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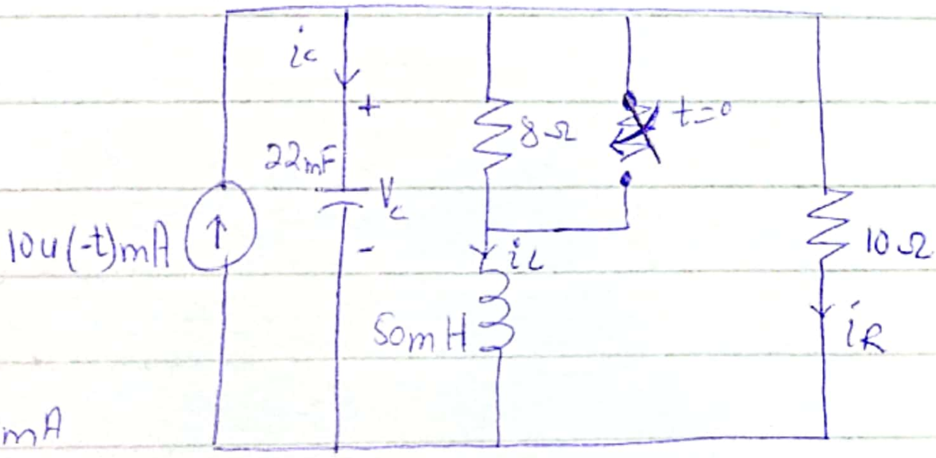
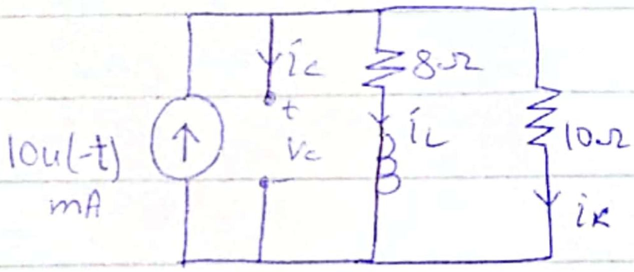
Bsee 19047

ENA - Assignment 1

Q:-1

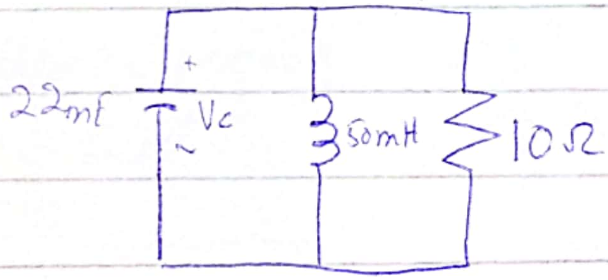


$i_R(0^-) = \frac{10 \times 8}{18}$ mA



$\boxed{i_R(0^-) = 4.44 mA}$

$i_L(0^-) = \frac{10 \times 10}{18}$ mA



$\boxed{i_L(0^-) = 5.55 mA}$

$\boxed{i_c(0^-) = 0A}$

$V_c(0^-) = 5.5mA \times 8$, $\boxed{V_c(0^-) = 44.5mV}$

$\boxed{i_L(0^+) = 5.5mA}$, $\boxed{V_c(0^+) = 44.5mV}$

$\boxed{i_R(0^+) = 4.44mA}$, $\boxed{i_c(0^+) = -10mA}$ (-4.4 -5.5 mA)

$\alpha = 2.27 s^{-1}$, $w_0 = \frac{1}{\sqrt{LC}}$, $w_0 = 30.15 \text{ rads}^{-1}$

$\alpha < w_0$, $w_d = \sqrt{w_0^2 - \alpha^2}$, $w_d = 30 \text{ rads}^{-1}$

$$V_c(t) = e^{-\alpha t}(B_1 \cos w_d t + B_2 \sin w_d t)$$

$$V_c(0) = 44.5m = 1(B_1)$$

$$B_1 = 0.0445$$

$$V_c(t) = e^{-2.27t}\left[0.0445 \cos(30t) + B_2 \sin(30t)\right] \text{ -①}$$

Taking differential on both sides.

$$\frac{dv}{dt} = e^{-2.27t}\left[-1.335 \sin(30t) + 30B_2 \cos(30t)\right] + -2.27 e^{-2.27t}\left[0.0445 \cos(30t) + B_2 \sin(30t)\right].$$

x by C on both sides.

