**TABLE 8.3**
(Continued)

```
LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)
CONSTRAINTS TO BE RELEASED ARE:

          CONSTRAINTS FROM GROUP 2

          CONSTR:  20   (1,V30,F10)-(2,V30,F10)=0;
          CONSTR:  21   (1,V31,F12)-(2,V31,F12)=0;
          CONSTR:  22   (1,V32,F12)-(2,V32,F12)=0;
          CONSTR:  23   (1,F8,F5)-(2,F8,F5)=0;
          CONSTR:  24   (1,F8,F6)-(2,F8,F6)=0;
          CONSTR:  25   (1,F9,F5)-(2,F9,F5)=0;
          CONSTR:  26   (1,F9,F8)-(2,F9,F8)=0;
          CONSTR:  27   (1,F9,F2)-(2,F9,F2)=0;
          CONSTR:  28   (1,F10,F2)-(2,F10,F2)=0;
          CONSTR:  29   (1,F10,F3)-(2,F10,F3)=0;
          CONSTR:  30   (1,F10,F4)-(2,F10,F4)=0;
          CONSTR:  31   (1,F10,F6)-(2,F10,F6)=0;
          CONSTR:  32   (1,F10,F8)-(2,F10,F8)=0;
          CONSTR:  33   (1,F11,F10)-(2,F11,F10)=0;
          CONSTR:  34   (1,F11,F2)-(2,F11,F2)=0;
          CONSTR:  35   (1,F11,F3)-(2,F11,F3)=0;
          CONSTR:  36   (1,F12,F8)-(2,F12,F8)=0;
          CONSTR:  37   (1,F12,F9)-(2,F12,F9)=0;
          CONSTR:  38   (1,F12,F11)-(2,F12,F11)=0;
          CONSTR:  39   (1,F12,F5)-(2,F12,F5)=0;
          CONSTR:  40   (1,F12,F3)-(2,F12,F3)=0;
          CONSTR:  41   (1,E26,E25)-(2,E26,E25)=0;
```

| | CUMULATIVE MULTIVARIATE STATISTICS | | | | UNIVARIATE INCREMENT | |
|---|---|---|---|---|---|---|
| STEP | PARAMETER | CHI-SQUARE | D.F. | PROBABILITY | CHI-SQUARE | PROBABILITY |
| 1 | CONSTR: 31 | 6.539 | 1 | .011 | 6.539 | .011 |
| 2 | CONSTR: 4 | 11.279 | 2 | .004 | 4.740 | .029 |
| 3 | CONSTR: 18 | 14.852 | 3 | .002 | 3.574 | .059 |
| 4 | CONSTR: 39 | 18.529 | 4 | .001 | 3.676 | .055 |
| 5 | CONSTR: 7 | 21.725 | 5 | .001 | 3.196 | .074 |
| 6 | CONSTR: 30 | 24.251 | 6 | .000 | 2.527 | .112 |
| 7 | CONSTR: 11 | 26.139 | 7 | .000 | 1.888 | .169 |
| 8 | CONSTR: 35 | 28.051 | 8 | .000 | 1.912 | .167 |
| 9 | CONSTR: 14 | 29.902 | 9 | .000 | 1.851 | .174 |
| 10 | CONSTR: 16 | 33.172 | 10 | .000 | 3.269 | .071 |
| 11 | CONSTR: 27 | 34.493 | 11 | .000 | 1.321 | .250 |
| 12 | CONSTR: 13 | 36.429 | 12 | .000 | 1.937 | .164 |

# 9

# Application 7: Testing for Latent Mean Differences Based on a First-Order CFA Model

In the years since the printing of my first EQS book in 1994, there has been a steady albeit moderate increase in reported findings from tests for multigroup equivalence. Review of the SEM literature, however, reveals that most tests for invariance have been based on the analysis of covariance structures COVS, as exemplified in chapters 7 and 8. Despite Sörbom's (1974) introduction of the mean and covariance structures (MACS) strategy in testing for latent mean differences 30 years ago, few studies have been designed to test for latent mean differences across groups based on real (as opposed to simulated) data (see, e.g., Aiken, Stein, & Bentler, 1994; Byrne, 1988b; Cooke et al., 2001; Little, 1997; Marsh & Grayson, 1994; Reise et al., 1993; and Widaman & Reise, 1997). This chapter introduces you to basic concepts associated with the analysis of latent mean structures and walks you through an application that tests for the invariance of latent means across two different cultural groups. Specifically, we test for differences in the latent means of four nonacademic self-concepts (SCs)—Physical SC (Appearance), Physical SC (Ability), Social SC (Peers), and Social SC (Parents)—across Australian and Nigerian adolescents; these constructs comprise the four nonacademic SC components of the Self-Description Questionnaire I (SDQ-I; Marsh, 1992). The present application is taken from a study by Byrne and Watkins (2003) but extends this previous work in two ways: (a) analyses are based on MACS rather than only on COVS, and (b) analyses address the issue of missing data with respect to the Nigerian sample (data were complete for the Australian sample).

## BASIC CONCEPTS UNDERLYING TESTS OF LATENT MEAN STRUCTURES

In the usual univariate or multivariate analyses involving multigroup comparisons, researchers are typically interested in testing whether the observed means representing the various groups are statistically significantly different from each other. Because these values are directly calculable from the raw data, they are considered to be observed values. In contrast, the means of latent variables (i.e., latent constructs) are unobservable; that is, they are not directly observed. Rather, these latent constructs derive their structure indirectly from their indicator variables, which in turn are directly observed and, hence, measurable. Testing for the invariance of mean structures conveys the notion that we intend to test for the equivalence of means related to each underlying construct or factor. Another way of saying this is that we intend to test for differences in the latent means (of factors for each group).

For all examples considered thus far, the analyses were based on covariance structures. In other words, only parameters representing regression coefficients, variances, and covariances have been of interest. Accordingly, the covariance structure of the observed variables constitutes the crucial parametric information; thus, a hypothesized model can be estimated and tested via the sample covariance matrix. One limitation of this level of invariance is that whereas the unit of measurement for the underlying factors (i.e., the factor loading) is identical across groups, the origin of the scales (i.e., the intercepts) is not. As a consequence, comparison of latent factor means is not possible, thereby leading Meredith (1993) to categorize this level of invariance as "weak" factorial invariance. This limitation, notwithstanding, evidence of invariant factor loadings nonetheless permits researchers to move on in testing further for the equivalence of factor variances, factor covariances, and pattern of these factorial relations, a focus of substantial interest to researchers more concerned with construct validity issues than in testing for latent mean differences. These subsequent tests continue to be based on the analysis of COVS.

In the analysis of COVS, it is implicitly assumed that all observed variables are measured as deviations from their means; in other words, *their* means are equal to zero. As a consequence, the intercept terms generally associated with regression equations are not relevant to the analyses. However, when the observed means take on nonzero values, the intercept parameter must be considered, thereby necessitating a reparameterization of the hypothesized model. Such is the case when one is interested in testing for the invariance of latent mean structures. An example taken from the EQS manual (Bentler, 2005) should help to clarify both the concept and the term "mean structures". First consider the following regression equation:

$$y = \alpha + \beta x + \varepsilon$$



where $\alpha$ is an intercept parameter. Although the intercept can assist in defining the mean of y, it does not generally equal the mean. Considering expectations of both sides of this equation and assuming that the mean of $\varepsilon$ is zero, the above expression yields:

$$\mu_y = \alpha + \beta\mu_x$$

where $\mu_y$ is the mean of y and $\mu_x$ is the mean of x. As such, y and its mean can now be expressed in terms of the model parameters $\alpha$, $\beta$, and $\mu_x$. It is this decomposition of the mean of y, the dependent variable, that leads to the term *mean structures*. More specifically, it serves to characterize a model in which the means of the dependent variables can be expressed or "structured" in terms of structural coefficients and the means of the independent variables. The previous equation illustrates how incorporating a mean structure into a model necessarily includes the new parameters $\alpha$ and $\mu_x$, the intercept and observed mean (of x), respectively. Thus, models with structured means merely extend the basic concepts associated with the analysis of covariance structures.

In each previous application involving the analysis of COVS, variances of dependent variables were never parameters in the model. The same dictum holds true in the analysis of mean structures; dependent variable means cannot be parameters in the model. However, as will become clear in the next sub-section, the intercepts of dependent variables actually do become parameters in the model—but only because they operate as the regression coefficients of a "constant" variable that EQS creates to carry out the analyses of mean structures. It functions as an independent variable within the context of the Bentler–Weeks schema. In summary, any model involving mean structures may include the following parameters:

- regression coefficients
- variances and covariances of the independent variables
- intercepts of the dependent variables
- means of the independent variables

## MODELING MEAN STRUCTURES IN EQS

### Basic Parameterization

It may be obvious that to accommodate two additional parameters in the model and yet retain the restriction that means and variances of dependent variables cannot be parameterized, the model must be restructured in some way. This is exactly what does take place for EQS to complete the analyses. Achievement of this task entails the utilization of two unique tricks: (a) creation of a constant variable that

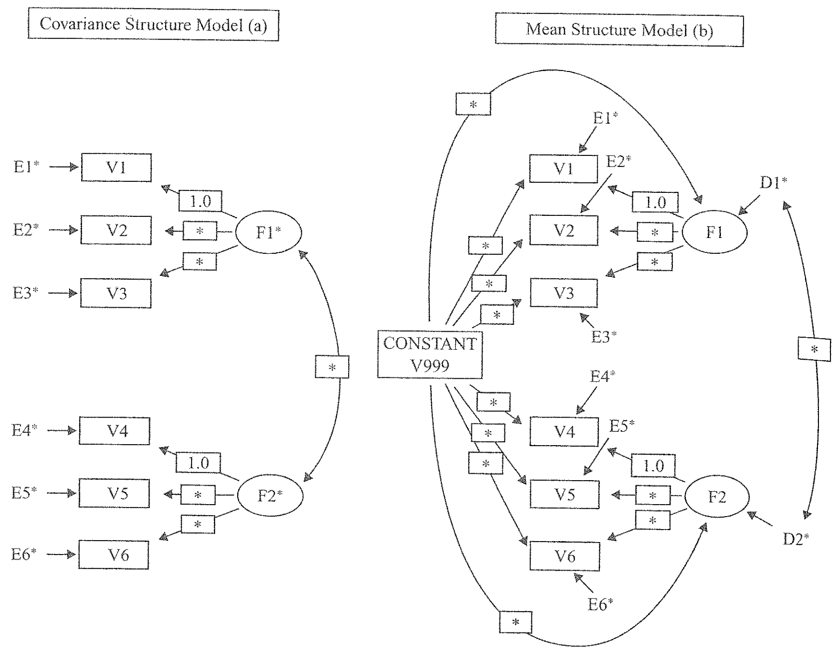


FIG. 9.1. Sample models of covariance and mean structures.

EQS designates V999,[1] and (b) reconceptualization of the independent variables as dependent variables, in the Bentler–Weeks sense. A simple CFA model presented in Fig. 9.1 illustrates these points and conceptualizes how the covariance structure model transforms into a mean structure model.

