Mathematics X Arithmetic Sequences

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It is the time to see new sights and make new insights. While starting Arithmetic Sequence of tenth standard we have some expectations. We hope this will enrich the thoughts of the learner and help them to develop a self study attitude.

Instructions

- * Attend VICTERS channel classes carefully
- * Do the worksheet without any help
- ★ Ask your doubts in the group
- \star The doubts can be send by photo or voice.

- 1) Consider the sequence $2,4,6,8\cdots$
 - a) Write two more terms of this sequence
 - b) Which is he smallest two digit term of this sequence?
 - c) Which is the largest two digit term of this sequence
 - d) Can the sum of any two terms of this sequence 75?
- 2) Consider the sequence $1, 3, 5, 7 \cdots$
 - a) Write next two more terms of this sequence
 - b) Which is the smallest two digit term of this sequence?
 - c) Which is the largest two digit term of this sequence
 - d) Can the sum of any two terms of this sequence $75\,$
- a) Write the sequence of numbers in which 1 comes in the one's place
 - b) which is the largest two digit term of this sequence?
 - c) How many numbers are there in this sequence below 100?

a)	Write the number of squares in each line as a sequence	е		
b)	Write the number of match sticks in each line as a sequ	ience		
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c) How many match sticks are used to make next line

4) Look at the picture . Squares are made by joining match sticks.

- 5) Find out the rule behind the formation of the sequence $1,4.9,16\cdots$
 - a) Write next three terms
 - b) What is the tenth term of this sequence
 - c) Is $900\ \mbox{a}$ term of this sequence ? If it is ,what is its position?
 - d) Is 1000a term of this sequence? Why?
- 6) There is a popular sequence named frbonacci sequence .lt is given below

$$1, 1, 2, 3, 5, 8, 13, 21 \cdots$$

Can you find out the rule behind the formation of this sequence

- a) write next two terms?
- b) Which is the largest two digit term of this sequence?

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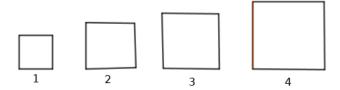
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Have you attended todays class in the channel? Questions given on the basis of this class . Your teacher will explain and clarify your doubts. Answer the questions and post it in the group

Note

- \star Numbers of the form $1,2,3\cdots$ are called counting numbers. All sequences are formed from counting numbers.
- * Sequences are generated from various situations Some are generated from geometrical patters
- * A number sequence will have a rule which help us to understand the sequence properly.

- 1) $10, 20, 30 \cdots$ is a squence in which 0 comes in the one's place
 - a) Write the sequence in which $\boldsymbol{1}$ comes in the one's place
 - b) Which is the largest two digit term of this sequence?
 - c) Which is the smallest two digit term of this sequence?
 - d) Which number is added repeatedly to get the terms of this sequence?
- 2) $1, 6, 11, 16 \cdots$ is the sequence in which 1 or 6 comes in the right end of the number
 - a) Write next five more terms of this sequence
 - b) Which is the largest two digit term of this sequence
 - c) How many numbers are there below 100 in this sequence ? Find it by actual counting
 - d) can you find it without counting?
- 3) Consider the numbers which gives the remainder $\boldsymbol{1}$ on dividing by $\boldsymbol{3}$
 - a) Write the sequence of these numbers
 - b) Which is the smallest two digit term of this sequence
 - c) How many numebrs are there up to 25 in this sequence ?
 - d) What are the possible remainders on dividing a number by $\boldsymbol{3}$
- 4) Look at the sequence of squares



- a) Draw a rough diagram of the sequence of equilateral triangles with sides 1 cm, 2 cm, $3 \text{cm} \cdots$
- b) Write the sequence of the perimetres of these triangles?
- c) What is the side of the triangle having perimetre just below $10\ \mathrm{in}$ this sequence
- d) What is the side of the triangle having perimetre just below $100\,\mathrm{in}$ this sequence
- 5) Write the remaiders on dividing the counting numbers from $1\ \mathrm{to}\ 10\ \mathrm{by}\ 3$

- a) What is the sum of the remainders?
- b) What is the sum of the remainders on dividing the numbers from $1\mbox{to}\ 10$ by $4\mbox{?}$
- c) What is the sum of the remainders on dividing the numbers from $1\mathrm{to}\ 100$ by $3\mathbf{?}$

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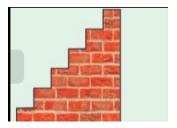
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Notes

- \star If a polygon has n sides its angle sum will be $(n-2)\times180^{\circ}$
- ★ If sides and angles of a polygon are equal it will be a regular polygon.
- * A sequence can be described more than one way. It depends on how we see the relation among the terms.

