} F1, V999 \\ F2, V999 \end{bmatrix} + \begin{bmatrix} \mathrm{D}_1 \\ \mathrm{D}_2 \end{bmatrix} \tag{2}$$

This equation clarifies that each factor is decomposed into a mean and a deviation from mean. The means carry information about average intercept and slope values; the residuals carry individual differences in intercept and slope. The specification of these parameters makes possible the estimation of interindividual differences in change. The key element in this specification is the Constant term because it provides the mechanism for transforming the covariance structure of the measurement (Level 1) model into the mean structure needed for analysis of the structural (Level 2) model. Specifically, in modeling and testing mean and covariance structures, as is the case here, analysis must be based on the moment matrix that is made possible by the inclusion of the Constant in the model specification. As discussed in chapters 9 and 10, this variable is always taken as an independent variable that has no variance and no covariance with other variables in the model.

Within the usual context of SEM specification, the model in Fig. 12.1 would convey the notion that both the Intercept and Slope factors are predicted by the Constant but with some degree of error as captured by the residual terms (D1, D2). Furthermore, these residuals are hypothesized to covary (D1, D2). However, in the special case of an LGC model, as noted previously, the residuals call for a different interpretation. Specifically, they represent individual differences in the intercept and slope parameters. These differences derive from deviations between the individual growth parameters and their respective population means (or average intercept and slope values).

We now reexamine Equation (2) albeit in more specific terms to clarify information bearing on possible differences in change across time. Within the context of the first construct, Perceived Ability in Math, interest focuses on five parameters that are key to determining between-person differences in change: two factor means (F1,V999 and F2,V999), two factor residual variances (D1 and D2), and one residual covariance (D1 and D2). The factor means represent the average population values for the intercept and slope and answer the question, "What is the population mean starting point and mean increment in Perceived Math Ability from Grades 8 through 10?" The factor residuals represent deviations of the individual intercepts and slopes from their population means, thereby reflecting population interindividual differences in the initial (Grade 8) Perceived Math Ability scores and the rate of change in these scores, respectively. Addressing the issue of variability, these key parameters answer the question, "Are there interindividual differences in the starting point and growth trajectories of Perceived Math Ability in the population?" Finally, the residual covariance represents the population covariance between any deviations in initial status and rate of change and answers the question, "Do students who start higher (or lower) in Perceived Math Ability tend to grow at higher (or lower) rates in that ability?"

Testing for Interindividual Differences in Change

Reviewing the hypothesized model in Fig. 12.1, we now focus on the assigned numerical values and asterisks. Numerical values are assigned only to the paths flowing from the Intercept and Slope factors to the observed variables; these paths of course represent fixed parameters in the model. The 1's specified for the paths flowing from the Intercept factor to each observed variable indicate that each is constrained to a value of 1.0. This constraint reflects the fact that the intercept value remains constant across time for each individual (Duncan, Duncan, Stryker, Li, & Alpert, 1999). The values of 0, 1, and 2 assigned to the Slope parameters represent Years 1, 2, and 3, respectively. These constraints address the issue of model identification; they also ensure that the second factor can be interpreted as a slope.

Three important points are of interest with respect to these fixed slope values. First, technically speaking, the first path (assigned a zero value) is really nonexistent and therefore has no effect. Although it would be less confusing to simply eliminate this parameter, it has become customary to include this path in the model, albeit with an assigned value of zero (Bentler, 2005). Second, these values represent equal time intervals (i.e., one year) between measurements; had data collection taken place at unequal intervals, the values would need to be calculated accordingly (e.g., 6 months = .5). Third, the choice of fixed values assigned to the Intercept and Slope factor loadings is somewhat arbitrary because any linear transformation of the time scale is usually permissible, and the specific coding of time chosen determines the interpretation of both factors. The Intercept factor is tied to a time scale (Duncan et al., 1999) because any shift in fixed loading values on the Slope factor necessarily modifies the scale of time bearing on the Intercept factor that, in turn, influences interpretations (Duncan et al., 1999). Relatedly, the variances and correlations among factors in the model change depending on the chosen coding. Here, the "initial value" coding is used, but there are other possibilities. Fourth, as indicated by the assigned asterisks, the paths flowing from the Constant to each factor is freely estimated. These parameters are typically unknown and, as noted previously, represent the average intercept (or starting point) and average linear growth coefficients. Finally, the variances of both the random measurement error and disturbance terms are estimated along with the disturbance covariances.

Before moving on, it may be instructive to examine Fig. 12.1 taking into account the points just presented. By path tracing in Fig. 12.1, starting with V999 and going down to the three Perceived Math Ability variables, we see that because of the fixed 1.0 Intercept (F1) loadings, the path V999 → F1 would reproduce precisely the mean of the Time 1 variable score. This score represents the mean initial level because the V999 → F2 path cannot be traced to the Time 1 variable. However, V999 → F2 is precisely the increment in means from Time 1 to Time 2 because its factor-loading multiplier is 1.0. The increment in means from Time 1 to Time 3 is 2.0 (times V999 → F2) due to the F2 → Time 3 loading. Hence, the mean slope, the V999 → F2 path, shows the mean change in Perceived Math Ability from occasion to occasion.

