DA_Homework_05_r09546042_TerryYang

April 11, 2021

1 Q1

import package

```
[1]: import pandas as pd
import numpy as np
from scipy import stats
```

read data

```
[2]: Female_data = pd.read_csv(r"D:\Program Files\Document\DA\Female.csv")

Male_data = pd.read_csv(r"D:\Program Files\Document\DA\Male.csv")

N = len(Female_data)

#log transformation

Female_data_log = np.log1p(Female_data)

Male_data_log = np.log1p(Male_data)

Var_Female_data = Female_data_log.var(ddof=1)

Var_Male_data = Male_data_log.var(ddof=1)
```

calculate statisics

```
[3]: #std deviation
s = np.sqrt((Var_Female_data + Var_Male_data)/2)
t = (Female_data_log.mean() - Male_data_log.mean())/(s*np.sqrt(2/N))
t2, p2 = stats.ttest_ind(Female_data_log,Male_data_log)
#
df = 2*N - 2
# p-value after comparison with the t
p = 1 - stats.t.cdf(t,df=df)
```

print result

```
[4]: print("p = " + str(2*p))
print("t = " + str(t2))
print("p(t) = " + str(2*p2))
```

```
p = [5.68158054e-05 \ 2.10973532e-05 \ 4.77621800e-08]

t = [4.43535567 \ 4.7374114 \ 6.52107442]
```

```
p(t) = [1.13631611e-04 \ 4.21947064e-05 \ 9.55243601e-08]
```

```
[5]: for i in range(0, len(p)):
    if 2*p[i] < p2[i]:
        print(Female_data.columns[i]+" is significantly different")
    else:
        print(Female_data.columns[i]+" is not significantly different")</pre>
```

Length is significantly different Width is significantly different Height is significantly different

2 Q2

import package

```
[6]: from PIL import Image
  import numpy as np
  from numpy import array
  from tkinter import _flatten
  from sklearn.linear_model import Lasso
  from sklearn.linear_model import Ridge
  import math
  import matplotlib.pyplot as plt
```

2.1 a.

read ORL data

find less mse under different lambda

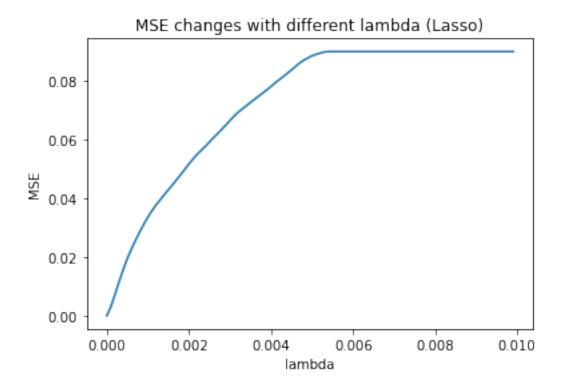
```
[8]: mse = []
x_axis = []
for i in range(0,100,1):
```

```
'''lasso '''
    lasso = Lasso(alpha = i/10000, normalize = True)
    lasso.fit(X, y)
    mse.append(np.mean((lasso.predict(X) - y) ** 2))
    x_axis.append(i/10000)
plt.xlabel("lambda")
plt.ylabel("MSE")
plt.title("MSE changes with different lambda (Lasso)")
plt.plot(x_axis,mse)
<ipython-input-8-eef449c5a4e7>:6: UserWarning: With alpha=0, this algorithm does
not converge well. You are advised to use the LinearRegression estimator
  lasso.fit(X, y)
C:\Users\TerryYang\anaconda3\envs\TENSORFLOW\lib\site-
packages\sklearn\linear_model\_coordinate_descent.py:530: UserWarning:
Coordinate descent with no regularization may lead to unexpected results and is
discouraged.
  model = cd_fast.enet_coordinate_descent(
C:\Users\TerryYang\anaconda3\envs\TENSORFLOW\lib\site-
```

packages\sklearn\linear_model_coordinate_descent.py:530: ConvergenceWarning: Objective did not converge. You might want to increase the number of iterations. Duality gap: 0.016627935728148246, tolerance: 0.0036000000000000003

