BLOCK 4

Master en Planification des Transports

L'Ecole Nationale Supérieure des Travaux Publics de Yaoundé

Applied Operations Research & Mathematics in

Transportation and Distribution Logistics

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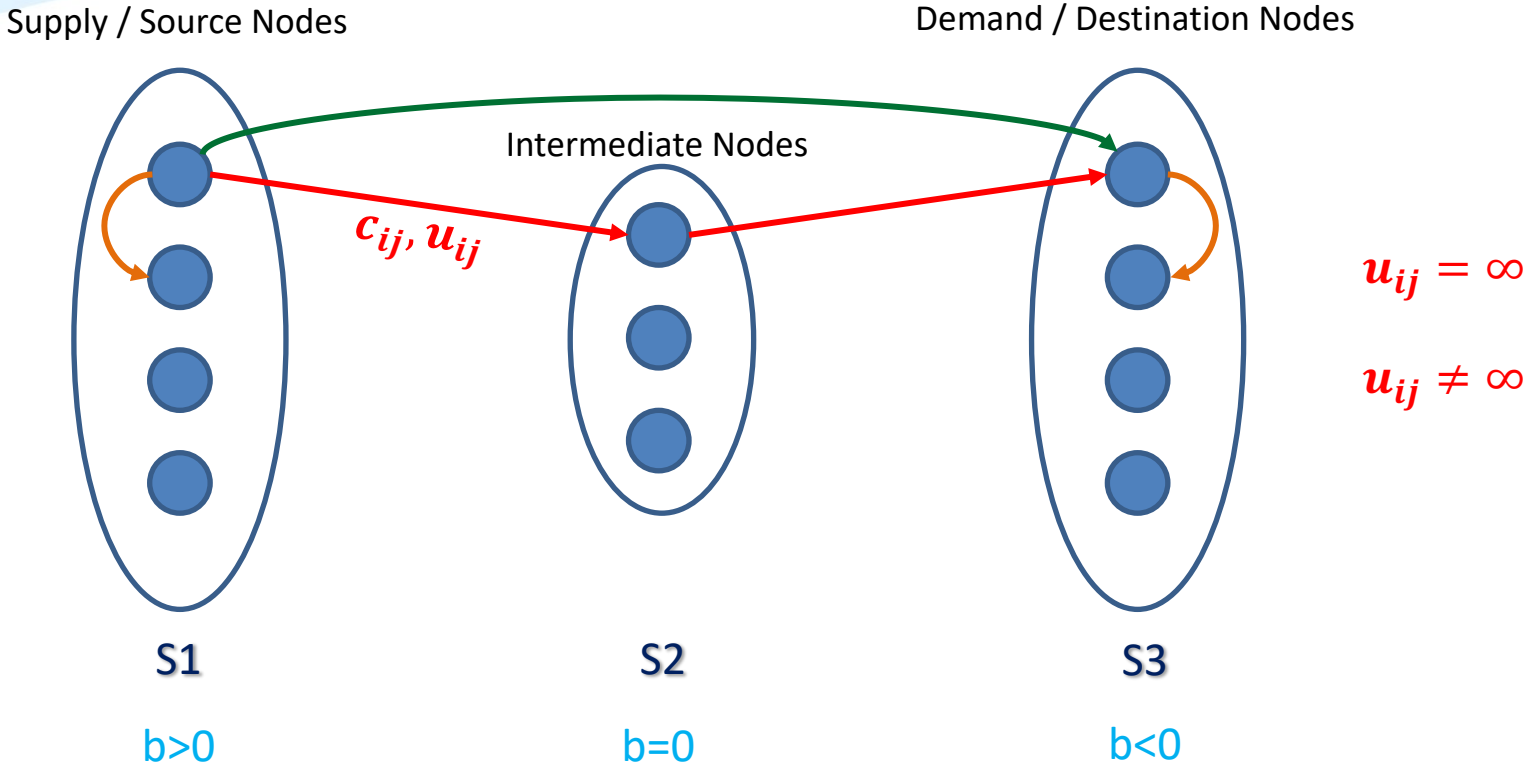
Classification of Transportation Problems

