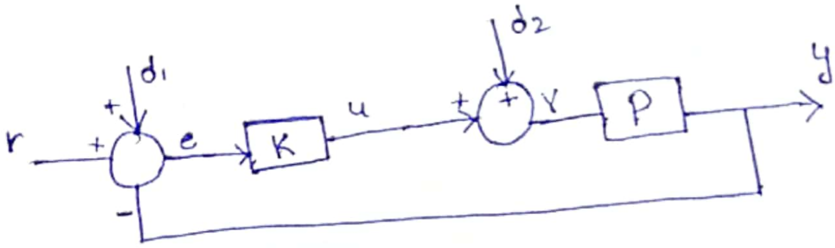
Rafay Aamir

BSce 19047

Control-Sys-Assignment 1

Q:-1

(a)



$y = Pv + d_2$

$y = Pu + d_2$

$y = P(Ke) + d_2$

$y = PK(r + d_1 - y) + d_2 \Rightarrow PKr + PKd_1 - PKy + d_2$

$y(1+PK) = PK(r+d_1) + d_2$

$y = \dfrac{PK(r+d_1)}{(1+PK)} + \dfrac{d_2}{(1+PK)}$ ——, $K = \dfrac{y - d_2}{P(r + d_1 - y)}$ —— (i)

(b) Yes we can, as the system is working under the feedback control loop so eventually output will become equals to the desired value, $(y = r)$

(c) $P = 5,\ d_1 = 0,\ d_2 = 3,\ r = 10,\ K?$

(i) $y = 99\%$ of r, $y = 9.9$

substituting values ing eq (1)

$K = \dfrac{9.9 - 3}{5(10 - 9.9)}$, $\boxed{K = 13.8}$

(ii) $y = 95\%$ of r, $y = 9.5$

$K = \dfrac{9.5 - 3}{5(\cancel{100}\ 10 - 9.5)}$, $\boxed{K = 2.6}$

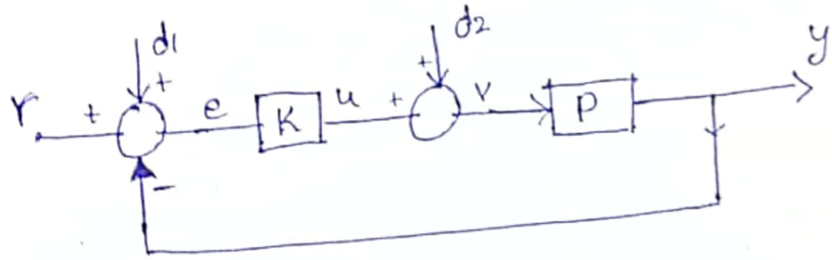
Rafay Aamir

Bsee 19047

Control systems A1

Q:-1

(a)



$$y = Pv$$

$$y = P(u + d_2)$$

$$y = P(Ke + d_2)$$

$$y = P(K(r + d_1 - y) + d_2) \longleftarrow (1)$$

$$y + PKy = P(Kr + Kd_1 + d_2)$$

$$y(1 + PK) = P(Kr + Kd_1 + d_2)$$

$$y = \frac{P(Kr + Kd_1 + d_2)}{1 + PK} \quad , \quad K = \frac{y - Pd_2}{P(r + d_1 - y)}$$

(b) Yes, we can, as the system is working under the feedback control loop so eventually the output will become equals to the desired value, $(y = r)$.

(c) $P = 5$, $d_1 = 0$, $d_2 = 3$, $r = 10$

(i) $K = ?$, $Y = 9.9$

$$K = \frac{9.9 - 5 \times 3}{5(10 - 9.9)} = \boxed{-10.2}$$

(ii) $K = ?$, $Y = 9.5$

$$K = \frac{9.5 - 5 \times 3}{5(10 - 9.05)} = \boxed{-2.2}$$

Q:- 2

(a) $\dddot{y} = 3u$

Solve

$$s^3 y(s) = 3u(s)$$

$$\frac{y(s)}{u(s)} = \frac{3}{s^3}$$

(b) $2\ddot{y} = 5\dot{u} + 9u.$

Solve

$$2s^2 y(s) = 5s\,u(s) + 9u(s)$$

$$\frac{y(s)}{u(s)} = \frac{5s+9}{2s^2}$$

(c) $\dot{y} = u + 1$

Solve

$$s\,y(s) = u(s) + \frac{1}{s}$$

$$\frac{y(s)}{u(s)} = \frac{(s+1)}{s^2}$$

(c) $\dot{y} = u + 1$

Solve

$$s(y(s)) = u(s) + 1u(s)$$

$$\frac{y(s)}{u(s)} = \frac{2}{s}$$

(c) $\dot{y} = u + 1$

Solve

$$s\,y(s) = u(s)$$

$$\frac{y(s)}{u(s)} = \frac{1}{s}$$

or the Transfer function is not possible to write as there is a constant value in the expression.

