# Introduction to cryptography

## 3. Hashing

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INFO-F-405
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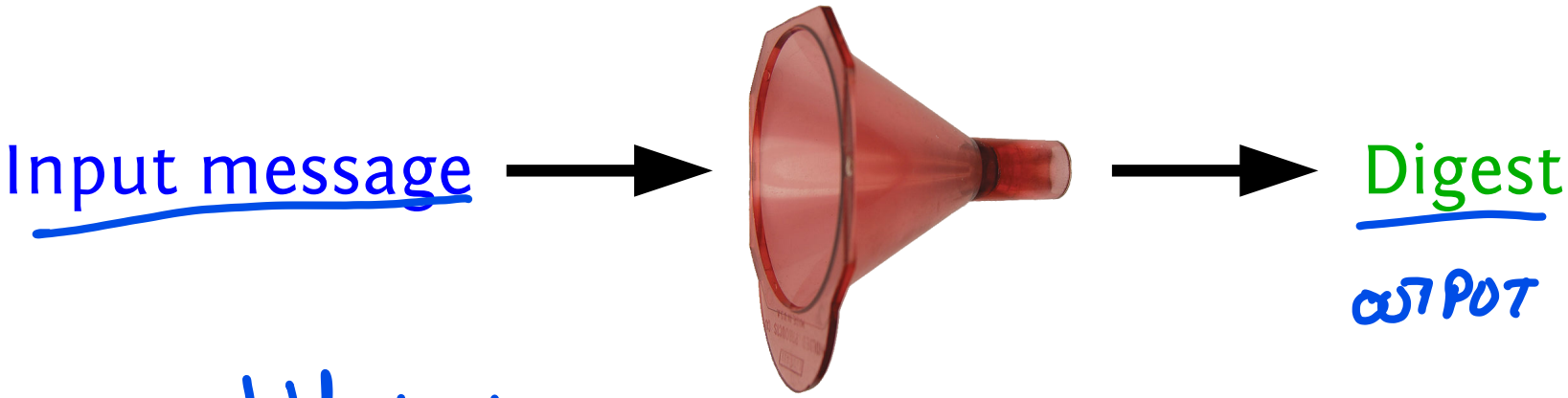
# Cryptographic hash functions

Hashing = no keys

$$h : \{0,1\}^* \to \{0,1\}^n$$




output

input the digest and

HASH

- Applications
  - *Signatures*: $\text{sign}_{\text{RSA}}(h(M))$ instead of $\text{sign}_{\text{RSA}}(M)$ — not the message in block cipher — on input sets that is finite and limited
  - *Key derivation*: master key $K$ to derived keys ($K_i = h(K\|i)$)
  - *Bit commitment, predictions*: $h$(what I know)
  - *Message authentication*: $h(K\|M)$
  - ...

multiple input can give the same output. Really hard to inverse

# Cryptographic hash functions

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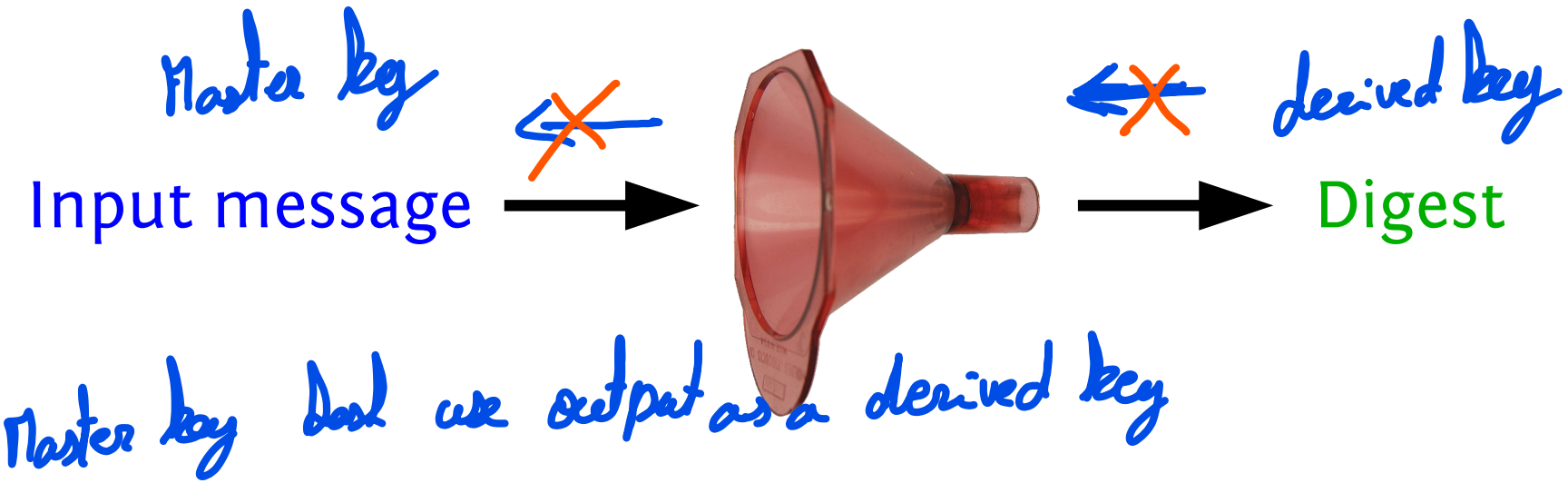


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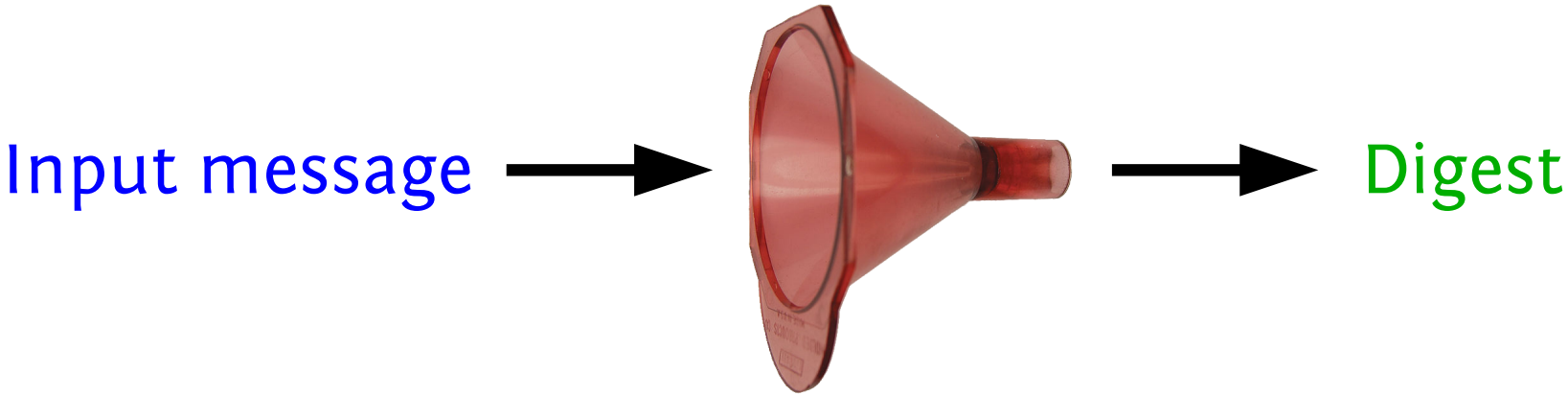


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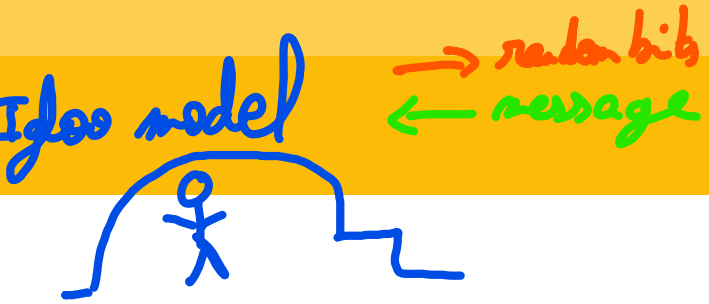
* on a tjrs le m digest

Le dire qu'on vote pour qqn et garder le fichier originel pour prouver en cas de recours qu'il

# Cryptographic hash functions

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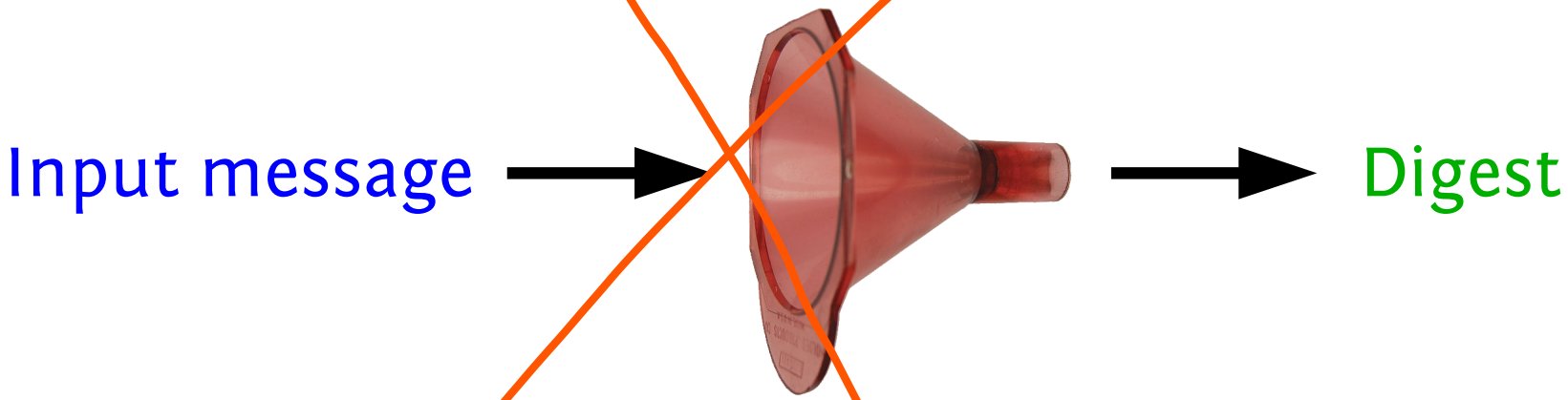


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  - ...

# Generalized: extendable output function (XOF)

$$h : \{0,1\}^* \to \{0,1\}^\infty$$

"XOF: a function in which the output can be extended to any length."

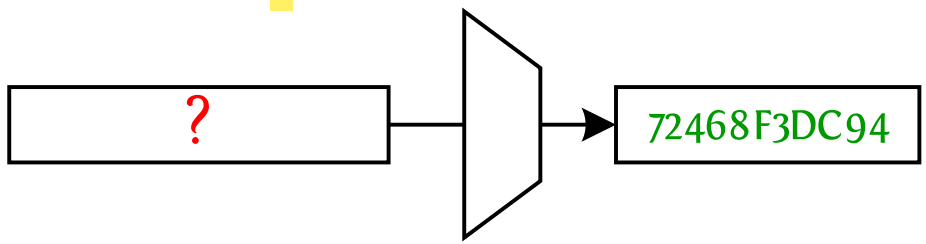
[Ray Perlner, SHA 3 workshop 2014]

- Applications
  - *Signatures*: full-domain hashing, mask generating function
  - *Key derivation*: as many/long derived keys as needed
  - *Stream cipher*: $C = P \oplus h(K \| \text{nonce})$

# Preimage resistance

HASHING: preimage resistance, second image resistance, collision resistance } 3 mains requirements

- Given $y \in \mathbf{Z}_2^n$, find $x \in \mathbf{Z}_2^*$ such that $h(x) = y$



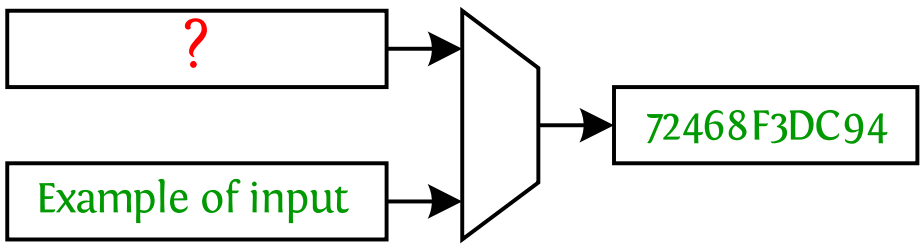

- If $h$ is a random function, about $2^n$ attempts are needed
- **Example**: given derived key $K_1 = h(K\|1)$, find master key $K$

→ 1 chance / $2^n$ chances to guess the correct input

# Second preimage resistance

Find another input that gives the same digest as another input that we have!