With a general understanding of LGC modeling and as it bears on the hypothesized multidimensional model in particular, we now direct our focus on analyses related to this model. We first review the EQS input file shown in Table 12.1.

TABLE 12.1
EQS Input for Hypothesized Model

```
/TITLE
 LGC Model of Perceived Math & Science Abilities "lgcmsG1"
 Based in Group 1 (Complete Data)
/SPECIFICATIONS
 DATA='c:\eqs61\files\books\data\LGC.Group1.ess';
 VARIABLES=46; CASES=500;
 METHODS=ML,ROBUST; MATRIX=RAW; ANALYSIS=MOMENT;
 DEL=467;
/LABELS
 V1=SUBJECT; V2=GENDER; V3=SCHTYPE; V4=MATGRDS1; V5=SCIGRDS1;
 V6=ENGGRDS1; V7=MATH04S1; V8=MATH05S1; V9=MATH06S1; V10=LANG04S1;
 V11=LANG05S1; V12=LANG06S1; V13=SCI04S1; V14=SCI05S1; V15=SCI06S1;
 V16=M_ABILS1; V17=L_ABILS1; V18=S_ABILS1; V19=MATGRDS2; V20=SCIGRDS2;
 V21=ENGGRDS2; V22=MATH04S2; V23=MATH05S2; V24=MATH06S2; V25=LANG04S2;
 V26=LANG05S2; V27=LANG06S2; V28=SCI04S2; V29=SCI05S2; V30=SCI06S2;
 V31=M_ABILS2; V32=L_ABILS2; V33=S_ABILS2; V34=MATGRDS3; V35=SCIGRDS3;
 V36=ENGGRDS3; V37=MATH04S3; V38=LANG04S3; V39=LANG05S3; V40=LANG06S3;
 V41=SCI04S3; V42=SCI05S3; V43=SCI06S3; V44=M_ABILS3; V45=L_ABILS3;
 V46=S_ABILS3;
/EQUATIONS
 V16 =  + 1F1  + 0F2  + 1E16;
 V31 =  + 1F1  + 1F2  + 1E31;
 V44 =  + 1F1  + 2F2  + 1E44;
 F1 =  + *V999 + 1D1;
 F2 =  + *V999 + 1D2;
 V18 =  + 1F3  + 0F4  + 1E18;
 V33 =  + 1F3  + 1F4  + 1E33;
 V46 =  + 1F3  + 2F4  + 1E46;
 F3 =  + *V999 + 1D3;
 F4 =  + *V999 + 1D4;
/VARIANCES
 V999 = 1.00;
 E16 = *;
 E31 = *;
 E44 = *;
 E18 = *;
 E33 = *;
 E46 = *;
 D1 = *;
 D2 = *;
 D3 = *;
 D4 = *;
/COVARIANCES
 D1,D2 = *; D3,D4 = *;
/PRINT
 FIT=ALL;
/LMTEST
 SET=PDD;
/END
```

THE EQS INPUT FILE

By now, model specifications in this file should be fairly straightforward, but we review the /EQUATIONS paragraph nonetheless. First, for each set of observed variables, note that Factor 1 (the Intercept) is fixed to a value of 1.0 at each of the three time points, whereas Factor 2 (the Slope) has fixed values of 0, 1, and 2 for Occasions 1, 2, and 3, respectively. Second, consistent with the path diagram, each of the four factors is regressed onto the Constant (V999) and these paths are freely estimated. In the /COVARIANCES paragraph, consistent with theory and underlying assumptions associated with LGC modeling, the disturbance terms related to the intercept and slope for both Perceived Math Ability and Perceived Science Ability are free to covary. Finally, review of the /LMTEST specification shows that the search for modification indexes is limited to possible correlations between pairs of disturbances (PDD). Realistically, this is the only modification that makes sense within the current context. We now discuss the related EQS output.

THE EQS OUTPUT FILE

Before reviewing goodness-of-fit of the hypothesized model, a brief overview of the related descriptive statistics is presented in Table 12.2. Not surprisingly, the initial reaction may be that nothing too exciting is going on here, and you will be quite correct in your perception. My interest in showing you these results is to draw your attention to the univariate and multivariate normality of the data. Univariately, variables are not too skewed or kurtotic, but there is some nonzero multivariate kurtosis. This is an opportunity to show that when data are fairly multivariate normal, any difference between the usual ML χ^2 statistic and the S-Bχ^2 statistic is minimal, as should be the case for the other fit statistics.

Turning to Table 12.3 in which the goodness-of-fit statistics are reported, we do find a slight difference between the usual $\chi^2_{(11)}$ (174.757) and corrected S-B$\chi^2_{(11)}$ (144.452) statistics. However, of primary concern is the obvious poor fit of the model, as indicated by the *CFI value of .800 (CFI = .815). Added to these model-fit results are an SRMR = .213 and the *RMSEA value of .156 (RMSEA = .173). Clearly, this model is misspecified in a very major way. For answers to this substantial misspecification, we review the LM Test statistics in Table 12.4.

