# Electrical Network Analysis (EE-241)

Assignment # 5, Spring 2021

Submission Deadline: **Thursday** July 01, 2021 Maximum Marks: 100

1. (a) State the complex frequency or frequenices **s** as well as **s*** associated with each function: [4]
   i. $f(t) = 5e^{-10t}\cos 50t$
   ii. $g(t) = (4e^{-2t} - e^{-t})\cos(5t - 93^o)$
   iii. $h(t) = 7e^{-9t}\sin(100t + 9^o)$
   iv. $i(t) = 2\sin 60t$

   (b) Use real constants $A, B, \theta, \phi$ etc. to construct the general form of a real time function characterized by the following frequency components: [3]
   i. $(10 - j3)\ s^{-1}$
   ii. $0.25\ s^{-1}$
   iii. $0\ s^{-1}, 1\ s^{-1}, -j\,\mathrm{s}^{-1}, (1+j)\ s^{-1}$

   (c) The following voltage sources $Ae^{Bt}\cos(Ct+\theta)$ are connected (one at a time) to a $280\,\Omega$ resistor. Calculate the resulting current at $t = 0, 0.5$ and $1\,\mathrm{s}$ assuming passive sign convention. [3]
   i. $A = 1\,\mathrm{V}, B = 0.2\,\mathrm{Hz}, C = 0, \theta = 45^o$.
   ii. $A = 285\,\mathrm{mV}, B = -1\,\mathrm{Hz}, C = 2\,\mathrm{rad/s}, \theta = -45^o$.

2. (a) For the circuit shown in Figure 4, the voltage source is chosen such that it can be represented by the complex frequency domain function $\mathbf{V}e^{st}$, with $\mathbf{V} = 2.5\underline{/-20^o}$ and $s = 1 + j100\ s^{-1}$. Calculate [6]
   i. $\mathbf{s}^*$
   ii. the time-domain representation of voltage $v(\mathrm{t})$
   iii. the current $i(\mathrm{t})$

   (b) With regard to the circuit depicted in Figure 4, determine the time-domain voltage $v(\mathrm{t})$ which corresponds to a frequency domain current of $5\underline{/30^o}\,\mathrm{A}$ for a complex frequency of $s = (-3+j)\,\mathrm{s}^{-1}$. [4]

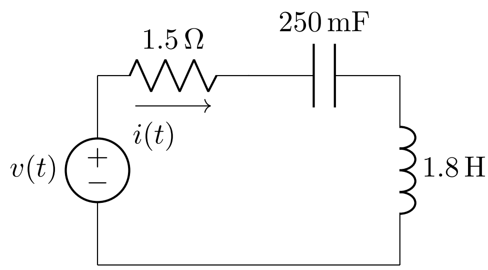


**Figure 1:** Circuit diagram for problem 2

3. (a) Find the **Laplace Transform** of $g(\mathrm{t})$, $h(\mathrm{t})$ and $k(\mathrm{t})$ shown in Figure 2. [6]

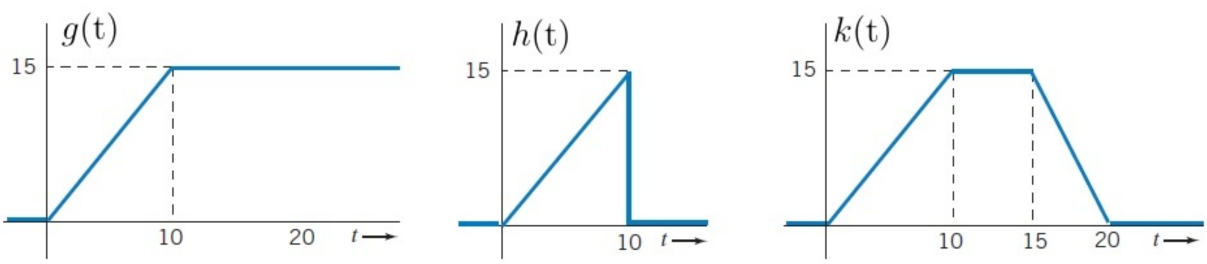


**Figure 2:** Functions for problem 3a

   (b) Show that the **Laplace Transform** of the square wave is $F(s) = \frac{1}{s(1+e^{-as})}$. [4]

4. (a) Obtain an expression for $\mathbf{G}(\mathrm{s})$ if $g(\mathrm{t})$ is given by [5]

   i. $[5u(t)]^2 - u(t)$

   ii. $3e^{-2t}u(t) + 5u(t)$

   iii. $tu(2t)$

   iv. $\delta(t) + u(t) - tu(t)$

   v. $2e^{-t}u(t) + 3u(t)$

   (b) Let $i_1 = 20e^{-3t}\cos 4t\,\mathrm{A}$ and $i_2 = 30e^{-3t}\sin 4t\,\mathrm{A}$ in the circuit of Figure 3. Work in the frequency domain to find $\mathbf{V}_x$ and then find $v_x$. [5]

