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Bsee 19047

ENA-Assignment #1

Q:-1

(a) $f(t) = 5u(t-1)$; evaluate at $t = -3, 0, +3$.

Solve $f(t)$ at $t = -3 : f(-3) = 5u(-3-1) = 5u(-4) = 5(0) \Rightarrow \boxed{f(-3) = 0}$

$$f(0) = 5u(0-1) = 5u(-1) = 5(0) ; \boxed{f(0) = 0}$$

$$f(3) = 5u(3-1) = 5u(2) = 5(1) ; \boxed{f(3) = 5} = f(3)$$

(b) $h(t) = 4u(1-t) + 2u(t+2)$; evaluate at $t = 0, +2$

$$h(0) = 4u(1) + 2u(2) \quad h(0) = 4 + 2 \quad \boxed{h(0) = 6}$$

$$h(+2) = 4u(-1) + 2u(4) \quad h(+2) = 2 \quad \boxed{h(2) = 2}$$

(c) $K(t) = 3u(t) - 2u(-t) + 0.8u(1-t)$

Evaluate at $t = 0.8$

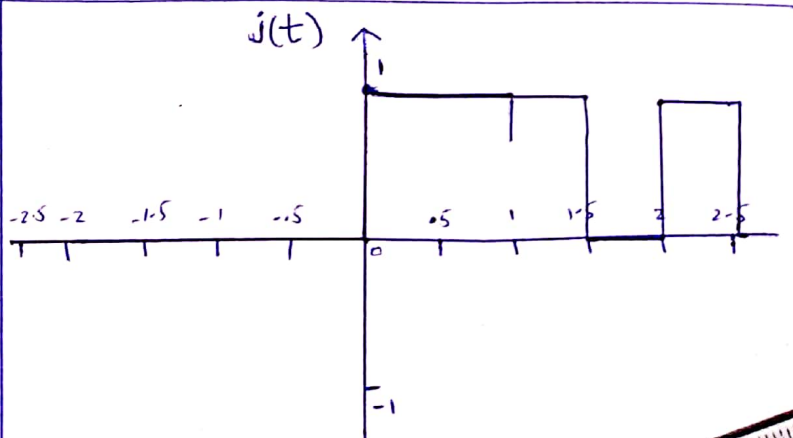
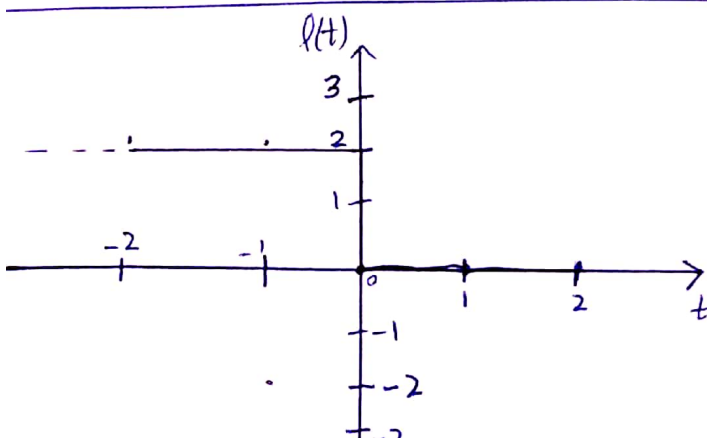
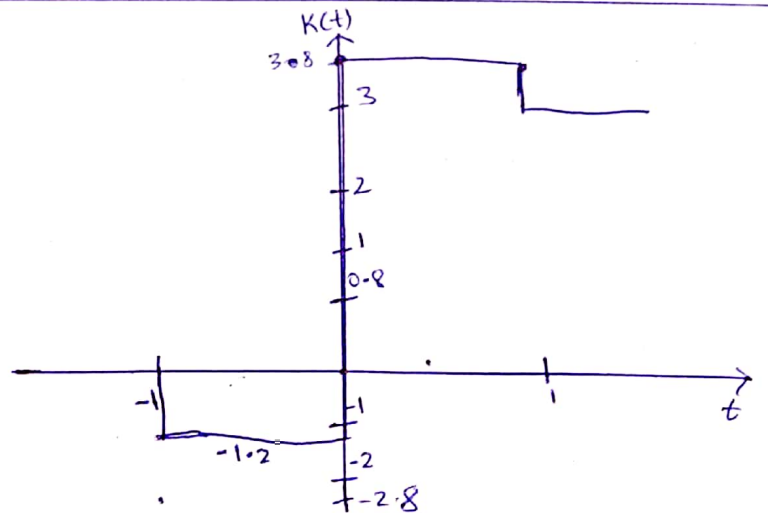
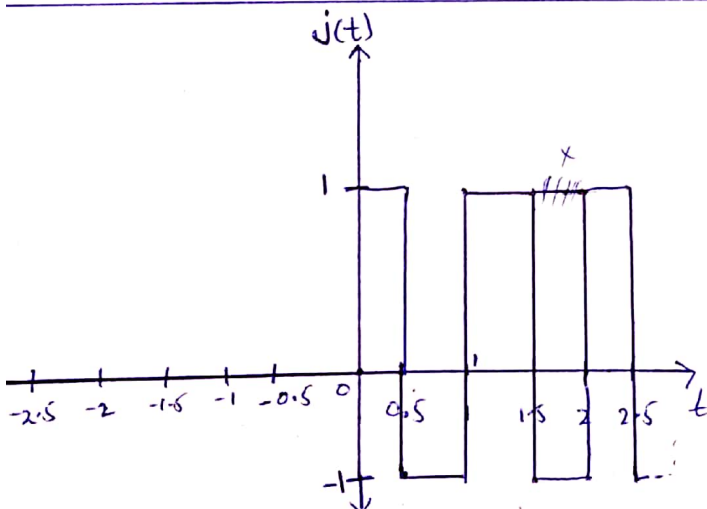
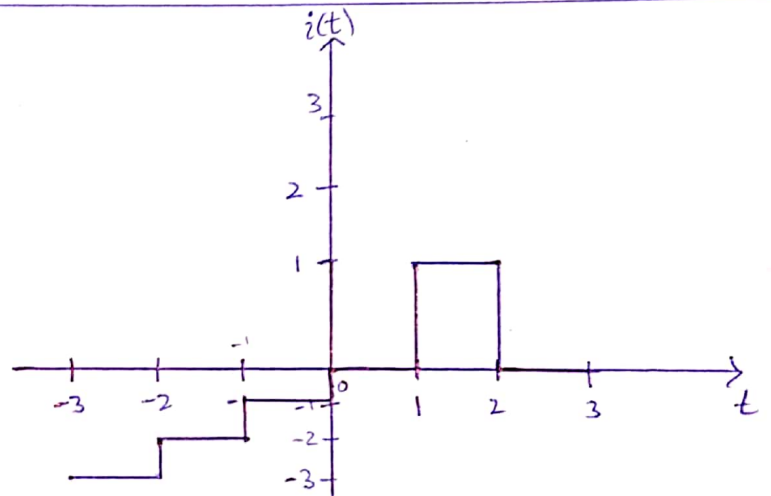
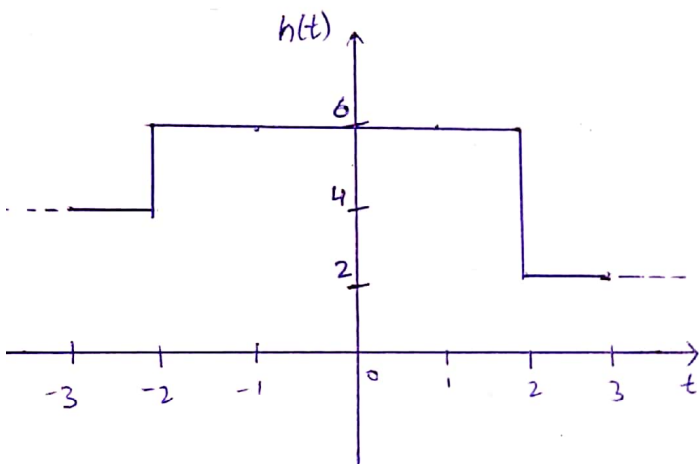
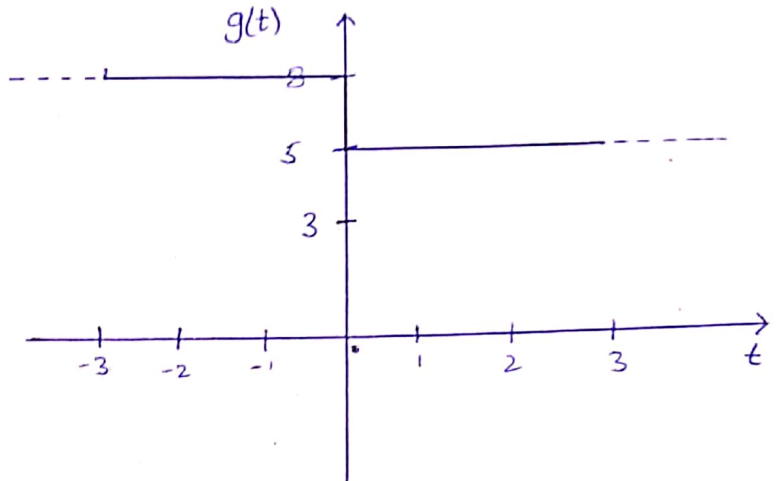
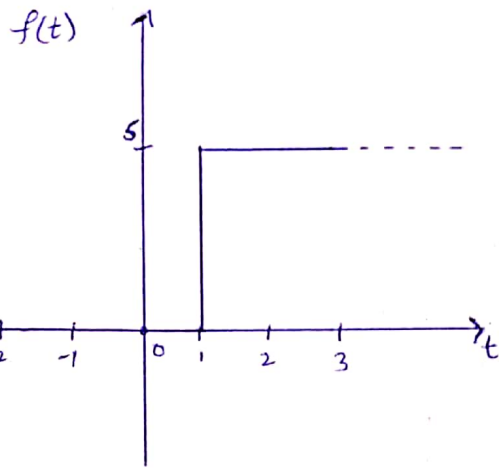
$$K(0.8) = 3u(0.8) - 2u(-0.8) + 0.8u(1-0.8)$$

