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Bsee 19047

Control Sys A # 4

Q:-1

(a) $G(s) = \dfrac{s+1}{s^2+2s+2} = \dfrac{s+1}{(s+1+j)(s+1-j)}$

① open loop poles :- $s=-1+j$, $s=-1-j$ (two poles)

② open loop zeros :- $s=-1$, (one zero)

③ no of branches :- 2-branches are there are two poles , # of asymptotes = 1

④ Asymptotes :- $\theta = \dfrac{(2K+1)\pi}{1}$, here $K = 1-1 = 0$, so, $\theta_0 = \pi$

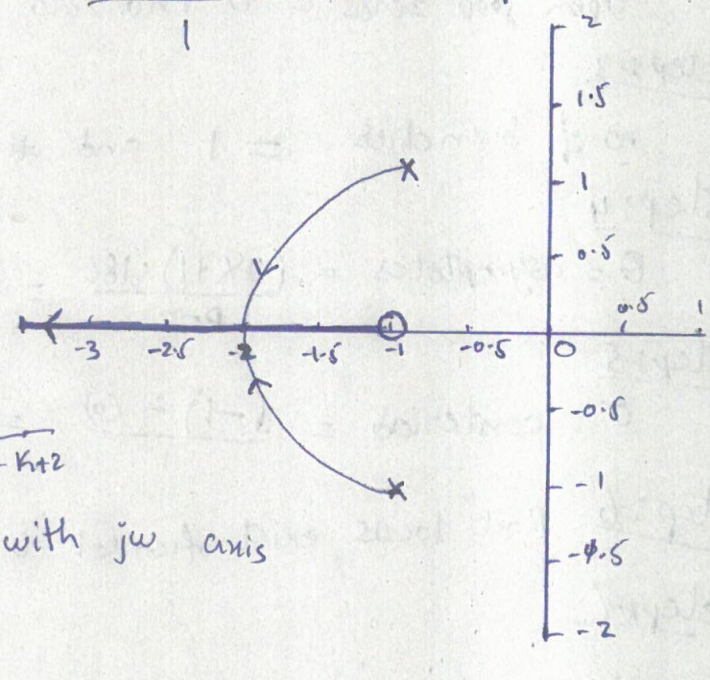
⑤ centroid :- $\sigma = \dfrac{\Sigma(\text{poles}) - \Sigma(\text{zeros})}{1}$, $\sigma = \dfrac{(-2)-(-1)}{1} = -1$

⑥ Root locus shown below

⑦ Angle of departure and ∠ of arrival

$\theta p_1 = 180° - (\theta p_2 - \theta z_1) = 180°$

$\theta p_2 = 180°$



⑧ Intersection with jw axis.

closed loop TF $= \dfrac{K(s+1)}{s^2+2s+2+K(s+1)} = \dfrac{K(s+1)}{s^2+(K+2)s+K+2}$

$K = -2$ so, it will not intersect with jw axis

$s^2 + (K+2)s + K+2$

s^2	1	K+2
s	K+2	0
s^0	K+2	0

⑨ Breakaway point / Break-in - point

$\dfrac{d}{ds}\left\{\dfrac{s+1}{s^2+2s+2}\right\} = 0$, $(s^2+2s+2)(1) - (s+1)(2s+2) = 0$

, $s^2+2s+2-2s^2-4s-2=0$

, $s^2+2s=0$, $s(s+2)=0$

$s=0$, $s=-2$

as the centroid is at -1 and RL does not exist at 0 so, break-in point will be $s = -2$

