Received: 21 October 2022 | Revised: 20 February 2023 | Accepted: 21 February 2023
DOI: 10.1002/mcda.1805

RESEARCH ARTICLE

WILEY

PROMETHEE γ: A new PROMETHEE based method for partial ranking based on valued coalitions of monocriterion net flow scores

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Abstract

Multicriteria decision aid consists of helping decision makers to compare (rank, choose, sort, etc.) different alternatives which are evaluated on conflicting criteria. In the last decades, numerous decision aid methods have been developed. Three main categories of decision aid methods are usually considered: the aggregating, interactive and outranking methods. While aggregation methods produce a complete ranking of the set of alternatives, outranking methods usually allow some pairs of alternatives to remain incomparable. This happens either if the two alternatives present some strong conflicting information or if there are not enough elements in the decision problem to state the preference or indifference between them. A well-known family of outranking procedures are the PROMETHEE methods. The aim of this work is first to provide an analysis of the incomparability relation produced by PROMETHEE I. From our point of view, some shortcomings of this incomparability relation are presented. Then, a new method based on the comparison of weighted coalitions of mono-criterion net flow score differences, called PROMETHEE γ, is proposed. PROMETHEE γ and PROMETHEE I are then further compared.

KEYWORDS
incomparability, multiple criteria analysis, outranking relations, PROMETHEE

1 | INTRODUCTION

Multicriteria decision aid consists in helping decision makers to compare a set of alternatives denoted $A=\{a_1,a_2,\ldots,a_n\}$ which are evaluated using a family $F=\{f_1,\cdots,f_k\}$ of k conflicting criteria. These last decades, numerous decision aid methods have been developed. They can be divided into three main families: aggregating, outranking and interactive methods (Vincke, 1992).

Aggregating methods are based on the assumption that decision makers attempt to maximize some aggregating function U such that (Dyer, 2005):

$$U(a_i)=U(f_1(a_i),\ldots,f_k(a_i)) \qquad \forall a_i \in A. \tag{1}$$

Once the aggregated score of each alternative has been computed, these can be exploited to build $\langle P,I\rangle$ or $\langle P,Q,I\rangle$ preference structures, with P, Q and I representing respectively the preference, weak preference and indifference relations (Tsoukias et al., 2016). In this work, we are not going to consider the weak preference relation but only consider $\langle P,I\rangle$ preference structures. These preference structures can be characterized by $R=P\cup I$, a complete and transitive relation on A.

An example of a simple preference structure that can be obtained with aggregating methods is:

$$\begin{aligned} a_iPa_j &\Leftrightarrow U(a_i)>U(a_j),\\ a_iIa_j &\Leftrightarrow U(a_i)=U(a_j). \end{aligned} \tag{2}$$

or, using the characteristic relation R:

$$a_iRa_j \Leftrightarrow U(a_i)\geq U(a_j). \tag{3}$$

The construction of a preference structure characterized by a complete relation is not always desired nor always possible (Roy and

Vincke, 1984). Indeed, some decision problems do not provide enough information to enable us to state a clear preference or indifference relation between some pairs of alternatives. Furthermore, stating the incomparability between pairs of alternatives can bring some interesting insights about the decision problem (Vincke, 1992). Outranking methods have been developed according to these considerations. These methods allow to build an incomplete outranking relations S characterizing preference structures $\langle P,I,J \rangle$ (with J being the incomparability relation).

In this work, we are going to focus on a specific family of outranking methods called PROMETHEE (Brans and Vincke, 1985). As attested by (Behzadian et al., 2010) and (Mareschal, 2020), these methods are widely used in practice. However, we shall see that PROMETHEE I possesses features that are, according to the authors, not desirable. These features are the following ones:

- In decision problems characterized by alternatives evaluated on ordinal scales (and with all the alternatives of the problem having different evaluations), incomparability cannot be modeled using PROMETHEE I;
- Such as in PROMETHEE II, the indifference relation of PROMETHEE I is defined in a very strict way. This leads to some counter-intuitive incomparability situations;
- The modifications of some preferential parameters in PROMETHEE I have counter-intuitive results on the preference structure produced.

For these reasons, a new method based on the comparison of weighted coalitions of mono-criterion net flow score differences, called PROMETHEE γ, will be presented. It will be shown that PROMETHEE γ overcomes these shortcomings and it will be further compared to PROMETHEE I.

The rest of this paper is organized as follows. In Section 2, some background information about outranking relations and about the incomparability relation is provided. Furthermore, a list of distinctive features is given that should, according to the authors, be expected from outranking relations. In Section 3, a short introduction to the PROMETHEE methods will be provided. In particular, the shortcomings of PROMETHEE I mentioned here above will be explained and illustrated. In Section 4, PROMETHEE γ will be presented. Finally, a comparison between this new method and PROMETHEE I will be performed in Section 5.

2 | OUTRANKING METHODS AND THE INCOMPARABILITY RELATIONS

There is no formal definition of the "outranking methods" family (Bouyssou et al., 2006). However, most outranking methods possess some of the following characteristics:

1. The quality of each alternative is assessed by performing pairwise comparisons between pairs of alternatives instead of assigning to each alternative a unique numerical evaluation on a common scale (Pirlot, 1997);
2. The outranking relation characterizing the preference structure can be built using the "concordance-discordance" principle (Bouyssou, 1996). According to this principle, an alternative a_i will outrank another alternative a_j (a_iSa_j) if "*given what is known about the decision-maker's preferences and given the quality of the evaluations of the actions and the nature of the problem, there are enough arguments to decide that a_i is at least as good as a_j, while there is no essential reason to refute that statement*" (Vincke, 1992; Roy, 1974);
3. The concordance-discordance principle is linked with, but not equivalent to, the notion of non compensatory outranking relations. A small disadvantage of a_i over a_j on some criteria can be compensated by a small advantage of a_i over a_j on some other criteria. However, large disadvantages cannot be compensated (Pirlot, 1997).

The particularity of outranking relations is that they usually lead to preference structures which are neither transitive nor complete (Bouyssou, 1996). These preference structures are characterized by an outranking relation S:

$$\begin{array}{lcccc} a_iPa_j & \Leftrightarrow & a_iSa_j & \wedge & \neg(a_jSa_i), \\ a_iIa_j & \Leftrightarrow & a_iSa_j & \wedge & a_jSa_i, \\ a_iJa_j & \Leftrightarrow & \neg(a_iSa_j) & \wedge & \neg(a_jSa_i). \end{array} \tag{4}$$

Some outranking methods, such as PROMETHEE, also produce valued outranking relations (denoted μ):

$$\mu : A \times A \mapsto [0,1]. \tag{5}$$

These outranking relations are generally complete and can have different significations according to the outranking method. They can for instance represent the credibility that an alternative is outranking another one or they can give an indication of how much one alternative is preferred to another.

Whether they are valuated or not, it is necessary to understand what incomparability represents and in particular what the consequences are of stating incomparability instead of indifference or preference to be able to exploit these outranking relations correctly. Philosopher R. Chang defined incomparability as follows: "*Two items are incomparable if it is false that any positive, basic, binary value relation holds between them with respect to a covering consideration, 'V'*" Chang (2015). In the domain of multicriteria decision aid, the covering consideration 'V' simply consists of all criteria considered. The positive, basic binary value relations being the preference and the indifference relations (as well as the weak preference relation when this relation is considered). This definition matches the ones provided by Roy and Vincke (1984); Roy (1985) whose preference, indifference and incomparability relation definitions can be seen in Table 1.

Authors have pointed out several causes that can lead to incomparability relations between pairs of alternatives in a multi-criteria

TABLE 1 Definition of the preference, indifference and incomparability relations Roy and Vincke (1984); Roy (1985).