Worksheet3

- 1) a) Draw the sequence of equilateral triangle, square, regular pentagon, regular hexagon \cdots .
 - b) Write the interior angle sum as a sequence
 - c) Write the sequence of one interior angle
 - d) Write the measure of exterior angle as a sequence
- 2) Look at the picture given below



The height from the floor to the first step is $10 \, \mathrm{cm}$. The heights of subsequent steps is $5 \, \mathrm{cm}$

- a) Write the sequence of heights attained by a boy while moving up through the steps
- b) What will be the height from the floor when the boy reaches the fifth step?
- c) The boy reaches the last step which is 20 th from the floor. What is the height from the floor at this position?
- d) The starts moving down from 20 th step. Write the sequence of heights

3) Let us think about a puzzle.

There are two electonic devices on the table. The device A makes BEEP sound in every 1 minute. The device B makes BEEP sound in every 62 seconds.

Both of the devices beeped at $10\ \mathrm{O'clock}$.

The times of beeping the first device is written as a sequence below

 $10:01:00,10:02:00,10:03:00,10:04:00\cdots$

- a) Write the sequence of the beeping time of the second device
- b) What is the nearest time at which the devices beeped together after $10\ {
 m O'clock}$

$$1, 6, 11, 16, 21, 26 \cdots$$

The teacher described this sequence as: 'it is the sequence of numbers having $1\ {\rm or}\ 6$ in one's place'.

a) Nasrin, a brilliant student said. It is the sequence of numbers starting from 1 and adding 5repeatedly.If you can agree with this , write the three more terms of this sequence using this opinion

- b) Riswan, another student said,it is 4 less than a multiple of 5. If you can agree with this write the tenth term of this sequence .
- c) We can see both statements are true. How many terms are there below 100 in this sequence?
- d) What is the 20 th term of this sequence?
- e) Write the algebraic expression of the \boldsymbol{n} th term of this sequence.
- 5) Look at the sequence $1,4,9,16,25\cdots$.These numbers can be written as $1^2,2^2,3^2\cdots$.
 - a) What is the remainder obtained by dividing $1^2, 2^2, 3^2$ by 3
 - b) What is the sum of the remainders obtained by dividing $1^2, 2^2, 3^2 \cdots 10^2$ by 3
 - c) What is the sum of the remainders obtained by dividing $1^2, 2^2, 3^2 \cdot \cdot \cdot 100^2$ by 3
 - d) Can a perfect square be a term of the sequence $2,5,8,11\cdots$. How can you realize this ?

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Notes

The numbers $1, 2, 3, 4 \cdots$ are called counting numbers or natural numbers. Different types of sequences can be formed by various operations on the natural numbers.

In all the sequences that we discuss here are closely related to the counting numbers. Knowing this relationship between the terms of the sequences and natural numbers help us to study the sequence properly. The relation between the terms of the sequence and natural numbers is called algebraic form of the sequence.

- 1) a) Multiply each number of the sequence $1,2,3\cdots$ by 3 , add 2 then write the resulting numbers as a sequence
 - b) What is the tenth term of this sequence?
 - c) At what position $32 \mathrm{comes}$ in this sequence ?
 - d) How many terms are there below 100 in this sequence?
- 2) a) Add 1 to the square of each of the numbers of the sequence $1,2,3,4\cdots$
 - b) What is the 20 th term of this sequence
 - c) Which is the smallest three digit term of this sequence?
 - d) How many terms are there below $100?\mbox{Which}$ is the largest among them?
 - e) Write the n th term of this sequence
- 3) a) Write the sequence of numbers starting from 3 and adding 2 repeatedly.
 - b) What should be added to 3 to get 17?
 - c) At what position 17 comes in the sequence?
 - d) How many times 2 is added to 3 for getting its n th term.
 - e) Prove that its n th term is 2n+1

