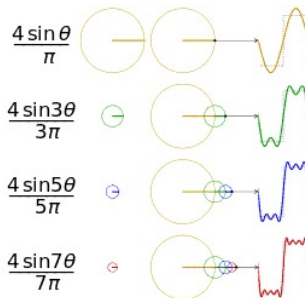


# COMS20011 – Data-Driven Computer Science

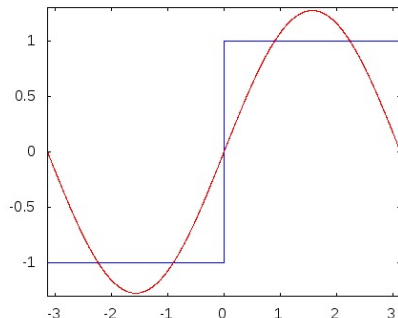


## Lecture Video MM07 – Fourier Series

March 2022

Majid Mirmehdi

# Next in DDCS



## Feature Selection and Extraction

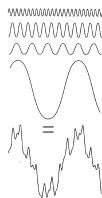
- Signal basics and **Fourier Series**
- 1D and 2D Fourier Transform
- Another look at features
- Convolutions

# Frequency Analysis



**Trigonometric Fourier Series:** Any *periodic* function can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficient. → *Jean Baptiste Joseph Fourier (1822).*

$$f(x) = \sum_{n=0}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right)$$



A function with period  $T$  is represented by two infinite sequences of coefficients.  $n$  is the no. of cycles/period.

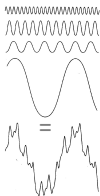
- The sines and cosines are the **Basis Functions** of this representation.  $a_n$  and  $b_n$  are the **Fourier Coefficients**.
- The sinusoids are harmonically related: each one's frequency is an integer multiple of the fundamental frequency of the input signal.

# Frequency Analysis



**Trigonometric Fourier Series:** Any periodic function can be expressed as the sum of sines and/or cosines of different frequencies, each multiplied by a different coefficient. → *Jean Baptiste Joseph Fourier (1822).*

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right)$$



A function with period  $T$  is represented by two infinite sequences of coefficients.  $n$  is the no. of cycles/period.

- The sines and cosines are the **Basis Functions** of this representation.  $a_n$  and  $b_n$  are the **Fourier Coefficients**.
- The sinusoids are harmonically related: each one's frequency is an integer multiple of the fundamental frequency of the input signal.
- $a_0$  is often referred to as the **DC term or the average of the signal**

# Fourier Series Solution

A *Fourier series* provides an equivalent representation of the function:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right)$$

The coefficients are:

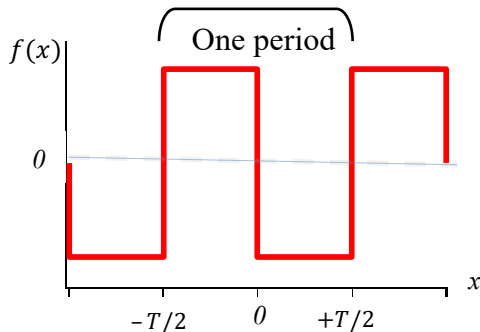
$$a_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \cos\left(\frac{2\pi nx}{T}\right) dx$$

$$b_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \sin\left(\frac{2\pi nx}{T}\right) dx$$

# Fourier Series Example: Square Wave

$f(x) \rightarrow$  a square wave

$$f(x) = \begin{cases} +1 & -\frac{T}{2} \leq x < 0 \\ -1 & 0 \leq x < \frac{T}{2} \end{cases}$$



Example periodic function on  $-T/2, +T/2$

# Fourier Series Example: Square Wave

$f(x) \rightarrow$  a square wave

$$a_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \cos(2\pi n x / T) dx$$

$$= \frac{2}{T} \int_{-T/2}^0 \cos(2\pi n x / T) dx - \frac{2}{T} \int_0^{+T/2} \cos(2\pi n x / T) dx = 0$$

