Multivariate data analysis Assignment 4

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1 Answers

1.1 Question 1

1.1.1 Part A

The **IPCA** has been applied and it is seen that irrespective of the no of constraints (the initial guess) we choose there is always a remarkable difference between the last 11 eigenvalues of the covariance matrix (for large number of constraints). The eigenvalues for different number of constraints are represented in **table 1** .An external iteration was run over the IPCA to find that for the number of constraints = 11 , all the last 11 eigenvalues of the scaled Data matrix tend to 1 a expected. When inspected by the IPCA for number of constraints = 11,12 ,they seem to be extremely similar in terms of the eigenvalues causing some confusion between the number of constraints (as both tend to 1 in the last 11 values),but if you plot the SCREE plot we can clearly assert that the number of constraints = 11(**since if the no of constraints were 12 ,the last 12 eigenvalues should tend to 1 which is not the case here**) ,**Hence we can easily assert the number of independent variables = 28 -11 =17**.We can algorithmically also quantify number of constraints ,by starting from $'N'$(in our case 28) ,checking if the last $'N'$ eigenvalues tend to 1 (apply some threshold),if not decrease it by 1 ,run the check again,loop until the IPCA constraint is satisfied.Please check **https://github.com/RAAKASH/CH5440** for the code for finding number of constraints using IPCA.

1.1.2 Part B

Now using the constraint matrix obtained by the IPCA , we get the regression matrix to be equal to the matrix displayed below. The maximum absolute difference between the actual , regressed coefficients are 0.0063.

$$Y_{dep} = \begin{bmatrix} -0.99884 & -0.9999 & -0.00305 & 1.001014 & -0.00031 & 0.998728 & 1.002387 & 0.995592 & 1.001228 & -1.00005 & 0.000711 & -0.00049 & -1.00089 & 0.998501 & 0.999952 & -0.99825 & 0.001261 \\ -0.00021 & 1.000035 & -0.00028 & 0.000162 & -1.00013 & 0.000307 & 0.001561 & 0.000627 & -0.00063 & -0.00198 & -0.00114 & 0.000659 & 5.46E-05 & 0.00086 & -0.00078 & -0.00038 & 0.000479 \\ 0.001284 & 0.000452 & -0.0038 & 1.001314 & -0.99936 & 0.999532 & 1.00051 & 0.994806 & 1.000816 & -0.99879 & -0.00048 & -0.00174 & -1.00206 & 0.999894 & 0.998618 & -0.99648 & 0.00021 \\ -0.99994 & -1.00289 & 0.999912 & 1.000977 & 0.001204 & 1.000193 & 1.002712 & 0.993742 & 1.000614 & -0.99938 & 0.000111 & -0.00197 & -1.00268 & 0.999341 & 0.998906 & -0.99489 & 0.000447 \\ -0.00083 & -0.00041 & -0.00338 & -0.00011 & 0.002042 & 1.00152 & -0.00032 & -0.00452 & 1.001663 & -0.99767 & -0.99876 & -0.99901 & -0.00098 & -0.0029 & 0.000162 & 0.002137 & -0.00186 \\ 0.000577 & 0.001917 & -0.00055 & 0.000121 & 0.000197 & 6.83E-05 & -0.00137 & -0.00105 & -0.00021 & 0.000136 & 1.000213 & -0.00161 & -0.00037 & 0.000368 & 1.000505 & -7.15E-06 & 0.999252 \\ 0.999203 & 0.001272 & -1.00094 & -1.00155 & 0.001212 & 0.001307 & -1.00322 & -0.99867 & 2.39E-05 & 0.002882 & 0.000435 & -0.99898 & 1.001102 & -0.9987 & 0.001614 & 0.998232 & 0.99624 \\ -0.00015 & 0.000345 & 1.000523 & 1.00095 & -0.00012 & -0.00015 & 0.001561 & 8.57E-05 & -0.0002 & -0.00198 & -0.00237 & -0.00145 & -0.00078 & -0.00082 & -0.00078 & 0.001351 & 0.001617 \\ 0.000748 & 0.001329 & -0.00114 & -0.00038 & 0.00054 & -0.00074 & -0.00051 & -0.00217 & 0.999706 & -0.99959 & -0.00135 & -0.00185 & -1.00133 & 1.000224 & 0.999512 & -0.99902 & 0.000552 \\ 0.000562 & 0.002082 & -0.0009 & -0.0006 & 0.00022 & 0.000334 & -0.00052 & 0.001406 & -0.00044 & -0.99938 & -0.00031 & -0.00132 & -3.15E-05 & 0.000531 & 0.998543 & -0.99991 & -0.00115 \\ -5.76E-05 & -0.00227 & 1.002314 & 1.001292 & -0.00075 & -0.0002 & 1.001396 & -0.00172 & 0.000809 & -0.00123 & -0.00075 & 0.000554 & -1.00074 & -0.00324 & -2.75E-06 & 0.002275 & 0.001519 \end{bmatrix} Y_{ind}$$

