

Application 5: Testing for the Factorial Invariance of a Measuring Instrument (First-Order CFA Model)

So far, previous applications have illustrated analyses based on single samples. In Part III, however, we focus on applications involving more than one sample in which the central concern is whether components of the measurement model and/or the structural model are invariant (i.e., equivalent) across particular groups. In seeking evidence of multigroup invariance, researchers are typically interested in finding the answer to one of five questions. First, do the items composing a particular measuring instrument operate equivalently across different populations (e.g., gender, age, ability, culture)? In other words, is the measurement model group-invariant? Second, is the factorial structure of a single instrument or theoretical construct equivalent across populations as measured either by items of a single assessment measure or by subscale scores from multiple instruments? Typically, this approach exemplifies a construct-validity focus. In such instances, invariance of both the measurement and structural models is of interest. Third, are certain paths in a specified causal structure invariant across populations? Fourth, are the latent means of particular constructs in a model different across populations? Finally, does the factorial structure of a measuring instrument replicate across independent samples drawn from the same population? This last question, of course, addresses the issue of cross-validation.

Applications presented in this and the next three chapters provide specific examples of how each question can be answered using SEM based on EQS. The applications illustrated in chapters 7 and 8 are based on the analysis of

covariance structures, whereas those in chapters 9 and 10 are based on the analysis of means and covariance structures; commonly used acronyms are the analyses of COVS and MACS, respectively. When analyses are based on COVS, only the variances and covariances of the observed variables are of interest; all single-group applications illustrated thus far have been based on the analysis of COVS. However, when analyses are based on MACS, the modeled data include both sample means and covariances. Details related to the MACS approach to invariance are addressed in chapter 9.

In the first multigroup application, hypotheses related to the invariance of a single measuring instrument are tested across two different panels of school teachers. Specifically, we test for equivalency of the factorial measurement (i.e., scale items) of the Maslach Burnout Inventory (MBI; Maslach & Jackson, 1986) and its underlying latent structure (i.e., relations among dimensions of burnout) across elementary and secondary school teachers. Purposes of the original study, from which this example is taken (Byrne, 1993), were (a) to test for the factorial validity of the MBI separately for each of three teacher groups;¹ (b) given findings of inadequate fit, to propose and test an alternative factorial structure; (c) to cross-validate this structure over independent samples within each teacher group; and (d) to test for the equivalence of item measurements and theoretical structure across the three teaching panels. Only analyses bearing on tests for invariance across calibration samples of elementary ($N = 580$) and secondary ($N = 692$) school teachers are central to this chapter. Before reviewing the model under scrutiny, however, an overview is provided of the general procedure involved in tests for invariance (or equivalence) across groups.

TESTING FOR MULTIGROUP INVARIANCE

Development of a procedure capable of testing for multigroup invariance derives from the seminal work of Jöreskog (1971b). Accordingly, Jöreskog recommended that all tests of invariance begin with a global test of the equality of covariance structures across the groups of interest. Expressed more formally, this initial step tests the null hypothesis (H_0), $\Sigma_1 = \Sigma_2 = \dots = \Sigma_G$, where Σ is the population variance-covariance matrix and G is the number of groups. Rejection of the null hypothesis argues for the nonequivalence of the groups and thus for the subsequent testing of increasingly restrictive hypotheses to identify the source of noninvariance. Conversely, if H_0 cannot be rejected, the groups are considered to have equivalent covariance structures and thus tests for invariance are not needed. Presented with such findings, Jöreskog recommended that group data be pooled and all subsequent investigative work be based on single-group analyses.

Although this omnibus test appears reasonable and fairly straightforward, it often leads to contradictory findings with respect to equivalencies across groups. For example, sometimes the null hypothesis is found to be tenable, yet subsequent tests of hypotheses related to the invariance of particular measurement or structural parameters must be rejected (see, e.g., Jöreskog, 1971b). Alternatively, the global null hypothesis may be rejected, yet tests for the invariance of measurement and structural invariance hold (see, e.g., Byrne, 1988a). Such inconsistencies in the global test for invariance stem from the fact that there is no baseline model for the test of invariant variance-covariance matrices, thereby making it substantially more restrictive than is the case for tests of invariance related to sets of model parameters. Indeed, any number of inequalities may possibly exist across the groups under study. Realistically then, testing for the equality of specific sets of model parameters appears to be the more informative and interesting approach to multigroup invariance.

In testing for equivalencies across groups, sets of parameters are put to the test in a logically ordered and increasingly restrictive fashion. Depending on the model and hypotheses to be tested, the following sets of parameters are most commonly of interest in answering questions related to multigroup invariance: (a) factor-loading paths, (b) factor covariances, and (c) structural regression paths. Historically, the Jöreskog tradition of invariance testing holds that the equality of these error variances and their covariances should also be tested. However, it is now widely accepted that to do so represents an overly restrictive test of the data. Indeed, Bentler (2005) contends that testing for the equality of error variances and covariances is probably of least interest and importance. Nonetheless, there may be particular instances where findings bearing on the equivalence or nonequivalence of these parameters can provide important information (e.g., scale items).

The Testing Strategy

Testing for factorial invariance encompasses a series of hierarchical steps that begin with the separate determination of a baseline model for each group. This model represents the one that best fits the data from the perspectives of both parsimony and substantive meaning. Addressing the somewhat tricky combination of model fit and model parsimony, it ideally represents one for which fit to the data and minimal parameter specification are optimal. Following completion of this preliminary task, tests for the equivalence of parameters are conducted across groups at each of several increasingly stringent levels. Jöreskog (1971b) argues that these tests should most appropriately begin with scrutiny of the measurement model. In particular, the pattern of factor loadings for each observed measure is tested for its equivalence across the groups. Once it is known which measures are group-invariant, these parameters are constrained equal while subsequent tests of the structural parameters are conducted. As each new set of parameters is tested,

¹Middle-school teachers composed the third group.

process of determining nonequivalence of measurement and structural parameters across groups involves the testing of a series of increasingly restrictive hypotheses.

