GEST-H501

Logistics & Quality Engineering

Review Session

- Forecasting
- Logistics Network Modelling
- Transportation & Distribution Logistics

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Forecasting

Forecasting - Quantitative Methods

- Moving Average
- Simple Exponential Smoothing
- Double Exponential Smoothing
- Triple Exponential Smoothing
- Forecast Error measurement (MAE, Bias and RMSE)

Moving Average

$$f_{t+1} = \frac{\sum_{i=t-n+1}^{t} \; d_i}{n}$$

f_{t+1} = forecast for period t+1;

n = number of periods used to calculate moving average;

d_i = actual demand in period i

Double exponential smoothing (with Damped Trend)

$$a_t = \alpha d_t + (1-\alpha)(a_{t-1} + \phi b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)(\phi b_{t-1})$$

- f_{t+1} : Forecast for the following period performed in the current period,
- a_t : Level estimation,
- b_t: Trend estimation
- λ: Number of periods for which we are making the forecast

ϕ: Dampening Factor

$$f_{t+1} = a_t + \phi b_t$$

$$f_{t+\lambda} = a_t + b_t \sum_{i=1}^{\lambda} \phi^i$$

Double exponential smoothing (with Damped Trend)

Initialization

Date	Demand	Forecast	Level (a)	Trend (b)
1	37	37	37	23
2	60			
3	85			
4	112			
5	132			
6	145			
7	179			
8	198			
9	212			
10	232			
11				
12				
13				

Alpha:	30%
Beta:	40%
Phi:	80%

$$a_t = \alpha d_t + (1-\alpha)(a_{t-1} + \phi b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)(\phi b_{t-1})$$

$$f_{t+1} = a_t + \phi b_t$$

Double exponential smoothing (with Damped Trend)

Date	Demand	Forecast	Level (a)	Trend (b)
1	37	37	37	23
2	60			
3	85			
4	112			
5	132			
6	145			
7	179			
8	198			
9	212			
10	232			
11				
12				
13				

Alpha:	30%
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Double exponential smoothing (with Damped Trend)

Date	Demand	Forecast	Level (a)	Trend (b)
1	37	37	37	23
2	60			
3	85			
4	112			
5	132			
6	145			
7	179			
8	198			
9	212			
10	232			
11				
12				
13				

Alpha:	30%
Beta:	40%
Phi:	80%

Triple exponential smoothing

Pattern: (Level + Trend)*Seasonality

$$a_t = \alpha\left(\frac{d_t}{s_{t-p}}\right) + (1-\alpha)(a_{t-1} + \phi b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)(\phi b_{t-1})$$

$$s_t = \gamma\left(\frac{d_t}{a_t}\right) + (1-\gamma)\, s_{t-p}$$

$$f_t = (a_{t-1} + \phi b_{t-1})\, s_{t-p}$$

$$f_{t+\lambda} = \left(a_t + b_t \sum_{i=1}^{\lambda} \phi^i\right) s_{t-p+\lambda}$$

Without damping factor:

$$f_{t+\lambda} = (a_t + \lambda\, b_t)\, s_{t-p+\lambda}$$

Triple exponential smoothing

Initialization

Date	Demand	Forecast	Level (a)	Trend (b)	Season (s)
1	14	14	5,5	3,2	8,5
2	10				
3	6				
4	2				
5	18				
6	8				
7	4				
8	1				
9	16				
10	9				
11	5				
12	3				
13	18				
14	11				
15	4				
16	2				
17	17				
18	9				
19	5				
20	1				
21					
22					
23					
24					

Alpha:	80%
Beta:	10%
Gamma:	40%

$$a_t = \alpha\left(\frac{d_t}{s_{t-p}}\right) + (1-\alpha)(a_{t-1} + \phi b_{t-1})$$

$$b_t = \beta(a_t - a_{t-1}) + (1-\beta)(\phi b_{t-1})$$

$$s_t = \gamma\left(\frac{d_t}{a_t}\right) + (1-\gamma)\, s_{t-p}$$

$$f_t = (a_{t-1} + \phi b_{t-1})\, s_{t-p}$$

Forecast Error Measurement

Date	Demand	Forecast
1	37	
2	60	60
3	85	83
4	112	107
5	132	134
6	145	158
7	179	175
8	198	198
9	212	220
10	232	237
11		255

Error	Absolute Error	Squared Error
-2	2	4
-5	5	23
2	2	3
13	13	161
-4	4	19
0	0	0
8	8	65
5	5	28

Alpha:	30%
Beta:	100%

$$RMSE = \sqrt{\frac{1}{n}\sum_{1}^{n}(f_t - d_t)^2}$$

$$MAE = \frac{1}{n}\sum_{1}^{n}|f_t - d_t|$$

$$Bias = \frac{1}{n}\sum_{1}^{n}(f_t - d_t)$$

RMSE	
MAE	
Bias	

Logistics Network Modelling

Logistics Network Modelling - Mathematical Models

- Capacitated facility location Model
- Allocating Demand to Production Facilities
- The capacitated Plant Location Model
- The capacitated Plant Location Model with Single Sourcing

Network Optimization Models

Capacitated Facility Location Model

Model Inputs (Parameters)

n = number of potential plant locations/capacity (each level of capacity will count as a separate location)

m = number of markets or demand points

D_j = annual demand from market j

K_i = potential capacity of plant i

F_i = annualized fixed cost of keeping plant i open

C_{ij}= cost of producing and shipping one unit from plant i to market j (cost includes production, inventory, transportation, and tariffs)

Model Outputs (Decision Variables)

y_i= 1 if plant i is open, 0 otherwise

x_{ij} = quantity shipped from plant i to market j

Objective Function and its Constraints

Fixed Costs

$$Min \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

Variable Cost

Demand Satisfied

$$\sum_{i=1}^{n} x_{ij} = D_j \quad for \quad j = 1, \dots, m$$

Supply can not be more than capacity

$$\sum_{i=1}^{n} x_{ij} \le K_i y_i \quad for \quad i = 1, \dots, n$$

Mixed Integer Programming

$$y_i \in \{0,1\} \qquad for\ i = 1, \dots, n, x_{ij} \ge 0$$

Network Optimization Models

Allocating Demand to Production Facilities

Model Inputs (Parameters)

n = number of factory locations

m = number of markets or demand points

D_j = *annual demand from market j*

K_i = *capacity of factory i*

C_{ij} = *cost of producing and shipping one unit from factory i to market j (cost includes production, inventory, and transportation)*

The goal is to allocate the demand from different markets to the various plants to minimize the total cost of facilities, transportation, and inventory. Define the decision variables:

Model Outputs (Decision Variables)

x_{ij}= quantity shipped from plant *i* to market *j*

Objective Function and its Constraints

$$\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

all demand is satisfied

$$\sum_{i=1}^{n} x_{ij} = D_j \quad \text{for} \quad j = 1, \ldots, m$$

Production <= Capacity

$$\sum_{j=1}^{m} x_{ij} \leq K_i \quad \text{for} \quad i = 1, \ldots, n$$

Network Optimization Models

The Capacitated Plant Location Model

- Merge the companies
- Solve using location-specific costs

Model Outputs (Decision Variables)

y_i = 1 if factory i is open, 0 otherwise

x_{ij} = quantity shipped from factory i to market j

Objective Function and its Constraints

Fixed Costs

$$\underbrace{Min \sum_{i=1}^{n} f_i y_i}_{\text{Fixed Costs}} + \underbrace{\sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij}}_{\text{Variable Cost}}$$

Variable Cost

$$\sum_{i=1}^{n} x_{ij} = D_j \quad \text{for} \quad j = 1, \ldots, m$$

$$\sum_{j=1}^{m} x_{ij} \leq K_i y_i \quad \text{for} \quad i = 1, \ldots, n$$

$$y_i \in \{0, 1\} \quad \text{for} \quad i = 1, \ldots, n, x_{ij} \geq 0$$

Network Optimization Models

The Capacitated Plant Location Model with Single Sourcing

- Market supply by only one Factory
- Modify decision variables

reducing complexity
improving coordination

Model Outputs (Decision Variables)

y_i = 1 if factory is located at site i, 0 otherwise

x_{ij} = 1 if market j is supplied by factory i, 0 otherwise

Objective Function and its Constraints

$$\text{Min} \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n} \sum_{j=1}^{m} D_j c_{ij} x_{ij}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \quad \text{for} \quad j = 1, \ldots, m$$

$$\sum_{j=1}^{m} D_j x_{ij} \le K_i y_i \quad \text{for} \quad i = 1, \ldots, n$$

$$x_{ij}, y_i \in \{0, 1\}$$

Ensures single sourcing

Inputs - Costs, Capacities, Demands (for HighOptic)