RL - exist

Q1 (b) $\quad G(s) = \dfrac{s^3 + 5s^2 + 9s + 12}{s^4 + 7s^3 + 2s^2 + s - 1}$

Solve

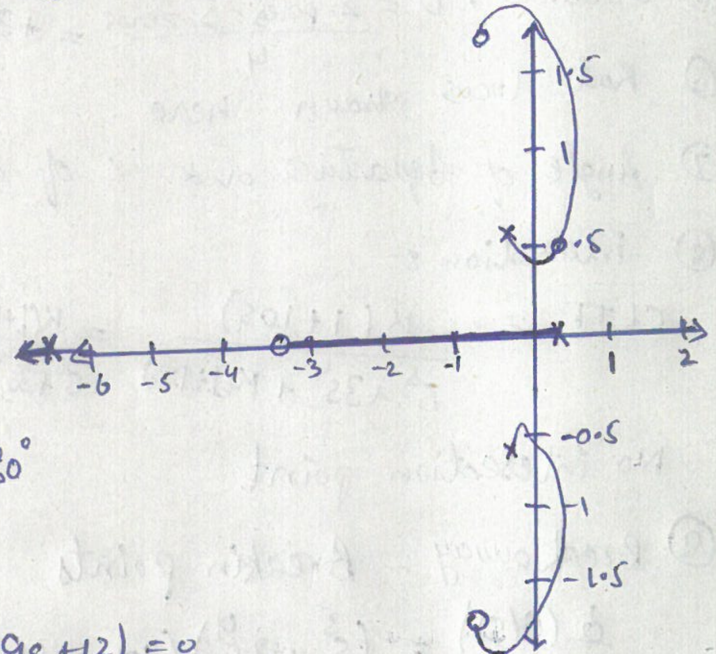
① open loop poles :→ $s = -6.74, 0.367, -0.303 + 0.54j, -0.303 - 0.54j$

② open loop zeros :→ $s = -3.39, -0.805 + 1.7j, -0.805 - 1.7j$

③ No of branches $= 4$, No of Asymptotes $= (4) - (3) = 1$

④ $\angle$ of Asymptotes $= \dfrac{(2k+1)\pi}{1}$, $k = 0$, $\theta_0 = \pi = 180^\circ$

⑤ Centroid :- $\sigma = ($



⑥ Root locus shown here

⑦ $\theta_{z_1} = 0^\circ$, $\theta_{z_2} = 52.4^\circ$

$\theta_{z_3} = -52.4^\circ$, $\theta_{p_1} = 180^\circ$, $\theta_{p_2} = 180^\circ$

$\theta_{p_3} = -36.62^\circ$, $\theta_{p_4} = 36.62^\circ$

⑧ intersection :-

$s^4 + 7s^3 + 2s^2 + s - 1 + K(s^3 + 5s^2 + 9s + 12) = 0$

$s^4 + (K+7)s^3 + (5K+2)s^2 + (9K+1)s + 12K - 1 = 0$

s^4	1	$5K+2$	$12K-1$
s^3	$K+7$	$9K+1$	0
s^2	A	$12K-1$	0
s	B	0	0
s^0	$12K-1$	0	0

Now to find K, $B = 0$ and $K = 2.39$

and $12K - 1 = 0$, $K = \frac{1}{12}$

$K = 0.15$, ~~2.09~~ 2.39.

$As^2 = \dfrac{(5K^2 + 28K + 13)}{K+7}$, $\quad s = \pm 0.58j, \pm 1.55j$

$A = \dfrac{(K+7)(5K+2) - 9K+1}{K+7}$

$A = \dfrac{5K^2 + 28K + 13}{K+7}$

$B = \dfrac{A(9K+1) - (K+7)(12K-1)}{A}$

⑨ Break-in - Break away point

$\dfrac{d}{ds}[G(s)] = 0$

$(s^4 + 7s^3 + 2s^2 + s - 1)(3s^2 + 10s + 9) - (s^3 + 5s^2 + 9s + 12)(4s^3 + 21s^2 + 4s + 1) = 0$

$s^6 + 10s^5 + 60s^4 + 172s^3 + 262s^2 + 38s + 3 = 0$

No break away point

$s + 3s$ $\quad s(s^{?} + 3s)$ $\quad s(s^4 + 3s^3)$

Q1(c)