model = cd_fast.enet_coordinate_descent(

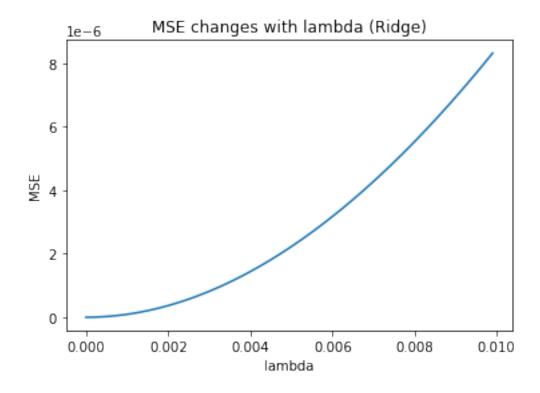
[8]: [<matplotlib.lines.Line2D at 0x20082aff970>]



```
[9]: mse = []
    x_axis = []
    for i in range(0,100,1):
        ridge = Ridge(alpha = i/10000,normalize = True)
        ridge.fit(X, y)
        mse.append(np.mean((ridge.predict(X) - y) ** 2))
        x_axis.append(i/10000)
plt.xlabel("lambda")
plt.ylabel("MSE")
plt.title("MSE changes with lambda (Ridge)")
plt.plot(x_axis,mse)
```

C:\Users\TerryYang\anaconda3\envs\TENSORFLOW\lib\sitepackages\sklearn\linear_model_ridge.py:187: LinAlgWarning: Ill-conditioned
matrix (rcond=4.06671e-19): result may not be accurate.
 dual_coef = linalg.solve(K, y, sym_pos=True,

[9]: [<matplotlib.lines.Line2D at 0x20082bd2700>]



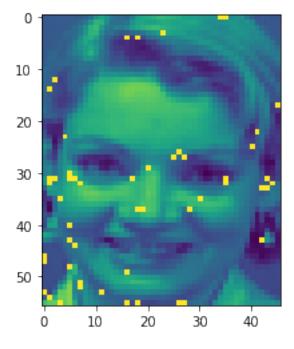
MSE is smaller when lambda is small, thus we choose a small value (0.001) as lambda

2.2 b.

Lasso regression under small lambda (0.001)

Lasso 52

plot the selected pixels on the image



3 Q3

import package

```
[12]: import pandas as pd
      import numpy as np
      import statsmodels.api as sm
      import scipy
     read data
[13]: data = pd.read_csv(r"D:\Program Files\Document\DA\economics of transportation_
      →equipment.csv")
      y = data.loc[:,"Value Added"]
      X = data.loc[:,["Year","Capital","Labor"]]
     using OLS regression
[14]: model = sm.regression.linear_model.OLS(y, X).fit()
      print(model.summary())
      X_Labor = X.loc[:,"Labor"].to_numpy()
      X_Capital = X.loc[:,"Capital"].to_numpy()
      y=y.to_numpy()
                                      OLS Regression Results
     Dep. Variable:
                             Value Added R-squared (uncentered):
     0.996
     Model:
                                       OLS
                                            Adj. R-squared (uncentered):
     0.995
     Method:
                             Least Squares
                                            F-statistic:
     931.6
     Date:
                          Sun, 11 Apr 2021
                                            Prob (F-statistic):
     1.79e-14
                                  18:58:23
     Time:
                                            Log-Likelihood:
     -120.00
     No. Observations:
                                        15
                                            AIC:
     246.0
     Df Residuals:
                                        12
                                             BIC:
     248.1
     Df Model:
                                         3
     Covariance Type:
                                 nonrobust
```

	coef	std err	t	P> t	[0.025	0.975]
Year Capital Labor	-39.6373 0.0064 0.0053	26.892 0.006 0.004	-1.474 1.163 1.212	0.166 0.268 0.249	-98.231 -0.006 -0.004	18.956 0.018 0.015
Omnibus: Prob(Omnibus): Skew: Kurtosis:		0.0	5.134 Durbin 0.077 Jarque 0.979 Prob(J 3.607 Cond.		:	1.864 2.628 0.269 2.21e+05

Warnings:

- [1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
- [2] The condition number is large, 2.21e+05. This might indicate that there are strong multicollinearity or other numerical problems.

```
C:\Users\TerryYang\anaconda3\envs\TENSORFLOW\lib\site-packages\scipy\stats\stats.py:1603: UserWarning: kurtosistest only valid for n>=20 ... continuing anyway, n=15
```

warnings.warn("kurtosistest only valid for n>=20 ... continuing "

define Cobb-Douglas function

```
[15]: def sm_model(B1): return np.sum( ((X_Capital * B1 + X_Labor * (1-B1)) - y) ** 2 )
```

optimize

```
[16]: res = scipy.optimize.minimize(sm_model, x0 = 0)
print('under constrain that Beta1 + Beta2 = 1\nestimate: Beta1 = ', res.x[0], \( \to '\), and Beta2 = ', (1- res.x[0]))
```

```
under constrain that Beta1 + Beta2 = 1 estimate: Beta1 = 3.7161686266426086, and Beta2 = -2.7161686266426086
```

4 Q4

import package

```
[17]: import numpy as np import matplotlib.pyplot as plt import pandas as pd
```

read AutoMPG dataset

```
[18]: data = pd.read_csv(r"C:\Users\TerryYang\pythonwork\pythonwork\Data Analytics⊔

→Homework\DA_Demo.csv")

X=data.drop(" car name",axis = 1).to_numpy()
```

define pca function

```
[19]: def pca(dataMat, use_cov, topNfeat=20, ):
    meanVals = np.mean(dataMat, axis=0)
    meanRemoved = dataMat - meanVals
    if use_cov == True:
        covMat = np.cov(meanRemoved, rowvar=0)
    else:
        covMat = np.corrcoef(meanRemoved, rowvar=0)
    eigVal, eigVect = np.linalg.eig(np.mat(covMat))
    meanRemoved_score_matrix = meanRemoved * eigVect
    original_matrix = (meanRemoved_score_matrix * eigVect.T) + meanVals
    return meanRemoved_score_matrix, original_matrix, eigVal, eigVect
```

define plotting function

```
[21]: def show_eigVal_picture(eigVals):
          x_axis=[]
          y = []
          y_culmulated = []
          total_val = eigVals.sum()
          y_culmulated.append(eigVals[0]/total_val)
          for i in range(0,len(eigVals)):
              x axis.append(i)
              y.append(eigVals[i]/total_val)
              if i !=0 :
                  y_culmulated.append(eigVals[i]/total_val+y_culmulated[i-1])
          fig, ax1 = plt.subplots()
          color = 'tab:blue'
          ax1.set_xlabel("number of PC adden")
          ax1.set_ylabel("Variance", color=color)
          ax1.bar(x_axis, eigVals, color=color)
          ax1.tick_params(axis='y', labelcolor=color)
          ax2 = ax1.twinx() # instantiate a second axes that shares the same x-axis
          color = 'tab:red'
```

```
ax2.set_ylabel("Cumulative Percentage", color=color) # we already handled_

→ the x-label with ax1

ax2.plot(x_axis,y_culmulated, color=color)

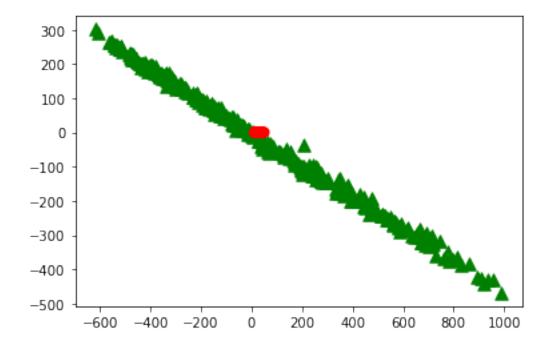
ax2.tick_params(axis='y', labelcolor=color)

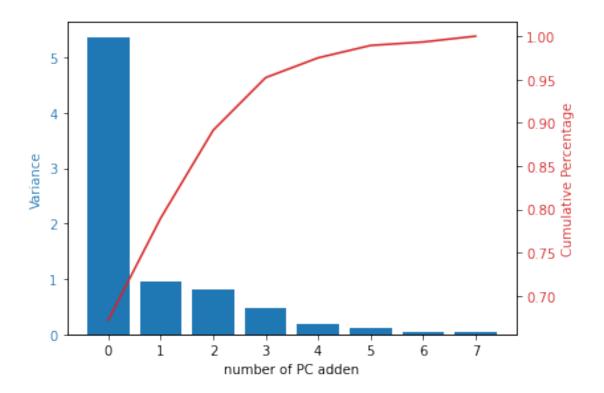
fig.tight_layout() # otherwise the right y-label is slightly clipped

plt.show()
```