$$= 3 \times 1 - 2 \times (0) + 0.8 \times 1$$

$$K(0.8) = 3 + 0.8$$

$$\boxed{K(0.8) = 3.8}$$

(d) sketch graph:- $-3 \leq t \leq 3$



Q:- 2

(a) $C = 3.1 \text{ nF}$, $R = 55 \text{ M}\Omega$, $W = 200 \text{ mJ}$

Solve

$$T = RC = 3.1 \text{ nF} \times 55 \text{ M}\Omega , T = 170 \text{ ms} \text{ --- (1)}$$

$$W(0) = \frac{1}{2} C V_0^2 , V_0 = \sqrt{\frac{2 \times W(0)}{C}} = \sqrt{\frac{2 \times 200 \text{ m}}{3.1 \text{ nF}}}$$

(i)

$$V_0 = 11359 \text{ V} \text{ --- (1)}$$

$$V(t) = V_0 e^{-t/T} = 11359 e^{-5.8t} \text{ V} = V(t)$$

(ii)

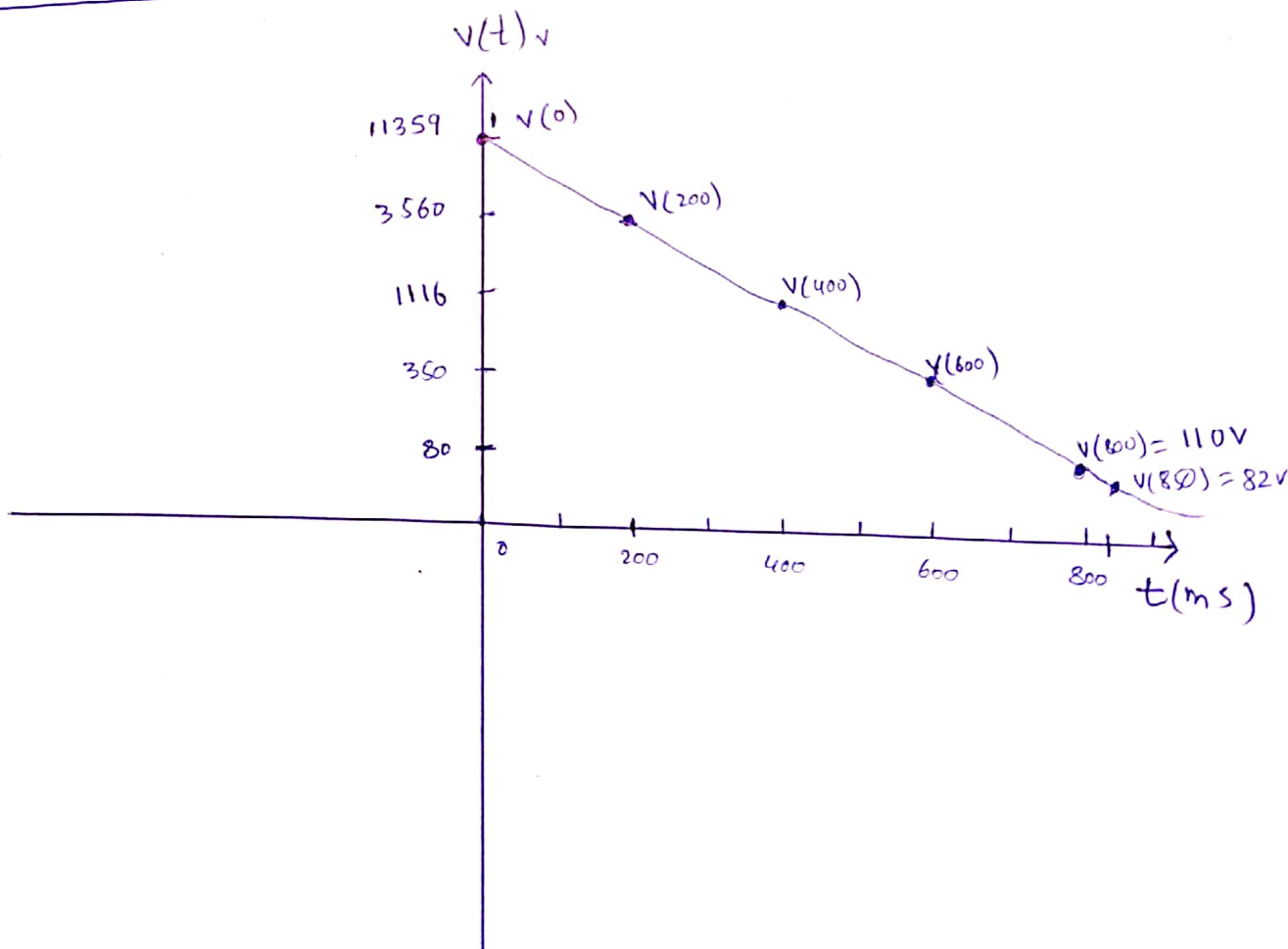
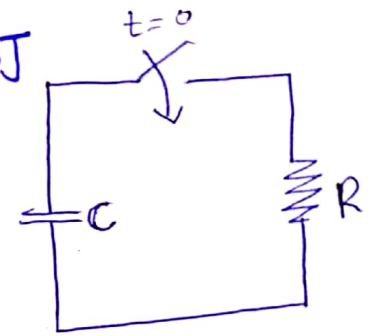
$$V(170 \text{ ms}) = 11359 e^{-1} , V(170 \text{ ms}) = 4178 \text{ V}$$