Supply City	Demand City – Production and Transportation Cost per 1000 Units: Atlanta	Boston	Chicago	Denver	Omaha	Portland	Fixed Cost ($)	Capa-city
Baltimore	1675	400	685	1630	1160	2800	7650	18
Cheyenne	1460	1940	970	100	495	1200	3500	24
Salt Lake	1925	2400	1425	500	950	800	5000	27
Memphis	380	1355	543	1045	665	2321	4100	22
Wichita	922	1646	700	508	311	1797	2200	31
Demand	10	8	14	6	7	11		

Decision Variables

Supply City	Demand City - Production Allocation (1000 Units): Atlanta	Boston	Chicago	Denver	Omaha	Portland	Plants (1=open)
Baltimore	0	0	0	0	0	0	
Cheyenne	0	0	0	6	7	0	1
Salt Lake	0	0	0	0	0	11	1
Memphis	0	0	0	0	0	0	
Wichita	0	0	0	0	0	0	

Constraints

Supply City	Excess Capacity
Baltimore	18
Cheyenne	11
Salt Lake	16
Memphis	22
Wichita	31

	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Unmet Demand	10	8	14	0	0	0

Objective Function

Cost =	$ 21.416

Optimal Demand Allocation for HighOptic

1. Using Data | Analysis | Solver, solve the demand allocation problem for HighOptic

Supply cities	i	Demand cities	j
Baltimore	1	Atlanta	1
Cheyenne	2	Boston	2
Salt Lake	3	Chicago	3
Memphis	4	Denver	4
Wichita	5	Omaha	5
		Portland	6

$$Min\ Z = 100X_{24} + 495X_{25} + 1200X_{26} + 500X_{34} + 950X_{35} + 800X_{36}$$

S.t:

$$X_{24} + X_{25} + X_{26} \le 24$$
$$X_{34} + X_{35} + X_{36} \le 27$$

$$X_{24} + X_{34} = 6$$
$$X_{25} + X_{35} = 7$$
$$X_{26} + X_{36} = 11$$

$$X_{ij} \ge 0; i \in \{1,2,3,4,5\}, j \in \{1,2,3,4,5,6\}$$

$$Min\ Z = 1675X_{11} + 400X_{12} + 685X_{13} + 380X_{41} + 1355X_{42} + 543X_{43} + 922X_{51} + 1646X_{52} + 700X_{53}$$

S.t:

$$X_{11} + X_{12} + X_{13} \leq 18$$
$$X_{41} + X_{42} + X_{43} \leq 22$$
$$X_{51} + X_{52} + X_{53} \leq 31$$

$$X_{11} + X_{41} + X_{51} = 10$$
$$X_{12} + X_{42} + X_{52} = 8$$
$$X_{13} + X_{43} + X_{53} = 14$$

$$X_{ij} \geq 0; i \in \{1,2,3,4,5\}, j \in \{1,2,3,4,5,6\}$$

Inputs - Costs, Capacities, Demands (for TelecomOne)

	Demand City *Production and Transportation Cost per 1000 Units*						**Fixed**	**Capa-**
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	**Cost ($)**	**city**
Baltimore	1.675	400	685	1.630	1.160	2.800	7.650	18
Cheyenne	1.460	1.940	970	100	495	1.200	3.500	24
Salt Lake	1.925	2.400	1.425	500	950	800	5.000	27
Memphis	380	1.355	543	1.045	665	2.321	4.100	22
Wichita	922	1.646	700	508	311	1.797	2.200	31
Demand	10	8	14	6	7	11		

Decision Variables

	Demand City - Production Allocation (1000 Units)						Plants
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	(1=open)
Baltimore	0	8	2	0	0	0	1
Cheyenne	0	0	0	0	0	0	0
Salt Lake	0	0	0	0	0	0	0
Memphis	10	0	12	0	0	0	1
Wichita	0	0	0	0	0	0	1

Constraints

Supply City	*Excess Capacity*
Baltimore	8
Cheyenne	24
Salt Lake	27
Memphis	0
Wichita	31

	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Unmet Demand	0	0	0	6	7	11

Objective Function

Cost = $ 28.836

Optimal Demand Allocation for TelecomOne

1. Using Data | Analysis | Solver, solve the demand allocation problem for HighOptic

$Min\ Z$
$$= 1675X_{11} + 400X_{12} + 685X_{13} + 1630X_{14} + 1160X_{15} + 2800X_{16} + 1460X_{21} + 1940X_{22} + 970X_{23} + 100X_{24} + 495X_{25} + 1200X_{26} + 1925X_{31} + 2400X_{32} + 1425X_{33} + 500X_{34} + 950X_{35} + 800X_{36} + 380X_{41} + 1355X_{42} + 543X_{43} + 1045X_{44} + 665X_{45} + 2321X_{46} + 922X_{51} + 1646X_{52} + 700X_{53} + 508X_{54} + 311X_{55} + 1797X_{56}$$
S.t:

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} \leq 18$$
$$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} \leq 24$$
$$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} \leq 27$$
$$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} \leq 22$$
$$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} \leq 31$$

$$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} = 10$$
$$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} = 8$$
$$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} = 14$$
$$X_{14} + X_{24} + X_{34} + X_{44} + X_{54} = 6$$
$$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} = 7$$
$$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} = 11$$

$$X_{ij} \geq 0; i \in \{1,2,3,4,5\}, j \in \{1,2,3,4,5,6\}$$

Inputs - Costs, Capacities, Demands (for TelecomOptic)

	Demand City Production and Transportation Cost per 1000 Units						**Fixed**	**Capa-**
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	**Cost ($)**	**city**
Baltimore	1.675	400	685	1.630	1.160	2.800	7.650	18
Cheyenne	1.460	1.940	970	100	495	1.200	3.500	24
Salt Lake	1.925	2.400	1.425	500	950	800	5.000	27
Memphis	380	1.355	543	1.045	665	2.321	4.100	22
Wichita	922	1.646	700	508	311	1.797	2.200	31
Demand	10	8	14	6	7	11		

Decision Variables

	Demand City - Production Allocation (1000 Units)						**Plants**
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	(1=open)
Baltimore	0	8	2	0	0	0	1
Cheyenne	0	0	0	6	0	0	1
Salt Lake	0	0	0	0	0	11	1
Memphis	10	0	12	0	0	0	1
Wichita	0	0	0	0	7	0	1

Constraints

Supply City	*Excess Capacity*					
Baltimore	8			*Total Available Capacity*		*122*
Cheyenne	18					
Salt Lake	16					
Memphis	0					
Wichita	24					
	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Unmet Demand	0	0	0	0	0	0

Objective Function

Cost =	$ 48.913

$$
\begin{aligned}
&Min\ Z \\
&= 1675X_{11} + 400X_{12} + 685X_{13} + 1630X_{14} + 1160X_{15} + 2800X_{16} \\
&+ 1460X_{21} + 1940X_{22} + 970X_{23} + 100X_{24} + 495X_{25} + 1200X_{26} \\
&+ 1925X_{31} + 2400X_{32} + 1425X_{33} + 500X_{34} + 950X_{35} + 800X_{36} \\
&+ 380X_{41} + 1355X_{42} + 543X_{43} + 1045X_{44} + 665X_{45} + 2321X_{46} \\
&+ 922X_{51} + 1646X_{52} + 700X_{53} + 508X_{54} + 311X_{55} + 1797X_{56} \\
&+ 7650y_1 + 3500y_2 + 5000y_3 + 4100y_4 + 2200y_5
\end{aligned}
$$

S.t:

$$
\begin{aligned}
X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} &\le 18y_1 \\
X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} &\le 24y_2 \\
X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} &\le 27y_3 \\
X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} &\le 22y_4 \\
X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} &\le 31y_5
\end{aligned}
$$

$$
\begin{aligned}
X_{11} + X_{21} + X_{31} + X_{41} + X_{51} &= 10 \\
X_{12} + X_{22} + X_{32} + X_{42} + X_{52} &= 8 \\
X_{13} + X_{23} + X_{33} + X_{43} + X_{53} &= 14 \\
X_{14} + X_{24} + X_{34} + X_{44} + X_{54} &= 6 \\
X_{15} + X_{25} + X_{35} + X_{45} + X_{55} &= 7 \\
X_{16} + X_{26} + X_{36} + X_{46} + X_{56} &= 11
\end{aligned}
$$

$$X_{ij} \ge 0,\ y_i \in \{0,1\};\ i \in \{1,2,3,4,5\},\ j \in \{1,2,3,4,5,6\}$$

Inputs - Costs, Capacities, Demands (for TelecomOptic)

