Lecture 4

Derivatives and edges

Administrative

A1 is out

- It is graded
- Due **Jan 24**

Administrative

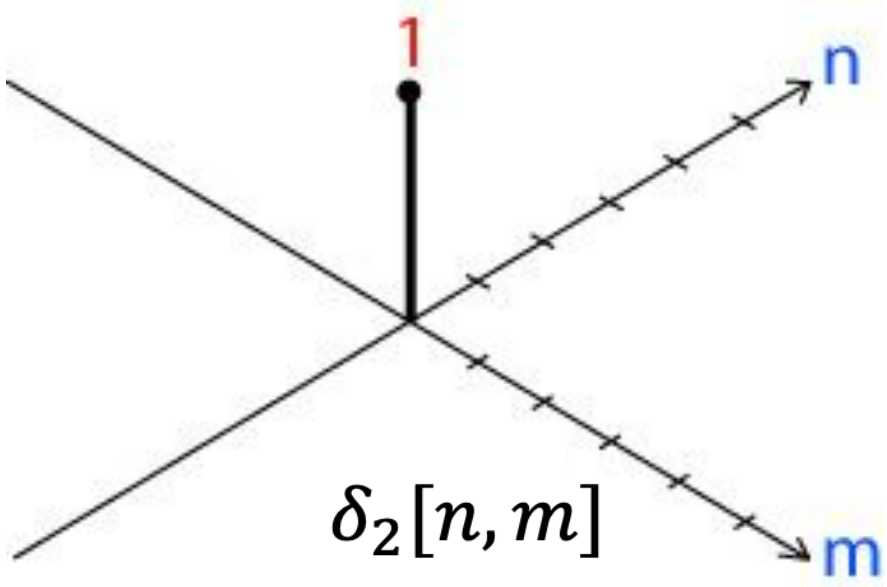
Recitations (2 options)

- Friday mornings 9:30-10:20am @ MGH 231
- Friday afternoons 12:30-1:20pm @ CSE2 G01

This week:
We will go over Python & Numpy basics

So far: 2D impulse function

- A special function
- 1 at the origin [0,0].
- 0 everywhere else



So far: We get the impulse response when we pass an impulse function through a LSI system

- The moving average filter equation again: $g[n,m] = \frac{1}{9}\sum_{k=-1}^{1}\sum_{l=-1}^{1} f[n-k, m-l]$

$$\delta_2[n,m] \xrightarrow{S} h[n,m]$$

Pass in an impulse function — Record its response

$$h[n,m] = \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & \underline{1/9} & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

So far: write down f as a sum of impulses

Let's say our input f is a 3x3 image:

f[0,0]	f[0,1]	f[1,1]
f[1,0]	f[1,1]	f[1,2]
f[2,0]	f[2,1]	f[2,2]

$=$

f[0,0]	0	0
0	0	0
0	0	0

$+$

0	f[0,1]	0
0	0	0
0	0	0

$+ \ldots +$

0	0	0
0	0	0
0	0	f[2,2]

$=$ f[0,0] $\times$

1	0	0
0	0	0
0	0	0

$+$ f[0,1] $\times$

0	1	0
0	0	0
0	0	0

$+ \ldots +$ f[2,2] $\times$

0	0	0
0	0	0
0	0	1

$$= f[0,0] \cdot \delta_2[n,m] + f[0,1] \cdot \delta_2[n,m-1] + \ldots + f[2,2] \cdot \delta_2[n-2,m-2]$$

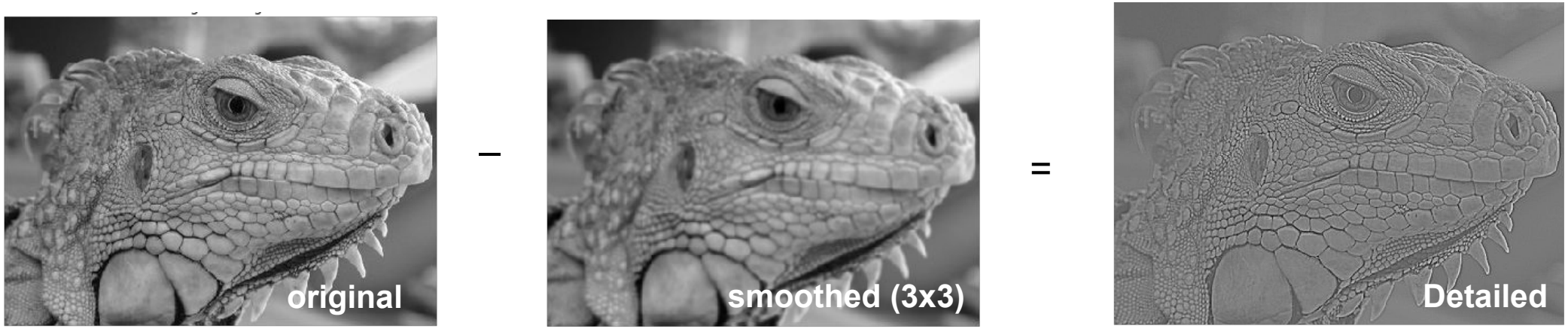
So far: We derived convolutions

- An LSI system is completely specified by its impulse response.
  - For any input f, we can compute the output g in terms of the impulse response h.

Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$

So far: We created a sharpening system by combining filters



Let's add it back to get a sharpening system:

(Cross) correlation – symbol: $**$

Cross correlation of two 2D signals f[n,m] and h[n,m]

$$f[n,m] ** h[n,m] = \sum_k \sum_l f[k,l]h[n+k,m+l]$$

Equivalent to a convolution without the flip

Today’s agenda

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

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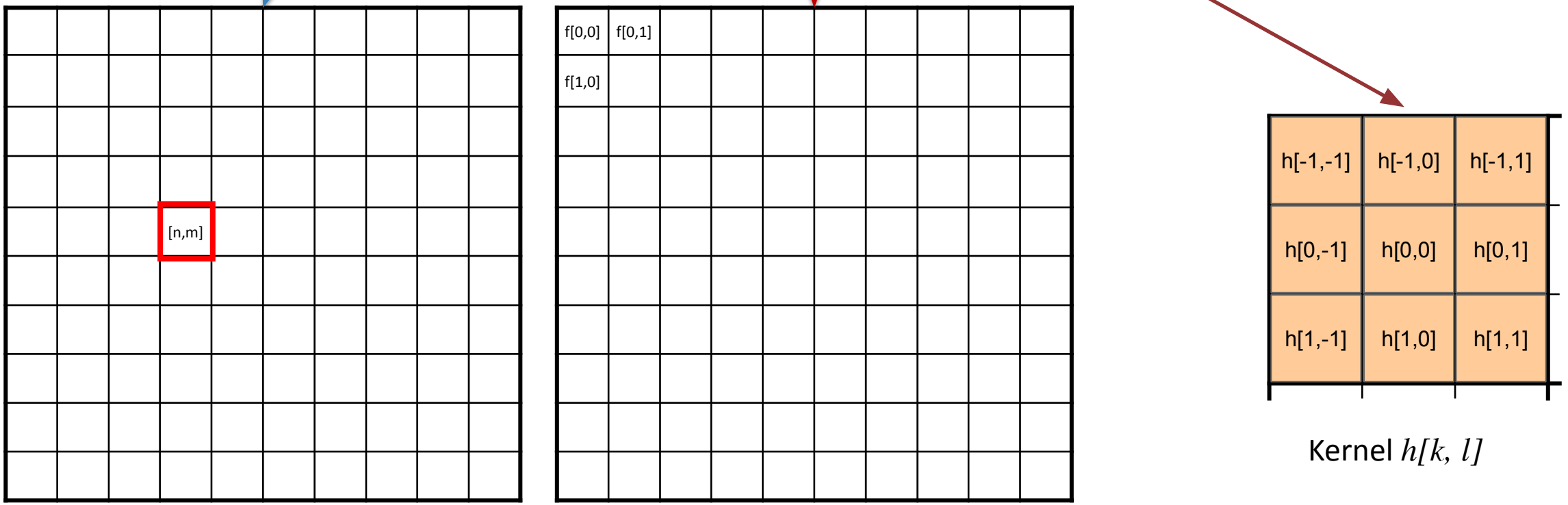
2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$



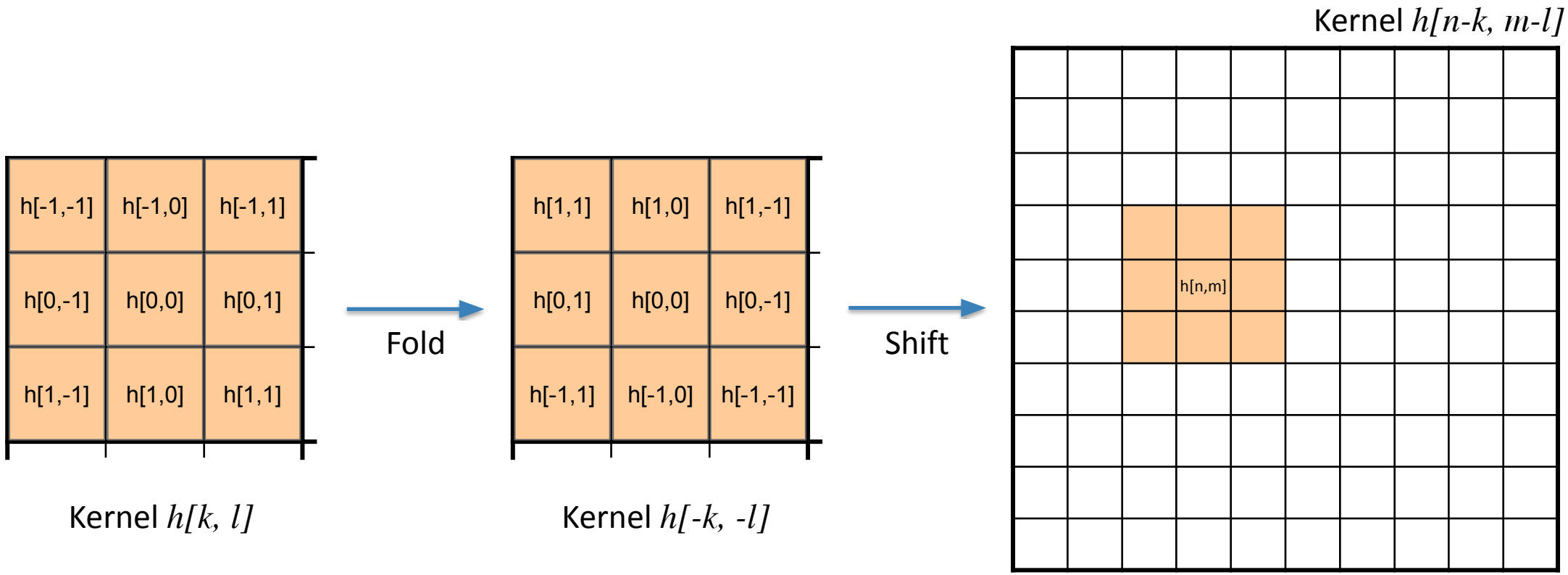
Output $f * h$

Image $f[k, l]$

Kernel $h[k, l]$

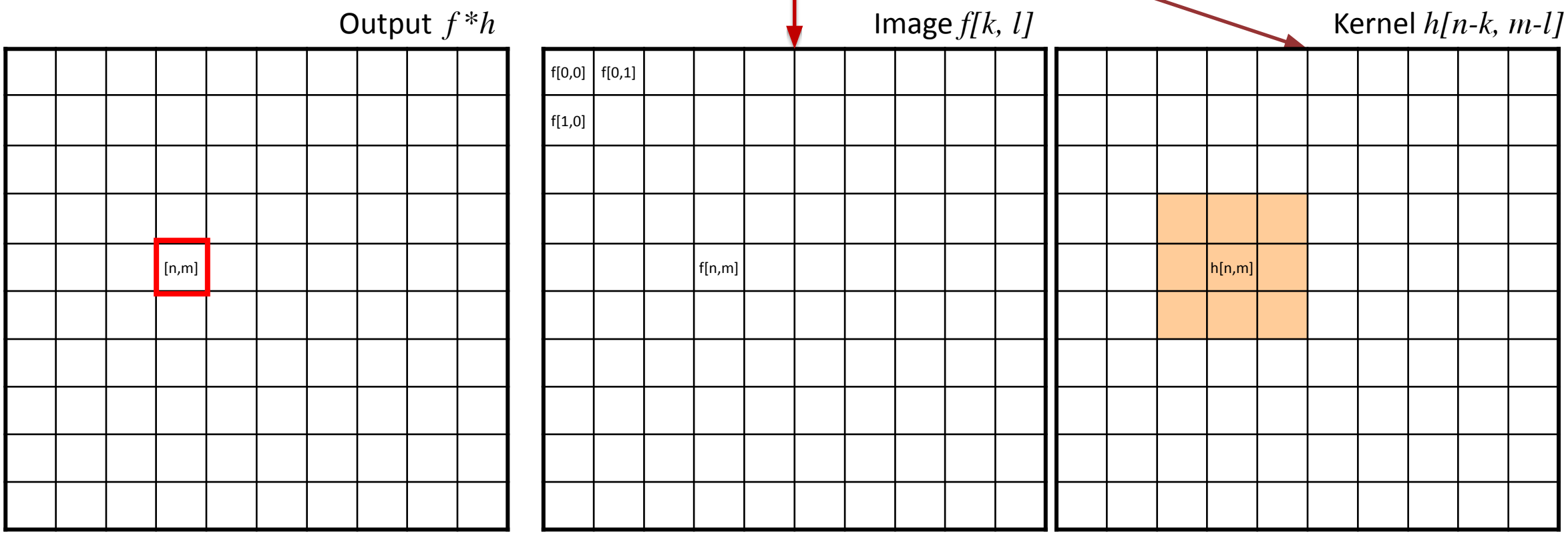
2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$



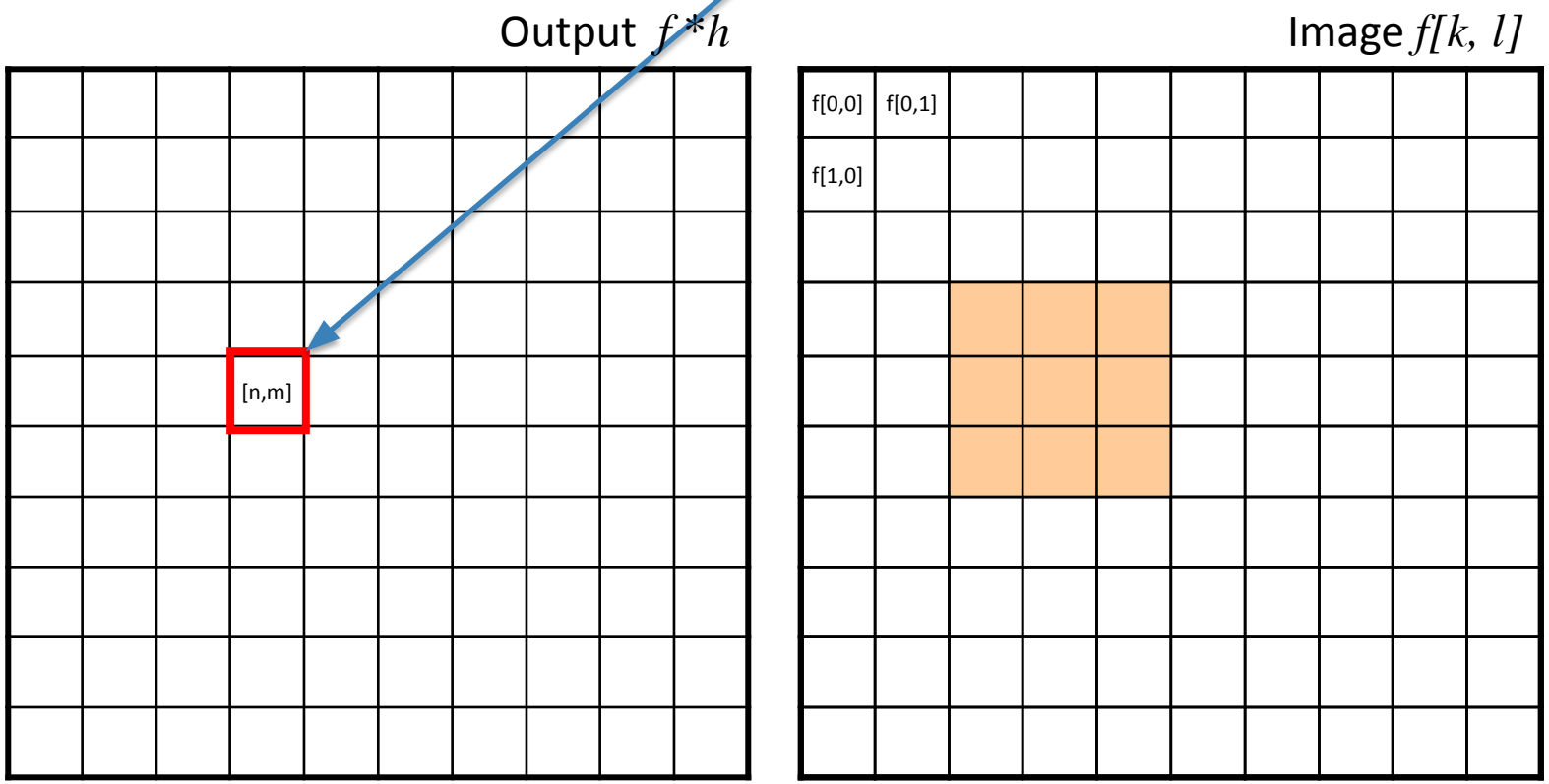
2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$



2D Discrete Convolution

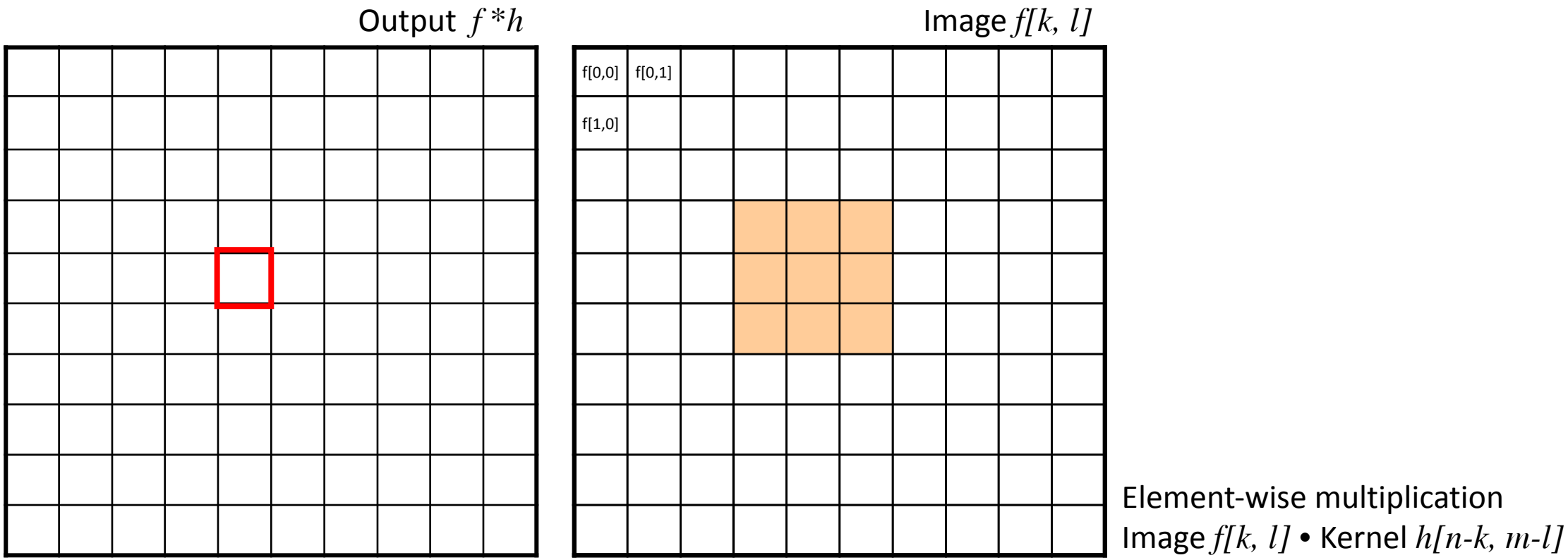
$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$



