BJT 02 - BJT at large signal

This project deals with the BJT models to be used at large signals.

BOM	NPN transistors : BC574B (NPN) and BC557B (PNP) Resistors: 2x 1 kΩ, 22 kΩ, 33 kΩ, 47 kΩ, 220 kΩ and 2x 470 kΩ

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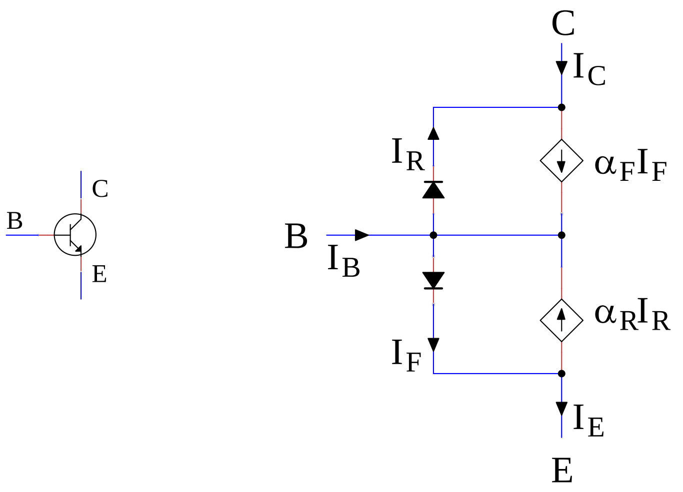
The injection model

We initially expected the BJT to work as two diodes: one for the BE junction and another for the BC junction. Now we know that when one junction is forward biased and the other is reverse biased, the reverse biased junction starts conducting current.

This behavior is usually modeled using the Ebers-Moll injection transistor model.



The polarity of the currents on the base I_B, collector I_C and emitter I_E have been defined so that they are all positive in the transistor Forward Active region of operation.

The model includes two diodes. The current of the diodes follow the traditional exponential equations and the saturation current of both diodes are different. On each diode its current depends on the voltage at the junction (from P to N) and the Thermal Voltage V_T.

$$I_F = I_{ES}\left(e^{V_{BE}/V_T} - 1\right) \qquad I_R = I_{CS}\left(e^{V_{BC}/V_T} - 1\right)$$

This would be the model of the BJT if it behaved like two diodes. As we know, having one junction forward biased generates current on the other junction. That is modeled with two controlled current sources $\alpha_F I_F$ and $\alpha_R I_R$ that are proportional to the previously defined diode currents. Both α_F and α_R are constant and lower than one. In the case of α_F, it is usually very close to one. The value of α_R is usually much lower.

It is interesting to know that the I_{ES} and I_{CS} saturation currents are related to the α values so that the following equation holds true for ideal transistors:

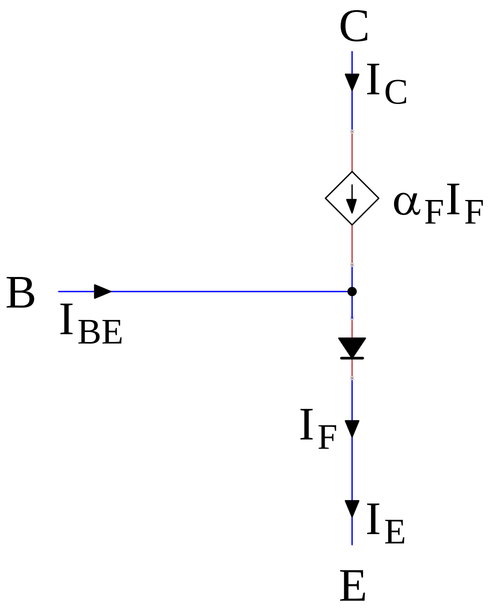
$$\alpha_F I_{ES} = \alpha_R I_{CS} = I_S$$

So, the model has, in fact, only three independent parameters: I_S, α_F and α_R.

The injection model is a good model because is closely related to the internal transistor physics, but from a circuit point of view it is not perfect: It includes a very complex base node that joins all four internal components and includes one parameter, α_F, that is very close to one but that cannot be approximated to one because that will imply a zero base current in the Forward Active region.

The transport model

From the injection model we can build the transport model. In order to simplify the calculations we will consider that only the forward diode can be forward biased. That removes from the model the I_R contributions in the diode and the current source.



We have rewritten I_B as I_{BE} because we are only considering the contribution of I_B for the forward biasing of the BE junction. Then we calculate I_C as function of I_B:

$$I_C = \alpha_F I_F \qquad I_{BE} = I_F - \alpha_F I_F = I_F(1 - \alpha_F)$$

From that:

$$I_F = \frac{I_{BE}}{1 - \alpha_F} \qquad I_C = \frac{\alpha_F}{1 - \alpha_F} I_{BE}$$

We now redefine the above relationship between I_C and I_{BE} as the β_F current gain:

$$I_C = \beta_F I_{BE} \qquad\qquad \beta_F = \frac{\alpha_F}{1 - \alpha_F}$$

The base current is proportional to I_F and I_F depends exponentially on the V_{BE} value:

$$I_{BE} = I_F(1-\alpha_F) = I_F = I_{ES}(1-\alpha_F)\left(e^{V_{BE}/V_T} - 1\right)$$

We can relate α_F and β_F and we can relate I_{ES} with I_S:

$$\alpha_F = \frac{\beta_F}{1+\beta_F} \qquad\qquad I_{ES} = \frac{I_S}{\alpha_F}$$

So that we can rewrite I_{BE} relating I_{ES} to I_S:

