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Sns. Assignment no:- 3

Q:-1

① Difference b/w fourier series and fourier transform.

Fourier series is the expansion of a periodic signal as a linear combination of sin and cosines whereas Fourier Transform is the process of converting or transforming a time-Domain signal into its frequency-Domain equivalent signal

$$x(t) = \sum_{k=-\infty}^{+\infty} (a_k e^{jk\omega_0 t}) \Rightarrow \text{Fourier-Series [continuous time]}$$

$$x[n] = \sum_{k=-\infty}^{+\infty} (a_k e^{jk\omega_0 n}) \Rightarrow \text{Fourier-Series [Discrete time]}$$

$$x(j\omega) = \int_{-\infty}^{\infty} (x(t) e^{-j\omega t}) dt \Rightarrow \text{Fourier-Transform}$$

$$x(j\omega) = \int_k (x(t) e^{-j\omega t}) dt \Rightarrow (\text{for Discrete or dis/non contin})$$

②

- Noise Cancellation
- Signal Processing
- Control Theory
- Telecommunication
- Radio

Q:-2

$$N=5, a_0=2, a_2=a_{-2}^* = 2e^{j\pi/6}$$
$$a_4=a_{-4}^* = e^{j\pi/3}$$

express  $x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\omega_k n + \phi_k)$

Solve

$$x[n] = A_0 + \sum_{k=1}^N A_k e^{j\omega_k n} = A_0 + \sum_{k=\pm 1}^{\pm 5} A_k e^{j\omega_k n}$$

plug-in the given values,

$$x[n] = 2 + (2e^{j\pi/6})(e^{j(2\pi/5)n}) + (2e^{-j\pi/6})(e^{-j(2\pi/5)n}) + (e^{j\pi/3})(e^{j(4\pi/5)n}) + (e^{-j\pi/3})(e^{-j(4\pi/5)n})$$

$$= 2 + 4 \cos\left(\frac{4\pi}{5}n + \frac{\pi}{6}\right) + 2 \cos\left(\frac{8\pi}{5}n + \frac{\pi}{3}\right)$$

~~$\because \cos(\phi + 90^\circ) = \sin(\phi)$~~

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$$x[n] = 2 + 4 \cos\left(\frac{4\pi}{5}n + \frac{\pi}{6} + \frac{\pi}{2}\right) + 2 \cos\left(\frac{8\pi}{5}n + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$x[n] = 2 + 4 \cos\left[\frac{4\pi}{5}n + \frac{2\pi}{3}\right] + 2 \cos\left[\frac{8\pi}{5}n + \frac{5\pi}{6}\right] \text{ A}$$

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Q:- 3  $N = 8$

$$a_k = \cos\left[\frac{k\pi}{4}\right] + \sin\left[\frac{3k\pi}{4}\right] \quad \because \omega = \frac{\pi}{4}$$

Solve

$$a'_k = \frac{1}{N} \sum_0^7 x[n] e^{-jk\omega n} \Rightarrow \frac{1}{8} \sum_0^7 x[n] e^{-jk n \frac{\pi}{4}}$$

$$a'_k = \frac{1}{8} x[0] e^0 + \frac{x[1]}{8} e^{-jk\pi/4} + \frac{x[2]}{8} e^{-2jk\pi/4} + \frac{x[3]}{8} e^{-3jk\pi/4} \\ + \frac{x[4]}{8} e^{-4jk\pi/4} + \frac{x[5]}{8} e^{-5jk\pi/4} + \frac{x[6]}{8} e^{-6jk\pi/4} + \frac{x[7]}{8} e^{-7jk\pi/4}$$

$$a'_k = \frac{x[0]}{8} + \frac{x[1] e^{-jk\pi/4}}{8} + \frac{x[2] e^{-2jk\pi/4}}{8} + \frac{x[3] e^{-3jk\pi/4}}{8} + \frac{x[4] e^{-4jk\pi/4}}{8} \\ + \frac{x[5] e^{-5jk\pi/4}}{8} + \frac{x[6] e^{-6jk\pi/4}}{8} + \frac{x[7] e^{-7jk\pi/4}}{8} \quad \text{--- (1)}$$

$$a_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4} \Rightarrow \frac{e^{jk\pi/4}}{2} + \frac{e^{-jk\pi/4}}{2} + \frac{e^{3jk\pi/4}}{2} - \frac{e^{-3jk\pi/4}}{2}$$

$$a_k = \frac{e^{jk\pi/4}}{2} + \frac{e^{-jk\pi/4}}{2} + \frac{e^{3jk\pi/4}}{2} - \frac{e^{-3jk\pi/4}}{2} \quad \text{--- (2)}$$

comparing eq (1) and (2)

$$x[0] = 0, \quad x[1] = 4, \quad x[3] = -4j, \quad x[5] = 4j, \quad x[7] = 4$$

as  $a_k$  is an odd function hence only odd values of  $n$  will be significant and for discrete

$$x[n] = 0 + 4[\delta[n-1]] + 4\delta[n-7] - 4j\delta[n-3] + 4j\delta[n-5]$$

$$x[n] = 4\delta[n-1] - 4j\delta[n-3] + 4j\delta[n-5] + 4\delta[n-7] \quad \text{Answer}$$

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