$$W(170 \text{ ms}) = \frac{1}{2} C V^2 = \frac{1}{2} (3.1 \text{ n}) \times (4178)^2 , W(170 \text{ ms}) = 27 \text{ mJ}$$

(iii)

$$V(2T) = 11359 e^{-2T/T}$$

$$V(2T) = 1537 \text{ V}$$



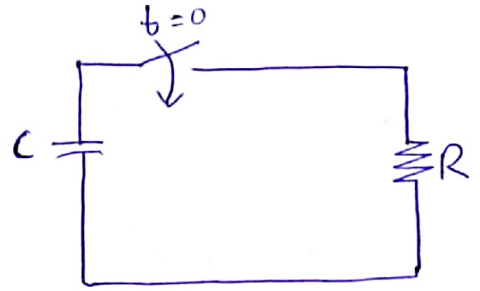
Q:-2

(b) $R = 1\Omega$, $C = 22\text{mF}$, $w = 891\text{mJ}$

Solve

$$\tau = RC = 22\text{ms} \quad \text{--- ②}$$

$$V_0 = \sqrt{\frac{2 \times (891\text{m})}{22\text{m}}}, \quad V_0 = 9\text{V} \quad \text{--- ①}$$



$$(i) \quad V(t) = V_0 e^{-t/\tau}, \quad = 9e^{-t/22\text{m}}$$

$$\boxed{V(t) = 9e^{-45t}\text{V}}$$

(ii) $t = 11\text{ms}$, 33ms , $w(t) = ?$

$$V(11\text{ms}) = 9e^{-45 \times 11\text{m}}, \quad V(11\text{m}) = 5.4\text{V}$$

$$w(11\text{ms}) = \frac{1}{2} CV(11\text{ms}) = \frac{1}{2} \times 22\text{m} \times 5.4 \quad \boxed{w(11\text{ms}) = 594\text{mJ}}$$

$$V(33\text{ms}) = 9e^{-45 \times 33\text{m}}, \quad V(33\text{ms}) = 2\text{V}$$

$$w(33\text{ms}) = \frac{1}{2} CV(33\text{m}) = \frac{1}{2} \times 22\text{m} \times 2 \quad \boxed{w(33\text{ms}) = 220\text{mJ}}$$

(i)

$$R = 100\text{k}\Omega$$

$$\tau = RC = 22\text{n} \times 100\text{k}\Omega, \quad \boxed{\tau = 2.2\text{ms}}$$

$$V_0 = 9\text{V}$$

$$\boxed{V(t) = 9e^{-455t}}, \quad \boxed{V(11\text{m}) = 60\text{mV}}, \quad \boxed{V(33\text{m}) = 2.7\mu\text{V}}$$

$$, \quad \boxed{w(11) = 0.66\text{mJ}}$$

$$\boxed{w(11) = 39\mu\text{J}}$$

Q2

(b)

(iii)

$$R_{eq} = 0.9999 \Omega \approx 1 \Omega$$

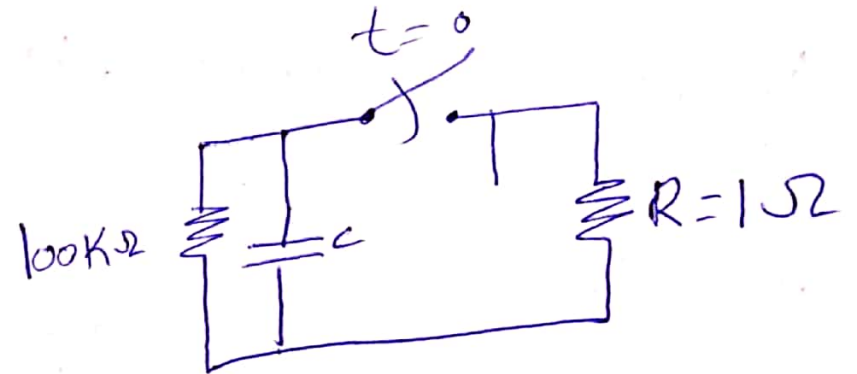
so there would be no

visible change in the values.

