Introduction to cryptography

2. Secret-key techniques

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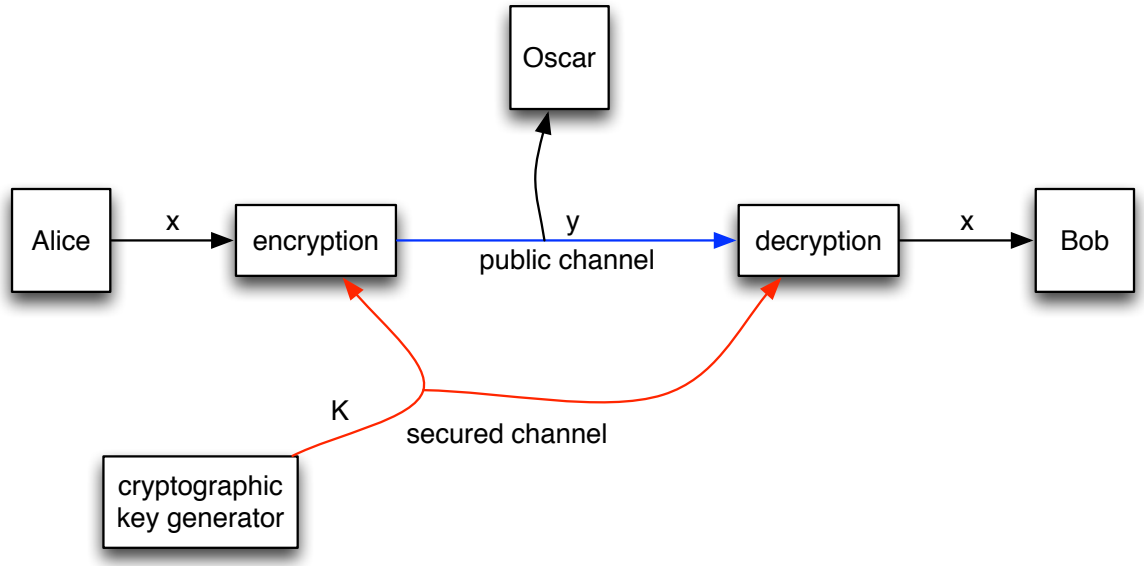
Setting



The secret key comes from, e.g.,

- hand-to-hand distribution
- smartcard
- a key establishment scheme

A layered approach

- Primitives
  - Keystream generators: e.g., RC4, Trivium, Salsa20
  - Block ciphers: e.g., DES, AES, Skinny, Saturnin
  - Permutations: e.g., KECCAK-f, Ascon, Gimli, XOODOO
- Modes of operations (and constructions) to build
  - an encryption scheme: e.g., stream cipher, CBC, CTR, sponge
  - an authentication scheme: e.g., CBC-MAC, sponge
  - an auth. enc. scheme: e.g., GCM, SIV, SpongeWrap
  - a hash function: e.g., Davies-Meyer + Merkle-Damgård, sponge (see next chapter)
  - one primitive from another: e.g.,
    - CTR turns a block cipher into a keystream generator
    - Even-Mansour turns a permutation into a block cipher

A layered approach

- Primitives
  - Require peer review and cryptanalysis
- Modes of operations (and constructions)
  - Can have generic attacks, i.e., attacks that work regardless of the underlying primitive
  - Are analyzed assuming the primitive is ideal (random)
    - to determine the generic attacks
    - to prove that no other generic attack exists

At the end of the day, this gives us secure schemes

- up to the generic attacks
- until someone finds a flaw in the primitive

A layered approach

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What is a stream cipher?

A stream cipher is an encryption scheme that uses a keystream generator to output bits gradually and on demand. The output bits are call the *keystream*.

- To encrypt, the keystream is XORed bit by bit with the plaintext.
- To decrypt, the keystream is XORed bit by bit with the ciphertext.

What is a stream cipher?

Keystream generator: $G : K \times \mathbb{N} \to \mathbb{Z}_2^\infty$

- inputs a secret key $k \in K$ and a diversifier $d \in \mathbb{N}$
- outputs a long (potentially infinite) keystream $(s_i) \in \mathbb{Z}_2^\infty$

To encrypt plaintext $(m_0, \ldots, m_{|m|-1})$:

$$c_i = m_i + s_i \quad (\text{in } \mathbb{Z}_2)$$

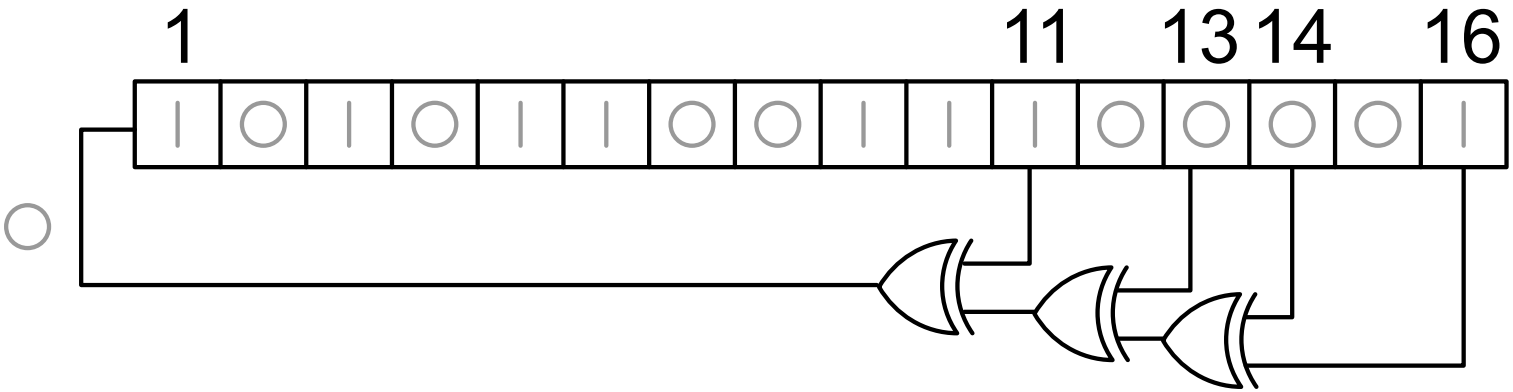
To decrypt ciphertext $(c_0, \ldots, c_{|c|-1})$:

$$m_i = c_i + s_i \quad (\text{in } \mathbb{Z}_2)$$

The diversifier d is public and must be a **nonce**, i.e., unique per encryption for a given key.

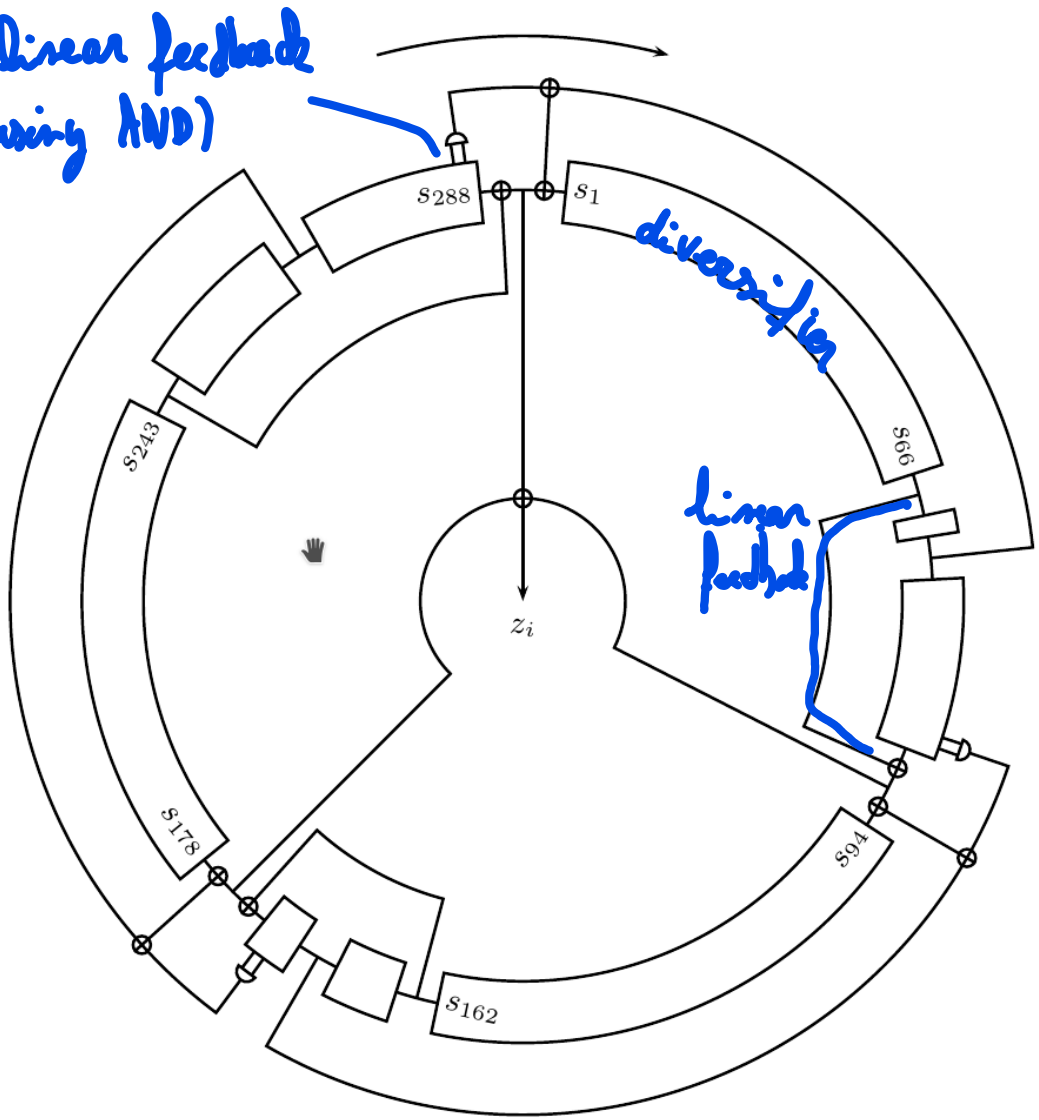
Linear feedback shift register (LFSR)

Loop of $2^{16}-1$ values, used in the 70,80's.



Feedback polynomial: $1 + D^{11} + D^{13} + D^{14} + D^{16}$

Trivium – is sometimes used (still now!)



[Christophe De Cannière, Bart Preneel, eSTREAM, 2005]

RC4

$S \in \mathbb{Z}_{256}^{256}$, an array of 256 bytes

Initialization($k \in \mathbb{Z}_{256}^{*}$: key and diversifier)

- for i from 0 to 255:
  - $S[i] \leftarrow i$
- $j \leftarrow 0$
- for i from 0 to 255:
  - $j \leftarrow (j + S[i] + k[i \bmod |k|]) \bmod 256$
  - swap values of $S[i]$ and $S[j]$

we use every byte of k

to make sure it $\in [0; 255]$

scan the entire array and swap often

Keystream generation

...

RC4

$S \in \mathbb{Z}_{256}^{256}$, an array of 256 bytes

Initialization($k \in \mathbb{Z}_{256}^{*}$: key and diversifier) ...

Keystream generation

- $i \leftarrow 0, j \leftarrow 0$
- loop as long as necessary:
  - $i \leftarrow (i + 1) \bmod 256$
  - $j \leftarrow (j + S[i]) \bmod 256$
  - swap values of $S[i]$ and $S[j]$
  - $s \leftarrow S[(S[i] + S[j]) \bmod 256]$
  - output s

RC4 was finally broken
→ Attacking RC4

$S[1] \neq 2$ 255/256
$S[2] = 0$ 1/256

255/256 no

2nd byte is 0 0 1/256

yes

2nd byte of output = 00

$\Rightarrow$ 2/256

$x \neq 2$

[presumably Ron Rivest, 1987; secret then anonymously published in 1994]

n	0	1	2	...	x	..
S[n]		x	0		y	

Iteration 1) $i = 1$ $j = S[i] = S[1] = x$
swap $S[1]$ and $S[x]$

$\Rightarrow$

n	0	1	2	....	x	...
S[n]		y	0		x	

Block cipher

A **block cipher** is a mapping $E : \mathbb{Z}_2^n \times \mathbb{Z}_2^m \to \mathbb{Z}_2^n$

- from a secret key $k \in \mathbb{Z}_2^m$ and an input block $x \in \mathbb{Z}_2^n$
- to an output block $y = E_k(x) \in \mathbb{Z}_2^n$

For each key k, it has an inverse: $x = E_k^{-1}(y)$.

Note: Despite its name, a block cipher is not an encryption scheme, except for the very restricted plaintext space $\mathbb{Z}_2^n$ but then it's not even IND-CPA.

Security notions (informally):

- Pseudo-random permutation (PRP): without knowing the secret key, it should be infeasible for an adversary having access to $E_k(\cdot)$ to distinguish it from a permutation randomly drawn from the set of all permutations on $\mathbb{Z}_2^n$
- Strong PRP (SPRP): same, but the adversary also gets access to $E_k^{-1}(\cdot)$

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most used, something that maps n bits to n bits with a key of n bits.

Block cipher

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- to an output block $y = E_k(x) \in \mathbb{Z}_2^n$

For each key k, it has an inverse: $x = E_k^{-1}(y)$. ← inverse of the key

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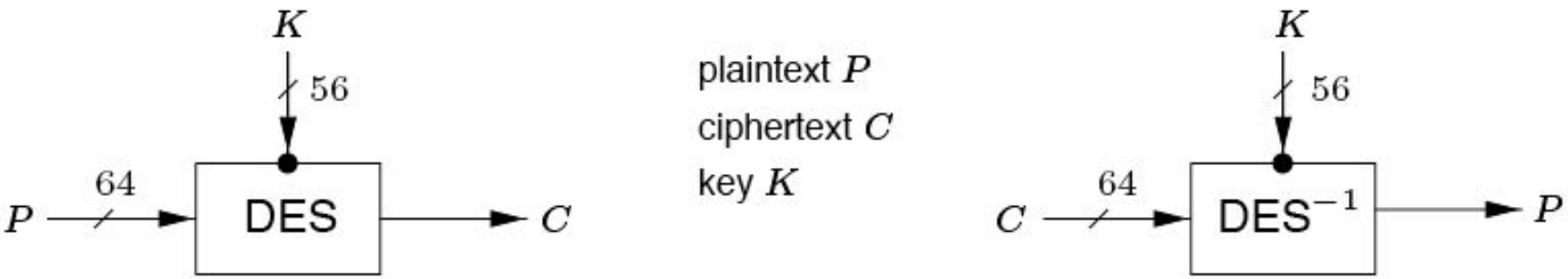
DES

Horst Feistel

Don Coppersmith

DES

DES : $\mathbb{Z}_2^{64} \times \mathbb{Z}_2^{56} \to \mathbb{Z}_2^{64}$
Lucifer in 1973 - DES is a standard in 1976

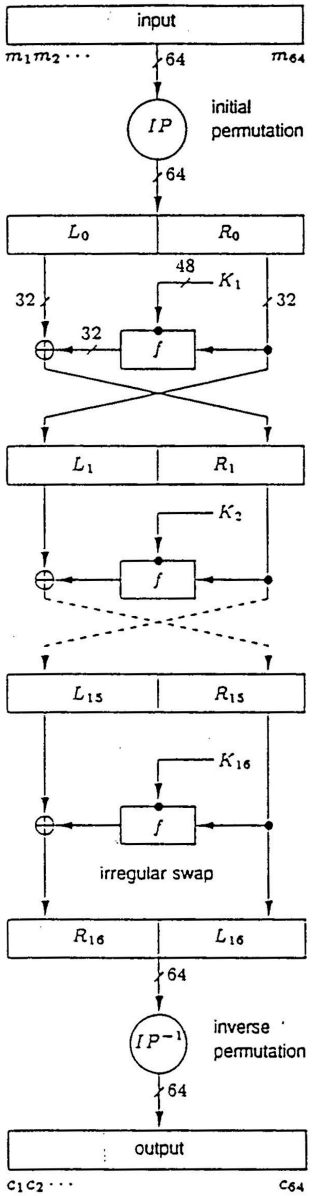


DES input-output.

Sorry, this is a bad figure; the input is often *not* plaintext, and the output often *not* ciphertext.

