
MULTIPLE LINEAR REGRESSION

Assignment Task:

Your task is to perform a multiple linear regression analysis to predict the price of Toyota corolla based on the given attributes.

Dataset Description:

The dataset consists of the following variables:

Age: Age in years

KM: Accumulated Kilometers on odometer

FuelType: Fuel Type (Petrol, Diesel, CNG)

HP: Horse Power

Automatic: Automatic (Yes=1, No=0)

CC: Cylinder Volume in cubic centimeters

Doors: Number of doors

Weight: Weight in Kilograms

Quarterly_Tax:

Price: Offer Price in EUROS

Tasks:

1. Perform exploratory data analysis (EDA) to gain insights into the dataset. Provide visualizations and summary statistics of the variables. Pre process the data to apply the MLR.

Answer:

Dataset overview

The dataset contains 1,436 rows and the following main variables: Price (EUR), Age_08_04 (years), KM (kilometers), Fuel_Type (Diesel/Petrol), HP, Automatic (0/1), cc (engine displacement), Doors, Cylinders, Gears, and Weight.

Key descriptive statistics (high level)

- Price: mean \approx 10,731, median 9,900; wide spread (min 4,350; max 32,500).
- Age_08_04: mean \approx 55.9 years (dataset uses this encoding), median 61; right/left shape depends on sample — check plot.
- KM: mean \approx 68,533 km, heavily right-skewed with max 243,000 (long tail).
- HP and Weight are meaningful positive predictors: HP mean \approx 101, Weight mean \approx 1,072 kg.
- cc: median \approx 1,600 but a suspicious max of 16,000 — likely data-entry error (should be 1600). Investigate and correct/remove if confirmed erroneous.
- Automatic is rare (\approx 5.6% = mostly manual).
-

Distributions & outliers

- KM and Price have skew and long tails; consider log transforms for modeling.
- Several extreme values detected (KM, cc, Price). Use IQR or z-score methods to flag them; do not blindly drop — verify domain plausibility.
- Fuel_Type shows Diesel and Petrol as main categories; use one-hot encoding for modeling.
-

Correlations & relationships

- Weight and HP positively correlated with Price.
- Age_08_04 and KM negatively correlated with Price (older cars and high km \rightarrow lower price).
- Variance Inflation Factor (VIF) showed multicollinearity among some features (Weight, HP, Doors, cc had elevated VIF), so prefer Ridge/Lasso or drop/transform correlated columns.

Preprocessing summary for MLR

- Convert types to numeric, encode Fuel_Type as dummies, ensure Automatic is binary int.
- Impute missing numeric values with median (none in current file but included for robustness).
- Optionally cap extreme cc (e.g., cap at 3000) or remove rows where cc is implausible (like 16000).
- Consider log(Price) and log/KM scaling for heteroscedasticity and skew reduction.
- Scale numeric features (StandardScaler) before Ridge/Lasso.

2.Split the dataset into training and testing sets (e.g., 80% training, 20% testing).

Answer:

```
# split_dataset.py
# Split cleaned Toyota Corolla dataset into train/test (80/20) and save splits to disk.
# Update DATA_DIR if needed.

import os
from pathlib import Path
import pandas as pd
from sklearn.model_selection import train_test_split

DATA_DIR = Path(r"D:\DATA SCIENCE\ASSIGNMENTS\6 MLR\MLR")
CLEANED_CSV = DATA_DIR / "ToyotaCorolla_MLR_cleaned.csv"
ORIG_CSV = DATA_DIR / "ToyotaCorolla - MLR.csv"

# Load cleaned file if present, otherwise load original and do minimal prep
if CLEANED_CSV.exists():
    df = pd.read_csv(CLEANED_CSV)
else:
    df = pd.read_csv(ORIG_CSV)
    # minimal preprocessing to ensure numeric & dummies if CLEANED not available
    for c in df.columns:
        if c not in ("Fuel_Type",):
            df[c] = pd.to_numeric(df[c], errors="coerce")
    if "Fuel_Type" in df.columns:
        df["Fuel_Type"] = df["Fuel_Type"].astype(str).str.strip()
        df = pd.get_dummies(df, columns=["Fuel_Type"], drop_first=True)
        df = df.dropna(subset=["Price"])

# Identify target and feature columns
TARGET = "Price"
exclude_cols = {TARGET, "Price_pos", "log_Price"} # exclude auxiliary cols if present
feature_cols = [c for c in df.columns if c not in exclude_cols]

X = df[feature_cols].drop(columns=[TARGET]) if TARGET in feature_cols else df[feature_cols]
y = df[TARGET]

# Ensure index alignment and no NA in X/y
```

```

mask = X.dropna().index.intersection(y.dropna().index)
X = X.loc[mask].copy()
y = y.loc[mask].copy()

# Train-test split (80% train / 20% test)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.20,
random_state=42)

# Save splits
out_dir = DATA_DIR / "splits"
out_dir.mkdir(parents=True, exist_ok=True)
X_train.to_csv(out_dir / "X_train.csv", index=False)
X_test.to_csv(out_dir / "X_test.csv", index=False)
y_train.to_csv(out_dir / "y_train.csv", index=False)
y_test.to_csv(out_dir / "y_test.csv", index=False)

# Print summary
print("Saved splits to:", out_dir)
print("Shapes -> X_train:", X_train.shape, "X_test:", X_test.shape, "y_train:",
y_train.shape, "y_test:", y_test.shape)

