

Mathematics

-Presented by

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CHAPTER 1

SETS

Mathematics, a set is defined as a collection of distinct, well-defined objects forming a group. There can be any number of items, be it a collection of whole numbers, months of a year, types of birds, and so on. Each item in the set is known as an element of the set. We use curly brackets while writing a set.

- Suppose two sets $A = \{a, b\}$ and $B = \{1, 2, 3\}$. Find $A \times B$ and $B \times A$. Here, $A = \{a, b\}$ $B = \{1, 2, 3\}$ Now, $A \times B = \{(a, 1), (a, 2), (a, 3), (b, 1), (b, 2), (b, 3)\}$ $B \times A = \{(1, a), (1, b), (2, a), (2, b), (3, a), (3, b)\}$

Roster Form

- Roster Form In Roster form, all the elements of a set are listed. For example, the set of natural numbers less than 5. Natural Number = 1, 2, 3, 4, 5, 6, 7, 8, Natural Number less than 5 = 1, 2, 3, 4 Therefore, the set is $N = \{ 1, 2, 3, 4 \}$ Set Builder Form The general form is, $A = \{ x : \text{property} \}$
- Example: Write the following sets in set builder form:
 $A = \{ 2, 4, 6, 8 \}$ Solution: $2 = 2 \times 1, 4 = 2 \times 2, 6 = 2 \times 3, 8 = 2 \times 4$ So, the set builder form is $A = \{ x : x = 2n, n \in N \text{ and } 1 \leq n \leq 4 \}$ Also, Venn Diagrams are the simple and best way for visualized representation of sets

- Example 2: Find $A \cup B$ and $A \cap B$ and $A - B$. If $A = \{a, b, c, d\}$ and $B = \{c, d\}$. Solution: $A = \{a, b, c, d\}$ and $B = \{c, d\}$
 $A \cup B = \{a, b, c, d\}$ $A \cap B = \{c, d\}$ and $A - B = \{a, b\}$
- Sets Formulas Some of the most important set formulas are:
 For any three sets A, B and C
 $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
 If $A \cap B = \emptyset$, then $n(A \cup B) = n(A) + n(B)$
 $n(A - B) + n(A \cap B) = n(A)$
 $n(B - A) + n(A \cap B) = n(B)$
 $n(A - B) + n(A \cap B) + n(B - A) = n(A \cup B)$
 $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

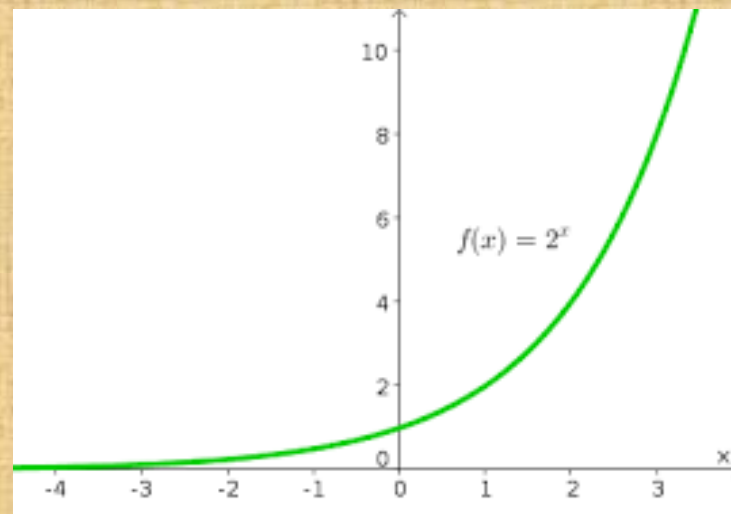
CHAPTER 2

Exponential and logarithm functions

- specify for which values of a the exponential function $f(x) = a^x$ may be defined,
- recognize the domain and range of an exponential function,
- identify a particular point which is on the graph of every exponential function,
- specify for which values of a the logarithm function $f(x) = \log_a x$ may be defined,
- recognize the domain and range of a logarithm function,
- identify a particular point which is on the graph of every logarithm function,
- understand the relationship between the exponential function $f(x) = e^x$ and t