	Demand City							
	Production and Transportation Cost per 1000 Units						**Fixed**	**Capa-**
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	**Cost ($)**	**city**
Baltimore	1.675	400	685	1.630	1.160	2.800	7.650	18
Cheyenne	1.460	1.940	970	100	495	1.200	3.500	24
Salt Lake	1.925	2.400	1.425	500	950	800	5.000	27
Memphis	380	1.355	543	1.045	665	2.321	4.100	22
Wichita	922	1.646	700	508	311	1.797	2.200	31
Demand	10	8	14	6	7	11		

Decision Variables

	Demand City - Production Allocation (1000 Units)						**Plants**
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	(1=open)
Baltimore	0	8	2	0	0	0	1
Cheyenne	0	0	0	6	7	11	1
Salt Lake	0	0	0	0	0	0	0
Memphis	10	0	12	0	0	0	1
Wichita	0	0	0	0	0	0	0

Constraints

Supply City	*Excess Capacity*					
Baltimore	8					
Cheyenne	0					
Salt Lake	0					
Memphis	0					
Wichita	0					
	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Unmet Demand	0	0	0	0	0	0

Objective Function

Cost =	$ 47.401

$$
\begin{aligned}
&Min\ Z \\
&= 1675X_{11} + 400X_{12} + 685X_{13} + 1630X_{14} + 1160X_{15} + 2800X_{16} \\
&+ 1460X_{21} + 1940X_{22} + 970X_{23} + 100X_{24} + 495X_{25} + 1200X_{26} \\
&+ 1925X_{31} + 2400X_{32} + 1425X_{33} + 500X_{34} + 950X_{35} + 800X_{36} \\
&+ 380X_{41} + 1355X_{42} + 543X_{43} + 1045X_{44} + 665X_{45} + 2321X_{46} \\
&+ 922X_{51} + 1646X_{52} + 700X_{53} + 508X_{54} + 311X_{55} + 1797X_{56} \\
&+ 7650y_1 + 3500y_2 + 5000y_3 + 4100y_4 + 2200y_5
\end{aligned}
$$

S.t:

$$
\begin{aligned}
10X_{11} + 8X_{12} + 14X_{13} + 6X_{14} + 7X_{15} + 11X_{16} &\leq 18y_1 \\
10X_{21} + 8X_{22} + 14X_{23} + 6X_{24} + 7X_{25} + 11X_{26} &\leq 24y_2 \\
10X_{31} + 8X_{32} + 14X_{33} + 6X_{34} + 7X_{35} + 11X_{36} &\leq 27y_3 \\
10X_{41} + 8X_{42} + 14X_{43} + 6X_{44} + 7X_{45} + 11X_{46} &\leq 22y_4 \\
10X_{51} + 8X_{52} + 14X_{53} + 6X_{54} + 7X_{55} + 11X_{56} &\leq 31y_5
\end{aligned}
$$

$$
\begin{aligned}
X_{11} + X_{21} + X_{31} + X_{41} + X_{51} &= 1 \\
X_{12} + X_{22} + X_{32} + X_{42} + X_{52} &= 1 \\
X_{13} + X_{23} + X_{33} + X_{43} + X_{53} &= 1 \\
X_{14} + X_{24} + X_{34} + X_{44} + X_{54} &= 1 \\
X_{15} + X_{25} + X_{35} + X_{45} + X_{55} &= 1 \\
X_{16} + X_{26} + X_{36} + X_{46} + X_{56} &= 1
\end{aligned}
$$

$$X_{ij},\ y_i \in \{0,1\};\ i \in \{1,2,3,4,5\}, j \in \{1,2,3,4,5,6\}$$

Inputs - Costs, Capacities, Demands (for TelecomOptic)

	Demand City Production and Transportation Cost per 1000 Units						Fixed	Capa-
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	Cost ($)	city
Baltimore	1.675	400	685	1.630	1.160	2.800	7.650	18
Cheyenne	1.460	1.940	970	100	495	1.200	3.500	24
Salt Lake	1.925	2.400	1.425	500	950	800	5.000	27
Memphis	380	1.355	543	1.045	665	2.321	4.100	22
Wichita	922	1.646	700	508	311	1.797	2.200	31
Demand	10	8	14	6	7	11		

Decision Variables

	Demand City Supplied (1 indicates Cities Supplied)						Plants
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland	(1=open)
Baltimore	0	0	0	0	0	0	0
Cheyenne	0	0	0	0	0	0	0
Salt Lake	0	0	0	1	0	1	1
Memphis	1	1	0	0	0	0	1
Wichita	0	0	1	0	1	0	1

Resulting Production Allocation

	Demand City - Production Allocation (1000 Units)					
Supply City	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Baltimore	0	0	0	0	0	0
Cheyenne	0	0	0	0	0	0
Salt Lake	0	0	0	6	0	11
Memphis	10	8	0	0	0	0
Wichita	0	0	14	0	7	0

Constraints

Supply City	*Excess Capacity*					
Baltimore	0					
Cheyenne	0					
Salt Lake	10					
Memphis	4					
Wichita	10					
	Atlanta	Boston	Chicago	Denver	Omaha	Portland
Demand	1	1	1	1	1	1

Objective Function

Cost =	$ 49.717

Transportation and Distribution Logistics

Transportation & Distribution Logistics

- Standard Transportation Problem
- Identifying a basic feasible solution (Construction methods)
  - North-west corner method
  - Min cost method
  - Penalty cost (Vogel's approximation)
- Optimal solution (Improving methods)
  - Stepping stone

Transportation & Distribution Logistics

- Minimum Spanning Tree - Prime’s Algorithm
- Shortest Path Problem - Dijkstra’s Algorithm
- Minimum Cost Flow - Network Simplex Method
- Travelling Salesman Problem - Nearest Neighbor

Standard Transportation Problem

$\min \; 4x_{11} + 6x_{12} + 8x_{13} + 8x_{14} + 6x_{21} + 8x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 7x_{32} + 6x_{33} + 8x_{34}$

Subject to:

$x_{11} + x_{12} + x_{13} + x_{14} \leq 40$

$x_{21} + x_{22} + x_{23} + x_{24} \leq 60$

$x_{31} + x_{32} + x_{33} + x_{34} \leq 50$

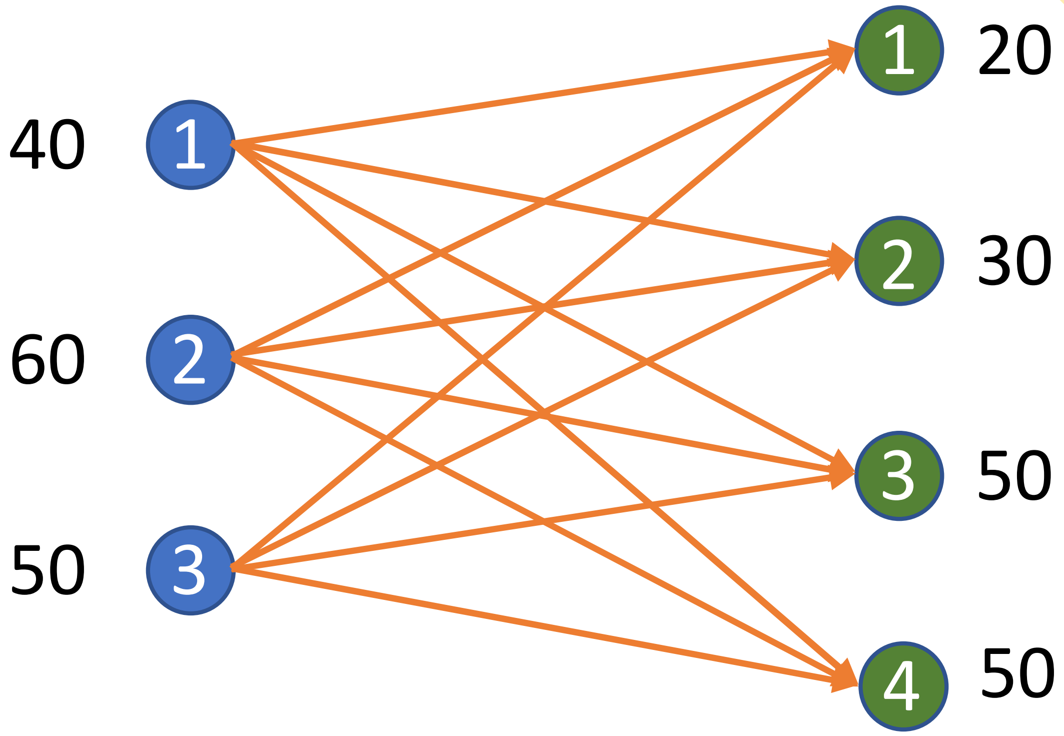
$x_{11} + x_{21} + x_{31} + x_{41} \geq 20$