```

The screenshot shows a Jupyter Notebook interface. The Explorer pane on the left lists files in a 'python apps' directory, including 'split_dataset.py'. The main editor displays the Python code for splitting the dataset. The Output pane at the bottom shows the execution results, including the shapes of the training and testing datasets.

3. Build a multiple linear regression model using the training dataset. Interpret the coefficients of the model. Build minimum of 3 different models.

Answer:

```

PS D:\python apps> & "D:/python apps/.venv/Scripts/python.exe" "d:/python
apps/toyota_ml_r_models.py"
Loading saved splits from: D:\DATA SCIENCE\ASSIGNMENTS\6 MLR\MLR\splits

Final feature set used: ['Age_08_04', 'KM', 'HP', 'Automatic', 'cc', 'Doors', 'Weight',
'Fuel_Type_Diesel', 'Fuel_Type_Petrol']
Train size: (1148, 9) Test size: (288, 9)

```

=== Model A - OLS (all features) summary ===
OLS Regression Results

```

=====
Dep. Variable:          Price  R-squared:
0.869
Model:                  OLS   Adj. R-squared:
0.868
Method:                 Least Squares  F-statistic:
842.1
Date:                   Tue, 30 Sep 2025  Prob (F-statistic):
0.00
Time:                   00:46:19  Log-Likelihood:
-9866.8
No. Observations:      1148  AIC:                1.975e+04
Df Residuals:          1138  BIC:                1.980e+04
Df Model:               9

```

Covariance Type: nonrobust

```

=====
              coef  std err          t    P>|t|    [0.025    0.975]
-----
const      -1.186e+04  1508.957    -7.858    0.000   -1.48e+04   -8896.289
Age_08_04   -120.8231    2.894   -41.744    0.000   -126.502   -115.144
KM           -0.0159    0.001   -10.849    0.000    -0.019    -0.013
HP          15.7772    3.985    3.959    0.000    7.957    23.597
Automatic    93.0820   176.442    0.528    0.598   -253.107   439.271
cc          -0.0302    0.091   -0.333    0.739   -0.208    0.148
Doors       -84.4835    44.153   -1.913    0.056   -171.115    2.148
Weight       26.0692    1.499   17.390    0.000    23.128    29.011
Fuel_Type_Diesel  4.2021   391.745    0.011    0.991   -764.422   772.826
Fuel_Type_Petrol 1453.6945  335.442    4.334    0.000   795.540   2111.849

```

```

=====
Omnibus:          216.690  Durbin-Watson:
2.027
Prob(Omnibus):    0.000  Jarque-Bera (JB):    2442.201
Skew:             -0.512  Prob(JB):
0.00
Kurtosis:         10.072  Cond. No.          3.07e+06
=====

```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 3.07e+06. This might indicate that there are strong multicollinearity or other numerical problems.

Model A - OLS (all features) performance:

MAE: 992.301
RMSE: 1491.411
R2: 0.8333

Coefficients:

feature	coef
const	-11856.9404
Age_08_04	-120.8231
KM	-0.0159
HP	15.7772

Automatic 93.0820
cc -0.0302
Doors -84.4835
Weight 26.0692
Fuel_Type_Diesel 4.2021
Fuel_Type_Petrol 1453.6945

--- Building Model B via backward elimination (p-value) ---
Dropping Fuel_Type_Diesel with p-value 0.9914
Dropping cc with p-value 0.7378
Dropping Automatic with p-value 0.6151

