

2020-21 Academic year Works

Mathematics X Arithmetic Sequences

1

It is the time to see new sights and make new insights. While starting Arithmetic Sequence of tenth standard we have some expectations. We hope this will enrich the thoughts of the learner and help them to develop a self study attitude.

Instructions

- ★ Attend VICTERS channel classes carefully
- ★ Do the worksheet without any help
- ★ Ask your doubts in the group
- ★ The doubts can be send by photo or voice.

Worksheet 1

1) Consider the sequence 2, 4, 6, 8...

- Write two more terms of this sequence
- Which is the smallest two digit term of this sequence?
- Which is the largest two digit term of this sequence
- Can the sum of any two terms of this sequence 75?

Answers

- 10, 12
- 10
- 98
- Terms are even numbers. The sum of two even numbers cannot be an odd number. So 75 cannot be the sum

2) Consider the sequence 1, 3, 5, 7...

- Write next two more terms of this sequence
- Which is the smallest two digit term of this sequence?
- Which is the largest two digit term of this sequence
- Can the sum of any two terms of this sequence 75

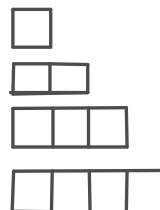
1

Answers

- 9, 11
- 11
- 99
- No. The terms are odd numbers. the sum of two odd numbers cannot be an odd number. .

- 3) a) Write the sequence of numbers in which 1 comes in the one's place
b) which is the largest two digit term of this sequence?
c) How many numbers are there in this sequence below 100?

4) Look at the picture . Squares are made by joining



match sticks.

- Write the number of squares in each line as a sequence
- Write the number of match sticks in each line as a sequence
- How many match sticks are used to make next line

Answers

- 1, 2, 3, 4...
- 4, 7, 10...
- 16

5) Find out the rule behind the formation of the sequence 1, 4, 9, 16...

- Write next three terms
- What is the tenth term of this sequence
- Is 900 a term of this sequence ? If it is ,what is its position?
- Is 1000 a term of this sequence? Why?

Answers

- a) 25, 36, 49
- b) $10^2 = 100$
- c) 900 is its 30 th term
- d) 1000 is not a perfect square. So it cannot be a term

- 6) There is a popular sequence named fibonacci sequence .It is given below

1, 1, 2, 3, 5, 8, 13, 21 . . .

Can you find out the rule behind the formation of this sequence

- a) write next two terms?
- b) Which is the largest two digit term of this sequence?

Answers

- a) $13 + 21 = 34, 21 + 34 = 55$
- b) 89

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2020-21 Academic year Works

Mathematics X Arithmetic Sequences

1

Have you attended today's class in the channel? Questions given on the basis of this class. Your teacher will explain and clarify your doubts. Answer the questions and post it in the group

Note

- ★ Numbers of the form $1, 2, 3, \dots$ are called counting numbers. All sequences are formed from counting numbers.
- ★ Sequences are generated from various situations. Some are generated from geometrical patterns.
- ★ A number sequence will have a rule which helps us to understand the sequence properly.

Worksheet 2

- 1) $10, 20, 30, \dots$ is a sequence in which 0 comes in the one's place
- Write the sequence in which 1 comes in the one's place
 - Which is the largest two digit term of this sequence?
 - Which is the smallest three digit term of this sequence?
 - Which number is added repeatedly to get the terms of this sequence?

Answers

- $11, 21, 31, \dots$
- 91
- 101
- 10

- 2) $1, 6, 11, 16, \dots$ is the sequence in which 1 or 6 comes in the right end of the number

- Write next five more terms of this sequence
- Which is the largest two digit term of this sequence
- How many numbers are there below 100 in this sequence? Find it by actual counting
- Can you find it without counting?

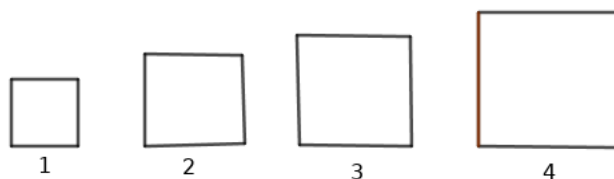
Answers

- 21, 26, 31, 36, 41
- 96
- 20
- There are 2 nos below 10. No. of terms below 100 is $2 \times 10 = 20$

- 3) Consider the numbers which give the remainder 1 on dividing by 3

- Write the sequence of these numbers
- Which is the smallest two digit term of this sequence
- How many numbers are there up to 25 in this sequence?
- What are the possible remainders on dividing a number by 3

- 4) Look at the sequence of squares



- Draw a rough diagram of the sequence of equilateral triangles with sides 1cm, 2cm, 3cm, ...
- Write the sequence of the perimeters of these triangles?
- What is the side of the triangle having perimeter just below 10 in this sequence
- What is the side of the triangle having perimeter just below 100 in this sequence

Answers

- a) Draw yourself
- b) 3, 6, 9...
- c) 3cm
- d) 33cm

- 5) Write the remainders on dividing the counting numbers from 1 to 10 by 3

- a) What is the sum of the remainders?
- b) What is the sum of the remainders on dividing the numbers from 1 to 10 by 4?
- c) What is the sum of the remainders on dividing the numbers from 1 to 100 by 3?

Answers

- a) 1, 2, 0, 1, 2, 0, 1, 2, 0, 1. sum= 10
- b) 1, 2, 3, 0, 1, 2, 3, 0, 1, 2 sum= 15
- c) 1, 2, 0, 1, 2, 0...
- sum= $33 \times 3 + 1 = 100$

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2020-21 Academic year Works

Mathematics X Arithmetic Sequences

3

Notes

- ★ If a polygon has n sides its angle sum will be $(n - 2) \times 180^\circ$
- ★ If sides and angles of a polygon are equal it will be a regular polygon.
- ★ A sequence can be described more than one way. It depends on how we see the relation among the terms.

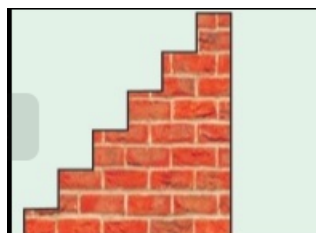
Worksheet3

- 1) a) Draw the sequence of equilateral triangle, square, regular pentagon, regular hexagon \dots .
- b) Write the interior angle sum as a sequence
- c) Write the sequence of one interior angle
- d) Write the measure of exterior angle as a sequence

Answers

- a) Draw yourself
- b) 180, 360, 540, 720 \dots
- c) 60, 90, 108, 120 \dots
- d) 120, 90, 72, 60 \dots

- 2) Look at the picture given below



The height from the floor to the first step is 10cm. The heights of subsequent steps is 5cm

- a) Write the sequence of heights attained by a boy while moving up through the steps
- b) What will be the height from the floor when the boy reaches the fifth step?
- c) The boy reaches the last step which is 20 th from the floor. What is the height from the floor at this position?
- d) The starts moving down from 20 th step. Write the sequence of heights

Answers

- a) 10, 15, 20, \dots
- b) 30. It is the 6 th multiple of 5 according to this sequence
- c) 105 cm
- d) 105, 100, 95, 90 \dots

- 3) Let us think about a puzzle.

