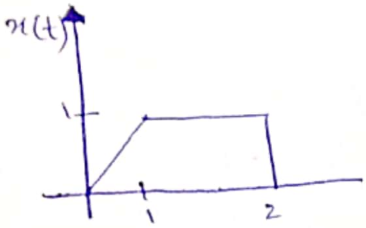
Rafay Aamir

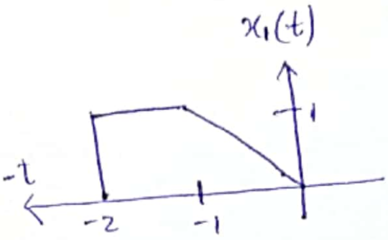
Bscee19047

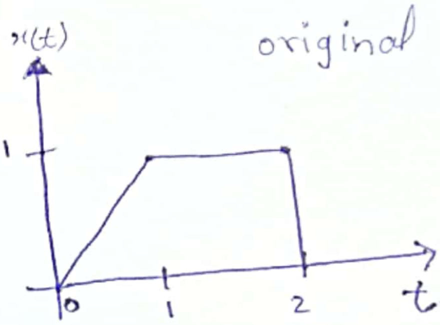
Communication sys A2

Q:-2

(a) $x_1(t) = x(-t)$







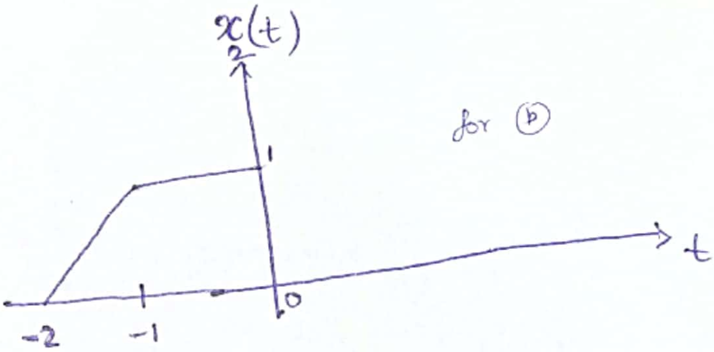
original

(b) $x_2(t) = x(2+t)$

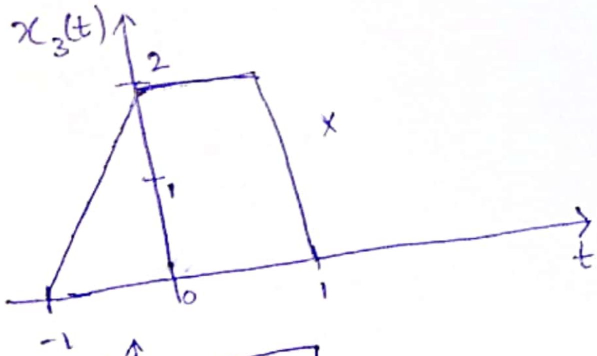
(c) $x_3(t) = 2x(t) + 1$

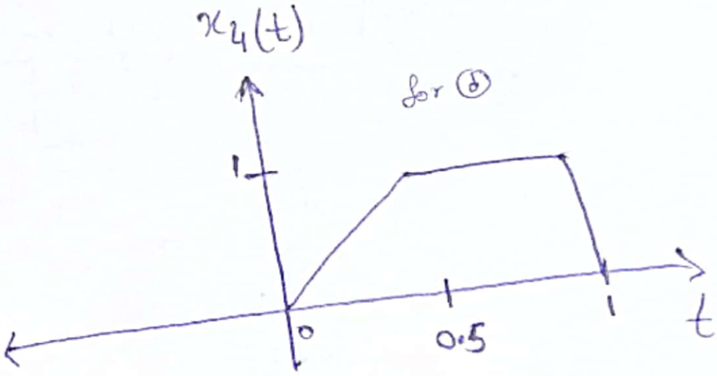
(d) $x_4(t) = x(2t)$

(e) $x_5(t) = x(t/4)$

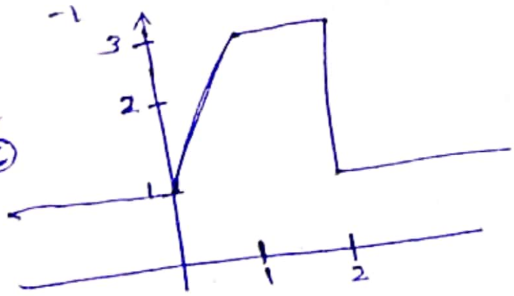


for (b)