TESTING FOR INVARIANCE ACROSS INDEPENDENT SAMPLES

The Hypothesized Model

The focus here is to test for the equivalence of the MBI across elementary and secondary school teachers. Details regarding the structure of this measuring instrument are discussed in chapter 4; thus, they will not be repeated here. Of key importance in testing for the invariance of the MBI across these two selected groups of teachers is its determined baseline model structure for each group, not the extent to which the instrument may have been validated for other populations. Once these models are established, they (if two different baseline models are ascertained) represent the hypothesized model under test.

Establishing Baseline Models

Because the estimation of baseline models involves no between-group constraints, the data can be analyzed separately for each group. However, in testing for invariance, equality constraints are imposed on particular parameters; therefore, the data for all groups must be analyzed simultaneously to obtain efficient estimates (Bentler, 2005; and Jöreskog & Sörbom, 1996). The pattern of fixed and free parameters nonetheless remains consistent with the baseline model specification for each group. However, measuring instruments are often group-specific in the way they operate; thus, it is possible that baseline models may not be completely identical across groups (Bentler, 2005; and Byrne, Shavelson, & Muthén, 1989). For example, it may be that the best-fitting model for one group includes an error covariance (see, e.g., Bentler, 2005) or a cross-loading² (see, e.g., Byrne, 1988b; and Reise, Widaman, & Pugh, 1993), whereas these parameters may not be specified for the other group. Presented with such findings, Byrne et al. (1989) showed that by implementing a condition of partial measurement invariance, multigroup analyses can still continue. As such, some but not all measurement parameters are constrained equal across groups in the testing for structural invariance or latent factor mean differences. A priori knowledge of such group differences, as illustrated in this chapter, is critical to the application of invariance-testing procedures.

In testing for the validity of the three-factor structure of the MBI (see Fig. 4.1), findings were consistent in revealing goodness-of-fit statistics for this initial model that were less than optimal for both elementary teachers ($S-B \chi^2_{(206)} = 804.084$; SRMR = .071; *CFI = .855; *RMSEA = .071; 90% C.I. .066, .076) and secondary teachers ($S-B \chi^2_{(206)} = 996.069$; SRMR = .080; *CFI = .833; *RMSEA = .075; 90% C.I. .070, .079). Three exceptionally large error covariances and one cross-loading contributed identically to the misfit of the model for both teacher panels. The error covariances involved Items 1 and 2, Items 6 and 16, and Items 10 and 11; the cross-loading involved the loading of Item 12 on Factor 1 (Emotional Exhaustion) in addition to its targeted Factor 3 (Personal Accomplishment). Indeed, these findings replicate those reported in chapter 4 for male elementary school teachers. (For a discussion related to possible reasons for these misfitting parameters, see chap. 4.) To review the extent to which these four parameters erode fit of the originally hypothesized model to data for elementary and secondary school teachers we turn to Table 7.1, which is an abbreviated list of results from the LM Test that include the Ordered Univariate Test Statistics and the χ^2 Univariate Increments associated with the Cumulative Multivariate Statistics.

Review of both the parameter change statistics and the univariate increments of the multivariate statistics for elementary school teachers, clearly shows that all four parameters are contributing substantially to model misfit. The error covariance between Item 6 and Item 16 exhibit the most profound effect. The content of both items, focuses on the extent to which working with people all day is stressful.³ These results, as they relate to secondary school teachers, exhibit precisely the same pattern, albeit the effect appears, to be even more pronounced than it was for elementary school teachers. However, there is one slight difference between the two groups of teachers regarding the impact of the four parameters on model misfit: Whereas the error covariance between Items 6 and 16 was found to be the most seriously misfitting parameter for elementary school teachers, the error covariance between Items 1 and 2 held this dubious honor for secondary school teachers. These items both focus on the extent to which teachers feel emotionally drained or used up from their work at the end of the day.

Given that both the cross-loading of Item 12 on Factor 1 and the error covariances can be substantively justified (for an elaboration, see chap. 4), they were subsequently specified as free parameters in the model for each teacher group and then the models were reestimated. Results from these analyses for both groups yielded model-fit statistics that were substantially and significantly improved

²This is the loading of an observed variable on a factor other than the one on which it was designed to load (i.e., its targeted factor).

³Unfortunately I am unable to present the actual item context here as the publisher of the MBI refused copyright permission to do so. As a result, I am able only to describe the context of these and other items exhibiting poor loadings (Items 11, 9, 19).

TABLE 7.1
Selected EQS Output for Initially Hypothesized Model: Modification Indexes

LAGRANGE MULTIPLIER TEST (FOR ADDING PARAMETERS)									
ORDERED UNIVARIATE TEST STATISTICS:									
NO	CODE	PARAMETER	CHI-SQUARE		PROB.	HANCOCK		STANDARDIZED CHANGE	MSEA
			CHI-SQUARE	PROB.		PARAMETER	D.F.		
1	2	6	E16, E5	180.06	.000	.904	.895	.595	
2	2	6	E2, E1	102.999	.000	1.000	.535	.494	
3	2	12	V12, F1	81.163	.000	1.000	-.400	-.241	
4	2	6	E11, E10	67.616	.000	1.000	.687	.828	
CUMULATIVE MULTIVARIATE STATISTICS									
UNIVARIATE INCREMENT									
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	CHI-SQUARE	PROB.	D.F.	PROB.	HANCOCK'S SEQUENTIAL
1	E16, E6	180.006	1	.000	180.006	.000	206	.904	
2	E2, E1	277.423	2	.000	97.416	.000	205	1.000	
3	V12, F1	358.585	3	.000	81.163	.000	204	1.000	
4	E11, E10	426.201	4	.000	67.616	.000	203	1.000	
5	E7, E4	466.869	5	.000	40.668	.000	202	1.000	
6	E19, E18	501.328	6	.000	34.459	.000	201	1.000	
7	V16, F2	533.006	7	.000	31.678	.000	200	1.000	
8	V14, F3	556.432	8	.000	23.426	.000	199	1.000	
Secondary Teachers									
LAGRANGE MULTIPLIER TEST (FOR ADDING PARAMETERS)									
ORDERED UNIVARIATE TEST STATISTICS:									
NO	CODE	PARAMETER	CHI-SQUARE		PROB.	HANCOCK	206 DF	STANDARDIZED DZED	MSEA
			CHI-SQUARE	PROB.		PARAMETER	CHANGE	CHANGE	
1	2	6	E2, E1	167.986	.000	.976	.618	.577	
2	2	6	E11, E10	142.389	.000	1.000	1.219	1.509	
3	2	6	E16, E6	130.671	.000	1.000	.692	.464	
4	2	12	V12, F1	118.034	.000	1.000	-.467	-.291	
CUMULATIVE MULTIVARIATE STATISTICS									
UNIVARIATE INCREMENT									
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	CHI-SQUARE	PROB.	D.F.	PROB.	HANCOCK'S SEQUENTIAL
1	E2, E1	167.986	1	.000	167.986	.000	205	.976	
2	E11, E10	310.374	2	.000	142.389	.000	205	1.000	
3	E16, E6	435.076	3	.000	124.702	.000	204	1.000	
4	V12, F1	553.110	4	.000	118.034	.000	203	1.000	
5	V11, F1	609.593	5	.000	56.483	.000	202	1.000	
6	E19, E9	652.274	6	.000	42.681	.000	201	1.000	
7	E20, E2	685.208	7	.000	32.934	.000	200	1.000	
8	V16, F2	715.198	8	.000	29.990	.000	199	1.000	