There are two electronic devices on the table. The device A makes BEEP sound in every 1 minute.

The device B makes BEEP sound in every 62 seconds.

Both of the devices beeped at 10 O'clock.
The times of beeping the first device is written as a sequence below
10 : 01 : 00, 10 : 02 : 00, 10 : 03 : 00, 10 : 04 : 00 ...

- a) Write the sequence of the beeping time of the second device
- b) What is the nearest time at which the devices beeped together after 10 O'clock

Answers

a)

10:00:00, 10:01:02, 10:02:04 ...

- b) The first device beeped at 10 : 31 : 00. The second device beeped at 10 : 30 : 60. It is 10 : 31 : 00 itself. So the devices beeped together at 10 : 31

- 4) On a day teacher wrote a sequence in the blackboard.

1, 6, 11, 16, 21, 26 ...

The teacher described this sequence as: 'it is the sequence of numbers having 1 or 6 in one's place'.

- a) Nasrin, a brilliant student said. It is the sequence of numbers starting from 1 and adding 5 repeatedly. If you can agree with this, write the three more terms of this sequence using this opinion
- b) Riswan, another student said, it is 4 less than a multiple of 5. If you can agree with this write the tenth term of this sequence
- c) We can see both statements are true. How many terms are there below 100 in this sequence?
- d) What is the 20 th term of this sequence ?

- e) Write the algebraic expression of the n th term of this sequence.

Answers

a) 31, 36, 41

b) It is 4 less than the tenth multiple of 5. It is $50 - 4 = 46$

c) 20 by simple reasoning

d) $5n - 4$

- 5) Look at the sequence 1, 4, 9, 16, 25 These numbers can be written as $1^2, 2^2, 3^2 \dots$

- a) What is the remainder obtained by dividing $1^2, 2^2, 3^2$ by 3
- b) What is the sum of the remainders obtained by dividing $1^2, 2^2, 3^2 \dots 10^2$ by 3
- c) What is the sum of the remainders obtained by dividing $1^2, 2^2, 3^2 \dots 100^2$ by 3
- d) Can a perfect square be a term of the sequence 2, 5, 8, 11 How can you realize this ?

Answers

- a) 1, 1, 0
- b) We can make three groups like $\boxed{1,1,0}$. The group sum is 2 and sum of the remainders is $3 \times 2 + 1 = 7$
- c) $33 \times 2 + 1 = 67$
- d) No, perfect squares leave the remainder 1 or 0 on dividing by 3. The terms of the given sequence leave the remainder 2 on dividing by 3. So this sequence contains no perfect squares.

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2020-21 Academic year Works

Mathematics X Arithmetic Sequences

4

Notes

The numbers $1, 2, 3, 4 \dots$ are called counting numbers or natural numbers. Different types of sequences can be formed by various operations on the natural numbers.

In all the sequences that we discuss here are closely related to the counting numbers. Knowing this relationship between the terms of the sequences and natural numbers help us to study the sequence properly. The relation between the terms of the sequence and natural numbers is called algebraic form of the sequence.

Worksheet 4

- 1) a) Multiply each number of the sequence $1, 2, 3 \dots$ by 3, add 2 then write the resulting numbers as a sequence
b) What is the tenth term of this sequence?
c) At what position 32 comes in this sequence?
d) How many terms are there below 100 in this sequence?

Answers

- a) $5, 8, 11, 14 \dots$
b) $3 \times 10 + 2 = 32$
c) 10 th position
d) $3 \times 32 + 2 = 98$. There are 32 numbers

- 2) a) Add 1 to the square of each of the numbers of the sequence $1, 2, 3, 4 \dots$

- b) What is the 20 th term of this sequence
c) Which is the smallest three digit term of this sequence?
d) How many terms are there below 100? Which is the largest among them?
e) Write the n th term of this sequence

Answers

- a) $2, 5, 10, 17 \dots$
b) $20^2 + 1 = 401$
c) $101, \rightarrow 10^2 + 1 = 101$
d) $9, 9^2 + 1 = 82$

- 3) a) Write the sequence of numbers starting from 3 and adding 2 repeatedly.
b) What should be added to 3 to get 17?
c) At what position 17 comes in the sequence?
d) How many times 2 is added to 3 for getting its n th term.
e) Prove that its n th term is $2n + 1$

Answers

- a) $3, 5, 7, \dots$
b) $7 \times 2 = 14$
c) 8th term
d) $(n - 1) \times 2 + 3 = x_n$
Where x_n stands for n th term
e) On simplifying, $x_n = 2n - 2 + 3 = 2n + 1$

- 4) a) Write the sequence in which first term is 5 and the number adding repeatedly is 3.
- b) How many times 3 is added to its first term to get tenth term
- c) Which is the tenth term of this sequence?
- d) Write the algebraic form or n th term of this sequence

Answers

- a) 5, 8, 11 \dots
- b) 9×3 should be added
- c) $5 + 9 \times 3 = 32$
- d) $x_n = 5 + (n - 1) \times 3 = 3n + 2$

- 5) a) Write the sequence of numbers which leaves the remainder 1 on dividing by 5
- b) Describe this sequence in other two ways(See 03as, the opinions of Nasrin and Riswan)
- c) Write the algebraic form of this sequence
- d) Write the 10 th term of this sequence using its algebraic form

Answers

- a) 1, 6, 11, 16 \dots
- b) It is a sequence starting from 1 and adding 5 repeatedly
It is a sequence of numbers in which the terms are 4 less than the multiples of 5
- c) $x_n = 5n - 4$
- d) $5 \times 10 - 4 = 46$

2020–21 Academic year Works

Mathematics X Arithmetic Sequences

5

Notes

We have seen sequences in general. In the previous worksheet we gave special importance to the sequence which starts from a number and adding another number repeatedly. Such sequences are called arithmetic sequences. In this worksheet we think about the algebra of the arithmetic sequence and some simple uses of algebraic form of the sequence. The starting number is called first term f or x_n

The number added repeatedly is called common difference, denoted by d .

The algebraic form or n th term of an arithmetic sequence will be $x_n = f + (n - 1) \times d$.

This can be written as $x_n = dn + (f - d)$

Worksheet 5

- 1) The first term of an arithmetic sequence is 2 and common difference 3.
- a) Write the sequence
 - b) How many times common difference 3 should be added to the first term to get 25th term?
 - c) What is the 25th term of this sequence?
 - d) Write the algebraic form of this sequence

Answers

- a) 2, 5, 8, 11...
- b) 24 times 3 should be added.
- c) $x_{25} = 2 + 24 \times 3 = 2 + 72 = 74$
- d) $x_n = dn + (f - d) = 3n - 1$

- 2) Consider the arithmetic sequence 1, 5, 9, 13...

- a) What is the common difference of this sequence?
- b) How many times common difference should be added to 1 for getting its tenth term?
- c) How many times 3 should be added to the first term 1 to get its n th term?
- d) Write the algebraic form of this sequence.

