國立政治大學 112 學年度碩士班暨碩士在職專班招生考試

電腦閱卷選擇題答案

系所組名稱：資訊科學系、資訊安全碩士學位學程

科目名稱：計算機數學

題號	答案	題號	答案
1	D		
2	C		
3	D		
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8	A		
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12	D		
13	D		
14	B		
15	C		
16	B		
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18	D		
19	B		
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23	A		
24	D		
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本次考試共 25 題單選題，每題 4 分。　　　選擇題請在答案卡上作答，否則不予計分。

1. If $\begin{bmatrix} 11 & 5 \\ 35 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ c & 1 \end{bmatrix}$ and $a, b, c \in R$, then $a + b + c =$?

(A) 7 (B) 8 (C) 9 (D) 10

2. How many of the following statements are true?

- If E is an elementary matrix, then $\det(E) = \pm 1$.
- For any $A, B \in M^{n\times n}(F)$, $\det(AB) = \det(A)\cdot\det(B)$.
- A matrix $A \in M^{n\times n}(F)$ has rank n if and only if $\det(A) \neq 0$.
- For any $A \in M^{n\times n}(F)$, $\det(A^t) = -\det(A)$.

(A)0 (B)1 (C)2 (D)3 (E)4

3. Let A be an $m \times n$ matrix whose null space has dimension k. Which conclusion is correct?

(A) The dimension of $NULL(A^T)$ is k.
(B) The dimension of row space of A is $m - k$.
(C) The dimension of column space of A is $m - k$.
(D) The dimension of row space of A is $n - k$.

4. How many of the following vector functions are linear transformations?

- $T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 \\ x + y \\ y^2 \end{bmatrix}$
- $T_2\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y \\ x + y + z \\ 0 \end{bmatrix}$
- $T_3\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} e^{x+y} \\ \sqrt{y} \end{bmatrix}$
- $T_4\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x+y \\ 10 \end{bmatrix}$

(A)0 (B)1 (C)2 (D)3 (E)4

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5. How many of the following statements are true?

- The Gram–Schmidt orthogonalization process allows us to construct an orthonormal set from an arbitrary set of vectors.
- An orthonormal basis must be an ordered basis.
- Every orthogonal set is linearly independent.
- Every orthonormal set is linearly independent

(A)0 (B)1 (C)2 (D)3 (E)4

6. Let $A = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$, please find A^{100}

(A) $\begin{bmatrix} -4^{100} & 1-4^{100} \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4^{100} & 1-4^{100} \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 4^{100} & 1-4^{100} \\ 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 4^{100} & 1+4^{100} \\ 0 & 1 \end{bmatrix}$

7.How many of the following statements are true?

- Every linear operator on an n-dimensional vector space has n distinct eigenvalues.
- Any two eigenvectors are linearly independent.
- Similar matrices always have the same eigenvalues.
- Similar matrices always have the same eigenvectors.

(A)0 (B)1 (C)2 (D)3 (E)4

For problems 8-10, please find a singular value decomposition for the following matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} = U\Sigma V$$

8. $U = ?$

(A) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (B) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$ (D) $\begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{\sqrt{2}}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} \\ \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

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9. $\Sigma = ?$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (B) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (C) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ (D) $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. $V = ?$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{\sqrt{2}} & \frac{3}{\sqrt{2}} \\ 0 & \frac{3}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{-3}{\sqrt{2}} \end{bmatrix}$

11. Determine whether each of these compound propositions is satisfiable.

(1) $(p \vee \neg q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$.

(2) $(p \rightarrow q) \wedge (p \rightarrow \neg q) \wedge (\neg p \rightarrow q) \wedge (\neg p \rightarrow \neg q)$.

(3) $(p \vee q) \wedge (\neg p \vee q) \wedge (\neg p \vee \neg q)$.

(A)(1) (B)(1), (2) (C)(2), (3) (D) (1), (3)

12. Let $S = \{a, \{a\}, \phi, \{\phi\}\}$, and $P(S)$ denote the power set of S. How many of the following statements are true?