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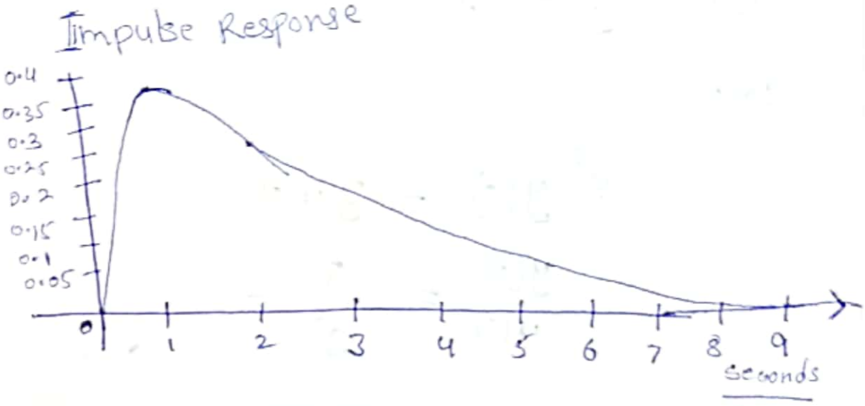
Q:- 3

ⓐ $G(s) = \frac{1}{(s+1)^2}$

solve By ILT

$$G(t) = t e^{-t} u(t)$$



ⓑ $G(s) = \frac{s+1}{s^2+3s+1}$

Solve

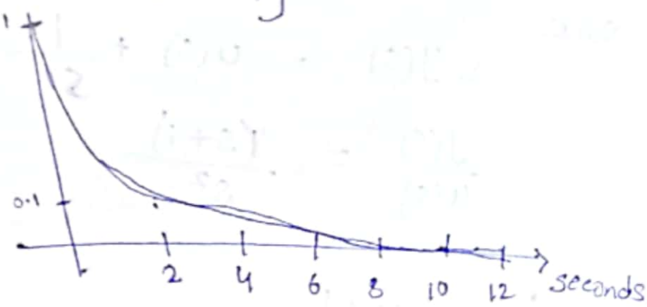
$$= \cancel{\frac{s+1}{s^2+2s+s+1}} \quad \frac{s+1}{s^2+3s+1} = \frac{s+\frac{3}{2}}{\left(s+\frac{3}{2}\right)^2 - \frac{5}{4}} - \frac{1}{2\left(\left(s+\frac{3}{2}\right)^2 - \frac{5}{4}\right)}$$

Inverse Laplace:-

$$= e^{-\frac{3t}{2}} \cosh\left(\frac{\sqrt{5}}{2} t\right) - \frac{1}{2} e^{-\frac{3t}{2}} \sinh\left(\frac{\sqrt{5}}{2} t\right)$$

$$G(t) = \left[e^{-\frac{3t}{2}} \cosh\left(\frac{\sqrt{5}\, t}{2}\right) - \frac{1}{2} e^{-\frac{3t}{2}} \sinh\left(\frac{\sqrt{5}\, t}{2}\right) \right] u(t)$$

Impulse response graph:-



ⓒ $G(s) = \frac{1}{s^2+3s-1}$

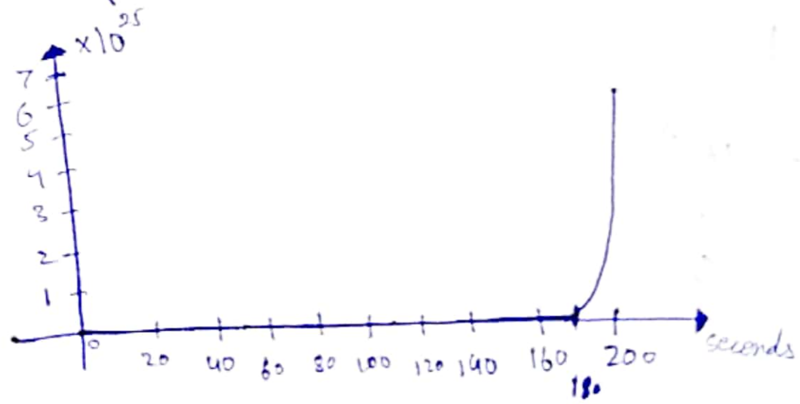
solve

$$G(s) = \frac{1}{\left(s+\frac{3}{2}\right)^2 - \frac{13}{4}}$$

inverse laplace

$$G(t) = e^{-\frac{3t}{2}} \sinh\left(\frac{\sqrt{13}\ t}{2}\right) u(t)$$

impulse response



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Q:-4

(a) $G(s) = \dfrac{5}{s^3 + 3s^2 + 2s + 1}$

Solve

$$\dddot{y} + 3\ddot{y} + 2\dot{y} + y = 5u$$

Let $x_1 = y$, $x_2 = \dot{y}$, $x_3 = \ddot{y}$, $\dot{x}_1 = x_2 = \dot{y}$, $\dot{x}_2 = x_3 = \ddot{y}$

and

$$\dot{x}_3 = \dddot{y} = 5u - 3x_3 - 2x_2 - x_1$$

$$\dot{x} = Ax + Bu \quad , \quad y = Cx + D$$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} , \; x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0$$