Table 1: Eigen Values of Scaled Data

No of constraints																												
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	254549.7	193685.6	52027.88	23835.14	13255.49	7169.184	5573.396	3897.343	3512.321	2287.36	1393.437	808.9546	529.8775	284.2879	117.6707	85.74084	40.39678	1.020289	0.999715	0.338973	0.212135	0.157099	0.113101	0.060983	0.041323	0.028873	0.007357	0.003266
8	2440445	1904851	262720.5	90031.73	59836.03	41657.91	16261.2	14121.3	9140.189	8932.344	4610.573	3745.168	3187.198	2537.063	1773.726	1380.947	1078.765	1.968504	1.777838	1.245541	1.014069	1.000246	0.994752	0.985838	0.976975	0.914269	0.839745	0.769493
9	1692483	1087175	181385.2	73742.15	45026.82	33901.78	23525.34	14611.64	12274.81	8680.533	6100.672	4098.773	2956.047	2713.998	1927.26	1624.604	1321.281	1.608931	1.335966	1.056626	1.024509	1.016572	1.005201	0.994775	0.981816	0.970764	0.947153	0.873186
10	1438833	868286.1	139240.6	77922.48	43918.21	29940.45	16987.49	14961.94	8497.481	7730.497	6720.386	4323.943	2946.74	2781.144	1853.315	1497.105	1338.07	1.639848	1.073975	1.05977	1.03146	1.024074	1.004737	0.987529	0.97946	0.97242	0.94034	0.891401
11	1456006	663480.2	123211.2	63942.4	44990.09	27318.84	15546.06	13754.23	8636.24	5983.178	4107.034	3545.049	2825.794	2102.572	1648.196	1557.111	1269.525	1.132552	1.083308	1.049144	1.04373	1.012928	0.995287	0.972993	0.97154	0.935778	0.915847	0.886894
12	2181281	633221.4	172075.2	61735.23	52698.38	27441.27	20189.5	11447.48	8410.23	6312.579	4077.298	2640.122	2101.92	1628.519	1333.472	1121.418	2.98903	1.067481	1.01709	0.999385	0.974269	0.918471	0.889526	0.820549	0.766676	0.581967	0.005476	0.003712
13	1297310	664852.7	134159.8	88669.34	39150.36	24802.74	9910.004	7164.716	5946.603	2113.526	1897.617	875.9524	30.36805	8.358088	5.180014	2.354346	1.928423	1.05722	1.000918	0.995137	0.968747	0.806197	0.749007	0.639782	0.003055	0.002368	0.001553	0.001158
14	24571995	4106717	366443	204285.3	49841.71	42193.44	16319.31	7280.776	4481.971	4180.834	1841.397	1048.778	14.02635	3.159927	2.208392	1.8054	1.351584	1.075453	1.042746	0.986722	0.975442	0.92525	0.885428	0.72986	0.001751	0.001297	0.001039	0.000695
15	1589868	605955.9	177408.5	60220.58	39363.72	25027.37	8763.141	6431.577	4227.04	1967.898	1432.034	465.4649	13.82531	2.34605	2.080241	1.618651	1.197982	1.072423	0.998873	0.988509	0.974361	0.913632	0.471699	0.36176	0.001716	0.000998	0.000923	0.000648
16	11000138	688694	78590.21	43211.44	29693.15	18091.62	7308.52	3133.169	2351.223	1729.22	28.4821	11.60514	2.364425	2.082871	1.947983	1.318371	1.095309	1.050371	1.000073	0.975023	0.962476	0.89816	0.300177	0.002087	0.001267	0.000945	0.000703	0.000598
17	943541.6	424280.8	46059.88	36937.91	19134.3	10878.98	3742.353	1816.8	27.21466	13.79704	11.84615	2.859325	2.063976	1.921962	1.340064	1.187172	1.000912	0.985515	0.939284	0.702639	0.686481	0.255003	0.001355	0.001159	0.000988	0.000891	0.000615	0.000559
18	1043884	369315.2	42705.77	37298.3	30071.6	19722.46	3506.632	2922.528	39.06529	15.98773	3.341445	2.336962	2.000976	1.765278	1.177623	1.115941	0.984452	0.96705	0.946379	0.665419	0.638972	0.343644	0.00121	0.000875	0.000776	0.000551	0.000471	0.000374
19	943072.2	461841.7	150769.1	52892.32	23809.36	17060.69	4753.882	2979.816	8.389482	3.645524	2.510906	1.860073	1.65519	1.500196	1.0381	0.994198	0.963825	0.920085	0.756636	0.446087	0.422	0.311659	0.000783	0.000583	0.0005	0.000374	0.000319	0.000211
20	824295.5	301002.6	36264.64	24358.36	16213.45	6800.682	3298.167	2098.655	3.532117	3.27331	2.251805	1.560298	1.368283	1.208976	0.997146	0.972318	0.805222	0.668404	0.47073	0.464444	0.378346	0.285235	0.000626	0.000487	0.000362	0.000282	0.000246	0.000148
21	831137.2	271815.2	31977.22	29827.07	11635.19	2748.986	28.14852	3.642402	3.338537	2.82064	1.897791	1.389509	1.229113	1.098098	0.996694	0.953328	0.784189	0.650326	0.499118	0.486675	0.360753	0.000532	0.000413	0.000318	0.000263	0.000239	0.000168	0.000152
22	3090750	277993.2	23514.19	16240.46	39.35435	11.98752	4.148959	3.509342	2.826426	2.090176	1.563929	1.242626	1.137881	1.087609	0.959782	0.725323	0.505399	0.450137	0.348867	0.001254	0.000679	0.000385	0.000359	0.000338	0.000252	0.000234	0.000164	0.000132
23	580220.1	119719.5	18033.57	11470.75	86.38459	4.135174	3.552489	3.250063	2.564348	1.472973	1.334935	1.180671	1.066746	0.950032	0.830567	0.653601	0.441093	0.343424	0.27491	0.000744	0.00039	0.000322	0.000252	0.00024	0.000217	0.000152	0.000146	0.000118
24	12476.34	277.2567	68.96814	19.41342	5.268111	4.385783	2.826744	2.663787	1.935733	1.24453	1.20512	1.018249	0.802025	0.676824	0.573313	0.36104	0.325768	0.000988	0.000405	0.000332	0.000293	0.000231	0.00021	0.000174	0.000148	0.000113	7.64E-05	6.51E-05
25	1965.045	90.28786	27.35527	5.832919	3.993917	3.78345	2.43087	2.078471	1.448825	1.103853	0.986309	0.790778	0.597678	0.470118	0.421924	0.315929	0.260821	0.001006	0.000243	0.000199	0.000186	0.000165	0.00012	0.000115	0.000103	7.12E-05	6.40E-05	3.72E-05
26	13157.71	61.1425	5.683275	4.742315	3.631121	3.114857	1.976254	1.686187	1.096995	0.963787	0.841198	0.608315	0.514851	0.35501	0.287342	0.258012	0.210824	0.001193	0.000208	0.000161	0.000138	9.73E-05	7.93E-05	7.16E-05	6.26E-05	5.23E-05	3.52E-05	2.77E-05
27	49.17695	9.942335	4.040479	3.197552	2.278011	1.692972	1.242671	0.942919	0.7939	0.667865	0.642195	0.47047	0.335091	0.272284	0.188584	0.169426	0.122614	0.000188	0.000136	0.00011	4.72E-05	4.24E-05	4.11E-05	2.78E-05	1.37E-05	1.24E-05	6.76E-06	4.91E-06
28	11.29686	7.501073	2.400703	1.766825	1.019702	0.890225	0.738842	0.658981	0.478133	0.348638	0.284982	0.187828	0.127744	0.114468	0.073321	0.056372	0.055023	8.28E-05	6.08E-05	3.62E-05	2.58E-05	2.12E-05	1.79E-05	1.27E-05	9.57E-06	6.96E-06	5.51E-06	3.65E-06

1.2 Question 2

1.2.1 Part A

Since we have lots of samples at hand we can afford to simply throw away points which have missing ,measurements (although it is best not to throw away the data point unless until the no of measurements not missing in the data point ¡ number of redundancies) .
Maximum Difference between regressed , actual values are 0.0082.

Table 2: Constraint Equations Coefficients

													Constrain model														
-0.00016	0.000683	-0.01465	-0.01501	-0.00014	-0.00412	-0.00276	0.000123	-0.00466	0.046863	0.00224	0.004314	0.003254	-0.00036	-0.0446	0.042589	-0.00201	0.000305	-0.00015	3.90E-05	0.000343	0.003451	0.001257	0.000785	0.012264	0.000477	0.042265	0.002863
-0.00174	0.00056	0.051412	0.050894	-0.00061	0.000409	0.009775	0.001861	-0.00173	0.013391	-0.00136	0.001408	-0.00764	-0.0004	-0.01476	0.012141	-0.00253	0.000609	-0.00049	-9.12E-05	-0.00055	-0.00035	0.000837	0.001778	-0.04113	0.002109	0.011814	-0.0079
-0.00022	-0.00017	-0.0021	-0.00196	0.000313	0.017677	0.00458	0.000353	0.016613	-0.01804	-0.03881	-0.01722	-0.00348	-0.00074	-0.02082	-0.00078	-0.02165	8.29E-05	0.000474	-0.00018	0.000272	-0.01784	0.021047	0.000568	0.006523	0.001089	-0.0015	-0.00418
-0.01514	-0.01167	-0.00212	0.007044	-0.0022	0.014258	0.024289	0.018965	0.013482	0.000633	0.008915	0.004888	-0.02353	0.018348	0.008302	-0.00425	0.003982	-0.00519	0.001784	-0.00399	-0.00464	-0.00044	-0.0093	0.005312	0.017233	0.00079	0.014044	-0.00516
-0.00643	0.003559	0.001048	0.000548	-0.00262	-0.0161	0.008845	0.00625	-0.01784	0.006208	-0.00086	0.022219	-0.0071	0.004514	-0.00714	-0.01603	-0.02305	0.00015	-0.00286	0.000291	0.000577	0.015067	0.015936	0.007165	0.008278	0.001706	-0.01156	-0.00268
-0.01013	-0.014	-0.00326	0.00264	0.005182	0.006227	-0.01563	0.012132	0.010101	-0.00294	-0.00203	0.005837	0.011792	0.016008	0.000863	-0.0088	-0.00791	-0.00387	0.007154	-0.002	-0.00303	0.002643	0.004669	0.003238	-0.01828	-0.00384	0.007183	0.027777
-0.00245	0.024977	-0.00067	-0.00198	-0.02238	0.002177	-0.00639	0.001971	0.00789	-0.00298	-0.00125	-0.00023	0.000695	0.00765	0.001705	-0.00271	-0.001	0.000875	-0.0229	0.000506	0.00119	-0.00475	-0.00351	0.004514	-0.00442	-0.00572	0.004959	0.008369
-0.00849	0.00046	-0.00032	0.002243	-0.00491	0.009063	0.004365	0.008106	-0.01017	0.001595	0.000123	-0.00092	0.014859	-0.01109	-0.00153	0.002528	0.000962	-0.00292	-0.0053	0.000376	-0.00191	-0.00458	-0.00465	0.003644	0.002114	0.019208	-0.00857	0.003746
-0.00394	0.004847	0.007528	-0.00583	0.010503	-0.00241	-0.00206	-0.00034	-0.00083	-5.84E-05	-0.00152	0.002085	0.000416	0.001239	-0.00147	-0.00212	-0.00362	0.009143	0.006226	0.004274	0.001932	-0.01292	-0.01139	0.015015	0.003821	-0.0016	-0.00089	0.00169
-0.00715	-0.00408	0.008563	-0.00425	0.001317	-8.90E-05	-0.00101	-0.00151	0.001211	-0.00067	-0.00025	-0.00141	-0.00032	-0.00017	0.000385	0.00078	0.00117	0.004111	-0.00734	0.008648	-0.01551	0.002856	0.003095	-0.00426	0.003214	-0.00132	0.000571	-0.00049
-0.00732	-0.00155	-0.0015	0.000676	0.005821	-0.00217	-0.00075	-0.00331	0.000349	0.000252	-0.00076	-0.00116	-0.00172	-0.00083	-0.00103	0.001436	0.000436	-0.01286	-0.00482	0.010662	0.006486	-0.00213	-0.00137	0.000949	-0.00145	-0.00252	0.000639	-0.00257