Binary relation	Definition
Preference	There exist clear and positive reasons to justify that one of the two alternatives is preferred over the other one;
Indifference	There exist clear and positive reasons to justify the equivalence of the two alternatives;
Incomparability	The alternatives are not comparable as there is a lack of clear and positive reasons to justify one of the above situations.

context. The two principal reasons being the lack of information to compare the two alternatives or when the two alternatives present strong conflicting information Tsoukias et al. (2016); Tsoukias and Vincke (1997); Vincke (1992); Roy (1985).

In this work, situations where a lack of information prevents the comparison of two alternatives will not be considered. We will therefore consider incomparability as a result of strong conflicting information. In this context, we will consider that two alternatives a_i and a_j should be considered as incomparable if there exist both strong reasons to state that a_i is better than a_j and also strong reasons to state that a_j is better than a_i **for some criterion or coalition of criteria**. Or, in other words, a_i and a_j will be considered as incomparable if both alternatives have significant advantages with respect to the other one **on a given coalition of criteria**.

According to these considerations, outranking methods should produce an incomparability relation respecting the following conditions:

1. As outranking methods are aimed at being non compensatory (or are least not totally compensatory), outranking methods should allow incomparability relations between pairs of "different" alternatives since both alternatives should have reasons to be preferred over the other one on specific coalitions of criteria. "Similar" alternatives where none of the two alternatives presents reasons to be preferred over the other one should be considered as indifferent.

Furthermore, for totally non-compensatory outranking methods:

2. Let us consider a decision problem for which a method produces an initial preference structure $\langle P^1, I^1, J^1 \rangle$ such that $\neg(a_i I^1 a_j)$. Suppose that the problem is changed such that the reasons for a_i to be preferred over a_j and/or the reasons for a_j to be preferred over a_i are reinforced. This increase can be due to a change in the preferential parameters of the problem (if the decision maker changes his mind), to a modification of the evaluation of an alternative (due to the discovery of a mistake in the initial evaluations for instance) or any other reason. Then the new preference structure obtained $\left\langle P^2, I^2, J^2 \right\rangle$ should be such that $\neg\left(a_i I^2 a_j\right)$;
3. Similarly, let us consider a problem leading to $\neg\left(a_i J^2 a_j\right)$. Suppose that this problem is changed such that the reasons for a_i to be preferred over a_j and/or the reasons for a_j to be preferred over a_i are alleviated. Then the new preference structure obtained $\left\langle P^2, I^2, J^2 \right\rangle$ should be such that $\neg\left(a_i J^2 a_j\right)$.

As already stated, these last two conditions are to be respected only for non compensatory methods.

Indeed, in the case of compensatory methods, even if a_i and a_j both have enough arguments to be mutually preferred one over the other and therefore to be considered as incomparable by a decision maker, this decision maker could still decide to state that a_i is preferred over a_j if the arguments in favor of a_i are significantly more important than the arguments in favor of a_j. In this situation, reducing the advantages of a_i over a_j could then lead to a situation of incomparability. This commentary will be further discussed in Sections 3 and 4.

Since these last two conditions apply only for non compensatory methods, and since it is not clear whether PROMETHEE is considered as non-compensatory or partially compensatory, is not clear whether PROMETHEE methods should respect these conditions. This is already a drawback of PROMETHEE in itself as it shows that the method is more difficult to interpret than usually suggested.

In the next section, the two main methods of the PROMETHEE family will be presented. It will be shown that the incomparability relation produced by PROMETHEE I does not respect the conditions stated here above. Furthermore, some other properties of PROMETHEE I that are, according to the authors, undesirable will be presented.

3 | PROMETHEE METHODS

3.1 | PROMETHEE II

In this section, a short introduction about the PROMETHEE II method is provided. For more details, the reader can refer to the description provided in (Brans and De Smet, 2016).

Without loss of generality, we will assume that the criteria of the decision problem must be maximized.

In a first step, PROMETHEE II starts by building a complete valued outranking relation π between each pair of alternatives from A. This outranking relation is also called pairwise preference index in the context of PROMETHEE and is computed as follows. First, the differences between the evaluations of each pair of alternatives on each criterion are computed:

$$d_c(a_i, a_j) = f_c(a_i) - f_c(a_j) \qquad \forall c = 1, \ldots, k. \tag{6}$$

In a second step, these differences are transformed into mono criterion preference indices $\mathscr{F}_c(d_c(a_i, a_j))$. $\mathscr{F}_c$ being a non-decreasing preference function in $[0,1]$. The exact forms of $\mathscr{F}_c$ are left to the decision maker to decide. The so-called linear preference function shown in Figure 1a is an example of such a function. To use it, the decision maker has to instantiate the values of two parameters, denoted q_c and p_c, which represent respectively an indifference and a

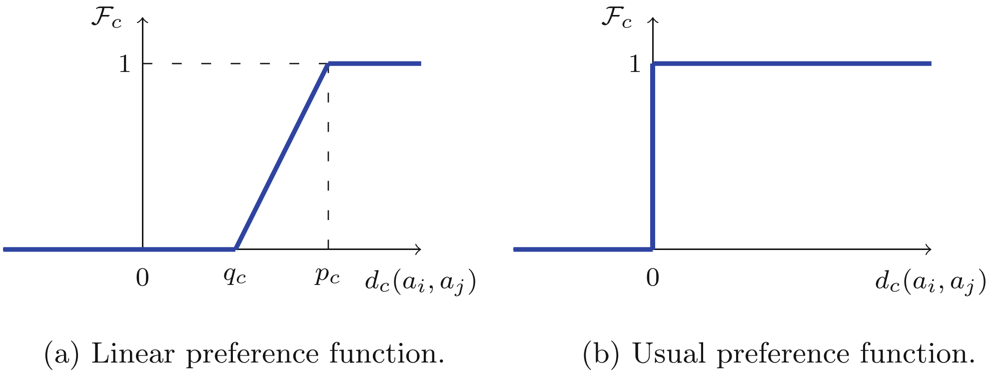


FIGURE 1 Two examples of preference functions.

strict preference threshold for criterion f_c. Another widely used preference function is the usual preference function shown in Figure 1b. For this function, no additional preferential parameter is necessary. $\mathscr{F}_c(d_c(a_i,a_j))$ represents how much a_i is preferred over a_j on criterion c. For simplicity reasons, $\mathscr{F}_c(d_c(a_i,a_j))$ will also be denoted π_{ij}^c in this work.

We suppose that the decision maker is able to provide positive and normalized weights denoted w_c for the different criteria. These weights are then used to compute the global pairwise preference index between a pair of alternatives and a_j denoted π_{ij}, as follows:

$$\pi_{ij} = \sum_{c=1}^{k} \pi_{ij}^c \cdot w_c. \tag{7}$$

The main idea of Promethee methods is to compute outranking flows scores which will represent how alternatives will behave in comparison to the whole set of alternatives. These outranking flow scores consist, for each alternative a_i, of a positive, a negative and a net flow score. The positive flow score represents how much a_i is preferred on average to the other alternatives:

$$\phi^+(a_i) = \frac{1}{n-1}\sum_{j=1}^{n} \pi_{ij}. \tag{8}$$

The negative flow score represents how much the other alternatives are preferred to a_i on average:

$$\phi^-(a_i) = \frac{1}{n-1}\sum_{j=1}^{n} \pi_{ji}. \tag{9}$$

The Promethee ii methods defines a unique score for each alternative defined as the difference between the positive and negative flow scores:

$$\phi(a_i) = \phi^+(a_i) - \phi^-(a_i) = \frac{1}{n-1}\sum_{j=1}^{n}\left(\pi_{ij} - \pi_{ji}\right). \tag{10}$$

A complete ranking can then be built according to this net flow scores:

$$\begin{aligned} &a_i P^{II} a_j \Leftrightarrow \phi(a_i) > \phi(a_j), \\ &a_i I^{II} a_j \Leftrightarrow \phi(a_i) = \phi(a_j). \end{aligned} \tag{11}$$

Unlike most other outranking methods, Promethee ii produces a ranking and therefore a $\langle P, I\rangle$ preference structure. If the decision maker does not require to build a complete but a partial ranking, another version of Promethee, Promethee i, can be used.