- 1. Exponential functions

- Consider a function of the form $f(x) = a^x$, where $a > 0$. Such a function is called an exponential function. We can take three different cases, where $a = 1$, $0 < a < 1$ and $a > 1$. If $a = 1$ then $f(x) = 1^x = 1$. So this just gives us the constant function $f(x) = 1$.
- This example demonstrates the general shape for graphs of functions of the form $f(x) = a^x$ when $a > 1$. What is the effect of varying a ? We can see this by looking at sketches of a few graphs of similar functions. $f(x) = a^x$

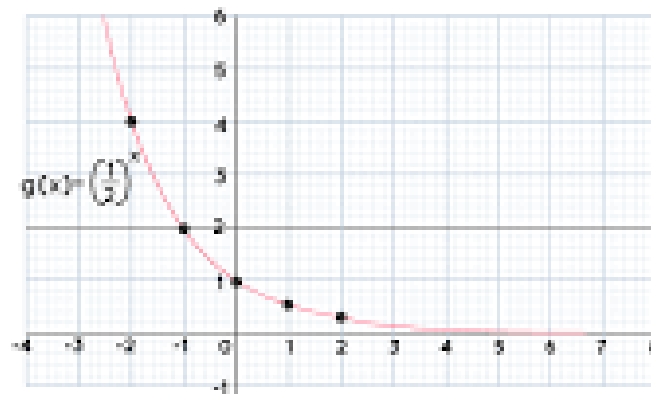


Graphing the function $g(x) = \left(\frac{1}{2}\right)^x$



$$g(x) = \left(\frac{1}{2}\right)^x$$

x	$g(x)$	
-2	4	$\left(\frac{1}{2}\right)^{-2} = 2^2 = 4$
-1	2	$\left(\frac{1}{2}\right)^{-1} = 2^1 = 2$
0	1	$\left(\frac{1}{2}\right)^0 = 1$
1	0.5	$\left(\frac{1}{2}\right)^1 = \frac{1}{2} = 0.5$
2	0.25	$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$



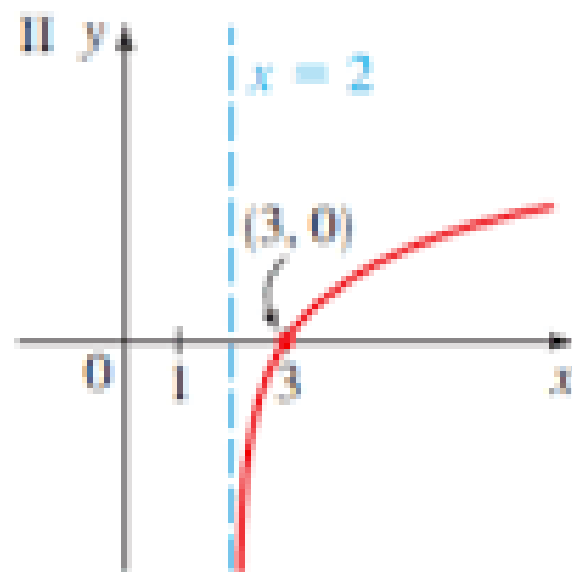
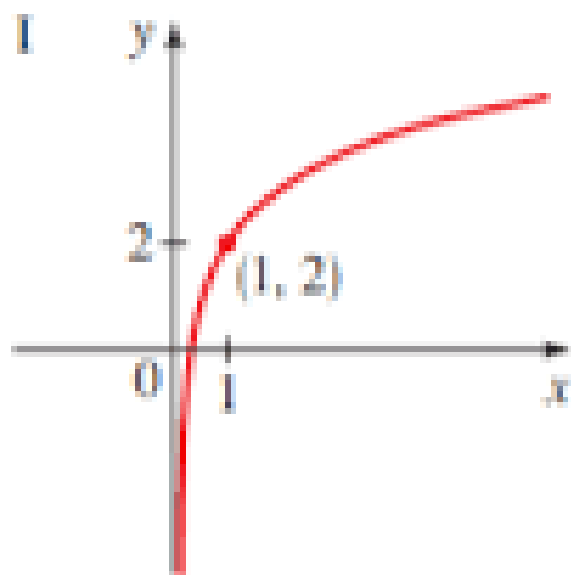
Logarithm functions