$$C\frac{dv}{dt} = - - \cdots$$

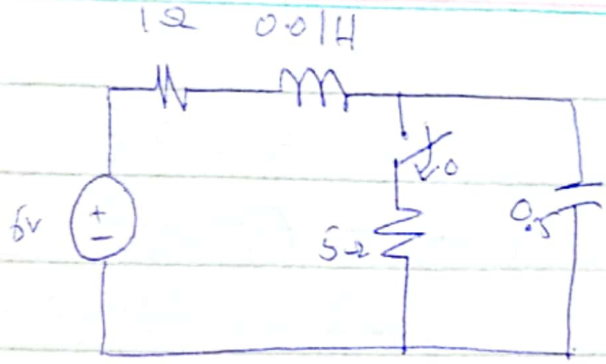
at $t = 0$, $C\frac{dv}{dt} = i_c(0) = -10mA$

$$-10mA = 30B_2 - (2.27 \times 0.0445)$$

$$B_2 = 3.03m = 0.003$$

$$\boxed{V_c(t) = e^{-2.27t}\left[0.0445 \cos(30t) + 0.003 \sin(30t)\right]}$$

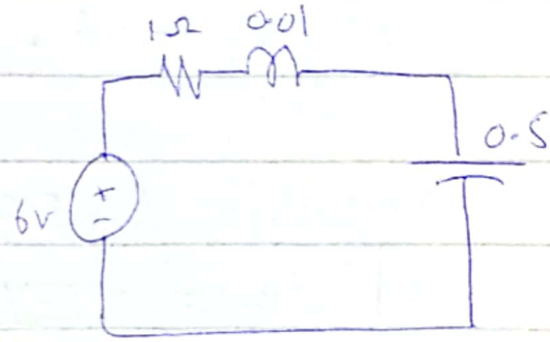
Q2



$\alpha = \frac{R}{2L} = 50\ s^{-1}$

$\omega_0 = \frac{1}{\sqrt{LC}} = 14.14\ rad s^{-1}$

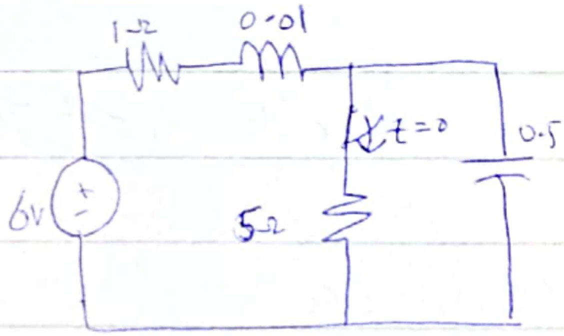
$\alpha > \omega_0$, overdamped.



$S = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$S_1 = -2.04, \quad S_2 = -97.95$

$V(t) = A_1 e^{-2.04t} + A_2 e^{-98t}$ — ①



$V_c(0^-) = \frac{6 \times 5}{6} = 5V$

$i_L(0^-) = 1A, \quad i_c(0^-) = 0A$

$i_c(0^+) = \cancel{6}\ \frac{V_c(0^-)}{1\Omega}, \quad i_c(0^+) = 5A$

Now

at $t = 0$

$A_1 + A_2 = 5$ — ②

differentiating ①

$C \times \frac{dv}{dt} = \left[-2.04 A_1 e^{-2.04t} - 98 A_2 e^{-98t}\right] 0.5$

at $t = 0$

$i(0) = 5 = -1.02 A_1 - \cancel{46.5}\ 49 A_2$ — ③

$A_1 = \cancel{-0.22}\ 5.21$

$A_2 = -0.22 \approx -0.21$

$$V_c(t) = 5.21e^{-2.04t} - 0.21e^{-98t}$$

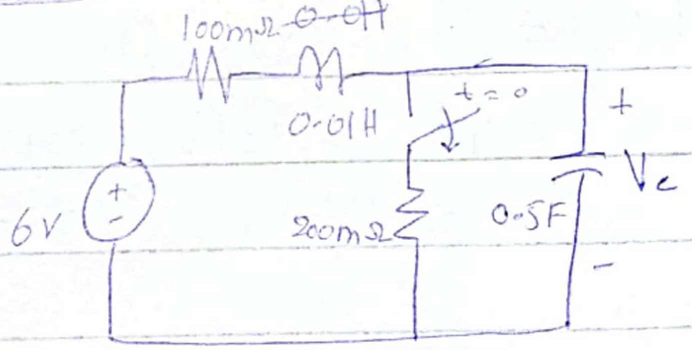
$$V_c(\infty) = 6V$$

$$\boxed{V_c(t) = 6 + 5.21e^{-2.04t} - 0.21e^{-98t}}$$

(b) $V_c(10m) = 6 + 5.21e^{-2.04\times10^{-3}} - 0.21e^{-98\times10^{-3}}$

$$\boxed{V_c(10m) = 5.02V}$$

$$\boxed{V_c(600m) = 5.706V}$$



(C)