from those for the initially hypothesized model (i.e., elementary: corrected $\Delta S-B\chi^2_{(4)} = 197.159$; secondary: corrected $\Delta S-B\chi^2_{(4)} = 268.560$). Additionally, all newly specified parameters were statistically significant. Model goodness-of-fit statistics were as follows:⁴

Elementary School Teachers: $S-B\chi^2_{(202)} = 471.180$; SRMR = .053; *CFI = .935; *RMSEA = .048, 90% C.I. .042, .054
 Secondary School Teachers: $S-B\chi^2_{(202)} = 587.190$; SRMR = .059; *CFI = .918; *RMSEA = .053, 90% C.I. .048, .057

Let's turn now to Table 7.2 where an abbreviated list of the LM Test statistics associated with this modified model (Model 2) is presented. Review of these results for elementary school teachers shows that the two highest incremental χ^2 values represent error covariances (i.e., E7,E4 and E19,E18). Although, admittedly, the third parameter in this list represents a cross-loading that exhibits a fairly substantial parameter change value (i.e., .864), I consider it best to leave this model in place as the baseline model for this group. I make this decision based on the fact that (a) the model represents a fairly well-fitting model, and (b) the addition of more error covariances that are not distinctively different from one another runs counter to the criterion of parsimony.

In reviewing the results for secondary school teachers, however, there is more work to do in establishing an appropriate baseline model. This decision is based on two factors: (a) the model does not yet reflect a satisfactorily good fit to the data, and (b) in reviewing the multivariate LM Test statistics, there are perhaps two additional parameters (i.e., V11,F1 and E19,E9) that require scrutiny, as evidenced by both the large χ^2 values and the substantial drop between these values (71.235 and 48.570) and those for the remaining three parameters shown here (34.638, 29.330, and 26.730). The cross-loading of Item 11 (which expresses the concern that the job is hardening the teacher emotionally) onto Factor 1 (Emotional Exhaustion) appears to be reasonable. Likewise, a covariance between the error terms associated with Item 9 and Item 19 is meaningful in the sense that it reflects some degree of overlap in content. Item 19 measures the extent to which respondents believe they have accomplished many worthwhile things in the job; likewise, Item 9 measures the extent to which they believe they are positively influencing other peoples lives through their work. Consequently, both parameters were freely estimated and this new model (Model 3) was reestimated.⁵ Results from the estimation of Model 3, for secondary school teachers, yielded goodness-of-fit statistics that represented a

⁴The decrease in the number of degrees of freedom from 206 to 202 represents the estimation of four additional parameters in the model, thereby using up 4 degrees of freedom.
⁵As a cautionary measure in adding these parameters to the model, each parameter was originally specified separately (i.e., two separate models). Provided with no aberrant differences in the results, I therefore suggest the inclusion of both parameters in this newly specified model.

TABLE 7.2
Selected EQS Output for Model 2: Modification Indexes

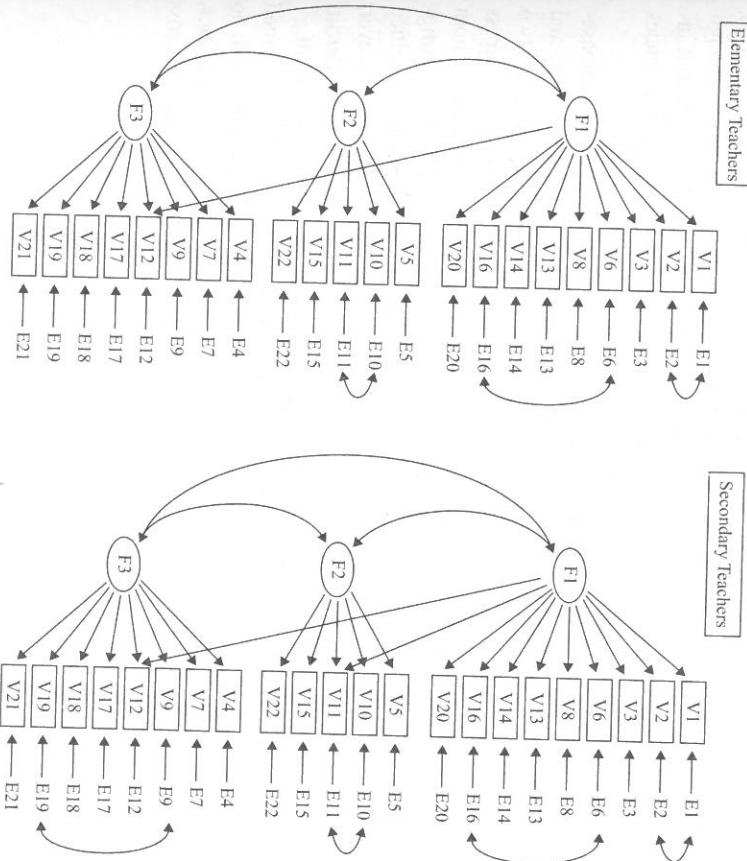
Elementary Teachers						
LAGRANGE MULTIPLIER TEST (FOR ADDING PARAMETERS)						
ORDERED UNIVARIATE TEST STATISTICS:						
NO	CODE	PARAMETER	CHI-SQUARE	D.F.	PROB.	HANCOCK
1	2	E7, E4	38.867	.000	1.000	206 DF PROB.
2	2	E6	38.659	.000	1.000	206 DF PROB.
3	2	E19, E18	24.387	.000	1.000	206 DF PROB.
4	2	E12, E3	23.940	.000	1.000	206 DF PROB.
5	2	E13, E12	20.444	.000	1.000	206 DF PROB.

