RAIA LK: A Two–Knob Topological Spine for 4D Physics

Executive Summary & Master Equation Set

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Executive Summary

Framework (Two Knobs). A 6D topological spine descends to 4D and leaves two observable knobs:

- a **flavor holonomy angle** θ (flavor textures);
- a **parity knob** measured as an *achromatic, near-isotropic* CMB polarization rotation φ, mapped to an axion-like displacement $\Delta\theta_a$.

Using fixed sign conventions and anomaly orientation, we define *positive* conversion factors $C_\gamma > 0$ and $c_g > 0$ so that

$$\varphi = C_\gamma \, \Delta\theta_a, \qquad \Xi = \frac{k \, c_g}{2} \, \Delta\theta_a,$$

which in turn control CMB EB/TB and the parity of the stochastic GW background.

Cross-locked predictions once $(\theta,\ \varphi)$ are set:

- *CMB*: achromatic, near-isotropic EB/TB rotation with consistent sign across EB and TB estimators.
- *GW (mHz)*: fixed-sign chirality $\Delta\chi(f) = \tanh \Xi(f)$ with $\text{sign}[\Delta\chi] = \text{sign}[\varphi]$ and saturation $|\Delta\chi| \to 1$ for $f \gtrsim f_\star$.
- *Unification*: $M_G \simeq 2 \times 10^{16}\,\text{GeV}$, $\alpha_G^{-1} \sim 37$; proton lifetime $\tau_p \sim (1.6\text{–}3.5) \times 10^{37}\,\text{y}$ (dim-6 exchange).
- *Neutrinos*: $\sum m_\nu \simeq 0.059$ eV and $m_{\beta\beta} \sim 1.4\text{–}3.7$ meV.
- *KK mode*: $m_{\text{KK}}^{(1)} \sim$ few-TeV (benchmark ~ 4 TeV), couplings model-dependent.

Falsifiers (kill criteria). (i) significant *frequency dependence* of φ; (ii) *large-angle anisotropy* of φ; (iii) GW chirality sign not matching $\text{sign}[\varphi]$.

Systematics & robustness. We include explicit nuisances for instrument angle, EB/TB leakage, Faraday $\propto \lambda^2$, and dipole/quadrupole anisotropy; we cross-check EB vs. TB; we separate

propagation vs. possible *source* chirality in the SGWB. We publish a single sign test statistic and a LISA/TAIJI sign-SNR criterion.

What to do with this: fit φ from EB/TB using the nuisance-aware likelihood, infer $\Delta\theta_a$, compute the knee $f_\star(\varphi)$, and test the GW *sign-lock* with the LISA/TAIJI parity-odd channel.

§1 Conventions

$\epsilon^{0123} = +1$; $\tilde{F}^{\mu\nu} = \frac{1}{2}\,\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$; $\tilde{R}^{\mu\nu}{}_{\rho\sigma} = \frac{1}{2}\,\epsilon_{\rho\sigma\alpha\beta}R^{\mu\nu\alpha\beta}$. All angles *inside equations* are in radians (degrees may be used in figures). Polarization follows the IAU/HEALPix basis: $(Q + \mathrm{i}U) \to e^{+2\mathrm{i}\psi}(Q + \mathrm{i}U)$.

§2 Parity–Axion Sector and Boundary Term

$$S_{\mathrm{axP}} = \int \mathrm{d}^4x\,\sqrt{-g}\;\frac{\theta_a}{16\pi^2 f_a(L)}\Big(\kappa_\gamma\,F_{\mu\nu}\tilde{F}^{\mu\nu} + \kappa_g\,R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu}{}_{\rho\sigma}\Big) + S^{\mathrm{bdy}}_{\mathrm{gCS}}, \tag{1}$$

$$S^{\mathrm{bdy}}_{\mathrm{gCS}} = \frac{\ell}{192\pi^2}\int_{\partial\mathcal{M}}\theta_a\,\mathrm{Tr}\Big(\omega\wedge\mathrm{d}\omega + \tfrac{2}{3}\,\omega\wedge\omega\wedge\omega\Big). \tag{2}$$

Axion–geometry tie from the 6D spine (integer orientation $k_{\mathrm{CS}} > 0$):

$$f_a(L) = \frac{k_{\mathrm{CS}}}{2\pi}\,\frac{f_0}{L}.$$

Positive observable maps (sign safe):

$$\varphi = C_\gamma\,\Delta\theta_a,\quad C_\gamma > 0,\qquad \xi(\eta) = c_g\,\theta_a'(\eta),\quad c_g > 0.$$

Anomaly-ratio origin (fixes $\mathrm{sign}[c_g/C_\gamma] > 0$):

$$C_\gamma = \zeta_\gamma\frac{\kappa_\gamma}{16\pi^2 f_a},\qquad c_g = \zeta_g\frac{\kappa_g}{16\pi^2 f_a},\qquad \frac{c_g}{C_\gamma} = \frac{\zeta_g}{\zeta_\gamma}\frac{\kappa_g}{\kappa_\gamma} > 0.$$