Review of LM Test Univariate Incremental χ^2 statistics shows that not including a covariance between the intercept terms related to Perceived Math Ability and Perceived Science Ability accounts for most of the misspecification. Although misspecified covariance between the two slopes is also noted here, the LMχ^2 value is substantially less than the value associated with the intercepts. Given the substantive reasonableness of this residual covariance—together with Willet and Sayer's (1996) caveat that in multiple-domain LGC models, covariation among

TABLE 12.2
Selected EQS Output for Hypothesized Model: Descriptive Statistics

SAMPLE STATISTICS BASED ON COMPLETE CASES

UNIVARIATE STATISTICS

VARIABLE	M_ABILS1	S_ABILS1	M_ABILS2	S_ABILS2	M_ABILS3
MEAN	5.1162	4.7442	4.9973	4.9138	4.7669
SKEWNESS (G1)	-.9833	-.4256	-.7506	-.7435	-.6518
KURTOSIS (G2)	.8756	-.1277	.0534	.4142	-.3769
STANDARD DEV.	1.2324	1.1952	1.3184	1.2623	1.4798

VARIABLE	S_ABILS3	V999
MEAN	4.9973	1.0000
SKEWNESS (G1)	-.8615	.0000
KURTOSIS (G2)	.6890	.0000
STANDARD DEV.	1.2399	.0000

MULTIVARIATE KURTOSIS

```
MARDIA'S COEFFICIENT (G2,P) =    10.1996
NORMALIZED ESTIMATE =            11.6270
```

TABLE 12.3
Selected EQS Output for Hypothesized Model: Goodness-of-Fit Statistics

GOODNESS OF FIT SUMMARY FOR METHOD = ML

```
CHI-SQUARE =       174.757 BASED ON      11 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS         .00000
```

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)

```
BENTLER-BONETT      NORMED FIT INDEX =       .808
BENTLER-BONETT NON-NORMED FIT INDEX =        .691
COMPARATIVE FIT INDEX (CFI)          =       .815
ROOT MEAN-SQUARE RESIDUAL (RMR)      =       .352
STANDARDIZED RMR                     =       .213
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)    =      .173
90% CONFIDENCE INTERVAL OF RMSEA  (          .150,          .195)
```

GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST

```
SATORRA-BENTLER SCALED CHI-SQUARE = 144.4517 ON 11 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS         .00000
```

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)

```
BENTLER-BONETT      NORMED FIT INDEX =       .791
BENTLER-BONETT NON-NORMED FIT INDEX =        .667
COMPARATIVE FIT INDEX (CFI)          =       .800
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)    =      .156
90% CONFIDENCE INTERVAL OF RMSEA  (          .134,          .179)
```

TABLE 12.4
Selected EQS Output for Hypothesized Model: Modification Indexes

	CUMULATIVE MULTIVARIATE STATISTICS				UNIVARIATE INCREMENT			
							HANCOCK'S SEQUENTIAL	
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	CHI-SQUARE	PROB.	D.F.	PROB.
1	D3,D1	112.022	1	.000	112.022	.000	11	.000
2	D4,D2	121.531	2	.000	9.510	.002	10	.485

TABLE 12.5
Selected EQS Output for Model 2: Goodness-of-Fit Statistics

```
GOODNESS OF FIT SUMMARY FOR METHOD = ML

CHI-SQUARE =       45.550 BASED ON      10 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00000

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT       NORMED FIT INDEX =        .952
BENTLER-BONETT NON-NORMED FIT INDEX =          .925
COMPARATIVE FIT INDEX (CFI)          =         .960
ROOT MEAN-SQUARE RESIDUAL (RMR)      =         .080
STANDARDIZED RMR                     =         .047
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)      =         .084
90% CONFIDENCE INTERVAL OF RMSEA  (            .060,           .110)

GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST

SATORRA-BENTLER SCALED CHI-SQUARE =     37.9880 ON        10 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS         .00004

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT       NORMED FIT INDEX =        .947
BENTLER-BONETT NON-NORMED FIT INDEX =          .921
COMPARATIVE FIT INDEX (CFI)          =         .958
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)      =         .075
90% CONFIDENCE INTERVAL OF RMSEA  (            .051,           .101)
```

the growth parameters across domains (as reflected in their residual terms) should be considered—Model 2 was modified to address these concerns. Goodness-of-fit results for this respecified model are shown in Table 12.5.

Of primary interest is the difference that the incorporation of just one residual covariance can make to a model! We now have a well-fitting model with a *CFI value of .958 (CFI = .960). Furthermore, the SRMR has dropped to .047 and the *RMSEA to .075 (RMSEA = .084). However, we need to check on the residual covariance of D4,D2 noted in our review of the LM Test statistics related to the hypothesized model (see Table 12.4). To check all estimated residual covariances, we also need to review the status of their statistical significance. Results related to both the estimates and LM Test values are reported in Table 12.6.