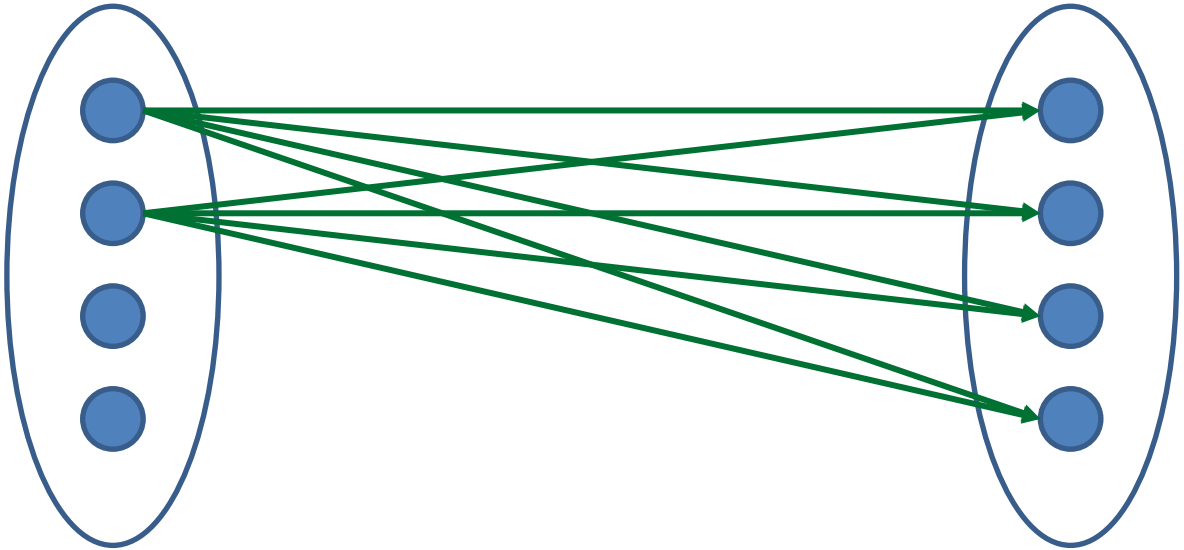


$u_{ij} = \infty$

$u_{ij} \neq \infty$

Classification of Transportation Problems

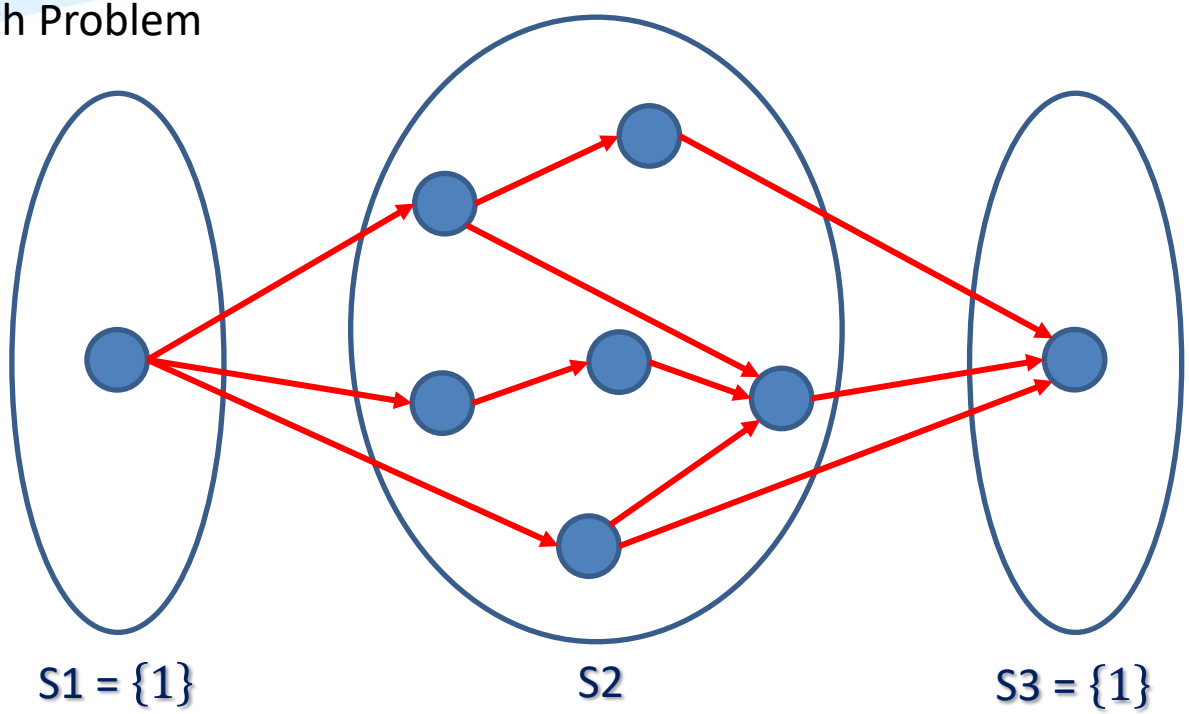
Transportation Problem



No intermediate point/facility

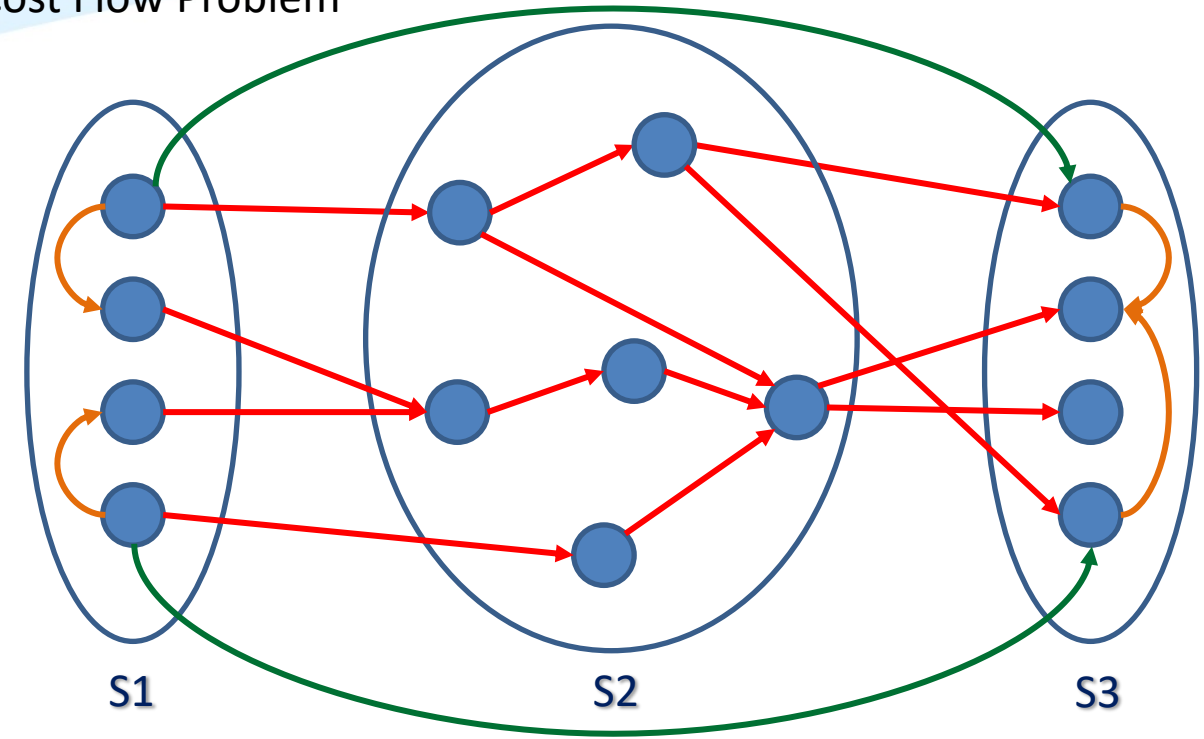
Classification of Transportation Problems

Shortest Path Problem



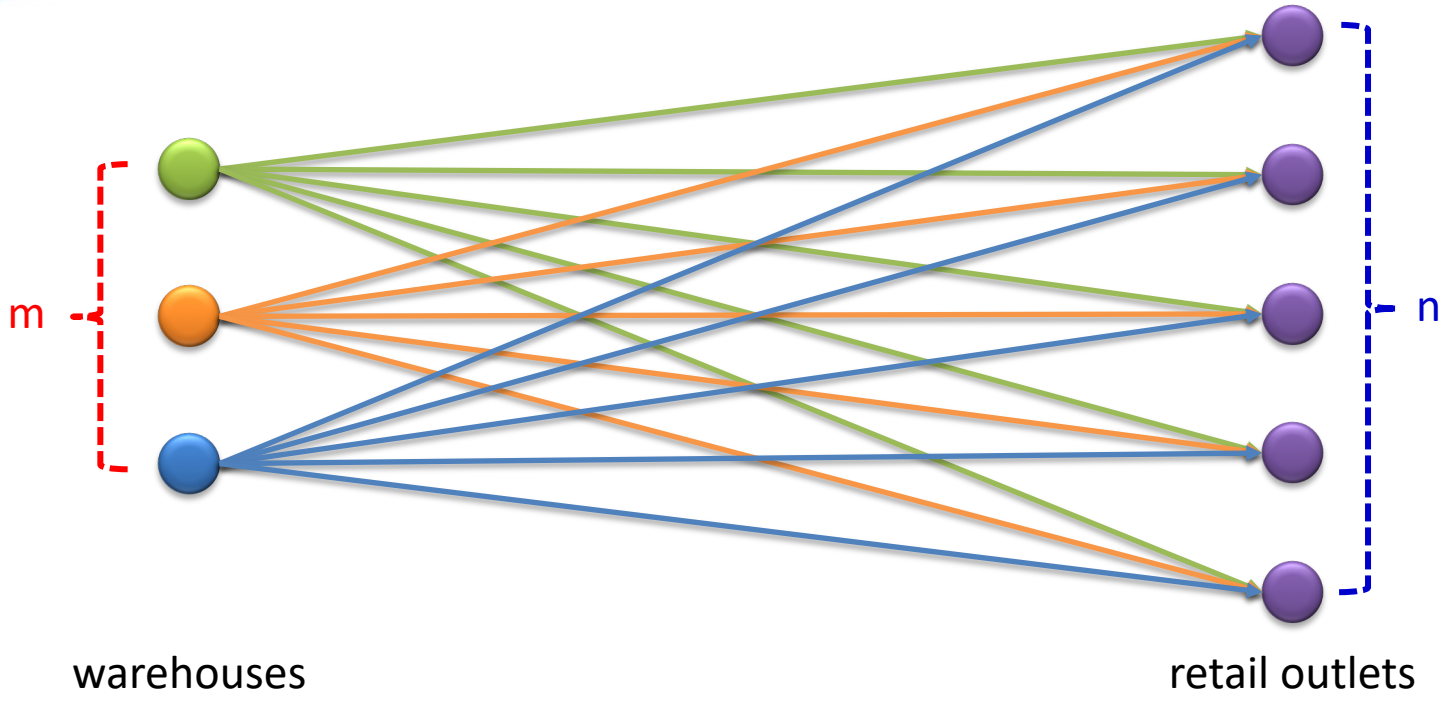
Classification of Transportation Problems

Minimum Cost Flow Problem



Classification of Transportation Problems

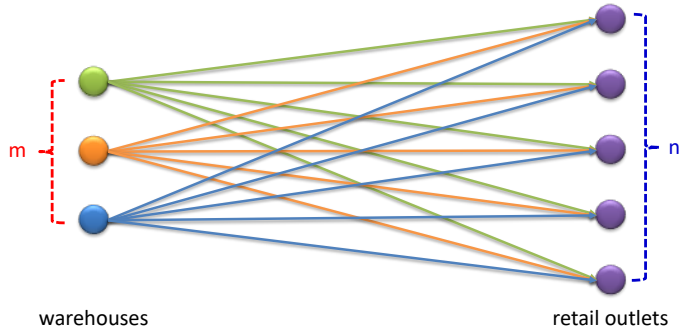
Standard Transportation Problem (An LP Formulation)



Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

- The total supply of the product from warehouse i is a_i , where $i = 1, 2, \ldots, m$.
- The total demand for the product at outlet j is b_j , where $j = 1, 2, \ldots, n$.
- The cost of sending one unit of the product from warehouse i to outlet j is equal to c_{ij} , where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. The total cost of a shipment is linear in the size of the shipment

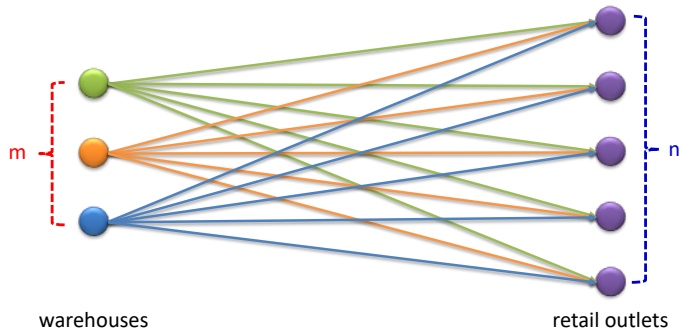


Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

The Decision Variables

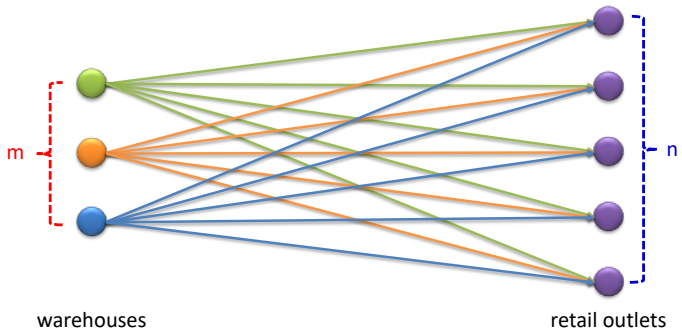
x_{ij} = the size of the shipment from warehouse i to outlet j, where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. This is a set of $m \times n$ variables.



Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

$$\text{Minimize} \quad \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$



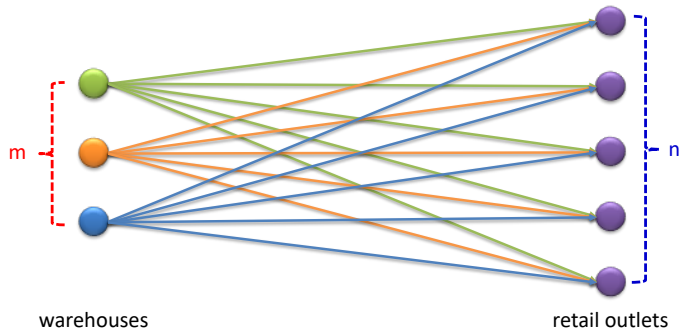
Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

The Constraints

Since the total supply from warehouse i is ai , the total outgoing shipment cannot exceed ai . That is, we must require

$$\sum_{j=1}^{n} x_{ij} \leq a_i , \quad \text{for } i = 1, 2, \ldots, m.$$



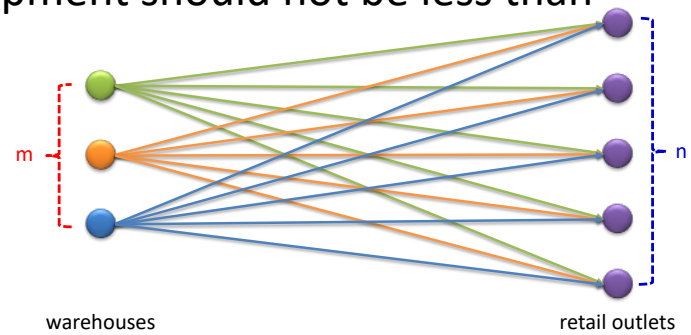
Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

The Constraints

Since the demand at outlet j is b_j , the total incoming shipment should not be less than b_j . That is, we must require

$$\sum_{i=1}^{m} x_{ij} \geq b_j , \quad \text{for } j = 1, 2, \ldots, n.$$



Classification of Transportation Problems

Standard Transportation Problem (An LP Formulation)

LP Formulation

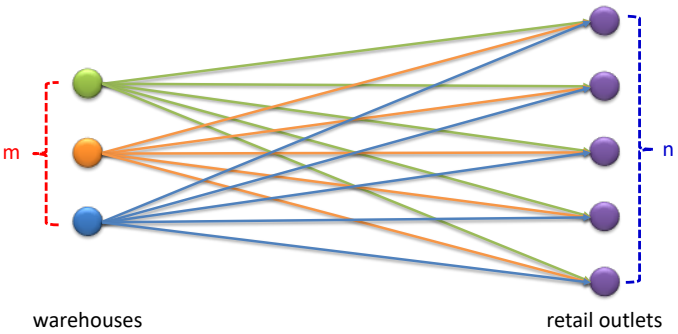
Minimize $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$

Subject to:

$$\sum_{j=1}^{n} x_{ij} \leq a_i \quad \text{for } i = 1, 2, \ldots, m$$

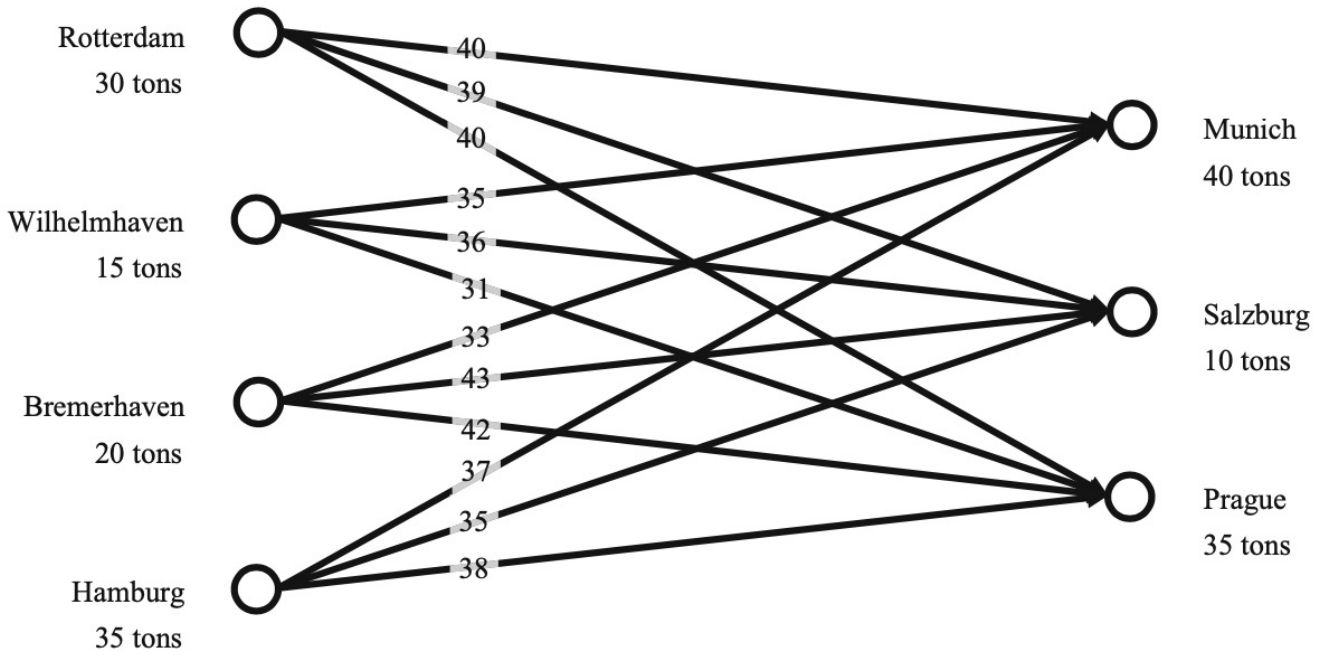
$$\sum_{i=1}^{m} x_{ij} \geq b_j \quad \text{for } j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0 \quad \text{for } i = 1, 2, \ldots, m \text{ and } j = 1, 2, \ldots, n$$



Classification of Transportation Problems

Standard Transportation Problem



Classification of Transportation Problems

Transportation Problem

$$\min\ 40x_{11} + 39x_{12} + 40x_{13} + 35x_{21} + 36x_{22} + 31x_{23} + 33x_{31} + 43x_{32} + 42x_{33} + 37x_{41} + 35x_{42} + 38x_{43}$$

Subject to:

$$x_{11} + x_{12} + x_{13} \leq 30$$

$$x_{21} + x_{22} + x_{23} \leq 15$$

$$x_{31} + x_{32} + x_{33} \leq 20$$

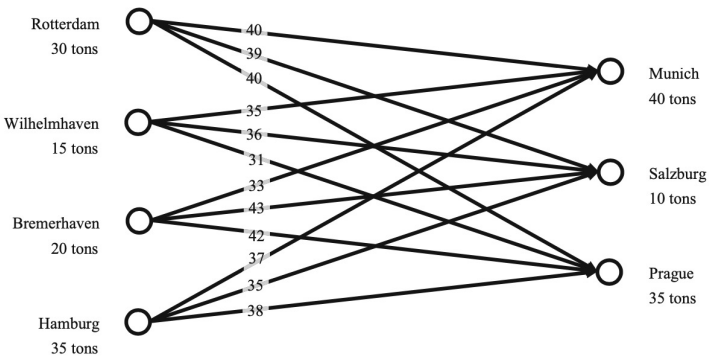
$$x_{41} + x_{42} + x_{43} \leq 35$$

$$x_{11} + x_{21} + x_{31} + x_{41} \geq 50$$

$$x_{12} + x_{22} + x_{32} + x_{42} \geq 30$$

$$x_{13} + x_{23} + x_{33} + x_{43} \geq 50$$

$$x_{ij} \geq 0$$



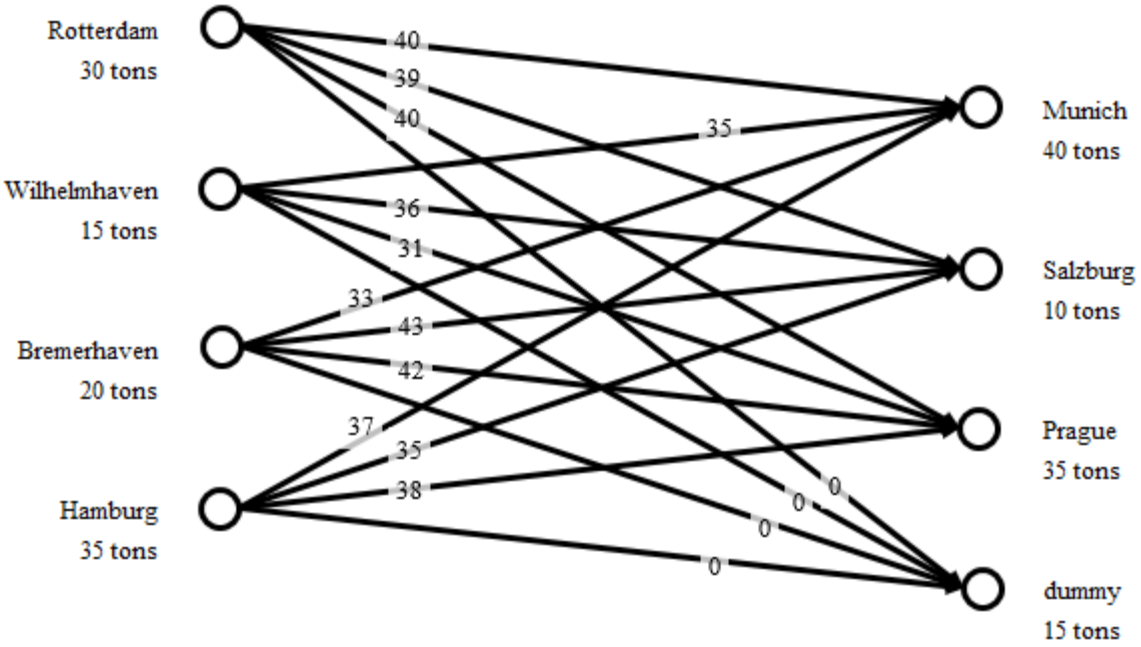
Classification of Transportation Problems

Transportation Problem

1. **Identifying a basic feasible solution (Construction methods)**
    - North-west corner method
    - Min cost method
    - Penalty cost (Vogel's approximation)
2. **Optimal solution (Improving methods)**
    - Stepping stone
    - MODI (Modified distribution)

