

## EE 365 Assignment # 3(Solution) [CLO 2]

Print out this cover page and attach above your hand-written, legible submission.

**Name:**

**Reg #:**

Total Points	P1 (10)	P2 (10)	P3 (10)	P4 (10)	P5 (10)	P6 (10)	P7 (10)	P8 (10)	P9 (10)	P10 (10)	Grand Total (100)
Obtained Points											

- You may consult TA during his office hours for any queries.
- Late Submission Policy advertised with Assignment 1 (uploaded on Piazza) applies.
- Please work out these ten (10) problems stated below
- “Agbo & Sadiku” refers to the text: [Principles of Modern Communication Systems by Agbo and Sadiku, Cambridge University Press, 2019](#). E.g., Problem 2.32 (Agbo & Sadiku) refers to Exercise 32 of Chapter 2 in this book.
- Show your work and explain reasoning. Solve the problems in the order they are given below.

### **Problems:**

- Problem 3.1 (Agbo & Sadiku)
- Problem 3.3 (Agbo & Sadiku)
- Problem 3.5 (Agbo & Sadiku)
- Problem 3.11 (Agbo & Sadiku)
- Problem 3.13 (Agbo & Sadiku)
- Problem 3.24 (Agbo & Sadiku)
- Problem 3.31 (Agbo & Sadiku)
- Problem 3.33 (Agbo & Sadiku)
- Problem 3.37 (Agbo & Sadiku)
- Problem 3.45 (Agbo & Sadiku)

(1)

3.1Ques:-

$$\text{Bandwidth} = 30 \text{ KHz}$$

$$v = f \lambda$$

(a) Without Modulation

$$\lambda = \frac{3 \times 10^8}{30 \times 10^3}$$

$$\lambda = 10 \text{ Km}$$

length of antenna  $L = \frac{\lambda}{10}$ 

$$\boxed{L = 1 \text{ Km}} \rightarrow$$

(b) With Modulation

$$\text{BW} = 30 \text{ K} \times 100 = 3 \times 10^6 \text{ Hz}$$

$$\lambda = \frac{3 \times 10^8}{3 \times 10^6}$$

$$\lambda = 100 \text{ m}$$

length of antenna  $L = \lambda/10$ 

$$\boxed{L = 10 \text{ m}} \rightarrow$$


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3.3

Qn02:-

message signal.