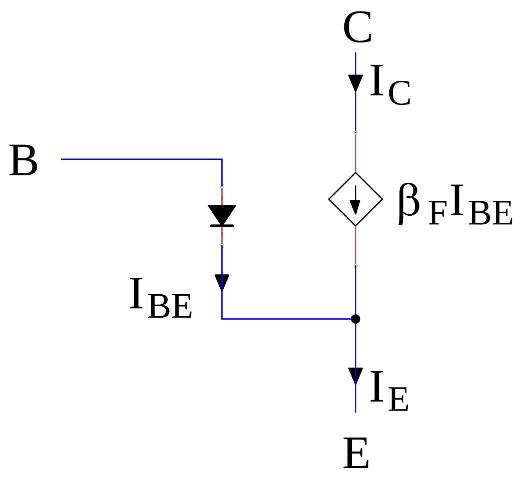
$$I_{BE} = I_{ES}(1-\alpha_F)\left(e^{V_{BE}/V_T} - 1\right) = \frac{I_S}{\alpha_F}(1-\alpha_F)\left(e^{V_{BE}/V_T} - 1\right)$$

And we can also relate α_F to β_F:

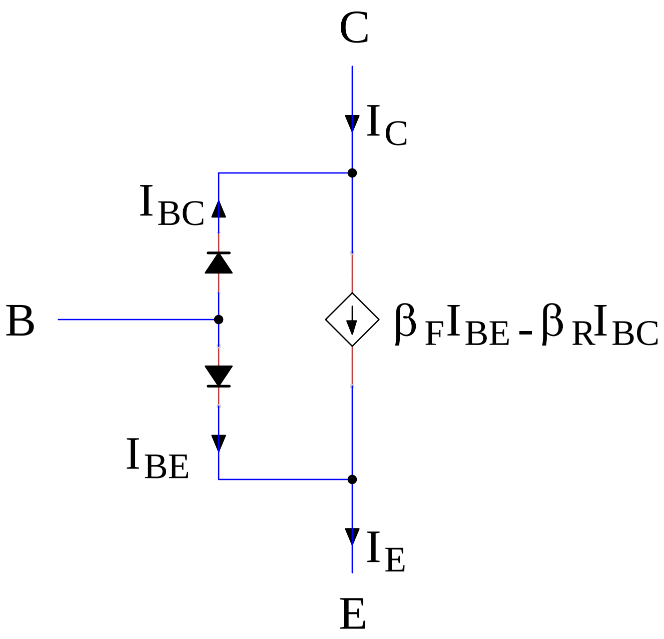
$$I_{BE} = I_S\frac{1-\alpha_F}{\alpha_F}\left(e^{V_{BE}/V_T} - 1\right) = I_S\frac{1-\frac{\beta_F}{1+\beta_F}}{\frac{\beta_F}{1+\beta_F}}\left(e^{V_{BE}/V_T} - 1\right)$$

$$I_{BE} = I_S\frac{\frac{1}{1+\beta_F}}{\frac{\beta_F}{1+\beta_F}}\left(e^{V_{BE}/V_T} - 1\right) = \frac{I_S}{\beta_F}\left(e^{V_{BE}/V_T} - 1\right)$$

As we know how I_C relates to I_{BE} and as I_{BE} depends exponentially on V_{BE}, we can rewrite the model as:



If we redo the calculations for the other diode we get a final transport model:

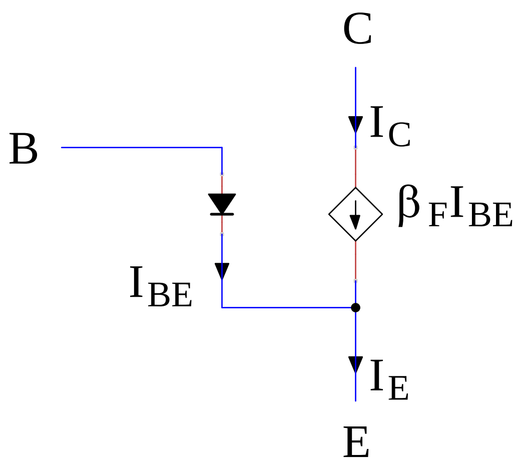


Where the I_{BE} and I_{BC} currents are defined by exponential equations:

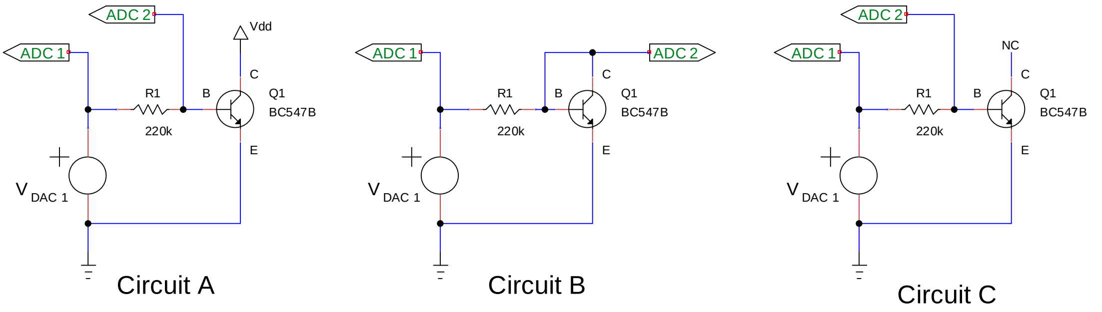
$$I_{BE} = \frac{I_S}{\beta_F}\left(e^{V_{BE}/V_T} - 1\right) \qquad\qquad I_{BC} = \frac{I_S}{\beta_R}\left(e^{V_{BC}/V_T} - 1\right)$$

The transport model is good from the circuit point of view because uses the β usually reported by the device manufacturers instead of α that is more useful to device designers than circuit designers. It also good because it uses one common I_S parameter for both exponential equations. Finally, the model simplifies the base node respect to the injection model.