- Given $x \in \mathbf{Z}_2^*$, find $x' \neq x$ such that $h(x') = h(x)$



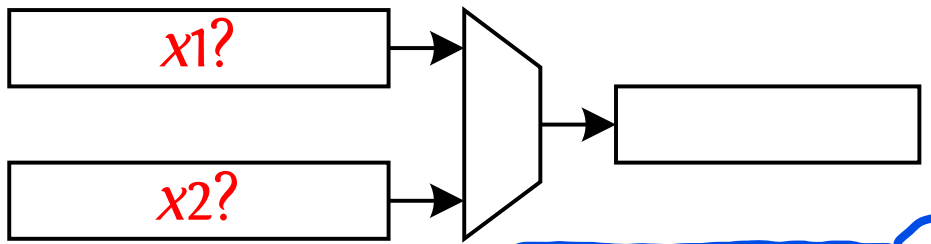

- If $h$ is a random function, about $2^n$ attempts are needed
  → same complexity as preimage
- **Example**: signature forging
  - Given $M$ and sign$(h(M))$, find $M' \neq M$ with equal signature

→ if we have one message + signature ($h(M)$)
hope to get the same signature with another message

# Collision resistance

Find $2^{\neq}$ inputs that gives the same output

- Find $x_1 \neq x_2$ such that $h(x_1) = h(x_2)$

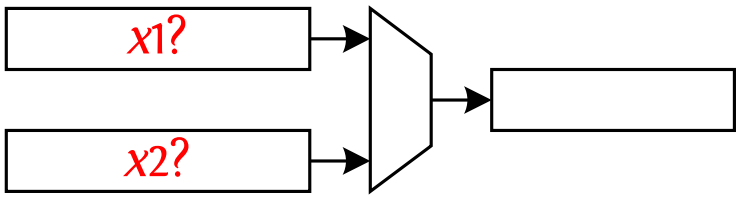



- If $h$ is a random function, about $2^{n/2}$ attempts are needed
  - Birthday paradox: among 23 people, probably two have same birthday
  - Scales as $\sqrt{|\text{range}|} = 2^{n/2}$

easier!

see §4.6 Dap$^2$

# Collision resistance (continued)




- **Example**: “secretary” signature forging
  - Set of good messages $\{M_i^{\text{good}}\}$
  - Set of bad messages $\{M_i^{\text{bad}}\}$
  - Find $h(M_i^{\text{good}}) = h(M_j^{\text{bad}})$
  - Boss signs $M_i^{\text{good}}$, but valid also for $M_j^{\text{bad}}$

[Yuval, 1979]

# Other requirements

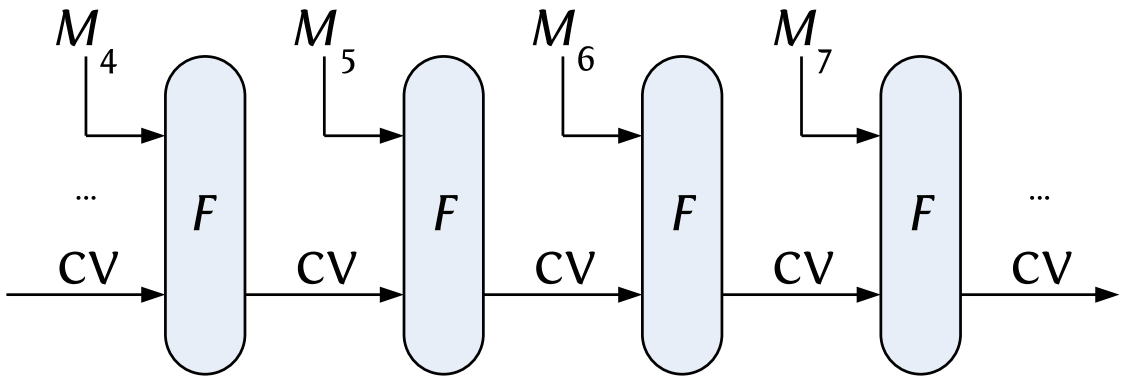
- Security claims by listing desired properties
  - Collision resistant
  - (Second-) preimage resistant
    - Multi-target preimage resistance
    - Chosen-target forced-prefix preimage resistance
  - Correlation-free
  - Resistant against length-extension attacks
  - ...
- But ever-growing list of desired properties
- A good hash function should behave like a **random mapping**...

# Security requirements summarized

- Hash or XOF $h$ with $n$-bit output
- Modern security requirements
  - $h$ behaves like a random mapping
  - ... up to security strength $s$
- Classical security requirements, derived from it

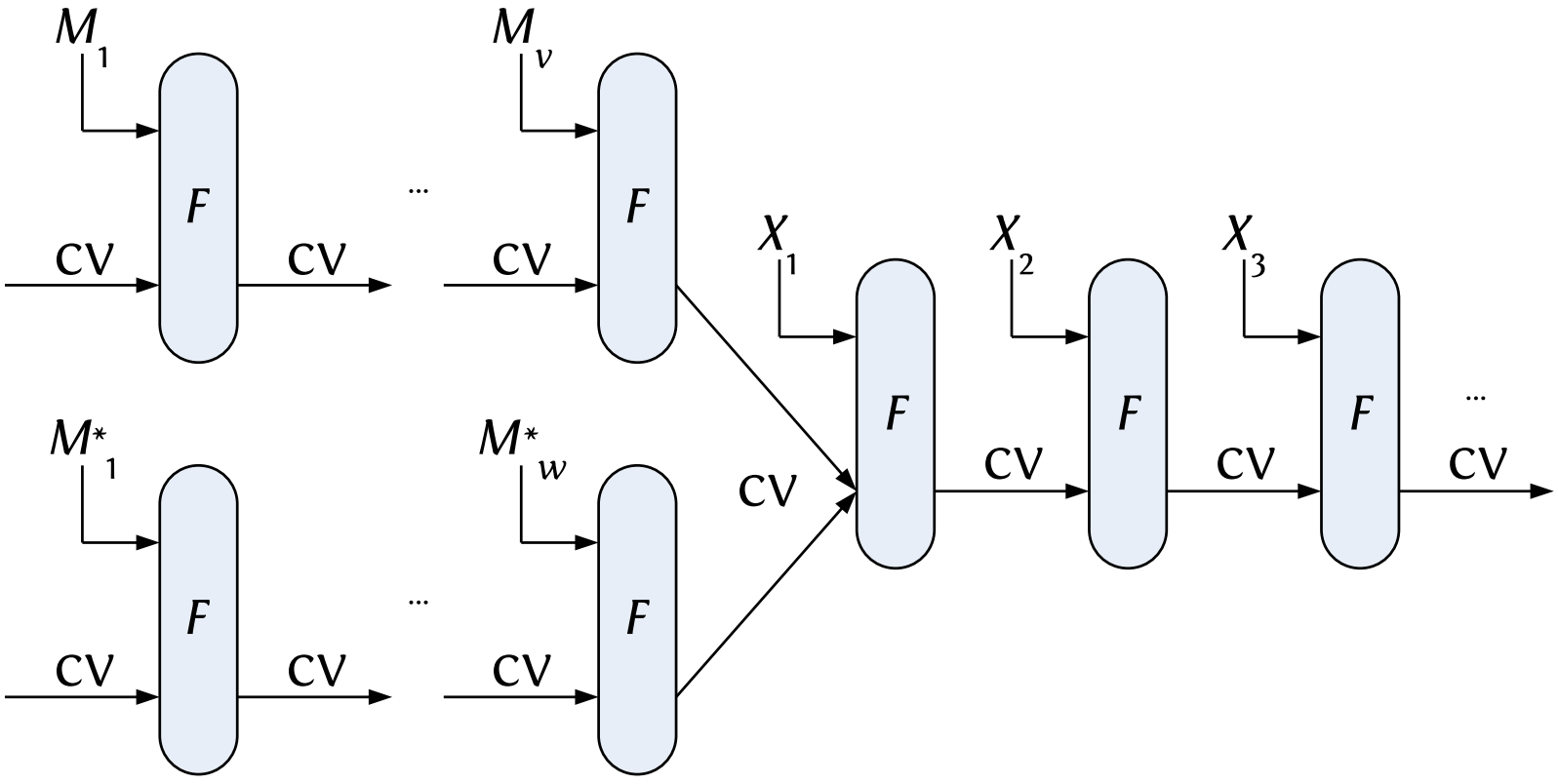
| | |
|---|---|
| Preimage resistance | $2^{\min(n,s)}$ |
| Second-preimage resistance | $2^{\min(n,s)}$ |
| Collision resistance | $2^{\min(n/2,s)}$ |

# Iterated functions




- All practical hash functions are iterated
  - Message $M$ cut into blocks $M_1, \ldots, M_l$
  - $q$-bit chaining value
- Output is function of final chaining value

# Internal collisions!




- Different inputs $M$ and $M^*$ giving the same chaining value
- Messages $M||X$ and $M^*||X$ always collide for any string $X$

Does not occur in a random mapping!

# Examples of hash functions

- MD5: $n = 128$
  - Published by Ron Rivest in 1992
  - Successor of MD4 (1990) ~ MD4 = Test ~ MD5 is used!
- SHA-1: $n = 160$
  - Designed by NSA, standardized by NIST in 1995
  - Successor of SHA-0 (1993)
- SHA-2: family supporting multiple lengths
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  - SHA-224, SHA-256, SHA-384 and SHA-512
- SHA-3: based on KECCAK
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  - SHA3-{224, 256, 384, 512}, SHAKE{128, 256}, ParallelHash{128, 256}, ...
- Other SHA-3 finalists
  - Blake (Aumasson et al.), Grøstl (Gauravaram et al.), JH (Wu), Skein (Ferguson et al.)

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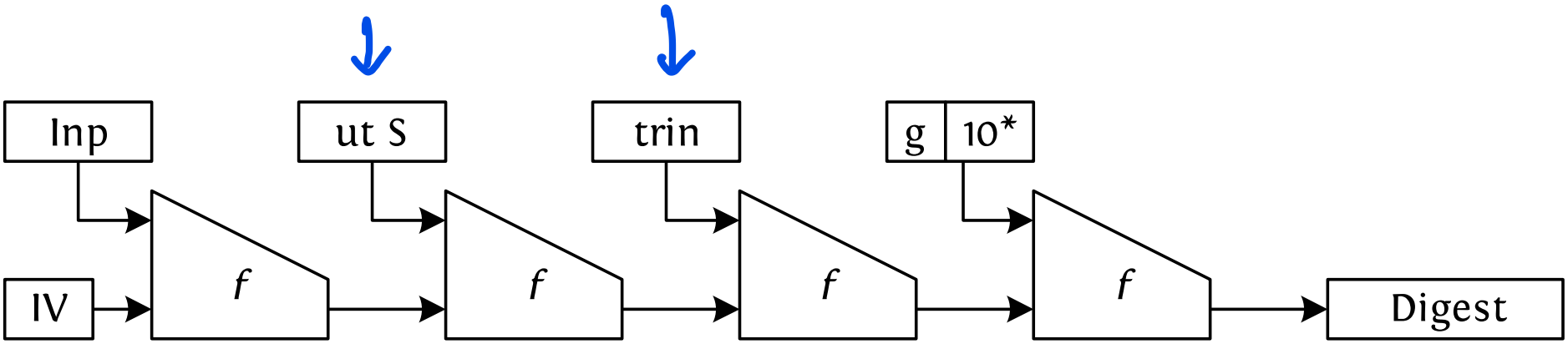
↳ 128 and 256 = s = security strength.

# Attacks on MD5, SHA-0 and SHA-1

- 2004: SHA-0 broken (Joux et al.)
- 2004: MD5 broken (Wang et al.)
- 2005: practical attack on MD5
  (Lenstra et al., and Klima)
- 2005: SHA-1 theoretically broken
  (Wang et al.)
- 2006: SHA-1 broken further
  (De Cannière and Rechberger)
- 2016: freestart collision on SHA-1
  (Stevens, Karpman and Peyrin)
- 2017: actual collision on SHA-1
  (Stevens, Bursztein, Karpman, Albertini and Markov)

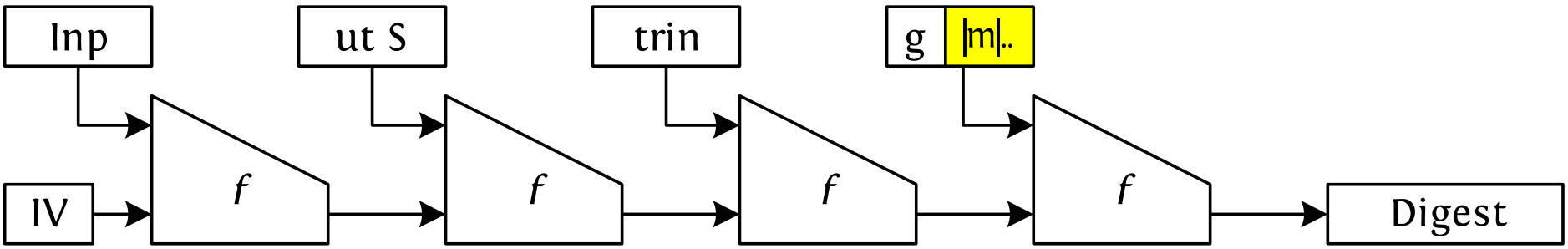
# Merkle-Damgård




- Uses a compression function from $n + m$ bits to $n$ bits
- Instances: MD5, SHA-1, SHA-2 ...
- Merkle-Damgård strengthening

[Merkle, CRYPTO'89], [Damgård, CRYPTO'89]