3.2 | Promethee I

Such as in Promethee ii, Promethee i starts by computing the pairwise preferences π between each pair of alternative as well as a positive and negative outranking flow score for each alternative such as indicated in equations 8 and 9. However, these scores are not aggregated in net flow scores but directly used to define the following preference structure:

$$\begin{aligned} &a_i P^I a_j \Leftrightarrow \begin{cases} \phi^+(a_i) \geq \phi^+(a_j) \wedge \phi^-(a_i) < \phi^-(a_j) \\ \vee \\ \phi^+(a_i) > \phi^+(a_j) \wedge \phi^-(a_i) \leq \phi^-(a_j) \end{cases} \\ &a_i I^I a_j \Leftrightarrow \phi^+(a_i) = \phi^+(a_j) \wedge \phi^-(a_i) = \phi^-(a_j) \\ &a_i J^I a_j \Leftrightarrow \begin{cases} \phi^+(a_i) < \phi^+(a_j) \wedge \phi^-(a_i) < \phi^-(a_j) \\ \vee \\ \phi^+(a_i) > \phi^+(a_j) \wedge \phi^-(a_i) > \phi^-(a_j) \end{cases} \end{aligned} \tag{12}$$

The preference structure produced by Promethee i can therefore be seen as the intersection of the two rankings produced by the positive and negative outranking flow scores respectively.

As seen in this section, even if Promethee methods and more specifically Promethee i are not based on the concordance-discordance principle, they are however often considered as non compensatory (Pirlot, 1997; Greco et al., 2021; Costa and Alves, 2020) or partially compensatory (Ishizaka and Resce, 2021) outranking methods since a very bad negative outranking flow score cannot be compensated by a very good positive outranking flow score (and vice-versa). However, as it will be seen hereunder, the intuition behind the incomparability relation developed in section 2 is not compatible with the incomparability relation obtained using Promethee i.

3.2.1 | Usual criterion

In this section, we will consider problems characterized by ordinal scales. In this context, computing differences of evaluations is meaningless. As a consequence, the only acceptable preference function is the usual one:

$$\pi_{ij}^{c} = \begin{cases} cll1 & if \quad f_c(a_i) > f_c(a_j), \\ 0 & else. \end{cases} \tag{13}$$

In this setting, the mono-criterion pairwise preference of one of the alternatives over the other is maximal as soon as its evaluation is strictly greater than the evaluation of the other alternative. It is therefore the type of preference function for which the mono-criterion preference indices take the highest possible values. We should therefore expect to get at least as much incomparability relations as with other types of preference functions (since strong monocriterion conflicts are more likely to happen than with other forms of preference functions). However, it will be shown hereunder that it is not the case. More specifically, we will show hereunder that it is impossible for two given alternatives to be considered as incomparable according to the PROMETHEE I method when using usual preference functions if all alternatives have different evaluations on any criterion. Since we suppose that all alternatives have different evaluations, we can compute the negative outranking flow score of any alternative a_i as follows:

$$\begin{aligned} \phi^-(a_i) &= \frac{1}{n-1}\sum_{a_x \in A}\sum_{c=1}^{k}\pi_{xi}^{c} \\ &= \frac{1}{n-1}\sum_{a_x \in A \setminus a_i}\sum_{c=1}^{k}\pi_{xi}^{c} \\ &= \frac{1}{n-1}\sum_{a_x \in A \setminus a_i}\sum_{c=1}^{k}\left(1-\pi_{ix}^{c}\right) \\ &= 1-\phi^+(a_i). \end{aligned}$$

and we can therefore deduce that:

$$\phi^+(a_i) > \phi^+(a_j) \Rightarrow \phi^-(a_i) < \phi^-(a_j).$$

In this setting, two pairs of alternatives can never be considered as incomparable according to PROMETHEE I. However, in PROMETHEE I, the use of ordinal criteria (with all alternatives having different evaluations) leads to the full compensations of the advantages and drawbacks on the different criteria.

3.2.2 | Particular cases of unexpected incomparability relations

Let us consider the decision problem represented in Table 2.

In this bi-criteria example, one can see that a_1 and a_4 are very different as they both have the best evaluation on one criterion and the worst evaluation on the second criterion. In addition, their difference of evaluation is higher than the strict preference threshold (p) on both criteria. On the other hand, a_2 and a_3 could be considered as more similar as they have small differences of evaluations on both criteria. However, when applying PROMETHEE I, a_2 and a_3 will be considered as incomparable while a_1 and a_4 are not. The respective positive and negative outranking flow scores of these alternatives can be seen in Table 3.

TABLE 2 Example of a decision problem with preference parameters.

A	$f_1(.)$	$f_2(.)$
a_1	10	0
a_2	4.75	5.25
a_3	5.25	4.75
a_4	0	10
a_5	4	1
q	0	0
p	3	3
w	0.4	0.6

TABLE 3 Positive and negative outranking flows of a_1 and a_4 (left) and a_2 and a_3 (right).

A	ϕ^+	ϕ^-	A	ϕ^+	ϕ^-
a_1	0.4	**0.5**	a_2	0.45	0.27
a_4	**0.6**	0.4	a_3	**0.46**	**0.28**

The bold values represent the largest value per column.

This can be related to the results found in (Dejaegere and De Smet, 2022) and (Dejaegere et al., 2022). Indeed, these work respectively show that rank reversal could happen only between pairs of alternatives having small differences of net flow scores and show that the net flow score procedure is respecting a *strict* form of monotonicity. Both results, as well as the example provided in Table 3 tend to indicate that the indifference relation in the context of PROMETHEE methods should be reconsidered.

3.2.3 | Increasing indifference threshold produces incomparability

In this section, we will consider that PROMETHEE I is applied using linear preference functions such as the one illustrated in Figure 1a. However, the same reasoning can be applied with any preference function which makes use of an indifference threshold.

Given the intuition of the incomparability relation as the situation where both alternatives have significant advantages over the other, the increase, for a given criterion c, of the indifference threshold q_c should not result in an increase of number of pairs of alternatives considered as incomparable. Indeed, increasing q_c can, for each pair of alternatives a_i and a_j, only decrease the pairwise preferences π_{ix}^{c} and π_{xj}^{c} for any $a_x \in A$ and similarly only decrease π_{jx}^{c} and π_{xi}^{c} for any $a_x \in A$ which should result only in the decrease of the arguments stating that

TABLE 4 Example of a decision problem with preference parameters.

A	$f_1(.)$	$f_2(.)$
a_1	8	2
a_2	3	8
a_3	5	5
a_4	1	9
q	1	1
p	6	6
w	$\frac{1}{2}$	$\frac{1}{2}$

TABLE 5 Positive and negative outranking flows after the application of PROMETHEE I for a_2 and a_3 when $q_2 = 1$.

A	ϕ^+	ϕ^-
a_2	0.266667	0.166667
a_3	0.200000	0.233333

TABLE 6 Positive and negative outranking flows after the application of PROMETHEE I for a_2 and a_3 when $q_2 = 3$.

A	ϕ^+	ϕ^-
a_2	0.200000	0.166667
a_3	0.133333	0.122222

a_i is preferred to a_j or that a_j is preferred to a_i respectively. However, this is not always the case as it will be shown in the example below.