DES is made of rounds

16 rounds + initial and inverse permutation
each round is exactly the same



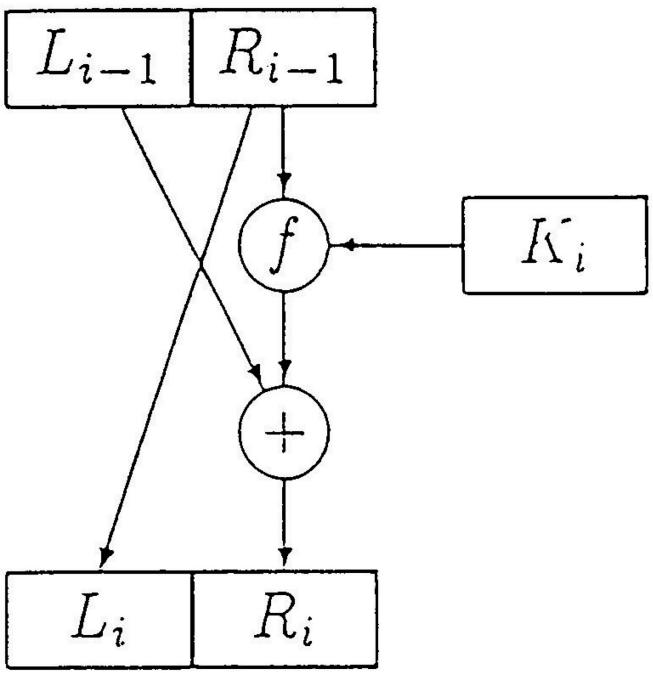
- An initial bit transposition (IP) is applied to the input: $L_0 \| R_0 = \mathrm{IP}(x)$, with $L_0, R_0 \in \mathbb{Z}_2^{32}$.
- Then 16 iterations of the **round function**:

$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus f(R_{i-1}, k_i),$$

  where $k_1, \ldots, k_{16} \in \mathbb{Z}_2^{48}$ are the 16 sub-keys.
- The inverse of IP is applied before returning the output: $y = \mathrm{IP}^{-1}(R_{16} \| L_{16})$.

DES is not it's own inverse

DES is a Feistel network



One round forward:

$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus f(R_{i-1}, k_i)$$

One round backward:

$$R_{i-1} = L_i$$
$$L_{i-1} = R_i \oplus f(L_i, k_i)$$

Advantages:

- E and E^{-1} share the same structure, subkeys in reverse order;
- f does not need to be invertible.

Inside DES' f function

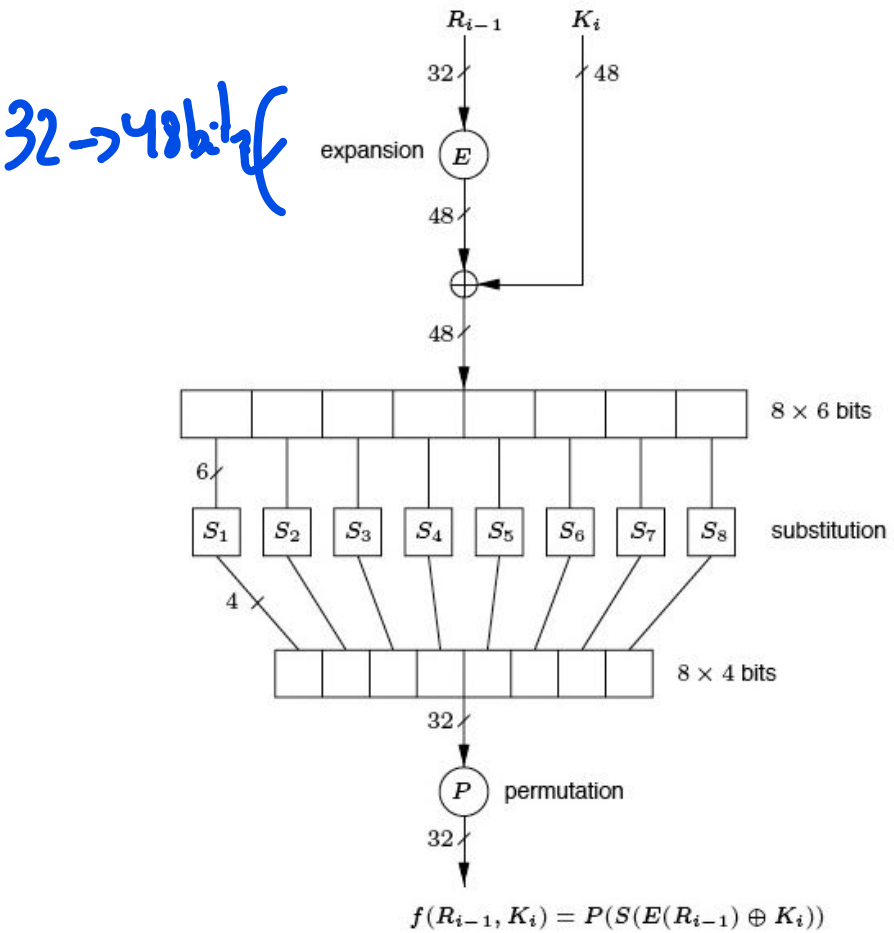


Figure 7.10: *DES inner function f.*

$$f : \mathbb{Z}_2^{32} \times \mathbb{Z}_2^{48} \to \mathbb{Z}_2^{32}$$

1. Expand R through E from 32 to $48 = 16 + 2 \times 16$ bits
2. Compute $E(R) \oplus K_i$
3. Cut in 6-bit blocs B_j
4. $C_j = S_j(B_j)$, 8 different S-Boxes
   $S_j : \mathbb{Z}_2^6 \to \mathbb{Z}_2^4$
5. Bit transposition
   $P : \mathbb{Z}_2^{32} \to \mathbb{Z}_2^{32}$

DES: Tables for bit transpositions and expansion *E*

IP							
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

IP^{-1}							
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

Table 7.2: *DES initial permutation and inverse (IP and IP^{-1}).*

E					
32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

P			
16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

Table 7.3: *DES per-round functions: expansion E and permutation P.*

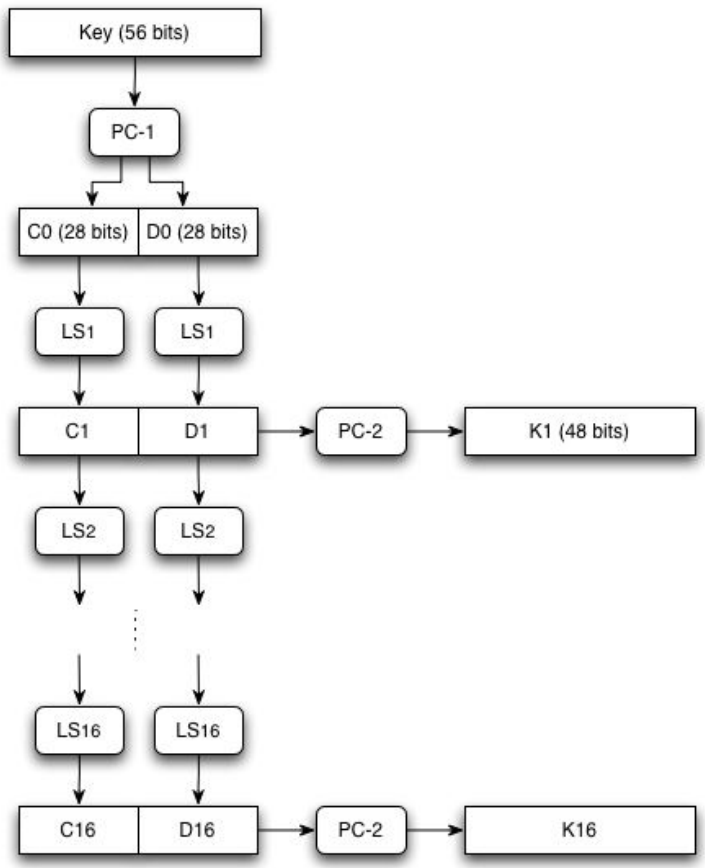
DES: Tables of the S-boxes

row	column number															
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
								S_1								
[0]	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
[1]	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
[2]	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
[3]	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13
								S_2								
[0]	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
[1]	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
[2]	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
[3]	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9
								S_3								
[0]	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
[1]	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
[2]	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
[3]	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12
								S_4								
[0]	7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
[1]	13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
[2]	10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
[3]	3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14
								S_5								
[0]	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
[1]	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
[2]	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
[3]	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
								S_6								
[0]	12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
[1]	10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
[2]	9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
[3]	4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13
								S_7								
[0]	4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
[1]	13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
[2]	1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
[3]	6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12
								S_8								
[0]	13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
[1]	1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
[2]	7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
[3]	2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

DES' key schedule

You can come backwards to generate the key

From the secret key $k \in \mathbb{Z}_2^{56}$ to round keys $K_i \in \mathbb{Z}_2^{48}$



1. Bit transposition $C_0 \| D_0 = \text{PC1}(k)$
   $\text{PC1} : \mathbb{Z}_2^{56} \to \mathbb{Z}_2^{28} \times \mathbb{Z}_2^{28}$
2. For $i = 1$ to 16:
   - $C_i = \text{LS}_i(C_{i-1})$ and $D_i = \text{LS}_i(D_{i-1})$, where LS_i is a circular shift to the left by one position when $i = 1, 2, 9, 16$ and by two positions otherwise
   - $K_i = \text{PC2}(C_i \| D_i)$, with
     $\text{PC2} : \mathbb{Z}_2^{56} \to \mathbb{Z}_2^{48}$

DES: Tables for PC1 and PC2

PC1						
57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
above for C_i; below for D_i						
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

PC2					
14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

Table 7.4: DES key schedule bit selections (PC1 and PC2).

DES non-ideal properties

Secrecy is of 2^{55} and not 2^{56} bc. see slides.
4b key search

- 4 weak keys k : E_k is an involution, i.e., $\forall x\ E_k(E_k(x)) = x$
- 6 pairs of semi-weak keys (k_1, k_2) such that $\forall x\ E_{k_1}(E_{k_2}(x)) = x$
- Complementarity property: $E_k(x) = y \Leftrightarrow E_{\overline{k}}(\overline{x}) = \overline{y}$
  $\Rightarrow$ reduces the exhaustive search by one bit

DES security

key size is a problem

- Exhaustive key search in 2^{55} → ≃ 1 day to crack
  feasible with dedicated hardware or FPGA
- Susceptible to **differential cryptanlysis** [Biham and Shamir, 1993]
- Susceptible to **linear cryptanlysis** [Matsui, 1993]

faster than exhaustive key search

attack method	data complexity		storage complexity	processing complexity
	known	chosen		
exhaustive precomputation	—	1	2^{56}	1 (table lookup)
exhaustive search	1	—	negligible	2^{55}
linear cryptanalysis	2^{43} (85%)	—	for texts	2^{43}
	2^{38} (10%)	—	for texts	2^{50}
differential cryptanalysis	—	2^{47}	for texts	2^{47}
	2^{55}	—	for texts	2^{55}

Table 7.7: *DES strength against various attacks.*

Differential cryptanalysis

Analyzes the propagation of differences through the rounds

$$f(x) = y$$
$$f(\delta \oplus x) = y + \Delta$$

(x varies, δ constant)

Eli Biham and Adi Shamir

Linear cryptanalysis

Analyzes the correlation between input and output parities

$$\left| \Pr[v^\top x = u^\top f(x)] - \frac{1}{2} \right|$$

(x varies, u and v constant)

Mitsuru Matsui

Triple-DES

Triple-key triple-DES (168-bit keys)

$$3\mathrm{DES}_{k_1\|k_2\|k_3} = \mathrm{DES}_{k_3} \circ \mathrm{DES}_{k_2}^{-1} \circ \mathrm{DES}_{k_1}$$

Double-key triple-DES (112-bit keys)

$$3\mathrm{DES}_{k_1\|k_2} = \mathrm{DES}_{k_1} \circ \mathrm{DES}_{k_2}^{-1} \circ \mathrm{DES}_{k_1}$$

Retro-compatibility with single DES

$$3\mathrm{DES}_{k\|k\|k} = \mathrm{DES}_{k} \circ \mathrm{DES}_{k}^{-1} \circ \mathrm{DES}_{k} = \mathrm{DES}_{k}$$

Triple-DES

-1 to have easy retro

Triple-key triple-DES (168-bit keys)

$$3\mathrm{DES}_{k_1\|k_2\|k_3} = \mathrm{DES}_{k_3} \circ \mathrm{DES}_{k_2}^{-1} \circ \mathrm{DES}_{k_1}$$

Double-key triple-DES (112-bit keys) → variant with $k_3 = k_1$

$$3\mathrm{DES}_{k_1\|k_2} = \mathrm{DES}_{k_1} \circ \mathrm{DES}_{k_2}^{-1} \circ \mathrm{DES}_{k_1}$$

→ more used than triple

Retro-compatibility with single DES

$$3\mathrm{DES}_{k\|k\|k} = \mathrm{DES}_{k} \circ \mathrm{DES}_{k}^{-1} \circ \mathrm{DES}_{k} = \mathrm{DES}_{k}$$

Triple-DES

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Retro-compatibility with single DES

$$3\mathrm{DES}_{k\|k\|k} = \mathrm{DES}_{k} \circ \mathrm{DES}_{k}^{-1} \circ \mathrm{DES}_{k} = \mathrm{DES}_{k}$$

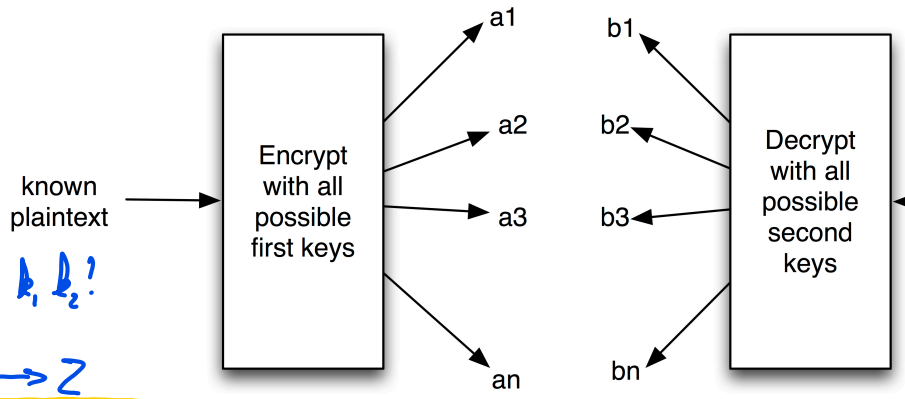
Why not Double-DES?

Why not $2\text{DES}_{k_1 \| k_2} = \text{DES}_{k_2} \circ \text{DES}_{k_1}$?

because the attacker could do a "meet in the middle" attack to find that $a_1 = b_1, a_2 = b_2 \ldots$

$\Rightarrow 2^{57}$ time!

2^{56} memory



$DES_{k_2}(DES_{k_1}(x)) = z$ k_1 k_2?

$x \rightarrow [DES_{k_1}] \rightarrow y \rightarrow [DES_{k_2}] \rightarrow z$

$\forall k_1$ Phase 1
$y = DES_{k_1}(x)$
store (k_1, y) in table
sorted by the value y

$\forall k_2$ Phase 2
$y = DES^{-1}_{k_2^*}(z)$
lookup (k_1^*, y)
$DES_{k_1^*}(DES^{-1}_{k_2^*}(x')) = z'$? retrieve the values k_1^*, k_2^* if work

$t \Rightarrow 2^{56} + 2^{56} = 2^{57}$ $m = 2^{56}$ entries!

The adversary looks for matches between the ai and the bj

Attack in time 2^{57} with 2^{56} memory.

Competition for a new standard

In September 1997, USA's National Institute of Standards and Technology (NIST) called for a new block cipher family, with

- block size $n = 128$ bits
- key sizes $m = 128$, 192 and 256 bits

Organized as an open competition, the winner would become the Advanced Encryption Standard (AES).