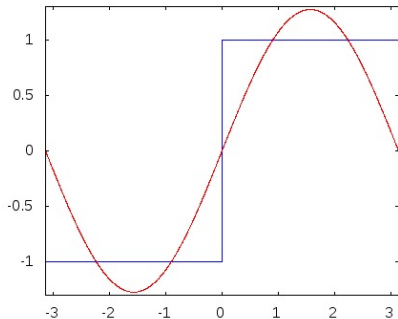
$$b_n = \frac{2}{T} \int_{-T/2}^{+T/2} f(x) \sin(2\pi n x / T) dx$$

$$= \begin{cases} \frac{4}{n\pi} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

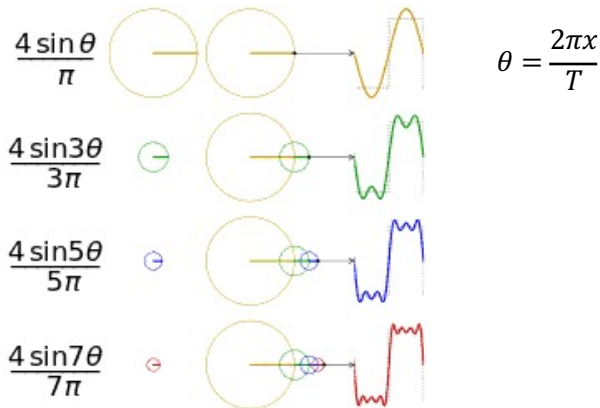
$$f(x) = \frac{4}{\pi} \cdot \sin \frac{2\pi x}{T} + \frac{4}{3\pi} \cdot \sin 3 \cdot \frac{2\pi x}{T} + \frac{4}{5\pi} \cdot \sin 5 \cdot \frac{2\pi x}{T} + \dots$$

$$f(x) = \begin{cases} +1 & -\frac{T}{2} \leq x < 0 \\ -1 & 0 \leq x < \frac{T}{2} \end{cases}$$

$n = 1, 3, 5, 7, \dots$



# Approximating the Square Wave

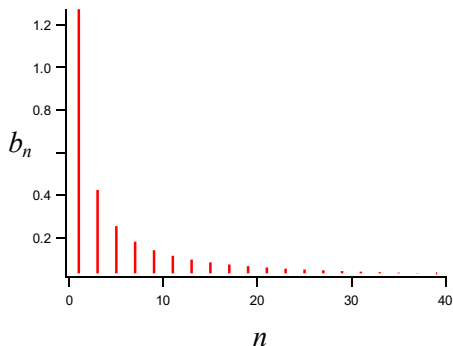


$$f(x) = \frac{4}{\pi} \cdot \sin \frac{2\pi x}{T} + \frac{4}{3\pi} \cdot \sin 3 \cdot \frac{2\pi x}{T} + \frac{4}{5\pi} \cdot \sin 5 \cdot \frac{2\pi x}{T} + \frac{4}{7\pi} \cdot \sin 7 \cdot \frac{2\pi x}{T} + \dots$$

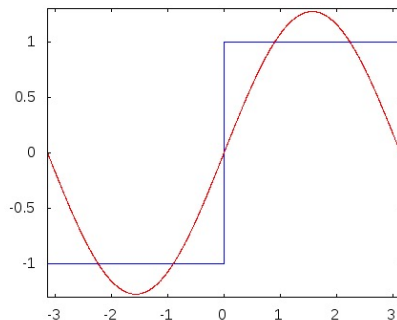


# Fourier Space/Domain for the Square Wave

- The set of *Fourier Space* coefficients  $b_n$  contain complete information about the function
- Although  $f(x)$  is periodic to infinity,  $b_n$  is negligible beyond a finite range
- Sometimes the Fourier representation is more convenient to use, or just view



## Next in DDCS



## Feature Selection and Extraction

- Signal basics and Fourier Series
- **1D and 2D Fourier Transform**
- Another look at features
- Convolutions