**Popular summary**

Quantum computers are one of the most promising technologies of current research and have the potential to sustainably shape industry and the economy. However, the biggest obstacle when building quantum computers is their low stability to effects of noise. One promising approach to overcome this drawback are topological quantum computers, which are robust against local noise and therefore less sensitive to the environment. Unfortunately, the computational power of quantum systems suitable for topological quantum computation is severely limited. This can be ameliorated by adding defects to the system, such as vacancies in a lattice. In our work, we create a framework to describe how quantum systems with defects, that are appropriable for topological quantum computation, evolve in time.

In the absence of external forces, we leave the total number of defects indefinite. Each lattice site is then described by a “maybe link” which can either be a regular occupied link or a defect, respectively. In the presence of external forces, the situation is more subtle. Here, we encode dynamical information in the system energy (Hamiltonian), which reduces the problem to a combinatorial one. We then study a specific one-dimensional example with our technique, which indeed reveals a model with higher computational power when adding defects.

All in all, this research enables physicists to make sense of impurities in imperfect experimental realizations of quantum systems suitable for topological quantum computation. Moreover, it allows the description of systems with high computational power, and thereby gives hope for realizing practical tasks on a topological quantum computer one day.