

Time Series Analytics - HW 01

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```
[1]: # import packages needed
import math
import matplotlib.pyplot as plt
import numpy as np
import random
```

Q1

```
[2]: def simulator(time_list, threshold):
    random.seed(seed_value)
    phi = random.uniform(0, 1.)
    output_list = []
    for index, time_unit in enumerate(time_list):
        value = math.cos(2 * math.pi * ((time_unit / 12) * phi)) + (time_list[-1] - time_unit) * threshold
        output_list.append(np.around(value, decimals=4))
    return output_list
```

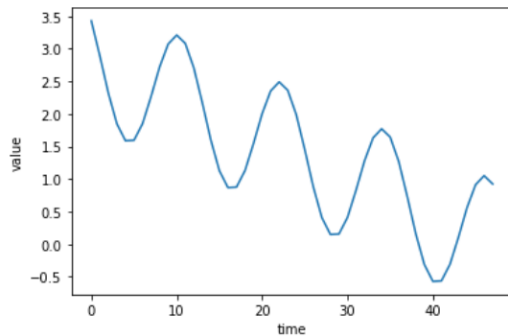
```
[3]: seed_value = 100
time_list = [i for i in range(48)]
threshold = 0.06

output_list = simulator(time_list=time_list,
                        threshold=threshold)

print(output_list, '\n')

plt.plot(output_list)
plt.xlabel('time')
plt.ylabel('value')
plt.show()
```

[3.4296, 2.8915, 2.3183, 1.8473, 1.5887, 1.5957, 1.8504, 2.2685, 2.7217, 3.0727, 3.2113, 3.0843, 2.7096, 2.1715, 1.5983, 1.1273, 0.8687, 0.8757, 1.1304, 1.5485, 2.0017, 2.3527, 2.4913, 2.3643, 1.9896, 1.4515, 0.8783, 0.4073, 0.1487, 0.1557, 0.4104, 0.8285, 1.2817, 1.6327, 1.7713, 1.6443, 1.2696, 0.7315, 0.1583, -0.3127, -0.5713, -0.5643, -0.3096, 0.1085, 0.5617, 0.9127, 1.0513, 0.9243]



Q2

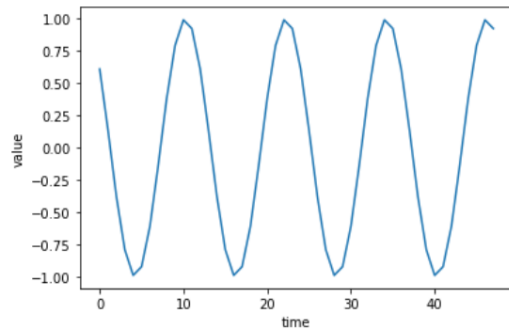
```
[4]: seed_value = 100
time_list = [i for i in range(48)]
threshold = 0

output_list = simulator(time_list=time_list,
                        threshold=threshold)

print(output_list, '\n')

plt.plot(output_list)
plt.xlabel('time')
plt.ylabel('value')
plt.show()
```

```
[0.6096, 0.1315, -0.3817, -0.7927, -0.9913, -0.9243, -0.6096, -0.1315, 0.3817, 0.7927, 0.9913, 0.9243, 0.6096, 0.1315, -0.3817, -0.7927, -0.9913,
-0.9243, -0.6096, -0.1315, 0.3817, 0.7927, 0.9913, 0.9243, 0.6096, 0.1315, -0.3817, -0.7927, -0.9913, -0.9243, -0.6096, -0.1315, 0.3817, 0.7927,
0.9913, 0.9243, 0.6096, 0.1315, -0.3817, -0.7927, -0.9913, -0.9243, -0.6096, -0.1315, 0.3817, 0.7927, 0.9913, 0.9243]
```



Q3 $V(X)=V(Y)$, ask $\text{COV}(X+Y, X-Y)$

$$\begin{aligned}\text{COV}(X+Y, X-Y) &= \text{COV}(X, X-Y) + \text{COV}(Y, X-Y) \\ &= \text{COV}(X, X) - \cancel{\text{COV}(X, Y)} + \cancel{\text{COV}(Y, X)} - \text{COV}(Y, Y) \\ &= \text{Var}(X) - \text{Var}(Y) \quad \# \end{aligned}$$

Q4 $E(X)=3, V(X)=9, E(Y)=4, V(Y)=16, \text{Corr}(X,Y)=0.25$

(a) $V(X+Y)$

$$\text{Corr}(X,Y) = \frac{\text{COV}(X,Y)}{\sqrt{V(X) \cdot V(Y)}} \Rightarrow 0.25 = \frac{\text{COV}(X,Y)}{\sqrt{9 \times 16}} \Rightarrow 0.25 \times 12 = \text{COV}(X,Y) \Rightarrow 3 = \text{COV}(X,Y)$$

$$V(X+Y) = V(X) + V(Y) + 2 \cdot \text{COV}(X,Y) = 9 + 16 + 6 = 31 \quad \#$$

(b) $\text{COV}[X, X+Y]$

$$\text{COV}(X, X+Y) = \text{COV}(X+Y, X) = \text{COV}(X, X) + \text{COV}(Y, X).$$

$$= \text{Var}(X) + 3 \Rightarrow 9 + 3 = 12 \quad \#$$

(c) $\text{Corr}(X+Y, X-Y)$

$$\text{Corr}(X+Y, X-Y) = \frac{\text{cov}(X+Y, X-Y)}{\sqrt{V(X+Y) \cdot V(X-Y)}} \Rightarrow \frac{(-7)}{\sqrt{28 \times 19}} = (-0.3034)$$

where $\text{cov}(X+Y, X-Y) = \text{cov}(X, X-Y) + \text{cov}(Y, X-Y)$

$$= \text{cov}(X, X) - \text{cov}(X, Y) + \text{cov}(Y, X) - \text{cov}(Y, Y)$$

$$= 9 - 16 = (-7)$$

and $V(X+Y) = 28$

while $V(X-Y) = V(X) + V(Y) - 2\text{Cov}(X, Y) = 9 + 16 - 6 = 19 \#$