# Merkle-Damgård

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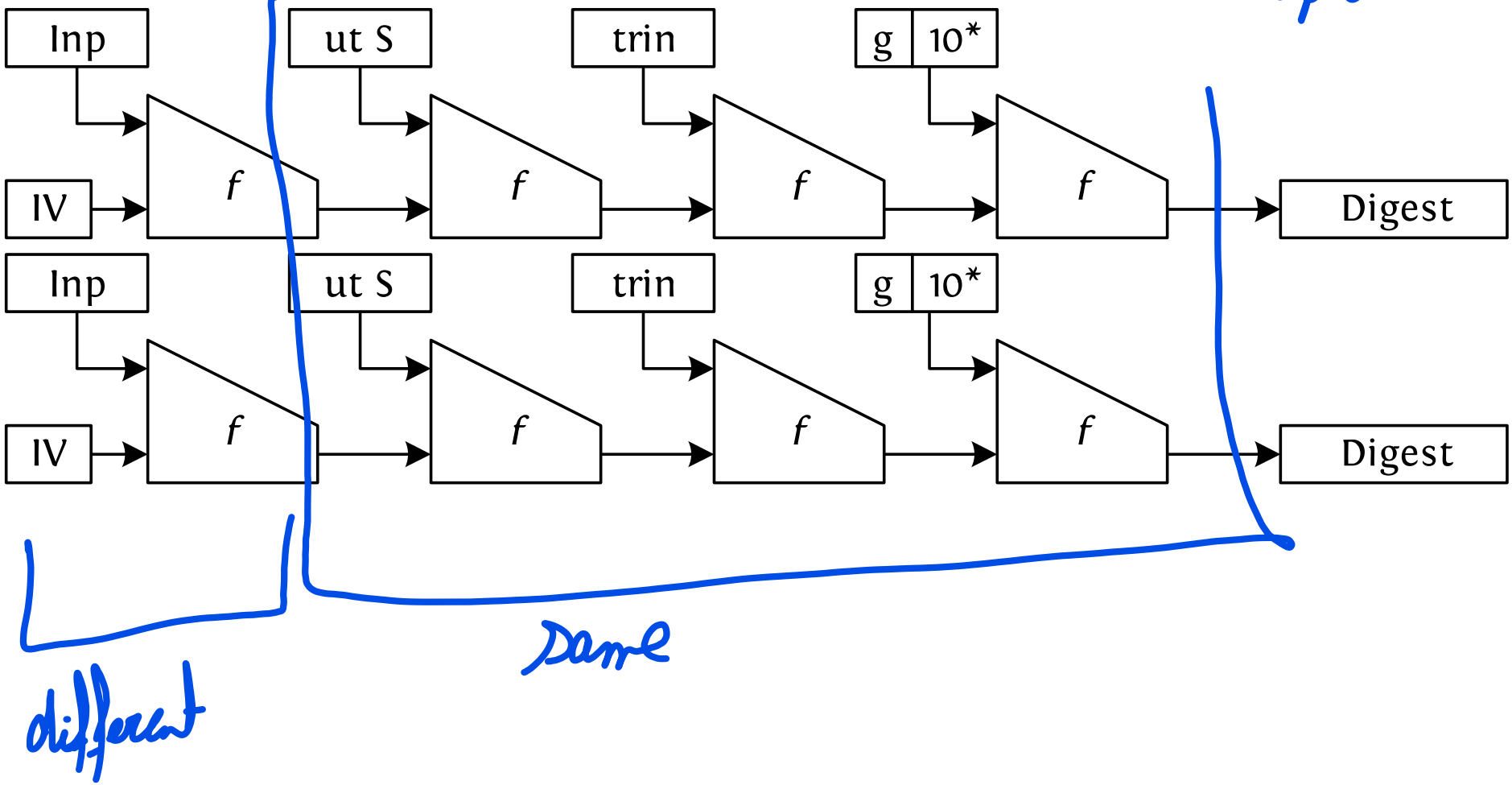
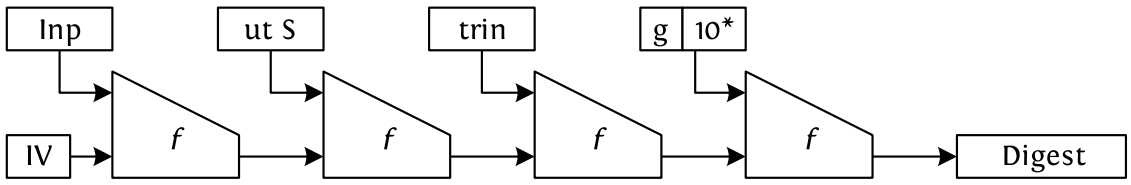
# Merkle-Damgård: preserving collision resistance

assume that all ut S and trin are the same → no collision at the output



# Merkle-Damgård: length extension




Recurrence (modulo the padding):

- $h(M_1) = f(\mathrm{IV}, M_1) = \mathrm{CV}_1$
- $h(M_1 \| \ldots \| M_i) = f(\mathrm{CV}_{i-1}, M_i) = \mathrm{CV}_i$

Forgery on naïve message authentication code (MAC):

- $\mathrm{MAC}(M) = h(\mathrm{Key} \| M) = \mathrm{CV}$
- $\mathrm{MAC}(M \| \mathrm{suffix}) = f(\mathrm{CV} \| \mathrm{suffix})$

Solution: **HMAC**

$$\mathrm{HMAC}(M) = h(\mathrm{Key}_{\mathrm{out}} \| h(\mathrm{Key}_{\mathrm{in}} \| M))$$

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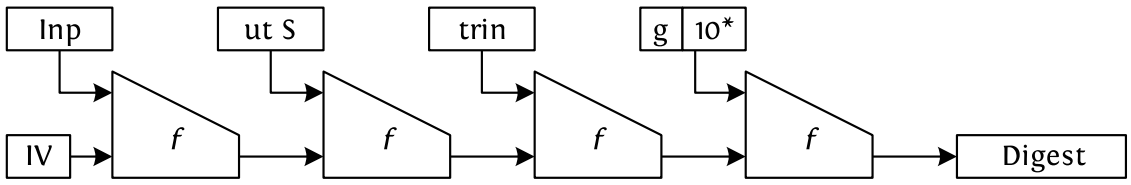


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Padding the key and the message not good

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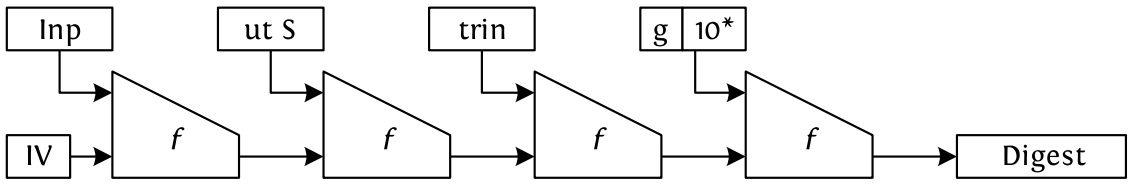


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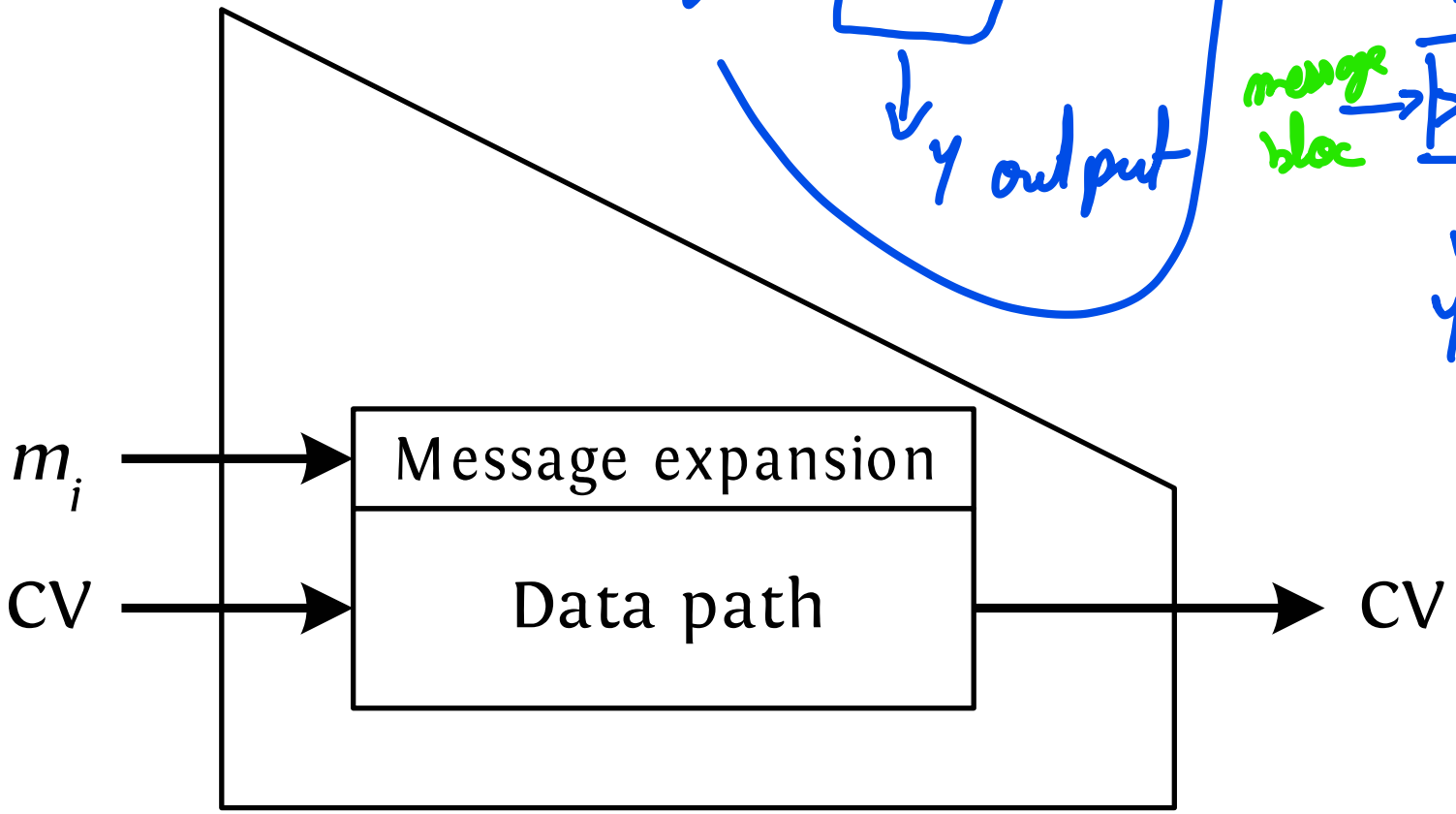
inner MAC