Solve

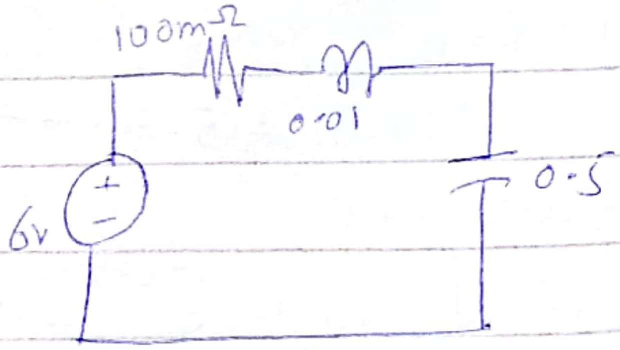
$\alpha = \frac{R}{2L} = 5$

$\omega_0 = \frac{1}{\sqrt{LC}} = 14.14$

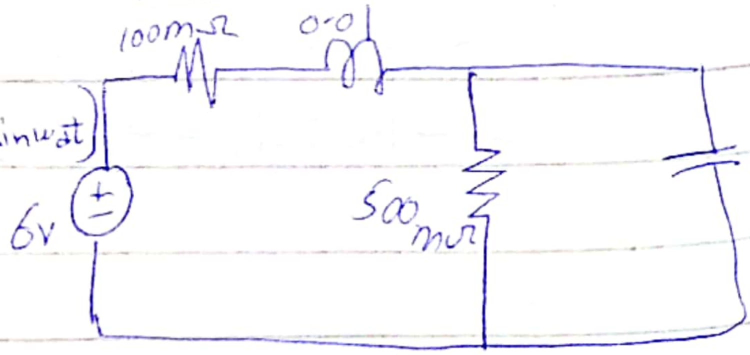
$\alpha < \omega_0$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$\omega_d = 13.2$



$V_c(t) = e^{-\alpha t}\left[B_1\cos\omega_d t + B_2\sin\omega_d t\right]$



$$V_c(t) = e^{-5t}\left[B_1\cos(13.2t) + B_2\sin(13.2t)\right] \quad \text{--- } ①$$

$$V_c(0^-) = \frac{6 \times 500m}{600m} = 5V = V_c(0^+)$$

$$i_c(0^-) = 0A \; , \; i_L(0^-) = \frac{5}{500m} = 10A = i_L(0^+)$$

$$i_c(0^+) = \frac{5}{100m} \; 50A \, ,$$

Now

at $t=0$, $B_1 = 5$

$$V_c(t) = e^{-5t}\left[5\cos(13.2t) + B_2\sin(13.2t)\right] \quad \text{--- } ②$$

differentiating

$$\frac{dv}{dt} = e^{-5t}\left[-66\sin(13.2t) + 13.2B_2\cos(13.2t)\right] + -5e^{-5t}\left[5\cos(13.2t) + B_2\sin(13.2t)\right]$$