Answers

- a) $d = 5 - 1 = 4$
- b) 9 times common difference should be added.
- c) $(n - 1)$ times 3 should be added 1
- d) $x_n = dn + (f - d) = 4n - 3$

3) Algebraic form of an arithmetic sequence is $3n + 7$.

- a) Write the sequence.
- b) What is the 20 th term of this sequence
- c) What should be added to 5 th term of this sequence to get 10 th term?

Answers

- a) Give the values 1, 2, 3 \dots to n in its algebraic form. Terms are 10, 13, 16 \dots
- b) $x_{20} = 3 \times 20 + 7 = 67$
- c) 5 times common difference should be added. It is $5 \times 3 = 15$

4) The algebraic form of an arithmetic sequence is $\frac{1}{2}n + 3$.

- a) Give a suitable value for n for getting its first integer term?
- b) Write the sequence of integer terms
- c) Integer terms make another arithmetic sequence . Write its algebraic form
- d) What is the largest two digit integer term of this sequence
- e) How many integer terms are there below 100

Answers

- a) $n = 2$
- b) Give the values 2, 4, 6, 8 \dots to n . The sequence is 4, 5, 6, 7 \dots
- c) $x_n = n + 3$
- d) 99
- e) 96

5) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7} \dots$ is an arithmetic sequence.

- a) Write the n th term of this sequence
- b) What value of n the sequence gets its first integer term?
- c) At what position 100 comes as a term of this sequence ?
- d) How many integer terms are there below 100 in this sequence ?

Answers

- a) $\frac{n}{7}$
- b) $n = 7$
- c) 700th position
- d) Give the values 7, 14, 21 \dots to n we get integer terms . Sequence of integer terms are 1, 2, 3 \dots . So there are 99 integer terms below 100.

2020-21 Academic year Works

Mathematics X
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6

Notes

We have already discussed algebra of arithmetic sequence and one of its properties. If f is the first term and d is the common difference, its algebraic form is $x_n = dn + (f - d)$. The difference between any two terms of an arithmetic sequence will be a multiple of common difference. Other words, the difference between any two terms of an arithmetic sequence is divisible by the common difference.

Worksheet 6

- 1) The algebraic form of an arithmetic sequence is $4n + 3$.
- a) What is its common difference?
 - b) Can the difference between any two terms of this sequence 176?
 - c) Which term would be obtained on adding 176 to the second term
 - d) Which term would be obtained on adding 176 to its tenth term

Answers

- a) 4
- b) 176 is divisible by 4. So 176 can be the difference between two terms
- c) $x_2 + 176 = x_2 + 44 \times 4 = x_{46}$
- d) $x_{10} + 44 \times 4 = x_{54}$

- 2) x, y, z are in an arithmetic sequence.
- a) If $y - x = k(z - x)$ then what is k ?
 - b) If $y - x = k(x - z)$ then what is k ?
 - c) If $x - y = k(x - z)$ then what is k ?
 - d) Prove that $y = \frac{x+y+z}{3}$

Answers

- a) $d = k \times 2d, k = \frac{1}{2}$
- b) $d = k \times -2d, k = -\frac{1}{2}$,
- c) $-d = k \times -2d, k = \frac{1}{2}$
- d) $y - x = z - y, 2y = x + z$. Therefore, $y + 2y = x + y + z, 3y = x + y + z, y = \frac{x+y+z}{3}$

3) Fifth term of an arithmetic sequence is 24 and its common difference is 3

- a) What is its first term?
- b) Write the algebraic form of the sequence
- c) Write some terms of this sequence in an order
- d) What is the difference between n th term and $n + 6$ th term of this sequence
- e) What is the difference between $m + n$ th term and n th term

Answers

a) $x_1 = x_5 - 4d = 24 - 12 = 12$

b) $x_n = dn + (f - d) = 3n + 9$

c) $12, 15, 18 \dots$

d) Difference is $6 \times 3 = 18$

e) Difference is $m \times 3 = 3m$

4) First term of an arithmetic sequence is f and common difference d .

- a) What is its fifth term?
- b) What is its tenth term?
- c) Five times 5 th term is equal to 10 times tenth term . Write this statement algebraically
- d) What is the fifteenth term of this sequence ?
- e) What is the product of first 15 terms ?

Answers

a) $x_5 = f + 4d$

b) $x_{10} = f + 9d$

c) $5 \times (f + 4d) = 10 \times (f + 9d)$

d) Simplifying , $f + 14d = 0, x_{15} = 0$

e) Since 15 th term is 0, product is 0

5) Fifth term of an arithmetic sequence is 10 and tenth term is 5.

- a) What is the common difference?
- b) What is its first term?
- c) What is its fifteenth term?
- d) What is the sum of product of 15 terms?

Answers

a) $5d = 5 - 10 = -5, d = -1$

b) $x_1 = x_5 - 4 \times d = 10 - 1 \times 4 = 14$

c) $x_{15} = f + 14d = 14 + 14 \times (-1) = 14 - 14 = 0$

d) 0

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Mathematics X
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7

Notes

Algebraic form of a sequence is the most important aspect for understanding the sequence properly. f is the first term and d is the common difference of an arithmetic sequence, its algebraic form will be $x_n = dn + (f - d)$. The difference between any two terms of an arithmetic sequence is always a multiple of its common difference. If the terms are natural numbers, the terms give the same remainder on dividing it by the common difference.

Worksheet

- 1) Consider the arithmetic sequence 1, 8, 15, 22...
- What is the common difference?
 - What will be the remainder when the terms are divided by its common difference?
 - Which is the first three digit term of this sequence?
 - Write the algebraic form of this sequence
 - How many terms are there below 100 in this sequence?

Answers

- $d = 8 - 1 = 7$
- 1
- 106
- $x_n = dn + (f - d) = 7n + (1 - 7) = 7n - 6$
- $7n - 6 < 100, 7n < 106, n < 15.1, n = 15$
15 terms below 100.

- 2) The algebraic form of an arithmetic sequence is $7n + 3$.
- What is the common difference?
 - What is the first term of this sequence?
 - What is the position of 171 in this sequence?
 - What is the position of 248 in this sequence?
 - How many terms are there from 171 to 248 in this sequence

Answers

- a) 7
- b) $7 \times 1 + 3 = 10$
- c) $7n + (10 - 7) = 171$
 $7n + 3 = 171, 7n = 168, n = 24$
 24th term is 171
- d) $7n + 3 = 248, 7n = 245, n = 37$
- e) $37 - 24 + 1 = 14$

3) $x - 1, x, x + 1, x + 2 \dots$ are in an arithmetic sequence.

- a) What is the common difference of this sequence ?
- b) Write the algebraic form of this sequence
- c) Write some more terms of this sequence
- d) If the product of first and third terms of this sequence is 15, what is its first term?

Answers

- a) $d = 1$
- b) $x_n = dn + (f - d) = 1 \times n + (x - 1 - 1) = n + (x - 2)$
- c) $x + 3, x + 4, x + 5 \dots$
- d) $(x - 1)(x + 1) = 16, x^2 - 1 = 15, x^2 = 16, x = 4$
 First term is 3.
 (Think about another value of x)

4) Consider the arithmetic sequence 97, 94, 91 \dots

- a) What is the common difference of this sequence ?
- b) Write the algebraic form of this sequence
- c) Which is smallest positive term of this sequence ?
- d) How many positive terms are there in this sequence ?