Classification of Transportation Problems

Standard Transportation Problem



Classification of Transportation Problems

Transportation Problem

4	6	8	8	40
6	8	6	7	60
5	7	6	8	50
20	30	50	50	

Classification of Transportation Problems

Transportation Problem

$$\min\ 4x_{11} + 6x_{12} + 8x_{13} + 8x_{14} + 6x_{21} + 8x_{22} + 6x_{23} + 7x_{24} + 5x_{31} + 7x_{32} + 6x_{33} + 8x_{34}$$

Subject to:

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 40$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq 60$$

$$x_{31} + x_{32} + x_{33} + x_{34} \leq 50$$

$$x_{11} + x_{21} + x_{31} + x_{41} \geq 20$$

$$x_{12} + x_{22} + x_{32} + x_{42} \geq 30$$

$$x_{13} + x_{23} + x_{33} + x_{43} \geq 50$$

$$x_{14} + x_{24} + x_{34} + x_{44} \geq 50$$

$$x_{ij} \geq 0$$

Classification of Transportation Problems

Transportation Problem

North-west corner method

4 (1) 20	6 (2) 20	8 —	8	40
6 |	8 (3) 10	6 (4) 50	7 —	60
5 |	7 |	6 |	8 (5) 50	50
20	30	50	50	

$TC = 4 \times 20 + 6 \times 20 + 8 \times 10 +$

$6 \times 50 + 8 \times 50 = 980$

Classification of Transportation Problems

Transportation Problem

Min cost method

4 (1) 20	6 (2) 20	8	8
6	8 1	6 (3) 50	7 10 (5)
5	7 10 (4)	6 1	8 (6) 40

~~40~~ ~~20~~ 0

~~60~~ 10

~~50~~ 40

~~20~~ 0 ~~30~~ ~~10~~ 0 ~~50~~ 0 ~~50~~ 40

TC = 20x4 + 20x6 + 50x6 + 10x7 +

+ 10x7 + 40x8 = 960

Classification of Transportation Problems

Transportation Problem

Penalty cost (Vogel's approximation)

	D1	D2	D3	D4	
S1	4 (1) 20	6 (2) 20	8 —	8 —	40
S2	6	8	6 (6) 10	7 (5) 50	60 10 0
S3	5	7 (3) 10	6 (4) 40	8 —	50 40 0
	20 0	30 10 0	50 10	50 0	

	P1	P2	P3	P4	P5
	2	2	—	—	—
		1	1	1	1
	1	1	1	2	—

P1	1	1	0	1
P2	—	1	0	1
P3	—	1	0	1
P4	—	—	0	1
P5	—	—	6	7

TC = 20×4 + 20×6 + 10×7 + 40×6 + 50×7 + 10×6 = 920

Classification of Transportation Problems

Transportation Problem

Exercise

N-W Corner

40 100	47	80	100
72 100	36 100	58	200
24	61 100	71 200	300
200 100	200 100	200 0	

Min-Cost

40	47 1	80 100	100
72	36 200	58	200
24 200	61 1	71 100	300
200	200	200	

40	47	80	100
72	36	58	200
24	61	71	300
200	200	200	

Classification of Transportation Problems

Transportation Problem

Exercise

40	47	80	100
72	36	58	200
24	61	71	300
200	200	200	

Classification of Transportation Problems

Transportation Problem

Stepping Stone Method

	4 20	6 20	8	8
40	6	8	6 50	7 10
60	5	7 10	6	8 40
50				
	20	30	50	50

$\bar{C}_{13} = C_{13} - C_{23} + C_{24} - C_{34} + C_{32} - C_{12} =$

$8 - 6 + 7 - 8 + 7 - 6 = 2$

$\bar{C}_{21} = 2$

$\bar{C}_{22} = 2$

$\bar{C}_{31} = 0$

$\bar{C}_{33} = -2$

Selected

Classification of Transportation Problems

Transportation Problem

Stepping Stone Method

4 20	6 20	8	8
6	8	6 $50-\theta$	7 $10+\theta$
5	7 10	6 $+\theta$	8 $40-\theta$

20 30 50 50

$\bar{C}_{13} = C_{13} - C_{23} + C_{24} - C_{34} + C_{32} - C_{12} =$
$8 - 6 + 7 - 8 + 7 - 6 = 2$

40 $\bar{C}_{21} = 2$

60 $\bar{C}_{22} = 2$

50 $\bar{C}_{31} = 0$

$\bar{C}_{33} = -2$ Selected

$\theta = \min\{40, 50\} = 40$

Classification of Transportation Problems

Transportation Problem

Stepping Stone Method

4 20	6 20	8	8	40
6	8	6 10	7 50	60
5	7 10	6 40	8	50
20	30	50	50	

$\bar{c}_{13} = 3$

$\bar{c}_{14} = 2$

$\bar{c}_{21} = 1$

$\bar{c}_{22} = 1$

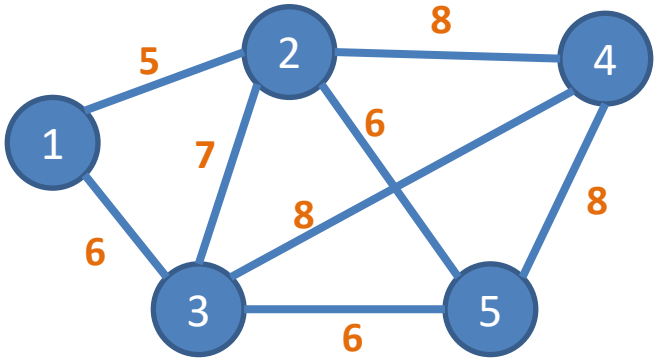
$\bar{c}_{31} = 0$

$\bar{c}_{34} = 1$

all $\bar{c}_{ij} \geq 0$ → optimal

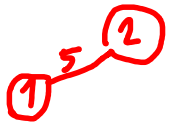
Classification of Transportation Problems

Minimum Spanning Tree – Prim's Algorithm



1

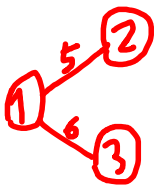
$S_1 = \{1,2\}$



$S_2 = \{3,4,5\}$

2

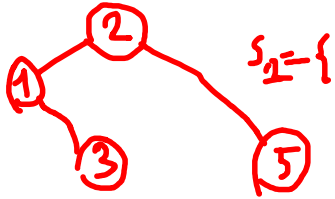
$S_1 = \{1,2,3\}$



$S_2 = \{4,5\}$

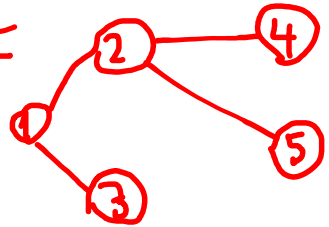
3

$S_1 = \{1,2,3,5\}$



$S_2 = \{4\}$

4



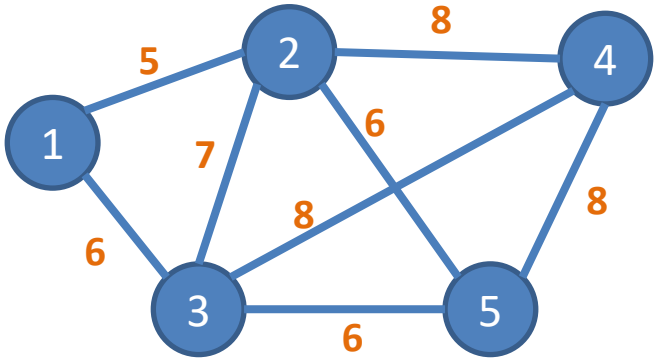
$S_1 = \{1,2,3,4,5\}$

$S_2 = \{\}$

TC = 25

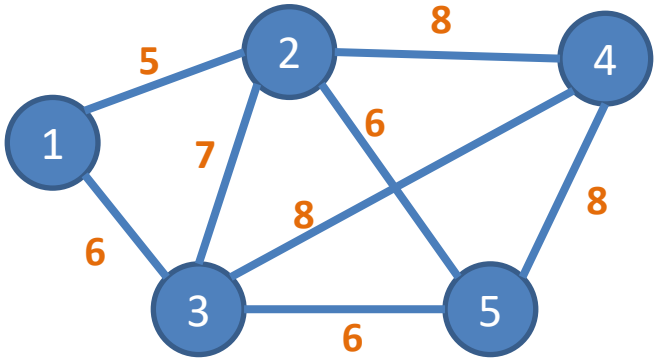
Classification of Transportation Problems

Minimum Spanning Tree – Prim's Algorithm



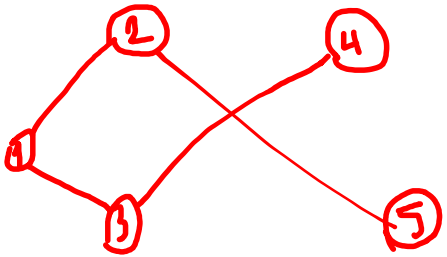
Classification of Transportation Problems