calling pca fuction using correlation matrix

[22]: meanRemoved_score_matrix, original_matrix, eigVals, eigVects = pca(X, False)
 show_reduction_picture(meanRemoved_score_matrix,original_matrix)
 show_eigVal_picture(eigVals)





```
print("Eigen Values:\n",eigVals)
  print("Eigen Vectors:\n",eigVects)
  print("Score Matrix:\n", X*eigVects)
Loading Matrix:
    [[-0.38586239 -0.07663269 0.29228579 0.09998251 0.74036644 0.38735165
           0.1151321 -0.19588516]
     \begin{bmatrix} -0.27786815 & 0.50150064 & 0.30732382 & -0.74328281 & -0.04739508 & -0.12086663 \\ \end{bmatrix} 
       -0.07951102 -0.0345266 ]
    [-0.21386777 -0.6904632
                                                                                                         0.5871892 -0.10601968 -0.30134385 -0.11002592
       -0.0542884
                                                          0.12501506]
    \lceil -0.2647309 -0.41690206 -0.63943514 -0.49280794 0.09773197 0.20293343 \rceil
       -0.03518826 0.22891382]
    [ 0.40157579 -0.21102
                                                                                                     -0.00089399 -0.32246785 -0.13127292 0.23585961
           0.30991105 - 0.72202073
    [ \ 0.40183594 \quad 0.11148007 \quad 0.23605571 \quad -0.11971643 \quad -0.08426839 \quad 0.6667096 \quad -0.08426839 \quad 0.6667096 \quad -0.08426839 \quad 0.6667096 \quad -0.08426839 \quad -0.0842689 \quad
           0.13477548 0.53504996]
    [ 0.41644435 -0.12632499
                                                                                                        0.07423622 -0.13581398  0.30331627 -0.00699705
       -0.82916553 -0.08422855]
    0.41774679 0.27878265]]
```

[5.3758723 0.94366326 0.81164365 0.48615594 0.18282657 0.11432193

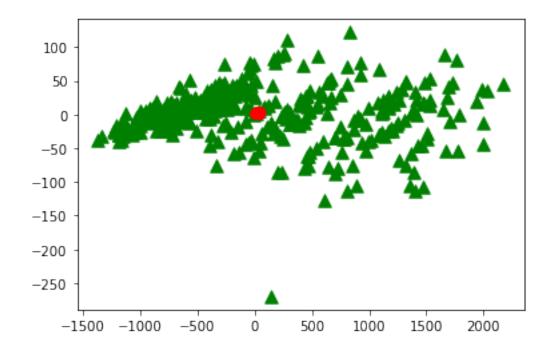
[23]: print("Loading Matrix:\n",eigVects)