↓

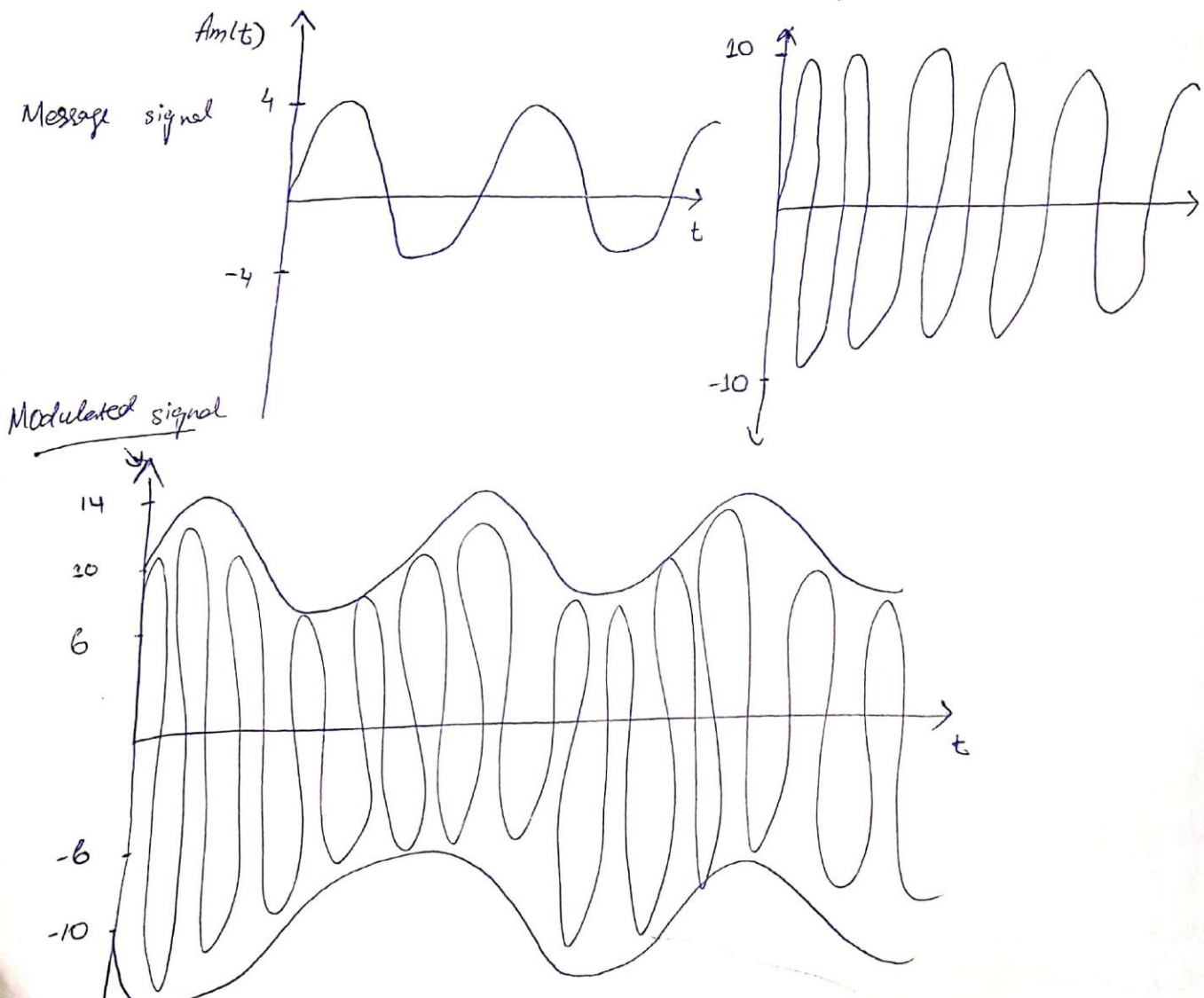
~~m(t)~~  $A_m \cos \omega_c t$

$$A_c(t) = 10 \cos \omega_c t$$

a)  $A_m = 4$

$$\mu = \frac{A_m}{A_c} = \frac{4}{10}$$