$$
Y_{dep} = \begin{bmatrix} \multicolumn{17}{c}{\text{Regression Matrix}} \\
-0.99939 & -1.00007 & -0.0004 & 0.998772 & 0.000664 & 0.998482 & 1.001835 & 0.992793 & 1.001676 & -0.99856 & 0.001902 & -0.00014 & -1.00075 & 1.000052 & 0.999441 & -0.99833 & -0.00056 \\
-0.00028 & 0.999722 & -0.00031 & 0.000766 & -0.99988 & 0.000169 & 0.001679 & 0.001819 & -0.00061 & -0.00158 & -0.00023 & 0.000838 & -0.00017 & -7.55E-05 & 9.32E-05 & -0.0007 & 8.38E-05 \\
0.001035 & 0.000521 & -0.0024 & 0.999916 & -0.99853 & 0.998823 & 0.999915 & 0.992037 & 1.001918 & -0.99651 & 0.001196 & -0.00062 & -1.00184 & 0.999043 & 0.99911 & -0.99698 & -0.00128 \\
-1.00004 & -1.0008 & 1.000299 & 0.998474 & 0.001649 & 1.000126 & 1.000503 & 0.991812 & 1.001506 & -0.99883 & 4.47E-05 & -0.00166 & -1.00238 & 0.999843 & 0.997246 & -0.99517 & -0.00054 \\
-0.0002 & -8.69E-05 & -0.00366 & -0.00023 & 0.002009 & 1.000608 & -0.00031 & -0.00455 & 1.002451 & -0.99817 & -0.99782 & -0.99871 & -0.0006 & -0.0032 & 0.000439 & 0.001988 & -0.00261 \\
9.14E-05 & 0.001598 & -0.00072 & -0.0003 & 0.000682 & 0.000241 & -0.00111 & -0.00252 & 0.000451 & 0.000558 & 0.998963 & -0.00108 & -0.00056 & -0.00032 & 1.000727 & 0.000682 & 0.999152 \\
0.999605 & 0.001457 & -1.00031 & -1.00082 & 0.001203 & -0.00012 & -1.00125 & -0.99895 & 0.000837 & 0.001936 & 0.000442 & -0.99877 & 1.001224 & -1.00027 & 0.001587 & 0.998053 & 0.997047 \\
-0.00022 & 0.001556 & 1.000674 & 1.000961 & -0.00018 & 0.000215 & 0.001165 & 0.001231 & 0.000185 & -0.00303 & -0.00392 & -0.00115 & -0.00068 & -0.00197 & -0.00118 & 0.000614 & 0.001347 \\
0.001103 & 0.001378 & -0.00106 & -0.00073 & 0.000345 & -0.00163 & -0.0003 & -0.0033 & 1.001024 & -0.99893 & -0.00133 & -0.00174 & -1.001 & 0.999364 & 0.999449 & -0.99833 & 0.001014 \\
0.000587 & 0.002202 & -0.00073 & -0.00078 & 0.000648 & -0.00047 & -0.00052 & -0.00024 & 0.000652 & -0.99892 & 5.75E-05 & -0.00159 & 1.46E-05 & 0.000534 & 0.998353 & -0.99923 & -0.00111 \\
-0.00025 & -0.00179 & 1.002253 & 1.000583 & -0.00128 & 0.000788 & 1.00068 & -0.00153 & 0.000184 & -0.00066 & -0.0024 & 0.000446 & -1.00073 & -0.00233 & -0.00037 & 0.00171 & 0.001888
\end{bmatrix} Y_{ind}
$$

1.2.2 Part B

This was using the mean imputed data matrix.
Maximum Difference between regressed , actual values are 0.6853.

Table 3: Constraint Equations Coefficients after mean imputation

Constrain model

-0.00226	-0.00569	0.004077	-0.00079	-0.00264	-0.00666	0.002536	-0.003	-0.01019	0.033326	0.00225	0.005117	-0.00218	0.002431	-0.04195	0.050043	0.003676	0.000137	-0.00017	0.000583	0.000131	0.005694	-7.74E-05	0.001777	0.001609	-0.00021	0.03915	0.00257
-0.00388	-0.0022	0.002079	0.003273	0.002802	-0.0177	0.007497	0.00708	-0.02172	0.00652	0.011779	0.017925	-0.00416	0.008482	0.018248	-0.014	0.022119	-0.00124	0.008497	-0.00145	-0.00071	0.021482	-0.01294	0.002149	-0.00233	-0.00086	-0.0067	0.003156
-0.00719	-0.02176	0.004293	0.007151	0.012129	0.015433	0.011159	0.010279	0.024167	-0.00455	0.001609	-0.00231	-0.01664	0.010044	0.005349	0.000954	0.001867	-0.00245	0.01497	-0.00672	-0.00186	-0.00464	-0.00536	0.000654	0.002055	-0.00468	0.01451	0.001024
-0.004	-0.00762	-0.00403	-0.00119	0.000922	0.006679	-0.02393	0.004198	0.005966	-0.00232	0.001009	-0.00091	0.024603	0.003611	0.002807	0.000533	0.002955	-0.00237	0.002922	-0.00118	-0.00138	-0.00152	-0.00244	-0.00016	-0.02215	0.000664	0.004127	0.02847
-0.01236	0.009831	0.005112	0.010981	-0.01796	0.008081	0.008697	0.012692	0.013241	-0.0042	0.00163	0.003631	-0.01081	0.012084	0.007453	-0.00971	0.007823	-0.00838	-0.01663	0.003202	-0.00339	-0.00049	-0.00554	0.002702	-0.00377	-0.00452	0.009283	0.003187
0.007778	0.002532	-0.00965	-0.01197	-0.00084	0.007142	-0.00035	-0.00525	0.002956	-0.00098	0.004702	-0.0155	0.007068	-0.00702	0.010994	0.01502	0.031081	0.001077	-0.00016	0.001992	-0.00029	-0.00945	-0.0226	-0.00704	0.008475	0.00204	0.00453	-0.00682
0.002129	0.011268	-0.03205	-0.03295	-0.00853	-0.00162	-0.0072	0.002353	0.006008	-0.0008	-0.00043	0.005321	0.000892	0.006436	0.003336	-0.0062	-0.00047	0.010339	-0.00303	-0.00424	-0.00468	0.000159	-0.00129	0.003551	0.027041	-0.00739	0.004512	0.009514
0.006856	0.008317	0.04836	0.032682	-0.00281	-0.00383	-0.00041	-0.00468	0.002338	-0.00133	-7.23E-06	-0.00175	-0.00315	-0.00043	0.000283	8.82E-05	0.003373	0.010371	-0.00068	-0.00019	-0.0028	-0.00369	-0.00452	0.001548	-0.03633	-0.00656	0.002152	-0.00336
3.65E-05	-0.00303	-0.00718	-0.01738	0.014138	-0.00935	-0.01466	-0.00936	0.004524	-0.00179	-0.0005	-0.00069	2.75E-05	0.002337	0.001037	-0.00154	-0.00171	-0.00106	0.002448	0.010691	0.000554	7.20E-05	0.001075	-0.00116	0.00284	-0.01421	0.001723	0.002729
0.013182	0.009591	-0.01067	-0.00121	-0.007	-0.00522	-0.00762	-0.00555	0.001797	-0.00046	0.000193	-0.00114	6.86E-05	0.001116	0.001957	0.001796	0.002081	-0.00357	-0.00061	-0.00544	0.015264	-3.53E-06	-0.0003	-0.00169	-0.0065	-0.00897	0.001742	0.000882
0.008701	-0.00634	-0.00443	0.00095	-0.00445	-0.0012	-0.00282	-0.00378	0.002943	-0.00149	-0.0004	-0.00359	-4.83E-05	-0.00201	0.000855	0.001982	0.003352	-0.00203	-0.00156	-0.0027	-0.00719	0.0131	0.013197	-0.01742	-0.00467	-0.00417	0.001717	-0.00215

