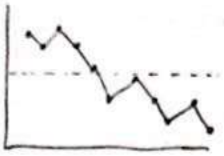
1.

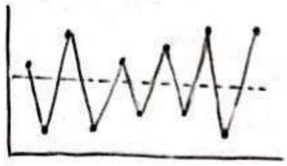
(a)



ρ_1: strongly positive (∵相鄰兩點位在 mean 同側)

ρ_2: moderately positive (∵兩點中間隔一點,大致位於 mean 的同側)

(b)



ρ_1: strongly negative (∵相鄰兩點位在 mean 的不同側)

ρ_2: strongly positive (∵兩點中間隔一點,幾乎位於 mean 的同側)

No need to know the scale of measurement for the series.

2. (a) $y_t = 3 + y_{t-1} + a_t - 0.75a_{t-1} \Rightarrow ARIMA(0,1,1)$, $p=0, d=1, q=1$

$E(\nabla y_t) = E(y_t - y_{t-1}) = E(3 + a_t - 0.75a_{t-1}) = 3$ #

$V(\nabla y_t) = E[(y_t - y_{t-1})^2] - [E(y_t - y_{t-1})]^2$

$= E[(3 + a_t - 0.75a_{t-1})(3 + a_t - 0.75a_{t-1})] - 9$

$= E(9 + a_t^2 + 0.5625a_{t-1}^2) - 9 = 1.5625\sigma_a^2$ #

(b) $y_t = 10 + 1.25y_{t-1} - 0.25y_{t-2} + a_t - 0.1a_{t-1} \Rightarrow ARIMA(1,1,1)$ $p=1$ $d=1$ $q=1$

$\Rightarrow y_t - y_{t-1} = 10 + 0.25(y_{t-1} - y_{t-2}) + a_t - 0.1a_{t-1}$

∴ $\nabla y_t = y_t - y_{t-1}$ 可視為 ARMA(1,1) with $\phi = 0.25$, $\theta = 0.1$

← ∵ stationary.

各取期望值 ∴ $E(\nabla y_t) = 10 + 0.25E(\nabla y_t) \Rightarrow E(\nabla y_t) = \frac{10}{0.75} = 13.33$ #

$V(\nabla y_t) = \gamma_0 = \left(\frac{1 - 2\theta\phi + \theta^2}{1 - \phi^2}\right)\sigma_a^2 = \frac{1 - 2\times0.25\times0.1 + 0.1^2}{1 - 0.25^2}\sigma_a^2$

$= 1.024\sigma_a^2$ #

(c) $y_t = 5 + 2y_{t-1} - 1.7y_{t-2} + 0.7y_{t-3} + a_t - 0.5a_{t-1} + 0.25a_{t-2}$

$\Rightarrow \nabla y_t - \nabla y_{t-1} + 0.7\nabla y_{t-2} = 5 + a_t - 0.5a_{t-1} + 0.25a_{t-2} \Rightarrow ARIMA(2,1,2)$

$p=2, d=1, q=2$

$E(\nabla y_t) - \underbrace{E(\nabla y_{t-1})}_{=E(\nabla y_t)} + 0.7\underbrace{E(\nabla y_{t-2})}_{=E(\nabla y_t)} = 5 \Rightarrow E(\nabla y_t) = \frac{5}{0.7} = 7.1429$ #

$\phi_1 = 1$

$\phi_2 = -0.7$

$\theta_1 = 0.5$

$\theta_2 = -0.25$

$(-0.49)\cdot V(\nabla y_t) = -1.4\gamma_1 + 0.2125\sigma_a^2$

$\Rightarrow V(\nabla y_t) = \frac{-1.4\gamma_1 + 0.2125\sigma_a^2}{-0.49}$ #

ARMA(2,2) : $(1-\phi_1 B-\phi_2 B^2)y_t = (1-\theta_1 B-\theta_2 B^2)a_t$

則 $V(y_t)=E[y_t y_t] = E\left[(\phi_1 y_{t-1}+\phi_2 y_{t-2}+a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})(\phi_1 y_{t-1}+\phi_2 y_{t-2}+a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})\right]$