§3 CMB Rotation: Achromatic, Near-Isotropic, Systematics-Aware

Achromatic prior with explicit nuisances:

$$\varphi(\nu,\hat{\boldsymbol{n}}) = \varphi_0 + \beta_{\mathrm{inst}} + \epsilon_\nu\ln(\nu/\nu_0) + \underbrace{R_{\mathrm{F}}\,\lambda^2}_{\text{Faraday}},\quad \lambda = c/\nu + \sum_{\ell=1}^{2}\sum_m A_{\ell m}\,Y_{\ell m}(\hat{\boldsymbol{n}}). \tag{3}$$

Small-angle EB/TB relations with leakage:

$$\widehat{C}^{EB}_\ell = 2\varphi_0\,(C^{EE}_\ell - C^{BB}_\ell) + \lambda_{EB}\,C^{EE}_\ell,\quad \widehat{C}^{TB}_\ell = 2\varphi_0\,C^{TE}_\ell + \lambda_{TB}\,C^{TT}_\ell. \tag{4}$$

EB vs. TB closure (internal null test; w_ℓ are analysis weights):

$$\Delta\varphi_{\mathrm{EB}} = \frac{\sum_\ell w^{\mathrm{EB}}_\ell\,C^{EB}_\ell}{2\sum_\ell w^{\mathrm{EB}}_\ell(C^{EE}_\ell - C^{BB}_\ell)},\quad \Delta\varphi_{\mathrm{TB}} = \frac{\sum_\ell w^{\mathrm{TB}}_\ell\,C^{TB}_\ell}{2\sum_\ell w^{\mathrm{TB}}_\ell C^{TE}_\ell}.$$

Likelihood and analytic posterior for $\Delta\theta_a$:

$$\boldsymbol{\varphi} = \mathbf{A}\,\Delta\theta_a + \boldsymbol{n},\quad \mathbf{A} = (C_\gamma,\ldots,C_\gamma)^T,\qquad \hat{\Delta\theta_a} = \frac{\mathbf{A}^T\mathbf{C}^{-1}\boldsymbol{\varphi}}{\mathbf{A}^T\mathbf{C}^{-1}\mathbf{A}},\quad \sigma^{-2}_{\Delta\theta_a} = \mathbf{A}^T\mathbf{C}^{-1}\mathbf{A},$$

with $\{\beta_{\mathrm{inst}},\epsilon_\nu,R_{\mathrm{F}},A_{\ell m},\lambda_{EB},\lambda_{TB}\}$ profiled/marginalized.

§4 GW Propagation Chirality, Knee, and Source Contamination

Helicity propagation (FRW, conformal time η):

$$h_A'' + 2\mathcal{H}h_A' + (k^2 - A\,k\,\xi)h_A = 0, \quad A = \{+1(R), -1(L)\}, \quad k = 2\pi f\, a_0, \quad \xi = c_g\,\theta_a'(\eta).$$

Optical-depth integral and chirality:

$$\Xi(f) = \frac{k}{2}\int_{\eta_*}^{\eta_0} \mathrm{d}\eta\,\xi(\eta) = \frac{k\,c_g}{2}\,\Delta\theta_a, \qquad \Delta\chi(f) = \tanh\Xi(f) = \tanh\Big[\frac{k\,c_g}{2}\,\Delta\theta_a\Big].$$

Saturation knee (links EB→GW):

$$|\Xi(f_\star)| = 1 \Rightarrow k_\star = \frac{2}{c_g\,\Delta\theta_a} \Rightarrow f_\star = \frac{k_\star}{2\pi a_0} = \frac{1}{\pi a_0\,c_g\,\Delta\theta_a} = \frac{C_\gamma}{\pi a_0\,c_g}\,\frac{1}{\varphi_0}.$$

Two-component template (allow helical sources):

$$\Delta\chi(f) = \underbrace{\tanh\Big[\tfrac{k\,c_g}{2}\,\Delta\theta_a\Big]}_{\text{propagation}} + \Delta\chi_{\rm src}(f),$$

fit $\Delta\chi_{\rm src}(f)$ (e.g. broken power law) and require propagation dominance near $f_\star$.