The same set of variables and factors are parameterized in two different configurations: in (a) as a covariance structure model and in (b) as a mean structure model. In the covariance structure model, there are three measured variables each regressed on two factors; the factors are hypothesized to be correlated. Consistent with EQS notation, the asterisks (*) represent parameters to be estimated; the first of each set of congeneric measures is fixed to 1.00 for purposes of model identification and latent variable scaling. In terms of the Bentler–Weeks conceptualization, this model has eight independent variables (i.e., two F's and six E's) and six dependent variables (six V's). Typically variances can be estimated for only the independent variables. Finally, with 21 elements in the sample covariance matrix ($6[6 + 1]/2$) and 13 estimable parameters, this model is overidentified with eight overidentifying restrictions.

[1]The rationale is that relative to the measured variables in the input file, the constant is always considered last.



Turning next to the mean structure model in Fig. 9.1 (b), we see a schematic representation that is vastly different from its covariance structure counterpart in (a). Let's now examine these differences more closely. First, you will see the inclusion in the model of a constant that EQS labels V999. The important aspect of this variable is that although it is an independent variable, in the Bentler–Weeks sense, it has no variance, no covariance with other variables in the model, and always remains fixed at a value of 1.0.

Second, note that the six observed variables and the two factors are regressed onto the constant. Each coefficient associated with these regression one-way arrows represents an intercept, which in turn expresses a mean value. Regression of the two factors onto the constant yields intercepts that represent the latent factor means; regression of the measured variables onto the constant represent the observed variable intercepts. Whenever there are no indirect effects of the constant on the measured variables, each intercept value represents a direct effect and should equal the observed mean.[2] On the other hand, if there are indirect effects in the model, as in Fig. 9.1 (b), the expected mean of a measured variable is represented by a total effect (i.e., direct effect + indirect effect). A brief example clarifies these points. Suppose the intercept for Factor 1 (V999 → F1) were fixed to zero. The expected mean value of V2 would then be determined solely by the direct effect of the constant (V999) on V2; this value would therefore equal the observed mean. By contrast, the expected mean value of V5 would be determined by both the direct effect of V999 on V5 and the indirect effect of V999 → F2 on V5, with these values in combination representing a total effect. Based on the tracing rule used in path analysis, the indirect effect on V5 would equal the product of paths V999 → F2 and F2 → V5. The decomposition of effects can be obtained as an option within the /PRINT paragraph of EQS.

As a consequence of regression onto the constant V999, the two factors are now dependent variables and, as such, cannot have variances and covariances. Instead, the residual of a variable that is a dependent variable by sole virtue of its regression onto the constant manifests the variance and covariance information for that variable. Thus, as shown in Fig. 9.1 (b), variances associated with D1 and D2 are estimated as well as the covariance between them.

Bentler (2005) has noted that in analyzing structured means models, there must always be fewer intercepts than there are measured variables. The intent of this requirement is to guard against possible underidentification related to the structured-means portion of the model. To fully comprehend this advice, let's reexamine Fig. 9.1 (b) more closely: It is actually composed of both a covariance structure as depicted in Fig. 9.1 (a) and a mean structure. Consider these two structures in terms of sample data. From all previous examples, we know that data for the covariance structure derive from the sample variance–covariance matrix. As noted

[2]Direct effects represent the impact of one variable on another, with no mediation by any other variable; indirect effects operate through at least one intervening variable (Bollen, 1989a).

previously, this portion of the model was overidentified with eight restrictions that provide degrees of freedom (df) in the model-testing process.

But what about the mean structure portion of the model in terms of model identification? It was discussed previously that to analyze a structured-means model, the user must input the observed mean value for each measured variable; these provide the data and are fundamental in determining the status of model identification. Thus, if the number of intercepts being estimated exceeds the amount of information coming into the structured means portion of the model as per the observed mean values, the model is underidentified. Indeed, the model in Fig. 9.1 (b) exemplifies this situation. There are six measured variables and, consequently, six pieces of information (i.e., six observed mean values) from which to estimate the means-related parameters. However, as indicated by the asterisks, the number of intercepts to be estimated is eight (i.e., six variable intercepts and two factor intercepts), thereby rendering the model underidentified. Thus, to estimate a mean structure as in Fig 9.1(b), two or more intercept restrictions must be imposed. The factor intercepts can be fixed to zero, yielding a just-identified mean structure that would yield identical covariance parameter results, as in Fig. 9.1(a) At the other extreme, all the V999 $\rightarrow$ V paths could be fixed to zero, in which case the means of the V's are explained solely by the means of the F's. In multisample models, as we shall see shortly, this type of situation can be resolved with the imposition of equality constraints across groups.

## Multigroup Parameterization

As with previous examples of multigroup invariance, applications based on structured-means models involve testing simultaneously across two or more groups. However, in testing for invariance based on the analysis of MACS, the multigroup specification is unique in two important ways. First, because the two (or more) groups under study are tested simultaneously, evaluation of the identification criterion is considered across groups. As a consequence, although the structured-means model may not be identified in one group, it can become so when analyzed across groups. This outcome occurs as a function of specified equality constraints across groups.

Second, the multigroup model demands the specification of an additional constraint to satisfy the need for factor identification. The reason for this added constraint derives from the fact that when the intercepts of the measured variables are constrained equal across groups (as they typically should be), the latent factor intercepts have an arbitrary origin. A standard way of addressing this situation is to constrain the latent factor intercepts of one group to zero (Bentler, 2005). This group then operates as a reference group against which latent means for the other group(s) are compared. Consequently, latent factor means are interpretable only in a relative sense. Statistical significance associated with the differences between the latent mean(s) for the reference group (i.e., fixed at 0.0) and those freely estimated for the other group(s) can be determined on the basis of the z-statistic.



# TESTING FOR LATENT MEAN DIFFERENCES OF A FIRST-ORDER CFA MODEL

## The Strategy

The approach to testing for differences in latent factor means follows the same pattern as the one outlined and applied in chapter 7. That is, we first establish a well-fitting baseline model for each group separately. This step is followed by a hierarchically ordered series of analyses that test for the invariance of particular sets of parameters across groups. The primary difference in the tests for invariance in chapter 7 (based on the analysis of COVS) and those based on the analysis of MACS illustrated in this chapter (and in chap. 10) is the additional tests for the equivalence of intercepts and latent factor means across groups. The difference between the present chapter and chapter 10, however, lies with the levels of invariance testing involved. We turn now to the hypothesized model under study and the related tests for invariance of a first-order CFA structure.

# THE HYPOTHESIZED MODEL

The application to be examined in this chapter bears on the equivalency of latent mean structures related to four nonacademic SC dimensions—Physical SC (appearance), Physical SC (ability), Social SC (peers), and Social SC (parents)—across Australian and Nigerian adolescents. These constructs comprise the four nonacademic SC components of the SDQ-I (Marsh, 1992). Although the data for Australian adolescents are complete ($n = 497$), those for Nigerian ($n = 463$) adolescents are incomplete. Provided with evidence of substantial multivariate kurtosis, as indicated by Mardia's normalized estimate of 80.70 for the Australians, and Yuan, Lambert, and Fouladi's (2004) normalized estimate of 71.20 for the Nigerians,[3] all analyses were based on the Robust statistics. The originally hypothesized model tested separately for each group is presented schematically in Fig. 9.2.

---

[3] When data are incomplete, there are typically many different patterns of missingness; thus, methods designed for one complete data set cannot be used. The Yuan, Lambert, and Fouladi (2004) coefficient is an extension of the Mardia (1970, 1974) test of multivariate kurtosis that can be used effectively with missing data. Essentially, it aggregates information across the missing data patterns to yield one overall summary statistic. Whenever a model is tested with missing data, EQS automatically computes and reports this coefficient along with its normalized estimate.

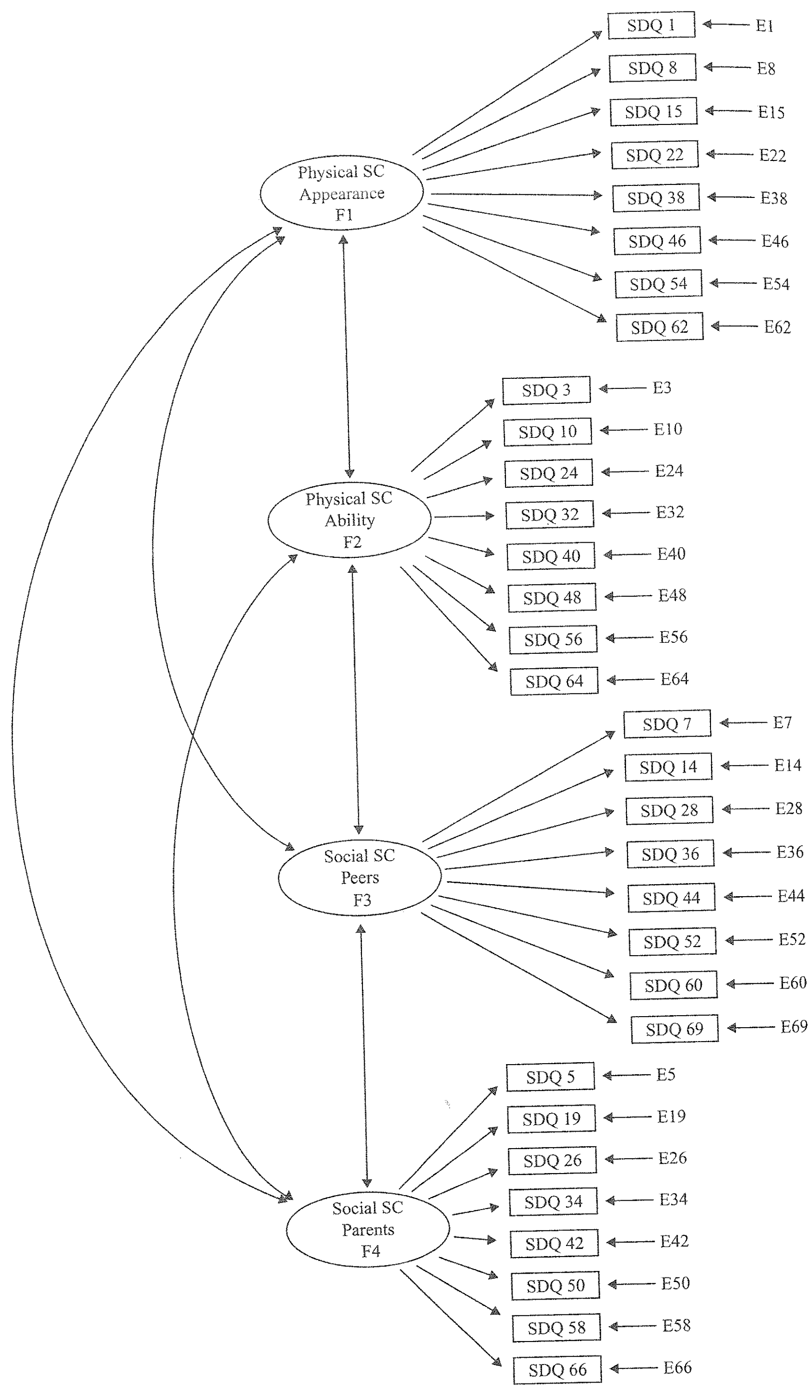


FIG. 9.2. Hypothesized model of factorial structure for the Self-
Description Questionnaire-I (SDQ-I; Marsh, 1992; non-academic



## Testing for Baseline Models

### *Australian Adolescents.*

Initial testing of the hypothesized model for this group yielded only a marginally good fit to the data (S-B$\chi^2_{(458)}$ = 1059.29; SRMR = .07; *CFI = .90; *RMSEA = .05; 90% C.I. = .047, .055). Review of the LM Test statistics revealed one cross-loading (F3 → SDQ38) and two error covariances (SDQ40/SDQ24: SDQ26/SDQ19) to be markedly misspecified. Of the three parameters, the cross-loading exhibited the largest standardized parameter change statistic, whereas the error covariance between Items 40 and 24 exhibited the largest univariate incremental $\chi^2$ statistic. These SDQ items are as follows:

- Item 19: I like my parents.
- Item 26: My parents like me.
- Item 24: I enjoy sports and games.
- Item 40: I am good at sports.
- Item 38: Other kids think I am good-looking.