$x_{12} + x_{22} + x_{32} + x_{42} \geq 30$

$x_{13} + x_{23} + x_{33} + x_{43} \geq 50$

$x_{14} + x_{24} + x_{34} + x_{44} \geq 50$

$x_{ij} \geq 0 \; ; i \in \{1,2,3\}, j \in \{1,2,3,4\}$

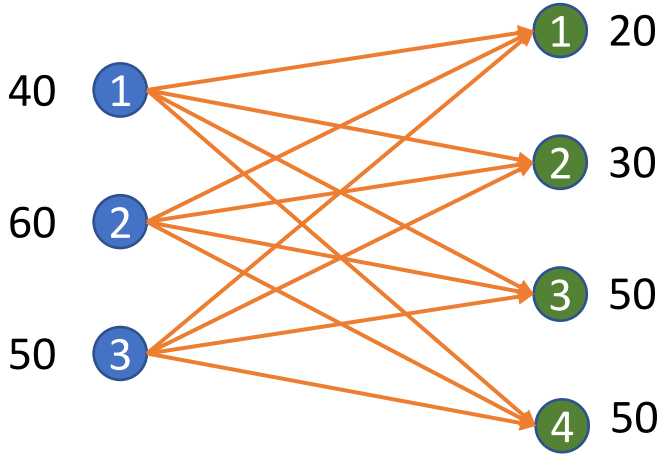


Standard Transportation Problem

$$\min\ 4x_{11} + 6x_{12} + 8x_{13} + 8x_{14} + 6x_{21} + 8x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 7x_{32}$$
$$+ 6x_{33} + 8x_{34}$$

Subject to:

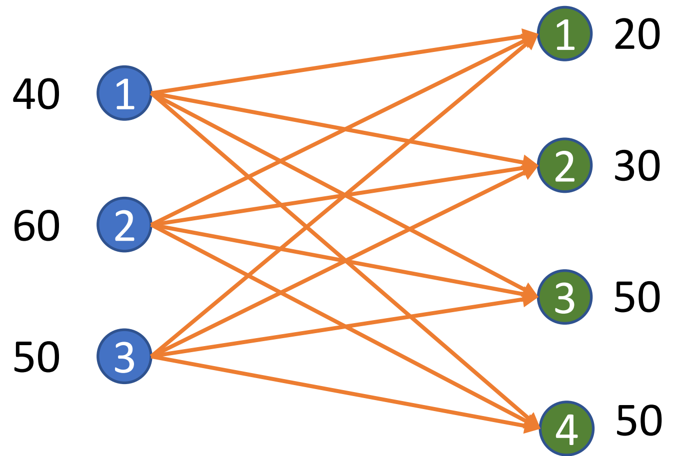
$$x_{11} + x_{12} + x_{13} + x_{14} \leq 40$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 60$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 50$$
$$x_{11} + x_{21} + x_{31} + x_{41} \geq 20$$
$$x_{12} + x_{22} + x_{32} + x_{42} \geq 30$$
$$x_{13} + x_{23} + x_{33} + x_{43} \geq 50$$
$$x_{14} + x_{24} + x_{34} + x_{44} \geq 50$$
$$x_{ij} \geq 0\ ;\ i \in \{1,2,3\}, j \in \{1,2,3,4\}$$



4	6	8	8	40
6	8	6	7	60
5	7	6	8	50
20	30	50	50	

Transportation Problem

4	6	8	8	40
6	8	6	7	60
5	7	6	8	50
20	30	50	50	



Transportation Problem

4	6	8	8	40
6	8	6	7	60
5	7	6	8	50
20	30	50	50	

Transportation Problem

				Supply					
					P1	P2	P3	P4	P5
4 20	6 20	8 —	8 —	40 ~~20~~ 0	2	2	—	—	—
6	8	6 10	7 50	~~60~~ ~~10~~ 0	0	1	1	1	1
5	7 10	6 40	8 —	~~50~~ ~~40~~ 0	1	1	1	2	—
~~20~~ 0	~~30~~ ~~10~~ 0	~~50~~ ~~10~~ 0	~~50~~ 0						
1	1	0	1	P1					
~~4~~ —	1	0	1	P2					
—	1	0	1	P3					
—	—	0	1	P4					
		6	7						

Transportation Problem

$x_{13} = 0$ > 0

4 20	6 20 θ	8 $\bar{c}_{13}$ θ	8 $\bar{c}_{14}$	40
6 $\bar{c}_{21}$	8	6 $50-\theta$ $-\theta$	7 $10+\theta$ $+\theta$	60
5	7 10 $+\theta$	6 θ	8 40 $-\theta$ $-\theta$	50
20	30	50	50	