=== Model B - OLS (backward selection) summary ===
OLS Regression Results

```
=====
Dep. Variable:          Price  R-squared:
0.869
Model:                  OLS  Adj. R-squared:
0.869
Method:                 Least Squares  F-statistic:
1266.
Date:                   Tue, 30 Sep 2025  Prob (F-statistic):
0.00
Time:                   00:46:19  Log-Likelihood:
-9867.0
No. Observations:       1148  AIC:                1.975e+04
Df Residuals:           1141  BIC:                1.978e+04
Df Model:                6
```

Covariance Type: nonrobust

```
=====
              coef  std err      t  P>|t|  [0.025  0.975]
-----
const      -1.201e+04  1458.296   -8.237   0.000  -1.49e+04  -9150.578
Age_08_04    -120.6190    2.849  -42.334   0.000  -126.209  -115.029
KM           -0.0160    0.001  -10.923   0.000   -0.019   -0.013
HP           15.3789    3.468    4.435   0.000    8.575   22.183
Doors        -86.5364   43.517   -1.989   0.047  -171.919   -1.154
Weight       26.1885    1.330   19.684   0.000   23.578   28.799
Fuel_Type_Petrol 1483.1709  222.361   6.670   0.000  1046.889  1919.453
=====
Omnibus:          219.794  Durbin-Watson:
2.027
Prob(Omnibus):    0.000  Jarque-Bera (JB):    2509.918
Skew:            -0.522  Prob(JB):
0.00
Kurtosis:         10.168  Cond. No.          2.97e+06
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 2.97e+06. This might indicate that there are strong multicollinearity or other numerical problems.

Model B - OLS (backward selection) performance:

MAE: 993.266
RMSE: 1493.734
R2: 0.8328

Coefficients:

feature	coef
const	-12011.8200
Age_08_04	-120.6190
KM	-0.0160
HP	15.3789
Doors	-86.5364
Weight	26.1885
Fuel_Type_Petrol	1483.1709

--- Building Model C: RidgeCV (with scaling) ---

Ridge chosen alpha: 104.81131341546852

Model C - RidgeCV performance:

MAE: 996.846
RMSE: 1460.784
R2: 0.8401

Ridge coefficients:

feature	ridge_coef
Age_08_04	-2061.3180
KM	-714.0607
HP	311.4659
Automatic	30.7296
cc	0.8155
Doors	-17.4864
Weight	1156.2779
Fuel_Type_Diesel	-18.0492
Fuel_Type_Petrol	243.2847

=== Summary comparison on test set ===

Model A - OLS (all features) performance:

MAE: 992.301
RMSE: 1491.411
R2: 0.8333

Model B - OLS (selected) performance:

MAE: 993.266
RMSE: 1493.734
R2: 0.8328

Model C - RidgeCV performance:

MAE: 996.846
RMSE: 1460.784
R2: 0.8401

Saved coefficient summary to: D:\DATA SCIENCE\ASSIGNMENTS\6
MLR\MLR\model_coefficients_summary.csv

Below I interpret the **baseline OLS coefficients you printed earlier** (the numbers come from the Model A run you posted). Use these lines in your report — they explain *how to read* the coefficients and what they mean practically.

- **Intercept (const) $\approx -11,860$** : the model's baseline predicted price when all numeric predictors are zero. Not directly meaningful here (cars with 0 km, 0 age, etc. don't exist), so ignore the intercept in practical interpretation.
- **Age_08_04 ≈ -120.82 ($p < 0.001$)**: Holding all other variables constant, each extra year of age is associated with a **decrease of about €121** in the car's price. This is expected: older cars sell for less.
- **KM ≈ -0.0159 ($p < 0.001$)**: Each additional kilometer reduces price by about **€0.016**, so +1,000 km \approx **-€16**. Effect is small per km but meaningful over large KM differences.
- **HP $\approx +15.78$ ($p < 0.001$)**: Each additional unit of horsepower increases price by about **€15.8**, holding other factors constant.
- **Automatic $\approx +93.08$ ($p \approx 0.60$)**: Automatic transmission appears to be associated with a slight increase (\sim €93), but this coefficient is not statistically significant (high p-value), so treat cautiously.
- **cc ≈ -0.030 ($p \approx 0.74$)**: Engine displacement shows a small negative coefficient and is not statistically significant — suggests no clear linear effect once HP/Weight are in the model (possible multicollinearity).
- **Doors ≈ -84.48 ($p \approx 0.056$)**: Each additional door associated with \sim €84 lower price (borderline significance). Interpretation depends on vehicle types (e.g., 2-door sport vs 4-door family).
- **Weight $\approx +26.07$ ($p < 0.001$)**: Heavier cars sell for more — each kg adds \sim €26 in predicted price (this number seems large; check units — if Weight is in hundreds, interpret per unit accordingly). (In your output it was 26.07 per 1 unit of Weight; verify weight units.)
- **Fuel dummies (e.g., Fuel_Type_Petrol $\approx +1453.7$, $p < 0.001$)**: Relative to the omitted fuel category (the baseline fuel type), petrol cars are predicted to have about **€1,454 higher** price, holding other features equal.

Important notes about interpretation:

- Coefficients are *ceteris-paribus* — they describe marginal effects holding other included variables constant.
- Large **condition numbers** / **high VIFs** in diagnostics indicate multicollinearity (some predictors convey overlapping information). Coefficient signs may be unstable — prefer Ridge or interpret with caution.
- For Model C (log-target) coefficients: coefficients are multiplicative — a coefficient β on an input means roughly a $100 \times \beta$ % change in price for a one-unit change in the predictor (for small β); interpret via $\exp(\beta)$ for exact multiplicative change.

Which model to pick?

- **If interpretability is priority**: use OLS model **A** or **B** (B has fewer variables if selection removes noisy predictors). But watch for multicollinearity (high VIFs).
- **If predictive performance and robustness to collinearity are priorities**: **Ridge (Model C)** is often preferable — it shrinks coefficients, reduces variance, and improves generalization.
- **If you need feature selection**: Lasso (not included above) can zero out coefficients.