Review of these results shows that although the residual covariances between the intercept and slope for Perceived Math Ability (D2,D1) and between the intercepts for both Perceived Math Ability and Perceived Science Ability (D3,D1)

TABLE 12.6
Selected EQS Output for Model 2: Residual Estimates and Modification Indexes

```
COVARIANCES AMONG INDEPENDENT VARIABLES
---------------------------------------
STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

                  E                            D
                 ---                          ---
                          I D2   -   F2                 -.278*I
                          I D1   -   F1                  .070 I
                          I                           -3.947@I
                          I                        (     .076)I
                          I                        (  -3.631@I
                          I                                   I
                          I D3   -   F3                  .515*I
                          I D1   -   F1                  .053 I
                          I                            9.677@I
                          I                        (     .055)I
                          I                        (   9.370@I
                          I                                   I
                          I D4   -   F4                 -.057*I
                          I D3   -   F3                  .061 I
                          I                             -.926 I
                          I                        (     .065)I
                          I                        (   -.871)I
                          I
```

	CUMULATIVE MULTIVARIATE STATISTICS				UNIVARIATE INCREMENT			
							HANCOCK'S SEQUENTIAL	
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	CHI-SQUARE	PROB.	D.F.	PROB.
1	D4,D2	9.962	1	.002	9.962	.002	10	.444

are statistically significant, the residual covariance between the intercept and slope for Perceived Science Ability (D4,D3) is nonsignificant. Turning next to the LM Test results, we observe that the residual covariance between the slopes of the two constructs remains a misspecified parameter in the model. Again, given the substantive rationality of a covariance between the slopes of two closely related subject areas, the model was respecified with this parameter (D4,D2) freely estimated. In addition, the nonsignificant residual covariance between the intercept and slope for Perceived Science Ability (D4,D3) was deleted from the model. Goodness-of-fit results for Model 3 are reported in Table 12.7.

Not surprisingly, with the incorporation of this second residual covariance, the fit of the model is further improved and now represents an exceptionally well-fitting model, as exemplified by a *CFI value of .970 (CFI = .971). These values are further supported by a SRMR value of .046 and a *RMSEA value of .063 (RMSEA = .072).

Having determined a well-fitting model, we are ready to review the substantive results of the analysis. These unstandardized estimates, as reported in the EQS output file, are presented in Table 12.8.

TABLE 12.7
Selected EQS Output for Model 3: Goodness-of-Fit Statistics

```
GOODNESS OF FIT SUMMARY FOR METHOD = ML

CHI-SQUARE =         35.530 BASED ON       10 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00010

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT       NORMED FIT INDEX =        .963
BENTLER-BONETT NON-NORMED FIT INDEX =          .947
COMPARATIVE FIT INDEX (CFI)          =         .971
ROOT MEAN-SQUARE RESIDUAL (RMR)      =         .078
STANDARDIZED RMR                     =         .046
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)    =       .072
90% CONFIDENCE INTERVAL OF RMSEA  (            .047,         .098)

GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST

SATORRA-BENTLER SCALED CHI-SQUARE =       29.9213 ON       10 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00088

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT       NORMED FIT INDEX =        .959
BENTLER-BONETT NON-NORMED FIT INDEX =          .944
COMPARATIVE FIT INDEX (CFI)          =         .970
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)    =       .063
90% CONFIDENCE INTERVAL OF RMSEA  (            .038,         .090)
```

Given that the measurement equations represented only fixed parameters, there is little of interest in this section. Of major importance, however, are the construct equations found next in the output file. The loading of the Constant (V999) onto each factor, of course, represents the latent factor score means. Here we see that all of the intercepts are statistically significant. Findings reveal the average score for Perceived Science Ability (4.760) is slightly lower than is the case for Perceived Math Ability (5.122). However, whereas adolescents' average Self-Perceived Math Ability decreased during a three-year period from Grades 8 through 10 (as indicated by the value of −0.164), average Self-Perceived Science Ability increased (0.125).

In the Variance section of the output file, all residual terms associated with the intercept and slope for each Perceived Ability domain (i.e., the D's) are statistically significant. These findings reveal strong interindividual differences in both the initial scores of Self-Perceived Ability in Math and Science at Time 1 and in their change over time, as the adolescents progressed from Grades 8 through 10. Such evidence of interindividual differences provides powerful support for further investigation of variability related to the growth trajectories. In particular, the incorporation of predictors in the model can serve to explain their variability. Of somewhat less importance substantively, albeit important methodologically, all random measurement error terms are also statistically significant.

We turn now to the Residual Covariances, review the within-domain residual covariance—that is, the covariance between the intercept and slope related to the

TABLE 12.8
Selected EQS Output for Model 3: Unstandardized Estimates

```
MEASUREMENT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.
 (ROBUST STATISTICS IN PARENTHESES)
M_ABILS1=V16 =   1.000 F1    + 1.000 E16
S_ABILS1=V18 =   1.000 F3    + 1.000 E18
M_ABILS2=V31 =   1.000 F1    + 1.000 F2    + 1.000 E31
S_ABILS2=V33 =   1.000 F3    + 1.000 F4    + 1.000 E33
M_ABILS3=V44 =   1.000 F1    + 2.000 F2    + 1.000 E44
S_ABILS3=V46 =   1.000 F3    + 2.000 F4    + 1.000 E46
 CONSTRUCT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.
 (ROBUST STATISTICS IN PARENTHESES)

   F1   =F1 =   5.122*V999   + 1.000 D1
                 .054
               94.000@
             (    .054)
             ( 94.000@

   F2   =F2 =   -.164*V999   + 1.000 D2
                 .032
               -5.102@
             (    .032)
             ( -5.102@

   F3   =F3 =   4.760*V999   + 1.000 D3
                 .051
               92.930@
             (    .051)
             ( 92.930@

   F4   =F4 =    .125*V999   + 1.000 D4
                 .030
                4.145@
             (    .030)
             (  4.145@

 VARIANCES OF INDEPENDENT VARIABLES
 ----------------------------------
 STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.
                   E                                      D
                  ---                                    ---
E16 -M_ABILS1              .207*I D1   -  F1               1.283*I
                           .099 I                           .127 I
                          2.083@I                        10.063@I
                     (     .098)I                    (     .150)I
                     (    2.108@I                    (    8.554@I
                                I                                I
```

