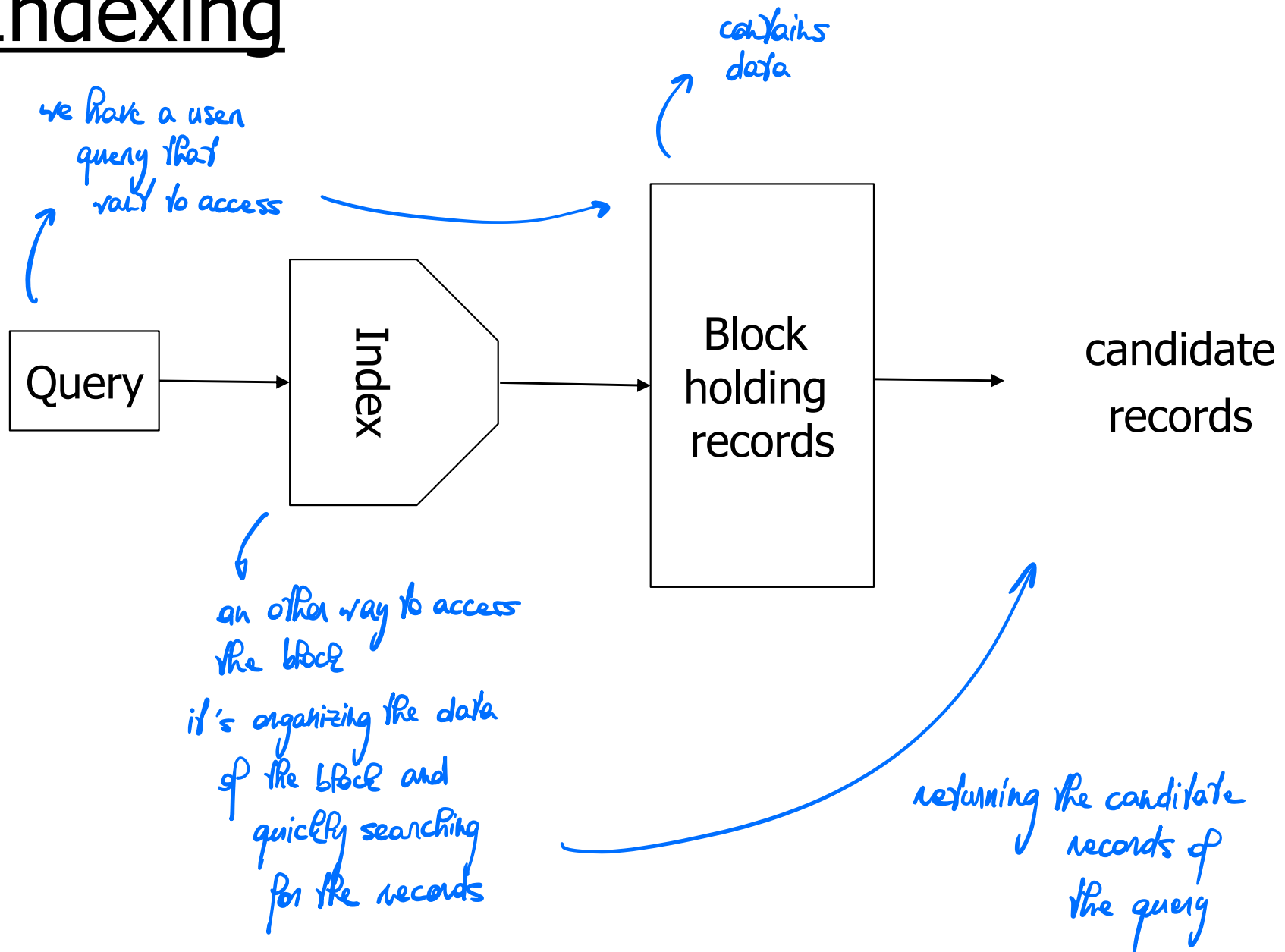


Indexing

Hector Garcia-Molina
Mahmoud Sakr

Indexing



Topics

- Conventional indexes
- B-trees
- Hashing schemes

Building
blocks



when accessing records,
we can't access subpages,
always take whole pages

page
↓ ~ 4 KB

the way our data is
organized in the storage
Sequential File

10	
20	

30	
40	

50	
60	

70	
80	

90	
100	

sorted by the key Dense Index

Sequential File

Dense Index = a pointer per key

→ sequential scan of the index

How to search for a key= 30 ?

How to search for a key= 25 ?

Can we use a dense index on a non-sequential file ? *yes*

→ we have a pointer that lead to the records

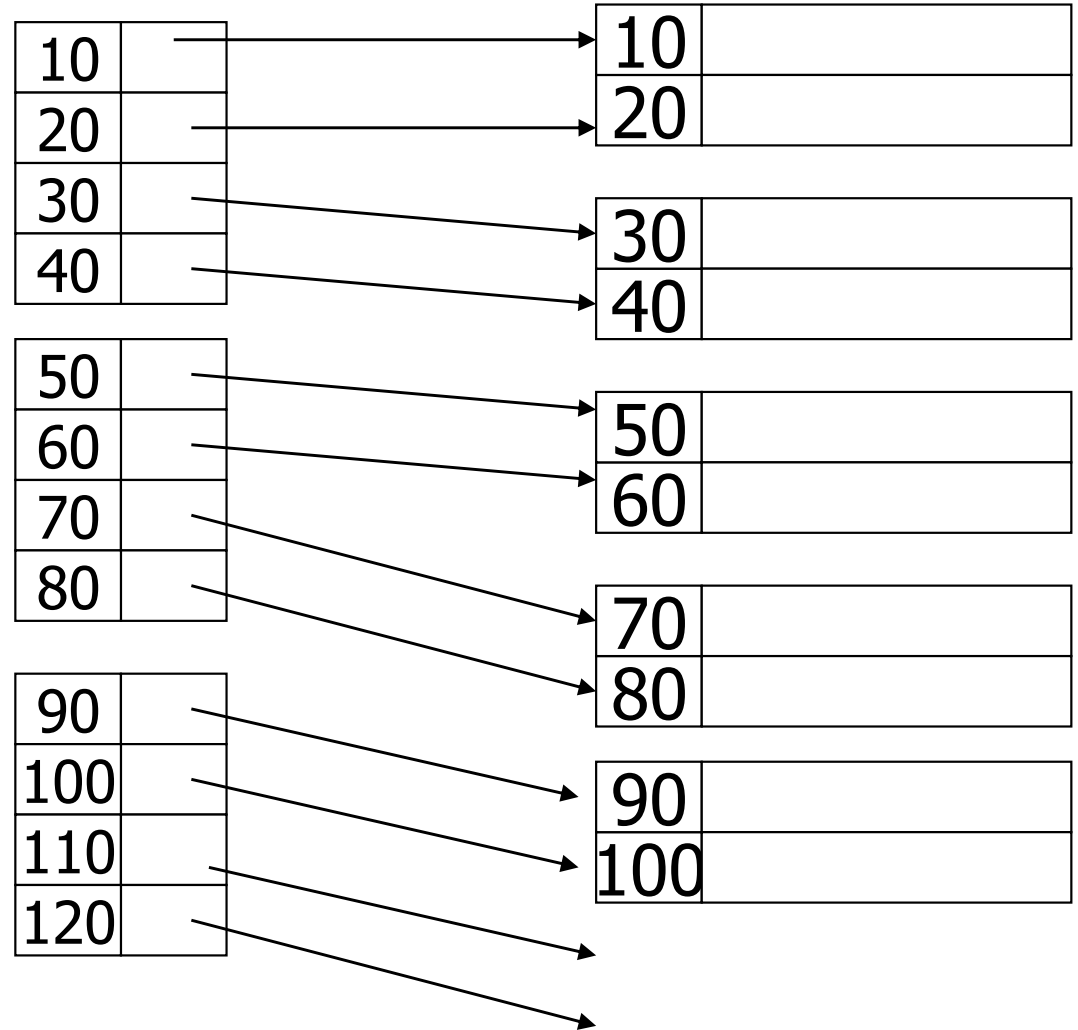
Why querying a dense index is more efficient than querying the sequential file ?

→ allow us to have a different order than the storage

⇒ we can sort the data in the index based on what we are looking for

→ search through a sorted list is way faster

→ Binary search



we are pointing blocks, not keys Sparse Index

Sequential File

Sparse index = a pointer per block
 ↳ less index entries

How to search for a key= 30 ?

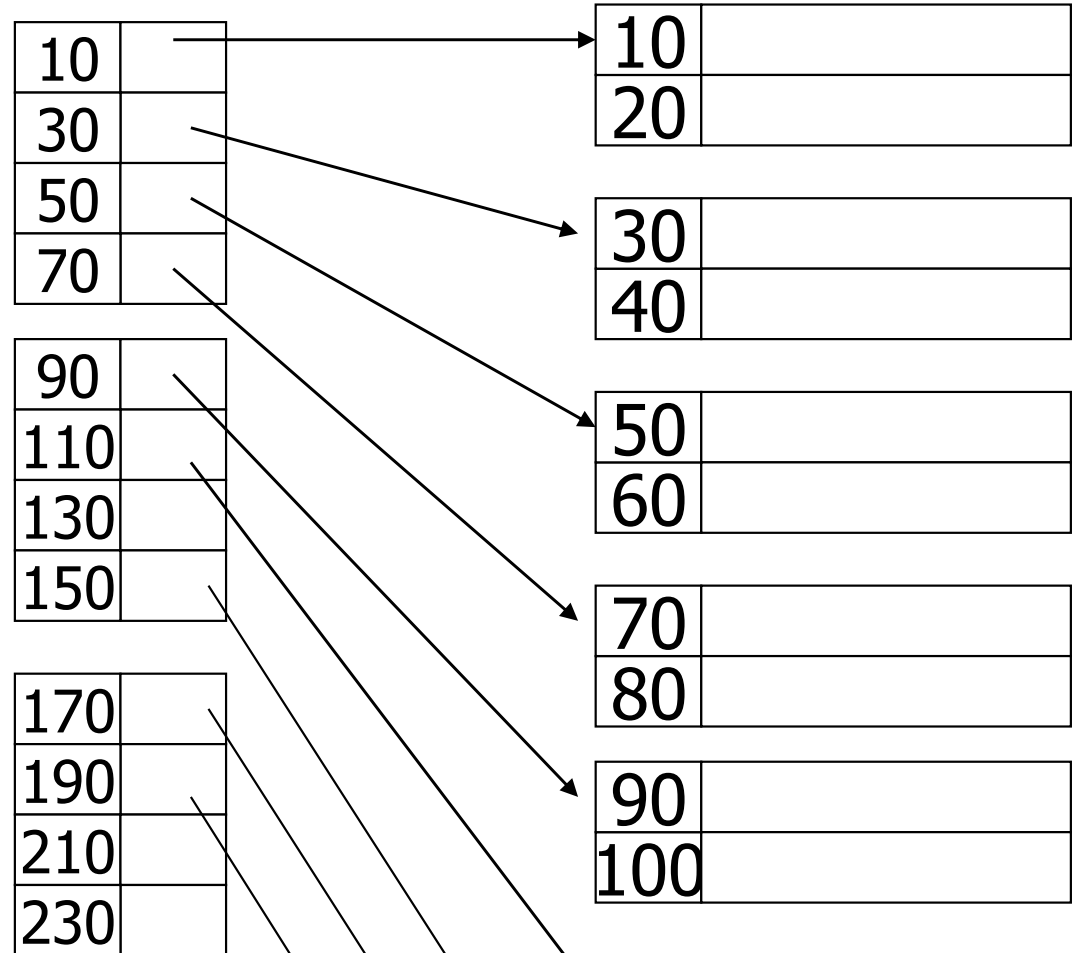
How to search for a key= 25 ?

Can we use a sparse index on a non-sequential file ? *No*

↳ for the sparse index to work, we need to be sure that the pages are sorted
 ↓
 if we are looking for 20 it must be between 10 and 30
 ⇒ sorted sequential file on the same key as the index

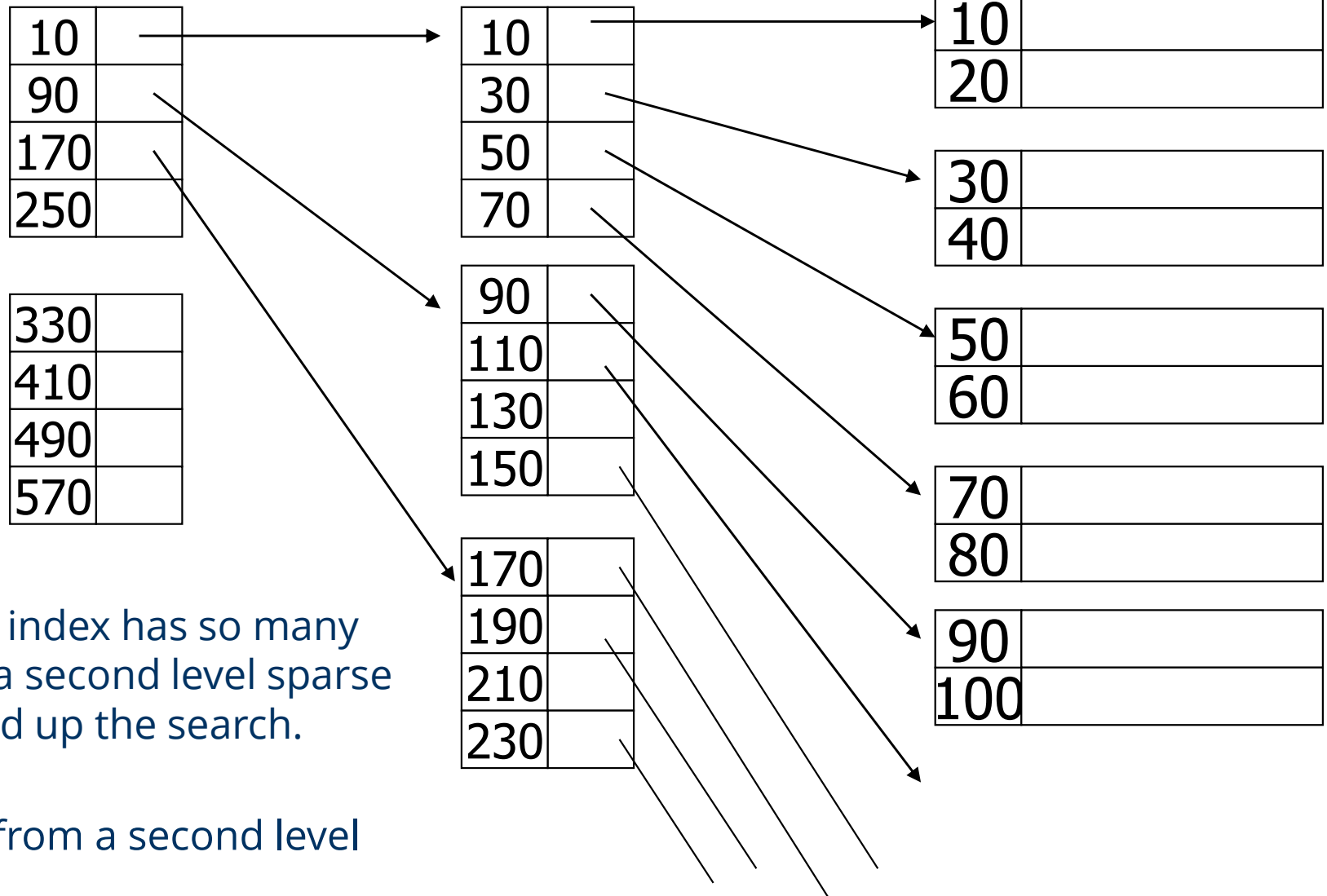
not in the index records ⇒ we look for it in the previous page

↳ we do not know if it is there or not, we have to go through the records of the page sequentially



Sparse 2nd level

Sequential File



If the first level index has so many pages, adding a second level sparse index can speed up the search.

Do we benefit from a second level **dense** index?

↳ no, we keep the same amount of entries ⇒ useless

Sparse vs. Dense Tradeoff

- Sparse: Less index space per record
can keep more of index in memory
- Dense: Can tell if any record exists
without accessing file

↓
but bigger size, we could be unable
to store the whole index in the memory

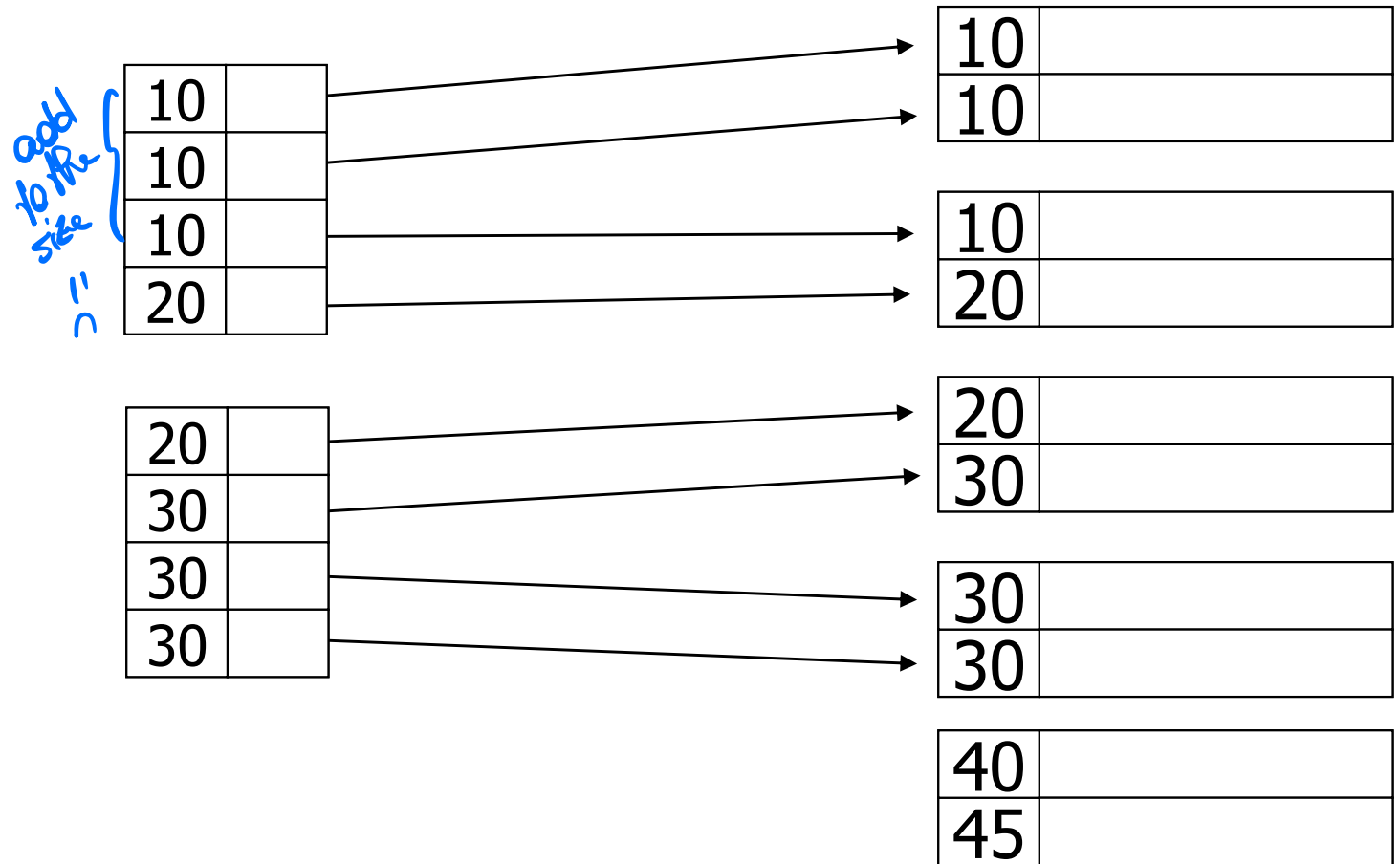
↓
reduce number
of access
(mean cost
in the database)

Duplicate keys

Dense index, one way to implement?

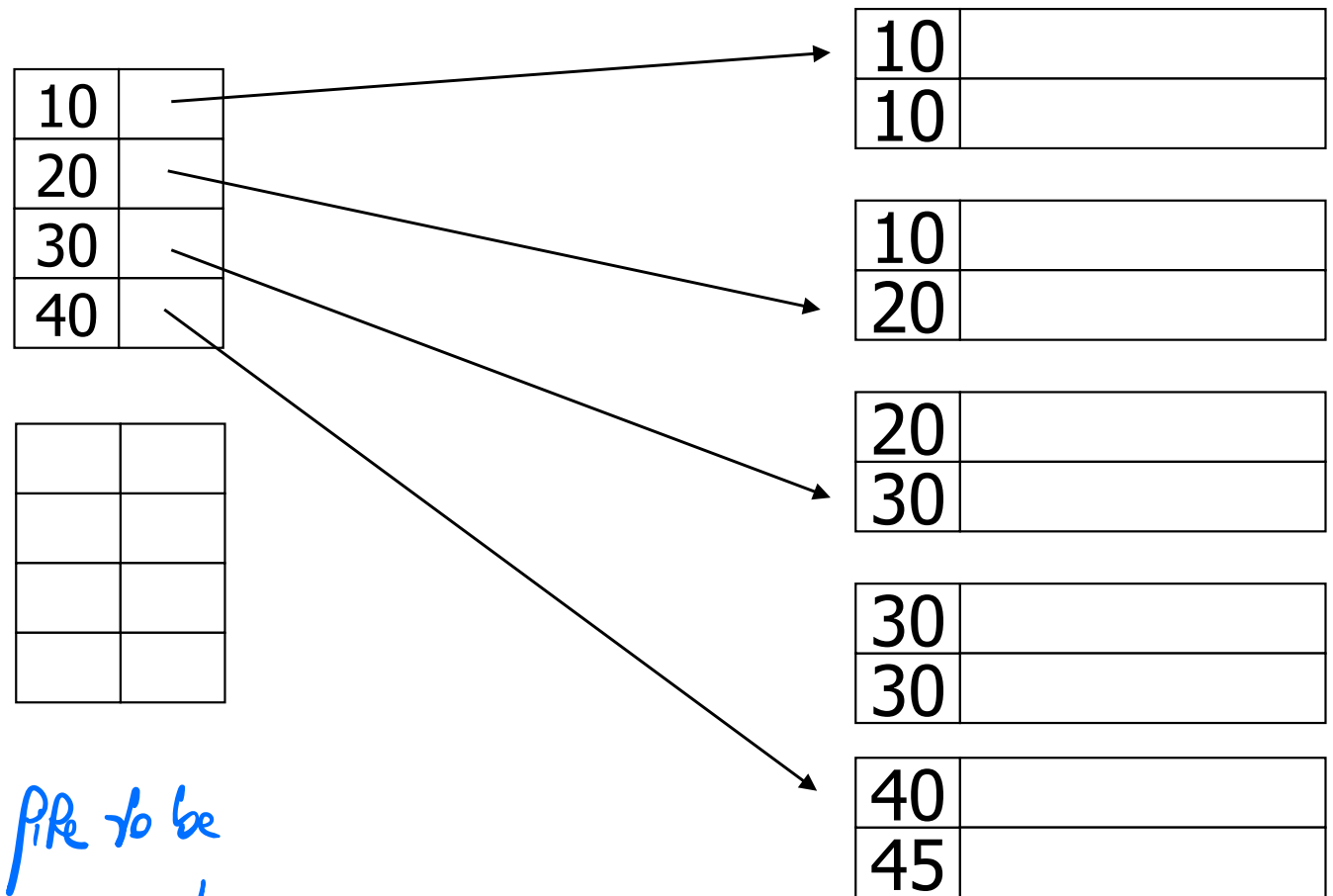
Dense index.

Can We do better ?



Duplicate keys

Dense index, better way?



*require the PIR to be
sorted in the same order
as the index*

Duplicate keys

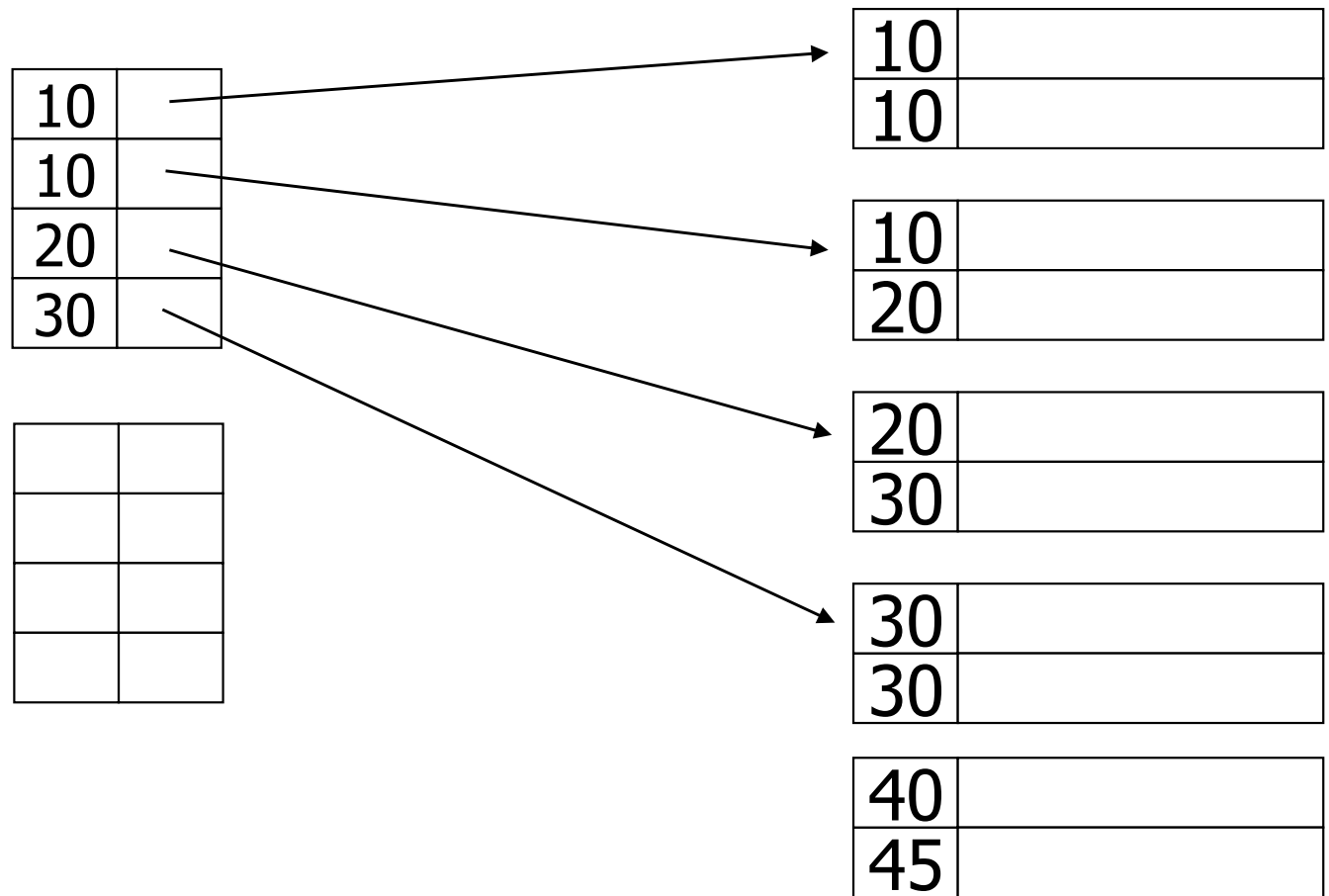
Sparse index, one way?

Sparse index - place
key from block

How to search for 30 ?

1. binary search the index
2. find page of the record
3. scan backward and forward

6
since store by pages,
can be same
in the previous and the
next page



Duplicate keys

Sparse index, another way?