In the **Forward Active** region the model simplifies to:



Now we will work with the models. Consider the following three circuits that can measure the current on resistor R1 as function of the V_{BE} voltage for different possible connections on the collector node.



1

Determine the $I_{R1}(V_{BE})$ current in all three cases from the transport model parameters I_S, β_F and β_R.

In which mode (Forward Active, Reverse Active, Saturation or Cut-off) will the BJT operate when the current I_{R1} flows on each circuit?.

The I_{R1} current vs V_{BE} voltage can be obtained by using the ***curveVI*** command of the DC module. Open a Python console, import the SLab module and connect to the board. Then import the DC module and obtain the I(V) curve for each circuit. In the case of circuit A we issue:

```
>>> import slab_dc as dc
>>> va,ia = dc.curveVI(0,3,r=220,returnData=True)
```

Call the ***curveVI*** command also for circuits B and C and store the data in variables vb, ib and vc, ic. You can draw together all three curves using the ***plotnn*** command:

```
>>> slab.plotnn([va,vb,vc],[ia,ib,ic],\
... "","Vbe (V)","IR1 (mA)",["A","B","C"])
```

All curves should have an exponential shape, but currents will be different at a given V_{BE} value. The higher current curve will be the first to rise and the lower current one will the last to rise.

2

Obtain the $I_{R1}(V_{BE})$ curve for all circuits and draw them together.

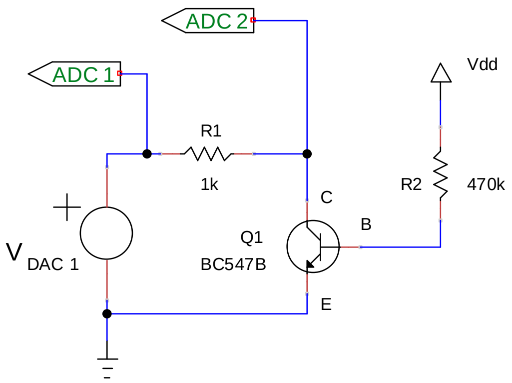
Sort the curves by their currents. Do the results make sense against the results obtained in 1?

As circuit B is the one that gives more current for a given V_{BE} (or less V_{BE} for a given current) it is the preferred way to implement a diode using a transistor and is called the **diode configuration** for the BJT.

Early Voltage

The transport model predicts that, in the forward active region, the collector current shall be independent on the V_{CE} voltage and depend only on the base current and the β_F gain. We will perform a measurement to check if that holds true.

The following circuit uses a R2 resistor to set a base current on the BJT and uses a R1 resistor to measure the collector current.



With the above configuration we can use the curveVI command of the DC module to obtain the $I_C(V_{CE})$ curve. In order to obtain more resolution we will average 100 readings for each measurement point and we will set the sweep voltage step to 20 mV.

```
>>> slab.setDCreadings(100)
>>> dc.curveVI(0,3,0.02)
```

The curve will take some time to be drawn as we are performing 15.000 individual measurements (100 readings at 150 points).

You should see the current rising in the saturation region up to a V_{CE} value of about 0.2 V. Above this voltage the current shall remain constant. As the β_F value is different in each particular BC547B transistor, the collector current will depend on the chosen device. This current also affects the voltage range in the horizontal V_{CE} axis.
If you get a V_{CE} range lower than 1.5 V, increase the R2 value as needed to get at least this range. The collector current, out of the saturation region should be near the 1 mA value. If you get a collector current below 0.5 mA, decrease the R2 value.

In the forward active region, out of the saturation region, the current should be horizontal. To see it better, zoom in the vertical axis on the active region for voltages over 0.4 V. You should see that the voltage is not constant but increases with increasing V_{CE} values.

3

Mount the circuit and obtain $I_C(V_{CE})$.

Check that I_C slightly increases as V_{CE} increases.

Take two I_C, V_{CE} value pairs, one at a voltage of about 0.5 V and another at a voltage of about 1.5 V.

What is happening is that when we change the V_{CE} value we are changing the reverse biasing of the BC junction. That affects the effective width of the base of the BJT, that is already very thin. The decrease of the effective width of the base increases the collector current and the current gain. This phenomenon is named the Early effect after its discoverer James M. Early.

This effect is modeled with an Early Voltage V_A that affects the dependence of the collector current and the current gain on the collector to emitter voltage.

$$I_C = I_{C0}\left(1+\frac{V_{CE}}{V_{AF}}\right) \qquad \beta_F = \beta_{F0}\left(1+\frac{V_{CE}}{V_{AF}}\right)$$

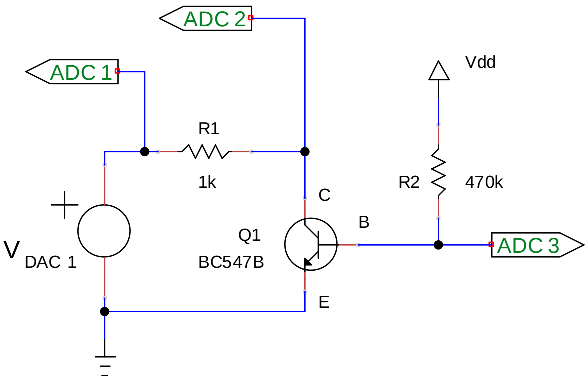
4

Obtain the expression that enables you to calculate the V_{AF} value from two set of I_C and V_{CE} values: I_{C1}, V_{CE1} and I_{C2}, V_{CE2}.

5

Use the result obtained in 1 to calculate the V_{AF} value from the measurements performed in 3.

