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Bsee 19047

## NA Assignment 2

Q:-1

$$f(x) = 5x^3 - 5x^2 + 6x - 2$$

(a) find real roots graphically.

from the graph we can predict that the function is 0 at

$x = 0.4$  or  $0.45$  so let's suppose

the real roots/root of  $f(x) = \frac{0.4 + 0.45}{2}$

$$\text{roots} = 0.425$$

from the graph the root of  $f(x)$  lies between  $0.4$  and  $0.425$  whereas from (Desmos) root of  $f(x)$  comes out to be  $0.4181$

(b)

$$x_l = 5.0, \quad x_u = 5.1$$

$$x_r = \frac{5 + 5.1}{2} = 5.05$$

$x_l, x_u$  cannot be considered as the valid values to compute, it might be a typo so I assume the  $x_l = 0, x_u = 1$

$$x_r = \frac{0 + 1}{2} = 0.5, \quad \epsilon = \left| \frac{0.425 - 0.5}{0.425} \right| \times 100 = 17.647\%$$

$$f(x_r) = 0.375$$

Now, let

$$x_u = x_l = 0.5, \quad x_l = 0 \\ x_r = \frac{0 + 0.5}{2} = 0.25, \quad \epsilon = 41.17\%$$

$$f(x_r) \leq 0, \text{ so } x_r = x_l = 0.25, \quad x_u = 1$$

using the same procedure

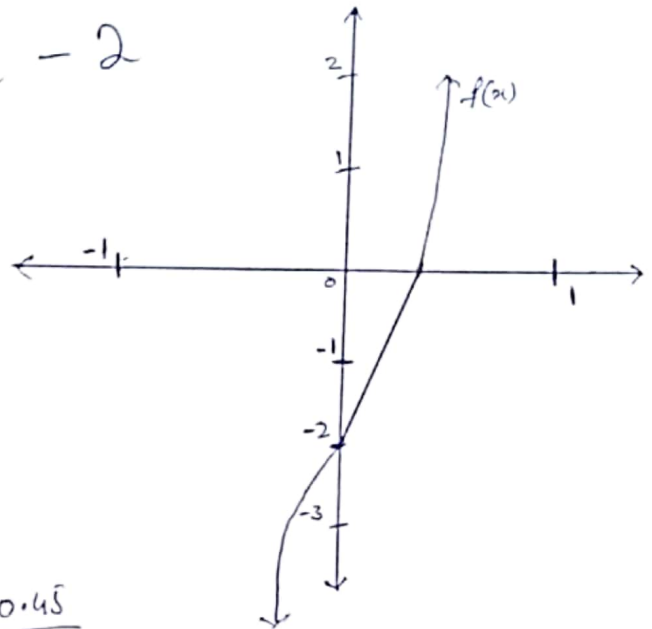
$$x_r = 0.625, \quad \epsilon = 48.68\%, \quad f(x_r) \geq 0$$

$$x_u = x_r = 0.625, \quad x_l = 0.25$$

$$x_r = \frac{0.25 + 0.625}{2} = 0.4375, \quad \epsilon = 4.6\%$$

so

$$\boxed{x_r = 0.4375}$$



Q:-2

$$f(x) = 0.7x^5 - 8x^4 + 44x^3 - 90x^2 + 82x - 25$$

Solve

- (a) real root of  $f(x)$  will be in the range of (0.5 to 0.6)

Actual root = 0.5794

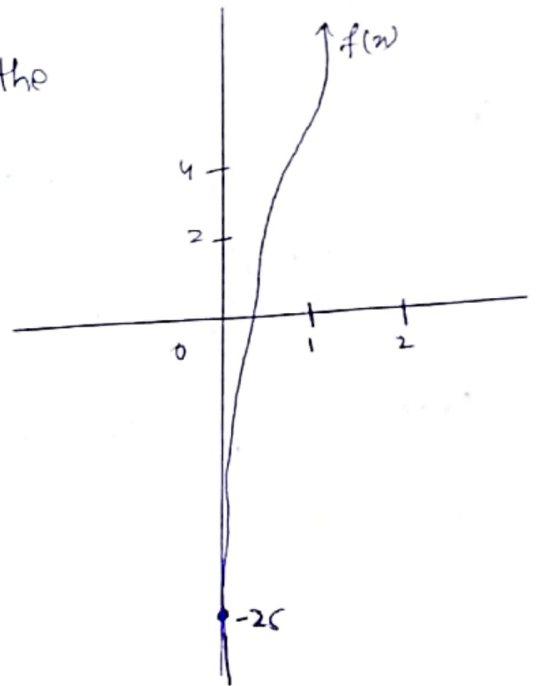
(b)

