Diode 02 - Diode at large signal

This project deals with models used to obtain the response of a diode against large signals.

BOM	Diode: 1N4148 Resistor: 1 kΩ

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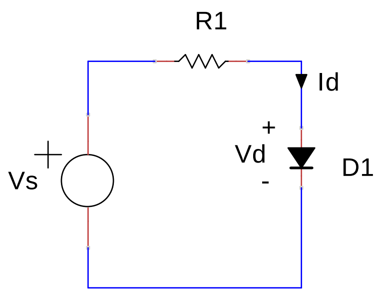
Diode calculation problems

As we know, a semiconductor diode is a component that can be modeled by an exponential function:

$$I_d(V_d) = I_S\left(e^{V_d/\eta\cdot V_T} - 1\right)$$

This model is a very good one for DC operation. You only need to know the value of the I_S and η parameters (V_T is calculated from the diode temperature, about 26 mV at 300 K) and it provides a good agreement with the real operation of the diode.

Unfortunately this is a difficult model to use for hand calculations. Let's try, for instance, to solve the following circuit:



Using some circuit theory we get:

$$V_S - I_d R1 - V_d = 0$$

Joining this equation with the diode equation we get a system:

$$\begin{cases} I_d = I_S\left(e^{V_d/\eta\cdot V_T} - 1\right) \\ V_S - I_d R1 - V_d = 0 \end{cases}$$

Then we get:

$$V_S - R1 \cdot I_S\left(e^{V_d/\eta\cdot V_T} - 1\right) - V_d = 0$$

We just need to isolate V_d to obtain the solution. Try that yourself.

You can't? Off course you can't. Nobody can.

You can solve the above equation numerically but you cannot isolate the Vd variable.

There is an easy numeric way: isolate the exponential:

$$e^{V_d/\eta \cdot V_T} = \frac{V_S - V_d}{R1 \cdot I_S} + 1$$

Then take logarithms on both sides and leave Vd alone on one side:

$$V_d = \eta \cdot V_T \cdot ln\left(\frac{V_S - V_d}{R1 \cdot I_S} + 1\right)$$

This is an interesting equation because it uses Vd as its input parameter and gets Vd also as its output. If we feed the equation with an initial seed Vd value and we input in the equation each solution we get, it converges to the desired solution for Vd.

Let's solve the equation for the 1N4148 diode and Vs = 2.5 V, R1 = 1 kΩ.

Recall the I_S and η values measured previously for the diode and open a python interpreter. Then set the known diode parameters and define the equation.

```
>>> import math

>>> Is = # Is value (mA)
>>> n =  # n value (close to 2)
>>> Vs = 2.5
>>> R1 = 1  # In k Ohm
>>> Vt = 0.026

>>> func = lambda x: n*Vt*math.ln(((Vs-x)/(R1*Is))+1)
```

Now we can apply the function to a seed 0 value:

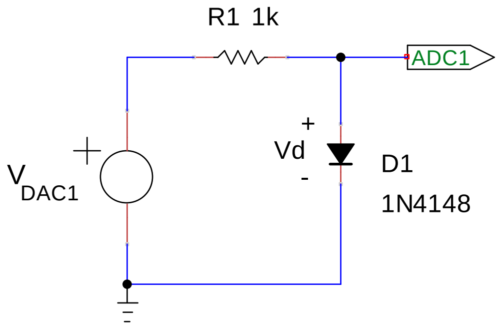
```
>>> v=func(0)
>>> v
```

Then, rinse and repeat:

```
>>> v=func(v)
>>> v
```

Repeat until the obtained value is equal to the previous one, at least on three decimal positions.

Now, we want to check how the result compares with real measurements.



Mount the above circuit, import the slab module, connect with the board and perform the measurement.

```
>>> slab.setVoltage(1,2.5)
>>> slab.readVoltage(1)
```

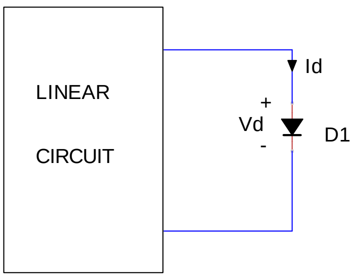
🛠	**1**	Calculate the Vd value using the proposed method. Compare the result to the circuit measured Vd value.