- 4) a) Write the sequence in which first term is 5 and the number adding repeatedly is 3.
 - b) How many times $\boldsymbol{3}$ is added to its first term to get tenth term
 - c) Which is the tenth term of this sequence?
 - d) Write the algebraic form or n th term of this sequence
- 5) a) Write the sequence of numbers which leaves the remainder 1 on dividing by 5
 - b) Describe this sequence in other two ways(See 03as, the opinions of Nasrin and Riswan)
 - c) Write the algebraic form of this sequence
 - d) Write the $10\ \mathrm{th}$ term of this sequence using its algebraic form

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Notes

We have seen sequences in general.In the previous worksheet we gave special importance to the sequence which starts from a number and adding another number repeatedly. Such sequences are called arithmetic sequences.In this worksheet we think about the algebra of the arithmetic sequence and some simple uses of algebraic form of the sequence. The starting number is called first term f or x_n

The number added repeatedly is called common difference, denoted by $d\boldsymbol{.}$

The algebraic form or n th term of an arithmetic sequence will be $x_n = f + (n-1) \times d$.

This can be written as $x_n = dn + (f - d)$

- 1) The first term of an arithmetic sequence is 2 and common difference 3.
 - a) Write the sequence
 - b) How many times common difference 3 should be added to the first term to get 25 th term?
 - c) What is the 25 th term of this sequence?
 - d) Write the algebraic form of this sequence
 - 2) Consider the arithmetic sequence $1, 5, 9, 13 \cdots$
 - a) What is the common difference of this sequence?
 - b) How many times common difference should be added to 1 for getting its tenth term?
 - c) How many times 3 should be added to the first term 1 to get its nth term?
 - d) Write the algebraic form of this sequence .
 - 3) Algebraic form of an arithmetic sequence is 3n + 7.
 - a) Write the sequence.
 - b) What is the 20 th term of this sequence
 - c) What should be added to 5 th term of this sequence to get 10 th term?
 - 4) The algebraic form of an arithmetic sequence is $\frac{1}{2}n+3$.
 - a) Give a suitable value for n for getting its first integer term?
 - b) Write the sequence of integer terms
 - c) Integer terms make another arithmetic sequence . Write its albebraic form
 - d) What is the largest two digit integer term of this sequence
 - e) How many integer terms are there below $100\,$

- 5) $\frac{1}{7}, \frac{2}{7}, \frac{3}{7} \cdots$ is an arithmetic sequence.
 - a) Write the \boldsymbol{n} th term of this sequence
 - b) What value of n the sequence gets its first integer term?
 - c) At what position $100\ \mathrm{comes}$ as a term of this sequence ?
 - d) How many integer terms are there below $100\ \mathrm{in}$ this sequence ?

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Notes

We have already discussed algebra of arithmetic sequence and one of its properties.If f is the first term and d is the common difference ,its algebraic form is $x_n=dn+(f-d)$. The difference between any two terms of an arithmetic sequence will be a mmultiple of common difference. Other words , the difference between any two terms of an arithmetic sequence is divisible by the common difference.

- 1) The algebraic form of an arithmetic sequence is 4n + 3.
 - a) What is its common difference?
 - b) Can the difference between any two terms of this sequence 176?
 - c) Which term would be obtained on adding $176\ \mathrm{to}$ the second term
 - d) Which term would be obtained on adding $176\ \mathrm{to}$ its tenth term
- 2) x,y,z are in an arithmetic sequence .
 - a) If y x = k(z x) then what is k?
 - b) If y x = k(x y) then what is k?
 - c) If x y = k(x z) then what is k?
 - d) Prove that $y = \frac{x+y+z}{3}$
- 3) Fifth term of an arithmetic sequence is 24 and its common difference is 3
 - a) What is its first term?
 - b) Write the algebraic form of the sequence
 - c) Write some ters of this sequence in an order
 - d) What is the difference between nth term and n+6th term of this sequence
 - e) What is the diffrence between $m+n{\rm th}$ term and $n{\rm th}$ term

- 4) First term of an arithmetic sequence is f and common difference d.
 - a) What is its fifth term?
 - b) What is its tenth term?
 - c) Five times $5\ \mathrm{th}$ term is equal to $10\ \mathrm{times}$ tenth term . Write this statement algebraically
 - d) What is the fifteenth term of this sequence?
 - e) What is the product of first $15\ \mathrm{terms}$?
- 5) Fifth term of an arithmetic sequence is 10 and tenth term is 5.
 - a) What is the common difference?
 - b) What is its first term?
 - c) What is its fifteenth term?
 - d) What is the sum of first 15 terms?