$$i(0) = C\frac{dv(0)}{dt}$$

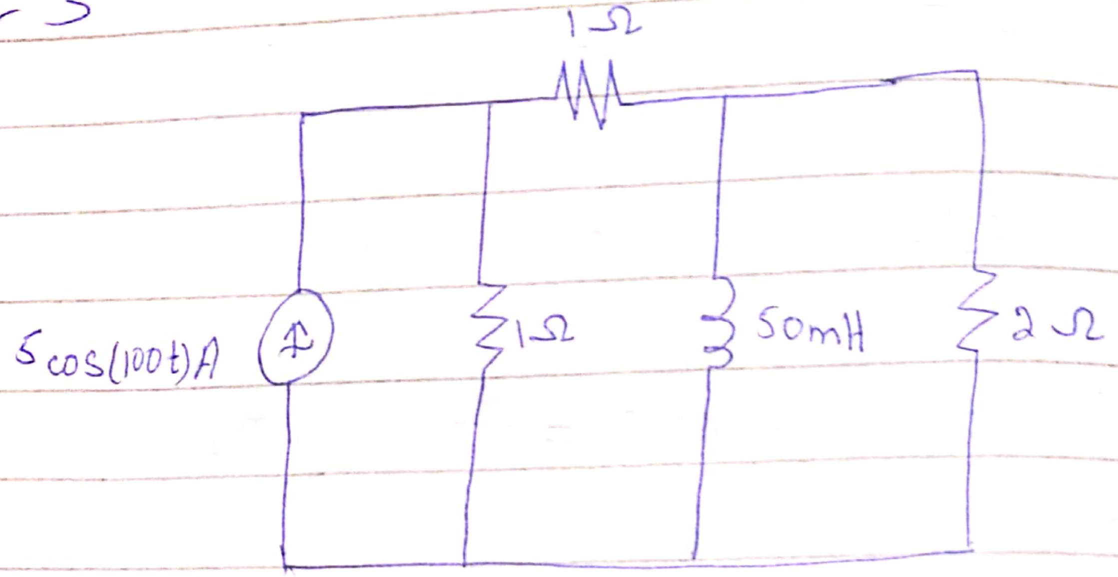
$$i(0) = 0.5\left[13.2B_2 - 5(5)\right] = 50$$

$$6.6B_2 - 12.5 = 50$$

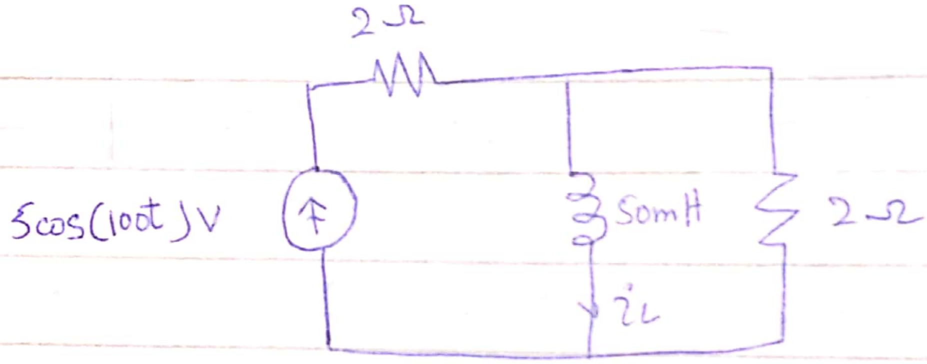
$$B_2 = 9.5$$

$$v(t) = e^{-5t}\left[5\cos(13.2t) + 9.5\sin(13.2t)\right]$$

Q:- 3



using source transformation.



$$V_L = 5\cos(100t) \times \frac{2}{4}$$

$$\boxed{V_L = 2.5\cos(100t)\ v}$$

$V_m = 2.5$, $\omega = 100\ rad\,s^{-1}$, $L = 50mH$

$$R_{eq} = \frac{2\times 2}{2+2} = 1\Omega$$

$$i_L(t) = \frac{V_m}{\sqrt{R_{eq}^2 + \omega^2 L^2}} \times \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

$$i_L(t) = \frac{2.5}{\sqrt{1^2 + (100^2)(50m)^2}} \times \cos\left((100)t - \tan^{-1}\frac{100 \times 50 \times 10^{-3}}{1}\right)$$

$$i_L(t) = 0.5\cos(100t - 78.7^\circ) \quad \text{—— } (1)$$

(b) $V_L = L\frac{di}{dt} = 500m \times \left[100 \times 0.5(-\sin(100t - 78.7^\circ))\right]$

$$V_L = -2.45\sin(100t - 78.7^\circ) \quad \text{—— } (2)$$

$$P = \frac{V^2}{R} = \frac{(-2.45\sin(100t - 78.7^\circ))^2}{2}$$

$$P = 3\sin^2(100t - 78.7^\circ)$$

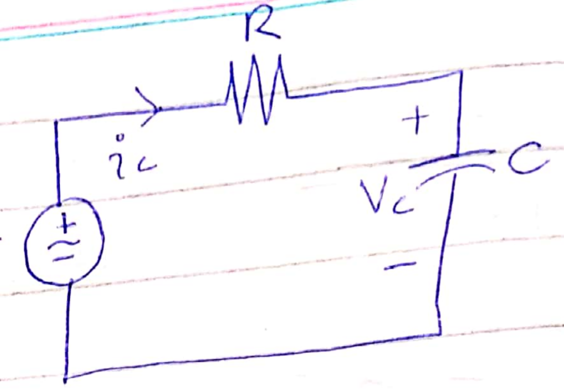
$$\sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$P = 3\left[\frac{1 - \cos 2(100t - 78.7^\circ)}{2}\right]$$