Answers

- a) $94 - 97 = -3$
- b) $x_n = dn + (f - d) = -3n + (97 - 3) = -3n + 100$
- c) $-3n + 100 > 0, -3n > -100, n < \frac{100}{3}, n < 33.3, n = 33$
 33 positive terms .
- d) 33th term is $-3 \times 33 + 100 = 1$

5) Consider the arithmetic sequence $-48, -44, -40 \dots$

- a) What is its common difference ?

- b) Write the algebraic form of this sequence
- c) What is the position of 0 in this sequence ?
- d) How many negative terms are there in this sequence ?

Answers

a) $d = -44 - (-48) = 4$

b) $x_n = dn + (f - d) = 4n + (-48 - 4) = 4n - 52$

c) $4n - 52 = 0, 4n = 52, n = 13$
13 th term is 0

d) 12

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2020–21 Academic year Works

Mathematics X Arithmetic Sequences

8

Notes

- ★ Let us write an arithmetic sequence having three terms.

3, 7, 11

On dividing the sum of the terms by its number of terms we get the middle term. If the number of terms is 5 or 7 or any other odd number the same result holds. That is, the sum of the terms divided by the number of terms will be the middle term.

- ★ What about the number of terms is even?

We can make them pairs. Make the terms equidistant from both ends as pairs and find the pair sum.

The pair sums are equal. You can verify it by writing the examples.

- ★ If the number of terms is odd we can find a term at the middle. The middle term will be half of the pair sum
- ★ To get the middle term just find either divide the sum of the terms by its number of terms or find half of the pair sum

Worksheet

1) Insert suitable number in the box which make the arithmetic sequence

- a) $\boxed{5}, \boxed{}, \boxed{11}, \boxed{}, \boxed{17}$
- b) $\boxed{13}, \boxed{9}, \boxed{}, \boxed{}, \boxed{-3}, \boxed{-7}$
- c) $\boxed{}, \boxed{}, \boxed{6}, \boxed{10}, \boxed{}, \boxed{}$
- d) $\boxed{7}, \boxed{15}, \boxed{}, \boxed{}, \boxed{39}, \boxed{47}$

Answers

- a) 5, 8, 11, 14, 17
- b) 13, 9, 5, 1, -3, -7
- c) -2, 2, 6, 10, 14, 18
- d) 7, 15, 23, 31, 39, 47

2) Angles of a right triangle are in an arithmetic sequence.

- a) Find the middle term of the sequence
- b) Write the angles of the triangle

Answers

a) $x_2 = \frac{180}{3} = 60$

b) $30^\circ, 60^\circ, 90^\circ$

3) Angles of a quadrilateral are in an arithmetic sequence

- a) Find the angle sum
- b) What is the sum of first term and fourth term?
- c) What is the sum of second term and third term?
- d) Suggest a suitable name of this quadrilateral

Answers

a) 360°

b) $x_1 + x_4 = 180^\circ$

c) $x_2 + x_3 = 180^\circ$

d) Since co interior angle sum is 180° , two sides are parallel. It is a trapezium

4) Angles of a pentagon are in an arithmetic sequence .

- a) What is the sum of the terms of this sequence
- b) What is the middle term ?
- c) If the smallest angle is 40° what is the difference between two adjacent angles
- d) Write the angles as a sequence

Answers

a) $(5 - 2) \times 180 = 540^\circ$

b) $x_3 = \frac{540}{5} = 108^\circ$

c) $108 - 40 = 2d, d = 34^\circ$

d) $40^\circ, 74^\circ, 108^\circ, 142^\circ, 176^\circ$

5) There is an arithmetic sequence having 9 terms. The sum of the first and 9 th terms is 32.

- a) What is the sum of second and eighth term?
- b) what is the fifth term?
- c) If the sixth term is 19 then what is its common difference?
- d) Write the first term of this sequence

Answers

a) $x_2 + x_8 = 32$

b) $\frac{32}{2} = 16^\circ$

c) $d = x_6 - x_5 = 3$

d) $x_1 = x_5 - 4d = 16 - 12 = 4$

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2020-21 Academic year Works

Mathematics X Arithmetic Sequences

9

Notes

This is the continuation of eighth work sheet. In this new work sheet we discuss problems to find the sequence , its algebraic form from two given terms.

Worksheet

- 1) 13 th term of an arithmetic sequence is 48. Its 17 th term is 64
- What is the common difference?
 - What is the 15 th term?
 - Find the first term of this sequence
 - Write the algebraic form of this sequence
 - What is the product of a number of terms from the beginning of the sequence .

Answers

- a) $x_{17} - x_{13} = 4d, 4d = 64 - 48 = 16$
 $d = 4$
- b) $x_{15} = \frac{48+64}{2} = 56$
or
 $x_{15} = x_{13} + 2d = 48 + 2 \times 4 = 48 + 8 = 56$
or
 $x_{15} = x_{17} - 2d = 64 - 8 = 56$
- c) $x_1 = x_{13} - 12 \times d = 48 - 12 \times 4 = 48 - 48 = 0$
- d) $x_n = dn + (f - d) = 4n - 4$
- e) Since first term is 0, the product is 0

- 2) The first term of an arithmetic sequence is $\frac{1}{3}$ and its common difference $\frac{1}{6}$
- Write the algebraic form of this sequence
 - At what position the first integer term comes in the sequence?
 - What are the term positions at which the integer terms occur in the sequence?
 - What should be added to its 10 th term to get 25 th term?

Answers

- a) $x_n = dn + (f - d)$
 $\frac{1}{6}n + (\frac{1}{3} - \frac{1}{6}) = \frac{1}{6}n + \frac{1}{6} = \frac{n+1}{6}$
- b) $x_5 = \frac{5+1}{6} = 1$. The fifth term is 1. It is the first integer term
- c) 5, 11, 17, 23... At these positions integer terms exist
- d) $x_{25} = x_{10} + 15 \times \frac{1}{6} = x_{10} + \frac{15}{6}$.
 $\frac{15}{6}$ should be added.

3) The sum of first 5 terms of an arithmetic sequence is 70. The sum of the first 11 terms is 286.

- a) What is its third term?
- b) What is its sixth term?
- c) What is the common difference of this sequence ?
- d) What is the first term of this sequence ?
- e) Write the algebraic form of this sequence

Answers

- a) $x_3 = \frac{70}{5} = 14$
- b) $x_6 = \frac{286}{11} = 26$
- c) $x_6 - x_3 = 26 - 14$
 $3d = 12, d = 4$
- d) $x_1 = x_3 - 2d = 14 - 2 \times 4 = 14 - 8 = 6$
- e) $x_n = dn + (f - d) = 4n + (6 - 4) = 4n + 2$

4) Seventh term of an arithmetic sequence is 21.

- a) What is the sum of sixth and eighth term of this sequence ?
- b) What is the sum of first and thirteenth term ?
- c) If the eighth term is 25, what is its common difference
- d) What is the first term of this sequence?
- e) Can the difference between any two terms 45?

