# Assignment 1

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# 1 Question 1

## 1.1 (1)

Timo is a cow.

## 1.2 (2)

Fido is not a dog.

## 1.3 (3)

Owing a cow is not enough to recognize a Person as a LivestockOwner.

## 1.4 (4)

0.(Zero)

# 2 Question 2

## 2.1 (1)

I will translate it into SHOIQ form:(this form is from wiki)

(1) ML is an AI course taught by ZZH, a professor working at NJU
inclusions:$\{ML\} \sqsubseteq AIcourse \sqcap (\exists teach^{-}.\{ZZH\})$,$\{ZZH\} \sqsubseteq Professor \sqcap (\exists workAt.\{NJU\})$

(2) NJU is a university whose members are a school or a department
inclusions:$\{NJU\} \sqsubseteq University \sqcap (\forall hasMember.(School \sqcup Department))$

(3) NJU has at least 30,000 students
inclusions:$\{NJU\} \sqsubseteq (\geq 30000 has.Student)$

(4) All members of AI School are undergraduates, graduates, or teachers

inclusions:$(\exists memberOf.\{AISchool\}) \sqsubseteq (Undergraduate \sqcup Graduate \sqcup Teacher)$

(5) The domain of the relation “citizenOf” consists of countries

inclusions:$(\exists citizenOf.\{\top\}) \sqsubseteq (Country)$

### 2.2 (2)

**Sentence 1:All members of AI School are undergraduates,graduates,or teachers**

$\forall x(memberOf(x, AISchool) \to Undergraduate(x) \vee Graduate(x) \vee Teacher(X))$

**Sentence 2:The domain of the relation ”citizenOf” consists of countries**

$\forall x(\exists y citizenOf(x, y) \to Country(x))$

## 3 Question 3

All the answer is follow the question:

- There is an ontology that has only finite models.

  Disprove.
  Here is a way to create infinite models.Firstly we assume we have an model:$I = \{\Delta^I, .^I\}$ And then we assume that:$\{a\} \subseteq \Delta^I$, then we create a new element called:$\{a'\}$,so we could have a new model: $I' = \{\Delta^{I'}, .^{I'}\}$,and then we get:$\Delta^{I'} = (\Delta^I/a) \cup \{a'\}$
  So if we create the new model in this form, we could create infinite models.

- Every ontology has either no model or infinite many models.

  Prove:
  If the ontology has no model, it must in this way:$\top \sqsubseteq \bot$,and from question 2.1 we could get that the number of models is infinite. So it has proved.

- A satisfiable class must always have a non-empty interpretation.

  Prove:
  From definition 2.14, the satisfiability said that:C is satisfiable with respect to $\tau$ iff $C^I \neq \emptyset$ for some model $I$ of $\tau$, the satisfiable model must have an interpretation, so proved.

- An unsatisfiable class may have a non-empty interpretation in some models.

  Disprove:
  If the unsatisfiable class have a non-empty interpretation in some models, from definition 2.14, this is also satisfy the definition of satisfiable class, so it’s contradictory.

- An unsatisfiable class will be a subclass of any other class.

  Prove:
  From question 2.4 we get that the unsatisfiable class is an emptyset.So an emptyset is always the subclass of any other class.

# 4 Question 4

All the answer is follow the questions:

1. $\exists r.(A \sqcup B)$:$\{d, f\}$
2. $\exists s.\exists s.\neg A$:$\{d, e\}$
3. $\neg A \sqcap \neg B$:$\{f, h, i\}$
4. $\forall r.(A \sqcup B)$:$\{d, f, g, h, i\}$
5. $\leq 1s.\top$:$\{e, f, g, h, i\}$

# 5 Question 5

## 5.1 (1)

All the answer is follow the questions:

