ETH zürich

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Visual Computing

Exercise 08: Light & Colors

Hand-out Date: 12 November 2024

Goals

- Understand the theoretical basics of color spaces.
- Working with the CIE-Chart.

Resources

The lecture slides and exercise slides are accessible via the Visual Computing Course Web Page.

Tasks

1. Color Modes

- How do you transform a specification in RGB into CMY?

  Since CMY and RGB color spaces are complementary to each other:

$$[C \quad M \quad Y] = [1 \quad 1 \quad 1] - [R \quad G \quad B]$$

- Why were color spaces such as RGB, CMY, YIQ, and HSL specified, and where are they being applied?

  RGB: Additive color space used for color monitors. Neighboring color pixels are mixed to one color by the human eye.

  CMY: Subtractive color space used in color printers where transparent color pigments are added to a white background.

  YIQ: Is based on the psycho physical properties of the human eye. Most of the color space resolution is used for color tones, which can best be distinguished by the human eye (like skin tones). Other color tones have a lower resolution. This color space is being used in the NTSC-Norm used in television.

  HSV: "Subjective color model". Different colors can be picked in an easier way. It is being used in art etc.

  HLS: Alternative color space to HSV, which is being used for the same applications.

- Provide the values for a medium gray in the following color modes: RGB, CMY, YIQ and HSV.

  RGB: [0.5, 0.5, 0.5]

  CMY: [0.5, 0.5, 0.5]

  CMYK: e.g. [0, 0, 0, 0.5]

  YIQ:

$$\begin{bmatrix} Y \\ I \\ Q \end{bmatrix} = \begin{bmatrix} 0.299 & 0.587 & 0.114 \\ 0.596 & -0.275 & -0.321 \\ 0.212 & -0.523 & 0.311 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix}$$

  HSV: [. . . ,0, 0.5]

  HLS: [. . . ,0.5, 0]

2. RGB Color Space and White Point Calibration

The RGB color space is a subspace of the XYZ color space. Assume the base vectors are directly related to the used phosphors often used in monitors, also known as ITU-R BT.709-standard. The color components x and y of the RGB base vectors are given in the table below.

	R	G	B
x	0.64	0.30	0.15
y	0.33	0.60	0.06

The white point is identified as (0.9505, 1.0000, 1.0890).

- Name one advantage and one disadvantage of the RGB color space. Furthermore, list one color space each, which does not have this advantage or disadvantage.

  Advantage and colorspace not having this advantage:

  - Useful for monitors and displays (in contrary to Lab, Luv, HLS, HSV, CMYK...).

  Disadvantage and colorspace not having this disadvantage:

  - Color mixing is not intuitive (in contrary to HLS, HSV).
  - Does not allow meaningful color distance measurements (in contrary to Lab, Luv).
  - Not useful for printing (in contrary to CMYK).
  - Not all visible colors can be mixed from primaries (in contrary to XYZ).

- Evaluate the z-component of the RGB-base vectors.

	R	G	B
z = 1 - x - y	0.03	0.10	0.79

- Provide the equation system for the white point calibration. Name the calibration parameters C_R, C_G and C_B.

  Transformation matrix with chromaticity values of the primaries:

$$\begin{bmatrix} 0.64 & 0.30 & 0.15 \\ 0.33 & 0.60 & 0.06 \\ 0.03 & 0.10 & 0.79 \end{bmatrix}$$

  C_R, C_G and C_B as additional calibration parameters:

$$\begin{bmatrix} 0.64C_R & 0.30C_G & 0.15C_B \\ 0.33C_R & 0.60C_G & 0.06C_B \\ 0.03C_R & 0.10C_G & 0.79C_B \end{bmatrix}$$

  Transformation of $(1,1,1)^T$ onto white point:

$$\begin{bmatrix} 0.64C_R & 0.30C_G & 0.15C_B \\ 0.33C_R & 0.60C_G & 0.06C_B \\ 0.03C_R & 0.10C_G & 0.79C_B \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9505 \\ 1.0000 \\ 1.0890 \end{bmatrix}$$