Answers

- a) $x_6 + x_8 = 21 \times 2 = 42$
- b) $x_1 + x_{13} = 42$
- c) $d = x_8 - x_7 = 25 - 21 = 4$
- d) $x_1 = x_7 - 6d = 21 - 6 \times 4 = 21 - 24 = -3$
- e) 45 is not a multiple of 4.
 \therefore 45 cannot be the difference between two terms

- 5) A wire of length 70cm cut into five pieces. When they are arranged in an order, its lengths become an arithmetic sequence
- a) What is the length of third piece in the arrangement?
 - b) If the shortest piece has length 8cm what is the difference between the lengths of two neighbouring pieces ?
 - c) What is the length of the longest piece
 - d) Write the sequence of lengths

Answers

- a) $l_3 = \frac{70}{5} = 14$ where l stands for the length
- b) $l_3 - l_1 = 6, 2d = 6, d = 3$. The difference between two neighbouring lengths is 3
- c) $l_5 = l_3 + 2d = 14 + 2 \times 3 = 20$
- d) 8, 11, 14, 17, 20

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Mathematics X Arithmetic Sequences

10

Notes

- a) Let us calculate the sum of a definite number of terms of this sequence. Simplest process of finding the sum is pairing.

Consider the arithmetic sequence 5, 8, 11, 14, 17, 20. In this sequence pairing can be done as (5, 20), (8, 17), (11, 14). The pair sum is 25. Sum of the terms of the sequence = $25 \times 3 = 75$.

$\text{sum} = (5 + 20) \times \frac{6}{2}$. Sum can be calculated by multiplying the pair sum by half of the number of terms

- b) Consider the sequence 5, 8, 11, 14, 17. There are 5 terms in this sequence. On pairing we get the middle term without a pair.

Now we get two pairs with the pair sum 22 and the middle term.

$$\begin{aligned}\text{sum} &= (5 + 17) \times 2 + 11 \\ &= (5 + 17) \times 2 + \frac{22}{2} \\ &= 22 \times 2 + \frac{22}{2} \\ &= 22\left(2 + \frac{1}{2}\right) \\ &= 22 \times \frac{5}{2}\end{aligned}$$

$$\text{sum} = (5 + 17) \times \frac{5}{2}$$

If the first term x_1 , last term is x_n then $\text{sum} = (x_1 + x_n) \times \frac{n}{2}$

Worksheet 10

- 1) Consider the arithmetic sequence 11, 15, 19 \dots 327
- Write the algebraic form of this sequence
 - How many terms are there in this sequence?
 - Calculate the sum of the terms
 - Can the sum of any 25 terms of this sequence be 2020. Why?

Answers

- a) $x_n = dn + (f - d)$, $x_n = 4n + (11 - 4) = 4n + 7$
- b) $4n + 7 = 327$, $4n = 320$, $n = 80$
- c) $\text{sum} = (x_1 + x_n) \times \frac{n}{2} = (11 + 327) \times \frac{80}{2} = 13520$
- d) All terms are odd numbers. 25 odd numbers cannot make the sum 2020 an even number.

2) Consider the sequence of numbers which leaves the remainder 3 on dividing by 4

- a) Write the sequence
- b) Write the algebraic form of this sequence
- c) Which is the term just below 100
- d) How many terms are there below 100 in this sequence?
- e) Calculate the sum of all terms below 100

Answers

- a) 3, 7, 11, ...
- b) $x_n = dn + (f - d) = 4n + (3 - 4) = 4n - 1$
- c) 99
- d) $4n - 1 = 99$, $4n = 100$, $n = 25$
- e) $\text{sum} = (x_1 + x_n) \times \frac{n}{2} = (3 + 99) \times \frac{25}{2} = 1275$

3) Tenth term of an arithmetic sequence is 36.

- a) What is the sum of first term and 19th term?
- b) Calculate the sum of first 19 terms of this sequence
- c) If the common difference is 3 what is its 3rd term
- d) Write the algebraic form of this sequence

Answers

- a) $2 \times 36 = 72$
- b) $\text{sum} = (x_1 + x_{19}) \times \frac{19}{2} = 72 \times \frac{19}{2} = 684$
- c) $x_3 = x_{10} - 7d = 36 - 7 \times 3 = 36 - 21 = 15$
- d) $x_n = dn + (f - d) = 3n + (9 - 3) = 3n + 6$

4) First term of an arithmetic sequence is 17. The alternate terms of this sequence are 17, 27, 37, ...

- a) What is the common difference?
- b) Write the algebraic form of this sequence
- c) Which is the term just below 100?

- d) How many terms are there below 100
 e) Calculate the sum of all terms below 100 in this sequence.

Answers

- a) $2d = 27 - 17 = 10, d = 5$
 b) $x_n = dn + (f - d) = 5n + (17 - 5) = 5n + 12$
 c) 97
 d) $5n + 12 = 97, 5n = 85, n = 17$
 e) $\text{sum} = (x_1 + x_n) \times \frac{n}{2} = (17 + 97) \times \frac{17}{2} = 961$

5) Write the sequence and make calculations as given below

- a) First term 2, common difference 4. Write the arithmetic sequence and calculate the sum of 2 terms
 b) First term 3, common difference 3. Write the arithmetic sequence and calculate the sum of three terms.
 c) First term 4, common difference 8. Calculate the sum of first 4 terms.
 d) First term 5, common difference 10. Write the arithmetic sequence and find the sum of first 5 terms
 e) If the first term of an arithmetic sequence is n and common difference $2n$ then prove that the sum of first n terms is n^3

Answers

- a) 2, 6
 $\text{sum} = 2 + 6 = 8$
 b) 3, 9, 15
 $\text{sum} = 3 + 9 + 15 = 27$
 c) 4, 12, 20, 28
 $\text{sum} = 4 + 12 + 20 + 28 = 64$
 d) 5, 15, 25, 35, 45
 $\text{sum} = 5 + 15 + 25 + 35 + 45 = 125$
 e) $n, 3n, 5n, 7n \dots (2n - 1)n$
 $x_1 = n, x_n = (2n - 1)n$
 $\text{sum} = (x_1 + x_n) \times \frac{n}{2} = n^3$

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Mathematics X
സമാന്തരശ്രേണികൾ

11

Notes

- ★ The relation between the counting numbers $1, 2, 3 \dots$ and the terms of a number sequence make its algebraic form
- ★ To write the algebraic form of a sequence we have to realize the rule which makes the sequence properly
- ★ Sequences can be formed from various situations .Algebraic form may not exist for all sequences
- ★ If a polygon has n sides , then its interior angle sum is $(n - 2) \times 180$.
- ★ If all angles and all sides are equal in a polyon it will be a regular polygon

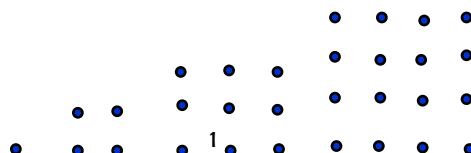
Worksheet11

- 1) As we know all angles and all sides of an equilateral triangle equal.
- Draw a rough diagram of a sequence of equilateral triangles with sides 1cm , 2cm, 3cm ,4cm
 - Write the sequence of perimetres
 - Describe the sequence of perimetres in another form
 - What is the perimeter of tenth triangle
 - Write the algebraic form of the sequence of perimetres

Answers

- Draw diagram
- $3, 6, 9, 12 \dots$
- This is the sequence of the multiples of 3
- $3 \times 10 = 30\text{cm}$
- $x_n = 3n$

- 2) This is the sequence of fine dots which make square pattern



- Write the number of dots in each square as a sequence.
- How many dots are there in sixth square
- What position in the sequence a square with 100 dots comes
- Write the algebraic form of this sequence
- How many dots are there in the square just below 1000?