```

138 )
139 print("\nRidge coefficients:")
140 print(ridge_coefs.to_string(index=False))
141
142 # ----- Compare models on test set -----
143 print("\n--- Summary comparison on test set ---")
144 print_metrics("Model A - OLS (all features)", y_test, preds_all)
145 print_metrics("Model B - OLS (selected)", y_test, preds_sel)
146 print_metrics("Model C - RidgeCV", y_test, preds_ride)
147
148 # Save model coefficient tables for reporting
149 coef_A = pd.DataFrame({'feature': ['const'] + list(X_train.columns), 'coef_A': np.round(ols_all.params.values,4)})
150 coef_B = pd.DataFrame({'feature': ['const'] + list(X0.columns), 'coef_B': np.round(ols_sel.params.values,4)})
151 coef_R = ridge_coefs
152
153 coef_out = Path("D:\DATA SCIENCE\ASSIGNMENTS\6 ML\MLK\").joinpath("model_coefficients_summary.csv")
154 coef_df = coef_A.merge(coef_B, on='feature', how='outer').merge(coef_R, on='feature', how='outer')
155 coef_df.to_csv(coef_out, index=False)
156 print("\nSaved coefficient summary to:", coef_out)
157

```

```

(.venv) PS D:\python apps\6 ML\MLK> python "D:\python apps\toyota_mlr_models.py"

--- Summary comparison on test set ---

Model A - OLS (all features) performance:
MAE: 992.381
RMSE: 1491.411
R2: 0.8333

Model B - OLS (selected) performance:
MAE: 993.266
RMSE: 1493.294
R2: 0.8328

Model C - RidgeCV performance:
MAE: 996.846
RMSE: 1488.784
R2: 0.8461

Saved coefficient summary to: D:\DATA SCIENCE\ASSIGNMENTS\6 ML\MLK\model_coefficients_summary.csv
(.venv) PS D:\python apps\6 ML\MLK>

```

4. Evaluate the performance of the model using appropriate evaluation metrics on the testing dataset.

Answer:

1) Metrics (quick)

- **MAE (Mean Absolute Error):** average absolute prediction error (units = €). Easy to interpret.
- **RMSE (Root MSE):** penalizes large errors more than MAE. Useful if you care about big misses.
- **R²:** proportion of variance explained (0–1). Higher is better.
- **Adjusted R²:** R² penalized for number of predictors (useful for model comparison).
- **MAPE (Mean Absolute Percentage Error):** average % error — be careful if targets can be near zero.
- **Cross-validated RMSE:** gives robustness estimate (optional).

2. Code used :

```

# evaluation_helpers.py
import numpy as np
import pandas as pd
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score

```

```

def adjusted_r2(r2, n, p):
    """Adjusted R2 where p = number of predictors (not counting intercept)."""
    if n - p - 1 == 0:
        return np.nan
    return 1 - (1 - r2) * (n - 1) / (n - p - 1)

```

```

def mape(y_true, y_pred):
    y_true, y_pred = np.array(y_true), np.array(y_pred)
    # avoid division by zero; ignore those elements where y_true==0
    mask = y_true != 0
    if mask.sum() == 0:
        return np.nan
    return np.mean(np.abs((y_true[mask] - y_pred[mask]) / y_true[mask])) * 100

```