Element-wise multiplication
Image $f[k, l]$ • Kernel $h[n-k, m-l]$

2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$



2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$

Output $f * h$

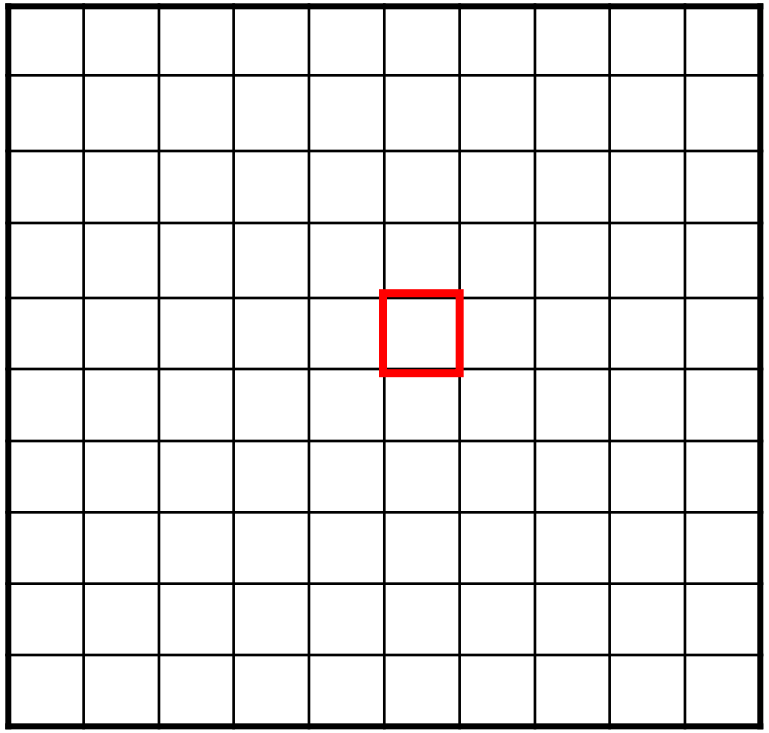
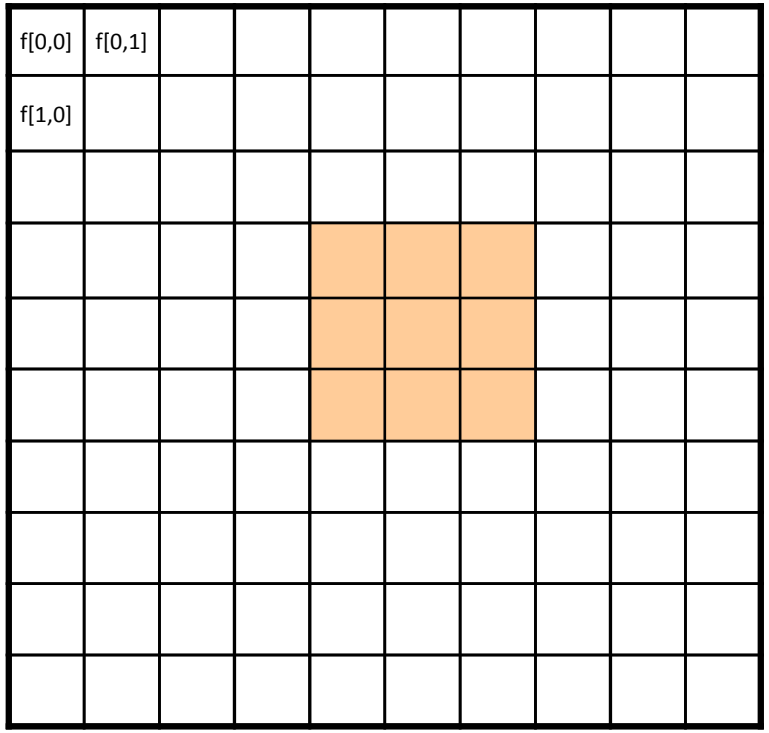


Image $f[k, l]$



Element-wise multiplication

Image $f[k, l]$ • Kernel $h[n\text{-}k, m\text{-}l]$

2D Discrete Convolution

$$f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$$

Output $f * h$

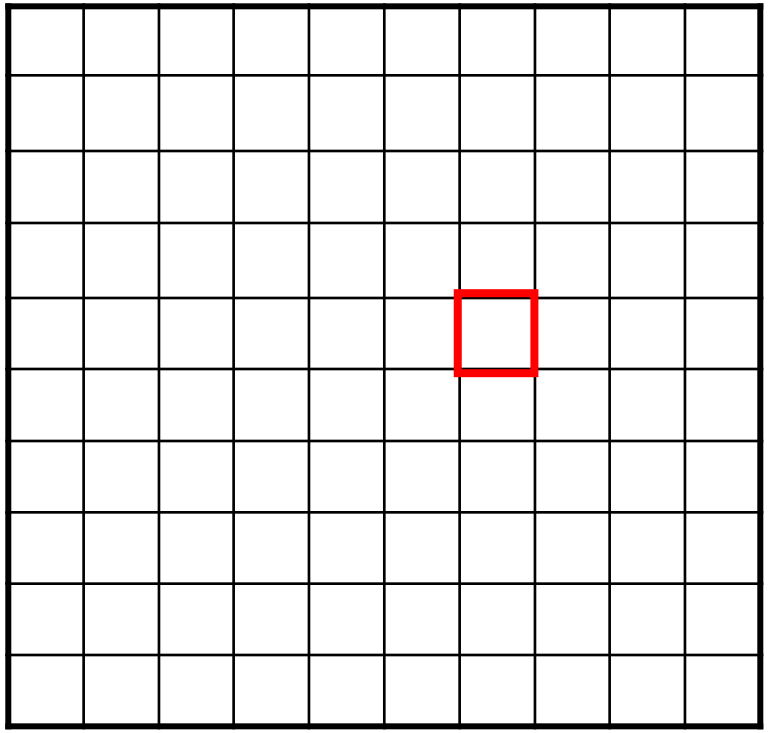
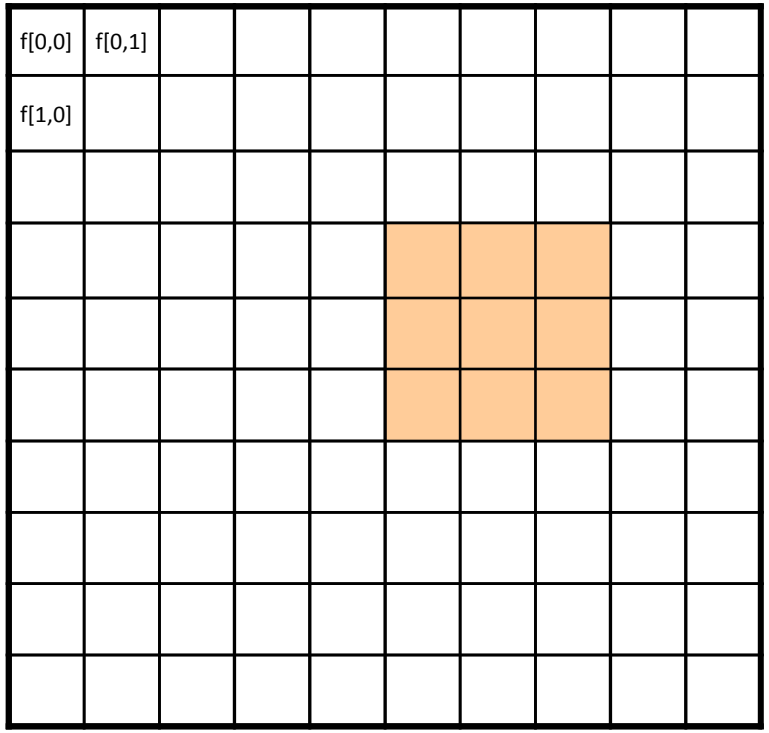


Image $f[k, l]$



Element-wise multiplication

Image $f[k, l]$ • Kernel $h[n\text{-}k, m\text{-}l]$

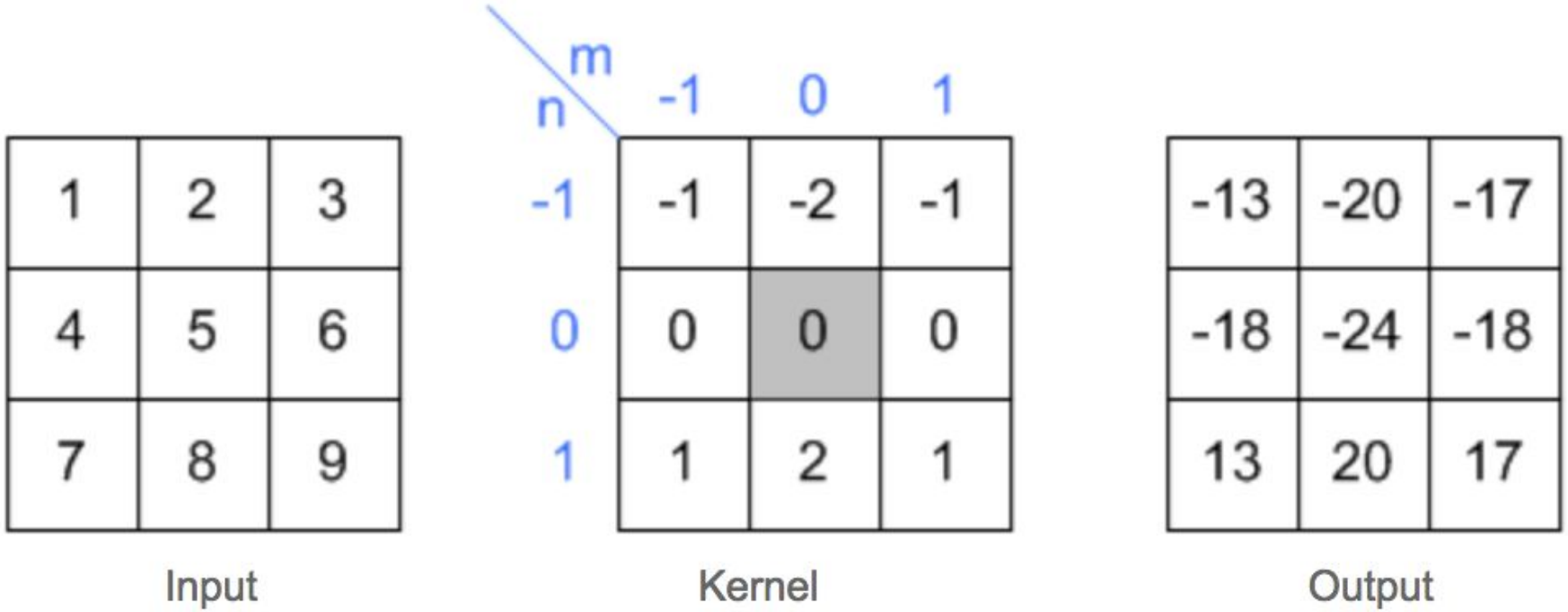
2D Discrete Convolution

- $f[n,m] * h[n,m] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} f[k,l] \cdot h[n-k, m-l]$

Algorithm:

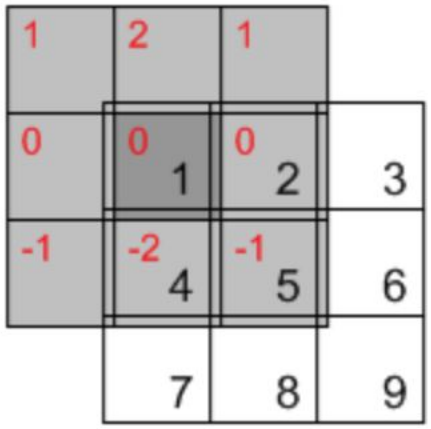
- Fold $h[k, l]$ about origin to form $h[-k, -l]$
- Shift the folded results by n, m to form $h[n - k, m - l]$
- Multiply $h[n - k, m - l]$ by $f[k, l]$
- Sum over all k, l, store result in output position $[n, m]$
- Repeat for every n, m

2D convolution example



Slide credit: Song Ho Ahn

2D convolution example



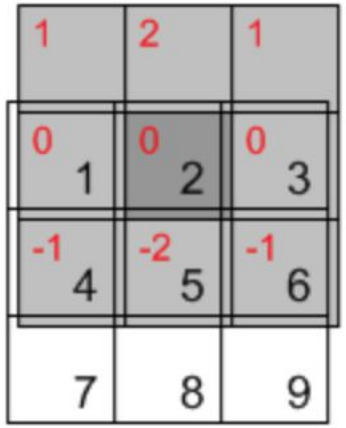
$$
\begin{aligned}
&= x[-1,-1]\cdot h[1,1] + x[0,-1]\cdot h[0,1] + x[1,-1]\cdot h[-1,1] \\
&\quad + x[-1,0]\cdot h[1,0] + x[0,0]\cdot h[0,0] + x[1,0]\cdot h[-1,0] \\
&\quad + x[-1,1]\cdot h[1,-1] + x[0,1]\cdot h[0,-1] + x[1,1]\cdot h[-1,-1] \\
&= 0\cdot 1 + 0\cdot 2 + 0\cdot 1 + 0\cdot 0 + 1\cdot 0 + 2\cdot 0 + 0\cdot(-1) + 4\cdot(-2) + 5\cdot(-1) = -13
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

Slide credit: Song Ho Ahn

2D convolution example



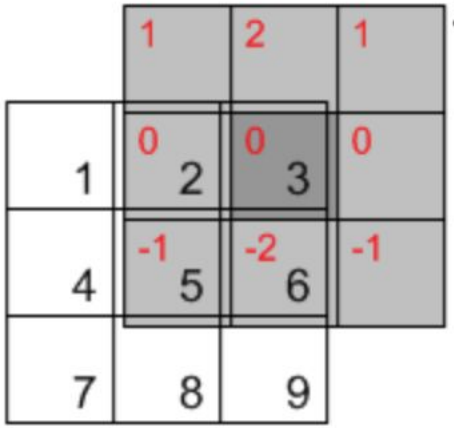
$$
\begin{aligned}
&= x[0,-1]\cdot h[1,1] + x[1,-1]\cdot h[0,1] + x[2,-1]\cdot h[-1,1] \\
&\quad + x[0,0]\cdot h[1,0] + x[1,0]\cdot h[0,0] + x[2,0]\cdot h[-1,0] \\
&\quad + x[0,1]\cdot h[1,-1] + x[1,1]\cdot h[0,-1] + x[2,1]\cdot h[-1,-1] \\
&= 0\cdot 1 + 0\cdot 2 + 0\cdot 1 + 1\cdot 0 + 2\cdot 0 + 3\cdot 0 + 4\cdot(-1) + 5\cdot(-2) + 6\cdot(-1) = -20
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

Slide credit: Song Ho Ahn

2D convolution example



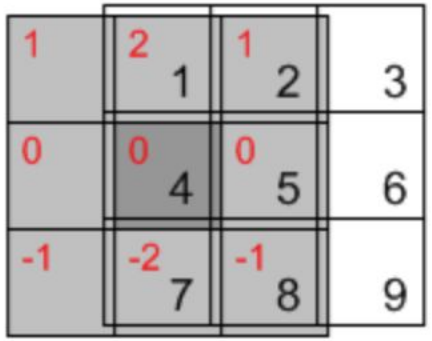
$$
\begin{aligned}
&= x[1,-1]\cdot h[1,1] + x[2,-1]\cdot h[0,1] + x[3,-1]\cdot h[-1,1] \\
&\quad + x[1,0]\cdot h[1,0] + x[2,0]\cdot h[0,0] + x[3,0]\cdot h[-1,0] \\
&\quad + x[1,1]\cdot h[1,-1] + x[2,1]\cdot h[0,-1] + x[3,1]\cdot h[-1,-1] \\
&= 0\cdot 1 + 0\cdot 2 + 0\cdot 1 + 2\cdot 0 + 3\cdot 0 + 0\cdot 0 + 5\cdot(-1) + 6\cdot(-2) + 0\cdot(-1) = -17
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

Slide credit: Song Ho Ahn

2D convolution example



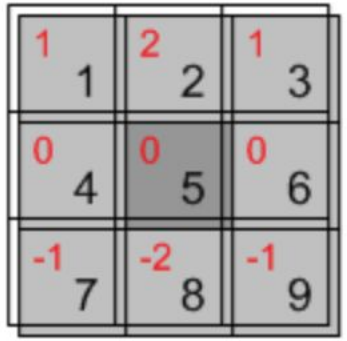
$$
\begin{aligned}
&= x[-1,0]\cdot h[1,1] + x[0,0]\cdot h[0,1] + x[1,0]\cdot h[-1,1] \\
&\quad + x[-1,1]\cdot h[1,0] + x[0,1]\cdot h[0,0] + x[1,1]\cdot h[-1,0] \\
&\quad + x[-1,2]\cdot h[1,-1] + x[0,2]\cdot h[0,-1] + x[1,2]\cdot h[-1,-1] \\
&= 0\cdot 1 + 1\cdot 2 + 2\cdot 1 + 0\cdot 0 + 4\cdot 0 + 5\cdot 0 + 0\cdot(-1) + 7\cdot(-2) + 8\cdot(-1) = -18
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

Slide credit: Song Ho Ahn

2D convolution example



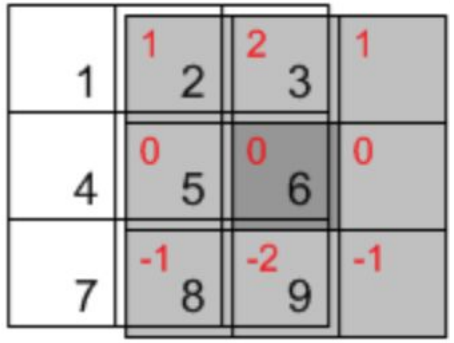
$$
\begin{aligned}
&= x[0,0]\cdot h[1,1] + x[1,0]\cdot h[0,1] + x[2,0]\cdot h[-1,1] \\
&\quad + x[0,1]\cdot h[1,0] + x[1,1]\cdot h[0,0] + x[2,1]\cdot h[-1,0] \\
&\quad + x[0,2]\cdot h[1,-1] + x[1,2]\cdot h[0,-1] + x[2,2]\cdot h[-1,-1] \\
&= 1\cdot 1 + 2\cdot 2 + 3\cdot 1 + 4\cdot 0 + 5\cdot 0 + 6\cdot 0 + 7\cdot(-1) + 8\cdot(-2) + 9\cdot(-1) = -24
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