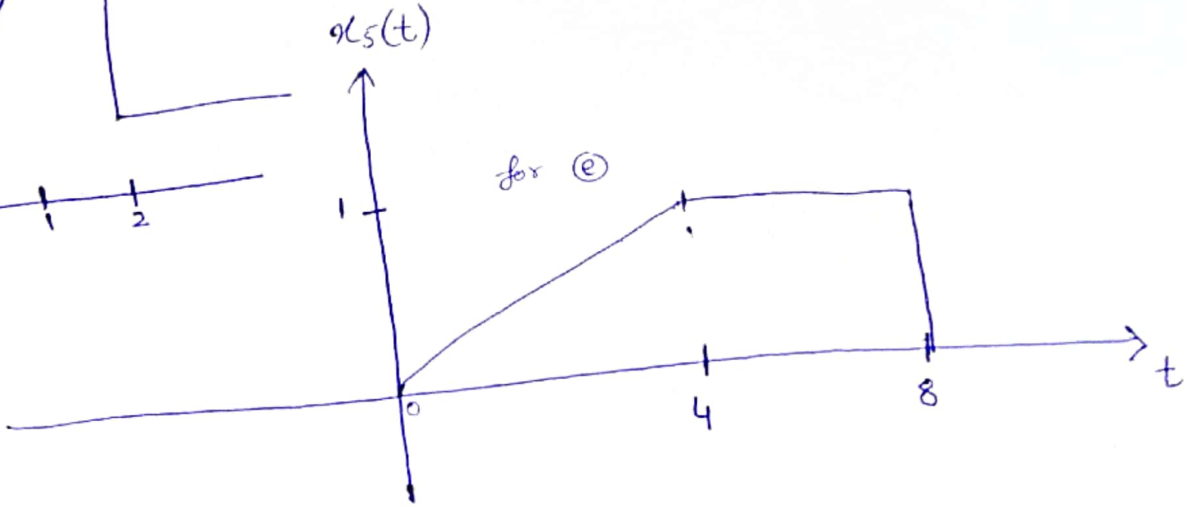




for (d)



for (c)



for (e)

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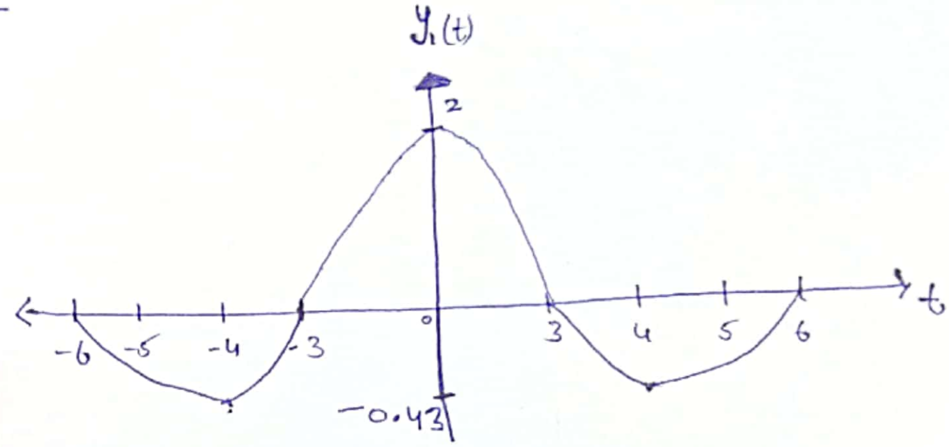
Q:-1