- We shall now look at logarithm functions. These are functions of the form $f(x) = \log_a x$ where $a > 0$. We do not consider the case $a = 1$, as this will not give us a valid function.
- What happens if $a > 1$? To examine this case, take a numerical example. Suppose that $a = 2$. Then
- $f(x) = \log_2 x$ means $2^{f(x)} = x$.
- An important point to note here is that, regardless of the argument, $2^{f(x)} > 0$. So we shall consider only positive arguments.
- $f(1) = \log_2 1$ means $2^{f(1)} = 1$ so $f(1) = 0$
- $f(2) = \log_2 2$ means $2^{f(2)} = 2$ so $f(2) = 1$
- $f(4) = \log_2 4$ means $2^{f(4)} = 4$ so $f(4) = 2$
- $f\left(\frac{1}{2}\right) = \log_2 \left(\frac{1}{2}\right)$ means $2^{f\left(\frac{1}{2}\right)} = \frac{1}{2} = 2^{-1}$ so $f\left(\frac{1}{2}\right) = -1$
- $f\left(\frac{1}{4}\right) = \log_2 \left(\frac{1}{4}\right)$ means $2^{f\left(\frac{1}{4}\right)} = \frac{1}{4} = 2^{-2}$ so $f\left(\frac{1}{4}\right) = -2$
- We can put these results into a table, and plot a graph of the function.

57–58 ■ Graphing Logarithmic Functions Match the logarithmic function with one of the graphs labeled I or II.

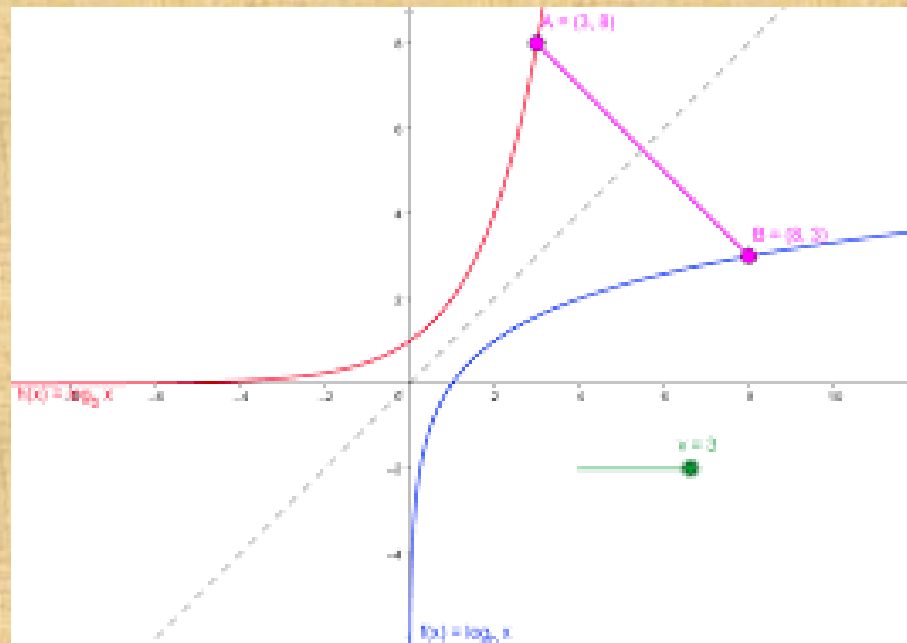
57. $f(x) = 2 + \ln x$

58. $f(x) = \ln(x - 2)$



The relationship between exponential functions and logarithm functions

- We can see the relationship between the exponential function $f(x) = e^x$ and the logarithm function $f(x) = \ln x$ by looking at their graphs.
- You can see straight away that the logarithm function is a reflection of the exponential function in the line represented by $f(x) = x$