$\bar{c}_{13} = +8 - 6 + 7 - 8 + 7 - 6 = 2$

$\bar{c}_{14} = 8 - 8 + 7 - 6 = 1$

$\bar{c}_{21} = +2$

$\bar{c}_{22} = +2$

$x_{34} = 0$

$\bar{c}_{31} = 0$

$\bar{c}_{33} = -1 \rightarrow \boxed{\theta = 40} \rightarrow x_{33} = 40$

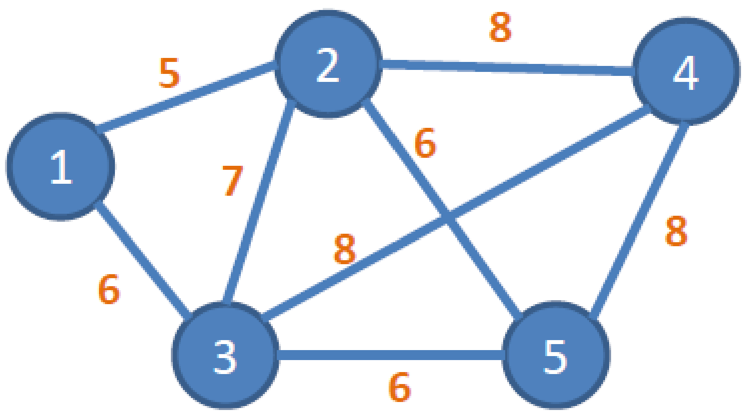
Transportation Problem

4 20	6 20	8 x	8 x	40
6 x	8 x	6 10	7 50	60
5 x	7 10	6 40	8 x	50
20	30	50	50	

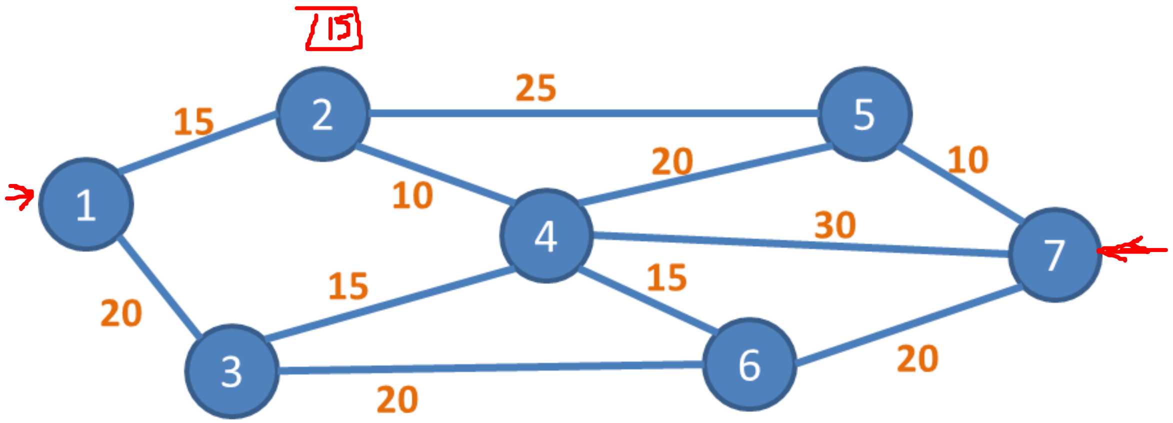
$\bar{c}_{13} =$

$\bar{c}_{ij} \geq 0 \longrightarrow$ optimal

Minimum Spanning Tree - Prime's Algorithm



Shortest Path Problem - Dijkstra's Algorithm

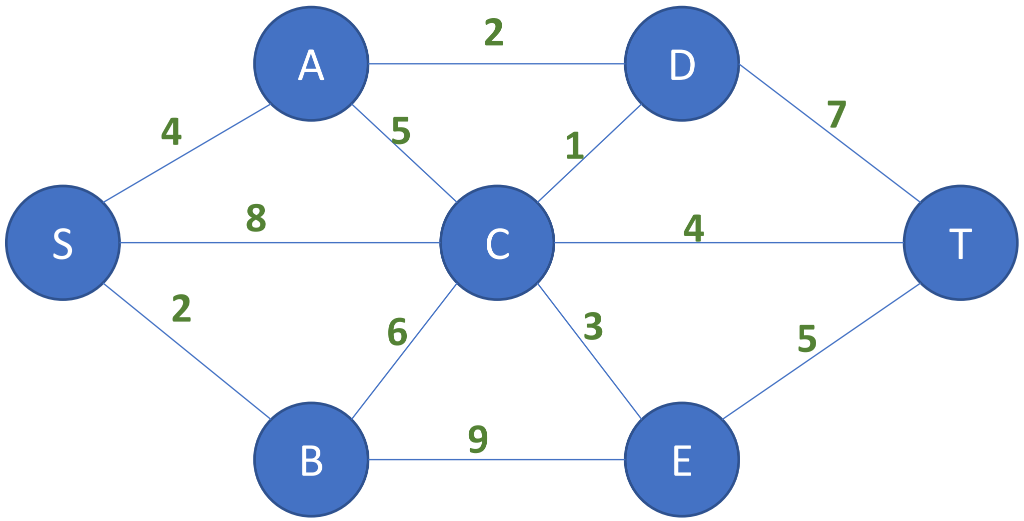


$$\min\ 15x_{12} + 20x_{13} + 10x_{24} + 15x_{34} + 25x_{25} + 20x_{36} +$$
$$20x_{45} + 30x_{47} + 15x_{46} + 10x_{57} + 20x_{67}$$

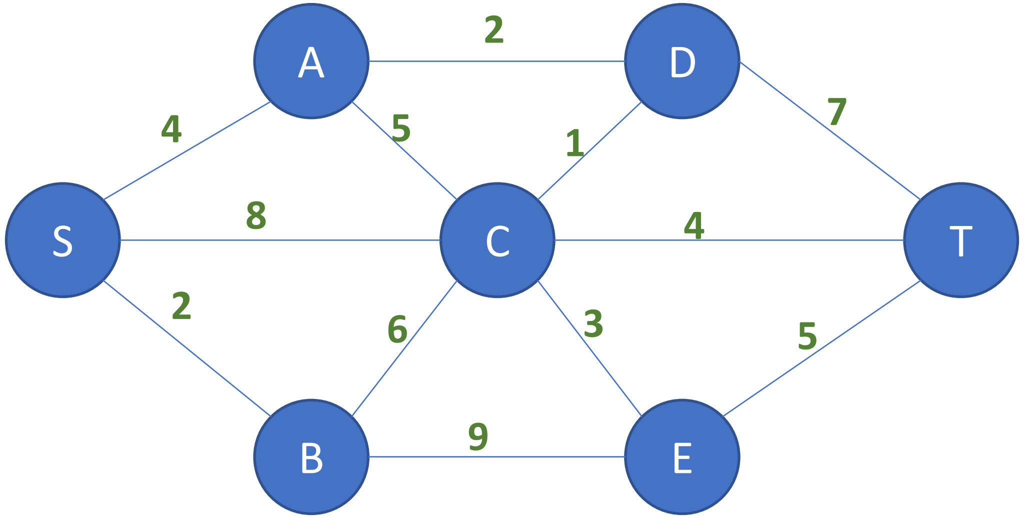
Subject to:

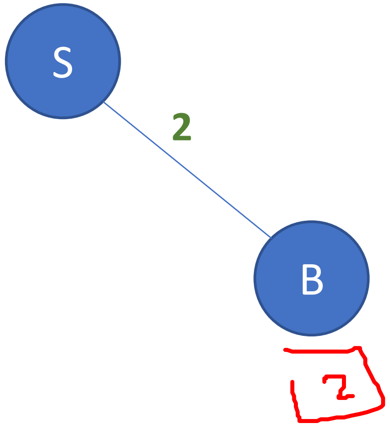
$$x_{12} + x_{13} = 1$$
$$-x_{12} + x_{24} + x_{25} = 0$$
$$-x_{13} + x_{34} + x_{36} = 0$$
$$-x_{24} - x_{34} + x_{45} + x_{46} + x_{47} = 0$$
$$-x_{25} - x_{45} + x_{57} = 0$$
$$-x_{36} - x_{46} + x_{67} = 0$$
$$-x_{67} - x_{47} - x_{57} = -1$$
$$x_{ij} = 0,1$$

Shortest Path Problem - Dijkstra's Algorithm

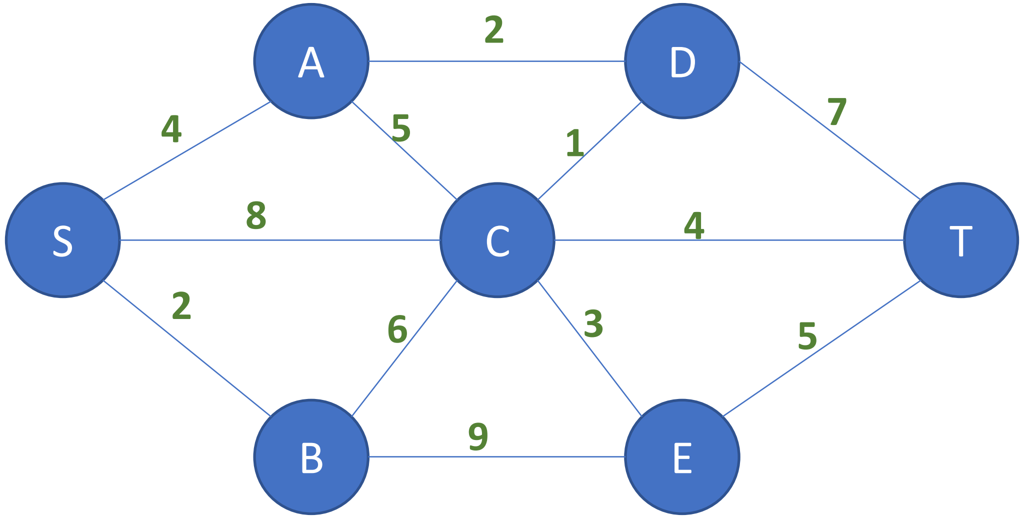


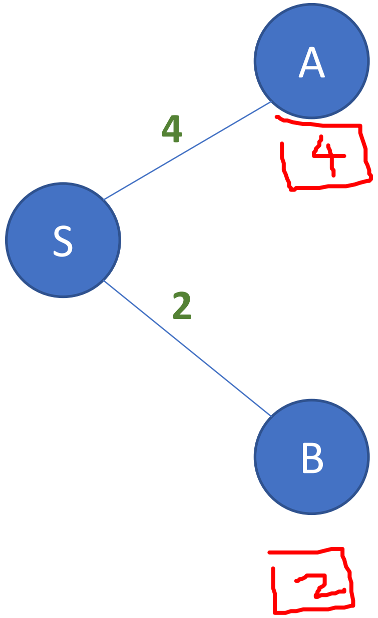
Shortest Path Problem - Dijkstra’s Algorithm



