HW05.

IE所碩一
趙上涵
R08546036.

Q1

Q2(a)

$\nabla y_t = y_t - y_{t-1} \Rightarrow (3+y_{t-1}+a_t-0.75a_{t-1})-(y_{t-1})$

$\Rightarrow 3+a_t-0.75a_{t-1} \Rightarrow E(3+a_t-0.75a_{t-1}) = 3 = E(\nabla y_t)$ #

$Var(\nabla y_t) = (1^2+(-0.75)^2)\sigma_e^2 \Rightarrow (1+\frac{9}{16})\sigma_e^2 = \frac{25}{16}\sigma_e^2$ #

Q2(b)

$\nabla y_t = 10+0.25y_{t-1}-0.25y_{t-2}+a_t-0.1a_{t-1}$.

$= 10+0.25(y_{t-1}-y_{t-2})+a_t-0.1a_{t-1}$ $\Rightarrow$ stationary & invertible.

$\Rightarrow E(\nabla y_t) = \frac{\theta_0}{1-\phi} = \frac{10}{1-0.25} \Rightarrow \frac{10}{0.75} = \frac{40}{3}$ #

$Var(\nabla y_t) = \frac{(1-2\phi\theta+\theta^2)}{1-\phi^2}\sigma_e^2 \Rightarrow \frac{(1-2(0.25)(0.1)+(0.1)^2}{1-(0.25)^2}\sigma_e^2 \Rightarrow 1.024\sigma_e^2$

Q2(c)

Factoring AR $\Rightarrow 1-2x+1.7x^2-0.7x^3 \Rightarrow (1-x)(1-x+0.7x^2)$

$\Rightarrow$ AR(2) models.

$\nabla y_t = 5 + \nabla y_{t-1} - 0.7 y_{t-2} + e_t - 0.5 e_{t-1} + 0.25 e_{t-2}$

$\Rightarrow$ ARIMA (2, 1, 2) Model with $\phi_1 = 1$, $\phi_2 = (-0.75)$, $\theta = 0.5$

$\theta_2 = (-0.25)$ & $\theta_0 = 5$. $\Rightarrow E(\nabla y_t) = \frac{5}{1-0.5} \Rightarrow \frac{5}{0.5} = 10$

Q3(a)

$E(y_t) = A + B_t \Rightarrow$ Different when different ts $\Rightarrow$ not stationary

Q3(b)

$\nabla y_t = (A + B_t + X_t) - (A + B_{t-1} + X_{t-1}) \Rightarrow B_t - B_{t-1} + X_t - X_{t-1}$

$\Rightarrow B + X_t - X_{t-1} \Rightarrow B + \nabla X_t \Rightarrow E(\nabla y_t) = B$

And $Cov(y_t, y_{t-k}) = 0$ $\Rightarrow$ Is stationary

(B is constant)

Q3(c)

$E(y_t) + E(A) + E(B_t) \Rightarrow$ still different from t

$\Rightarrow$ Not Stationary

Q3(d)

$\Rightarrow E(\nabla y_t) = E(\beta)$ (the same)

$Cov(y_t, y_{t-k}) \Rightarrow Cov(\beta + \nabla X_t, \beta + \nabla X_{t-k}) = Var(\beta)$

(β is random variable)

$\Rightarrow$ Is still stationary ##

Q4

$Var(\nabla y_t) = Var(y_t) + Var(y_{t-1}) - 2Cov(y_t, y_{t-1})$

$\Rightarrow 2(1-\rho_1)Var(y_t) \Rightarrow$ if $\rho_1 < \frac{1}{2} \Rightarrow (1-\rho_1) > 0.5$

$\Rightarrow$ is larger than y_t ##