Q1 $y_t = a + bt + C_t + X_t$ with C_t is deterministic,

and periodic with period s,

with $\{X_t\}$ is a $SARIMA(p,0,q)\times(P,1,Q)_s$.

$\Rightarrow W_t = y_t - y_{t-s}$

$\Rightarrow (\cancel{a} + bt + C_t + X_t) - (\cancel{a} + b(t-s) + C(t-s) + X(t-s))$

$\Rightarrow bs + C_t - C_{t-s} + \nabla_s X_t \Rightarrow bs + \nabla_s X_t$

$\therefore \{W_t\}$ is $ARMA(p,q)$ with a constant term bs #

Q2 (a)

$$y_t = 0.5 y_{t-1} + y_{t-4} - 0.5 y_{t-5} + a_t - 0.3 a_{t-1}$$

$$\Rightarrow (y_t - y_{t-4}) = 0.5 (y_{t-1} - y_{t-5}) + a_t - 0.3 a_{t-1}$$

$\Rightarrow$ ARIMA$(1, 0, 1) \times (0, 1, 0)_4$, with $\phi_1 = 0.5$

$\theta_1 = 0.3$ #

season = 4

Q2 (b)

$$y_t = y_{t-1} + y_{t-12} - y_{t-13} + a_t - 0.5 a_{t-1} - 0.5 a_{t-12} + 0.25 a_{t-13}$$

$$\Rightarrow (y_t - y_{t-1}) - (y_{t-12} - y_{t-13}) = \nabla \nabla_{12} y_t$$

$$= a_t - 0.5 a_{t-1} - 0.5 a_{t-12} + (0.5 \times 0.5) a_{t-13}$$

$\Rightarrow$ ARIMA$(0, 1, 1) \times (0, 1, 1)_{12}$, with $\theta_1 = 0.5$

$\Theta = 0.5$

season = 12

Q3(a)

Characteristic function : $(1-1.6B+0.7B^2)(1-0.8B^{12})$

$\Phi = 0.8 \Rightarrow$ Seasonal part is stationary

Non Seasonal Part :

$$\phi_1 = 1.6\ ,\ \phi_2 = (-0.7)$$

$$\begin{cases} \phi_1+\phi_2 = 0.9 < 1 \\ \phi_2-\phi_1 = (-0.23) < 1 \\ |\phi_2| = 0.7 < 1 \end{cases}$$

$\Rightarrow$ Complete Model is Stationary.

Q3(b)

$\Rightarrow$ SARIMA $(2,0,0)\times(1,0,0)_{12}$ #

$$\Rightarrow y_t = 1.6\,y_{t-1} - 0.7\,y_{t-2} + 0.8\,y_{t-12} - 1.6(0.8)\,y_{t-13} + 0.7(0.8)\,y_{t-14} + a_t.$$

$$\Rightarrow y_t = 1.6\,y_{t-1} - 0.7\,y_{t-2} + 0.8\,y_{t-12} - 1.28\,y_{t-13} + 0.56\,y_{t-14} + a_t$$

#

Q4 (a) $y_t = y_{t-4} + a_t$; with $y_t = a_t$, $t = 1, 2, 3, 4$

$\Rightarrow E(y_t) = 0$, $t = 4k + r$ with $r = 1, 2, 3$ and $k = 0, 1, 2, 3$

(r is quater ; k is year).

$$y_t = y_{t-4} + a_t = (y_{t-8} + a_{t-4}) + a_t = y_{t-12} + a_{t-8} + a_{t-4} + a_t$$

$$= \underbrace{a_t + a_{t-4} + a_{t-8} + \cdots + a_{r+4} + a_r}_{(k+1)}$$

$\Rightarrow Var(y_t) = (k+1)\sigma_a^2$ #

Q4 (b)

let $s > t \Rightarrow s = 4j + i$, where $\begin{cases} i = 1, 2, 3, 4 \\ j = 0, 1, 2, 3 \end{cases}$

$Cov(y_t, y_s) = Cov(a_t + a_{t-4} + a_{t-8} + \cdots + a_r,$
$a_s + a_{s-4} + a_{s-8} + \cdots a_i)$

if $r \neq i \Rightarrow$ there would be no overlap $\Rightarrow Cov(y_t, y_s) = 0$.

if $r = i \Rightarrow Cov(y_t, y_s) = Var(y_t)$.

For $s > t$

$$Cov(y_t, y_s) \begin{cases} (k+1)\sigma_a^2 & ; \text{if } r = i \\ 0 & ; \text{else} \end{cases} \Rightarrow Corr(y_t, y_s) = \begin{cases} \sqrt{\dfrac{k+1}{j+1}} & ; \begin{matrix} s = 4j + r \\ t = 4k + r \end{matrix} \\ 0 & ; \text{else} \end{cases}$$

#

Q4(c) $y_t = y_{t-4} + a_t \Rightarrow y_t - y_{t-4} = a_t$.

$\Rightarrow$ SARIMA$(0,0,0) \times (0,1,0)_4$ #