CUMULATIVE MULTIVARIATE STATISTICS						
UNIVARIATE INCREMENT						
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	HANCOCK'S SEQUENTIAL D.F.	HANCOCK'S SEQUENTIAL D.F.
1	E7, E4	38.867	1	.000	38.867 .000	38.867 .000
2	E19, E18	73.644	2	.000	34.777 .000	202 1.000
3	V14, F3	98.031	3	.000	24.387 .000	200 1.000
4	E12, E3	121.544	4	.000	23.513 .000	199 1.000
5	E13, E12	136.775	5	.000	15.230 .000	198 1.000

Secondary Teachers						
LAGRANGE MULTIPLIER TEST (FOR ADDING PARAMETERS)						
ORDERED UNIVARIATE TEST STATISTICS:						
NO	CODE	PARAMETER	CHI-SQUARE	D.F.	PROB.	HANCOCK
1	2	V11, F1	71.235	.000	1.000	206 DF PROB.
2	2	E19, E9	48.570	.000	.470	206 DF PROB.
3	2	E15, E11	38.034	.000	.371	206 DF PROB.
4	2	E15, E5	36.869	.000	.333	206 DF PROB.
5	2	E20, E16	29.335	.000	.323	206 DF PROB.

CUMULATIVE MULTIVARIATE STATISTICS						
UNIVARIATE INCREMENT						
STEP	PARAMETER	CHI-SQUARE	D.F.	PROB.	HANCOCK'S SEQUENTIAL D.F.	HANCOCK'S SEQUENTIAL D.F.
1	V11, F1	71.235	1	.000	71.235 .000	202 1.000
2	E19, E9	119.806	2	.000	48.570 .000	201 1.000
3	E18, E7	154.443	3	.000	34.638 .000	200 1.000
4	E20, E16	183.773	4	.000	29.330 .000	199 1.000
5	E15, E5	210.503	5	.000	26.730 .000	198 1.000

FIG. 7.1. Hypothesized multigroup model of factorial structure of the Maslach Burnout Inventory (Maslach & Jackson, 1986).



satisfactorily good fit to the data ($S-B\chi^2_{(200)} = 500.332$; SRMR = .054; *CFI = .936; *RMSEA = .047, 90% C.I. .042, .052). This final model is determined to be the baseline model for secondary school teachers.

With these baseline models established, we are now ready to test hypotheses bearing on the equivalence of the MBI across elementary and secondary school teachers. The baseline models are shown in Fig. 7.1, which provides the foundation against which the series of increasingly stringent hypotheses related to MBI structure are tested.

Testing for Configural Invariance

This initial step in testing for invariance requires only that the same number of factors and factor-loading pattern be the same across groups. As such, no equality constraints are imposed on the parameters. That is, the same parameters estimated in the baseline model for each group separately are again estimated in this

multigroup model. Goodness-of-fit related to this multigroup parameterization should be indicative of a well-fitting model. Of importance here is that although the factor structure is similar, it is not identical. More specifically, given that invariant latent factors have not been identified in the two groups, group differences on any parameters in the model cannot be tested.

In essence, the model being tested is a multigroup representation of the baseline models. Accordingly, it incorporates the baseline models for elementary and secondary school teachers within the same file. This multigroup model serves two important functions: First, it allows for invariance tests to be conducted across the two groups simultaneously. In other words, parameters are estimated for both groups at the same time. Second, in testing for invariance, the fit of this configural model provides the baseline value against which all subsequently specified invariance models are compared. In contrast to single-group analyses, however, this multigroup analysis yields only one set of fit statistics for overall model fit. When ML estimation is used, the χ^2 statistics are summative; thus, the overall χ^2 value for the multigroup model should equal the sum of the χ^2 values obtained when the baseline model is tested separately for each group of teachers (with no cross-group constraints imposed). If estimation is based on the robust statistics, the S-B χ^2 statistic for each group separately is not necessarily summative across the groups.

The EQS Input File

To fully comprehend the specification of parameters representing the configural model, we turn Table 7.3 where the related multigroup input file is presented. In reviewing this file, you will see that it actually contains two sets of model parameters: The first part of the file specifies the model for elementary school teachers, whereas the second part specifies the model for secondary school teachers. Closer examination of each baseline model reveals the post hoc addition of parameters for each teacher group. Thus, for Group 1 (Elementary Teachers), note the cross-loading of V12 onto F1 in addition to the three error covariances (E2,E1; E16,E6; and E11,E10). For Group 2 (Secondary Teachers), there are the same four specifications but in addition, the cross-loading of V11 onto F1 and the error covariance between Items 19 and 9 (E19,E9). Basically, this initial multigroup file simply represents the input file for one group stacked on top of the input file for the other; however, there is one very critical piece of information that must be added to the /SPECIFICATION paragraph for the first group: the statement “GROUPS = 2;”. This command tells the program how many groups are involved in the analysis. By default, if this information is not specified, the program assumes there is only one group and all action ceases when the analyses for Group 1 are completed.