- August 1998: 15 candidates
- August 1999: 5 finalists
  - MARS [IBM, including Don Coppersmith]
  - RC6 [Ron Rivest, Matt Robshaw, Ray Sidney and Yiqun Lisa Yin]
  - Rijndael [Joan Daemen and Vincent Rijmen]
  - Serpent [Ross Anderson, Eli Biham and Lars Knudsen]
  - Twofish [Bruce Schneier, John Kelsey, Doug Whiting, David Wagner, Chris Hall and Niels Ferguson]
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Rijndael

Joan Daemen and Vincent Rijmen

Rijndael – AES

NIST standardized 3 instances of Rijndael with block size 128 bits and key sizes $\in \{128, 192, 256\}$:

AES-128 : $\mathbb{Z}_2^{128} \times \mathbb{Z}_2^{128} \to \mathbb{Z}_2^{128}$	10 rounds
AES-192 : $\mathbb{Z}_2^{128} \times \mathbb{Z}_2^{192} \to \mathbb{Z}_2^{128}$	12 rounds
AES-256 : $\mathbb{Z}_2^{128} \times \mathbb{Z}_2^{256} \to \mathbb{Z}_2^{128}$	14 rounds

Rijndael – $\mathbb{Z}_2^{128} \leftrightarrow \mathrm{GF}(2^8)^{4\times 4}$

Input $x \in \mathbb{Z}_2^{128}$ is mapped to an array of 4×4 bytes:

$$\begin{pmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{pmatrix} \leftarrow \begin{pmatrix} x_0 & x_4 & x_8 & x_{12} \\ x_1 & x_5 & x_9 & x_{13} \\ x_2 & x_6 & x_{10} & x_{14} \\ x_3 & x_7 & x_{11} & x_{15} \end{pmatrix}$$

where x_i is the i^{th} byte of x. And vice-versa for the output.

Each byte $s_{i,j}$ represents an element of the finite field $\mathrm{GF}(2^8)$.

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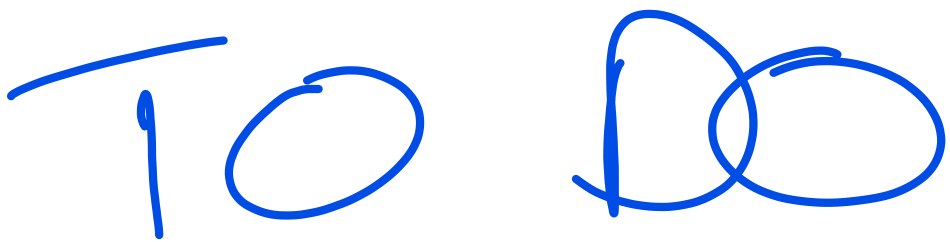

$\mathrm{GF}(2^8)$ in Rijndael

linearity, distributivity, etc.

- $\mathrm{GF}(2^8)$ is the finite field (Galois Field) of size $2^8 = 256$.
- Rijndael uses the representation $\mathrm{GF}(2)[x]/(x^8 + x^4 + x^3 + x + 1)$.
- A byte with arithmetic value $s = \sum_{n=0}^{7} s_i 2^i$ represents the polynomial $s = \sum_{n=0}^{7} s_i x^i$ in $\mathrm{GF}(2)[x]$.
  E.g., 0x83 represents $x^7 + x + 1$.
- Addition is like a bitwise XOR operations
  E.g., $(x^7 + x + 1) + (x^6 + x) = (x^7 + x^6 + 1) \Leftrightarrow \texttt{0x83} \oplus \texttt{0x42} = \texttt{0xC1}$
- Multiplication is done modulo $x^8 + x^4 + x^3 + x + 1$
  E.g., $x(x^7 + x + 1) = x^8 + x^2 + x = (x^8 + x^2 + x) + (x^8 + x^4 + x^3 + x + 1) = x^4 + x^3 + x^2 + 1$

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→ convention, it could have been other values

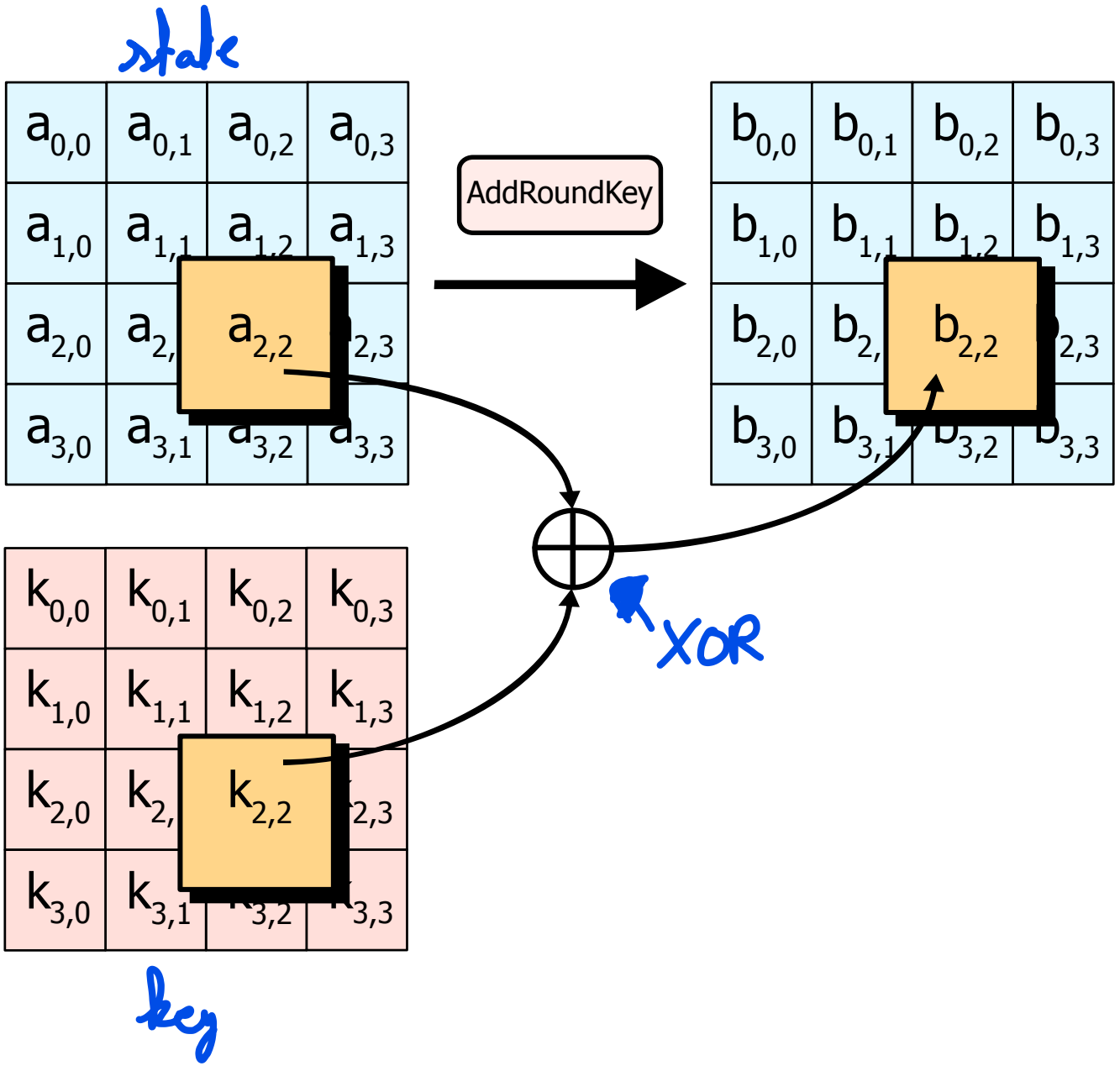
Rijndael data path

Input $x \in \mathbb{Z}_2^{128}$ and $k \in \mathbb{Z}_2^{128 \text{ (or 192 or 256)}}$

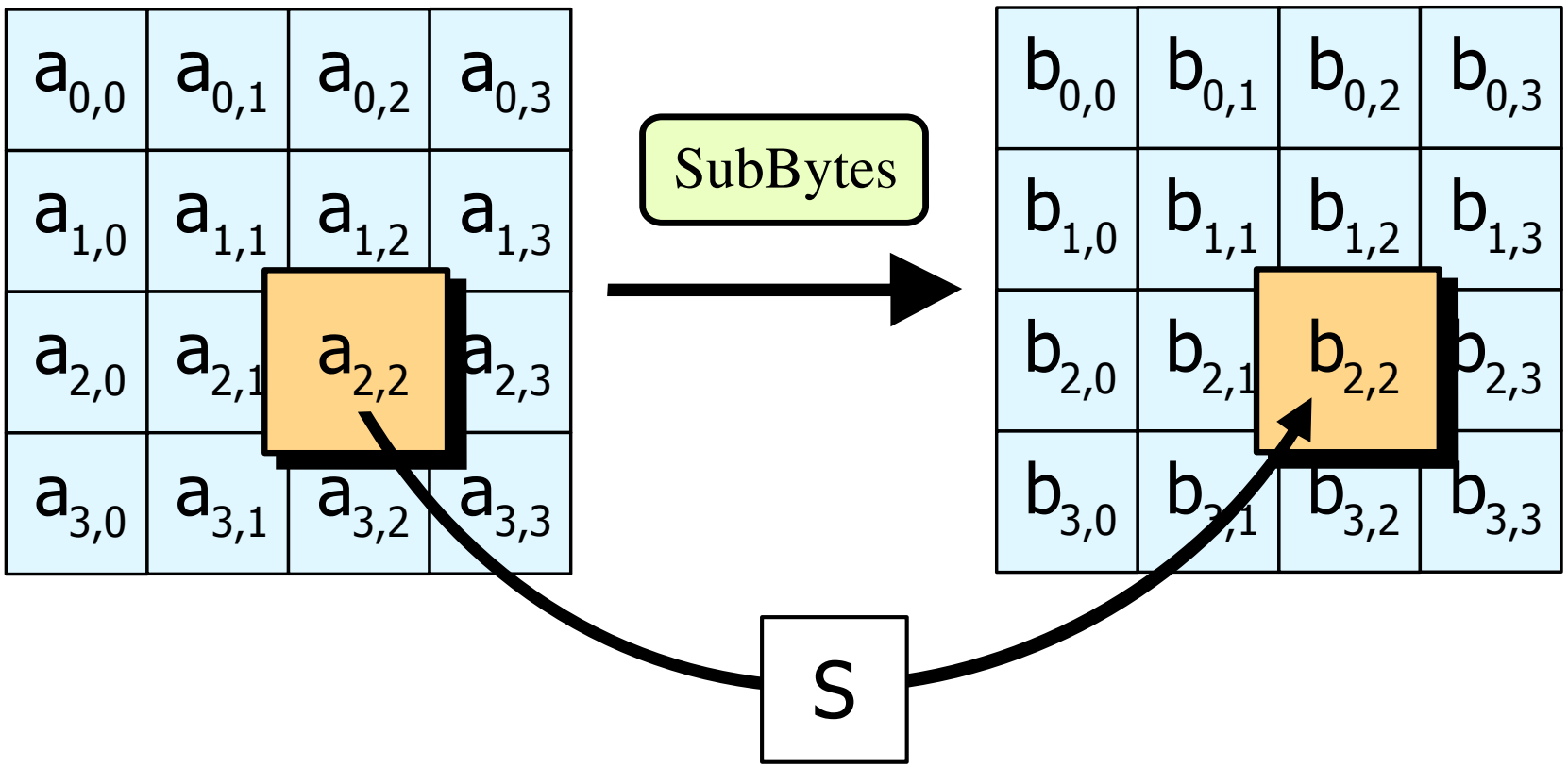
- Key schedule: $(K_i) \leftarrow \texttt{KeyExpansion}(k)$
- state $\leftarrow x$ (but represented as $\mathrm{GF}(2^8)^{4\times 4}$)
- $\texttt{AddRoundKey}(\text{state}, K_0)$
- For each round $i = 1$ to 9 (or 11 or 13):
  - $\texttt{SubBytes}(\text{state})$
  - $\texttt{ShiftRows}(\text{state})$
  - $\texttt{MixColumns}(\text{state})$
  - $\texttt{AddRoundKey}(\text{state}, K_i)$
- And for the last round:
  - $\texttt{SubBytes}(\text{state})$
  - $\texttt{ShiftRows}(\text{state})$
  - $\texttt{AddRoundKey}(\text{state}, K_{10 \text{ (or 12 or 14)}})$

Output $y \leftarrow$ state back in $\mathbb{Z}_2^{128}$

Rijndael – AddRoundKey



Rijndael – SubBytes



Rijndael – SubBytes

The 0 doesn't have an inverse → 0 ⇒ 63

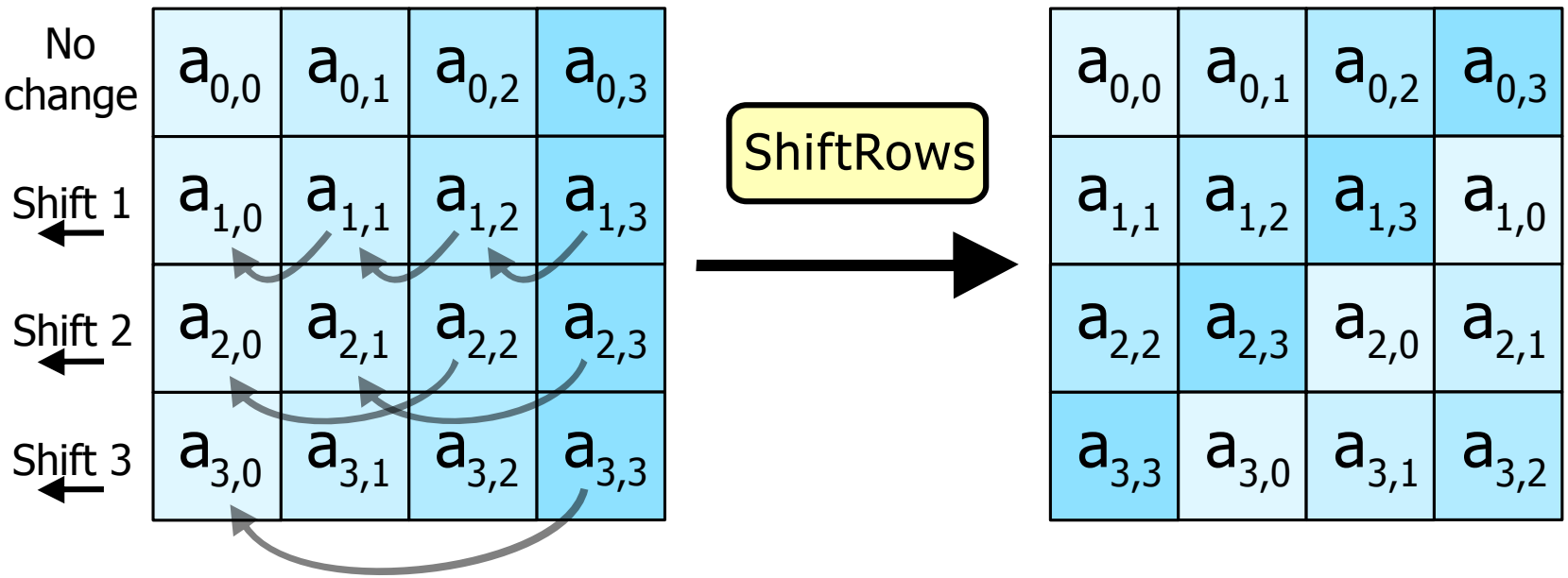
		y															
		0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
x	0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
	1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
	2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
	3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
	4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
	5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
	6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
	7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
	8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
	9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
	a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
	b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
	c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
	d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
	e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
	f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Figure 7. S-box: substitution values for the byte xy (in hexadecimal format).