$$P = \frac{3}{2} - \frac{3}{2}\cos 2(100t - 78.7^\circ) \quad A$$

Q:- 4

(a)



Applying KVL

$$i_c R + V_c = V_m \cos\omega t$$

$$i_c = C\frac{dv}{dt}$$

$$RC\frac{dv}{dt} + V_c = V_m \cos\omega t$$

÷ by RC on b/s

$$\frac{dv}{dt} + \frac{V_c}{RC} = \frac{V_m}{RC}\cos\omega t$$

$$V_c = A\cos(\omega t + \phi) \text{ ——— } (1)$$

~~$V_c = A\cos\omega t - As$~~

$$V_c = A\cos(\omega t)\cos\phi - A\sin(\omega t)\sin\phi$$

$$\frac{dV_c}{dt} = -A\omega\cos\phi\sin(\omega t) - A\omega\sin\phi\cos(\omega t)$$

$$-A\omega\cos\phi\sin(\omega t) - A\omega\sin\phi\cos(\omega t)$$

$$+\frac{1}{RC}\left[A\cos\phi\cos\omega t) - A\sin\phi\sin\omega t\right] = \frac{V_m}{RC}\cos\omega t$$

$$-A\omega \sin\phi + \frac{A}{RC}\cos\phi = \frac{V_m}{RC}$$

$$-ARC\omega \sin\phi + A\cos\phi = V_m$$

$$V_m = -A\omega RC\sin\phi + A\cos\phi$$

$$V_m = A(\cos\phi - \omega RC\sin\phi)$$

$$A = \frac{V_m}{\cos\phi - \omega RC\sin\phi} \quad \text{———} \quad (2)$$

substituting A in (1)

$$V_c = \frac{V_m}{\cos\phi - \omega RC\sin\phi}\left[\cos(\omega t + \phi)\right]$$

$$\phi = \tan^{-1}(-\omega RC)$$

$$V_c(t) = \frac{V_m \times \left[\cos(\omega t + \phi)\right]}{\cos(\tan^{-1}(-\omega RC)) - \omega RC\sin(\tan^{-1}(-\omega RC))}$$

(b) $V_s = 20 \sin 100t$, $R = 10\Omega$, $C = 100\mu F$

solve

$V_m = 20$, $R = 10\Omega$, $C = 100\mu$,

$\omega = 100$

using eq

$V_c(t) =$

$\phi = \tan^{-1}(-\omega RC) = -5.71^\circ$

$$V_c(t) = \frac{V_m}{\cos\phi - \omega RC \sin\phi} \times \cos(\omega t + \phi)$$

$$V_c(t) = \frac{20 \times \cos(\omega t + \phi)}{\cos(-5.71) - 100 \times 10 \times 100\mu \sin(-5.71)}$$

$$V_c(t) = \frac{20 \cos(\omega t - 5.71^\circ)}{0.99 - 9.9m}$$

$$\boxed{V_c(t) = 20.4 \cos(100t - 5.71^\circ)}$$

(c) $V_s = 20\sin(100t)$

$V_s = 20\cos(100t - 90^\circ)$

$V_s = 20e^{j(100t-90^\circ)}$ —— (1)

$-20e^{j(100t-90)} + Ri_c + V_c = 0$

$-20e^{j(100t-90)} + 10i_c + V_c = 0$ —— (2)

$i_c = C\frac{dV_c}{dt}$

$-20e^{j(100t-90)} + 10C\frac{dV_c}{dt} + V_c = 0$ —— (3)

$V_c = V_m e^{j(100t-90)}$

$-20e^{j(100t-90)} + 0.001\frac{d}{dt}\left(V_m e^{j(100t-90)}\right) + V_m e^{j(100t-90)} = 0$

$-20e^{j(100t-90)} + 0.001\left[100jV_m e^{j(100t-90)}\right] + V_m e^{j(100t-90)} = 0$