TABLE 7.3

EQS Input File for Test for Invariance of MBI: The Configural Model
TESTING FOR INVARIANCE OF THE MBI FOR ELEMENTARY/SECONDARY TEACHERS "MBIINVehls1"
/TITLE

GROUP 1: ELEMENTARY TEACHERS

/SPECIFICATIONS

DATA='C:\EQS6\files\books\data\mbienvehls1.ess';

VARIABLES=22; CASES= 580; GROUPS = 2;

METHODS=ML,ROBUST; MATRIX=RAW; FO='(22F1,0)';

/LABELS

V1=MB11; V2=MB12; V3=MB13; V4=MB14; V5=MB15; V6=MB16; V7=MB17; V8=MB18;

V9=MB19; V10=MB110; V11=MB111; V12=MB112; V13=MB113; V14=MB114;

V15=MB115; V16=MB116; V17=MB117; V18=MB118; V19=MB119; V20=MB120;

V21=MB121; V22=MB122;

F1=EE; F2=DP; F3=PA;

/EQUATION

V1 = F1 + E1;

V2 = *F1 + E2;

V3 = *F1 + E3;

V6 = *F1 + E6;

V8 = *F1 + E8;

V13 = *F1 + E13;

V14 = *F1 + E14;

V16 = *F1 + E16;

V20 = *F1 + E20;

V5 = F2 + E5;

V10 = *F2 + E10;

V11 = *F2 + E11;

V15 = *F2 + E15;

V22 = *F2 + E22;

V4 = F3 + E4;

V7 = *F3 + E7;

V9 = *F3 + E9;

V12 = *F3 + *F1 + E12;

V17 = *F3 + E17;

V18 = *F3 + E18;

V19 = *F3 + E19;

V21 = *F3 + E21;

/VARIANCES

F1 to F3 = *;

E1 to E22 = *;

/COVARIANCES

F1 to F3 = *;

E2,E1 = *, E16,E6 = *, E11,E10 = *,

/PRINT

Fit = All;

/END

The EQS Output File

TABLE 7.3
(Continued)

```

/TITLE
TESTING FOR INVARIANCE OF THE MBI FOR ELEMENTARY/SECONDARY TEACHERS 'MBIINVehsl',
GROUP 2: SECONDARY TEACHERS

/SPECIFICATIONS
DATA='C:\EQS61\files\books\data\mbisec1.ess';
VARIABLES=22; CASES=692; DEL=.115;
METHODS=ML,ROBUST; MATRIX=RAW; FO='(22F1.0)';

/LABELS
V1=MB11; V2=MB12; V3=MB13; V4=MB14; V5=MB15; V6=MB16; V7=MB17; V8=MB18;
V9=MB19; V10=MB110; V11=MB111; V12=MB112; V13=MB113; V14=MB114;
V15=MB115; V16=MB116; V17=MB117; V18=MB118; V19=MB119; V20=MB120;

V21=MB121; V22=MB122;

F1=EE; F2=DP; F3=PA;

/EQUATION
V1=F1+E1;
V2=*F1+E2;
V3=*F1+E3;
V6=*F1+E6;
V8=*F1+E8;
V13=*F1+E3;
V14=*F1+E14;
V16=*F1+E16;
V20=*F1+E20;
V5=F2+E5;
V10=*F2+E10;
V11=*F2+*F1+E11;
V15=*F2+E15;
V22=*F2+E22;
V4=F3+E4;
V7=*F3+E7;
V9=*F3+E9;
V12=*F3+*F1+E12;
V17=*F3+E17;
V18=*F3+E18;
V19=*F3+E19;
V21=*F3+E21;

/VARIANCES
F1 to F3=.*;
E1 to E22=.*;
/COVARANCES
F1 to F3=.*;
E2,E1=*,E16,E6=*,E11,E10=.*;
E19,E9=.*;
/PRINT
Fit=All;
/END

```

The next step addresses the question of equality with respect to the measurement model. That is, only the invariance of factor loadings and measurement error variances–covariances are of interest. Typically, however, primary interest focuses on the factor loadings because tests for the equivalence of error variances–covariances are now widely considered excessively stringent. Nonetheless, from a psychometric perspective, when invariance hypotheses bear on a

TABLE 7.4
Selected EQS Output for the Configural Model: Goodness-of-Fit Statistics

GOODNESS OF FIT SUMMARY FOR METHOD = ML		
CHI-SQUARE =	1221.325	BASED ON 402 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS		.00000
FIT INDICES		
BENTLER-BONETT	NORMED FIT INDEX =	.899
BENTLER-BONETT NON-NORMED FIT INDEX =		.919
COMPARATIVE FIT INDEX (CFI) =		.929
ROOT MEAN-SQUARE RESIDUAL (RMR) =		.111
STANDARDIZED RMR =		.053
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) =		.040
90% CONFIDENCE INTERVAL OF RMSEA (.037, .043) =		
GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST		
SATORRA-BENTLER SCALDED CHI-SQUARE =	971.9910	ON 402 DEGREES OF FREEDOM
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS		.00000
FIT INDICES		
BENTLER-BONETT	NORMED FIT INDEX =	.896
BENTLER-BONETT NON-NORMED FIT INDEX =		.926
COMPARATIVE FIT INDEX (CFI) =		.936
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) =		.033
90% CONFIDENCE INTERVAL OF RMSEA (.031, .036) =		

measuring instrument, it seems important to know whether additionally specified parameters—such as cross-loadings and error covariances—hold across groups (i.e., they are group-equivalent). These parameterizations speak for the quality of such items in the measurement of their related latent factors. In testing for invariance of the MBI measurement model across elementary and secondary school teachers, then, I include the one cross-loading (V12,F1) and three error covariances (E2,E1; E16,E6; E11,E10) found commonly specified for each group.

Testing for the equivalence of factor loadings entails specification of equality constraints for all freely estimated factor loadings that are similarly specified in both baseline models. The key conditions here require the factor loadings to be (a) freely estimated (as opposed to fixed at some value), and (b) consistently specified across groups. The first criterion addresses the issue of model identification and latent variable scaling (see chap. 2); the second addresses the issue of partial measurement invariance (addressed later in this chapter). For clarification of these points, the reader may review either the graphic representation of the model under study (see Fig. 7.1) or the EQS input file (see Table 7.3). The fixed parameters in the model for both groups are the factor loadings representing Item 1 (V1,F1), Item 5 (V1,F2), and Item 4 (V4,F3). Review of the input file shows that these parameters are not accompanied by an asterisk (*), indicating that they are fixed to a value of 1.0 (by default) by the program. On the issue of consistent specification, we note that whereas one cross-loading (*V11,F1) and one error covariance (*E19,E9) were specified for secondary school teachers, it was not so for elementary school teachers. Thus, in testing for factor-loading invariance, equality constraints are placed on all those that are freely estimated except the one that is unique to secondary school teachers. Likewise, in testing for the invariance of the error covariances, equality constraints are specified for all except the one involving Items 19 and 9 (E19,E9). The portion of the input file where these constraints are specified in shown in Table 7.5.