$$V(t) = 9e^{-45t} \text{ V}$$

$$W(11\text{m}) = 594\text{mJ}$$

$$W(33\text{m}) = 220\text{mJ}$$



Q2

(c) $C = 10\text{mF}$,

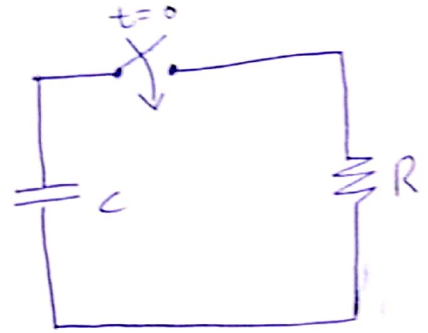
(i) $t = ?$ $R = 1\Omega$

$$\tau = RC = 1 \times 10\text{mF}$$

$$\tau = 10\text{ms}$$

(ii) $R = 100\Omega$

$$\tau = RC = 100 \times 10\text{mF}, \tau = 1\text{s}$$



Q:-3

(a)

$$V_C(0^-) = \frac{100 \times 2.7\text{K}}{3.7\text{K}}$$

$$V_C(0^-) = 73\text{V}$$

$$V_O(0^-) = V_O(0^+) = \frac{100 \times 2}{8}$$

$$V_O(0^-) = 25\text{V}$$

$$V_O[V_C(0^+) = 73\text{V}]$$

$$V_O(0^+) = \frac{73 \times 82}{9}, V_O(0^+) = 65\text{V}$$

$$V_O(0^+) = 16.2\text{V}$$

$$V_C(t) = 73e^{-t/\tau}$$

$$\tau = R_{eq}C = 2\text{ms}$$

$$V_C(t) = 73e^{-500t}$$

V

$$V_C(10\text{ms}) = 491\text{mV}$$

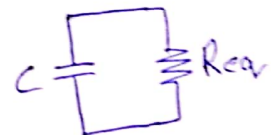
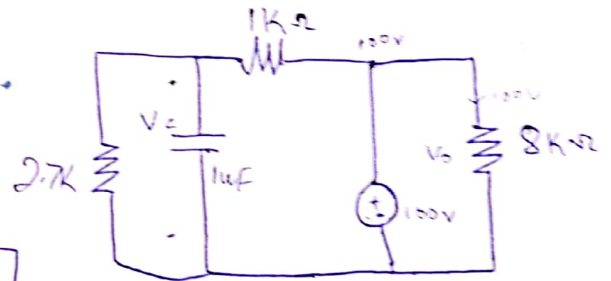
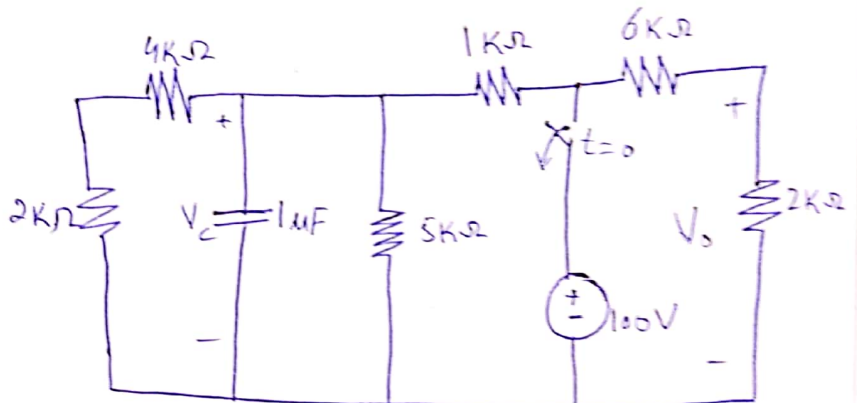
$$V_C(12\text{ms}) = 180\text{mV}$$

$$V_O(10\text{ms}) = 437\text{mV}$$

$$V_O(12\text{ms}) = 160\text{mV}$$

$$V_O(10\text{ms}) = 109\text{mV}$$

$$V_O(12\text{ms}) = 40\text{mV}$$



$$R_{eq} = 9\text{K} \parallel 2.7\text{K}$$

$$R_{eq} = 2\text{K}\Omega$$

Q3

(b)

$$i_x(0^-) = i_L(0^-) = 0A, v_L(0^-) = 0V$$

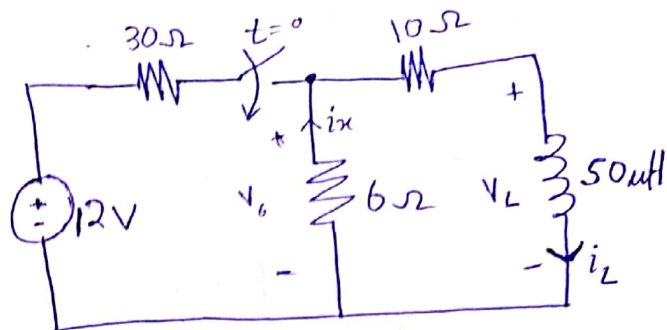
$$V_6(0^+) = \frac{12 \times 6}{36} = 2V$$

$$i_x(0^+) = -2 \times 6\Omega = \boxed{i_x(0^+) = -12A}$$

$$i_L(0^+) = 2 \times 10\Omega = \boxed{i_L(0^+) = 20A}$$

$$V_L = L \frac{di}{dt}, V_L(0^+) = L \times 20 = 50\mu H \times 20$$

$$\boxed{V_L(0^+) = 1mV}$$



Q3

(c)

using nodal analysis

$$\frac{V_c - 10}{6K} + \frac{V_c}{9K} + \frac{1.5i_1}{27K} = 0$$

$$i_1 = 3V_c - 30 + 2V_c + \frac{27i_1}{27K} = 0$$

$$5V_c - 30 + 24i_1 = 0 \quad \text{--- (1)}$$

$$i_1 = \frac{V_c}{9K} \quad \text{--- (2) substituting 2 in 1}$$

$$5V_c - 30 + \frac{27V_c}{9K} = 0, \quad 45KV_c - 270K + 27V_c = 0$$

$$\boxed{V_c(0^-) = 6V}, \quad \boxed{V_c(0^+) = 6V}, \quad T =$$

$$3V_c - 30 + 2V_c + 27i_1 = 0$$

$$i_1 = \frac{V_c}{9K}, \quad 5V_c - 30 + 3V_c = 0$$

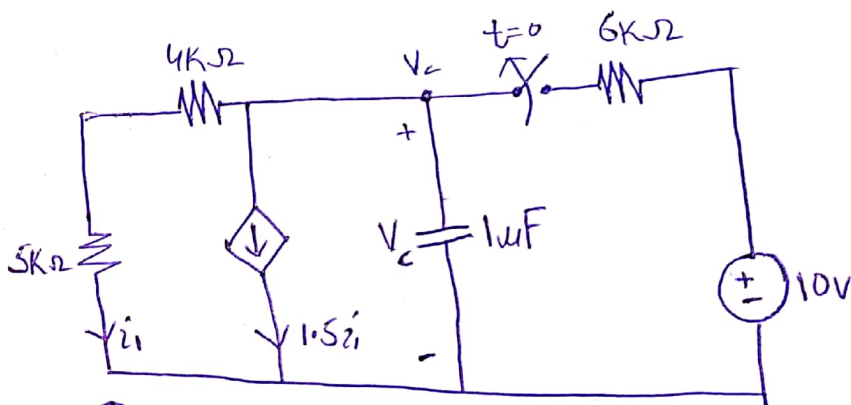
$$\boxed{V_c = 3.75V} = V_c(0^+) = V_c(0^-)$$

$$T = R_{eq}C, \quad i_c = 2.5i_1, \quad i_c = \frac{2.5 \times V_c}{9K}, \quad R_{eq} = \frac{V_c}{i_c} = \frac{V_c \times 9K}{2.5 \times V_c}$$