Given that the cross-loading of Item 38 on Factor 3 (Social [Peers]) seemed reasonable, this parameter was added to the model first. Based on a comparison of *CFI values, this respecification resulted in a trivial improvement in model fit (S-B$\chi^2_{(457)}$ = 1003.701; SRMR = .063; *CFI = .905; *RMSEA = .049; 90% C.I. = .045, .053). Analogously, because the overlap of content between Items 19 and 26 was obvious and that between Items 24 and 40 seemed highly possible, the model was subsequently respecified and reestimated with these two error covariances included. This reparameterization resulted in a further slight improvement in model fit (S-B$\chi^2_{(455)}$ = 903.88; SRMR = .06; *CFI = .91; *RMSEA = .05; 90% C.I. = .045, .053). Although review of the LM Test statistics suggested the addition of a second cross-loading to the model (F1 → SDQ32), this parameter was not incorporated for two reasons: (a) considerations of parsimony, and (b) the questionable meaningfulness of this cross-loading across males and females. The content of this item reads as "I have good muscles." This content was considered possibly gender-specific, thereby arguing against its model specification. Thus, the baseline model considered the most appropriate and of reasonably good model fit for the Australian data consisted of the one cross-loading and two error covariances noted previously.

### *Nigerian Adolescents*

Given the incomplete nature of SDQ-I responses for these adolescents, it was necessary that missingness be addressed in the analyses of these data. Although many approaches can be taken in dealing with missing data, the one considered most efficient and is therefore the most highly recommended is that of ML estimation (see, e.g., Arbuckle, 1996; Enders & Bandalos, 2001; Gold & Bentler,

**TABLE 9.1**
EQS Input: Test of Hypothesized Model for Nigerian Adolescents

```
/SPECIFICATIONS
DATA='C:\EQS61\files\books\data\nigerom.ess';
VARIABLES= 79; CASES=465;
METHODS=ML,ROBUST; MATRIX=RAW; ANALYSIS=MOMENT;
MISSING=ML; SE=FISHER;
```

2000; and Schafer & Graham, 2002). Nonetheless, Bentler (2005) notes that when the amount of missing data is extremely small, there may be some conditions in which some of the more commonly used methods, such as listwise and pairwise deletion, hot deck, and mean imputation, may suffer only marginal loss of accuracy and efficiency compared with the ML method pairwise. (For an abbreviated review of the issues, advantages and disadvantages of various approaches to handling missing data, see Byrne, 2001; for a more extensive and comprehensive treatment of these topics, see Arbuckle, 1996, and Schafer & Graham, 2002; for a comparison of missing data methods, see Enders & Bandalos, 2001; and for a review of ML methods, see Enders, 2001.) In testing for the validity of the hypothesized model for Nigerian adolescents, analyses were based on the ML estimation approach to missing data. The /SPECIFICATIONS paragraph of the related input file is shown in Table 9.1.

Review of this command paragraph shows three specifications that have not appeared in previous input files. The first is ANALYSIS=MOMENT. When analyses are based on MACS, the data to be modeled include both sample means and sample covariances. This information is typically contained in the moment matrix, although the format by which it is contained varies differently among SEM programs. For example, whereas LISREL organizes these in a moment matrix that combines means and covariances as cross-products of sample data, EQS analyzes means and covariances separately. In EQS, this is done by adding a "constant" variable to the input, with the recognition that a coefficient for regressing any variable on a constant is an intercept of that variable. For independent variables, the intercept is that variable's model-based mean; for other variables, the total effect of the constant on that variable is the variable's model-based mean (Bentler, 2005). The statistical theory assumes that the sample means and covariances are being analyzed in all groups; that is, the variables have not been standardized.

The second specification to note is MISSING=ML.[4] EQS uses the expectation maximization (EM) type of ML estimation procedure. Based on the work of Jamshidian and Bentler (1999), this approach provides optimal results when the data are multivariate normal; when this condition does not hold (i.e., data are non-normally distributed), a correction to the test statistics and standard errors

[4]MISSING=COMPLETE is equivalent to listwise deletion of missing data. It is the default condition for EQS and, therefore, has not been used in previous application input files.



must be made. Yuan and Bentler (2000) provided these corrections such that the Yuan–Bentler scaled statistic (Y-B$\chi^2$) is analogous to the S-B$\chi^2$ when data are both incomplete and non-normally distributed. Consistent with computation of the S-B$\chi^2$, the Y-B$\chi^2$ requires the specification of ME=ML, ROBUST, which is shown in Table 9.1.

The final specification of note is that of SE=FISHER, which indicates that the Fisher information matrix is used to compute the standard errors. This is the sensible standard error option if sample size is not too small; otherwise, the SE=OBSERVED option would be chosen.

Let's return now, to findings related to testing of the hypothesized model for Nigerian adolescents. In contrast to the Australian group, these results revealed a fairly well-fitting model (Y-B$\chi^2_{(458)}$ = 729.88; SRMR = .05; *CFI = .93; *RMSEA = .04; 90% C.I. = .031, .041). Review of the LM Test statistics revealed one parameter that could be regarded as misspecified: an error covariance between Items 26 and 19. This covariance replicates the same finding for Australian adolescents. Consequently, the model was subsequently respecified and reestimated with this parameter freely estimated. The respecification led to a slight improvement in model fit, thereby resulting in a fairly good fit to the data (Y-B$\chi^2_{(457)}$ = 699.54; SRMR = .05; *CFI = .94; *RMSEA = .03; 90% C.I. = .029, .039). Given no further clear evidence of badly specified parameters, this model was deemed the most appropriate baseline model for Nigerian adolescents.

With an established baseline model for both groups of adolescents, we now proceed in testing for the validity of the multigroup model in which both baseline models are tested simultaneously to determine evidence of invariance. This multigroup model is shown in Fig. 9.3.

## Testing Validity of the Configural Model

As discussed in chapter 7, the model under test in this step of the invariance-testing process is a multigroup model in which no parameter constraints are imposed. The configural model simply incorporates the baseline models for both groups and allows for their simultaneous analyses. However, there is an important difference between the testing of the configural model in this chapter compared with the one in chapter 7 that arises solely as a result of the structure of the data. In this chapter, the Nigerian data comprise incomplete scores; consequently, this missingness must be considered in all analyses. In the current version of the program (EQS6.1), this requisite demands that analyses be based on mean as well as covariance structures. Conversely if the data for both groups had been based on complete data, tests for invariance related to both the configural and measurement models would have followed the same strategy illustrated in chapter 7. Because it is likely to be easier for you to follow specifications in the input file of the configural model if you can visualize the parameters in the related path diagram, I present this baseline model in Fig. 9.4. However, due to space restrictions, only the baseline model

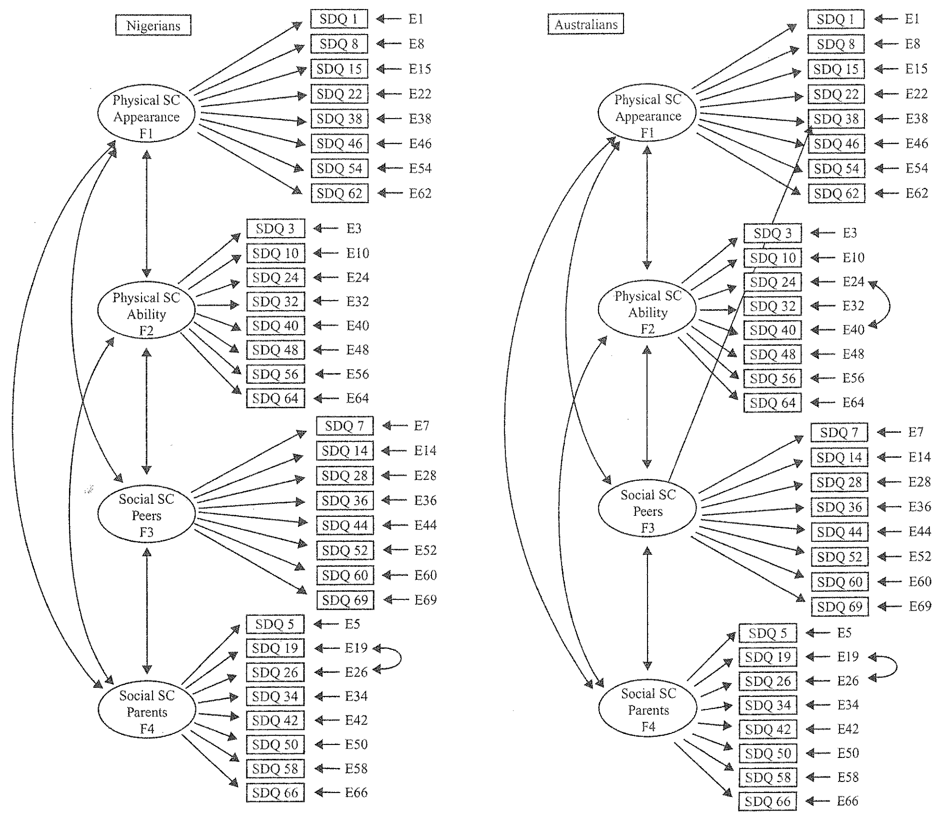


FIG. 9.3. Baseline models of SDQ-I structure for Nigerian and Australian adolescents.