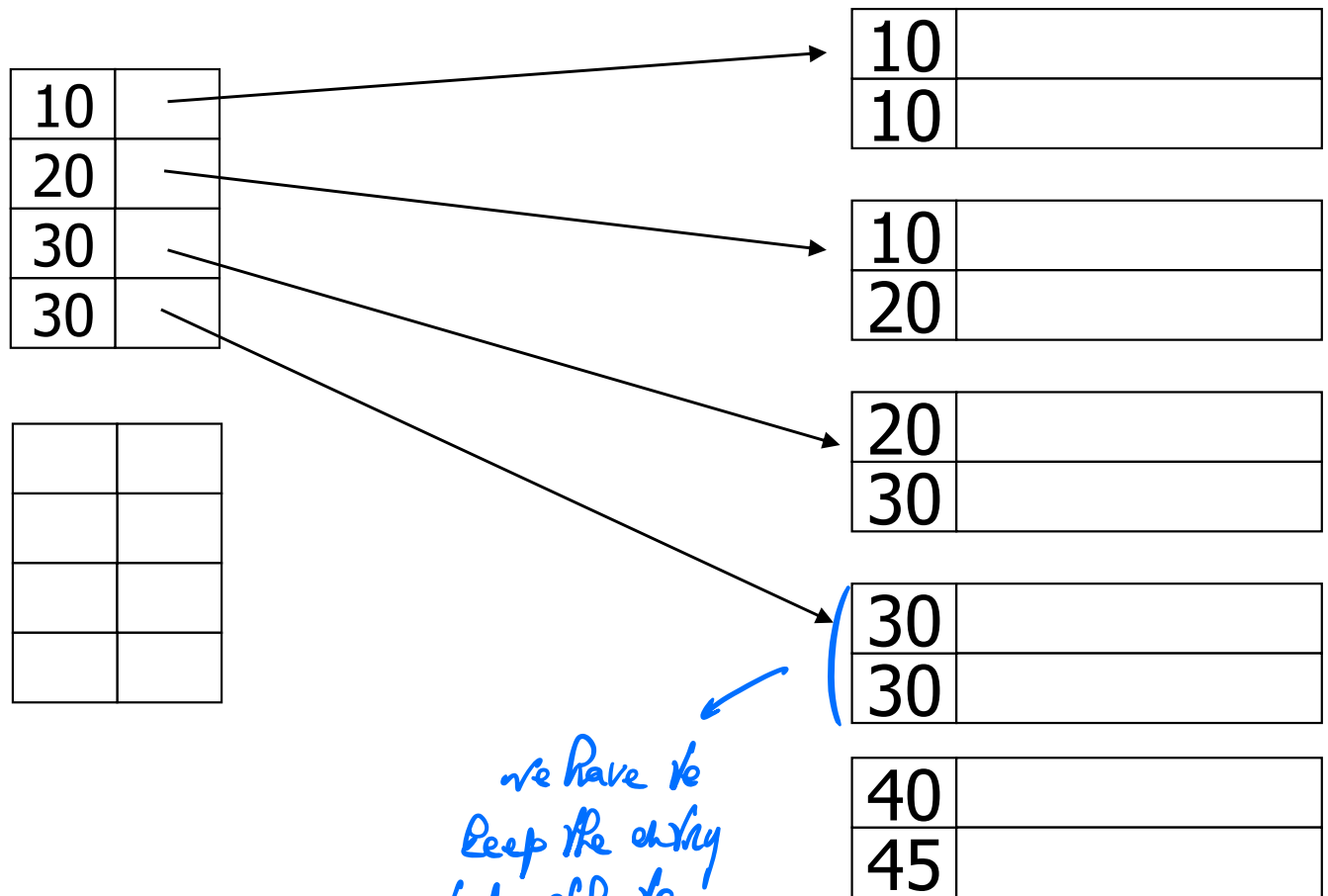
Sparse index - place first **new** key from block

How to search for 30 ?

↳ no need to scan back

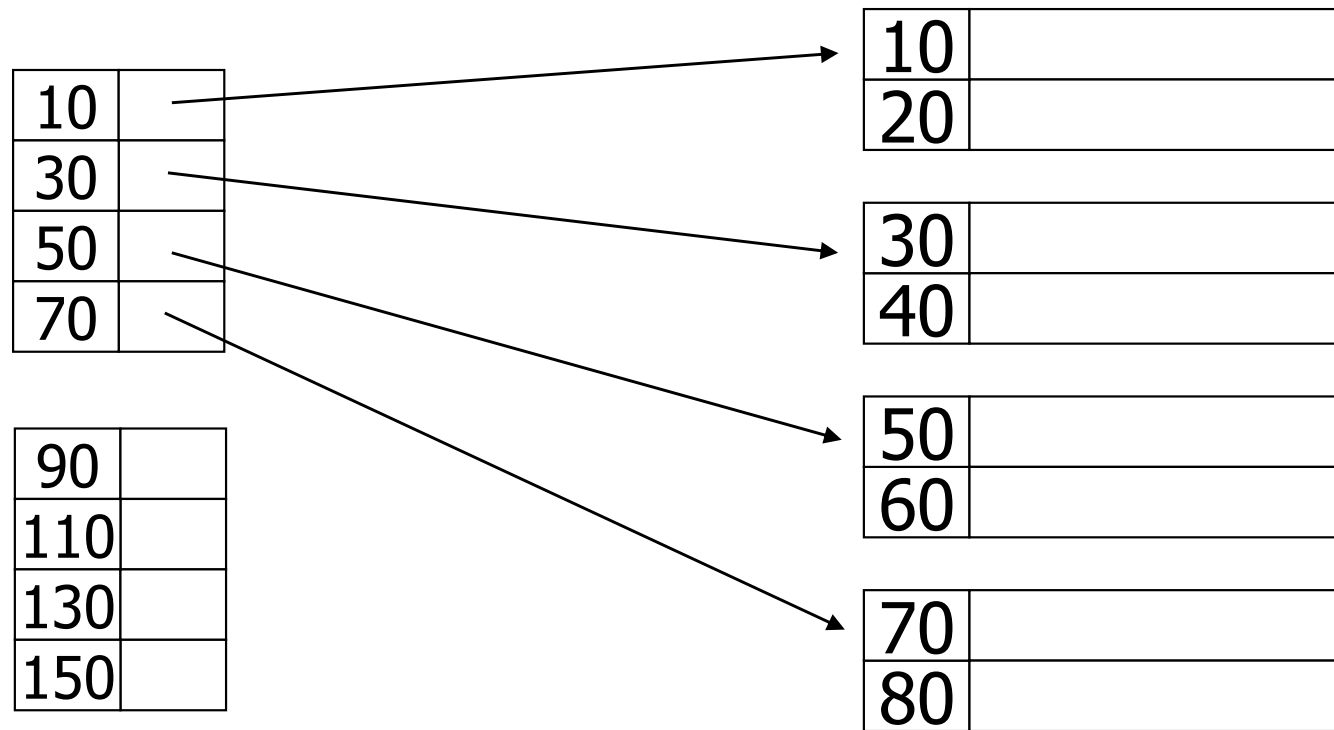
How to search for 35 ?

↳ go to the previous pointer and scan the page for the key



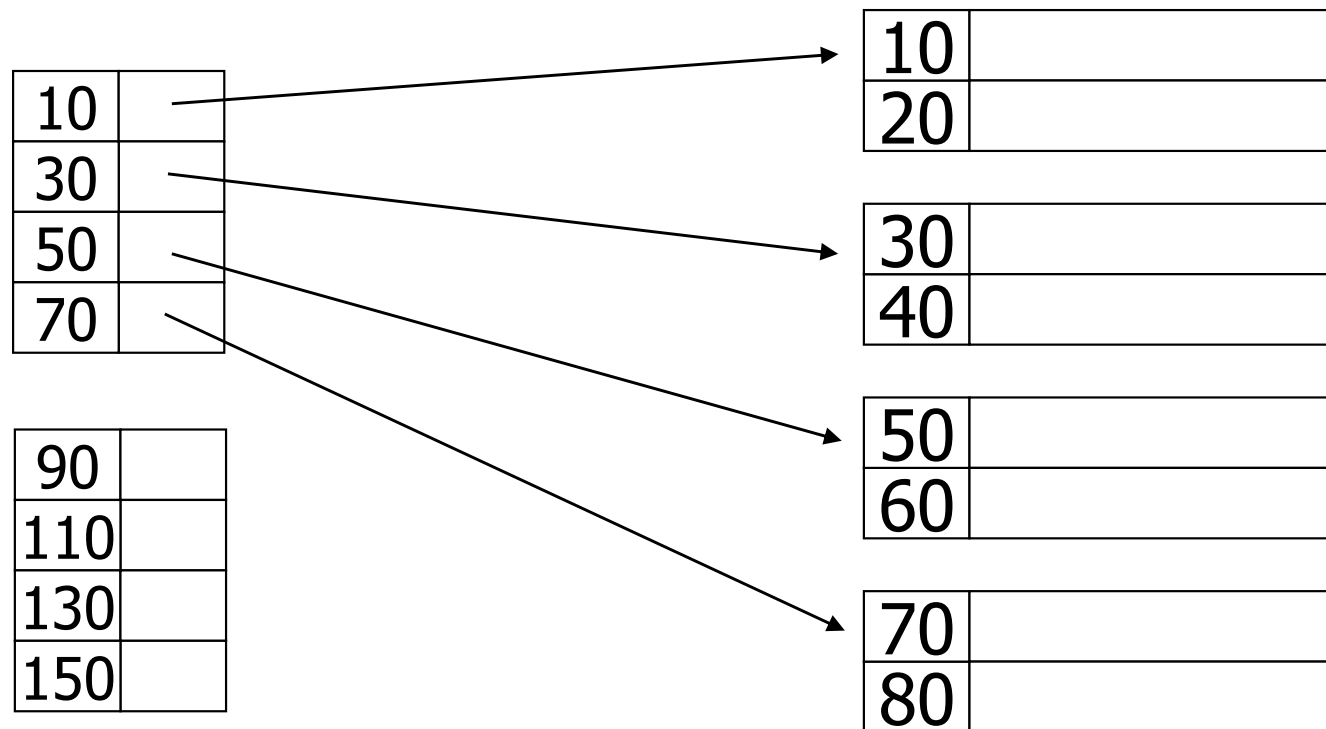
we have to keep the entry to be able to access it even though there is no new key