As both the collector current and the gain increase in the same way, the base current does not change. We can check that by measuring a modified version of the previous circuit that also measures the base voltage.



The we use the ***dcSweep*** command to check the measure all ADCs for DAC 1 values from 0 V to 3 V.

```
>>> d1,a1,a2,a3,a4=slab.dcSweep(1,0,3,0.02)
```

Then we can obtain the V_{CE}, I_C, I_B and β_F magnitudes:

```
>>> vce=a2
>>> ic=(a1-a2)
>>> ib=(slab.vdd-a3)/470
>>> beta=ic/ib
```

And we can show I_C, I_B and β_F against V_{CE}:

```
>>> slab.plot11(vce,ic)
>>> slab.plot11(vce,ib)
>>> slab.plot11(vce,beta)
```

🛠	**6**	Add the ADC3 connection to the previous circuit and perform the DC sweep. Obtain the curves of I_C, I_B and β_F against V_{CE}. Check that I_C, and β_F show the same dependence on V_{CE} whereas I_B is practically independent on V_{CE}. If you obtain V_{AF} from the I_C or the β_F curve the result should be the same.

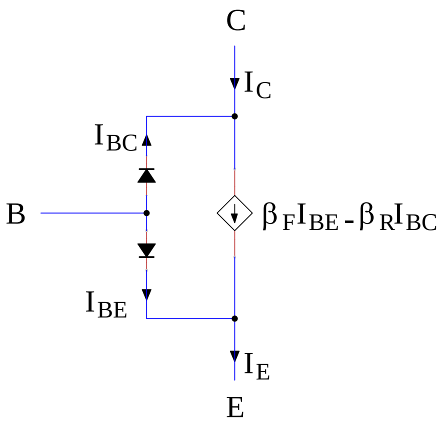
The Early effect is very small effect because Early voltages are in the 100 V range for small signal transistors. That means that we only get about a 1% collector current increase for each volt increase of V_{CE}. That's why we usually neglect this effect in hand calculations. This effect, however cannot always be neglected. In particular, we cannot neglect it when what we are taking into account is the dependence of the I_C current on the V_{CE} voltage.

For BJT circuits biased by using resistors, the Ohm's law on the biasing resistors has usually a greater effect on the collector current than the Early effect and it can usually be neglected. However, for BJT circuits biased using current sources, as is usual inside integrated circuits, the Early effect cannot usually be neglected as it could be the main effect that enables us to obtain the V_{CE} voltage variations.

In the same way that there is an Early effect for the forward active operation of the BJT, there is also an Early effect for the reverse active operation mode. This effect is modeled by a V_{AR} parameter, but, as BJTs usually are not designed to operate in the reverse region and also as the Early effect can usually be neglected, this parameter is not usually needed.

Simplifying the model

At this point we have a quite complex transport model for the BJT.



The model equations take into account the forward and reverse Early effect. If you want to simplify it neglecting the Early effect, just make V_{AF} and V_{AR} infinite.

$$I_{BE} = \frac{I_S}{\beta_F}\left(1+\frac{V_{CE}}{V_{AF}}\right)\left(e^{V_{BE}/V_T}-1\right) \qquad I_{BC} = \frac{I_S}{\beta_R}\left(1+\frac{V_{EC}}{V_{AR}}\right)\left(e^{V_{BC}/V_T}-1\right)$$

$$\beta_F = \beta_{F0}\left(1+\frac{V_{CE}}{V_{AF}}\right) \qquad \beta_R = \beta_{R0}\left(1+\frac{V_{EC}}{V_{AR}}\right)$$

We will need this model in the future, but, in most cases, it is a too complex model to perform hand calculations.

In order to simplify the model we can use a constant voltage model instead of an exponential model for the two BE and BC diodes. Recall that this model considers two states ON and OFF for the diode with the following equations:

$$\begin{cases} V_d = V_\gamma & I_d \geq 0 & ON\ \ State \\ I_d = 0 & V_d \leq V_\gamma & OFF\ State \end{cases}$$

From previous experiments, like in circuit A at 🛠2, we have seen that the base current in the forward active region starts to grow at a V_{BE} voltage of about 0,65 V. As we don't need too much precision in hand calculations we simplify the BE diode with a threshold $V_{\gamma BE}$ voltage of 0,7 V.

From previous experiments, we also have seen that the BJT goes from forward active mode to saturation mode at V_{CE} voltages of about 0,2 V. We, then, define a saturation voltage $V_{CE\ Sat}$ to be about 0,2 V.

As saturation means that both junctions are forward biased, their voltages, using a simple constant voltage diode model will be:

$$V_{BE} = V_{\gamma BE} \qquad V_{BC} = V_{\gamma BC}$$

From the voltages V_{BE}, V_{BC} and V_{CE} on the transistor it always hold true:

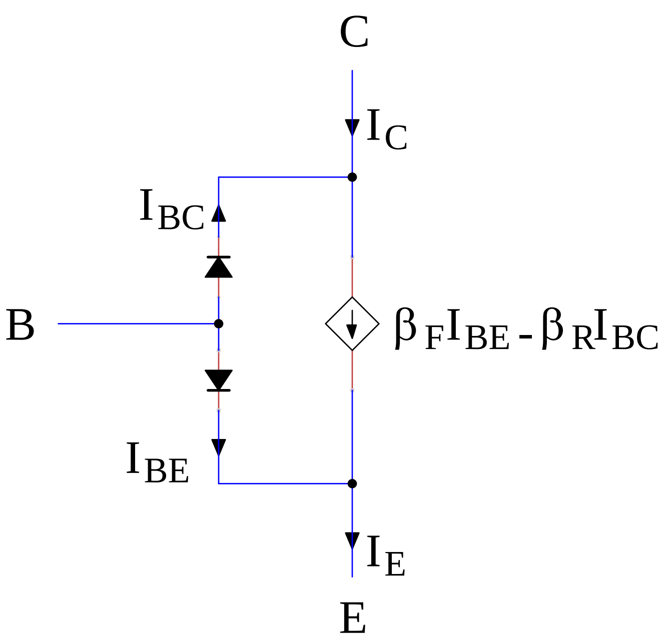
$$V_{BC} = V_{BE} - V_{CE}$$

So, in the saturation region:

$$V_{BC} = V_{BE} - V_{CE} = V_{\gamma BE} - V_{CE\,Sat} = V_{\gamma BC}$$

So the BC junction threshold voltage $V_{\gamma BE}$ will be about 0,5 V.