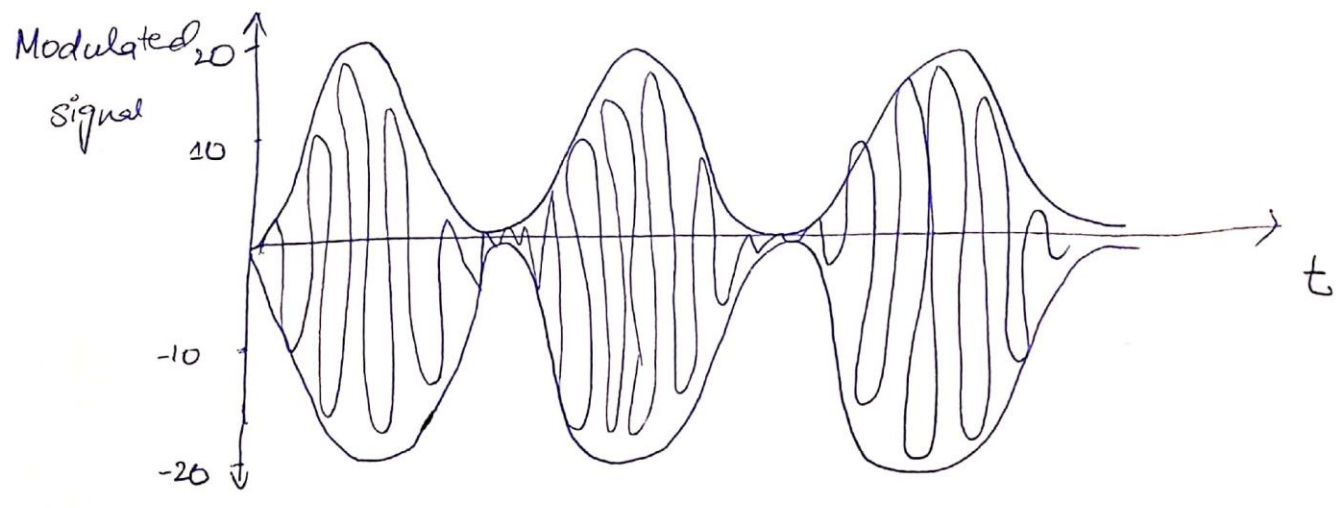
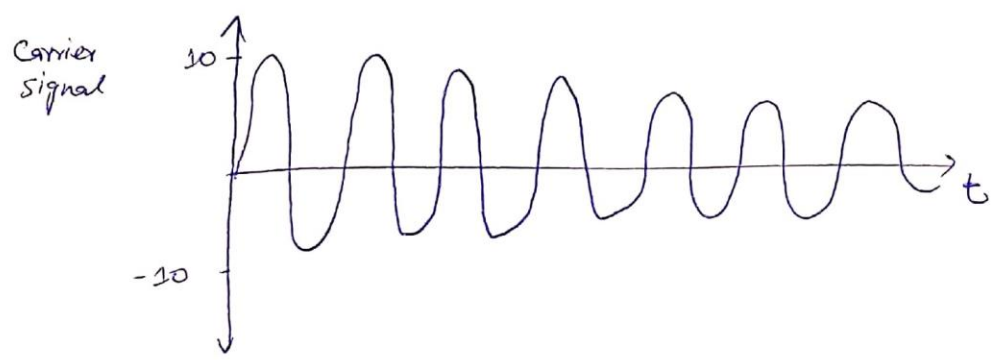
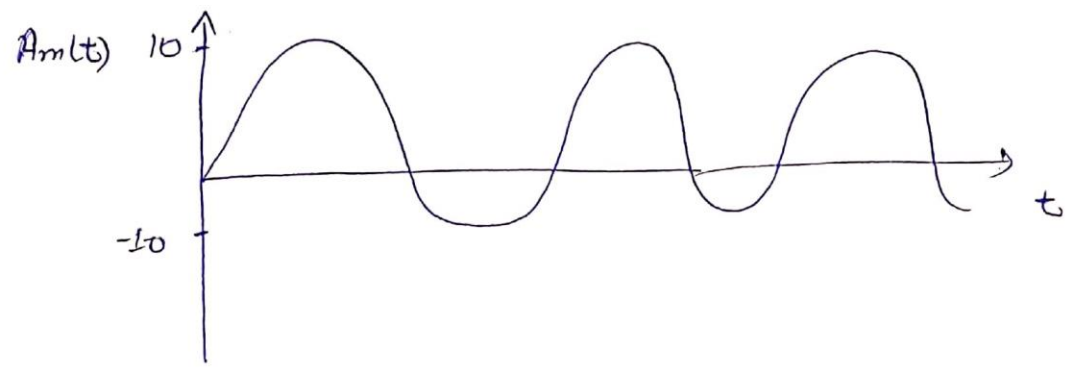
$\mu = 0.4$  modulation index.