Let us consider the decision problem represented in Table 4.

As shown in Table 5, the application of PROMETHEE I in this decision problem would lead to a preference of a_2 over a_3 as both the positive outranking flow of a_2 is larger than the one of a_3 and its negative outranking flow is smaller than the one of a_3.

However, as it can be seen in Table 6, this preference relation is transformed into an incomparability relation when q_2 is increased from 1 to 3. Indeed, in this setting, both the negative and positive outranking flows of a_2 are larger than the ones of a_3.

In order to circumvent the shortcomings listed in this section, a new variation of the PROMETHEE method will be proposed and studied in Section 4.

4 | PROMETHEE γ

Let us first note that the preference relation produced by using the net flow scores has already been studied. The pairwise preference aggregation (Marchant, 1996) and the net flow procedure (Bouyssou, 1992; Dejaegere et al., 2022) have been axiomatically characterized. Furthermore, (Mareschal et al., 2008) have shown that the net flow score is the score minimizing the sum of the squared deviations from the difference of pairwise preferences of the alternatives. Keyser and Peeters (1996) have shown that, such as most other outranking methods, PROMETHEE methods suffer from the rank reversal phenomenon: when one alternative is removed or added to the dataset, the respective order in the ranking of two other alternatives can be reversed. This phenomenon has however been mitigated in (Mareschal et al., 2008; Verly and Smet, 2013) and more recently in (Dejaegere and De Smet, 2022). We can therefore assume that even if PROMETHEE II should not be applied for any decision problem, there exists a solid foundation on the computation of the net flow scores.

For these reasons, a proposition of new variant of PROMETHEE called PROMETHEE γ extending the notion of net flow scores in order to model incomparabilities will be detailed in this section. This method will work by computing, for each pair of alternatives a_i and a_j, some aggregated pairwise preference indicators γ_{ij} and γ_{ji}. In accordance with the main ideas of the PROMETHEE methods (and its notion of flows), these indicators will not only depend on the mutual pairwise comparison of the concerned alternatives, but also on how they behave pairwise with all the other alternatives of the problem. Furthermore, it will be shown that these indicators reflect the net flow scores as the difference between γ_{ij} and γ_{ji} is equal to their difference of net flow scores.

This will be performed at the cost of the decision maker having to select three (at most) additional preference parameters. The exact method will be detailed hereunder.

4.1 | PROMETHEE γ presentation

For any possible type of preference function, the net flow score of each alternative obtained with PROMETHEE II can be rewritten:

$$\begin{aligned}\phi(a_i) &= \frac{1}{n-1}\sum_{j=1}^{n}\left(\sum_{c=1}^{k}\pi_{ij}^c\cdot w_c - \sum_{c=1}^{k}\pi_{ji}^c\cdot w_c\right)\\ &= \sum_{c=1}^{k} w_c\cdot\frac{1}{n-1}\sum_{j=1}^{n}\left(\pi_{ij}^c-\pi_{ji}^c\right)\\ &= \sum_{c=1}^{k} w_c\cdot\phi^c(a_i)\end{aligned}$$

with $\phi^c(a_i) = \frac{1}{n-1}\sum_{j=1}^{n}\left(\pi_{ij}^c - \pi_{ji}^c\right)$ representing the mono-criterion net flow of a_i on criterion c.

When comparing a_i and a_j, we will compute the two new preference indices γ_{ij} and γ_{ji} based on the mono-criterion net flow scores:

$$\gamma_{ij} = \sum_{f^c(a_i) > f^c(a_j)} w_c\cdot\left(\phi^c(a_i) - \phi^c(a_j)\right). \tag{14}$$

γ_{ij} (respectively γ_{ji}) represents the global advantages of a_i over a_j in the whole data set. It is the importance of the coalition of weights for which a_i is better than a_j with the weights being themselves weighed by the difference of mono-criterion net flow scores. The value of γ lies in the $[0,2]$ interval. When using usual criteria of even weights, γ_{ij} can be interpreted as the sum of the number of alternatives having their evaluations between a_i and a_j for all criteria where

a_i is better than a_j. This makes the interpretation of the difference between these two indicators as a difference of Borda scores for the net flow score procedure provided in Marchant (1996) still applicable. Therefore, having high values for both γ_{ij} and γ_{ji} is an indication of strong conflicting information. In this case, this pleads in favor of the incomparability of a_i and a_j.

We can show that the difference between these preferences is consistent with the difference of net flow scores obtained with PROMETHEE II:

$$\gamma_{ij} - \gamma_{ji} = \sum_{f^c(a_i) > f^c(a_j)} w_c \cdot \left(\phi^c(a_i) - \phi^c(a_j)\right) \tag{15}$$

$$- \sum_{f^c(a_j) > f^c(a_i)} w_c \cdot \left(\phi^c(a_j) - \phi^c(a_i)\right) \tag{16}$$

$$= \sum_{f^c(a_i) > f^c(a_j)} w_c \cdot \left(\phi^c(a_i) - \phi^c(a_j)\right) \tag{17}$$

$$+ \sum_{f^c(a_j) > f^c(a_i)} w_c \cdot \left(\phi^c(a_i) - \phi^c(a_j)\right) \tag{18}$$

$$= \sum_c w_c \cdot \left(\phi^c(a_i) - \phi^c(a_j)\right) \tag{19}$$

$$= \phi(a_i) - \phi(a_j). \tag{20}$$

In order to define the preference relation between the two alternatives, four different indicators $I_{ij}, J_{ij}, P_{ij}, P_{ji}$ will be computed using γ_{ij} and γ_{ji}. These indicators respectively represent the arguments in favor of stating that a_i and a_j should be considered indifferent, incomparable, that a_i should be preferred over a_j or that a_j should be preferred over a_i. They are computed as follows:

$$\begin{aligned} I_{ij} &= T_I - max(\gamma_{ij}, \gamma_{ji}), \\ P_{ij} &= (\gamma_{ij} - \gamma_{ji})/P_f, \\ P_{ji} &= (\gamma_{ji} - \gamma_{ij})/P_f, \\ J_{ij} &= min(\gamma_{ij}, \gamma_{ji}) - T_J \end{aligned} \tag{21}$$

with T_I, T_J and P_f being respectively a global indifference threshold, a global incomparability threshold and a global preference factor that must be chosen by the decision maker. Some comments about the selection of these factors will be provided in Section 4.2. Let us point out however that, in order to make sense, T_I should be lower or equal to T_J.

The idea is then to assign each pair of alternatives to the relation having the highest indicator:

$$\begin{aligned} I_{ij} > max(J_{ij}, P_{ij}, P_{ji}) &\Rightarrow a_i I^{\gamma} a_j, \\ J_{ij} > max(I_{ij}, P_{ij}, P_{ji}) &\Rightarrow a_i J^{\gamma} a_j, \\ P_{ij} > max(I_{ij}, J_{ij}, P_{ji}) &\Rightarrow a_i P^{\gamma} a_j, \\ P_{ji} > max(I_{ij}, J_{ij}, P_{ij}) &\Rightarrow a_j P^{\gamma} a_i. \end{aligned} \tag{22}$$

However, in order to obtain a preference structure, each pair of alternative should be assigned to exactly one of the preference relations. This is not the case yet in Equation (22) due to the possibility of equality between several of these indicators. For this reason, the preference structure will be built such that:

$$\begin{aligned} a_i I^{\gamma} a_j &\Leftrightarrow I_{ij} \geq max(P_{ij}, P_{ji}) \vee (P_{ij}, P_{ji}, J_{ij} \leq 0), \\ a_i J^{\gamma} a_j &\Leftrightarrow \neg(a_i I^{\gamma} a_j) \wedge J_{ij} \geq max(P_{ij}, P_{ji}), \\ a_i P^{\gamma} a_j &\Leftrightarrow \neg(a_i I^{\gamma} a_j) \wedge \neg(a_i J^{\gamma} a_j) \wedge P_{ij} \geq P_{ji}, \\ a_j P^{\gamma} a_i &\Leftrightarrow \neg(a_i I^{\gamma} a_j) \wedge \neg(a_i J^{\gamma} a_j) \wedge P_{ji} \geq P_{ij}. \end{aligned} \tag{23}$$

Equation (23) leads to the same preference relations as Equation (22) when there is no equality between indicators. In the case of an equality, indifference will have the highest priority while strict preference will have the lowest priority. Furthermore, if $\gamma_{ij} = \gamma_{ji}$, the preference relation will be defined as an indifference relation as long as both γ_{ij} and γ_{ji} do not exceed T_j.