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Notes

Algebraic form of a sequence is the most important aspect for understanding the sequence properly. f is the first term and d is the common difference of an arithmetic sequence , its algebraic form will be $x_n=dn+(f-d)$. The difference between any two terms of an arithmetic sequence is always a multiple of its common difference. If the terms are natural numbers, the terms give the same remainder on dividing it by the common difference.

- 1) Consider the arithmetic sequence $1, 8, 15, 22 \cdots$
 - a) What is the common difference?
 - b) What will be the remainder when the terms are divided by its common difference?
 - c) Which is the first three digit term of this sequence?
 - d) Write the algebraic form of this sequence
 - e) How many terms are there below $100 \mathrm{in}$ this sequence?
- 2) The algebraic form of an arithmetic sequence is 7n+3.
 - a) What is the common difference?
 - b) What is the first term of this sequence?
 - c) What is the position of 171in this sequence?
 - d) What is the position of 248 in this sequence?
 - e) How many terms are there from $171\ \mathrm{to}\ 248$ in this sequence
- 3) $x-1, x, x+1, x+2 \cdots$ are in an arithmetic sequence.
 - a) What is the common difference of this sequence?
 - b) Write the algebraic form of this sequence
 - c) Write some more terms of this sequence
 - d) If the product of first and third terms of this sequence is 15, what is its first term?

- 4) Consider the arithmetic sequence $97,94,91\cdots$
 - a) What is the common difference of this sequence?
 - b) Write the algebraic form of this sequence
 - c) Which is smallest positive term of this sequence?
 - d) How many positive terms are there in this sequence?
- 5) Consider the arithmetic sequence $-48, -44, -40 \cdot \cdot \cdot$
 - a) What is its common difference?
 - b) Write the algebraic form of this sequence
 - c) What is the position of $\boldsymbol{0}$ in this sequence ?
 - d) How many negative terms are there in this sequence?

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Notes

★ Let us write an arithmetic sequence having three terms.

3, 7, 11

On dividing the sum of the terms by its number of terms we get the middle term. If the number of terms is 5 or 7 or any other odd number the same result holds. That is , the sum of the terms divided by the number of terms will be the middle term.

* What about the number of terms is even?
We can make them pairs. Make the terms equidistant from both ends as pairs and find the pair sum.

The pair sums are equal. You can verify it by writing the examples.

- \star If the number of terms is odd we can find a term at the middle. The middle term will be half of the pair sum
- \star To get the middle term just find either divide the sum of the terms by its number of terms or find half of the pair sum

- 1) Insert suitable number in the box which make the arithmetic sequence
 - a) $\boxed{5}$, $\boxed{11}$, $\boxed{17}$
 - b) [13], [9], [], [-3], [-7]

 - d) 7, 15, __, __, 39, 47
- 2) Angles of a right triangle are in an arithmetic sequence.
 - a) Find the middle term of the sequence
 - b) Write the angles of the triangle
- 3) Angles of a quadrilateral are in an arithmetic sequence
 - a) Find the angle sum
 - b) What is the sum of first term and fourth term?
 - c) What is the sum of second term and third term?
 - d) Suggest a suitable name of this quadrilateral

- 4) Angles of a pentagon are in an arithmetric sequence .
 - a) What is the sum of the terms of this sequence
 - b) What is the middle term?
 - c) If the smallest angle is 40° what is the difference between two adjacent angles
 - d) Write the angles as a sequence
- 5) There is an arithmetic sequence having 9 terms. The sum of the first and 9 th terms is 32.
 - a) What is the sum of second and eighth term?
 - b) what is the fifth term?
 - c) If the sixth term is 19 then what is its common difference?
 - d) Write the first term of this sequence

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Notes

This is the continuation of eighth work sheet. In this new work sheet we discuss problems to find the sequence, its algebraic form from two given terms.