Deletion from sparse index



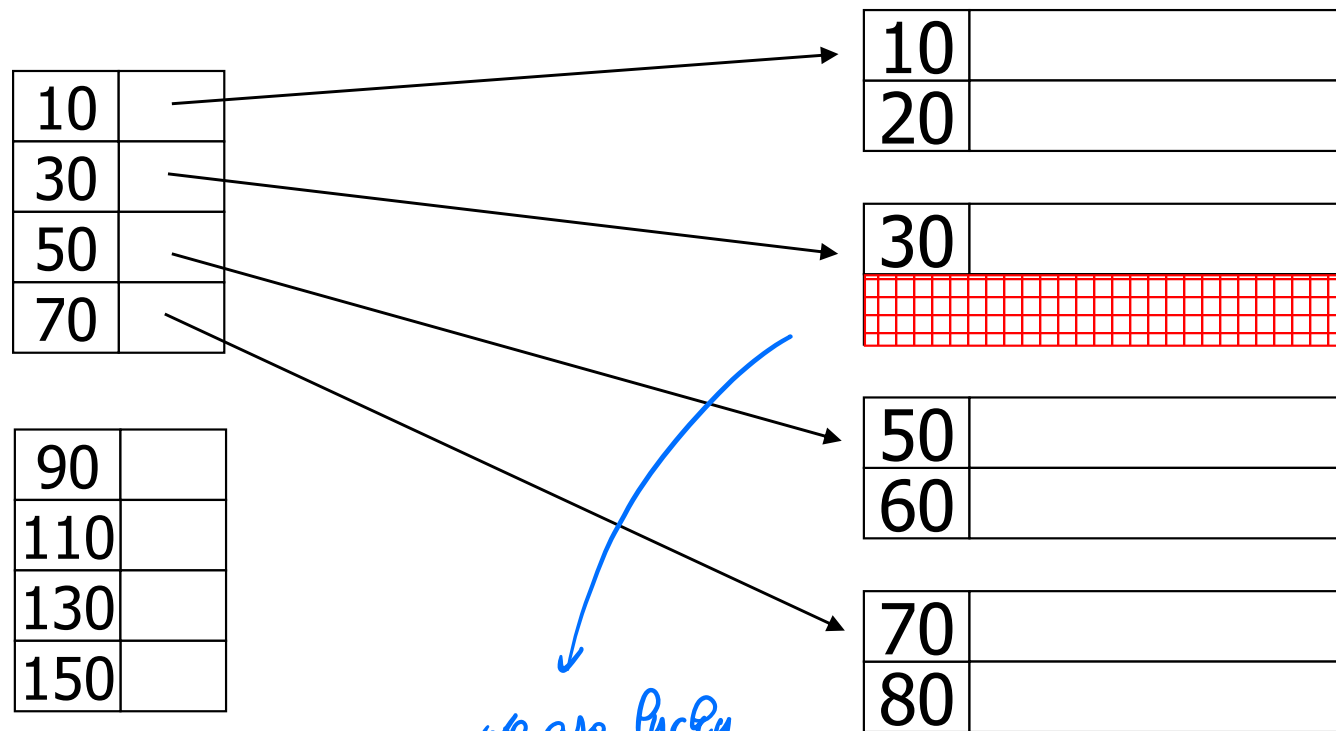
Deletion from sparse index

– delete record 40



Deletion from sparse index

– delete record 40

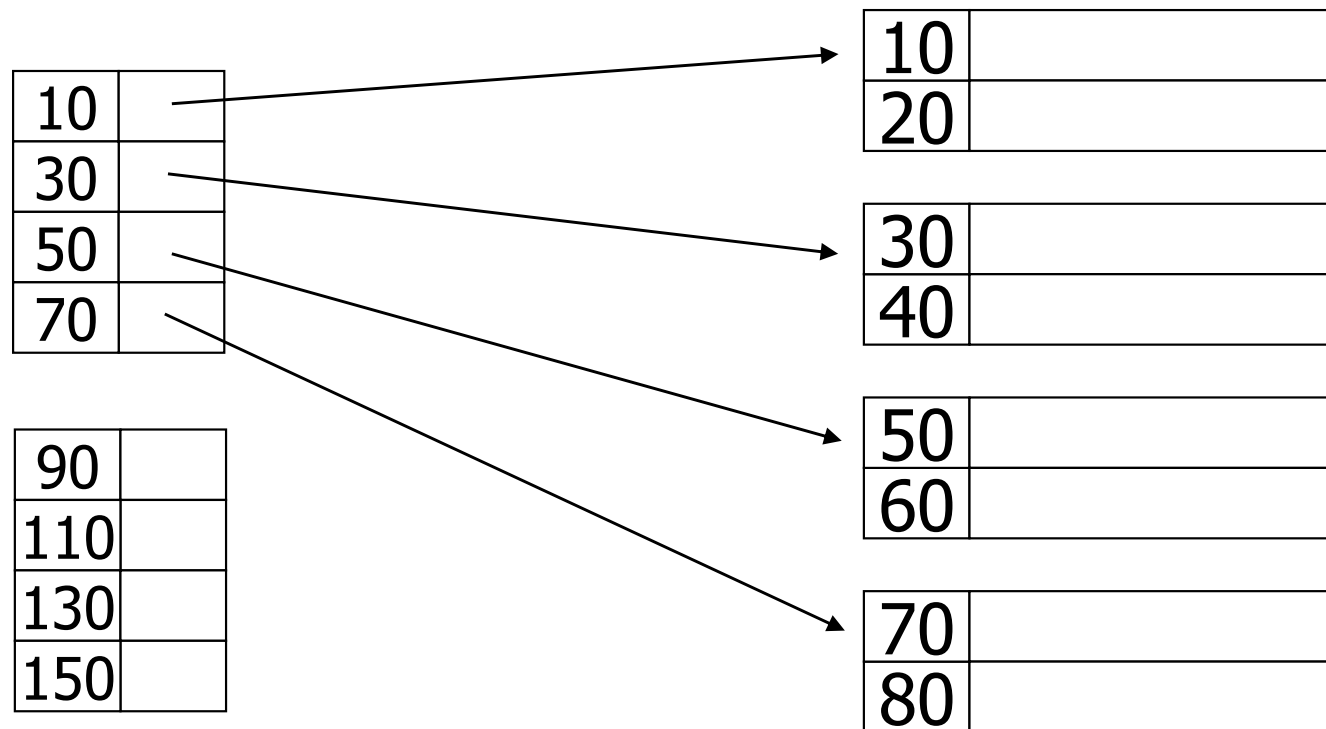


*we are lucky
it was not a
key of the index
⇒ just suppress it*

*+ we could want to
keep the data sequential
in the file
(shifting up)*

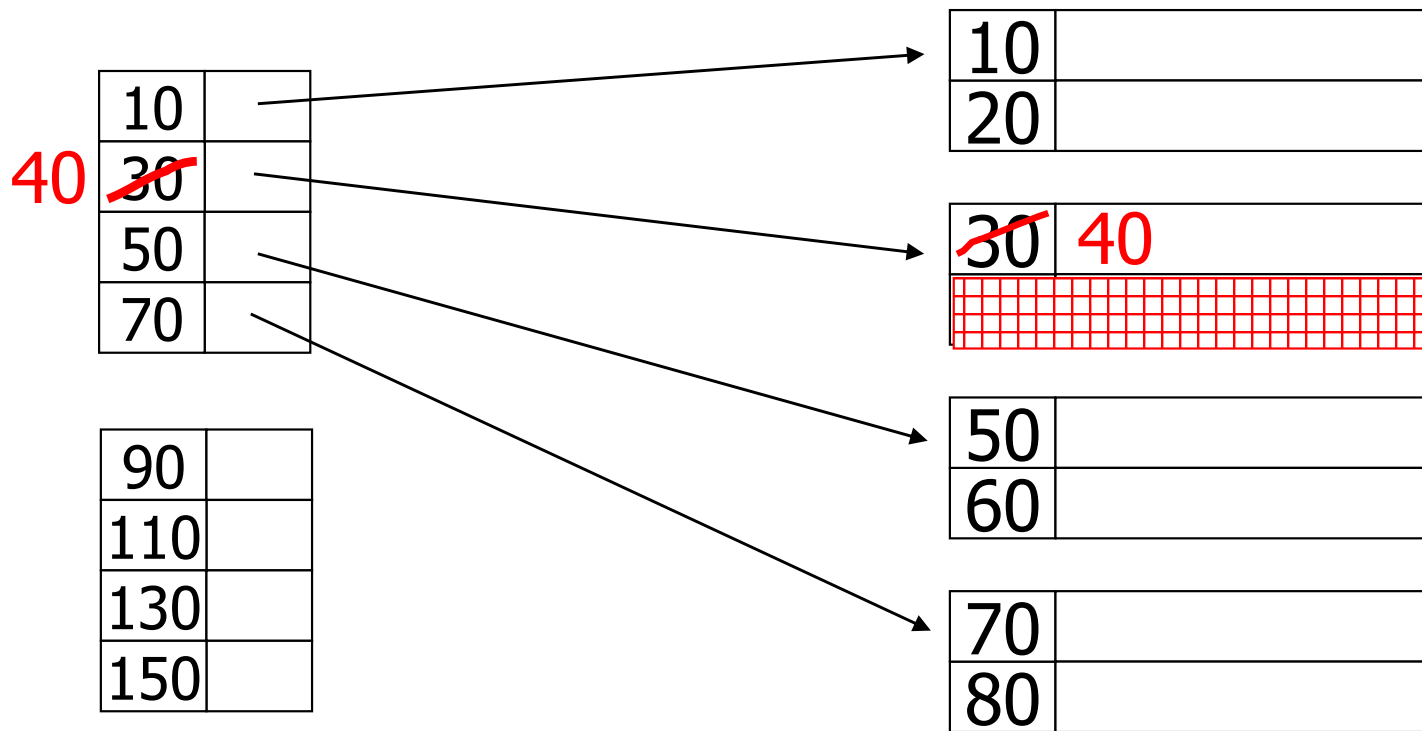
Deletion from sparse index

– delete record 30



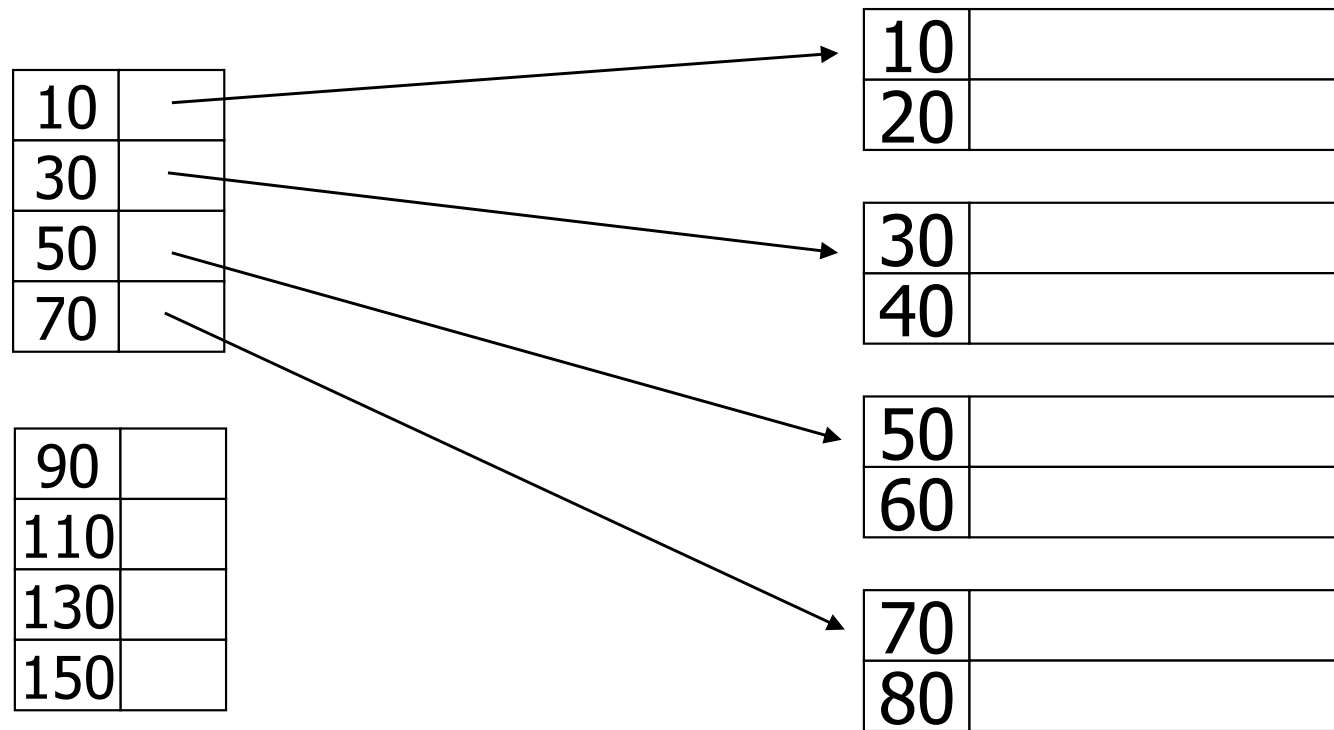
Deletion from sparse index

– delete record 30



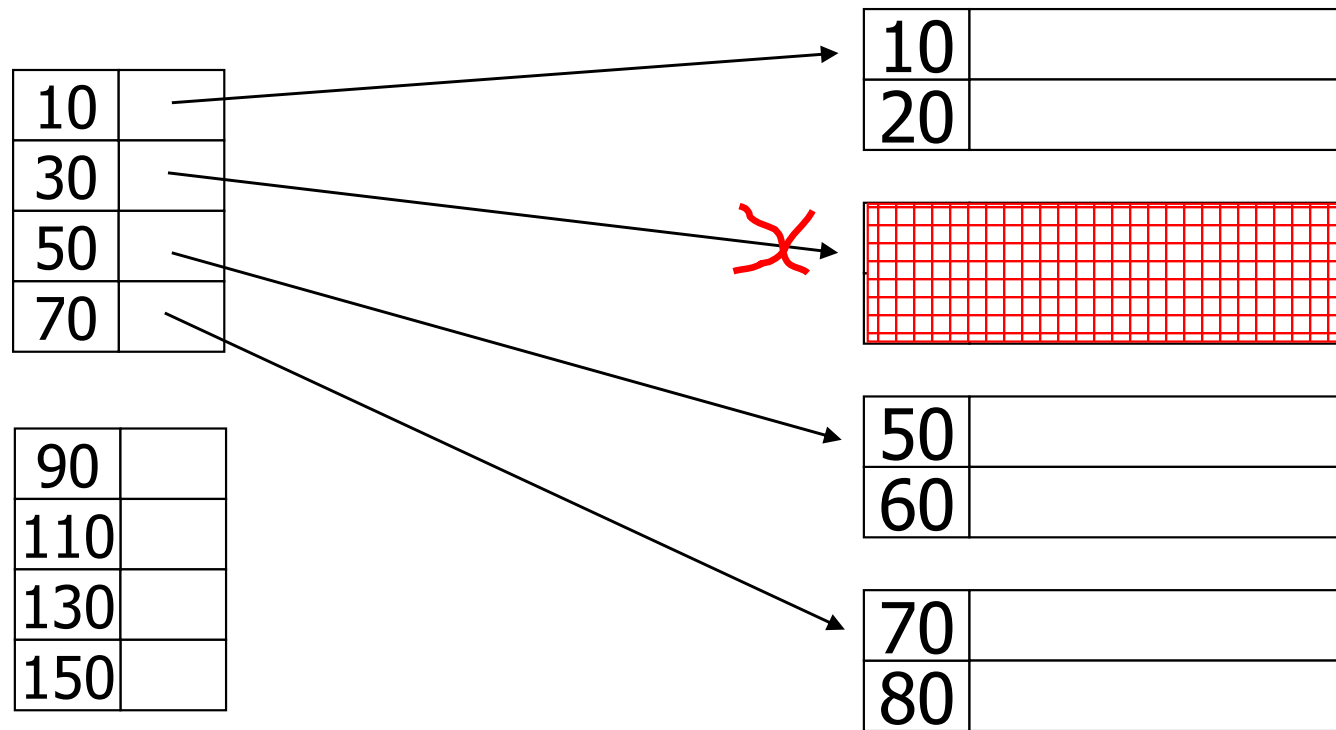
Deletion from sparse index

– delete records 30 & 40



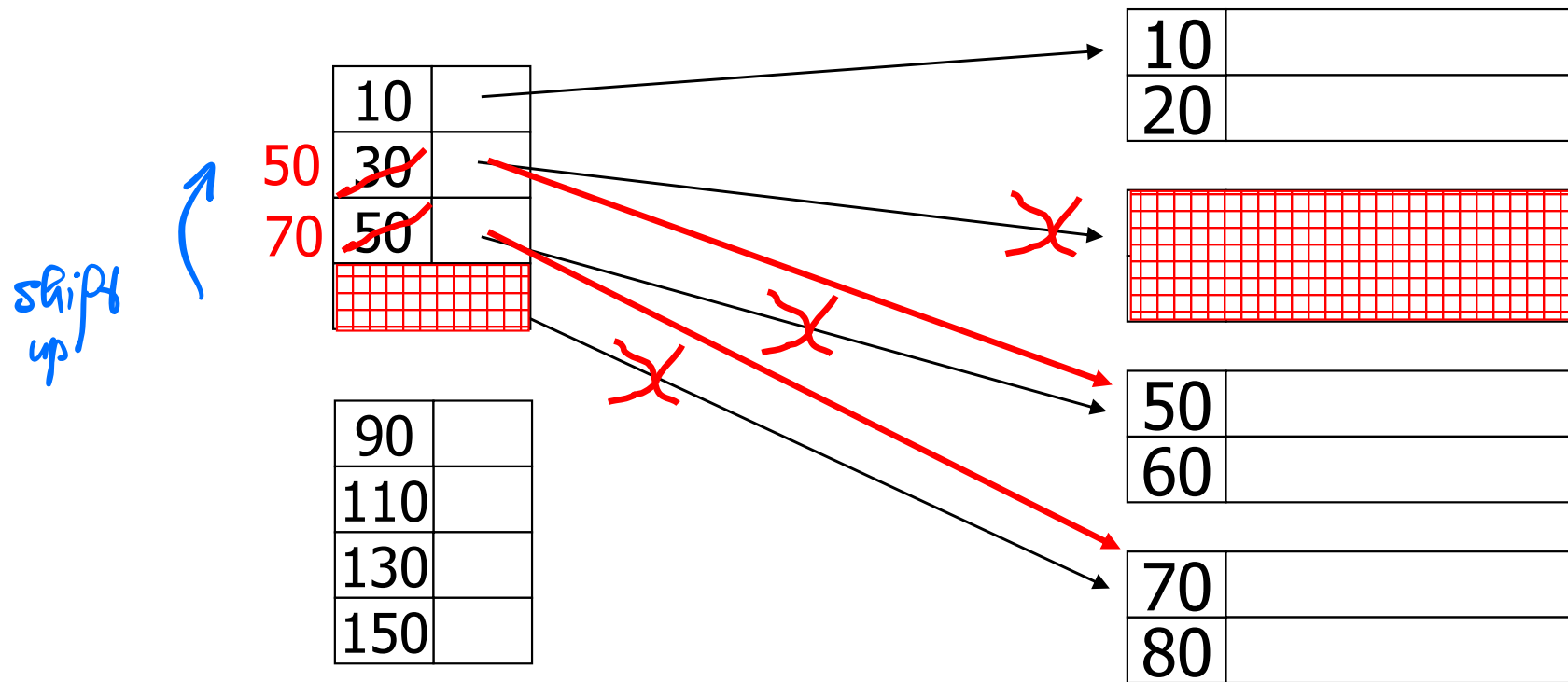
Deletion from sparse index

– delete records 30 & 40

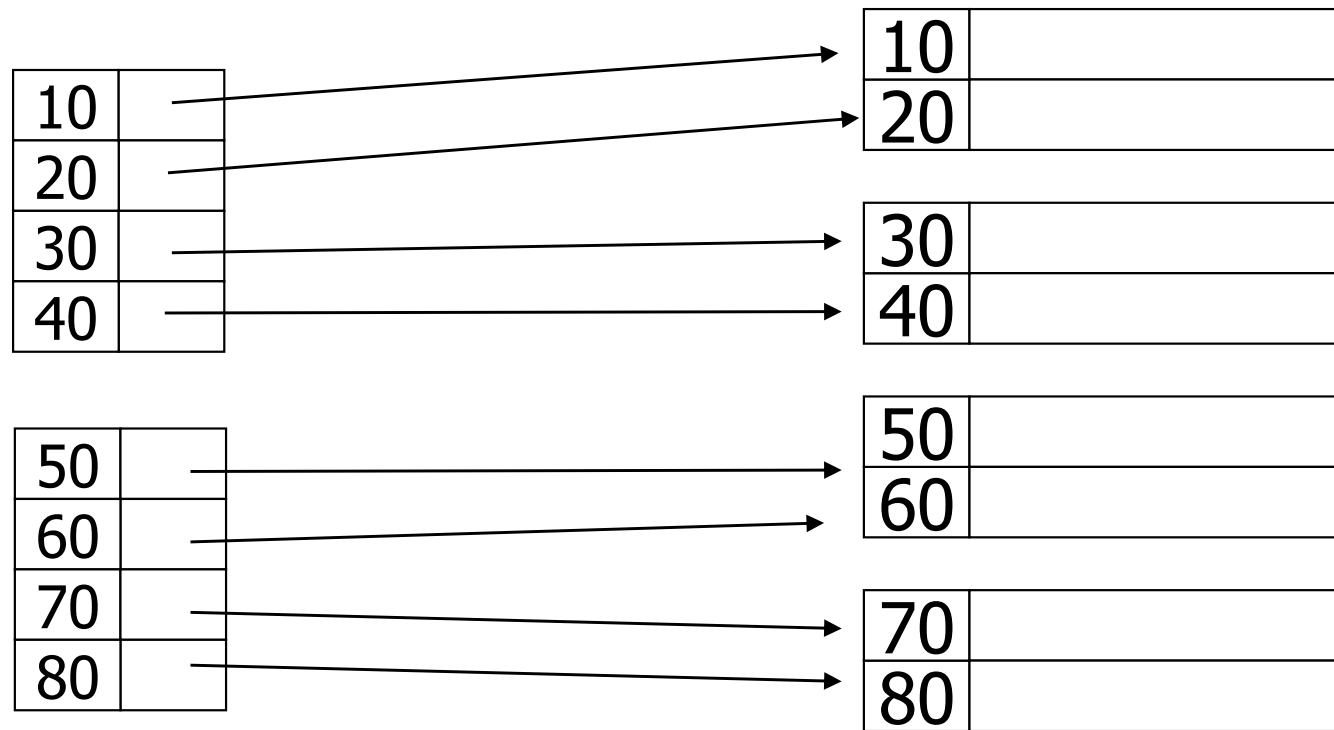


Deletion from sparse index

– delete records 30 & 40

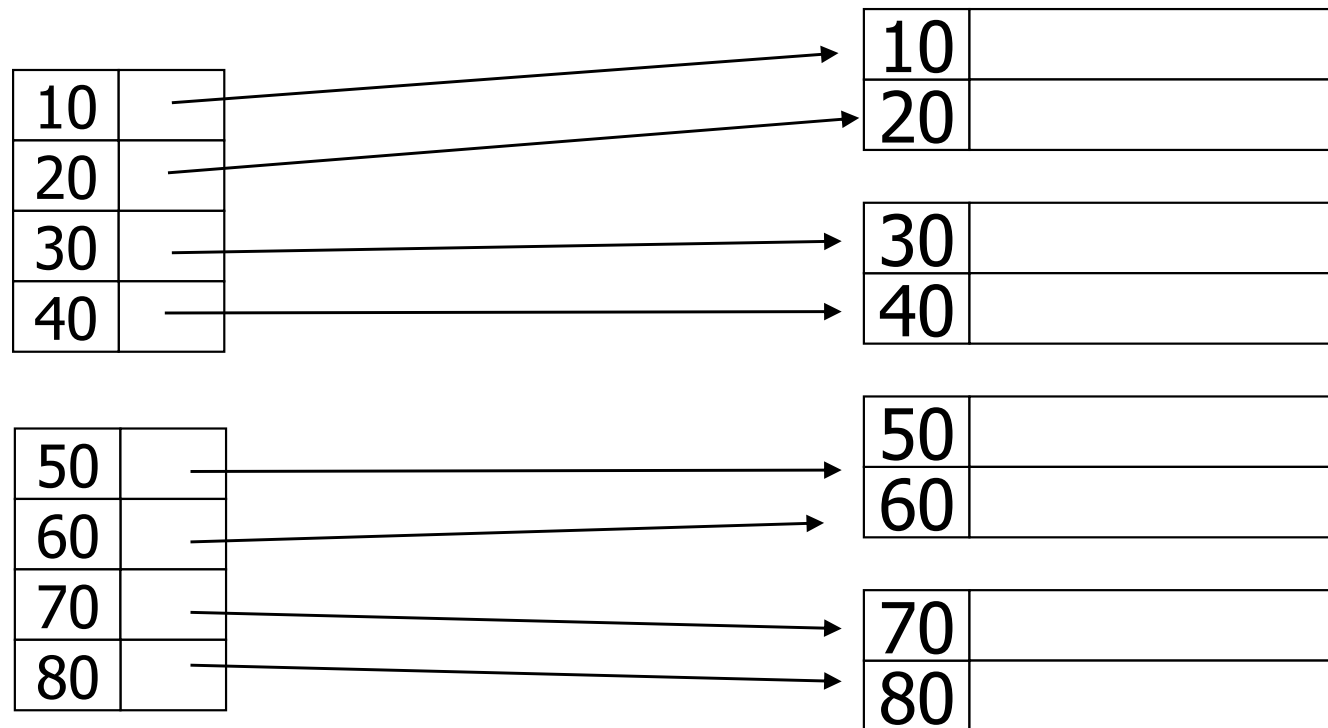


Deletion from dense index



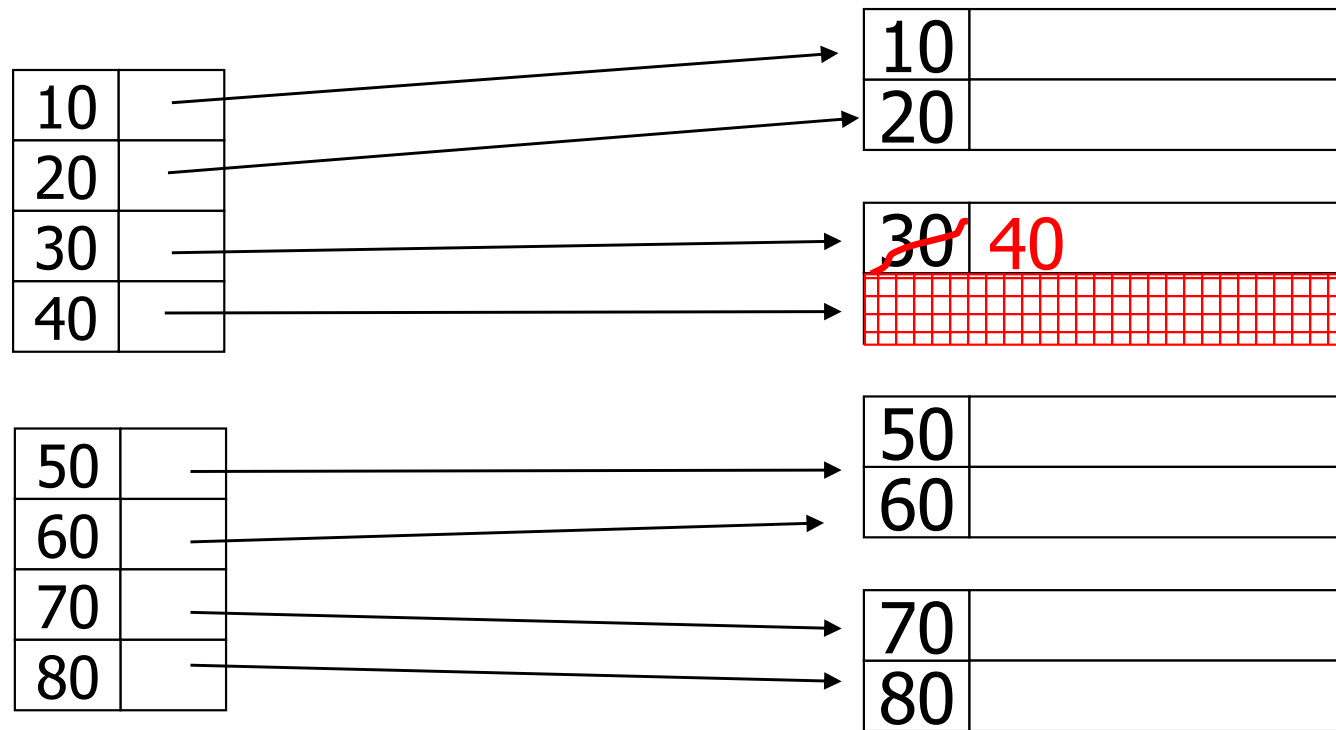
Deletion from dense index

– delete record 30



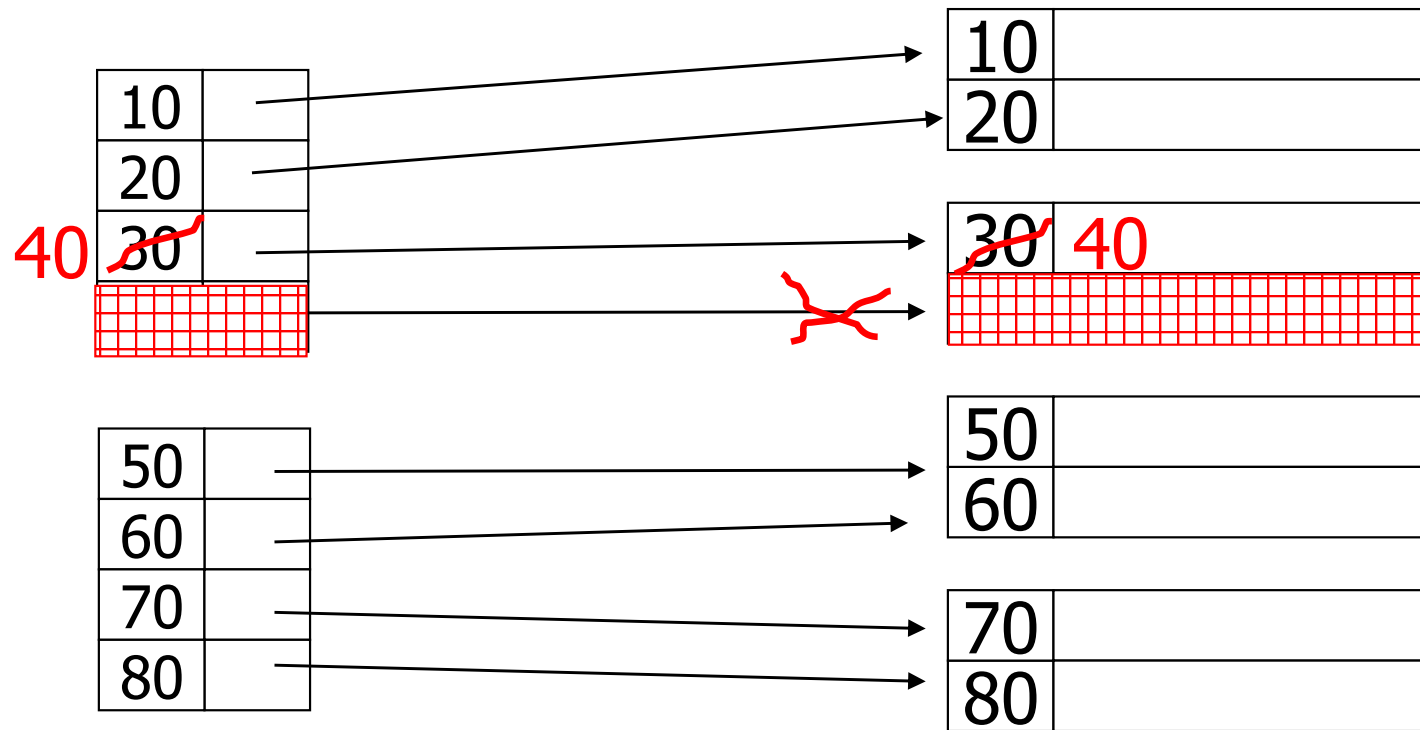
Deletion from dense index

– delete record 30

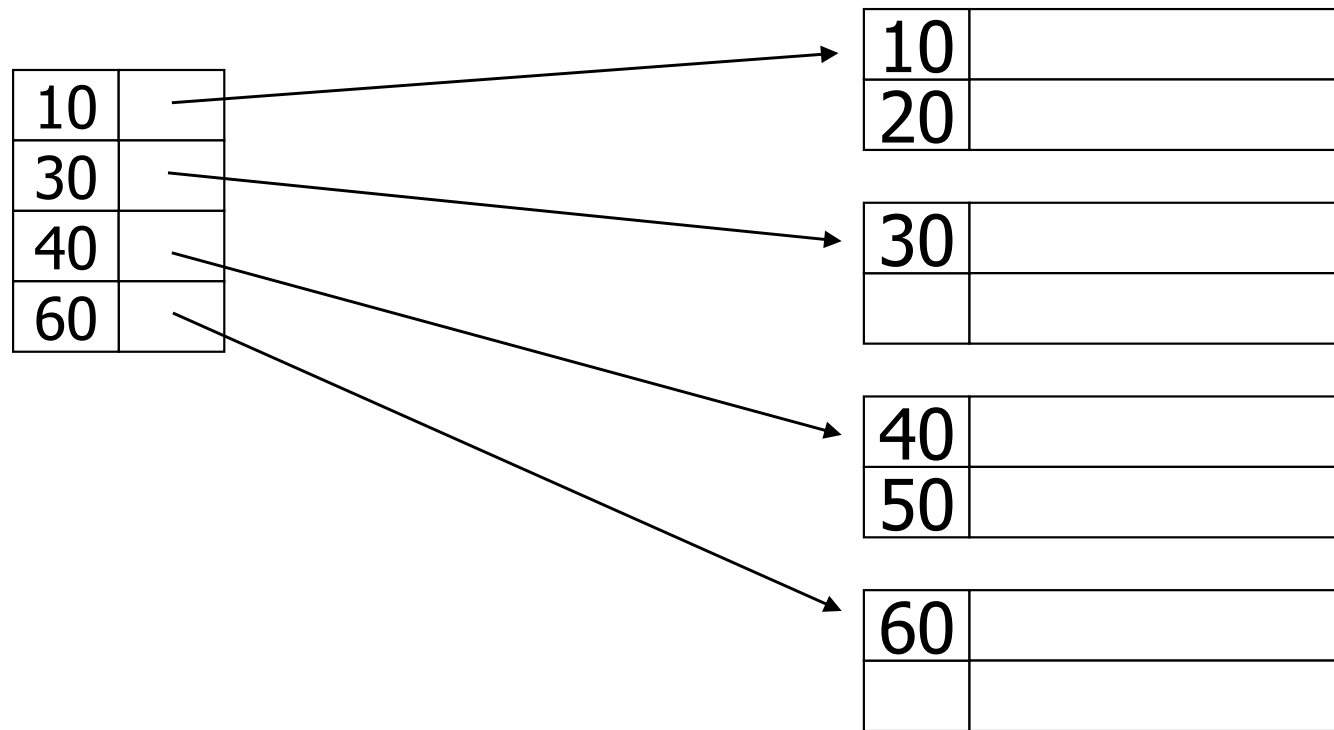


Deletion from dense index

– delete record 30

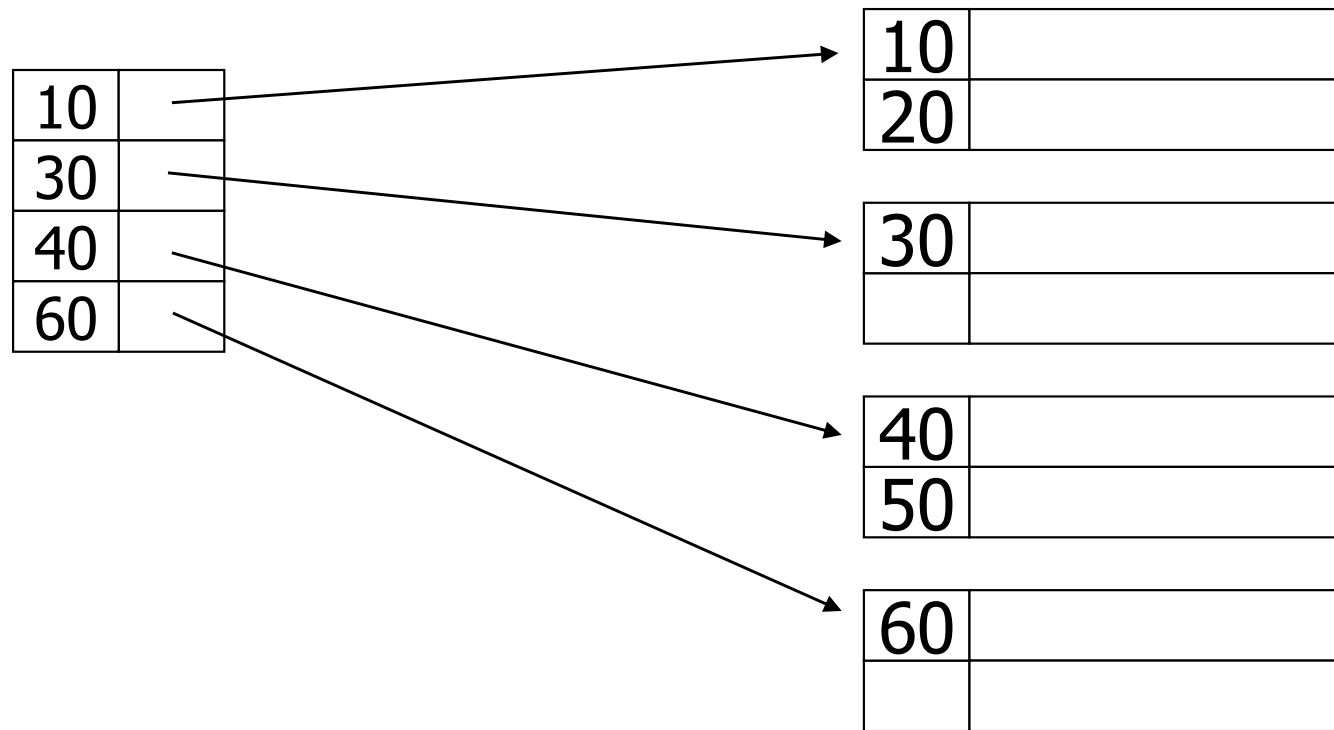


Insertion, sparse index case



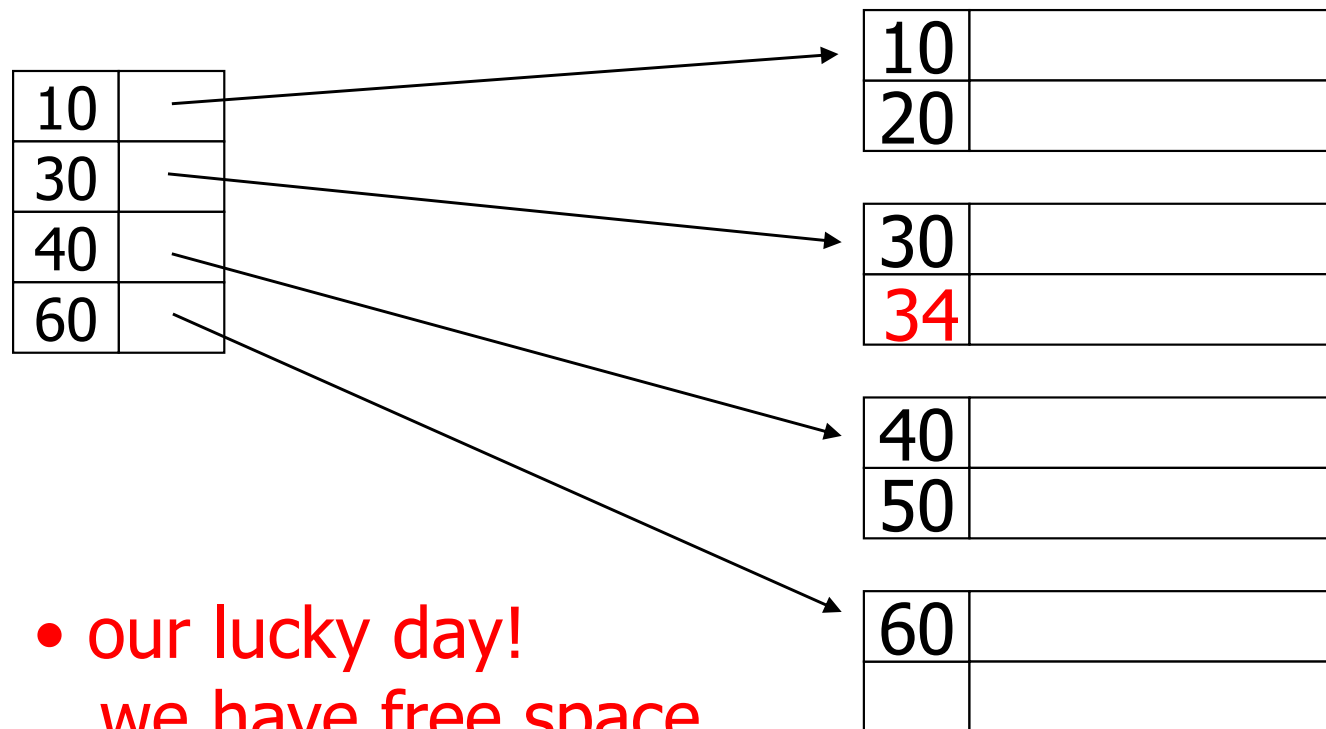
Insertion, sparse index case

– insert record 34



Insertion, sparse index case

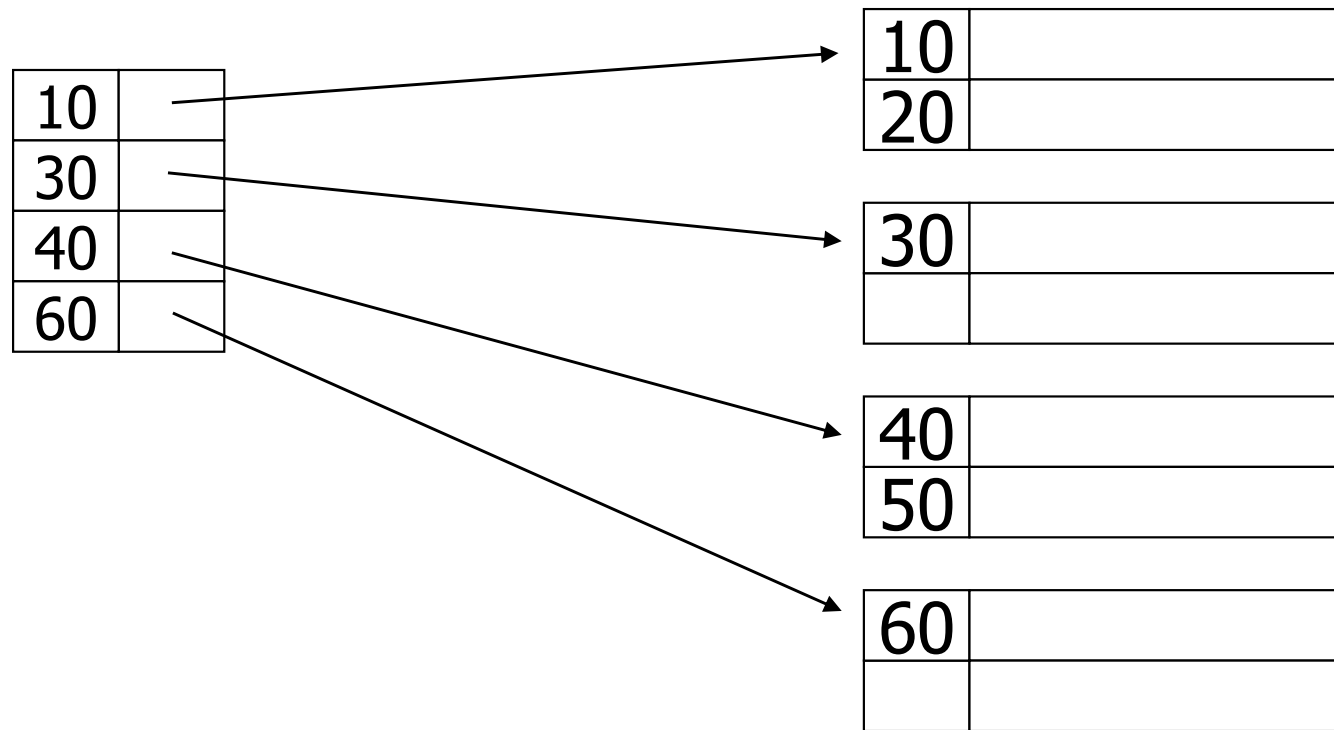
– insert record 34



- our lucky day!
we have free space
where we need it!