$-20e^{j(100t-90)} + 0.1jV_m e^{j(100t-90)} + V_m e^{j(100t-90)} = 0$

$\cancel{-20 + 1.1V_m = 0}$

$\underline{V_m = 18.18}$ $V_m[0.1j + 1] = 20$

$V_m = \frac{20}{0.1j+1}$

$V_m = \frac{20}{\sqrt{(1^2)+(0.1^2)}} \angle -\tan^{-1}\left[\frac{0.1}{1}\right] = 19.9 \angle -5.7$

$$V_m = 19.9 e^{-j5.7t}$$

$$V_c(t) = 19.9 e^{-j5.7} \times e^{j(100t-90)}$$

$$V_c(t) = 19.9 e^{j(100t-95.7)}$$

$$\boxed{V_c(t) = 19.9 \cos(100t - 95.7^\circ)}$$

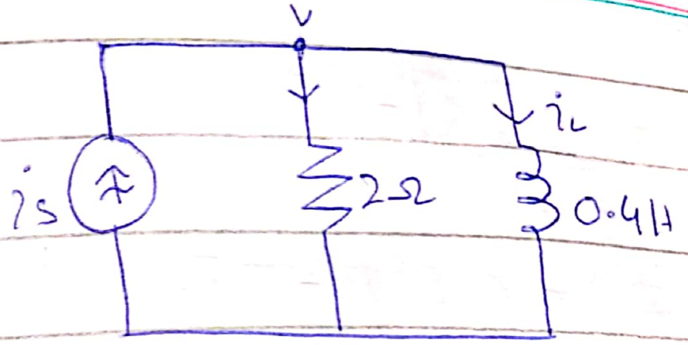
$$i_c(t) = c \frac{dv}{dt}$$

$$= 100\mu \left[-19.9 \times 100 \sin(100t - 95.7^\circ) \right]$$

$$\boxed{i_c(t) = 0.199 \sin(100t - 95.7^\circ)}$$

Q:- 5

(a)



$i_s = 2\cos 20t\,A$

$i_s = 2e^{j20t}\,A$

Applying KCL

$$-2e^{j20t} + \frac{V}{2} + i_L = 0$$

$$V = L\frac{di}{dt}$$

$$-2e^{j20t} + \frac{L}{2}\frac{di}{dt} + i_L = 0$$

$$i_L = I_m e^{j20t} \quad \text{——— (1)}$$

$$-2e^{j20t} + \frac{L}{2}\frac{d}{dt}\left(I_m e^{j20t}\right) + I_m e^{j20t} = 0$$