(Continued)

TABLE 12.8
(Continued)

```
E18 -S_ABILS1            .824*I D2  -  F2                .256*I
                         .070 I                          .056 I
                       11.720@I                         4.556@I
                     (   .070)I                     (   .059)I
                     ( 11.796@I                     (  4.306@I
                              I                               I
E31 -M_ABILS2            .680*I D3  -  F3                .630*I
                         .061 I                          .065 I
                       11.173@I                         9.742@I
                     (   .075)I                     (   .065)I
                     (  9.089@I                     (  9.640@I
                              I                               I
E33 -S_ABILS2            .784*I D4  -  F4                .093*I
                         .064 I                          .031 I
                       12.200@I                         2.970@I
                     (   .077)I                     (   .035)I
                     ( 10.213@I                     (  2.617@I
                              I                               I
E44 -M_ABILS3            .971*I                               I
                         .130 I                               I
                        7.498@I                               I
                     (   .141)I                               I
                     (  6.911@I                               I
                              I                               I
E46 -S_ABILS3            .615*I                               I
                         .102 I                               I
                        6.005@I                               I
                     (   .109)I                               I
                     (  5.671@I                               I
                              I                               I

   COVARIANCES AMONG INDEPENDENT VARIABLES
   ---------------------------------------

   STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.

               E                                   D
              ---                                 ---

                              I D2  -  F2               -.249*I
                              I D1  -  F1                .070 I
                              I                        -3.540@I
                              I                     (   .076)I
                              I                     ( -3.277@I
                              I                               I
                              I D3  -  F3                .483*I
                              I D1  -  F1                .054 I
                              I                         8.872@I
                              I                     (   .057)I
                              I                     (  8.454@I
                              I                               I
                              I D4  -  F4                .056*I
                              I D2  -  F2                .017 I
                              I                         3.232@I
                              I                     (   .020)I
                              I                     (  2.871@I
                              I                               I
```


same construct. As expected, the estimated covariance between the intercept and slope for Perceived Math Ability is statistically significant. The negative estimate of −.249 suggests that for adolescents whose Self-Perceived Math Ability scores were high in Grade 8, their rate of increase in scores during the three-year period from Grades 8 through 10 was lower than for adolescents whose Self-Perceived Math Ability scores were lower at Time 1. In other words, Grade 8 students who perceived themselves as being less able than their peers in math made the greater gains. A negative correlation between initial status and possible gain is an old phenomenon in psychology known as the Law of Initial Values.

Turning to the first between-domain residual covariance (D3,D1), we observe a strong association between the intercepts of Perceived Ability in Math and in Science (.483), a finding that appears reasonable. In contrast, the residual covariance between the slopes for Perceived Ability in Math and in Science, although statistically significant, is small (.056). Nonetheless, review of the standardized coefficients in Table 12.9 shows this correlation ($r = .37$), together with the other between-domain residual ($r = .54$) and within-domain residual ($r = -.44$), to be moderately high.

TABLE 12.9
Selected EQS Output for Model 3: Standardized Solution

```
STANDARDIZED SOLUTION:                                          R-SQUARED

M_ABILS1=V16 =    .928 F1     + .373 E16                          .861
S_ABILS1=V18 =    .658 F3     + .753 E18                          .433
M_ABILS2=V31 =    .863 F1     + .386 F2    + .629 E31             .605
S_ABILS2=V33 =    .647 F3     + .248 F4    + .721 E33             .480
M_ABILS3=V44 =    .750 F1     + .670 F2    + .653 E44             .574
S_ABILS3=V46 =    .625 F3     + .479 F4    + .617 E46             .619
  F1     =F1  =   .000*V999  +1.000 D1                            .000
  F2     =F2  =   .000*V999  +1.000 D2                            .000
  F3     =F3  =   .000*V999  +1.000 D3                            .000
  F4     =F4  =   .000*V999  +1.000 D4                            .000

CORRELATIONS AMONG INDEPENDENT VARIABLES
---------------------------------------

           E                                 D
          ---                               ---

                        I D2  -  F2              -.435*I
                        I D1  -  F1                    I
                        I                              I
                        I D3  -  F3               .537*I
                        I D1  -  F1                    I
                        I                              I
                        I D4  -  F4               .366*I
                        I D2  -  F2                    I
                        I                              I
```