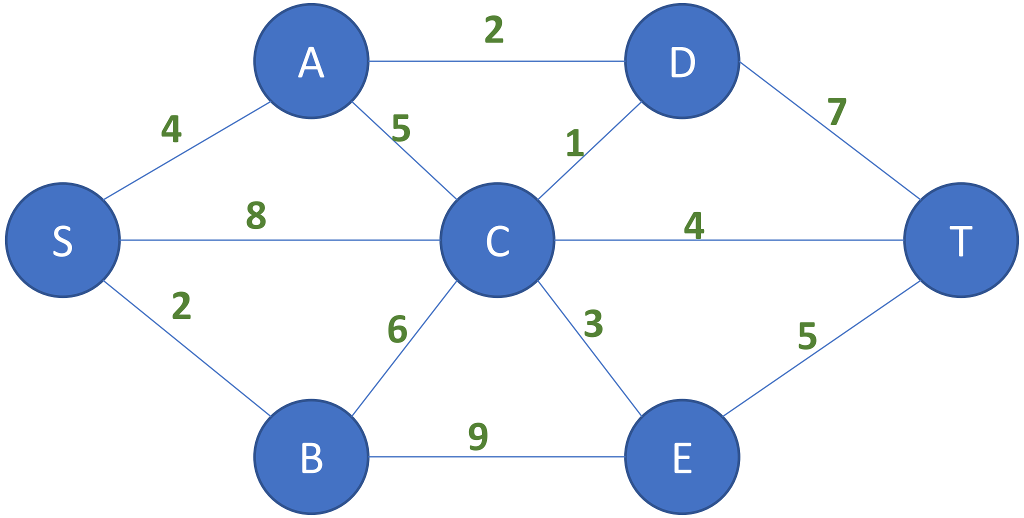


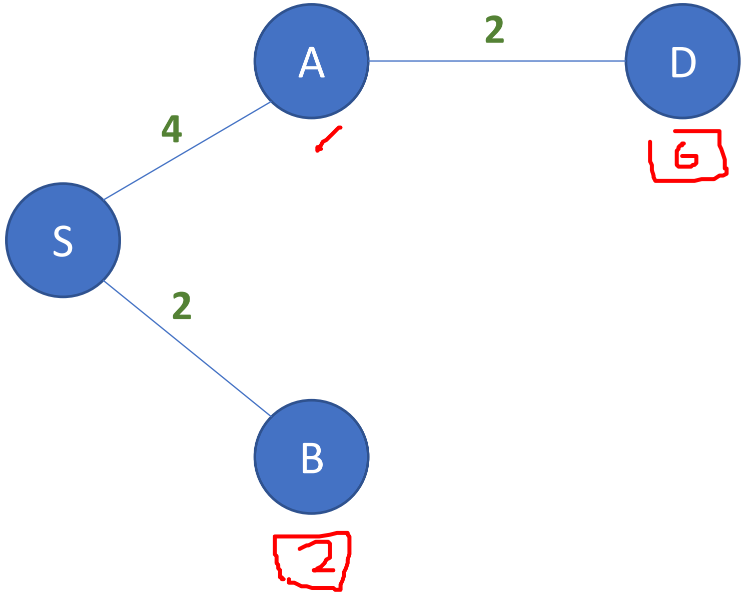
Shortest Path Problem - Dijkstra's Algorithm