$$
Y_{dep} = \begin{bmatrix} \multicolumn{17}{c}{\text{Regression Matrix}} \\
-1.00497 & -0.86954 & 0.007147 & 0.867889 & 0.114558 & 0.958148 & 0.89913 & 0.992762 & 1.19586 & -0.75971 & 0.069387 & 0.070651 & -0.99208 & 0.691056 & 0.947804 & -1.13614 & -0.04614 \\
0.129452 & 1.171346 & -0.20158 & 0.050746 & -1.25865 & -0.02742 & 0.019248 & 0.089336 & -0.10885 & -0.01353 & -0.05078 & 0.052825 & 0.111255 & 0.031475 & 0.013527 & -0.16515 & 0.199941 \\
0.013996 & 0.116117 & -0.01954 & 0.962669 & -0.97828 & 0.947396 & 1.016858 & 0.990092 & 1.155975 & -0.74167 & 0.019085 & 0.142477 & -0.9435 & 0.595376 & 1.00636 & -1.15841 & 0.013396 \\
-0.97324 & -0.68583 & 0.980275 & 0.837033 & 0.079603 & 0.913138 & 0.924544 & 0.946236 & 1.216948 & -0.78543 & 0.013947 & 0.110203 & -0.93652 & 0.60438 & 0.916094 & -1.26844 & -0.03803 \\
-0.03707 & -0.20944 & 0.004214 & -0.01718 & 0.204443 & 0.989102 & -0.18585 & 0.001977 & 1.23856 & -0.71361 & -0.31468 & -0.76154 & 0.066304 & -0.14262 & 0.160876 & -0.087 & -0.44039 \\
0.023658 & 0.240299 & -0.04372 & -0.04863 & -0.1942 & 0.009767 & 0.163726 & 0.164153 & -0.11721 & -0.05625 & 0.348268 & -0.04878 & 0.066559 & 0.112043 & 0.830088 & -0.11336 & 1.2856 \\
0.990629 & -0.09264 & -1.1138 & -0.95793 & -0.03147 & 0.050943 & -0.96433 & -0.78682 & -0.06677 & -0.02394 & -0.00938 & -0.91854 & 1.089622 & -0.71208 & 0.035266 & 0.980412 & 0.844527 \\
0.017308 & -0.01987 & 1.110991 & 0.967786 & -0.02644 & -0.00591 & 0.027234 & -0.00181 & -0.01003 & -0.01895 & 0.008623 & -0.01376 & 0.001396 & -0.00834 & -0.0252 & -0.0061 & 0.0081 \\
0.001886 & 0.159258 & -0.01931 & -0.08031 & 0.0327 & 0.011412 & -0.01047 & 0.152453 & 1.139813 & -0.75579 & -0.00888 & 0.129512 & -0.92002 & 0.644483 & 0.969623 & -1.2205 & -0.05821 \\
0.023757 & 0.190264 & -0.17547 & -0.05588 & 0.046218 & 0.004955 & -0.08388 & 0.091973 & 0.06216 & -0.73458 & -0.01169 & 0.021924 & 0.079473 & -0.01558 & 1.036089 & -1.29156 & -0.06395 \\
0.012565 & 0.008712 & 1.090206 & 0.941589 & 0.035864 & -0.02013 & 1.032342 & 0.004225 & 0.020437 & 0.017117 & -0.00147 & -0.02329 & -1.01614 & -0.03997 & -0.04762 & -0.0036 & -0.02122
\end{bmatrix} Y_{ind}
$$

1.2.3 Part C

Now using PCA as a tool to measure the missing measurements we get the following below results :
The Maximum difference between regressed,actual values are 0.0067.

$$Y_{dep} = \overset{\text{Regression Matrix}}{\begin{bmatrix} -0.9984 & -0.99891 & -0.00308 & 1.000656 & -0.00047 & 0.998202 & 1.00169 & 0.994773 & 1.001697 & -1.00004 & 0.00156 & -0.00022 & -1.00075 & 0.997392 & 0.999609 & -0.99839 & 0.00124 \\ -0.00034 & 0.999489 & 5.46E-07 & 0.000364 & -1.00022 & 0.000451 & 0.001859 & 0.000643 & -0.00086 & -0.00212 & -0.001 & 0.000644 & -6.54E-05 & 0.000952 & -0.00062 & -2.11E-05 & 0.000514 \\ 0.001486 & 0.000695 & -0.00384 & 1.001079 & -0.99957 & 0.999247 & 1.000008 & 0.994224 & 1.001169 & -0.99869 & 0.000231 & -0.00128 & -1.00196 & 0.998872 & 0.998678 & -0.99637 & 0.000212 \\ -0.99947 & -1.00155 & 0.999102 & 1.000515 & 0.001082 & 0.999672 & 1.00178 & 0.993339 & 1.001301 & -0.99931 & 0.000713 & -0.00145 & -1.00238 & 0.997815 & 0.998597 & -0.99523 & 0.000372 \\ -0.00076 & -0.00051 & -0.00349 & -0.00024 & 0.002125 & 1.001503 & -0.00086 & -0.00454 & 1.001895 & -0.99767 & -0.99794 & -0.99874 & -0.00093 & -0.0032 & 0.000166 & 0.002365 & -0.00224 \\ 0.000581 & 0.00212 & -0.00056 & -0.00022 & 0.000204 & 4.49E-05 & -0.00154 & -0.00138 & -0.00012 & 0.000208 & 0.999722 & -0.00147 & -0.00035 & 0.000121 & 1.000389 & -4.10E-05 & 0.999341 \\ 0.999016 & 0.000793 & -1.00028 & -1.0018 & 0.001336 & 0.001487 & -1.00316 & -0.99823 & 9.70E-05 & 0.0025 & -0.00022 & -0.99883 & 1.001199 & -0.99827 & 0.001403 & 0.998436 & 0.995929 \\ -0.00022 & 0.000272 & 1.000222 & 1.001159 & -2.05E-05 & -1.00E-04 & 0.001298 & 6.36E-05 & -0.00019 & -0.00172 & -0.00225 & -0.00138 & -0.00099 & -0.00108 & -0.00046 & 0.001487 & 0.001641 \\ 0.000842 & 0.001449 & -0.00142 & -0.00041 & 0.000404 & -0.00086 & -0.00059 & -0.00222 & 0.999945 & -0.99943 & -0.00104 & -0.00149 & -1.00119 & 0.999675 & 0.999658 & -0.99921 & 0.000591 \\ 0.000684 & 0.002305 & -0.00112 & -0.00071 & 0.000192 & 0.000297 & -0.00057 & 0.001431 & -0.00034 & -0.9993 & -4.34E-05 & -0.00139 & 1.96E-06 & 0.000381 & 0.998402 & -1.00003 & -0.00103 \\ -5.50E-05 & -0.00191 & 1.001659 & 1.001266 & -0.00063 & -0.00023 & 1.000848 & -0.00189 & 0.001021 & -0.00108 & -0.00063 & 0.000604 & -1.00083 & -0.0036 & -1.84E-05 & 0.002236 & 0.001565 \end{bmatrix}} Y_{ind}$$

1.2.4 Part A using IPCA

It can be yet again distinctly seen that No of constraints = 11 from the table below,which is an expected result. The maximum absolute difference between