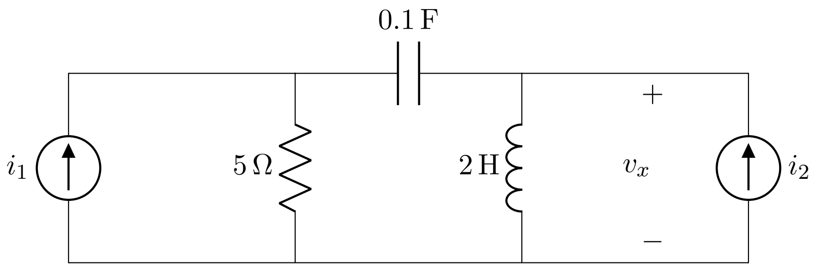


**Figure 3:** Circuit diagram for problem 4b

5. (a) Prove that $\frac{d^2v}{dt^2} \Leftrightarrow s^2\ \mathbf{V}(\mathrm{s})\ - sv(0^-) - v'(0^-)$ [2]

   (b) Prove that $\int_{0^-}^{t} v(x)dx \Leftrightarrow \frac{\mathbf{V}(s)}{s}$ [2]

   (c) Prove that $f(t-a)u(t-a) \Leftrightarrow e^{-as}\ \mathbf{F}(\mathrm{s})$ [2]

   (d) Show that $\lim_{t\to 0^+} f(t) = \lim_{s\to\infty}\ [\mathbf{sF(s)}]$ [2]

   (e) Show that $\lim_{t\to\infty} f(t) = \lim_{s\to 0}\ [\mathbf{sF(s)}]$ [2]

6. (a) Apply the **initial or the final-value theorem** as appropriate to determine $f(0^+)$ and $f(\infty)$ for the following functions:

   i. $\frac{1}{s^2(s+4)^2(s+6)^3} - \frac{2s^2}{s}+9$ [2]

   ii. $\frac{4s^2+1}{(s+1)^2(s+2)^2}$ [2]

   (b) Find the **Inverse Laplace Transform** of the functions $\frac{s+3}{s^2+7s+10}$ and $\frac{10}{(s^2+6s+10)(s+2)}$ [3+3]

7. (a) Assuming zero initial conditions in Figure 4, find $i(\mathrm{t})$ and $v(\mathrm{t})$.

   i. In time-domain. [3]

   ii. Working initially in the s-domain and then converting back to time domain. [3]

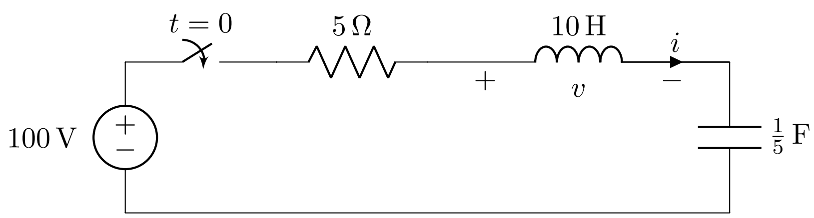


**Figure 4:** Circuit diagram for problem 7a

   (b) In the circuit of Figure 5 , $v(0^-) = 1.2\,\mathrm{V}$ and $i(0^-) = 0.4\,\mathrm{A}$. Find $v(t)$ and $i(t)$ for $t > 0$. [4]

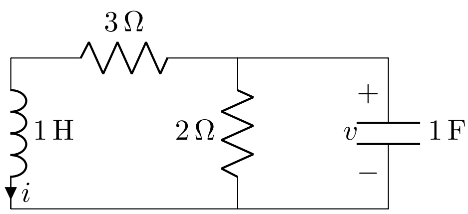


**Figure 5:** Circuit diagram for problem 7b

8. (a) Using either **Thevenin's** or **Norton's Theorem**, determine an equivalent network for the terminals $a - b$ in the Figure 6 for zero initial conditions. What will be the current in a 1 F capacitor if connected between terminals $a - b$? [2]

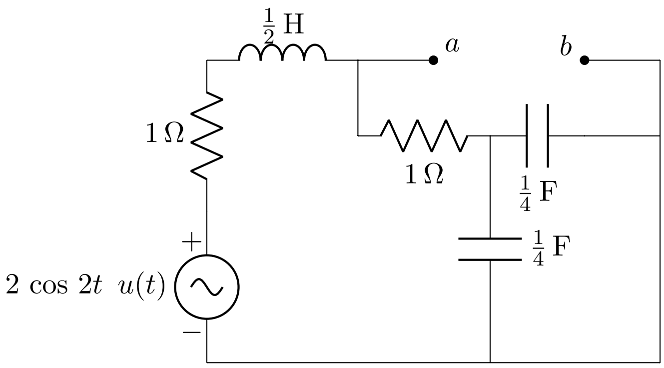


**Figure 6:** Circuit diagram for problem 8a

(b) For the circuit of Figure 7, the switch $S$ is closed at $t = 0$, a steady state having previously existed. Find the expression for the current in the resistor $R$ for $t > 0$. You may use any Network Theorem. [4]