GENDER AS A TIME-INVARIANT PREDICTOR OF CHANGE

As discussed previously, when provided with evidence of interindividual differences, we can then ask whether and to what extent one or more predictors might explain this heterogeneity. For our purposes here, we ask whether statistically significant heterogeneity in the individual growth trajectories (i.e., intercept and slope) of Perceived Ability in Math and in Science can be explained by gender as a time-invariant predictor of change. As such, we might ask two questions: "Do self-perceptions of ability in math and science differ for adolescent boys and girls at Time 1 (Grade 8)?" and "Does the rate at which self-perceived ability in math and science change over time differ for adolescent boys and girls?" To answer these questions, the predictor variable Gender must be incorporated into the Level 2 (or structural) part of the model. This predictor model represents an extension of the final best-fitting multiple-domain model (Model 3) and is shown schematically in Fig. 12.2.

Of importance regarding the path diagram displayed in Fig. 12.2 are the newly added components. The first of these is the regression path leading from the Constant to the predictor variable of Gender. Essentially, this path represents the intercept on the variable of Gender. However, this path can be more intuitively understood by reviewing the following standard regression equation:

$$y = a + b1x1 + b2x2 + \cdots + e \qquad (3)$$

where the intercept "a" is the coefficient for the regression of a variable on a constant. As such, we can rewrite Equation (3) equivalently as follows:

$$y = a1 + b1x1 + b2x2 + \cdots + e \qquad (4)$$

where "1" is the constant "variable."

The regression of a variable on a constant is an intercept when the variable is an independent variable; otherwise, it is the mean of the variable. Relatedly, the path flowing from the Constant to the variable of Gender in Fig. 12.2 essentially represents a simpler version of the standard regression equation that can be stated as follows:

$$y = a1 + e \qquad (5)$$

where, y = Gender; a = the path leading from the Constant to Gender, with the Constant equal to 1; and E47 = e.

The second set of new components comprises the regression paths that flow from Gender to the Intercept and Slope factors associated with each Perceived Ability domain. These regression paths are of primary interest in this predictor