CHAPTER 3

Euler's Theorem

- 1. Homogeneous Function
- A function f of two independent variables x, y is said to be a homogeneous function of degree n if it can be put in either of the following two forms :

$$\frac{\partial f}{\partial z} = 3x^2y + 5xy^2 + 16z^3$$

Now

$$\begin{aligned} x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} + z \frac{\partial f}{\partial z} &= 6xyz + 5y^2z + 3x^2z + 10xyz + 3x^2y + 5xy^2 + 16z^3 \\ &= 4(3x^2yz + 5xy^2z + 4z^4) \\ &= 4 f(x, y, z) \end{aligned}$$

This verify Euler's theorem.

Generalization of Euler's theorem-

If $f(x_1, x_2, x_3, \dots, x_m)$ be a homogeneous function of variables $x_1, x_2, x_3, \dots, x_m$ of degree n , then

$$x_1 \frac{\partial f}{\partial x_1} + x_2 \frac{\partial f}{\partial x_2} + \dots + x_m \frac{\partial f}{\partial x_m} = n f$$

Key Words: Homogeneous function, degree of function.

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- 3. Euler's Theorem on Homogeneous Function of Three Variables

1. Homogeneous Function

Example : The function

$$f(x, y) = \frac{x^4 + y^4}{x - y}$$

is a homogeneous function of degree 3, since

$$f(x, y) = \frac{x^4 + y^4}{x - y} = \frac{x^4 \left[1 + \left(\frac{y}{x} \right)^4 \right]}{x \left[1 - \left(\frac{y}{x} \right) \right]} = x^3 \frac{\left[1 + \left(\frac{y}{x} \right)^4 \right]}{\left[1 - \left(\frac{y}{x} \right) \right]} = x^3 \phi \left(\frac{y}{x} \right)$$

or alternatively,

$$f(tx, ty) = \frac{(tx)^4 + (ty)^4}{tx - ty} = \frac{t^4(x^4 + y^4)}{t(x - y)} = t^3 \frac{(x^4 + y^4)}{(x - y)} = t^3 f(x, y).$$

Similarly, the function

$$f(x, y, z) = \frac{x^3 + y^3 + z^3}{x + y + z}$$

is a homogeneous function of degree 2, since

$$f(x, y, z) = \frac{x^3 + y^3 + z^3}{x + y + z} = \frac{x^3 \left[1 + \left(\frac{y}{x} \right)^3 + \left(\frac{z}{x} \right)^3 \right]}{x \left[1 + \left(\frac{y}{x} \right) + \left(\frac{z}{x} \right) \right]} = x^2 \frac{\left[1 + \left(\frac{y}{x} \right)^3 + \left(\frac{z}{x} \right)^3 \right]}{\left[1 + \left(\frac{y}{x} \right) + \left(\frac{z}{x} \right) \right]} = x^2 \phi \left(\frac{y}{x}, \frac{z}{x} \right)$$

$$f(tx, ty, tz) = \frac{(tx)^3 + (ty)^3 + (tz)^3}{tx + ty + tz} = \frac{t^3(x^3 + y^3 + z^3)}{t(x + y + z)} = t^2 \frac{(x^3 + y^3 + z^3)}{(x + y + z)} = t^2 f(x, y, z).$$

2. Euler's Theorem on Homogeneous Function of Two Variables

Statement : If u be a homogeneous function of degree n in two independent variables x, y , then

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu.$$

Proof : Let

$$u = A_1 x^{\alpha_1} y^{\beta_1} + A_2 x^{\alpha_2} y^{\beta_2} + A_3 x^{\alpha_3} y^{\beta_3} + \dots + A_n x^{\alpha_n} y^{\beta_n} \quad \dots(1)$$

where $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \alpha_3 + \beta_3 = \dots = \alpha_n + \beta_n = n$