① Open loop poles $= s = 0, 0, 0, 0, -3$

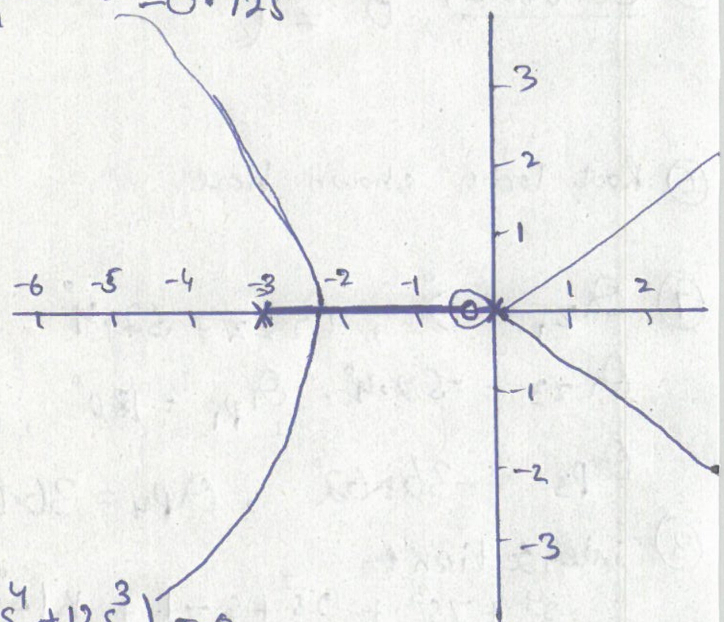
② Open loop zeros $= s = -0.1$

③ No of branches $= 5 = 5$, No of Asymptotes $= 5 - 1 = 4$

④ Asymptotes : $\theta = \frac{(2K+1)\pi}{4}$, $K = 0, 1, 2, 3$, $\theta_0 = 45°$, $\theta_1 = 135°$, $\theta_2 = 225°$, $\theta_3 = 315°$

⑤ Centroid :- $\sigma = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{4} = \cancel{+3+} \frac{-3+0.1}{4} = -0.725$

⑥ Root locus shown here.



⑦ Angle of departure and $\angle$ of arrival

⑧ intersection :-

$$CLTF = \frac{K(1+10s)}{s^5 + 3s^4 + K(1+10s)} = \frac{K(1+10s)}{s^5 + 3s^4 + 10Ks + K}$$

No intesection point

⑨ Breakaway - Breakin points

$$\frac{d}{ds}(G(s)) = (s^5 + 3s^4)(10) - (1 + 10s)(5s^4 + 12s^3) = 0$$

$$10s^5 + 30s^4 - 50s^5 - 5s^4 - 12s^3 - 120s^4 = 0$$

$$40s^5 + 95s^4 + 12s^3 = 0$$

$$s = 0, 0, 0, -23.812, -23.812$$

s^5	40	12
s^4	95	
s^3		

s^5	1	0	10K
s^4	3	0	K
s^3	ε	$\frac{29K}{3}$	0
s^2	$\frac{-29K}{\varepsilon}$	K	0
s			
s^0			

Q:-2

ⓐ $G(s) = \frac{1}{(s^2+4s+4)(s+3)}$, $C(s) = \frac{5K}{s+5}$

CLTF $= \frac{C(s)G(s)}{1+C(s)G(s)} \Rightarrow \frac{5K}{(s^2+4s+4)(s+3)(s+5)+5K} = \frac{5K}{s^4+12s^3+51s^2+92s+(60+5K)}$

Routh Harwitz Table

s^4	1	51	60+5K
s^3	12	92	0
s^2	$\frac{130}{3}$	60+5K	0
s	$\frac{9800-180K}{130}$	0	0
s^0	60+5K	0	0

$\frac{\frac{130}{3}\times 92 - 12(60+5K)}{\frac{130}{3}}$

$\frac{9800-180K}{130}$

$K<0$ for last row of RHT

$\frac{9800-180K}{130} = 0$

$180K = 9800$

$K = 54.44$, at which Root locus cuts the $j\omega$ axis-

Gain of Root locus, $K = 54.44$.

