Assignment: Hypothesis Test — Weekly Operating Cost

1. Background & data

- Theoretical cost model: W=1000+5XW = 1000 + 5XW=1000+5X.
- Units produced X∼N(μX=600, σX=25)X \sim N(\mu_X=600,\ \sigma_X=25)X∼N(μX=600, σX=25).
- Sample: n=25n = 25n=25 restaurants, sample mean weekly cost x¯=3050\bar{x} = 3050x¯=3050 (Rs).
- Because WWW is linear in XXX with multiplier 5, population standard deviation for WWW is σW=5×σX=5×25=125\sigma_W = 5 \times \sigma_X = 5 \times 25 = 125σW=5×σX=5×25=125.

Compute theoretical mean weekly cost under model:

μW=1000+5×600=1000+3000=4000 (Rs)\mu_W = 1000 + 5\times600 = 1000 + 3000 = 4000 \text{ (Rs)}μW=1000+5×600=1000+3000=4000 (Rs)

2. Hypotheses

We test the restaurant owners' claim that costs are **higher** than the model predicts. That gives a **one-sided (right-tailed)** test.

- Null hypothesis H0H_0H0: μ=4000\mu = 4000μ=4000 (no increase)
- Alternative H1H_1H1: μ>4000\mu > 4000μ>4000 (mean weekly cost is higher)

3. Test statistic

Because the population standard deviation σW\sigma_WσW is known (125), use the Z-test:

Z=x¯−μσ/n.Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}.Z=σ/nx¯−μ.

Plugging values:

σ/n=12525=1255=25.\sigma/\sqrt{n} = \frac{125}{\sqrt{25}} = \frac{125}{5} = 25.σ/n =25125=5125=25. Z=3050−400025=−95025=−38.0.Z = \frac{3050 - 4000}{25} = \frac{-950}{25} = -38.0.Z=253050−4000=25−950=−38.0.

4. Critical value (α = 0.05, right-tailed)

For a right-tailed test at significance level α=0.05\alpha=0.05α=0.05, the critical z-value is:

z0.95≈1.645.z_{0.95} \approx 1.645.z0.95≈1.645.

Decision rule: Reject H0H_0H0 if Z>1.645Z > 1.645Z>1.645.

5. Decision

Calculated Z=−38.0Z = -38.0Z=−38.0. This is far **less** than the critical value 1.6451.6451.645. Therefore, **we fail to reject** the null hypothesis in the direction the owners claimed (i.e., there is no evidence μ>4000\mu > 4000μ>4000). In fact, the sample mean is hugely lower than the model's mean — the data provide strong evidence the sample mean is **not** higher (it's dramatically lower).

6. Conclusion (plain language)

Using the sample provided, there is **no evidence** to support the restaurant owners' claim that weekly operating costs are higher than the theoretical model suggests. The sample mean (Rs. 3,050) is much lower than the model mean (Rs. 4,000), producing a test statistic Z=−38.0Z=-38.0Z=−38.0, which lies in the **left** tail rather than the right tail required to support the claim. If the owners actually believe costs are higher, either the sample mean is misreported or there is confusion about which quantity is being compared. With the given numbers, the conclusion is that costs are not higher — they appear far lower in this sample.

Short instructor-ready summary

Hypotheses: H0:μ=4000H_0: \mu = 4000H0:μ=4000 vs H1:μ>4000H_1: \mu > 4000H1:μ>4000.

Test statistic: Z=3050−4000125/25=−38.0.Z = \dfrac{3050 - 4000}{125/\sqrt{25}} = -38.0.Z=125/253050−4000=−38.0.

Critical value: z0.95≈1.645z_{0.95} \approx 1.645z0.95≈1.645.

Decision: Fail to reject H0H_0H0.

Conclusion: There is no evidence that weekly operating costs are higher than the model predicts; with the given sample the mean cost is much lower than the model value.