In the end we get the same model circuit as in the transport model.



But this time, the diodes are modeled using constant voltages instead of exponential functions:

$$\begin{cases} V_{BE} = V_{\gamma BE} & I_{BE} \geq 0 & ON\ \ State \\ I_{BE} = 0 & V_{BE} \leq V_{\gamma BE} & OFF\ State \end{cases}$$

$$\begin{cases} V_{BC} = V_{\gamma BC} & I_{BC} \geq 0 & ON\ \ State \\ I_{BC} = 0 & V_{BC} \leq V_{\gamma BC} & OFF\ State \end{cases}$$

Saturation voltage is defined as the difference between the junction threshold voltages:

$$V_{CE\,Sat} = V_{\gamma BE} - V_{\gamma BC}$$

Typical values for the voltage parameters are:

$V_{\gamma BE}$	$V_{\gamma BC}$	$V_{CE\ Sat}$
0,7 V	0,5 V	0,2 V

If we add the β_F and β_R parameters we get a set of 5 parameters that define the operation of this model.

$V_{\gamma BE}$	$V_{\gamma BC}$	$V_{CE\ Sat}$	β_F	β_R

As the three voltage parameters are related, we can always calculate $V_{\gamma BC}$ from $V_{\gamma BE}$ and $V_{CE\ Sat}$ so the list of primary parameter is reduced to 4:

$V_{\gamma BE}$	$V_{CE\ Sat}$	β_F	β_R

As the model features two diodes and each diode can be in two possible states, there are four possible states for the transistor.

BE Junction	BC Junction	BJT State
OFF	OFF	Cut-Off
ON	OFF	Forward Active
ON	ON	Saturation
OFF	ON	Reverse Active

In the case of the constant voltage diode model we built a table that includes the diode equations for each mode of operation and the conditions to check to verify that we have selected the correct model. In order to solve a circuit, we make a hypothesis on the device state, we use the device equations for this state to solve the circuit, and finally we use the verification inequations to check that the chosen hypothesis is correct.
We can do the same on the BJT.

Hypothesis	**Equations**	**Verification**
Cut-Off	$I_B = 0$ $I_C = 0$	$V_{BE} \leq V_{\gamma BE}$ $V_{BC} \leq V_{\gamma BC}$
Forward Active	$V_{BE} = V_{\gamma BE}$ $I_C = \beta_F I_B$	$I_B \geq 0$ $V_{CE} \geq V_{CE\ Sat}$
Reverse Active	$V_{BC} = V_{\gamma BC}$ $I_E = \beta_R I_B$	$I_B \geq 0$ $V_{BE} \leq V_{\gamma BE}$
Saturation	$V_{BE} = V_{\gamma BE}$ $V_{CE} = V_{CE\ Sat}$	$I_B \geq 0$ $I_C \leq \beta_F I_B$ $-I_E \leq \beta_R I_B$

Note that having $I_B \geq 0$ is a required condition for all cases except Cut-Off. As we force $I_B = 0$ at Cut-Off, this condition is also true in this region.

In order to solve a circuit we make a hypothesis about the BJT state. Then we use the equations to solve the circuit and finally we check that all the verification inequations hold true. If any inequation is false, we need to make a different hypothesis and start again.

In the **Cut-Off** region, as both diodes are in OFF state, the base current must be zero from the model, also the collector and emitter currents need to be zero. In order to check that this hypothesis is true we only need to check that both diodes have voltages below their threshold value.

In the **Forward Active** region, as only the BE diode is in ON state, the base current I_B is equal to the I_{BE} current and I_{BC} is zero. Due to that, the collector I_C current is $\beta_F I_B$. To check that the BE diode is in ON state we only need to check that I_B is positive. To check that the BC diode is OFF we only need to check that V_{BC} is below its threshold $V_{\gamma BC}$. That condition, however, is equivalent in this case to have a V_{CE} voltage above $V_{CE\,Sat}$.

The **Reverse Active** region is similar to the Forward Active. We only interchange the diodes BE and BC operating modes.

In the **Saturation** region as both diodes are in ON state, all BJT voltage relations are known. V_{BE} is $V_{\gamma BE}$, V_{BC} is $V_{\gamma BC}$ and V_{CE} is $V_{CE\,Sat}$. As the three voltages are related by a voltage Kirchoff's law, we only need to set two of these three voltages.
Checking the saturation condition is tricky, however. In order to check that both diodes are in ON state we need to guarantee that both diode currents are positive:

$$I_{BE} \geq 0 \qquad\qquad I_{BC} \geq 0$$

Calculating those currents is not easy. So we will try to find other equivalent conditions.
First, in order to be both I_{BE} and I_{BC} positive, I_B needs to be positive. This condition, however, is not enough as one current could be positive and the other be negative so that the total base current is positive.