Slide credit: Song Ho Ahn

2D convolution example



$$
\begin{aligned}
&= x[1,0]\cdot h[1,1] + x[2,0]\cdot h[0,1] + x[3,0]\cdot h[-1,1] \\
&\quad + x[1,1]\cdot h[1,0] + x[2,1]\cdot h[0,0] + x[3,1]\cdot h[-1,0] \\
&\quad + x[1,2]\cdot h[1,-1] + x[2,2]\cdot h[0,-1] + x[3,2]\cdot h[-1,-1] \\
&= 2\cdot 1 + 3\cdot 2 + 0\cdot 1 + 5\cdot 0 + 6\cdot 0 + 0\cdot 0 + 8\cdot(-1) + 9\cdot(-2) + 0\cdot(-1) = -18
\end{aligned}
$$

-13	-20	-17
-18	-24	-18
13	20	17

Output

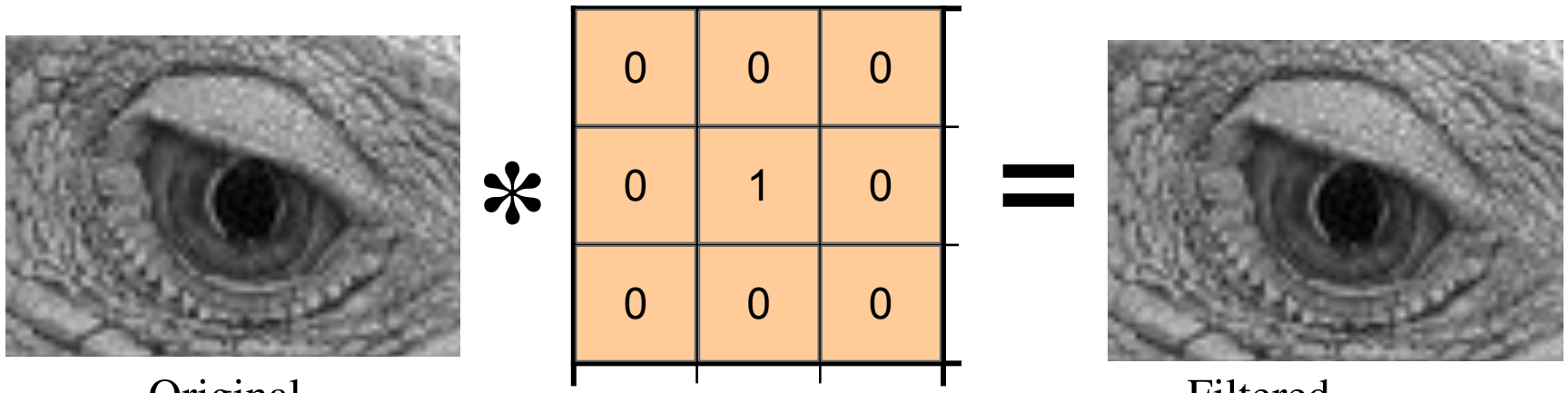
Slide credit: Song Ho Ahn

Practice with convolution

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = ?$$

Practice with convolution

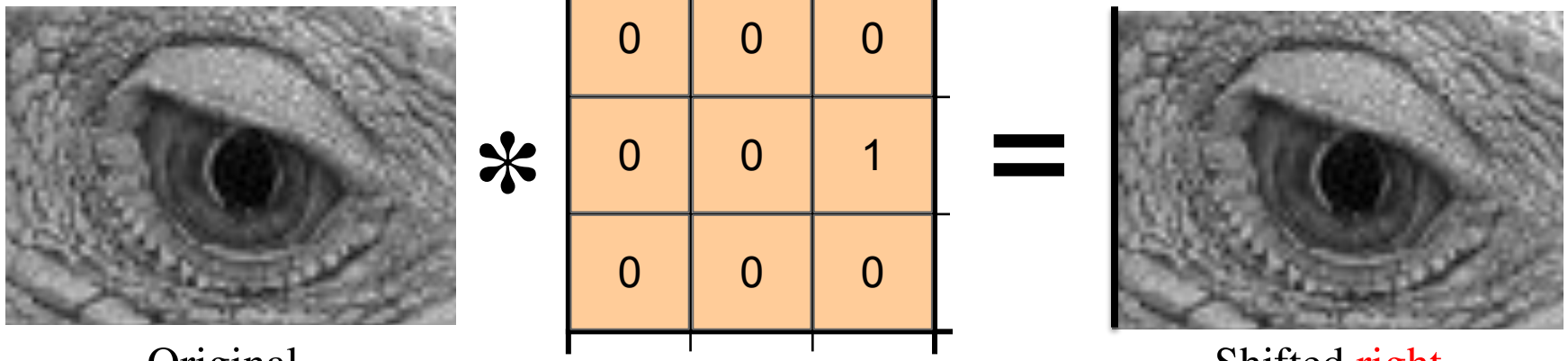


Practice with convolution

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \; ?$$

Practice with convolution



Original

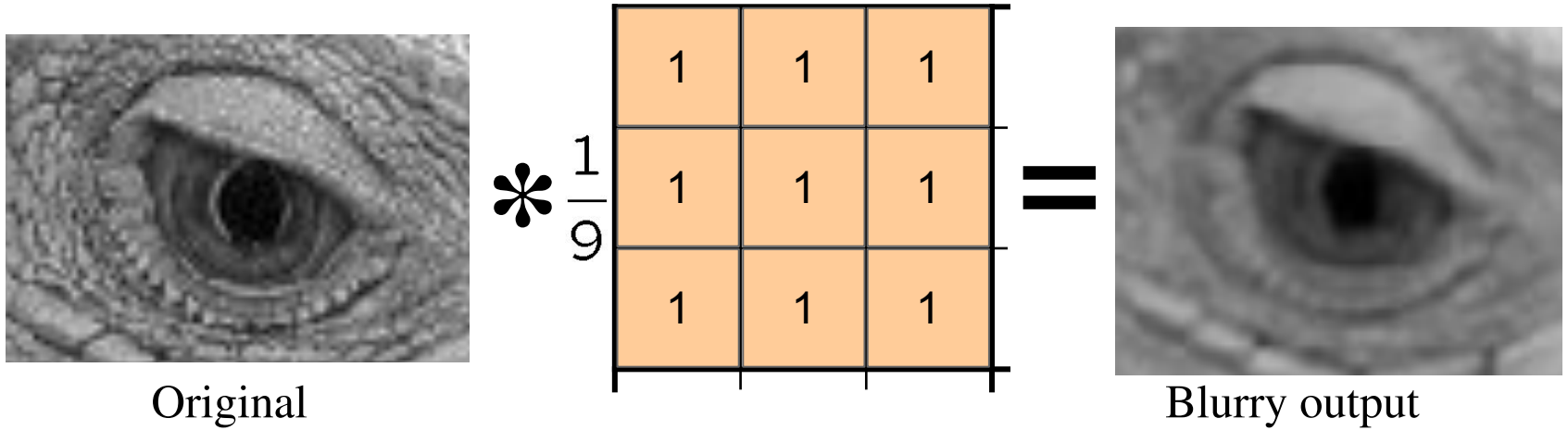
Shifted right
By 1 pixel

Practice with convolution

Original

$$* \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \; ?$$

Practice with convolution



What happens if a system contains multiple filters?

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

(Note that filter sums to 1)

What happens if a system contains multiple filters?

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

What does blurring take away?

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

What does blurring take away?

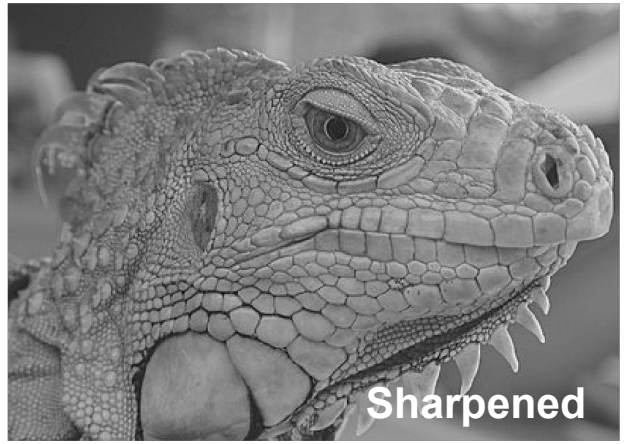
–

=

Let’s add it back to get a sharpening system:

+

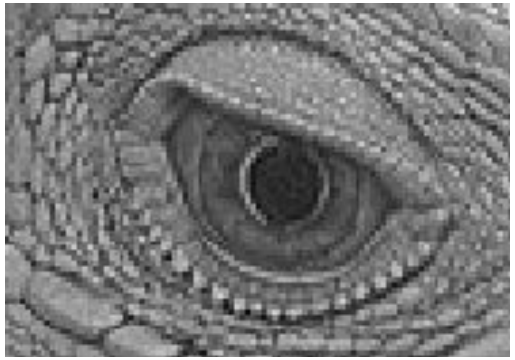
=



Convolution in 2D – Sharpening filter

Original

Sharpening system



Sharpening system: Accentuates differences with local average

Implementation detail: Image support and edge effect

- A computer will only convolve **finite support signals.**
  - That is: images that are zero for n,m outside some rectangular region
- numpy's convolution performs 2D convolution of finite-support signals.

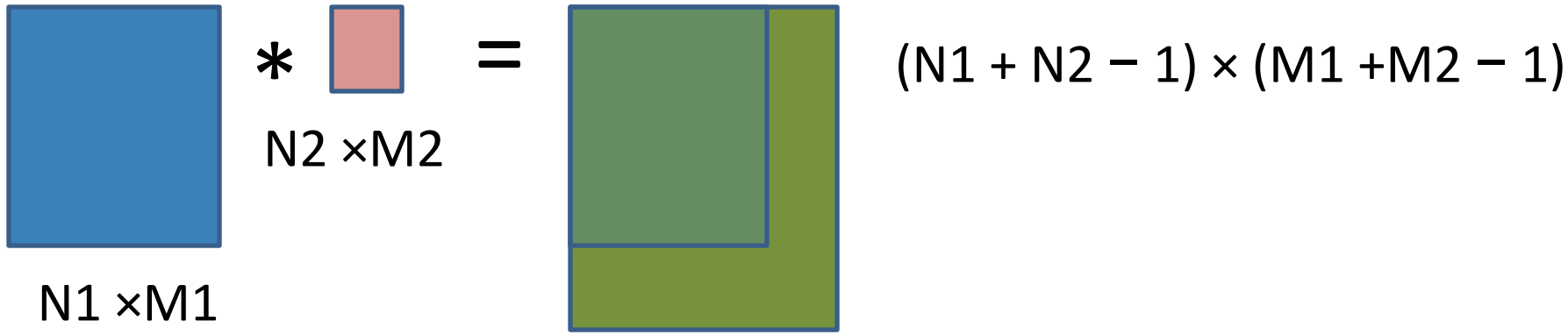
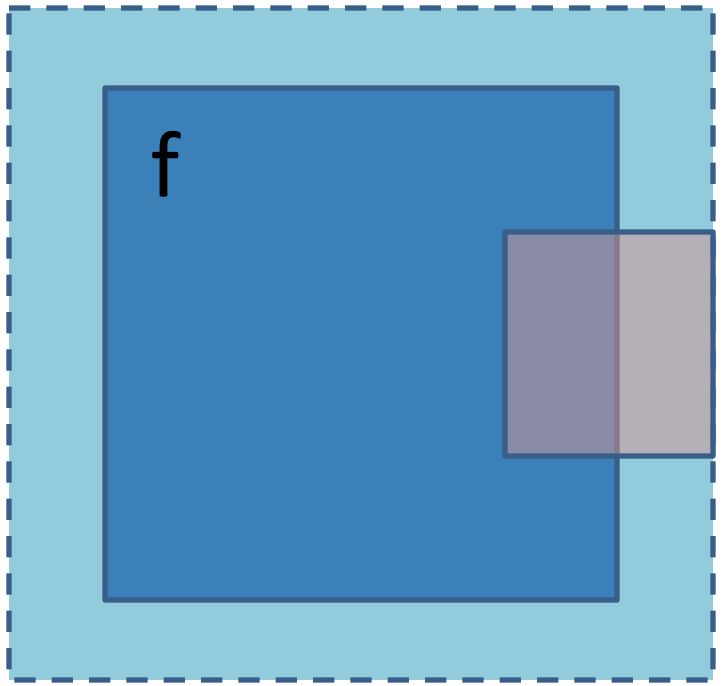


Image support and edge effect

- A computer will only convolve **finite support signals.**
- What happens at the edge?



- zero “padding”
- edge value replication
- mirror extension
- more (beyond the scope of this class)

Today's agenda

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

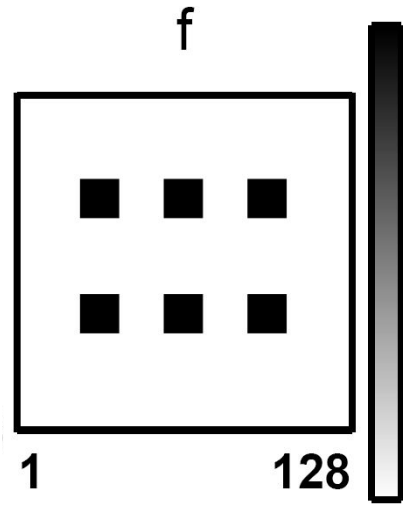
(Cross) correlation – symbol: $**$

Cross correlation of two 2D signals f[n,m] and h[n,m]

$$f[n,m] ** h[n,m] = \sum_k \sum_l f[k,l]h[n+k,m+l]$$

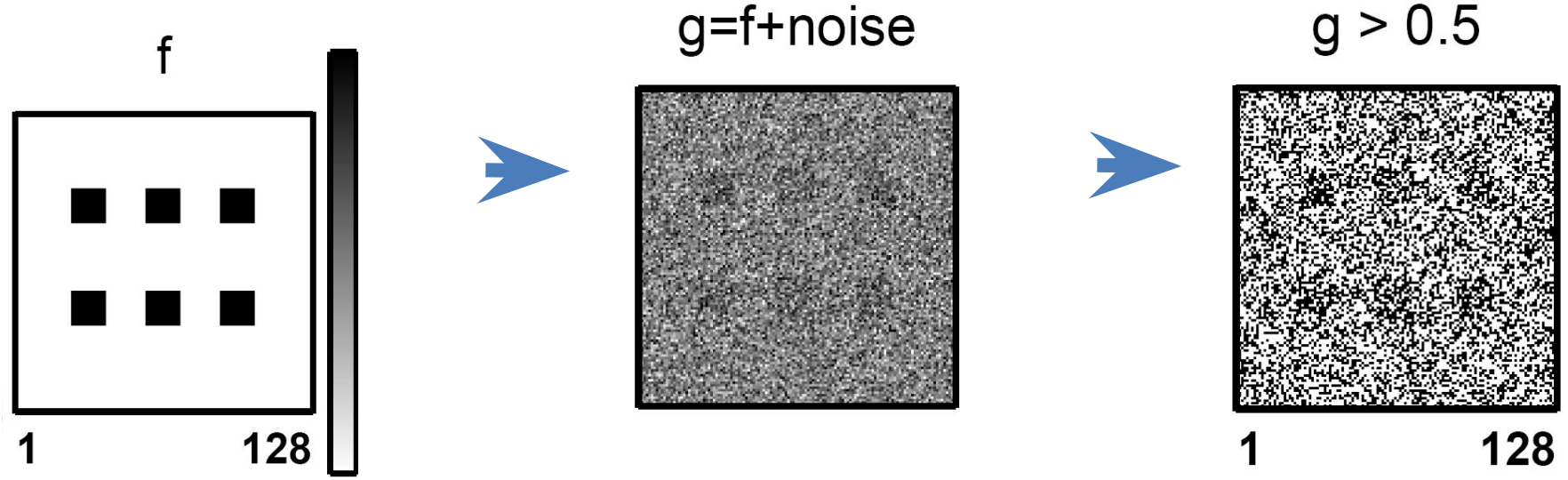
- Equivalent to a convolution without the flip
- Use it to measure 'similarity' between f and h.