Table 4: Eigen Values using IPCA

No of constraints																												
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1245785	420607.2	109791.4	49976.81	17382.89	2977.029	661.5882	251.6126	102.9969	43.30421	38.77712	27.60231	18.25553	7.963071	7.417333	5.238914	2.269415	0.983336	0.912638	0.669241	0.444821	0.018144	0.010764	0.005142	0.002476	0.00199	0.001778	0.000814
8	3809468	1573611	546606.6	169376.2	75175.48	62495.86	46332.67	37713.29	22299.51	12799.71	9489.558	6013.174	3482.515	3068.028	2705.271	2038.273	1719.632	2.876055	2.133878	1.362661	1.035938	1.0171	1.00466	0.996193	0.985452	0.973574	0.927456	0.896491
9	1679825	769512.8	299358.9	90764.54	61133.25	44662.05	26292.55	18131.37	9557.706	6681.818	5297.414	4139.959	3000.612	2429.872	2217.737	1850.203	1646.06	2.198934	1.343495	1.044258	1.024667	0.996325	0.988013	0.961674	0.947662	0.897455	0.876078	0.684443
10	24485963	741126.3	502409.9	98145.16	47512.51	31329.21	20608.17	16455.11	15084.63	7696.819	5681.098	5185.674	3043.595	2694.51	2278.304	2001.731	1777.486	2.017588	1.074743	1.052038	1.046528	1.032857	0.999029	0.968998	0.946741	0.901525	0.862507	0.789572
11	1384239	710518.9	124687.2	70504.56	44443.8	27512.22	15273.16	14541.05	8469.411	6082.201	4096.845	3407.874	2820.528	2147.195	1667.113	1590.22	1267.113	1.171748	1.083497	1.072777	1.044834	1.017851	1.007153	0.9818	0.973646	0.916532	0.893738	0.836425
12	2601110	701062.4	189293.1	76913.65	52472.06	29798.11	23892.98	12542.71	8060.494	4800.631	3331.379	3071.797	2297.592	1753.839	1344.838	1004.883	2.988376	1.085069	1.055117	1.024571	1.013801	0.985282	0.937227	0.911472	0.818234	0.721068	0.005277	0.003668
13	1726277	730031.1	279656.9	112214.5	39843.18	30895.23	13075.44	10723.19	6302.232	1956.525	1779.833	627.5548	30.29274	7.976816	4.997622	2.341778	1.928849	1.091583	1.018184	1.004829	0.970311	0.888315	0.829886	0.550209	0.002975	0.002309	0.001509	0.001133
14	1397244	604712.3	102710.6	43559.51	36171.7	18115.05	10240.09	4830.49	4234.032	3117.256	2455.732	1656.373	13.94109	2.936791	2.172842	1.796493	1.34898	1.07232	1.040203	0.988242	0.963105	0.909249	0.704826	0.492624	0.001735	0.001215	0.00101	0.000686
15	1702986	627991.7	109633.4	60689.55	38622.74	31291.59	15227.77	7172.807	4993.852	1919.477	1410.886	330.2208	14.01131	2.30213	2.071833	1.638933	1.221695	1.062438	1.011305	0.990072	0.951305	0.836388	0.643438	0.327295	0.001725	0.000976	0.000912	0.00065
16	1837015	659588.8	109485.5	51629.33	32634.18	21829.58	8023.227	3787.758	2150.708	1887.02	28.93049	11.46312	2.398415	2.039727	1.929394	1.335785	1.095613	1.03205	1.009856	0.979308	0.926779	0.720244	0.434026	0.002102	0.001258	0.000931	0.000677	0.000603
17	1043816	530901.9	61624.98	43507.94	21443.55	13090.96	4392.366	1873.108	27.57464	13.19402	11.89754	2.828116	2.07877	1.874373	1.352925	1.200483	1.021519	0.992335	0.984914	0.947208	0.573863	0.401571	0.00136	0.001155	0.000985	0.000867	0.0006	0.000561
18	955871.3	428613.8	110819.4	40513.84	26575.61	21389.83	3675.595	2944.387	41.92608	15.82198	3.33726	2.327392	2.00849	1.734342	1.198314	1.114374	0.994555	0.971492	0.929036	0.672834	0.518228	0.350726	0.001242	0.000871	0.000766	0.000545	0.00048	0.000356
19	785538.4	302315.6	43203.78	23432.79	17293.46	11665.06	3587.269	2500.418	8.943549	3.61746	2.488429	1.856566	1.690307	1.492822	1.057685	0.972282	0.967303	0.918574	0.782215	0.437238	0.412378	0.276379	0.000783	0.000592	0.000489	0.00039	0.0003	0.000224
20	758344.2	265971.6	34733.03	17205	14312.49	5590.105	2617.494	1843.029	3.627074	3.221191	2.208676	1.550183	1.38258	1.186295	1.008553	0.952867	0.798846	0.688947	0.404624	0.375652	0.355073	0.258753	0.000623	0.000478	0.000366	0.000273	0.000244	0.000147
21	766553.8	328597.9	46623.72	17993.09	10719.53	2125.678	28.75312	3.654567	3.368253	2.812299	1.848775	1.387937	1.236269	1.113263	0.991304	0.928392	0.768638	0.670666	0.525375	0.37423	0.332707	0.000503	0.000413	0.000337	0.000266	0.000241	0.000175	0.000149
22	1605909	310596.7	32719.95	15302.01	45.54891	12.04847	4.172368	3.536746	2.795142	2.114373	1.499094	1.267464	1.136132	1.062245	0.976381	0.714179	0.510476	0.404445	0.360441	0.001258	0.000653	0.000403	0.000361	0.00033	0.000252	0.000232	0.000167	0.000131
23	393411.7	118738.6	29013.32	9731.344	87.76679	4.147183	3.560093	3.286034	2.518546	1.48463	1.28645	1.201079	1.036958	0.969013	0.828958	0.647089	0.459582	0.348376	0.236307	0.000739	0.000403	0.000324	0.000253	0.000248	0.000213	0.000152	0.000137	0.000118
24	841.9586	340.4399	64.48694	28.7677	5.433339	4.132608	2.704831	2.519496	2.105031	1.319241	1.110669	0.969704	0.86092	0.720042	0.596664	0.462045	0.313284	0.000975	0.000663	0.000333	0.000278	0.000228	0.00017	0.000148	0.000123	9.42E-05	7.71E-05	5.45E-05
25	7074.781	171.0432	25.84865	5.540163	4.062696	3.46576	2.208318	2.093584	1.552705	1.14382	1.003077	0.810615	0.686228	0.579459	0.420357	0.346182	0.251176	0.000908	0.000563	0.000219	0.000191	0.000179	0.00011	8.38E-05	7.37E-05	6.09E-05	4.66E-05	3.42E-05
26	5204.812	65.77041	5.738483	4.736235	3.610523	3.089868	1.974312	1.701082	1.128207	0.925118	0.828701	0.607144	0.499304	0.347048	0.282977	0.252154	0.210163	0.00118	0.000205	0.00016	0.000138	9.57E-05	7.44E-05	7.25E-05	6.14E-05	4.80E-05	3.34E-05	2.80E-05
27	51.77031	9.947146	4.124087	3.128462	2.293734	1.652379	1.274223	0.961601	0.766165	0.691259	0.644022	0.458558	0.328138	0.261159	0.183966	0.163015	0.121464	0.000188	0.000139	0.000109	4.63E-05	3.92E-05	3.65E-05	2.66E-05	1.39E-05	1.19E-05	6.69E-06	4.87E-06
28	11.48846	7.517637	2.414608	1.693401	0.988632	0.897101	0.741539	0.62984	0.441985	0.338956	0.270945	0.178379	0.118481	0.105226	0.066807	0.05508	0.052654	7.76E-05	5.65E-05	3.46E-05	2.40E-05	1.97E-05	1.69E-05	1.25E-05	9.30E-06	6.82E-06	5.48E-06	3.66E-06

the regression matrix,actual matrix : 0.0075.

The regression matrix is as below for number of constraints = 11:

$$Y_{dep} = \overset{\text{Regression Matrix}}{\left[\begin{array}{rrrrrrrrrrrrrrrrrr}
-0.99939 & -1.00039 & -0.00048 & 0.999075 & 0.000417 & 0.998491 & 1.002202 & 0.993471 & 1.001493 & -0.99886 & 0.00175 & -3.94E-05 & -1.0006 & 1.000265 & 0.99963 & -0.99833 & -0.00046 \\
-0.00026 & 1.000012 & -0.00043 & 0.000725 & -0.9998 & 0.000133 & 0.00161 & 0.001796 & -0.00054 & -0.00159 & -0.00028 & 0.00079 & -0.00017 & -0.00018 & -3.65E-06 & -0.00079 & 2.87E-05 \\
0.001126 & 0.00057 & -0.00264 & 1.000187 & -0.9988 & 0.998763 & 1.000228 & 0.992747 & 1.001785 & -0.99684 & 0.000954 & -0.00057 & -1.00165 & 0.999145 & 0.999178 & -0.99712 & -0.00115 \\
-1.0001 & -1.00127 & 1.000853 & 0.998658 & 0.001395 & 1.000111 & 1.001056 & 0.992472 & 1.001308 & -0.99917 & -6.08E-05 & -0.0015 & -1.00221 & 1.000036 & 0.99734 & -0.99525 & -0.00048 \\
-0.00019 & 5.84E-05 & -0.00367 & -0.00023 & 0.001855 & 1.000663 & -0.00018 & -0.00443 & 1.002374 & -0.99852 & -0.99842 & -0.99876 & -0.0006 & -0.0032 & 0.000255 & 0.002095 & -0.00241 \\
6.28E-05 & 0.001339 & -0.00073 & -0.00023 & 0.000813 & 0.000226 & -0.00123 & -0.00268 & 0.00049 & 0.000675 & 0.999512 & -0.00107 & -0.00063 & -0.00033 & 1.000965 & 0.000795 & 0.998985 \\
0.999607 & 0.001441 & -1.00076 & -1.00089 & 0.001379 & -1.40E-05 & -1.00172 & -0.99968 & 0.000917 & 0.00208 & 0.000569 & -0.99894 & 1.000959 & -1.00034 & 0.001675 & 0.998495 & 0.99708 \\
-0.00023 & 0.0015 & 1.000944 & 1.001051 & -0.00022 & 0.000148 & 0.001319 & 0.001236 & 0.000197 & -0.00307 & -0.00389 & -0.00112 & -0.00065 & -0.00203 & -0.00121 & 0.000584 & 0.001363 \\
0.001107 & 0.001319 & -0.00111 & -0.00069 & 0.000338 & -0.00164 & -0.00032 & -0.00337 & 1.000961 & -0.99915 & -0.00134 & -0.00178 & -1.00103 & 0.999602 & 0.99948 & -0.99822 & 0.000984 \\
0.00057 & 0.002088 & -0.00071 & -0.00072 & 0.000655 & -0.00048 & -0.0005 & -0.00029 & 0.000676 & -0.99916 & 7.22E-05 & -0.00154 & -2.37E-05 & 0.000459 & 0.998389 & -0.99903 & -0.00115 \\
-0.00024 & -0.00184 & 1.002741 & 1.000756 & -0.00139 & 0.000606 & 1.001282 & -0.00153 & 0.000196 & -0.00078 & -0.00244 & 0.000479 & -1.00066 & -0.00245 & -0.0005 & 0.001613 & 0.002033
\end{array}\right]} Y_{ind}$$