ⓐ $y_1(t) = 2\,\text{sinc}\left(\frac{\pi t}{3}\right)$

Solve

$$y_1(t) = 2\,\frac{\sin\frac{\pi t}{3}}{\frac{\pi t}{3}}$$

t	$y_1(t)$
0	2
± 2	0.86
± 3	0
± 4	-0.43
± 5	-0.34
± 6	0



ⓑ $y_2(t) = \Delta(t) - \Pi(t-2)$

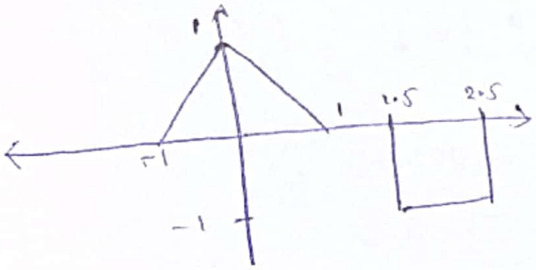
Solve

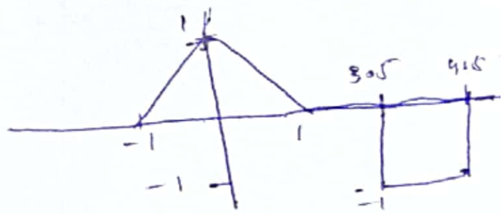
$$\Delta(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 < t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$y_2(t) = \begin{cases} 1+t & -1 \le t \le 0 \\ 1-t & 0 \le t \le 1 \\ -1 & 1.5 < t < 2.5 \end{cases}$$

$$\Pi(t) = \begin{cases} 1 & 1.5 < t < 2.5 \\ 0 & \text{everywhere} \end{cases}$$

t	$y_2(t)$
0	1
1	0
1.5	-1
2.4	-1
2.6	0
-1	0



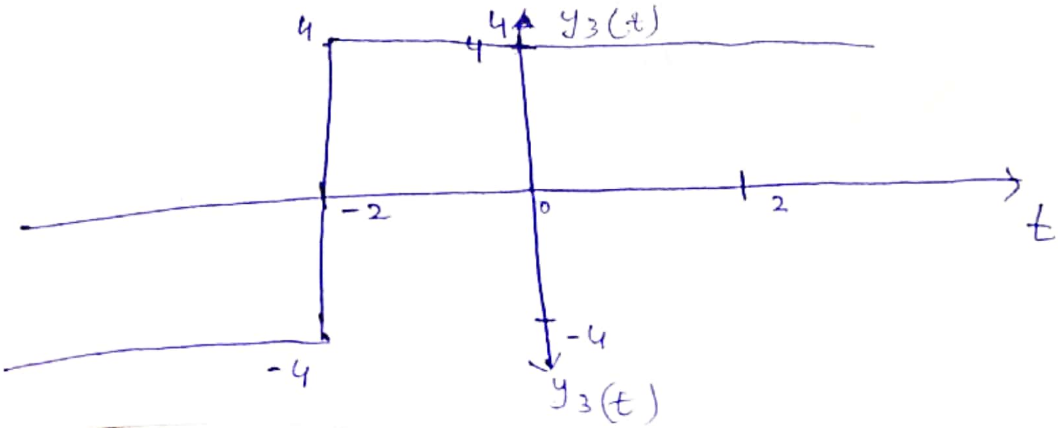


ⓒ $y_3(t) = 4\,\text{sgn}(t+2)$

$$\text{sgn}(t) = \begin{cases} -1 & -\infty \le t \le 0 \\ 0 & t = 0 \\ 1 & 0 < t < +\infty \end{cases}$$

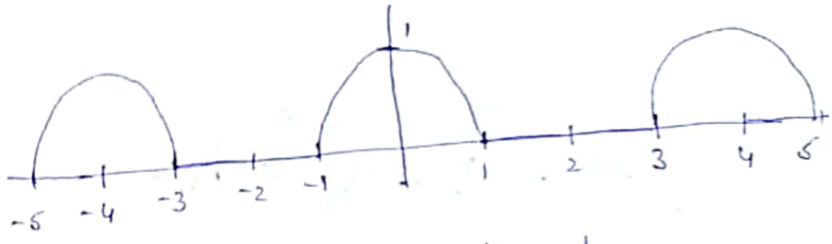
Solve

$$4\,\text{sgn}(t+2) = \begin{cases} -4 & -\infty < t < -2 \\ 0 & t = -2 \\ 4 & -2 < t < \infty \end{cases}$$



Q3 (2.14)

Solve



Time period $(T) = 4$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{2}$$

$b_n = 0$ as the function is even

$$f(t) = \begin{cases} 0 & -2 < t < -1 \\ \cos\frac{\pi t}{2} & -1 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt = \frac{1}{2}\int_0^1 \cos\frac{\pi t}{2}\,dt + \frac{1}{2}\int_1^2 0\,dt$$

$$a_0 = \frac{1}{2}\left[\frac{\sin\frac{\pi t}{2}}{\pi/2}\right]\Bigg|_0^1 = \frac{1}{2}\times\frac{2}{\pi}[1-0], \quad a_0 = \frac{1}{\pi}$$

$$\cancel{a_n = \frac{1}{2}\int_0^1 \left\{\cos\frac{\pi}{2}(n+1)t + \cos\frac{\pi}{2}\right.}$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{2}$$