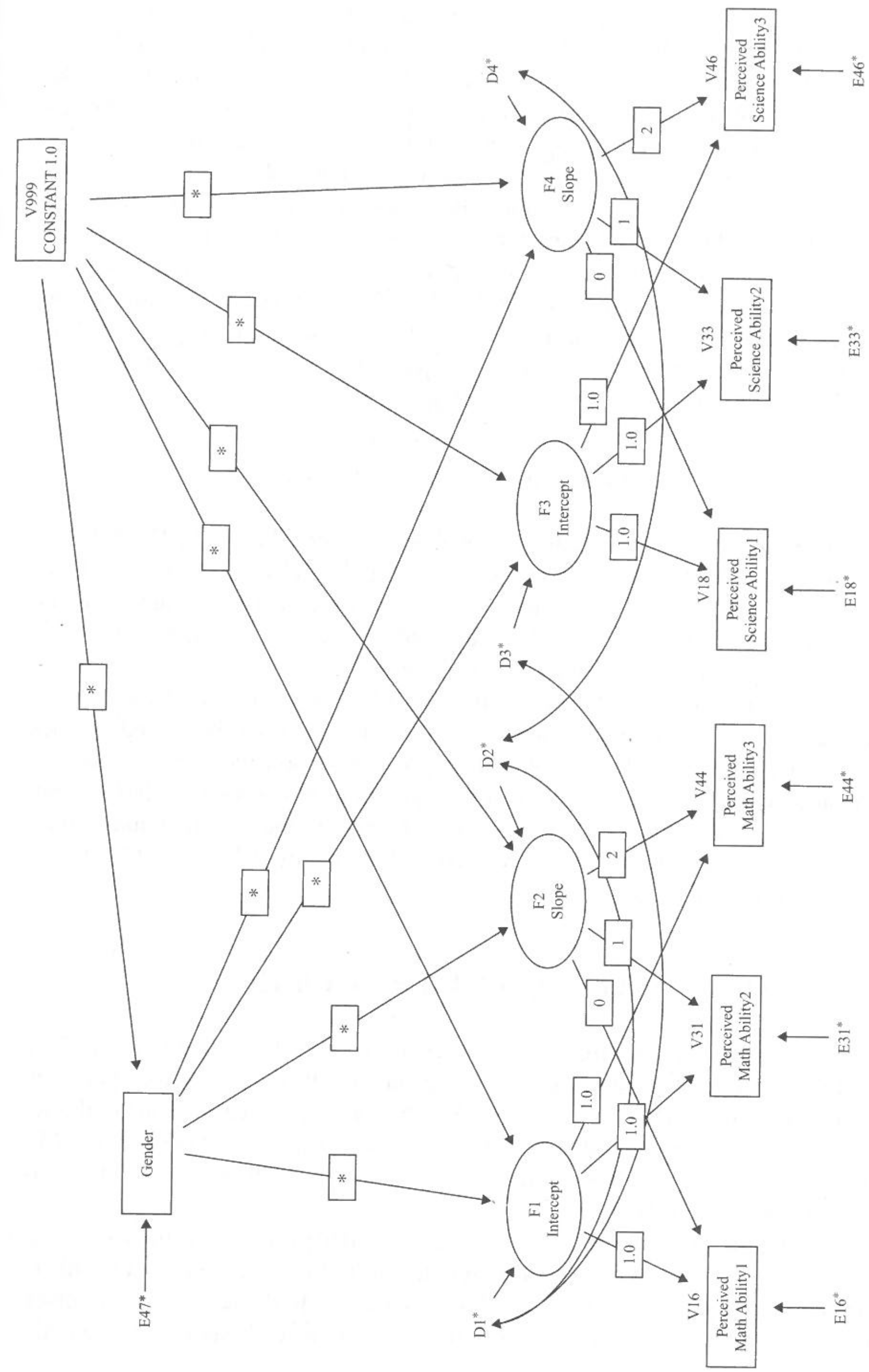

FIG. 12.2. Hypothesized multiple-domain latent growth curve predictor model.

model because they hold the key to the question of whether the trajectory of Perceived Ability in Math and in Science differs for adolescent boys and girls. Also noteworthy is the fact that with the addition of a predictor variable to the model, interpretation of the residuals necessarily changes; these residuals now represent variation remaining in the intercepts and slopes after all variability in their prediction by Gender has been explained (Willett & Keiley, 2000). Rephrased within a comparative framework, in the multiple-domain model in which no predictors were specified, the residuals represent deviations between the factor intercepts and slopes and their population means. In contrast, for this current model in which a predictor variable is specified, the residual variances represent deviations from their conditional population means. As such, these residuals represent the adjusted values of factor intercepts and slopes after partialling out the linear effect of the Gender predictor variable (Willett & Keiley, 2000).

THE EQS INPUT FILE

The EQS input file for this predictor model is presented in Table 12.10. Although most of this file remains the same as for the previous model (with no predictor), two important aspects related to parameter estimations for this predictor model should be highlighted. First, specifications are based on the final best-fitting multiple-domain model. As a result, the /COVARIANCES paragraph contains specifications for the two between-domain residual covariances (D1,D3 and D2,D4) albeit no specification for a within-domain residual covariance for Perceived Science Ability (D3,D4). Second, the /EQUATIONS paragraph now incorporates the Gender variable. In particular, observe that the regression path flows from the Constant (V999) to Gender (V47) and the paths flow from both the Constant and Gender to the intercept and slope for both Perceived Math Ability (F1, F2) and Perceived Science Ability (F3, F4).

THE EQS OUTPUT FILE

We now review the results from the testing of this model. Shown first in Table 12.11 are the goodness-of-fit statistics. It is interesting that there is evidence of an even better fitting model (*CFI = .975; CFI = .975) than was the case for the multiple-domain model with no predictor (note that the CFI value based on both the ML and robust χ^2 values is now identical). The SRMR is .039 and the *RMSEA is .056 (RMSEA = .062).

Because the content of major importance related to the predictor model focuses on the variable of Gender, the results presented in Table 12.12 are pertinent only to the construct equations that contain this information. In the results for Perceived Math Ability, we see that Gender was found to be a statistically significant predictor

TABLE 12.10
EQS Input for Predictor Model

```
/TITLE
 LGC Model of Perceived Math & Science Abilities "msG1gen"
 With Gender as Predictor
 Based on Group 1 (Complete Data)
 Based on Model 3
/SPECIFICATIONS
 DATA='c:\eqs61\files\books\data\LGC.Group1.ess';
 VARIABLES=46; CASES=500;
 METHODS=ML,ROBUST; MATRIX=RAW; ANALYSIS=MOMENT;
 DEL=467;
/LABELS
 V1=SUBJECT;  V2=GENDER; V3=SCHTYPE; V4=MATGRDS1; V5=SCIGRDS1;
 V6=ENGGRDS1;  V7=MATH04S1; V8=MATH05S1; V9=MATH06S1; V10=LANG04S1;
 V11=LANG05S1;  V12=LANG06S1; V13=SCI04S1; V14=SCI05S1; V15=SCI06S1;
 V16=M_ABILS1;  V17=L_ABILS1; V18=S_ABILS1; V19=MATGRDS2; V20=SCIGRDS2;
 V21=ENGGRDS2;  V22=MATH04S2; V23=MATH05S2; V24=MATH06S2; V25=LANG04S2;
 V26=LANG05S2;  V27=LANG06S2; V28=SCI04S2; V29=SCI05S2; V30=SCI06S2;
 V31=M_ABILS2;  V32=L_ABILS2; V33=S_ABILS2; V34=MATGRDS3; V35=SCIGRDS3;
 V36=ENGGRDS3;  V37=MATH04S3; V38=LANG04S3; V39=LANG05S3; V40=LANG06S3;
 V41=SCI04S3; V42=SCI05S3; V43=SCI06S3; V44=M_ABILS3; V45=L_ABILS3;
 V46=S_ABILS3;
/EQUATIONS
 V47 = *V999 + E47;
 V16 =  + 1F1  + 0F2  + 1E16;
 V31 =  + 1F1  + 1F2  + 1E31;
 V44 =  + 1F1  + 2F2  + 1E44;
 F1 =  + *V999 + *V47 + 1D1;
 F2 =  + *V999 + *V47 + 1D2;
 V18 =  + 1F3  + 0F4  + 1E18;
 V33 =  + 1F3  + 1F4  + 1E33;
 V46 =  + 1F3  + 2F4  + 1E46;
 F3 =  + *V999 + *V47 + 1D3;
 F4 =  + *V999 + *V47 + 1D4;
/VARIANCES
 V999 = 1.00;
 E16 = *;
 E31 = *;
 E44 = *;
 E18 = *;
 E33 = *;
 E46 = *;
 D1 = *;
 D2 = *;
 D3 = *;
 D4 = *;
/COVARIANCES
 D1,D2 = *; D1,D3 = *; D2,D4 = *;
/PRINT
 FIT=ALL;
/END
```