$$R_{eq} = 3.6K\Omega$$

$$T = (3.6K) \times (1\mu F)$$

$$\boxed{T = 3.6ms}$$



$$V(t) = V_0 e^{-t/\tau} = 3.75 e^{-277t} \text{ ————— (ii)}$$

$$V(3m) = 3.75 e^{-277 \times 3m}$$

$$\boxed{V(3ms) = 1.6V} = 1.63V$$

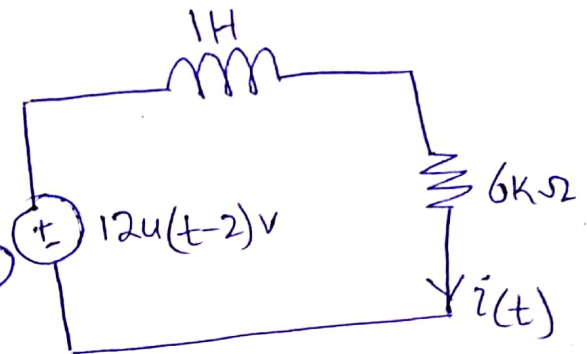
Q:-4

(a)

at $t \geq 2^+$, $v = 12V$
and before $t = 2$, $v = 0$ Hence,

$$i(t) = i(0^-) = i(0^+) = i(2^-) = 0A \quad \text{--- (1)}$$

$$T = \frac{L}{R} = \frac{1}{6K\Omega}, \quad \boxed{T = 166\mu s}$$



$$i(2^+) = 0A.$$

Now $i(t) = i_0 - i_0 e^{-t/\tau} : i_0 = \frac{V_0}{R} = \frac{12}{6K} = 2mA$

$$i(t) = (2m - 2me^{-t/166\mu s}) \times 12 u(t-2)$$

$$\cancel{i(t) = (2m - 2me^{-6t})(12 u(t-2))} \quad \text{--- (2)}$$

$$\cancel{i(4ms) = (2m - 2me^{-0.024})(1)} \quad \text{--- (2)}$$

$$\boxed{\cancel{i(4ms) = 1.04mA}}$$

$$i(t) = (2m - 2me^{-6024t})(u(t-2)) \quad \text{--- (2)}$$

$$i(4ms) = (2m - 2me^{-24})(1)$$

$$\boxed{i(4ms) = 2mA}$$

Q:- 4

(b) For $R = 10\Omega$

$$\tau = LR, \quad \boxed{\tau = 400\text{ms}}$$

for $t \leq 0$, $i(t) = 0\text{A}$, $v(t) = 0\text{V}$

$$V(t) = 12u(t) - 12(t-1)v \quad \text{--- (1)}$$

at $t = 0^+$

$$V(0^+) = 12\text{V} \quad \text{--- (2)}$$

~~for $t \rightarrow$~~ for $0 < t < 1$

~~$i(0^+)$~~

$$i(0^+) = \frac{12}{10}, \quad \boxed{i(0^+) = 1.2\text{A}}$$

$$w(10\Omega) = \frac{1}{2}Li^2 = \frac{1}{2}(4)(1.2)^2, \quad \boxed{w(10\Omega) \Rightarrow 2.88\text{J}}$$

For $R = 1\Omega$

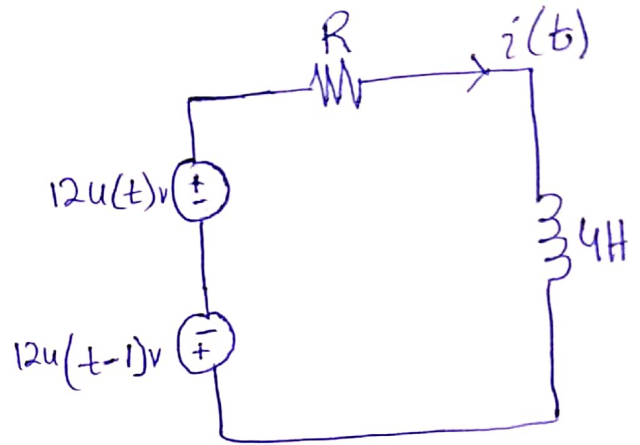
$$\tau = 4\text{s}$$

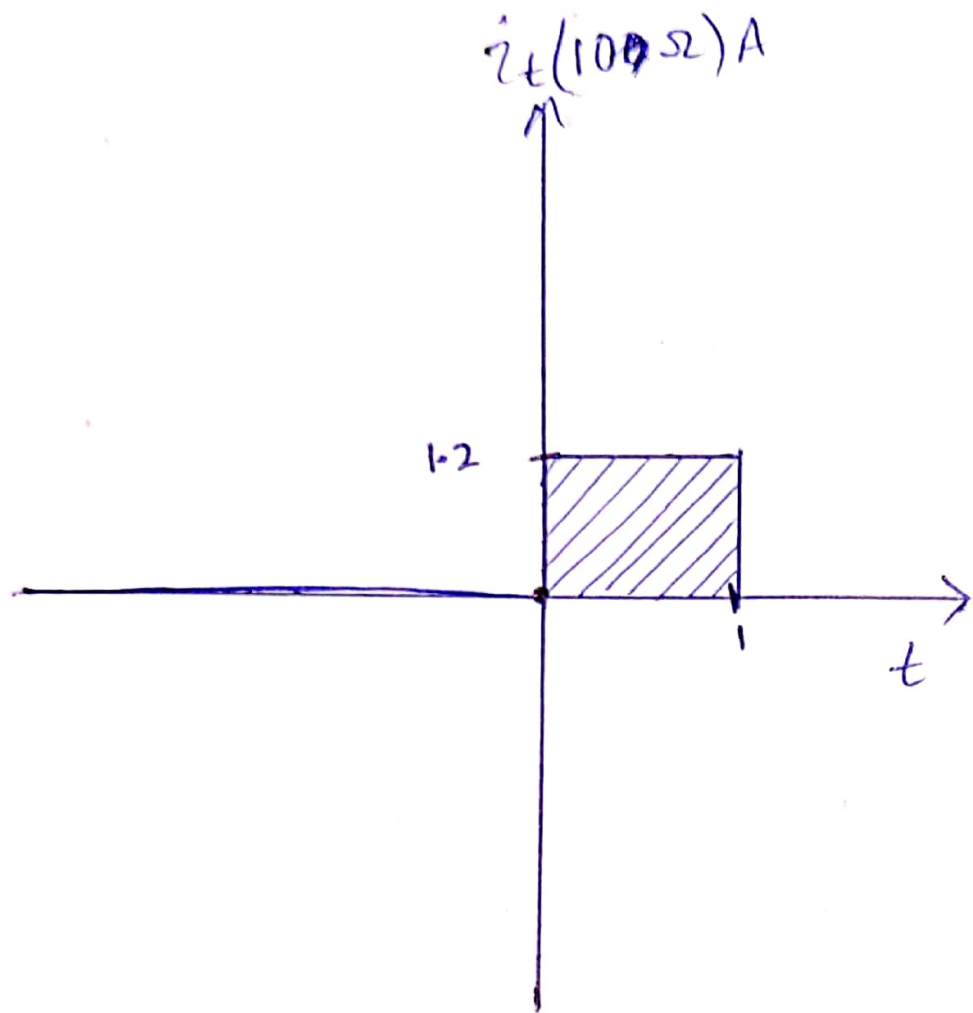
As the time constant with $R = 1\Omega$ is 4s and the circuit only works for less than one second.