- $(Q \sqcap \geq 2r.P)^{\mathcal{I}}$:$\emptyset$
- $(\forall r.Q)^{\mathcal{I}}$:$\{b, c, d, e\}$
- $(\neg\exists r.Q)^{\mathcal{I}}$:$\{b, c, e\}$
- $(\forall r.\top \sqcap \exists r^{-}.P)^{\mathcal{I}}$:$\{b, d, e\}$
- $(\exists r^{-}.\bot)^{\mathcal{I}}$:$\emptyset$

## 5.2 (2)

All the answer is follow the questions:

- $\mathcal{I} \models A \equiv \exists r.B$:True
- $\mathcal{I} \models A \sqcap B \sqsubseteq \top$:True
- $\mathcal{I} \models \exists r.A \sqsubseteq A \sqcap B$:True
- $\mathcal{I} \models \top \sqsubseteq B$:False
- $\mathcal{I} \models B \sqsubseteq \exists r.A$:False

# 6 Question 6

All the answer is follow the questions:

- if $C \sqsubseteq D$ holds, then $\exists r.C \sqsubseteq \exists r.D$ holds.

  For this question the proof as follow:
  $(\exists r.C)^I = \{d \in \Delta^I | \exists e \in \Delta^I : (d,e) \in r^I and\ e \in C^I\}$
  $\sqsubseteq \{d \in \Delta^I | \exists e \in \Delta^I : (d,e) \in r^I and\ e \in C^I\} \sqcup \{d \in \Delta^I | \exists e \in \Delta^I : (d,e) \in r^I and\ e \in D^I / C^I\}$
  $= \{d \in \Delta^I | \exists e \in \Delta^I : (d,e) \in r^I and\ e \in D^I\}$
  $= (\exists r.D)^I$

- $\exists r.C$ is equivalent to $\leq 1r.\top$.

  Disprove as follow:
  If we have model like: $r^I = \{(a,b)\},\ C^I = \{a\},\ \Delta^I = \{a,b\}$
  So we get $(\exists r.C)^I$ is empty, but the $\leq 1r.\top$ is $\{a,b\}$
  So $\exists r.C$ is not equivalent to $\leq 1r.\top$

- $\leq 0r.\top$ is equivalent to $\forall r.\bot$.

  For this question the proof as follow:
  $(\leq 0r.\top)^I = \{d \in \Delta^I | \{e \in \Delta^I : (d,e) \in r^I and\ e \in T\} \leq 0\}$
  $= \{d \in \Delta^I | \{e \in \Delta^I : (d,e) \in r^I and\ e \in T\} = \emptyset\}$
  $= \{d \in \Delta^I | \{e \in \Delta^I : (d,e) \in r^I\} = \emptyset\}$
  $= \{d \in \Delta^I | there\ is\ no\ relation\ (d,e) \in r^I\}$

  $(\forall r.\bot)^I = \{d \in \Delta^I | for\ all\ e \in \Delta : (d,e) \in r^I \to e \in \bot\}$
  $= \{d \in \Delta^I | for\ all\ e \in \Delta : (d,e) \in r^I \to e \in \emptyset\}$
  $= \{d \in \Delta^I | there\ is\ no\ relation\ (d,e) \in r^I\}$

- $\forall r.(A \sqcup B)$ is equivalent to $(\forall r.A) \sqcup (\forall r.B)$.

  Disprove as follow:
  If we have model:$\Delta^I = \{a,b,c\}, A^I = \{a\}, B^I = \{b\}, r^I = \{(c,a),(c,b)\}$
  As the interpretation of $(\forall r.(A \sqcup B))^I$ goes: $\{d \in \Delta^I | for\ all\ e \in \Delta : (d,e) \in r^I \to e \in (A \cup B)\} = \{a,b,c\}$
  But if we get the interpretation of $(\forall r.A)^I \sqcup (\forall r.B)^I$ we get the answer goes:$\{a,b\}$
  So $\forall r.(A \sqcup B)$ is not equivalent to $(\forall r.A) \sqcup (\forall r.B)$

- $\exists r.(A \sqcup B)$ is equivalent to $(\exists r.A) \sqcup (\exists r.B)$.

