gradient descent

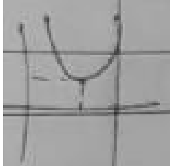
$$x_{n+1} = x_n - \alpha f'(x_n)$$

Taylor expansion in linear order

$$f(x_{n+1}) = f(x_n) + f'(x_n)(x_{n+1} - x_n)$$

$$f(x_{n+1}) = f(x_n) + f'(x_n)(x_n - \alpha f'(x_n) - x_n)$$

$$f(x_{n+1}) - f(x_n) = -f'(x_n)^2 \alpha$$



$$f'(x_n) = \sqrt{\frac{f(x_n) - f(x_{n+1})}{\alpha}}$$

as $n \to \infty$

$$f(x_n) \approx f(x_{n+1})$$

also clearly

$$f(x_n) - f(x_{n+1}) \geq 0$$

$\Rightarrow f(x_{n+1})$ gradually decreases

so $f(x)$ has local minima

$$\therefore \lim_{n \to \infty} f'(x_n) \approx 0$$

$\Rightarrow$ slope is 0

$\Rightarrow$ gradient converges at point of local minima of function

or

$$\sum_{i=0}^{n} f(x_{i+1}) - f(x_i) = -\left(f'(x_0)^2 + f'(x_1)^2 + \dots + f'(x_n)^2\right)\alpha$$

$$\underbrace{f(x_{n+1}) - f(x_n)}_{\text{difference is finite}} = -\sum_{0}^{n} f'(x_i)^2 \alpha$$

↳ ~~ultima~~ $f'(x_i) \downarrow$ as $i \uparrow$

$\Rightarrow$ ultimately $f'(x_n) = 0$