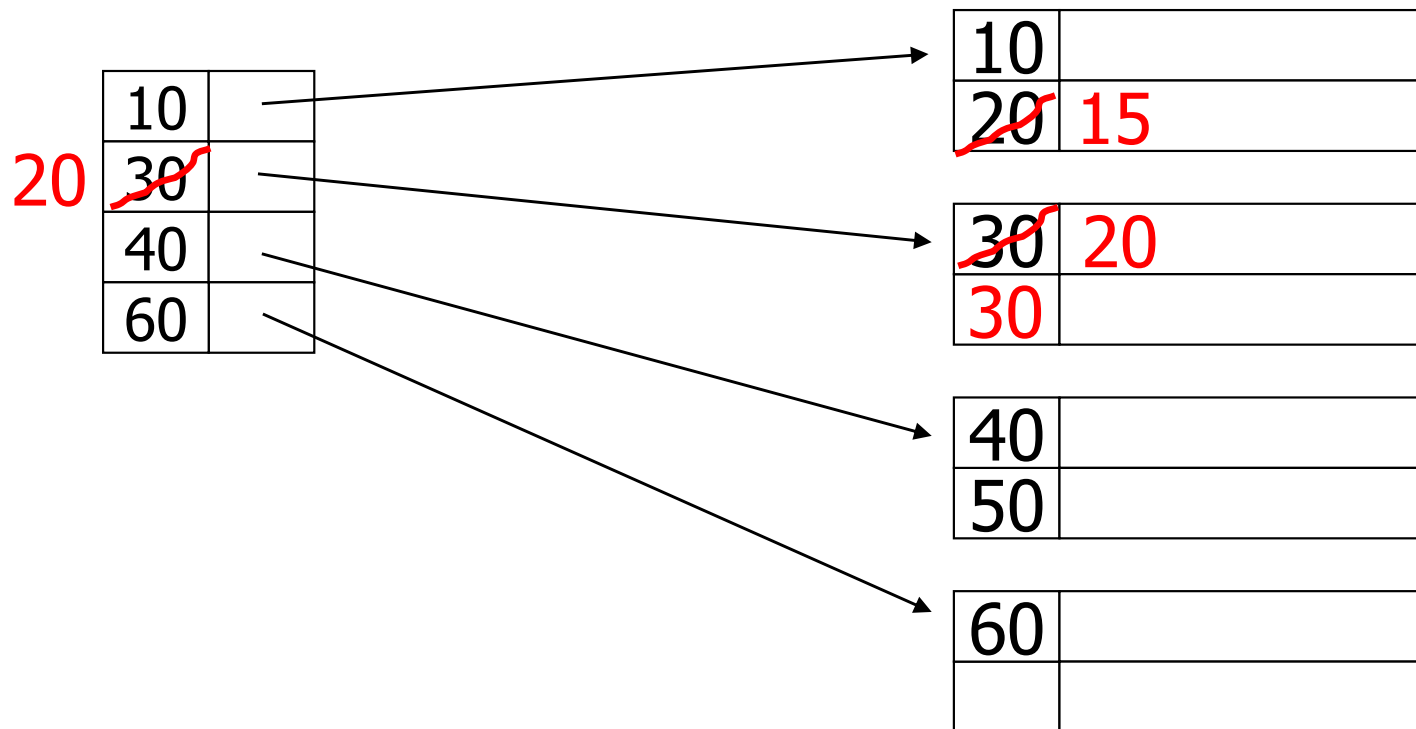
Insertion, sparse index case

– insert record 15



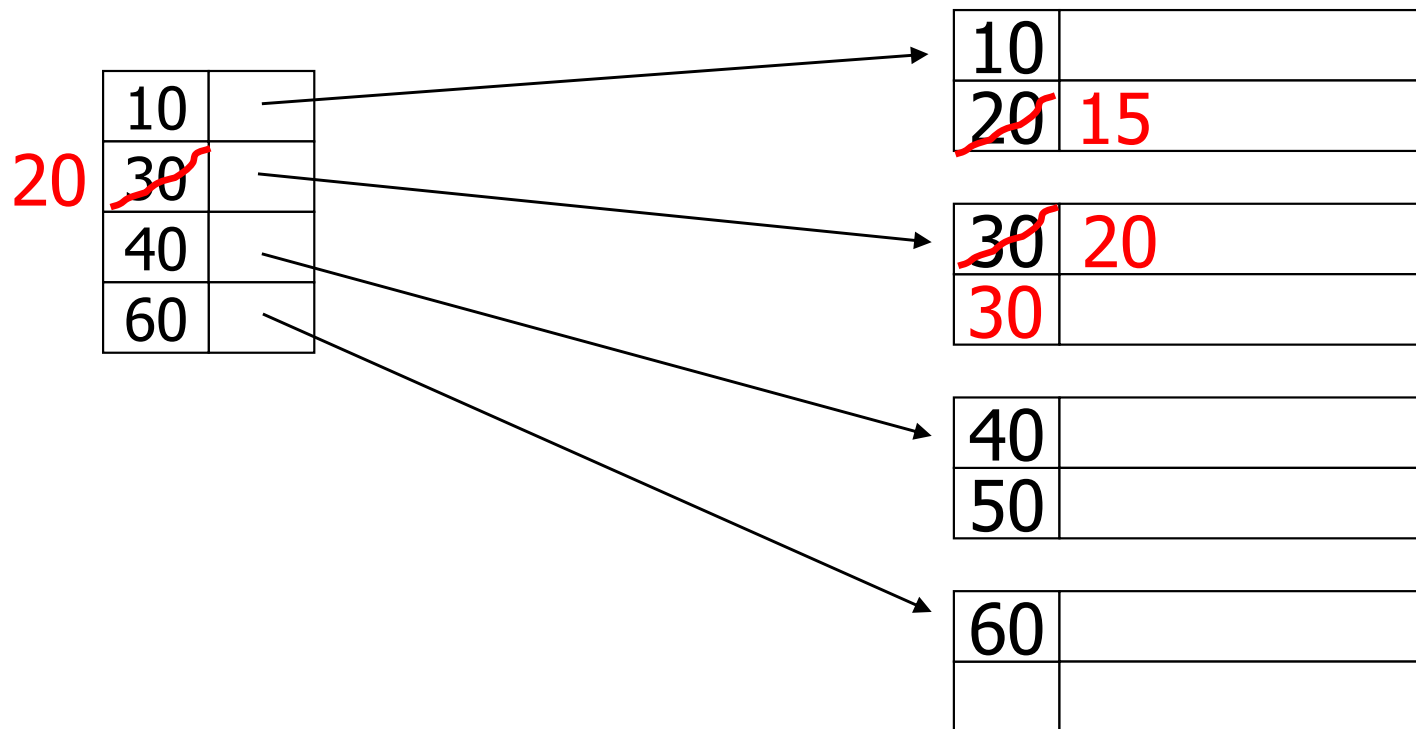
Insertion, sparse index case

– insert record 15



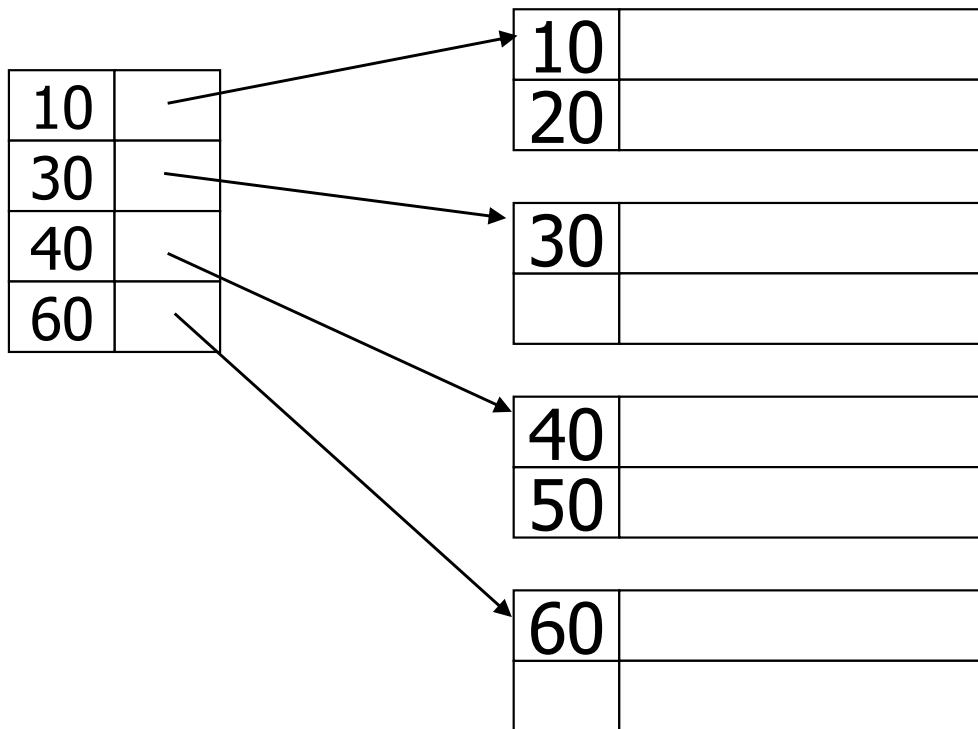
Insertion, sparse index case

– insert record 15



Insertion, sparse index case

– insert record 25



1. follow the index
↳ have to be after 10
2. we go to the page
3. we scan, we need
to go to the next
page
4. insert 25 and shift
30
5. replace the index key
by 25

Insertion, dense index case

- Similar
- Often more expensive . . .

Secondary indexes

*data is not stored sorted
by the key*



Sequence
field

30	
50	

20	
70	

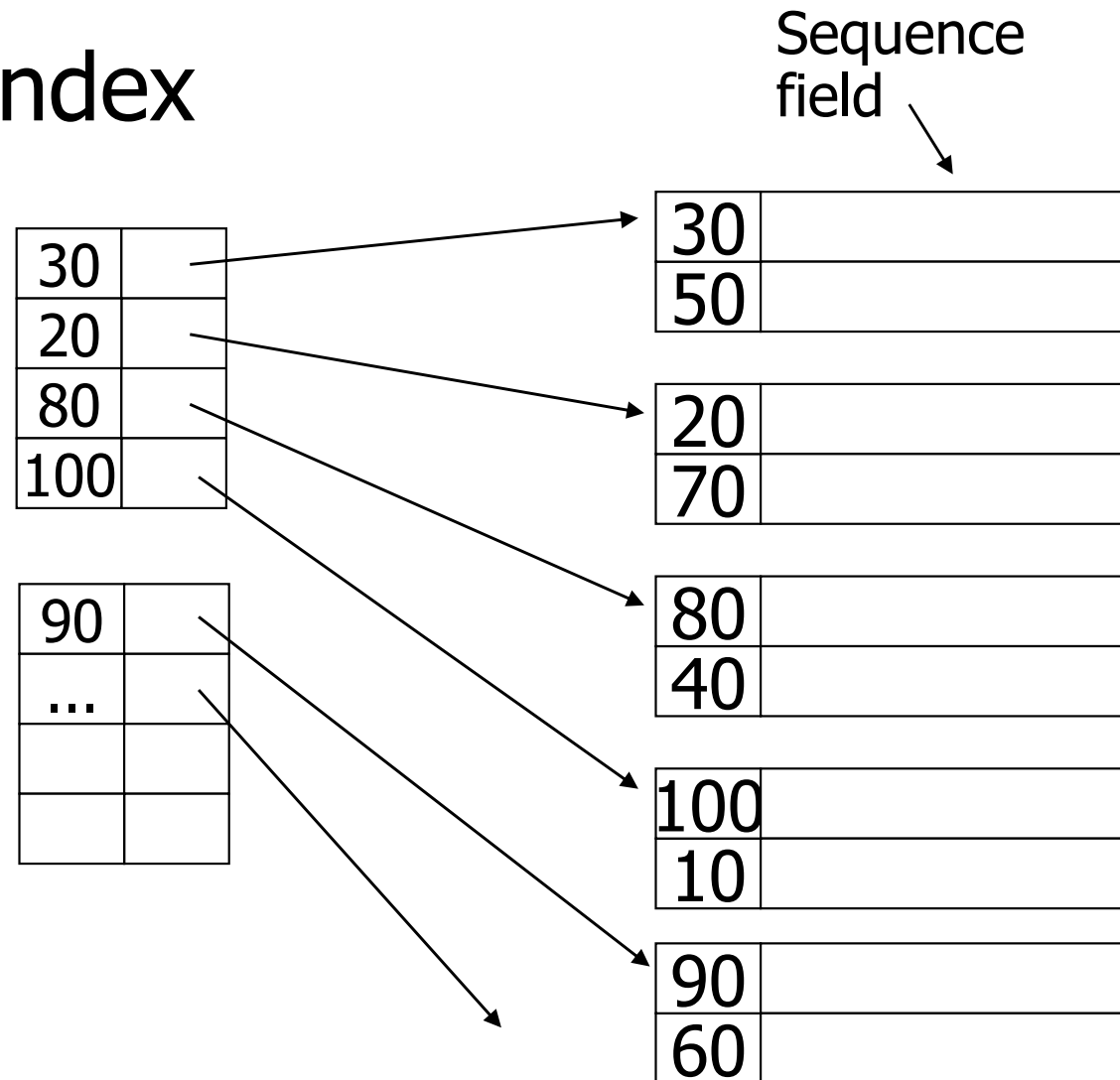
80	
40	

100	
10	

90	
60	

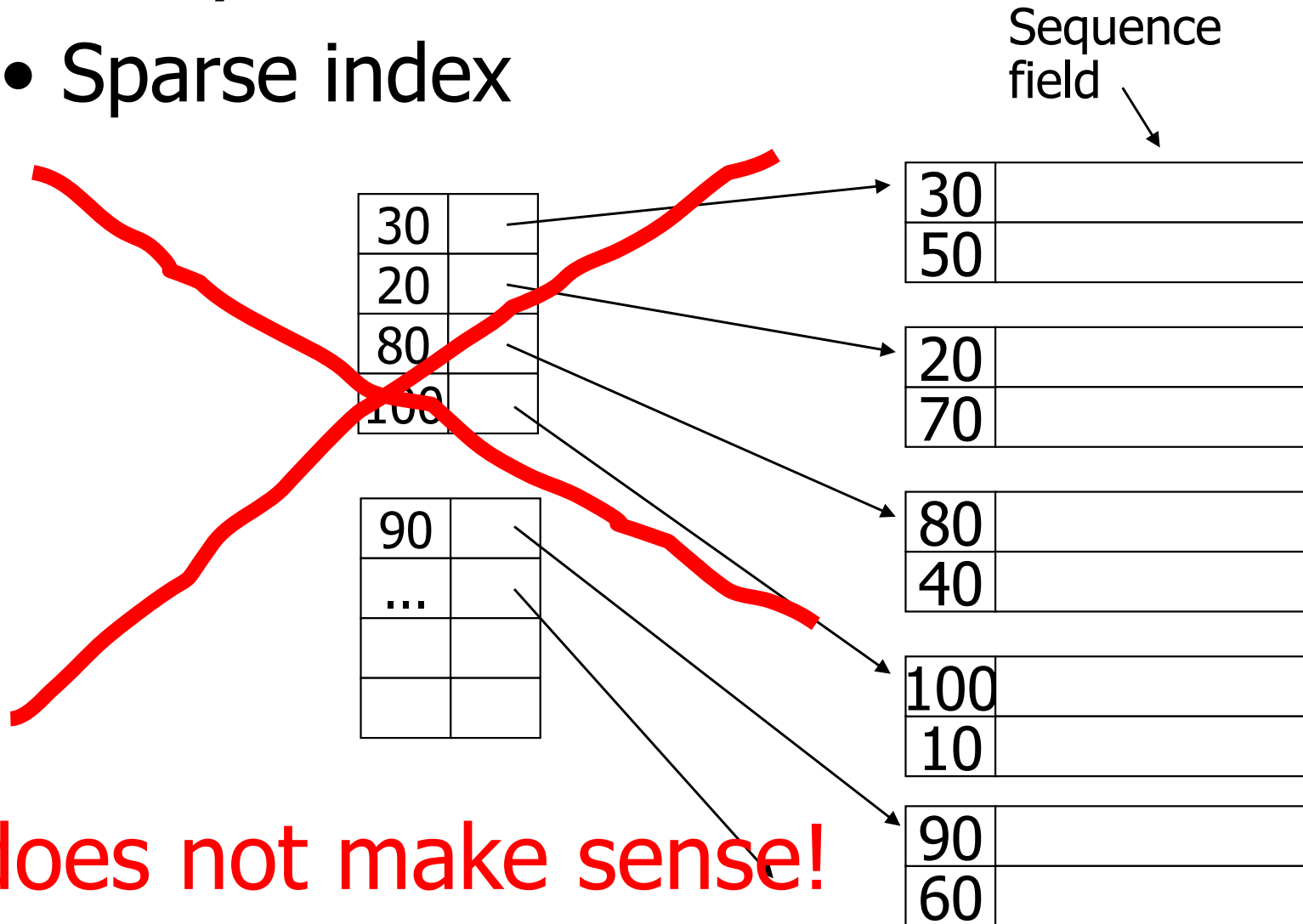
Secondary indexes

- Sparse index



Secondary indexes

- Sparse index




does not make sense!

*valid not because file
is not sorted*

Secondary indexes

- Dense index

Sequence
field



30	
50	

20	
70	

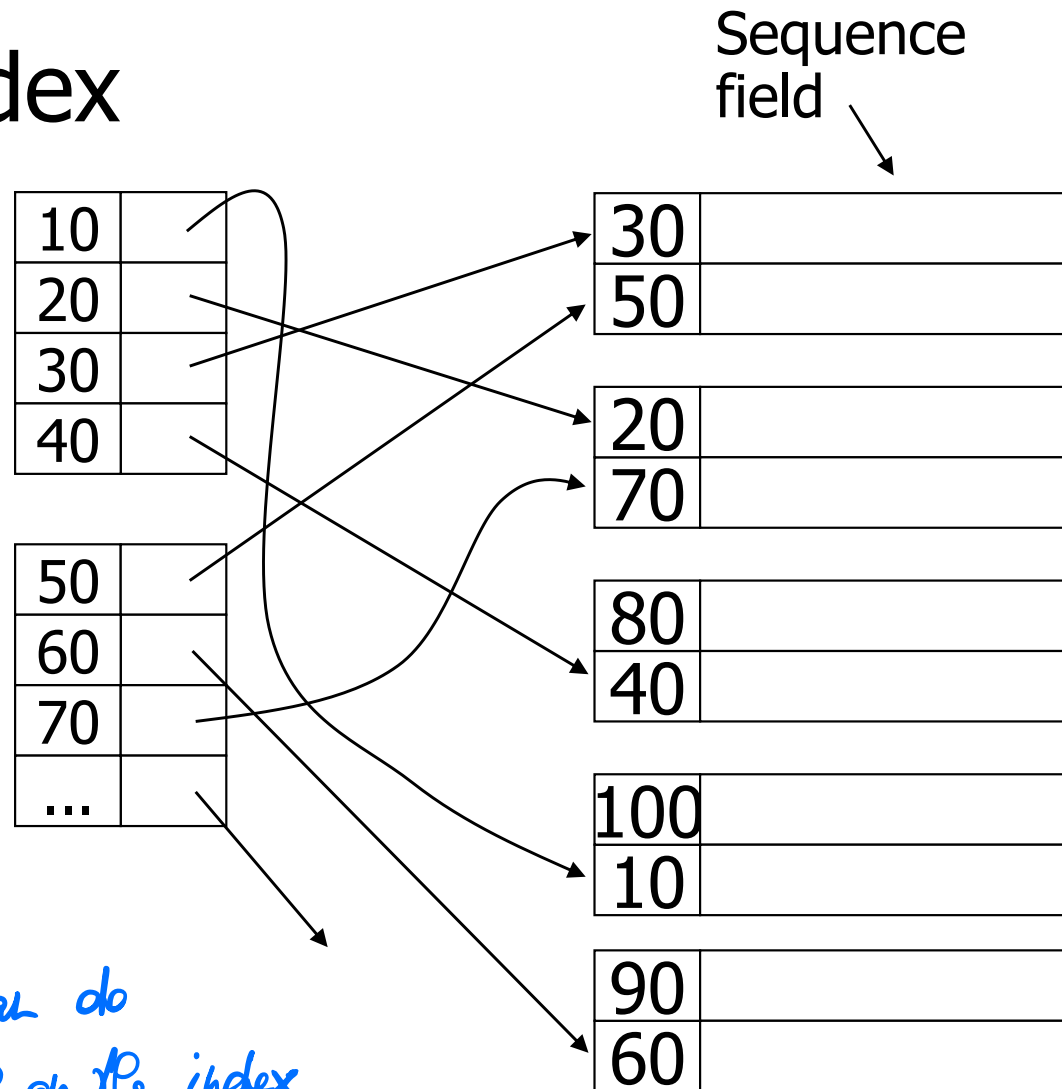
80	
40	

100	
10	

90	
60	

Secondary indexes

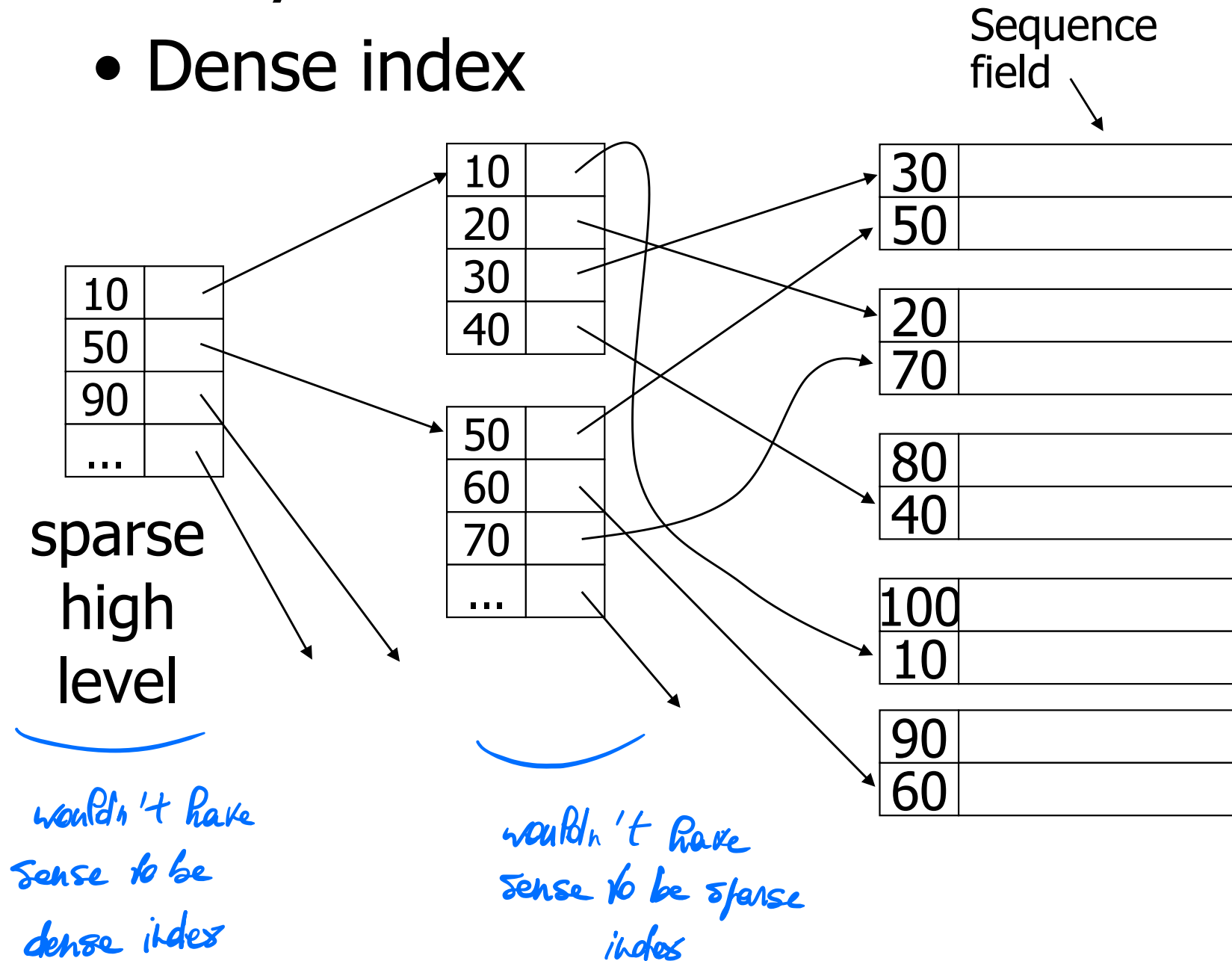
- Dense index



now we can do
binary search on the index
while we couldn't on the sequence
field

Secondary indexes

- Dense index



With secondary indexes:

- Lowest level is dense
- Other levels are sparse

Duplicate values & secondary indexes

20	
10	

20	
40	

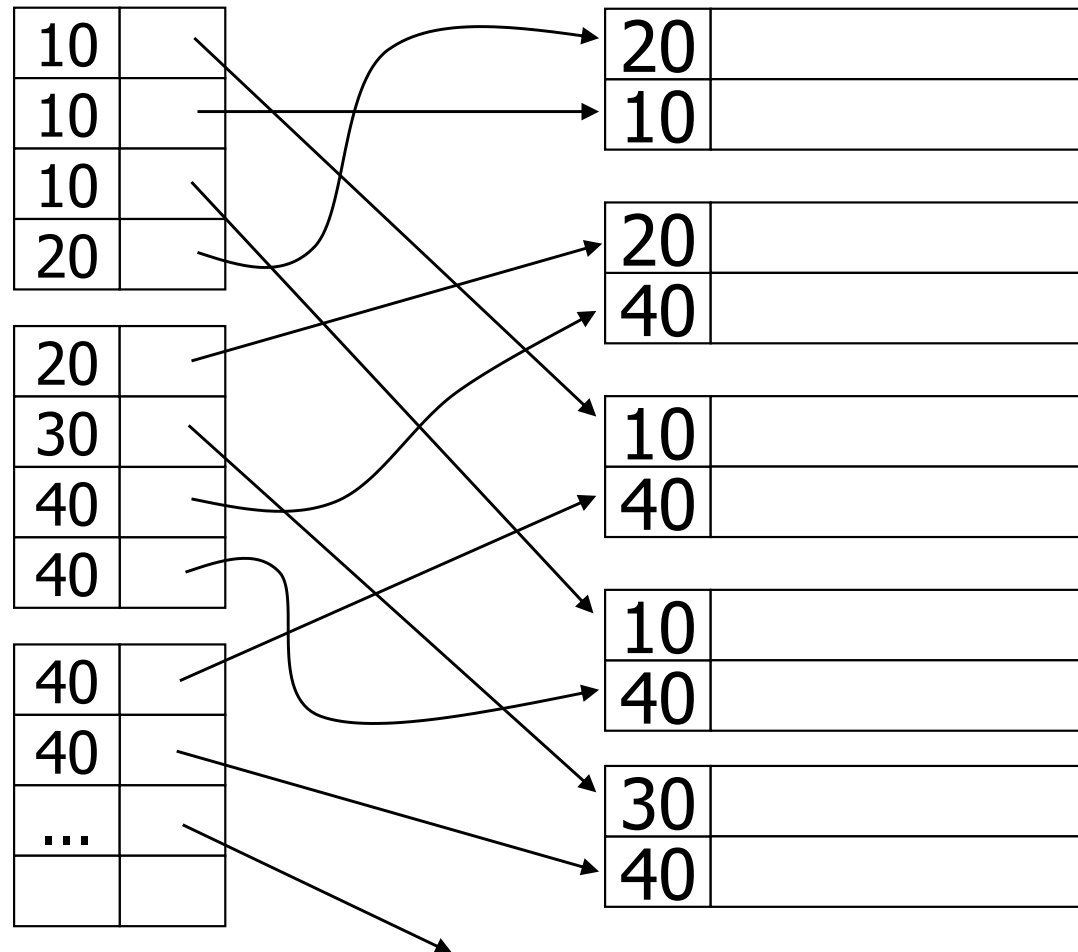
10	
40	

10	
40	

30	
40	

Duplicate values & secondary indexes

one option...