Sign-lock and detection. With $c_g/C_\gamma > 0$,

$$\mathrm{sign}[\Delta\chi(f)] = \mathrm{sign}[\varphi_0].$$

Single-number sign test:

$$\mathcal{T}_{\rm sign} \equiv \frac{\widehat{\Delta\varphi}}{\sigma_\varphi} \times \frac{\widehat{\Delta\chi}}{\sigma_{\Delta\chi}} \quad (\text{expect } \mathcal{T}_{\rm sign} > 0).$$

LISA/TAIJI sign-SNR and threshold:

$$\mathrm{SNR}^2_{\rm sign} = T\int \mathrm{d}f\,\frac{\big[\gamma_V(f)\,\Delta\chi(f)\,\Omega_{\rm GW}(f)\big]^2}{\mathcal{N}(f)}, \qquad \text{claim sign if } \mathrm{SNR}_{\rm sign} \geq 3,$$

$$\Omega^{\rm min}_{\rm GW}(f) = \frac{1}{|\Delta\chi(f)|}\sqrt{\frac{\mathcal{N}(f)}{T}}\,\frac{1}{|\gamma_V(f)|}.$$

§5 Unification Map and Stiffness

Let $t = \ln(M_G/M_Z)$. Using pairwise differences (two-loop form factors suppressed here for brevity):

$$\Delta^{\rm meas}_{ij} \equiv \alpha_i^{-1}(M_Z) - \alpha_j^{-1}(M_Z) - (\Delta_i - \Delta_j), \quad \Delta^{\rm th}_{ij}(t) \equiv \frac{b_i - b_j}{2\pi}\,t + \big[B_i^{(2)}(t) - B_j^{(2)}(t)\big].$$

Solve $\Delta^{\rm meas}_{12} = \Delta^{\rm th}_{12}$ and $\Delta^{\rm meas}_{23} = \Delta^{\rm th}_{23}$ for (t, α_G^{-1}). Publish sensitivities (Jacobian) and proton lifetime scaling:

$$\mathbf{J} \equiv \begin{pmatrix} \partial\ln M_G/\partial\Delta_{12} & \partial\ln M_G/\partial\Delta_{23} \\ \partial\ln\alpha_G^{-1}/\partial\Delta_{12} & \partial\ln\alpha_G^{-1}/\partial\Delta_{23} \end{pmatrix}, \qquad \tau_p \propto M_G^4\,\alpha_G^{-2}.$$

§6 First KK Mode (Portable)

$$m_{\rm KK}^{(1)} \simeq x_1\, k\, e^{-kL}, \qquad x_1 = \mathcal{O}(1\text{–}3) \text{ (state boundary conditions).}$$

Typical coupling scaling for collider estimates (RS-like localization):

$$g_{{\rm KK}-f} \sim g_4\, e^{-ky_f} \quad \Rightarrow \quad \sigma(pp \to {\rm KK}) \propto g_{{\rm KK}-q}^2.$$

§7 PQ Quality (Gravity-Safe)

If the leading PQ-violating operator has dimension n (e.g. $n = 12$),

$$\bar\theta \sim \frac{f_a^n}{M_{\rm Pl}^{n-4}\,\Lambda_{\rm QCD}^4} \quad \text{(evaluate at } f_a(L)\text{; discrete } \textit{gauge} \text{ origin assumed),}$$

and report the resulting $\bar\theta \ll 10^{-10}$.

§8 Flavor Addendum ($|n| = 2$ harmonic, observable-facing)

$$s_{13} \simeq (1-\varepsilon)\,\frac{\lambda}{\sqrt{2}}, \qquad \varepsilon \simeq 0.06, \quad \delta s_{13} = -\varepsilon\,\frac{\lambda}{\sqrt{2}}.$$

Induced residuals:

$$\delta J_{\rm CP} = K\,(1-3s_{13}^2)\,\delta s_{13}, \quad K = c_{12}s_{12}\,c_{23}s_{23}\sin\delta, \qquad \delta\theta_{12} \simeq \kappa_{12}\varepsilon, \quad \delta\theta_{23} \simeq \kappa_{23}\varepsilon.$$

Clean Falsifiers (Kill-Criteria)

1. **Chromaticity kill**: $|\epsilon_\nu| > \epsilon_\nu^{\max}$ or significant $R_{\rm F} \neq 0$ (Faraday).
2. **Anisotropy kill**: $\sum_{\ell=1}^{2}\sum_m |A_{\ell m}|^2 > A_{\max}$.
3. **Sign-flip kill**: one-sided $p(\mathcal{T}_{\rm sign} \le 0) < p_{\rm crit}$ (e.g., $p_{\rm crit} = 0.01$).

Tiny Calculator I/O (for Reproducibility)

Inputs: $\{\varphi_0, \epsilon_\nu, R_{\rm F}, A_{\ell m}, \beta_{\rm inst}, \lambda_{EB}, \lambda_{TB}, C_\gamma, c_g\}$ and optional flavor knob θ.
Outputs: $\{\Delta\theta_a, \Delta\chi(f), f_\star, M_G, \alpha_G^{-1}, \tau_p, m_{\beta\beta}, m_{\rm KK}^{(1)}\}$ plus $\mathcal{T}_{\rm sign}$ and $\Omega_{\rm GW}^{\min}(f)$.

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