1.2.5 Part B using IPCA

It is to be carefully noted that the check to see if the data point was useful before mean imputing wasn't performed hence the chances that we coud fing the number of constraints using IPCA method are dull. So going by the algorithm that was mentioned in the first question to find the number of constraints ,number of constraints 13,12,11 are close (10,9 aren't because the number of last eigenvalues tending to 1 are greater than the number of constraints). But the algorithm manages to predicts the number of constraints as 11.
But the number of constraints have been taken as 11. The absolute difference between regression matrix and the actual value is 0.2769 .

Table 5: IPCA of mean Imputed Data

No of constraints																												
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	851.9099	289.4148	167.531	50.25814	26.07101	17.37399	14.71584	5.55346	1.874482	1.736837	1.546786	1.156356	1.074578	1.019061	0.823978	0.790501	0.692641	0.463298	0.349296	0.208237	0.177785	0.125204	0.115849	0.08065	0.063765	0.033862	0.016073	0.004169
8	1036.185	311.7209	138.0764	65.64787	40.64329	36.56067	32.41427	10.39456	9.089825	4.618141	4.147707	3.167232	2.233108	1.633509	1.322649	1.038096	0.961675	0.930168	0.888746	0.823854	0.74036	0.602863	0.558909	0.495906	0.460572	0.445667	0.328438	0.201781
9	3130.721	780.8898	148.7507	97.51678	91.82129	45.69925	40.00026	19.52326	15.05942	12.99024	8.851532	6.649867	5.735785	4.404912	3.959234	2.919508	2.399451	1.692772	1.216102	1.033088	1.029437	1.006878	1.000688	0.994096	0.967809	0.9615	0.946841	0.865077
10	1148.91	394.6596	113.9813	90.12675	69.88816	32.50784	28.75073	14.6404	12.98716	10.01813	7.489861	6.20804	4.474597	3.931084	3.3204	2.47451	2.069836	1.327529	1.057318	1.050368	1.044057	1.024619	1.005542	0.995063	0.987574	0.972257	0.94839	0.914812
11	1115.781	546.8784	269.7344	92.81511	74.55805	41.65762	32.06124	14.58007	12.64875	9.748881	7.696088	5.108277	4.269845	3.870805	3.385373	2.328226	2.063881	1.147823	1.054152	1.049459	1.040288	1.018813	1.000193	0.978166	0.970173	0.947561	0.919306	0.867421
12	1335.036	401.6982	106.1128	88.59918	65.10343	38.64774	31.36019	14.98132	11.90874	10.45888	7.332157	5.245847	4.290726	3.436205	3.046434	2.181648	1.27299	1.155053	1.067039	1.03898	1.030126	1.023388	0.976163	0.961615	0.944271	0.934575	0.844691	0.751108
13	5632.371	423.4075	188.725	112.3126	71.09192	33.20058	26.03324	15.5794	12.14455	8.541466	5.206062	4.430153	3.602316	3.35649	2.960475	1.406416	1.289954	1.131682	1.080351	1.041789	1.012883	0.98992	0.974927	0.952731	0.859656	0.826821	0.743537	0.689332
14	2763.362	445.5539	351.2791	106.5868	73.16054	44.7095	22.02503	17.70273	10.19939	9.687165	6.355602	3.901077	3.668939	2.670865	1.656552	1.315126	1.228995	1.107415	1.04385	1.01423	0.99318	0.963833	0.947079	0.899683	0.807355	0.71045	0.629335	0.563273
15	1312.585	433.0495	208.7512	110.1067	70.27222	45.57607	22.02643	14.78854	10.29907	8.510655	6.361259	3.778894	3.153141	1.72719	1.581511	1.217111	1.165302	1.085576	1.055399	0.998871	0.976546	0.918964	0.898187	0.839028	0.727474	0.628719	0.554538	0.507042
16	1041.728	417.0327	103.714	89.01598	59.02796	29.07346	19.87413	13.87322	10.5672	6.845209	3.851298	3.295955	1.738127	1.652751	1.472849	1.220471	1.081959	1.057649	1.042102	0.989397	0.964125	0.930454	0.878186	0.856059	0.633668	0.595582	0.473871	0.412749
17	4077.028	473.2482	241.6901	105.7065	77.96515	38.99838	21.13905	14.52745	8.531241	6.34915	4.003353	1.992028	1.657284	1.551709	1.303387	1.143131	1.091024	1.037552	1.000277	0.982983	0.93433	0.919712	0.867429	0.629577	0.556298	0.504464	0.427439	0.390383
18	1653.109	388.2548	194.1349	117.8657	37.40735	23.8358	16.02605	7.68969	6.617751	4.170553	2.051956	1.729481	1.648862	1.52939	1.225594	1.123629	1.046969	0.994991	0.941577	0.882743	0.849923	0.747304	0.647911	0.571592	0.523333	0.421807	0.390908	0.24695
19	801.2864	326.6644	78.93762	58.48169	21.74064	19.55603	12.6274	7.102769	6.003156	2.411401	1.903955	1.610217	1.567227	1.426297	1.091596	1.062146	0.928185	0.88925	0.866545	0.757425	0.719112	0.642162	0.573914	0.507184	0.414948	0.397327	0.357775	0.196058
20	487.9755	281.6422	78.52785	27.73816	19.07311	14.97649	12.5962	6.389011	2.908574	2.225809	1.704852	1.524433	1.415692	1.335697	1.033102	1.00091	0.953743	0.87042	0.78153	0.635764	0.545968	0.519767	0.420315	0.411578	0.367347	0.319137	0.255374	0.190973
21	2386.304	436.1282	39.01124	26.06956	17.15194	14.03761	9.798487	2.901487	2.762471	1.960903	1.609904	1.357137	1.254781	1.233275	1.026681	0.927776	0.887804	0.71791	0.647022	0.58777	0.498885	0.392485	0.344169	0.342912	0.293089	0.252455	0.216531	0.181427
22	1289.716	217.2823	35.66574	29.46933	15.01055	13.079	3.238456	2.618893	2.502569	1.812764	1.351503	1.243908	1.173624	1.108146	0.941943	0.840484	0.804763	0.65156	0.575802	0.492035	0.376724	0.348013	0.305602	0.268337	0.245498	0.212934	0.179815	0.156949
23	823.0368	257.32	41.97689	25.36567	14.71125	3.73471	2.67672	2.547864	2.307892	1.409021	1.213653	1.18237	1.114844	0.997741	0.906293	0.782397	0.674437	0.610938	0.514863	0.485935	0.353911	0.307892	0.256123	0.231521	0.191912	0.166309	0.137498	0.049529
24	435.341	160.0545	28.75226	17.90296	4.452646	3.629525	2.472089	2.296336	1.77343	1.223225	1.133245	1.043144	0.806966	0.754006	0.675176	0.609878	0.538417	0.417832	0.36568	0.310135	0.235216	0.220364	0.210599	0.18902	0.162116	0.132265	0.111834	0.047324
25	462.5365	61.13895	29.07642	5.153502	3.599956	3.178427	2.227132	1.956523	1.424582	1.045556	0.994408	0.850679	0.714246	0.589421	0.561619	0.538915	0.452258	0.337755	0.247659	0.226593	0.168532	0.150465	0.141781	0.120478	0.103577	0.094785	0.072143	0.04901
26	85.75076	49.71167	5.244193	4.427846	2.99032	2.689303	1.996644	1.587679	1.058414	0.934536	0.821261	0.716943	0.589785	0.478823	0.416912	0.410423	0.334203	0.281623	0.181079	0.151361	0.137001	0.123211	0.108123	0.087918	0.08182	0.06831	0.048196	0.034071
27	46.29162	9.519655	3.656749	2.941242	2.213854	1.650934	1.230514	0.987799	0.822781	0.735641	0.688749	0.593225	0.376191	0.315991	0.303481	0.217365	0.200118	0.118893	0.110147	0.067843	0.051845	0.049365	0.039192	0.03233	0.024217	0.020407	0.01752	0.013952
28	11.12544	7.313205	2.170083	1.612433	0.99292	0.930618	0.836939	0.681042	0.545189	0.402179	0.296697	0.242321	0.154006	0.136036	0.107363	0.09032	0.0834	0.079334	0.036886	0.031353	0.026436	0.022011	0.01878	0.018459	0.014332	0.012984	0.011964	0.007267