Gain of controller, $5K = 272.222$

ⓑ $G(s) = \frac{1}{(s-2)^4}$, $C(s) = K_1 + K_2 s$

CLTF :- $\frac{K_1+K_2 s}{(s-2)^4+K_1+K_2 s} \Rightarrow \frac{K_1+K_2 s}{s^4+(-8s^3)+24s^2+(K_2-32)s+K_1+16}$

s^4	1	24	K_1+16
s^3	-8	K_2-32	0
s^2	$\frac{K_2+160}{8}$	K_1+16	0
s	A	0	0
s^0	K_1+16	0	0

$A = \frac{\left[\frac{K_2+160}{8}\right][K_2-32] - (-8)(K_1+16)}{\frac{K_2+160}{8}}$

$= \frac{(K_2+160)(K-32)+64(K_1+16)}{K_2+160}$

$(K_2+160)(K_2-32)+64(K_1+16) = 0$

$K_2^2 + 128K_2 - 5120 + 64K_1 + 1024 = 0$

solve

$$s^3 + 3ps^2 + 7ps - 9p = 0$$

$$s^3 + p(3s^2 + 7s - 9) = 0$$

Transfer function is $= \dfrac{3s^2 + 7s - 9}{s^3}$

Poles $= s_1 = s_2 = s_3 = 0$

zeros $= z_1 = 0.9216$, $z_2 = -3.255$

There will be (3) Three branches and one asymptote

$$\theta = \frac{(2K+1)\pi}{2-3} = -\pi = \pi$$

$$\sigma = \frac{\Sigma(P) - \Sigma(z)}{3-2} = 2.334 \text{ (centroid)}$$

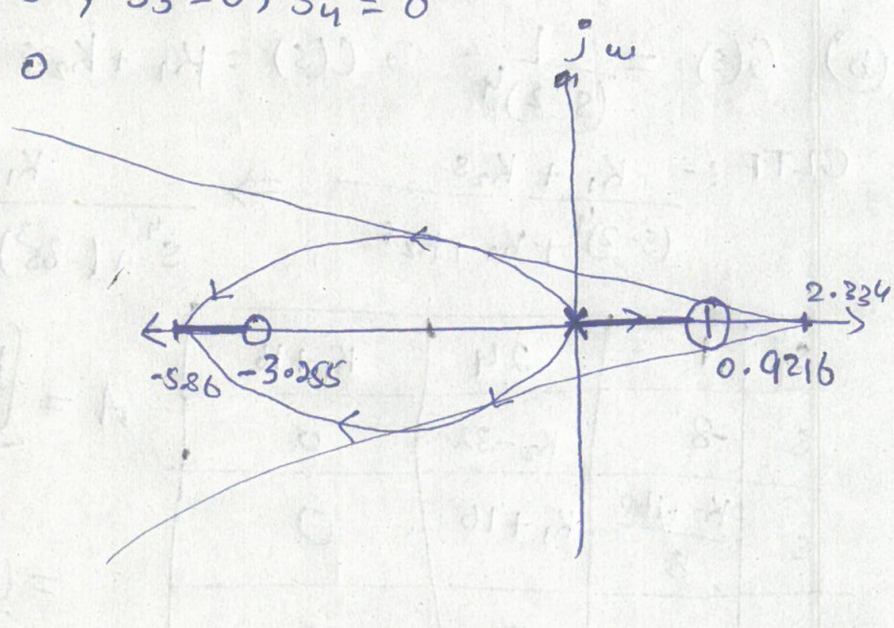
Root locus exists on real axis at $(-\infty, -3.255)$ & $[0, 0.9216]$

