MT240:Complex Variables and Transforms

Homework 1

Due Date: 15th April,2021

Max Marks : 100

Spring 2021

Tips to avoid plagiarism:

- Do not copy the solutions of your classmates.
- You are encouraged to discuss the problems with your classmates in whatever way you like but, make sure to REPRODUCE YOUR OWN SOLUTIONS in what you submit for grading.
- Keep your work in a secure place.

Problem 1

Consider the complex numbers $z_1 = z_o + i\bar{z}_o \ and \ z_2 = \bar{z}_o / z_o$, where $z_o = -4 - 4\sqrt{3}i$

(i) Evaluate z_1 and plot it in the complex plane.

(ii) What is $|z_1|$ and $\arg z_1$?

Repeat (i) and (ii) for z_2 .

Problem 2

Find all the complex solutions of the following algebraic polynomials and plot the solutions on the complex plane.

a) $z^5 = 16(\sqrt{3} - i)$
b) $z^8 - 1 = 0$
c) $z^2 - 2z + i = 0$
d) $z^3 - 3z^2 + 6z - 4 = 0$

Problem 3

Represent in polar form.

(i) -5
(ii) $3i$
(iii) $-4 + 4i$
(iv) $\left(\sqrt{2} + \frac{i}{3}\right) / (-\sqrt{8} - \frac{2i}{3})$

Rafay Aamir

Bsee 19047

CVT – Assignment 1

Problem 1

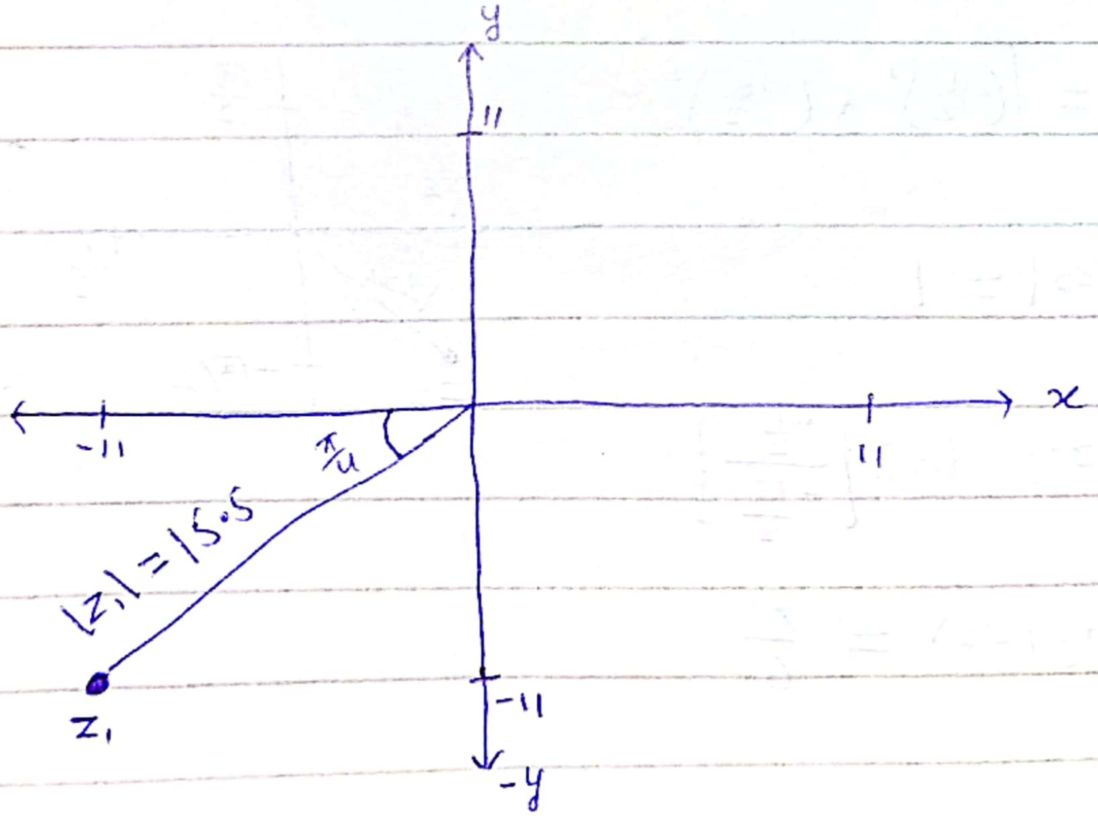
$z_1 = z_0 + i\bar{z}_0 \qquad z_2 = \bar{z}_0 / z_0 \qquad z_0 = -4 - 4\sqrt{3}i$

(i) Evaluate z_1 and plot.

$$z_1 = -4 - 4\sqrt{3}i + i(-4 + 4\sqrt{3}i)$$

$$= -4 - 4\sqrt{3}i - 4i - 4\sqrt{3}$$

$$z_1 = -4 - 4\sqrt{3} + (-4\sqrt{3} - 4)i \approx -11 - 11i$$



(ii) $|z_1|$ and $\arg(z_1)$

$$|z_1| = \sqrt{(-11)^2 + (-11)^2} = 15.5$$

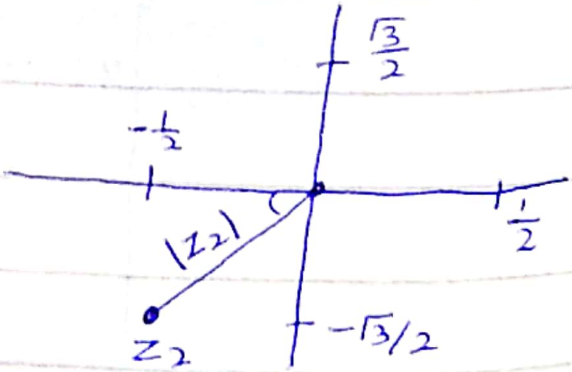
$$\arg(z_1) = \tan^{-1}\left[\frac{-11}{-11}\right] = \frac{\pi}{4}$$

For z_2

(i) $z_2 = \bar{z}_0 / z_0 = \dfrac{-4+4\sqrt{3}i}{-4-4\sqrt{3}i} \times \dfrac{-4+4\sqrt{3}i}{-4+4\sqrt{3}i}$

$$z_2 = \frac{(16-48-32\sqrt{3}i)}{64} = \frac{-1}{2} - \frac{\sqrt{3}i}{2}$$

$$|z_2| = \sqrt{\left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{-\sqrt{3}}{2}\right)^2}$$



$$|z_2| = 1$$

$$\arg(z_2) = \tan^{-1}\left[\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}\right]$$

$$\arg(z_2) = \frac{\pi}{6}$$

Problem 2

(a) $z^5 = 16(\sqrt{3} - i)$

$$z^5 = 16\sqrt{3} - 16i \quad \text{——} \quad A$$

$$r = \sqrt{(16\sqrt{3})^2 + (-16)^2} = 32$$

$$z^n = \sqrt[n]{r}\left[cis\left(\frac{\theta + 2\pi K}{n}\right)\right]$$

$$z^5 = 2\left[cis\left(\frac{\theta + 2\pi K}{5}\right)\right] \quad \text{——} \quad (1)$$

using eq (A).

$$\theta = \tan^{-1}\left[\frac{-16}{16\sqrt{3}}\right], \quad \theta = -\frac{\pi}{6}$$

as x is positive and y(i) is negative so it wold be in 2nd quadrant. so $\theta = 2\pi - \pi/6$, $\theta = \frac{11\pi}{6}$

$$z^5 = 2\,cis\left[\frac{\frac{11\pi}{6} + 2\pi K}{5}\right] \quad \text{——} \quad (2)$$

Here $K = 0, 1, 2, 3, 4$.

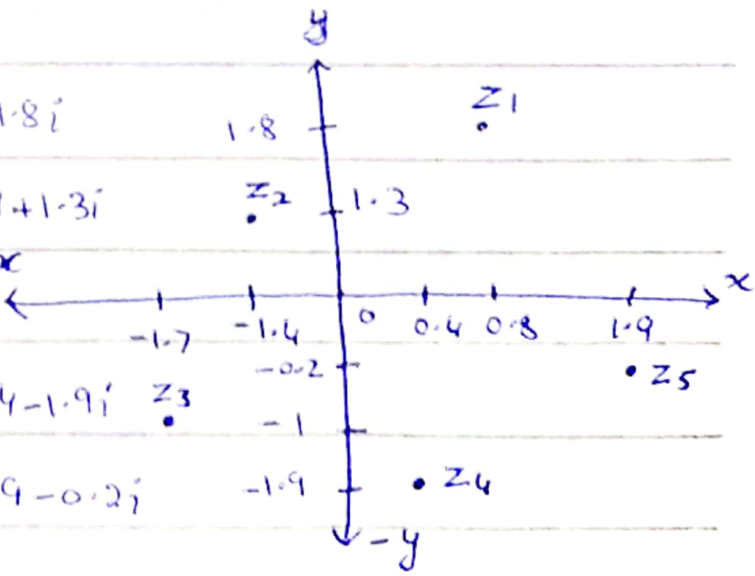
$K=0$, $z_1 = 2\left[cis\left[\frac{11\pi}{30}\right]\right] = 0.8 + 1.8i$

$K=1$, $z_2 = 2\left[cis\left[\frac{23\pi}{30}\right]\right] = -1.4 + 1.3i$

$K=2$, $z_3 = 2\left(cis\left[\frac{35\pi}{30}\right]\right) = -1.7 - i$

$K=3$, $z_4 = 2\left(cis\left(\frac{47\pi}{30}\right)\right) = 0.4 - 1.9i$

$K=4$, $z_5 = 2\left(cis\left[\frac{59\pi}{30}\right]\right) = 1.9 - 0.2i$



(b) $z^8 - 1 = 0$

Solve $\quad z^8 = 1 + 0i$

$r = 1 \quad , \quad \theta = \cancel{\frac{\pi}{2}}\, 0$

$$z^8 = cis\left[\frac{0 + 2k\pi}{8}\right] \quad \text{——— } (1)$$

Here $\quad n = 8, \quad k = 0, 1, 2, 3, 4, 5, 6, 7$

$k = 0, \; z_1 = cis(0) \quad = \quad 1 + 0i$

$k = 1, \; z_2 = cis\left(\frac{\pi}{4}\right) \quad = \quad 0.7 + 0.7i$

$k = 2, \; z_3 = cis\left(\frac{\pi}{2}\right) \quad = \quad 0 + i$

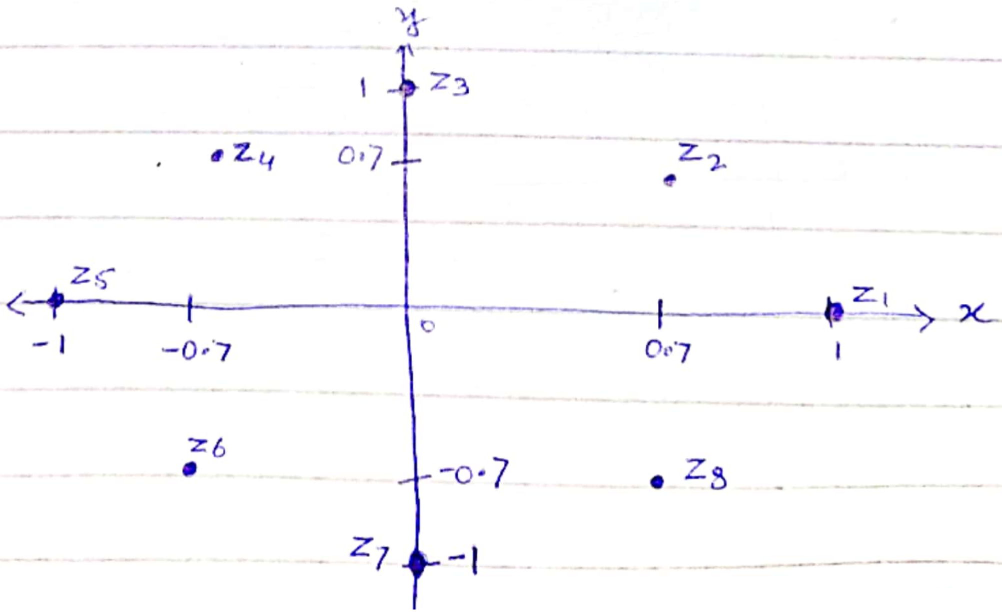
$k = 3, \; z_4 = cis\left(\frac{3\pi}{4}\right) \quad = \quad -\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = -0.7 + 0.7i$

$k = 4, \; z_5 = cis(\pi) \quad = \quad -1 + 0i$

$k = 5, \; z_6 = cis\left(\frac{5\pi}{4}\right) \quad = \quad -0.7 - 0.7i$

$k = 6, \; z_7 = cis\left(\frac{3\pi}{2}\right) \quad = \quad 0 - i$

$k = 7, \; z_8 = cis\left(\frac{7\pi}{4}\right) \quad = \quad \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0.7 - 0.7i$



(c) $z^2 - 2z + i = 0$

Solve $$z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(i)}}{2(1)}$$

$$z = 1 \pm \sqrt{1-i} \text{ ——— } (1)$$

let $l^2 = 1 - i$

$r_1 = \sqrt{2}$, $\theta = \frac{7\pi}{4}$

$$l = \sqrt[4]{2}\left[\operatorname{cis}\left[\frac{\frac{7\pi}{4} + 2k\pi}{2}\right]\right]$$

$z^2 - 2z + i = 0$, $z^2 - 2z + 1 - 1 + i = 0$

$\cancel{(z-2)^2}$ $(z-1)^2 + 1 + i$

$(z-1)^2 - 1 + i = 0$

(d) $z^3 - 3z^2 + 6z - 4 = 0$

Solve

$$\begin{array}{r|l} & z^2 - 2z + 4 \\ \hline z-1 & z^3 - 3z^2 + 6z - 4 \\ & -z^3 + z^2 \\ \hline & -2z^2 + 6z - 4 \\ & 2z^2 - 2z \\ \hline & 4z - 4 \\ & -4z + 4 \\ \hline & 0 \end{array}$$

$(z-1)(z^2 - 2z + 4) = 0$

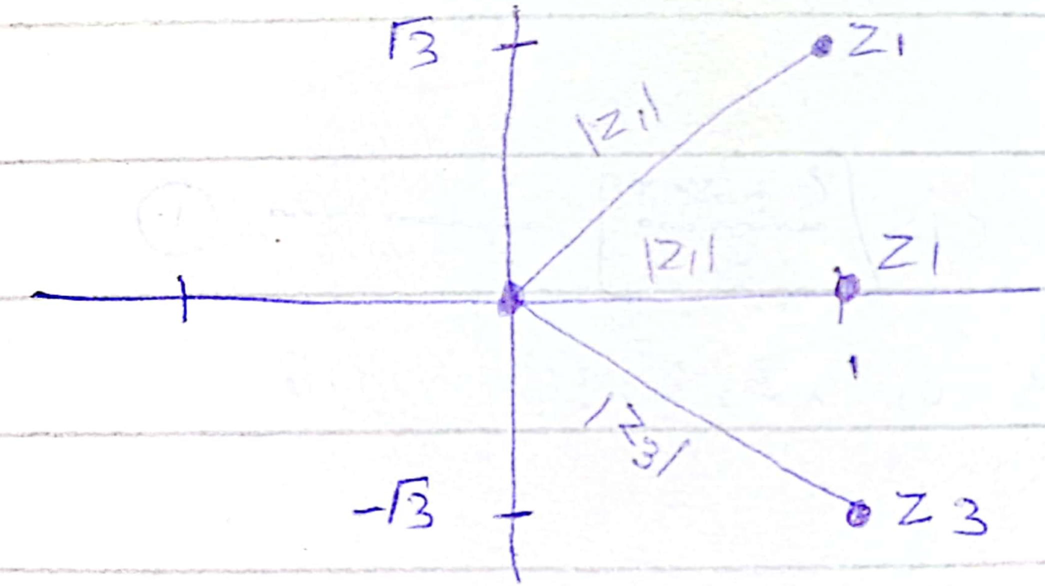
$z - 1 = 0$, $z^2 - 2z + 4 = 0$

Now applying quadratic formula

$$z = \frac{+2 \pm \sqrt{4 - 16}}{2}$$

$$z = 1 \pm \sqrt{-3}$$

$$\boxed{\begin{aligned} z_1 &= 1 + 0i \\ z_2 &= 1 + \sqrt{3}\, i \\ z_3 &= 1 - \sqrt{3}\, i \end{aligned}}$$



√3
-√3
|z1|
|z1|
|z3|
z1
z1
z3
1

Q:-3

(a) $-5+0i$

$r = 5$, $\theta = 0$

so,

$z = 5 \text{ cis}[0]$

(b) $3i + 0$

$r = 3$, $\theta = \frac{\pi}{2}$

so

$z = 3 \text{ cis}\left[\frac{\pi}{2}\right]$

(c) $-4-4i$

$r = 4\sqrt{2}$, $\theta = \frac{\pi}{4}$

so

$z = 4\sqrt{2} \text{ cis}\left[\frac{\pi}{4}\right]$

(d) $\left(\sqrt{2} + \frac{i}{3}\right) / \left(-\sqrt{8} - \frac{2i}{3}\right) = \frac{1}{2} + 0i$

$\cancel{r = 0}$ $r = \frac{1}{2}$, $\theta = 0$

$z = \frac{1}{2} \text{ cis}(0)$

(c) $z^2 - 2z + i = 0$

Solve

$z - \sqrt{z^2 - 2z + i}$

$$Z = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(i)}}{2}$$

$$z = 1 \pm \sqrt{1 - i} \quad \text{———} \quad (1)$$

Let $y = \sqrt{1 - i}$, $y^2 = 1 - i$

$r = \sqrt{2}$, $\theta = \frac{7\pi}{4}$

$$y^n = \sqrt[n]{2}\left[\operatorname{cis}\left[\frac{\frac{7\pi}{4} + 2\pi K}{n}\right]\right]$$

$n = 2$, $K = 0, 1$

$K = 0$, $y_1 = 2 \operatorname{cis} \frac{7\pi}{8}$

$K = 1$, $y_2 = 2 \operatorname{cis} \frac{15\pi}{8}$

substituting y in (1)

$z = 1 \pm y_1$, $z = 1 \pm y_2$

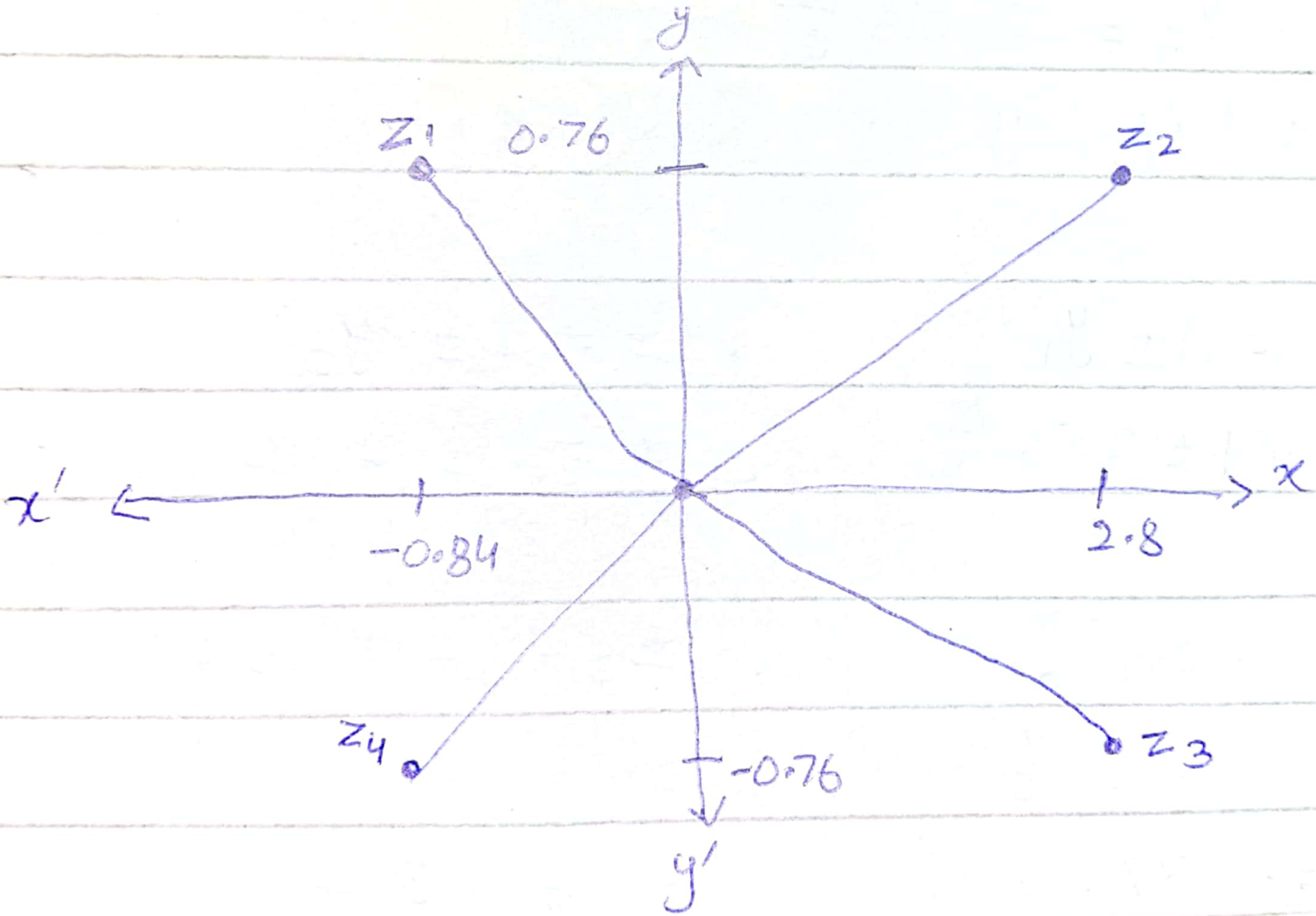
$z = 1 \pm 2\operatorname{cis} \frac{7\pi}{8}$, $z = 1 \pm 2\operatorname{cis} \frac{15\pi}{8}$

$z_1 = 1 + 2\operatorname{cis} \frac{7\pi}{8}$, $z_3 = 1 + 2 \operatorname{cis} \frac{15\pi}{8}$

$z_2 = 1 - 2\operatorname{cis} \frac{7\pi}{8}$, $z_4 = 1 - 2 \operatorname{cis} \frac{15\pi}{8}$

$z_1 = -0.84 + 0.7i$, $z_3 = 2.8 - 0.76i$

$z_2 = 2.8 + 0.7i$, $z_4 = -0.84 - 0.76i$



y
z_1
0.76
z_2
x'
x
-0.84
2.8
z_4
z_3
-0.76
y'