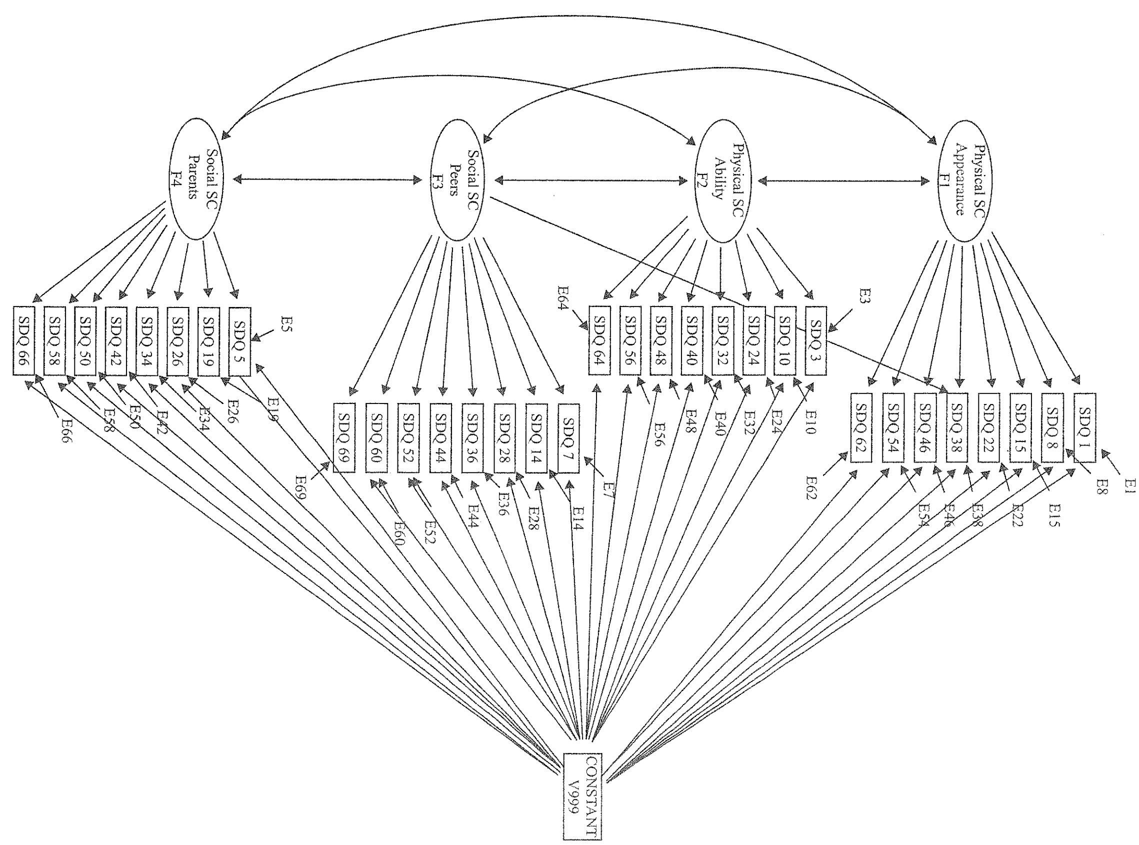


FIG. 9.4. Mean structure model of SDQ-I structure showing only observed variable intercepts.

for Australian adolescents is presented; in the interest of clarity, the two error covariances specified for the Australians are not shown.

The rectangle located at right center of the model is labeled "Constant" and designated "V999." Because intercepts are coefficients for regression on a constant, its addition to the model allows for the introduction of structured means. The constant (in EQS) is an independent variable that has no variance or covariances with other variables in the model and always remains fixed to a value of 1.0. The regression paths radiating out from the Constant to each of the observed variables represent the intercepts. The remaining structure of this model, of course will be familiar to you.

## THE EQS INPUT FILE

Let's turn now, then, to the input file related to the configural model which is presented in Table 9.2. Again, to conserve space, duplicate specifications are not shown. However, the /EQUATIONS paragraph is included for both groups to eliminate any confusion in the presentation of subsequent output in which the numbered variables (V's) are presented. Because the Nigerian data comprise three additional variables that are incorporated into the data file prior to the SDQ items, the EQS numbers assigned to the V's necessarily differ for Australian and Nigerian files; for example, whereas the SDQ1 item is labeled V1 for Australians, it is labeled V4 for the Nigerians. Other than this differential specification, the key specifications here and for all subsequent analyses in this chapter are (a) notification that there are two groups to be analyzed (GROUPS=2), and (b) commands related to the fact that data are incomplete, as shown in Table 9.1. Although data for Australian adolescents are complete, specifications regarding method of analyses must be consistent across the groups in a multisample model. As a result, it is necessary to specify METHODS=ML, SE=FISHER, and ANALYSIS=MOMENT anyway. Nonetheless, as you will see later, the program automatically detects that the data for the Australians are complete. Finally, with the use of raw data, there is no need to specify the observed mean values in the input file; if the data are not in this form, this input is required in the analysis of MACS.

Evaluation of this model focuses on its goodness-of-fit to the multisample data, with this value serving as the baseline against which all subsequent models are compared. Results from the testing of this model for Australian and Nigerian adolescents revealed a modestly well-fitting model (i.e., Y-B$\chi^2_{(912)}$ = 1644.55; SRMR = .068; *CFI = .92; *RMSEA = .03; 90% C.I. = .027, .031). Because one of the groups has incomplete data, the overall robust statistic is based on the Yuan–Bentler scaled value. The 912 df represent the summed df associated with the final baseline model for Australian (455 df) and Nigerian (457 df) adolescents.

**TABLE 9.2**
Selected EQS Input: Testing for Validity of the Configural Model

```
/TITLE
Testing for Invariance of SDQ Nonacademic SCs "sdqinvan1"
GROUP1: Australian Adolescents
CONFIGURAL MODEL
/SPECIFICATIONS
DATA='C:\EQS61\files\books\data\ausdata.ess';
VARIABLES=77; CASES=497; GROUPS=2;
METHODS=ML,ROBUST; MATRIX=RAW;
MISSING=ML; SE=FISHER; ANALYSIS=MOMENT;
/LABELS
V1=SDQ1; V2=SDQ2; V3=SDQ3; V4=SDQ4; V5=SDQ5; V6=SDQ6; V7=SDQ7; V8=SDQ8; V9=SDQ9;
V10=SDQ10; V11=SDQ11; V12=SDQ12; V13=SDQ13; V14=SDQ14; V15=SDQ15; V16=SDQ16; V17=SDQ17;
V18=SDQ18; V19=SDQ19; V20=SDQ20; V21=SDQ21; V22=SDQ22; V23=SDQ23; V24=SDQ24; V25=SDQ25;
V26=SDQ26; V27=SDQ27; V28=SDQ28; V29=SDQ29; V30=SDQ30; V31=SDQ31; V32=SDQ32; V33=SDQ33;
V34=SDQ34; V35=SDQ35; V36=SDQ36; V37=SDQ37; V38=SDQ38; V39=SDQ39; V40=SDQ40; V41=SDQ41;
V42=SDQ42; V43=SDQ43; V44=SDQ44; V45=SDQ45; V46=SDQ46; V47=SDQ47; V48=SDQ48; V49=SDQ49;
V50=SDQ50; V51=SDQ51; V52=SDQ52; V53=SDQ53; V54=SDQ54; V55=SDQ55; V56=SDQ56; V57=SDQ57;
V58=SDQ58; V59=SDQ59; V60=SDQ60; V61=SDQ61; V62=SDQ62; V63=SDQ63; V64=SDQ64; V65=SDQ65;
V66=SDQ66; V67=SDQ67; V68=SDQ68; V69=SDQ69; V70=SDQ70; V71=SDQ71; V72=SDQ72; V73=SDQ73;
V74=SDQ74; V75=SDQ75; V76=SDQ76; V77=GEN;
/EQUATIONS
V1 = *V999+F1+E1;
V8 = *V999+*F1+E8;
V15 = *V999+*F1+E15;
V22 = *V999+*F1+E22;
V38 = *V999+*F1+*F3+ E38;
V46 = *V999+*F1+E46;
V54 = *V999+*F1+E54;
V62 = *V999+*F1+E62;
V3 = *V999+F2+E3;
V10 = *V999+*F2+E10;
V24 = *V999+*F2+E24;
V32 = *V999+*F2+E32;
V40 = *V999+*F2+E40;
V48 = *V999+*F2+E48;
V56 = *V999+*F2+E56;
V64 = *V999+*F2+E64;
V7 = *V999+F3+E7;
V14 = *V999+*F3+E14;
V28 = *V999+*F3+E28;
V36 = *V999+*F3+E36;
V44 = *V999+*F3+E44;
V52 = *V999+*F3+E52;
V60 = *V999+*F3+E60;
V69 = *V999+*F3+E69;
V5 = *V999+F4+E5;
V19 = *V999+*F4+E19;
V26 = *V999+*F4+E26;
V34 = *V999+*F4+E34;
V42 = *V999+*F4+E42;
V50 = *V999+*F4+E50;
V58 = *V999+*F4+E58;
V66 = *V999+*F4+E66;
/VARIANCES
E1 =*; E3 =*; E5 =*; E7 =*; E8 =*; E10 =*; E14 =*; E15 =*; E19 =*; E22 =*; E24 =*; E26 =*; E28 =*; E32 =*;
E34 =*; E36 =*; E38 =*; E40 =*; E42 =*; E44 =*; E46 =*; E48 =*; E50 =*; E52 =*; E54 =*; E56 =*; E58 =*;
E60 =*; E62 =*; E64 =*; E66 =*; E69 =*;
F1 to F4 = *;
/COVARIANCES
F1 to F4 = *;
E40,E24 = *; E26,E19 = *;
/END
```

TABLE 9.2
(Continued)

```
/TITLE
GROUP2: NIGERIAN ADOLESCENTS
/SPECIFICATIONS
DATA='C:\EQS61\files\books\data\nigerom.ess';
VARIABLES= 79; CASES=465;
METHODS=ML,ROBUST; MATRIX=RAW;
MISSING=ML; SE=FISHER; ANALYSIS=MOMENT;
/LABELS
V1=ID; V2=SEX; V3=AGE; V4=SDQ1; V5=SDQ2; V6=SDQ3; V7=SDQ4; V8=SDQ5; V9=SDQ6; V10=SDQ7;
V11=SDQ8; V12=SDQ9; V13=SDQ10; V14=SDQ11; V15=SDQ12; V16=SDQ13; V17=SDQ14; V18=SDQ15;
V19=SDQ16; V20=SDQ17; V21=SDQ18; V22=SDQ19; V23=SDQ20; V24=SDQ21; V25=SDQ22; V26=SDQ23;
V27=SDQ24; V28=SDQ25; V29=SDQ26; V30=SDQ27; V31=SDQ28; V32=SDQ29; V33=SDQ30; V34=SDQ31;
V35=SDQ32; V36=SDQ33; V37=SDQ34; V38=SDQ35; V39=SDQ36; V40=SDQ37; V41=SDQ38; V42=SDQ39;
V43=SDQ40; V44=SDQ41; V45=SDQ42; V46=SDQ43; V47=SDQ44; V48=SDQ45; V49=SDQ46; V50=SDQ47;
V51=SDQ48; V52=SDQ49; V53=SDQ50; V54=SDQ51; V55=SDQ52; V56=SDQ53; V57=SDQ54; V58=SDQ55;
V59=SDQ56; V60=SDQ57; V61=SDQ58; V62=SDQ59; V63=SDQ60; V64=SDQ61; V65=SDQ62; V66=SDQ63;
V67=SDQ64; V68=SDQ65; V69=SDQ66; V70=SDQ67; V71=SDQ68; V72=SDQ69; V73=SDQ70; V74=SDQ71;
V75=SDQ72; V76=SDQ73; V77=SDQ74;V78=SDQ75; V79=SDQ76;
/EQUATIONS
V4 = *V999+F1+E4;
V11 = *V999+*F1+E11;
V18 = *V999+*F1+E18;
V25 = *V999+*F1+E25;
V41 = *V999+*F1+E41;
V49 = *V999+*F1+E49;
V57 = *V999+*F1+E57;
V65 = *V999+*F1+E65;
V6 = *V999+F2+E6;
V13 = *V999+*F2+E13;
V27 = *V999+*F2+E27;
V35 = *V999+*F2+E35;
V43 = *V999+*F2+E43;
V51 = *V999+*F2+E51;
V59 = *V999+*F2+E59;
V67 = *V999+*F2+E67;
V10 = *V999+F3+E10;
V17 = *V999+*F3+E17;
V31 = *V999+*F3+E31;
V39 = *V999+*F3+E39;
V47 = *V999+*F3+E47;
V55 = *V999+*F3+E55;
V63 = *V999+*F3+E63;
V72 = *V999+*F3+E72;
V8 = *V999+F4+E8;
V22 = *V999+*F4+E22;
V29 = *V999+*F4+E29;
V37 = *V999+*F4+E37;
V45 = *V999+*F4+E45;
V53 = *V999+*F4+E53;
V61 = *V999+*F4+E61;
V69 = *V999+*F4+E69;
/VARIANCES
  .
  .
  .
/COVARIANCES
F1 to F4 = *;
E29,E22 = *;
/PRINT
Fit = all;
/END
```