Differentiating both sides of equation (1) partially w. r. t. x , we get

$$\frac{\partial u}{\partial x} = A_1 (\alpha_1 x^{\alpha_1-1}) y^{\beta_1} + A_2 (\alpha_2 x^{\alpha_2-1}) y^{\beta_2} + A_3 (\alpha_3 x^{\alpha_3-1}) y^{\beta_3} + \dots + A_n (\alpha_n x^{\alpha_n-1}) y^{\beta_n}$$

$$\text{This} \Rightarrow x \frac{\partial u}{\partial x} = A_1 \alpha_1 x^{\alpha_1} y^{\beta_1} + A_2 \alpha_2 x^{\alpha_2} y^{\beta_2} + A_3 \alpha_3 x^{\alpha_3} y^{\beta_3} + \dots + A_n \alpha_n x^{\alpha_n} y^{\beta_n} \quad \dots(2)$$

Now, differentiating both sides of equation (1) partially w. r. t. y , we get

$$\frac{\partial u}{\partial y} = A_1 x^{\alpha_1} (\beta_1 y^{\beta_1-1}) + A_2 x^{\alpha_2} (\beta_2 y^{\beta_2-1}) + A_3 x^{\alpha_3} (\beta_3 y^{\beta_3-1}) + \dots + A_n x^{\alpha_n} (\beta_n y^{\beta_n-1})$$

$$\text{This} \Rightarrow y \frac{\partial u}{\partial y} = A_1 \beta_1 x^{\alpha_1} y^{\beta_1} + A_2 \beta_2 x^{\alpha_2} y^{\beta_2} + A_3 \beta_3 x^{\alpha_3} y^{\beta_3} + \dots + A_n \beta_n x^{\alpha_n} y^{\beta_n} \quad \dots(3)$$

Adding equations (2) and (3), we get

$$\begin{aligned}
 x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} &= (\alpha_1 + \beta_1) A_1 x^{\alpha_1} y^{\beta_1} + (\alpha_2 + \beta_2) A_2 x^{\alpha_2} y^{\beta_2} + (\alpha_3 + \beta_3) A_3 x^{\alpha_3} y^{\beta_3} \\
 &\quad + \dots + (\alpha_n + \beta_n) A_n x^{\alpha_n} y^{\beta_n} \\
 &= n A_1 x^{\alpha_1} y^{\beta_1} + n A_2 x^{\alpha_2} y^{\beta_2} + n A_3 x^{\alpha_3} y^{\beta_3} + \dots + n A_n x^{\alpha_n} y^{\beta_n} \\
 &\quad (\because \alpha_1 + \beta_1 = \alpha_2 + \beta_2 = \alpha_3 + \beta_3 = \dots = \alpha_n + \beta_n = n) \\
 &= n (A_1 x^{\alpha_1} y^{\beta_1} + A_2 x^{\alpha_2} y^{\beta_2} + A_3 x^{\alpha_3} y^{\beta_3} + \dots + A_n x^{\alpha_n} y^{\beta_n}) \\
 &= n u \text{ (using equation (1))}
 \end{aligned}$$

i.e.,

$$\boxed{x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = n u}$$

Example

Example 1 : Verify Euler's Theorem when $u = \frac{x(x^3 - y^3)}{x^3 + y^3}$.

Solution : According to Euler's Theorem, if u be a homogeneous function of degree n in two independent variables x, y , then

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu.$$

Given that

$$u = \frac{x(x^3 - y^3)}{x^3 + y^3} \quad \dots\dots(1)$$

$$\text{i.e., } u = \frac{x^4 \left[1 - \left(\frac{y}{x} \right)^3 \right]}{x^3 \left[1 + \left(\frac{y}{x} \right)^3 \right]} = x \frac{\left[1 - \left(\frac{y}{x} \right)^3 \right]}{\left[1 + \left(\frac{y}{x} \right)^3 \right]} = x \phi \left(\frac{y}{x} \right), \text{ where } \phi \text{ is a function of } \frac{y}{x}.$$

$$\left[\begin{matrix} x \\ y \end{matrix} \right] \left[\begin{matrix} x \\ y \end{matrix} \right]$$