(Cross) correlation – example



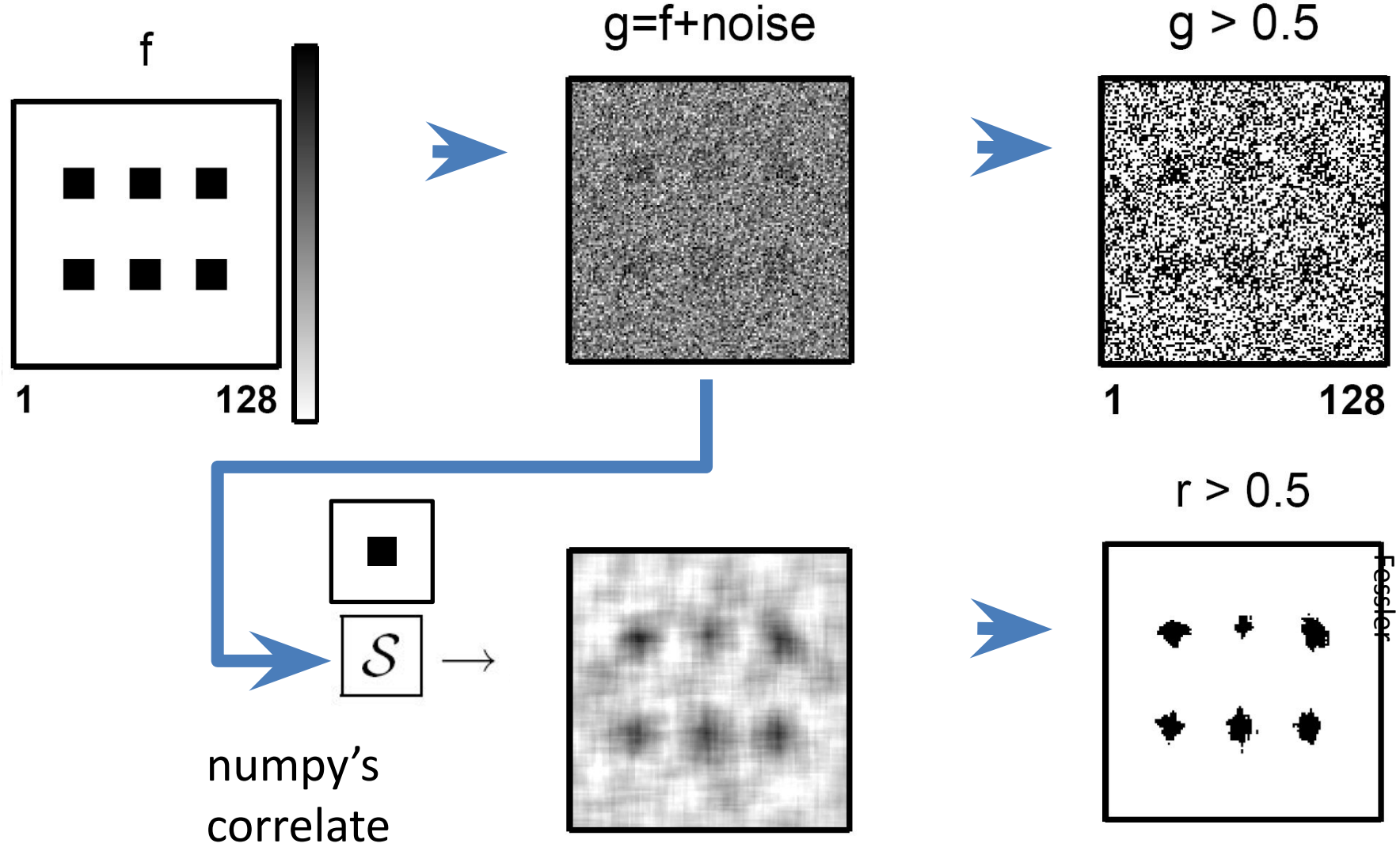
Courtesy of J. Fessler

(Cross) correlation – example



Courtesy of J. Fessler

(Cross) correlation – example



Courtesy of J. Fessler

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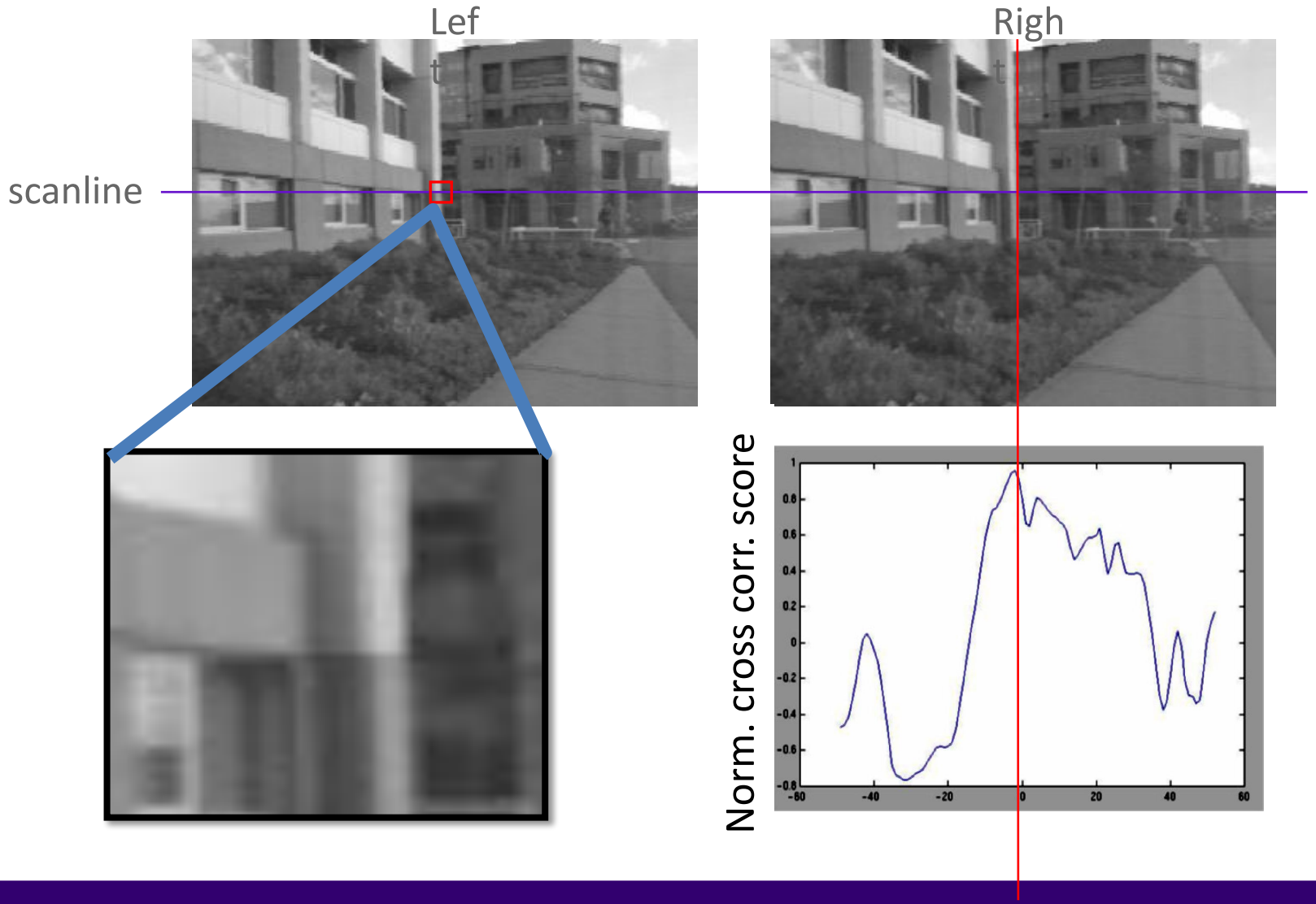
(Cross) correlation – example



Cross Correlation Application: Vision system for TV remote control

- uses template matching

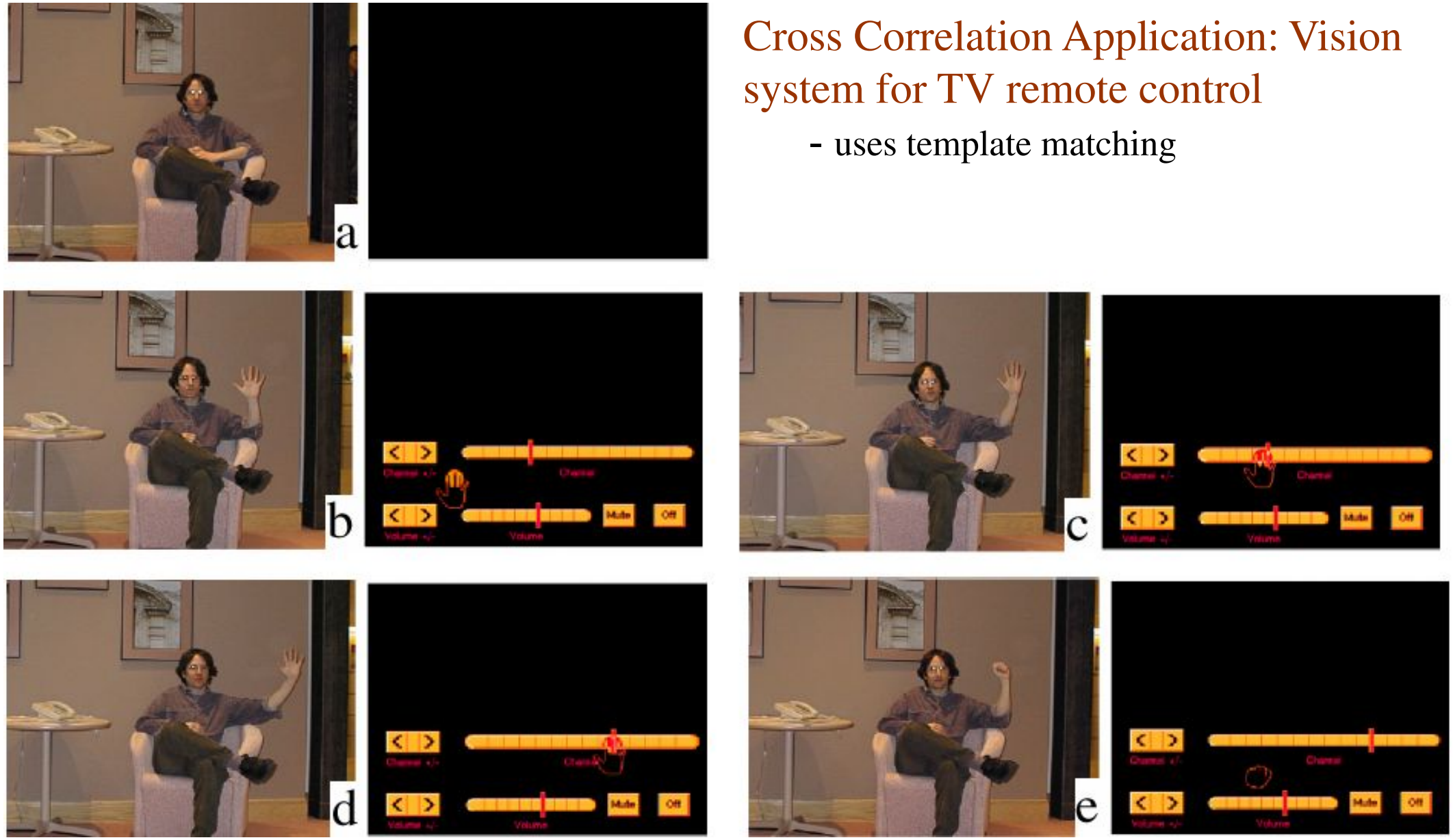


Figure from "Computer Vision for Interactive Computer Graphics," W.Freeman et al, IEEE Computer Graphics and Applications, 1998 copyright 1998, IEEE

Properties of cross correlation

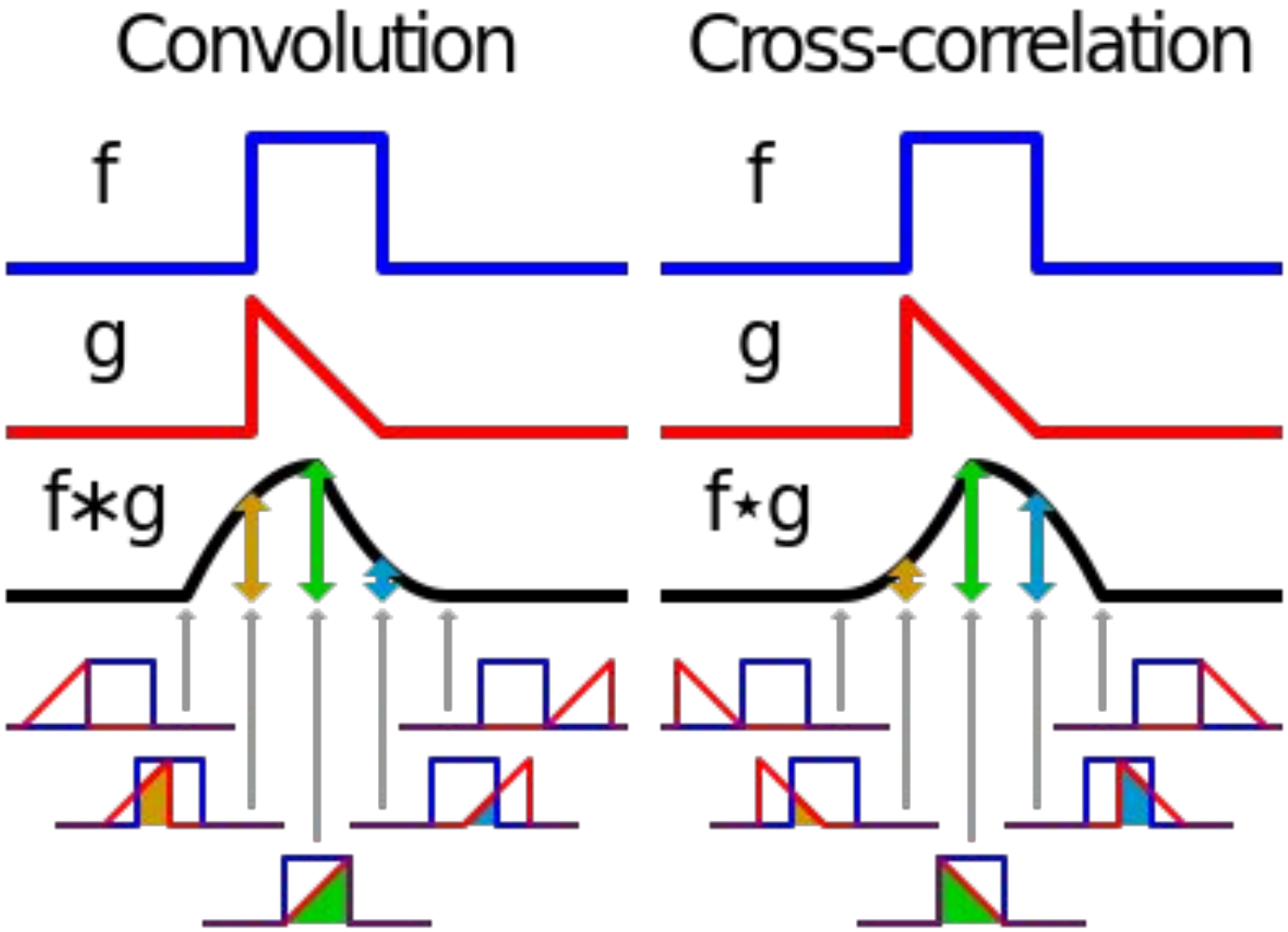
- Associative property:

$$(f ** h_1) ** h_2 = f ** (h_1 ** h_2)$$

- Distributive property:

$$f ** (h_1 + h_2) = (f ** h_1) + (f ** h_2)$$

The order doesn't matter! $h_1 ** h_2 = h_2 ** h_1$


Convolution
Cross-correlation
f
g
f∗g
f
g
f⋆g

Convolution vs. (Cross) Correlation

- When is correlation equivalent to convolution?
- In other words, Q. when is f**g = f*g?

Convolution vs. (Cross) Correlation

- A **convolution** is an integral that expresses the amount of overlap of one function as it is shifted over another function.
  - convolution is a **filtering** operation

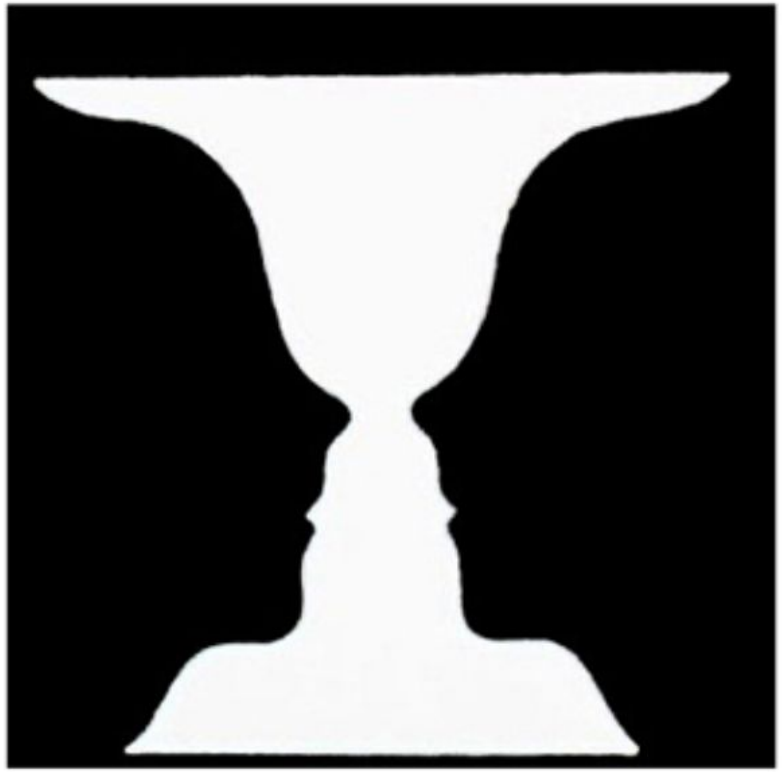
- **Correlation** compares the ***similarity*** *of* ***two*** *sets of* ***data.*** Correlation computes a measure of similarity of two input signals as they are shifted by one another. The correlation result reaches a maximum at the time when the two signals match best .
  - correlation is a measure of relatedness of two signals

What we will learn today

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

Some background reading:
Forsyth and Ponce, Computer Vision, Chapter 8

Q. What do you see?

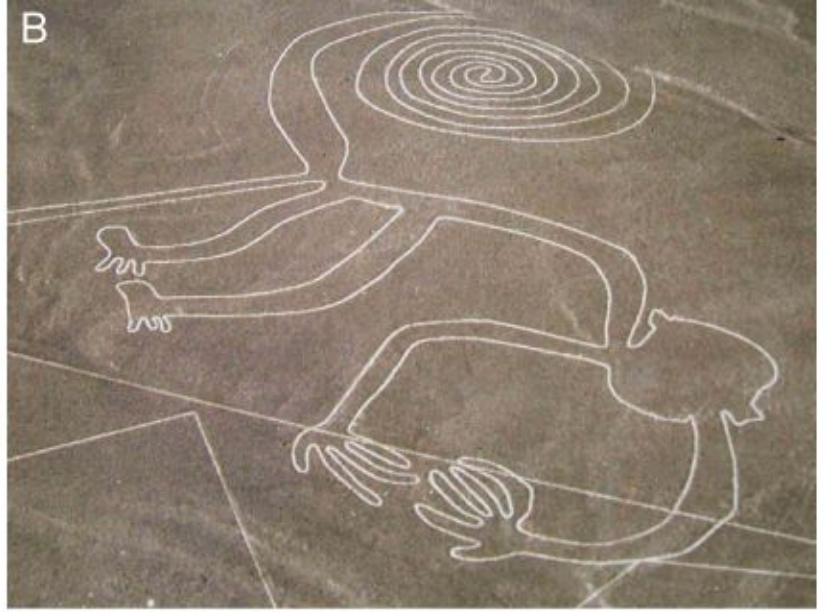


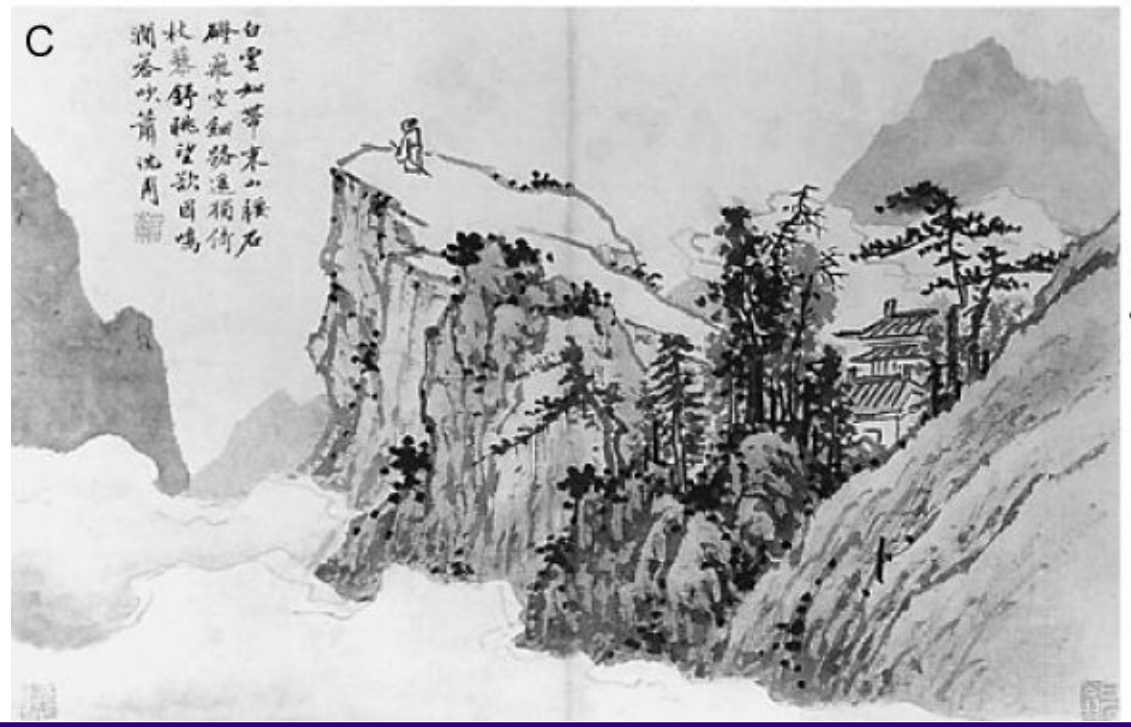
(A) Cave painting at Chauvet, France, about 30,000 B.C.;