- $a \in S$
- $\{a\} \subseteq S$
- $\{\{a\}\} \subseteq S$
- $\phi \in S$
- $\phi \subseteq S$
- $\phi \in P(S)$
- $\phi \subseteq P(S)$
- $\{\phi\} \in P(S)$
- $\{\phi\} \subseteq P(S)$

(A)6 (B)7 (C)8 (D) 9

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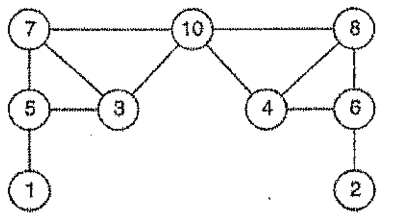
13. Let set $A = \{1,2,3,4\}$. Define a relation of R of A as $R = \{(1,3), (1,4), (2,3), (2,4), (3,1), (3,4)\}$. Which of the following properties does this relation have?
(1) symmetric
(2) asymmetric
(3) antisymmetric
(4) reflexive
(5) irreflexive
(6) transitive
(A)(1), (4), (6) (B)(2), (5), (6) (C)(3), (5) (D)(5)

14. Consider a graph



How many of the following statements are true?
- It is bipartite.
- It has the longest simple path of length 8.
- It has an Euler circuit.
- It doesn't have an Euler circuit.

(A)0 (B)1 (C)2 (D)3

15. How many of the following statements are true?
- A graph G has a spanning tree if G is connected.
- A graph $G = (V, E)$ with $|E| = m$ satisfying $2m = \sum_{v \in V} \deg(v)$.
- A graph $G = (V_1, V_2, E)$ is bipartite, when G has a Hamilton cycle, $|V_1| = |V_2|$.
- A graph $G = (V_1, V_2, E)$ is bipartite, when G has a Hamilton cycle, $||V_1| - |V_2|| \leq 1$.

(A)1 (B)2 (C)3 (D)4

For problems 16-18, please solve the linear recurrence relation $a_n + 6a_{n-1} + 9a_{n-2} = (-3)^n$ with $a_0 = 2$ and $a_1 = 3$, and let $a_n = (i + jn + kn^2) \cdot (-3)^n$.
16. i =? (A)1 (B)2 (C)-2 (D)3

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17. j =? (A) $-\frac{7}{2}$ (B) $\frac{7}{2}$ (C) $-\frac{5}{2}$ (D) $\frac{5}{2}$

18. k =? (A) $\frac{3}{2}$ (B) $-\frac{3}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$

For problems 19-20, please find $X = 101^{-1}$ modulo 4620.
Let $X = 100 \cdot a + b$.
19. a =? (A)15 (B)16 (C)17 (D)18
20. b =? (A)0 (B)1 (C)2 (D)3

For problems 21-22, suppose E and F are events in a sample space with $p(E) = \frac{1}{3}$, $p(F) = \frac{1}{2}$, and $p(E|F) = \frac{2}{5}$. Find $p(F|E) = \frac{a}{b}$.
21. a =? (A)1 (B)2 (C)3 (D)4
22. b =? (A)2 (B)3 (C)4 (D)5

23. Which the following statement is false?
(A) If NFA with k states accepts any character at all, then it cannot accept a string of length $< k$
(B) The set for all the string that does not belong to a particular regular language L, is also a regular language
(C) The result of subset operation of a regular language set can still be regular
(D) Any kind of NFA can always convert to a DFA

24. Let N be an NFA with n states, let k be the number of states of a minimal DFA which is equivalent to N. Which one of the following is necessarily true?
(A) $k \geq n^2$
(B) $k \geq 2^n$
(C) $k \leq n^2$
(D) $k \leq 2^n$

25. Which of the following is not context-free language?
(A) $L1 : \{ 0^p1^q0^r | \ p = q$ and $pqr \geq 0\}$
(B) $L2 : \{ 0^p1^q0^r | \ p = q = r$ and $pqr \geq 0\}$
(C) $L1 : \{ 0^p1^q0^r | \ p = q$ or $q = r$ and $pqr \geq 0\}$
(D) all of above are context-free language

備 註	一、作答於試題上者，不予計分。 二、試題請隨卷繳交。