The EQS Input File

In testing for group invariance, equality constraints are specified in a paragraph labeled /CONSTRAINTS, which must be included in the file for the last group only. Thus, in the present case, this paragraph appears as a component of the specified model for secondary school teachers. Examination of the /CONSTRAINTS paragraph in Table 7.5 reveals that one equality specification is required for each parameter being constrained equal across groups, with the parameter relative to

Group 1 being specified within the first parentheses and the parameter relative to Group 2 in the second parentheses. Note also that (a) all constrained parameters are estimable; and (b) given the same loading pattern for elementary and secondary school teachers, the cross-loading of Item 12 on the Emotional Exhaustion factor (V12,F1) is also tested for its equivalence across groups. Finally, unlike previous LMTEST paragraphs observed thus far, the one shown in Table 7.5 involves no

TABLE 7.5

EQS Input for Test for Invariance of the Measurement Model: Equality Constraints

TITLE
TESTING FOR INVARIANCE OF THE MBI FOR ELEMENTARY/SECONDARY TEACHERS "MBIINvchs1"
GROUP2: SECONDARY TEACHERS

```
/PRINT
Fit = All;
/CONSTRAINTS
(V12,F1) = (2,V2,F1);
(V3,F1) = (2,V3,F1);
(V6,F1) = (2,V6,F1);
(V8,F1) = (2,V8,F1);
(V13,F1) = (2,V13,F1);
(V14,F1) = (2,V14,F1);
(V16,F1) = (2,V16,F1);
(V20,F1) = (2,V20,F1);
(V10,F2) = (2,V10,F2);
(V15,F2) = (2,V15,F2);
(V22,F2) = (2,V22,F2);
(V7,F3) = (2,V7,F3);
(V9,F3) = (2,V9,F3);
(V12,F3) = (2,V12,F3);
(V12,F1) = (2,V12,F1);
(V17,F3) = (2,V17,F3);
(V18,F3) = (2,V18,F3);
(V19,F3) = (2,V19,F3);
(V21,F3) = (2,V21,F3);
(E2,E1) = (2,E2,E1);
(E16,E6) = (2,E16,E6);
(E11,E10) = (2,E11,E10);



---



```

SET command related to particular parameters. This omission exists because with multigroup invariance, the purpose of the LM Test is to test the null hypothesis that each specified constraint is true in the population.

The EQS Output File

Let's turn now to the results of this initial test for invariance, which are presented in Table 7.6. Shown first are the goodness-of-fit statistics describing the entire multigroup model. Conventionally, it is argued that invariance holds if goodness-of-fit related to this model is deemed adequate (Widaman & Reise, 1997) and if there is minimal difference in fit from that of the configural model. Likewise, these same criteria hold in the testing of all subsequent invariance models. Review of

TABLE 7.6
Selected EQS Output for Testing Invariance of Measurement Model: Goodness-of-Fit Statistics

GOODNESS OF FIT SUMMARY FOR METHOD = ML		LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)	
CONSTRAINTS TO BE RELEASED ARE:	CONSTRAINTS FROM GROUP 2	CONSTRAINTS TO BE RELEASED ARE:	CONSTRAINTS FROM GROUP 2
CHI-SQUARE = 1268.803 BASED ON 424 DEGREES OF FREEDOM			
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .00000			
FIT INDICES			
BENTLER-BONETT NORMED FIT INDEX = .895	CONSTR: 1 (1, V2, F1) - (2, V3, F1) = 0;		
BENTLER-BONETT NON-NORMED FIT INDEX = .921	CONSTR: 2 (1, V6, F1) - (2, V6, F1) = 0;		
COMPARATIVE FIT INDEX (CFI) = .927	CONSTR: 3 (1, V6, F1) - (2, V6, F1) = 0;		
ROOT MEAN-SQUARE RESIDUAL (RMR) = .118	CONSTR: 4 (1, V8, F1) - (2, V8, F1) = 0;		
STANDARDIZED RMR	CONSTR: 5 (1, V13, F1) - (2, V13, F1) = 0;		
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) = .058	CONSTR: 6 (1, V14, F1) - (2, V14, F1) = 0;		
90% CONFIDENCE INTERVAL OF RMSEA (.037, .042)	CONSTR: 7 (1, V16, F1) - (2, V16, F1) = 0;		
	CONSTR: 8 (1, V20, F1) - (2, V20, F1) = 0;		
	CONSTR: 9 (1, V10, F2) - (2, V10, F2) = 0;		
	CONSTR: 10 (1, V15, F2) - (2, V15, F2) = 0;		
GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST	CONSTR: 11 (1, V22, F2) - (2, V22, F2) = 0;		
SATORRA-BENTLER SCALED CHI-SQUARE = 1010.4789 ON 424 DEGREES OF FREEDOM	CONSTR: 12 (1, V7, F3) - (2, V7, F3) = 0;		
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .00000	CONSTR: 13 (1, V9, F3) - (2, V9, F3) = 0;		
FIT INDICES	CONSTR: 14 (1, V12, F3) - (2, V12, F3) = 0;		
BENTLER-BONETT NORMED FIT INDEX = .892	CONSTR: 15 (1, V12, F1) - (2, V12, F1) = 0;		
BENTLER-BONETT NON-NORMED FIT INDEX = .928	CONSTR: 16 (1, V17, F3) - (2, V17, F3) = 0;		
COMPARATIVE FIT INDEX (CFI) = .934	CONSTR: 17 (1, V18, F3) - (2, V18, F3) = 0;		
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) = .033	CONSTR: 18 (1, V19, F3) - (2, V19, F3) = 0;		
90% CONFIDENCE INTERVAL OF RMSEA (.030, .036)	CONSTR: 19 (1, V21, F3) - (2, V21, F3) = 0;		
	CONSTR: 20 (1, E2, E1) - (2, E2, E1) = 0;		
	CONSTR: 21 (1, E16, E6) - (2, E16, E6) = 0;		
	CONSTR: 22 (1, E11, E10) - (2, E11, E10) = 0;		

the goodness-of-fit results reported in Table 7.6 show that despite the imposition of equality constraints on all appropriate factor loadings, as well as three error covariances, the multigroup model underwent some deterioration in model fit (corrected $\Delta S\text{-}B\chi^2 = 38.350$, $p < .01$; $\Delta^*CFI = .02$). Overall, however, the multigroup model still exhibits a good fit to the data ($*CFI = .934$; $*RMSEA = .033$; 90% C.I. = .030, .036).