In practice however, the situation of equality between the indicators is very rare and the intuition between Equation (22) is enough to understand the method.

4.2 | Interpretation of the thresholds

Hereunder, some additional comments about the different thresholds are provided.

4.2.1 | PROMETHEE γ as a non compensatory method: Interpretation of T_i and T_j

If the decision maker wants to use PROMETHEE γ in a totally uncompensatory manner, it is sufficient to assign an arbitrarily high value to P_f. Indeed, if P_f is arbitrarily high, P_{ij} and P_{ji} will be small enough such that the alternatives will be considered as incomparable or indifferent as soon as J_{ij} or I_{ij} is greater than 0. Equation (23) can then be rewritten as:

$$\begin{aligned} I_{ij} = T_I - max(\gamma_{ij}, \gamma_{ji}) \geq 0 &\Rightarrow a_i I^{\gamma} a_j \\ J_{ij} = min(\gamma_{ij}, \gamma_{ji}) - T_J \geq 0 &\Rightarrow a_i J^{\gamma} a_j \\ I_{ij} < 0 \wedge J_{ij} < 0 \wedge \gamma_{ij} > \gamma_{ji} &\Rightarrow a_i P^{\gamma} a_j \\ I_{ij} < 0 \wedge J_{ij} < 0 \wedge \gamma_{ji} > \gamma_{ij} &\Rightarrow a_j P^{\gamma} a_i \end{aligned} \tag{24}$$

In this situation, it is easy to give an interpretation to the global indifference and incomparability thresholds. Indeed, if both γ_{ij} and γ_{ji} are lower than T_i then the two alternatives will be considered as indifferent. Similarly, if both γ_{ij} and γ_{ji} are higher than T_j then the two alternatives will be considered as incomparable.

To illustrate the preference relation between pairs of alternatives, the values of the indices $(\gamma_{ij}, \gamma_{ji})$ can be plotted on an orthogonal graph. The graph will be divided into four regions according to the new parameters of method T_I, T_J and P_f.

An example of such a division can be seen in Figure 2 in the case where the decision maker wants to use PROMETHEE γ in a non compensatory manner. Pairs of alternatives (a_i, a_j) for which the couple

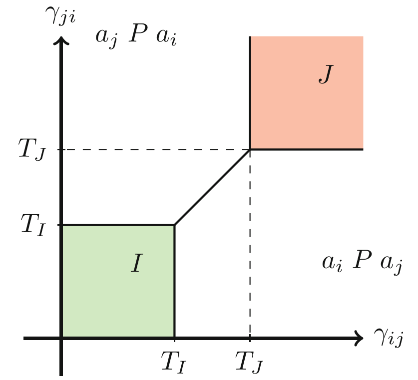


FIGURE 2 Division of the graph of γ_{ij} and γ_{ji} according to the different parameters of PROMETHEE γ when P_f is arbitrarily high.

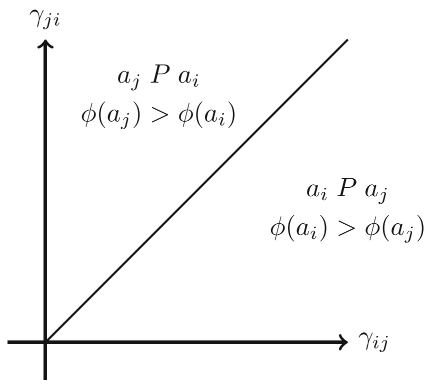


FIGURE 3 Division of the graph of γ_{ij} and γ_{ji} according to the different parameters of PROMETHEE γ with $P_f = 0$.

$(\gamma_{ij},\gamma_{ji})$ lies in the green (respectively red) region will be considered according to PROMETHEE γ as indifferent (respectively incomparable).

4.2.2 | PROMETHEE γ as a totally compensatory method

As opposed to the non compensatory version, if the decision maker wants to use PROMETHEE γ in a fully compensatory manner, than P_f can be assigned a value of 0. In this way, P_{ij} would have an infinite value as soon as $\gamma_{ij} > \gamma_{ji}$ leading to a preference of a_i over a_j. In this setting PROMETHEE γ and PROMETHEE II will be identical methods. This case is represented on Figure 3.

4.2.3 | PROMETHEE γ as a partially compensatory method: Interpretation of P_f

Finally, if the decision maker would like to use the method in a partially compensatory manner, then the appropriate value of P_f has to be selected. Indeed, the P_f factor represents the ratio of importance between the "intensity of preference" $|\gamma_{ij} - \gamma_{ji}|$ and I_{ij} or J_{ij}. The smaller P_f, the larger P_{ij} (or P_{ji}) will be, leading to more possible preference between a_i and a_j even when J_{ij} or I_{ij} are positive. Similarly, the larger P_f, the smaller P_{ij} (or P_{ji}) will be, leading to less possible preference between a_i and a_j when J_{ij} or I_{ij} are positive.

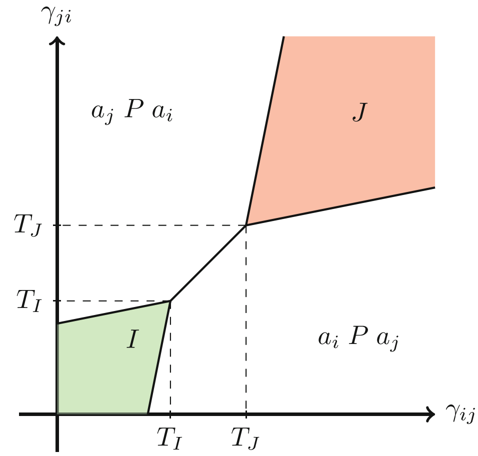


FIGURE 4 Division of the graph of γ_{ij} and γ_{ji} according to the different parameters of PROMETHEE γc.

This situation is represented in Figure 4.

In this figure, we can verify that as for the non compensatory version of PROMETHEE γc, both γ_{ij} and γ_{ji} need to be lower than T_I in order to have an indifference relation and that they both need to be higher than T_J to have an incomparability relation. However, even when this is the case (both γ_{ij} and γ_{ji} are higher than T_J for instance) it is possible for an alternative to be preferred over the other one. The preference or the incomparability is defined according to the values of the γ indices and the value of the P_f parameter which defines the slopes of the lines delimiting the incomparability or indifference regions.

The parameter P_f is probably the additional parameter which would be the most difficult to define for a decision maker. For this reason, we would suggest to assign a default value of $P_f = 1$ for all decision makers who are not comfortable with this parameter (in this work, all tests have been performed with $P_f = 1$). It remains of course possible to perform some sensitivity analysis on P_f (as well as the other parameters). Furthermore decision makers who would feel more comfortable with this parameter could select different values.

Finally, in this work we will generally consider $T_I = T_J$. This constraint will lead to situations similar to the one represented in Figure 5.