$$-2e^{j20t} + \frac{0.4}{2}\left(j20 I_m e^{j20t}\right) + I_m e^{j20t} = 0$$

$$-2e^{j20t} + \cancel{0.2}\, 4j I_m e^{j20t} + I_m e^{j20t} = 0$$

$$-2 + 4j I_m + I_m = 0$$

$$I_m\{4j + 1\} = 2$$

$$I_m = \frac{2}{4j+1} \quad \text{——— (2)}$$

Scanned with CamScanner

$$I_m = \frac{2}{\sqrt{4^2+1^2}} \angle -\tan^{-1}\left(\frac{4}{1}\right)$$

$$I_m = 0.5 \angle -75.96^\circ$$

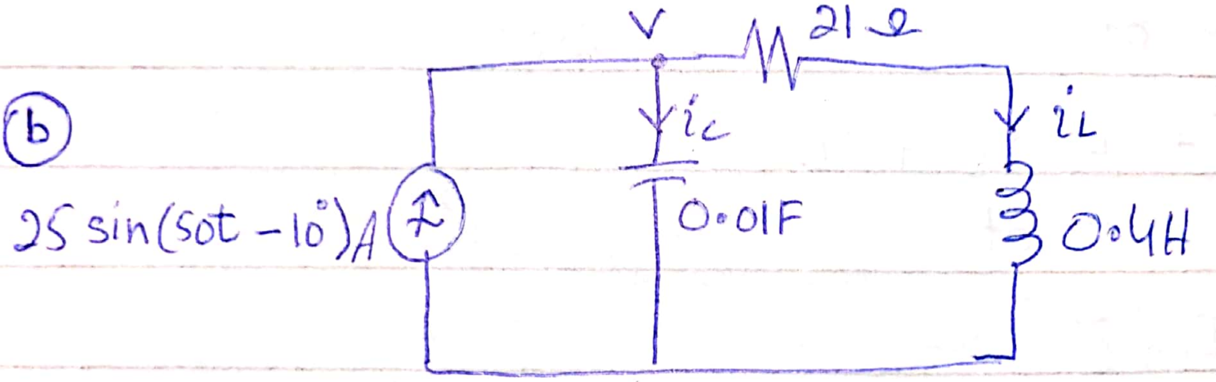
$$I_m = 0.5\, e^{-j76^\circ}$$

substituting in ①

$$i_L(t) = 0.5\, e^{-j76^\circ} \times e^{j20t}$$

$$\boxed{i_L(t) = 0.5 \cos(20t - 76^\circ)}$$

(b)



$i_s = 25\sin(50t - 10^\circ)$ ∵ sin leads cos by 90°

$i_s = 25\cos(50t - 100^\circ) = 25 e^{j(50t-100)}$

As

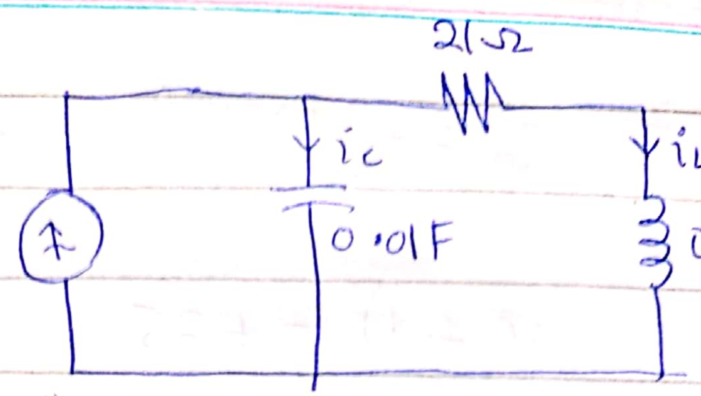
$$-25e^{j(50t-100)} + i_c + i_L = 0$$

$$-25e^{j(50t-100)} + C\frac{dv}{dt} + i_L = 0$$

$V_c =$

(b)

$25\sin(50t-10^\circ)\,V$



Solve

$$V_s = 25\sin(50t-10^\circ)\,v$$

$$V_s = 25\cos(50t-100^\circ) \qquad \because \text{sin leads cos by } 90^\circ$$

$$V_s = 25e^{j(50t-100)} \quad \text{——} \quad (1)$$

Now Applying KCL

$$-25e^{j(50t-100)} + i_c + i_L = 0$$

$$-25e^{j(50t-100)} + C\frac{dv}{dt} + i_L = 0 \quad \text{——} \quad (2)$$

And $\quad 21 i_L + 0.4\frac{di}{dt} = V$

$$-25e^{j(50t-100)} + 0.01\cancel{\times 0.4}\frac{d}{dt}\left(21 i_L + 0.4\frac{di}{dt}\right) + i_L = 0$$

$$-25e^{j(50t-100)} + 0.21\frac{di_L}{dt} + 0.004\frac{d^2 i}{dt^2} + i_L = 0 \quad \text{—} \quad (3)$$

$$i_c(t) = I_m e^{j(50t-100)} \quad \text{——} \quad (4)$$

substituting (4) in (3)

$$-25e^{j(50t-100)} + 0.21\frac{d}{dt}\left(I_m e^{j(50t-100)}\right) + 0.004\frac{d^2}{dt^2}\left(I_m e^{j(50t-100)}\right) +$$

$$I_m e^{j(50t-100)}$$

$$-25e^{j(50t-100)} + j10.5 I_m e^{j(50t-100)} - 10 I_m e^{j(50t-100)} + I_m e^{j(50t-100)}$$

$$I_m (j10.5 - 10 + 1) = +25$$

$$I_m = \frac{25}{10.5j - 9}$$

$$I_m = \frac{25}{\sqrt{(10.5)^2 - 9^2}} \angle -\tan^{-1}\left[\frac{10.9}{9}\right]$$

$$I_m = 4.6 \angle -50.5$$

$$I_m = 4.6 e^{-j50.5} \quad \text{———} \quad (1)$$

substituting I_m in (4)

$$i(t) = 4.6 e^{-j50.5} \times e^{j(50t-100)}$$

$$\boxed{i(t) = 4.6 \cos(50t - 150.5) \text{ A}}$$