```
def evaluate_models(y_test, preds_dict, p_counts=None):
    """
    y_test: array-like true values
    preds_dict: dict of {'model_name': y_pred_array}
    p_counts: optional dict {'model_name': p} where p is number of predictors (excl
    intercept)
    """
    rows = []
    n = len(y_test)
    for name, y_pred in preds_dict.items():
        y_pred = np.array(y_pred)
        mae = mean_absolute_error(y_test, y_pred)
        rmse = np.sqrt(mean_squared_error(y_test, y_pred))
        r2 = r2_score(y_test, y_pred)
        p = None
        adjr2 = None
        if p_counts and name in p_counts:
            p = p_counts[name]
            adjr2 = adjusted_r2(r2, n, p)
        rows.append({
            'model': name,
            'n_test': n,
            'p': p if p is not None else "",
            'MAE': round(mae, 3),
            'RMSE': round(rmse, 3),
            'R2': round(r2, 4),
            'Adj_R2': round(adjr2, 4) if adjr2 is not None else "",
            'MAPE_%': round(mape(y_test, y_pred), 3)
        })
    df = pd.DataFrame(rows).sort_values('RMSE')
    return df
```

```
# Example usage (replace these names with your variables):
# preds = {
#     "Model A - OLS": y_pred_A,
#     "Model B - OLS (sel)": y_pred_B,
#     "Model C - log-back": y_pred_C,
#     "Ridge": y_pred_ridge,
#     "Lasso": y_pred_lasso
# }
# pcounts = {"Model A - OLS": X_train.shape[1], "Model B - OLS (sel)": Xb.shape[1],
# ...}
# table = evaluate_models(y_test, preds, p_counts=pcounts)
# print(table.to_string(index=False))
```

3) Example interpretation (use your numbers)

You already ran models earlier and printed metrics. Using those results:

- **Model A — Baseline OLS:**
 - $MAE \approx 992$ €, $RMSE \approx 1,491$ €, $R^2 \approx 0.833$
 - Interpretation: on average predictions are off by ~€1k; RMSE shows larger errors (about €1.5k), and the model explains ~83% of variance — strong fit for a linear model.
- **Model B — Low-VIF OLS (reduced):**
 - $MAE \approx 2,148$ €, $RMSE \approx 2,987$ €, $R^2 \approx 0.331$

- Interpretation: much worse predictive performance — removing predictors to reduce multicollinearity cost a lot of explanatory power. Good for diagnosing collinearity but not for prediction.
- **Model C — Log-target + KM^2 :**
 - If your evaluate_models shows lower RMSE and similar/higher R^2 than Model A, then the log transform improved heteroscedasticity and produced more stable predictions. (Report the exact numbers from the table.)
- **Ridge/Lasso (regularized models):**
 - If Ridge gives slightly lower RMSE than OLS, it indicates multicollinearity was inflating variance and shrinkage improved generalization.
 - If Lasso zeros coefficients and gives comparable RMSE, it's useful for feature selection — but watch for underfitting if RMSE rises.

Decision rule:

- Prefer the model with **lowest RMSE** and **reasonable MAE** (application dependent).
- Consider parsimony and statistical significance: if two models have similar RMSE, pick the simpler one (fewer features) or the one with better-behaved residuals.
- Also check residual diagnostics (normality, heteroscedasticity, influential points) before finalizing.

4) Extra checks you should run (recommended)

- **Residuals plot:** `plt.scatter(y_pred, y_test - y_pred)` to visually check heteroscedasticity.
- **Q-Q plot** of residuals for normality.
- **Prediction intervals** (if needed) from `statsmodels OLS`.
- **Cross-validated RMSE** (use `cross_val_score` with negative MSE and take `sqrt`) to estimate model stability.

Example cross-validation snippet:

```
from sklearn.model_selection import cross_val_score
from sklearn.linear_model import Ridge
cv_rmse = np.sqrt(-cross_val_score(Ridge(alpha=ridge_cv.alpha_), X_train_s,
y_train, scoring="neg_mean_squared_error", cv=5))
print("CV RMSE (Ridge):", cv_rmse.mean(), "±", cv_rmse.std())
```

5. Apply Lasso and Ridge methods on the model.

Answer :

Code used:

```
# toyota_ridge_lasso.py
# Apply RidgeCV and LassoCV to ToyotaCorolla dataset, evaluate and save
coefficients/results.
# Save at: D:\DATA SCIENCE\ASSIGNMENTS\6 MLR\MLR\toyota_ridge_lasso.py
# Requires: pandas, numpy, scikit-learn, statsmodels

import os
```

```

from pathlib import Path
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
from sklearn.linear_model import RidgeCV, LassoCV
from sklearn.metrics import mean_absolute_error, mean_squared_error, r2_score

DATA_PATH = r"D:\DATA SCIENCE\ASSIGNMENTS\6
MLR\MLR\ToyotaCorolla_MLR_cleaned.csv"
if not Path(DATA_PATH).exists():
    DATA_PATH = r"D:\DATA SCIENCE\ASSIGNMENTS\6 MLR\MLR\ToyotaCorolla
- MLR.csv"

df = pd.read_csv(DATA_PATH)
if 'Price' not in df.columns:
    raise SystemExit("Target column 'Price' not found in CSV.")

# prepare X, y (drop helper columns if present)
drop_cols = ['Price_pos', 'log_Price', 'KM_pos']
X = df.drop(columns=[c for c in drop_cols if c in df.columns] + ['Price'],
errors='ignore')
y = pd.to_numeric(df['Price'], errors='coerce')
X = X.apply(pd.to_numeric, errors='coerce')

# align and drop NA rows
mask = X.dropna().index.intersection(y.dropna().index)
X = X.loc[mask].copy()
y = y.loc[mask].copy()

# train-test split
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.20,
random_state=42)

# scale numeric features
scaler = StandardScaler()
X_train_s = scaler.fit_transform(X_train)
X_test_s = scaler.transform(X_test)

# common alpha grid
alphas = np.logspace(-4, 4, 100)

# RidgeCV (with built-in CV)
ridge_cv = RidgeCV(alphas=alphas, cv=5).fit(X_train_s, y_train)
ridge_alpha = ridge_cv.alpha_
ridge_coef = ridge_cv.coef_
ridge_intercept = ridge_cv.intercept_
y_pred_ridge = ridge_cv.predict(X_test_s)
ridge_mae = mean_absolute_error(y_test, y_pred_ridge)
ridge_rmse = np.sqrt(mean_squared_error(y_test, y_pred_ridge))
ridge_r2 = r2_score(y_test, y_pred_ridge)

# LassoCV (with built-in CV)
lasso_cv = LassoCV(alphas=None, cv=5, max_iter=10000,
random_state=42).fit(X_train_s, y_train)
lasso_alpha = lasso_cv.alpha_

```

```

lasso_coef = lasso_cv.coef_
lasso_intercept = lasso_cv.intercept_
y_pred_lasso = lasso_cv.predict(X_test_s)
lasso_mae = mean_absolute_error(y_test, y_pred_lasso)
lasso_rmse = np.sqrt(mean_squared_error(y_test, y_pred_lasso))
lasso_r2 = r2_score(y_test, y_pred_lasso)

# results DataFrame
results = pd.DataFrame({
    'model': ['RidgeCV', 'LassoCV'],
    'alpha': [ridge_alpha, lasso_alpha],
    'MAE': [round(ridge_mae,3), round(lasso_mae,3)],
    'RMSE': [round(ridge_rmse,3), round(lasso_rmse,3)],
    'R2': [round(ridge_r2,4), round(lasso_r2,4)]
})

# coefficients table
coef_df = pd.DataFrame({
    'feature': X_train.columns,
    'ridge_coef': np.round(ridge_coef, 6),
    'lasso_coef': np.round(lasso_coef, 6)
})

# save outputs
out_dir = Path(r"D:\DATA SCIENCE\ASSIGNMENTS\6
MLR\MLR\ridge_lasso_results")
out_dir.mkdir(parents=True, exist_ok=True)
results.to_csv(out_dir / "ridge_lasso_metrics.csv", index=False)
coef_df.to_csv(out_dir / "ridge_lasso_coefficients.csv", index=False)

# print summary
print("Ridge alpha:", ridge_alpha)
print("Lasso alpha:", lasso_alpha)
print("\nEvaluation metrics:")
print(results.to_string(index=False))
print("\nTop coefficients (sorted by absolute Ridge coef):")
print(coef_df.assign(abs_ridge=lambda df:
df.ridge_coef.abs()).sort_values('abs_ridge',
ascending=False).head(20).to_string(index=False))

# Save trained models (optional - requires joblib)
try:
    import joblib
    joblib.dump(ridge_cv, out_dir / "ridge_cv_model.joblib")
    joblib.dump(lasso_cv, out_dir / "lasso_cv_model.joblib")
    print("\nSaved models to:", out_dir)
except Exception:
    print("\njoblib not available — models not saved. Install joblib to save models.")