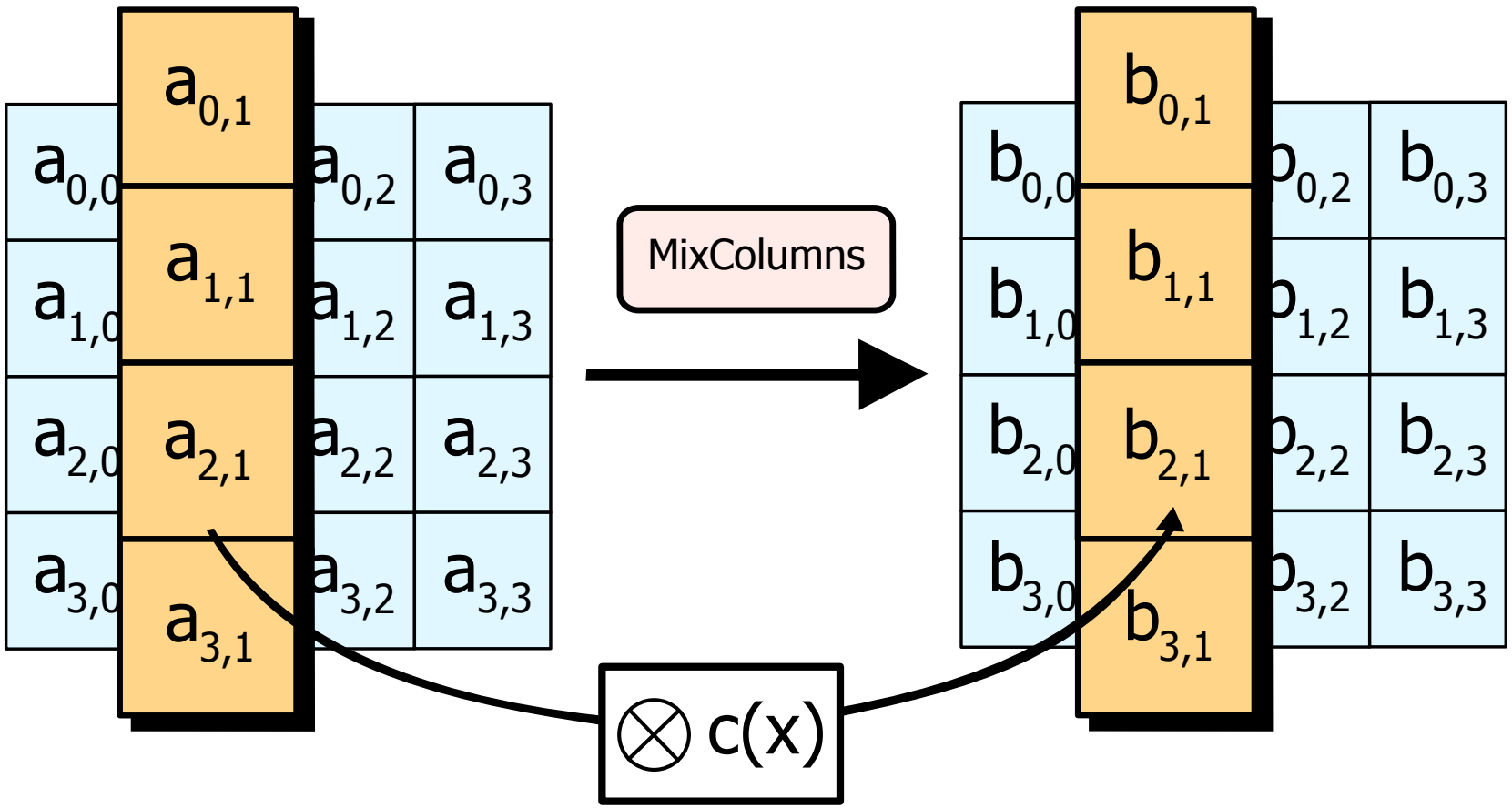
The Rijndael S-box is based on the multiplicative inverse in GF(2^8).

we could create a table for the inverse 63 ⇒ 0

Rijndael – ShiftRows



Rijndael – MixColumns



Rijndael – MixColumns

Each column undergoes the following matrix multiplication:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

The inverse of MixColumns uses the inverse matrix:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

Rijndael – MixColumns

Each column undergoes the following matrix multiplication:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

The inverse of MixColumns uses the inverse matrix:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

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Each column undergoes the following matrix multiplication:

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

The inverse of MixColumns uses the inverse matrix: see slide

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{pmatrix} \begin{pmatrix} s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix}$$

Rijndael inverse data path

MOST USED not broken

Input $y \in \mathbb{Z}_2^{128}$ and $k \in \mathbb{Z}_2^{128\text{ (or 192 or 256)}}$

- Equivalent key schedule: $(K'_i) \leftarrow \texttt{EqKeyExpansion}(k)$
- state $\leftarrow y$ (but represented as $\mathrm{GF}(2^8)^{4\times 4}$)
- $\texttt{AddRoundKey}(\text{state}, K'_{10\text{ (or 12 or 14)}})$
- For each round $i = 9$ (or 11 or 13) down to 1:
  - $\texttt{InvSubBytes}(\text{state})$
  - $\texttt{InvShiftRows}(\text{state})$
  - $\texttt{InvMixColumns}(\text{state})$
  - $\texttt{AddRoundKey}(\text{state}, K'_i)$
- And for the last round:
  - $\texttt{InvSubBytes}(\text{state})$
  - $\texttt{InvShiftRows}(\text{state})$
  - $\texttt{AddRoundKey}(\text{state}, K'_0)$

Output $x \leftarrow$ state back in $\mathbb{Z}_2^{128}$

What is a mode of operation?

Going from a block cipher to

- an encryption scheme that works for plaintexts of any lengths
- an authentication scheme that works for messages of any lengths

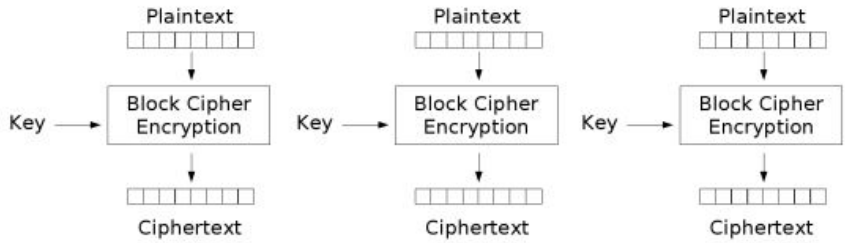
A bad example: Electronic codebook (ECB)

Input: secret key $k \in \mathbb{Z}_2^m$, plaintext $p \in \mathbb{Z}_2^*$, output: ciphertext $c \in (\mathbb{Z}_2^n)^*$

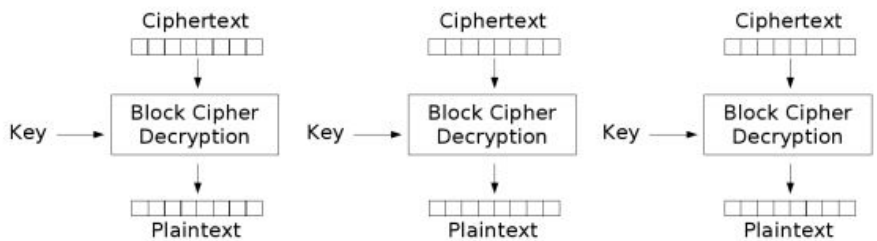
- Pad p with 10^* to reach a multiple of the block size n
- Cut $p \| 10^*$ into blocks p_i of size n
- Process each block independently through E_k

$$c_i = E_k(p_i)$$

And similarly with E_k^{-1} for decryption.

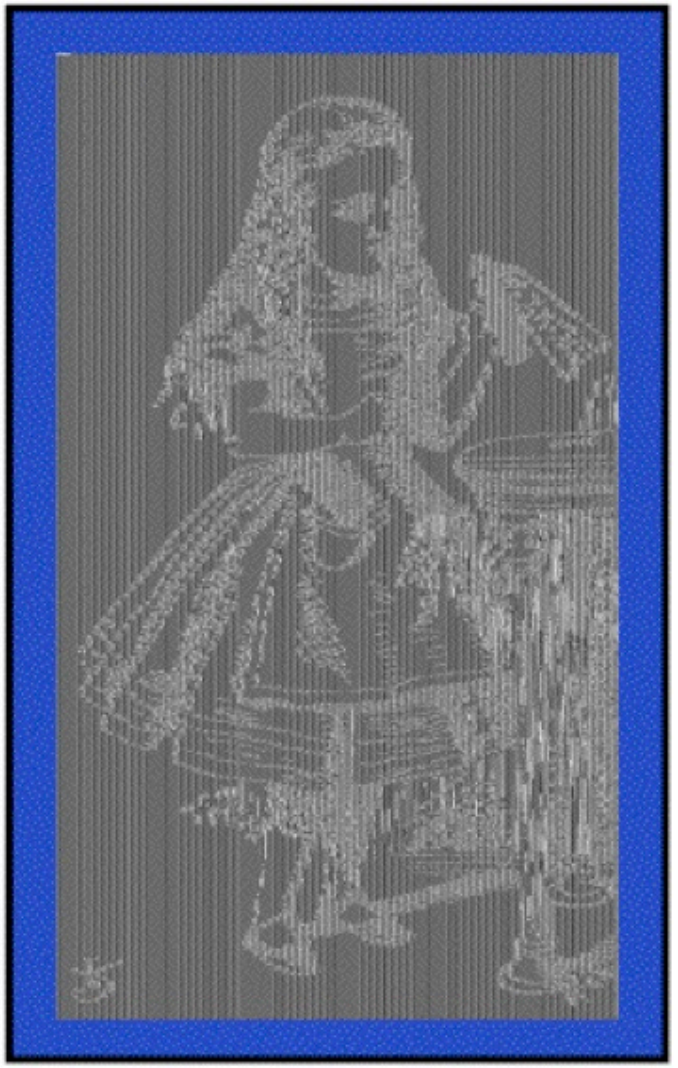


Electronic Codebook (ECB) mode encryption

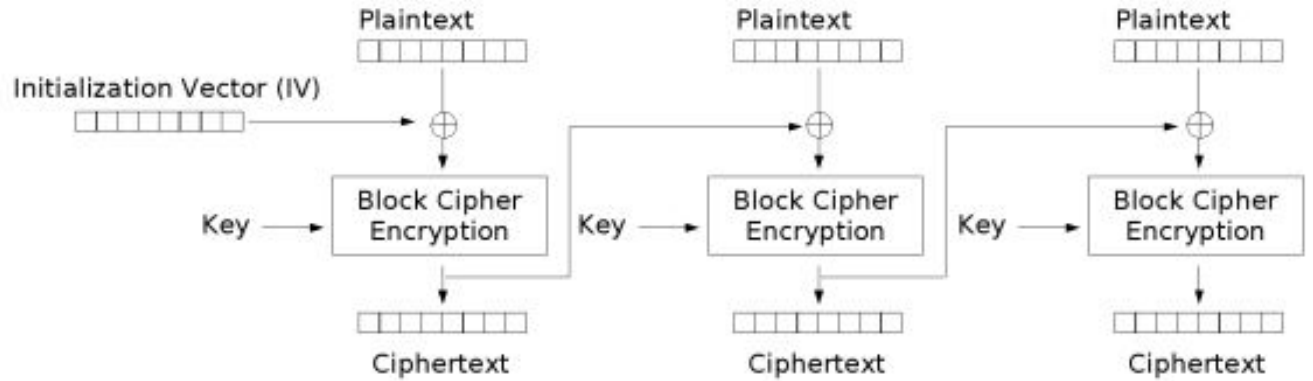


Electronic Codebook (ECB) mode decryption

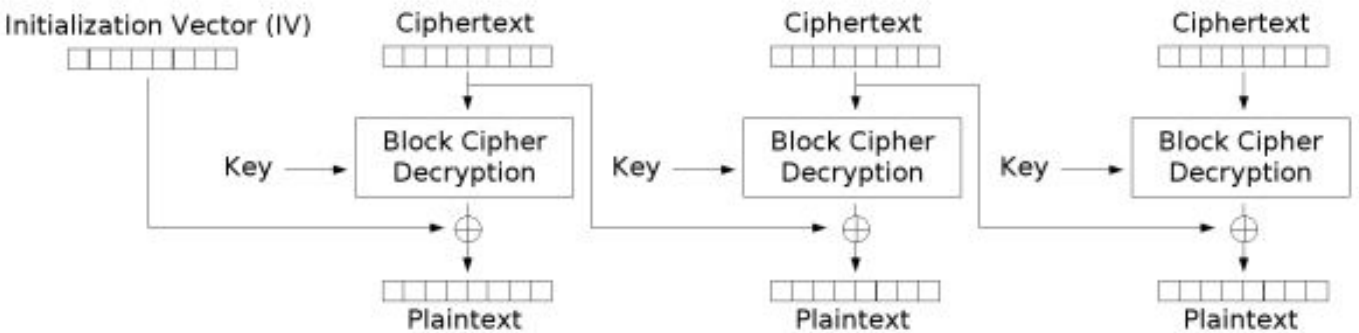
A bad example: Electronic codebook (ECB)



Ciphertext block chaining (CBC)



Cipher Block Chaining (CBC) mode encryption



Cipher Block Chaining (CBC) mode decryption

Ciphertext block chaining (CBC) ✓

Input: secret key $k \in \mathbb{Z}_2^m$, plaintext $p \in \mathbb{Z}_2^*$ and diversifier $d \in \mathbb{N}$
Output: ciphertext $c \in (\mathbb{Z}_2^n)^*$

- Pad p with 10^* and cut into blocks $(p_1, p_2, \dots)$
- Define $c_0 = E_k(d)$ and then for $i \geq 1$

$$c_i = E_k(p_i \oplus c_{i-1})$$

To decrypt:

$$p_i = E_k^{-1}(c_i) \oplus c_{i-1}$$

Here again, the diversifier d must be a **nonce**.

The limits of CBC

si 2 ciphertext sont les mêmes pour une même clé

What happens if $c_i = c'_j$ for some ciphertext blocks encrypted under the same key? => PROBLEM

$$c_i = c'_j$$
$$E_k(p_i \oplus c_{i-1}) = E_k(p'_j \oplus c'_{j-1})$$
$$p_i \oplus c_{i-1} = p'_j \oplus c'_{j-1}$$
$$p_i \oplus p'_j = c_{i-1} \oplus c'_{j-1}$$

Information is revealed on the plaintext!

How likely is $c_i = c'_j$?

The limits of CBC

What happens if $c_i = c'_j$ for some ciphertext blocks encrypted under the same key? ⇒ PROBLEM

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$$\boxed{p_i \oplus p'_j = c_{i-1} \oplus c'_{j-1}}$$

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The limits of CBC

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$$\begin{aligned} c_i &= c'_j \\ E_k(p_i \oplus c_{i-1}) &= E_k(p'_j \oplus c'_{j-1}) \\ p_i \oplus c_{i-1} &= p'_j \oplus c'_{j-1} \\ p_i \oplus p'_j &= c_{i-1} \oplus c'_{j-1} \end{aligned}$$

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Intermezzo: the birthday paradox

↳ permits to evaluate how likely $c_i = c_j$'

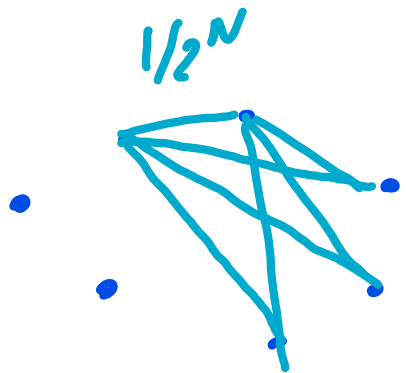
Say we have 2^n objects and we make L draws with replacement. How much does L need to be to have a good chance to get a collision?

↳ 23 personnes dans une pièce 50% de chance que 2 aient la même birthdate

Sketch:

- After L draws, there are $\binom{L}{2} = \frac{L(L-1)}{2} \approx \frac{L^2}{2}$ pairs
- For each pair, the probability of getting the same object is 2^{-n}
- The probability of collision is therefore $\approx \frac{L^2}{2^{n+1}}$
- When $L = \sqrt{2^n} = 2^{n/2}$, this probability $\approx \frac{1}{2}$

Answer: $L \approx 2^{n/2}$.



L bit strings

$\frac{L(L-1)}{2}$ pairs

$\frac{L^2}{2^{N+1}} \simeq \frac{1}{2} \iff L = \sqrt{2^N}$

$\boxed{L = 2^{N/2}}$

Intermezzo: the birthday paradox

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Answer: $L \approx 2^{n/2}$.

Intermezzo: the birthday paradox

Say we have 2^n objects and we make L draws with replacement. How much does L need to be to have a good chance to get a collision?