$$= E\left[\phi_1^2 y_{t-1}^2 + \phi_1\phi_2 y_{t-1}y_{t-2} - \theta_1\phi_1 y_{t-1}a_{t-1} - \theta_2\phi_1 y_{t-1}a_{t-2} + \phi_1\phi_2 y_{t-1}y_{t-2} + \phi_2^2 y_{t-2}^2 - \theta_2\phi_2 y_{t-2}a_{t-2} + a_t^2 - \phi_1\theta_1 a_{t-1}y_{t-1} + \theta_1^2 a_{t-1}^2 - \phi_1\theta_2 y_{t-1}a_{t-2} - \theta_2\phi_2 a_{t-2}y_{t-2} + \theta_2^2 a_{t-2}^2\right]$$

$$=(\phi_1^2+\phi_2^2)V(y_t) + 2\phi_1\phi_2 E(y_t y_{t-1}) + (1+\theta_1^2+\theta_2^2)\sigma_a^2 - 2(\theta_1\phi_1+\theta_2\phi_2)\sigma_a^2 - 2\theta_2\phi_1(\phi_1-\theta_1)\sigma_a^2$$

$$\Rightarrow (1-\phi_1^2-\phi_2^2)V(y_t) = 2\phi_1\phi_2\gamma_1 + \left(1+\theta_1^2+\theta_2^2-2\theta_1\phi_1-2\theta_2\phi_2-2\theta_2\phi_1^2+2\theta_1\theta_2\phi_1\right)\sigma_a^2$$

NOTE: $E[y_t a_t] = E[a_t(\phi_1 y_{t-1}+\phi_2 y_{t-2}+a_t-\theta_1 a_{t-1}-\theta_2 a_{t-2})] = \sigma_a^2$

$E[y_t, a_{t-1}] = E[a_{t-1}(\phi_1 y_{t-1}+\phi_2 y_{t-2}+\cancel{a_t}-\theta_1 a_{t-1}-\cancel{\theta_2 a_{t-2}})] = \phi_1\sigma_a^2-\theta_1\sigma_a^2 = (\phi_1-\theta_1)\sigma_a^2$

3. $y_t = A + Bt + x_t$, A, B is constant.

(a) $E(y_t) = A+Bt$ is dependent with t, Therefore, y_t is NOT stationary.

(b) $\nabla y_t = y_t - y_{t-1}$

$E(\nabla y_t) = E[(\cancel{A}+\cancel{Bt}+x_t)-(\cancel{A}+\cancel{B(t-1)}+x_{t-1})] = B$ independent with t.

$\mathrm{cov}(\nabla y_t, \nabla y_{t-k}) = \mathrm{cov}(\cancel{B}+x_t-x_{t-1}, \cancel{B}+x_{t-k}-x_{t-k-1}) = 0$ independent with t

$\therefore$ ∇y_t is stationary.

$y_t = A+Bt+X_t$, A, B is r.v.

(c) $E(y_t) = E(A+Bt+X_t) = E(A)+tE(B)$ is dependent with t $\Rightarrow$ y_t is NOT stationary.

(d) $E(\nabla y_t) = E[(A+Bt+X_t)-(A+B(t-1)+X_{t-1})] = E[B+X_t-X_{t-1}] = E(B)$ independent with t

$\mathrm{cov}(\nabla y_t, \nabla y_{t-k}) = \mathrm{cov}(B+X_t+X_{t-1}, B+X_{t-k}+X_{t-k-1}) = V(B)$ independent with t

$\therefore$ ∇y_t is stationary.

4. $\rho_1 < 0.5 \Rightarrow \dfrac{\mathrm{cov}(y_t, y_{t-1})}{V(y_t)} < \dfrac{1}{2} \Rightarrow 2\mathrm{cov}(y_t, y_{t-1}) < V(y_t)$

$\Rightarrow 2V(y_t) - 2\mathrm{cov}(y_t, y_{t-1}) > V(y_t)$

$\Rightarrow V(\nabla y_t) > V(y_t)$ #

註: $V(\nabla y_t) = E[(y_t-y_{t-1})(y_t-y_{t-1})]$

$= V(y_t) - 2\mathrm{cov}(y_t, y_{t-1}) + V(y_t) = 2V(y_t) - 2\mathrm{cov}(y_t, y_{t-1})$