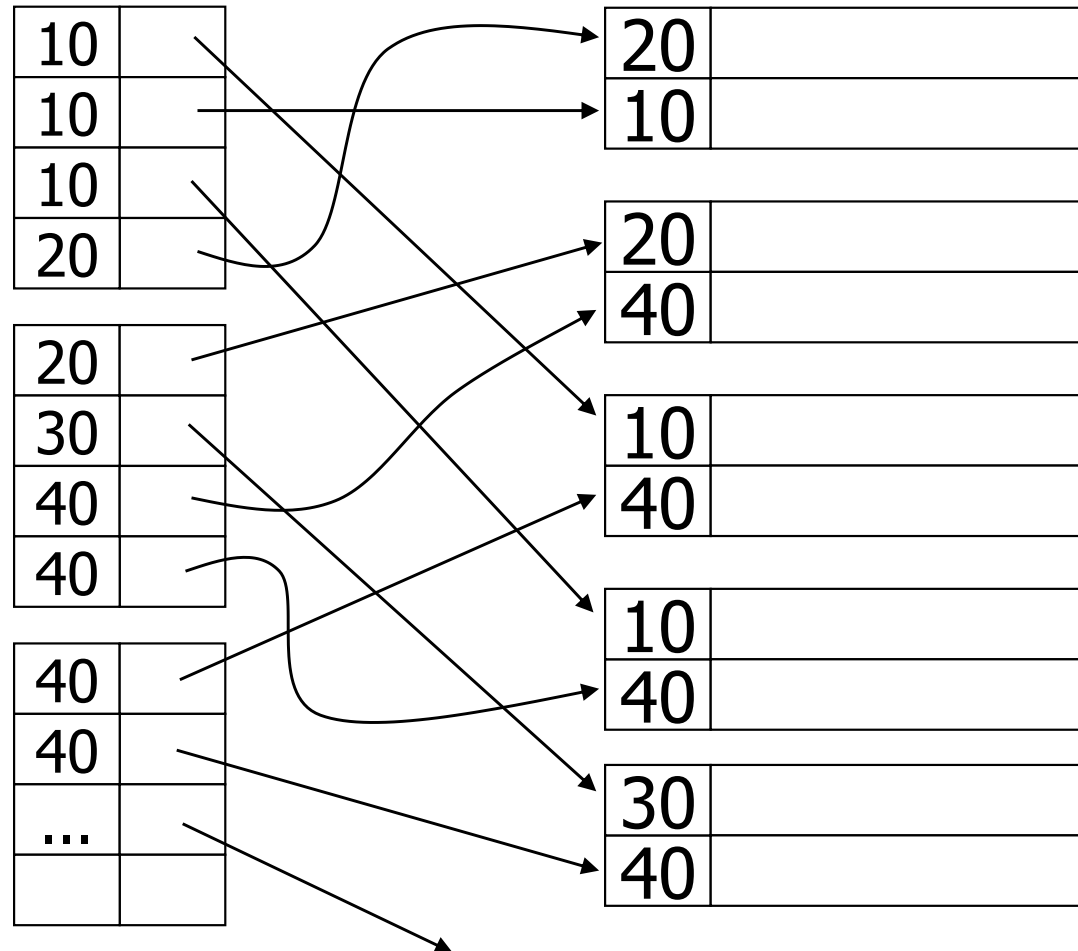
Duplicate values & secondary indexes

one option...

Problem:

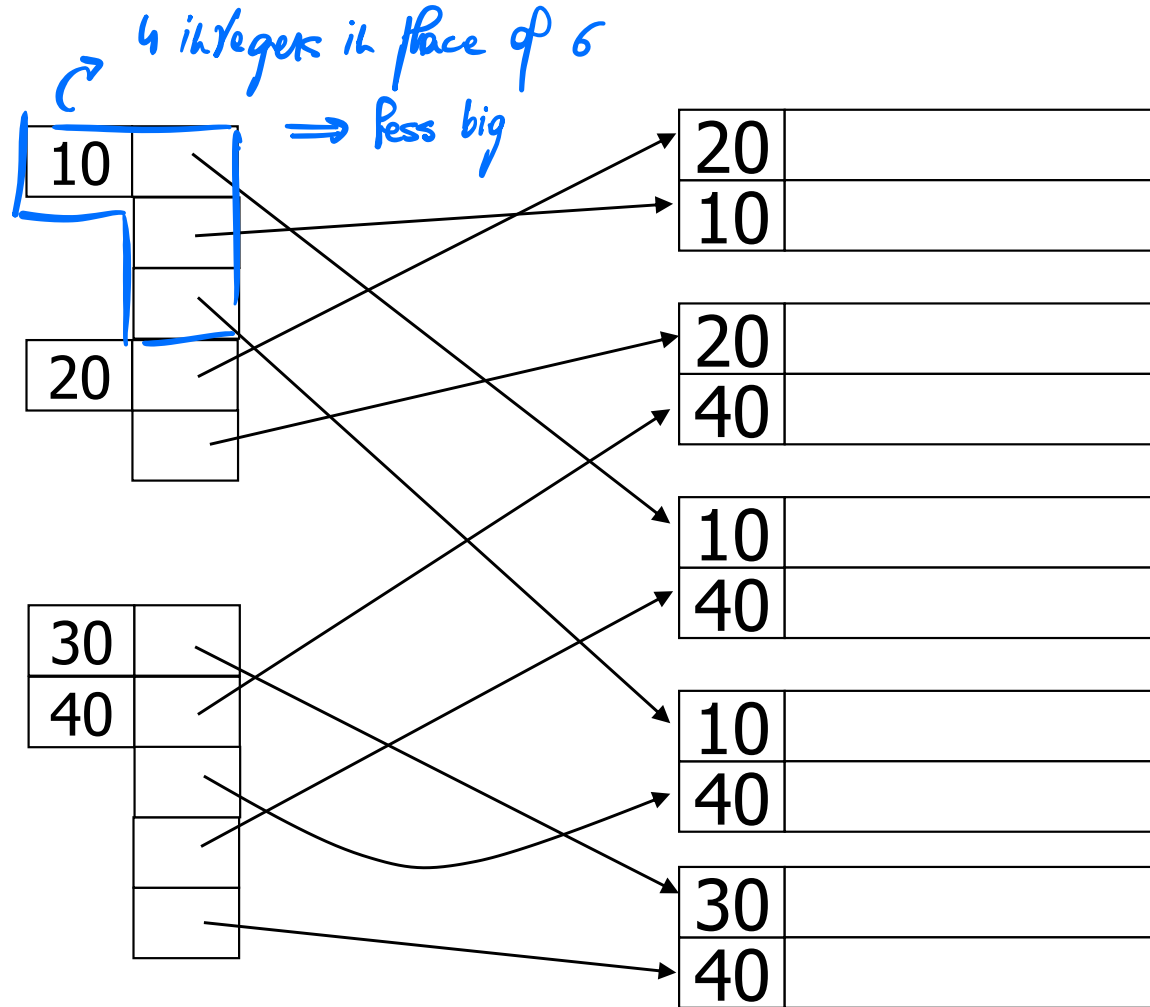
excess overhead!

- disk space
- search time



Duplicate values & secondary indexes

another option...

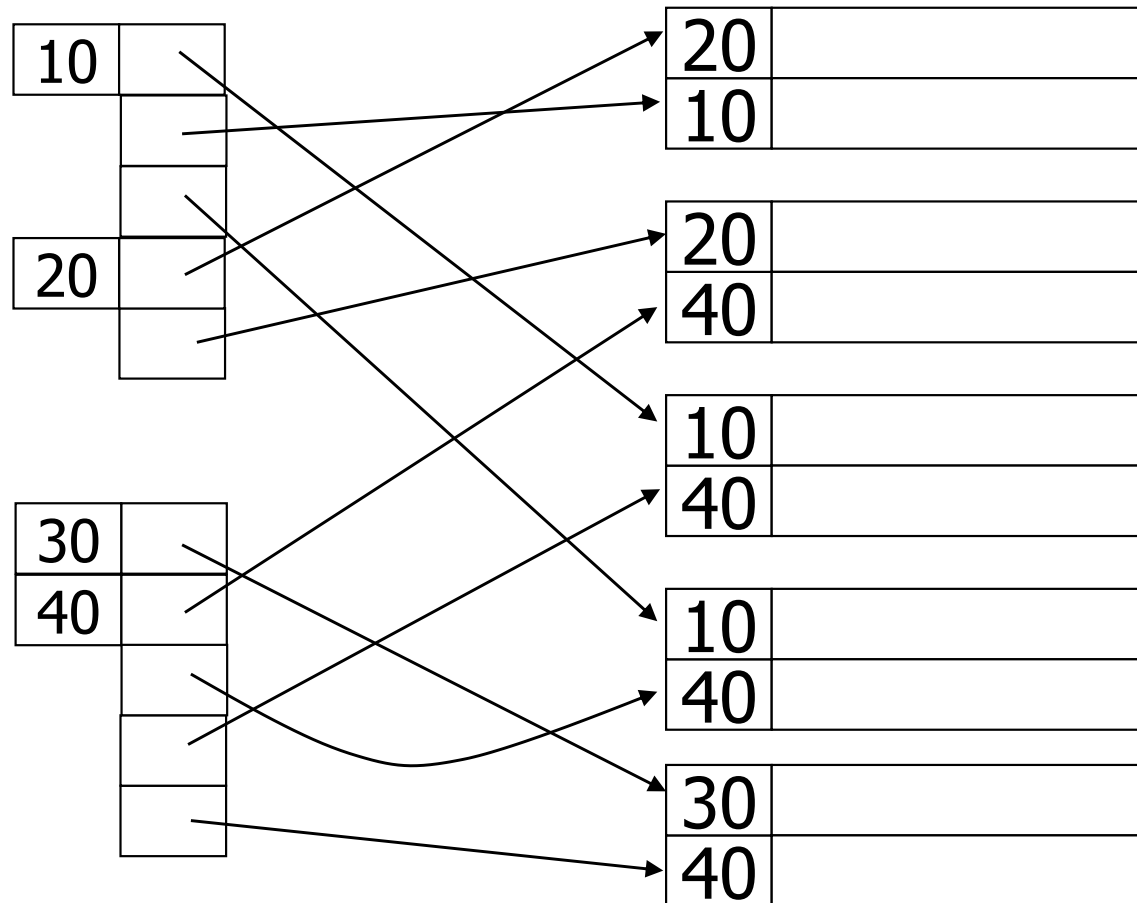


Duplicate values & secondary indexes

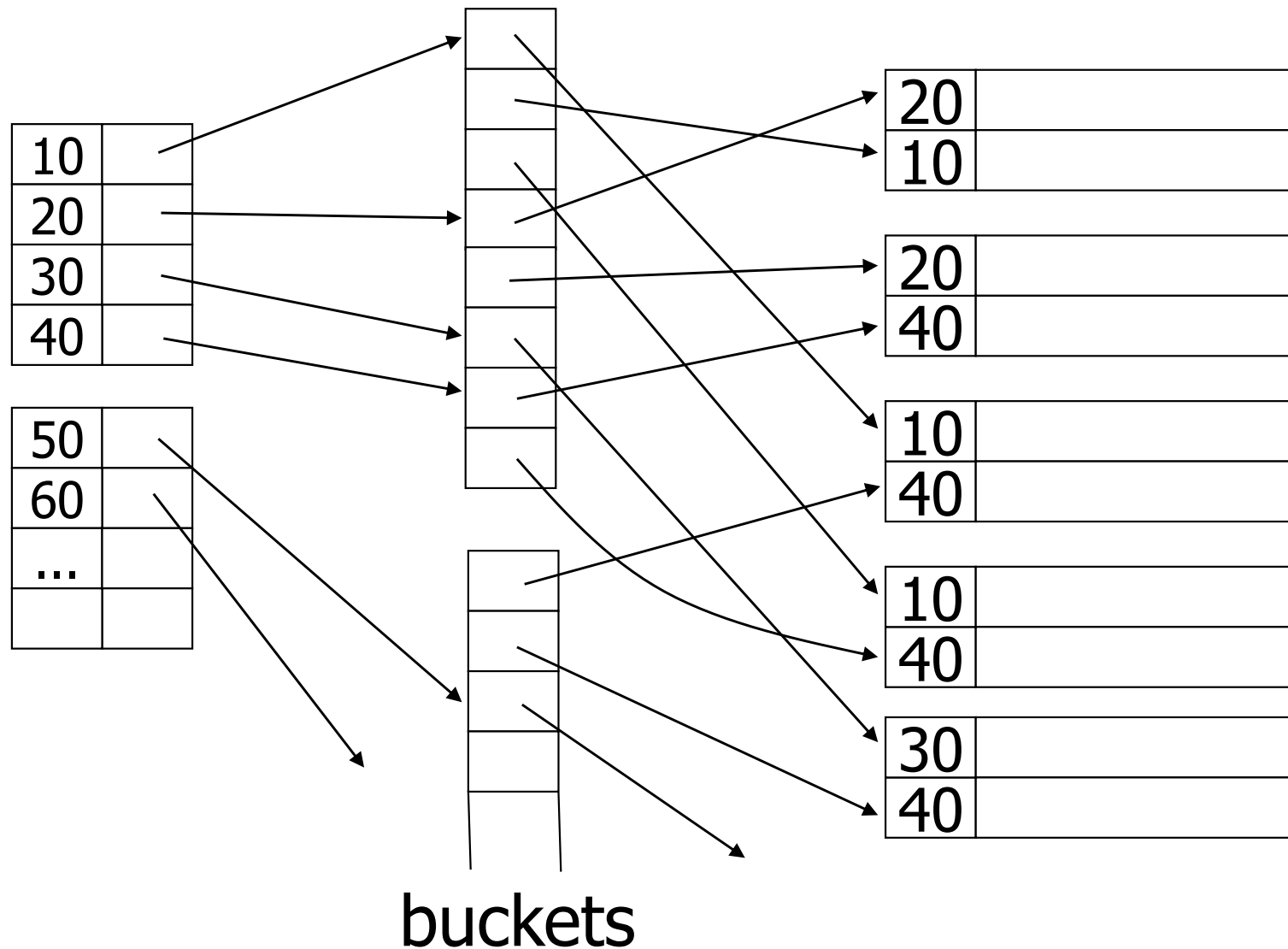
another option...

Problem:
variable size
records in
index!

↳ we need fixed
size records to do
binary search



Duplicate values & secondary indexes



↳ still divided in pages

Why “bucket” idea is useful

Indexes

Records

Name: primary

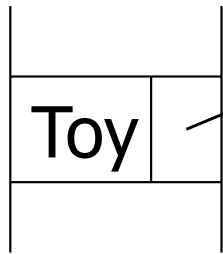
EMP (name,dept,floor,...)

Dept: secondary

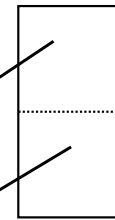
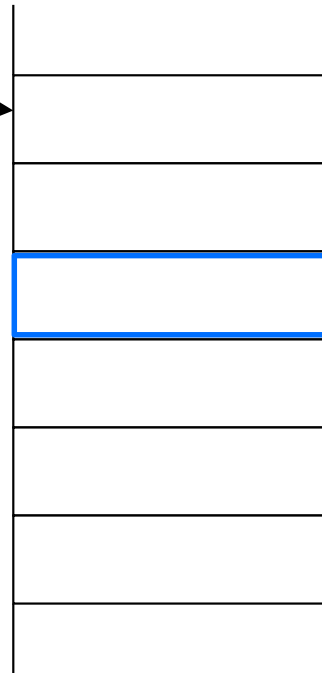
Floor: secondary

Query: Get employees in
(Toy Dept) \wedge (2nd floor)

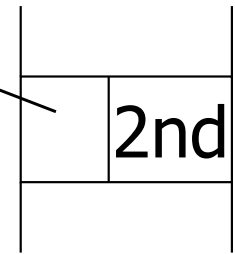
Dept. index



EMP

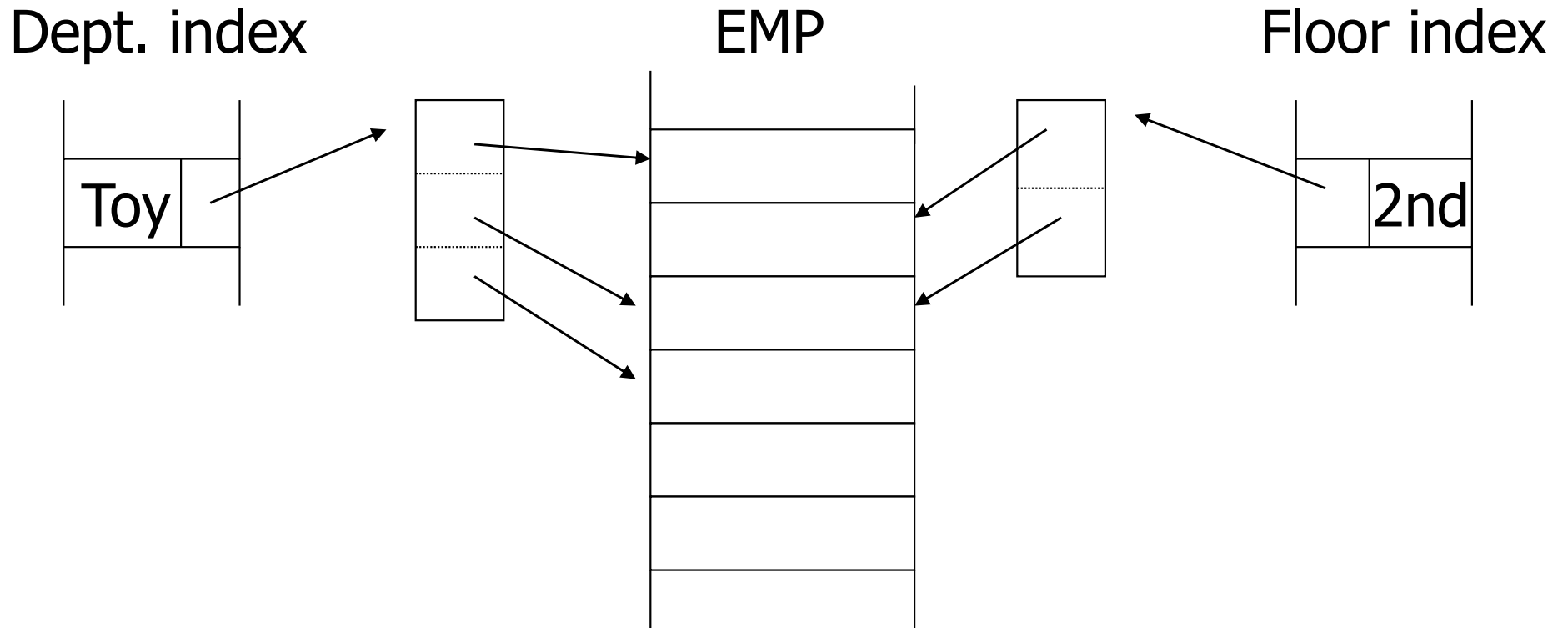


Floor index



because we have the buckets
we can compare the pointers
 \Rightarrow do not need to access the records

Query: Get employees in
(Toy Dept) \wedge (2nd floor)



→ Intersect toy bucket and 2nd Floor bucket to get set of matching EMP's

Summary so far

- Conventional index
 - Basic Ideas: sparse, dense, multi-level...
 - Duplicate Keys
 - Deletion/Insertion
 - Secondary indexes

Note: these are building blocks
not used in practice

Conventional indexes

Advantage:

- Simple
- Index is sequential file
good for scans

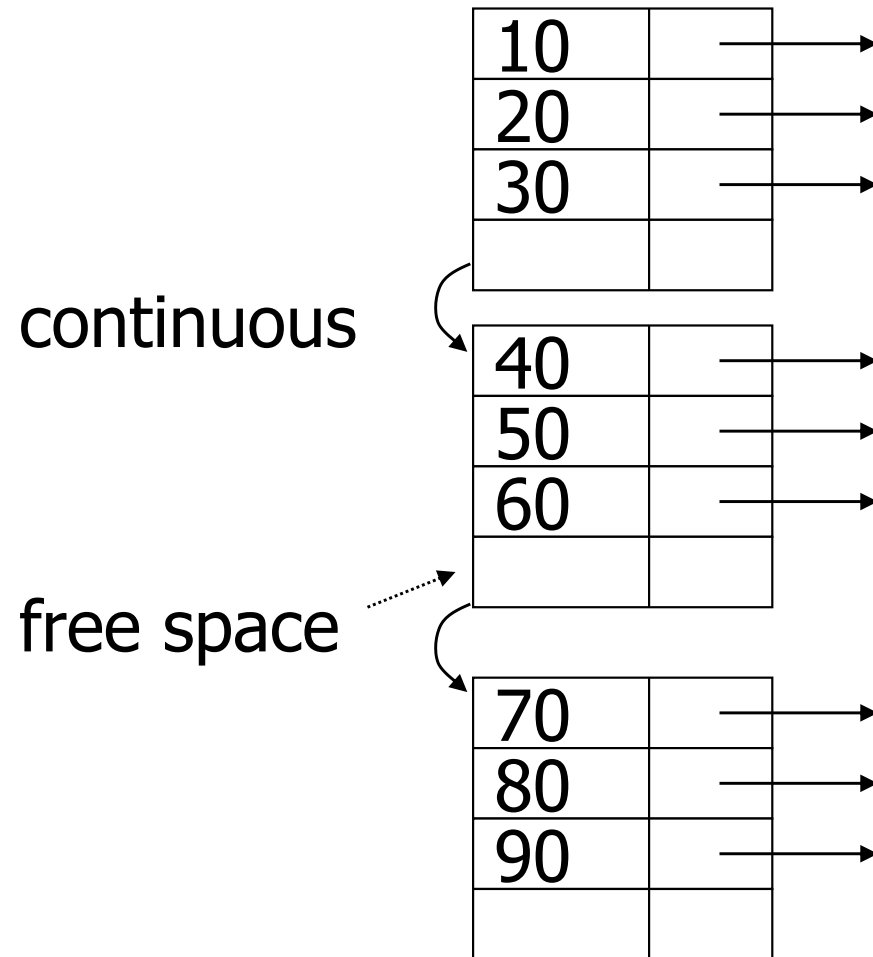
Disadvantage:

- Inserts expensive, or
- Lose sequentiality & balance

↳ the idea is to have indexes that are sorted
conventional indexes were sorted lists of key
⇒ maybe using other data structure would be better
↳ binary trees $\log(n)$ insertion and search

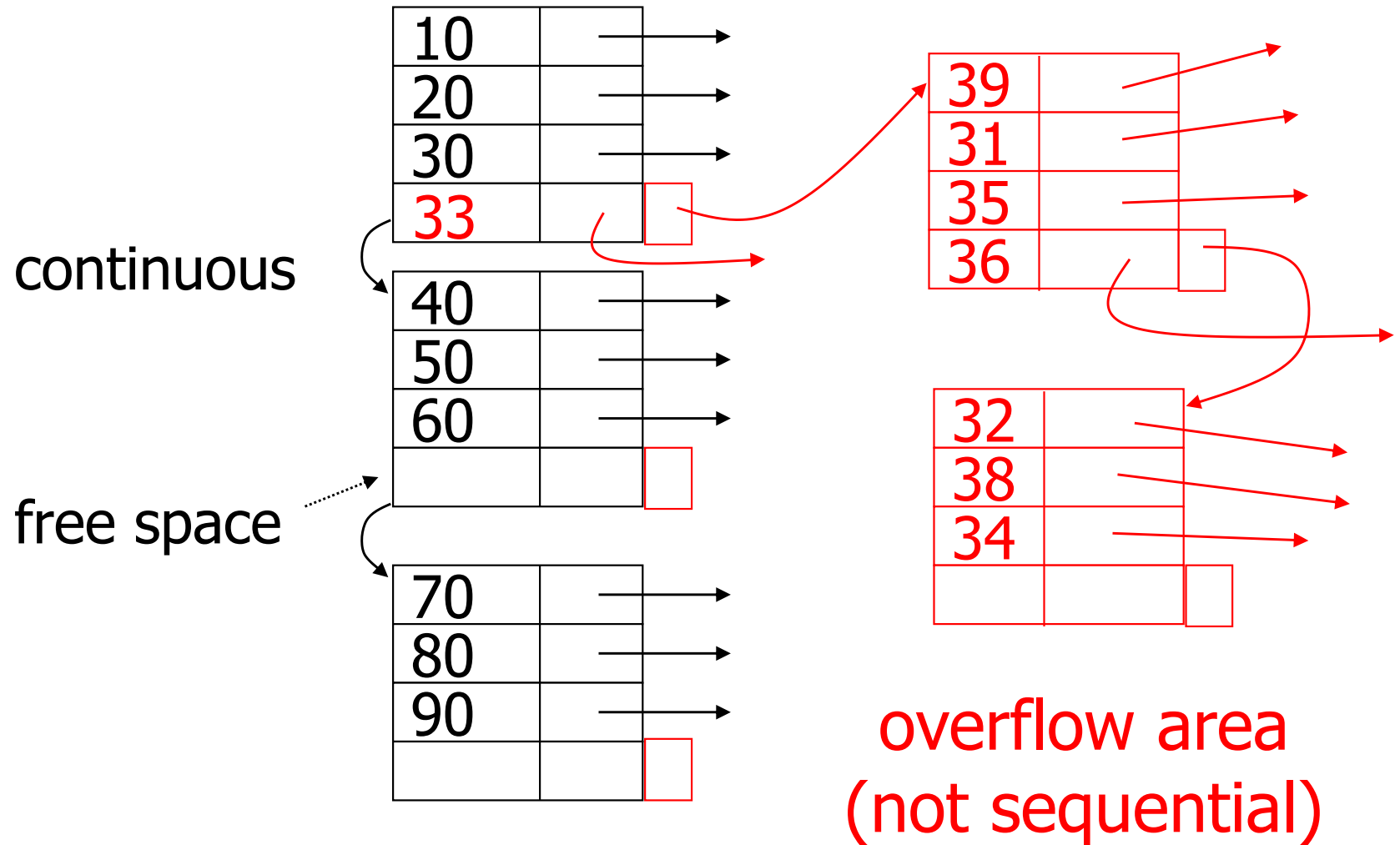
Example

Index (sequential)



Example

Index (sequential)



Outline:

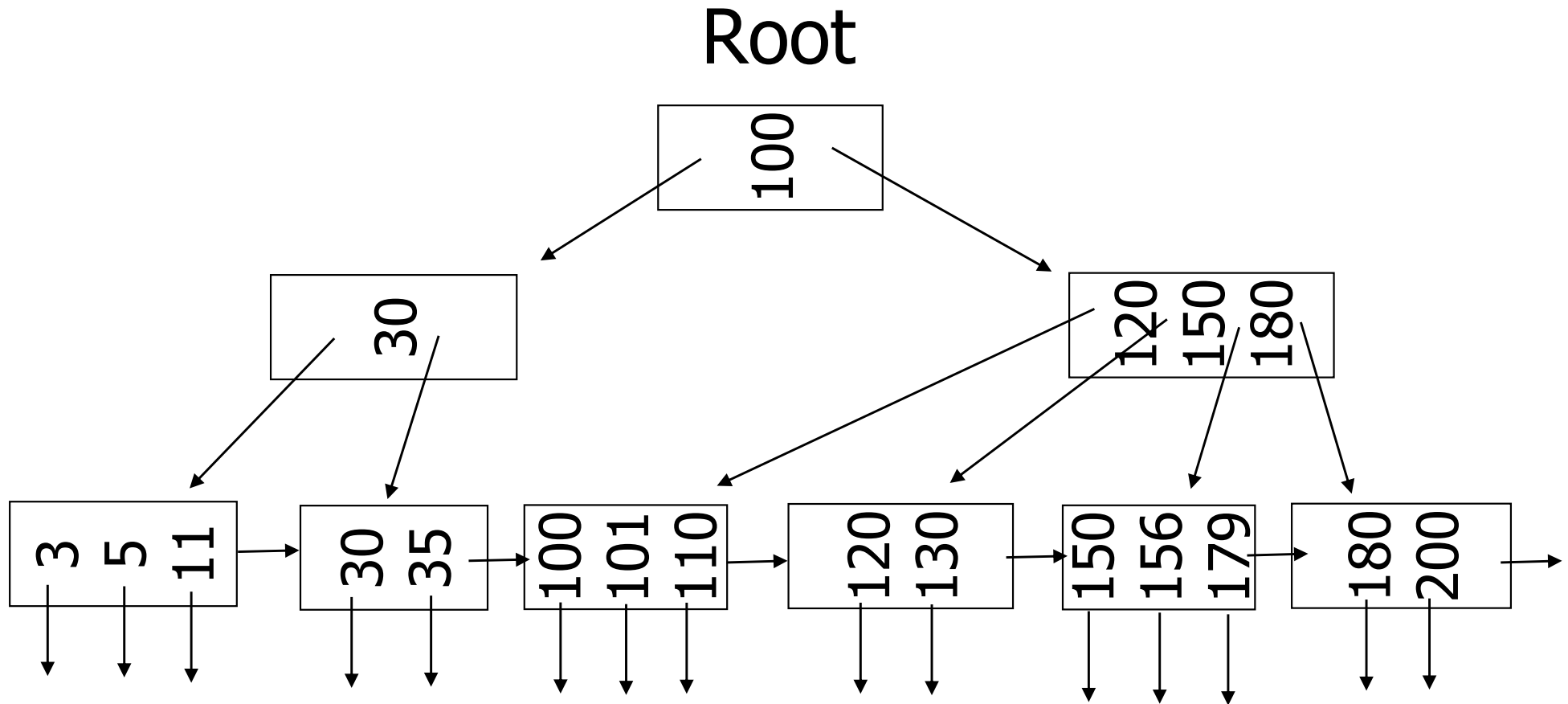
- Conventional indexes
- B-Trees \Rightarrow NEXT
- Hashing schemes