To determine which (if any) parameters were found not to be equivalent across elementary and secondary school teachers, we turn to Table 7.7. As shown here, the program first echoes the constraints specified and then presents the results. Although the program yields both univariate and multivariate tests of hypotheses, only the multivariate results are presented. Associated with each constraint is a cumulative multivariate LM Test χ^2 and an incremental univariate χ^2 value, along with their probability values. To locate parameters that are noninvariant across groups, we look for probability values associated with the incremental univariate χ^2 values that are $<.05$. Review of these values, as reported in Table 7.7, reveals three parameters that are not operating equivalently across elementary and secondary school teachers (i.e., Constraints 10, 22, and 11). A check of these specified constraints reveals two factor loadings (i.e., V15,F2 and V22,F2) and one commonly specified error covariance (i.e., E11,E10) to be noninvariant across the two teacher groups.

TABLE 7.7
Selected EQS Output for Testing Invariance of Measurement Model: LM Test Statistics

CUMULATIVE MULTIVARIATE STATISTICS				UNIVARIATE INCREMENT		
STEP	PARAMETER	CHI-SQUARE	D.F.	PROBABILITY	CHI-SQUARE	PROBABILITY
1	CONSTR: 10	5.374	1	.020	5.374	.020
2	CONSTR: 22	12.330	2	.002	6.956	.008
3	CONSTR: 11	18.259	3	.000	5.929	.015
4	CONSTR: 16	21.928	4	.000	3.669	.055
5	CONSTR: 21	24.719	5	.000	2.792	.095
6	CONSTR: 12	26.350	6	.000	1.631	.202
7	CONSTR: 15	28.111	7	.000	1.761	.185
8	CONSTR: 5	30.703	8	.000	2.592	.107
9	CONSTR: 3	32.636	9	.000	1.934	.164
10	CONSTR: 9	33.889	10	.000	1.252	.263

It is interesting that all items found to be noninvariant are designed to measure the factor of Depersonalization. Thus, it appears that whereas the MBI may be a viable measure of the emotional exhaustion and personal accomplishment components of teacher burnout, item content designed to measure the depersonalization component is being differentially interpreted by elementary and secondary school teachers. Having determined evidence of noninvariance related to these three parameters, the question is: Can we still continue to test for invariance of the structural model (i.e., interfactor relations)? The answer to this question is "yes" if certain conditions can be met. To move ahead in testing for the invariance of the structural

TESTING FOR INVARIANCE

TABLE 7.8
Selected EQS Input for Testing Invariance of Structural Model: Equality Constraints

TITLE
TESTING FOR INVARIANCE OF THE MBI FOR ELEMENTARY/SECONDARY TEACHERS "MBIINVehs1"
GROUP 2: SECONDARY TEACHERS
CONSTRAINTS
/PRINT
Fit = All;
(V2,F1) = (2,V2,F1);
(V3,F1) = (2,V3,F1);
(V6,F1) = (2,V6,F1);
(V8,F1) = (2,V8,F1);
(V13,F1) = (2,V13,F1);
(V14,F1) = (2,V14,F1);
(V16,F1) = (2,V16,F1);
(V20,F1) = (2,V20,F1);
(V10,F2) = (2,V10,F2);
(V7,F3) = (2,V7,F3);
(V9,F3) = (2,V9,F3);
(V12,F3) = (2,V12,F3);
(V12,F1) = (2,V12,F1);
(V17,F3) = (2,V17,F3);
(V18,F3) = (2,V18,F3);
(V19,F3) = (2,V19,F3);
(V21,F3) = (2,V21,F3);
(E2,E1) = (2,E2,E1);
(E6,E6) = (2,E6,E6);
(F1,F2) = (2,F1,F2);
(F1,F3) = (2,F1,F3);
(F2,F3) = (2,F2,F3);
/LMTEST
/END

The EQS Input File

In reviewing this section of the input file in Table 7.8, there are two important points: (a) equality constraints related to the factor loadings for Item 15 (V15,F2) and Item 22 (V22,F2) are now absent, as is the constraint related to the error covariance between Items 11 and 10 (E11,E10); and (b) equality constraints are now specified for the three factor covariances (see the last three lines of the /CONSTRAINTS paragraph). Of importance here is that the equality of these structural parameters is tested while concomitantly maintaining the equality of specified measurement parameters across groups. Thus, it is easy to see why the invariance-testing criteria become increasingly stringent as a researcher progresses from tests of the measurement model to tests of the structural model.

The EQS Output File

Results related to this analysis are presented in Tables 7.9 and 7.10. Shown first in Table 7.9 are the goodness-of-fit statistics that reflect a model that still represents a good fit to the data and with negligible difference in fit from that of the configural model.⁶ Indeed, comparison with the configural model yields a nonsignificant difference in S-B χ^2 values (corrected $\Delta S\text{-}B\chi^2_{(22)} = 23.224$, $p > .05$); likewise, the difference in *CFI values was minimal ($\Delta^*\text{CFI} = .01$). However, further insight into these results comes from the LM Test statistics shown in Table 7.10. Review of the univariate χ^2 incremental values reveals two with probabilities $<.05$. These probabilities are associated with Constraint 20, which represents, the factor covariance between Emotional Exhaustion and Depersonalization (F2,F1), and Constraint 10, which represents, the loading of Item 7 on the Personal Accomplishment

(V7,F3). However, although these parameters have been pinpointed as not operating equivalently across elementary and secondary school teachers, the $\Delta S\text{-}B\chi^2$ test yielded a difference value that was not significant. Presented with these findings, it seems appropriate to adhere to results of the $\Delta S\text{-}B\chi^2$ test. As such, it is concluded that structural relations among the three factors of burnout, as measured by the modified MBI items determined in this study, are invariant across the two groups of teachers.