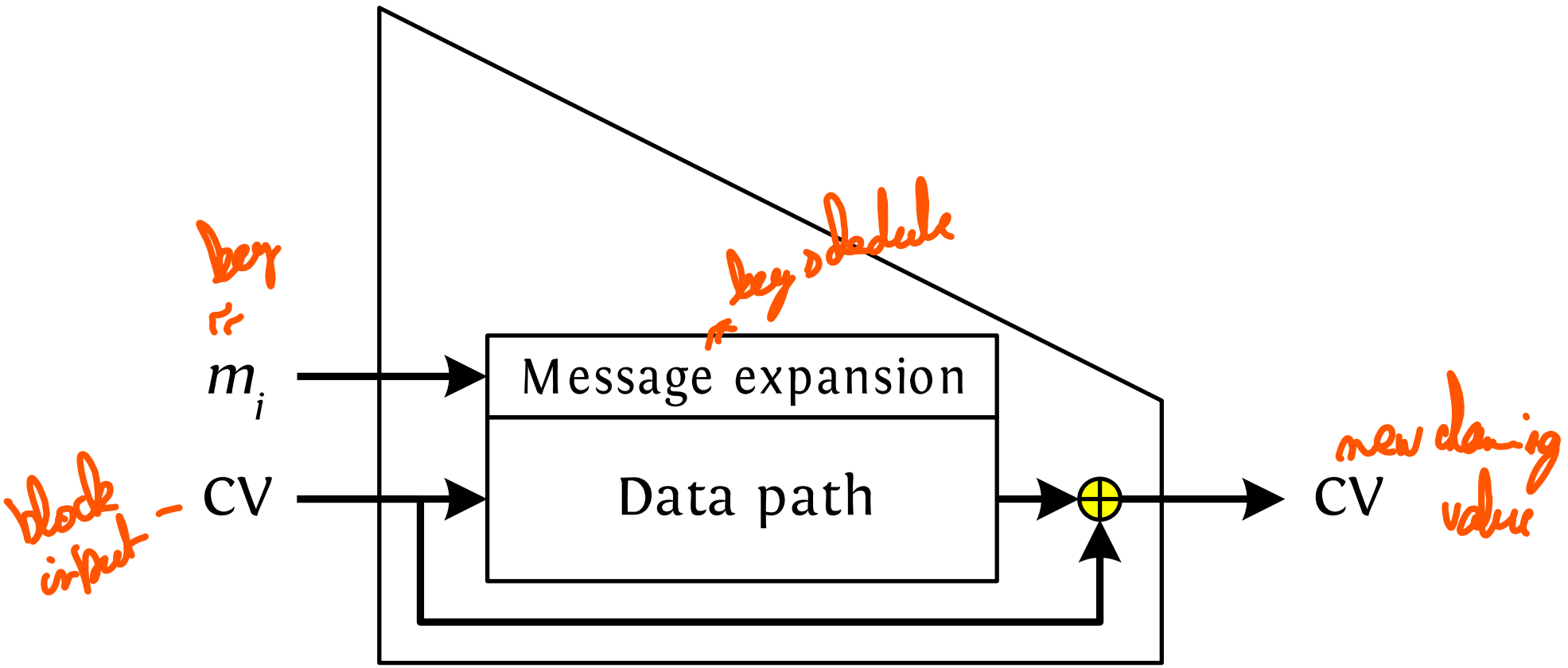
# Davies-Meyer



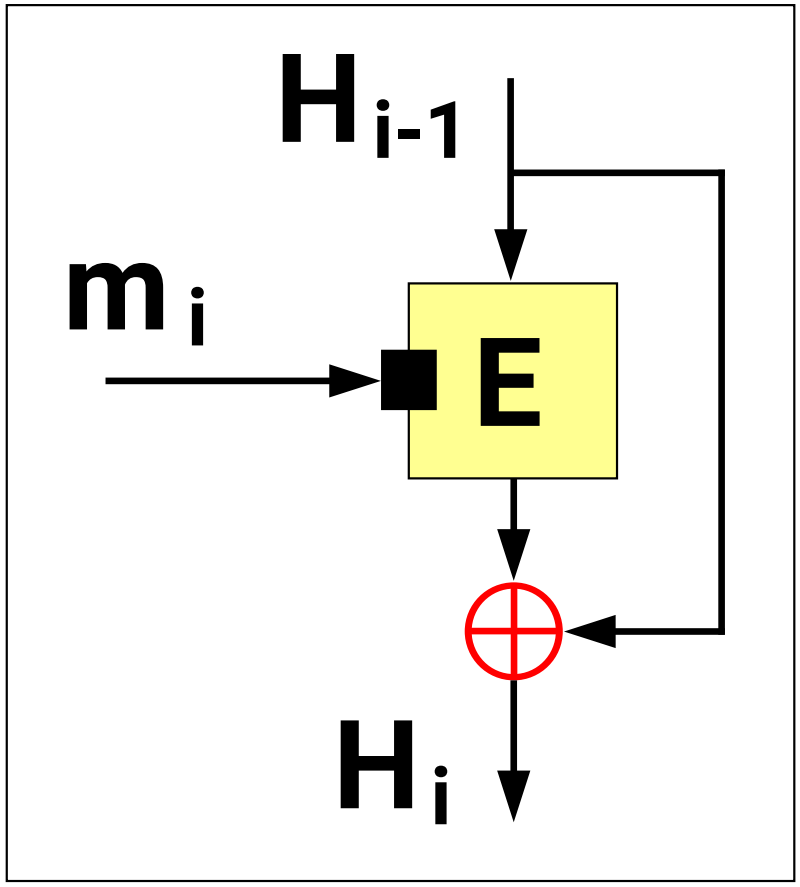

Block cipher
x input
key K
y output
Davies Meyer
x CV before
message bloc
y CV after

Problem : it is not one way !

# Davies-Meyer



# Other constructions using block ciphers




Same idea

Davies-Meyer

[Matyas et al., IBM Tech. D. B., 1985], [Quisquater et al., Eurocrypt'89]

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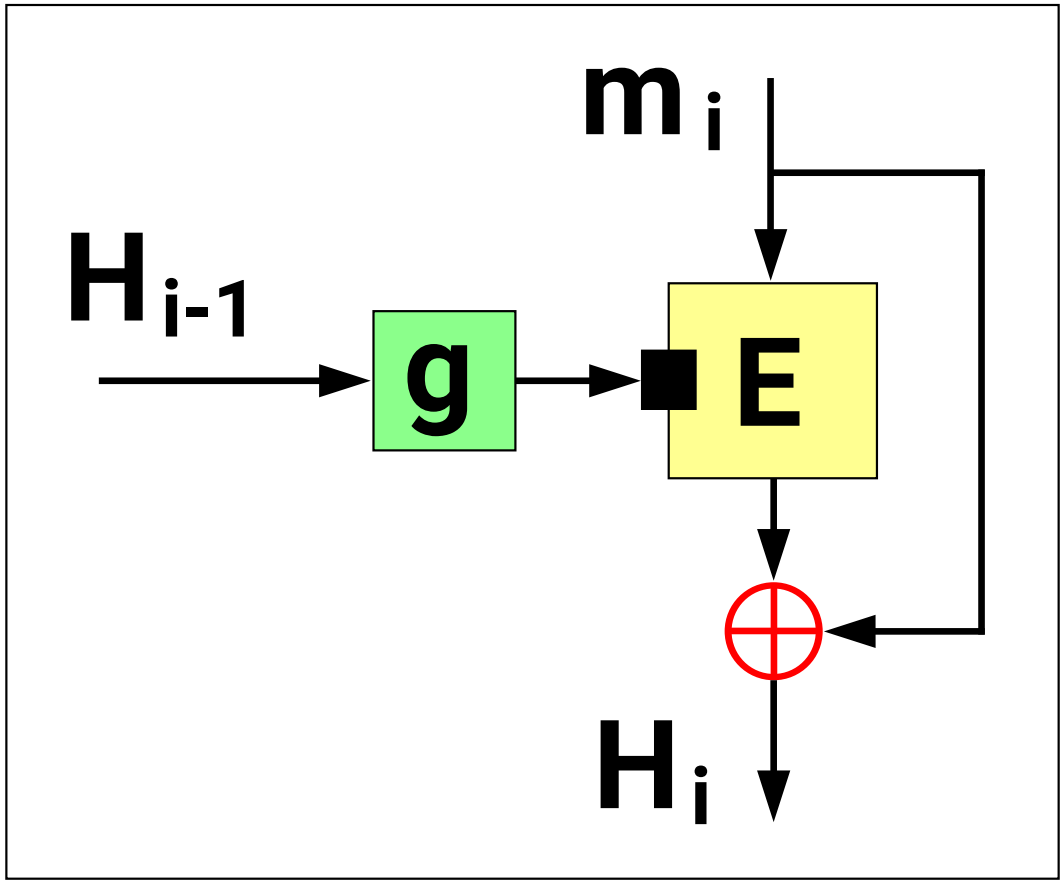


Matyas-Meyer-Oseas

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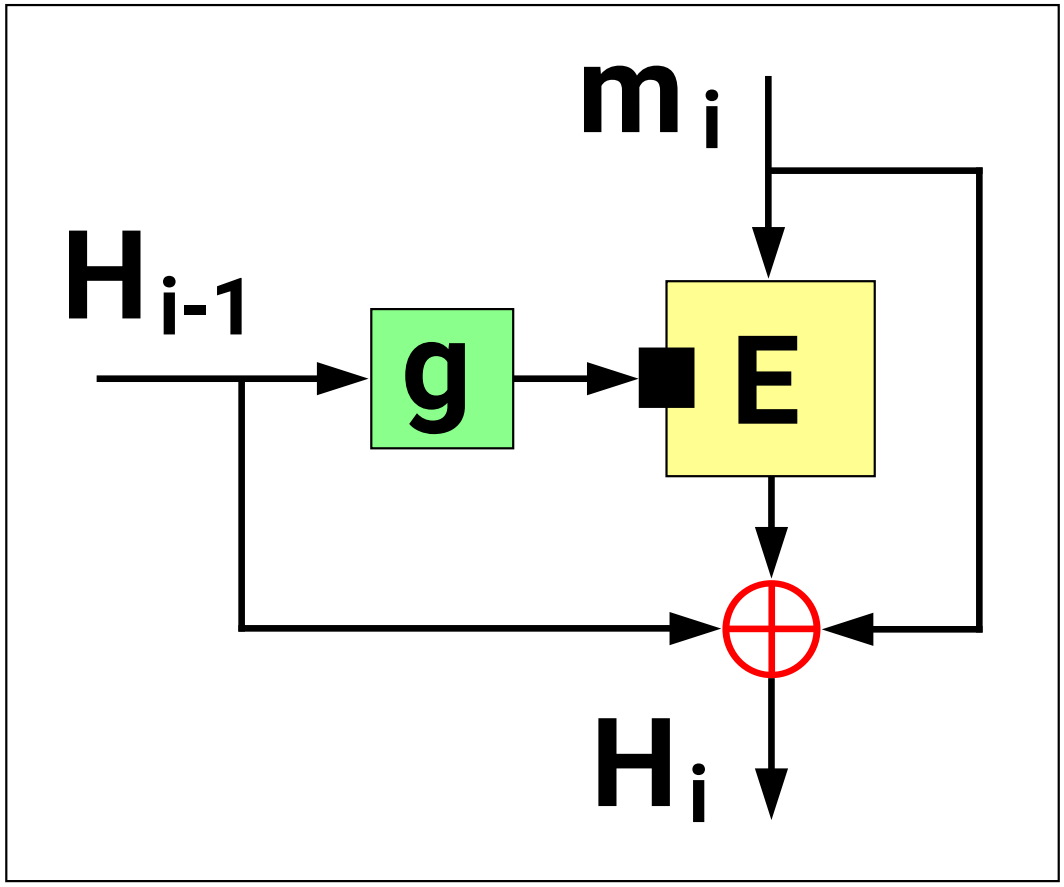


Miyaguchi-Preneel

[Miyaguchi et al., NTT Rev., 1990], [Preneel, PhD th., 1993]

# Inside SHA-1

- Uses Davies-Meyer with
  - data path $n = 160 = 5 \times 32$
  - message expansion $m = 512 = 16 \times 32$ → 16 words of 32 bits
- State initialized with $(A, B, C, D, E) =$
  $(67452301, \mathtt{EFCDAB89}, \mathtt{98BADCFE}, 10325476, \mathtt{C3D2E1F0})$
- Message block $(w_0, \ldots, w_{15})$ expanded as
  $w_t = (w_{t-3} \oplus w_{t-8} \oplus w_{t-14} \oplus w_{t-16}) \lll 1 \quad (16 \leq t \leq 79)$
- Data path with 80 steps...

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$$(\texttt{67452301}, \texttt{EFCDAB89}, \texttt{98BADCFE}, \texttt{10325476}, \texttt{C3D2E1F0})$$

- Message block $(w_0, \ldots, w_{15})$ expanded as expand 16 words to 80 words

$$w_t = (w_{t-3} \oplus w_{t-8} \oplus w_{t-14} \oplus w_{t-16}) \lll 1 \quad (16 \leq t \leq 79)$$

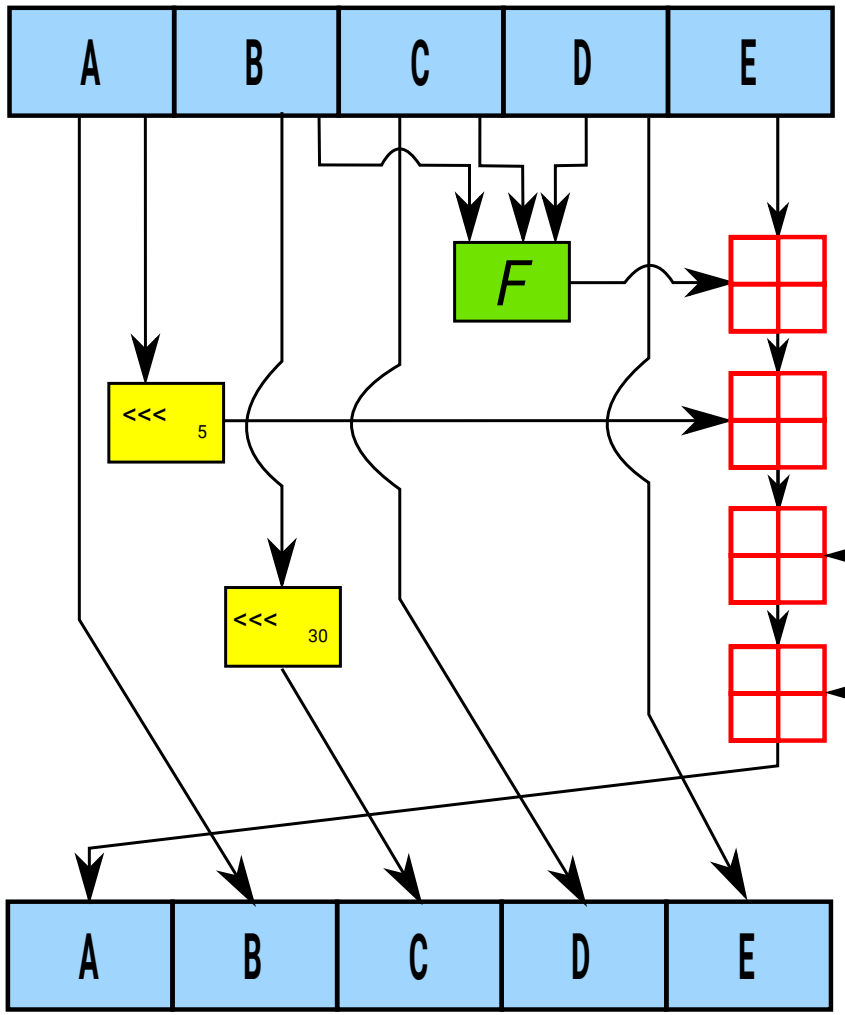
a LINEAR expansion

- Data path with 80 steps...