- NEXT: Another type of index
 - Give up on sequentiality of index
 - Try to get “balance”

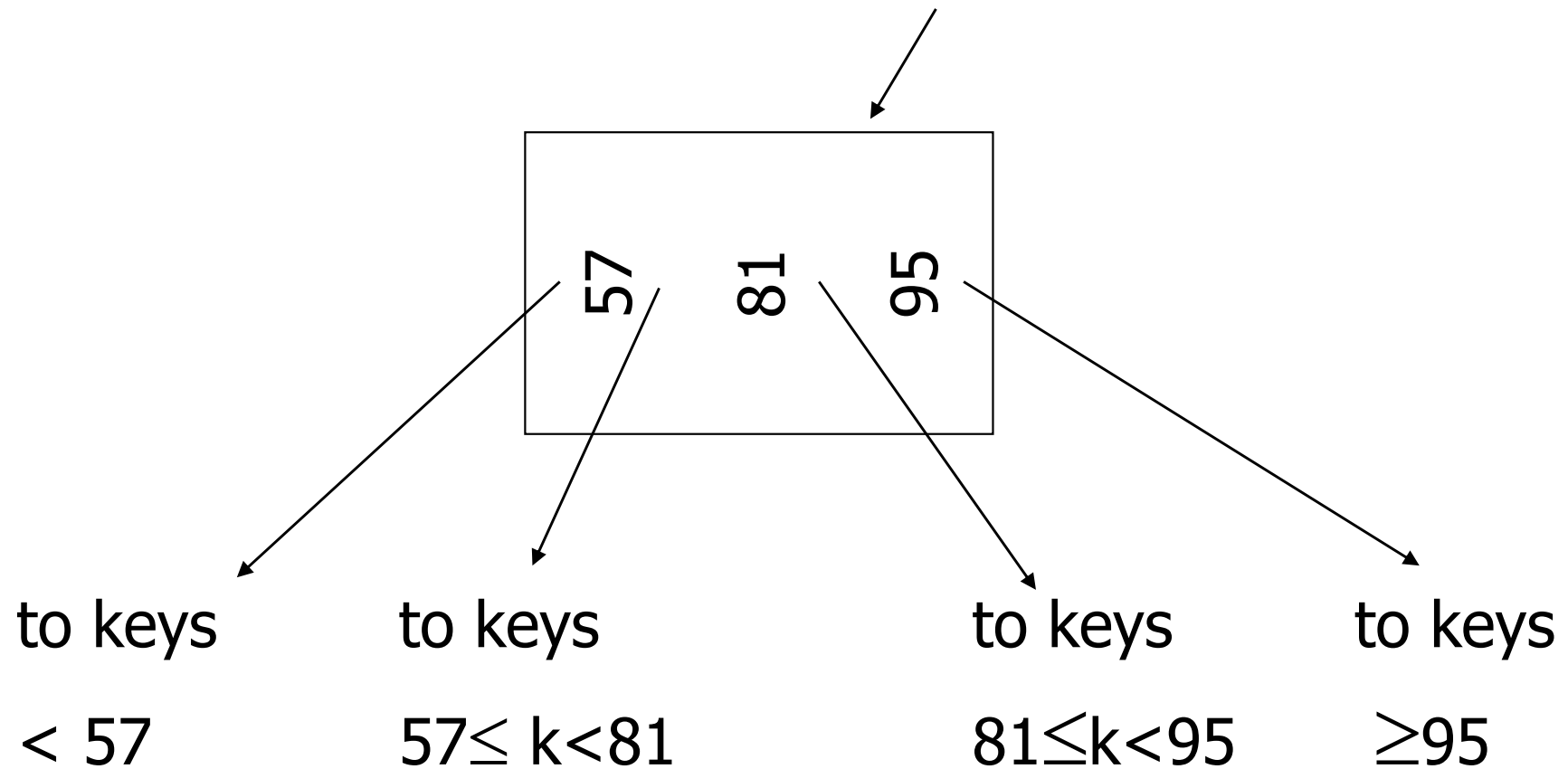
→ unbalanced trees
are far from the
 $\log(n)$ insertion
and search

B+Tree Example

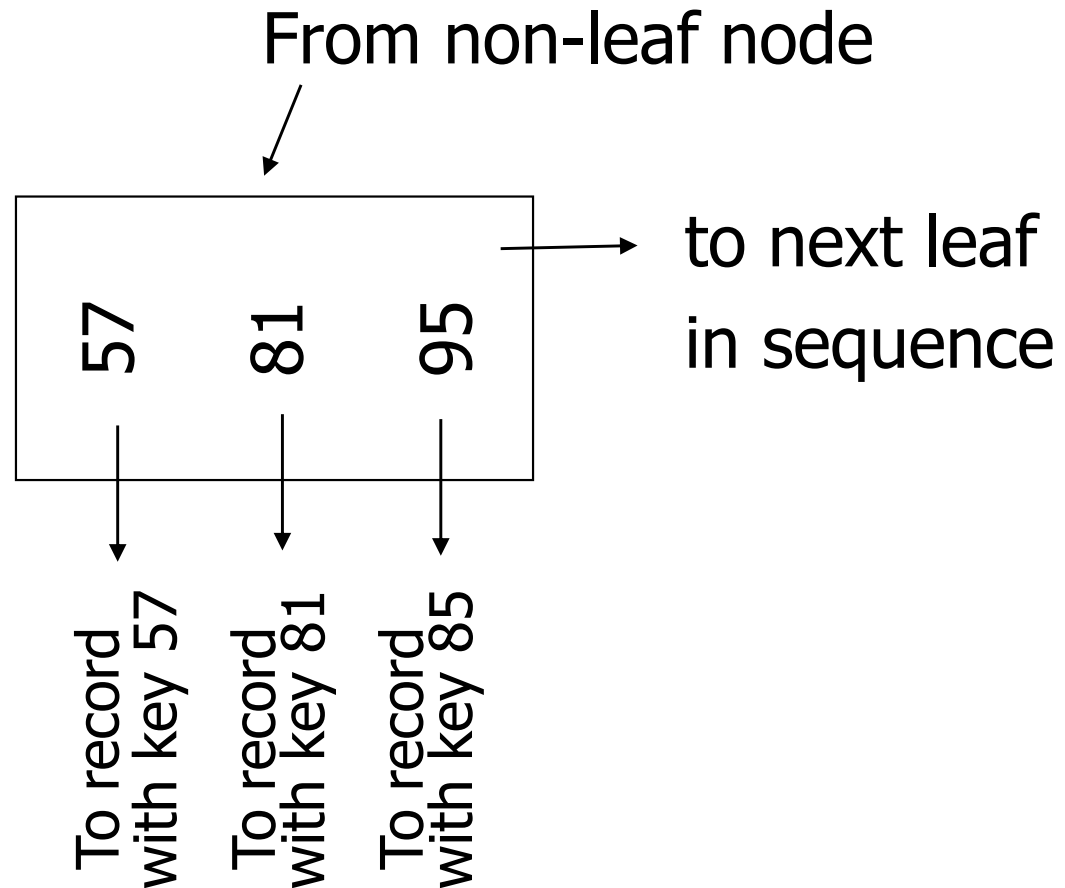
n=3



Sample non-leaf



Sample leaf node:



Size of nodes: $\left\{ \begin{array}{l} n+1 \text{ pointers} \\ n \text{ keys} \end{array} \right.$ (fixed)

Don't want nodes to be too empty

- Use at least

Non-leaf: $\lceil (n+1)/2 \rceil$ pointers

Leaf: $\lfloor (n+1)/2 \rfloor$ pointers to data

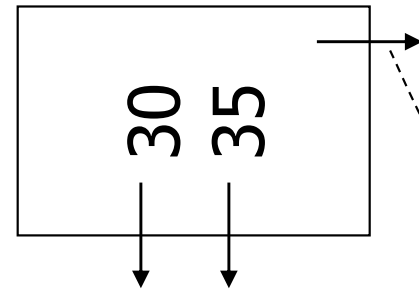
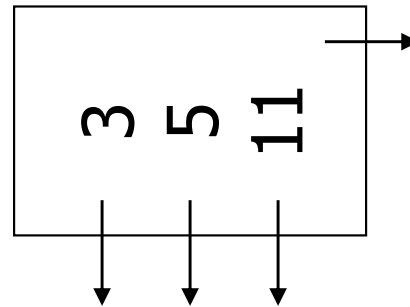
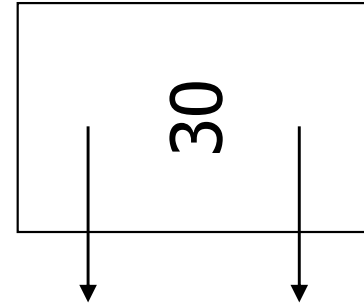
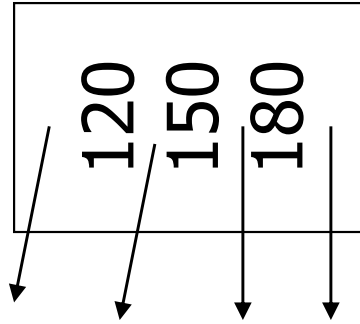
$n=3$

Non-leaf

Full node

min. node

Leaf



counts even if null


B+tree rules tree of order n

- (1) All leaves at same lowest level
(balanced tree)
- (2) Pointers in leaves point to records
except for "sequence pointer"

↳ point to the next leaf

(3) Number of pointers/keys for B+tree

	Max ptrs	Max keys	Min ptrs→data	Min keys
Non-leaf (non-root)	$n+1$	n	$\lceil (n+1)/2 \rceil$	$\lceil (n+1)/2 \rceil - 1$
Leaf (non-root)	$n+1$	n	$\lfloor (n+1)/2 \rfloor$	$\lfloor (n+1)/2 \rfloor$
Root	$n+1$	n	1	1