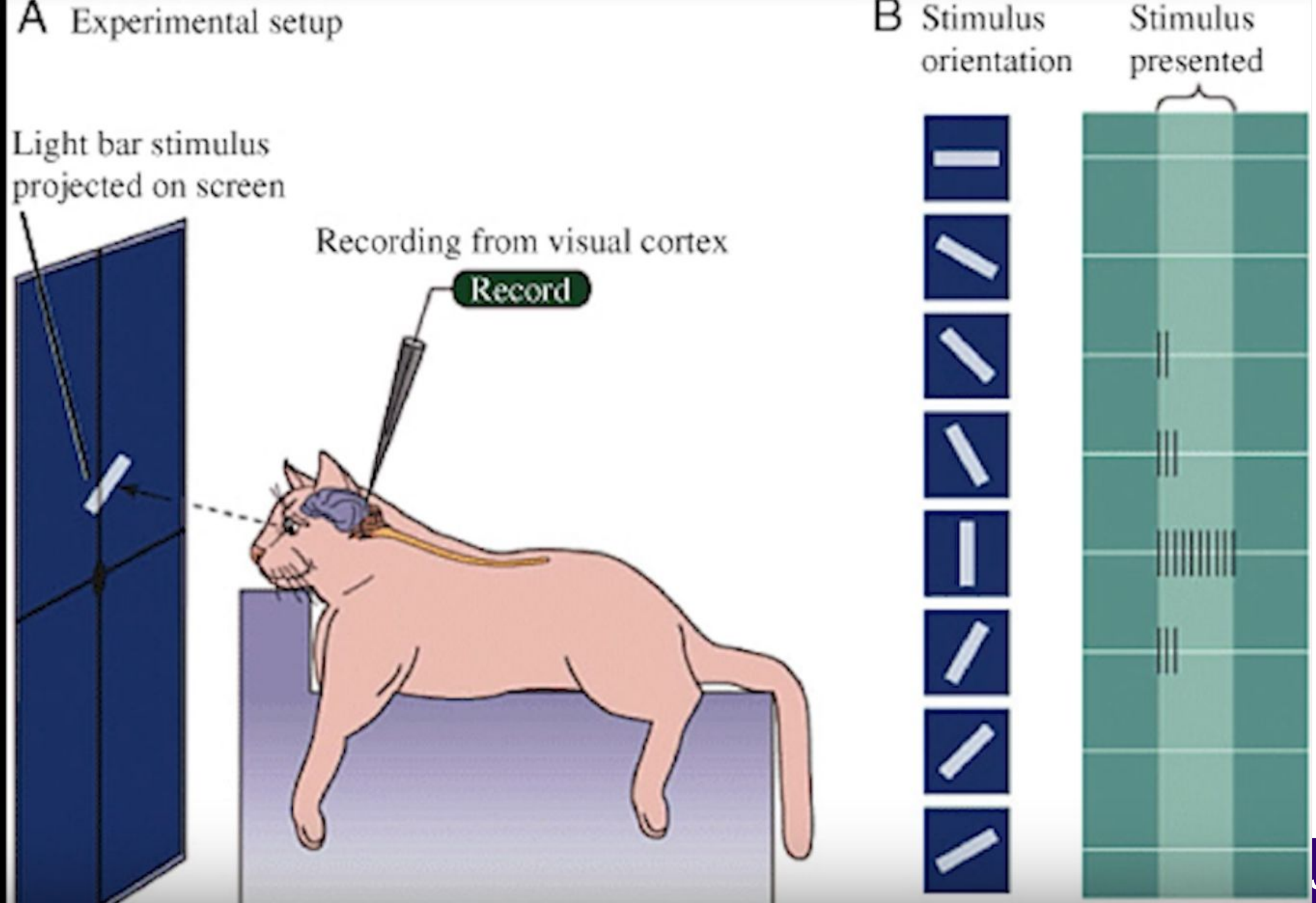
(B) Aerial photograph of the picture of a monkey as part of the Nazca Lines geoglyphs, Peru, about 700 – 200 B.C.;

(C) Shen Zhou (1427-1509 A.D.): Poet on a mountain top, ink on paper, China;

(D) Line drawing by 7-year old I. Lleras (2010 A.D.).

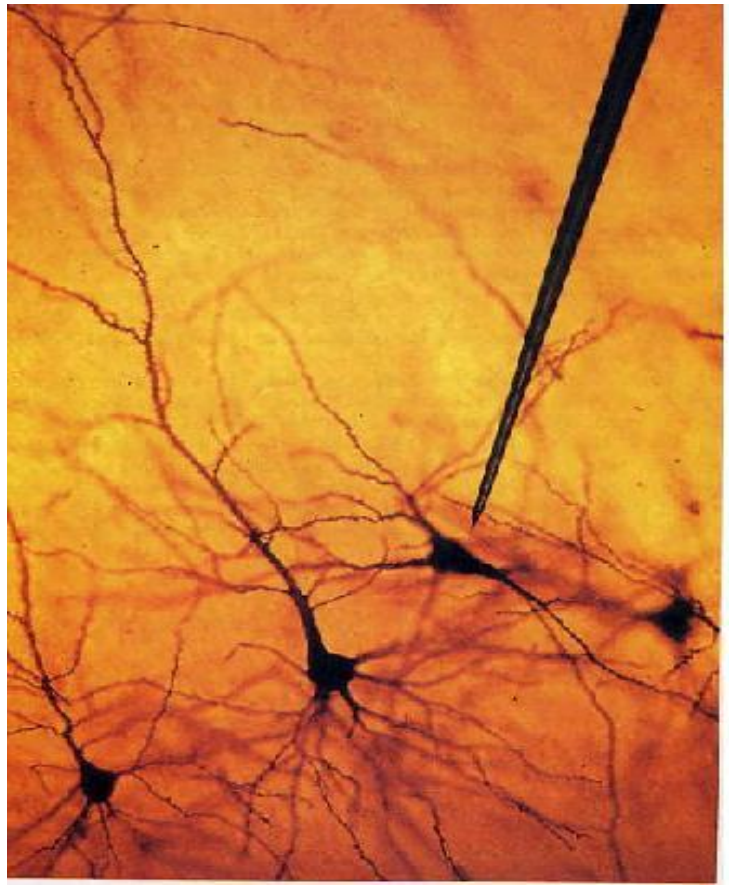




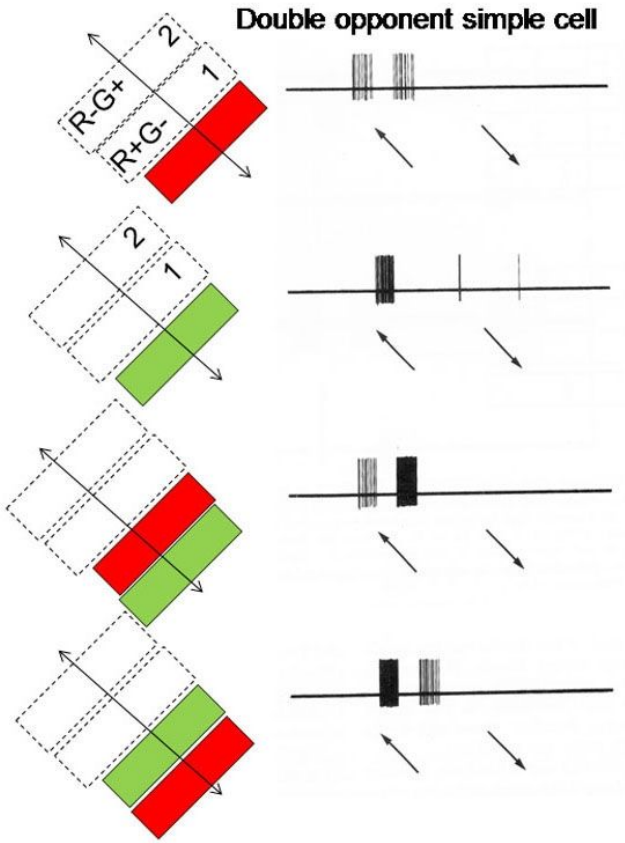


Hubel & Wiesel, 1960s

We know edges are special from human (mammalian) vision studies



Hubel & Wiesel, 1960s



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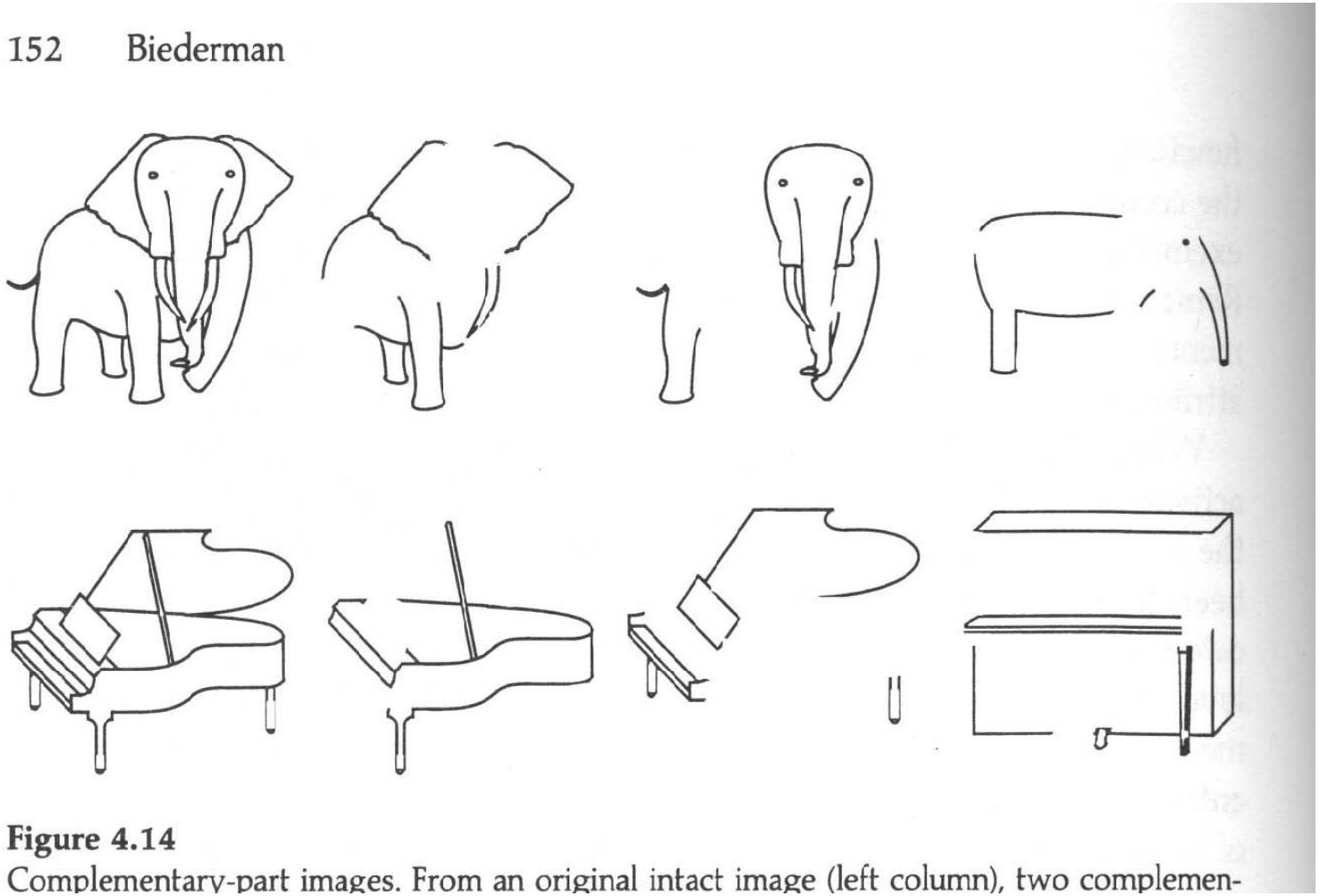
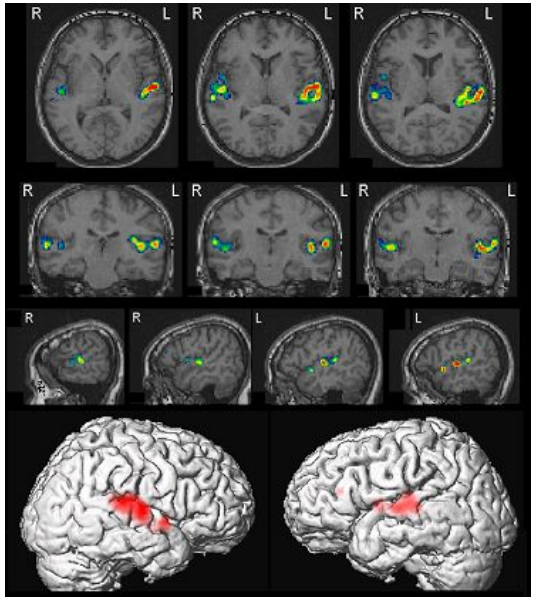
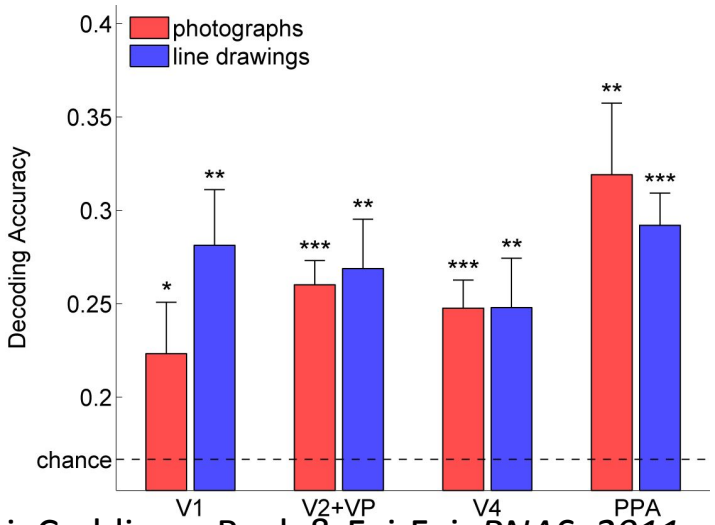


Figure 4.14
Complementary-part images. From an original intact image (left column), two complemen-



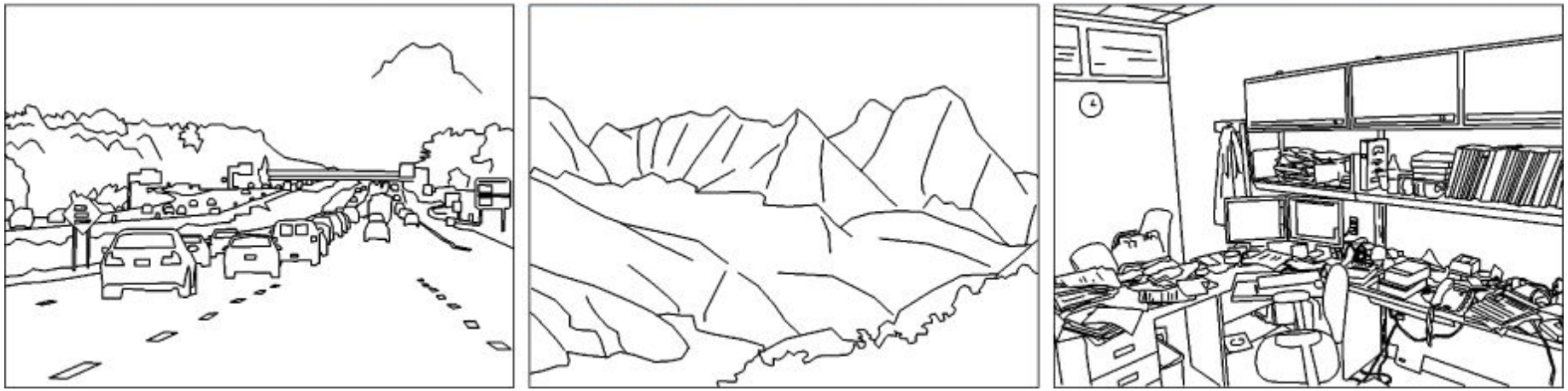


Walther, Chai, Caddigan, Beck & Fei-Fei, *PNAS, 2011*

Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels

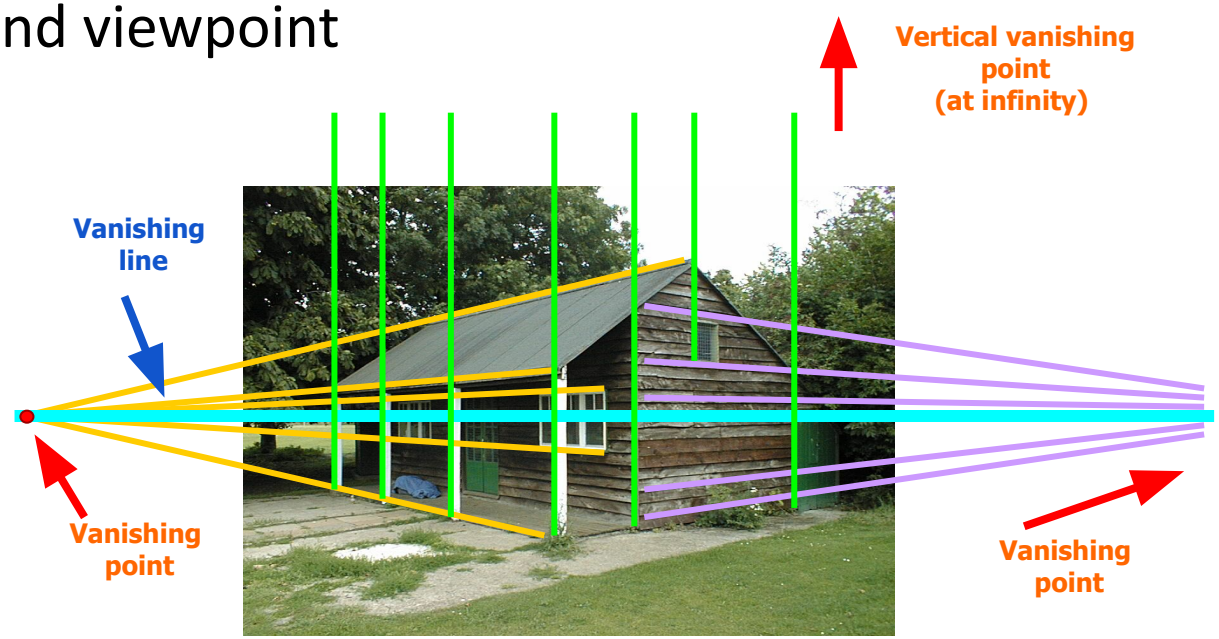
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)



Why do we care about edges?