Minimum Spanning Tree – Kruskal's Algorithm



ARC	W	
1-2	5	✓
1-3	6	✓
2-5	6	✓
3-5	6	✗
2-3	7	✗
3-4	8	✓
4-5	8	✗
2-4	8	✗

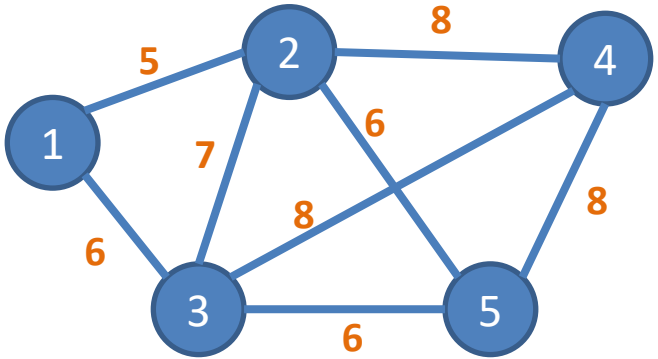
Sorted from min to Max



TC = 25

Classification of Transportation Problems

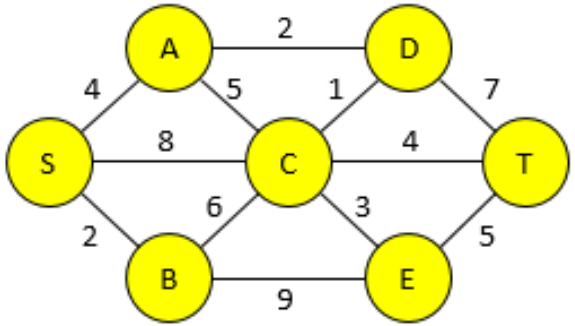
Minimum Spanning Tree – Kruskal's Algorithm



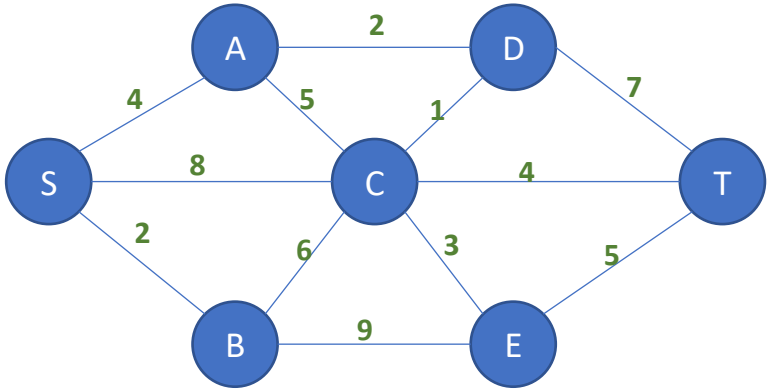
Classification of Transportation Problems

Shortest Path Problem – Dijkstra's Algorithm

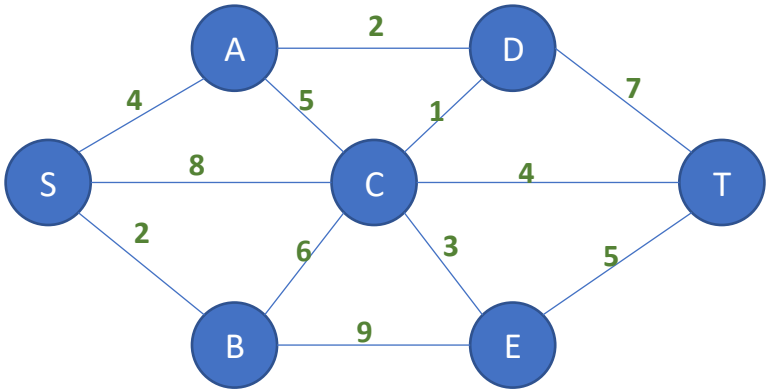
Exercise

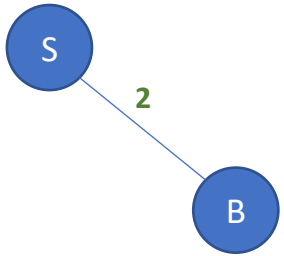


Shortest Path Problem - Dijkstra's Algorithm

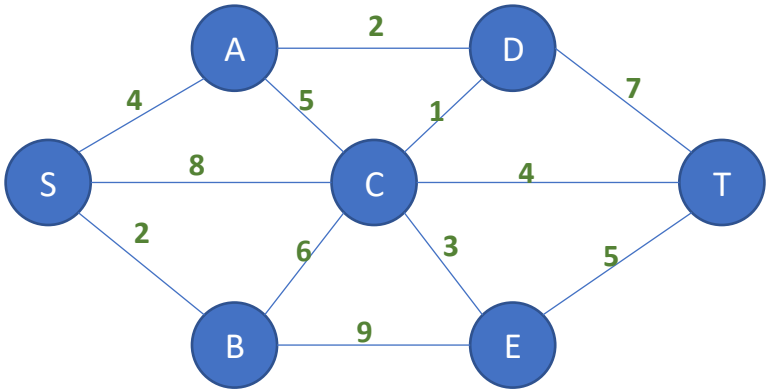


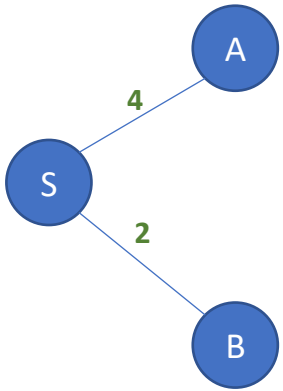
Shortest Path Problem - Dijkstra's Algorithm



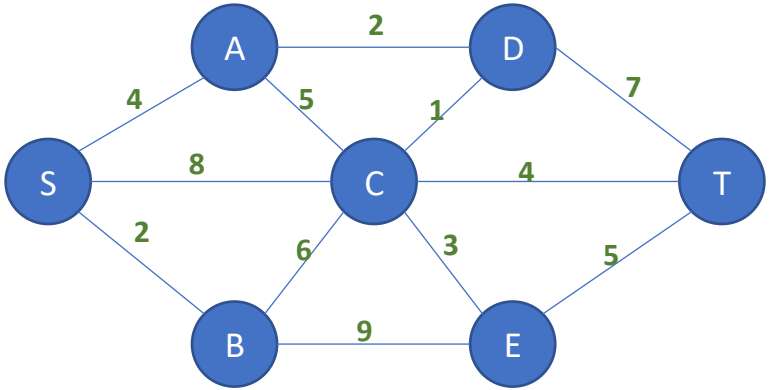


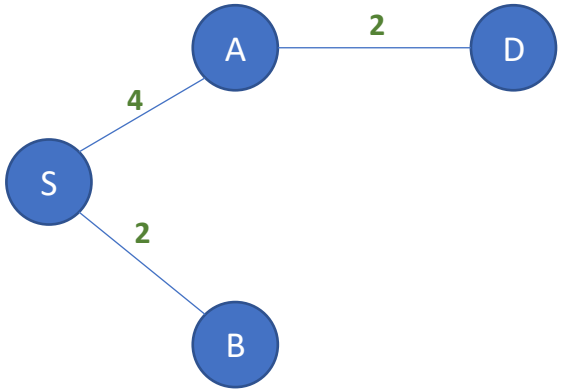
Shortest Path Problem - Dijkstra's Algorithm



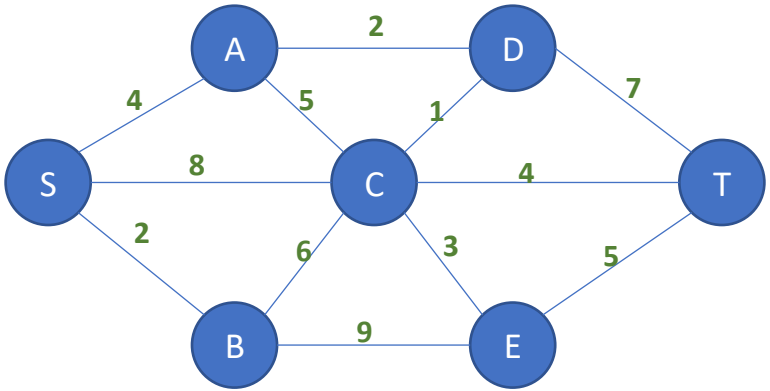


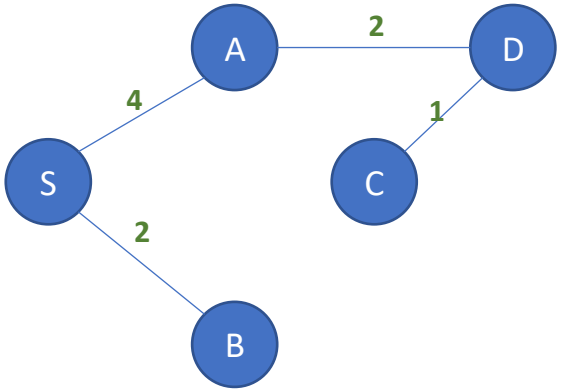
Shortest Path Problem - Dijkstra's Algorithm



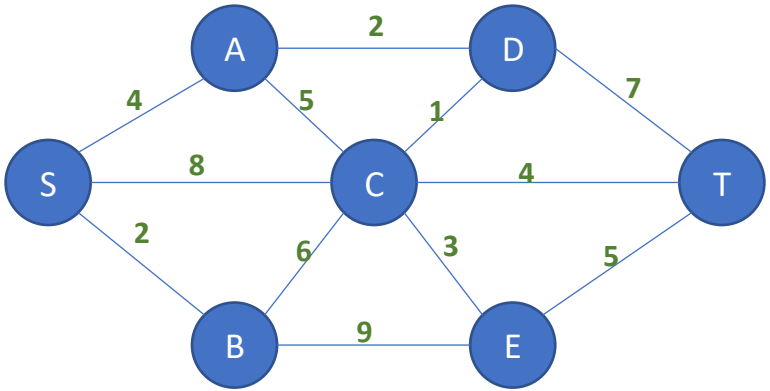


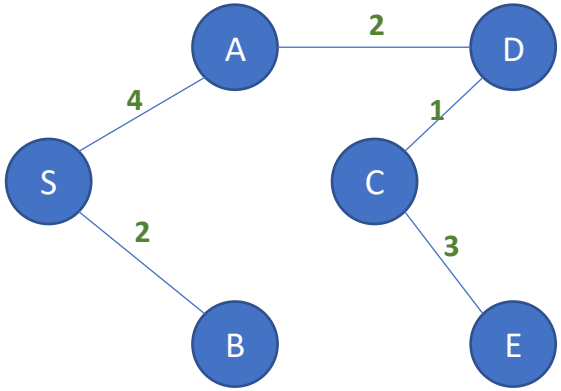
Shortest Path Problem - Dijkstra's Algorithm