rule broken
only for the root

Insert into B+tree

→ whole idea of using
a binary tree
↳ for cost insertion

(a) simple case

– space available in leaf

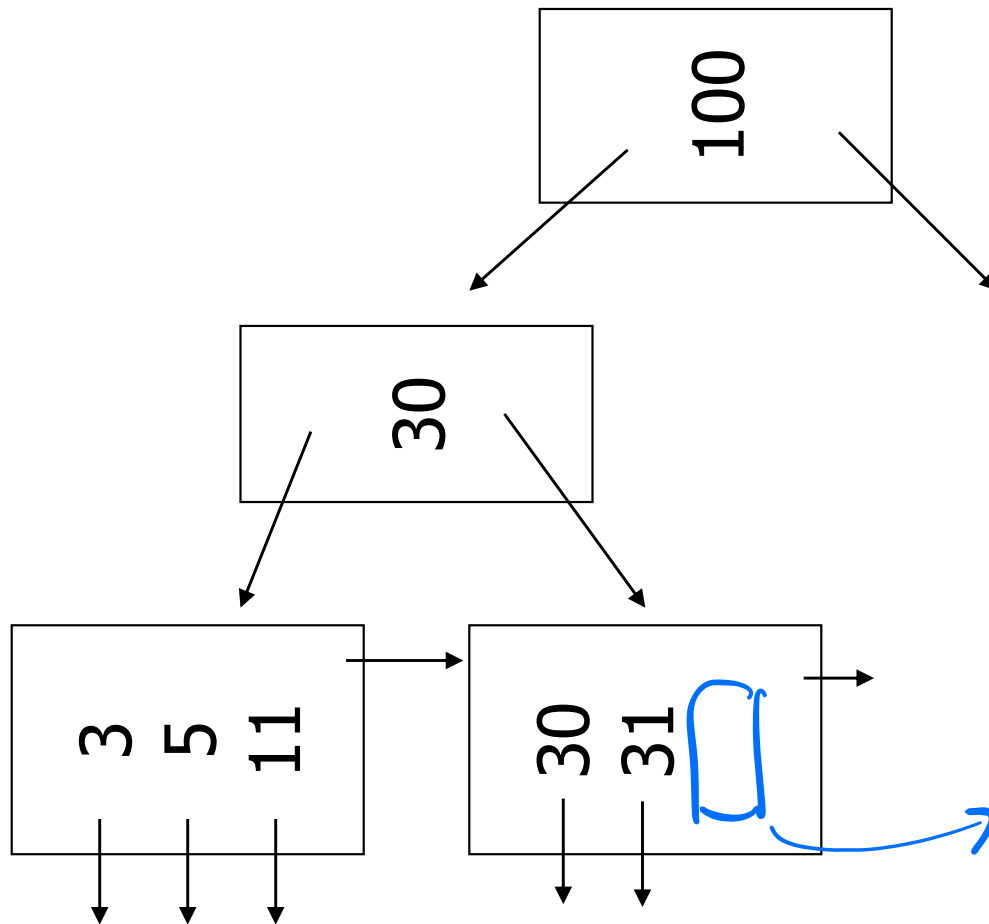
(b) leaf overflow

(c) non-leaf overflow

(d) new root

(a) Insert key = 32

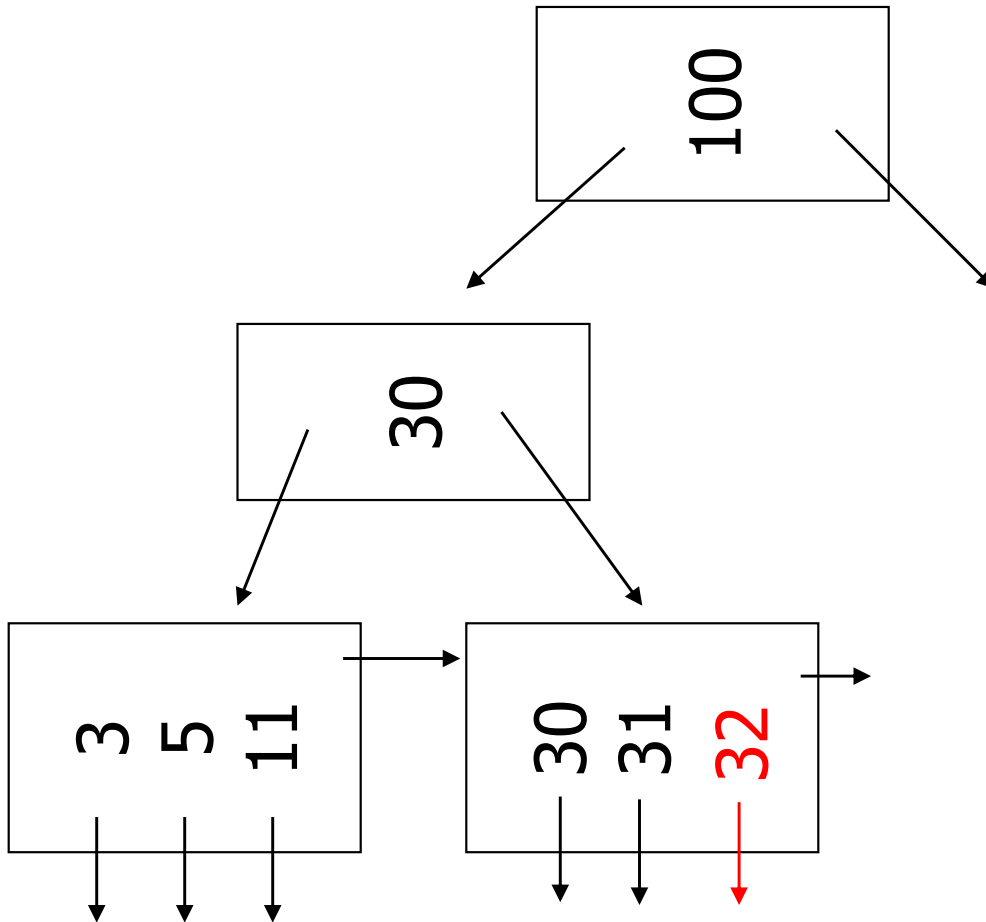
n=3



there is space
we use it

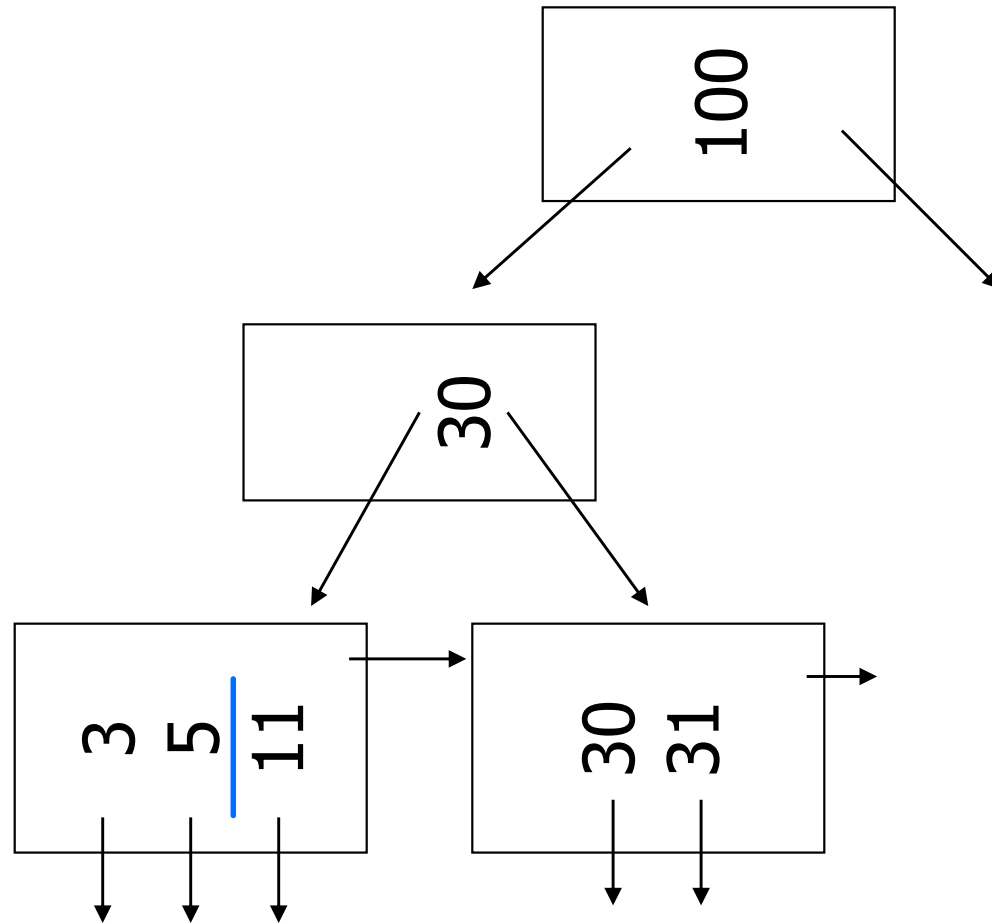
(a) Insert key = 32

n=3



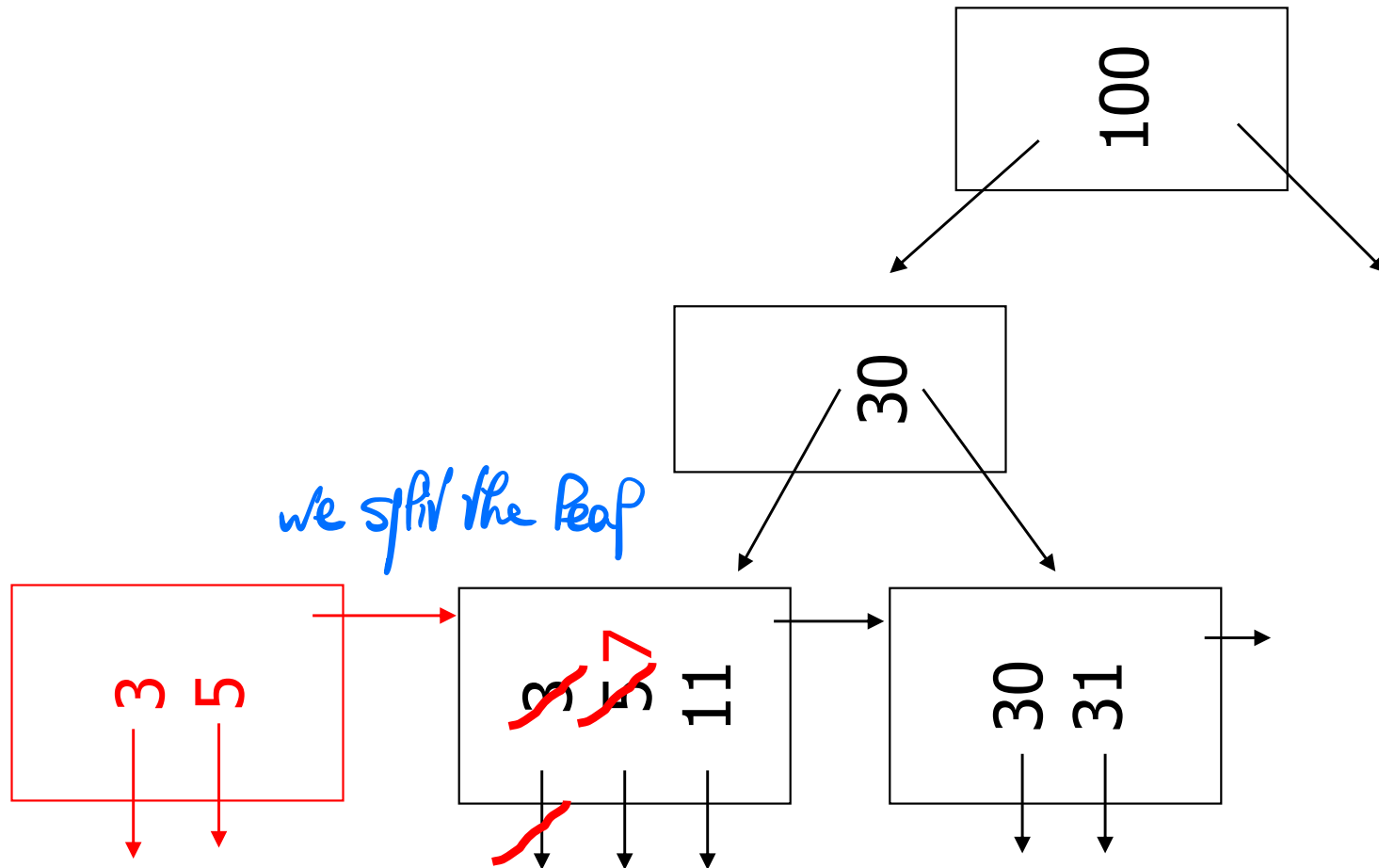
(a) Insert key = 7

n=3



(a) Insert key = 7

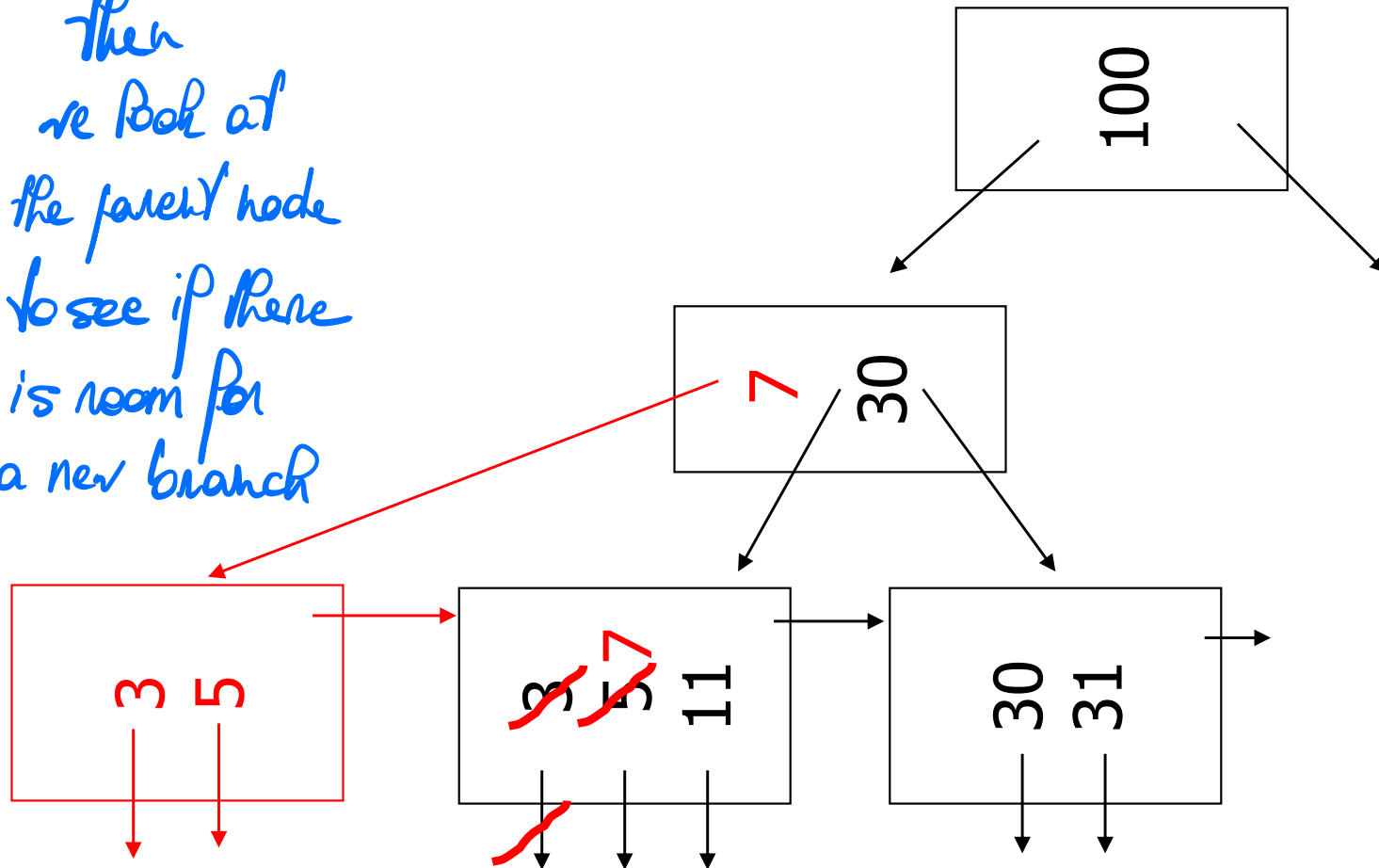
n=3



(a) Insert key = 7

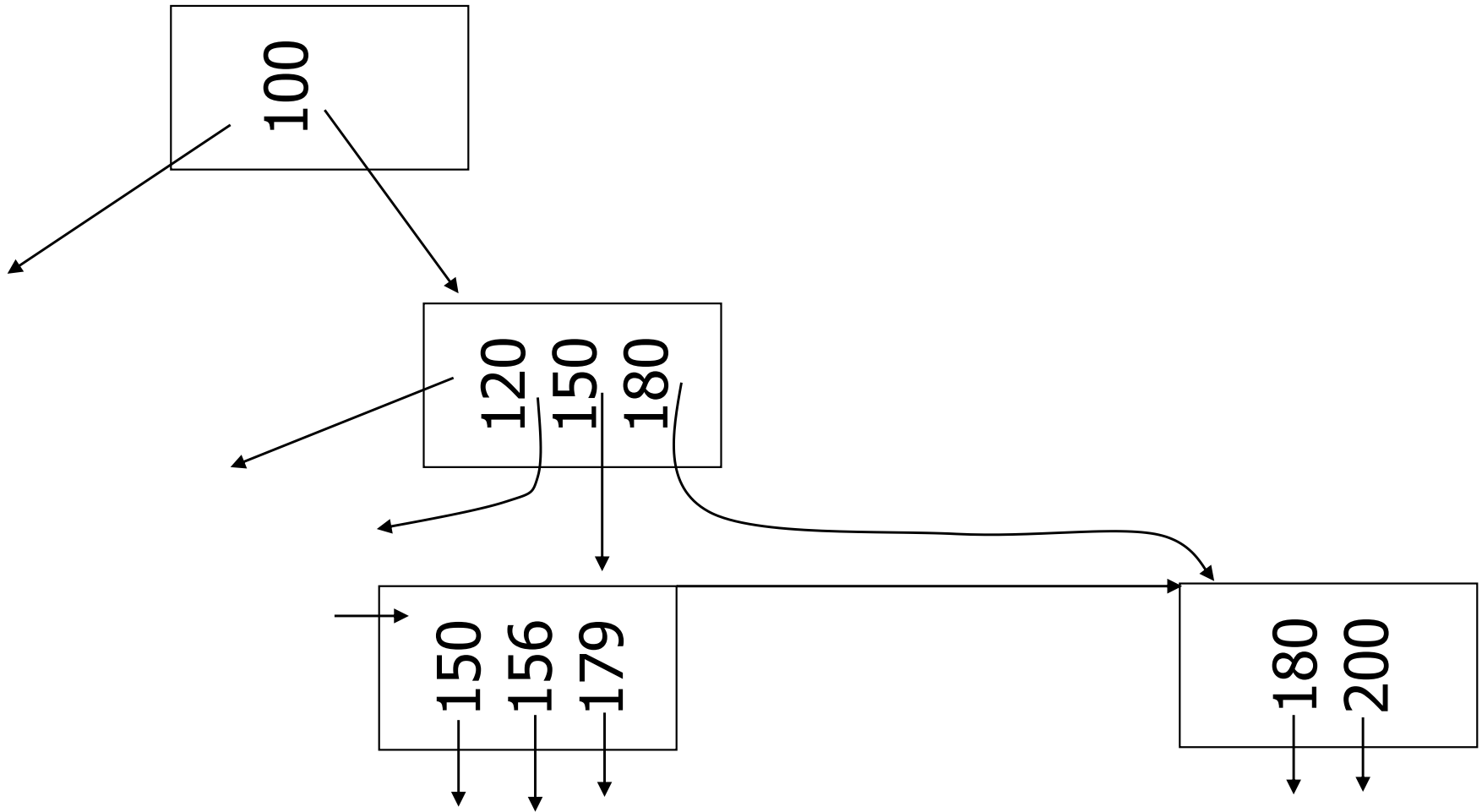
n=3

then
we look at
the parent node
to see if there
is room for
a new branch



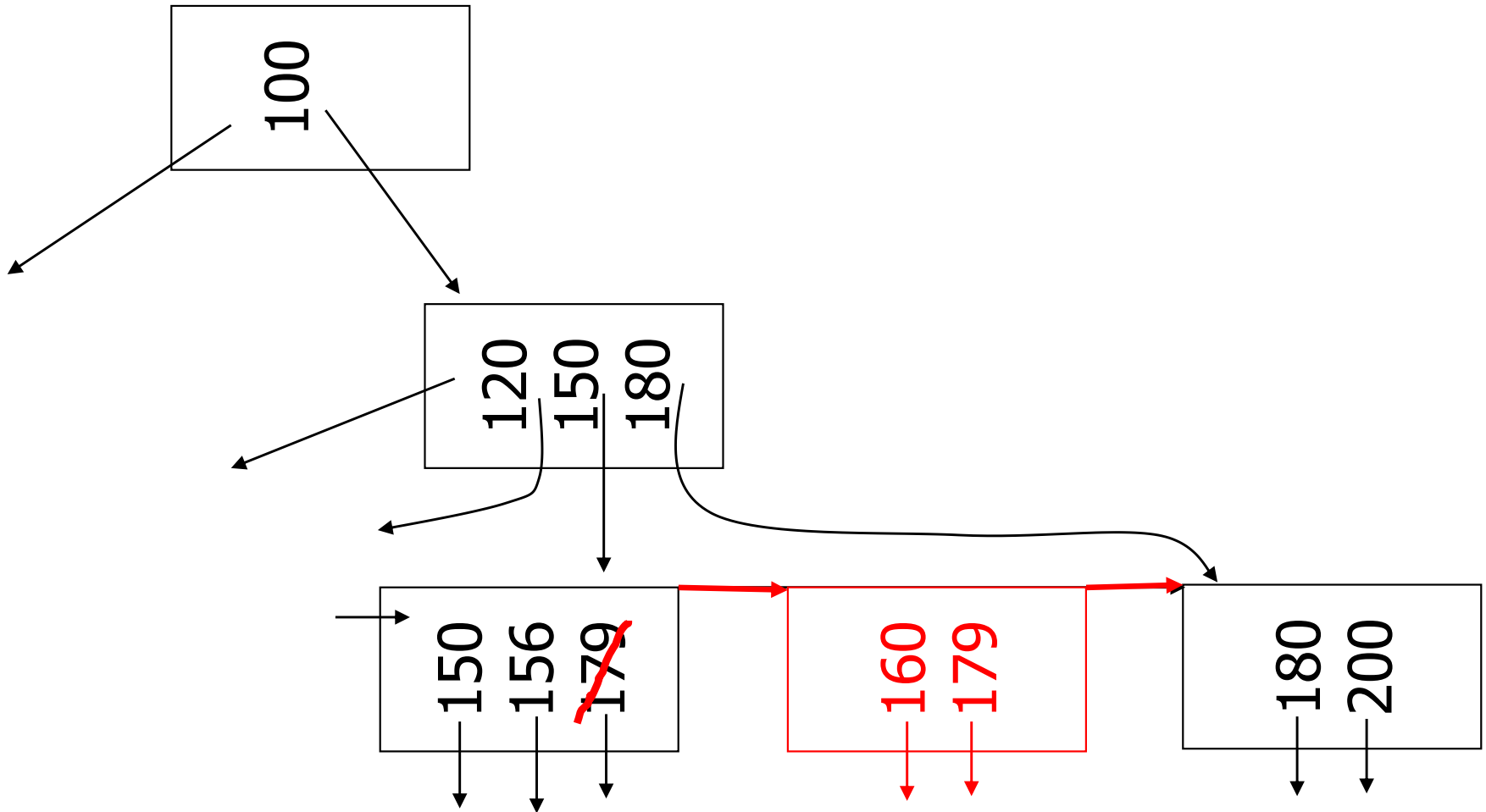
(c) Insert key = 160

n=3



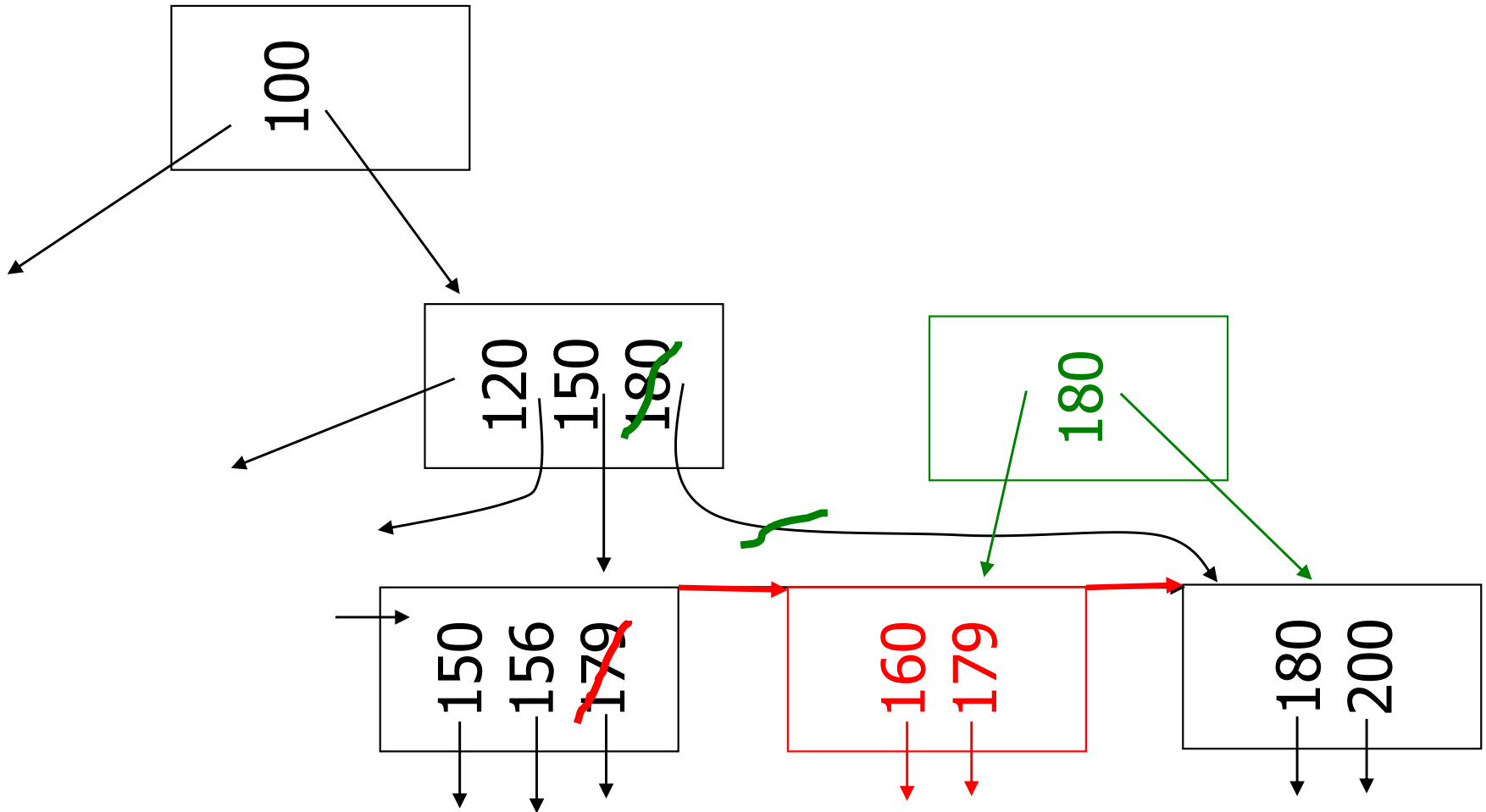
(c) Insert key = 160

n=3



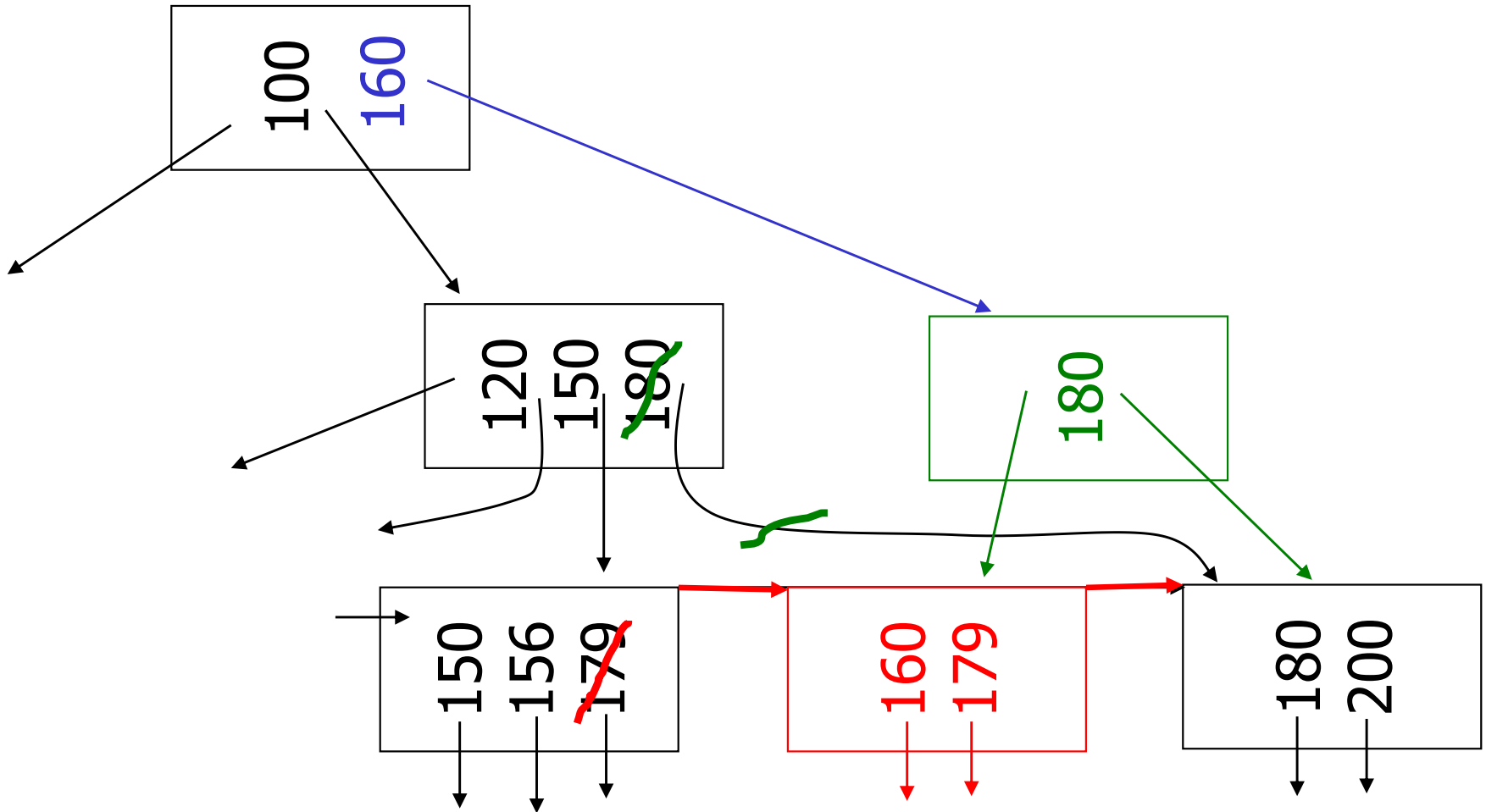
(c) Insert key = 160

n=3



(c) Insert key = 160

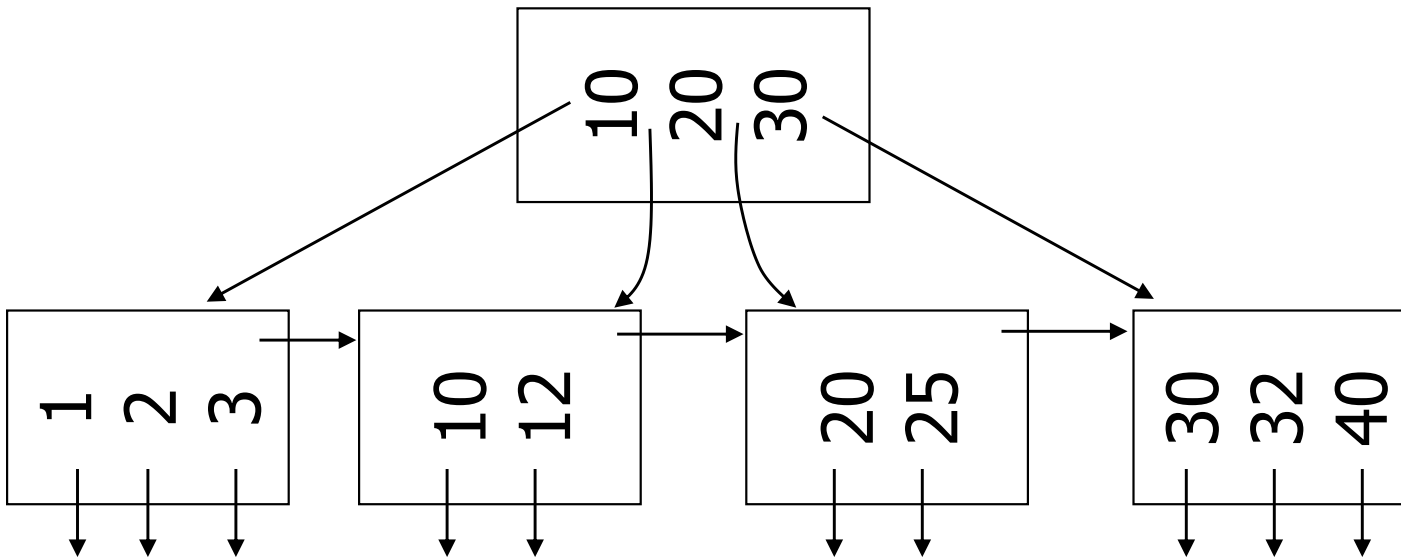
n=3



(d) New root, insert 45

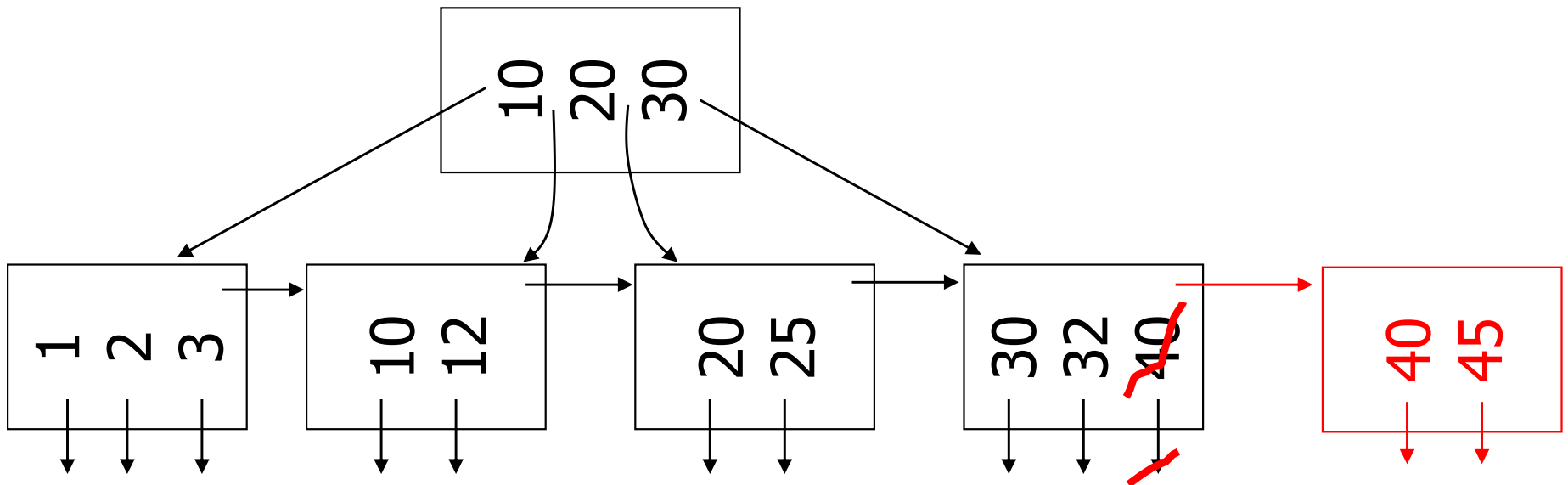
n=3

root is full



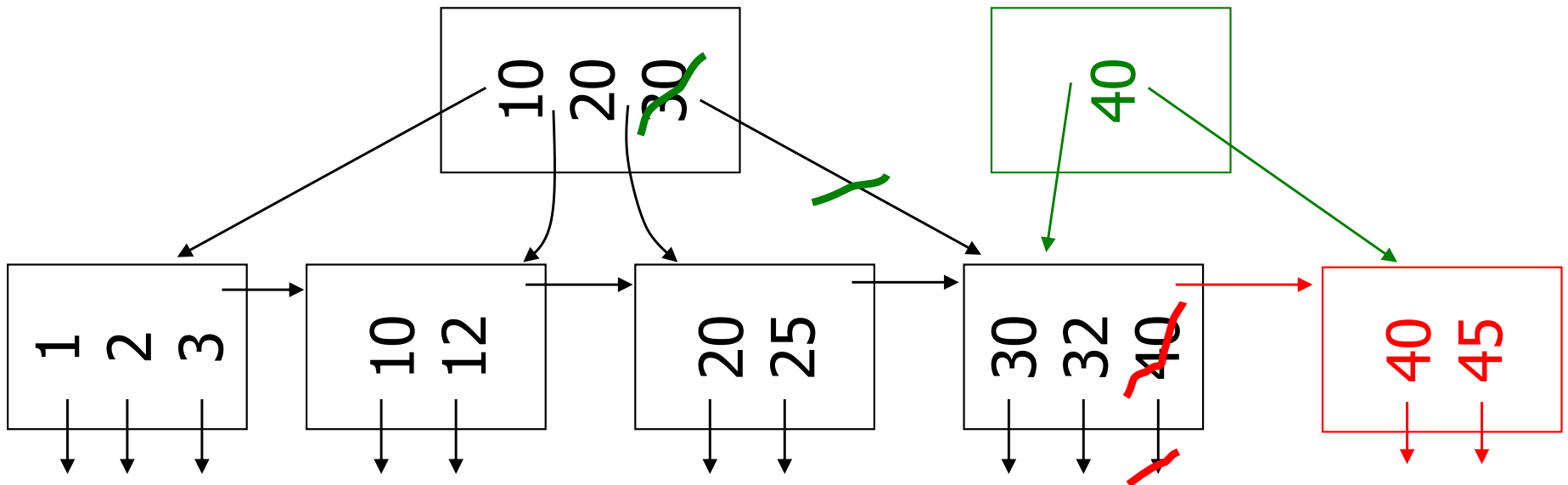
(d) New root, insert 45

n=3



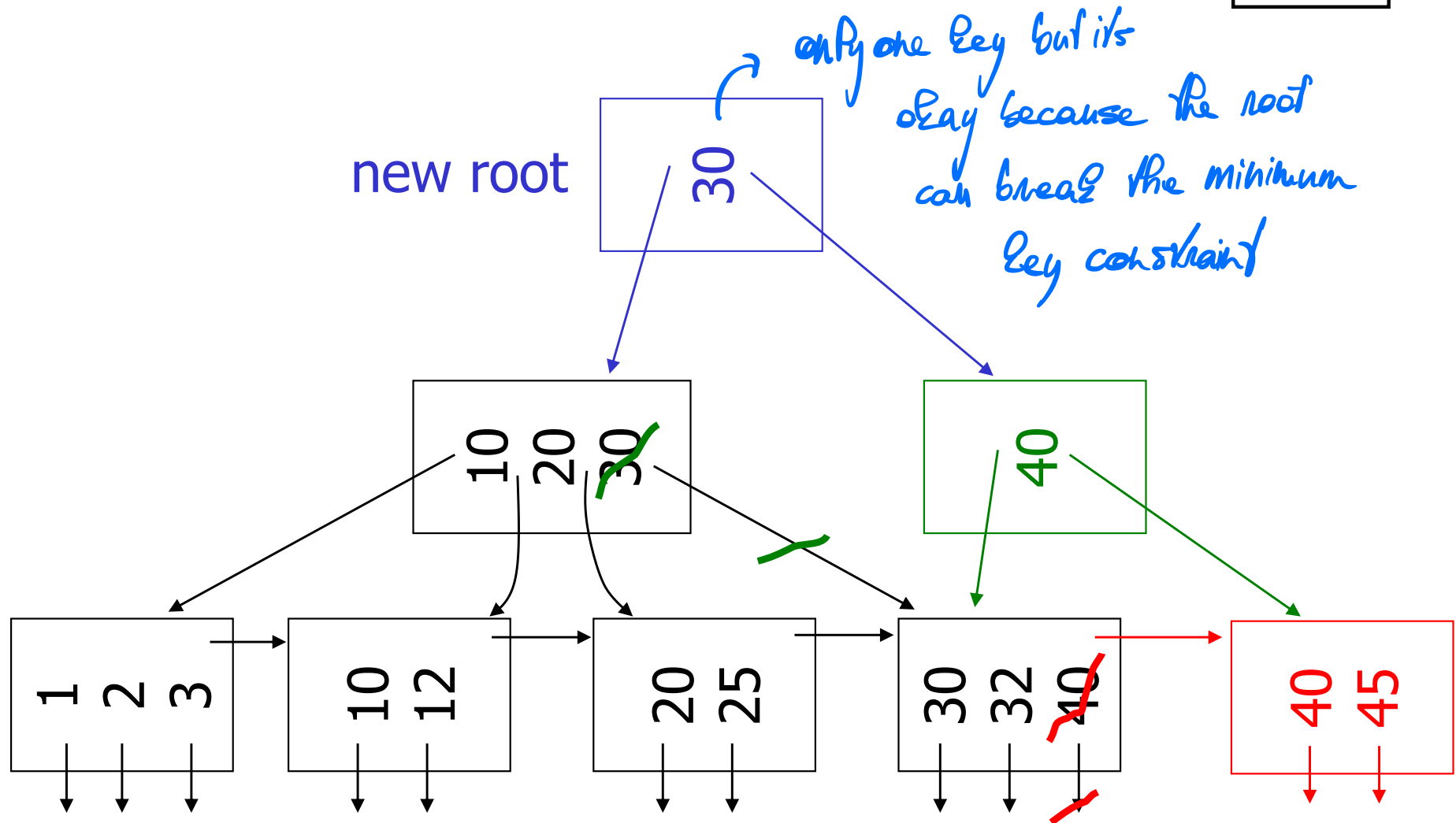
(d) New root, insert 45

n=3



(d) New root, insert 45

n=3



Deletion from B+tree

go fast through it
↳ not implemented
in reality because
too hard and too
expensive

(a) Simple case - no example

(b) Coalesce with neighbor (sibling)