We see that we can solve the circuit, but it is not easy and we don't get a closed formula for the circuit variables.
Moreover, the numerical solution works because the solution converges, but if you had isolated the Vd variable in a different way like:

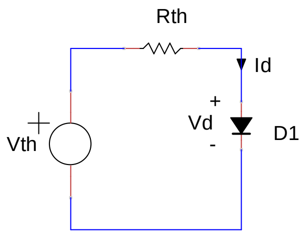
$$V_d = V_S - R1 \cdot I_S\left(e^{V_d/\eta \cdot V_T} - 1\right)$$

It would not work as this equation diverges. Try it yourself.

You could argue that we have only solver a two component circuit, but this method can be used always as long as the diode is the only non linear element in the circuit. You can always make the Thevenin equivalent of the circuit as seen from the diode to obtain a circuit like the one we have solved.



Diode connected to linear circuit



Thevenin equivalent of the linear circuit

There is no problem for computers to use numeric methods to obtain the diode solution, but for hand calculation it is cumbersome. Let's try a different way.

Graphical solution

If we recall the circuit we are trying to solve, we know that it is defined by two equations:

$$\begin{cases} I_d = I_S\left(e^{V_d/\eta \cdot V_T} - 1\right) \\ V_S - I_d R1 - V_d = 0 \end{cases}$$

The first equation is the non linear diode equation and the second equation is a linear equation that represents the rest of the circuit. We can isolate Id on this equation:

$$\begin{cases} I_d = I_S\left(e^{V_d/\eta \cdot V_T} - 1\right) \\ I_d = \dfrac{V_S - V_d}{R1} \end{cases}$$

The solution to the system is the Vd value that makes both equations provide the same Id value. As the second equation is linear and non time dependent, it can be represented by a straight line. In order to obtain this solution graphically we only need to draw this line on top of the diode curve.

To do that, we will first recall the diode curve we saved in the previous project.

```
>>> vd,id = slab.load("1N4148")
```

Now, for our considered case we can obtain the solution of the linear part of the circuit:

```
>>> id2 = (2.5-vd)/1
```

Finally we can plot together the diode and the line:

```
>>> slab.plot1n(vd,[id,id2],"","Vd (V)","Id (mA)")
```

Zoom in on the point where the curve and the line cross to find the solution and compare it with the numeric and measured values.

🛠	**2**	Use the graphical method to obtain Vd. Compare the result to the numeric calculations and the previous measurement.

The graphical method was a very useful method when we didn't have computers to do the numeric calculations as you only needed to draw a line over the diode curve provided by the manufacturer.

Graphical method in other components

The method can be used with other non linear two terminal components, just replace the diode curve with the non linear component curve. For some components, however, there can be some problems with this method. In our case the diode curve and the line only crosses in one point that is the solution for the system. For some non linear components there can be more that one crossing point and you need to use other methods to determine which point is the good one.

Make it easy

The graphical method is a step forward to solving the diode equation, especially if you don't have a computer at hand, but it is not fast and, also, has an inherent problem shared with the numerical solution: it only works if there is only one diode in the circuit.

You can find numerical solution for more than one diode, but they get quite complicated. The best numerical solution for more than one diode is to use a circuit simulator like one of the spice variants.

One way to simplify diode calculations is to use a simpler diode model. When you change the model you have a compromise: on one hand you ease the diode calculations, on the other hand you get less precise solutions.

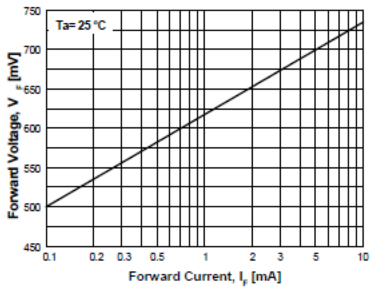


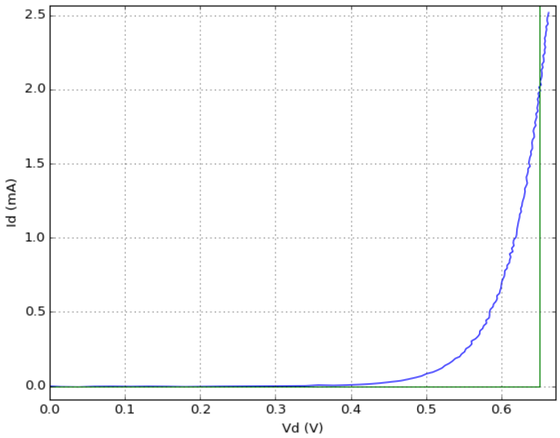
Figure 4. Forward Voltage vs Forward Current
VF - 0.1 to 10mA

The constant voltage diode mode makes use of the fact that the diode voltage has a very small change of voltage for a broad current range. For instance, if we get the 1N4148 current as an example, as seen on the above figure, we can see that the diode voltage changes from 0.6 V to 0.7 V for current values from 0.7 mA to 5 mA. So, we can suppose that the voltage is constant at a value of 0.65V as long as there is current in the diode. Most people, however, pick a value of 0.7 V for the diode mean forward voltage so that will be our chosen value.

