18 July 2019 09:08

Gauging defects in quantum spin systems

Kinematics (a) Total Hilbert Space
(b) "Topological" subspace

Question: What is the Hilbert space for a system of zero or one qubits.

"Single-particle hilbert space" $\cong$ $\mathfrak{h} \cong \mathbb{C}^2$

(pre Fock space)
$$\mathcal{F}(\mathfrak{h}) = \bigoplus_{N=0}^{\infty} \mathfrak{h}^{\otimes N}$$
$$= \underset{\substack{\uparrow \\ N=0 \\ \text{qubits}}}{\mathbb{C}} \oplus \underset{\substack{\uparrow \\ N=1 \\ \text{qubit}}}{\mathbb{C}^2} \oplus (\mathbb{C}^2 \otimes \mathbb{C}^2) \oplus \cdots$$

Indefinite # of qubits

$$\mathcal{F}_{\leq 1}(\mathfrak{h}) \cong \mathbb{C} \oplus \mathbb{C}^2$$

"Gauging" defects.

Quantum Spin System:

site 1: $\mathbb{C}^2$, site 2: $\mathbb{C}^2$, ...

$$\mathcal{H}_N \cong (\mathbb{C}^2)^{\otimes N} \otimes \mathbb{C}(|\underset{1\,2\,3}{1\,1\,1}\,1 \cdots \underset{N}{1}\rangle)$$
There is a qubit at site j

What if there is a "defect" at site j?

$\circ \ \circ \ \circ \ \circ \ \cdots \ \underset{j-1}{\circ} \ \underset{j}{\circ} \ \underset{j+1}{\circ} \ \circ \ \cdots$

$$\mathcal{H}_{N-1}(j) \equiv (\mathbb{C}^2)^{\otimes N-1} \otimes_{\mathbb{C}} (\mathbb{C}\,|1\,1\,1 \cdots \underset{j-1}{1}\underset{j}{0}\underset{j+1}{1} \cdots \rangle)$$

"Gauge" defects: $\Rightarrow$ Allow defects to 'move' $\Rightarrow$ Need more data!

Need to say where defects "are"

$$\mathcal{H}_{\text{defect}} = \bigoplus_{x_1=0,x_2 \cdots}^{1} (\mathbb{C}^2)^{\otimes \#\text{ of 1s in } \underline{x}} \otimes (\mathbb{C}\,|x_1 x_2 \cdots x_N\rangle)$$

$\wr\wr$

$$\mathcal{H}_{\text{defect}} = \bigotimes_{j=1}^{N} \mathcal{F}_{\leq 1}(\mathfrak{h})$$
$$= \bigotimes_{j=1}^{N} (\mathbb{C} \oplus \mathbb{C}^2)$$
$$= (\mathbb{C} \oplus \mathbb{C}^2) \otimes (\mathbb{C} \oplus \mathbb{C}^2) \otimes \cdots$$
$$= \mathbb{C} \oplus \underbrace{\mathbb{C}^2_1 \oplus \mathbb{C}^2_2 \oplus \cdots}_{\mathcal{H}_1} \oplus \underbrace{(\mathbb{C}^2 \otimes \mathbb{C}^2) \oplus \cdots}_{\binom{N}{2}} \oplus \cdots$$

$\mathcal{H}_1$ space of states for a qubit which can be on one of N positions

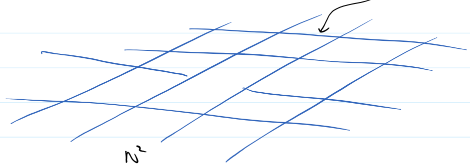
$\circ_1 \ \circ_2 \ \circ_3 \ \circ_{\cdots} \cdots \cdots \ \circ$

$$\mathcal{H}_1 \cong \mathbb{C}^N \otimes \mathbb{C}^2$$
$$\cong \mathbb{C}^{2N}$$

position of qubit

Gauging defects in (Toric code)

qubits



$$\mathcal{H}_{\text{defect}} \cong \bigotimes_{j=1}^{N^2} (\mathbb{C} \oplus \mathbb{C}^2)$$