# THE EQS OUTPUT FILE

In addition to the goodness-of-fit statistics, EQS provides a tremendous amount of helpful and important information related to missing data, some of which is introduced here. Although this material was presented in the separate model output for the Nigerians, it was not, of course in the output for the Australians. However, in the multigroup configural model, specification related to missing data must be the same across the two groups regardless of the fact that the data are complete for the Australians. The related output is presented in Table 9.3 for the Australians and in Table 9.4 for the Nigerians.

In the EQS output for the Australians, we see that there were no cases with missing data; therefore, data from all 497 cases were used in the analysis. The output for the Nigerians is a totally different picture. Here we note first a warning that, of the 465 cases in the data set, two were skipped because all variables were missing. Thus, the sample size is now 463 rather than 465.

Next we are advised that, of the 463 cases, there are 93 with missing data, thereby resulting in 48 different patterns of missing data. The program presents a summary of these various data patterns. Let's examine a few of these entries. Following the column headings, we determine that in the first row, there are 370 (79.91%) cases with no missing data. In the second row, there is one case with 13 missing scores on Variables 4 and 13 through 24 inclusive. In the third row, there are two cases with missing data on 12 variables (V13–V24) inclusive. In the fourth row, there is one case with missing data on four variables (V21–V24) inclusive, and so on. In reviewing these missing data patterns, it is evident that both the amount and type of data available across all cases vary immensely. Following this missing data pattern summary, EQS prints a pair-wise-present covariance matrix showing the sample sizes for each pair of cases. Also provided in the output are three sets of estimates of means and covariance matrices: (a) ML estimates of means

TABLE 9.3
Selected EQS Output: Missing Data Specification for Australian Adolescents

```
MULTIPLE POPULATION ANALYSIS, INFORMATION IN GROUP 1

MAXIMUM LIKELIHOOD SOLUTION (NORMAL DISTRIBUTION THEORY)

 NUMBER OF CASES USED                   =  497
 NUMBER OF CASES WITH POSITIVE WEIGHT   =  497
 NUMBER OF CASES WITH MISSING DATA      =    0
 NUMBER OF MISSING PATTERNS IN THE DATA =    1

 IN THE SUMMARY OF MISSING PATTERNS, M REPRESENTS A MISSING VALUE

                                  VARIABLES
    #         #       %           1         2         3
MISSING     CASES   CASES   1234567890123456789012345678901 2
-------     -----   -----

      0       497  100.00
```

**TABLE 9.4**
Selected EQS Output: Missing Data Specification for Nigerian Adolescents

```
*** WARNING *** THESE CASES ARE SKIPPED BECAUSE ALL VARIABLES ARE MISSING--
    265   272

MULTIPLE POPULATION ANALYSIS, INFORMATION IN GROUP 2

 MAXIMUM LIKELIHOOD SOLUTION (NORMAL DISTRIBUTION THEORY)

 NUMBER OF CASES USED                       =  463
 NUMBER OF CASES WITH POSITIVE WEIGHT       =  463
 NUMBER OF CASES WITH MISSING DATA          =   93
 NUMBER OF MISSING PATTERNS IN THE DATA =       48

 IN THE SUMMARY OF MISSING PATTERNS, M REPRESENTS A MISSING VALUE

                                  VARIABLES
      #        #        %              1         2         3
MISSING    CASES    CASES     12345678901234567890123456789012
-------    -----    -----
      0      370    79.91
     13        1      .22        M        MMMMMMMMMMMM
     12        2      .43                 MMMMMMMMMMMM
      4        1      .22                         MMMM
      1        3      .65                         M
      2        1      .22                         M      M
      1        2      .43                                M
      1        3      .65                             M
      1        2      .43                 M
      3        1      .22                 M        M         M
      1        2      .43                          M
      1        3      .65                                M
      2        1      .22                                MM
      1        1      .22                                 M
      1        1      .22                                   M
      2        1      .22              M                     M
      1        4      .86              M
      3        1      .22              MM            M
      4       10     2.16             MMM            M
      5        1      .22             MMM            M       M
      6        1      .22     M       MMM            M      M
      2        6     1.30               M            M
      3        1      .22           M   M            M
      1        3      .65           M
      2        1      .22           M                   M
      1        2      .43                               M
      1        2      .43                M
      1        3      .65                      M
      1        1      .22                       M
      1        2      .43     M
      2        1      .22     M      M
      1        1      .22                   M
      1        1      .22                                 M
      2        1      .22          M                      M
      1        1      .22          M
```



**TABLE 9.4**
(Continued)

```
      #        #        %              1         2         3
MISSING    CASES    CASES     12345678901234567890123456789012
-------    -----    -----
      1        1      .22          M
      1        2      .43                     M
      1        2      .43        M
      2        1      .22      M M
      1        3      .65                               M
      2        1      .22         M                     M
      2        1      .22                               M    M
      1        1      .22       M
      1        1      .22                    M
      1        1      .22                  M
      4        1      .22           M    M   M               M
      8        9     1.94                             MMMMMMMM
      6        1      .22      MMMM   M             M
```

**TABLE 9.5**
Selected EQS Output: GLS Tests of Missingness for Nigerian Adolescents

```
GLS TEST OF HOMOGENEITY OF MEANS

  CHI-SQUARE =       1796.254 BASED ON      1388 DEGREES OF FREEDOM
  PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00000

GLS TEST OF HOMOGENEITY OF COVARIANCE MATRICES

  CHI-SQUARE =      18983.662 BASED ON      8821 DEGREES OF FREEDOM
  PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00000

GLS COMBINED TEST OF HOMOGENEITY OF MEANS/COVARIANCES

  CHI-SQUARE =      20779.916 BASED ON     10209 DEGREES OF FREEDOM
  PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS          .00000
```

and covariance matrix based on the saturated (unstructured) model, (b) imputed estimates of means and sample covariance matrix based on the saturated model, and (c) imputed means and sample covariance matrix based on the structured model. The residual matrix used to evaluate fit is computed as (b)-(c). Readers are referred (to the manual [Bentler, 2005] for more details related to these matrices).

In addition to this output, EQS also provides information related to the extent to which the various patterns of data can be considered as samples from a single population with one mean ($\mu$) and one covariance matrix ($\Sigma$). This information derives from three GLS tests of homogeneity: means, covariance matrices, and means and covariances combined (Kim & Bentler, 2002). These results are shown in Table 9.5.

In a cursory overview of these reported findings, you might be somewhat astounded at the large number of df. However, given that these tests (particularly the combined test reported here) reflect the many means and covariances from numerous patterns of data but with relatively few estimated common means and covariances, the df are exceedingly large. Probability values >0.05 provide support that the various patterns of missing data represent samples from a single population. Review of results from the three tests clearly show that the various patterns of data for Nigerian adolescents differ among themselves in terms of means and covariances, as evidenced by the probability values of 0.0 associated with each. Thus, technically, some of the missing data patterns provide different estimates of $\mu$ and $\Sigma$. However, given that the bulk of the data comprises 80% of the cases and the many patterns have few cases, the results of these tests are not discussed further. EQS currently provides no suggestions on which patterns might be deviant, and Bentler (2005) suggests that there are times that this test can be ignored.

## Testing for Invariance of Factor Loadings

As illustrated in chapter 7, in testing for the invariance of factor loadings across groups, equality constraints were imposed on all factor loadings except (a) those that were fixed to 1.00 for identification and scaling,[5] and (b) the cross-loading specific to Australian adolescents. This test yielded a marginally acceptable fit to the multisample data (i.e., Y-B$\chi^2_{(940)}$ = 1810.03; SRMR = 0.07; *CFI = 0.91; *RMSEA = 0.03; 90% C.I. = 0.029, 0.033). However, these results are not surprising given the size of the model and the fact that the baseline model for Australian adolescents fit the data less well than for the Nigerian adolescents.

# THE EQS OUTPUT FILE

Results arising from this test for invariant factor loadings are shown in Table 9.6. Review of the LM Test statistics related to this model reveals four univariate incremental $\chi^2$ values with probabilities less than 0.05. A check of these parameters against the list of constraints to be released shows that they represent two items (24 and 40) measuring Factor 2 (Physical SC, ability) and two items (19 and 26) measuring Factor 4 (Social SC, Parents). These same items were cited previously with respect to their error covariances relative to the baseline models. Nonetheless, their content is repeated as follows:

- Item 24: I enjoy sports and games.
- Item 40: I am good at sports.
- Item 19: I like my parents.
- Item 26: My parents like me.

[5]Actually, EQS prompts the user with an error message if equality constraints on these fixed parameters are imposed.