$V(0^+) = 12\text{V}$, $i(0^+) = \frac{12}{1} = 12\text{Amp}$. then

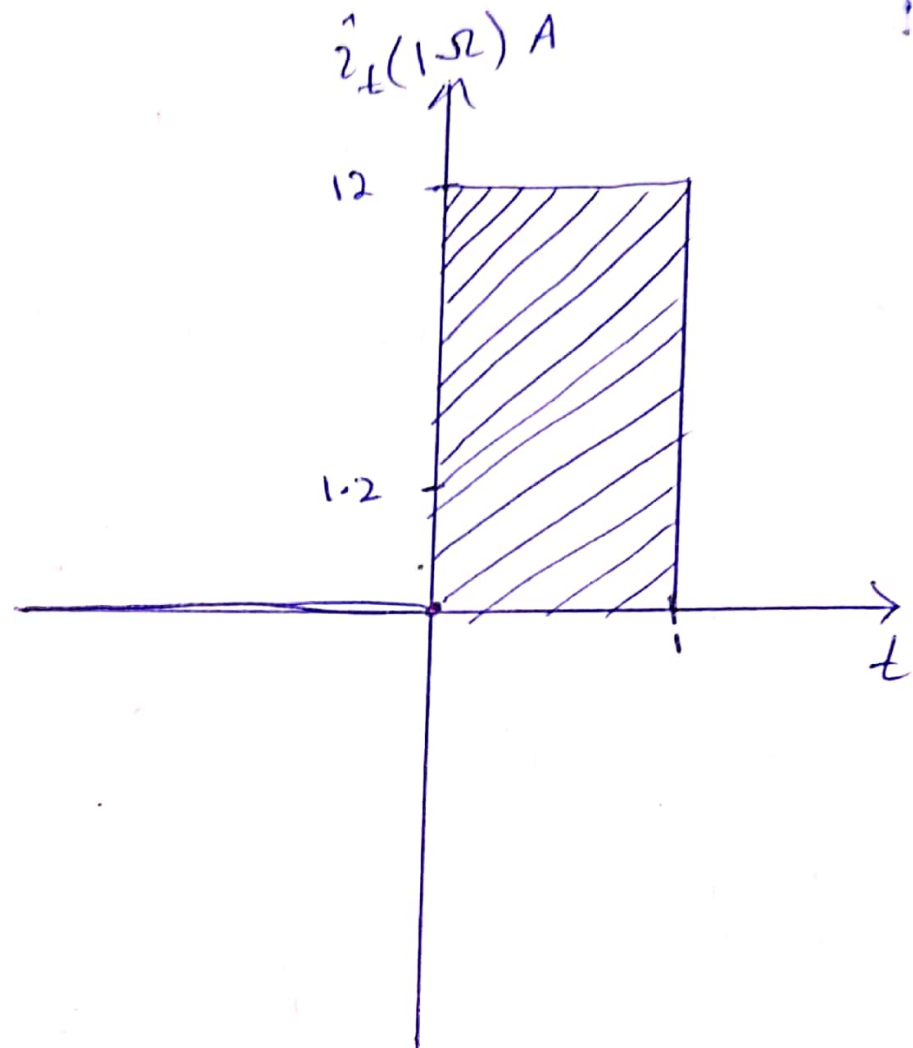
$$w(1\Omega) = \frac{1}{2}(4)(12)^2, \quad \boxed{w(1\Omega) = 288\text{J}}$$

The 1Ω resistor circuit will store 100 times more energy.





$$i(t) = (1.2 e^{-2.5t}) (u(t)) (u(t-1))$$



$$i(t) = (12 e^{-0.25t}) (u(t)) (u(t-1))$$

Q:- 3

(b)

$$i_x(0^-) = 0A$$

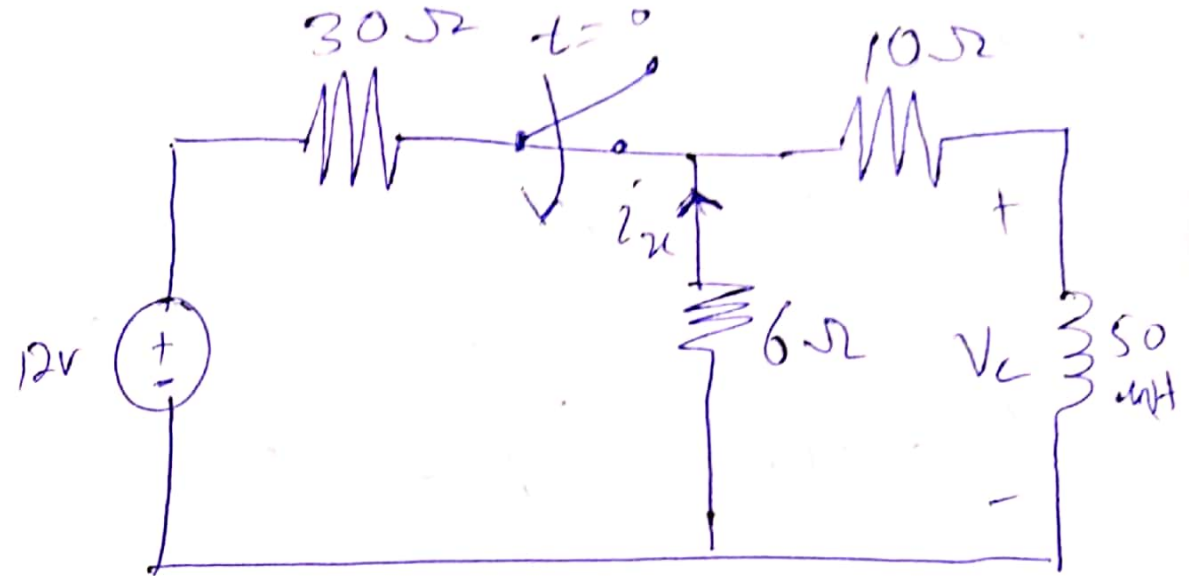
$$i_L(0^-) = 0A$$

$$i_L(0^+) = 0A$$

$$i_x(0^+) = \frac{12}{36} = 333mA$$

$$V_L(0^-) = 0V$$

$$V_L(0^+) = 0V$$



Q:-5

(a)