Topological subspace

Go back to regular lattice

$$\mathcal{H}_{\text{Toric}} \cong \bigotimes_{j=1}^{N^2} \mathbb{C}^2$$

Step 1 $\downarrow$ Build Toric codeham.

$$H = -\sum A_+ + B_\square$$

Step 2 Compute ground-eigenspace

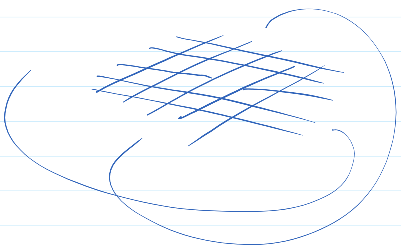
$$\downarrow$$
$$P_{\text{Grnd}} = |1\rangle\langle 1| + |e\rangle\langle e| + |m\rangle\langle m| + |e{\times}m\rangle\langle e{\times}m|$$

Step 3 Projects onto

$$V_\pi \subset \mathcal{H}$$

$$V_\pi = \text{Span}\{|1\rangle, |e\rangle, |m\rangle, |e{\times}m\rangle\} \cong \mathbb{C}^2 \otimes \mathbb{C}^2$$

Step 4: To every "Topology" we associate a potentially different subspace of $\mathcal{H}$.



$\longrightarrow 1 \longrightarrow 2 \longrightarrow 3 \longrightarrow \quad V_{\pi, \text{defect}} \cong \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2$

~~Gauge edge defects for (Toric) code~~

$$\mathcal{H}^{\text{full}}_{\text{defect}} = \bigotimes_{\text{edges}} (\mathbb{C} \oplus \mathbb{C}^2)$$

$$= (\mathbb{C}^2)^{\otimes N^2} \oplus \left(\underbrace{\bigoplus_{\text{edges}}}_{N^2} (\mathbb{C}^2)^{\otimes (N^2 - 1)} \right) \oplus \cdots$$

Set up equivalence relation,

We identify states which have same # of defects

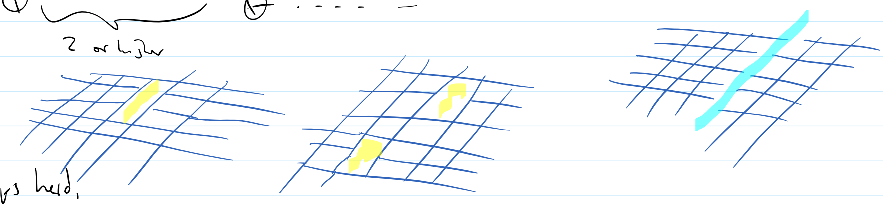
$|\phi\rangle \in (\mathbb{C}^2)^{\otimes(N^2-1)}$ for edge j missing

$\wr$

$|\phi\rangle \in \cdots$ for edge k missing

$\mathcal{H}^{full}_{defect}/\sim \implies$ Smaller Hilbert space.

$$\mathcal{H}^{full}_{defect}/\sim = (\mathbb{C}^2)^{\otimes N^2} \oplus (\mathbb{C}^2)^{\otimes(N^2-1)} \oplus \underbrace{\quad ? \quad}_{2 \text{ or higher}} \oplus \cdots -$$



Working out "$\sim$" properly $\longrightarrow$ Category helps here!

$$V_{Tops} = V_{\Pi} \oplus V_{\Pi \setminus \text{puncture}} \oplus V_{\Pi \setminus 2p} \cdots$$

Question: How to talk about "superposition" of defect and "not defect"?

Back to spin system chain

$\circ \quad \circ \quad \circ \quad \cdots \quad \circ$

$$\mathcal{H}_N \simeq (\mathbb{C}^2)^{\otimes N}$$

$$\mathcal{H}_{defect}(j) \simeq (\mathbb{C}^2)^{\otimes(N-1)}$$

Total Hilbert space

$$\mathcal{H}_{full} \simeq \underset{\underset{|\Omega\rangle}{\uparrow}}{\mathcal{H}_N} \oplus \underset{\underset{|\Phi\rangle}{\uparrow}}{\mathcal{H}_{defect}} \oplus \cdots$$

$$|\Omega\rangle + |\Phi\rangle \in \mathcal{H}_{full}$$