$$x_l = 0.5, \quad x_u = 1.0$$

$$x_r = \frac{0.5 + 1}{2} = 0.75, \quad \epsilon = 30\%$$

$$x_u = 0.75$$

$$x_r = \frac{0.5 + 0.75}{2} = 0.625, \quad \epsilon = 7.87\%$$



(c)  $x_l = 0.5, x_u = 1.0$

$$f(x_l) = -1.478, \quad f(x_u) = 3.7$$

$$x_r = 1 - \frac{(3.7)(0.5 - 1)}{-1.478 - 3.7} = 0.6427, \quad \epsilon = \left| \frac{0.5794 - 0.6427}{0.5794} \right| \times 100 = 10.92$$

$$f(x_r) = 0.9184$$

$$f(x_l) \times f(x_r) = -1.3574$$

Now as  $f(x_l) \times f(x_r) < 0$  so, we have to take  $x_u = x_r$

$$x_l = 0.5, \quad x_u = 0.6427$$

$$f(x_l) = -1.478, \quad f(x_u) = 0.9184$$

$$x_r = 0.6427 - \frac{(0.9184)(0.5 - 0.6427)}{-1.478 - 0.9184}$$

$$x_r = 0.588$$

$$\epsilon = \left| \frac{0.5794 - 0.588}{0.5794} \right| \times 100 = 1.4843\%$$

$$f(x_r) = 0.137, \quad f(x_l) f(x_r) = -0.2025 < 0$$

So let  $x_u = x_r$

$$x_1 = 0.5, \quad x_4 = 0.588$$

$$f(x_1) = -1.478, \quad f(x_4) = 0.137$$

$$x_r = 0.588 - \frac{(0.137)(0.5 - 0.588)}{-1.478 - (0.137)} = 0.5805$$

$$\epsilon = \left| \frac{0.5794 - 0.5805}{0.5794} \right| \times 100 = 0.19588\% < 0.2\%$$

Hence,  $x_r = 0.5805$  can be considered as the real root of  $f(x)$

Q:- 3

$$f(x) = x^{3.5} = 80$$

(a)  $f(x) = x^{3.5} - 80$   
let  $f(x) = 0$ ,  $x^{3.5} - 80 = 0$ ,  $x^{3.5} = 80$

$$\log_{10}(x) = \frac{\log_{10}(80)}{3.5} = 0.54374$$

$$\log_{10}(x) = 0.54374$$

$$x = 3.4947$$

$$\boxed{x_r = 3.4947}$$

$$\textcircled{b} \quad \epsilon_s = 2.5\% \quad f(x) = x^{3.5} - 80$$

$$x_l = 2, \quad x_u = 5 \quad \text{Actual value} = 3.4975$$

$$f(x_l) = -68.68, \quad f(x_u) = 199.5$$

$$x_r = 5 - \frac{(199.5)(2-5)}{-68.68 - 199.5}, \quad x_r = 2.76828, \quad \epsilon = 20.85\%$$

$$f(x_r) = -44.7, \quad \text{as } f(x_l)f(x_r) > 0 \quad \text{so}$$

$$x_l = x_r, \quad x_u = 5$$

$$x_r = 5 - \frac{(199.5)(2.76828-5)}{-44.7 - (+199.5)} = 3.1768, \quad \epsilon = 9.17\%$$

$$f(x_r) = -22.857$$

$$\text{as } f(x_l)f(x_r) > 0, \quad \text{so } x_l = x_r, \quad x_u = 5$$

$$x_l = 3.1768$$

$$x_r = 5 - \frac{(199.5)(3.1768-5)}{(-22.857) - 199.5} = 3.3642, \quad \epsilon = 3.8113\%$$

$$f(x_r) = -10.163$$

$$f(x_l)f(x_r) > 0, \quad x_l = x_r = 3.3642, \quad x_u = 5$$

$$x_r = 5 - \frac{(199.5)(3.3642-5)}{-10.163 - 199.5} = 3.4434, \quad \epsilon = 1.5468\%$$

$$\epsilon < \epsilon_s$$

so,  $x_r = 3.4434$  can be considered as the real root of  $(x^{3.5} = 80)$  or  $(x^{3.5} - 80)$