Given below is the regression matrix :

$$Y_{dep} = \begin{array}{c}\text{Regression Matrix}\\ \begin{bmatrix}
-1.00562 & -1.0395 & -7.91E-03 & 0.877425 & 4.92E-02 & 1.034431 & 0.817321 & 1.135959 & 1.079853 & -1.08107 & 1.14E-01 & 5.76E-02 & -1.02211 & 0.915512 & 0.979285 & -0.97337 & -7.26E-02 \\
2.52E-02 & 1.096929 & -1.79E-01 & 1.02E-02 & -0.97586 & 9.10E-03 & -6.74E-02 & -4.81E-02 & -2.13E-02 & 8.39E-02 & -1.48E-02 & -1.14E-01 & -5.10E-03 & 4.35E-02 & 1.83E-02 & -4.58E-02 & -3.38E-02 \\
5.03E-03 & 4.92E-02 & -8.46E-02 & 0.953742 & -1.01486 & 1.029494 & 0.919275 & 1.16183 & 1.072066 & -1.07957 & -6.26E-02 & 1.39E-01 & -0.97272 & 0.72312 & 1.010497 & -1.015 & -2.39E-02 \\
-0.98443 & -0.86139 & 0.957057 & 0.854473 & -7.62E-04 & 0.998896 & 0.862396 & 1.107531 & 1.105991 & -1.12864 & 1.39E-02 & 1.31E-01 & -0.95813 & 0.789256 & 0.952935 & -1.11616 & -4.27E-02 \\
2.90E-02 & -2.04E-02 & -1.33E-02 & -6.89E-04 & -2.62E-02 & 1.024132 & 4.81E-02 & 1.85E-01 & 1.090631 & -1.11072 & -1.05461 & -0.92866 & 7.24E-02 & -9.47E-02 & -4.30E-02 & 2.91E-02 & -9.83E-02 \\
-1.58E-02 & -7.25E-02 & -9.81E-03 & 2.00E-02 & -7.50E-03 & -8.00E-03 & -2.50E-02 & -6.82E-02 & -4.93E-02 & 3.84E-02 & 1.090172 & -4.60E-02 & -3.91E-02 & 1.35E-01 & 1.040864 & 6.96E-02 & 1.039606 \\
1.047546 & -6.76E-02 & -1.05954 & -0.8718 & -2.90E-02 & -2.77E-02 & -0.82393 & -0.98223 & -2.86E-02 & -5.55E-04 & -3.58E-02 & -1.12336 & 1.015684 & -0.86646 & -7.52E-05 & 1.172937 & 0.99281 \\
0.017794 & -0.01387 & 1.065882 & 0.967703 & -0.01641 & -0.00049 & 0.001238 & -0.00314 & -0.00484 & -0.0223 & 0.035467 & -0.00834 & -0.00179 & -0.00348 & -0.01974 & -0.00396 & -0.00726 \\
2.16E-02 & 6.65E-02 & -3.15E-02 & -3.69E-02 & -2.34E-02 & 1.69E-03 & 2.13E-02 & 8.30E-02 & 1.067502 & -1.08125 & -0.03937 & 8.33E-02 & -0.97625 & 0.823722 & 0.953005 & -1.0311 & -5.79E-03 \\
2.01E-02 & -8.61E-03 & -7.62E-02 & 2.21E-02 & -8.20E-03 & 7.00E-03 & 1.48E-02 & 2.61E-02 & 4.52E-02 & -1.0599 & -0.0266 & 1.11E-02 & 7.23E-03 & -4.32E-02 & 1.002119 & -0.96648 & -4.39E-03 \\
0.023401 & 0.034195 & 0.977764 & 0.934223 & 0.056294 & 0.001097 & 0.935552 & -0.02909 & 0.018825 & 0.03599 & 0.000621 & -0.06421 & -1.05289 & -0.03359 & -0.03547 & 0.051333 & -0.02093
\end{bmatrix}\end{array} Y_{ind}$$

1.2.6 Part C IPCA

Following the same explanation as the previous subsection,it can be seen that the no of constraints = 11, the results are as follows.

