Process Optimization HomeWork 1

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August 19, 2016

Problem Data

- Lengths of pipes :
  1. $L_1 = 300m$
  2. $L_2 = 500m$
  3. $L_3 = 400m$
- Flow Rate in pipes :
  1. $Q_1 = 9m^3/min$
  2. $Q_2 = 3m^3/min$
  3. $Q_3 = 2m^3/min$
- Minimum Heads :
  1. Minimum pressure at node B = $79m$
  2. Minimum pressure at node C = $87m$
  3. Minimum pressure at node D = $83m$

Question and Answers

1. Write down expressions for the heads at nodes 1, 2 and 3 in terms of D1,D2,D3.
   Ans :

$$H_A = 100m \tag{1}$$
$$H_B = H_A - \frac{7.78 * 10^{12}}{D_1^{4.87}} \tag{2}$$
$$H_C = H_A - \frac{7.78 * 10^{12}}{D_1^{4.87}} - \frac{1.70 * 10^{12}}{D_2^{4.87}} \tag{3}$$
$$H_D = H_A - \frac{7.78 * 10^{12}}{D_1^{4.87}} - \frac{6.42 * 10^{11}}{D_3^{4.87}} \tag{4}$$

2. Write down expression for the total cost, i.e., sum of costs of links I, II and III.
   Ans :

$$TotalCost(C) = \sum_{i=1}^{n=3} 1.2654 L_i D_i^{1.327} \tag{5}$$
$$TotalCost(C) = 379.62 D_1^{1.327} + 632.70 D_2^{1.327} + 506.16 D_3^{1.327} \tag{6}$$

3. Formulate the appropriate optimization problem to minimize the costs while ensuring that the heads at nodes 1, 2 and 3 are greater than the minimum values.
   Ans :
   Objective Function : (Total Cost).

Decision Variables :D_1, D_2, D_3 .
Constraint Equations: $0 \leq H_A \leq 100, 79 \leq H_B, 87 \leq H_C, 83 \leq H_D$.

There are repetitive constraints which needs to be removed . The optimization problem we obtain after such a process is as follows :

$$\min(\textbf{TotalCost})$$

Such that the below constraints are satisfied

$$87 \leq H_A \leq 100$$
$$87 \leq H_B \leq 100$$
$$87 \leq H_C \leq 100$$
$$83 \leq H_D \leq 100$$

4. At the optimum, what do you expect the values of the head at nodes 1, 2 and 3 to be? Give reasons.
   Ans : The Values of H_C, H_D can be easily reasoned to be equal to the minimal Head Values because there are no more pipes connected to them ,in other words they are the extreme nodes and hence their head values can't influence the diameters of any other pipes. Similarly the H_A pipe takes the maximum Head Value. The only decision variable to be found therefore is H_B.

5. Find the values of the Heads,Diameters at optimum.
   Ans : Following from the above answer ,we can translate the optimization problem to a single decision variable H_B.
   Therefore the new optimization problem is as follows

$$TotalCost(C) = 379.62\left(\frac{7.78 * 10^12}{100 - H_B}\right)^{\frac{1.327}{4.87}} + 632.70\left(\frac{1.70 * 10^12}{H_B - 87}\right)^{\frac{1.327}{4.87}} + 506.16\left(\frac{6.42 * 10^12}{H_B - 83}\right)^{\frac{1.327}{4.87}}$$

   $min(TotalCost)$, such that $87 \leq that H_B \leq 100$.

   By **Golden section method** we get the below table.

Xa	91.956	91.956	93.852	93.852	93.852	94.300	94.300	94.300	94.406	94.471	94.471	94.496	94.511	94.511	94.511	94.515	94.515	94.515	94.516	94.516	94.516
Xb	99.990	96.921	96.921	95.749	95.024	95.024	94.748	94.577	94.577	94.577	94.536	94.536	94.536	94.527	94.521	94.521	94.519	94.517	94.517	94.517	94.516

   Hence The values of the heads are as follows :
   $H_A = 100m, H_B = 94.5162m, H_C = 87m, H_D = 83m$.
   The corresponding values of the Diameters are as follows $D_1 = 312.792mm, D_2 = 214.540mm, D_3 = 258.199mm$.
   Cost at the optimum state is $Rs1.5744 * 10^6$

Pipeline Network

Pipeline problem

Enter your roll number in the box below and press the button titled Click

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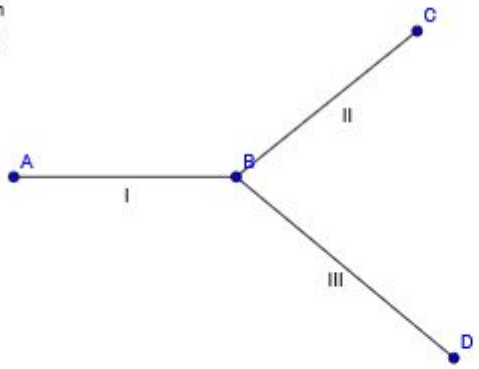
Roll number ME14B149

Minimum pressure at node B =79 m
Minimum pressure at node C=87 m
Minimum pressure at node D=83m



Source code : *https://github.com/RAAKASH/CH5170.git*