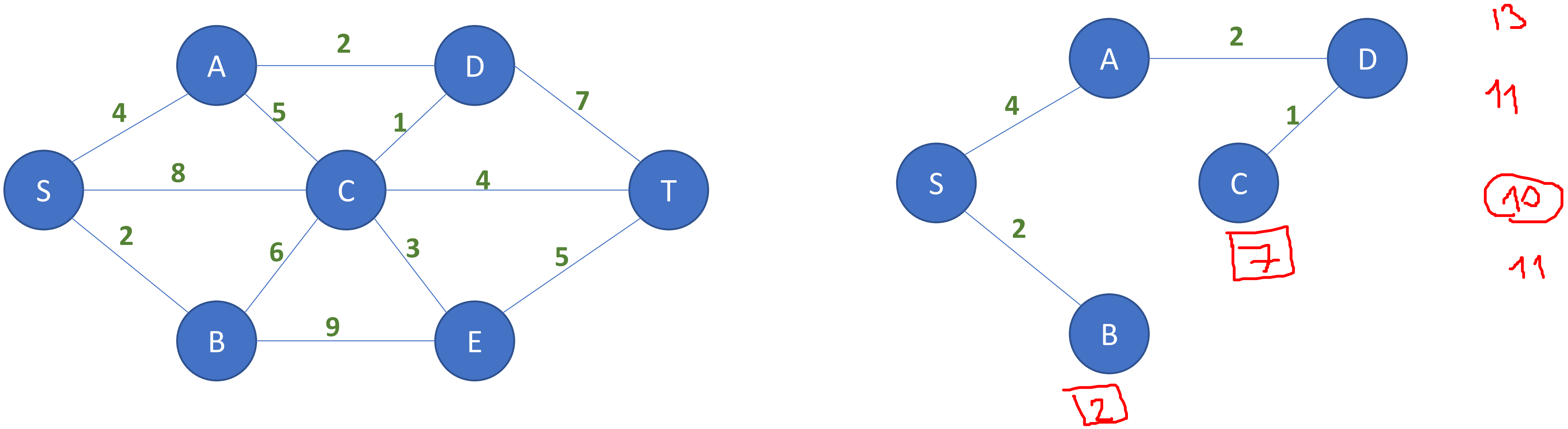


Shortest Path Problem - Dijkstra’s Algorithm

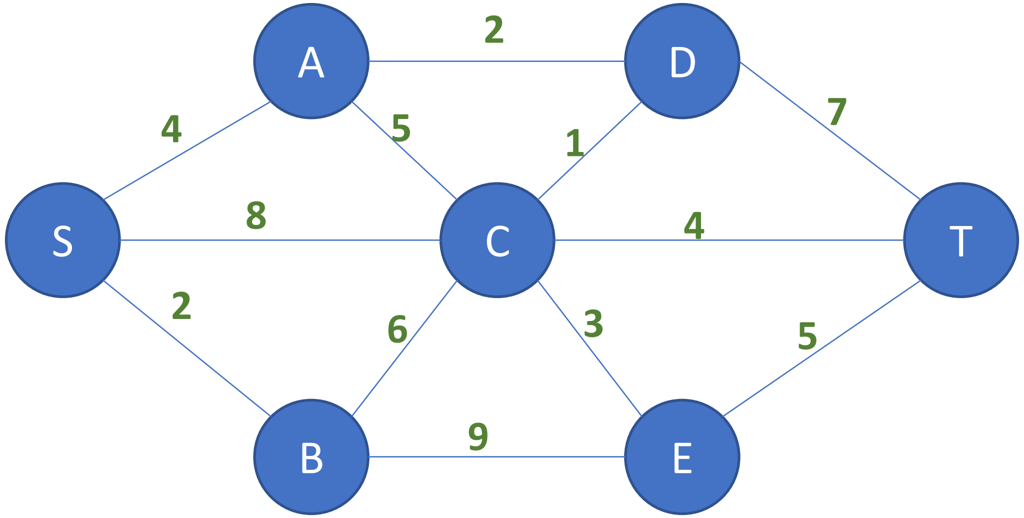


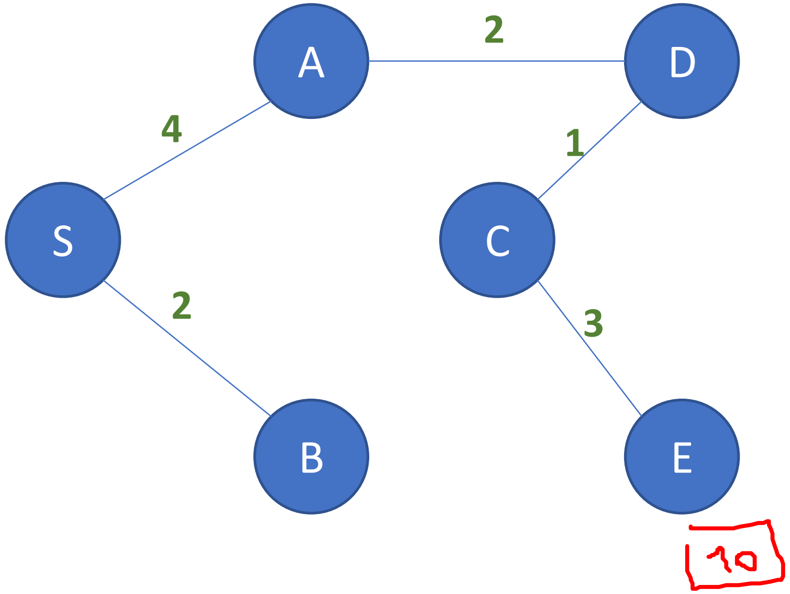


Shortest Path Problem - Dijkstra's Algorithm

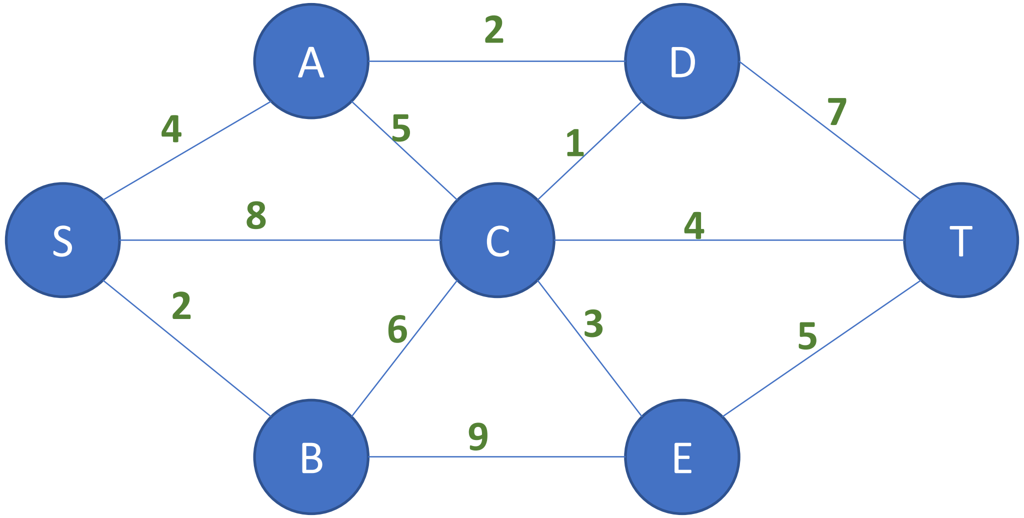


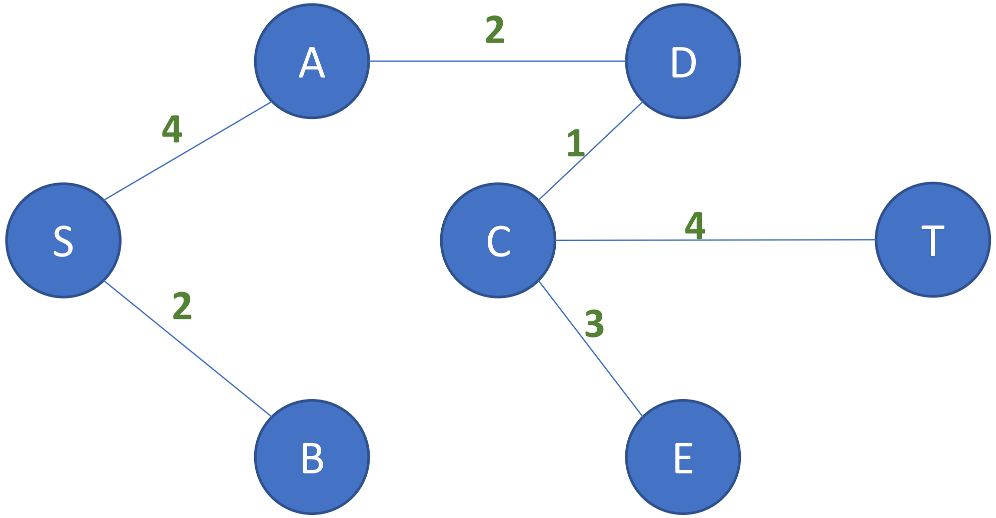
Shortest Path Problem - Dijkstra's Algorithm



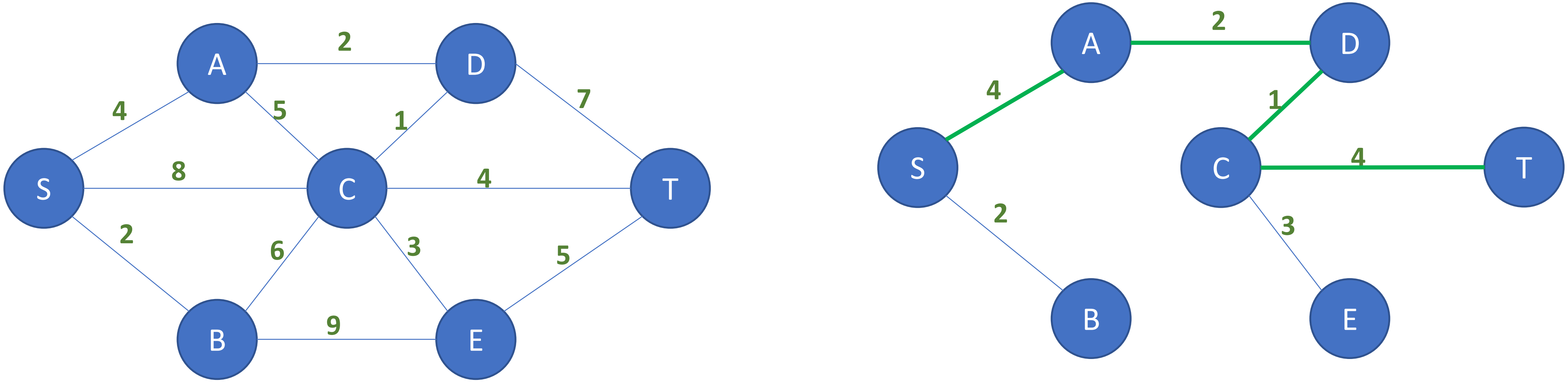


Shortest Path Problem - Dijkstra’s Algorithm

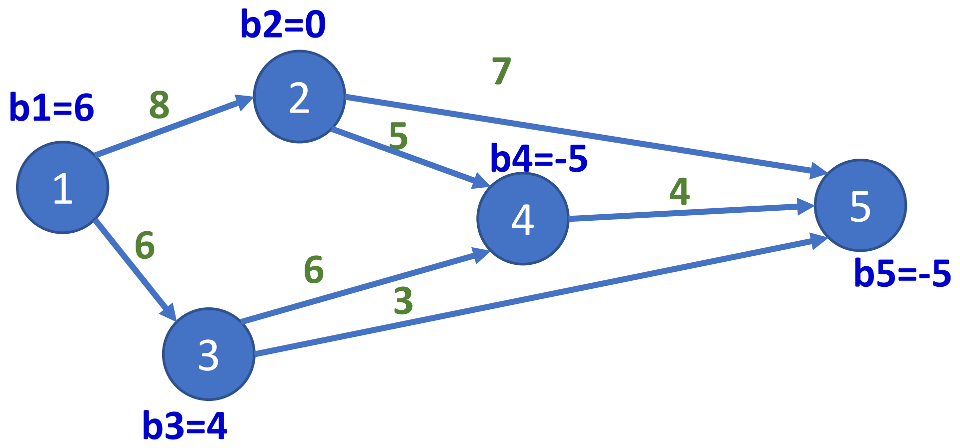




Shortest Path Problem - Dijkstra's Algorithm



Minimum Cost Flow - Network Simplex Method



$$\min\ 8x_{12} + 6x_{13} + 5x_{24} + 6x_{34} + 7x_{25} + 3x_{35} + 4x_{45}$$

Subject to:

$$x_{12} + x_{13} = 6$$

$$-x_{12} + x_{24} + x_{25} = 0$$

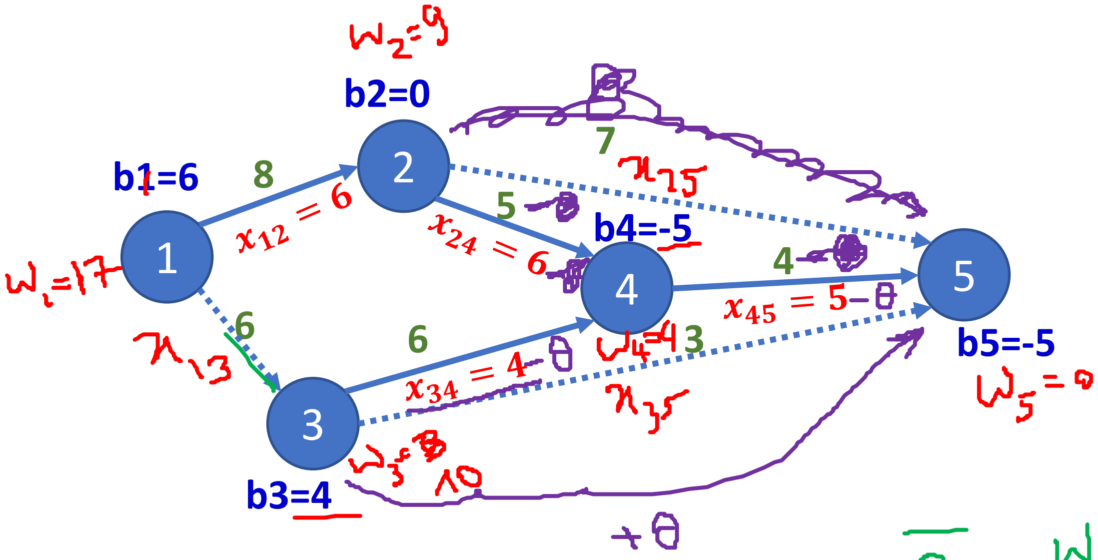
$$-x_{13} + x_{34} + x_{35} = 4$$

$$-x_{24} + x_{34} - x_{45} = -5$$

$$-x_{25} - x_{35} - x_{45} = -5$$

$$x_{ij} \geq 0$$

Minimum Cost Flow - Network Simplex Method



$\bar{c}_{ij} = w_i - w_j - c_{ij}$

Non-basic Variables: $\bar{c}_{ij} \neq 0$

Basic Variables: $\bar{c}_{ij} = 0$

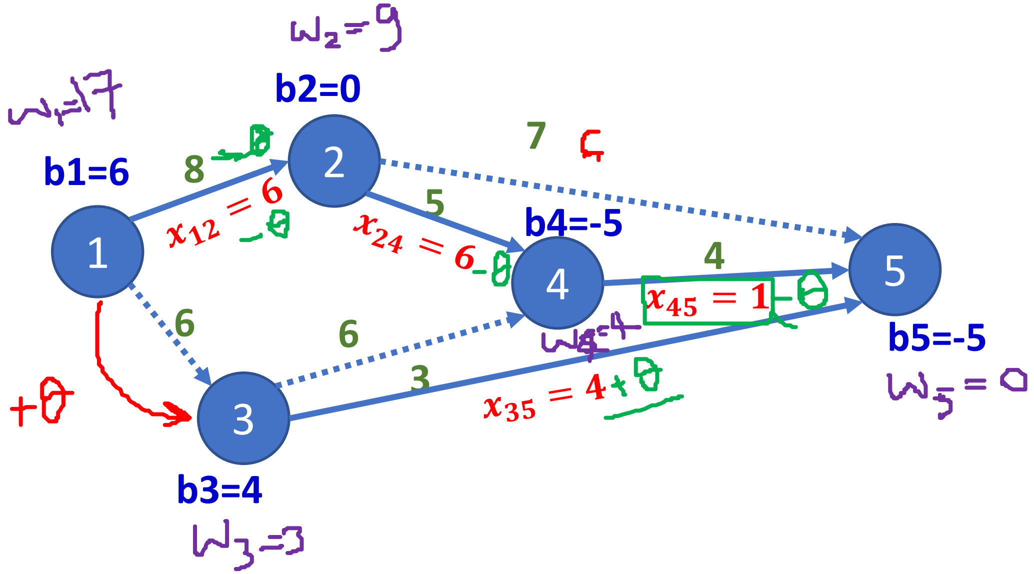
$w_3 - w_4 - c_{34} = 0$ $\quad w_3 - 4 - 6 = 0$ $\quad w_3 = 10$

$\bar{c}_{13} = w_1 - w_3 - c_{13} = 17 - 10 - 6 = +1$

$\bar{c}_{35} = \boxed{-7}$

$\bar{c}_{25} = 2$

Minimum Cost Flow - Network Simplex Method



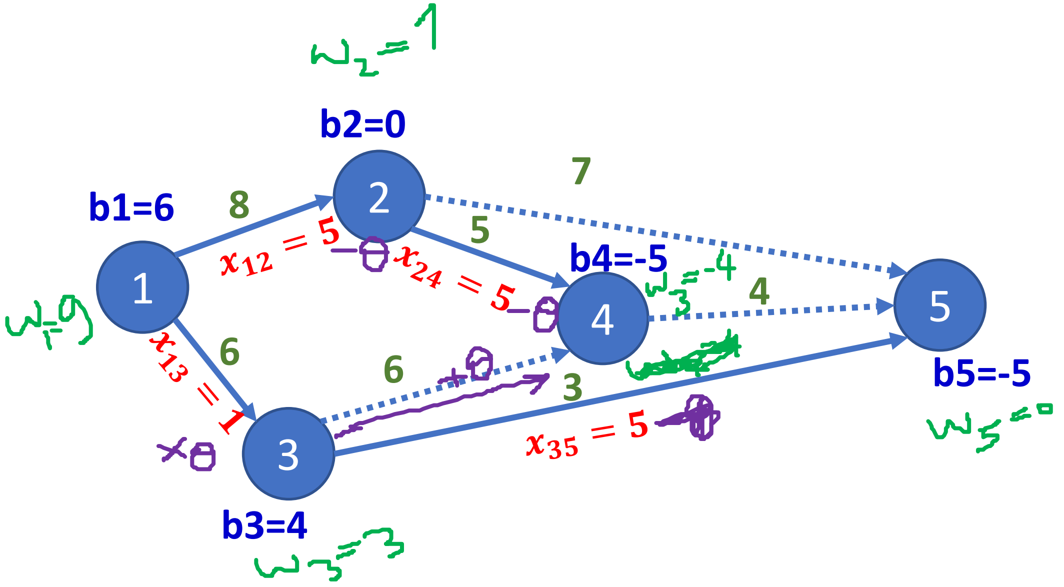
$\bar{c}_{25}=\cancel{7}\,2=9-7-0$

$\bar{c}_{13}=8 \leftarrow$

$\bar{c}_{34}=-7$

$x_{45} \longrightarrow$ $\theta=1$ $x_{13}=1$

Minimum Cost Flow - Network Simplex Method

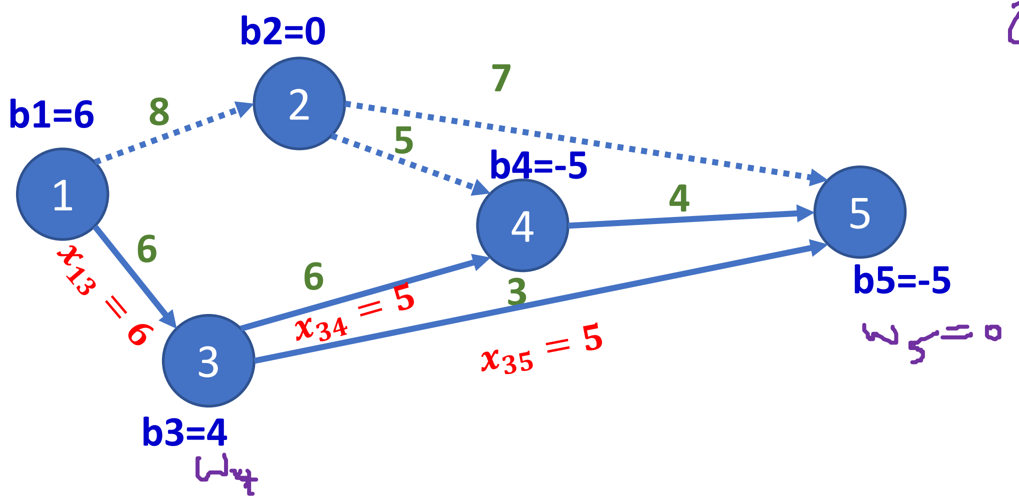


$\bar{c}_{15} = -6$

$\bar{c}_{34} = 1 \longleftarrow$

$\bar{c}_{45} = 0$

Minimum Cost Flow - Network Simplex Method



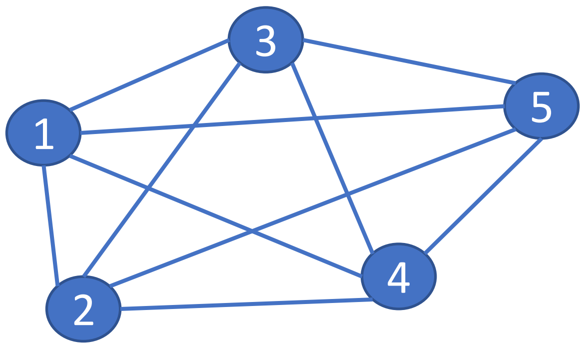
$\bar{c}_{25} \leq 0$

$\bar{c}_{24} \leq 0$

$\bar{c}_{12} \leq 0$

Travelling Salesman Problem - Nearest Neighbor

	1	2	3	4	5
1		10	8	9	7
2	10		10	5	6
3	8	10		8	9
4	9	5	8		6
5	7	6	9	6	



Travelling Salesman Problem - Nearest Neighbor

	1	2	3	4	5
1		10	8	9	7
2	10		10	5	6
3	8	10		8	9
4	9	5	8		6
5	7	6	9	6