Answers

- 1, 4, 9, 16, 25...
- 36
- 10th position
- $x_n = n^2$
- 961 dots . It is 31^2

- 3) Manju wrote a sequence .

3, 7, 11, 15...

Teacher said: This is the sequence starting from 3 and adding 4 repeatedly.

Manju gave another hint. Multiples of 4 are in the mind while writing this sequence .

- Describe the sequence using the hint given above
- What is the 10 th term of this sequence
- Write the algebraic form of this sequence
- Using the algebraic form find its 100 th term

Answers

- It is the sequence of numbers 1 less than the multiples of 4
- $x_{10} = 4 \times 10 - 1 = 39$
- $x_n = 4n - 1$
- $x_{100} = 4 \times 100 - 1 = 399$

- 4) Consider the sequence $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7} \dots$

- Write the next term of this sequence
- Which is the first integer term of this sequence
- Write the algebraic form of this sequence
- At what position 2 comes in the sequence?
- What are the positions where the integer terms occur in the sequence

Answers

- a) $\frac{5}{7}$
- b) $\frac{7}{7} = 1$
- c) $x_n = \frac{n}{7}$
- d) 14 th position
- e) 7, 14, 21, 28...

5) We have seen the sequence of interior angle sum of polygons in the class. The sequence is given below

180, 360, 540, 720...

- a) This is the sequence of the multiples of 180. Write the algebraic form of this sequence
- b) What is the fifth term of this sequence.
- c) If the polygons are regular, all interior angles are equal. Write the sequence of interior angles
- d) Think about the algebraic form of the sequence of interior angles and send your answer to the teacher.

Answers

- a) $x_n = 180n$
- b) $x_5 = 180 \times 5 = 900$
- c) 60, 90, 108, 120... This is obtained by dividing the angle sum by the number of sides.
- d) Look at the pattern given below

$$60 = \frac{180 \times 1}{3} = \frac{180 \times 1}{1 + 2}$$

$$90 = \frac{180 \times 2}{4} = \frac{180 \times 2}{2 + 2}$$

$$108 = \frac{180 \times 3}{5} = \frac{180 \times 3}{3 + 2}$$

$$120 = \frac{180 \times 4}{6} = \frac{180 \times 4}{4 + 2}$$

Proceeding like this we get

$$x_n = \frac{180n}{n + 2}$$

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Mathematics X Arithmetic Sequences

12

Notes

- ★ $1, 2, 3, \dots$ is the sequence of natural numbers. Algebraic form of this sequence is $x_n = n$.
- ★ The sequence of counting numbers is an arithmetic sequence
- ★ Let us calculate the sum of these numbers $1, 2, 3, 4, 5, 6$. For this we make pairs from both sides.
The sum of the numbers in each pair $(1, 6), (2, 5), (3, 4)$ is 7
 $1 + 2 + 3 + 4 + 5 + 6 = 7 \times 3 = (6 + 1) \frac{6}{2}$
- ★ Sum of natural numbers from 1 to 7 is $(7 + 1) \times \frac{7}{2}$
- ★ Sum of natural numbers from 1 to n is $(n + 1) \times \frac{n}{2}$

Worksheet 12

1) Complete the addition :

- a) Calculate the sum of counting numbers from 1 to 10
- b) Calculate the sum of counting numbers from 1 to 100
- c) Calculate the sum of the 48 terms of the sequence $3, 4, 5, \dots$ obtained by adding 1 repeatedly
- d) Calculate the sum of counting numbers from 10 to 20

Answers

- a) Sum = $(n + 1) \times \frac{n}{2} = (10 + 1) \times \frac{10}{2} = 55$
- b) Sum = $(100 + 1) \times \frac{100}{2} = 5050$
- c) Sum = $(1 + 2 + 3 + 4 \dots 50) - (1 + 2) = (50 + 1) \times \frac{50}{2} - 3 = 1272$
- d) Sum of numbers from 10 to 20 = sum from 1 to 20 – sum from 1 to 9
 $= (20 + 1) \times \frac{20}{2} - (9 + 1) \times \frac{9}{2} = 165$

2) If $a' = a + 1, a = a - 1$

- a) Find $1' - 1$
- b) Find $2' - 2$
- c) Find $(1' - 1) + (2' - 2) + (3' - 3) + \dots + (100' - 100)$
- d) Find $1' + 2' + 3' \dots 100'$

Answers

- a) $1' - '1 = (1 + 1) - (1 - 1) = 2$
- b) $2' - '2 = (2 + 1) - (2 - 1) = 2$
- c) $2 \times 100 = 200$
- d) $1' + 2' + 3' \cdots 100' = 2 + 3 + 4 + \cdots 101 = 1 + 2 + 3 + \cdots 100 = 5050$

3) The sum of first n counting numbers is $(n + 1) \times \frac{n}{2}$. Use this to complete the following calculations

- a) Consider the sequence of the multiples of 3
3, 6, 9 \cdots . Which is the largest two digit term of this sequence
- b) Calculate the sum of all terms of this sequence below 100
- c) Find the sum of all even numbers below 100
- d) Calculate the sum of all multiples of 7 below 100

Answers

- a) 99
- b) $3 + 6 + 9 + \cdots 99 = 3(1 + 2 + 3 + \cdots 33) = 3 \times (33 + 1) \times \frac{33}{2} = 1683$
- c) $2 + 4 + 6 + \cdots 98 = 2(1 + 2 + 3 + \cdots 49) = 2 \times (49 + 1) \times \frac{49}{2} = 2450$
- d) $7 + 14 + 21 + \cdots 98 = 7(1 + 2 + 3 \cdots 14) = 7 \times (14 + 1) \times \frac{14}{2} = 735$

4) Consider the sequence 1, 2, 3, 4 \cdots

- a) Make pairs as (1, 2), (3, 4), (5, 6) \cdots , find the sum of numbers in each pair and write the sums as a sequence
- b) Is this an arithmetic sequence ?why?
- c) Write the algebraic form of this sequence
- d) What is the common difference of the sequence obtained by adding three numbers in the group
- e) What is the difference between the sum of the first n natural numbers and sum of next n natural numbers.