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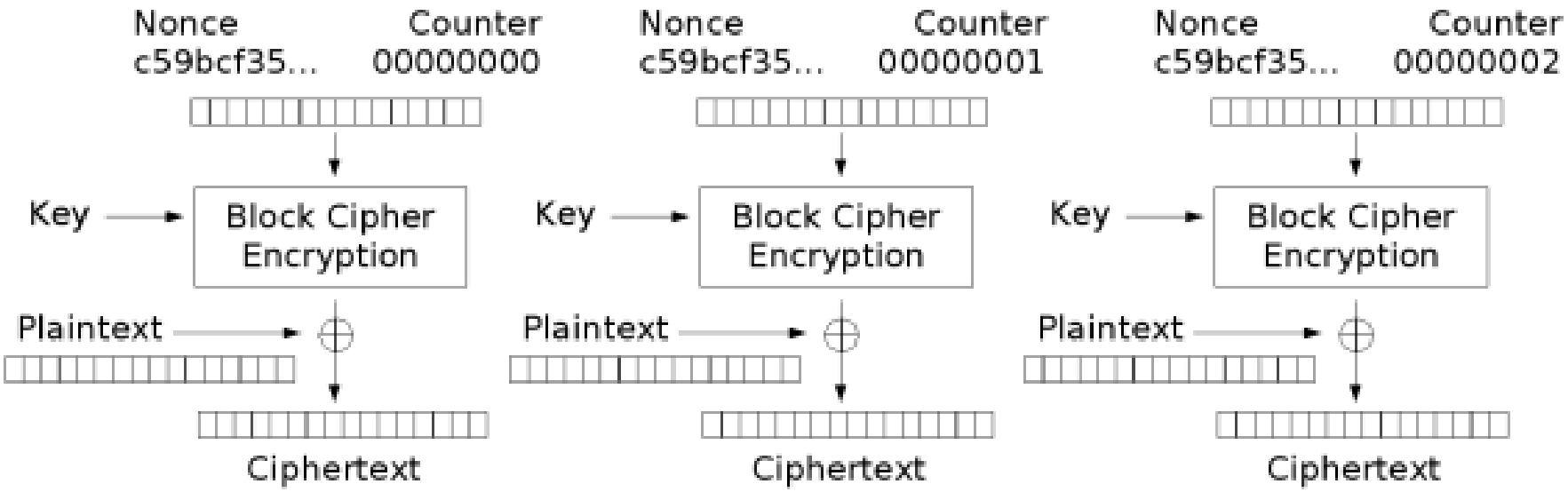
The limits of CBC

How likely is $c_i = c'_j$?

- This becomes very likely as the number of blocks encrypted under the same key reaches the order of $2^{n/2}$, with n the block size.
- This is more of a problem for DES (2^{32} blocks) than for AES (2^{64} blocks).

LOTS of data

Counter (CTR)



Counter (CTR) mode encryption

Counter (CTR)

Input: secret key $k \in \mathbb{Z}_2^m$, plaintext $p \in \mathbb{Z}_2^*$ and diversifier $d \in \mathbb{N}$
Output: ciphertext $c \in \mathbb{Z}_2^*$

- Cut into blocks of n bits $(p_1, p_2, \ldots, p_x)$, except for the last one that can be shorter
- Generate the keystream $k_i = E_k(d \| i)$

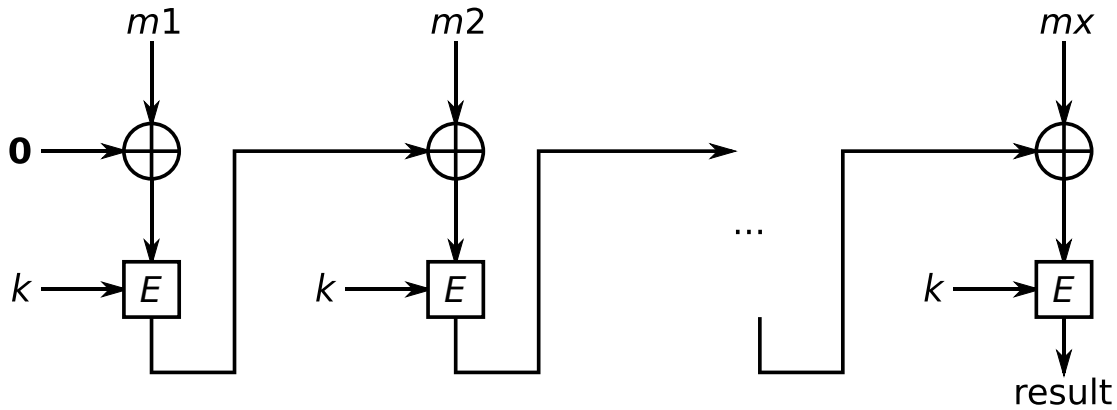
$$c_i = k_i \oplus p_i$$

Note: For the last block, the keystream block k_x is truncated to the size of p_x.

To decrypt, generate the same keystream and

$$p_i = k_i \oplus c_i$$

CBC-MAC



Input: secret key $k \in \mathbb{Z}_2^m$, message $m \in \mathbb{Z}_2^*$
Output: MAC $\in \mathbb{Z}_2^n$

- Prepend m with its length
- Pad $\text{len}(m) \| m$ with 10^* and cut into blocks $(m_1, m_2, \ldots, m_x)$
- Define $z_0 = 0^n$ and compute

$$z_i = E_k(m_i \oplus z_{i-1})$$

- Output MAC $= z_x$

Authenticated encryption

Best generic method:

- Encrypt (with k_1), then MAC the ciphertext (with k_2)

Specific modes (using only one key k):

- CCM combines CTR and CBC-MAC
- GCM combines CTR and a polynomial MAC in $\mathrm{GF}(2^{128})$
- ... and many many more ...

Permutation

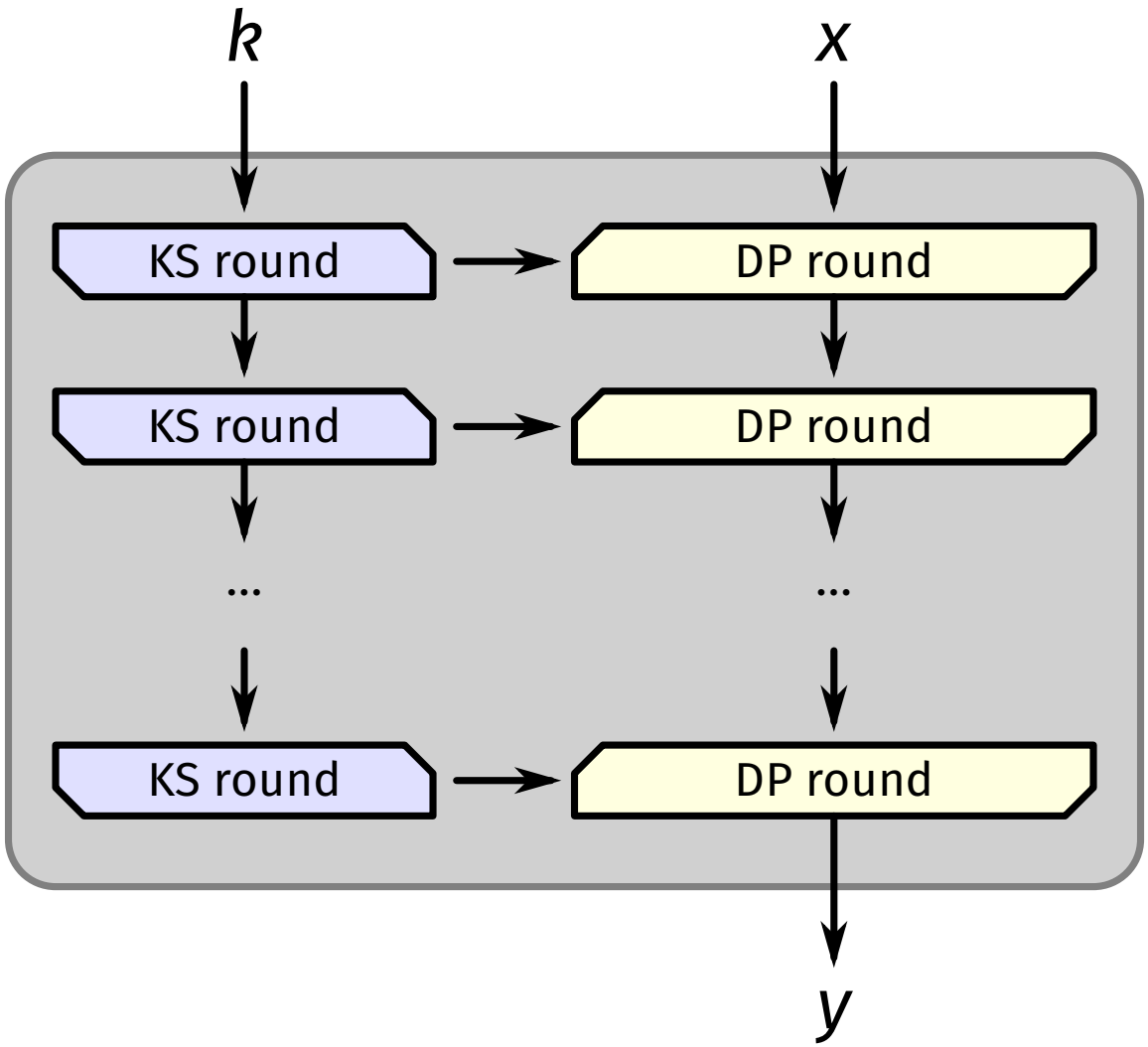
A cryptographic **permutation** is a bijective mapping

$$f : \mathbb{Z}_2^b \to \mathbb{Z}_2^b$$

- from an input block $x \in \mathbb{Z}_2^b$
- to an output block $y = f(x) \in \mathbb{Z}_2^b$

It has an inverse: $x = f^{-1}(y)$.

From block ciphers to permutations



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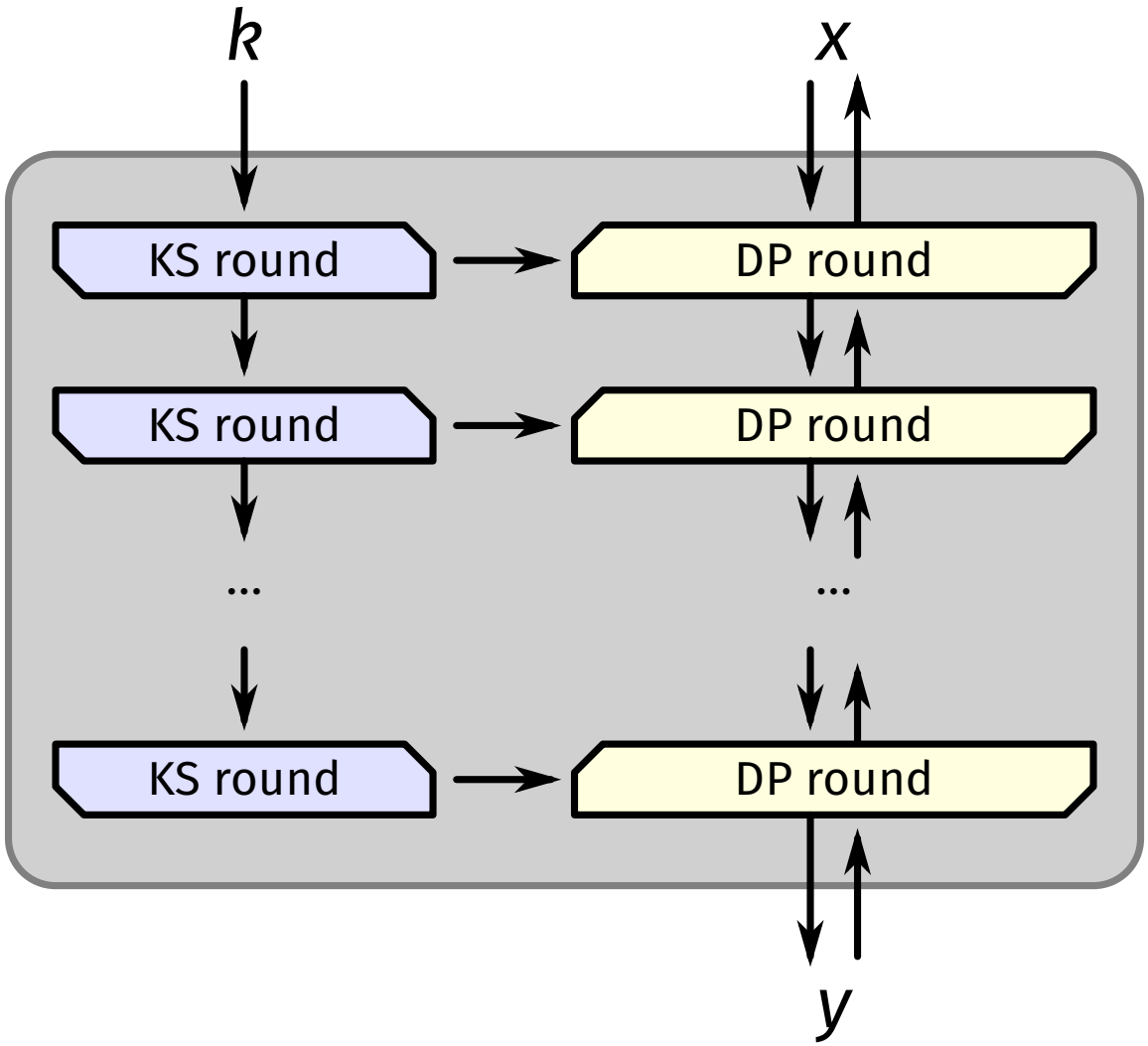

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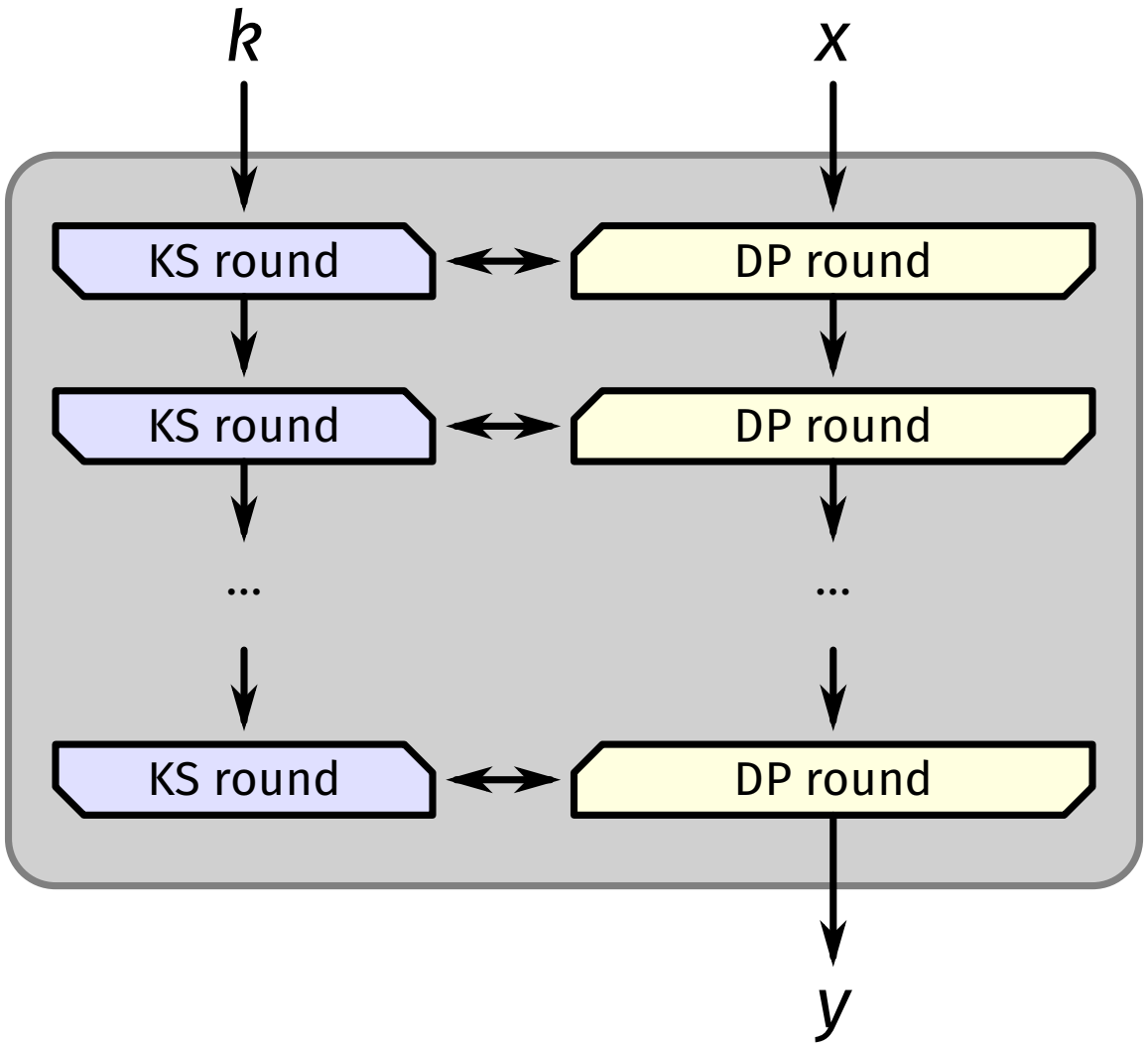