This \Rightarrow The given function u is a homogeneous function of degree 1 in two independent variables x, y . Therefore Euler's Theorem will be verified if we can prove that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u.$$

Taking logarithm of both sides of equation (1), we get

$$\log u = \log x + \log (x^3 - y^3) - \log (x^3 + y^3) \quad \dots\dots(2)$$

Now, differentiating both sides of equation (2) partially w. r. t. x , we get

$$\frac{1}{u} \frac{\partial u}{\partial x} = \frac{1}{x} + \frac{1}{x^3 - y^3} (3x^2) - \frac{1}{x^3 + y^3} (3x^2)$$

$$\text{This } \Rightarrow \frac{1}{u} \left(x \frac{\partial u}{\partial x} \right) = 1 + \frac{3x^3}{x^3 - y^3} - \frac{3x^3}{x^3 + y^3} \quad \dots\dots(3)$$

Similarly, differentiating both sides of equation (2) partially w. r. t. y , we get

$$\frac{1}{u} \frac{\partial u}{\partial y} = 0 + \frac{1}{x^3 - y^3} (-3y^2) - \frac{1}{x^3 + y^3} (3y^2)$$

$$\text{This } \Rightarrow \frac{1}{u} \left(y \frac{\partial u}{\partial y} \right) = -\frac{3y^3}{x^3 - y^3} - \frac{3y^3}{x^3 + y^3} \quad \dots\dots(4)$$

Adding equations (3) and (4), we get

$$\begin{aligned}\frac{1}{u}\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right) &= 1 + \frac{3(x^3-y^3)}{x^3-y^3} - \frac{3(x^3+y^3)}{x^3+y^3} \\ &= 1 + 3 - 3 \\ &= 1\end{aligned}$$

$$\text{This } \Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = u$$

\Rightarrow Euler's Theorem is verified for the given function.

Example 4 : If $u = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0$.

Solution : Given that

$$u = \cos^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}.$$

$$\text{This } \Rightarrow \cos u = \frac{x+y}{\sqrt{x} + \sqrt{y}} = \frac{x \left[1 + \frac{y}{x} \right]}{\sqrt{x} \left[1 + \sqrt{\frac{y}{x}} \right]} = x^{\frac{1}{2}} \frac{\left[1 + \frac{y}{x} \right]}{\left[1 + \sqrt{\frac{y}{x}} \right]} = x^{\frac{1}{2}} \phi \left(\frac{y}{x} \right), \text{ where } \phi \text{ is a function of } \frac{y}{x}.$$

$\Rightarrow \cos u$ is a homogeneous function of degree $\frac{1}{2}$ in two independent variables x, y .

Let $v = \cos u$ (1)

Then v is a homogeneous function of degree $\frac{1}{2}$ in two independent variables x, y . Therefore, by Euler's Theorem,

$$x \frac{\partial v}{\partial x} + y \frac{\partial v}{\partial y} = \frac{1}{2} v.$$

$$\text{This } \Rightarrow x \left(-\sin u \frac{\partial u}{\partial x} \right) + y \left(-\sin u \frac{\partial u}{\partial y} \right) = \frac{1}{2} \cos u$$

$$\left(\because \frac{\partial v}{\partial x} = -\sin u \frac{\partial u}{\partial x}, \frac{\partial v}{\partial y} = -\sin u \frac{\partial u}{\partial y} \text{ and } v = \cos u, \text{ by equation (1)} \right)$$

$$\Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = -\frac{1}{2} \frac{\cos u}{\sin u}$$

$$= -\frac{1}{2} \cot u$$

$$\Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + \frac{1}{2} \cot u = 0.$$

About Auther



**Shaipali Maurya is Student of Second Semester student.
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