$$a_n = \frac{4}{T}\int_0^{T/2} (f(t)) \cos n\omega t\,dt = \int_0^1 \cos\frac{\pi t}{2} \cos\frac{n\pi t}{2}\,dt$$

$$a_n = \frac{1}{2}\left[\int_0^1 \left(\cos\frac{\pi}{2}(n+1)t + \cos\frac{\pi t}{2}(n-1)\right) dt\right.$$

$$a_n = \frac{1}{2}\left[\frac{\sin\frac{\pi}{2}(n+1)t}{\frac{\pi(n+1)}{2}} + \frac{\sin\frac{\pi t}{2}(n-1)}{\frac{\pi(n-1)}{2}}\right]_0^1$$

$$a_n = \left[\frac{\sin\frac{\pi}{2}(n+1)}{\pi(n+1)} + \frac{\sin\frac{\pi}{2}(n-1)}{\pi(n-1)}\right]$$

$$a_1 = \frac{1}{2}$$

for $n =$ odd, $n = 3, 5, \dots$ $(n+1)$ and $(n-1)$ are both even so

$$a_{3,5,\dots} = 0$$

for $n =$ even, $n = 2, 4, 6, \dots$ $(n+1)$ and $(n-1)$ are both odd so

$$\sin\frac{\pi}{2}(n+1) = -\sin\frac{\pi}{2}(n-1) = \cos\frac{n\pi}{2} = (-1)^{n/2}$$

$$a_n = \frac{(-1)^{n/2}}{\pi(n+1)} - \frac{(-1)^{n/2}}{\pi(n-1)} = \frac{-2(-1)^{n/2}}{\pi(n^2-1)}$$

$$f(t) = a_0 + a_n^{\cos\theta} + b_n \sin\theta$$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi t}{2} - \frac{2}{\pi}\sum_{n=even}\left[\frac{(-1)^{n/2}}{(n^2-1)}\cos\frac{n\pi t}{2}\right]$$

$$f(t) = \frac{1}{\pi} + \frac{1}{2}\cos\frac{\pi t}{2} - \frac{2}{\pi}\sum_{n\to even}^{\infty}\left[\frac{(-1)^{n/2}\times\cos\frac{n\pi t}{2}}{(n^2-1)}\right]$$

Q:-4 2.19

Solve

$$f(t) = [40 - 20\sin(2\pi t + \tfrac{\pi}{6})]\cos 5\pi t$$

$$f(t) = 40\cos 5\pi t - 20\sin(2\pi t + \tfrac{\pi}{6})\cos 5\pi t$$

$$f(t) = 40\cos(5\pi t + 0) - 10(2\sin(2\pi t + \tfrac{\pi}{6})\cos 5\pi t)$$

$$f(t) = 40\cos(5\pi t + 0) - 10\left[\sin(2\pi t + \tfrac{\pi}{6} + 5\pi t) + \sin(2\pi t + \tfrac{\pi}{6} - 5\pi t)\right]$$

$$f(t) = 40\cos(5\pi t + 0) - 10\sin(7\pi t + \tfrac{\pi}{6}) - 10\sin(-3\pi t + \tfrac{\pi}{6})$$

$$f(t) = 40\cos(5\pi t + 0^\circ) - 10\cos(7\pi t + \tfrac{\pi}{6} - \tfrac{\pi}{2}) - 10\,\overset{\cos}{\cancel{\sin}}(-3\pi t + \tfrac{\pi}{6} - \tfrac{\pi}{2})$$

$$f(t) = 40\cos(5\pi t + 0^\circ) - 10\cos(7\pi t - 60^\circ) - 10\,\overset{\cos}{\cancel{\sin}}(-3\pi t - 60^\circ)$$

$$f(t) = 40\cos(5\pi t + 0^\circ) - 10\cos(7\pi t - 60^\circ) - 10\cos(-3\pi t - 60^\circ)$$

$a_1 = 40$, $\omega_1 = 5\pi$, $\theta_1 = 0^\circ$

$a_2 = -10$, $\omega_2 = 7\pi$, $\theta_2 = -60^\circ$

$a_3 = -10$, $\omega_3 = -3\pi$, $\theta_3 = -60^\circ$

$$f(t) = 40\cos(5\pi t + 0^\circ) - 10\cos(7\pi t - 60^\circ) \mp 10\cos(3\pi t + 60^\circ)$$