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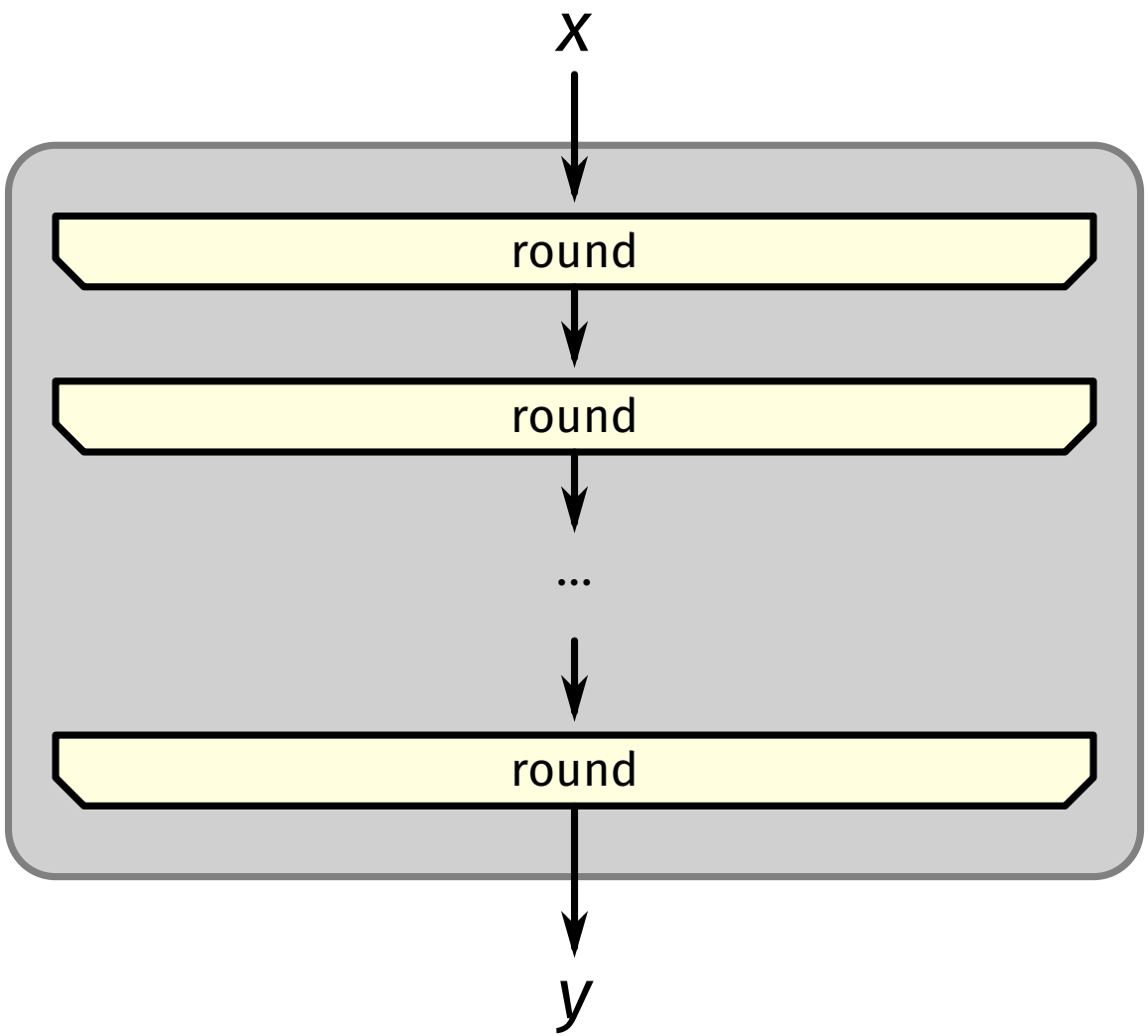
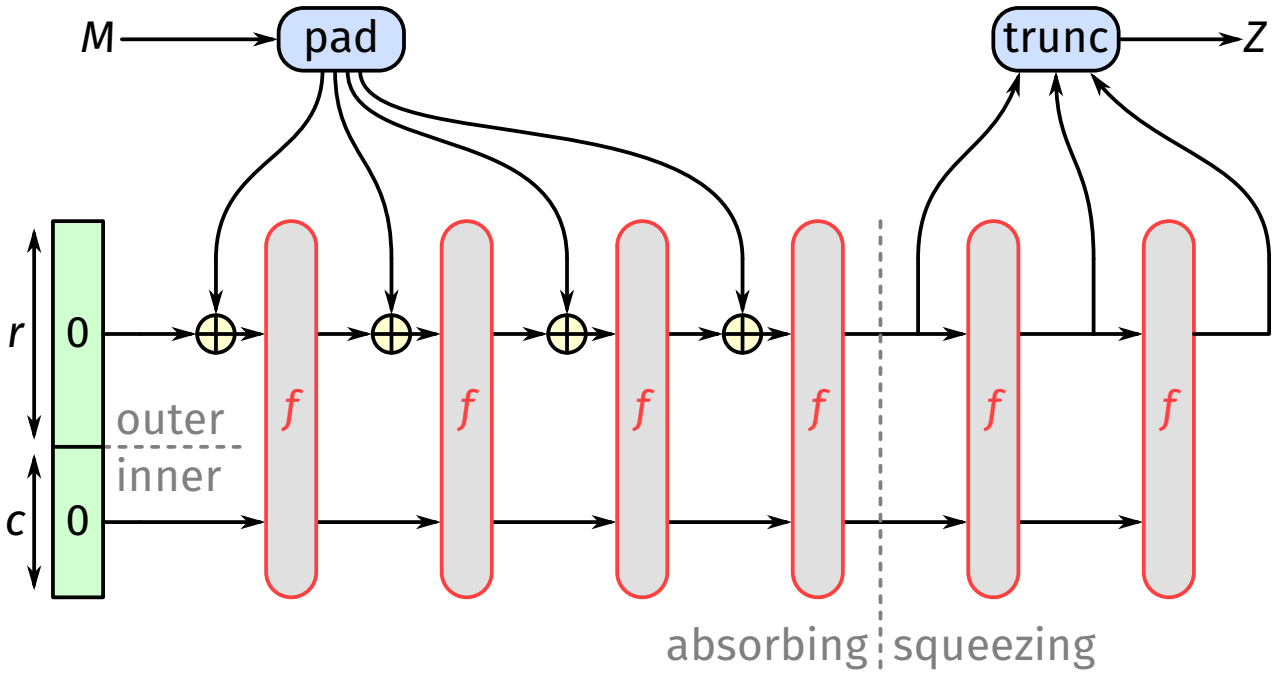


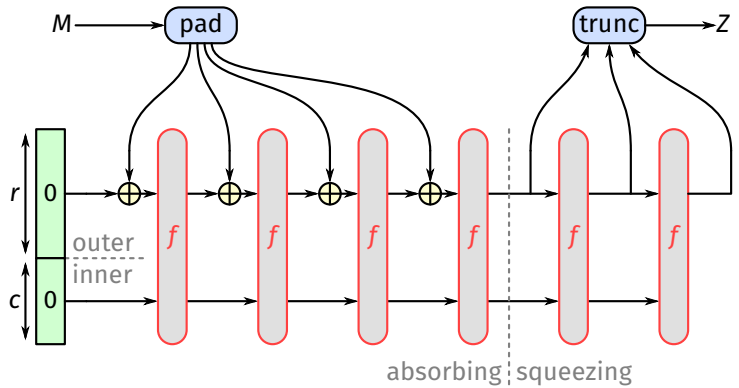
The sponge construction



- Calls a b-bit permutation f, with $b = r + c$
  - r bits of *rate*
  - c bits of *capacity* (security parameter)
- **Sponge function**: concrete instance with a given f, r, c

[Guido Bertoni, Joan Daemen, Michaël Peeters and GVA, 2007]

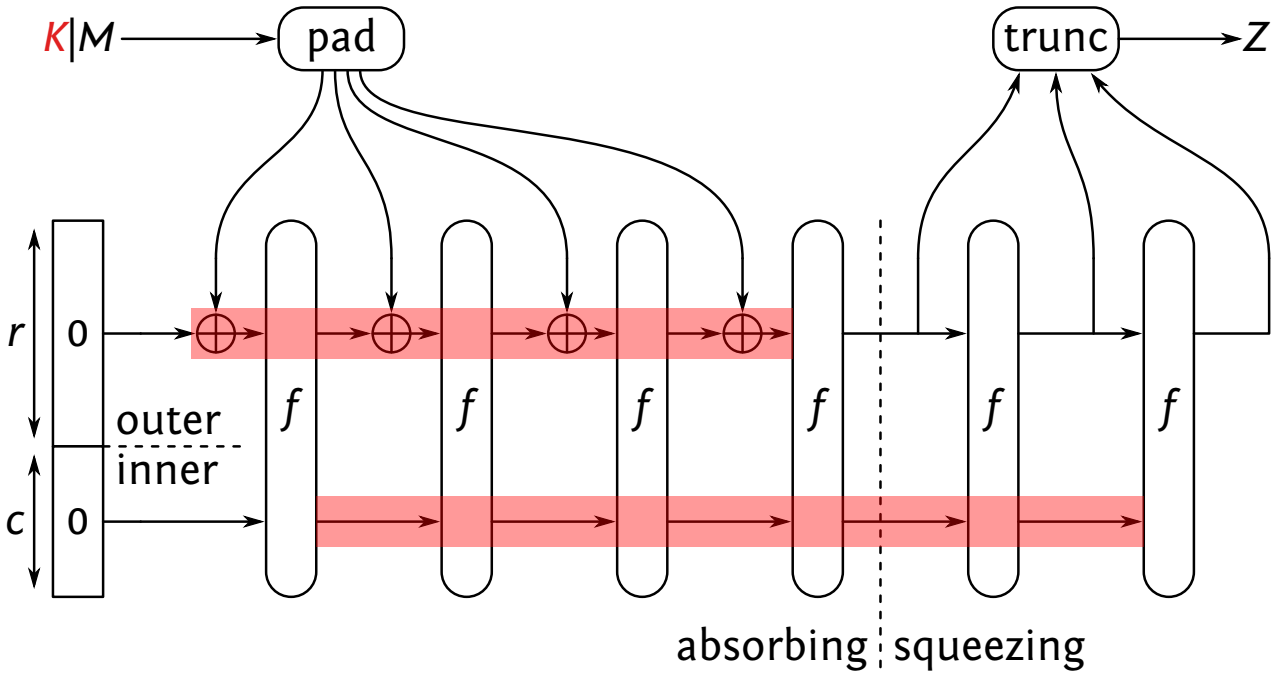
The sponge construction



A sponge function implements a mapping from $\mathbb{Z}_2^*$ to $\mathbb{Z}_2^\infty$ (truncated at an arbitrary length)

- $s \leftarrow 0^b$
- $M \| 10^*1$ is cut into r-bit blocks
- For each M_i do (absorbing phase)
  - $s \leftarrow s \oplus (M_i \| 0^c)$
  - $s \leftarrow f(s)$
- As long as output is needed do (squeezing phase)
  - Output the first r bits of s
  - $s \leftarrow f(s)$

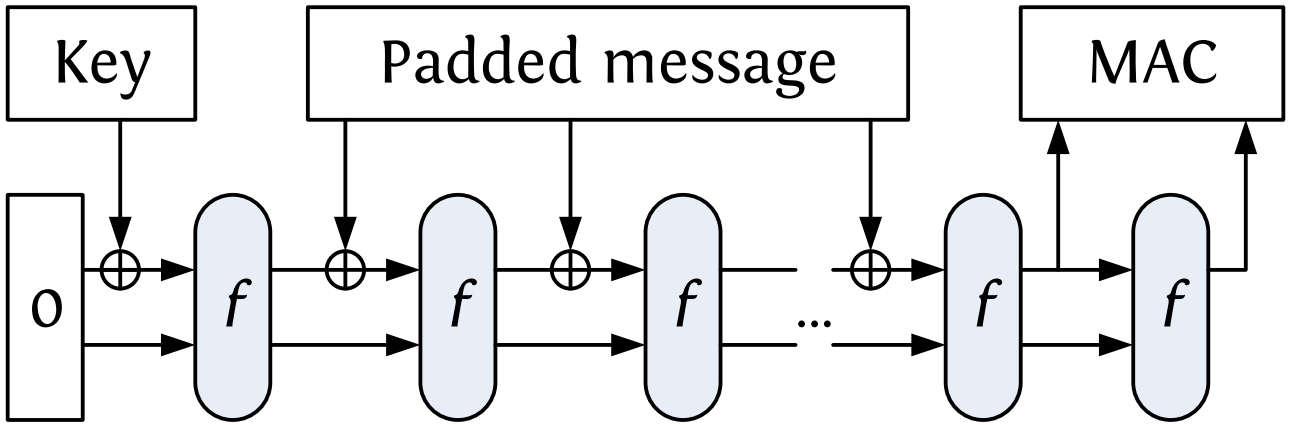
Keyed sponge



$$Z \leftarrow \text{sponge}_K(M) = \text{sponge}(K \| M)$$

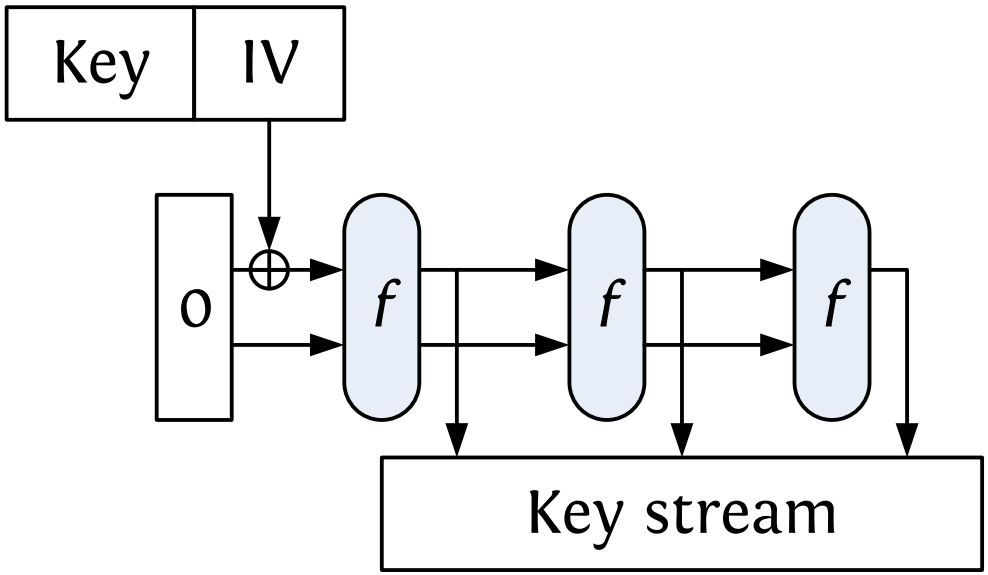
A keyed sponge function implements a mapping from $\mathbb{Z}_2^m \times \mathbb{Z}_2^*$ to $\mathbb{Z}_2^\infty$ (truncated). The input is prefixed with the secret key K.

