EE 365 Assignment #1 Solution

(**Spring 2022**)

Problem 1 Wavelengths in Radio Applications (10 points)

Radio waves propagate in free space (and in our atmosphere) at the speed of electromagnetic waves (*e.g.*, light waves) – an EM wave velocity of $v = 2.99792 \times 10^8$ meters per second. **For this problem use $v = 3.00 \times 10^8$ meters per second (m/sec).** An important wave parameter for electromagnetic waves is the wavelength λ which is inversely related to the wave frequency f (cycles per second in units of Hertz). The relationship is (as you know) velocity equals wavelength times frequency ($v = \lambda \cdot f$).

The reason wavelength λ is important is because the wavelength is approximately the spatial resolving dimension of radar and antenna sizes scale with wavelength (*e.g.*, long wavelengths requires large antennas where \the antenna will be of the order of the wavelength in size for best transmission and receiption).

To get a feel for the size of the free space wavelength λ for various radio communication systems, fill out the table below: [Express all **wavelengths in meters**.]

Radio Application	Frequency Band	Wavelength Range
AM broadcast radio	535 kHz to 1605 kHz	560.7 meters to 186.9 meters
FM broadcast radio	88 MHz to 108 MHz	3.409 meters to 2.778 meters
VHF Civil Aviation Band *(example)*	108 MHz to 136 MHz	2.778 meters to 2.206 meters *(example)*
GSM Cellular (Uplink)	890 MHz to 915 MHz	0.3371 meter to 0.3279 meter
Wi-Fi 802.11b/g/n	2.400 GHz to 2.497 GHz	0.1250 meter to 0.1201 meter
K-band Radar Sensor	24.125 GHz (narrowband)	0.01244 meter

Wi-Fi 802.11ad	60 GHz	find

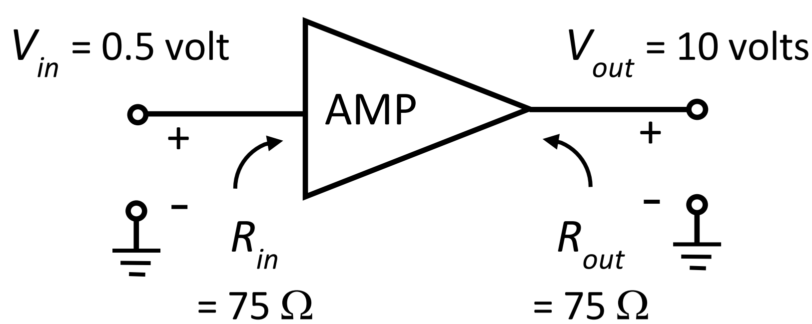
Problem 2 Height of an AM Braodcast Antenna (10 points)

In problem 1 you found that radio wavelengths cover a very broad span of values. For example, in broadcast AM radio (which has been around since the 1920s), the wavelengths covering its band are very large. The photo of a AM broadcast antenna is designed to be one-quarter of a wavelength ($\lambda/4$). Pakistan Broadcasting Corporation (PBC)'s Radio station in Rawalpindi broadcasts at frequency f = 1152 kHz. Given tht its broadcast antenna is a quarter wavelength, what is the height of PBC's antenna expressed in feet? (Note that 1 meter = 3.2808 feet).

For a radio station broadcasting at f = 1152 kHz the wavelength lambda= 260.24 meters = 853.79 feet (because 1 meter = 3.2808 feet). Therefore, a quarter wavelength lambda/4 = 213.44 feet high.

Problem 3 Voltage Gain & Power Gain (10 points)

Electrical engineers often specify, or characterize, circuit blocks and/or networks in terms of voltage gain and power gain. Voltage and power gains can be expressed either numerically or in decibels (<u>see Handout #1 for a discussion of decibels</u>). In this problem you are presented with the amplifier circuit shown diagrammatically shown below with input and output resistances and voltages levels as labeled.