$a_3 = -10$, $\omega_3 = 3\pi$, $\theta_3 = 60^\circ$

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Q:-6 (2.31)

Solve

$$P(t) = \begin{cases} \cos\frac{\pi t}{\tau} & |t| < \tau/2 \text{ or } -\tau/2 < t < \frac{\tau}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$F(P(t)) = \int_{-\infty}^{\infty} P(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} \left(\cos\frac{\pi t}{\tau}\right) e^{-j\omega t}\, dt$$

$$= \int_{-\tau/2}^{\tau/2} \left[\frac{e^{j\frac{\pi t}{\tau}} + e^{-j\frac{\pi t}{\tau}}}{2}\right] e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-\tau/2}^{\tau/2} e^{j(\pi/\tau - \omega)t}\, dt + \frac{1}{2}\int_{-\tau/2}^{\tau/2} e^{-j\left(\frac{\pi t}{\tau} + \omega t\right)}\, dt$$

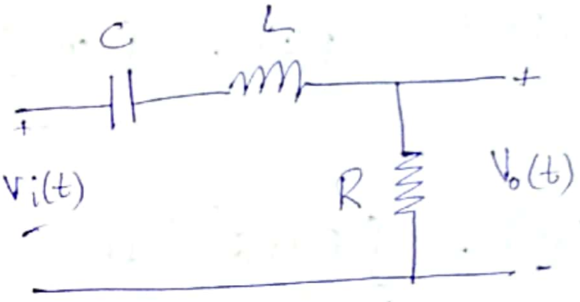
$$= \frac{1}{j2\left(\frac{\pi}{\tau} - \omega\right)}\left(e^{j\left(\frac{\pi}{\tau} - \omega\right)\frac{\tau}{2}} - e^{-j\left(\frac{\pi}{\tau} - \omega\right)\tau/2}\right) \cdot\, - \frac{1}{j2\left(\frac{\pi}{\tau} + \omega\right)} \times \left[e^{-j\left(\frac{\pi}{\tau} + \omega\right)\tau/2} - e^{+j\left(\frac{\pi}{\tau} + \omega\right)\frac{\tau}{2}}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{j\tau/2(\pi/\tau - \omega)} - e^{-j\tau/2(\pi/\tau - \omega)}}{\frac{\pi}{\tau} - \omega} - \frac{e^{-j\frac{\tau}{2}\left(\frac{\pi}{\tau} + \omega\right)} + e^{j\frac{\tau}{2}(\pi/\tau + \omega)}}{\frac{\pi}{\tau} + \omega}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{j\left(\pi/2 - \frac{\omega\tau}{2}\right)} - e^{-j\left(\frac{\pi}{2} - \frac{\omega\tau}{2}\right)}}{\frac{\pi}{\tau} - \omega} - \frac{e^{-j\left(\frac{\pi}{2} + \frac{\omega\tau}{2}\right)} + e^{j\left(\frac{\pi}{2} + \frac{\omega\tau}{2}\right)}}{\frac{\pi}{\tau} + \omega}\right]$$

Q:- 8 (2.42)

Solve



$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$V_o = \frac{V_i R}{Z} = \frac{V_i R}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$\frac{V_o}{V_i} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R\omega C}{R\omega C + j(\omega^2 LC - 1)}$$

$$\frac{V_o}{V_i} = \frac{jRC\omega}{jRC\omega + j(\omega^2 LC - 1)} = \frac{jRC\omega}{jRC\omega - \omega^2 LC + 1}$$

at $\omega = 0$

$$\frac{V_o(\omega=0)}{V_i(\omega=0)} = 0$$

at $\omega = \infty$

$$\frac{V_o(\infty)}{V_i(\omega)} = \lim_{\omega \to \infty} \frac{j\omega RC}{jRC\omega - \omega^2 LC + 1} = 0$$

As the gain is zero for lower and higher frequencies so, it seems like it is a band pass filter.