Keyed sponge for authentication



$\text{sponge}_K(M)$ gives a MAC on M under key K

Keyed sponge as a stream cipher

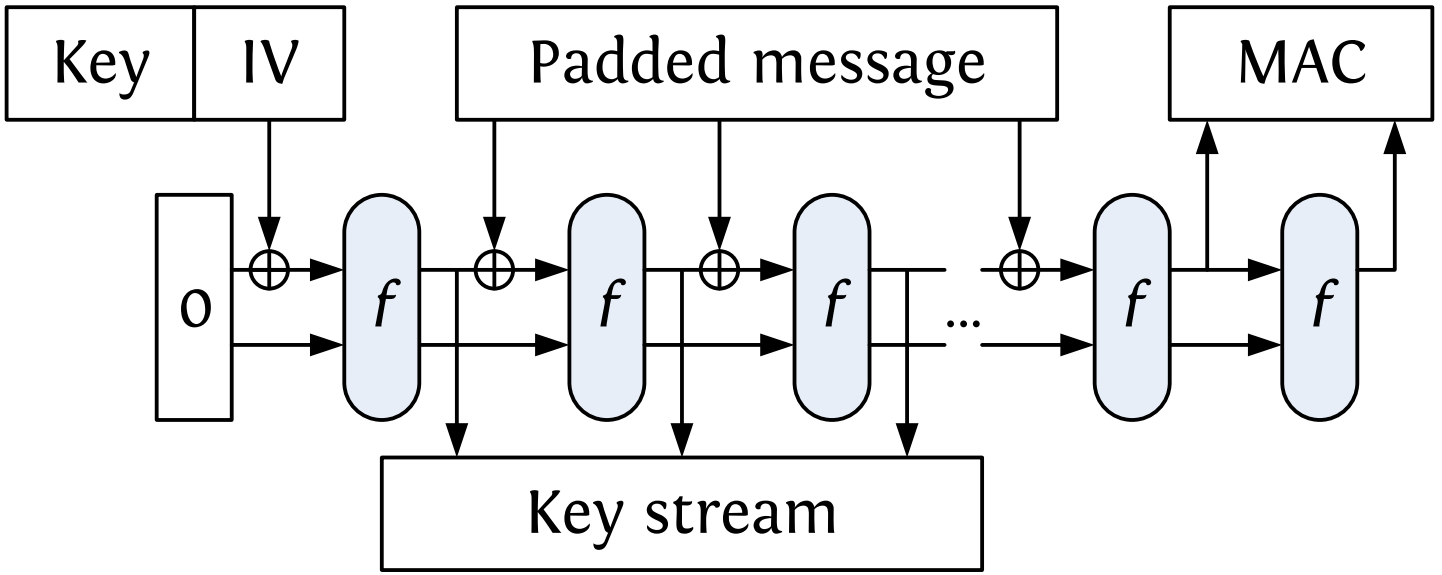


$s_i \leftarrow \text{sponge}_K(d)$ produces keystream from key K and diversifier d

$$c_i = m_i + s_i \quad (\text{in } \mathbb{Z}_2)$$

Note: IV = d

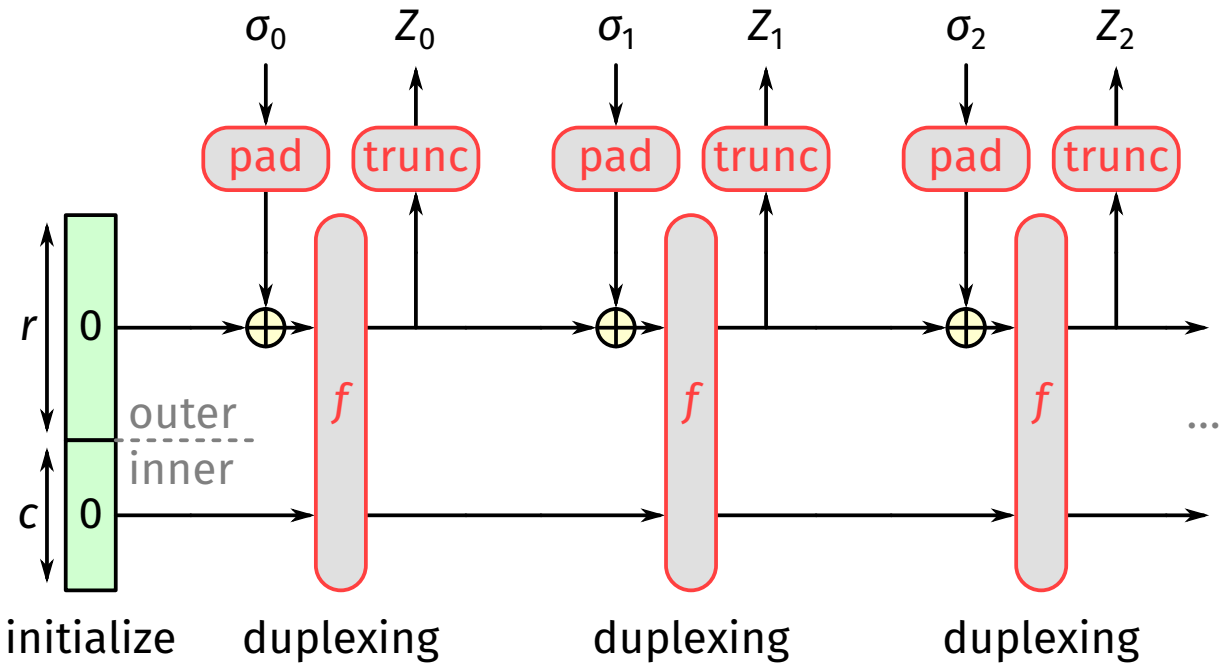
Authenticated encryption: spongeWrap



Variant of the sponge construction:

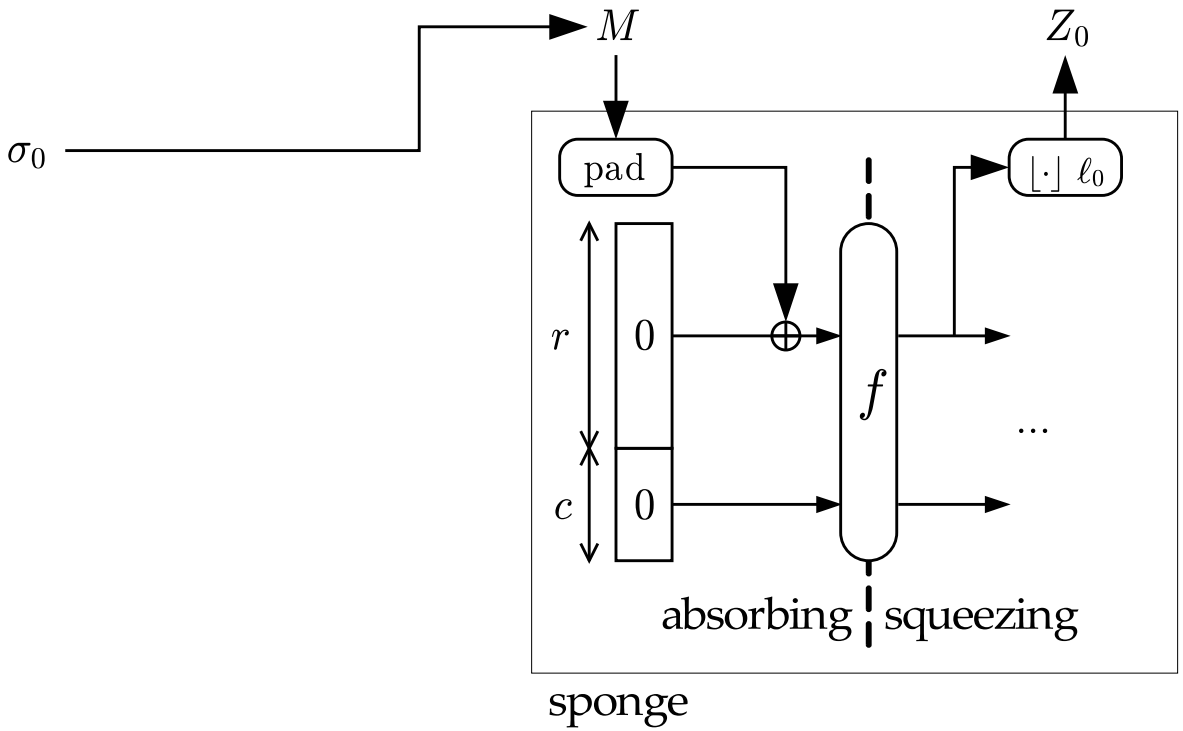
- Keystream: $s_i \leftarrow \text{sponge}_K(d \| \text{previous plaintext blocks})$
- MAC: $\text{sponge}_K(d \| \text{plaintext})$

The duplex construction



- Object: $D = \textsc{duplex}[f, \text{pad}, r]$
- Requesting ℓ-bit output $Z = D.\text{duplexing}(\sigma, \ell)$
  - input σ and output Z limited in length
  - Z depends on all previous inputs

Generating duplex responses with a sponge



$$Z_0 = \mathsf{sponge}(\sigma_0, \ell_0)$$

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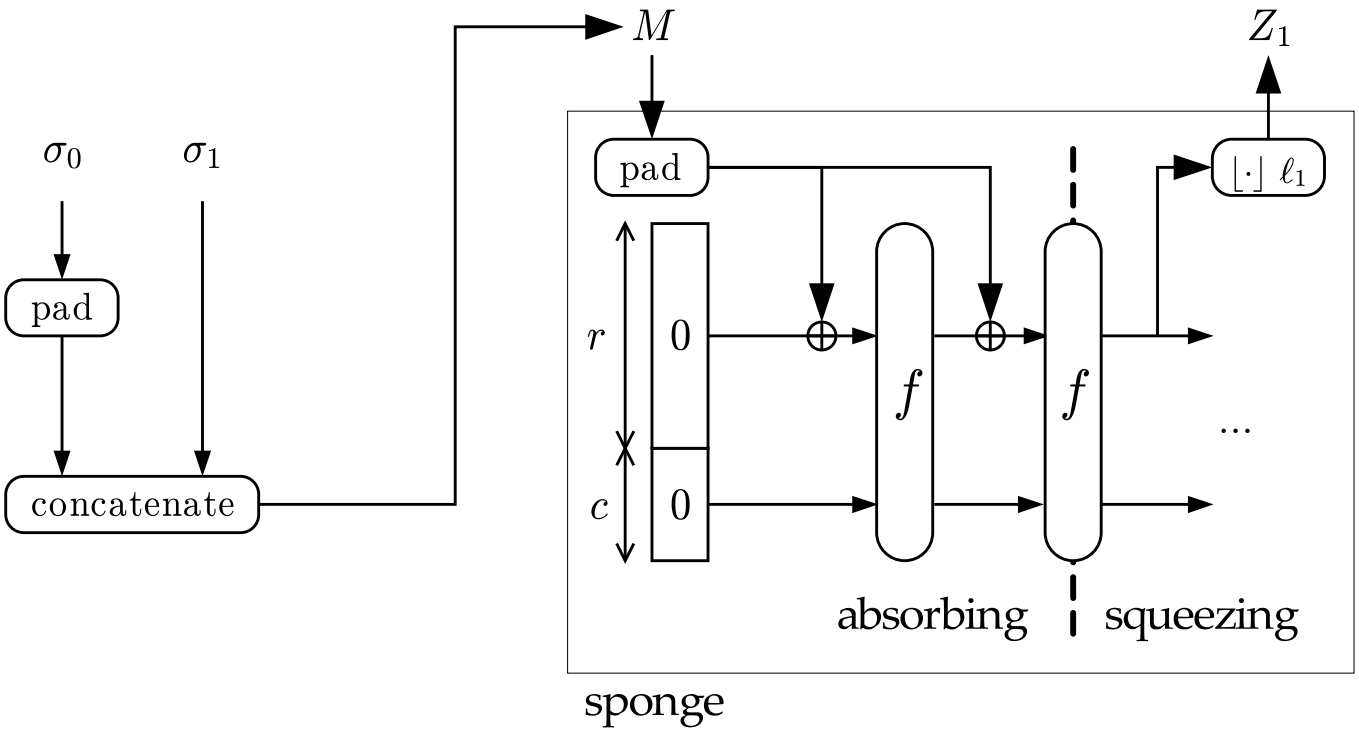
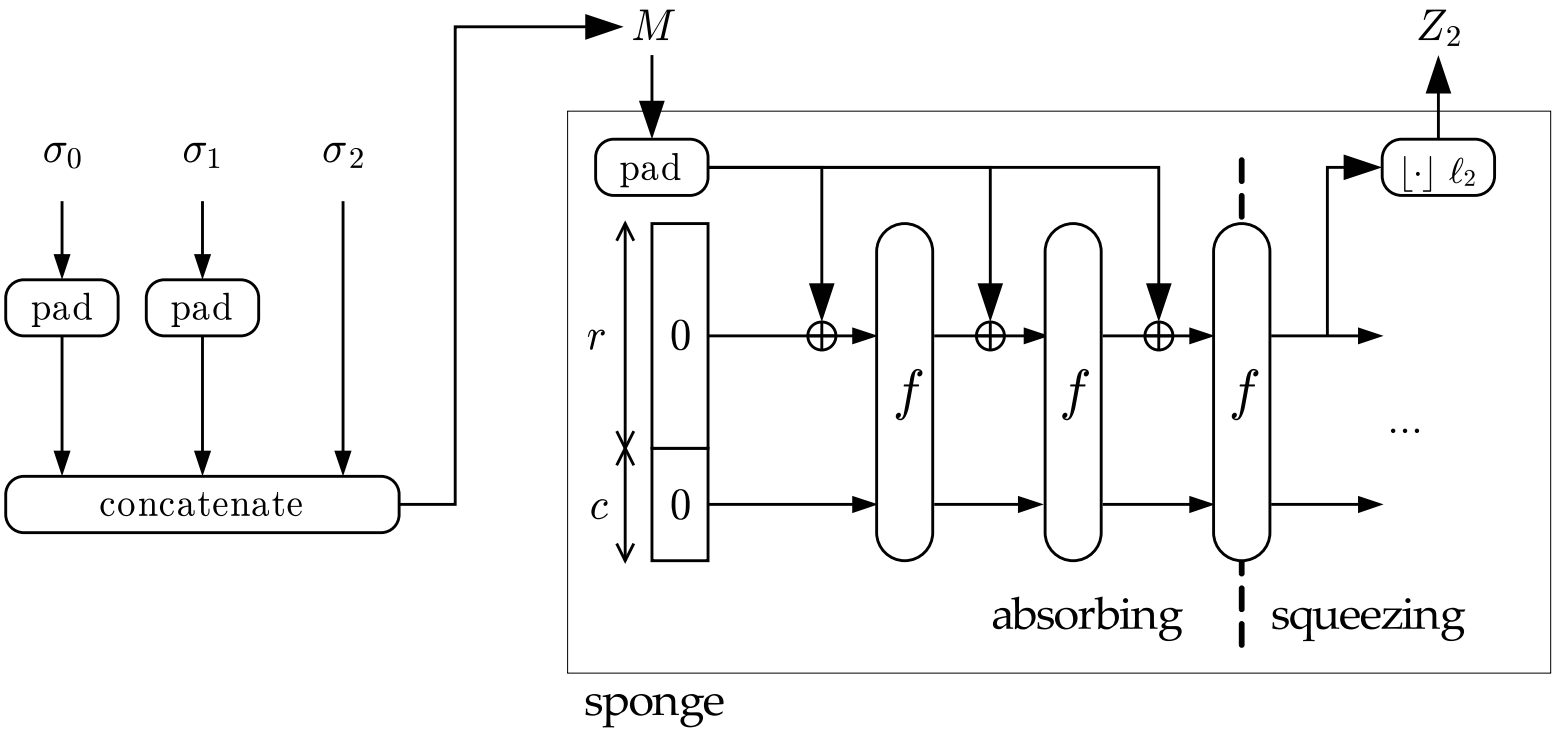


$$Z_1 = \mathsf{sponge}(\mathsf{pad}(\sigma_0)||\sigma_1, \ell_1)$$

Generating duplex responses with a sponge



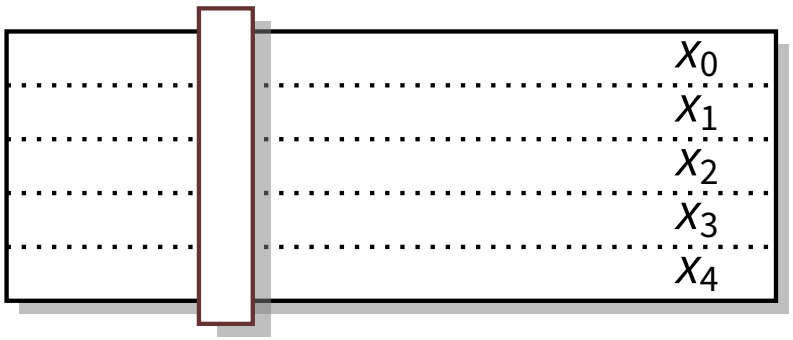
$$Z_2 = \mathsf{sponge}(\mathsf{pad}(\sigma_0)||\mathsf{pad}(\sigma_1)||\sigma_2, \ell_2)$$