In this chapter, we tested for the invariance of MBI factorial structure across elementary and secondary school teachers based on the analysis of COWS. First, a baseline model for each teacher group was established separately; these analyses revealed the best-fitting model to include one cross-loading (i.e., V12,F1) and three error covariances (i.e., E2,E1; E16,E6; and E11,E10) for both teacher groups. For secondary teachers, however, this model also included an additional cross-loading (i.e., V11,F1) and one error covariance (i.e., E19,E9). Following the establishment

⁶Given that three previously specified parameters from Model 2 were not estimated but three newly

TABLE 7.9
Selected EQS Output for Testing Invariance of Structural Model: Goodness-of-Fit Statistics

GOODNESS OF FIT SUMMARY FOR METHOD = ML					
CHI-SQUARE =	1249.170	BASED ON	424 DEGREES OF FREEDOM		
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .00000					
FIT INDICES					
BENTLER-BONETT NORMED FIT INDEX =	.897				
BENTLER-BONETT NON-NORMED FIT INDEX =	.923				
COMPARATIVE FIT INDEX (CFI) =	.929				
ROOT MEAN-SQUARE RESIDUAL (RMR) =	.127				
STANDARDIZED RMR	.060				
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) =	.039				
90% CONFIDENCE INTERVAL OF RMSEA (.037,				
	.042)				
GOODNESS OF FIT SUMMARY FOR METHOD = ROBUST					
SATORRA-BENTLER SCALED CHI-SQUARE =	996.0734	ON	424 DEGREES OF FREEDOM		
PROBABILITY VALUE FOR THE CHI-SQUARE STATISTIC IS .00000					
FIT INDICES					
BENTLER-BONETT NORMED FIT INDEX =	.893				
BENTLER-BONETT NON-NORMED FIT INDEX =	.930				
COMPARATIVE FIT INDEX (CFI) =	.935				
ROOT MEAN-SQUARE ERROR OF APPROXIMATION (RMSEA) =	.033				
90% CONFIDENCE INTERVAL OF RMSEA (.030,				
	.035)				

of these baseline models and based on partial-measurement invariance, the equivalence of common parameters composing both the measurement (factor loadings and error covariances) and structural (factor covariances) models across groups was tested. Based on results from implementation of the LM Test, only two factor loadings (i.e., V15,F2 and V22,F2) and one error covariance (i.e., E11,E10) were determined to be noninvariant across elementary and secondary school teachers. In the interest of completeness regarding the topic of multigroup invariance, there are two aspects of these analyses that require further elaboration. Although both issues are topics of considerable debate in the literature, they nonetheless need to be addressed: these are partial measurement and appropriate evaluative criteria in determining evidence of invariance.

OTHER CONSIDERATIONS IN TESTING FOR MULTIPLE-GROUP INVARIANCE

The Issue of Partial-Measurement Invariance

Perhaps the first paper to discuss the issue of partial-measurement invariance was that of Byrne et al., 1989, which addressed the difficulty commonly encountered in

TABLE 7.10
Selected EQS Output for Testing Invariance of Structural Model: LM Test Statistics

LAGRANGE MULTIPLIER TEST (FOR RELEASING CONSTRAINTS)					
CONSTRAINTS TO BE RELEASED ARE:					
CONSTRAINTS FROM GROUP 2					
CONSTR: 1	(1,V2,F1)-(2,V2,F1)=0;				
CONSTR: 2	(1,V3,F1)-(2,V3,F1)=0;				
CONSTR: 3	(1,V6,F1)-(2,V6,F1)=0;				
CONSTR: 4	(1,V8,F1)-(2,V8,F1)=0;				
CONSTR: 5	(1,V13,F1)-(2,V13,F1)=0;				
CONSTR: 6	(1,V14,F1)-(2,V14,F1)=0;				
CONSTR: 7	(1,V15,F1)-(2,V15,F1)=0;				
CONSTR: 8	(1,V20,F1)-(2,V20,F1)=0;				
CONSTR: 9	(1,V10,F2)-(2,V10,F2)=0;				
CONSTR: 10	(1,V7,F3)-(2,V7,F3)=0;				
CONSTR: 11	(1,V9,F3)-(2,V9,F3)=0;				
CONSTR: 12	(1,V12,F3)-(2,V12,F3)=0;				
CONSTR: 13	(1,V12,F1)-(2,V12,F1)=0;				
CONSTR: 14	(1,V17,F3)-(2,V17,F3)=0;				
CONSTR: 15	(1,V18,F3)-(2,V18,F3)=0;				
CONSTR: 16	(1,V19,F3)-(2,V19,F3)=0;				
CONSTR: 17	(1,V21,F3)-(2,V21,F3)=0;				
CONSTR: 18	(1,E2,E1)-(2,E2,E1)=0;				
CONSTR: 19	(1,E16,E6)-(2,E16,E6)=0;				
CONSTR: 20	(1,F1,F2)-(2,F1,F2)=0;				
CONSTR: 21	(1,F1,F3)-(2,F1,F3)=0;				
CONSTR: 22	(1,F2,F3)-(2,F2,F3)=0;				
CUMULATIVE MULTIVARIATE STATISTICS					
STEP	PARAMETER	CHI-SQUARE	D.F.	PROBABILITY	CHI-SQUARE PROBABILITY
1	CONSTR: 20	4.821	1	.028	4.821 .028
2	CONSTR: 10	9.735	2	.008	4.914 .027
3	CONSTR: 14	11.838	3	.008	2.103 .147
4	CONSTR: 19	13.520	4	.009	1.682 .195
5	CONSTR: 13	14.676	5	.012	1.156 .282
6	CONSTR: 5	16.417	6	.012	1.741 .187
7	CONSTR: 3	17.930	7	.012	1.512 .219
8	CONSTR: 9	19.361	8	.013	1.431 .232
9	CONSTR: 18	20.390	9	.016	1.029 .310
10	CONSTR: 11	21.007	10	.021	.617 .432

testing for multigroup invariance whereby certain parameters in the measurement model (typically, factor loadings) are found to be noninvariant across the groups of interest. At the time of writing that paper, researchers were generally under the impression that faced with such results, one should not continue to test for invariance of the structural model. Byrne and colleagues (1989) showed that as