- 1) 13 th term of an arithmetic sequence is 48. Its 17 th term is 64
 - a) What is the common difference?
 - b) What is the 15 th term?
 - c) Find the first term of this sequence
 - d) Write the algebraic form of this sequence
 - e) What is the product first some terms of this sequence
- 2) The first term of an arithmetic sequence is $\frac{1}{3}$ and its common difference $\frac{1}{6}$
 - a) Write the algebraic form of this sequence
 - b) At what position the first integer term comes in the sequence?
 - c) What are the term positions at which the integer terms occur in the sequence?
 - d) What should be added to its $10\ \mathrm{th}\ \mathrm{term}\ \mathrm{to}\ \mathrm{get}\ 25\ \mathrm{th}\ \mathrm{term?}$
- 3) The sum of first 5 terms of an arithmetic sequence is 70. The sum of the first 11 terms is 286.
 - a) What is its third term?
 - b) What is its sixth term?
 - c) What is the common difference of this sequence?
 - d) What is the first term of this sequence?
 - e) Write the algebraic form of this sequence
- 4) Seventh term of an arithmetic sequence is 21.
 - a) What is the sum of sixth and eighth term of this sequence?
 - b) What is the sum of first and thirteenth term?
 - c) If the eighth term is 25, what is its common difference

- d) What is the first term of this sequence?
- e) Can the difference between any two terms 45?
- 5) A wire of length $70 \, \mathrm{cm}$ cut into five pieces. When they are arranged in an order, its lengths become an arithmetic sequence
 - a) What is the length of third piece in the arrangement?
 - b) If the shortest piece has lenght 8 cm what is the difference between the lengths of two neighbouring pieces?
 - c) What is the lenght of the longest piece
 - d) Write the sequence of lengths

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Notes

a) Let us calculate the sum of a definite number of terms of this sequence. Simplest process of finding the sum is pairing.

Consider the arithmetic sequence 5,8,11,14,17,20. In this sequence pairing can be done as (5,20),(8,17),(11,14). The pair sum is $25.\mathrm{Sum}$ of the terms of the sequence $=25\times 3=75$.

 $\text{sum}=(5+20)\times\frac{6}{2}.\text{Sum}$ can be calculated by multiplying the pair sum by half of the number of terms

b) Consider the sequence 5,8,11,14,17. There are 5terms in this sequence . On pairing we get the middle term without a pair.

Now we get two pairs with the pair sum 22 and the middle term.

$$sum = (5 + 17) \times 2 + 11$$

$$= (5 + 17) \times 2 + \frac{22}{2}$$

$$= 22 \times 2 + \frac{22}{2}$$

$$= 22(2 + \frac{1}{2})$$

$$= 22 \times \frac{5}{2}$$

$$\mathrm{sum}{=}\left(5+17\right)\times \tfrac{5}{2}$$

If the first term x_1 , last term is x_n then sum $=(x_1+x_n) imes \frac{n}{2}$

- 1) Consider the arithmetic sequence $11, 15, 19 \cdots 327$
 - a) Write the algebraic form of this sequence
 - b) How many terms are there in this sequence?
 - c) Calcualte the sum of the terms
 - d) Can the sum of any 25 terms of this sequence 2020. Why?

- 2) Consider the sequence of numbers which leaves the remainder 3 on dividing by 4
 - a) Write the sequence
 - b) Write the algebraic form of this sequence
 - c) Which is the term just below 100
 - d) How many terms are there below 100in this sequence?
 - e) Calculate the sum of all terms below $100\,$
- 3) Tenth term of an arithmetic sequence is 36.
 - a) What is the sum of first term and 19 th term?
 - b) Calculate the sum of first 19 terms of this sequence
 - c) If the common difference is 3 what is its 3 rd term
 - d) Write the algebraic form of this sequence
- 4) First term of an arithmetic sequence is 7. The alternate terms of this sequence are $17, 27, 37 \cdots$
 - a) What is the common difference?
 - b) Write the algebraic form of this sequence
 - c) Which is the term just below 100?
 - d) How many terms are there below $100\,$
 - e) Calculate the sum of all terms below 100 in this sequence.
- 5) Write the sequence and make calculations as given below
 - a) First term 2,common difference 4. Write the arithmetic sequence and calculate the sum of 2 terms
 - b) First term 3, common difference 3 . Write the arithmetic sequence and calculate the sum of three terms.
 - c) First term 4, common difference 8 .Calculate the sum of first 4 terms.
 - d) First term 5,common difference 10. Write the arithmetic sequence and find the sum of first 5 terms
 - e) If the first term of an arithmetic sequence is n and common difference 2n then prove that the sum of first n terms is n^3