If we calculate the Kirchoff's current law on the collector node we get:

$$I_C + I_{BC} = \beta_F I_{BE} - \beta_R I_{BC}$$

We also know from the Kirchoff's law on the base:

$$I_B = I_{BE} + I_{BC}$$

So we can get:

$$I_C = \beta_F I_{BE} - (\beta_R + 1) I_{BC} \qquad I_{BE} = I_B - I_{BC}$$

$$I_C = \beta_F (I_B - I_{BC}) - (\beta_R + 1) I_{BC}$$

$$\beta_F I_B - I_C = (\beta_F + \beta_R + 1) I_{BC}$$

$$I_{BC} = \frac{\beta_F I_B - I_C}{\beta_F + \beta_R + 1}$$

One of the conditions to check is to have a positive I_{BC} current. As both β_F and β_R are positive, having a positive I_{BC} current is equivalent to the following equation being true:

$$I_C \leq \beta_F I_B$$

Note that as I_B must be positive, if I_C is negative, this inequation is always true.

In a similar way we can calculate the Kirchoff's current law on the emitter node and obtain:

$$I_{BE} + \beta_F I_{BE} - \beta_R I_{BC} = I_E$$

Adding the Kirchoff's law on the base node we obtain:

$$(1 + \beta_F) I_{BE} - \beta_R I_{BC} = I_E \qquad I_{BC} = I_B - I_{BE}$$

$$(1 + \beta_F) I_{BE} - \beta_R (I_B - I_{BE}) = I_E$$

$$(1 + \beta_F + \beta_R) I_{BE} - \beta_R I_B = I_E$$

$$I_{BE} = \frac{I_E + \beta_R I_B}{1 + \beta_F + \beta_R}$$

The second condition to check is to have a positive I_{BE} current. Having a positive I_{BE} current is equivalent to the following equation being true:

$$I_E + \beta_R I_B \geq 0$$

Or in an equivalent form:

$$-I_E \leq \beta_R I_B$$

Note that as I_B must be positive, if I_E is positive, this inequation is always true.

It is intuitive that in order to be in the saturation region, the collector or emitter currents must be below the ones we have in the forward active region or the reverse active regions. The above calculations give us a mathematical confirmation of this intuition.

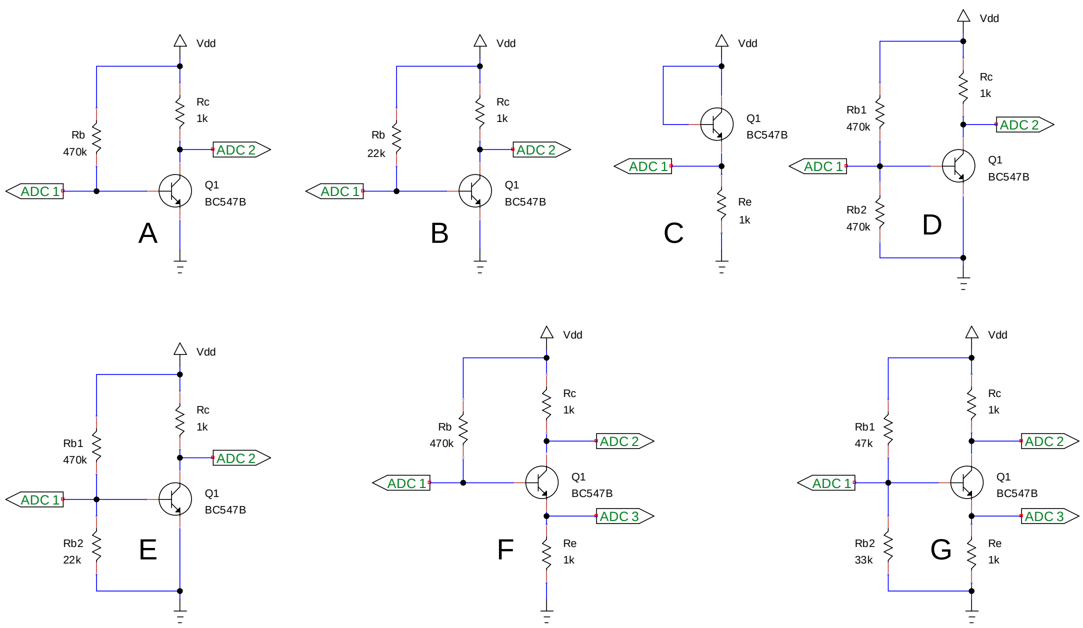
So, now we have a two constant voltage diode model for the BJT transistor. We will call this model the **Simple BJT Model**. The model has four regions of operation: Cut-Off, Forward Active, Reverse Active and Saturation. For each region there are two equations and two inequations (or three in the saturation region) that enables us to verify that we really are in this region.

Checking the Simple BJT Model

We will try to check this simple BJT model on the BC547B device. We will consider the following set of parameters for this transistor:

$V_{\gamma BE}$	$V_{CE\ Sat}$	β_F	β_R
0,7 V	0,2 V	200	2

Now consider the following seven BJT circuits.



We will use on the calculations the Vdd voltage of the SLab hardware board we are using. The circuits are designed to operate on a 3,3 V supply. If your Vdd voltage is much greater that 3,3 V use 3,3 V on the calculations.

	7	Use the Simple BJT Model to solve all seven circuits so that you can calculate the expected value we will read on the ADC nodes. Indicate the BJT mode of operation for each circuit.

Now we can test if the measurements agree with the calculations. Mount each circuit and measure the indicated ADC nodes. If you have considered Vdd to be 3,3V because the hardware board has a higher Vdd voltage, use the DAC1 output set at 3,3V as the Vdd node for the circuits.

You can use the ***dcPrint*** command to obtain the voltage at all four ADCs.