TABLE 12.11
Selected EQS Output for Predictor Model

```
GOODNESS OF FIT SUMMARY FOR METHOD = ML

CHI-SQUARE =        35.279 BASED ON      12 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00042

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT     NORMED FIT INDEX =       .964
BENTLER-BONETT NON-NORMED FIT INDEX =       .947
COMPARATIVE FIT INDEX (CFI)         =       .975
ROOT MEAN-SQUARE RESIDUAL (RMR)     =       .066
STANDARDIZED RMR                    =       .039
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)     =       .062
90% CONFIDENCE INTERVAL OF RMSEA  (         .039,        .087)

GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST
SATORRA-BENTLER SCALED CHI-SQUARE =      30.5457 ON      12 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS      .00231

FIT INDICES (BASED ON COVARIANCE MATRIX ONLY, NOT THE MEANS)
-----------
BENTLER-BONETT     NORMED FIT INDEX =       .962
BENTLER-BONETT NON-NORMED FIT INDEX =       .947
COMPARATIVE FIT INDEX (CFI)         =       .975
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA)     =       .056
90% CONFIDENCE INTERVAL OF RMSEA  (         .031,        .81)
```

of both initial status (.551) and rate of change (−.145). Given a coding of "0" for females and "1" for males, these findings suggest that whereas Self-Perceived Ability in Math was on average higher for boys than for girls at Time 1 by .551, the rate of change in this perception for boys from Grades 8 through 10 was slower than it was for girls by a value of −.145. (The negative coefficient indicates that boys had the lower slope.)

Results again revealed Gender as a significant predictor of Perceived Science Ability in Grade 8, with boys showing higher scores on average by a value of .233 than girls. On the other hand, rate of change was indistinguishable between boys and girls as indicated by its nonsignificant estimate.

In conclusion, I draw from the work of Willett and Sayer (1994, 1996) in highlighting several important features captured by the LGC modeling approach to the investigation of change. First, the methodology can accommodate from 3 to 30 waves of longitudinal data equally well. Willett (1988, 1989) showed, however, that the more waves of data collected, the more precise is the estimated growth trajectory and the higher is the reliability for the measurement of change. Second, there is no requirement that the time lag between each wave of assessments be equivalent. Indeed, LGC modeling can easily accommodate irregularly spaced measurements, but with the caveat that all subjects are measured on the same set of occasions. Third, individual change can be represented by either a linear or a nonlinear growth trajectory. Although linear growth is typically assumed by default, this assumption is easily tested and the model respecified to address

TABLE 12.12
Selected EQS Output for Predictor Model

```
CONSTRUCT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.
(ROBUST STATISTICS IN PARENTHESES)

F1    =F1   =      .551*V47    +   4.851*V999 + 1.000 D1
                   .106              .075
                  5.178@           64.938@
              (    .107)       (     .080)
              (   5.149@       (   60.367@
F2    =F2   =    -.145*V47     -    .092*V999 + 1.000 D2
                   .064              .045
                 -2.280@           -2.057@
              (    .064)       (     .045)
              (  -2.283@       (   -2.042@
F3    =F3   =      .233*V47    +   4.645*V999 + 1.000 D3
                   .102              .071
                  2.289@           65.027@
              (    .103)       (     .072)
              (   2.269@       (   64.344@
F4    =F4   =      .013*V47    +    .118*V999 + 1.000 D4
                   .060              .042
                   .213             2.804@
              (    .061)       (     .042)
              (    .211)       (    2.788@
```

curvilinearity if necessary. Fourth, in contrast to traditional methods used in measuring change, LGC models allow not only for the estimation of measurement error variances, but also for their autocorrelation and fluctuation across time in the event that tests for the assumptions of independence and homoscedasticity are untenable. Fifth, multiple predictors of change can be included in the LGC model. They may be fixed, as in the specification of Gender in this chapter, or time-varying (see, e.g., Byrne in press; and Willett & Keiley, 2000). Finally, the three key statistical assumptions associated with this application of LGC modeling (i.e., linearity, independence of measurement error variances, and homoscedasticity of measurement error variances), although not demonstrated in this chapter, can be easily tested via a comparison of nested models (see e.g., Byrne & Crombie, 2003).

To close this chapter on LGC modeling, readers should be aware that multilevel models provide an alternate way to study change with structural models. Singer and Willett (2003) provide an excellent discussion that focuses primarily on the Higher Linear Modeling approach.