# Inside SHA-1: data path

F = bitwise function

this round 80 times




add a fct BCD

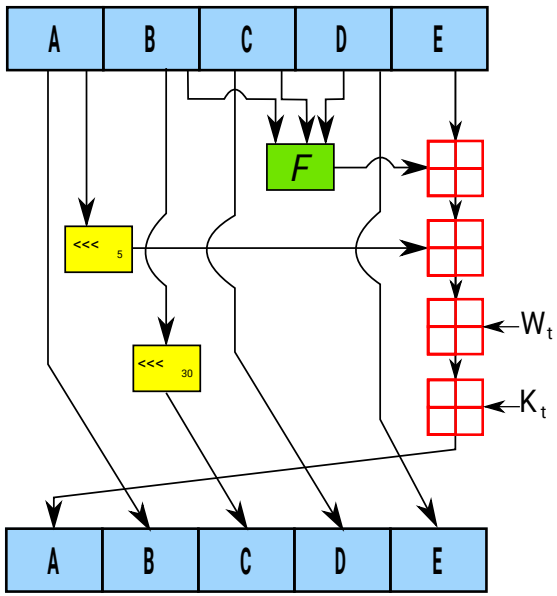
to msg block

to constant

≈ Feistel Network! (shift of the function and A replaced by output)

# Inside SHA-1: data path details

t = # turns




details for F

| | | |
|---|---|---|
| $0 \leq t \leq 19$ | $f(B,C,D) = (B \odot C) \oplus (\bar{B} \odot D)$ | $K_t = 5A827999$ |
| $20 \leq t \leq 39$ | $f(B,C,D) = B \oplus C \oplus D$ | $K_t = 6ED9EBA1$ |
| $40 \leq t \leq 59$ | $f(B,C,D) = (B \odot C) \oplus (B \odot D) \oplus (C \odot D)$ | $K_t = 8F1BBCDC$ |
| $60 \leq t \leq 79$ | $f(B,C,D) = B \oplus C \oplus D$ | $K_t = CA62C1D6$ |

selection function

XOR

majority function

# Collision in SHA-1

SHA-1 broken (2005), updated to increase complexity

→ broken again (collision seen in 2017) → several months on a lot of GPU

- February 23, 2017: first collision on SHA-1 published
- Estimated complexity: $2^{63} \ll 2^{80}$

[Stevens, Bursztein, Karpman, Albertini and Markov]

# Collision in SHA-1

$$\text{SHA-1}(P\|M_1^{(1)}\|M_2^{(1)}\|S) = \text{SHA-1}(P\|M_1^{(2)}\|M_2^{(2)}\|S)$$

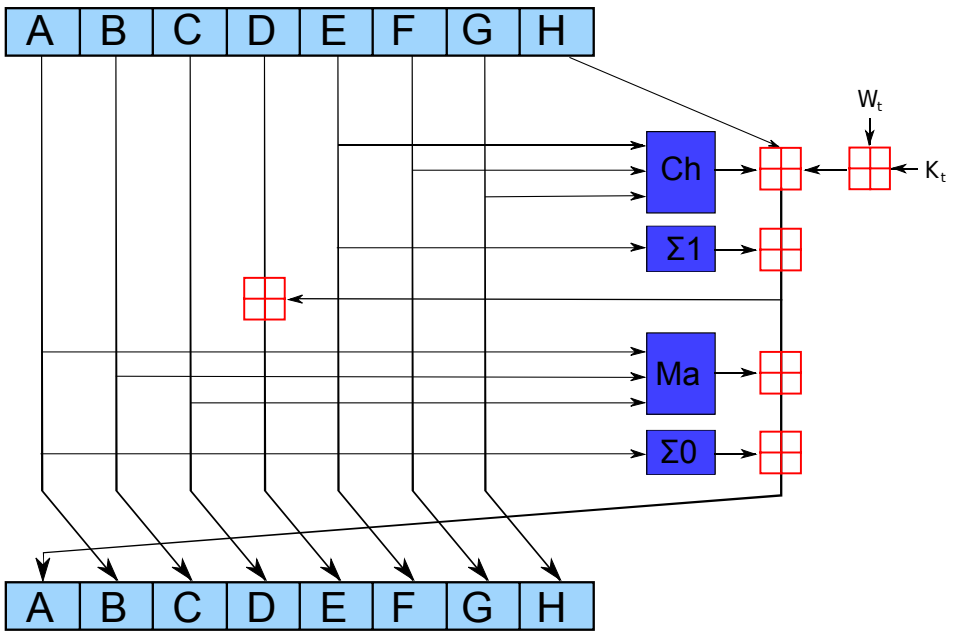
| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CV_0$ | 4e | a9 | 62 | 69 | 7c | 87 | 6e | 26 | 74 | d1 | 07 | f0 | fe | c6 | 79 | 84 | 14 | f5 | bf | 45 |
| $M_1^{(1)}$ | 7f | 46 | dc | 93 | a6 | b6 | 7e | 01 | 3b | 02 | 9a | aa | 1d | b2 | 56 | 0b | | | | |
| | 45 | ca | 67 | d6 | 88 | c7 | f8 | 4b | 8c | 4c | 79 | 1f | e0 | 2b | 3d | f6 | | | | |
| | 14 | f8 | 6d | b1 | 69 | 09 | 01 | c5 | 6b | 45 | c1 | 53 | 0a | fe | df | b7 | | | | |
| | 60 | 38 | e9 | 72 | 72 | 2f | e7 | ad | 72 | 8f | 0e | 49 | 04 | e0 | 46 | c2 | | | | |
| $CV_1^{(1)}$ | 8d | 64 | d6 | 17 | ff | ed | 53 | 52 | eb | c8 | 59 | 15 | 5e | c7 | eb | 34 | f3 | 8a | 5a | 7b |
| $M_2^{(1)}$ | 30 | 57 | 0f | e9 | d4 | 13 | 98 | ab | e1 | 2e | f5 | bc | 94 | 2b | e3 | 35 | | | | |
| | 42 | a4 | 80 | 2d | 98 | b5 | d7 | 0f | 2a | 33 | 2e | c3 | 7f | ac | 35 | 14 | | | | |
| | e7 | 4d | dc | 0f | 2c | c1 | a8 | 74 | cd | 0c | 78 | 30 | 5a | 21 | 56 | 64 | | | | |
| | 61 | 30 | 97 | 89 | 60 | 6b | d0 | bf | 3f | 98 | cd | a8 | 04 | 46 | 29 | a1 | | | | |
| $CV_2$ | 1e | ac | b2 | 5e | d5 | 97 | 0d | 10 | f1 | 73 | 69 | 63 | 57 | 71 | bc | 3a | 17 | b4 | 8a | c5 |

| | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $CV_0$ | 4e | a9 | 62 | 69 | 7c | 87 | 6e | 26 | 74 | d1 | 07 | f0 | fe | c6 | 79 | 84 | 14 | f5 | bf | 45 |
| $M_1^{(2)}$ | 73 | 46 | dc | 91 | 66 | b6 | 7e | 11 | 8f | 02 | 9a | b6 | 21 | b2 | 56 | 0f | | | | |
| | f9 | ca | 67 | cc | a8 | c7 | f8 | 5b | a8 | 4c | 79 | 03 | 0c | 2b | 3d | e2 | | | | |
| | 18 | f8 | 6d | b3 | a9 | 09 | 01 | d5 | df | 45 | c1 | 4f | 26 | fe | df | b3 | | | | |
| | dc | 38 | e9 | 6a | c2 | 2f | e7 | bd | 72 | 8f | 0e | 45 | bc | e0 | 46 | d2 | | | | |
| $CV_1^{(2)}$ | 8d | 64 | c8 | 21 | ff | ed | 52 | e2 | eb | c8 | 59 | 15 | 5e | c7 | eb | 36 | 73 | 8a | 5a | 7b |
| $M_2^{(2)}$ | 3c | 57 | 0f | eb | 14 | 13 | 98 | bb | 55 | 2e | f5 | a0 | a8 | 2b | e3 | 31 | | | | |
| | fe | a4 | 80 | 37 | b8 | b5 | d7 | 1f | 0e | 33 | 2e | df | 93 | ac | 35 | 00 | | | | |
| | eb | 4d | dc | 0d | ec | c1 | a8 | 64 | 79 | 0c | 78 | 2c | 76 | 21 | 56 | 60 | | | | |
| | dd | 30 | 97 | 91 | d0 | 6b | d0 | af | 3f | 98 | cd | a4 | bc | 46 | 29 | b1 | | | | |
| $CV_2$ | 1e | ac | b2 | 5e | d5 | 97 | 0d | 10 | f1 | 73 | 69 | 63 | 57 | 71 | bc | 3a | 17 | b4 | 8a | c5 |

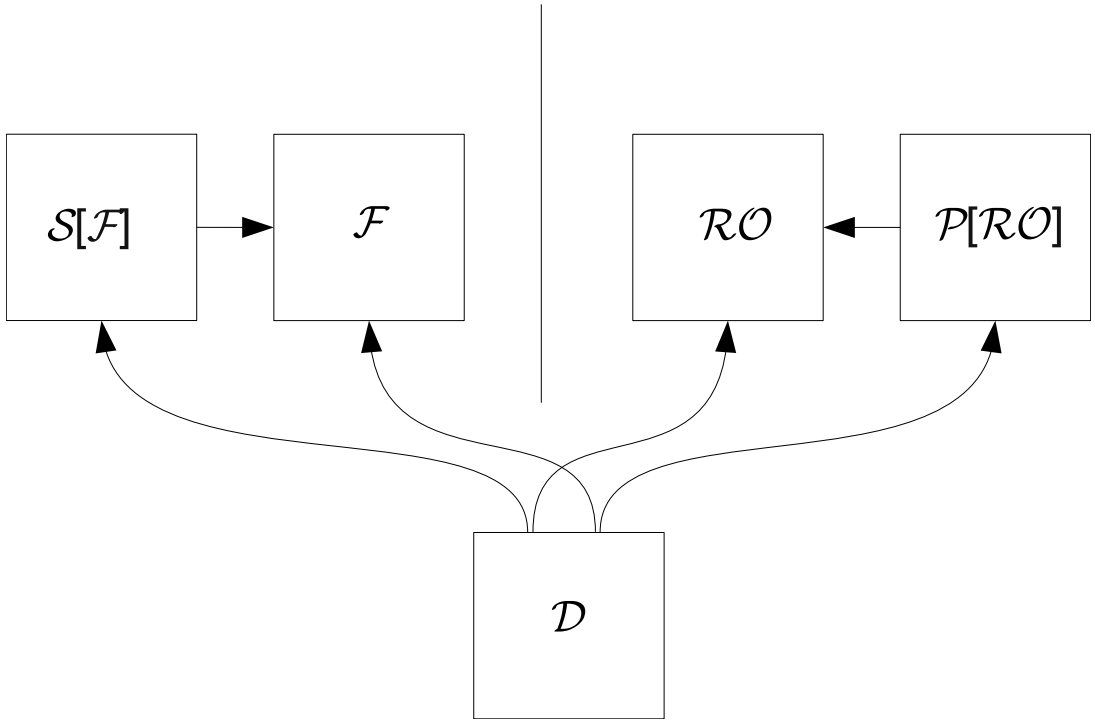
# From SHA-1 to SHA-2

Changes from SHA-1 to SHA-2:

- Two compression functions
  - SHA-{224, 256}: $n = 256 = 8 \times 32$ and $m = 512 = 16 \times 32$
  - SHA-{384, 512}: $n = 512 = 8 \times 64$ and $m = 1024 = 16 \times 64$
- Non-linear message expansion
- Stronger data path mixing



# Generic security: indifferentiability [Maurer et al. (2004)]




Applied to hash functions in [Coron et al. (2005)]

- distinguishing mode-of-use from ideal function ($\mathcal{RO}$)
- covers adversary with access to primitive $\mathcal{F}$ at left
- additional interface, covered by a *simulator* at right

# Consequences of indifferentiability

↦ allows to distinguish the mode from the UP.