Assume the amplifier is impedance matched at output and input. Calculate:

(a) The voltage gain ratio.

$$\frac{V_{out}}{V_{in}} = \frac{10\,(\text{volts})}{0.5\,(\text{volt})} = 20$$

(b) The voltage gain expressed in decibles (dB).

$$20 \cdot \log_{10}\left(\frac{V_{out}}{V_{in}}\right) = 20 \cdot \log_{10}(20) = 26\text{ dB}$$

(c) The power gain ratio.

$$P_{in} = \frac{(0.5\,V)^2}{75\,\Omega} = 3.33 \times 10^{-3}\text{ W} \quad \text{and} \quad P_{out} = \frac{(10\,V)^2}{75\,\Omega} = 1.333\text{ W}; \quad \frac{P_{out}}{P_{in}} = 400$$

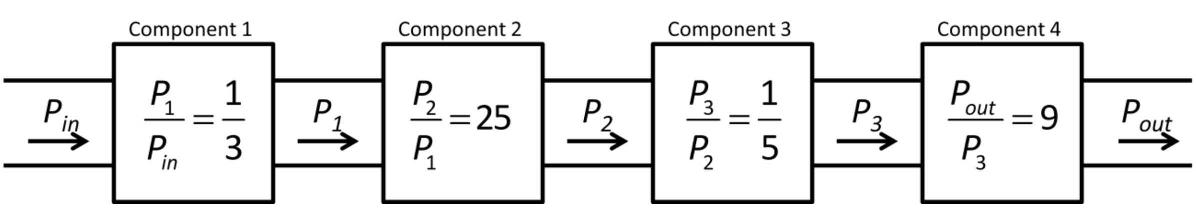
(d) The power gain ratio expressed in decibels (dB).

$$10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10 \cdot \log_{10}(400) = 26\text{ dB}$$

Added note to the curious student: Cable line amplifiers (CATV) in cable television distribution systems typically use 75 ohm coaxial cable (rather than 50 ohm coaxial cables) because a 77 ohm coxial cable provides the lowest poss per length of line and 75 ohm cable is therefore lower loss than a 50 ohm cable. The highest peak power carrying capability in a coxial cable is a cable with a 30 ohm characterisitic impedance. Thus, a 50 ohm coxial cable is a compromise between 30 ohm and 77 ohm characteristic impedances. That is why 50 ohm coaxial cables are so widely used.

Problem 4 Voltage Gain & Power Gain continued (10 points)

We have four circuit components cascaded together as shown in the block diagram below.



(a) Express the gains and losses in decibels (use the second column in the table below):

Numerical power ratio	Power ration in dB
P_1 / P_{in} = 1/3	-4.77 dB
P_2 / P_1 = 25	+13.98 dB
P_3 / P_2 = 1/5	-6.99 dB
P_{out} / P_3 = 9	+9.54 dB

(b) What is ratio of (P_{out}/P_{in}) (both numerically and in decibels)?

$$\frac{P_{out}}{P_{in}} = \left(\frac{1}{3}\right)(25)\left(\frac{1}{5}\right)(9) = 3 \cdot 5 = 15$$

$$10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 10 \cdot \log_{10}(15) = 11.76\ \text{dB}$$

or -4.77 dB + 13.98 dB -6.99 dB +9.54 dB = 11.76 dB

(c) If P_{in} = 30 mW, what is P_{out} in watts (W)?

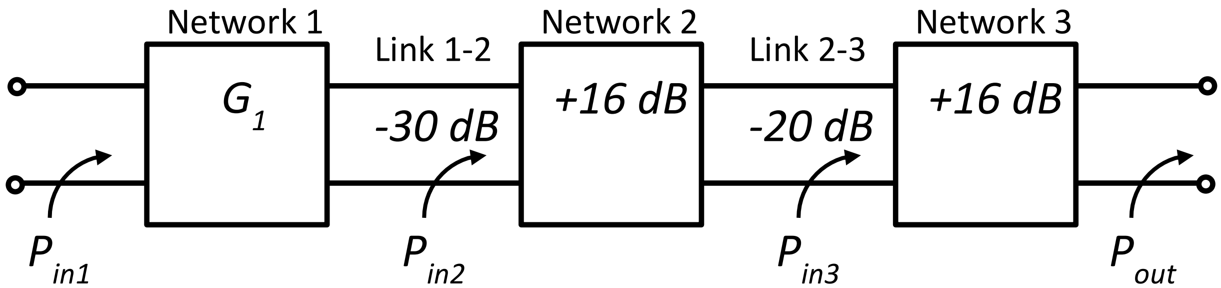
P_{out} = 15 × P_{in} = 15 × 30 mW = 450 mW = 0.45 W