# Quick note: to inspect non-zero lasso features
nonzero_lasso = coef_df[coef_df['lasso_coef'] != 0].sort_values('lasso_coef',
key=lambda s: s.abs(), ascending=False)
print(f"\nLasso selected {len(nonzero_lasso)} non-zero features. Top ones:")
print(nonzero_lasso.head(10).to_string(index=False))

```

Interview Questions:

1. What is Normalization & Standardization and how is it helpful?

Answer:

- **Standardization** (z-score): $x' = (x - \text{mean})/\text{std}$ — results in mean 0 and sd 1. Useful when features have different units and when algorithms assume centered data (e.g., regularized regression, PCA, k-NN).
- **Normalization** (min–max scaling): $x' = (x - \text{min})/(\text{max} - \text{min})$ — scales features to [0,1] or custom range. Useful when you need bounded features (e.g., in NN activations, or when features must be comparable in magnitude).
- **Why helpful?** It ensures features contribute comparably to model training, speeds up convergence, prevents features with large numeric ranges from dominating distance-based or gradient-based algorithms, and is required before regularization in many workflows.

2. What techniques can be used to address multicollinearity in multiple linear regression?

Answer:

- **Remove correlated predictors** (drop one of a highly-correlated pair).
- **Principal Component Regression (PCR)** or **PCA** to form orthogonal components.
- **Regularization**: Ridge regression reduces variance by shrinking coefficients (L2); Lasso (L1) performs variable selection.
- **Variance Inflation Factor (VIF)** diagnostics: remove predictors with very high VIF.
- **Centering**: subtract variable means (helpful for interaction terms but not removing collinearity).
- **Domain knowledge**: combine correlated variables into a composite score.

Final Output :

```
PS D:\python apps> & "D:/python apps/.venv/Scripts/python.exe" "d:/python apps/toyotoa_mlr_notebook.py"
```

OLS Regression Results

```
=====
Dep. Variable:          Price  R-squared:
0.869
Model:                  OLS   Adj. R-squared:
0.868
Method:                 Least Squares  F-statistic:
842.1
Date:                  Tue, 30 Sep 2025  Prob (F-statistic):
0.00
Time:                  00:35:02  Log-Likelihood:
-9866.8
No. Observations:      1148  AIC:                1.975e+04
Df Residuals:          1138  BIC:                1.980e+04
Df Model:              9
Covariance Type:       nonrobust

=====
               coef  std err          t      P>|t|    [0.025    0.975]
-----
```

const	-1.186e+04	1508.957	-7.858	0.000	-1.48e+04	-8896.289
Age_08_04	-120.8231	2.894	-41.744	0.000	-126.502	-115.144
KM	-0.0159	0.001	-10.849	0.000	-0.019	-0.013
HP	15.7772	3.985	3.959	0.000	7.957	23.597
Automatic	93.0820	176.442	0.528	0.598	-253.107	439.271
cc	-0.0302	0.091	-0.333	0.739	-0.208	0.148
Doors	-84.4835	44.153	-1.913	0.056	-171.115	2.148
Weight	26.0692	1.499	17.390	0.000	23.128	29.011
Fuel_Type_Diesel	4.2021	391.745	0.011	0.991	-764.422	772.826
Fuel_Type_Petrol	1453.6945	335.442	4.334	0.000	795.540	2111.849

=====

Omnibus:	216.690	Durbin-Watson:	
2.027			
Prob(Omnibus):	0.000	Jarque-Bera (JB):	2442.201
Skew:	-0.512	Prob(JB):	
0.00			
Kurtosis:	10.072	Cond. No.	3.07e+06

=====

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 3.07e+06. This might indicate that there are strong multicollinearity or other numerical problems.

[Model A (OLS)] MAE: 992.301 | RMSE: 1491.411 | R2: 0.833

VIF:

feature	VIF
Weight	224.435093
HP	98.649372
Fuel_Type_Petrol	56.652082
Doors	21.078771
Age_08_04	15.817494
cc	14.914158
Fuel_Type_Diesel	11.350510
KM	8.632256
Automatic	1.112641

OLS Regression Results

=====

Dep. Variable:	Price	R-squared:	
0.323			
Model:	OLS	Adj. R-squared:	
0.322			
Method:	Least Squares	F-statistic:	
273.3			
Date:	Tue, 30 Sep 2025	Prob (F-statistic):	9.21e-98
Time:	00:35:02	Log-Likelihood:	
-10811.			
No. Observations:	1148	AIC:	2.163e+04
Df Residuals:	1145	BIC:	2.164e+04
Df Model:	2		

Covariance Type: nonrobust

=====

	coef	std err	t	P> t	[0.025	0.975]

const	1.453e+04	186.506	77.904	0.000	1.42e+04	1.49e+04
KM	-0.0547	0.002	-23.339	0.000	-0.059	-0.050
Automatic	-179.8788	381.869	-0.471	0.638	-929.120	569.362
=====						
Omnibus:		285.157	Durbin-Watson:			
1.963						
Prob(Omnibus):		0.000	Jarque-Bera (JB):			
714.531						
Skew:		1.310	Prob(JB):		6.94e-156	
Kurtosis:		5.842	Cond. No.		3.43e+05	
=====						

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The condition number is large, 3.43e+05. This might indicate that there are strong multicollinearity or other numerical problems.

[Model B (OLS low-VIF)] MAE: 2148.086 | RMSE: 2987.497 | R2: 0.331

OLS Regression Results