- Extract information, recognize objects

- Recover geometry and viewpoint



Source: J. Hayes

Origins of edges

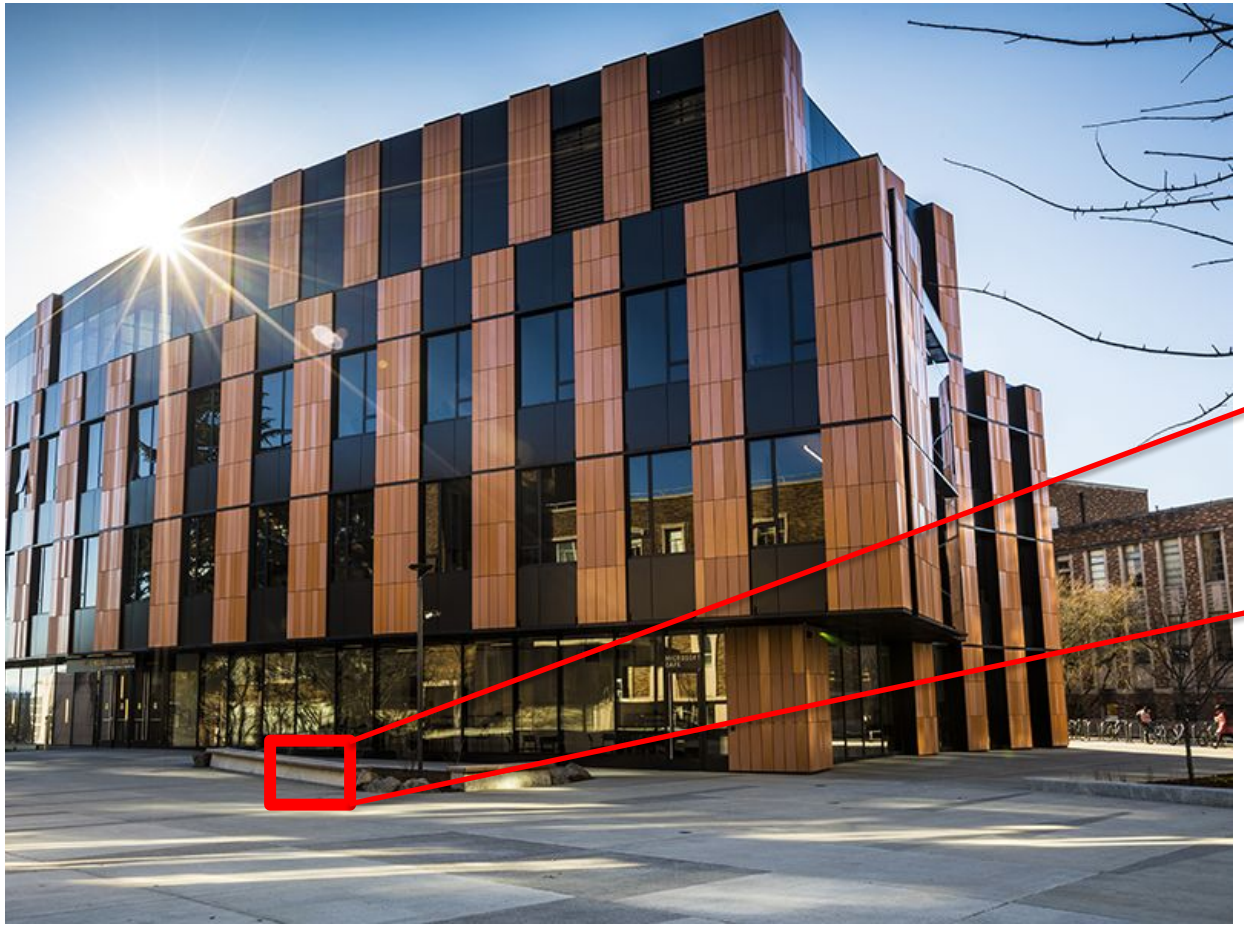
surface normal discontinuity

depth discontinuity

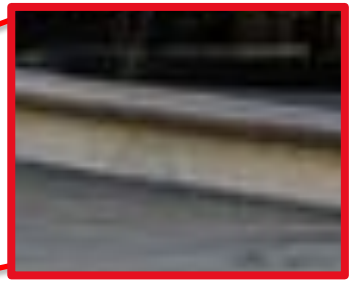
surface color discontinuity

illumination discontinuity

Closeup of edges



Surface normal discontinuity



Closeup of edges

Depth discontinuity

Closeup of edges

Surface color discontinuity

What we will learn today

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

Review: Derivatives in 1D - example

$$y = x^2 + x^4$$

Q. What is the dy/dx?

Review: Derivatives in 1D - example

$$y = x^2 + x^4$$

$$\frac{dy}{dx} = 2x + 4x^3$$

Derivatives in 1D - example

$$y = x^2 + x^4 \qquad y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = 2x + 4x^3$$

Q. What is the dy/dx?

Derivatives in 1D - example

$$y = x^2 + x^4 \qquad y = \sin x + e^{-x}$$

$$\frac{dy}{dx} = 2x + 4x^3 \qquad \frac{dy}{dx} = \cos x + (-1)e^{-x}$$

Approximating derivatives using numerical differentiation

$$\frac{df}{dx} = \lim_{\triangle x=0} \frac{f[x+\triangle x] - f[x]}{\triangle x} = f'(x) = f_x$$

Approximating derivatives using numerical differentiation

Change in *f* at *x*

$$\frac{df}{dx} = \lim_{\triangle x=0} \frac{f[x+\triangle x] - f[x]}{\triangle x} = f'(x) = f_x$$

Change in *x*

In discrete derivatives with images, smallest value of x is 1 pixel

$$\frac{df}{dx} = \lim_{\triangle x=0} \frac{f[x+\triangle x] - f[x]}{\triangle x} = f'(x) = f_x$$

$$= \frac{f[x+1] - f[x]}{1}$$

$$= f[x+1] - f[x]$$

This is called a forward derivative

But change at x can be measured in many different ways

$$\frac{df}{dx} = f[x] - f[x-1]$$

Backward

But change at x can be measured in many different ways

$$\frac{df}{dx} = f[x] - f[x-1]$$ Backward

$$= f[x+1] - f[x]$$ Forward

But change at x can be measured in many different ways

$$\frac{df}{dx} = f[x] - f[x-1]$$ Backward

$$= f[x+1] - f[x]$$ Forward

$$= \frac{1}{2}(f[x+1] - f[x-1])$$ Central

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n, m] = ??$$

Q. What is the equation in width (2nd) dimension?

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n, m] = f[n, m] - f[n, m - 1]$$

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n, m] = f[n, m] - f[n, m-1]$$

Q. Let's write this as a filter

Remember the moving average filter:

$h[\cdot, \cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Let's write this as a filter

$h[\cdot,\cdot]$

?	?	?
?	?	?
?	?	?

Remember the moving average filter:

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1	1	1
1	1	1
1	1	1

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Q. Let's write this as a filter

$h[\cdot,\cdot]$

?	?	?
?	1	?
?	?	?

Remember the moving average filter:

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1	1	1
1	1	1
1	1	1

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?	?	?
?	1	?
?	?	?

Remember the moving average filter:

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$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Let's write this as a filter

$h[\cdot,\cdot]$

0	?	?
?	1	?
?	?	?

Remember the moving average filter:

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

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$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Let's write this as a filter

$h[\cdot,\cdot]$

0	?	?
?	1	?
?	?	?

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1	1	1
1	1	1
1	1	1

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0	0	0
?	1	?
?	?	?

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1	1	1
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0	0	0
?	1	?
?	?	?

Remember the moving average filter:

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Let's write this as a filter

$h[\cdot,\cdot]$

0	0	0
?	1	?
0	0	0

Remember the moving average filter:

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Last ones: What are these two?

$h[\cdot,\cdot]$

0	0	0
?	1	?
0	0	0

Remember the moving average filter:

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation

$$g[n,m] = f[n,m] - f[n,m-1]$$

Q. Last ones: What are these two?

$h[\cdot,\cdot]$

0	0	0
0	1	-1
0	0	0

Remember the moving average filter:

$h[\cdot,\cdot]$

$\frac{1}{9}$

1	1	1
1	1	1
1	1	1

Designing filters that perform differentiation

- Using Backward differentiation:

0	1	-1

$$g[n, m] = f[n, m] - f[n, m-1]$$

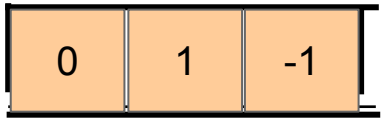
Remember the moving average filter:

$$h[\cdot, \cdot]$$

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Designing filters that perform differentiation

- Using Backward differentiation:



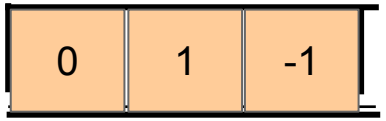
$$g[n,m] = f[n,m] - f[n,m-1]$$

- Using Forward differentiation:

Q. What is the formula?

Designing filters that perform differentiation

- Using Backward differentiation:



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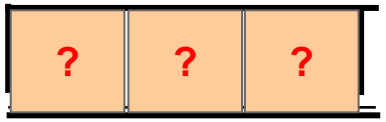
- Using Forward differentiation:

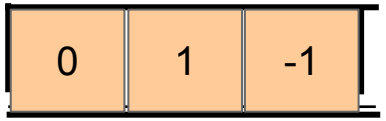


$$g[n,m] = f[n,m+1] - f[n,m]$$

Q. What is the filter look like?

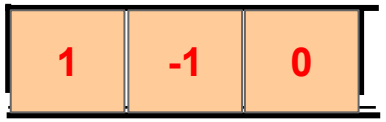
Designing filters that perform differentiation

- Using Backward differentiation:



$$g[n,m] = f[n,m] - f[n,m-1]$$

- Using Forward differentiation:



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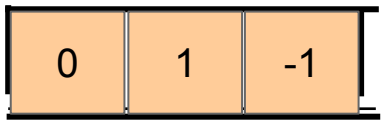
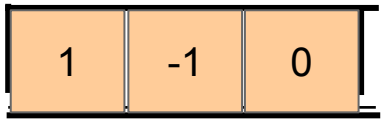
Designing filters that perform differentiation

- Using Backward differentiation:



$$g[n,m] = f[n,m] - f[n,m-1]$$

- Using Forward differentiation:



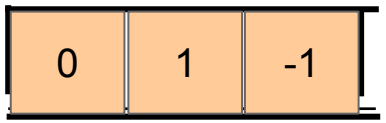
$$g[n,m] = f[n,m+1] - f[n,m]$$

- Using Central differentiation:

Q. What is the formula?

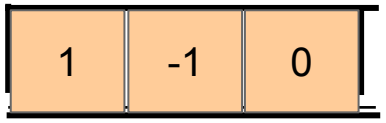
Designing filters that perform differentiation

- Using Backward differentiation:



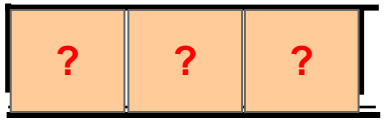
$$g[n,m] = f[n,m] - f[n,m-1]$$

- Using Forward differentiation:



$$g[n,m] = f[n,m+1] - f[n,m]$$

- Using Central differentiation:

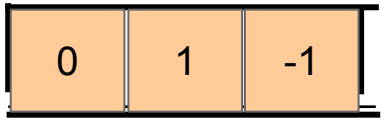


$$g[n,m] = f[n,m+1] - f[n,m-1]$$

Q. What is the filter?

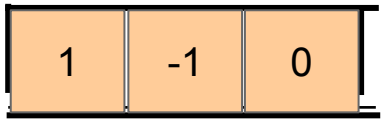
Designing filters that perform differentiation

- Using Backward differentiation:



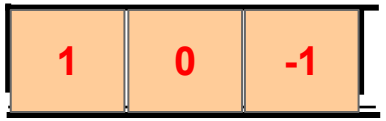
$$g[n,m] = f[n,m] - f[n,m-1]$$

- Using Forward differentiation:



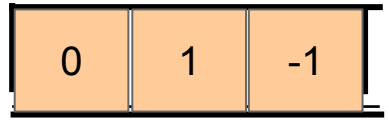
$$g[n,m] = f[n,m+1] - f[n,m]$$

- Using Central differentiation:



$$g[n,m] = f[n,m+1] - f[n,m-1]$$

Derivative in width dimension for one row



Using backward differentiation:

$$g[n, m] = f[n, m] - f[n, m-1]$$

$$f[0, :] = [10, \ \ 15, \ \ 10, \ \ 10, \ \ 25, \ \ 20, \ \ 20, \ \ 20]$$

Derivative in width dimension for one row

Using backward differentiation:

0	1	-1

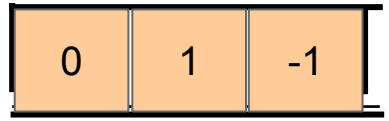
$$g[n,m] = f[n,m] - f[n,m-1]$$

$$f[0,:] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0,:] = [\ ? \qquad\qquad\qquad\qquad\qquad\qquad]$$

Derivative in width dimension for one row

Using backward differentiation:

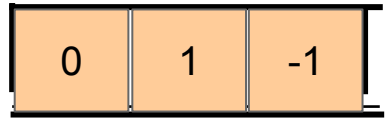


$$g[n, m] = f[n, m] - f[n, m-1]$$

$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad ? \quad \quad \quad \quad \quad \quad \quad]$$

Derivative in width dimension for one row



Using backward differentiation:

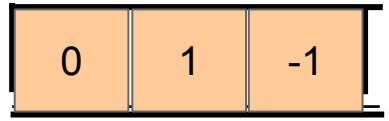
$$g[n, m] = f[n, m] - f[n, m-1]$$

$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad 5, \quad ? \quad \quad \quad \quad \quad \quad \quad]$$

Derivative in width dimension for one row

Using backward differentiation:



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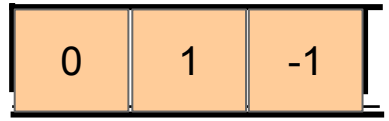
$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad 5, \quad -5, \quad ? \quad]$$

Derivative in width dimension for one row



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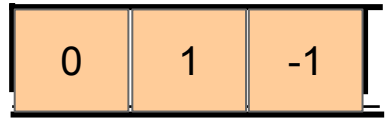
$$g[n, m] = f[n, m] - f[n, m-1]$$

$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad 5, \quad -5, \quad 0, \quad ? \quad]$$

Derivative in width dimension for one row



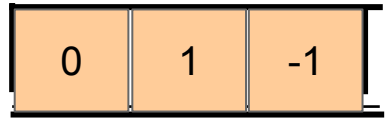
Using backward differentiation:

$$g[n, m] = f[n, m] - f[n, m-1]$$

$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad 5, \quad -5, \quad 0, \quad 15, \quad ? \quad ? \quad ?]$$

Derivative in width dimension for one row



Using backward differentiation:

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$$f[0, :] = [10, \quad 15, \quad 10, \quad 10, \quad 25, \quad 20, \quad 20, \quad 20]$$

$$\frac{df}{dm}[0, :] = [10, \quad 5, \quad -5, \quad 0, \quad 15, \quad -5, \quad 0, \quad 0]$$

Discrete derivation in 2D:

$$\text{Given function } f[n, m]$$

$$\text{Gradient filter } \nabla f[n, m] = \begin{bmatrix} \frac{df}{dn} \\ \frac{df}{dm} \end{bmatrix} = \begin{bmatrix} f_n \\ f_m \end{bmatrix}$$

Discrete derivation in 2D:

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$$\text{Gradient magnitude } |\nabla f[n,m]| = \sqrt{f_n^2 + f_m^2}$$

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$$\text{Gradient magnitude } |\nabla f[n,m]| = \sqrt{f_n^2 + f_m^2}$$

$$\text{Gradient direction } \theta = \tan^{-1}\left(\frac{f_m}{f_n}\right)$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} ? & ? & ? & ? & ? \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ ? & ? & ? & ? & ? \\ & & & & \\ & & & & \\ & & & & \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 0 & 0 & 0 & 0 & 0 \\ ? & ? & ? & ? & ? \\ & & & & \\ & & & & \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ ? & ? & ? & ? & ? \\ \\ \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ ? & ? & ? & ? & ? \end{bmatrix}$$

2D discrete derivative - example

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -10 & -10 & -20 & -20 & -20 \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} ? & & & & \\ ? & & & & \\ ? & & & & \\ ? & & & & \\ ? & & & & \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & ? \\ 10 & ? \\ 10 & ? \\ 10 & ? \\ 10 & ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & ? \\ 10 & 10 & ? \\ 10 & 10 & ? \\ 10 & 10 & ? \\ 10 & 10 & ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 10 & ? \\ 10 & 10 & 10 & ? \\ 10 & 10 & 10 & ? \\ 10 & 10 & 10 & ? \\ 10 & 10 & 10 & ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \\ 10 & 10 & 10 & 0 & ? \end{bmatrix}$$

Let's do the other one

$$f[n,m] = \begin{bmatrix} 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \\ 10 & 10 & 20 & 20 & 20 \end{bmatrix}$$

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$$

$$g[n,m] = \begin{bmatrix} 10 & 10 & 10 & 0 & -20 \\ 10 & 10 & 10 & 0 & -20 \\ 10 & 10 & 10 & 0 & -20 \\ 10 & 10 & 10 & 0 & -20 \\ 10 & 10 & 10 & 0 & -20 \end{bmatrix}$$

2D discrete derivative filters

Q. What does this filter do?

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

2D discrete derivative filters

$$h[n,m] = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

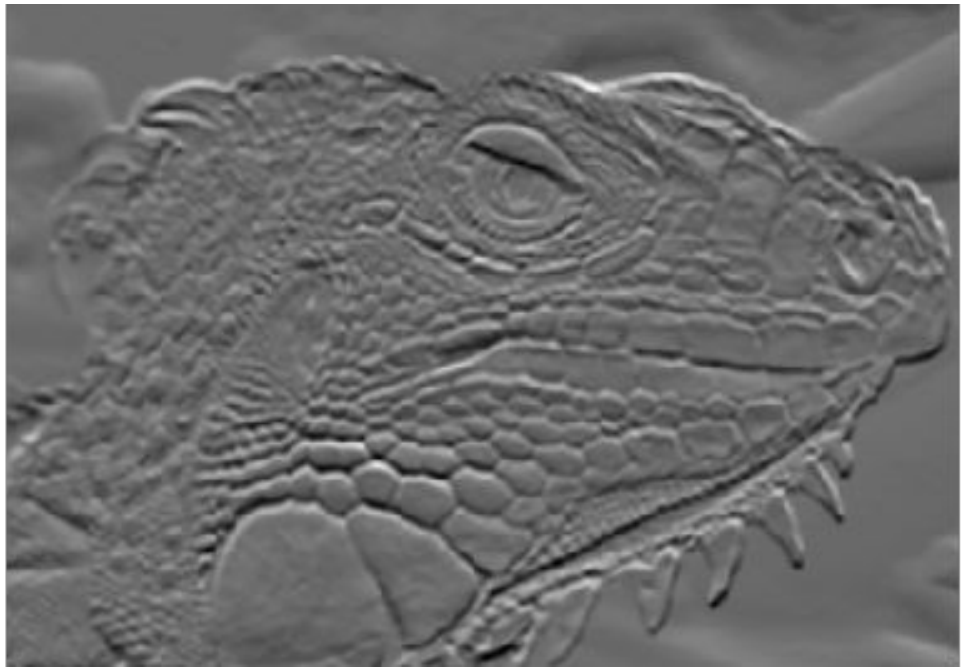
Q. What does this filter do?