4.3 | Practical examples

A practical representation of the preference relations assigned to pairs of alternatives according to γ_{ij} and γ_{ji} is provided in Figure 6. This figure represents 4 data sets of different sizes. The different data sets are detailed in Appendix Appendix A. For each dataset, PROMETHEE has been applied with linear preference functions with the parameters p_c and q_c being respectively equal to the third and first quartile of the difference of evaluations between all pairs of alternatives for the criterion $f_c()$.

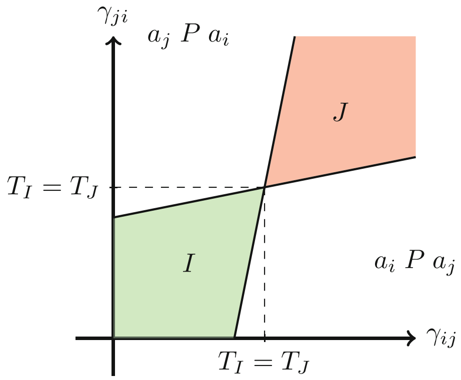


FIGURE 5 Division of the graph of γ_{ij} and γ_{ji} according to the different parameters of PROMETHEE γ with $T_I = T_J$.

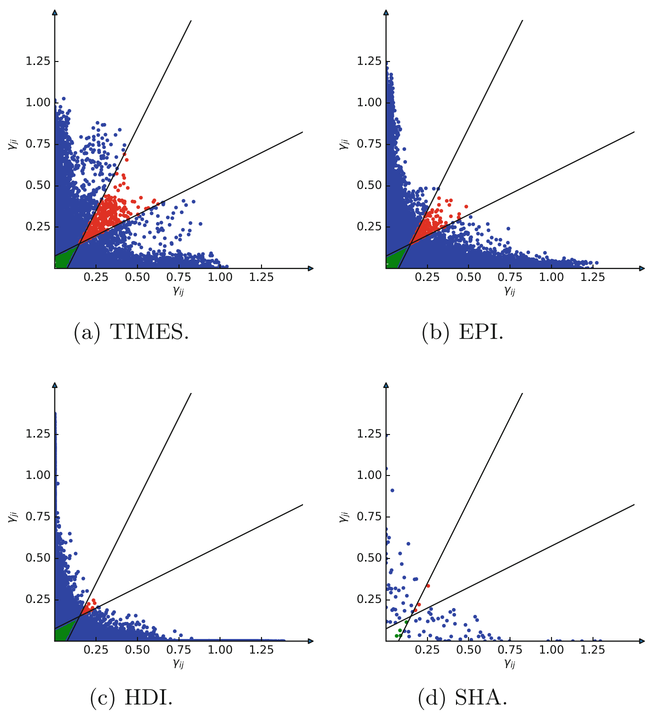


FIGURE 6 $T_I = T_J = 0.15$ and $P_f = 1$, red, green and blue dots respectively represent couples of alternatives for which an incomparable, indifference or preference relation holds.

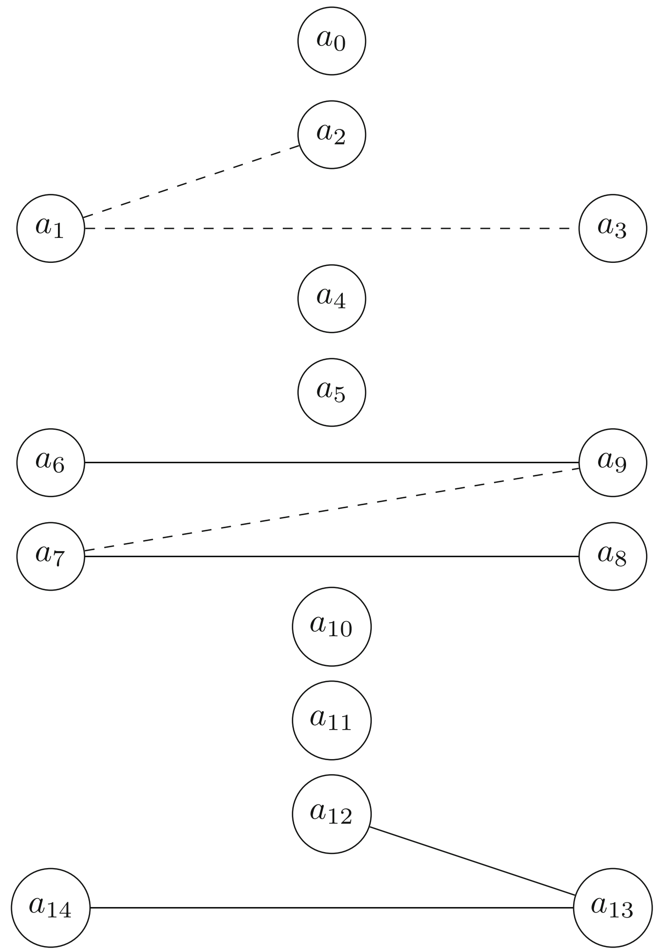


FIGURE 7 Top 15 alternatives of the Shangai data set. $T_I = T_J = 0.15$ and $P_f = 1$, indifferent alternatives are linked by a plain edge while incomparable alternatives are linked by a dashed edge. If no edge exists between two alternatives, the higher one in the figure is preferred over the lower one.

In Figure 7, the result of PROMETHEE γ applied on a relatively small data set is illustrated.

It should be noted that, such as in many outranking methods Bouyssou and Pirlot (2005), neither the indifference nor the preference relation produced with PROMETHEE γ are guaranteed to be transitive. However, as the difference of γ between pairs of alternatives is equal to their difference of net flow scores, we know that the preference relation is acyclic.

At this point, we can already conclude that some of the initial objectives of the method presented in Section 2 are satisfied. Indeed, the choice is left to the decision maker whether he/she wants to use PROMETHEE γ in a non compensatory manner or not according to the value of P_f. If the method is used in a non compensatory manner:

1. Increasing pairwise preferences of one alternative over another can only increase the values of the indicators γ which could only create new global preference or incomparability relations (or have no effect at all);
2. Decreasing pairwise preferences of one alternative over another can only decrease the values of the indicators γ which could only create new preference or indifference relations (or have no effect at all).

Concerning objective 1, we can already notice that the method admits both pairs of indifferent and of incomparable alternatives. However, it is difficult to quantify whether the indifferent alternatives are indeed "similar", while the incomparable alternatives are indeed "different". To overcome this difficulty, a comparison of the relationships produced using PROMETHEE γ and PROMETHEE I will be performed in Section 5.

5 | COMPARISON BETWEEN THE PROMETHEE I AND THE PROMETHEE γ METHODS

In order to compare the results of the new method with the results of the PROMETHEE I method, some experiments have been performed on the data sets described in Appendix Appendix A. This appendix describes the origin of the datasets as well as the values chosen for the classical PROMETHEE parameters (weights, type of preference functions, p and q). These parameters will be identical when using PROMETHEE γ and PROMETHEE I.

5.1 | Proportion of different relations

The first experiments consist in the building of a confusion matrix. This matrix will indicate the frequency of each preference relation obtained both with PROMETHEE γ and PROMETHEE I respectively for each couple of alternatives. This experiment aims at quantifying the pairs of alternatives for which using the PROMETHEE I method or using the PROMETHEE γ method would produce a different output preference relations. Before computing the confusion matrix, it can be noted that no preference can be inverted between the two methods:

$$\begin{aligned} rla_iP^\gamma a_j &\Rightarrow \neg\left(a_jP^I a_i\right) \\ a_iP^I a_j &\Rightarrow \neg\left(a_jP^\gamma a_i\right) \end{aligned} \tag{25}$$

Indeed, this can be seen as a direct consequence of equation (20).

The confusion matrix obtained by using the TIMES data set is shown in Table 7.