```

=====
Dep. Variable:          Price  R-squared:
0.851
Model:                  OLS   Adj. R-squared:
0.849
Method:                 Least Squares  F-statistic:
647.0
Date:                   Tue, 30 Sep 2025  Prob (F-statistic):
0.00
Time:                   00:35:02  Log-Likelihood:
857.88
No. Observations:       1148  AIC:
-1694.
Df Residuals:           1137  BIC:
-1638.
Df Model:                10

```

Covariance Type: nonrobust

	coef	std err	t	P> t	[0.025	0.975]
const	8.2188	0.133	62.009	0.000	7.959	8.479
Age_08_04	-0.0108	0.000	-39.924	0.000	-0.011	-0.010
KM	-2.846e-07	3.34e-07	-0.851	0.395	-9.41e-07	3.71e-07
HP	0.0017	0.000	4.730	0.000	0.001	0.002
Automatic	0.0312	0.015	2.018	0.044	0.001	0.062
cc	1.484e-06	7.97e-06	0.186	0.852	-1.42e-05	1.71e-05
Doors	0.0049	0.004	1.270	0.204	-0.003	0.013
Weight	0.0013	0.000	10.022	0.000	0.001	0.002
Fuel_Type_Diesel	0.0297	0.034	0.865	0.387	-0.038	0.097
Fuel_Type_Petrol	0.0829	0.029	2.815	0.005	0.025	0.141
KM_k	-2.839e-10	3.34e-10	-0.849	0.396	-9.4e-10	3.72e-10
KM_k_sq	-7.279e-06	1.65e-06	-4.418	0.000	-1.05e-05	-4.05e-06

```
=====
Omnibus:          240.482  Durbin-Watson:
2.043
Prob(Omnibus):    0.000  Jarque-Bera (JB):    1301.243
Skew:            -0.854  Prob(JB):    2.75e-283
Kurtosis:        7.928  Cond. No.    1.17e+18
=====
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

[2] The smallest eigenvalue is 5.2e-24. This might indicate that there are strong multicollinearity problems or that the design matrix is singular.

[Model C (log-target)] MAE: 877.232 | RMSE: 1303.276 | R2: 0.873
are

strong multicollinearity problems or that the design matrix is singular.

[Model C (log-target)] MAE: 877.232 | RMSE: 1303.276 | R2: 0.873

strong multicollinearity problems or that the design matrix is singular.

[Model C (log-target)] MAE: 877.232 | RMSE: 1303.276 | R2: 0.873

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[Model C (log-target)] MAE: 877.232 | RMSE: 1303.276 | R2: 0.873

[Model C (log-target)] MAE: 877.232 | RMSE: 1303.276 | R2: 0.873

Ridge alpha: 104.81131341546852

[RidgeCV] MAE: 996.846 | RMSE: 1460.784 | R2: 0.840

[RidgeCV] MAE: 996.846 | RMSE: 1460.784 | R2: 0.840

Lasso alpha: 55.53161298181698

[LassoCV] MAE: 996.544 | RMSE: 1450.672 | R2: 0.842

Ridge/Lasso coefficients:

feature	ridge_coef	lasso_coef
Age_08_04	-2061.318036	-2252.505697
KM	-714.060735	-629.675028
HP	311.465871	272.358128
Automatic	30.729612	0.000000
cc	0.815452	-0.000000
Doors	-17.486415	-0.000000
Weight	1156.277887	1132.837277
Fuel_Type_Diesel	-18.049211	-0.000000
Fuel_Type_Petrol	243.284658	288.642249