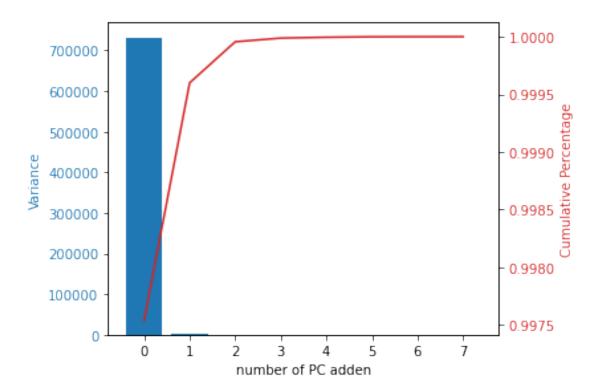
Eigen Values:

```
0.03196954 0.05354682]
Eigen Vectors:
 [[-0.38586239 -0.07663269 0.29228579 0.09998251 0.74036644 0.38735165
   0.1151321 -0.19588516]
 [-0.27786815 \quad 0.50150064 \quad 0.30732382 \quad -0.74328281 \quad -0.04739508 \quad -0.12086663
  -0.07951102 -0.0345266 ]
 [-0.21386777 -0.6904632
                             0.5871892 -0.10601968 -0.30134385 -0.11002592
  -0.0542884 0.12501506
  \begin{bmatrix} -0.2647309 & -0.41690206 & -0.63943514 & -0.49280794 & 0.09773197 & 0.20293343 \end{bmatrix} 
  -0.03518826 0.22891382]
 [ 0.40157579 -0.21102
                            -0.00089399 -0.32246785 -0.13127292 0.23585961
   0.30991105 - 0.72202073
 [ \ 0.40183594 \ \ 0.11148007 \ \ 0.23605571 \ -0.11971643 \ -0.08426839 \ \ 0.6667096
   0.13477548 0.53504996]
  \hbox{ [ 0.41644435 -0.12632499 \ 0.07423622 -0.13581398 \ 0.30331627 \ -0.00699705] } 
  -0.82916553 -0.08422855]
  \begin{bmatrix} 0.4023885 & -0.13842878 & 0.07223935 & -0.21603551 & 0.48261485 & -0.53092548 \end{bmatrix} 
   0.41774679 0.27878265]]
Score Matrix:
 [[ 1565.05683964 -819.02399361
                                       89.92164146 ... 908.31368256
    850.00772014 -2476.09449247]
                                     100.64964589 ... 974.66159067
 [ 1674.21598042 -859.99859581
    877.31613094 -2596.97829107]
 [ 1550.63202373 -803.41770537
                                      96.15958036 ...
                                                      905.32952002
    822.54364543 -2417.45151126]
 [ 979.96668612 -555.93894392
                                      78.48036526 ...
                                                        599.83940236
    611.04553259 -1615.74532333]
 [ 1103.92118365 -626.84985144
                                      70.24633882 ...
                                                        674.31560949
    724.37293978 -1853.03804918]
 [ 1141.49057482 -646.99940547
                                      71.16065031 ... 700.05380021
    755.36522758 -1920.34516473]]
```