Answers

- a) 3, 7, 11 \cdots
- b) The difference between any two adjacent terms are equal. Sequence has the common difference 4. So it is an arithmetic sequence
- c) $x_n = dn + (f - d) = 4n - 1$
- d) $3^2 = 9$
- e) n^2

- 5) Nasrin has drawn a circle in her note book and marked 11 fine dots on the circle. She joined first dot to all other 10 dots to make chords.
- a) How many chords can be drawn by joining second dot to the remaining dots in the same circle.
 - b) How many chords can be drawn using the third dot?
 - c) How many chords can be drawn using the fourth dot?
 - d) Find the total number of chords that can be drawn in this circle joining all 11 dots.

Answers

- a) 9
- b) 8
- c) 7
- d) $1 + 2 + 3 + \cdots + 10 = 55$

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Mathematics X Arithmetic Sequences

13

Notes

a) $1, 3, 5, 7, 9, \dots$ is the sequence of odd numbers.
Algebraic form of this sequence is $x_n = 2n - 1$.

b) The sum the first n odd numbers can be understand from the pattern given below

$$1 = 1 = 1^2$$

$$1 + 3 = 4 = 2^2$$

$$1 + 3 + 5 = 9 = 3^2$$

$$1 + 3 + 5 + 7 = 16 = 4^2$$

$$1 + 3 + 5 + 7 + 9 = 25 = 5^2$$

c) The sum of the first n odd numbers is n^2

Worksheet13

1) Answer the following questions

- a) Find the sum of first 10 odd numbers
- b) Find the sum of first 25 odd numbers
- c) How many odd numbers from 1 makes the sum 1225?
- d) $1 + 3 + 5 + 7 + \dots + 2n - 1$ is in between 900 and 1000. What is n ?

Answers

a) $s_n = n^2, s_{10} = 10^2 = 100$

b) $s_n = n^2, s_{25} = 25^2 = 625$

c) ആദ്യത്തെ n ഒറ്റസംഖ്യകളുടെ തുക $= n^2$.
 $n = \sqrt{1225} = 35$

d) Perfect square in between 900 and 1000 is 961. $n^2 = 961, n = 31$

2) Have you heard of triangular numbers? The sequence of triangular numbers is given below

$$1, 3, 6, 10, 15, 21 \dots$$

n th term of this sequence can be obtained from the pattern

$$1 = 1$$

$$3 = 1 + 2$$

$$6 = 1 + 2 + 3$$

$$10 = 1 + 2 + 3 + 4$$

$$15 = 1 + 2 + 3 + 4 + 5$$

- What is the 10th triangular number?
- Write the algebraic form of the sequence of triangular numbers
- Which is the largest two digit triangular number?
- Find the 50 th triangular number

Answers

a) $T_{10} = 1 + 2 + 3 \dots + 10 = (10 + 1) \frac{10}{2} = 55$

b) $(n + 1) \times \frac{n}{2}$

c) $T_{13} = (13 + 1) \times \frac{13}{2} = 91$

d) $T_{50} = (50 + 1) \times \frac{50}{2} = 1275$

3) Look at the pattern given below

```

      1
     3 5 7
    9 11 13 15 17
   19 21 23 25 27 29 31
  -----

```

- Write the sequence of number of numbers in each line
- Write the algebraic form of this sequence
- How many numbers are there in 20 th line ?
- Which number comes in the right end of 20 th line ?
- Which number comes in the left end of 20 th line ?
- How many numbers are needed to make 20 lines of this pattern.

Answers

- a) $1, 3, 5, 7 \dots$
- b) $x_n = dn + (f - d) = 2n - 1$
- c) $x_{20} = 2 \times 20 - 1 = 39$
- d) അവസാനസംഖ്യകളുടെ ശ്രേണി: $1, 7, 17, 31 \dots$
 $x_n = 2n^2 - 1, x_{20} = 2 \times 20^2 - 1 = 799$.
ഇരുപതാംവരിയിലെ അവസാനസംഖ്യ = 799
- e) $2 \times 19^2 - 1 + 2 = 723$. (Add 2 to the number in the right end of 19 th line)
- f) $20^2 = 400$ are needed

4) Look at the pattern given below

```

      1
     2 3 4
    5 6 7 8 9
-----

```

- a) Write the sequence of number of numbers in each line
- b) Write the algebraic form of this sequence
- c) How many numbers are there in 20 th line ?
- d) Which number comes in the right end of 20 th line ?
- d) Which number comes in the left end of 20 th line ?
- e) Calculate the sum of all numbers upto the end of 20 th line

Answers

- a) $1, 3, 5, 7 \dots$
- b) $x_n = 2n - 1$
- c) 39
- d) $20^2 = 400$
(Sequence of numbers in the right end : $1, 4, 9, 16 \dots$)
- e) $19^2 + 1 = 362$
- f) $1, 2, 3, 4 \dots 400$ Sum of numbers : $= (400 + 1) \times \frac{400}{2} = 80200$

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Mathematics X
സമാന്തരശ്രേണികൾ

13

Notes

- ★ Knowing the first term and common difference we can write the sequence . When $(n - 1)$ times common difference d is added to the first term we get n th term of this

$$x_n = f + (n - 1)d$$

If $2d$ is added to the first term we get third term. If $3d$ is added to the first term we get 4 th term. , $4d$ is added to the fifth term we get fifth term

- ★ If common difference is added to a term of an arithmetic sequence we get the term after it. If common difference is subtracted from a term we get the term before it
- ★ If a definite number of times common difference is added to a term of a sequence we get another term of this sequence
- ★ The difference between any two term of an arithmetic sequence will be a multiple of its common difference .That is , the difference is divisible by its common difference
- ★ Using this property we can check wheather a number a term of this sequence
- ★ The difference of two terms of an arithmetic sequence is proportional to its position difference. The proportionality constant is the common difference

Worksheet13

- 1) The first term of an arithmetic sequence is 3 and common difference 2.
 - a) Write the sequence
 - b) How many times common difference should be added to its first term to its tenth term
 - c) What is the tenth term of this sequence?
 - d) What is the 101 th term of this sequence?
 - e) Is 100 a term of this sequence ? How can we realize it?

Answers

- a) $3, 5, 7, 9 \dots$
- b) 9 times common difference should be added
- c) $x_{10} = 3 + 9 \times 2 = 3 + 18 = 21$
- d) $x_{101} = 3 + 100 \times 2 = 203$
- e) All terms are odd numbers . Even number 100 cannot be the term

2) Consider the arithmetic sequence $3, 5, 7, 9 \dots$

- a) What is the common difference ?
- b) 4 times common difference is added to the first term. What is the position of the term obtained?
- c) What is the tenth term of this sequence?
- d) Write the algebraic form of this sequence

Answers

- a) $d = 5 - 3 = 2$
- b) We get 5 th term
- c) $x_{10} = x_1 + 9d = 3 + 9 \times 2 = 21$
- d) $x_n = f + (n - 1)d = 3 + (n - 1) \times 2 = 2n + 1$
Note $: x_n = dn + (f - d)$ can be used to find n th term

3) $24, \square, 34, 39, \square \dots$ is a number sequence .If it is an arithmetic sequence

- a) What is its common difference
- b) Write two missing terms of this sequence
- c) Write three more terms of this sequence
- d) What should be added to the first term to get its tenth term
- e) Write the algebraic form of this sequence

Answers

- a) $2d = 34 - 24 = 10$
 $d = 5$
- b) Number in the first box is $\square 29$. Number in the second box $39 + 5 = 44$
- c) $49, 54, 59$
- d) 9 common difference $= 9 \times 5 = 45$ should be added
- e) $x_n = dn + (f - d) = 5n + 19$

4) 15 th term of an arithmetic sequence is 40 and its 20 th term is 60

- a) What is the common difference?
- b) What is the first term?
- c) What is the tenth term of this sequence?
- d) Write the n th term or algebraic form of this sequence

Answers

- a) $5d = 60 - 40 = 20, d = 4$
- b) $x_1 = x_{15} - 14 \times 4 = 40 - 14 \times 4 = 40 - 56 = -16$
- c) $x_{10} = f + 9d = -16 + 9 \times 4 = -16 + 36 = 20$
- d) $x_n = dn + (f - d) = 4n - 20$

5) Consider the arithmetic sequence 1, 5, 9, 13...