$$h[n,m] = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

Q. Which filter was applied?

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

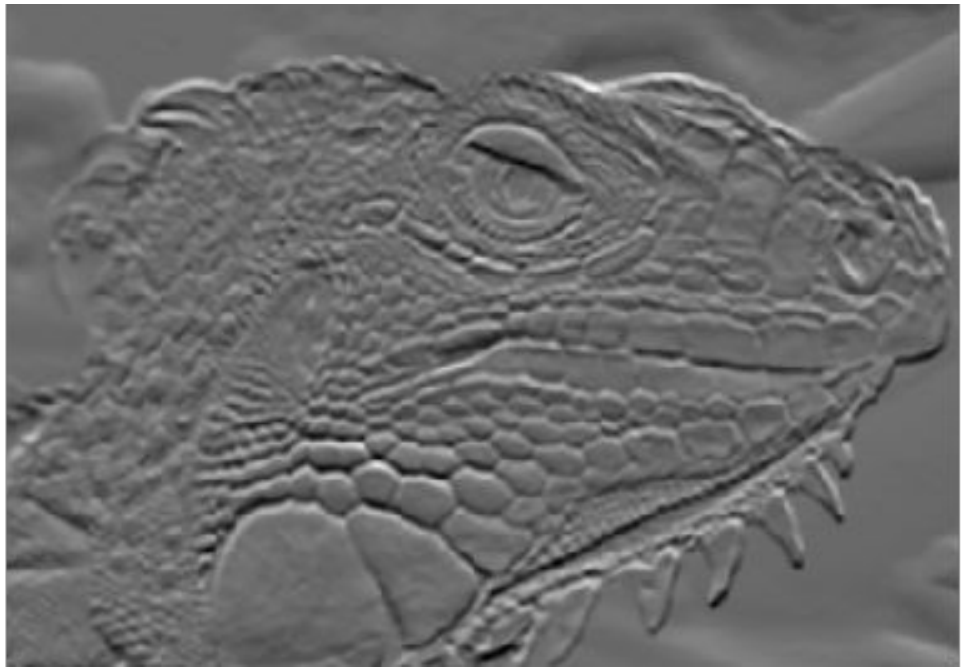
A **B**

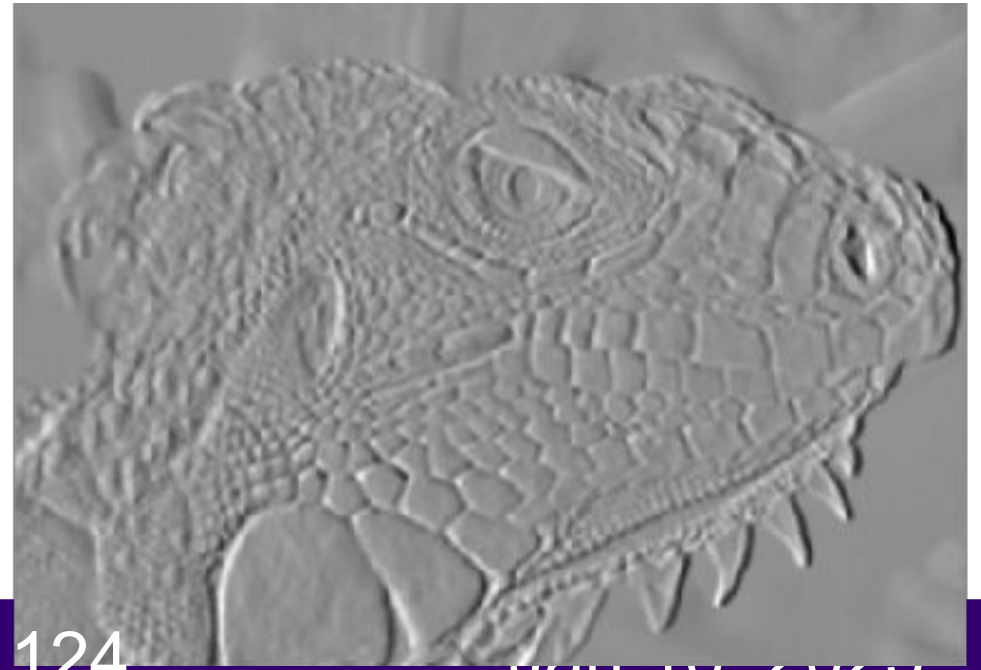


Q. Which filter was applied?

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

A **B**





What we will learn today

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

Characterizing edges

An edge is a place of rapid change in the image intensity function

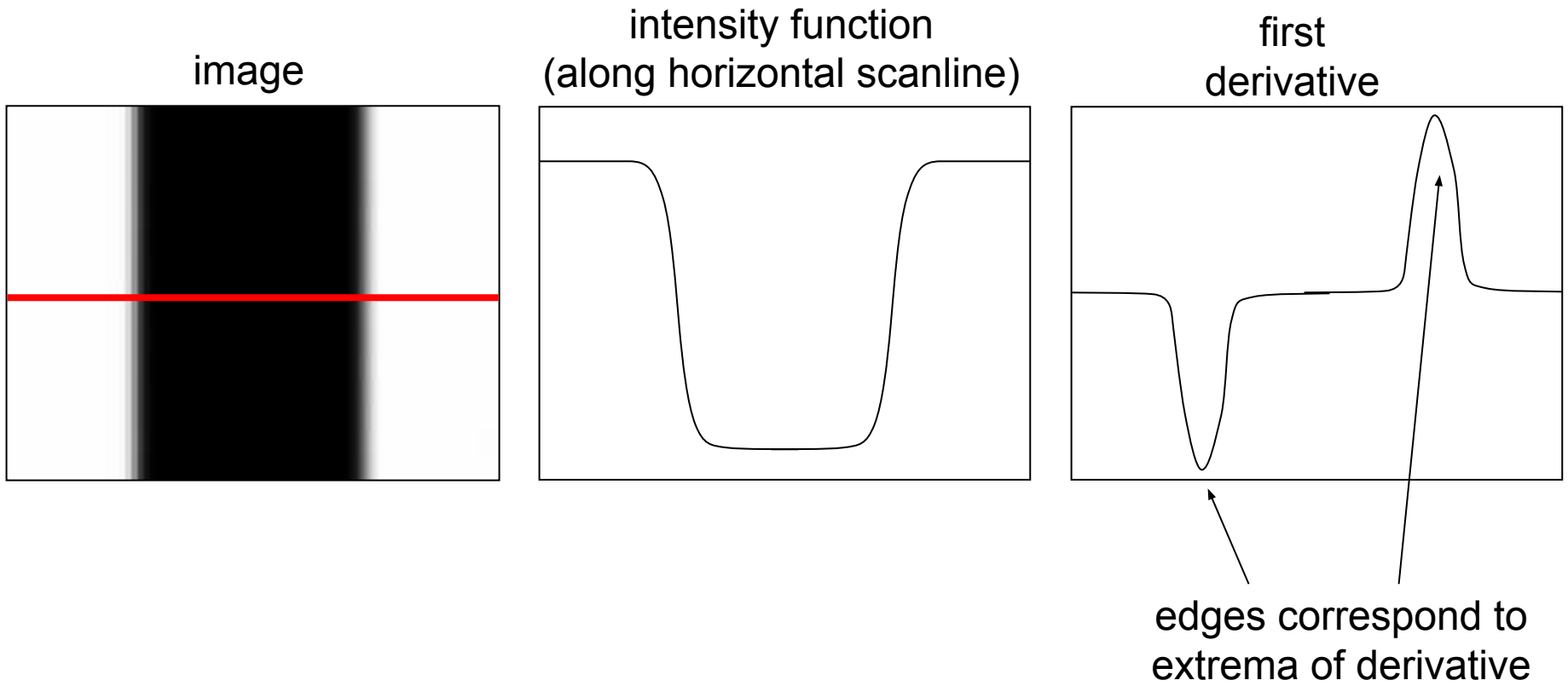
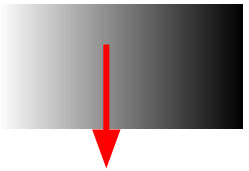


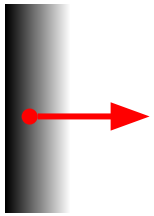
Image gradient

The gradient of an image:

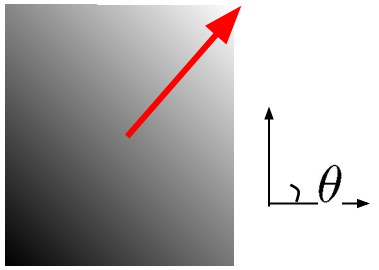


$$\nabla_n f[n,m] = \begin{bmatrix} \frac{df}{dn} & 0 \end{bmatrix}$$

$$\nabla f[n,m] = \begin{bmatrix} \frac{df}{dn} & \frac{df}{dm} \end{bmatrix}$$



$$\nabla_m f[n,m] = \begin{bmatrix} 0 & \frac{df}{dm} \end{bmatrix}$$

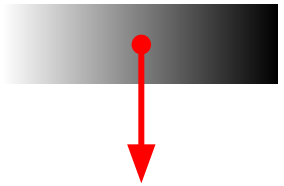


The gradient vector points in the direction of most rapid increase in intensity

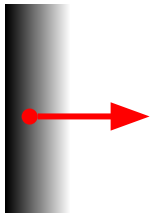
$$\theta = \tan^{-1}\left(\frac{f_m}{f_n}\right)$$

Image gradient

The gradient of an image:

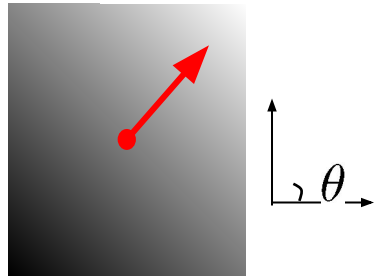


$$\nabla_n f[n,m] = \begin{bmatrix} \frac{df}{dn} & 0 \end{bmatrix}$$



$$\nabla_m f[n,m] = \begin{bmatrix} 0 & \frac{df}{dm} \end{bmatrix}$$

$$\nabla f[n,m] = \begin{bmatrix} \frac{df}{dn} & \frac{df}{dm} \end{bmatrix}$$



The gradient vector points in the direction of most rapid increase in intensity

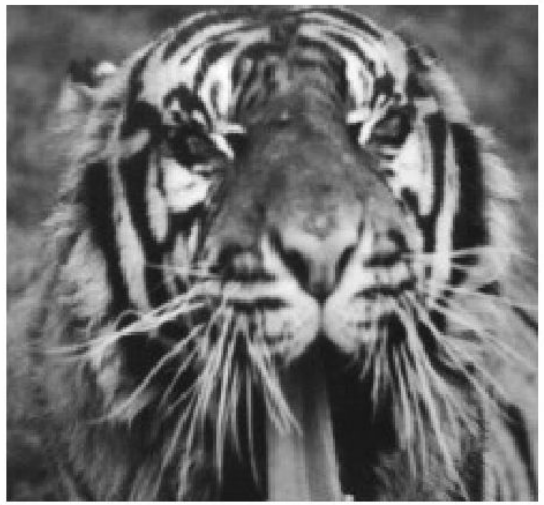
$$\theta = \tan^{-1}\left(\frac{f_m}{f_n}\right)$$