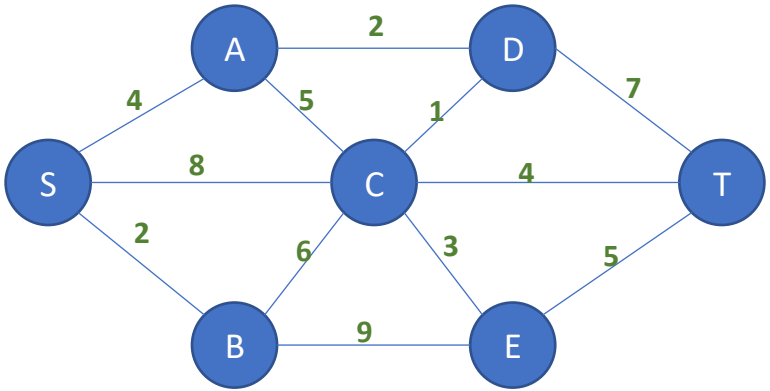


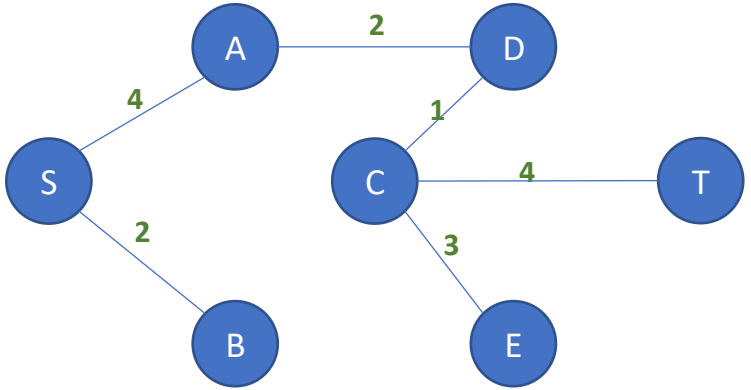
Shortest Path Problem - Dijkstra's Algorithm



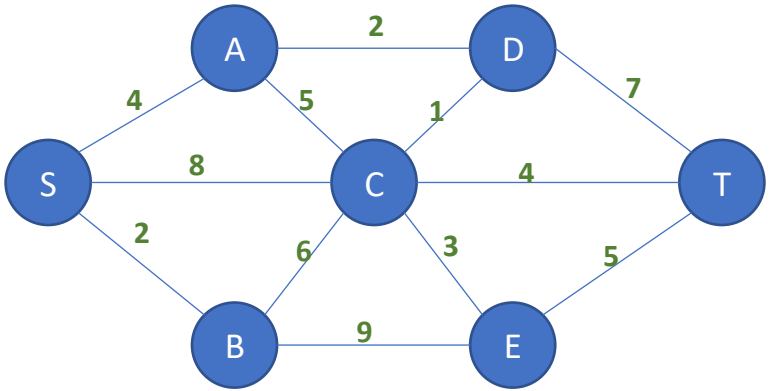


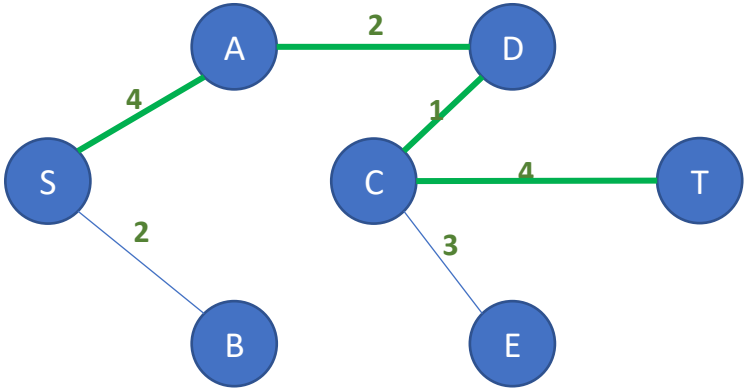
Shortest Path Problem - Dijkstra's Algorithm





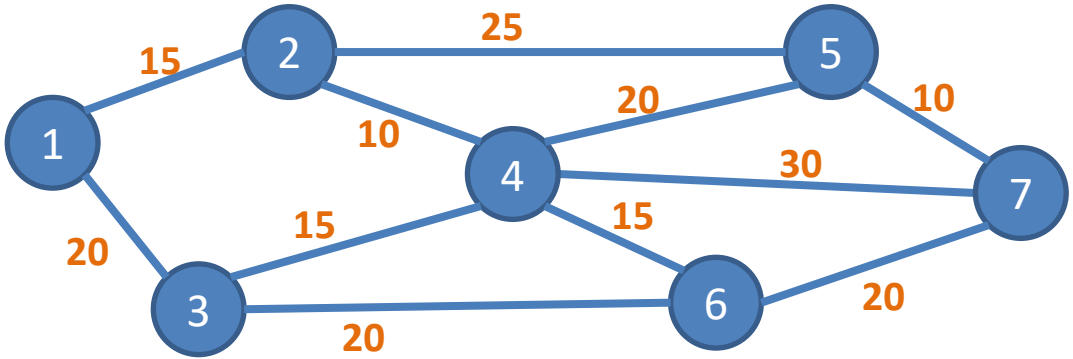
Shortest Path Problem - Dijkstra's Algorithm





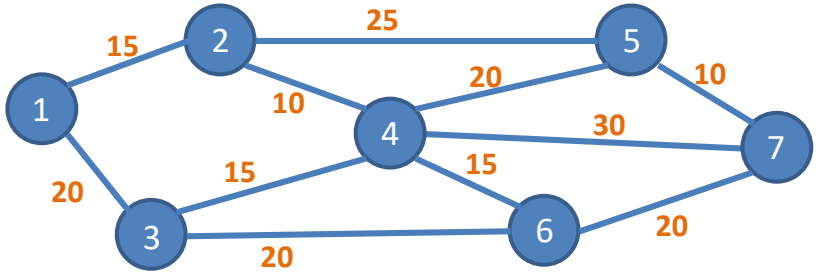
Classification of Transportation Problems

Shortest Path Problem – Dijkstra's Algorithm



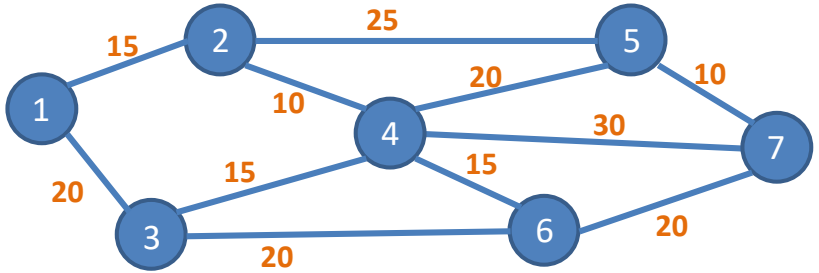
Classification of Transportation Problems

Shortest Path Problem – Dijkstra's Algorithm



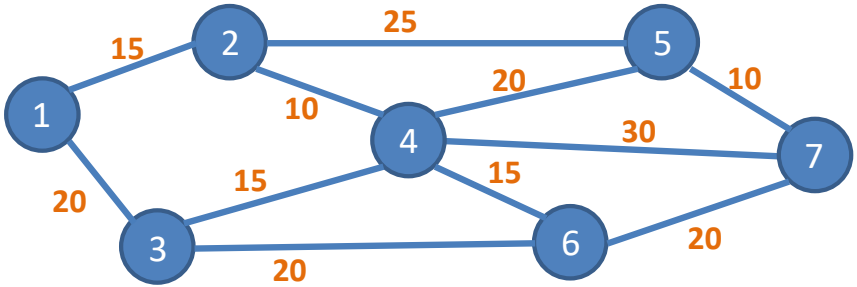
Classification of Transportation Problems

Shortest Path Problem – Dijkstra's Algorithm



Classification of Transportation Problems

Shortest Path Problem



$$\min\ 15x_{12} + 20x_{13} + 10x_{24} + 15x_{34} + 25x_{25} + 20x_{36} + 20x_{45} + 30x_{47} + 15x_{46} + 10x_{57} + 20x_{67}$$