**TABLE 9.6**
Selected EQS Output: Test for Invariant Factor Loadings - LM Test Statistics

```
LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)

CONSTRAINTS TO BE RELEASED ARE:

        CONSTRAINTS FROM GROUP 2

        CONSTR:    1   (1,V8,F1)-(2,V11,F1)=0;
        CONSTR:    2   (1,V15,F1)-(2,V18,F1)=0;
        CONSTR:    3   (1,V22,F1)-(2,V25,F1)=0;
        CONSTR:    4   (1,V38,F1)-(2,V41,F1)=0;
        CONSTR:    5   (1,V46,F1)-(2,V49,F1)=0;
        CONSTR:    6   (1,V54,F1)-(2,V57,F1)=0;
        CONSTR:    7   (1,V62,F1)-(2,V65,F1)=0;
        CONSTR:    8   (1,V10,F2)-(2,V13,F2)=0;
        CONSTR:    9   (1,V24,F2)-(2,V27,F2)=0;
        CONSTR:   10   (1,V32,F2)-(2,V35,F2)=0;
        CONSTR:   11   (1,V40,F2)-(2,V43,F2)=0;
        CONSTR:   12   (1,V48,F2)-(2,V51,F2)=0;
        CONSTR:   13   (1,V56,F2)-(2,V59,F2)=0;
        CONSTR:   14   (1,V64,F2)-(2,V67,F2)=0;
        CONSTR:   15   (1,V14,F3)-(2,V17,F3)=0;
        CONSTR:   16   (1,V28,F3)-(2,V31,F3)=0;
        CONSTR:   17   (1,V36,F3)-(2,V39,F3)=0;
        CONSTR:   18   (1,V44,F3)-(2,V47,F3)=0;
        CONSTR:   19   (1,V52,F3)-(2,V55,F3)=0;
        CONSTR:   20   (1,V60,F3)-(2,V63,F3)=0;
        CONSTR:   21   (1,V69,F3)-(2,V72,F3)=0;
        CONSTR:   22   (1,V19,F4)-(2,V22,F4)=0;
        CONSTR:   23   (1,V26,F4)-(2,V29,F4)=0;
        CONSTR:   24   (1,V34,F4)-(2,V37,F4)=0;
        CONSTR:   25   (1,V42,F4)-(2,V45,F4)=0;
        CONSTR:   26   (1,V50,F4)-(2,V53,F4)=0;
        CONSTR:   27   (1,V58,F4)-(2,V61,F4)=0;
        CONSTR:   28   (1,V66,F4)-(2,V69,F4)=0;
```

| | CUMULATIVE MULTIVARIATE STATISTICS | | | | UNIVARIATE INCREMENT | |
|---|---|---|---|---|---|---|
| STEP | PARAMETER | CHI-SQUARE | D.F. | PROBABILITY | CHI-SQUARE | PROBABILITY |
| 1 | CONSTR: 9 | 18.163 | 1 | .000 | 18.163 | .000 |
| 2 | CONSTR: 11 | 28.415 | 2 | .000 | 10.252 | .001 |
| 3 | CONSTR: 23 | 34.953 | 3 | .000 | 6.538 | .011 |
| 4 | CONSTR: 22 | 44.684 | 4 | .000 | 9.730 | .002 |
| 5 | CONSTR: 19 | 48.570 | 5 | .000 | 3.886 | .049 |
| 6 | CONSTR: 7 | 51.831 | 6 | .000 | 3.261 | .071 |
| 7 | CONSTR: 4 | 54.606 | 7 | .000 | 2.774 | .096 |
| 8 | CONSTR: 20 | 57.245 | 8 | .000 | 2.639 | .104 |
| 9 | CONSTR: 13 | 59.272 | 9 | .000 | 2.028 | .154 |
| 10 | CONSTR: 24 | 60.738 | 10 | .000 | 1.465 | .226 |

From a substantive perspective, the challenging question to be answered (if it is even possible) is: What is it about the content of these items that cause them to be differentially valid across the two cultural groups? Although space limitations prevent a full exploration of possible reasons, an elaboration of the topic as it relates to the current data for both cultural groups is presented in Byrne and Watkins (2003) and Byrne (2003).

## Testing Invariance of Common Error Covariance

As discussed in chapter 7, testing for the invariance of error variance is considered extremely stringent and unnecessary (see, e.g., Widaman & Reise, 1997). Nonetheless, given that the error covariance between Items 26 and 19 is an important parameter in the baseline models for both Australian and Nigerian adolescents, I consider it important from a psychometric perspective to test for its invariance across the two groups. Before testing for this equivalency, however, the issue of the four noninvariant factor loadings needs to be addressed; accordingly, these constraints are released, thereby allowing the parameters to be freely estimated.

Results related to the testing of this model yielded a slightly better fit to the data than was the case for the last model (i.e., Y-B$\chi^2_{(937)}$ = 1728.78; SRMR = .06; *CFI = .92; *RMSEA = .03; 90% C.I. = .027, .032). The drop from 940 df to 937 df is accounted for by the estimation of the four freely estimated factor loadings, which is offset by the added equality constraint placed on one error covariance, thus leading to a reduction of 3 df.

EQS output showing LM Test statistics is presented in Table 9.7. As you will readily see, the error covariance under test was found to be invariant across the two groups; given the previous findings of noninvariant factor loadings related to Items 19 and 26, this result is somewhat surprising. However, this analysis revealed two additional factor loadings to be noninvariant across groups: Item 62 measuring Factor 1 (Physical SC, Appearance) and Item 52 measuring Factor 3 (Social SC, Peers). The content of these items is as follows:

- Item 62: I have nice features like nose, and eyes, and hair.
- Item 52: I have more friends than most other kids.

## Testing for Invariance of Intercepts

Thus far in this chapter, all analyses have been based on the analysis of COVS. As discussed previously, researchers interested in testing for the invariance of the factor covariances—a focus of substantial interest in construct validity work—would continue their investigation within the framework of COVS. However, when interest focuses on differences in latent factor means, the next step in the process necessarily involves testing for the invariance of the intercepts. These tests must



**TABLE 9.7**
Selected EQS Output: Test for Invariant Common Error Covariance - LM Test Statistics

```
LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)

CONSTRAINTS TO BE RELEASED ARE:

        CONSTRAINTS FROM GROUP 2

        CONSTR:    1   (1,V8,F1)-(2,V11,F1)=0;
        CONSTR:    2   (1,V15,F1)-(2,V18,F1)=0;
        CONSTR:    3   (1,V22,F1)-(2,V25,F1)=0;
        CONSTR:    4   (1,V38,F1)-(2,V41,F1)=0;
        CONSTR:    5   (1,V46,F1)-(2,V49,F1)=0;
        CONSTR:    6   (1,V54,F1)-(2,V57,F1)=0;
        CONSTR:    7   (1,V62,F1)-(2,V65,F1)=0;
        CONSTR:    8   (1,V10,F2)-(2,V13,F2)=0;
        CONSTR:    9   (1,V32,F2)-(2,V35,F2)=0;
        CONSTR:   10   (1,V48,F2)-(2,V51,F2)=0;
        CONSTR:   11   (1,V56,F2)-(2,V59,F2)=0;
        CONSTR:   12   (1,V64,F2)-(2,V67,F2)=0;
        CONSTR:   13   (1,V14,F3)-(2,V17,F3)=0;
        CONSTR:   14   (1,V28,F3)-(2,V31,F3)=0;
        CONSTR:   15   (1,V36,F3)-(2,V39,F3)=0;
        CONSTR:   16   (1,V44,F3)-(2,V47,F3)=0;
        CONSTR:   17   (1,V52,F3)-(2,V55,F3)=0;
        CONSTR:   18   (1,V60,F3)-(2,V63,F3)=0;
        CONSTR:   19   (1,V69,F3)-(2,V72,F3)=0;
        CONSTR:   20   (1,V34,F4)-(2,V37,F4)=0;
        CONSTR:   21   (1,V42,F4)-(2,V45,F4)=0;
        CONSTR:   22   (1,V50,F4)-(2,V53,F4)=0;
        CONSTR:   23   (1,V58,F4)-(2,V61,F4)=0;
        CONSTR:   24   (1,V66,F4)-(2,V69,F4)=0;
        CONSTR:   25   (1,E26,E19)-(2,E29,E22)=0;
```

| | CUMULATIVE MULTIVARIATE STATISTICS | | | | UNIVARIATE INCREMENT | |
|---|---|---|---|---|---|---|
| STEP | PARAMETER | CHI-SQUARE | D.F. | PROBABILITY | CHI-SQUARE | PROBABILITY |
| 1 | CONSTR: 17 | 7.633 | 1 | .006 | 7.633 | .006 |
| 2 | CONSTR: 7 | 13.612 | 2 | .001 | 5.979 | .014 |
| 3 | CONSTR: 11 | 17.447 | 3 | .001 | 3.835 | .050 |
| 4 | CONSTR: 18 | 20.544 | 4 | .000 | 3.098 | .078 |
| 5 | CONSTR: 4 | 23.672 | 5 | .000 | 3.128 | .077 |
| 6 | CONSTR: 22 | 26.107 | 6 | .000 | 2.435 | .119 |
| 7 | CONSTR: 1 | 28.211 | 7 | .000 | 2.103 | .147 |
| 8 | CONSTR: 15 | 30.429 | 8 | .000 | 2.218 | .136 |
| 9 | CONSTR: 24 | 31.986 | 9 | .000 | 1.557 | .212 |
| 10 | CONSTR: 12 | 33.514 | 10 | .000 | 1.529 | .216 |

be based on the analysis of MACS. Meredith (1993) argued that only tests based on the analysis of MACS allow for "strong" tests for invariance; those based on the analysis of COVS are only capable of testing for "weak" forms of invariance. At first blush, Meredith's categorization of invariance tests as "weak" versus "strong" would appear to cast a rather negative shadow over such tests based only on the analysis of COVS, with the implication that the latter are in some way less worthy than those based on the analysis of MACS. Realistically, this perceived implication does not hold because the analysis of only COVS may be the most appropriate approach to take in addressing the issues and interests of a particular study.

Testing for the invariance of intercepts across groups entailed equality constraints being placed on all invariant factor loadings, the common error covariance (E26,E19), and all observed variable intercepts—regardless of whether the factor loading for a variable is fixed to 1.0 for model identification and latent variable scaling or freely estimated due to its noninvariance across groups. It may be helpful to review the modeling of these parameters as shown in Fig. 9.4.

Results from this test yielded a modestly well-fitting model (i.e., Y-B$\chi^2_{(967)}$ = 2483.21; SRMR = .11; *CFI = .92; *RMSEA = .04; 90% C.I. = .038, .042). Review of the LM Test statistics, however, revealed 25 of the 32 intercepts to be noninvariant; these results are shown in Table 9.8.

The question is: How does a researcher interpret and make sense of these noninvariant intercepts? Chan (2000) presented one useful approach to the interpretation of noninvariant findings by weaving together application based on the analysis of MACS with interpretation based on Item Response Theory (IRT) analysis. (For an application based on this approach, see Byrne and Stewart [in press]). In testing for the invariance of a measuring instrument within the framework of MACS based on CFA modeling, interest focuses on the extent to which the factor loadings and intercepts are equivalent across groups. In testing for such equality based on IRT modeling, analyses seek to identify differential item functioning (DIF), with interest focusing on two characteristics (termed *parameters* in IRT lexicon) associated with each item: the level of item difficulty and the level of item discrimination—both of which describe the link between a test item and its underlying latent factor (Widaman & Reise, 1997). Within the framework of MACS, Chan (2000) and Ferrando (1996) (see also Widaman & Reise, 1997) show that the item intercept corresponds to the item difficulty parameter, whereas the item factor loading is analogous to the item discrimination parameter. Interpreted within the CFA framework, the higher an intercept value (or item difficulty level), the more attractive the item is in the sense that its average response level reflects its stronger endorsement. In contrast, the higher a factor-loading value (or item discrimination level), the more concrete (i.e., less ambiguous) an item is perceived to be (Chan, 2000; and Ferrando, 1996).