  For this question the proof as follow:
  Firstly:we prove $\exists r.(A \sqcup B) \sqsubseteq (\exists r.A) \sqcup (\exists r.B)$
  As the interpretation goes: $\exists r.(A \sqcup B) = \{d \in \Delta^I | there\ is\ e \in \Delta^I : (d,e) \in r^I\ and\ e \in (A \cup B)\}$
  $\subseteq \{d \in \Delta^I | there\ is\ e \in \Delta^I :\ (d,e) \in r^I\ and\ e \in A\} \cup \{d \in \Delta^I | there\ is\ e \in \Delta^I :\ (d,e) \in r^I\ and\ e \in B\}$

$= \{d \in \Delta^I | there\ is\ e \in \Delta^I : (d,e) \in r^I\ and\ e \in Aor \in B\}$

Secondly:we prove:$(\exists r.A) \sqcup (\exists r.B) \sqsubseteq \exists r.(A \sqcup B)$
As the interpretation goes:$\{d \in \Delta^I | there\ is\ e \in \Delta^I : (d,e) \in r^I\ and\ e \in A\} \cup \{d \in \Delta^I | there\ is\ e \in \Delta^I : (d,e) \in r^I\ and\ e \in B\}$
$\subseteq \{d \in \Delta^I | there\ is\ e \in \Delta^I : (d,e) \in r^I\ and\ e \in Aor \in B\}$

So this question has proved.

# 7 Question 7

Here is the proof:

We assume a model as follow:$\Delta^I = \{a,b,c\}$,$Person^I = \{a,b,c\}$,$Parent^I = \{a,b\}$,$Mother^I = \{a\}$,and relationship:$hasChild^I = \{(a,c),(b,c)\}$.

Firstly we could get that:$(\exists \mathsf{hasChild.Person})^I$ equals to $\{a,b\}$, and $Person^I$ also equals to $\{a,b\}$, so we get:$\mathsf{Parent} \sqsubseteq \exists \mathsf{hasChild.Person}$

Secondly we get that:$Mother^I = \{a\}$,so $\mathsf{Mother} \sqsubseteq \mathsf{Parent}$.

At this time, we have proved $\mathcal{I} \models \mathcal{T}$.

Then,$Parent^I = \{a,b\}$ which is definitely not belongs to $Mother^I = \{a\}$.

So we have proved $\mathcal{I} \not\models \mathsf{Parent} \sqsubseteq \mathsf{Mother}$.

# 8 Question 8

Let $\mathcal{T}$ be an $\mathcal{ALC}$ TBox, which is a finite set of concept inclusions. Let $X$ and $Y$ be complex $\mathcal{ALC}$ concepts (note that a complex concept can also be an atomic concept). Show that:

- $X \sqsubseteq_{\mathcal{T}} Y$ if and only if $X \sqcap \neg Y$ is not satisfiable with respect to $\mathcal{T}$.

  Prove:
  $\Rightarrow$: If $X \sqsubseteq_{\mathcal{T}} Y$,so $X^I \subseteq Y^I$, so $X^I \cap \neg Y^I$ is *emptyset*, so there is no model, so is not satisfiable with respect to $\mathcal{T}$
  $\Leftarrow$: If $X \sqcap \neg Y$,so $X^I \cap \neg Y^I$ is *emptyset* is satisfiable,so $X^I \subseteq Y^I$,so $X \sqsubseteq_{\mathcal{T}} Y$.

- $X$ is satisfiable with respect to $\mathcal{T}$ if and only if $X \not\sqsubseteq \bot$.

  Prove:
  $\Rightarrow$: If $X$ is satisfiable with respect to $\mathcal{T}$,from definition 2.14 we get $X^I \neq \emptyset$, so $X \not\sqsubseteq \bot$.
  $\Leftarrow$: If $X \not\sqsubseteq \bot$, this means there exists an interpretation or model to satisfy X, so from definition X is satisfiable with respect to $\tau$

## 9 Question 9

## 10 Question 10