Our work establishes a fruitful connection between physics and mathematics. In particular, we describe the dynamics of defects in physical systems such as quantum spin chains by using the mathematical concepts of category theory and tube algebras. By this approach, we are able to simplify highly complex combinatorial problems to a graphical calculus. This pushes the field of quantum phases of matter into a new direction, where purely mathematical concepts can be incorporated to facilitate the analysis of defects.