Now from the next questions and onwards i will be writing the SSR directly just to save time

(b) $G(s) = \dfrac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$

Solve

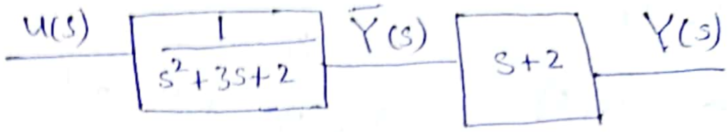
$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = \omega^2 u$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & \cancel{-\omega^2} \; -2\zeta\omega \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega^2 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

ⓒ $G(s) = \dfrac{s+1}{s^2+3s+2}$

Solve



$G(s) = \dfrac{1}{s^2+3s+2}\,(s+1)$

$\ddot{\bar{y}} + 3\dot{\bar{y}} + 2\bar{y} = u$

$x_1 = \bar{y}\ , \quad x_2 = \dot{\bar{y}}\ , \quad \dot{x}_1 = x_2 = \dot{\bar{y}}\ , \quad \dot{x}_2 = \ddot{\bar{y}} = u - 2\bar{y} - 3\dot{\bar{y}}$

$\dot{x}_2 = u - 2x_1 - 3x_2$

And

$y = \cancel{5}\dot{\bar{y}} + 2\bar{y} = \cancel{5}$

$y = \dot{\bar{y}} + 2\bar{y} = 2x_1 + x_2$

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\begin{bmatrix} u \end{bmatrix}$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0$$

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Q:- 5

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 9u$$

Solve

$$A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix}, \quad D = 9$$

As we know $G(s) = C(sI - A)^{-1} B + D$ $\because$ why it is independent of (D)?

$$G(s) = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix} \left(s \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 5 \\ 2 & 1 & 0 \\ 4 & 3 & 1 \end{bmatrix} \right)^{-1} \times \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + 9$$

$$G(s) = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix} \underbrace{\left(\begin{bmatrix} s-1 & 0 & 5 \\ 2 & s-1 & 0 \\ 4 & 3 & s-1 \end{bmatrix} \right)^{-1}}_{Z} \times \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + 9$$

$$|Z| = (s-1)^3 + 30 - 20s + 20 \quad = s^3 - 1 - 3s^2 + 3s - 20s + 50$$

$$|Z| = s^3 - 3s^2 - 17s + 49 \quad \text{——— (1)}$$

$$\operatorname{Adj}(Z) = \begin{bmatrix} s^2 - 2s + 1 & 15 & 5 - 5s \\ 2 - 2s & s^2 - 2s - 19 & 10 \\ 10 - 4s & 3 - 3s & s^2 - 2s + 1 \end{bmatrix}$$

$$Z^{-1} = \frac{1}{s^3 - 3s^2 - 17s + 49} \times \begin{bmatrix} s^2 - 2s + 1 & 15 & 5 - 5s \\ 2 - 2s & s^2 - 2s - 19 & 10 \\ 10 - 4s & 3 - 3s & s^2 - 2s + 1 \end{bmatrix}$$

$$G(s) = \begin{bmatrix} 0 & 0 & 7 \end{bmatrix} \times \frac{1}{s^3 - 3s^2 - 17s + 49} \times \begin{bmatrix} s^2 - 2s + 1 & 15 & 5 - 5s \\ 2 - 2s & s^2 - 2s - 19 & 10 \\ 10 - 4s & 3 - 3s & s^2 - 2s + 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 3 \end{bmatrix} + 9$$

$$= \begin{bmatrix} 0 & 0 & 7 \end{bmatrix} \times \frac{1}{s^3 - 3s^2 - 17s + 49} \begin{bmatrix} 3s^2 - 21s + 18 \\ 0 - 6s + 36 \\ 3s^2 - 18s + 33 \end{bmatrix} + 9$$

$$G(s) = \frac{1}{s^3 - 3s^2 - 17s + 49} \times \left[0 + 0 + 21s^2 - 126s + 231 \right] + 9$$

$$G(s) = \frac{21s^2 - 126s + 231}{s^3 - 3s^2 - 17s + 49} + 9$$

$$G(s) = \frac{9s^3 + -6s^2 - 297s + 672}{s^3 - 3s^2 - 17s + 49}$$