  Equation system:

$$\begin{bmatrix} 0.64 & 0.30 & 0.15 \\ 0.33 & 0.60 & 0.06 \\ 0.03 & 0.10 & 0.79 \end{bmatrix} \begin{bmatrix} C_R \\ C_G \\ C_B \end{bmatrix} = \begin{bmatrix} 0.9505 \\ 1.0000 \\ 1.0890 \end{bmatrix}$$

We have:

$$\begin{bmatrix} C_R \\ C_G \\ C_B \end{bmatrix} = \begin{bmatrix} 0.6445 \\ 1.1919 \\ 1.2031 \end{bmatrix}$$

- Suppose $C_R = 0.6445$, $C_G = 1.1919$ and $C_B = 1.2031$ are given as a solution. Evaluate the transformation matrix from the linear color space RGB into the color space XYZ.

$$\begin{bmatrix} 0.4125 & 0.3576 & 0.1805 \\ 0.2127 & 0.7151 & 0.0722 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix}$$

3. Color Space Transformation

In this exercise we focus on the transformation from colors in the sRGB-color space into the broadly known color spaces used in television, namely PAL and NTSC.

- In order to be compatible with old black and white systems, the first channel of the PAL-color space (also known as YUV-color space) is the Y-coordinate of the XYZ-color space. Since the Y-coordinate

$$Y = 0.2126 * R + 0.7152 * G + 0.0722 * B$$

contains a major green component, Cb and Cr are chosen in a way such that they contain a major blue respectively red component:

$$C_B = B–Y, C_R = R–Y.$$

Finally norming the Cb- and the Cr-channels leads to the YUV- or PAL-color space:

$$U = 0.49C_B, V = 0.88C_R$$

Provide the transformation matrix from the sRGB-color space into the YUV-color space.

First row of the transformation matrix stems from the definition of the Y-coordinate:

$$\begin{bmatrix} Y \\ U \\ V \end{bmatrix} = \begin{bmatrix} 0.2126 & 0.7152 & 0.0722 \\ \dots & \dots & \dots \\ \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$

C_B and C_R expressed by R, G and B:

$$C_B = -0.2126R–0.7152G + 0.9278B$$
$$C_R = 0.7874R–0.7152G - 0.0722B$$

Correct scaling with 0.49 and 0.88:

$$U = -0.1047R–0.3523G + 0.4570B$$
$$V = 0.6907R–0.6274G - 0.0633B$$

- The YIQ- or NTSC-color space is used as the US television standard. It is created from the PAL color space, by swapping the U- and V-coordinates followed by a rotation by 33 degrees around Y-axis. Evaluate the transformation matrix for the conversion from PAL to NTSC. You do not have to evalu¬ate trigonometric expressions.

  Matrix swap coordinates:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

  Matrix for rotation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(33°) & -sin(33°) \\ 0 & sin(33°) & cos(33°) \end{bmatrix}$$

  Tranformation matrix:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(33°) & -sin(33°) \\ 0 & sin(33°) & cos(33°) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -sin(33) & cos(33) \\ 0 & cos(33) & sin(33) \end{bmatrix}$$

- The YIQ-channels are splitting the bandwidth proportional to 8:5:2 during the transfer of NTSC color signals. Why is such an uneven bandwidth being used?
  - To take advantage of the limited bandwidth in an optimal way, channels for which the human perception is more sensitive are getting more bandwidth.
  - Based on psycho physical properties of the human visual system.

4. CIE-Chart

- Which properties does a mixed color in the CIE-Chart have to its primaries?

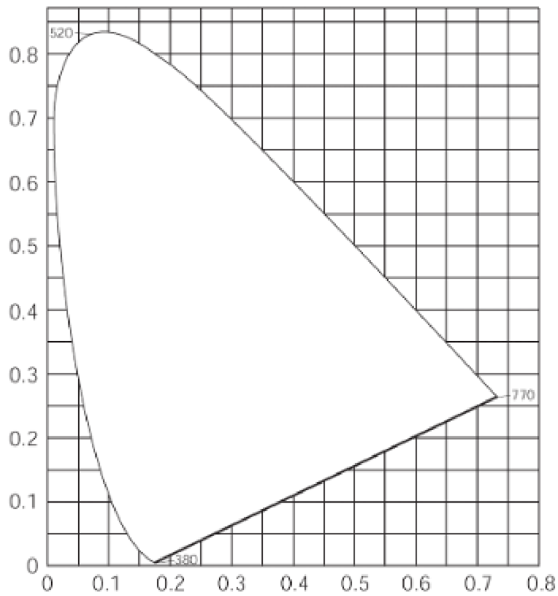
  The mixed color lies in the convex hull spanned by the primaries.