## Q4

### a) Code

```
% Bisection Method in MATLAB
a=input('Enter function with right-hand-side equals to zero:');
f=inline(a);
xl=input('Enter the initial guess (Xl) :');
xu=input('Enter the initial guess (Xu) :');
%Es=input('Enter the estimated error (Es) :');
if f(xl)*f(xu)<0
else
fprintf('The guess is incorrect! Enter new guesses!\n');
xl=input('Enter the initial guess (Xl) :');
xu=input('Enter the initial guess (Xu) :');
end
for i=1:4
xr=(xl+xu)/2;
if f(xr)*f(xl)>0
xl=xr;
else
xu=xr;
end

if f(xl)*f(xr)<0
xu=xr;
else
xl=xr;
end

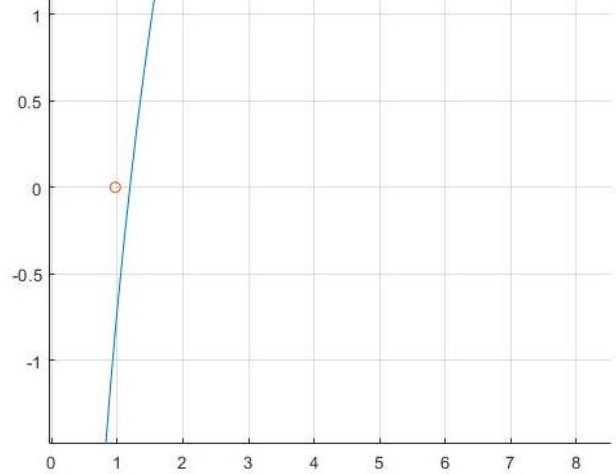
xnew(1)=0;
xnew(0)=xr;
%if abs((xnew(i)-xnew(i-1))/xnew(i))<Es,break,end
end
x=0.1:100;
y=log(x.^4)-0.7;
hold on
plot(x,y)
grid on
scatter(xr,0)
str = ['After three iterations of the bisection method the required
real positive root of the equation is: ', num2str(xr), '\n']
```

### a) Command window

```
>> Q4_a
Enter function with right-hand-side equals to zero:x^4
-0.7
Enter the initial guess (Xl) :0.5
Enter the initial guess (Xu) :2

str =

'After three iterations of the bisection method the
required real positive root of the equation is: 0.96875'
```



### b) Code

```
%False-position method
clc
syms x;
y = input('Enter function with right-hand-side equals to zero:');
xl=input('Enter the initial guess (Xl) :');
xu=input('Enter the initial guess (Xu) :');

fa = double(subs(y,x,xl));
fb = double(subs(y,x,xu));

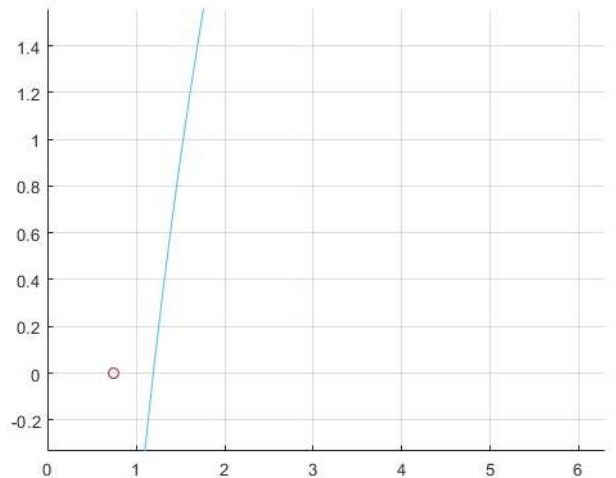
if fa*fb > 0
disp('Given initial values do not bracket the root:');
else
c = xl - (xl-xu) * fa/(fa-fb);
fc = eval(subs(y,x,c));
fprintf('nInitial values: %f, %f, %f, %f\n', xl, xu, c, fc);
for i=1:4
fprintf('%f, %f, %f, %f\n', xl, xu, c, fc);
if fa*fc < 0
xu = c;
fb = eval(subs(y,x,xu));
else
xl = c;
fa = eval(subs(y,x,xl));
end
c = xl - (xl-xu) * fa/(fa-fb);
fc = eval(subs(y,x,c));
end
fprintf('nAfter three iterations of the False_position method
the required real positive root of the equation is: %f\n', c);
hold on
grid on
x=0.1:100;
f=log(x.^4)-0.7;
plot(0.1:100,f)
scatter(c,0)
end
```

### c) Command Window

```
Enter function with right-hand-side equals to zero: x^4
-0.7
Enter the initial guess (Xl) :.5
Enter the initial guess (Xu) :2

a          b          c          f(c)
0.500000    2.000000    0.560000    -0.601655
0.560000    2.000000    0.614484    -0.557425
0.614484    2.000000    0.663188    -0.506560
0.663188    2.000000    0.706029    -0.451520
```

After three iterations of the **False position method** the required real positive root of the equation is: **0.743121**



## Comparison graph