Pr(undesired property on a given mode/construction) = Pr(UP ∧ mode behaves as RO) + Pr(UP ∧ ¬ mode behaves as RO)
≤ Pr(UP on a RO) + Pr(distinguish mode from RO)
= $t/2^n + t^2/2^{c+1}$ see slide below adv

→ 2 most generic attacks =

**Theorem 2.** *Let $\mathcal{H}$ be a hash function, built on underlying primitive $\pi$, and RO be a random oracle, where $\mathcal{H}$ and RO have the same domain and range space. Denote by $\boldsymbol{Adv}^{\mathrm{pro}}_{\mathcal{H}}(q)$ the advantage of distinguishing $(\mathcal{H}, \pi)$ from $(RO, S)$, for some simulator $S$, maximized over all distinguishers $\mathcal{D}$ making at most $q$ queries. Let* atk *be a security property of $\mathcal{H}$. Denote by $\boldsymbol{Adv}^{\mathrm{atk}}_{\mathcal{H}}(q)$ the advantage of breaking $\mathcal{H}$ under* atk*, maximized over all adversaries $\mathcal{A}$ making at most $q$ queries. Then:*

$$\boldsymbol{Adv}^{\mathrm{atk}}_{\mathcal{H}}(q) \leq \boldsymbol{Pr}^{\mathrm{atk}}_{RO}(q) + \boldsymbol{Adv}^{\mathrm{pro}}_{\mathcal{H}}(q), \qquad (1)$$

$t/2^n$ $t^2/2^{c+1}$

*where $\boldsymbol{Pr}^{\mathrm{atk}}_{RO}(q)$ denotes the success probability of a generic attack against $\mathcal{H}$ under* atk*, after at most $q$ queries.*

[Andreeva, Mennink, Preneel, ISC 2010]

# Limitations of indifferentiability

- Only about the mode
  - No security proof with a concrete primitive
- Only about single-stage games [Ristenpart et al., Eurocrypt 2011]
  - Example: hash-based storage auditing

$$Z = h(\mathsf{File} \| C)$$

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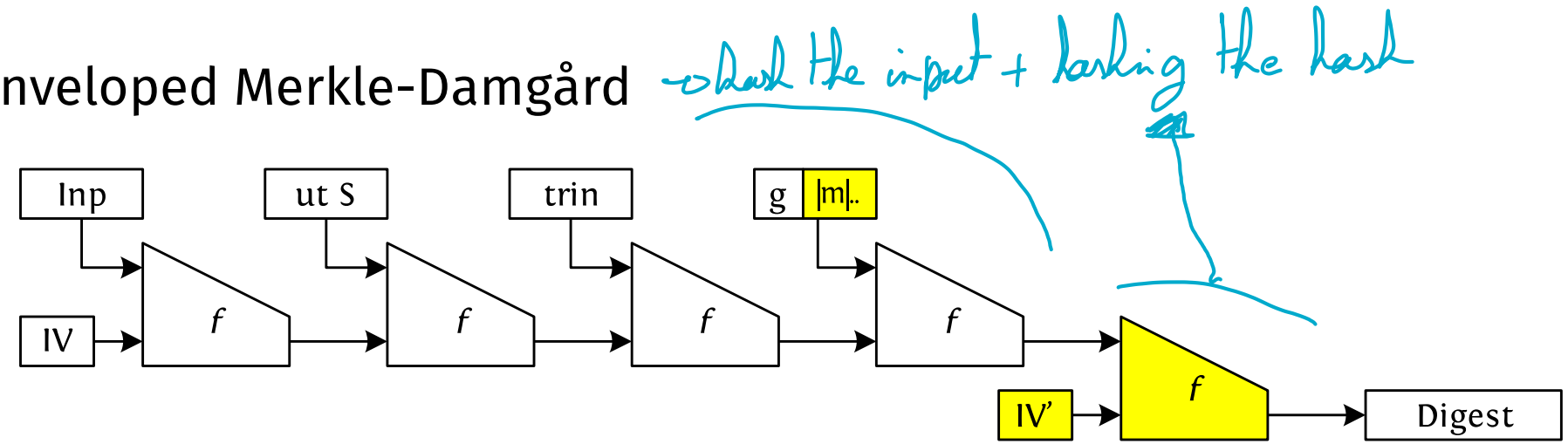
$$Z = h(\mathsf{File} \| C)$$

# Making Merkle-Damgård indifferentiable

## Enveloped Merkle-Damgård

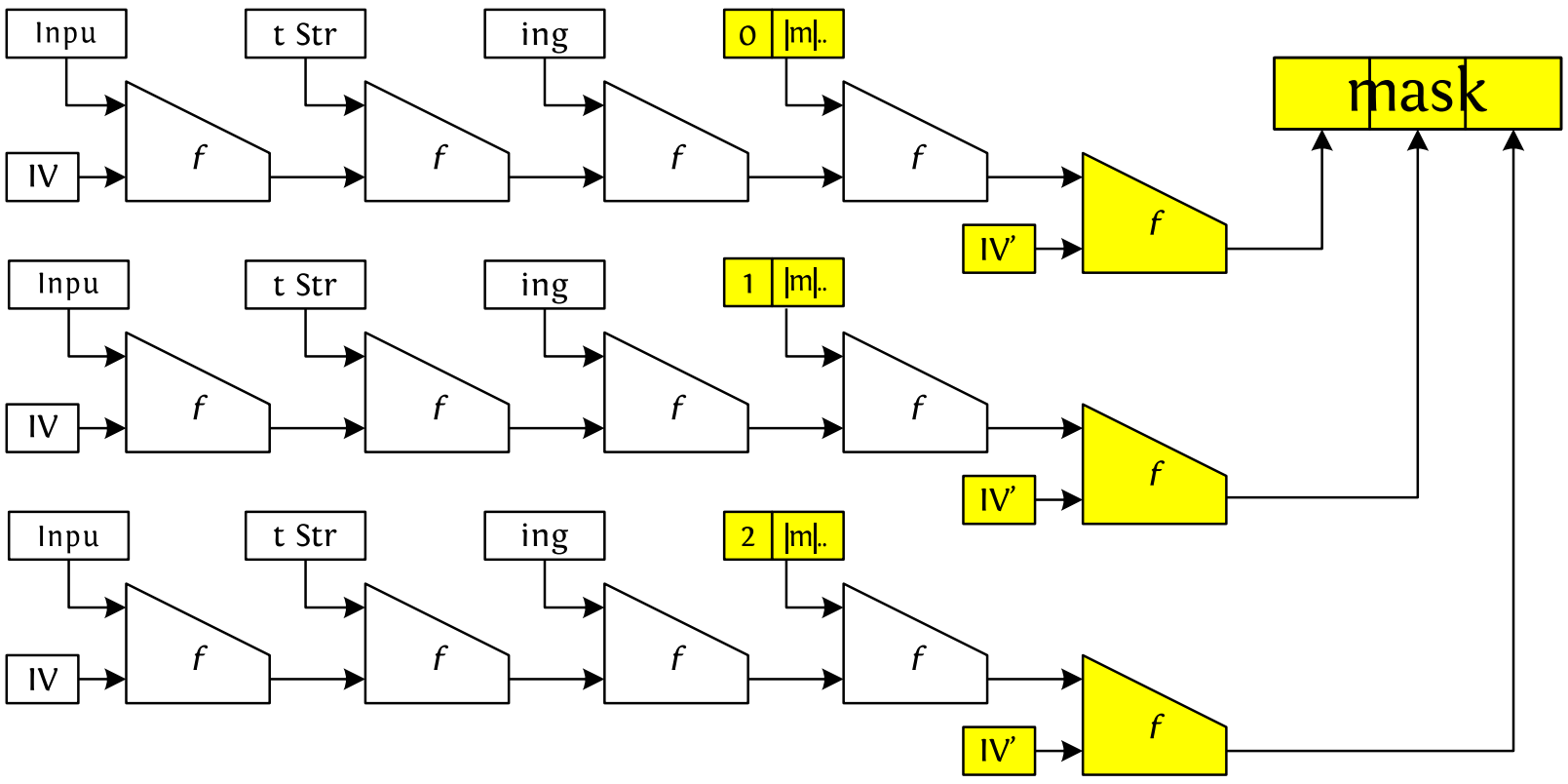



[Bellare and Ristenpart, Asiacrypt 2006]

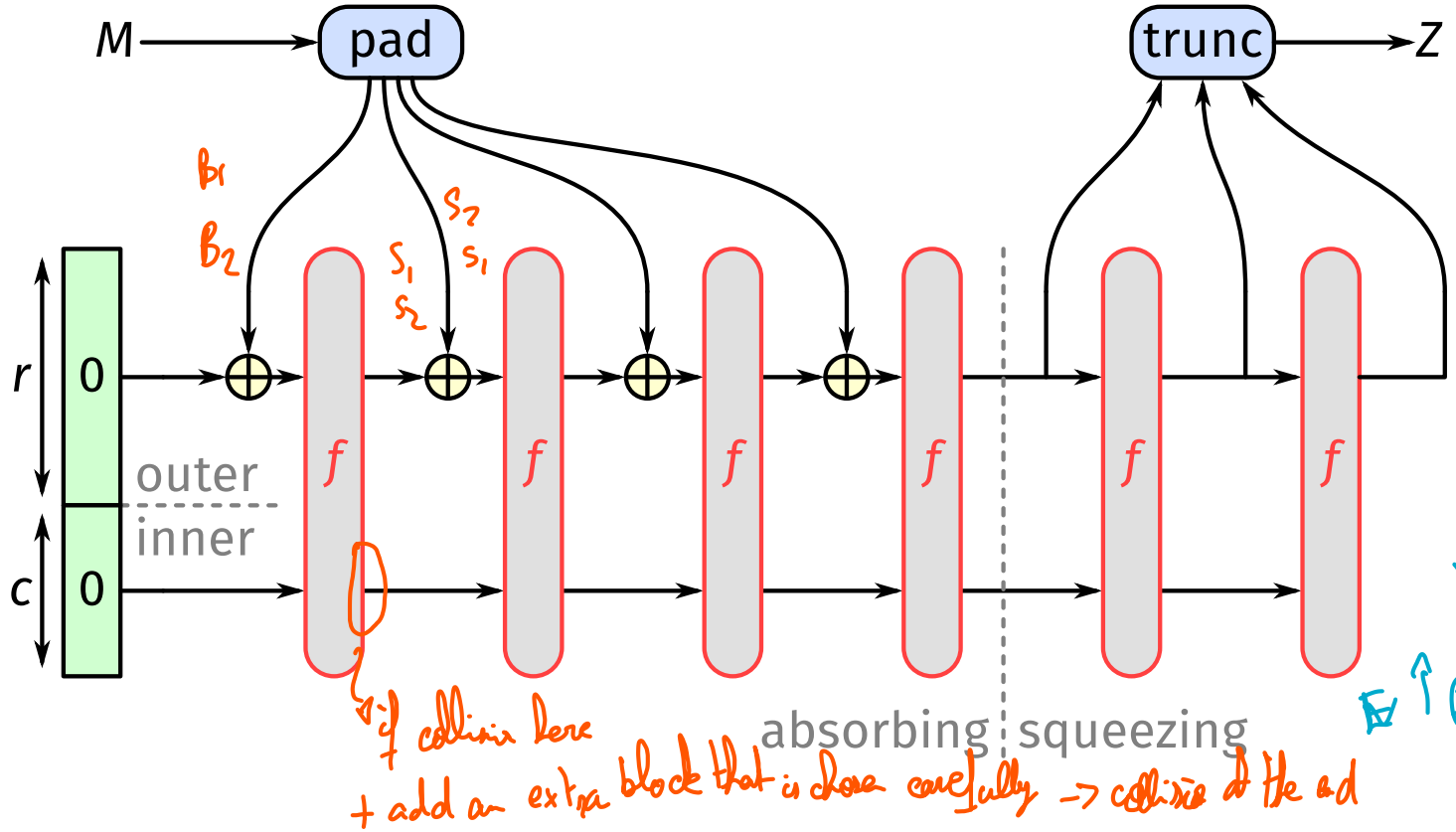
extendable output function

# Making Merkle-Damgård suitable for XOFs

## Mask generating function construction “MGF1”



# The sponge construction




- Calls a $b$-bit permutation $f$, with $b = r + c$
  - $r$ bits of *rate*
  - $c$ bits of *capacity* (security parameter)
- Natively implements a XOF

# Generic security of the sponge construction

**Theorem (Bound on the $\mathcal{RO}$-differentiating advantage of sponge)**

$t$ = # calls (time to evaluate)

$$\mathsf{Adv} \leq \frac{t^2}{2^{c+1}}$$

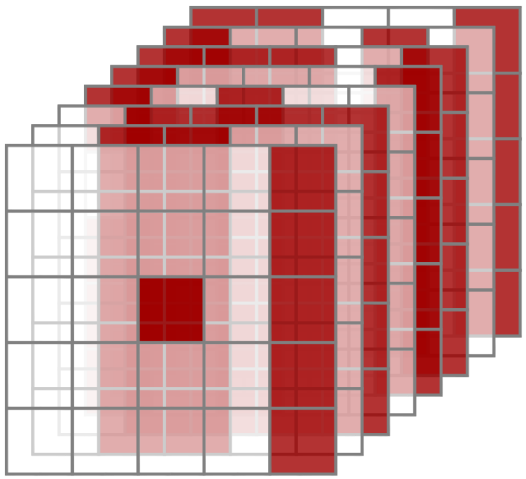
Adv: *differentiating advantage of random sponge from random oracle*
*t: time complexity (# calls to f)* *c: capacity* *[Eurocrypt 2008]*

| | |
|---|---|
| Preimage resistance | $2^{\min(n,c/2)}$ |
| Second-preimage resistance | $2^{\min(n,c/2)}$ |
| Collision resistance | $2^{\min(n/2,c/2)}$ |
| Any other attack | $2^{\min(\mathcal{RO},c/2)}$ (*) |

(*) This means the minimum between $2^{c/2}$ and the complexity of the attack on a random oracle.