Presented with evidence of noninvariance related to factor loadings and intercepts, Cooke, Kosson, and Michie (2001) contended that of the two, noninvariant factor loadings are by far the more serious. They further argue that group differences in intercepts need not preclude the usefulness of these items in measuring their underlying constructs. As a result, we now move on to test for differences in the latent factor means, forcing the intercepts to be identical across groups.

**TABLE 9.8**

Selected EQS Output: Test for Invariant Intercepts - LM Test Statistics

```
LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)

CONSTRAINTS TO BE RELEASED ARE:
        CONSTRAINTS FROM GROUP 2
        CONSTR:    1   (1,V8,F1)-(2,V11,F1)=0;
        CONSTR:    2   (1,V15,F1)-(2,V18,F1)=0;
        CONSTR:    3   (1,V22,F1)-(2,V25,F1)=0;
        CONSTR:    4   (1,V38,F1)-(2,V41,F1)=0;
        CONSTR:    5   (1,V46,F1)-(2,V49,F1)=0;
        CONSTR:    6   (1,V54,F1)-(2,V57,F1)=0;
        CONSTR:    7   (1,V10,F2)-(2,V13,F2)=0;
        CONSTR:    8   (1,V32,F2)-(2,V35,F2)=0;
        CONSTR:    9   (1,V48,F2)-(2,V51,F2)=0;
        CONSTR:   10   (1,V56,F2)-(2,V59,F2)=0;
        CONSTR:   11   (1,V64,F2)-(2,V67,F2)=0;
        CONSTR:   12   (1,V14,F3)-(2,V17,F3)=0;
        CONSTR:   13   (1,V28,F3)-(2,V31,F3)=0;
        CONSTR:   14   (1,V36,F3)-(2,V39,F3)=0;
        CONSTR:   15   (1,V44,F3)-(2,V47,F3)=0;
        CONSTR:   16   (1,V60,F3)-(2,V63,F3)=0;
        CONSTR:   17   (1,V69,F3)-(2,V72,F3)=0;
        CONSTR:   18   (1,V34,F4)-(2,V37,F4)=0;
        CONSTR:   19   (1,V42,F4)-(2,V45,F4)=0;
        CONSTR:   20   (1,V50,F4)-(2,V53,F4)=0;
        CONSTR:   21   (1,V58,F4)-(2,V61,F4)=0;
        CONSTR:   22   (1,V66,F4)-(2,V69,F4)=0;
        CONSTR:   23   (1,E26,E19)-(2,E29,E22)=0;
        CONSTR:   24   (1,V1,V999)-(2,V4,V999)=0;
        CONSTR:   25   (1,V8,V999)-(2,V11,V999)=0;
        CONSTR:   26   (1,V15,V999)-(2,V18,V999)=0;
        CONSTR:   27   (1,V22,V999)-(2,V25,V999)=0;
        CONSTR:   28   (1,V38,V999)-(2,V41,V999)=0;
        CONSTR:   29   (1,V46,V999)-(2,V49,V999)=0;
        CONSTR:   30   (1,V54,V999)-(2,V57,V999)=0;
        CONSTR:   31   (1,V62,V999)-(2,V65,V999)=0;
        CONSTR:   32   (1,V3,V999)-(2,V6,V999)=0;
        CONSTR:   33   (1,V10,V999)-(2,V13,V999)=0;
        CONSTR:   34   (1,V24,V999)-(2,V27,V999)=0;
        CONSTR:   35   (1,V32,V999)-(2,V35,V999)=0;
        CONSTR:   36   (1,V40,V999)-(2,V43,V999)=0;
        CONSTR:   37   (1,V48,V999)-(2,V51,V999)=0;
        CONSTR:   38   (1,V56,V999)-(2,V59,V999)=0;
        CONSTR:   39   (1,V64,V999)-(2,V67,V999)=0;
        CONSTR:   40   (1,V7,V999)-(2,V10,V999)=0;
        CONSTR:   41   (1,V14,V999)-(2,V17,V999)=0;
        CONSTR:   42   (1,V28,V999)-(2,V31,V999)=0;
        CONSTR:   43   (1,V36,V999)-(2,V39,V999)=0;
        CONSTR:   44   (1,V44,V999)-(2,V47,V999)=0;
        CONSTR:   45   (1,V52,V999)-(2,V55,V999)=0;
        CONSTR:   46   (1,V60,V999)-(2,V63,V999)=0;
        CONSTR:   47   (1,V69,V999)-(2,V72,V999)=0;
        CONSTR:   48   (1,V5,V999)-(2,V8,V999)=0;
        CONSTR:   49   (1,V19,V999)-(2,V22,V999)=0;
        CONSTR:   50   (1,V26,V999)-(2,V29,V999)=0;
        CONSTR:   51   (1,V34,V999)-(2,V37,V999)=0;
        CONSTR:   52   (1,V42,V999)-(2,V45,V999)=0;
        CONSTR:   53   (1,V50,V999)-(2,V53,V999)=0;
        CONSTR:   54   (1,V58,V999)-(2,V61,V999)=0;
        CONSTR:   55   (1,V66,V999)-(2,V69,V999)=0;
```

(Continued)

**TABLE 9.8**
(Continued)

| | | CUMULATIVE MULTIVARIATE STATISTICS | | | UNIVARIATE INCREMENT | |
|---|---|---|---|---|---|---|
| STEP | PARAMETER | CHI-SQUARE | D.F. | PROBABILITY | CHI-SQUARE | PROBABILITY |
| 1 | CONSTR: 40 | 70.621 | 1 | .000 | 70.621 | .000 |
| 2 | CONSTR: 35 | 121.755 | 2 | .000 | 51.134 | .000 |
| 3 | CONSTR: 33 | 153.070 | 3 | .000 | 31.315 | .000 |
| 4 | CONSTR: 39 | 185.873 | 4 | .000 | 32.803 | .000 |
| 5 | CONSTR: 34 | 202.710 | 5 | .000 | 16.837 | .000 |
| 6 | CONSTR: 43 | 218.708 | 6 | .000 | 15.998 | .000 |
| 7 | CONSTR: 46 | 234.300 | 7 | .000 | 15.592 | .000 |
| 8 | CONSTR: 24 | 245.381 | 8 | .000 | 11.082 | .001 |
| 9 | CONSTR: 29 | 257.586 | 9 | .000 | 12.204 | .000 |
| 10 | CONSTR: 26 | 269.816 | 10 | .000 | 12.230 | .000 |
| 11 | CONSTR: 27 | 282.710 | 11 | .000 | 12.895 | .000 |
| 12 | CONSTR: 28 | 300.923 | 12 | .000 | 18.212 | .000 |
| 13 | CONSTR: 25 | 315.878 | 13 | .000 | 14.955 | .000 |
| 14 | CONSTR: 31 | 343.879 | 14 | .000 | 28.001 | .000 |
| 15 | CONSTR: 30 | 376.739 | 15 | .000 | 32.860 | .000 |
| 16 | CONSTR: 36 | 388.656 | 16 | .000 | 11.918 | .001 |
| 17 | CONSTR: 37 | 400.649 | 17 | .000 | 11.993 | .001 |
| 18 | CONSTR: 44 | 410.047 | 18 | .000 | 9.398 | .002 |
| 19 | CONSTR: 45 | 417.974 | 19 | .000 | 7.928 | .005 |
| 20 | CONSTR: 47 | 425.674 | 20 | .000 | 7.699 | .006 |
| 21 | CONSTR: 41 | 436.185 | 21 | .000 | 10.511 | .001 |
| 22 | CONSTR: 53 | 442.955 | 22 | .000 | 6.771 | .009 |
| 23 | CONSTR: 52 | 449.414 | 23 | .000 | 6.459 | .011 |
| 24 | CONSTR: 51 | 455.743 | 24 | .000 | 6.329 | .012 |
| 25 | CONSTR: 1 | 458.960 | 25 | .000 | 3.217 | .073 |
| 26 | CONSTR: 42 | 461.493 | 26 | .000 | 2.533 | .112 |
| 27 | CONSTR: 49 | 464.118 | 27 | .000 | 2.626 | .105 |
| 28 | CONSTR: 50 | 469.093 | 28 | .000 | 4.974 | .026 |
| 29 | CONSTR: 11 | 471.238 | 29 | .000 | 2.145 | .143 |
| 30 | CONSTR: 9 | 474.105 | 30 | .000 | 2.867 | .090 |

## Testing the Invariance of Latent Factor Means

Previously in this chapter (see Fig. 9.1), we reviewed a simple CFA model, presented first as a COVS structure and then as a MACS structure. Analogously, in Fig. 9.5, the MACS model as it relates to the cross-cultural application is presented. However, for reasons of clarity and simplicity, I present only the baseline model for the Australian group. Also included in the model shown in Fig. 9.5 is a variety of symbols associated with particular parameters, which I hope will be helpful to you in visualizing the test for latent factor mean differences.

Before explaining the symbols in this model, however, we briefly review three major changes that occur in the transition from a COVS model to a MACS model to test for latent mean differences. First, given that latent factor means are represented



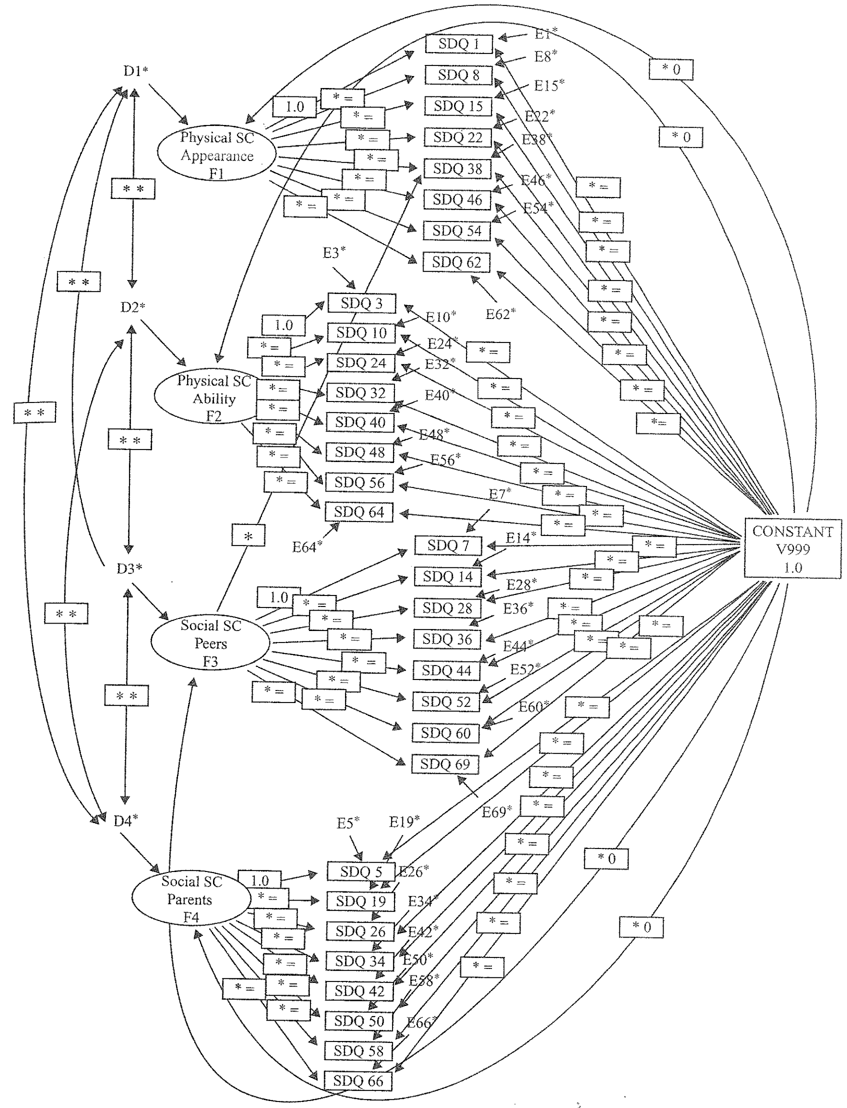


FIG. 9.5. Mean structure model representing test for latent factor mean differences.