Subject to:

$$x_{12} + x_{13} = 1$$

$$-x_{12} + x_{24} + x_{25} = 0$$

$$-x_{13} + x_{34} + x_{36} = 0$$

$$-x_{24} - x_{34} + x_{45} + x_{46} + x_{47} = 0$$

$$-x_{25} - x_{45} + x_{57} = 0$$

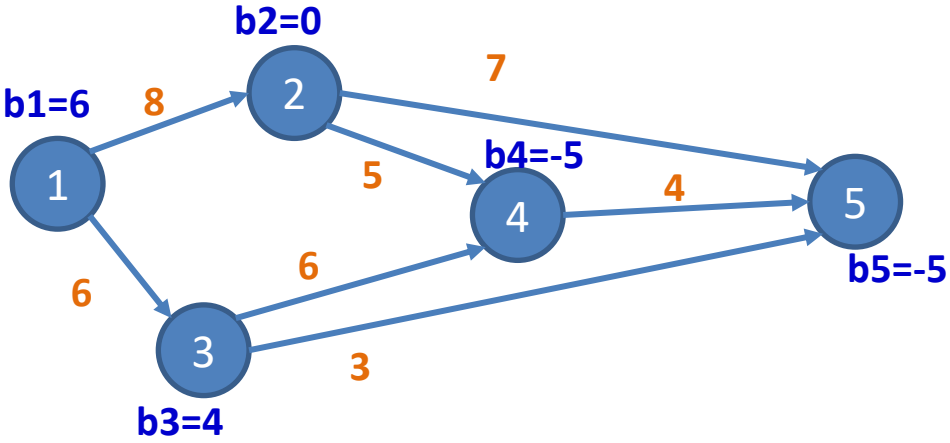
$$-x_{36} - x_{46} + x_{67} = 0$$

$$-x_{67} - x_{47} - x_{57} = -1$$

$$x_{ij} = 0,1$$

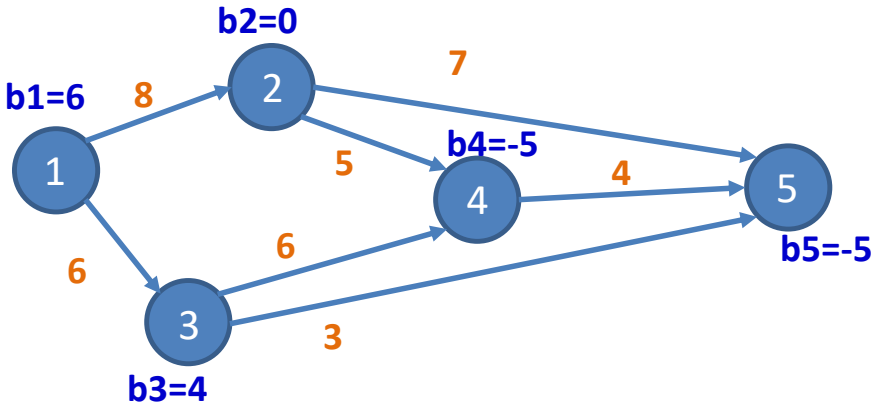
Classification of Transportation Problems

Minimum Cost Flow Problem



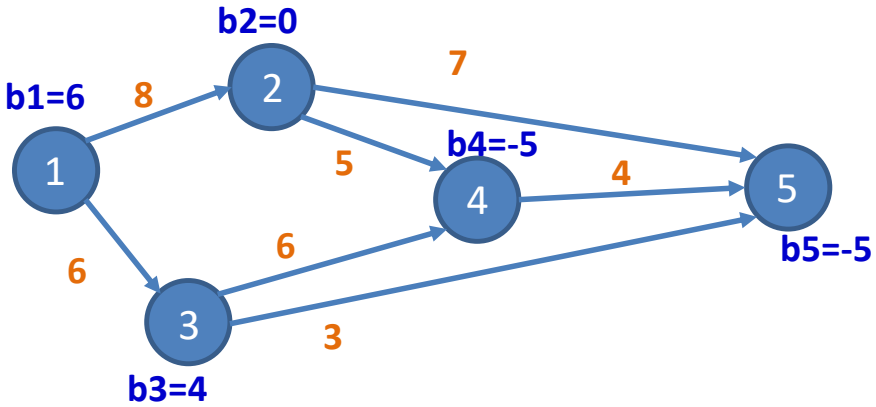
Classification of Transportation Problems

Minimum Cost Flow Problem



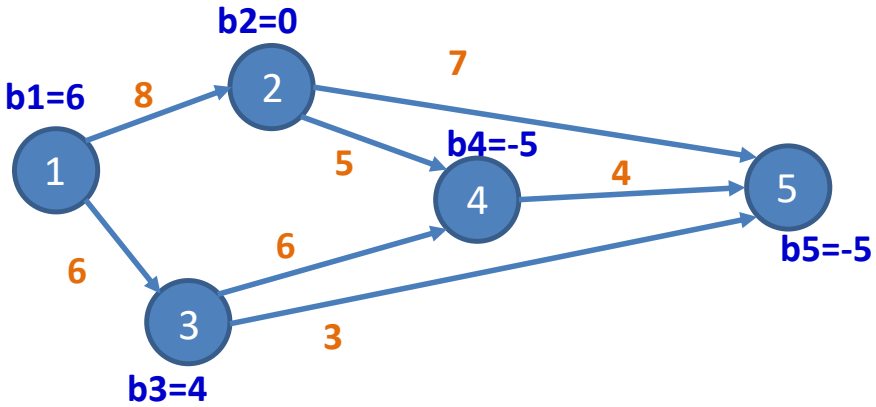
Classification of Transportation Problems

Minimum Cost Flow Problem



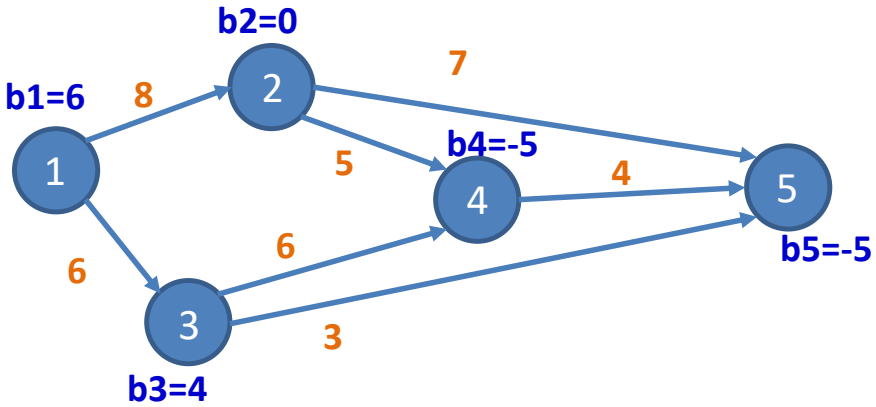
Classification of Transportation Problems

Minimum Cost Flow Problem



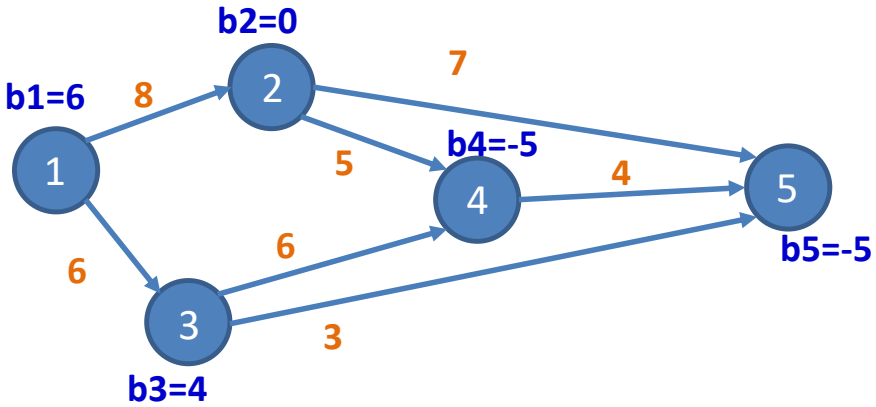
Classification of Transportation Problems

Minimum Cost Flow Problem



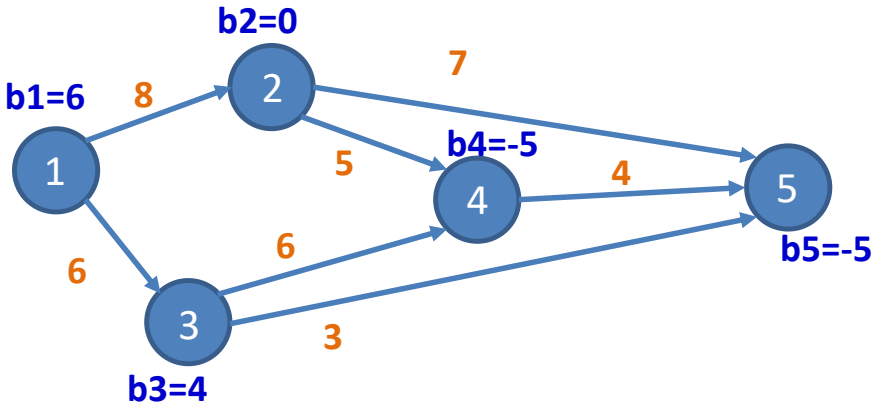
Classification of Transportation Problems

Minimum Cost Flow Problem



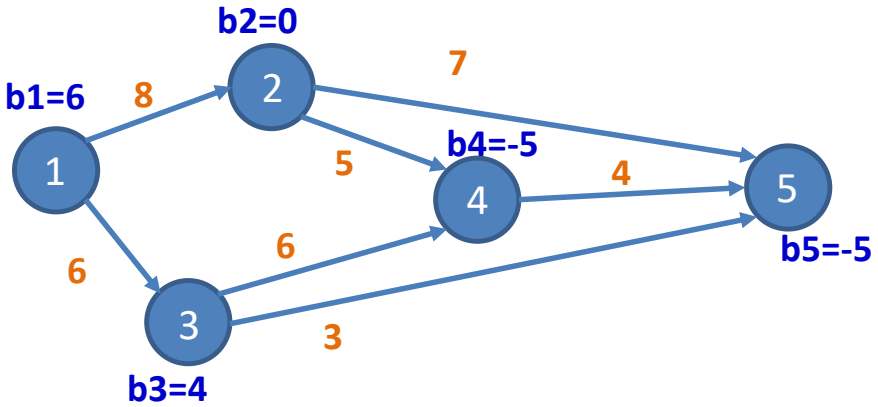
Classification of Transportation Problems