calling pca fuction using covariance matrix

[24]: meanRemoved_score_matrix, original_matrix, eigVals, eigVects = pca(X, True) show_reduction_picture(meanRemoved_score_matrix, original_matrix) show_eigVal_picture(eigVals)





```
[25]: print("Loading Matrix:\n",eigVects)
print("\nEigen Values:\n",eigVals)
```

```
print("\nEigen Vectors:\n",eigVects)
print("\nScore Matrix:\n",X*eigVects)
Loading Matrix:
 [[-7.59590581e-03 1.75785097e-02 4.19212264e-02 8.31000630e-01
 -5.49535507e-01 -5.61451547e-02 3.97118180e-02 2.43984110e-02]
 [-5.51527155e-04 3.24675463e-03 -1.23716014e-02 2.00685150e-02
 -5.30801776e-02 -8.77898223e-03 -9.25502182e-01 -3.74145010e-01]
 [-1.33689886e-03 2.39497354e-02 4.41269251e-02 5.51778482e-01
  8.24191632e-01 1.12006207e-01 -3.03888666e-02 -1.60382361e-02]
 [-1.35281211e-03 3.48299933e-02 7.68839939e-02 -2.08463123e-02
 -1.25527681e-01 9.88134411e-01 -9.09879495e-03 1.37739900e-02]
 [ 9.92644743e-01 1.20867105e-01 2.65874637e-03 3.53993832e-03
  -3.93041220e-03 -3.52931159e-03 1.85727735e-04 -1.31438408e-04]
 [ 3.89660894e-02 -2.98328518e-01 -9.47540130e-01 6.34424404e-02
  5.73169239e-03 8.63249865e-02 8.51000586e-03 8.19890724e-03]
 [ 1.14338202e-01 -9.45572572e-01 3.03873075e-01 9.22239231e-03
  3.41321063e-03 1.06909276e-02 -9.19061696e-04 -1.63889805e-02]
 [ 1.79257460e-03 -1.33223878e-02 7.28065126e-03 -4.31502411e-03
  9.17751826e-03 -1.53912563e-02 -3.75218753e-01 9.26626917e-01]]
Eigen Values:
 [7.32193919e+05 1.51443435e+03 2.61673181e+02 2.32569592e+01
 5.54405192e+00 2.85720860e+00 3.60437639e-01 2.58474909e-01]
Eigen Vectors:
 [[-7.59590581e-03 1.75785097e-02 4.19212264e-02 8.31000630e-01
  -5.49535507e-01 -5.61451547e-02 3.97118180e-02 2.43984110e-02]
 [-5.51527155e-04 3.24675463e-03 -1.23716014e-02 2.00685150e-02
 -5.30801776e-02 -8.77898223e-03 -9.25502182e-01 -3.74145010e-01]
 [-1.33689886e-03 2.39497354e-02 4.41269251e-02 5.51778482e-01
  8.24191632e-01 1.12006207e-01 -3.03888666e-02 -1.60382361e-02]
 [-1.35281211e-03 3.48299933e-02 7.68839939e-02 -2.08463123e-02
 -1.25527681e-01 9.88134411e-01 -9.09879495e-03 1.37739900e-02
 [ 9.92644743e-01 1.20867105e-01 2.65874637e-03 3.53993832e-03
 -3.93041220e-03 -3.52931159e-03 1.85727735e-04 -1.31438408e-04]
 [ 3.89660894e-02 -2.98328518e-01 -9.47540130e-01 6.34424404e-02
  5.73169239e-03 8.63249865e-02 8.51000586e-03 8.19890724e-03]
 [ 1.14338202e-01 -9.45572572e-01 3.03873075e-01 9.22239231e-03
  3.41321063e-03 1.06909276e-02 -9.19061696e-04 -1.63889805e-02]
 [ 1.79257460e-03 -1.33223878e-02 7.28065126e-03 -4.31502411e-03
  9.17751826e-03 -1.53912563e-02 -3.75218753e-01 9.26626917e-01]]
Score Matrix:
 -3.97390688e+00 2.09453383e+00]
 [ 3.71207552e+03 6.84249866e+01 -3.55220523e+01 ... 2.31815936e+01
```

-3.79505984e+00 1.57184533e+00]

two results are very different between covariance and correlation methods, thus PCA is not scale-invariant

5 Q5

import package

```
[34]: import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
from numpy import array
from tkinter import _flatten
```

read ORL data

define PCA function

```
def pca(dataMat, use_cov, topNfeat=20, ):
    meanVals = np.mean(dataMat, axis=0)
    meanRemoved = dataMat - meanVals
    if use_cov == True:
        covMat = np.cov(meanRemoved, rowvar=0)
    else:
        covMat = np.corrcoef(meanRemoved, rowvar=0)
    eigVal, eigVect = np.linalg.eig(np.mat(covMat))
    meanRemoved_score_matrix = meanRemoved * eigVect
    original_matrix = (meanRemoved_score_matrix * eigVect.T) + meanVals
    return meanRemoved_score_matrix, original_matrix, eigVal, eigVect
```