by the factor intercepts (i.e., the path leading from the Constant [V999] to each factor), the factors thus become dependent variables in the model; consequently, their variances and covariances are no longer estimable. Because the factors are dependent variables in the model, they each have a related disturbance term (D) associated with them, and these residuals operate as proxies in carrying their variances and covariances. Second, in testing for differences in the latent factor means, the factor intercepts for one group must be fixed to 0.0. In the present case, these intercepts were specified as free for the Australian group and fixed to 0.0 for the Nigerian group; the latter then served as the reference group against which evidence of statistical significance was determined. Finally, specification of the LM Test is not included in the input file.

We now examine the parameters of the model shown in Fig. 9.5. There are four factors (F1–F4), each correlated with the other as indicated by the curved arrows connecting their related disturbance terms (D1–D4). There are eight observed measures regressed on each of the four factors, with the first loading of each congeneric set of measures fixed at 1.0 for purposes of identification and latent variable scaling. Finally, associated with each observed measure is an error term (E). To minimize the graphics in portraying the MACS model shown in Fig. 9.5, the error covariances are not included. However, it is important to note that these parameters were, of course, still maintained in the original baseline models, with the error covariance between Items 26 and 19 and between Items 40 and 24 estimated for the Australian group, albeit with the one error covariance (E26,E19) being constrained equal across groups.

The explanation of symbols presented in Fig. 9.5. depicting the various constraints in the model is meant only as an aid to a simultaneous portrayal of the model representing each group of adolescents; it is not a conventional path diagram. The symbols denoting these constraints are explained as follows:

| | |
|---|---|
| * | A parameter to be freely estimated in one group. |
| ** | A parameter to be freely estimated in each group. |
| *= | A parameter that although freely estimated is constrained equal across groups. |
| *0 | A parameter to be freely estimated in one group but fixed to zero in the other group. |

So now, let's once again review Fig. 9.5, this time taking all paths and assigned symbols into account. Interpretation of the model can be summarized as follows:

- As indicated by the assigned * *'s, the variances of the D's are freely estimated in each group, as are the covariances among them.
- To minimize clutter in the diagram, the variances of the E's are assigned *'s. However, they are freely estimated in each group.
- As indicated by the assigned *, the cross-loading of Item 38 (SDQ 38) on F3 is freely estimated only for the Australian group.



- As indicated by the assigned * =, all factor loadings are constrained equal across groups except for those factor loadings fixed to 1.00.
- As indicated by the assigned * =, all intercepts for the observed measures are constrained equal across groups.
- As indicated by the assigned * 0, the four factor intercepts are freely estimated in one group (Australians) and constrained equal to zero in the other group (Nigerians); the latter is therefore regarded as the "reference" group.
- As noted previously, variance associated with the constant (V999) is not estimated; the constant remains fixed to 1.00.

Before turning to the EQS input file related to this model, it is worthwhile to review two important points with respect to MACS models. First, the number of estimated intercepts must be less than the number of measured variables. In a multigroup model, this is controlled by the imposition of equality constraints across the groups. In the present case, review of the structured-means portion of the model in Fig. 9.5 reveals 64 measured variables (32 for each group). Relatedly, there are 64 observed variable intercepts; of these, 32 are freely estimated for the Australian group and 32 are constrained equal across the Nigerian group. In addition, there are four intercept parameters for factors being estimated. Thus, these model parameters are overidentified, with 28 df.

Second, in the present application, we are primarily interested in the comparison of latent mean values and, thus, in values representing the factor intercepts only. Nonetheless, a brief explanation of effects related to the observed variable intercepts is helpful. When there are no indirect effects, the variable intercepts should equal the actual observed means if the model is plausible. Relatedly, because the factor intercepts are fixed to zero for the Nigerian group (the reference group), there are consequently no indirect effects; thus, the expected means should approximate the observed sample means for that group. For the Australian group, expected means for the measured variables derive from the indirect effect of the constant via the related factor and its loading plus the direct effect of the constant on the variable itself.

## THE EQS INPUT FILE

Table 9.9 shows selected portions of the input file used in testing for differences in the latent factor means. Given that this input basically replicates the one for testing the configural model shown in Table 9.2, only the portions of the file relevant to the present analysis are included.

In the /EQUATIONS paragraph for Group 1, Australian adolescents, there are four entries for the factor intercepts; these represent the latent factor means and are therefore estimated for the Australian group. Under the /VARIANCES paragraph, only variances related to the disturbance terms (D1–D4) are shown. In the /COVARIANCES paragraph, the specification of their covariances, along with

**TABLE 9.9**
Selected EQS Input: Test for Latent Mean Differences

```
/TITLE
Testing for Invariance of SDQ Nonacademic SCs "sdqinvan5"
GROUP1: Australian Adolescents
TESTING FOR LATENT MEAN DIFFERENCES
  - ALL INTERCEPTS INVARIANT
  - 6 NONINVARIANT FACTOR LOADINGS RELEASED
  - 1 ERROR CORR (E26,E19) INVARIANT
/SPECIFICATIONS
   .
   .
/LABELS
   .
   .
/EQUATIONS
V1 = *V999+F1+E1;
   .
   .
F1 = *V999+D1;
F2 = *V999+D2;
F3 = *V999+D3;
F4 = *V999+D4;
/VARIANCES
   .
   .
D1 to D4 = *;
/COVARIANCES
D1 to D4 = *;
E40,E24 = *; E26,E19 = *;
/END
/TITLE
GROUP2: NIGERIAN ADOLESCENTS
/SPECIFICATIONS
   .
   .
/LABELS
   .
   .
/EQUATIONS
V4 = *V999+F1+E4;
   .
   .
F1 = 0.0 V999+D1;
F2 = 0.0 V999+D2;
F3 = 0.0 V999+D3;
F4 = 0.0 V999+D4;
/VARIANCES
   .
   .
D1 to D4 = *;
/COVARIANCES
D1 to D4 = *;
E29,E22 = *;
/PRINT
Fit = all;
/CONSTRAINTS
   .
   .
/END
```



the two error covariances specific to this group, are shown. The main point to be made with specifications related to the Nigerian group lies within the /EQUATIONS paragraph, where we see the factor intercepts are fixed to zero. Of course, the constraints specified for this model are consistent with those specified for the configural model (see Table 9.2).

## THE EQS OUTPUT FILE

To answer the primary question of whether the latent factor means for the two cultural groups are significantly different, interest focuses on the construct equations. Given that the Nigerian group was designated as the reference group—thus, their factor means were fixed to zero—we concentrate solely on estimates as they relate to the Australian group. These equations are presented in Table 9.10. The key parameters in answering this question are the factor intercepts as they represent the latent mean values. Because analyses were based on the robust statistics, these estimates are interpreted in terms of the robust standard errors and the resulting z-statistics. Accordingly, these results indicate that whereas the means of Factor 1 (Physical SC, Appearance) and Factor 3 (Social SC, Peers) for Australian

**TABLE 9.10**
Selected EQS Output: Group 1 Latent Mean Estimates

```
CONSTRUCT EQUATIONS WITH STANDARD ERRORS AND TEST STATISTICS
STATISTICS SIGNIFICANT AT THE 5% LEVEL ARE MARKED WITH @.
(ROBUST STATISTICS IN PARENTHESES)

  F1   =F1   =    -1.136*V999   +   1.000 D1
                    .060
                 -18.868@
              (     .058)
              ( -19.714@
  F2   =F2   =      .040*V999   +   1.000 D2
                    .062
                    .648
              (     .059)
              (     .673)
  F3   =F3   =     -.275*V999   +   1.000 D3
                    .036
                  -7.693@
              (     .034)
              (  -8.088@
  F4   =F4   =     -.076*V999   +   1.000 D4
                    .044
                  -1.730
              (     .042)
              (  -1.811)
```

adolescents were significantly different from those for Nigerian adolescents, the means for Factor 2 (Physical SC, Ability) and Factor 4 (Social SC, Parents) were not. More specifically, in light of the negative signs associated with these statistically significant values, the findings convey the notion that Physical SC as it relates to appearance, and Social SC as it relates to the peer group for Australian adolescents, appear to be less positive on average than is the case for Nigerian adolescents. That is, Australian youth report being more negative about their appearance and social self-concepts than Nigerian youth. Such a mean difference effect cannot be found when only covariances are analyzed.

The interesting question now, of course, is why these results should be what they are. Most appropriately, interpretation must be made within the context of theory and empirical research. However, when data are based on two vastly different cultural groups, such as this case, the task can be particularly challenging. Clearly, knowledge of the societal norms, values, and identities associated with each culture is an important requisite in any speculative interpretation of these findings, a task that is not undertaken herein.

# 10

# Application 8: Testing for Latent Mean Differences Based on a Second-Order CFA Model

As in chapter 9, analyses in this chapter are based on mean and covariance structures (MACS). However, whereas the previous application was based on a first-order CFA model, the work in the present chapter focuses on a higher order CFA model. Building on the work of chapter 9, we test for latent mean differences in both the lower and higher order factors of an assessment measure. In reality, however, the work of this chapter addresses an impossible situation and provides alternative approaches to how a researcher might deal with the situation. Essentially, the major interest of this chapter is to obtain a mean structure for a second-order multiple-group model. However, such models are generally not identified (Lubke, Dolan, & Kelderman, 2001). To obtain any meaningful results, additional specialized constraints have to be added to ordinary mean structure models. Although there is no truly satisfactory solution, the discussion of the issues and an illustration of several approaches on how such constraints might be considered and implemented may be fruitful.

This application is taken from a study by Byrne and Stewart (in press) in which we tested (a) for equivalence of the Beck Depression Inventory II (BDI-II; Beck, Steer, & Brown, 1996) across Hong Kong ($n = 1460$) and American ($n = 451$) nonclinical adolescents, and (b) for latent mean differences in the related underlying constructs. In this chapter, we focus on the latter and test for the extent to which the latent mean scores on three lower order factors (Negative Attitude, Performance Difficulty, Somatic Elements) and one higher order factor (General