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Q:- 7

Solve

$$X(j\omega) = \frac{4 + j\omega}{-\omega^2 + j2\omega + 3}$$

ⓐ $x(t)\, e^{-j2t}$

$$F\left(x(t)\, e^{-2jt}\right) = X(\omega + 2)$$

$$X(\omega+2) = \frac{4 + j(\omega+2)}{-(\omega+2)^2 + j2(\omega+2) + 3} = \frac{4 + j(\omega+2)}{-\omega^2 - 4 - 4\omega + j2\omega + j4 + 3}$$

$$X(\omega+2) = \frac{4 + j(\omega+2)}{-(\omega^2 + 4\omega + 1) + 2j(\omega+2)}$$

ⓑ $x(t) \sin\pi(t-1)$

$$F\left(x(x) \sin\pi(t-1)\right) = -\frac{\left[X(\omega-\pi) - X(\omega+\pi)\right]}{2j}$$

$$= \frac{j}{2}\left[\frac{4 + j(\omega-\pi)}{-(\omega-\pi)^2 + j2(\omega-\pi) + 3} - \frac{4 + j(\omega+\pi)}{-(\omega+\pi)^2 + 2j(\omega+\pi) + 3}\right]$$

$$= \frac{1}{2}\left[\frac{4j - \omega + \pi}{-(\omega-\pi)^2 + j2(\omega-\pi) + 3} - \frac{4j - (\omega+\pi)}{-(\omega+\pi)^2 + 2j(\omega+\pi) + 3}\right]$$

$$= \frac{1}{2}\left[\frac{4j - (\omega-\pi)}{-(\omega-\pi)^2 + 2j(\omega-\pi) + 3} - \frac{4j - (\omega+\pi)}{-(\omega+\pi)^2 + 2j(\omega+\pi) + 3}\right]$$

ⓒ $x(t) * \delta(t-2) = x(t-2)$

$$F\left(x(t) * \delta(t-2)\right) = F\left[x(t-2)\right] = e^{-2j\omega} \cdot X(\omega)$$

$$= \left(e^{-2j\omega}\right)\left(\frac{4 + j\omega}{-\omega^2 + 2j\omega + 3}\right)$$

(d) $\int_{-\infty}^{t} x(\tau)\, d\tau$

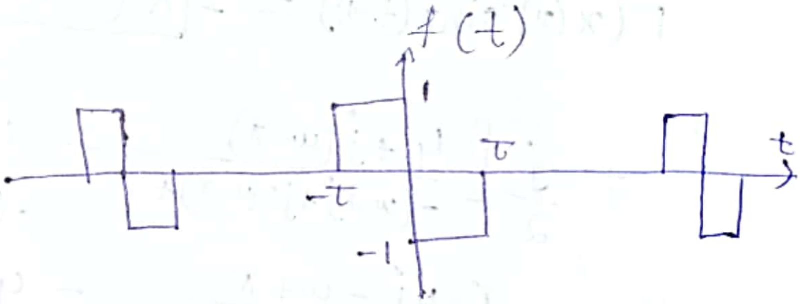
$$F\left(\int_{-\infty}^{T} x(\tau)\, d\tau\right) = \frac{X(\omega)}{j\omega} + \pi X(0)\delta(\omega)$$

$$X(\omega) = \frac{4+j\omega}{-\omega^2+2j\omega+3}, \quad X(0) = \frac{4}{3}$$

$$F\left(\int_{-\infty}^{T} x(\tau)\, d\tau\right) = \frac{4+j\omega}{j\omega(-\omega^2+2j\omega+3)} + \frac{4\pi}{3}\delta(\omega)$$

$$= \frac{-4j+\omega}{-\omega^3+2j\omega^2+3\omega} + \frac{4\pi}{3}\delta(\omega)$$

Q:-5



$$a_0 = \frac{1}{T}\int_{-\tau}^{\tau} f(t)\, dt$$

$$a_0 = \frac{1}{T}\left[\int_{-T}^{0} dt - \int_{0}^{T} dt\right]$$

$$a_0 = 0$$

$$a_n = \frac{1}{T}\left[\int_{-T}^{0} e^{-j\omega nt}\, dt - \int_{0}^{T} e^{-j\omega nt}\, dt\right]$$

$$a_n = \frac{1}{T}\left[\frac{1-e^{-j\omega nT}}{-j\omega n} + \frac{1-e^{-j\omega nT}}{-j\omega n}\right]$$

$$a_n = \frac{\left(2-2e^{-j\frac{2\pi nT}{\tau}}\right)}{(T)\left(-j\frac{(2\pi)}{T}n\right)} = \frac{\left(e^{\frac{-j2\pi nT}{\tau}}-1\right)}{\cancel{j\omega}\ jn\pi}$$

$$f(t) = \sum_{n=-\infty}^{\infty} \frac{\left(e^{\frac{-j2\pi nT}{\tau}}-1\right)\left(e^{\frac{jn\pi T}{\tau}}\right)}{jn\pi}$$