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SnS - Assignment 4

Q:-1

(a) $x[n] = y[n] - a\,y[n-1]$

Solve

when $|a| < 1$

The frequency response of this system can be found by using $H(e^{jw}) = \dfrac{Y(e^{jw})}{X(e^{jw})} = \dfrac{\sum_{K=0}^{M} (b_K e^{-jK\omega})}{\sum_{K=0}^{N} (a_K e^{-jK\omega})}$ — (z)

$$H(e^{jw}) = \frac{1}{1 - a e^{-jw}} \quad — \quad (1)$$

eq (1) is the fourier transform of the sequence $a^n u[n]$

the impulse response of this system is

$$\boxed{h[n] = a^n u[n]}$$

(b) $2x[n] = y[n] - \frac{3}{4} y[n-1] + \frac{1}{8} y[n-2]$

Solve from eq (z)

$$H(e^{jw}) = \frac{2}{1 - \frac{3}{4} e^{-jw} + \frac{1}{8} e^{-2jw}} = \frac{2}{\left(1 - \frac{e^{-jw}}{2}\right)\left(1 - \frac{e^{-jw}}{4}\right)}$$

$$H(e^{jw}) = \frac{4}{1 - \frac{e^{-jw}}{2}} - \frac{2}{1 - \frac{e^{-jw}}{4}}$$

$$\boxed{h[n] = 4\left[\frac{1}{2}\right]^n u[n] - 2\left[\frac{1}{4}\right]^n u[n]}$$

(c) $x[n] = \left[\frac{1}{4}\right]^n u[n]$

Solve

$$Y(e^{j\omega}) = H(e^{j\omega})X(e^{j\omega}) = \left[\frac{2}{\left(1-\frac{e^{-j\omega}}{2}\right)\left(1-\frac{e^{-j\omega}}{4}\right)}\right]\left[\frac{1}{1-\frac{e^{-j\omega}}{4}}\right]$$

$$Y(e^{j\omega}) = \frac{2}{\left(1-\frac{e^{-j\omega}}{2}\right)\left(1-\frac{e^{-j\omega}}{4}\right)^2}$$

$$= \frac{-4}{1-\frac{1}{4}e^{-j\omega}} - \frac{2}{\left(1-\frac{1}{4}e^{-j\omega}\right)^2} + \frac{8}{1-\frac{1}{2}e^{-j\omega}}$$

$$y[n] = \left\{-4\left[\frac{1}{4}\right]^n - 2(n+1)\left[\frac{1}{4}\right]^n + 8\left[\frac{1}{2}\right]^n\right\}u[n]$$ Answere

Q:-2

$$x[n] = \sin\left(\frac{\pi}{3}n + \frac{\pi}{4}\right)$$

$\omega = \frac{\pi}{3} = \frac{2\pi}{N}$, $N = 6$

$$x[n] = \sin\left(\frac{\pi n}{3} + \frac{\pi}{4}\right) = \frac{e^{j\left(\frac{\pi n}{3}+\frac{\pi}{4}\right)}}{2j} - \frac{e^{-j\left(\frac{\pi n}{3}+\frac{\pi}{4}\right)}}{2j}$$

$$x[n] = \frac{\left(e^{j\frac{\pi n}{3}}\right)\left(e^{j\pi/4}\right)}{2j} - \frac{\left(e^{-j\frac{\pi n}{3}}\right)\left(e^{-j\pi/4}\right)}{2j}$$

$$a_1 = \frac{e^{j\frac{\pi}{4}}}{2j}, \quad a_{-1} = \frac{-e^{-j\pi/4}}{2j}$$

ω from $-\pi$ to π, we obtain

$$X(e^{j\omega}) = 2\pi a_1 \delta\left(\omega - \frac{2\pi}{6}\right) + 2\pi a_{-1} \delta\left(\omega + \frac{2\pi}{6}\right)$$

$$= \cancel{2}\pi\left[\frac{e^{j\pi/4}}{\cancel{2}j}\right]\delta\left(\omega - \frac{2\pi}{6}\right) - \cancel{2}\pi\left[\frac{e^{-j\pi/4}}{\cancel{2}j}\right]\delta\left(\omega + \frac{2\pi}{6}\right)$$

$$= \frac{\pi e^{j\frac{\pi}{4}}}{j}\left[\delta\left(\omega - \frac{2\pi}{6}\right)\right] - \left[\delta\left(\omega + \frac{2\pi}{6}\right)\right]\frac{e^{-j\frac{\pi}{4}}\pi}{j}$$

$$= \frac{\pi}{j}\left[\left(e^{j\pi/4}\right)\left(\delta\left(\omega - \frac{2\pi}{6}\right)\right) - \left(e^{-j\frac{\pi}{4}}\right)\left(\delta\left(\omega + \frac{2\pi}{6}\right)\right)\right]$$

Q:- 3

$$X_1(e^{j\omega}) = e^{-j\omega} \sum_{K=1}^{10} (\sin K\omega)$$

Solve

let $Y_1(e^{j\omega}) = \sum_{K=1}^{10} (\sin K\omega)$

$$X_1(e^{j\omega}) = e^{-j\omega} Y_1(e^{j\omega})$$

(i) Imaginary (purely)

(ii) Neither even Nor odd