- Which meaning does the connection between 770nm and 380nm have in this chart?

  It is called the purple line. Colors lying on the purple line are not spectral colors. They are created by mixing the two spectral colors red (at 770nm) and blue (at 380nm).

- The figure below shows a CIE-Chart. Add following primaries into the chart.

	x	y	Y
C1	0.1	0.8	12
C2	0.6	0.3	26
C3	0.2	0.05	10



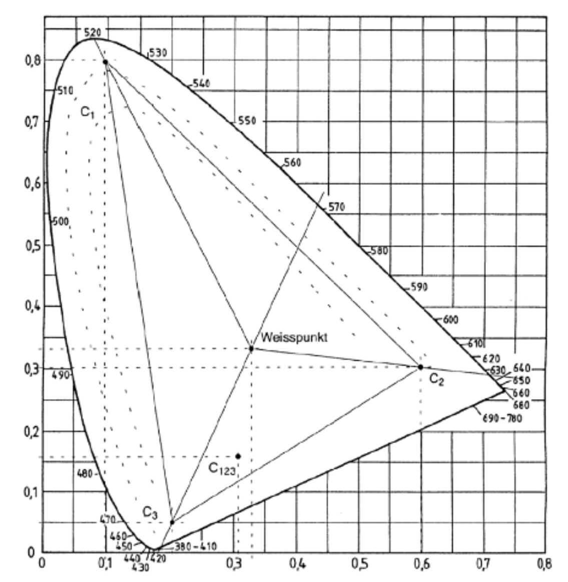
  Solution: see values in figure above.

- Determine the dominant wavelengths λ_1, λ_2 and λ_3 of the 3 primaries.

  Straight containing white point and primary intersected with the border: λ_1: 520 nm, λ_2: 635 nm, λ_3: not 568 nm, meaning all complementary wavelengths to 568 nm.

- For each primary, draw the isoline of constant saturation passing through it into the chart.

  Note that lines from the white point through the primaries are always gaining on saturation and therefore the isolines have to cross those.



- Determine the primary C_{123}, which is the sum (in the XYZ-space) of the 3 primaries C_1, C_2 and C_3. Add it to the chart.

$$y = \frac{Y}{X+Y+Z} \tag{1}$$

$$x = \frac{X}{X+Y+Z} \tag{2}$$

From the first relation (1) follows:

$$X + Y + Z = Y/y, \tag{3}$$

and from this including (2):

$$X = x\frac{Y}{y} \tag{4}$$

Then, from (3) and (4) follows:

$$Z = \frac{Y}{y} - x\frac{Y}{y} - Y.$$

So, X and Z can be determined for the three primaries C_1, C_2 and C_3:

$$C_1 : X_1 = 1.5, Z_1 = 1.5$$
$$C_2 : X_2 = 52, Z_2 = 8.666$$
$$C_3 : X_3 = 40, Z_3 = 150$$

From this follows:

$$X_{123} = X_1 + X_2 + X_3 = 93.5$$
$$Y_{123} = Y_1 + Y_2 + Y_3 = 48$$
$$Z_{123} = Z_1 + Z_2 + Z_3 = 160.1666$$

Inserting into (1) and (2) provides mixed primary x=0.308, y=0.159, Y=48.

- Can all spectral colors with full saturation be mixed from three linearly independent primaries?

  By mixing 3 linearly independent primaries in the CIE-Chart all colors inside and on the border of the spanning triangle can be created. Since the CIE-Chart is horseshoe shaped it is never possible to create another spectral color by mixing three spectral colors.