$$5\ \text{dB} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) \Rightarrow 10^{\frac{5}{10}} = 10^{\log_{10}(P_{out}/P_{in})} \quad \text{so} \quad 3.162 = \frac{P_{out}}{500\ \text{mW}}$$

$$P_{out} = 3.162 \times 500\ \text{mW} = 1.58\ \text{W}$$

Problem 5 Communication Link: Gain & Loss (10 points)

We have three circuit components cascaded together as shown on the block diagram below. The links (Link 1-2 and Link 2-3) are long stretches of transmission lines between the networks. Link 1-2 has a loss of 30 dB and Link 2-3 has a loss of 20 dB. We don't know the power gain G_1 of Network 1, but we are told that with an input power of P_{in1} = 500 mW fed into Network 1, the power flowing into the input of Network 2 is P_{in2} = 100 mW.



Calculate the following quantities:

(a) The output power (in milliwatts) from Network 1.

$$P_{in2} = P_{out1} \times L_{Loss\ of\ Link\ 1-2} = P_{out1} \times (0.001) = 100\ \text{mW}$$

because a -30 dB loss is numerically 1/1000 = 0.001

Thus, $P_{out1} = 1000 \times 100\ \text{mW} = 100\ \text{W}$ =100,000 mW

(b) The power gain (in decibels) of Network 1 (power gain denoted by G_1).

$$P_{out1} = P_{in1} \times G_1 = 500\ \text{mW} \times G_1 = 100\ \text{W}; \quad G_1 = \frac{100\ \text{W}}{0.5\ \text{W}} = 200$$

$$G_1 = 10 \cdot \log_{10}(200) = 23\ \text{dB}$$

(c) The overall power gain, or power loss, of the entire chain (*i.e.*, calculate P_{out}/P_{in1}). Express power gain in decibels.

$$G_{overall} = G_1 - Loss_{1-2} + G_2 - Loss_{2-3} + G_3 \quad \text{(dB)} = 23 - 30 + 16 - 20 + 16\ \text{dB} = +5\ \text{dB}$$

$$10 \times \log_{10}(G_{overall}) = 5 \quad \Rightarrow \quad G_{overall} = 3.162$$

Note: This example illustrates the advantage of using gain and loss in decibels.

(d) The output power P_{out} in watts.

$$P_{out} = G_{overall} \times P_{in1} = (3.162) \times 500\ \text{mW} = 1.581\ \text{mW} = 1.581\ \text{W}$$

Problem 5a (5 points)
(This is problem 1.11 in Abgo and Sadiku – page 14)

Evaluate the bandwidth of a channel with capacity 36,000 bits/sec and a signal-to-noise ratio of 30 dB.

$$\left(\frac{S}{N}\right)_{dB} = 30\text{dB} \longrightarrow \left(\frac{S}{N}\right) = 10^3 = 1000$$

$$C = B\log_2\left(1 + \frac{S}{N}\right) \longrightarrow B = \frac{C}{\log_2 1001} = \frac{36,000}{\dfrac{\log_{10} 1001}{\log_{10} 2}} = 3.6118 \text{ kHz}$$

Problem 5b Bandwidth of a Channel (5 points)
(This is problem 1.10 in Abgo and Sadiku – page 14)

Calculate the bandwidth required of a channel capacity of 25 kbps (kilobits/sec) when the signal-to-noise ratio is numerically 500.

$$25 \text{ kbps} = \text{Blog}_2(1 + 500) \longrightarrow B - \frac{25k}{\log_2 501} = 2.78 \text{ kHz}$$