(b)  $A_m = 10$

Modulation Index  $\Rightarrow u = \frac{A_m}{A_c} = \frac{10}{10}$

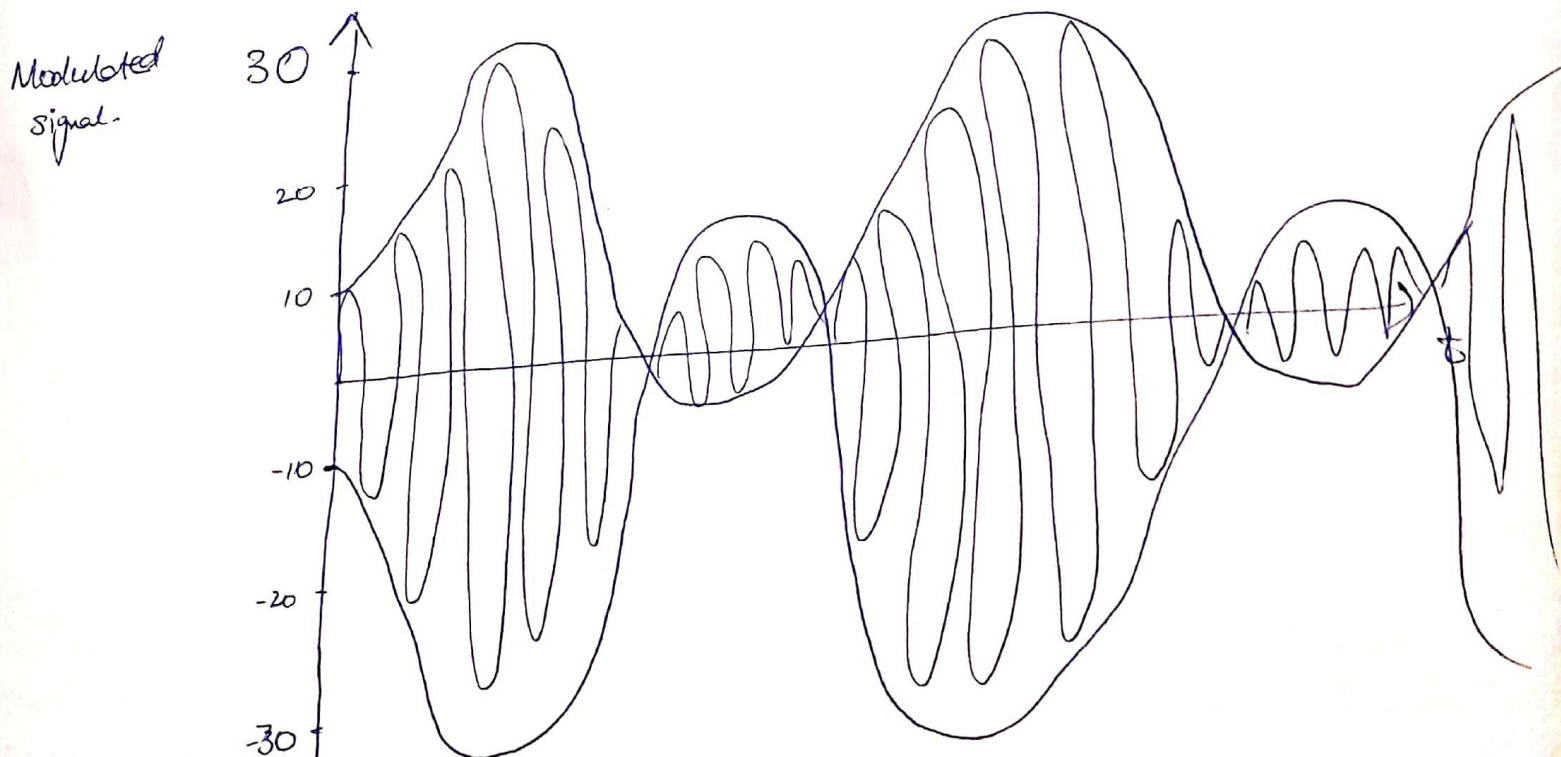
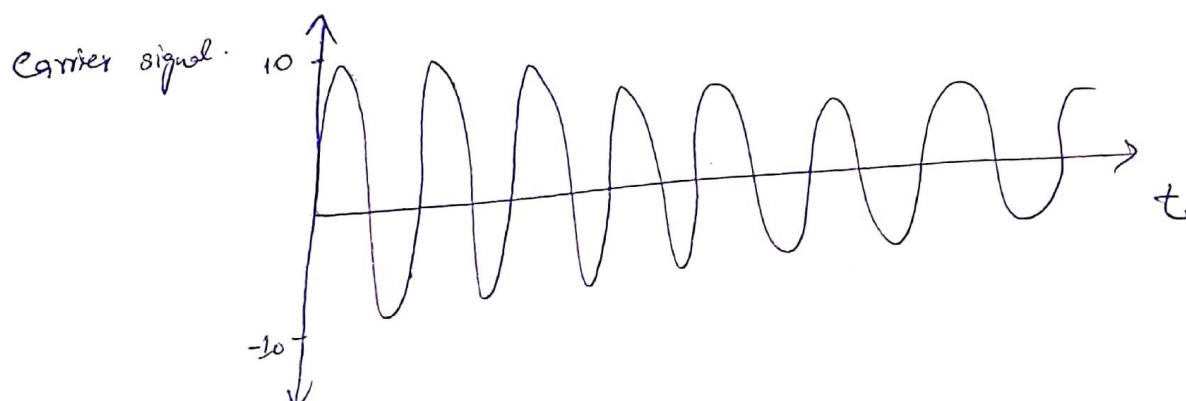
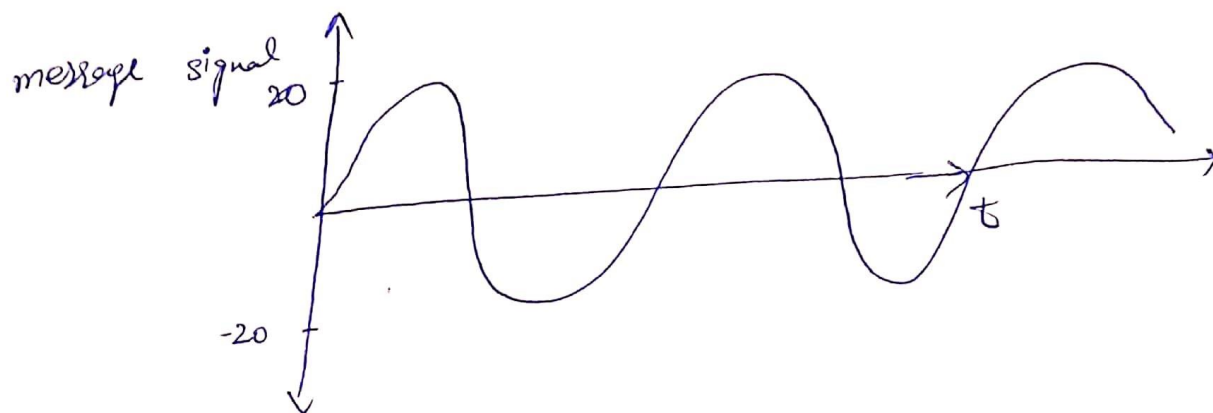
$u = 1$



(c)  $A_m = 20$

Modulation Index  $\Rightarrow$

$$\mu = \frac{20}{10} = 2$$





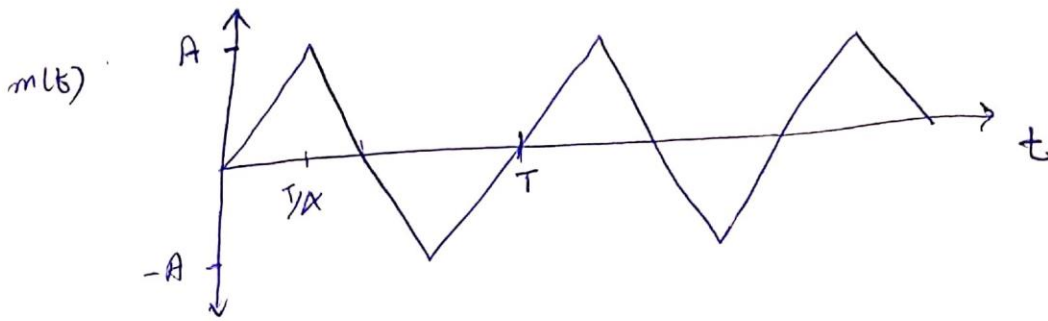
3.5

3

Qno 3:-

Carrier =  $4 \sin \omega_c t$   
Signal

Message signal  $\Rightarrow$  Triangular waveform



~~Qno 4~~

(referred to example 3.3)

$$m(t) = \frac{4A}{T} t, \quad 0 \leq t \leq \frac{T}{4}$$

$$P_m = \frac{4}{T} \int_0^{T/4} m^2(t) dt \Rightarrow \frac{4}{T} \int_0^{T/4} \frac{16A^2}{T^2} t^2 dt$$

$m(t)$

$$P_m = \frac{4}{T} \times \frac{16A^2}{T^2} \left[ \int_0^{T/4} t^2 dt \right]$$

$$P_m = \frac{64A^2}{T^3} \left[ \frac{t^3}{3} \right]_0^{T/4}$$

$$P_m = \frac{64A^2}{T^3} \frac{T^3}{64(3)}$$

$$P_m = \frac{A^2}{3}$$

$$P_c = \frac{A_c^2}{9}$$

a)  $m_p = 2$

$$u = \frac{2}{4}$$

$$u = 0.5$$

Modulation  
Index.

$$\eta = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{P_m}{P_c + P_m} \times 100$$

$$= \frac{m_p^2 / 3}{A_c^2 / 2 + \frac{m_p^2}{3}} \times 100$$

$$\eta =$$

$$\eta = \frac{P_m}{A_c^2 + P_m} \times 100$$

$$\eta = \frac{m_p^2 / 3}{A_c^2 + m_p^2 / 3} \times 100$$

$$\eta = \frac{1.33}{16 + 1.33} \times 100$$

$$\eta = 7.67 \%$$

(b)  $m_p = 4$

$$u = \frac{4}{4}$$

$$u = 1$$

Modulation  
Index.

$$\eta = \frac{P_m}{A_c^2 + P_m} \times 100$$

$$\eta = \frac{4^2 / 3}{4^2 + 4^2 / 3} \times 100$$

$$\eta = 25 \%$$

3.11

(4)

Qno 4

$$i_D = \beta (4V_D + V_D^2)$$

$$x(t) = R i_D$$

$$\therefore R = 1$$

$$x(t) = i_D$$

$$V_D = (A_c \cos \omega_c t + (\alpha + m(t)))$$

$$x(t) = \beta \left( 4(A_c \cos \omega_c t + (\alpha + m(t))) + (A_c \cos \omega_c t + (\alpha + m(t)))^2 \right)$$

$$x(t) = \beta \left( 4(A_c \cos \omega_c t + (\alpha + m(t))) + A_c^2 \cos^2 \omega_c t + (\alpha + m(t))^2 + 2A_c \cos \omega_c t (\alpha + m(t)) \right)$$

$$x(t) = \beta \left( 4(A_c \cos \omega_c t + (\alpha + m(t))) + \frac{A_c^2}{2} (\cos 2\omega_c t + 1) + (\alpha + m(t))^2 + 2A_c \cos \omega_c t (\alpha + m(t)) \right)$$

\* Bandpass filter of the cut-off frequency.

$$y(t) = \beta 4 A_c \cos \omega_c t + 2\beta A_c \cos(\omega_c t) (\alpha + m(t))$$

$$y(t) = 2\beta A_c \cos(\omega_c t) (2 + (\alpha + m(t))) \Rightarrow$$

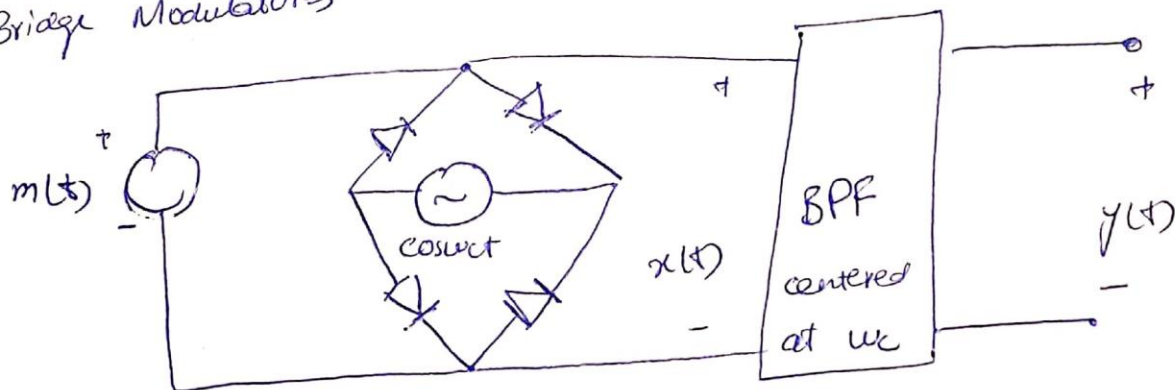


3.24

Qno 6:-

Circuit Diagram :-

Shunt Bridge Modulator  $\Rightarrow$



Referred to 3.35 r

$$x(t) = \frac{1}{2} m(t) + \frac{2}{\pi} \left[ m(t) \cos \omega_c t - \frac{1}{3} m(t) \cos 3\omega_c t + \frac{1}{5} m(t) \cos 5\omega_c t + \dots \right]$$

$$y(t) = \phi_{DSB-SC} = \frac{2}{\pi} m(t) \cos(\omega_c t) \quad \text{for}$$

3.31

(5)

Qno 7:-

$$BW = 10 \text{ kHz}$$

(a)  $\phi(t) = m(t) \cos(2\pi \times 10^5 t)$

Local oscillator  $\Rightarrow 2 \cos(1.2\pi \times 10^5 t) + 4 \cos(2.8\pi \times 10^5 t)$

$$x(t) = m(t) \cos(2\pi \times 10^5 t) \left[ 2 \cos(1.2\pi \times 10^5 t) + 4 \cos(2.8\pi \times 10^5 t) \right]$$

$$x(t) = m(t) \left[ 2 \cos(2\pi \times 10^5 t) \cos(1.2\pi \times 10^5 t) + 4 \cos(2\pi \times 10^5 t) \cos(2.8\pi \times 10^5 t) \right]$$

$$x(t) = m(t) \left[ \cos(3.2\pi \times 10^5 t) + \cos(0.8\pi \times 10^5 t) + 2 \cos(4.8\pi \times 10^5 t) + 2 \cos(0.8\pi \times 10^5 t) \right]$$

$$x(t) = m(t) \left[ 3 \cos(0.8\pi \times 10^5 t) + 2 \cos(4.8\pi \times 10^5 t) + \cos(3.2\pi \times 10^5 t) \right]$$

(b) Largest amplitude Term  $\Rightarrow 3m(t) \cos(0.8\pi \times 10^5 t)$

$$\omega_c = 0.8\pi \times 10^5$$

$$F_{\text{high}} = (0.8 \times \pi \times 10^5) / 2\pi + 10 \text{ kHz} = 50 \text{ kHz}$$

$$F_{\text{low}} = (0.8 \times \pi \times 10^5) / 2\pi - 10 \text{ kHz} = 30 \text{ kHz}$$

3.33

Qn08:-

$$m_1(t) = A_1 \cos(8\pi \times 10^3 t)$$

$$m_2(t) = A_2 \cos(12\pi \times 10^3 t)$$

$$\text{carrier} \rightarrow 2 \sin(2\pi \times 10^5 t)$$

a)  $\phi_{\text{QAM}}(t) = ?$

$$\phi_{\text{QAM}}(t) = A_1 \cos(8\pi \times 10^3 t) 2 \sin(2\pi \times 10^5 t) - A_2 \cos(12\pi \times 10^3 t) 2 \cos(2\pi \times 10^5 t)$$

$$= A_1 \left[ \sin(2.08\pi \times 10^5 t) - \sin(-1.92\pi \times 10^5 t) \right] - A_2 \left[ \cos(2.12\pi \times 10^5 t) + \cos(1.88\pi \times 10^5 t) \right]$$

$$\phi_{\text{QAM}}(t) = A_1 \left[ \sin(2.08\pi \times 10^5 t) + \sin(1.92\pi \times 10^5 t) \right] - A_2 \left[ \cos(2.12\pi \times 10^5 t) + \cos(1.88\pi \times 10^5 t) \right]$$

(b)  $z_1(t) = \phi_{\text{QAM}}(t) \times 2 \sin((2\pi \times 10^5 t) + \alpha)$

$$z_1(t) = A_1 \left[ \cos(-8\pi \times 10^3 t + \alpha) - \cos(4.08\pi \times 10^5 t + \alpha) + \cos(8\pi \times 10^3 t + \alpha) - \cos(3.92\pi \times 10^5 t + \alpha) \right] \\ - A_2 \left[ \sin(4.12\pi \times 10^5 t + \alpha) + \sin(-12\pi \times 10^3 t + \alpha) + \sin(3.88\pi \times 10^5 t + \alpha) + \sin(12\pi \times 10^3 t + \alpha) \right]$$

LPF:-

$$y_1(t) = A_1 \left[ \cos(-8\pi \times 10^3 t + \alpha) + \cos(8\pi \times 10^3 t + \alpha) \right] - A_2 \left[ \sin(-12\pi \times 10^3 t + \alpha) + \sin(12\pi \times 10^3 t + \alpha) \right]$$

$$y_1(t) = 2A_1 (\cos \alpha) \cos(8\pi \times 10^3 t) - 2A_2 (\sin \alpha) \cos(12\pi \times 10^3 t)$$

3.37

(6)

Qn09:-

$$\begin{aligned}\text{Carrier} &\Rightarrow A_c \sin \omega_c t \\ &\Rightarrow A_c \sin(10\omega_m t)\end{aligned}$$

$$\therefore \omega_c = 10\omega_m$$

(a)  $m(t) = A_m \sin(\omega_m t)$

$$m_h(t) = -A_m \cos(\omega_m t)$$

$$\Rightarrow \phi_{\text{USB}}(t) = m(t) A_c \sin(10\omega_m t) + A_c \cos(10\omega_m t) m_h(t)$$

$$\phi_{\text{USB}}(t) = A_m A_c \sin(10\omega_m t) \sin(\omega_m t) - A_c A_m \cos(10\omega_m t) \cos(\omega_m t)$$

$$\phi_{\text{USB}}(t) = \frac{A_c A_m}{2} [\cos(9\omega_m t) - \cos(11\omega_m t)] - \frac{A_c A_m}{2} [\cos(11\omega_m t) + \cos(9\omega_m t)]$$

$$\boxed{\phi_{\text{USB}}(t) = -A_c A_m \cos(11\omega_m t)}$$

(b)  $m(t) = A_1 \sin(\omega_m t) + A_2 \cos(3\omega_m t)$

$$m_h(t) = -A_1 \cos(\omega_m t) + A_2 \sin(3\omega_m t)$$

$$\boxed{\phi_{\text{USB}}(t) = m(t) A_c \sin(10\omega_m t) + A_c \cos(10\omega_m t) m_h(t)}$$



$$\phi_{USB}(t) = A_c \sin(10\omega_m t) \left[ A_1 \sin(\omega_m t) + A_2 \cos(3\omega_m t) \right] \\ + A_c \cos(10\omega_m t) \left[ -A_1 \cos(\omega_m t) + A_2 \sin(3\omega_m t) \right]$$

$$= A_c \left[ \frac{A_1}{2} (\cos(9\omega_m t) - \cos(11\omega_m t)) + \frac{A_2}{2} (\sin(13\omega_m t) + \sin(7\omega_m t)) \right] \\ + A_c \left[ -\frac{A_1}{2} (\cos(11\omega_m t) + \cos(9\omega_m t)) + \frac{A_2}{2} (\sin(13\omega_m t) - \sin(7\omega_m t)) \right]$$

$$\phi_{USB}(t) = -A_1 A_c \cos(11\omega_m t) + A_2 A_c \sin(13\omega_m t)$$



3.45

(7)

Qn 10:-

$$\omega_c = 10^6 \text{ rad/s}$$

$$\omega_m = 0.4 \times 10^5 \text{ rad/s}$$

$$m(t) = 4 \cos \omega_m t$$

$$\text{Carrier} = \cos \omega_c t$$

$$\phi_{\text{DSB}}(t) = 4 \cos(\omega_c t) \cos(\omega_m t)$$

$$\phi_{\text{DSB}}(t) = 2 \left( \cos((\omega_c + \omega_m)t) + \cos((\omega_c - \omega_m)t) \right)$$

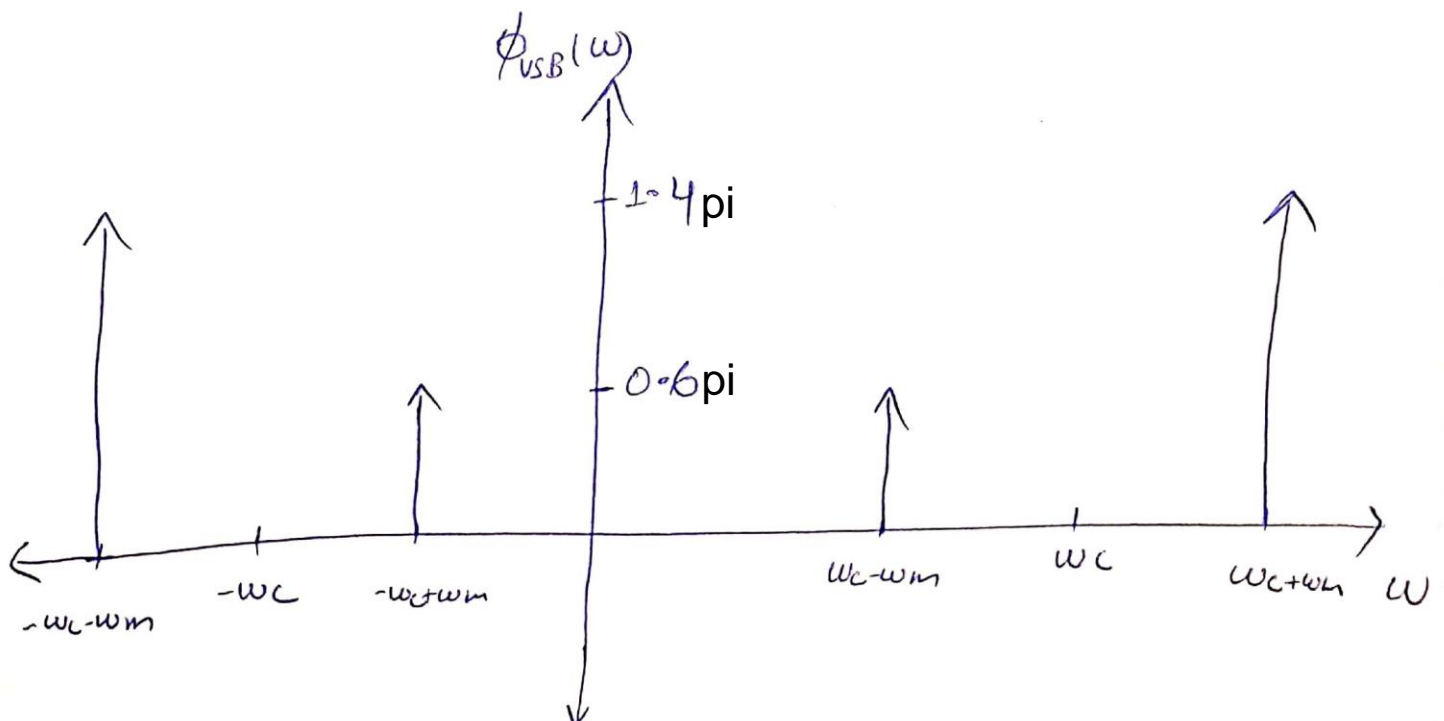
$$\omega_c + \omega_m = 10.4 \times 10^5 \text{ rad/s}$$

$$\omega_c - \omega_m = 9.6 \times 10^5 \text{ rad/s}$$

$$H_u(\omega_c + \omega_m) = 0.7$$

$$H_u(\omega_c - \omega_m) = 0.3$$

$$\phi_{\text{VSB}}(t) = 1.4 \cos((\omega_c + \omega_m)t) + 0.6 \cos((\omega_c - \omega_m)t)$$



Qno5:

(7)

Problem 3.13

a)

$$B = 5 \text{ kHz} \quad f_c = 500 \text{ kHz}$$

$$C = 20 \text{ nF}$$

as we know

$$RC = \sqrt{\frac{1}{Bf_c}}$$

$$R = \frac{1}{C} \sqrt{\frac{1}{Bf_c}} = \frac{1}{20 \times 10^{-9}} \sqrt{\frac{1}{5 \times 10^3 \times 500 \times 10^3}}$$

$$= 1 \text{ k}\Omega$$

b)

$$\phi_{AM}(t) = (A_c + A_m \cos \omega_m t) \cos \omega_c t = A_c \left[ 1 + \frac{A_m}{A_c} \cos \omega_m t \right] \cos \omega_c t$$

$$= A_c [1 + \mu \cos \omega_m t] \cos \omega_c t$$

(8)

The AM envelope is  $E(t) = A_c [1 + \mu \cos \omega_m t]$

the capacitor voltage is  $V_c(t) = E(t)$

Ensuring that  $V_c(t)$  never lies above

$E(t)$  requires that  $\left| \frac{dV_c(t)}{dt} \right| \geq \left| \frac{dE(t)}{dt} \right|$

$E(t)$  &  $V_c(t)$  have same peak value

$$E \Rightarrow V_c(t) = E e^{-t/RC} \approx E \left( 1 - \frac{t}{RC} \right)$$

$$V_c(t) = E e^{-t/RC} \approx E \left( 1 - \frac{t}{RC} \right) \quad (\text{Taylor series approximation of } e^{-x})$$

$$\left| \frac{dV_c(t)}{dt} \right| = \frac{E}{RC} = \frac{A_c [1 + \mu \cos \omega_m t]}{RC} \geq \left| \frac{dE(t)}{dt} \right|$$

$$= \left| -\mu \omega_m A_c \sin \omega_m t \right|$$

$$\therefore \frac{1}{RC} \geq \frac{\mu \omega_m A_c \sin \omega_m t}{A_c [1 + \mu \cos \omega_m t]}$$

(9)

Differentiating the first derivative to zero, its maximum occurs @

$$u = -\cos \omega_m t.$$

$$\text{Thus } \frac{1}{RC} \geq \frac{\omega_m}{\sqrt{1-u^2}}$$

$$R \leq \frac{\omega_m}{\sqrt{1-u^2} C}$$

i) for  $u=0.5$   $C=20\text{ nF}$

$$R_{\max} = \frac{\sqrt{1-0.5^2}}{0.5(2\pi \times 5 \times 10^3)(20 \times 10^{-9})} = 2.76 \text{ k Ohm}$$

ii)  $u=0.95$   $C=20\text{ nF}$

$$R_{\max} = \frac{\sqrt{1-0.95^2}}{0.95(2\pi \times 5 \times 10^3)(20 \times 10^{-9})} = 523.1 \Omega$$