Minimum Cost Flow Problem



Classification of Transportation Problems

Minimum Cost Flow Problem



$$\min\ 8x_{12} + 6x_{13} + 5x_{24} + 6x_{34} + 7x_{25} + 3x_{35} + 4x_{45}$$

Subject to:

$$x_{12} + x_{13} = 6$$

$$-x_{12} + x_{24} + x_{25} = 0$$

$$-x_{13} + x_{34} + x_{35} = 4$$

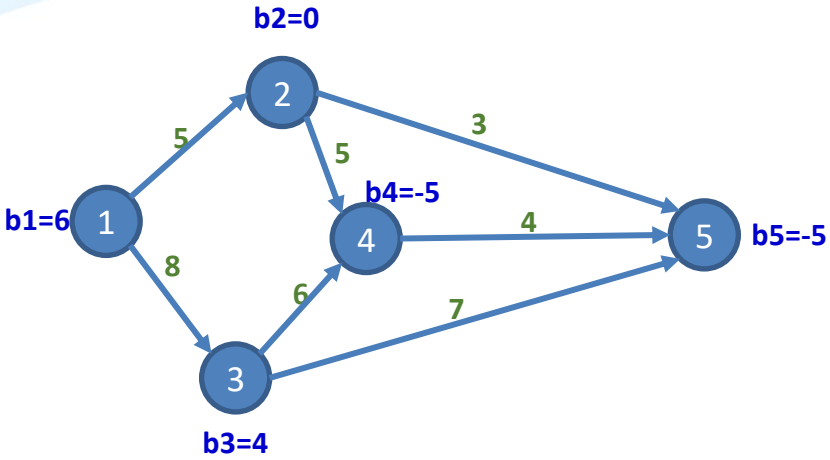
$$-x_{24} + x_{34} - x_{45} = -5$$

$$-x_{25} - x_{35} - x_{45} = -5$$

$$x_{ij} \geq 0$$

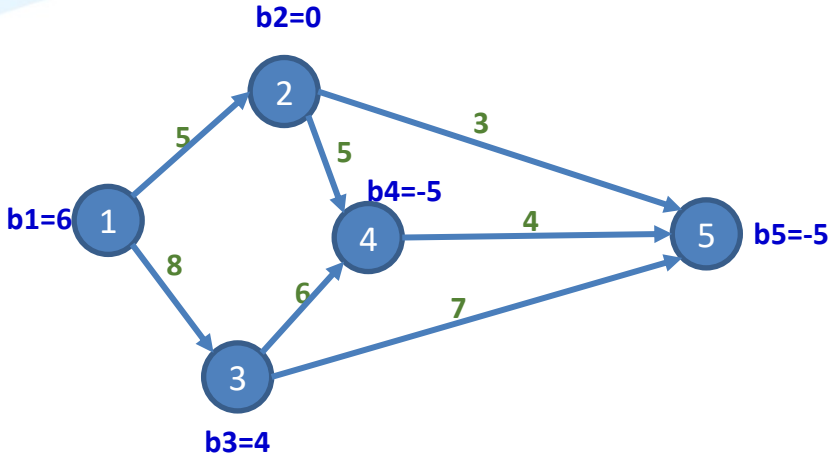
Classification of Transportation Problems

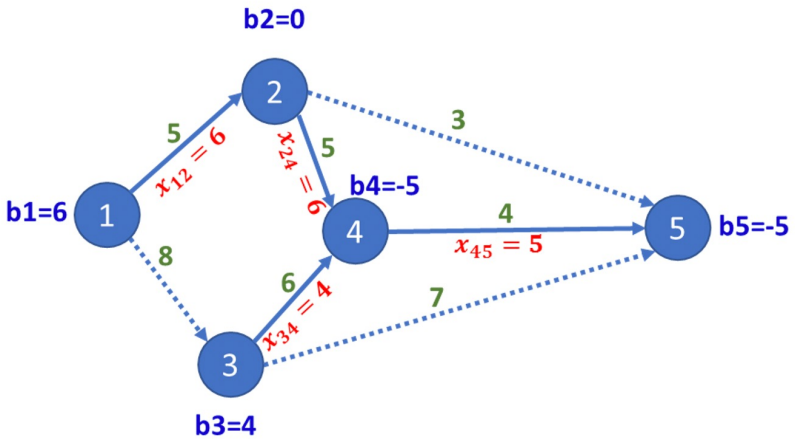
Minimum Cost Flow Problem



Classification of Transportation Problems

Minimum Cost Flow Problem





Classification of Transportation Problems

Traveling Salesman Problem

Nearest neighbour

		1	2	3	4	5
1	1		10	8	9	7
3	2	10		10	5	6
5	3	8	10		8	9
4	4	9	5	8		6
2	5	7	6	9	6	

1 → 5 → 2 → 4 → 3

Classification of Transportation Problems

Traveling Salesman Problem – Nearest Neighbour

	1	2	3	4	5
1		10	8	9	7
2	10		10	5	6
3	8	10		8	9
4	9	5	8		6
5	7	6	9	6	

Classification of Transportation Problems

Vehicle Routing Problem

Parameters: n = number of points (1 - depot, $2, \dots, n$ - clients)
d_{ij} = distance from point i to point j
D_i = demand of client i
C = capacity of each truck

Variables: x_{ij} = 1 if a truck goes from node i to node j (binary)
f_{ij} = number of units in a truck going from node i to node j

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$$

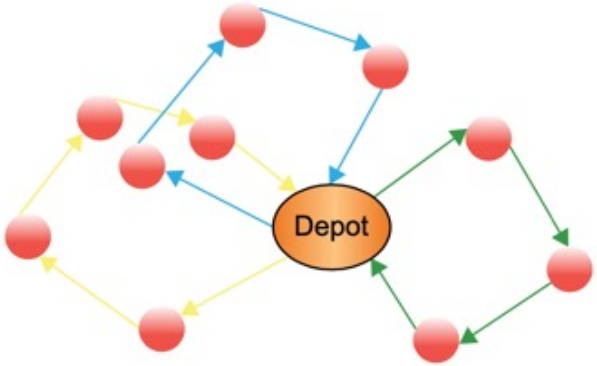
$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i = 2, \dots, n$$

$$\sum_{j=1}^{n} x_{ji} = 1 \quad \forall i = 2, \dots, n$$

$$\sum_{j=1}^{n} f_{ji} - \sum_{j=1}^{n} f_{ij} = D_i \quad \forall i = 2, \dots, n$$

$$0 \le f_{ij} \le C x_{ij} \quad \forall i, j = 1, \dots, n$$

$$x_{ij} \in \{0,1\} \quad \forall i, j = 1, \dots, n$$



Classification of Transportation Problems

Vehicle Routing Problem

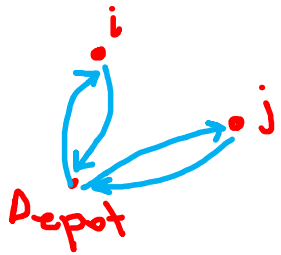
	0	1	2	3	4	5	6
0	-	20	18	14	16	12	19
1		-	22	18	30	26	28
2			-	32	20	22	21
3				-	20	22	21
4					-	30	22
5						-	26
6							-

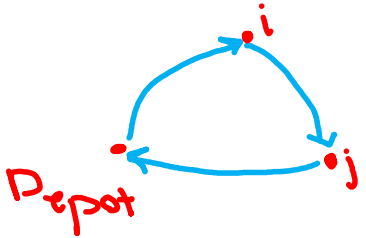
q= 4, 6, 3, 5, 3, 6

Q= 15

Total Distance

$TD = 2d_{oi} + 2d_{oj}$





$TD = d_{oi} + d_{oj} + d_{ij}$

$S_{ij} = d_{oj} + d_{oj} - d_{ij}$

Saving

Classification of Transportation Problems

Vehicle Routing Problem

	0	1	2	3	4	5	6
0	-	20	18	14	16	12	19
1		-	22	18	30	26	28
2			-	32	20	22	21
3				-	20	22	21
4					-	30	22
5						-	26
6							-

q= 4, 6, 3, 5, 3, 6 Q=15

i - j	S_{ij}	i - j	S_{ij}
1-2	16	1-4	6
1-3	16	1-5	6
2-6	16	5-6	5
2-4	14	3-5	4
3-6	12	4-6	3
4-6	11	2-3	0
3-4	10	4-5	-2
2-5	8		

Classification of Transportation Problems

Vehicle Routing Problem

	0	1	2	3	4	5	6
0	-	20	18	14	16	12	19
1		-	22	18	30	26	28
2			-	32	20	22	21
3				-	20	22	21
4					-	30	22
5						-	26
6							-

q= 4, 6, 3, 5, 3, 6 Q=15

i - j	S_{ij}	i - j	S_{ij}
✓ 1-2	16	x 1-4	6
✓ 1-3	16	x 1-5	6
x 2-6	16	✓ 5-6	5
x 2-4	14	x 3-5	4
x 3-6	12	✓ 4-6	3
x 4-6	11	+ 2-3	0
x 3-4	10	x 4-5	-2
x 2-5	8		

Vehicle 1:
1-2
1-3

Vehicle 2:
5-6
4-6

Classification of Transportation Problems

Vehicle Routing Problem

q= 4, 6, 3, 5, 3, 6 Q=15

	0	1	2	3	4	5	6
0	-	20	18	14	16	12	19
1		-	22	18	30	26	28
2			-	32	20	22	21
3				-	20	22	21
4					-	30	22
5						-	26
6							-

Vehicle 1

1-2

1-3

Route 1

Depot-2-1-3-Depot

Total distance of Vehicle 1 = 72

Vehicle 2

5-6

4-6

Route 2

Depot-5-6-4-Depot

Total distance of Vehicle 2 = 86

Total distance of all vehicles = 158