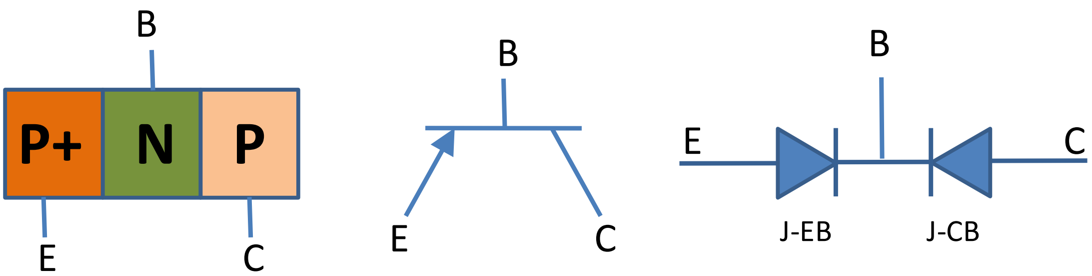
```
>>> slab.dcPrint()
```

Due to the high uncertainty on the BJT β_F value the measurements will not exactly match the calculations. If you use a measured current gain for the BJT you are using, the agreement will improve.
Other source of error is the constant voltage assumption on the ON state diodes. In practice, V_{gBE} will not be exactly 0,7V and the saturation region, although starts at a V_{CE} voltage of about 0,2 V, can have V_{CE} voltages as small as zero volts if the current is low enough.

	8	Measure all seven circuits. Do the results agree with the calculations performed in 7?

The PNP models

In all this document we have only considered the NPN transistor. Now it is time to talk a little about its PNP sibling. The PNP transistor is complementary to the NPN because its internal semiconductor regions are interchanged.

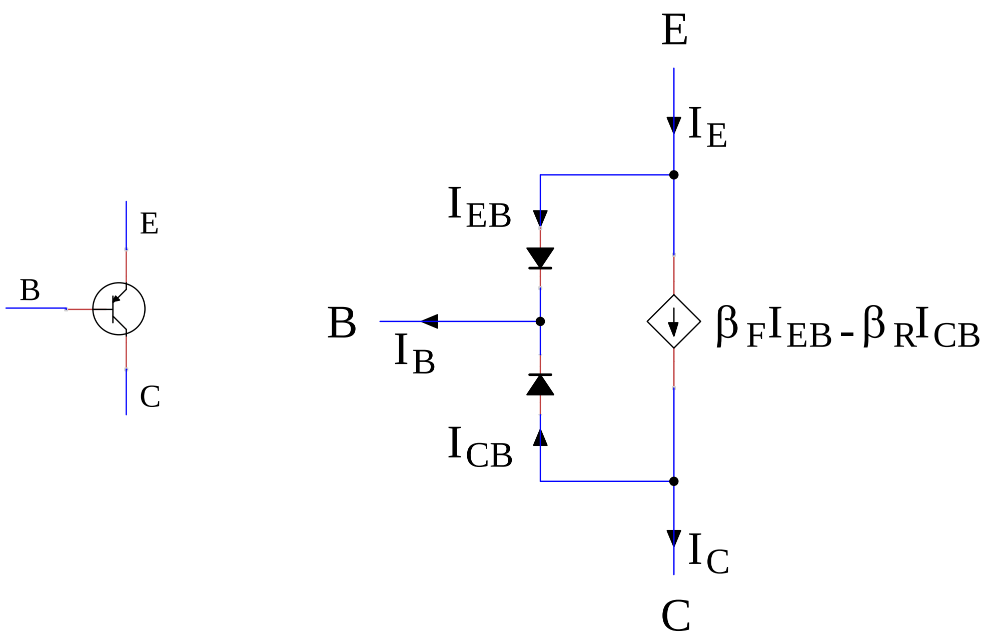


So, now, we have two junctions that are from emitter to base and from collector to base instead of the NPN case where the junctions were from base to emitter and collector.

As in the NPN case, the currents are defined so they are all positive in the Forward Active region of operation. Due to the diode configuration the current must exit the base. In a similar way, the current must enter the emitter when it is forward biased. Due to Kirchoff's current law, collector current must exit the collector.

Traditional circuit diagrams locate the positive supply rail in the upper side of the schematics and the negative supply rail in the lower side. In this fashion, the PNP transistor is usually drawn with the emitter at the top as the V_{EC} voltage is positive in the Forward Active region.

The following figure shows the transport model for the PNP transistor. It is the same model of the NPN one if we use the opposite polarities to all voltages and currents.



The model equations, if we take the Early effect into account, are:

$$I_{EB} = \frac{I_S}{\beta_F}\left(1 + \frac{V_{EC}}{V_{AF}}\right)\left(e^{V_{EB}/V_T} - 1\right) \qquad I_{CB} = \frac{I_S}{\beta_R}\left(1 + \frac{V_{CE}}{V_{AR}}\right)\left(e^{V_{CB}/V_T} - 1\right)$$

$$\beta_F = \beta_{F0}\left(1 + \frac{V_{EC}}{V_{AF}}\right) \qquad \beta_R = \beta_{R0}\left(1 + \frac{V_{CE}}{V_{AR}}\right)$$

For the model equations that neglect the Early effect, just use infinite V_{AF} and V_{AR} voltages.

