MT240:Complex Variables and Transforms

Homework 5

Due Date: July 8 ,2021
Max Marks : 100

Spring 2021

Tips to avoid plagiarism:

- Do not copy the solutions of your classmates.
- You are encouraged to discuss the problems with your classmates in whatever way you like but, make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Cite all the online sources that you get help from.
- Keep your work in a secure place.

Problem 1 [20 marks]

Find a Mobius Transformation that fulfills each of the following conditions and express in the form

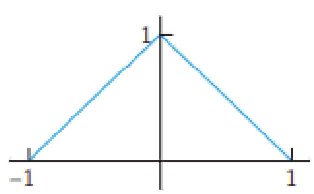
$$f(z) = \frac{az+b}{cz+d}, \quad for\ a,b,c,d \in \mathbb{C}$$

(a) $f(1) = 0\,,\ f(0) = 1,\ f(-1) = \infty$
(b) $f(0) = 0\,,\ f(1) = 1 + i,\ f(2i) = \infty$
(c) $f(1) = 0\,,\ f(\infty) = 1,\ f(-1) = \infty$
(d) $f(0) = -i\,,\ f(1) = \infty,\ f(\infty) = 1$

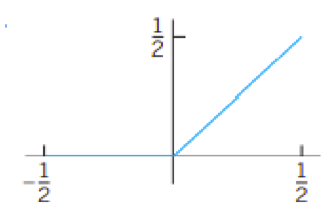
Problem 2 [40 marks]

Is the given function even or odd or neither even nor odd? Find its Fourier series. Show details of your work.

(a)



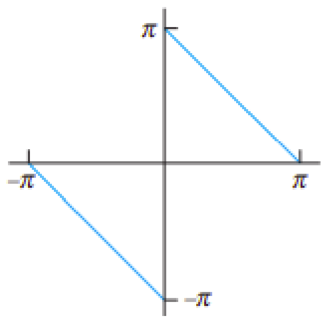
(b)



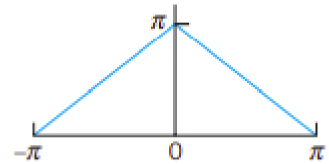
Problem 3 [40 marks]

Find the Fourier series of the given function $f(x)$, which is assumed to have the period 2π. Show the details of your work. Sketch the partial sums up to that including $\cos 5x$ and $\sin 5x$

(a)



(b)



Rafay Aamir

CVT Assignment: 5

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100

Problem:: 1

$$f(z) = \frac{az+b}{cz+d}$$

(a) $f(1) = 0$, $f(0) = 1$, $f(-1) = \infty$

Solve:- Let $w = f(z) = \frac{az+b}{cz+d}$, $w' = \frac{a(cz+d) - c(az+b)}{(cz+d)^2}$

$$w' = \frac{ad-bc}{(cz+d)^2} \quad \text{—— (1)} \quad \text{where } ad-bc \neq 0$$

using the given conditions.

$z_1 = 1$, $z_2 = 0$, $z_3 = -1$

$w_1 = 0$, $w_2 = 1$, $w_3 = \infty = \frac{1}{0}$

using

$$\frac{w-w_1}{w-w_3} \times \frac{w_2-w_3}{w_2-w_1} = \frac{z-z_1}{z-z_3} \times \frac{z_2-z_3}{z_2-z_1} \quad \text{—— (E)}$$

Substituting values

$$\frac{w-0}{w-\infty} \times \frac{1-\infty}{1-0} = \frac{z-1}{z+1} \times \frac{0+1}{0-1}$$

$$\frac{w}{w-\frac{1}{0}} \times \frac{1-\frac{1}{0}}{1} = \frac{z-1}{-z-1} \quad \text{—— (2)}$$

$$\frac{1}{0} = \frac{z-1}{-z-1} \quad , \quad \boxed{z = -1} \; , \; \boxed{w = \frac{-a+b}{-c+d}}$$

from eq ②

$$\frac{w}{w-\infty} \times \frac{1-\infty}{1} = \frac{z-1}{-z-1}$$

$$\frac{w}{\cancel{-\infty}} \times \frac{\cancel{-\infty}}{1} = \frac{z-1}{-z-1}, \quad \boxed{w = f(z) = \frac{z-1}{-z-1}}$$

Translation :- $w = z - 1$

Rotation :- $w = z$, $a = 1$, $|a| = 1$

Linear Transform :- $w = z - 1$

Inverse in the unit circle :- $w = \frac{1}{z}$

ⓑ $f(0) = 0$, $f(1) = 1+i$, $f(2i) = 0$

Solve Here $z_1 = 0$, $z_2 = 1$, $z_3 = 2i$

$w_1 = 0$, $w_2 = 1+i$, $w_3 = \infty$

Substituting values in eq Ⓔ

$$\frac{w-0}{w-\infty} \times \frac{(1+i)-\infty}{1+i-0} = \frac{z-0}{z-2i} \times \frac{1-2i}{1-0}$$

$$\frac{w}{-\infty} \times \frac{-\infty}{1+i} = \frac{z}{z-2i} \times (1-2i)$$

$$w = \frac{z}{z1-2i} \times (1-2i)(1+i) \Rightarrow \frac{z(4-i)}{z-2i}$$

$$w = \frac{4z - zi}{z-2i} = \frac{z[4-i]}{z-2i}$$

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(c) $f(1) = 0$, $f(\infty) = 1$, $f(-1) = \infty$

Solve

$z_1 = 1$, $z_2 = \infty$, $z_3 = -1$

$w_1 = 0$, $w_2 = 1$, $w_3 = \infty$

Substituting values in eq (E)

$$\frac{w-0}{w-\infty} \times \frac{1-\infty}{1-0} = \frac{z-1}{z+1} \times \frac{\infty+1}{\infty-1}$$

$$\frac{w}{-\infty} \times \frac{-\infty}{1} = \frac{z-1}{z+1} \times 1$$

$$\boxed{w = \frac{z-1}{z+1}} = f(z)$$

Translations → $w = z-1$

Rotations → $w = z$, $a = 1$, $|a| = 1$

Linear Translation → $w = z-1$

Invesion in the unit circle :- $w = \frac{1}{z}$

(d) $f(0) = -i \quad , f(1) = \infty \quad , f(\infty) = 1$

Solve

$z_1 = 0 \quad , z_2 = 1 \quad , z_3 = \infty$

$w_1 = -i \quad , w_2 = \infty \quad , w_3 = 1$

Substituting values in eq (E)

$$\frac{w+i}{w-1} \times \frac{\infty - 1}{\infty + i} = \frac{z-0}{z-\infty} \times \frac{1-\infty}{1-0}$$

$$\frac{w+i}{w-1} = z \quad , w+i = \cancel{zw + wi} = zw - z$$

$$w+i = zw - z$$

$$zw - w = z + i$$

$$w(z-1) = z+i$$

$$\boxed{w = f(z) = \frac{z+i}{z-1}}$$

Translations :- $w = z + i$

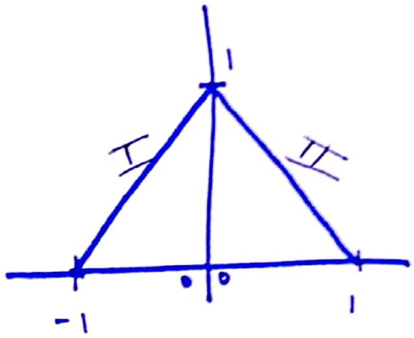
Rotations :- $w = az = z \,,\, a = 1 \,,\, |a| = 1$

Linear Translations :- $w = az + b = z + i$

Inversion in the unit circle :- $w = \frac{1}{z}$

Problem :: 2

ⓐ :: Solve



for (I) , $y = \left[\frac{1-0}{0+1}\right]x + 1$

$$y = x+1 \quad —— ①$$

for (II) , $y = \left[\frac{0-1}{1-0}\right]x + 1$

$$y = -x+1 \quad —— ②$$

combining eq ① and ② $\boxed{y = 1 - |x|}$ $-1 < x < 1$

$P = 2$,

$$a_0 = \frac{1}{P}\int_{-1}^{1}(1-|x|)\,dx$$

$$a_0 = \frac{\cancel{2}}{\cancel{2}}\int_{0}^{1}(1-x)\,dx \;,\; a_0 = \left[x - \frac{x^2}{2}\right]_0^1 \Rightarrow \left[1 - \frac{1}{2}\right]$$

$$\boxed{a_0 = \frac{1}{2}}$$

$$a_n = \frac{2}{P}\int_{-\phi}^{1}(1-|x|)\cos\left[\frac{2\pi n x}{P}\right]dx$$

$$a_n = \int_{-1}^{1}(1-|x|)\cos(\pi n x)\,dx$$

$$a_n = 2\int_{0}^{1}(1-x)\cos(\pi n x)\,dx$$

$$a_n = 2\int_0^1 \cos(\pi n x)\,dx - 2\int_0^1 x\cos(n\pi x)\,dx$$

$$a_n = 2\left[\frac{\sin \pi n x}{\pi n}\right]_0^1 - 2\left[\frac{x\sin(n\pi x)}{\pi n} - \int \frac{\sin(n\pi x)}{n\pi}\,dx\right]_0^1$$

$$a_n = 2\left[\frac{\sin(\pi n x)}{\pi n}\right]_0^1 - 2\left[\frac{x\sin(n\pi x)}{\pi n} + \frac{\cos(n\pi x)}{n^2\pi^2}\right]_0^1$$

Applying limits

$$a_n = 2\left[\cancel{\frac{\sin \pi n}{\pi n}}\right] - 2\left[\cancel{\frac{\sin \pi n}{\pi n}} + \frac{\cos \pi n}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right]$$

$$a_n = \frac{2}{n^2\pi^2} - \frac{2\cos \pi n}{n^2\pi^2}$$

$$a_n = \frac{1}{n^2\pi^2}\left[2 - 2\cos \pi n\right]$$

$$a_n = \frac{2}{n^2\pi^2}(1 - \cos \pi n) \quad \text{—— } (1)$$

$$a_1 = \frac{4}{\pi^2}, \quad a_2 = 0$$

$$b_n = \frac{2}{p}\int_{-1}^{1}(1-|x|)\sin\left(\frac{2n\pi x}{p}\right)dx$$

$$b_n = \int_{-1}^{1}(1-|x|)\sin(n\pi x)\,dx$$

$$b_n = 2\int_0^1 (1-x)\sin(n\pi x)\,dx$$

$$b_n = 2\int_0^1 \sin(n\pi x)\,dx - 2\int_0^1 x \sin(n\pi x)\,dx$$

$$b_n = 2\left[\frac{-\cos n\pi x}{n\pi}\right]_0^1 - 2\left[\frac{-x\cos(n\pi x)}{n\pi} + \int \frac{\cos(n\pi x)\,dx}{n\pi}\right]_0^1$$

$$b_n = -\frac{2}{n\pi}\left[\cos(n\pi x)\right]_0^1 - 2\left[\frac{-x\cos(n\pi x)}{n\pi} + \frac{\sin(n\pi x)}{n^2\pi^2}\right]_0^1$$

Applying limits

$$b_n = -\frac{2}{n\pi}\left[\cos n\pi - 1\right] - 2\left[\frac{-\cos n\pi}{n\pi} + \frac{\sin n\pi}{n^2\pi^2}\right]$$

$$b_n = -\frac{2\cos n\pi}{n\pi} + \frac{2}{n\pi} + \frac{2\cos n\pi}{n\pi} - \frac{2\sin n\pi}{n^2\pi^2}$$

$$b_n = \frac{2}{n\pi}\left[1 - \frac{\sin \pi n}{n\pi}\right] \quad \text{--- } (2)$$

$$b_1 = \frac{2}{\pi}, \quad b_2 = \frac{1}{\pi}$$

Fouries series is written as

$$f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x \cdots\cdots$$

Now substituting values from above data

$$f(x) = \frac{1}{2} + \frac{4}{\pi^2}\cos x + \frac{2\sin x}{\pi} + 0 + \frac{\sin 2x}{\pi}$$

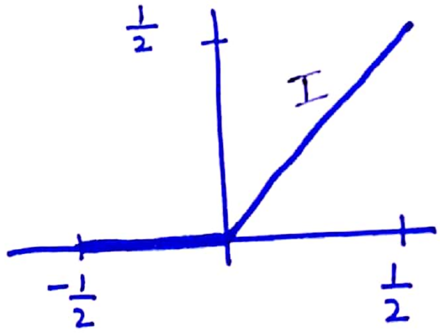
$$\boxed{f(x) = \frac{1}{2} + \frac{4\cos x}{\pi^2} + \frac{2\sin x}{\pi} + \frac{\sin 2x}{\pi} \cdots}$$

The fourier series of the given function*

Problem :- 2

(b) for (I)

$$y = \left[\frac{\frac{1}{2} - 0}{\frac{1}{2} - 0}\right] x + 0$$



$y = x \quad :- \quad 0 < x < \frac{1}{2} \qquad p = 1$

$y = 0 \quad :- \quad \text{otherwise}$

Solve

$$a_0 = \frac{1}{p}\int_0^{1/2} x\,dx \Rightarrow \int_0^{1/2} x\,dx$$

$$a_0 = \left[\frac{x^2}{2}\right]_0^{1/2}, \quad a_0 = \frac{1}{2}\left[\frac{1}{4}\right] = \boxed{\frac{1}{8} = a_0}$$

$$a_n = \frac{2}{p}\int_0^{\frac{1}{2}} x\cos\left[\frac{2n\pi x}{p}\right] dx$$

$$a_n = 2\int_0^{1/2} x\cos(2n\pi x)\,dx$$

$$a_n = 2\left[\frac{x\sin(2n\pi x)}{2n\pi} - \int \frac{\sin(2n\pi x)\,dx}{2n\pi}\right]_0^{1/2}$$

$$a_n = 2\left[\frac{x\sin(2n\pi x)}{2n\pi} + \frac{\cos(2n\pi x)}{4n^2\pi^2}\right]_0^{1/2}$$

Applying limit

$$a_n = 2\left[\frac{\sin(n\pi)}{4n\pi} + \frac{\cos(n\pi)}{4n^2\pi^2} + \frac{1}{4n^2\pi^2}\right]$$

$$a_n = \frac{1}{2n\pi}\left[\sin(n\pi) + \frac{\cos(n\pi)}{n\pi} + \frac{1}{n\pi}\right] \quad \text{— (1)}$$

$$a_1 = \frac{1}{2\pi}\left[0 + \frac{1}{\pi} + \frac{1}{\pi}\right] = \frac{1}{\pi^2}$$

$$a_2 = \frac{1}{4\pi}\left[0 - \frac{1}{2\pi} + \frac{1}{2\pi}\right] = 0$$

$$b_n = \frac{2}{P}\int_0^{1/2} x \sin\left[\frac{2n\pi x}{P}\right] dx = 2\int_0^{1/2} x \sin(2n\pi x)\,dx$$

$$b_n = 2\left[-\frac{x\cos(2n\pi x)}{2n\pi} + \int \frac{\cos(2n\pi x)}{2n\pi}\,dx\right]_0^{1/2}$$

$$b_n = 2\left[-\frac{x\cos(2n\pi x)}{2n\pi} + \frac{\sin(2n\pi x)}{4n^2\pi^2}\right]_0^{1/2}$$

Applying limit

$$b_n = 2\left[-\frac{\cos(n\pi)}{4n\pi} + \frac{\sin(n\pi)}{4n^2\pi^2}\right]$$

$$b_1 = \frac{1}{2\pi}, \quad b_2 = \cancel{0} - \frac{1}{4\pi} = -\frac{1}{4\pi}$$

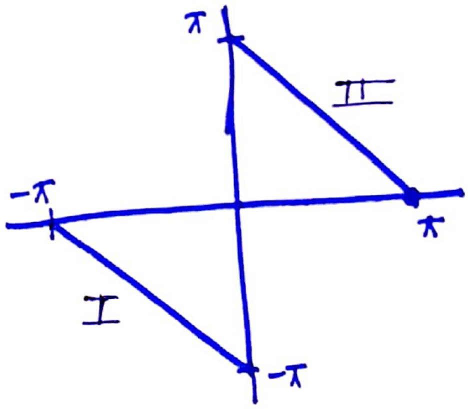
fourier series $f(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x \cdots$

substituting values

$$f(x) = \frac{1}{8} + \frac{\cos x}{\pi^2} + \frac{\sin x}{2\pi} + 0 - \frac{\sin 2x}{4\pi}$$

$$\boxed{f(x) = \frac{1}{8} + \frac{\cos x}{\pi^2} + \frac{\sin x}{2\pi} - \frac{\sin 2x}{4\pi} \cdots}$$

Problem : 3



ⓐ for (I)

$$y = \left[\frac{-\pi - 0}{0 + \pi}\right] x - \pi$$

$$y = -x - \pi \quad \text{--- (1)}$$

for (II)

$$y = \left[\frac{\pi - 0}{\pi - 0}\right] x + \pi$$

$$y = x + \pi \quad \text{--- (2)}$$

$$y = \begin{cases} -x - \pi & \text{if } \cancel{0 < x < \pi} \ -\pi < x < 0 \\ x + \pi & \text{if } 0 < x < \pi \end{cases}$$

$P = 2\pi$

$$a_0 = \frac{1}{P}\int_{-\pi}^{0} (-x - \pi)\, dx + \frac{1}{P}\int_{0}^{\pi} (x + \pi)\, dx$$

$$a_0 = \frac{1}{2\pi}\left[\left[-\frac{x^2}{2} - \pi x\right]_{-\pi}^{0} + \left[\frac{x^2}{2} + \pi x\right]_{0}^{\pi}\right]$$

$$\cancel{a_0 = \frac{1}{2\pi}\left[\frac{\mp\pi^2}{2} - \pi^2\right.} \qquad a_0 = \frac{1}{2\pi}\left[-\left(\frac{x^2}{2} + \pi x\right)_{-\pi}^{0} + \left[\frac{x^2}{2} + \pi x\right]_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\left[-\frac{\pi^2}{2} + \pi^2\right] + \left[\frac{\pi^2}{2} + \pi^2\right]\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{\pi^2}{2} + \pi^2 + \frac{\pi^2}{2} + \pi^2\right] = \frac{3\pi}{2}$$

$$\boxed{a_0 = \frac{3\pi}{2}}$$

$$a_n = \frac{2}{p}\int_{-\pi}^{0}(-x-\pi)\cos\left[\frac{2\pi n x}{p}\right] + \frac{2}{p}\int_{0}^{\pi}(x+\pi)\cos\left(\frac{2\pi n x}{p}\right)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}\underbrace{(-x-\pi)}_{\text{odd}}\underbrace{\cos(nx)}_{\text{even}}\,dx + \frac{1}{\pi}\int_{0}^{\pi}\underbrace{(x+\pi)}_{\text{odd}}\underbrace{\cos(nx)}_{\text{even}}\,dx$$

$a_n = 0$ for all values of (n) — (1)

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0}(-x-\pi)\sin(nx)\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x+\pi)\sin(nx)\,dx$$

$$b_n = \frac{1}{\pi}\left[\frac{(x+\pi)\cos(nx)}{n} - \int\frac{\cos nx}{n}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{(-x-\pi)\cos(nx)}{n} + \int\frac{\cos nx}{n}\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{(x+\pi)\cos nx}{n} - \frac{\sin nx}{n^2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{(-x-\pi)\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[-\frac{\sin n\pi}{\pi^2}\right] + \frac{1}{\pi}\left[\frac{(-2\pi)\cos n\pi}{\cancel{\pi}\, n} + \frac{\pi}{n} + \frac{\sin n\pi}{n^2}\right] \quad \times$$

$$b_n = -\frac{\sin n\pi}{n^2\pi} - \frac{2\cos n\pi}{n} + \frac{1}{n} + \frac{\sin n\pi}{\pi n^2} \quad \times$$

$$b_n = \frac{1}{n}\left[1 - 2\cos n\pi\right] \text{ — (2)} \quad \times$$

$$b_n = \frac{1}{\pi}\left[\frac{(x+\pi)\cos nx}{n} - \frac{\sin nx}{n^2}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{(-x-\pi)\cos nx}{n} + \frac{\sin nx}{n^2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n} - \frac{\sin n\pi}{n^2}\right] + \frac{1}{\pi}\left[\frac{-2\pi\cos n\pi}{n} + \frac{\pi}{n} + \frac{\sin n\pi}{n^2}\right]$$

$$b_n = \frac{1}{n} - \frac{\sin n\pi}{n^2\pi} - \frac{2\cos n\pi}{n} + \frac{1}{n} + \frac{\sin n\pi}{n^2\pi}$$

$$b_n = \frac{2}{n}(1 - \cos n\pi) \quad \text{——} \quad (2)$$

$b_1 = 2(2) = 4$, $b_2 = 0$, $b_3 = \frac{4}{3}$

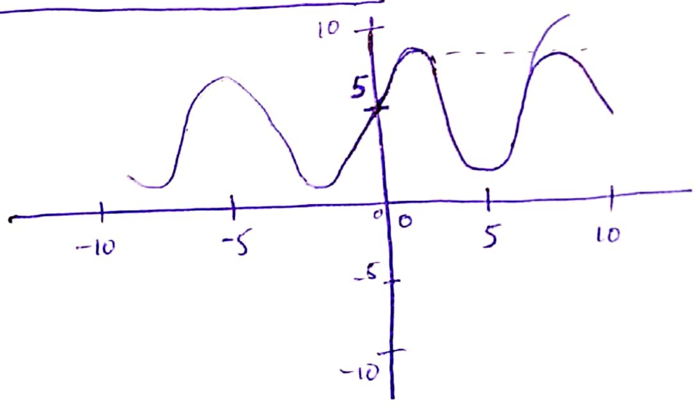
$b_4 = 0$, $b_5 = \frac{4}{5}$

fourier series :- $f(x) = a_0 + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + b_4 \sin 4x + b_5 \sin 5x$

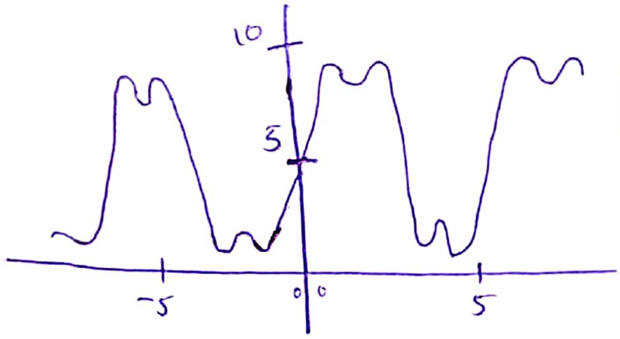
$a_0 = \frac{3\pi}{2}$

$$\boxed{f(x) = a_0 + 4\sin x + \frac{4}{3}\sin 3x + \frac{4}{5}\sin 5x}$$

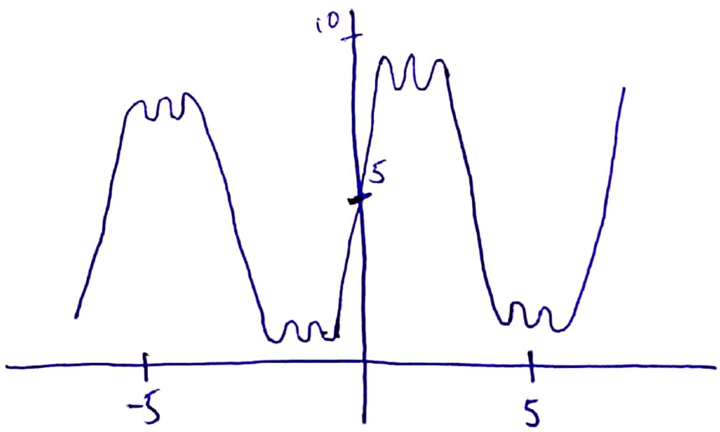
$S_1 = a_0 + 4\sin x$



$S_2 = a_0 + 4\sin x + \frac{4}{3}\sin 3x$



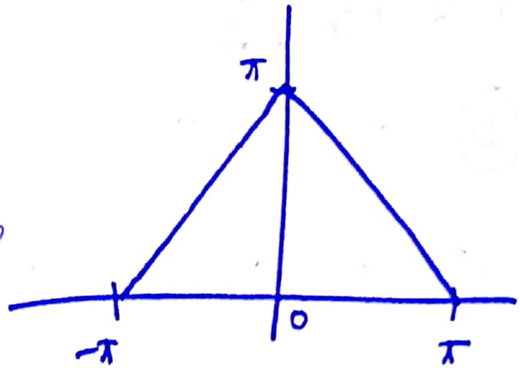
$S_3 = a_0 + 4\sin x + \frac{4}{3}\sin 3x + \frac{4}{5}\sin 5x$



Problem :• 3

(b) Solve

$$f(x) = \begin{cases} x+\pi & -\pi < x < 0 \\ -x+\pi & 0 < x < \pi \end{cases}$$



$$P = 2\pi$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(x+\pi)\,dx + \int_{0}^{\pi}(-x+\pi)\,dx\right.$$

$$a_0 = \frac{1}{2\pi}\left\{\left[\frac{x^2}{2}+\pi x\right]_{-\pi}^{0} + \left[-\frac{x^2}{2}+\pi x\right]_{0}^{\pi}\right.$$

$$a_0 = \frac{1}{2\pi}\left[-\frac{\pi^2}{2}+\pi^2\right] + \frac{1}{2\pi}\left[-\frac{\pi^2}{2}+\pi^2\right]$$

$$a_0 = \frac{1}{2\pi}\left[-\pi^2+2\pi^2\right) \quad , \quad \boxed{a_0 = \frac{\pi}{2}}$$

$$a_n = \frac{2}{P}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}(x+\pi)\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(-x+\pi)\cos nx\,dx$$

$$a_n = \frac{1}{\pi}\left[-\frac{\cos n\pi}{n^2} - \frac{\cos n\pi}{n^2} + \frac{2}{n^2}\right]$$

$$a_n = \frac{2}{n^2\pi}\left[1-\cos n\pi\right] \quad \text{———} \quad (1)$$

$$a_1 = \frac{4}{\pi}, \quad a_2 = 0, \quad a_3 = \frac{4}{9\pi}, \quad a_4 = 0, \quad a_5 = \frac{4}{25\pi}$$

$$b_n = \frac{2}{P}\int_{-\pi}^{\pi} f(x)\,\sin\left[\frac{2\pi n x}{P}\right] dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{0} (x+\pi)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} (-x+\pi)\sin nx\,dx$$

$$b_n = \frac{1}{\pi}\left[\left[\frac{\sin nx - nx\cos nx}{n^2}\right]_{-\pi}^{0} - \frac{\pi\cos nx}{n}\Bigg|_{-\pi}^{0} - \frac{\sin nx - nx\cos nx}{n^2}\Bigg]_{0}^{\pi} - \pi\frac{\cos nx}{n}\Bigg|_{0}^{\pi}\right]$$

$$b_n = \frac{1}{\pi}\left[-\frac{n\pi\cos n\pi}{n^2} - \cancel{\frac{\pi}{n}} + \frac{\pi\cos nx}{n} + \frac{n\pi\cos nx}{n^2} - \frac{\pi\cos n\pi}{n} + \cancel{\frac{\pi}{n}}\right]$$

$\boxed{b_n = 0}$ for all values of (n).

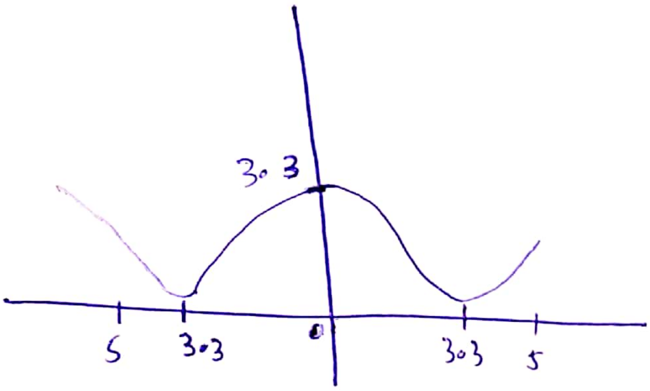
$b_1 = b_2 = b_3 = b_4 = b_5 = 0 = b_n$

Fourier series :-

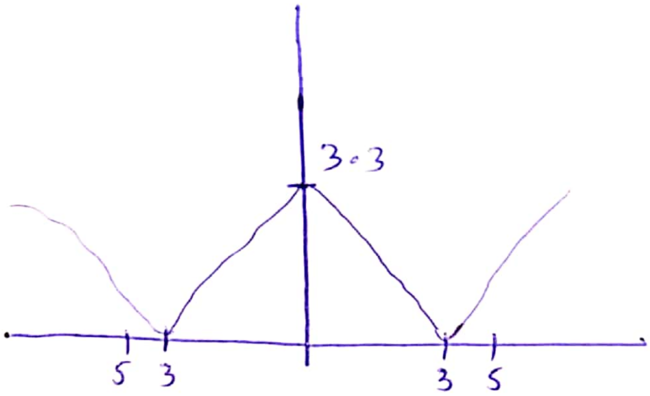
$$f(x) = a_0 + a_1\cos x + a_3\cos 3x + a_5\cos 5x$$

$$\boxed{f(x) = \frac{\pi}{2} + \frac{4\cos x}{\pi} + \frac{4\cos 3x}{9\pi} + \frac{4\cos 5x}{25\pi}}$$

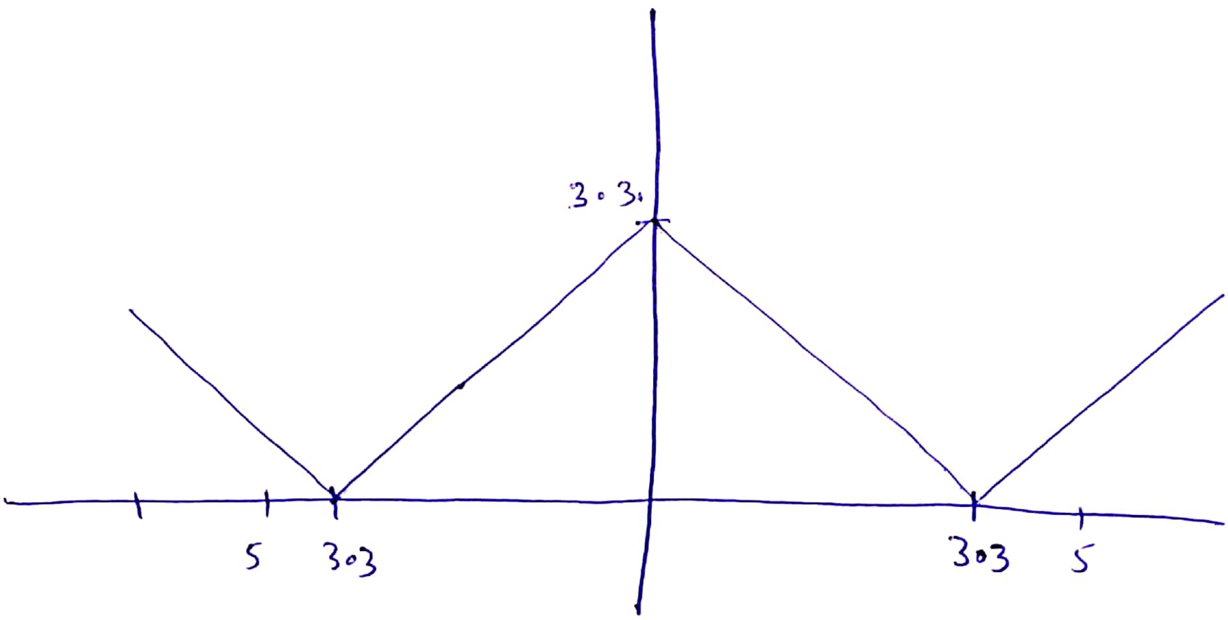
$$S_1 = \frac{\pi}{2} + \frac{4\cos x}{\pi}$$



$$S_2 = \frac{\pi}{2} + \frac{4\cos x}{\pi} + \frac{4\cos 3x}{9\pi}$$



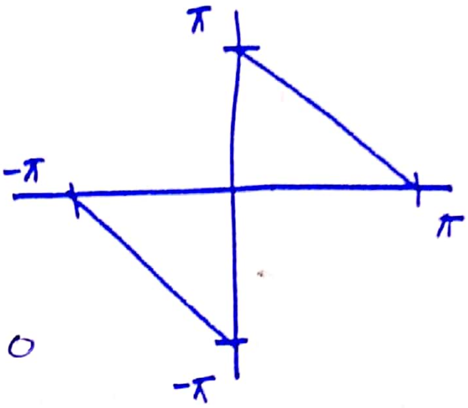
$$S_3 = \frac{\pi}{2} + \frac{4\cos x}{\pi} + \frac{4\cos 3x}{9\pi} + \frac{4\cos 5x}{25\pi}$$



Problem :: 3

ⓐ Solve

$P = 2\pi$



$$f(x) = \begin{cases} -x-\pi & -\pi \le x \le 0 \\ -x+\pi & 0 \le x < \pi \end{cases}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$a_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0}(-x-\pi)\,dx + \int_{0}^{\pi}(-x+\pi)\right] dx$$

$$a_0 = \frac{1}{2\pi}\left[\left[-\frac{x^2}{2}-\pi x\right]_{-\pi}^{0} + \left[-\frac{x^2}{2}+\pi x\right]_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{\pi^2}{2}-\pi^2 + \left(-\frac{\pi^2}{2}+\pi^2\right)\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0}\underbrace{(-x-\pi)}_{\text{odd}}\underbrace{\cos nx}_{\text{even}}\,dx + \frac{1}{\pi}\int_{0}^{\pi}\underbrace{(x-\pi)}_{\text{odd}}\underbrace{\cos nx}_{\text{even}}\,dx$$

$\cancel{a_n = \frac{1}{\pi}\Big[}$ $\boxed{a_n = 0}$

$a_1 = a_2 = a_3 = a_4 = a_5 = 0$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(-x-\pi)\sin nx\,dx + \frac{1}{\pi}\int_{0}^{\pi}(x-\pi)\sin nx\,dx$$

$t = -x,\ dt = -dx \quad \therefore x = -\pi$

$$b_n = \frac{1}{\pi}\left[-\int_{0}^{\pi} t\sin nt\,dt + \pi\int_{0}^{\pi}\sin nt\,dt - \int_{0}^{\pi} x\sin nx\,dx + \pi\int_{0}^{\pi}\sin nx\,dx\right]$$

$$b_n = \frac{1}{\pi}\left\{-\int_{0}^{\pi} x\sin nx\,dx + \pi\int_{0}^{\pi}\sin nx\,dx + \pi\int_{0}^{\pi}\sin nx\,dx\right.$$

$$= \frac{2}{\pi}\left[-\int_{0}^{\pi} x\sin nx\,dx + \pi\int_{0}^{\pi}\sin nx\,dx\right]$$

$$= \frac{2}{\pi}\left[\left[\frac{-1\cos nx}{n} + \frac{1}{n^2}\sin nx\,dx\right]_0^{\pi} - \frac{1}{n}\cos nx\Big|_0^{\pi}\right]$$

$$= \frac{2}{\pi}\left[\pi\left[\frac{(-1)^n}{n}\right] + \frac{\pi(-1)^{n+1}+1}{n}\right.$$

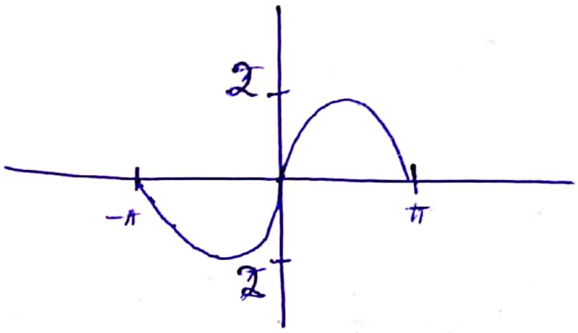
$$b_n = \frac{2}{n} \quad \text{--------} \quad (2)$$

$b_1 = 2,\ b_2 = 1,\ b_3 = \frac{2}{3},\ b_4 = \frac{1}{2},\ b_5 = \frac{2}{5}$

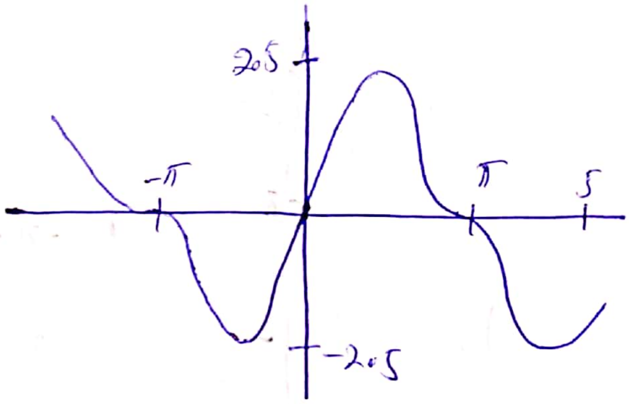
$$f(x) = 2\sin x + \sin 2x + \frac{2}{3}\sin 3x + \frac{\sin 4x}{2} + 2\,\frac{\sin 5x}{5}$$

Scanned with CamScanner

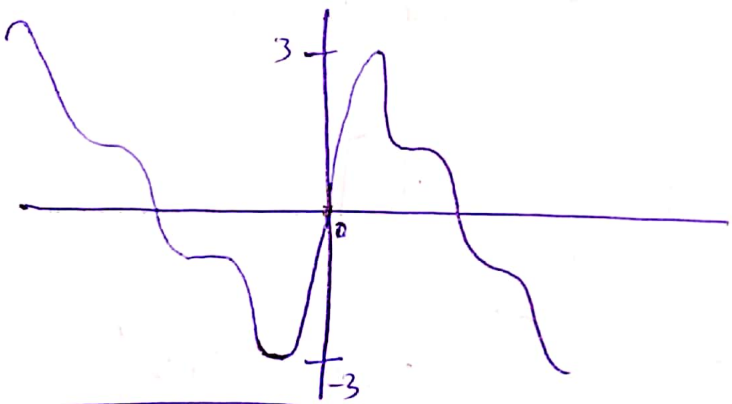
$S_1 = 2\sin x$



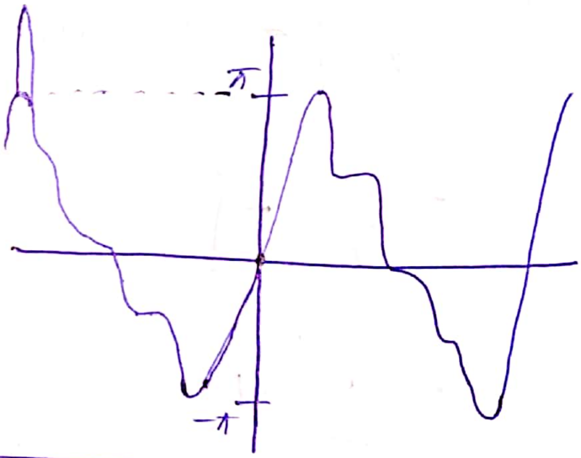
$S_2 = 2\sin x + \sin 2x$



$S_3 = 2\sin x + \sin 2x + \frac{2}{3}\sin 3x$



$S_4 = S_3 + \frac{\sin 4x}{2}$



$S_5 = S_4 + \frac{2\sin 5x}{5}$