$$\frac{d}{ds}[G(s)] = \frac{s^3(6s+7) - (3s^2 + 7s - 9)(3s^2)}{(s^3)^2} = 0$$

$$-3s^4 - 14s^3 + 21s^2 = 0$$

$s_1 = 1.29$, $s_2 = -5.86$, $s_3 = 0$, $s_4 = 0$

RL cuts $j\omega$ axis at 0



(b) $G(s) = \dfrac{s+1}{9s^2+3s+1}$

Solve

$9s^2+3s+1 = 0 \quad , \quad 9s^2 + (3s+1) = 0$

$G(s) = \dfrac{s^2}{3s+1}$ as $z>s$ so replacing numirator with deno

$G(s) = \dfrac{3s+1}{s^2}$

poles $= s = -1/3$, zeros $= z_1 = z_2 = 0$

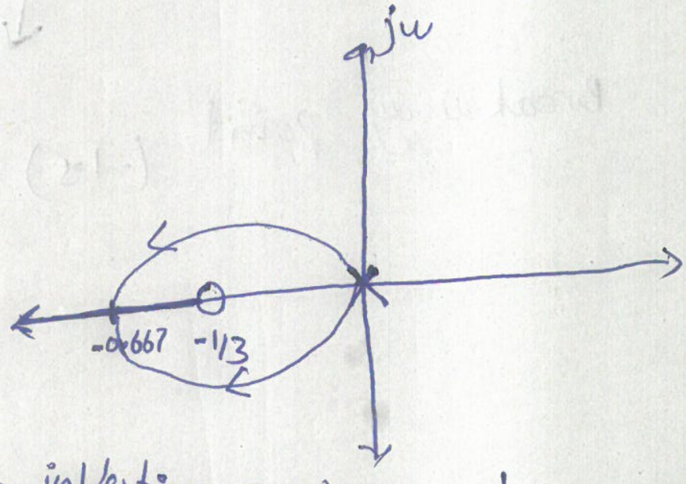
Branches $= 1$,

$\theta_0 = \dfrac{(2K+1)\pi}{1-2} = -\pi$, $\sigma = \dfrac{\Sigma(P) - \Sigma(Z)}{1-2} = 1/3$ (centroid)

Breakaway/Breakin points.

$\dfrac{d}{ds}\left[\dfrac{3s+1}{s^2}\right] = 0$, $s^2(3) - (3s+1)(2s) = 0$, $S_1 = 0$, $S_2 = -0.667$

RL cuts $j\omega$ axis at $x = s = 0$ and $[-\infty, \frac{-1}{3}]$ on real axis RL exists



(c) $G(s) = \dfrac{1}{Ps^2+3Ps-P}$

Solve

$Ps^2 + 3Ps - P = 0$

$0 + P(s^2 + 3s - 1) = 0$

$G(s) = \dfrac{s^2+3s-1}{1}$ $z >$ poles so inverting $(G(s)) = \dfrac{1}{s^2+3s-1}$

poles $= s_1 = 0.3027$, $s_2 = -3.3$

zeros = no zero

$\theta_0 = \dfrac{(2K+1)\pi}{2} = 90°$, $\theta_1 = 270° = -90°$

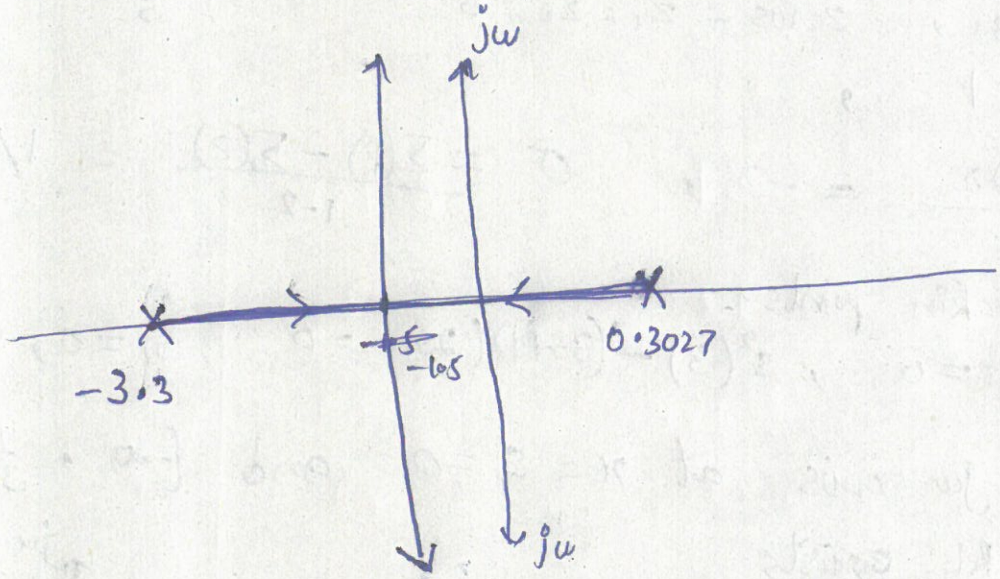
$\sigma = \dfrac{\Sigma(P) - \Sigma(Z)}{2} = -1.5$

RL exists from -3.3 to 0.3027

Breakaway / Breakin points

$s = -1.5$

Pole will cut the jw axis before breaking out



Breakaway point (-1.5)