For the Simple BJT Model, the circuit is the same as in the transport model but we substitute the exponential equations for the diodes for a simple constant voltage diode model. The equation table turn out to be:

Hypothesis	Equations	Verification
Cut-Off	$I_B = 0$ $I_C = 0$	$V_{EB} \le V_{\gamma EB}$ $V_{CB} \le V_{\gamma CB}$
Forward Active	$V_{EB} = V_{\gamma EB}$ $I_C = \beta_F I_B$	$I_B \ge 0$ $V_{EC} \ge V_{EC\,Sat}$
Reverse Active	$V_{CB} = V_{\gamma CB}$ $I_E = \beta_R I_B$	$I_B \ge 0$ $V_{EB} \le V_{\gamma EB}$
Saturation	$V_{EB} = V_{\gamma EB}$ $V_{EC} = V_{EC\,Sat}$	$I_B \ge 0$ $I_C \le \beta_F I_B$ $-I_E \le \beta_R I_B$

Note that the current equations are the same, as we have reversed the currents in the model, and the voltage equations are also the same is we reverse their polarities.

As in the NPN case, the Kirchoff's laws relate the three PNP voltages and currents:

$$I_E = I_B + I_C \qquad\qquad V_{CB} = V_{EB} - V_{EC}$$

In particular, the threshold voltages and the saturation voltage are related:

$$V_{\gamma CB} = V_{\gamma EB} - V_{EC\,Sat}$$

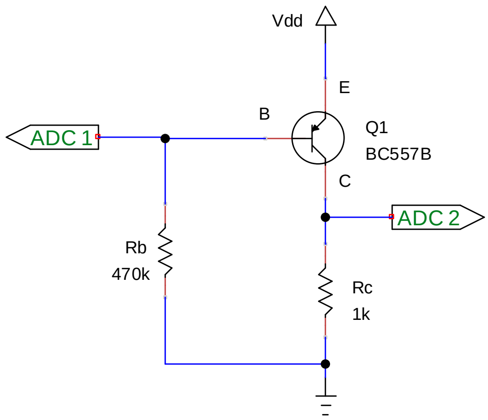
Like in the NPN case, the model has four independent parameters:

$V_{\gamma EB}$	$V_{EC\,Sat}$	β_F	β_R

Let's test the model.

Measuring the PNP

Build the following circuit that uses the BC557B PNP transistor.



We will need to know the BJT pinout, so go and find this BJT datasheet.

	9	Find the BC557B datasheet. Take note of the transistor pinout and the typical β_F (h_{FE}) current gain.

We can obtain the voltages at the base and the collector by reading ADC 1 and 2 voltages. We can also obtain the EB forward biased junction voltage and the V_{EC} voltage.

```
>>> vb=slab.readVoltage(1)
>>> vb
>>> vc=slab.readVoltage(2)
>>> vc
>>> veb=slab.vdd-vb
>>> veb
>>> vec=slab.vdd-vc
>>> vec
```

Now we can obtain the current, in mA, for the base and the collector.

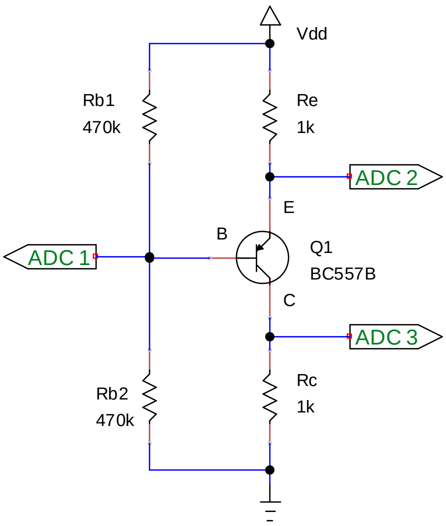
```
>>> ib=vb/470
>>> ib
>>> ic=vc
>>> ic
```

Finally we can also obtain the current gain.

```
>>> beta=ic/ib
>>> beta
```

🛠	**10**	Measure the BC557B circuit and obtain V_B, V_C, V_{EB} and V_{EC}. Obtain also the I_B and I_C currents and the β_F current gain. Can we guarantee that we are in the Forward Active region? Is the current gain inside the range indicated on the BJT datasheet?

Now, a final exercise for this project. Consider the following circuit.



📋	**11**	Analyze the circuit using the Simple BJT Model and obtain the voltages on the ADC nodes. Use the β_F value obtained in 🛠10.

🛠	**12**	Build and measure the circuit. Do the results agree with the calculations?

Last comments

In this document we have proposed several DC BJT models. The transport model, after we add the Early effect, is a quite precise model for the BJT operation. This model, however, is too complex to use in hand calculations.

From this model we have obtained the Simple BJT Model. This new model supposes that both junction diodes can only be on two states, ON or OFF, and that the current is zero on the OFF state and the voltage is $V\gamma$ on the ON state. This model is related to a table that gives us two equations for each possible BJT state: Cut-Off, Forward Active, Reverse Active and Saturation. The table also includes the inequations needed to check that we really are in one particular state.

We have used the simple model to solve several circuits and check the theoretical result against measurements. It is interesting to note that the deviation of the calculations from the measurements depends more on the current gain uncertainty than on the use of a simple model. Using the full transport model won't give us much better results if we are not sure of the exact current gain of the BJT we are measuring.

Finally we have developed both the transport and the simple models for the PNP sibling of the NPN transistor. We see that the models are the same as in the NPN case if we reverse voltage and current polarities.

The Simple Model is the foundation for solving, at DC, any circuit that includes BJTs. As the model assumes that the V_{BE} voltage is constant in the Forward Active region, it will give gross errors when the circuit operation depends on the V_{BE} variation. This is especially true in high gain amplifiers, so we will need to develop a small signal model for the BJT as we did previously for the diode.

References

SLab Python References

Those are the reference documents for the SLab Python modules. They describe the commands that can be carried out after importing each module.
They should be available in the **SLab/Doc** folder.

TinyCad

Circuit images on this document have been drawn using the free software TinyCad
https://sourceforge.net/projects/tinycad/

SciPy

All the functions plots have been generated using the Matplotlib SciPy package.
https://www.scipy.org/