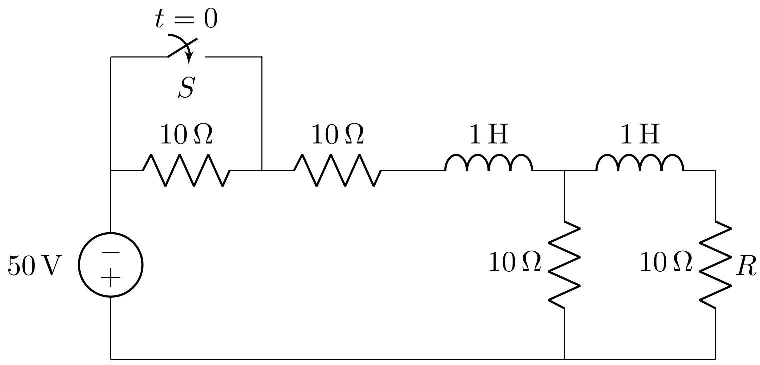


**Figure 7:** Circuit diagram for problem 8b

(c) For the circuit shown in Figure 8, obtain an expression for $v_x(t)$. [4]

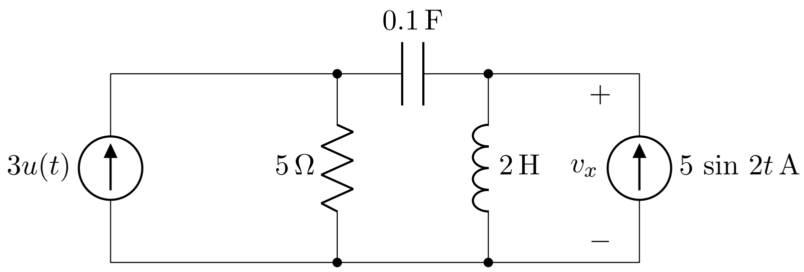


**Figure 8:** Circuit diagram for problem 8c

9. (a) Let $h(t) = 2e^{-3t}u(t)$ and $x(t) = u(t) - \delta(t)$. Find $y(t) = h(t) \star x(t)$ by [2]

   i. using convolution in the time domain

   ii. finding $H(s)$ and $X(s)$ and then obtaining inverse Laplace Transform of $H(s).X(s)$

   (b) If a network is found to have the transfer function $H(s) = \frac{s}{s^2+8s+7}$, determine the s-domain output voltage for $v_{in}$(t) equal to [4]

   i. $3u(t)$ V

   ii. $25e^{-2t}u(t)$ V

   iii. $4u(t+1)$ V

   iv. 2 sin $5t$ V

(c) A particular network is known to be characterized by the transfer function $H(s) = \frac{s+1}{s^2+23s+60}$. Determine the critical frequencies of the output if the input is [4]

i. $2u(t) + 4\delta(t)$

ii. $-5e^{-t}u(t)$

iii. $4te^{-2t}u(t)$

iv. $5\sqrt{2}e^{-10t}cos\ \ 5t\ u(t)$

10. (a) For each of the two networks shown in Figure 9, write the **Transfer Functions** $H$(s) and determine their **poles** and **zeros**. [2]

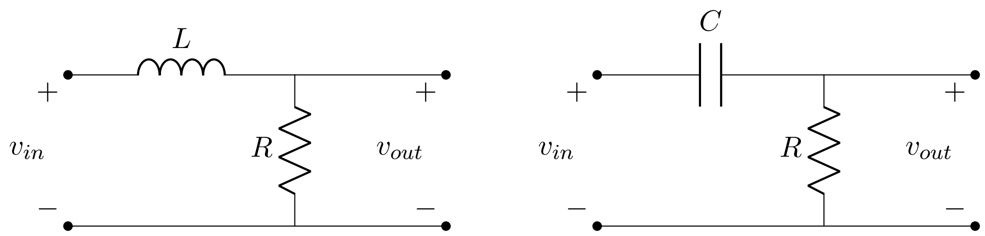


**Figure 9:** Circuit diagrams for problem 10a

(b) The **Transfer Function** of a circuit is $H(\mathrm{s}) = \frac{-5s}{s^2+15s+50}$. Determine the impulse response and step response of this circuit. [4]

(c) Design a circuit which produces the transfer function $H(s) = \frac{V_{out}}{V_{in}}$ equal to [4]

i. $5(s+1)$

ii. $\frac{5}{s+1}$

iii. $5\frac{s+1}{s+2}$

iv. $3\frac{s+50}{(s+75)^2}$