Table 6: IPCA of PCA imputed data

No of constraints																												
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	1498158	81932.31	36812.55	24190.65	7802.63	3590.324	2224.25	1397.858	565.9345	388.1862	100.6815	58.00265	26.96667	16.47852	9.54058	7.00103	2.459826	1.31141	0.861545	0.057622	0.040531	0.008245	0.003356	0.00275	0.001507	0.001085	0.001046	0.000838
8	1523623	797055.7	141301.7	73017.04	48024.55	26922.36	18530.69	15220.03	11689.27	7145.506	5723.52	4801.319	3290.482	3130.335	2182.198	1817.518	1731.501	1.567003	1.348064	1.18269	1.022083	1.005504	1.000754	1.000139	0.994052	0.977637	0.964883	0.69465
9	1606339	1005978	166158.8	79245.19	45452.17	32993.65	20100.67	13831.54	10675.69	8441.697	5995.844	3947.332	3064.043	2585.063	1870.161	1593.424	1373.526	1.595051	1.271623	1.045897	1.032774	1.015698	1.00001	0.997292	0.980926	0.976414	0.955036	0.871508
10	1620962	701811	125470.1	67391.05	48715.29	28339.32	15759.59	14126.52	9212.957	6374.182	4309.535	3664.743	2875.796	2283.093	1713.426	1692.578	1311.712	1.284766	1.084158	1.067325	1.040058	1.023843	1.003913	0.990915	0.976595	0.951651	0.940212	0.921329
11	1455217	681194.9	123755.1	66282.02	45742.49	28189.06	15624.62	14233.98	8886.013	6049.956	4213.553	3644.859	2852.395	2152.239	1675.191	1596.741	1317.274	1.132421	1.094024	1.049942	1.033821	1.016285	0.985902	0.969158	0.962511	0.941675	0.921915	0.892347
12	3231674	651059.3	209816	66643	53058.58	26179.34	21299.68	12262.09	9725.041	6557.052	4299.993	3220.922	2553.741	1859.043	1713.653	1177.953	2.988367	1.082504	1.030723	0.995606	0.965545	0.930876	0.912038	0.839416	0.790587	0.562176	0.005366	0.003648
13	1322805	772062.3	119709.2	99645.77	39750.2	26701.72	9973.8	7174.898	5896.134	2152.993	1923.358	1009.146	30.44817	8.356767	5.189567	2.352808	1.928493	1.064627	1.011149	0.99507	0.95292	0.823546	0.728286	0.63388	0.00301	0.002354	0.0015	0.001134
14	2025671	1200745	174802	106708.6	59674.25	40655.33	16609.23	6482.287	4485.453	3885.822	1856.779	1162.98	13.776	2.96813	2.191837	1.775086	1.323488	1.067666	1.058307	0.988139	0.955249	0.910717	0.78043	0.689698	0.001691	0.001221	0.001007	0.000673
15	1284844	732706.8	111086.2	55139.98	34501.59	26777.16	9567.065	6187.72	3621.1	2014.801	1484.63	487.2086	13.84509	2.347312	2.080881	1.615435	1.200125	1.069972	1.011448	0.996927	0.955293	0.914717	0.476244	0.345663	0.001684	0.000965	0.000919	0.000634
16	4302039	681998.2	81146.1	45965.38	38821.13	25382.52	13561.07	3411.201	2482.037	1671.457	28.53974	11.61638	2.364458	2.084874	1.948439	1.313053	1.094463	1.036133	1.003706	0.991277	0.947793	0.889155	0.287737	0.002036	0.00124	0.000922	0.000692	0.000585
17	1028680	465189.7	54194.27	39303.56	22695.69	10739.98	3969.646	1890.06	27.22424	13.80205	11.84232	2.859956	2.064137	1.924119	1.341208	1.182757	1.003234	0.977699	0.950644	0.74308	0.717779	0.249516	0.001335	0.00113	0.000965	0.000869	0.000605	0.000547
18	1070915	361817.4	39420.41	31336.85	24966.68	14540.44	3224.434	2850.486	39.08287	15.99706	3.341389	2.33656	2.001197	1.76422	1.169592	1.101666	0.972944	0.968179	0.775366	0.675347	0.644522	0.3028	0.001179	0.000855	0.000763	0.00054	0.00046	0.000368
19	962836.3	478507.4	157351.6	55892.78	23517.69	16249.19	4959.823	3058.315	8.394304	3.646439	2.511697	1.861075	1.65509	1.504181	1.044433	0.985252	0.956113	0.921698	0.7597	0.433422	0.420834	0.304006	0.000765	0.000572	0.000487	0.000368	0.000309	0.000208
20	826056.4	285560.3	34089.81	22591.59	14337.78	6494.236	2974.17	2073.683	3.533212	3.273575	2.252172	1.560457	1.36801	1.21427	1.003242	0.958843	0.80636	0.666572	0.453443	0.447473	0.37721	0.26046	0.000613	0.000476	0.000356	0.000275	0.000239	0.000146
21	873909.1	295918.7	32788.09	17624.22	11323.64	2703.62	28.14396	3.642368	3.338908	2.821028	1.89863	1.38944	1.222317	1.090335	0.97389	0.787425	0.744692	0.672134	0.521139	0.494342	0.361149	0.000521	0.000405	0.000311	0.000258	0.000236	0.000164	0.000149
22	1866312	302081.1	25376.17	16014.73	39.31514	11.98666	4.149366	3.509612	2.826056	2.090418	1.564336	1.242749	1.136877	1.080908	0.901007	0.725613	0.513796	0.458775	0.349259	0.001227	0.000659	0.000375	0.000352	0.000333	0.000247	0.000231	0.000159	0.00013
23	795115.6	131856.2	18421.59	12137.41	86.41757	4.134886	3.553845	3.250686	2.564049	1.473126	1.335341	1.181179	1.073397	0.941305	0.83075	0.653527	0.441053	0.343586	0.277535	0.000728	0.000383	0.000316	0.000246	0.000237	0.000214	0.000147	0.000141	0.000116
24	12760.65	277.4579	69.02471	19.41147	5.268418	4.385631	2.826608	2.663919	1.936223	1.244545	1.205074	1.018149	0.80222	0.676741	0.573186	0.360945	0.32581	0.000965	0.000397	0.000322	0.000289	0.000226	0.000207	0.000168	0.000142	0.000111	7.55E-05	6.31E-05
25	1968.404	90.29408	27.35688	5.83298	3.994099	3.783169	2.430934	2.078734	1.448909	1.104167	0.986063	0.790783	0.597691	0.469811	0.42201	0.315875	0.260802	0.000982	0.00024	0.000196	0.000181	0.000161	0.000117	0.000111	0.000101	6.98E-05	6.23E-05	3.65E-05
26	13224.22	61.14282	5.683165	4.74247	3.630771	3.114952	1.976499	1.6863	1.09705	0.963956	0.841167	0.608406	0.514766	0.354884	0.287332	0.257938	0.210868	0.001164	0.000203	0.000158	0.000135	9.50E-05	7.73E-05	6.99E-05	5.98E-05	5.17E-05	3.46E-05	2.72E-05
27	49.17652	9.942433	4.040476	3.197576	2.277724	1.693152	1.242828	0.942926	0.794059	0.667779	0.642217	0.470344	0.335073	0.272227	0.188547	0.169354	0.122671	0.000183	0.000133	0.000108	4.59E-05	4.14E-05	3.94E-05	2.72E-05	1.34E-05	1.22E-05	6.66E-06	4.82E-06
28	11.29687	7.501181	2.400585	1.766877	1.019596	0.890189	0.738918	0.659065	0.478172	0.348649	0.284901	0.187785	0.127775	0.114467	0.073321	0.05638	0.054991	8.12E-05	5.95E-05	3.57E-05	2.52E-05	2.05E-05	1.74E-05	1.21E-05	9.33E-06	6.90E-06	5.38E-06	3.59E-06

The absolute difference between regression matrix and the actual value is 0.0060 .

Given below is the regression matrix :

Regression Matrix

$$Y_{dep} = \begin{bmatrix}
-0.99841 & -0.99925 & -0.00314 & 1.000942 & -0.00071 & 0.998212 & 1.002066 & 0.995461 & 1.001512 & -1.00028 & 0.001426 & -9.51E-05 & -1.00059 & 0.997601 & 0.999828 & -0.99846 & 0.001339 \\
-0.00032 & 0.99978 & -1.30E-04 & 0.000325 & -1.00014 & 0.000418 & 0.001771 & 0.000631 & -0.0008 & -0.00213 & -0.00105 & 0.000597 & -7.07E-05 & 0.000842 & -0.00071 & -9.85E-05 & 0.000456 \\
0.001568 & 0.000743 & -0.00407 & 1.001331 & -0.99984 & 0.999187 & 1.000318 & 0.994947 & 1.001036 & -0.99897 & 2.74E-06 & -0.00121 & -1.00176 & 0.998951 & 0.998781 & -0.99656 & 3.43E-04 \\
-0.99954 & -1.00203 & 0.999682 & 1.00067 & 0.000835 & 0.999663 & 1.002329 & 0.994009 & 1.001104 & -0.99957 & 0.000621 & -0.00127 & -1.00219 & 0.998009 & 0.99871 & -0.99537 & 0.000439 \\
-0.00075 & -0.00036 & -0.00353 & -0.00026 & 0.001978 & 1.001568 & -0.00077 & -0.00443 & 1.001794 & -0.99797 & -0.99853 & -0.99882 & -0.00094 & -0.00316 & 1.60E-05 & 0.002457 & -0.00203 \\
0.000552 & 0.001879 & -0.00057 & -0.00016 & 0.000349 & 3.19E-05 & -0.00167 & -0.00154 & -6.59E-05 & 0.000311 & 1.000264 & -0.00146 & -0.00041 & 9.64E-05 & 1.000607 & 7.89E-05 & 0.999147 \\
0.99903 & 0.000796 & -1.00078 & -1.00185 & 0.001522 & 0.001603 & -1.00366 & -0.99896 & 1.62E-04 & 0.00262 & -1.05E-04 & -0.99905 & 1.000913 & -0.9983 & 0.00148 & 0.998925 & 0.995949 \\
-0.00023 & 0.000219 & 1.000519 & 1.001215 & -5.08E-05 & -1.56E-04 & 0.001441 & 7.97E-05 & -0.00018 & -0.00176 & -0.00223 & -0.00136 & -0.00095 & -0.00113 & -0.0005 & 0.001439 & 0.001649 \\
0.000846 & 0.001402 & -0.00148 & -0.00038 & 0.0004 & -0.00086 & -0.00062 & -0.00227 & 0.999879 & -0.99961 & -0.00104 & -0.00151 & -1.00121 & 0.999886 & 0.999716 & -0.99914 & 0.000557 \\
0.000671 & 0.002217 & -0.00112 & -0.00066 & 0.0002 & 0.000289 & -0.00057 & 0.001398 & -0.00033 & -0.9995 & -2.14E-05 & -0.00134 & -2.62E-05 & 0.000308 & 0.998448 & -0.99988 & -0.00107 \\
-5.53E-05 & -0.00198 & 1.002179 & 1.001412 & -0.00074 & -0.00041 & 1.001456 & -0.0019 & 0.001031 & -0.00118 & -0.00065 & 0.00065 & -1.00075 & -0.00372 & -1.47E-04 & 0.002118 & 0.00171
\end{bmatrix} Y_{ind}$$

1.2.7 Consolidation of Results for question 2

Table 7: Final Results table

Final Results	Auto Scale	IPCA
Part A	0.0082	0.0075
Part B	0.6853	0.2769
Part C	0.0067	0.0060

IPCA performs better than auto-scaling as expected in a general fashion for the all cases as expected.With regards to whether I can find the number of constraints for all cases , yes the program that I have implemented finds the correct number of constraints all the time.