It can be seen from this table that PROMETHEE I does not produce any indifference relation. This is not the case for PROMETHEE γ. This table also shows that around 85% of the pairs of alternatives have the same output preference relation according to the two methods. These results should however be considered carefully as these quantities are obviously very dependent on the additional parameters of PROMETHEE γ.

5.2 | Distances between indifferent and incomparable pairs of alternatives

The second experiment consists in computing the L_1 distances between pairs of indifferent or incomparable alternatives. In order for the distances to be meaningful despite the difference of nature of the criteria, these distances will not be computed in the evaluation space with an alternative a_i being represented by the point $(f_1(a_i), f_2(a_i), \dots, f_k(a_i))$ but will be computed in the space of mono-criterion net flow scores. Alternative a_i will therefore be represented by the point $(\phi^1(a_i), \phi^2(a_i), \dots, \phi^k(a_i))$. To simplify the notations, we will denote $\overline{X}$ the vector of distances for each pair of alternatives belonging to the binary relation X. Different statistics about these distances are shown in Table 8.

TABLE 7 Percentage of couples of alternatives for each possible relation produced by PROMETHEE I and PROMETHEE γ (with $T_I = T_J = 0.15$ and $P_f = 1$) with the TIMES data set.

	I^I	J^I	P^I
I^γ	0	2.95	2.51
J^γ	0	4.36	1.41
P^γ	0	7.03	81.74

The first two columns show minimum and mean distances between pairs of indifferent alternatives (not applicable to PROMETHEE I). The following two columns show the distance between pairs of incomparable alternatives. Finally the last two columns give the variance of the distance of the corresponding indicators.

These results show that the couples which are considered as indifferent by the PROMETHEE γ method are less different than the ones which are considered as incomparable. Furthermore, the couples of alternatives considered as incomparable by the PROMETHEE γ method are also more different than the ones considered incomparable by PROMETHEE I. Statistical tests (Welch's t-test) have been performed and confirm the intuition given by the tables and confirming the fulfillment of objective 1 detailed in Section 2.

5.3 | Practical differences on a small dataset

In this section, the small example of the Shangai dataset already used to illustrate PROMETHEE γ in Section 4.3 will be used again to analyze the differences between PROMETHEE γ and PROMETHEE I. In Figure 8, the results of the application of PROMETHEE γ (already illustrated on Figure 7) are put side by side with the results of the application of PROMETHEE I with this same dataset.

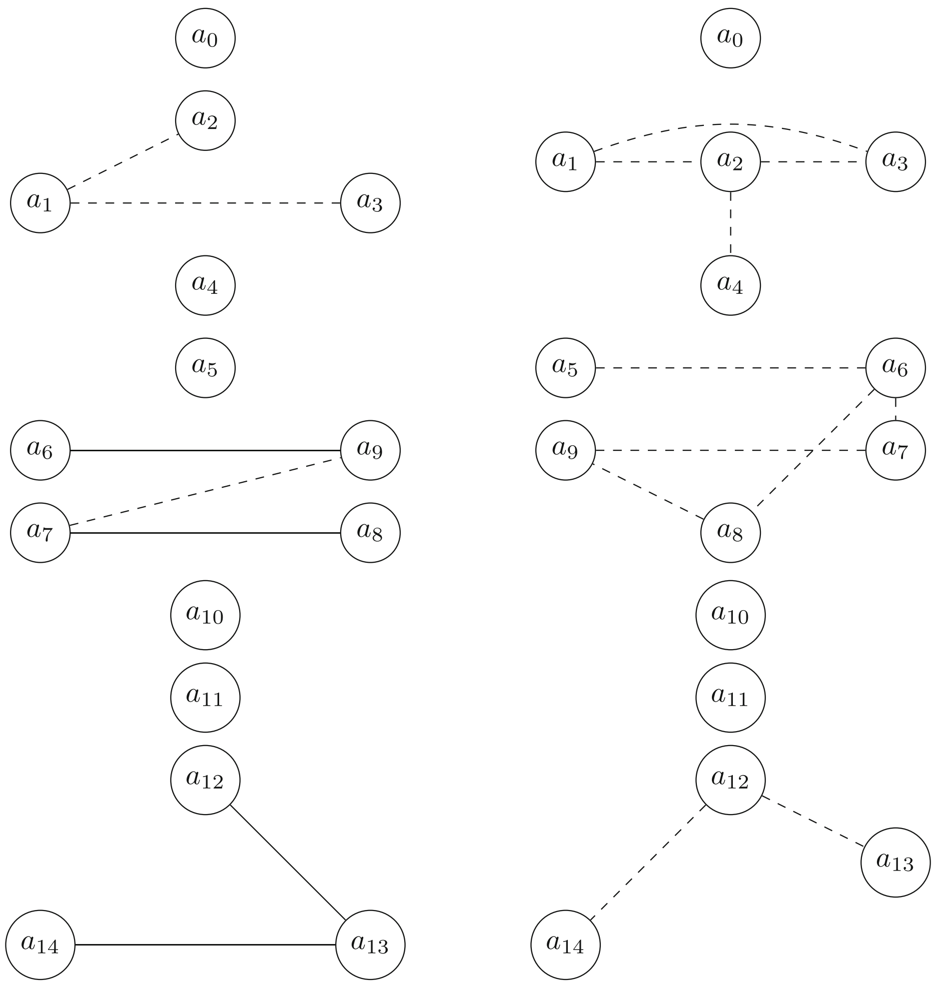
The figure shows that there are some differences between the results provided with the PROMETHEE γ and the PROMETHEE I methods. The differences are listed hereunder:

- $a_2P^\gamma a_3$ while $a_2J^I a_3$;
- $a_2P^\gamma a_4$ while $a_2J^I a_4$;
- $a_5P^\gamma a_6$ while $a_5J^I a_6$;
- $a_6I^\gamma a_9$ while $a_6P^I a_9$;
- $a_6P^\gamma a_7$ while $a_6J^I a_7$;
- $a_6P^\gamma a_8$ while $a_6J^I a_8$;
- $a_7I^\gamma a_8$ while $a_7P^I a_8$;
- $a_9P^\gamma a_8$ while $a_9J^I a_8$;
- $a_{12}I^\gamma a_{13}$ while $a_{12}J^I a_{13}$;
- $a_{12}P^\gamma a_{14}$ while $a_{12}J^I a_{14}$;
- $a_{13}I^\gamma a_{14}$ while $a_{13}P^I a_{14}$.

Some of these differences will be illustrated hereunder. For some of these situations, such as the relation between a_2 and a_3 represented in Figure 9, it is not possible to assess without a real decision maker whether the situation should be represented by an incomparability or by a preference relation.

TABLE 8 Distances in TIMES data set (with $T_I = T_J = 0.15$ and $P_f = 1$).

	min (I)	avg (I)	min (J)	avg (J)	var (J)	var (I)
PROMETHEE γ	0.166	1.08	1.43	3.13	0.61	0.228
PROMETHEE I			0.261	2.34	1.05	



(a) PROMETHEE γ.

(b) PROMETHEE I.

FIGURE 8 Comparison of PROMETHEE γ and PROMETHEE I on a small dataset. Straight lines represent indifference and dashed lines represent incomparability. If no line is joining two alternatives, then the highest one is preferred.