Solve:-

$$\tau = RC = 50 \text{ ms}$$

$$(i) V(0^-) = 4 \times 5 = \boxed{20 \text{ V}}$$

$$(ii) V(0^+) = 4 \times 5 = \boxed{20 \text{ V}}$$

$$(iii) V(40 \text{ ms}) = 20 e^{-\frac{40 \text{ ms}}{50 \text{ ms}}}$$

$$V(40 \text{ ms}) = 20 - 8.98$$

$$\boxed{V(40 \text{ ms}) = 11 \text{ V}}$$

$$(iv) R_{eq} = 5 \parallel 10 = 3.33 \Omega$$

$$\tau = 33.3 \text{ ms}$$

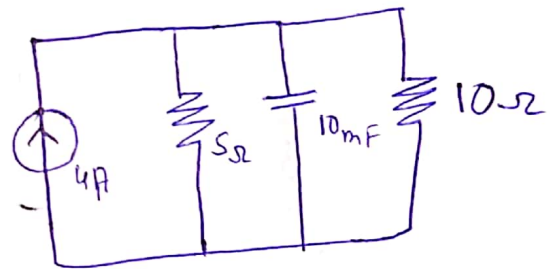
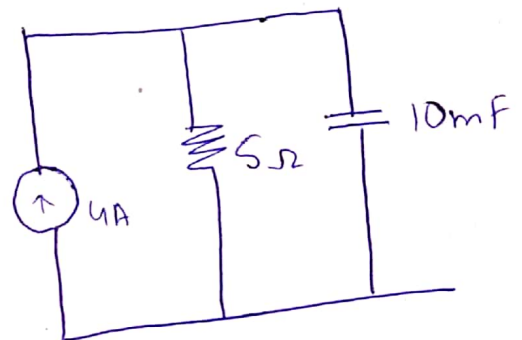
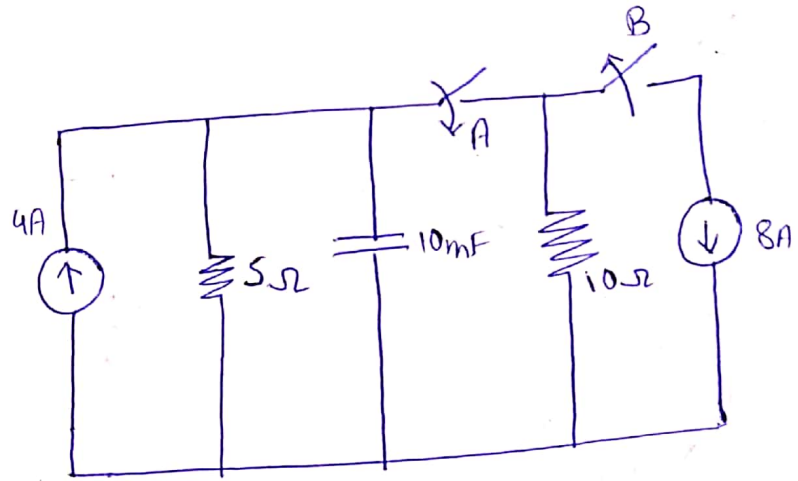
$$\boxed{V_c(40 \text{ ms}) = 11 \text{ V}}$$

$$(v) t = 50 \text{ ms}$$

$$V_c(50 \text{ ms}) = 11 + [20 - 11] e^{-\frac{50 \text{ ms}}{33 \text{ ms}}}$$

$$V_c(50 \text{ ms}) = 11 + 17.5$$

$$\boxed{V_c(50 \text{ ms}) = 28.5 \text{ V}}$$



Q:-5

(a) $\tau = RC = 5 \times 10 \text{m}$
 $\tau = 50 \text{ms}$

(i)

A_3
 $V(0^-) = 0 \text{V}$

~~(ii) $V(0^+) = 0 \text{V}$~~

(i) $V(0^-) = ?$

$V(0^-) = 4 \times 5 = 20 \text{V} \text{ --- (1)}$

(ii) $V(0^+) = 20 \text{V} \text{ --- (2)}$

(iii) $V(40 \text{ms}) = 20e^{-t/\tau} + 20$
 $= 20 - 20e^{-\frac{40 \text{ms}}{50 \text{ms}}}$

$V(40 \text{ms}) = 11 \text{V}$

$V(40 \text{ms}) = 11 + 9e^{-\frac{40 \text{ms}}{50 \text{ms}}}$

$V(40 \text{ms}) = 15 \text{V}$

(iv) $V(40^+)$

$V(40^+) = 15 \text{V}$

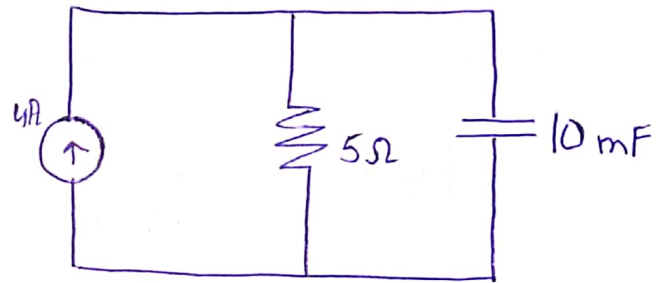
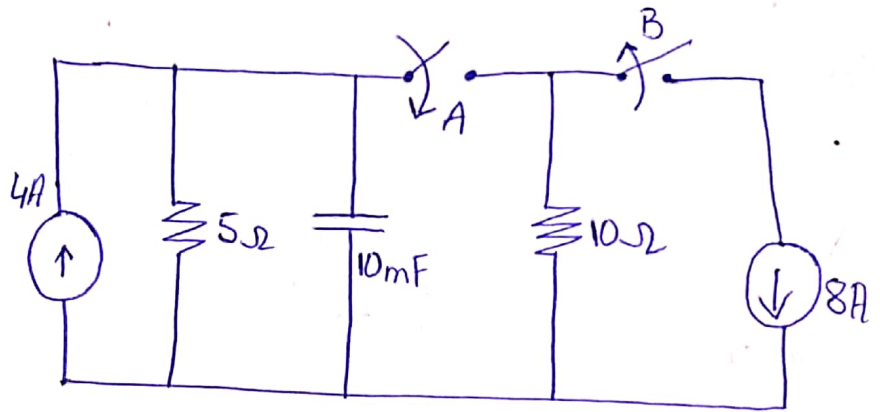
(v) $V(50 \text{ms}) = ?$