$\Rightarrow t << 2^{\min(n,c/2)}$

# Keccak-$f$



- The seven permutation army:
  - 25, 50, 100, 200, 400, 800, 1600 bits
  - toy, lightweight, fastest
  - standardized in [FIPS 202]
- Repetition of a simple round function
  - that operates on a 3D state
  - $(5 \times 5)$ lanes
  - up to 64-bit each

# KECCAK-*f* in pseudo-code

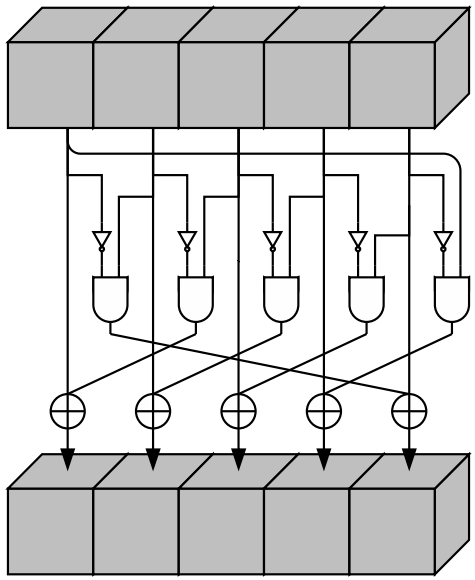
```
KECCAK-F[b](A) {
  forall i in 0…nr-1
    A = Round[b](A, RC[i])
  return A
}

Round[b](A,RC) {
  θ step
  C[x] = A[x,0] xor A[x,1] xor A[x,2] xor A[x,3] xor A[x,4], forall x in 0…4
  D[x] = C[x-1] xor rot(C[x+1],1),                          forall x in 0…4
  A[x,y] = A[x,y] xor D[x],                                 forall (x,y) in (0…4,0…4)

  ρ and π steps
  B[y,2*x+3*y] = rot(A[x,y], r[x,y]),                       forall (x,y) in (0…4,0…4)

  χ step
  A[x,y] = B[x,y] xor ((not B[x+1,y]) and B[x+2,y]),        forall (x,y) in (0…4,0…4)

  ι step
  A[0,0] = A[0,0] xor RC

  return A
}
```

https://keccak.team/keccak_specs_summary.html

# $\chi$, the nonlinear mapping in KECCAK-$f$

Take a bit, copy its value but if 01 ...

Invertable



- “Flip bit if neighbors exhibit 01 pattern”
- Operates independently and in parallel on 5-bit rows
- Cheap: small number of operations per bit
- Algebraic degree 2, inverse has degree 3

# $\theta$, mixing bits

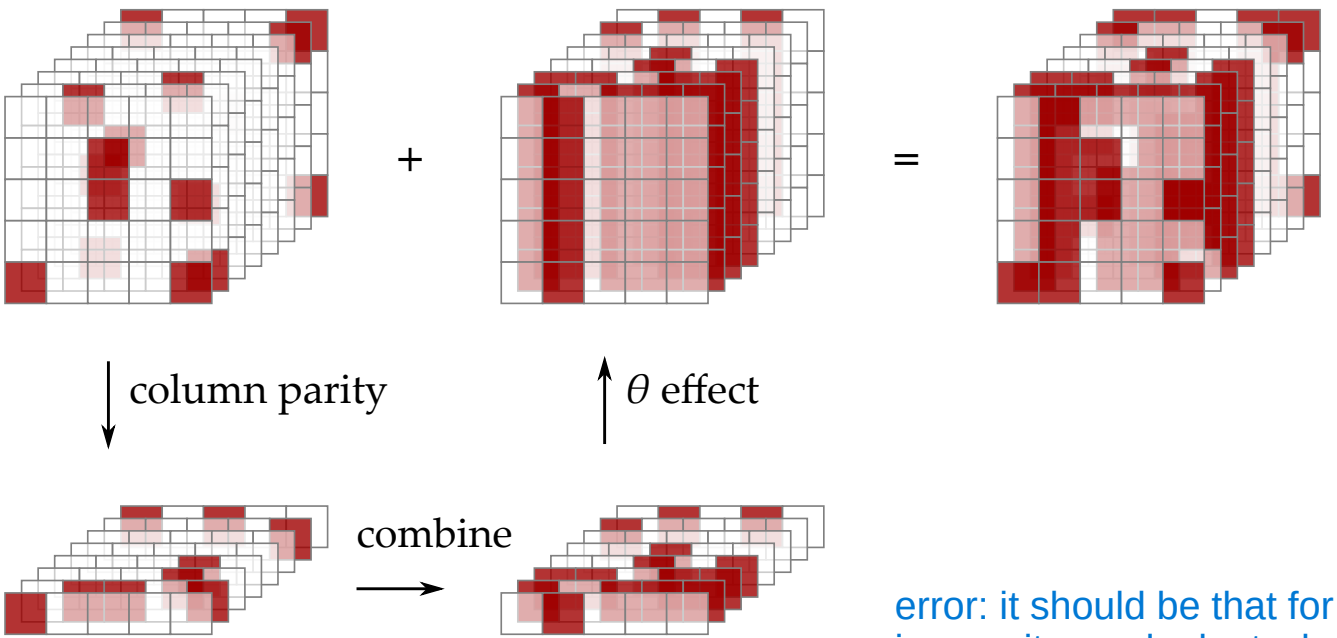
- Compute parity $c_{x,z}$ of each column
- Add to each cell parity of neighboring columns:

$$b_{x,y,z} = a_{x,y,z} \oplus c_{x-1,z} \oplus c_{x+1,z-1}$$

- Cheap: two XORs per bit

look at every column, make the sum of the parity of all the columns

red = 1
white = 0




le prof ne s'en souvient pas

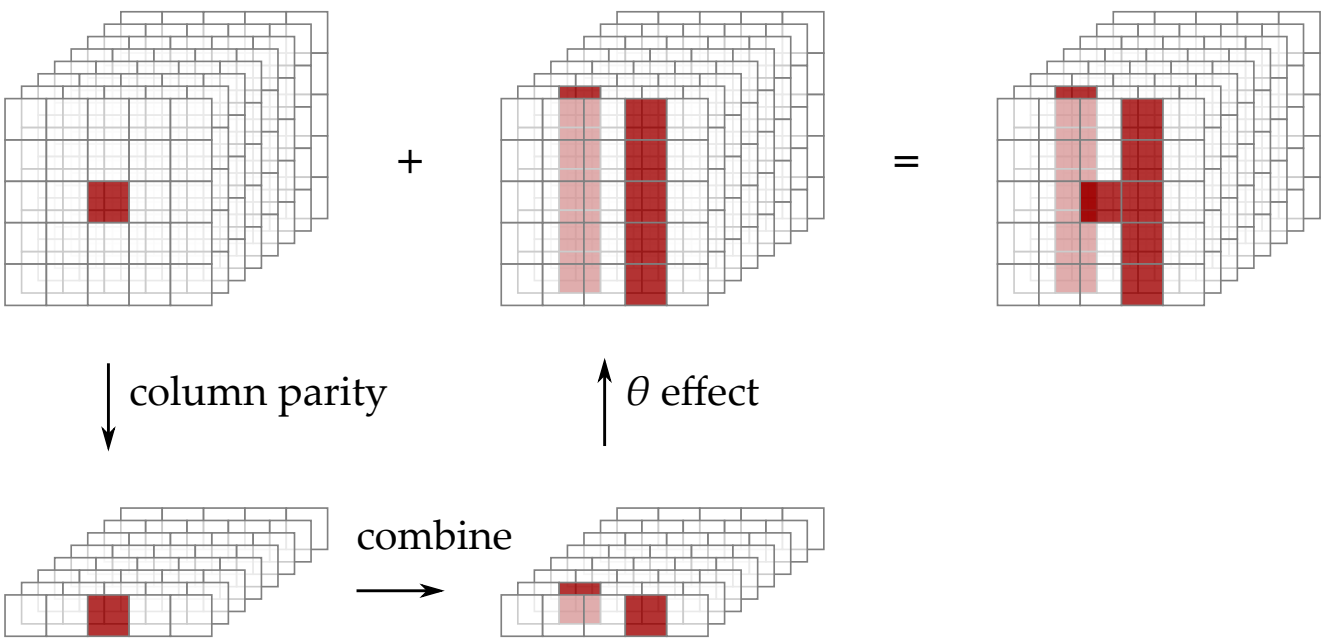
error: it should be that for each bit there is a parity, we look at what is to the left and sum the parity of itself+the leftone

# $\theta$, mixing bits

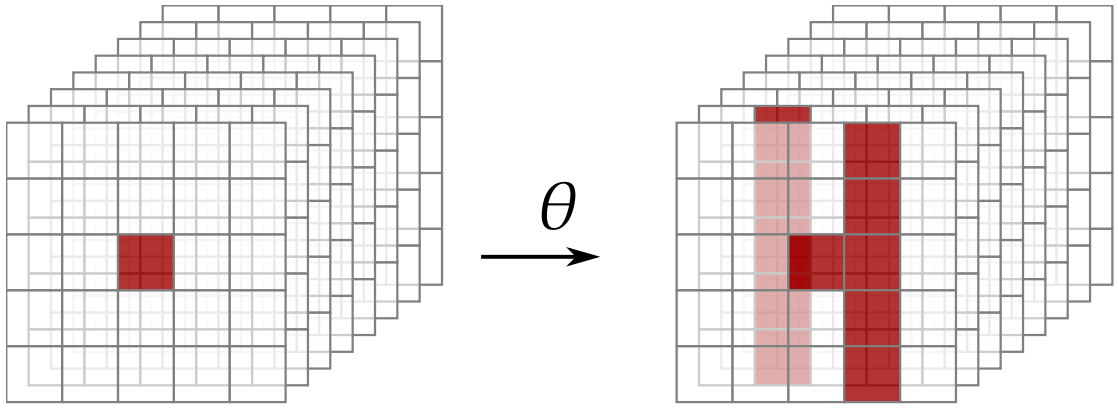
- Compute parity $c_{x,z}$ of each column
- Add to each cell parity of neighboring columns:

$$b_{x,y,z} = a_{x,y,z} \oplus c_{x-1,z} \oplus c_{x+1,z-1}$$

- Cheap: two XORs per bit cheap! -> good diffusion



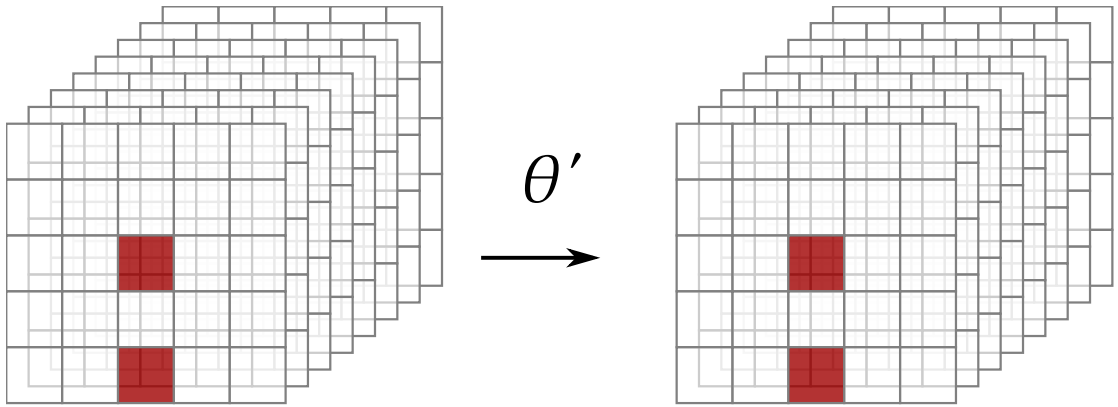
# Diffusion of $\theta$




parity = 1

$$1 + \left(1 + y + y^2 + y^3 + y^4\right)\left(x + x^4 z\right)$$
$$\left(\mathrm{mod}\left\langle 1 + x^5, 1 + y^5, 1 + z^w \right\rangle\right)$$

# Diffusion of $\theta$ (kernel)

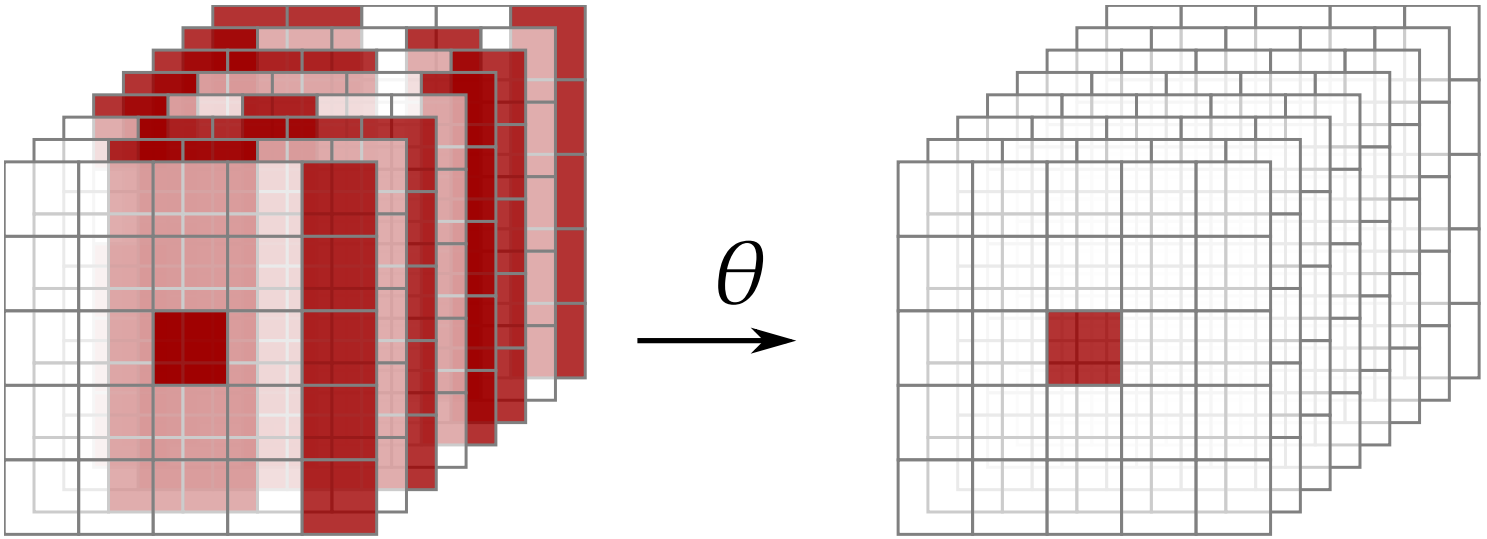



because 2 bits -> the column on the right will be 1+1=0->no influence

Column on the right doesn't change

$$1 + \left(1 + y + y^2 + y^3 + y^4\right)\left(x + x^4 z\right)$$
$$\left(\mathrm{mod}\left\langle 1 + x^5, 1 + y^5, 1 + z^w \right\rangle\right)$$