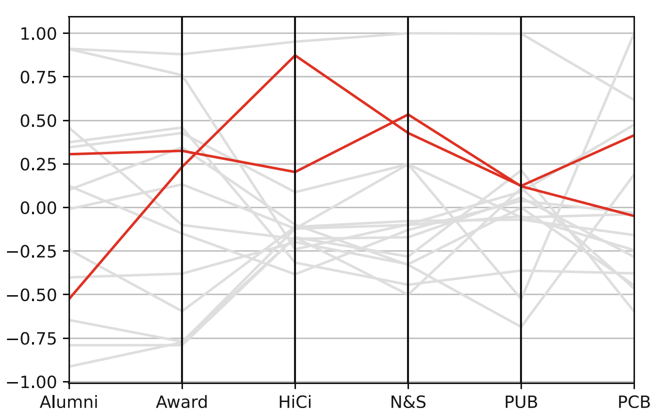
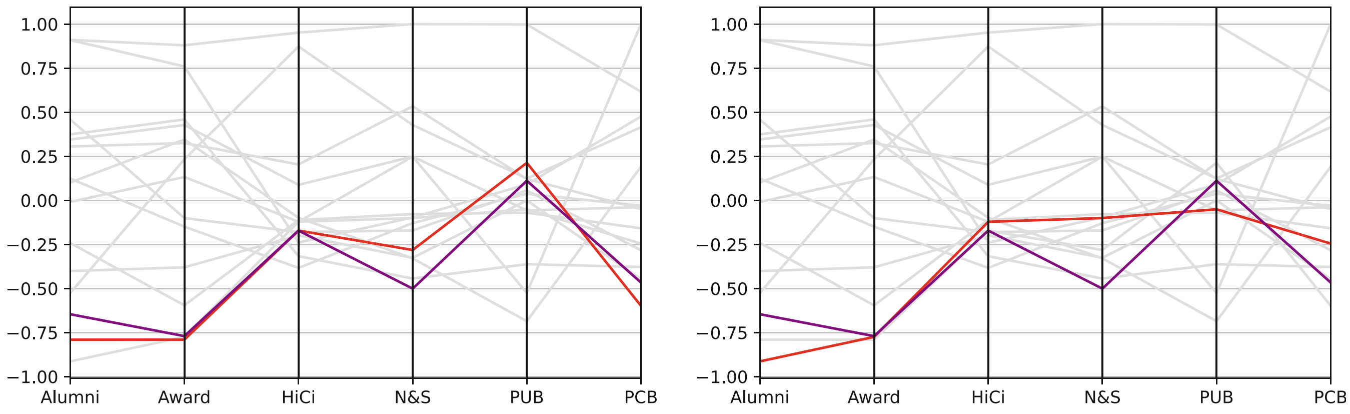


FIGURE 9 Mono-criterion net flow scores of the 15 alternatives for each criterion of the SHA dataset. Mono-criterion net flow scores of alternatives a_2 and a_3 are highlighted in red.

On Figure 9 the mono-criterion net flow scores of a_2 and a_3 are represented in red for all the criteria of the SHA dataset (see Appendix A for more details). The mono-criterion net flow scores of all the other alternatives are represented in gray. As it can be seen, alternatives a_2 and a_3 seem to be relatively different. Unfortunately, on its own, this figure does not allow to assess whether this difference is significant enough to state that they should be incomparable.

Some cases however seem to offer additional insights. It is for instance the case when observing the net flow scores of alternatives a_{12}, a_{13} and a_{14}. In Figure 10a it can be seen that alternatives a_{13} and a_{14} are very similar and that they should, in most cases, be assessed as indifferent by the decision makers instead of the preference of a_{13} over a_{14} as defined with PROMETHEE I. However, if the decision maker agrees with this preference, we believe that the situation between a_{12} and a_{14} (illustrated in Figure 10b) should also lead to a preference (instead of an incomparability as defined with PROMETHEE I).



(a) Comparison of a_{13} in red and a_{14} in purple.

(b) Comparison of a_{12} in red and a_{14} in purple.

FIGURE 10 Comparison of mono-criterion net flow scores for alternatives a_{12}, a_{13} and a_{14}.

6 | CONCLUSION

In this work, a study of the incomparability preference relation has been proposed in the context of the outranking method PROMETHEE I. It has been shown that this method reveals some unexpected or undesirable results. A new method called PROMETHEE γ has been proposed to overcome these shortcomings. It is based on the computation of new pairwise indicators that are believed to reflect the conflicting nature of pairs of alternatives better than the positive and negative outranking flow scores while still providing the same information as the net flow scores. An exploitation procedure for these indicators has been proposed. It has been shown that this exploitation procedure produces an incomparability relation which we consider more in adequation with the non-compensatory nature of outranking methods. PROMETHEE γ is therefore aimed at replacing PROMETHEE I for decision problems where decision makers want the incomparability relation to represent strong conflicting information on coalitions of criteria. However, this method comes at the cost, for the decision maker, of having to choose three additional preference parameters. Therefore, some additional elicitation or other procedure should be further developed in order to help the decision maker in determining these parameters before PROMETHEE γ can be considered as usable in practical decision problems. Furthermore, it has been pointed out that PROMETHEE γ produces a preference structure whose strict preference relation is not transitive. As in other outranking methods, it is not believed that this is a significant drawback as the strict preference is however acyclic and the preference structure therefore still possesses a hierarchical structure.

DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

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How to cite this article: Dejaegere, G., & De Smet, Y. (2023). PROMETHEE γ: A new PROMETHEE based method for partial ranking based on valued coalitions of monocriterion net flow scores. *Journal of Multi-Criteria Decision Analysis*, *30*(3-4), 147–160. https://doi.org/10.1002/mcda.1805

APPENDIX A: Datasets description

Four datasets have been used in this work: the *Academic Ranking of World Universities*, also called *Shangai Ranking* (SHA) SHA (2014), the *Times Higher Education World University Rankings* (TIMES) TIMES (2021), the *Human Development Index* (HDI) HDI (2020) and the *Environmental Performance Index 2020* (EPI) Wendling et al. (2020). Hereunder, a description of these datasets can be found.

As it can be seen, the weights used are the ones used in the initial datasets when possible.

SHA: Only the 15 best universities have been kept. They are evaluated on the following criteria: number of alumni, number of staff winning Nobel Prizes and Fields Medals, number of highly cited researchers, number of articles published in journals of Nature and Science, number of articles indexed in Science Citation Index - Expanded and Social Sciences Citation Index, and per capita performance of a university with their respective weights of $0.1, 0.2, 0.2, 0.2, 0.2$ and 0.1.

HDI: the criteria selected to compare the countries are “Life expectancy at birth” “Expected years of schooling”, “Mean years of schooling” as well as “Gross national income per capita” with their weights being respectively of $1/3, 1/6, 1/6$ and $1/3$.

TIMES: the criteria selected to compare the universities are “Teaching”, “Research”, “Citations”, “Industry Income” and “International Outlook” with their weights being respectively of $0.3, 0.3, 0.3, 0.07$ and 0.03.

EPI: the criteria selected to compare the countries are “Air Quality”, “Sanitation and Drinking Water”, “Heavy Metals”, “Waste Management”, “Biodiversity and Habitat”, “Ecosystem Services”, “Climate Change”, “Pollution Emissions”, “Water Resources” and “Agriculture” (the “Fisheries” criterion has not been used due to data not being usable for countries not performing any fishing activity). The weights of these criteria are respectively $20/94, 16/94, 2/94, 2/94, 3/94, 3/94, 24/94, 3/94, 6/94$ and $15/94$ (the “Fisheries” account for $6/100$ in the original dataset). Furthermore, “Kiribati”, “Marshall Islands”, “Tonga” and “Samoa” countries have neither been considered due to their missing value for the “Ecosystem Services” indicator.

For each dataset, PROMETHEE has been applied with linear preference functions. Furthermore, except for the SHA dataset, the parameters p and q are respectively equal to the third and first quartile of the difference of evaluations between all pairs of alternatives for the given criterion. For the *SHA* dataset, values of 5 and 25 have been selected for p and q (the evaluations on all criteria lie between 0 and 100).