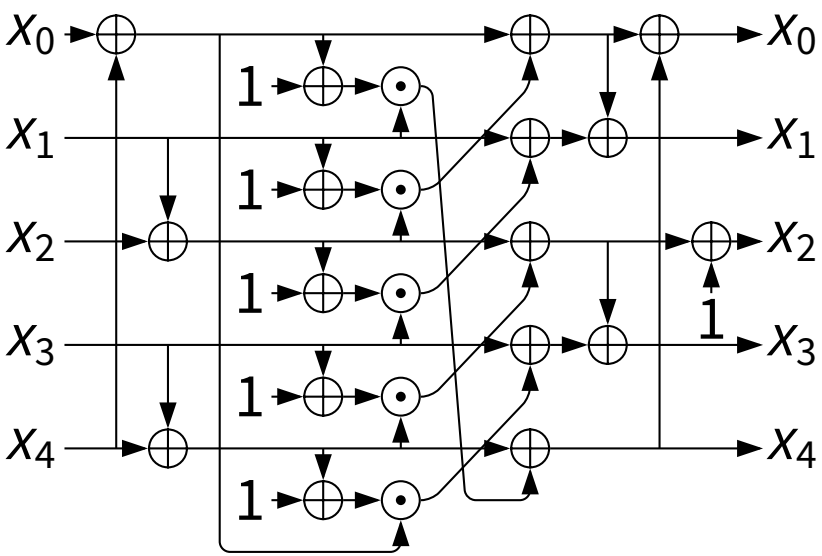
Ascon

- Ascon: permutation-based authenticated encryption scheme
- Designed by Christoph Dobraunig, Maria Eichlseder, Florian Mendel and Martin Schläffer in 2014
- One of the algorithms in the CAESAR portfolio

Permutation width: $b = 320 = 5 \times 64$ bits

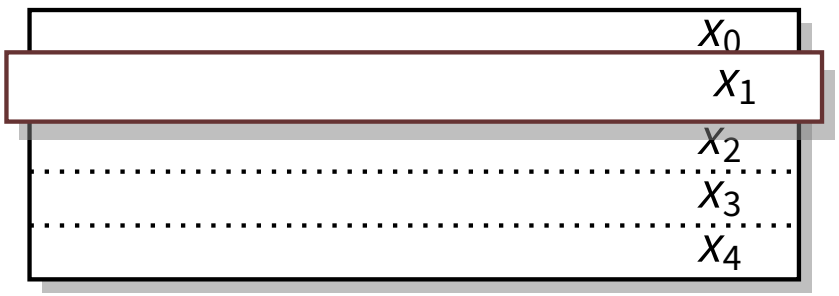
Ascon: non-linear step





- S-box applied to all 64 columns
- Central: $x_i \leftarrow x_i + (x_{i+1} + 1)x_{i+2}$ (=KECCAK S-box)
- Linear mixing before and after

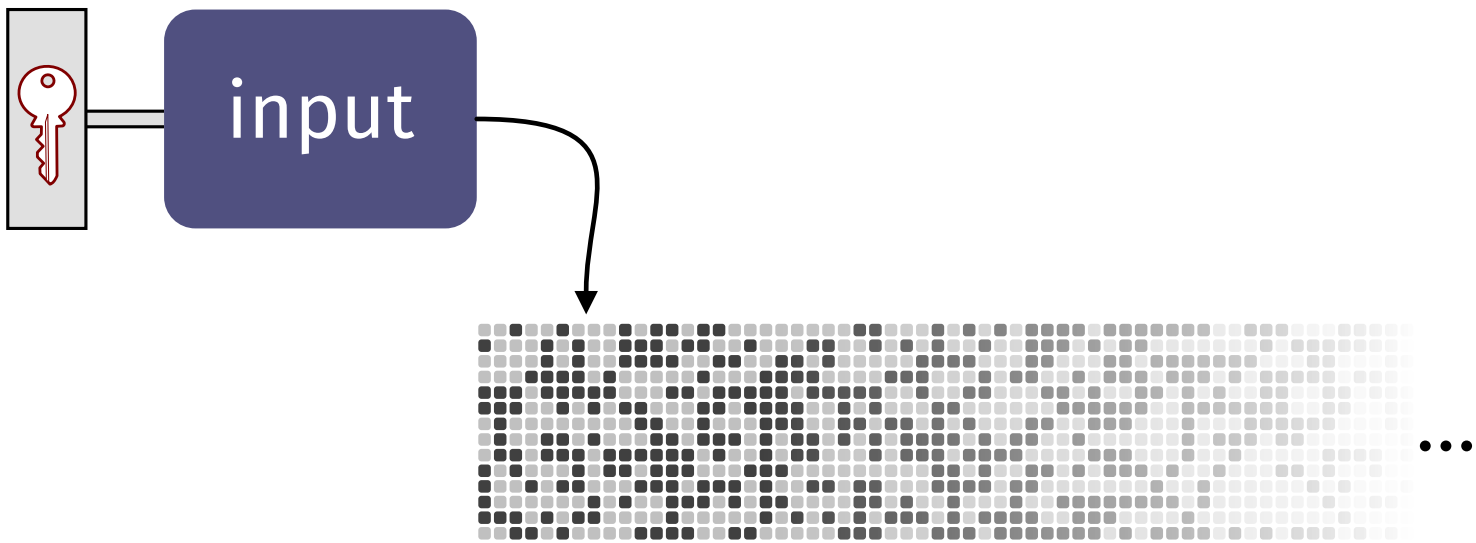
Ascon: linear step



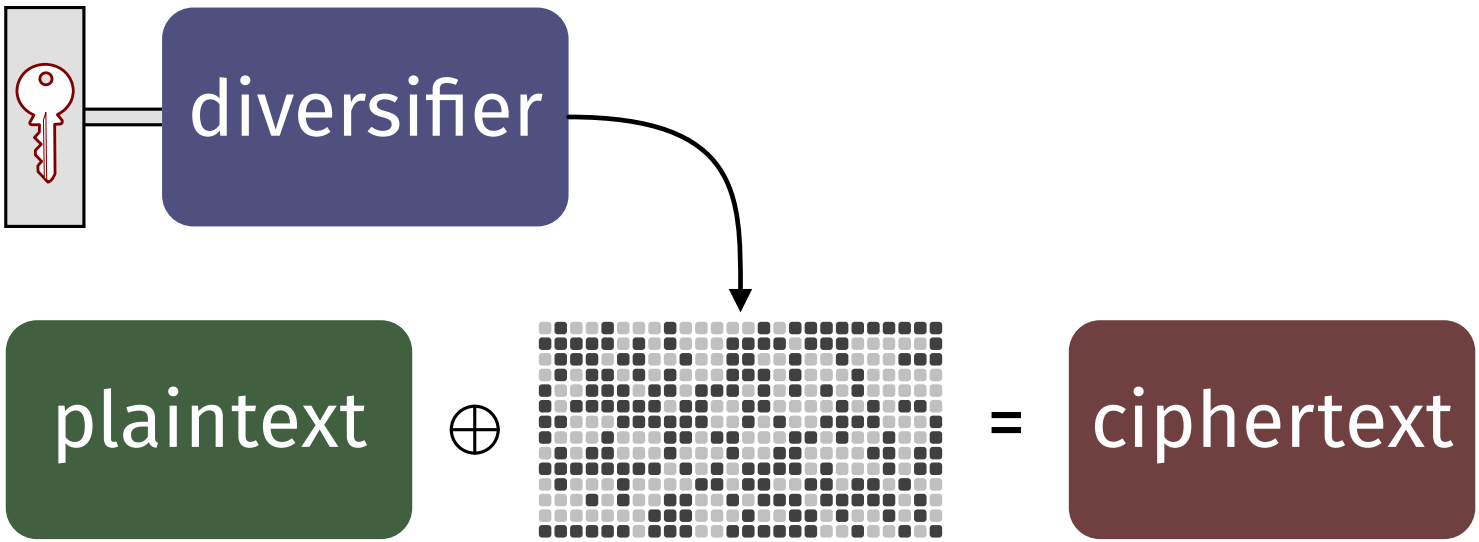
$$x_0 \leftarrow x_0 \oplus (x_0 \ggg 19) \oplus (x_0 \ggg 28)$$
$$x_1 \leftarrow x_1 \oplus (x_1 \ggg 61) \oplus (x_1 \ggg 39)$$
$$x_2 \leftarrow x_2 \oplus (x_2 \ggg 1) \oplus (x_2 \ggg 6)$$
$$x_3 \leftarrow x_3 \oplus (x_3 \ggg 10) \oplus (x_3 \ggg 17)$$
$$x_4 \leftarrow x_4 \oplus (x_4 \ggg 7) \oplus (x_4 \ggg 41)$$

- Intra-word linear diffusion
- Applied on each 64-bit word independently

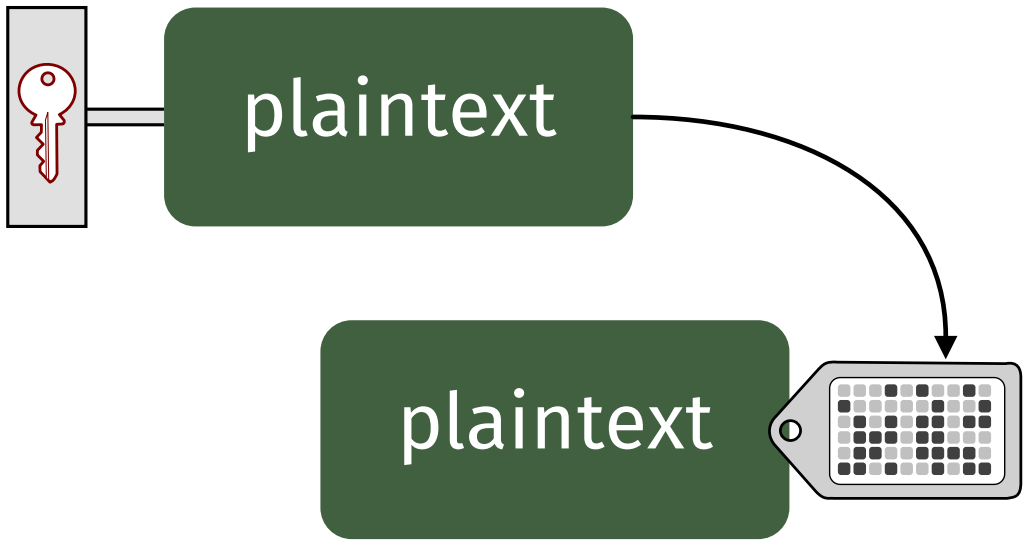
Pseudo-random function (PRF)



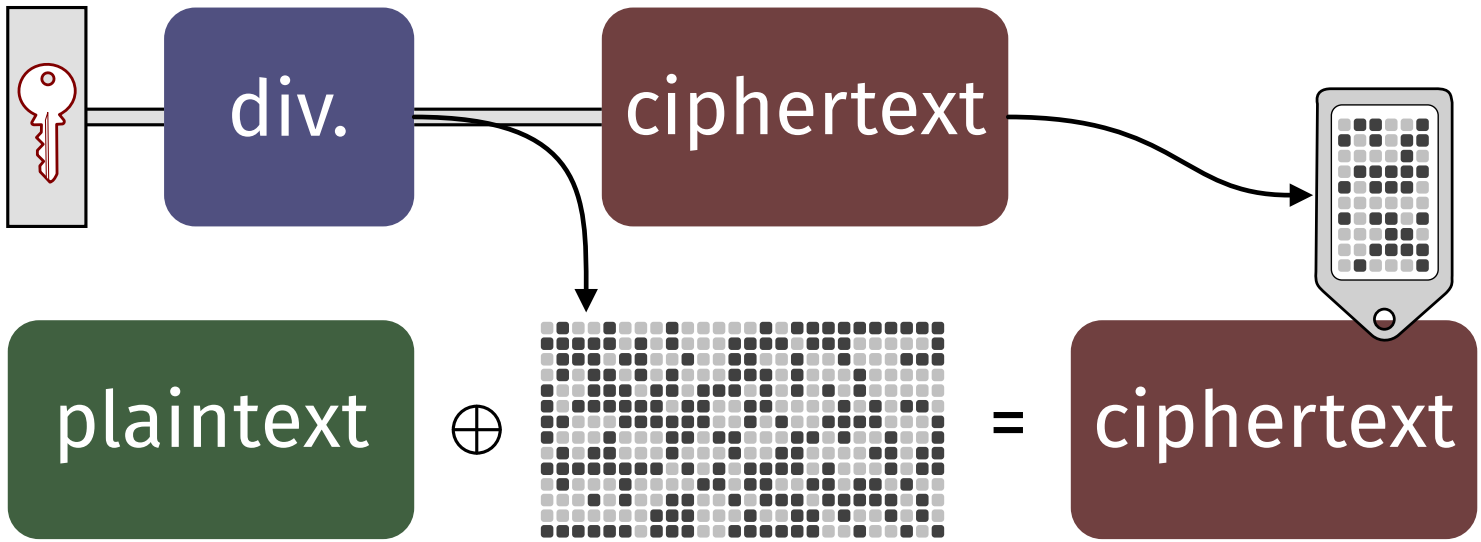
Stream cipher



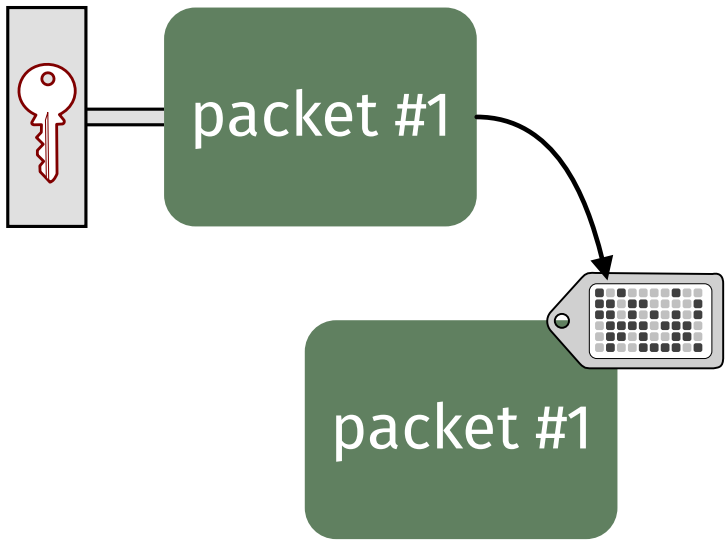
Message authentication code (MAC)



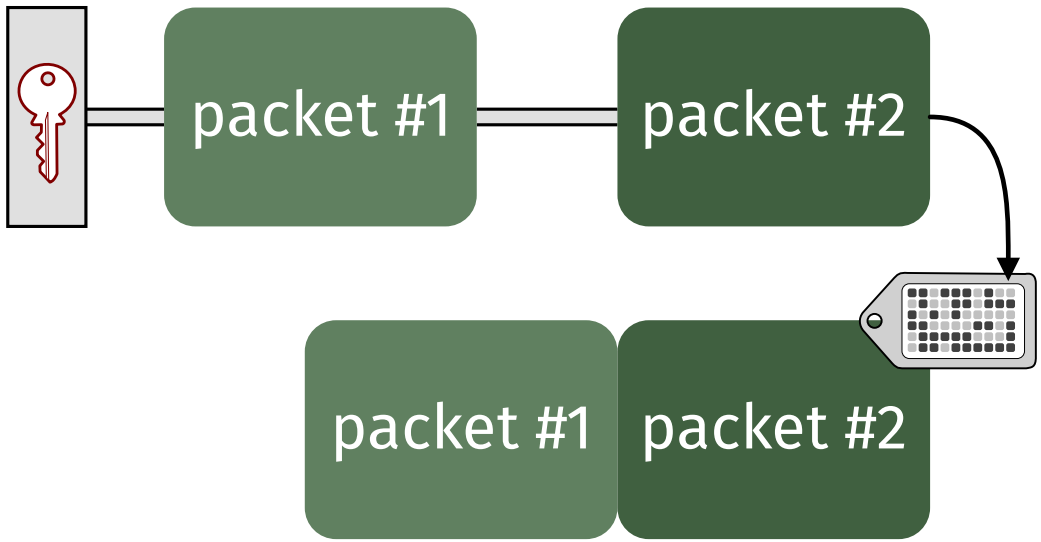
Authenticated encryption



Incremental MACs



Incremental MACs



Incremental MACs