We can define the average diode voltage when it conducts as Vγ and define the diode operation depending in two possible states: ON and OFF. That gives us the **constant voltage model** equations:

$$\begin{cases} V_d = V_\gamma & I_d \geq 0 & ON\ \ State \\ I_d = 0 & V_d \leq V_\gamma & OFF\ State \end{cases}$$

The following figure shows the 1N4148 curve, in blue, together with the constant voltage model, in green, if we define Vγ as 0,7V.



In order to use this model we need to know if the diode is in the ON or the OFF state. Unfortunately, as we have not yet solved the circuit, we don't know the state. To solve the problem we make a hypothesis, solve the circuit, and check if the hypothesis suits the model. If not, we know that the hypothesis is false and we need to use the other hypothesis.

For the constant voltage diode model the hypothesis, equations and checks are:

Hypothesis	Diode equation	Hypothesis check
ON	Vd = Vγ	Id ≥ 0
OFF	Id = 0	Vd ≤ Vγ

The method goes as follows:

1. Select one hypothesis
2. Solve the circuit using the diode equation for that hypothesis. Solve also the variable needed to check the hypothesis.
3. Check the hypothesis. If it fails, return to 2 selecting another hypothesis.

As the constant voltage diode model has only two hypotheses, if the selected one is false, the other needs to be true. It is advisable, however, to always check the hypothesis to detect possible calculation errors.

We will solve the previously defined circuit wit V_s = 2.5 V and R1 = 1 kΩ using the constant voltage diode model with $V\gamma$ = 0.7 V.

	3	Solve the circuit using the constant voltage diode model. Try first to use the OFF hypothesis and change to the ON hypothesis if it fails. Which is the good hypothesis? What are the obtained values for Vd and Id?

	4	Compare the result to the circuit measurements.

You should get a good agreement with the measurements, especially in the Id value. This model depends on having a circuit that does not depend on knowing the exact Vd voltage. So, if we connect the diode to a linear circuit, the higher the Thevenin open circuit voltage is, the better the model will work.

Constant voltage diode model for more than one diode

If you have more than one diode in the circuit, you can also use the constant voltage diode model. The hypotheses, however, are made for all the diodes at the same time. For two diodes, as each one has two possible states (ON, OFF) there are four possible hypotheses (OFF;OFF), (OFF, ON), (ON,OFF), (ON, ON). For three diodes there are nine hypotheses and so on.

If only one diode fails the check of one hypothesis you need to reject the hypothesis and you don't get information on the state of any diode. You cannot assume that the diodes that checked ok the hypothesis are in the correct state if any other diode fails the hypothesis.

In order to ease the problem it is good to apply some common sense to select the hypothesis to try. For instance, if several diodes are in series, the only possible hypotheses are all ON or all OFF.

The constant voltage diode model, due to its simplicity, is the most used diode model for hand calculations. In general, if we need a more exact model we use a circuit simulation package.

Last comments

This was a short project to introduce the **constant voltage diode model** that is usually used for hand calculations of circuits that include diodes.
We have seen also other methods to calculate circuits with diodes, like the graphical method or the numeric method but those methods are cumbersome and only work with only one diode in the circuit.
In general, for large signal DC calculations, we will use the **constant voltage diode model** if we need fast not too exact solutions and we will revert to circuit analysis packages for the case that the precision of the model is not enough.

References

SLab Python References

Those are the reference documents for the SLab Python modules. They describe the commands that can be carried out after importing each module.

They should be available in the **SLab/Doc** folder.

TinyCad

Circuit images on this document have been drawn using the free software TinyCad https://sourceforge.net/projects/tinycad/

SciPy

All the functions plots have been generated using the Matplotlib SciPy package. https://www.scipy.org/

LTSpice

Circuit simulator provided for free from Linear Technology. You can use the simulator to obtain numerical solutions of complex circuits.

http://www.linear.com/designtools/software/