- a) What is the common difference?
- b) Write the algebraic form of this sequence
- c) Is 101 a term of this sequence ?
- d) If it is a term ,what is the position of this term?

Answers

- a) $d = 4$
- b) $x_n = dn + (f - d) = 4n - 3$
- c) $101 - x_1 = 100$.
100 is divisible by the common difference 4. 101 is a term.
- d) $4n - 3 = 101, 4n = 104, n = 26$
 $x_{26} = 101$

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Mathematics X Arithmetic Sequence

15

Notes

Knowing the algebraic form of the sequence we can find the sum of the first n terms of the sequence. If the n th term of an arithmetic sequence is $an + b$ we can find the sum of first n terms as follows.

$$x_1 = a \times 1 + b$$

$$x_2 = a \times 2 + b$$

$$x_3 = a \times 3 + b$$

$$x_4 = a \times 4 + b$$

$$x_n = a \times n + b$$

$$\begin{aligned} x_1 + x_2 + x_3 + \cdots + x_n &= a(1 + 2 + 3 + \cdots + n) + nb \\ &= a(n+1) \times \frac{n}{2} + nb \end{aligned}$$

Worksheet15

- 1) First term of an arithmetic sequence is 3 and the common difference 4,
 - a) Write the algebraic form of this sequence
 - b) Calculate the sum of first 14 terms of this sequence
 - c) Can the sum of any 25 terms of this sequence 2020. How can we realize it
 - d) Write the sum of first n terms of this sequence

Answers

- a) $x_n = dn + (f - d) = 4n + (3 - 4) = 4n - 1$
- b) $\text{Sum} = 4(1 + 2 + 3 + \cdots + 14) - 1 \times 14 = 4 \times (14 + 1) \times \frac{14}{2} - 14 = 406$
- c) All terms are odd numbers. Sum of 25 odd numbers cannot be an even number . 2020 cannot be the sum
- d) $S_n = 4(1 + 2 + 3 + \cdots + n) - 1 \times n = 4 \times (n + 1) \times \frac{n}{2} - n = 2n^2 + 2n - n = 2n^2 + n$

- 2) The algebraic form of the sum of first n terms of a sequence is $n^2 + n$.

- a) Write the sequence

- b) Write the algebraic form of the sequence
- c) Can the sum of any number of terms 2021? How can we realize it
- d) How many terms are there below 100 in this sequence?
- e) Calculate the sum of all terms below 100 in this sequence

Answers

- a) $x_1 = 1^2 + 1 = 2, x_1 + x_2 = 2^2 + 2 = 6$
 $x_2 = 6 - 2 = 4, d = x_2 - x_1 = 4 - 2 = 2$
 Sequence : 2, 4, 6 ...
- b) $x_n = 2n$
- c) All terms are even numbers . Some of some even numbers cannot be an odd number.
 2021 cannot be the sum
- d) $2n = 98, n = 49$ There are 49 terms below 100
- e) Sum $= 2(1 + 2 + 3 + \dots + 49) = 2 \times (49 + 1) \times \frac{49}{2} = 49 \times 50 = 2450$

Note : There is a direct formula for calculating the sum of first n even numbers. It is $n(n+1)$

- 3) Consider an arithmetic sequence of algebraic form $3n + 2$
- a) What is the common difference of this sequence ?
- b) What is its first term?
- c) Find the sum of first n terms of this sequence
- d) Calculate the sum of first 10 terms of this sequence?
- e) Calculate the sum of the terms from 10 th to 20 th of this sequence

Answers

- a) 3
- b) $3 \times 1 + 2 = 5$
- c) $3 \times (n + 1) \times \frac{n}{2} + 2n = \frac{3}{2}n^2 + \frac{7}{2}n$
- d) $\frac{3}{2} \times 10^2 + \frac{7}{2} \times 10 = 185$
- e) Sum of 10th term to 20 th term = sum of first 20 terms – sum of first 9 terms
 $s_{20} = 670, s_9 = 153$ Sum is $670 - 153 = 517$

- 4) Look at the pattern given below

			2			
		4	6	8		
10	12	14	16	18		

- Write the number of numbers in each line as a sequence
- How many numbers are there in 30 th line ?
- Which number comes in the right end of 30 th line?
- Which number comes in the left end of 30 th line?
- Calculate the sum of all numbers in 30 lines of this pattern

Answers

- $1, 3, 5, 7 \dots$
- $x_n = 2n - 1, x_{30} = 2 \times 30 - 1 = 59$
- Sequence of numbers at the right end is : $2, 8, 18, 32 \dots$
 $x_n = 2n^2, x_{30} = 2 \times 30^2 = 1800$
- First number in the 30 th line is 2 more than last number of 29 th line . It is $2 \times 29^2 + 2 = 2 \times 841 + 2 = 1684$
- Sum = $2 + 4 + 6 + \dots + 1800 = 2(1 + 2 + 3 + \dots + 900) = 2 \times (900 + 1) \times \frac{900}{2} = 810900$

- 5) You have already studied the calculation of the sum of first n natural numbers . Look at the pattern given below

$$1^3 = 1$$

$$1^3 + 2^3 = 9 = 3^2 = (1 + 2)^2$$

$$1^3 + 2^3 + 3^3 = 36 = 6^2 = (1 + 2 + 3)^2$$

We can see an order in this calculation. This will help us to write more lines below . Answer the following questions

- How many cubic numbers are there from 1 to 8000?
- Find $1^3 + 2^3 + 3^3 + 4^3$
- Find the sum of the cubes of all natural numbers from 1 to 6
- We know that the sum of all natural numbers from 1 to 10 is 55. Calculate $1^3 + 2^3 + 3^3 \dots + 10^3$
- Write a formula to find the sum $1^3 + 2^3 + 3^3 \dots + n^3$

Answers

- $20^3 = 8000$. 8000 വരെ 20 ഘനസംഖ്യകളുണ്ട്
- $(1 + 2 + 3 + 4)^2 = 10^2 = 100$
- $(1 + 2 + 3 + 4 + 5 + 6)^2 = 21^2 = 441$
- $55^2 = 3025$
- $s_n = \left(\frac{n(n+1)}{2}\right)^2$

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