# Diffusion of $\theta^{-1}$




$$1 + \left(1 + y + y^2 + y^3 + y^4\right)\mathbf{Q},$$

$$\text{with } \mathbf{Q} = 1 + \left(1 + x + x^4 z\right)^{-1} \bmod \left\langle 1 + x^5, 1 + z^w \right\rangle$$

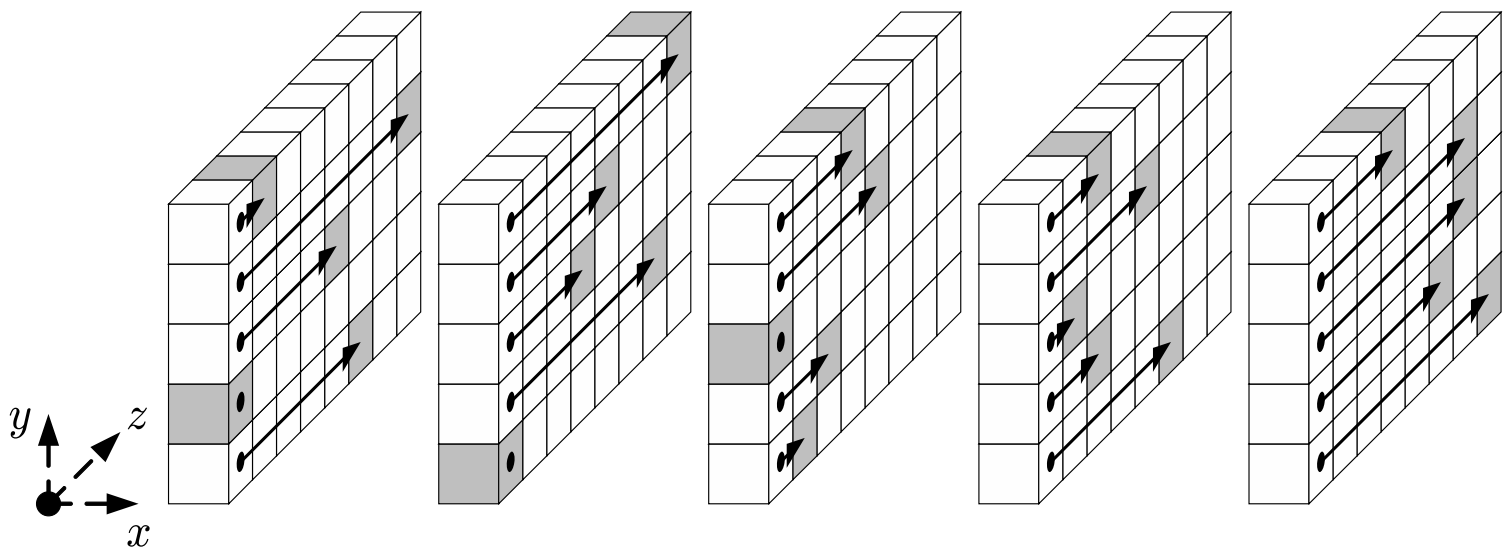
- **Q** is dense, so:
  - Diffusion from single-bit output to input very high
  - Increases resistance against LC/DC and algebraic attacks

# $\rho$ for inter-slice dispersion

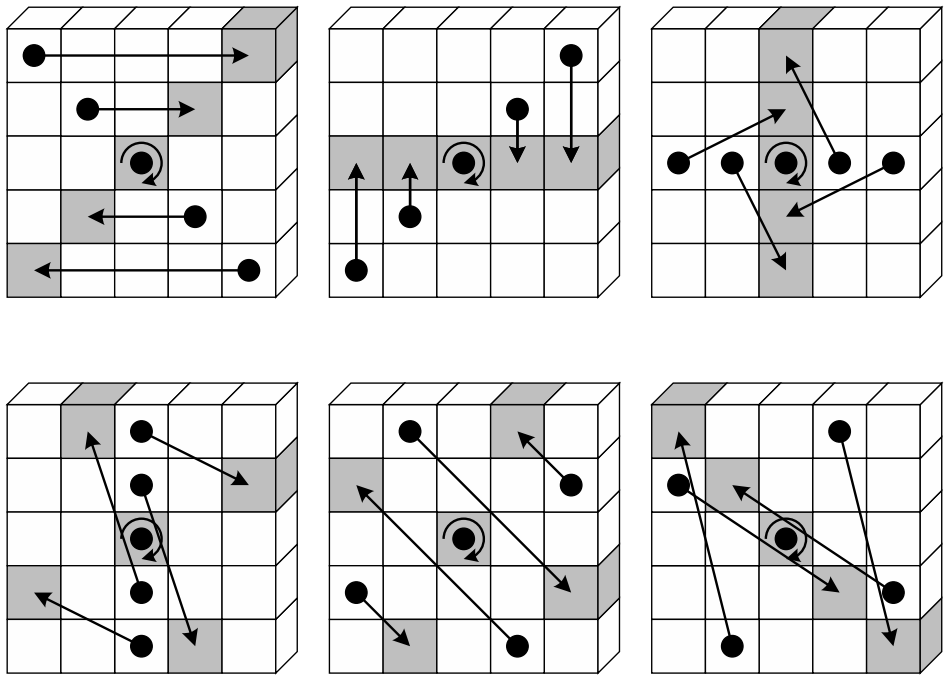
- We need diffusion between the slices ...
- $\rho$: cyclic shifts of lanes with offsets : shift every lane by a ≠ amount

$$i(i+1)/2 \bmod 2^\ell, \text{ with } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}^{i-1} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

- Offsets cycle through all values below $2^\ell$



# $\pi$ for disturbing horizontal/vertical alignment



$$a_{x,y} \leftarrow a_{x',y'} \text{ with } \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x' \\ y' \end{pmatrix}$$

# $\iota$ to break symmetry

iota

- XOR of round-dependent constant to lane in origin
- Without $\iota$, the round mapping would be symmetric
  - invariant to translation in the $z$-direction
  - susceptible to *rotational* cryptanalysis
- Without $\iota$, all rounds would be the same
  - susceptibility to *slide* attacks
  - defective cycle structure
- Without $\iota$, we get simple fixed points (000 and 111)

# Keccak-$f$ summary

- Round function:

$$\mathsf{R} = \iota \circ \chi \circ \pi \circ \rho \circ \theta$$

- Number of rounds: $12 + 2\ell$
  - Keccak-$f[25]$ has 12 rounds
  - Keccak-$f[1600]$ has 24 rounds

# NIST FIPS 202 (August 2015)

- Four drop-in replacements to SHA-2
- Two *extendable output functions* (XOF)

| **XOF** | **SHA-2 drop-in replacements** |
|---|---|
| $\textsc{Keccak}[c = 256](M \Vert 11 \Vert 11)$ | |
| | first 224 bits of $\textsc{Keccak}[c = 448](M \Vert 01)$ |
| $\textsc{Keccak}[c = 512](M \Vert 11 \Vert 11)$ | |
| | first 256 bits of $\textsc{Keccak}[c = 512](M \Vert 01)$ |
| | first 384 bits of $\textsc{Keccak}[c = 768](M \Vert 01)$ |
| | first 512 bits of $\textsc{Keccak}[c = 1024](M \Vert 01)$ |
| **SHAKE128** and **SHAKE256** | **SHA3-224** to **SHA3-512** |

- Toolbox for building other functions

# NIST SP 800-185 (December 2016)

Customized SHAKE (**cSHAKE**) more used

- $H(x) = \text{cSHAKE}(x, \text{name}, \text{customization string})$
- E.g., $\text{cSHAKE128}(x, N, S) = \text{KECCAK}[c = 256](\text{encode}(N, S) \| x \| \texttt{00})$
- $\text{cSHAKE128}(x, N, S) \triangleq \text{SHAKE128}$ when $N = S = \text{“”}$

**KMAC**: message authentication code (no need for HMAC-SHA-3!)

$$\text{KMAC}(K, x, S) = \text{cSHAKE}(\text{encode}(K) \| x, \text{”KMAC”}, S)$$

**TupleHash**: hashing a sequence of strings $\mathbf{x} = x_n \circ x_{n-1} \circ \cdots \circ x_1$

$$\text{TupleHash}(\mathbf{x}, S) = \text{cSHAKE}(\text{encode}(\mathbf{x}), \text{”TupleHash”}, S)$$

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- E.g., $\text{cSHAKE128}(x, N, S) = \text{KECCAK}[c = 256](\text{encode}(N, S) \| x \| \texttt{00})$
- $\text{cSHAKE128}(x, N, S) \triangleq \text{SHAKE128}$ when $N = S = \text{“”}$

**KMAC**: message authentication code (no need for HMAC-SHA-3!)

HMAC for SHA-2

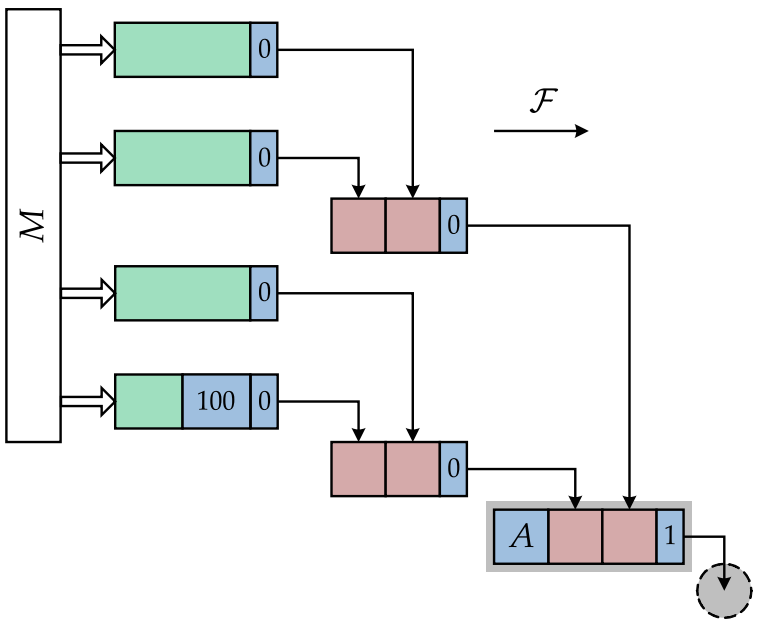
$$\text{KMAC}(K, x, S) = \text{cSHAKE}(\text{encode}(K) \| x, \text{”KMAC”}, S)$$

**TupleHash**: hashing a sequence of strings $\mathbf{x} = x_n \circ x_{n-1} \circ \cdots \circ x_1$

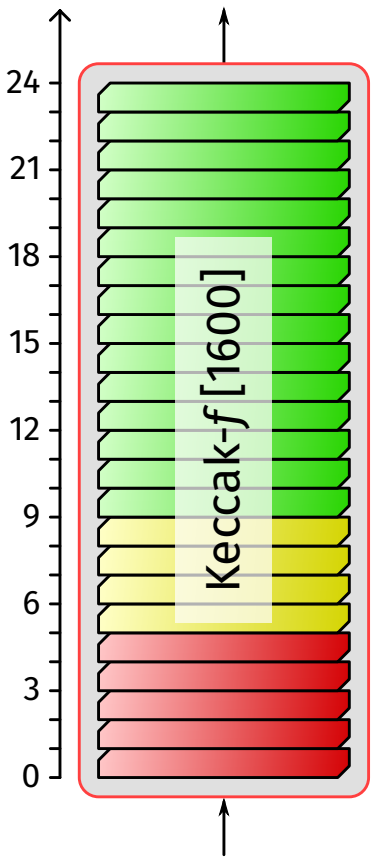
$$\text{TupleHash}(\mathbf{x}, S) = \text{cSHAKE}(\text{encode}(\mathbf{x}), \text{”TupleHash”}, S)$$

# NIST SP 800-185 (December 2016)

**ParallelHash**: faster hashing with parallelism



# Status of KECCAK cryptanalysis




- Collision attacks up to 5 rounds
  - Also up to 6 rounds, but for non-standard parameters ($c = 160$)

  [Song, Liao, Guo, CRYPTO 2017]
- Distinguishers
  - 7 rounds (practical time)
    [Huang et al., EUROCRYPT 2017]
  - 8 rounds ($2^{128}$ time)
    [Dinur et al., EUROCRYPT 2015]
  - 9 rounds ($2^{64}$ time)
    [Suryawanshi et al., AFRICACRYPT 2020]
- Lots of third-party cryptanalysis available at:
  `https://keccak.team/third_party.html`

# KangarooTwelve