$\tau = 33 \text{ms}$

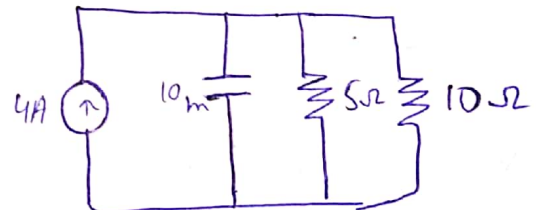
$V(t) = 15 - 15e^{-t/33 \text{ms}}$

$V(50 \text{ms}) = 15 - 15e^{-1.5} \text{V}$

$V(50 \text{ms}) = 11.7 \text{V}$



i, ii, iii



$R_{eq} = 3.3 \Omega$

Q:-5

(b)

$$\tau = RC = 50 \text{ ms}$$

(i) $V(0^-) = 20 \text{ V}$

$$w(0^-) = \frac{1}{2} C V^2$$

$$w(0^-) = \frac{(10 \text{ m})(20)^2}{2}$$

$$w(0^-) = 2 \text{ J}$$

(ii) $w(0^+) = 2 \text{ J}$

(iii) $w(200 \text{ ms}) = ?$

$$V(200 \text{ ms}) = 20 - 20 e^{-\frac{200 \text{ m}}{50 \text{ m}}}$$

$$V(200 \text{ ms}) = 19.6 \text{ V}$$

$$w(200 \text{ ms}) = \frac{10 \text{ m} \times (19.6)^2}{2}$$

$$w(200 \text{ ms}) = 1.927 \text{ J}$$

(iv) $w(400 \text{ ms}) = ?$

$$V_C(\infty) = \frac{4}{3.3} = 1.212$$

$$V_C(\infty) = 13.3 \text{ V}$$

$$V_C(400 \text{ ms}) = 13.3 + (20 - 13.3) e^{-\frac{400}{50}}$$

$$V_C(400 \text{ ms}) = 13.3 \text{ V}$$

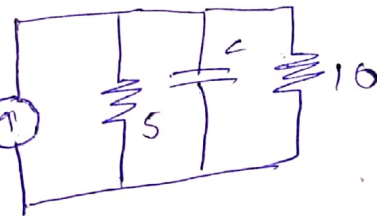
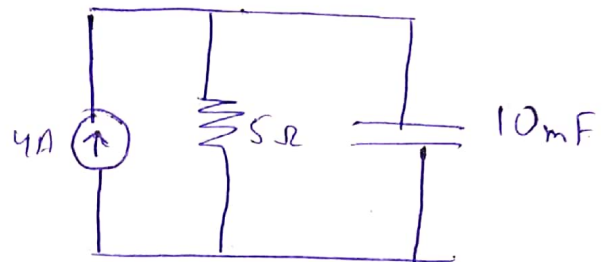
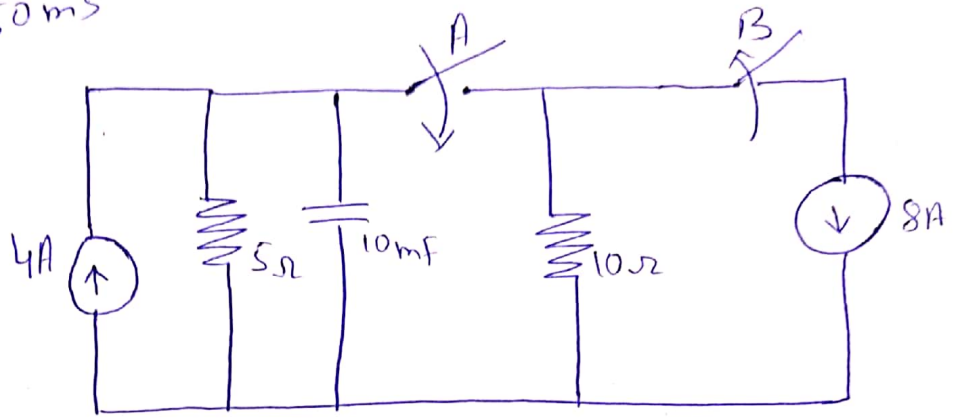
$$w(400 \text{ ms}) = 884 \text{ mJ}$$

(v) $w(400^+ \text{ ms}) = 884 \text{ mJ}$

(vi) $V(700 \text{ ms}) = 13.3 + (20 - 13.3) e^{-\frac{700}{33}} = 33.3 \text{ V}$

$$w(700 \text{ ms}) = \frac{10 \text{ m} \times (33.3)^2}{2}$$

$$w(700 \text{ ms}) = 5.54 \text{ J}$$



$$R_{eq} = 3.3 \Omega$$

$$\tau = 33 \text{ ms}$$

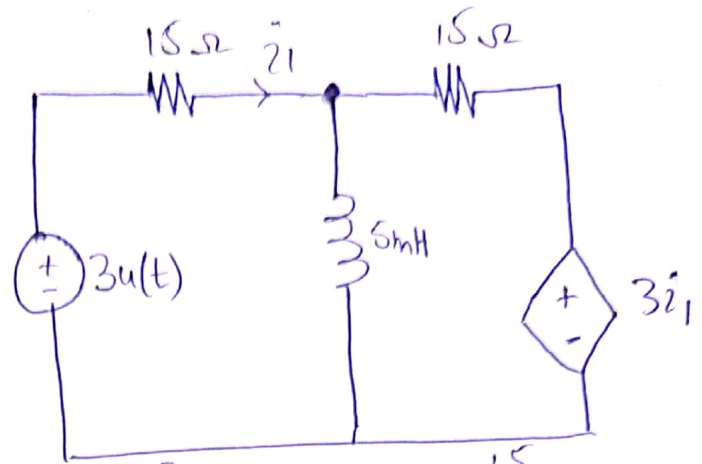
Q:-4
(c)

$$(15+15)i_1 + 3i_1 = 3$$

$$33i_1 = 3$$

$$i_1 = 3/33, \quad i_1(0) = 90.9 \mu\text{A}$$

$$i_1(0) = 90.9 \mu\text{A}$$

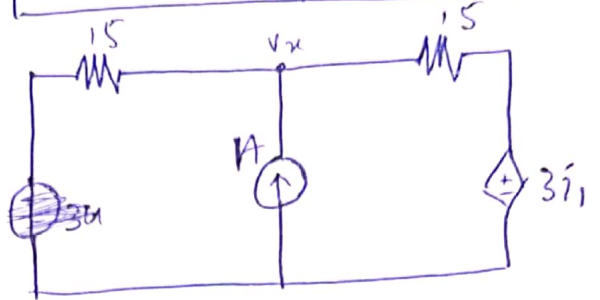


Now using nodal analysis.

$$\frac{V_x}{15} + \frac{V_x - 3(90.9 \mu\text{A})}{15} - 1 = 0$$

$$\frac{2V_x}{15} - 1.018 = 0$$

$$V_x = 7.636 \text{ V}, \quad V_L = R_L = \frac{V_x}{1} = 7.636 \Omega$$



$$T = L/R = \frac{5\text{m}}{7.636}, \quad T = 0.65 \text{ ms}$$

$$i_1(\infty) = 3/15 = 0.2 \text{ A}$$

$$i_1(0) = 90.9 \mu\text{A}$$

$$i_1(t) = i_0 e^{-t/T}$$

$$i_0 = 90.9 \mu\text{A} - 0.2, \quad i_0 = -0.1 \text{ A}$$

$$i_1(t) = -0.1 e^{-t/T} + 0.2 \text{ A}$$

$$i_1(t) = 0.0909 - 0.1 e^{-t/T}$$

$$i_1(t) = (200 - 100 e^{-t/T}) \text{ mA}$$