define analyse function

```
[41]: def analyse_data(dataMat):
         Printed = 0
         meanVals = np.mean(dataMat, axis=0)
         meanRemoved = dataMat-meanVals
         covMat = np.cov(meanRemoved, rowvar=0)
         eigvals, eigVects = np.linalg.eig(np.mat(covMat))
         eigValInd = np.argsort(eigvals)
         topNfeat = 2576
         eigValInd = eigValInd[:-(topNfeat+1):-1]
         cov all score = complex(sum(eigvals)).real
         sum cov score = 0
         for i in range(0, len(eigValInd)):
             line_cov_score = complex(eigvals[eigValInd[i]]).real
             sum_cov_score += line_cov_score
             if 60 > (sum_cov_score/cov_all_score*100).real > 50 and Printed == 0:
                print('Principal components %s, Variance percentage %s%%, Cumulated ∪
      →percentage %s%%' % (format(i+1, '2.0f'), format(line_cov_score/
      Printed = 1
             elif 70 > (sum_cov_score/cov_all_score*100).real > 60 and Printed == 1:
                print('Principal components %s, Variance percentage %s%%, Cumulated ∪
      →percentage %s%%' % (format(i+1, '2.0f'), format(line_cov_score/
      →cov_all_score*100, '4.2f'), format(sum_cov_score/cov_all_score*100, '4.1f')))
                Printed = 2
             elif 80 > (sum_cov_score/cov_all_score*100).real > 70 and Printed == 2:
                print('Principal components %s, Variance percentage %s%%, Cumulated ∪
      →percentage %s%%' % (format(i+1, '2.0f'), format(line_cov_score/
      Printed = 3
             elif 90 > (sum_cov_score/cov_all_score*100).real > 80 and Printed == 3:
                print('Principal components %s, Variance percentage %s%%, Cumulated ∪
      →percentage %s%%' % (format(i+1, '2.0f'), format(line_cov_score/
      →cov_all_score*100, '4.2f'), format(sum_cov_score/cov_all_score*100, '4.1f')))
                Printed = 4
             elif (sum_cov_score/cov_all_score*100).real > 90 and Printed == 4:
                print('Principal components %s, Variance percentage %s%%, Cumulated ∪
      →percentage %s%%' % (format(i+1, '2.0f'), format(line_cov_score/
      →cov_all_score*100, '4.2f'), format(sum_cov_score/cov_all_score*100, '4.1f')))
                Printed = 5
```

5.1 a(1).

call function (X=400x2576)

```
[42]: meanRemoved_score_matrix, original_matrix, eigVals, eigVects = pca(X, True) analyse_data(meanRemoved_score_matrix)
```

```
Principal components 5, Variance percentage 5.42%, Cumulated percentage 51.0% Principal components 9, Variance percentage 2.07%, Cumulated percentage 61.7% Principal components 16, Variance percentage 0.95%, Cumulated percentage 70.8% Principal components 32, Variance percentage 0.40%, Cumulated percentage 80.3% Principal components 76, Variance percentage 0.13%, Cumulated percentage 90.1%
```

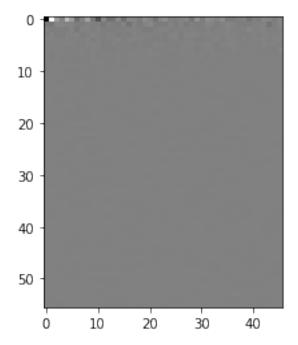
5.2 b(1).

plot PC1 image

```
[31]: first_PC = X@eigVects
    first_PC=first_PC[0]
    first_PC_array=first_PC.reshape(56,46).real

min_first_PC_array = np.min(first_PC_array)
    range_first_PC_array = np.max(first_PC_array) - np.min(first_PC_array)
    for i, j in enumerate(first_PC_array):
        first_PC_array[i] = 255 * ((j - min_first_PC_array) / range_first_PC_array)

imgplot = plt.imshow(first_PC_array, cmap='gray', vmin=0, vmax=255)
    plt.show()
```



5.3 a(2). call function (X=2576x400)

[32]: meanRemoved_score_matrix, original_matrix, eigVals, eigVects = pca(X.T, True) analyse_data(meanRemoved_score_matrix)

```
2, 10.55%, 57.0%
3, 4.57%, 61.5%
6, 2.07%, 70.5%
15, 0.66%, 80.2%
47, 0.17%, 90.1%
```

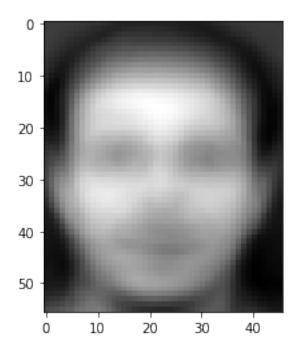
5.4 b(2).

plot PC1 image

```
[33]: first_PC = X.T@eigVects
    first_PC=first_PC.T[0]
    first_PC_array=first_PC.reshape(56,46).real

min_first_PC_array = np.min(first_PC_array)
    range_first_PC_array = np.max(first_PC_array) - np.min(first_PC_array)
    for i, j in enumerate(first_PC_array):
        first_PC_array[i] = 255 * ((j - min_first_PC_array) / range_first_PC_array)

imgplot = plt.imshow(first_PC_array, cmap='gray', vmin=0, vmax=255)
    plt.show()
```



[]: