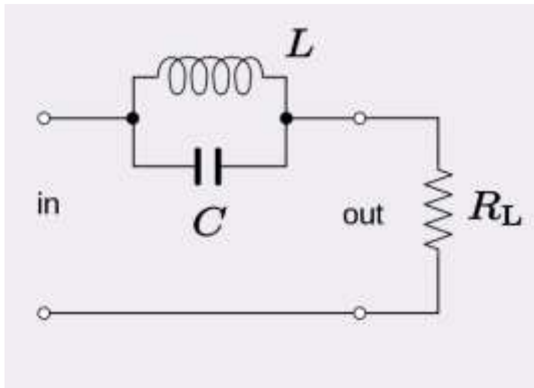
Control Systems Assignment # 2

Problem 1

Develop a mathematical model in state space for the following circuit using first principles.



Write down the variables and constants, define which variables are state variables and which variables are input and output variables.

Solution: Apply KVL:

$$V_{in} = \frac{1}{Cs + \frac{1}{Ls}} I + R_L I$$

$$\rightarrow \frac{V_{in}}{R_L I} = \frac{Ls}{R_L L C s^2 + R_L} + 1$$

$$\rightarrow \frac{V_{in}}{V_{out}} = \frac{Ls + R_L L C s^2 + R_L}{R_L L C s^2 + R_L}$$

$$\rightarrow G(s) = \frac{V_{out}}{V_{in}} = \frac{s^2 + \frac{1}{LC}}{s^2 + \frac{1}{R_L C} s + \frac{1}{LC}}$$

$$\rightarrow \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{1}{R_L C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$\rightarrow y = \begin{bmatrix} 0 & -\frac{1}{R_L C} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + u$$

where, $x_1 = \bar{y}, x_2 = \dot{x_1},\ y = \ddot{\bar{y}} + \frac{1}{LC}\bar{y} = \dot{x_2} + \frac{1}{LC}x_1$

Problem 2

Construct a mathematical (state space) model for the spread of an epidemic disease. Consider the state variables as follows

1. Number of people susceptible to the disease (S)
2. Number of people infected by the disease (I)
3. Number of people recovered from the disease after being infected (R)

Consider the input variable to be vaccination (V) and treatment (T). The output is the number of infected people (I).

Solution: Let us assume that susceptible people get infected with rate μ and if a susceptible individual is vaccinated, it is counted as a recovered individual.

$$\dot{S} = -\mu S - \alpha V$$
$$\dot{I} = \mu S - \beta T$$
$$\dot{R} = \alpha V + \beta T$$

Problem 3

Find the equilibrium points and linearize the following systems

a) $\dot{x} = 5x^2 + 3x + 9$

Solution: For equilibrium points, put $\dot{x} = 0 \rightarrow 5x^2 + 3x + 9 = 0 \rightarrow x = -0.3 \pm 1.3j$ (No real equilibrium point)

b) $\dot{x} = -x^3 - x$

Solution: Only one real equilibrium point i.e. $x = 0$ therefore, the linear system around the equilibrium point is $\dot{x} = -x$

c) $\dot{x} = \cos(x) + \sin(x), (-\pi \leq x \leq \pi)$

Solution: there are two equilibrium points in the given range i.e.

$$x = 2.3562\ (135^o) and\ x = -0.7854(-45^o)$$

Therefore, the two linear systems are

$$\dot{x} = -1.4142x\ (at\ x^* = 135^o), \dot{x} = 1.4142x\ (at\ x^* = -45^o)$$