(c) Re-distribute keys

(d) Cases (b) or (c) at non-leaf

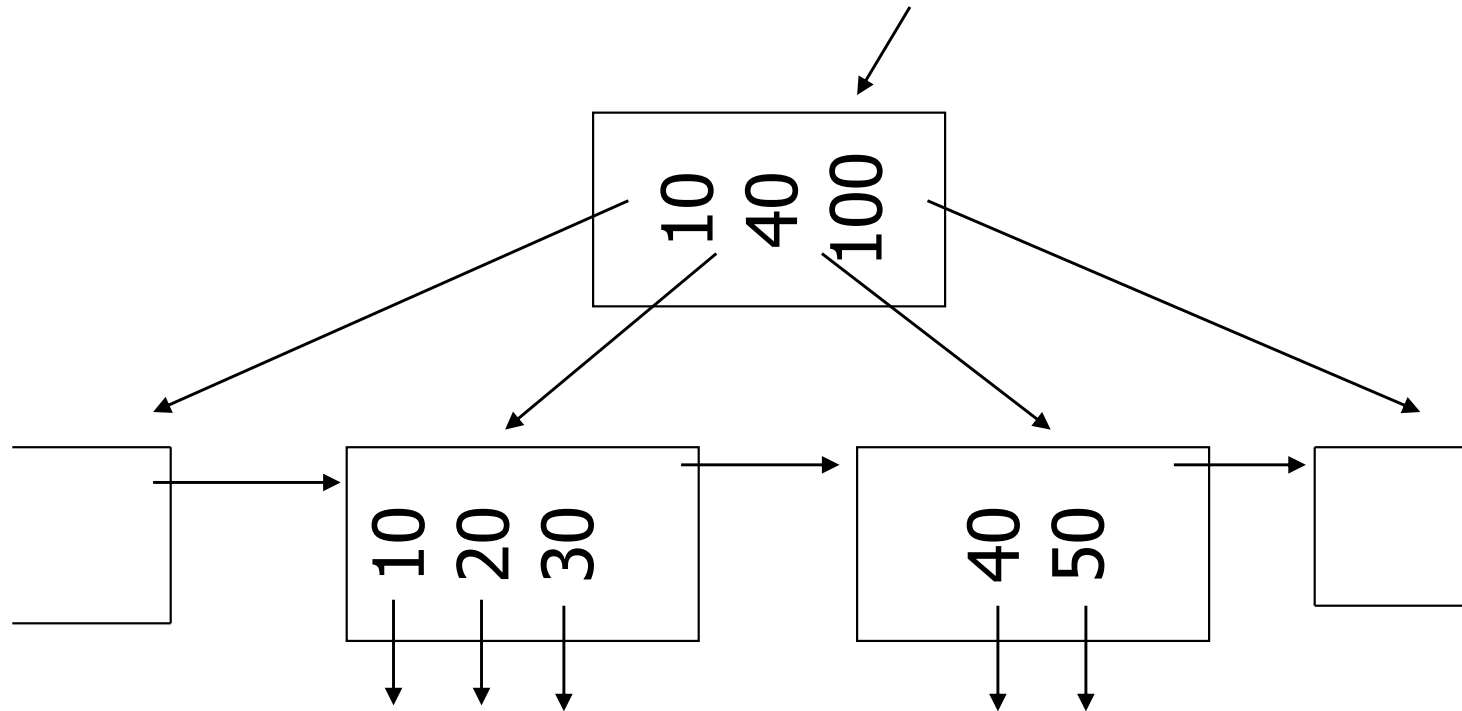
when there are
too many fragments
we relabel the tree

we only mark
the values as
deleted and go
next

(b) Coalesce with sibling

– Delete 50

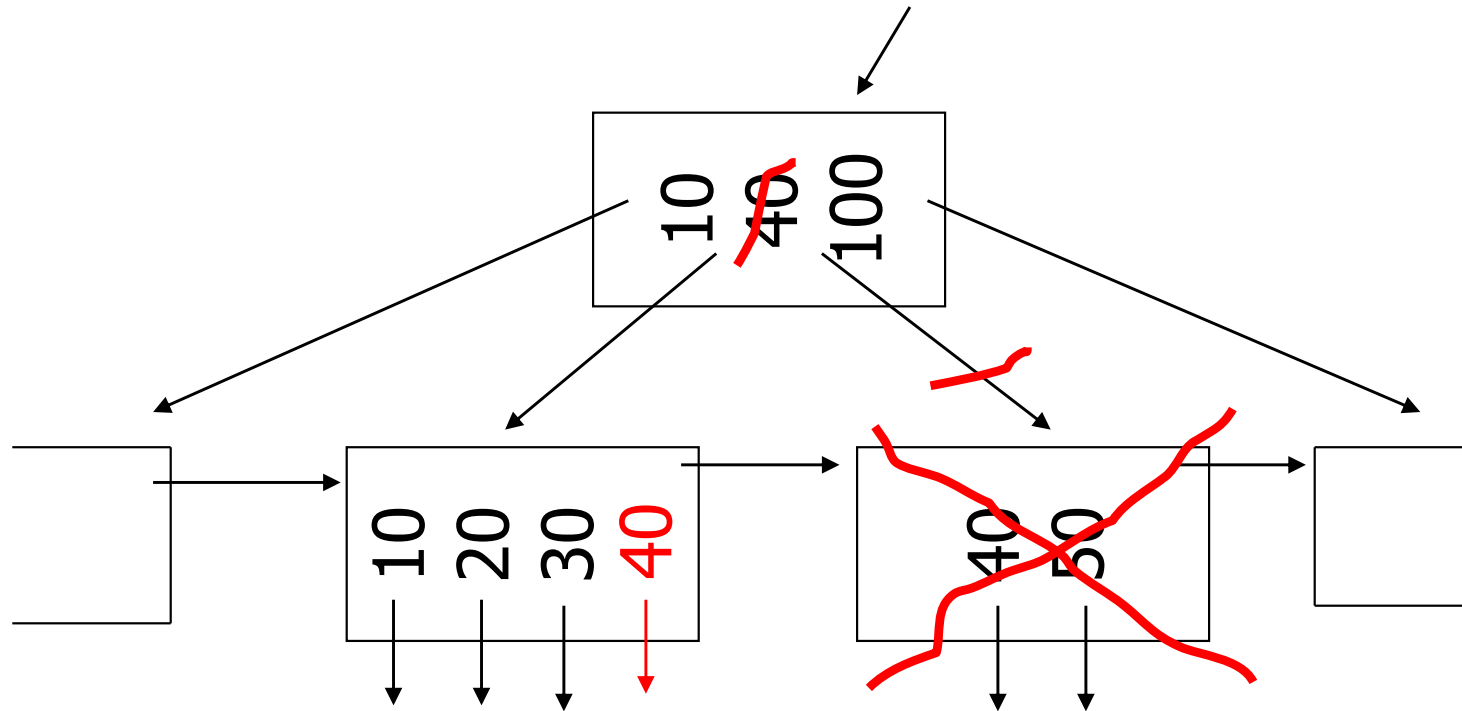
$n=4$



(b) Coalesce with sibling

– Delete 50

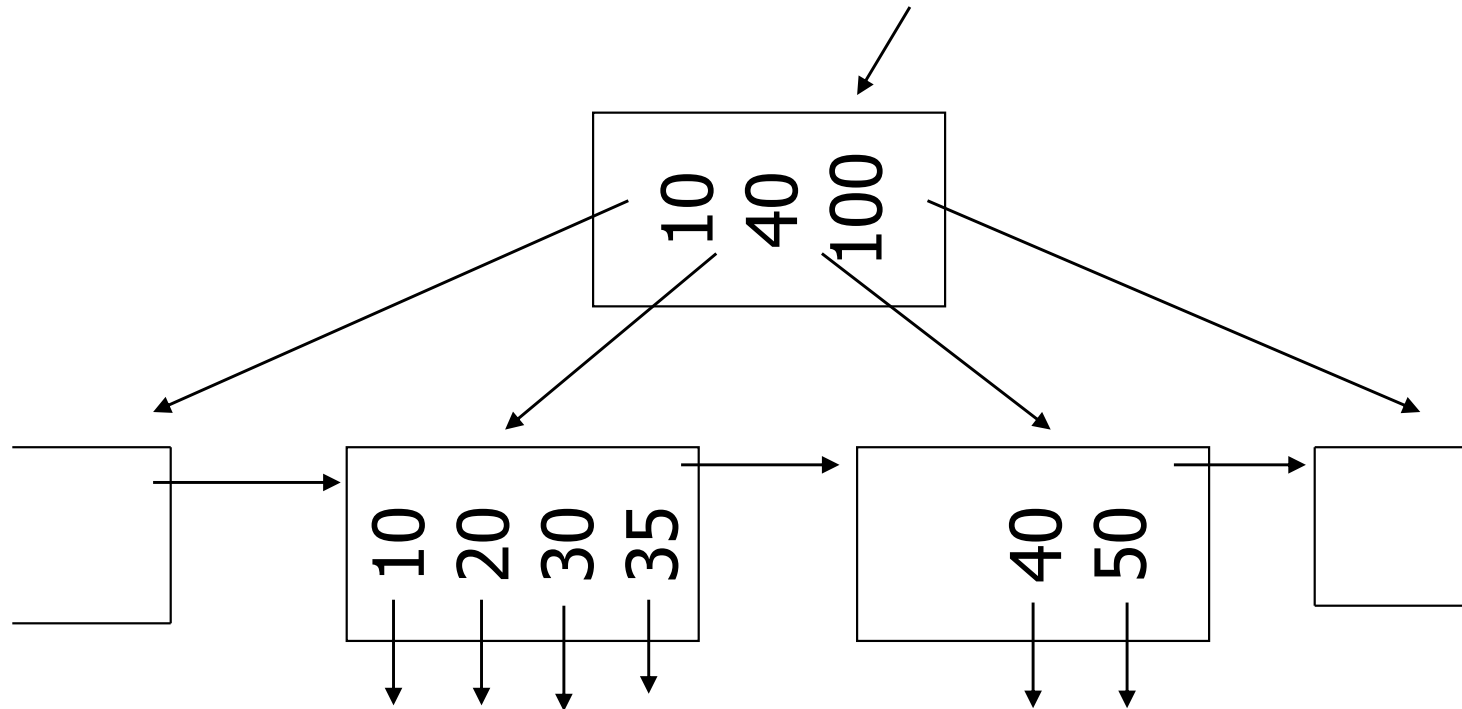
n=4



(c) Redistribute keys

– Delete 50

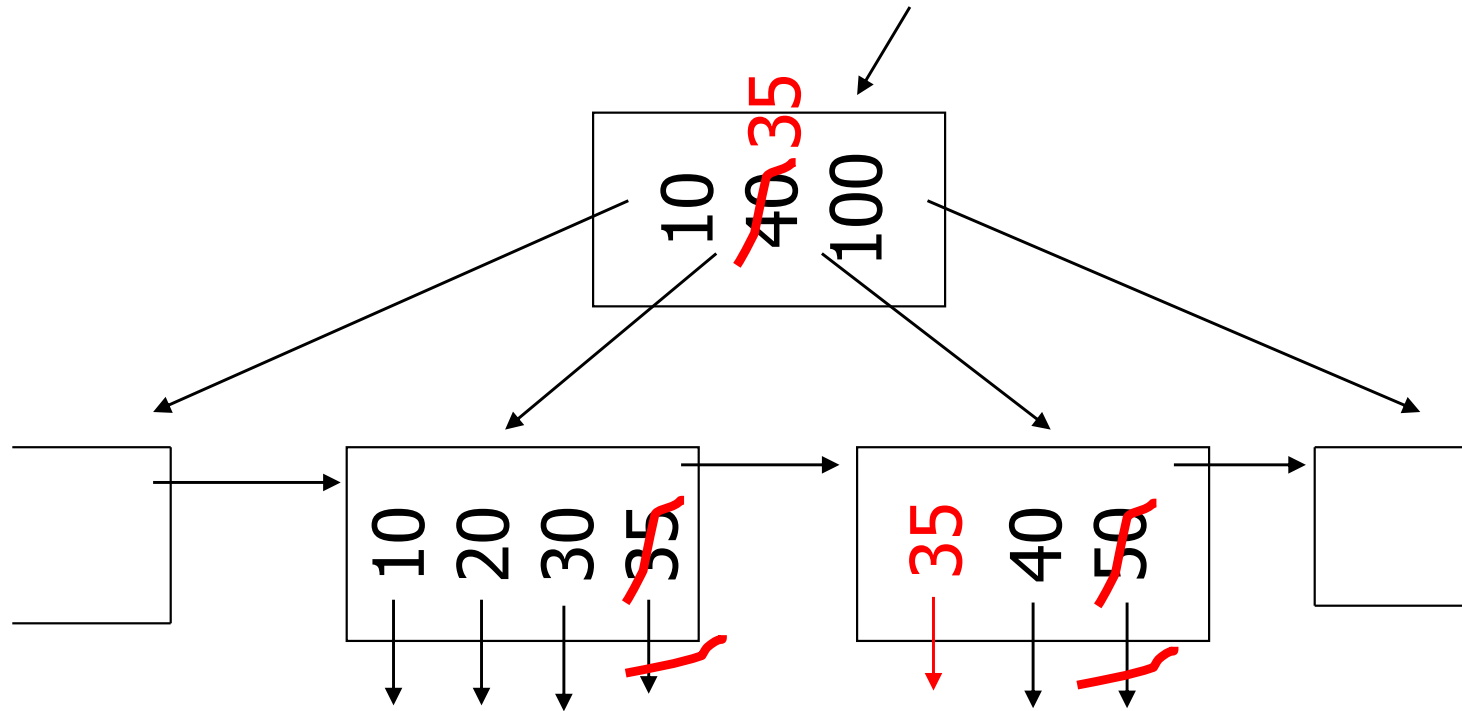
$n=4$



(c) Redistribute keys

– Delete 50

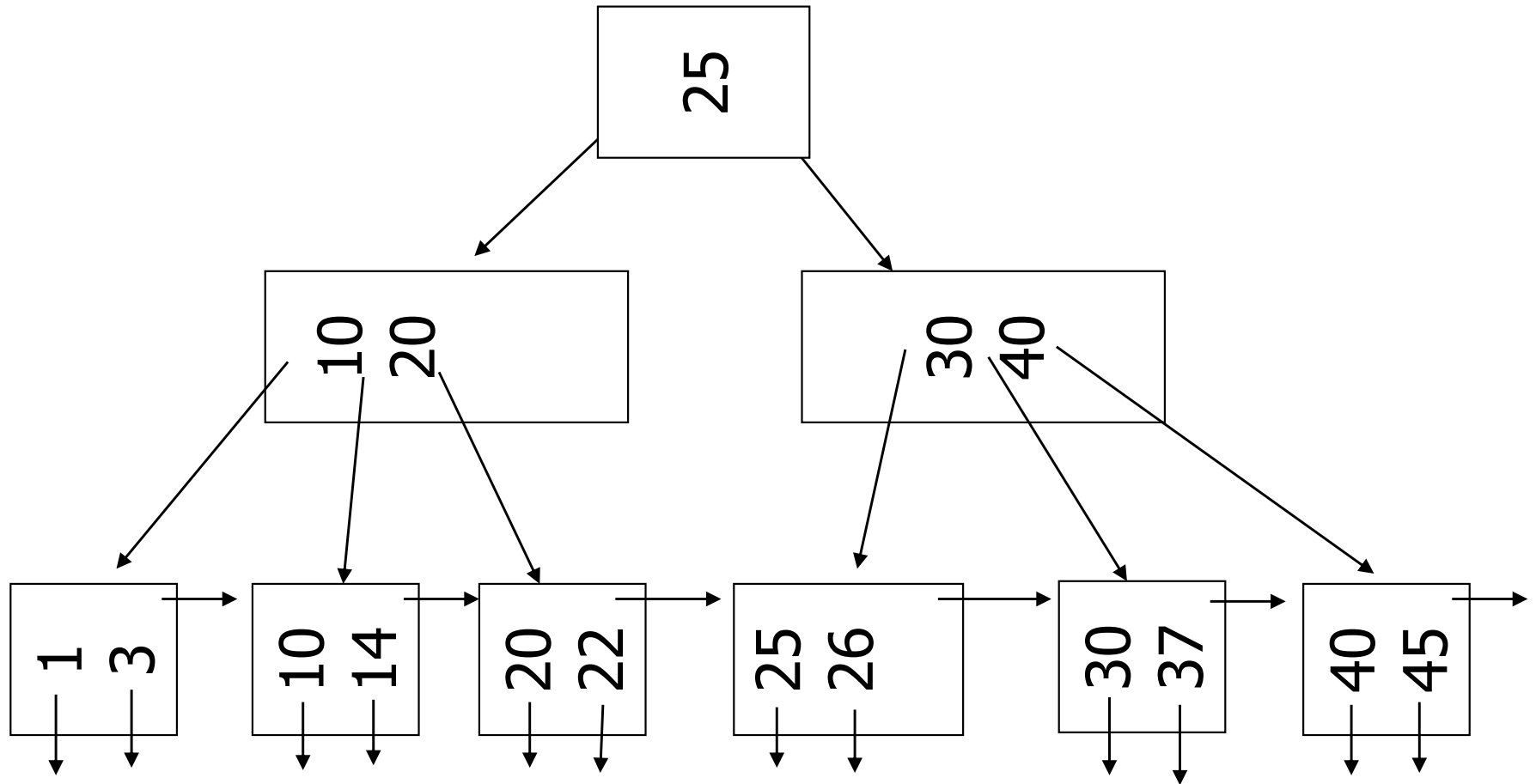
n=4



(d) Non-leaf coalesce

– Delete 37

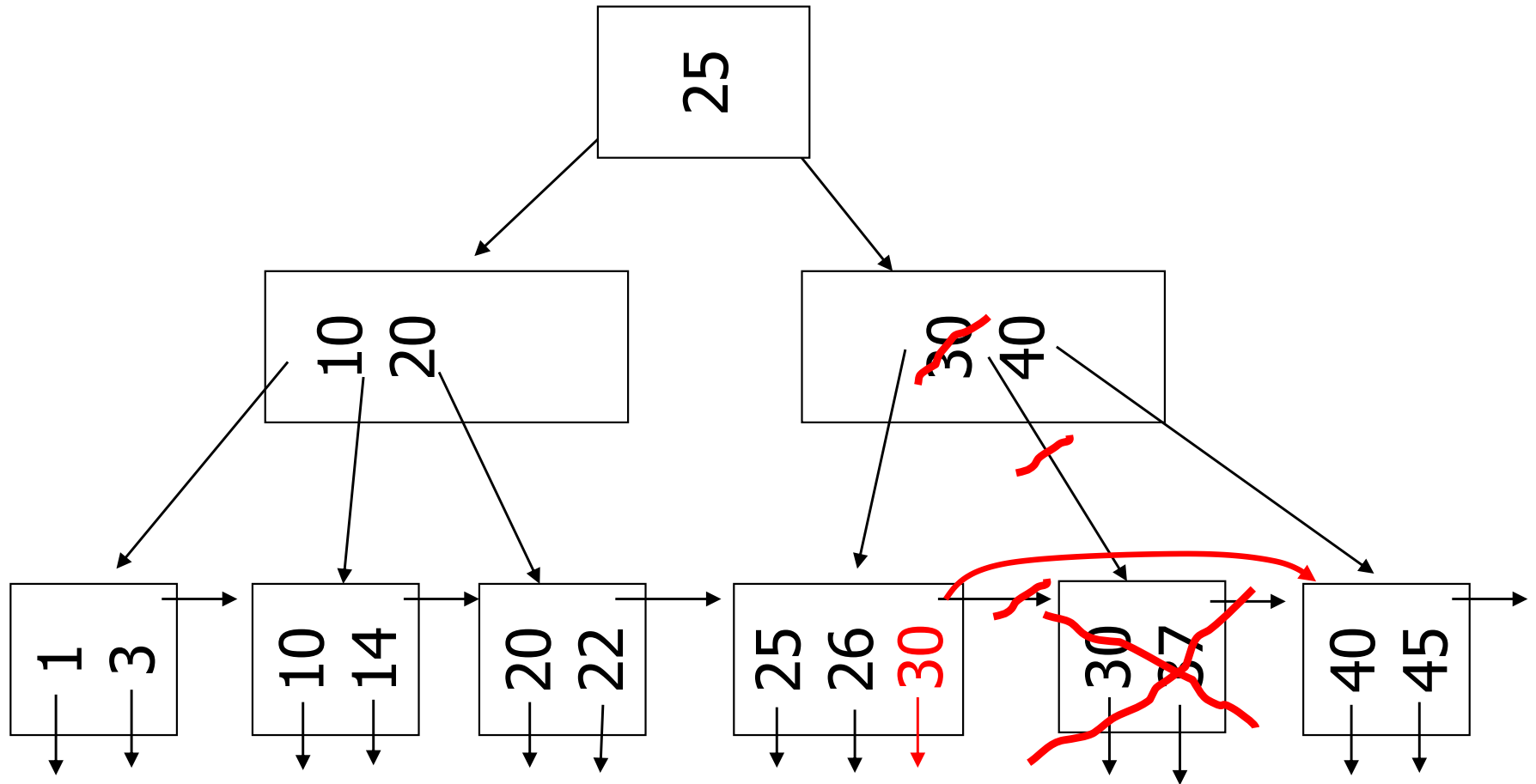
$n=4$



(d) Non-leaf coalesce

– Delete 37

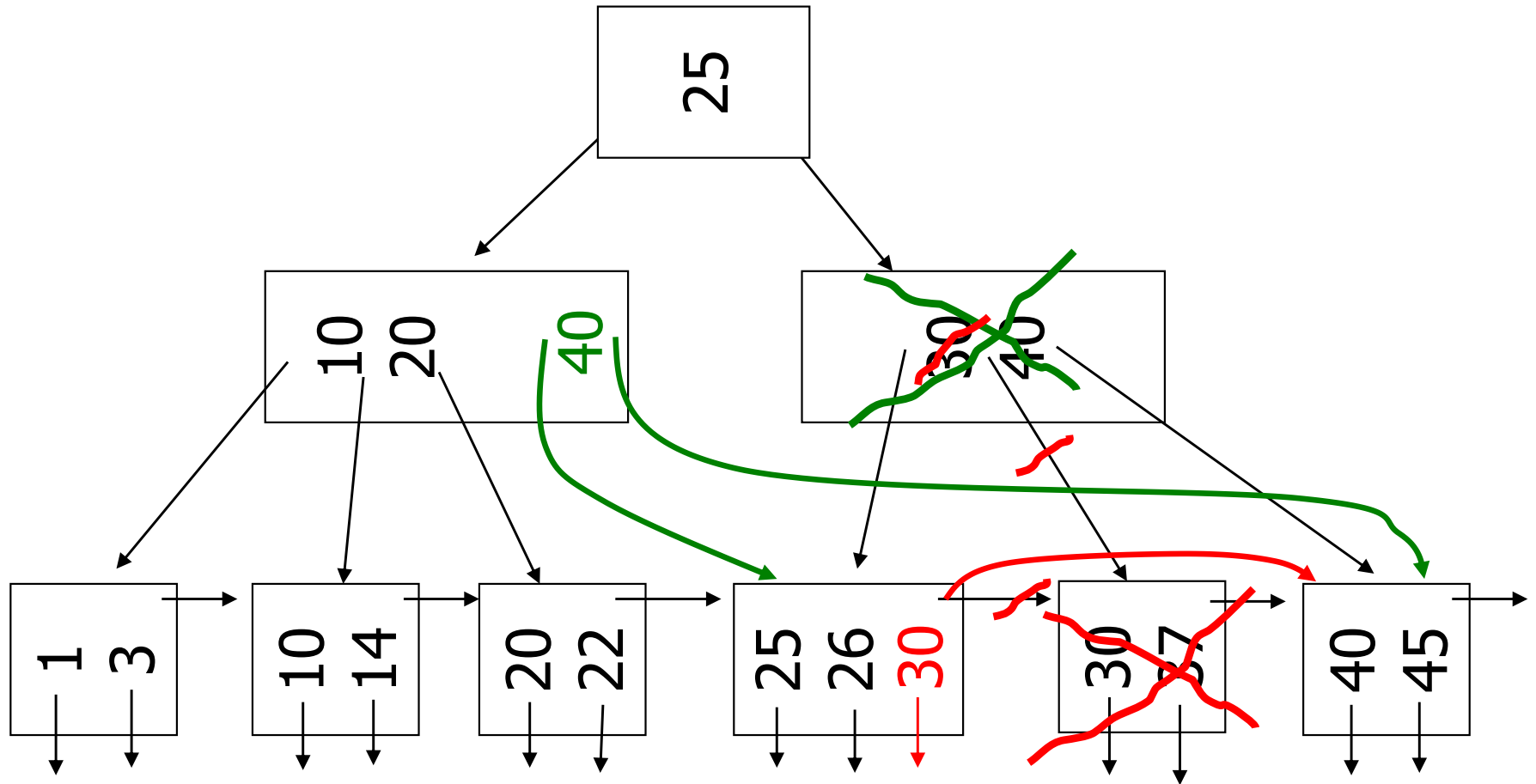
n=4



(d) Non-leaf coalesce

– Delete 37

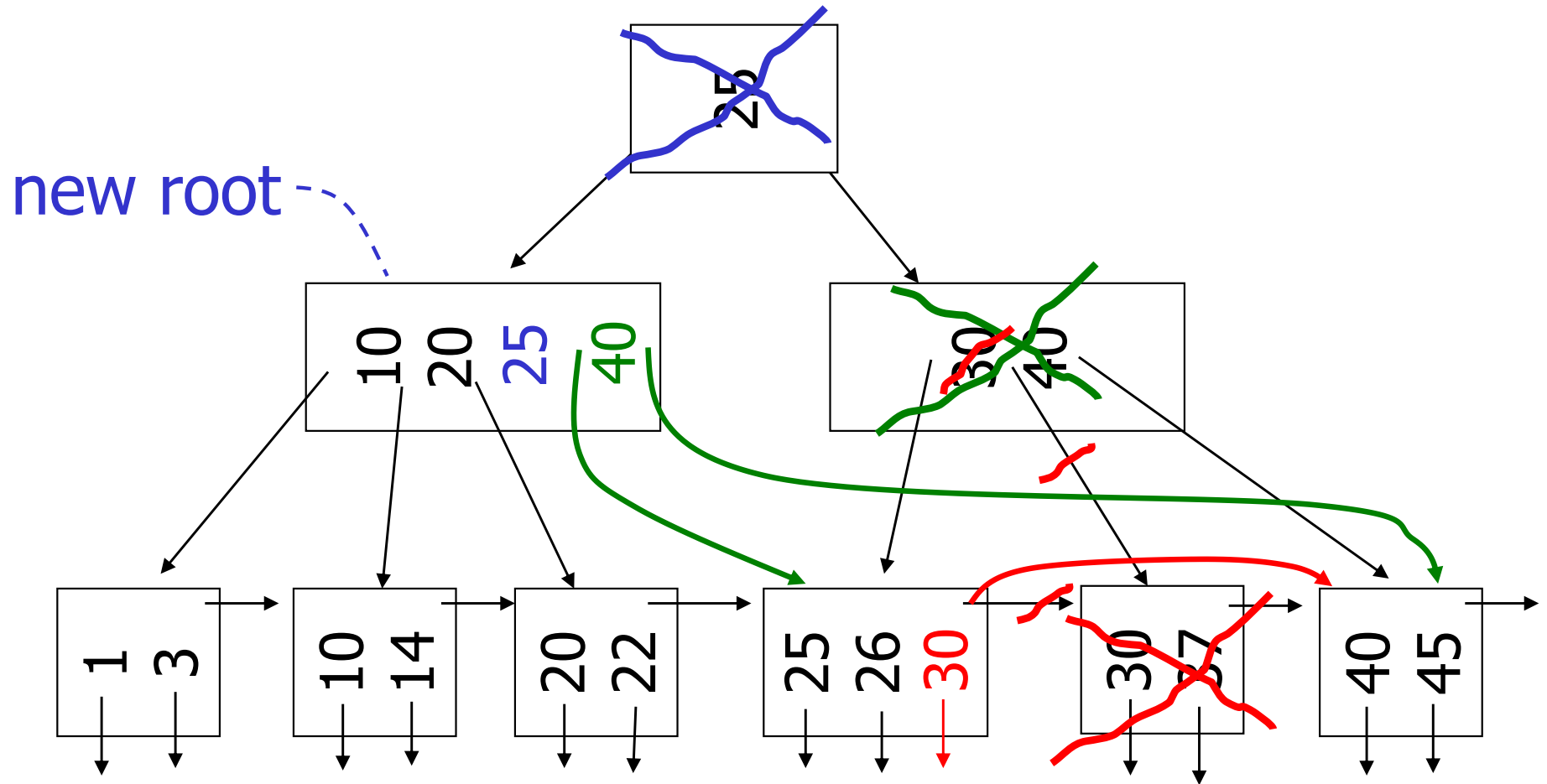
n=4



(d) Non-leaf coalesce

– Delete 37

n=4



B+tree deletions in practice

- Often, coalescing is not implemented
 - Too hard and not worth it!

Outline/summary

- Conventional Indexes
 - Sparse vs. dense
 - Primary vs. secondary
- B trees
- Hashing schemes (recommended reading, not mandatory)

The slides in this lecture are taken from:

- Hector Garcia-Molina, CS 245: Database System Principles, Notes 4: Indexing.

Reading

- Héctor García-Molina, Jeffrey Ullman, and Jennifer Widom. Database Systems: The Complete Book