The *edge strength* is given by the gradient magnitude

$$|\nabla f[n,m]| = \sqrt{f_n^2 + f_m^2}$$

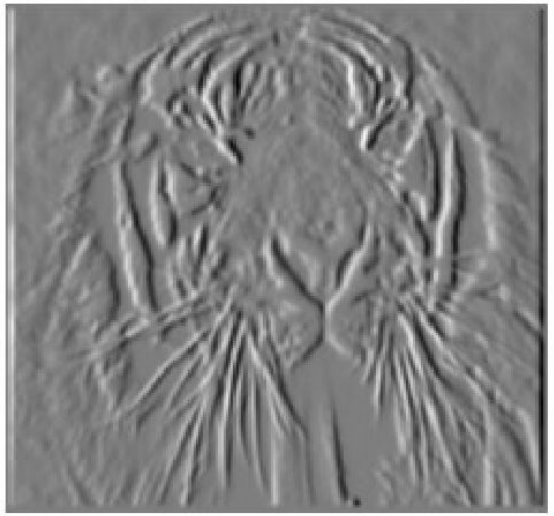
Finite differences: example

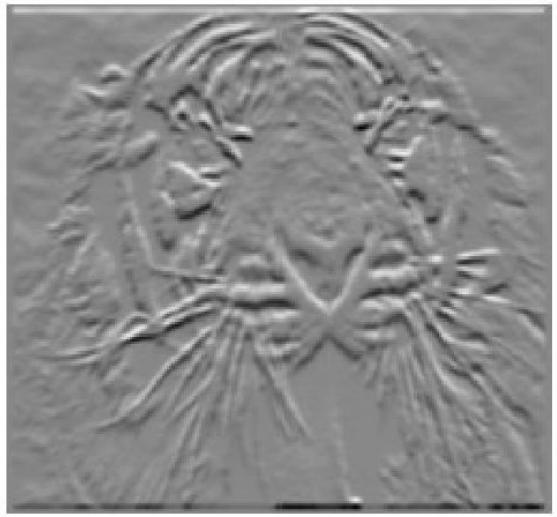
Original Image



Gradient magnitude

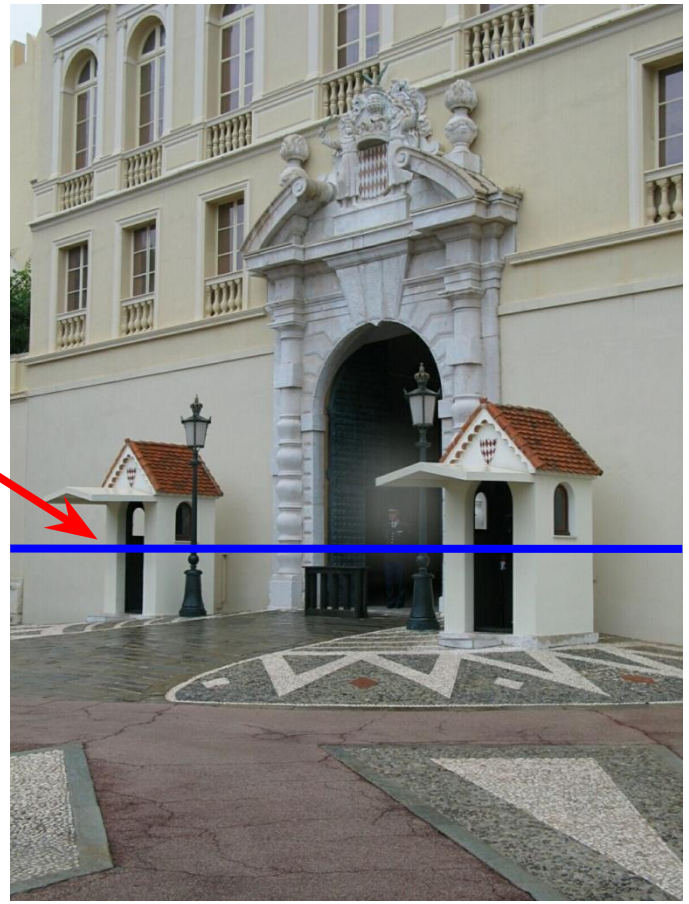
width-direction

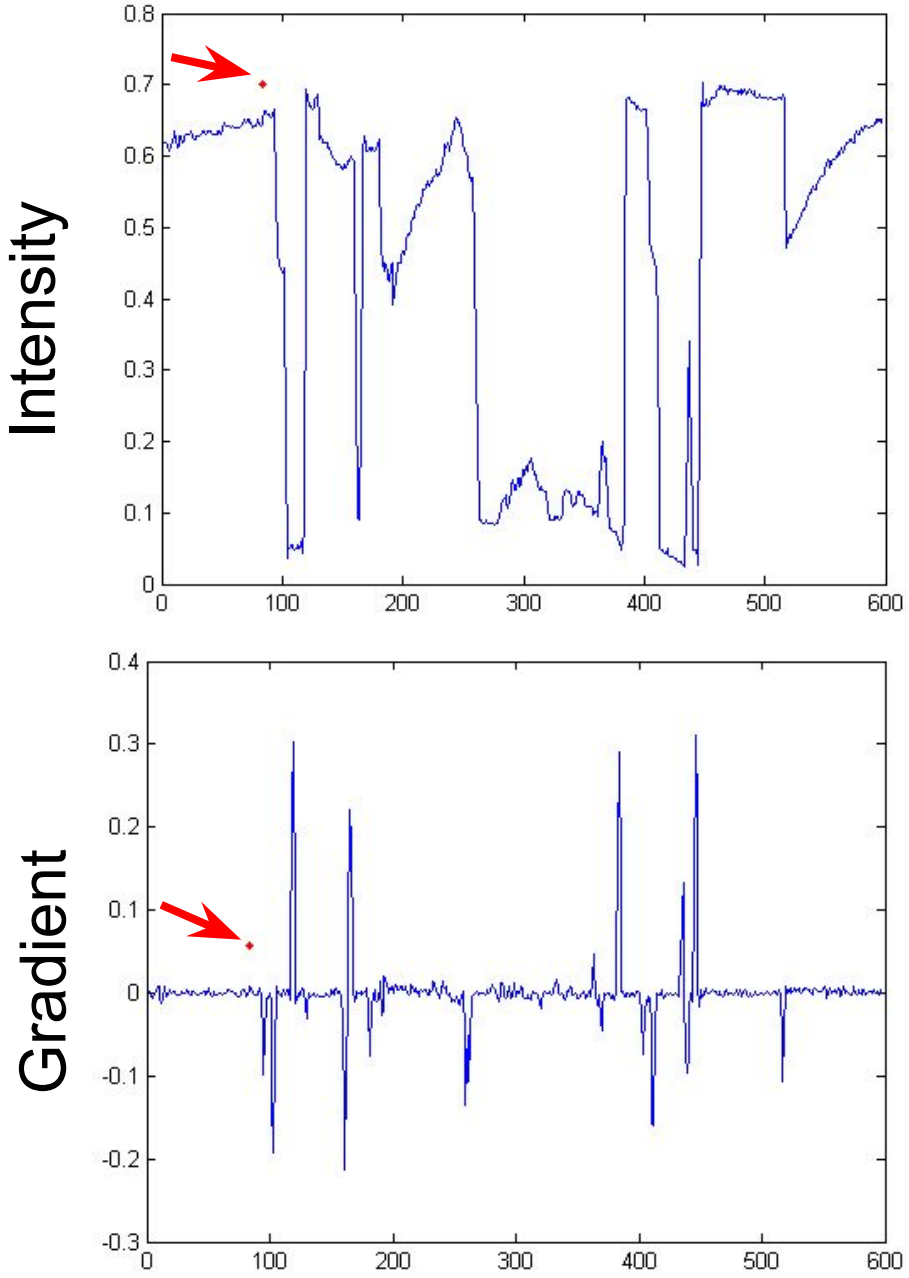




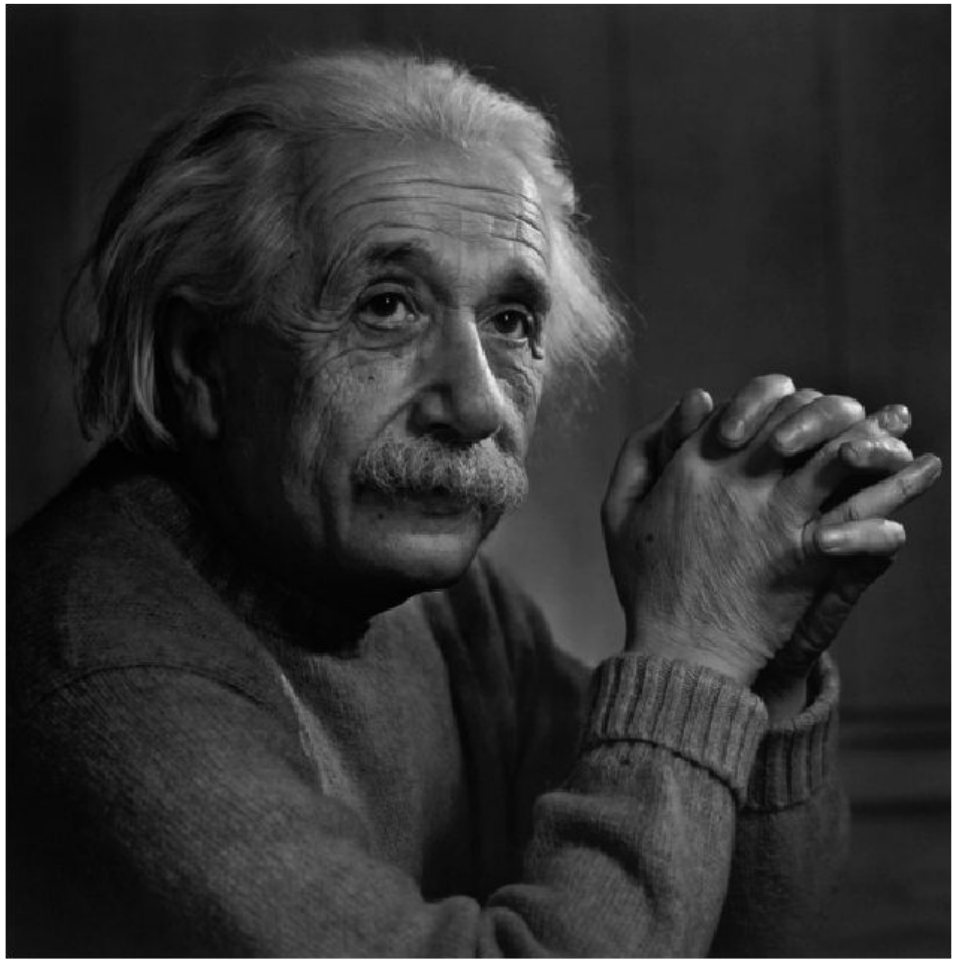
height-direction

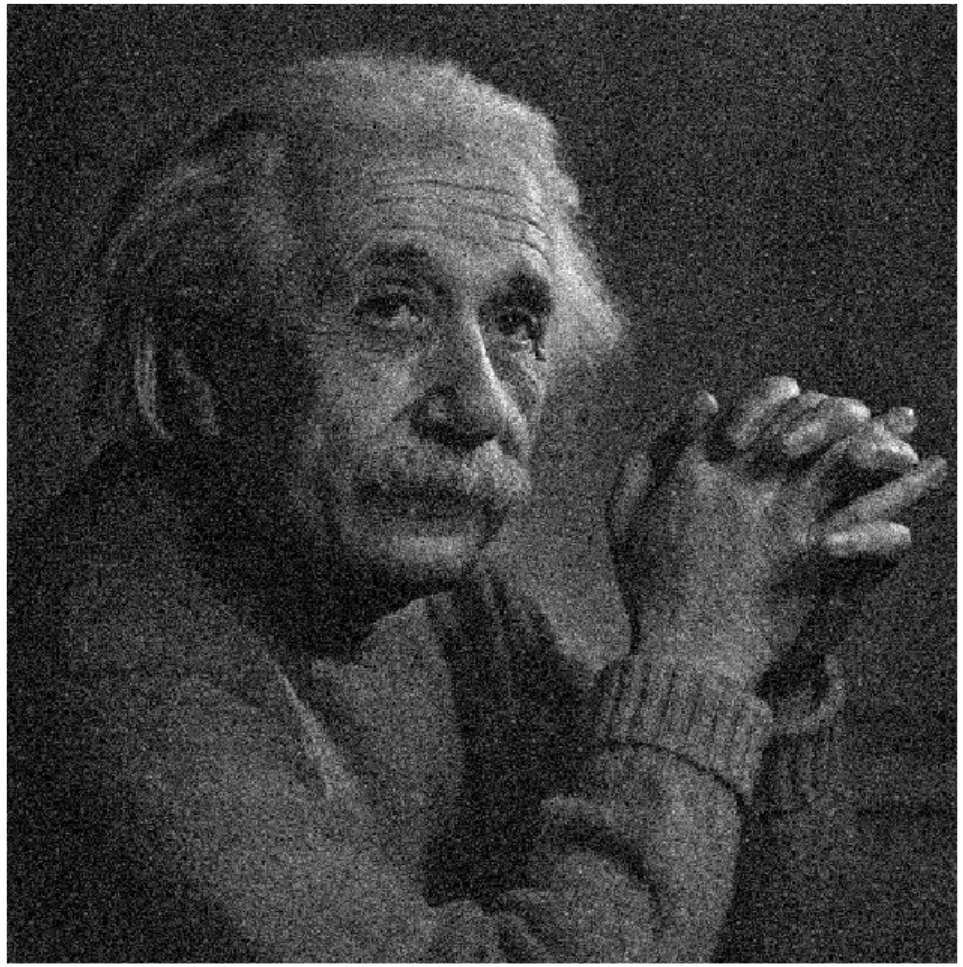
Intensity profile





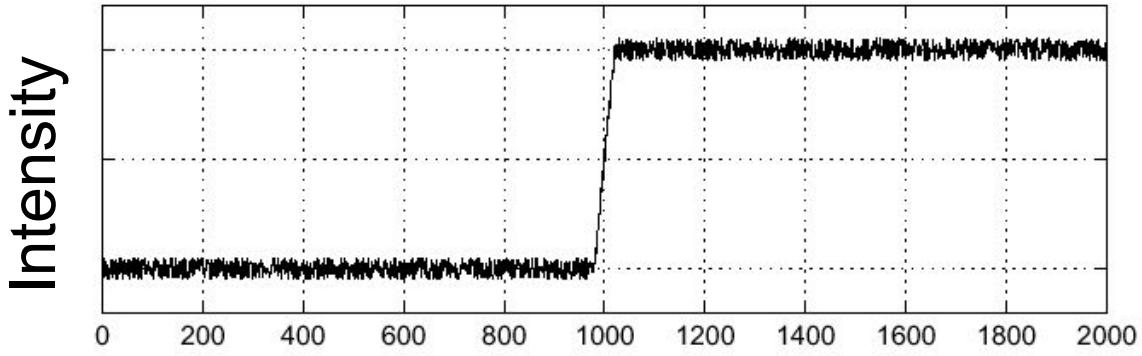
Q. What will happen if we use this edge detector on a noisy pixels?





Effects of noise

- Consider a single row or column of the image
  - Plotting intensity as a function of position gives a signal



Source: S. Seitz

Effects of noise

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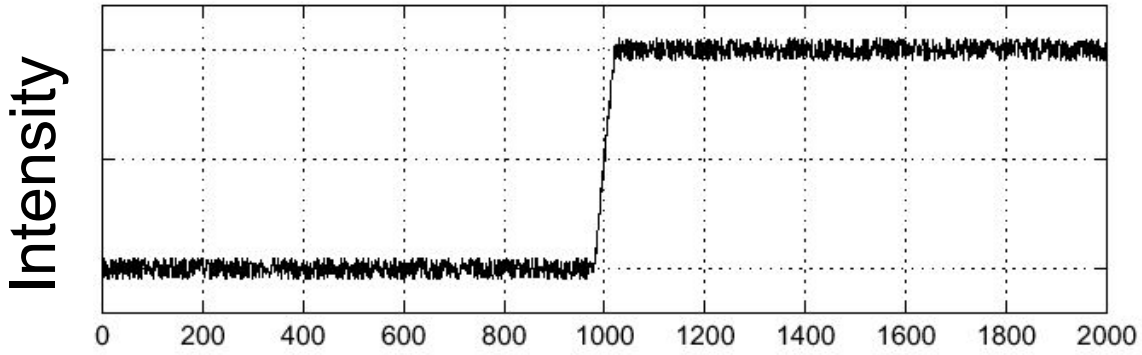
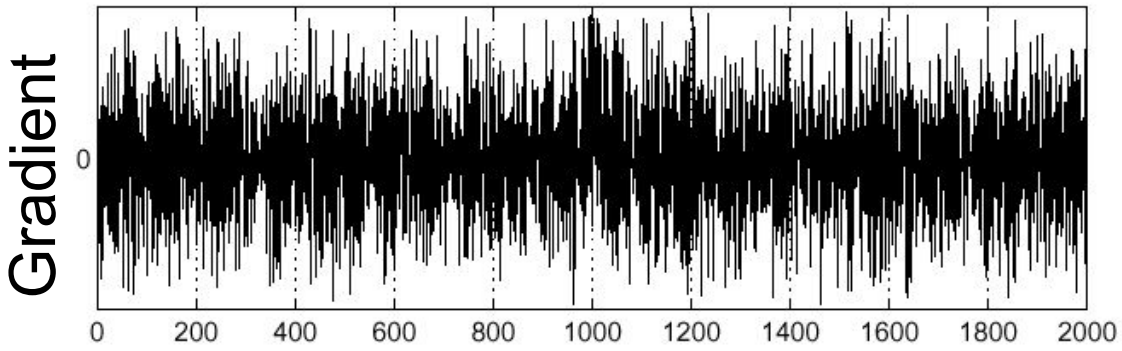




Where is the edge?

Source: S. Seitz

Effects of noise

- Finite difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- Q. What is a potential quick fix for noisy images?

Source: D. Forsyth

Effects of noise

- Finite difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response
- Q. What is a potential quick fix for noisy images?
- Smoothing the image should help, by forcing pixels different to their neighbors (=noise pixels?) to look more like neighbors

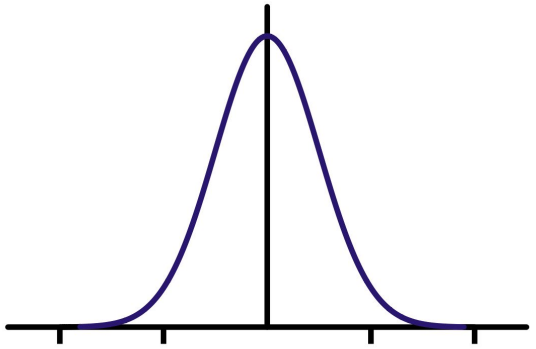
Source: D. Forsyth

Smoothing with different filters

- Mean smoothing

$$\frac{1}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} \qquad \frac{1}{3}\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

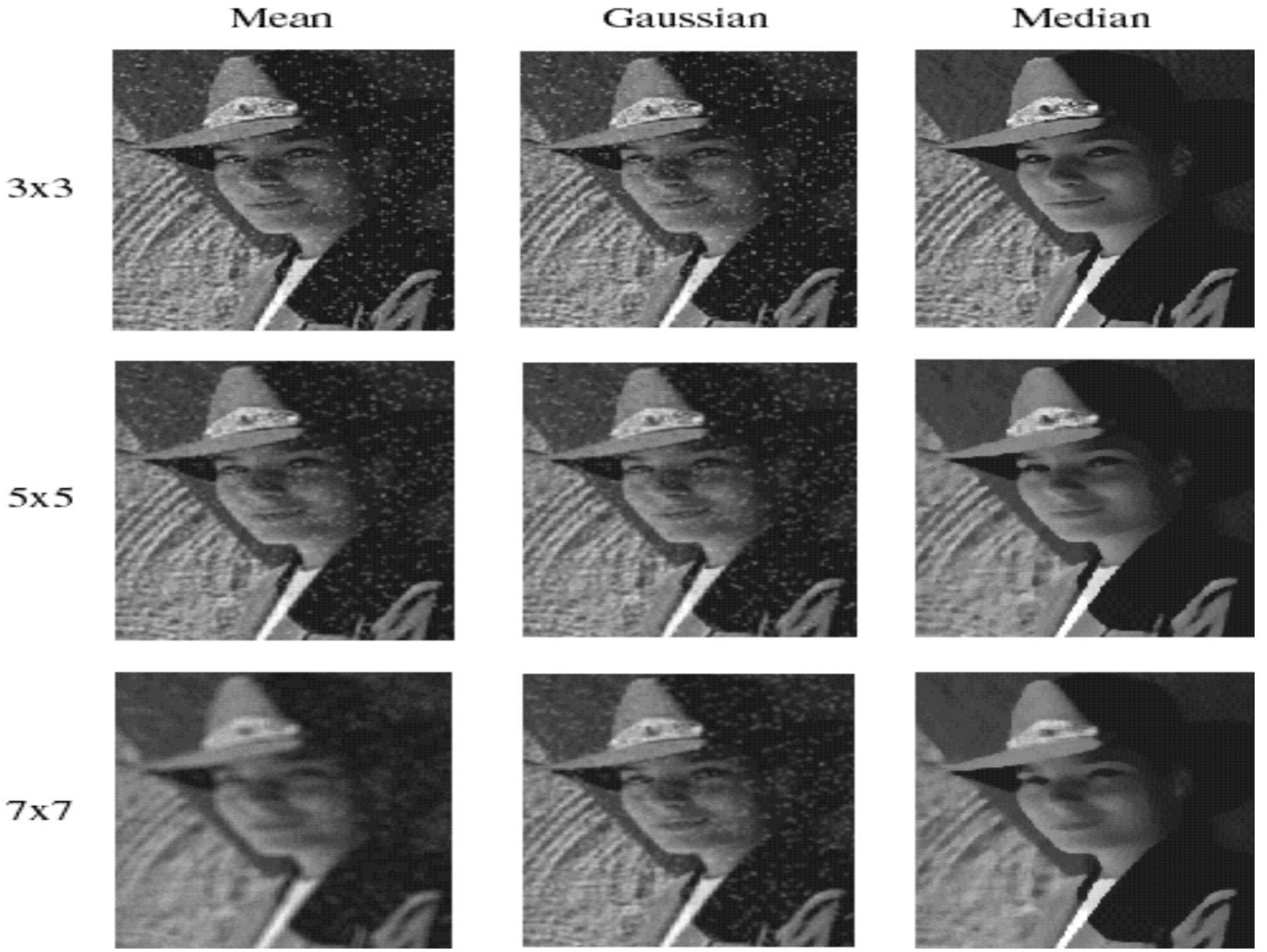
- Gaussian (smoothing * derivative)



$$\frac{1}{4}\begin{bmatrix}1\\2\\1\end{bmatrix} \qquad \frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix}$$

Slide credit: Steve Seitz

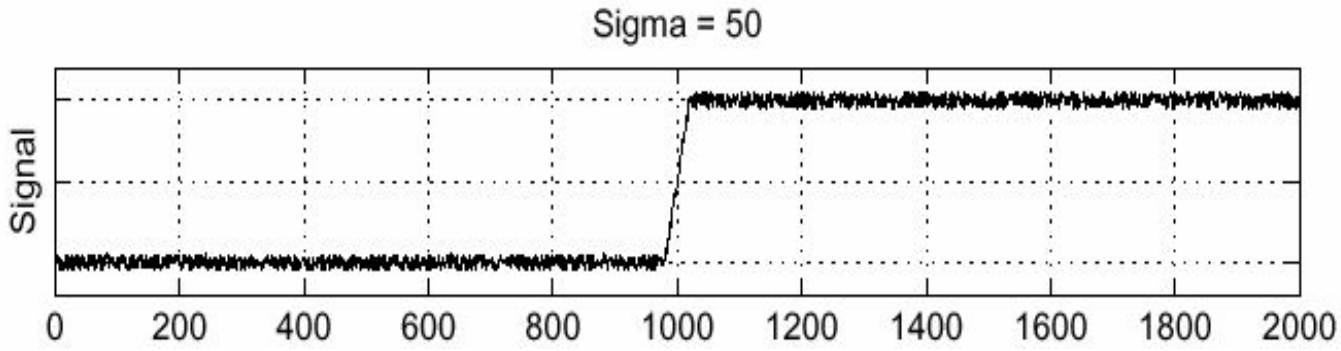
Smoothing with different filters



Slide credit: Steve Seitz

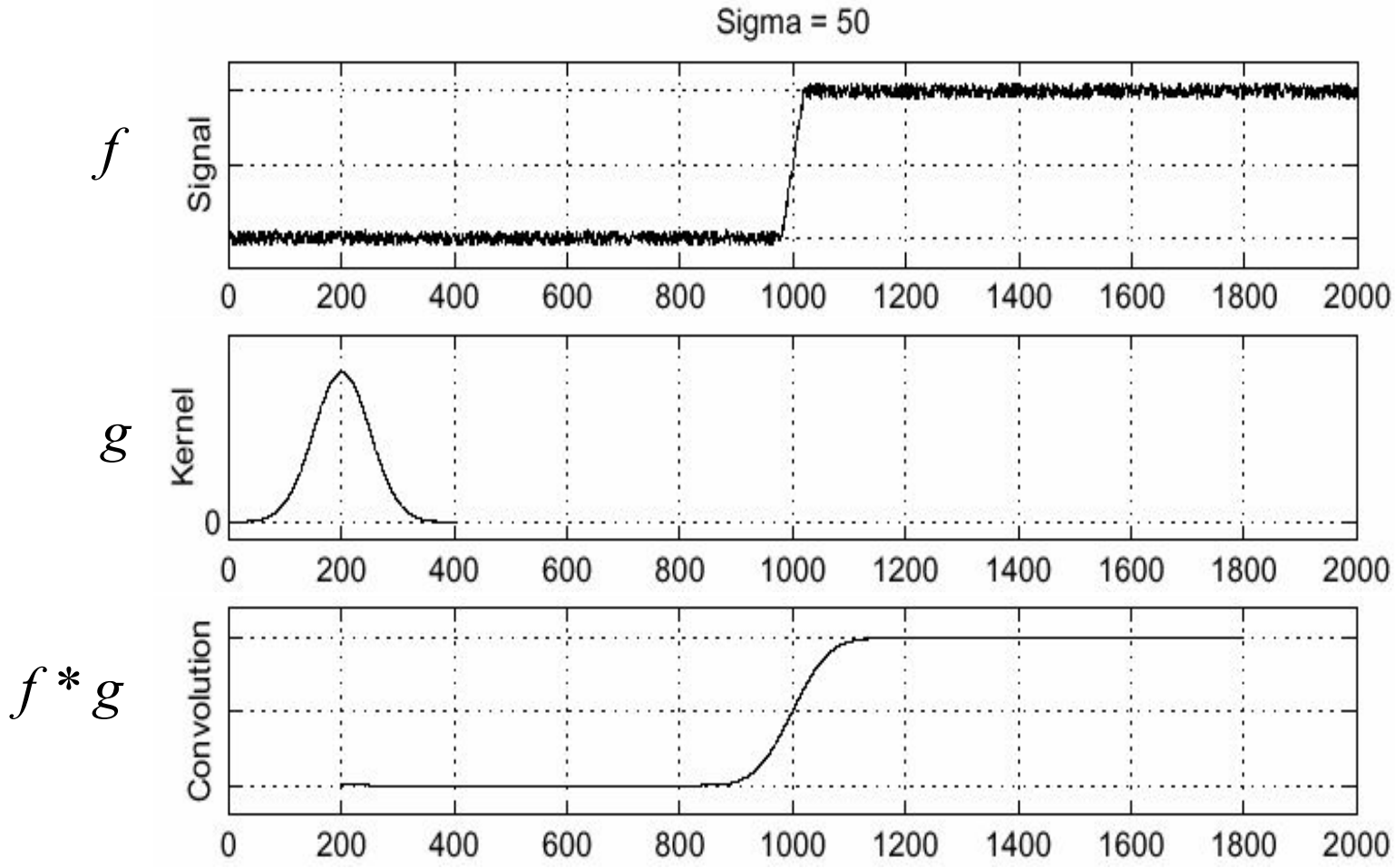
Solution: input function

f



Source: S. Seitz

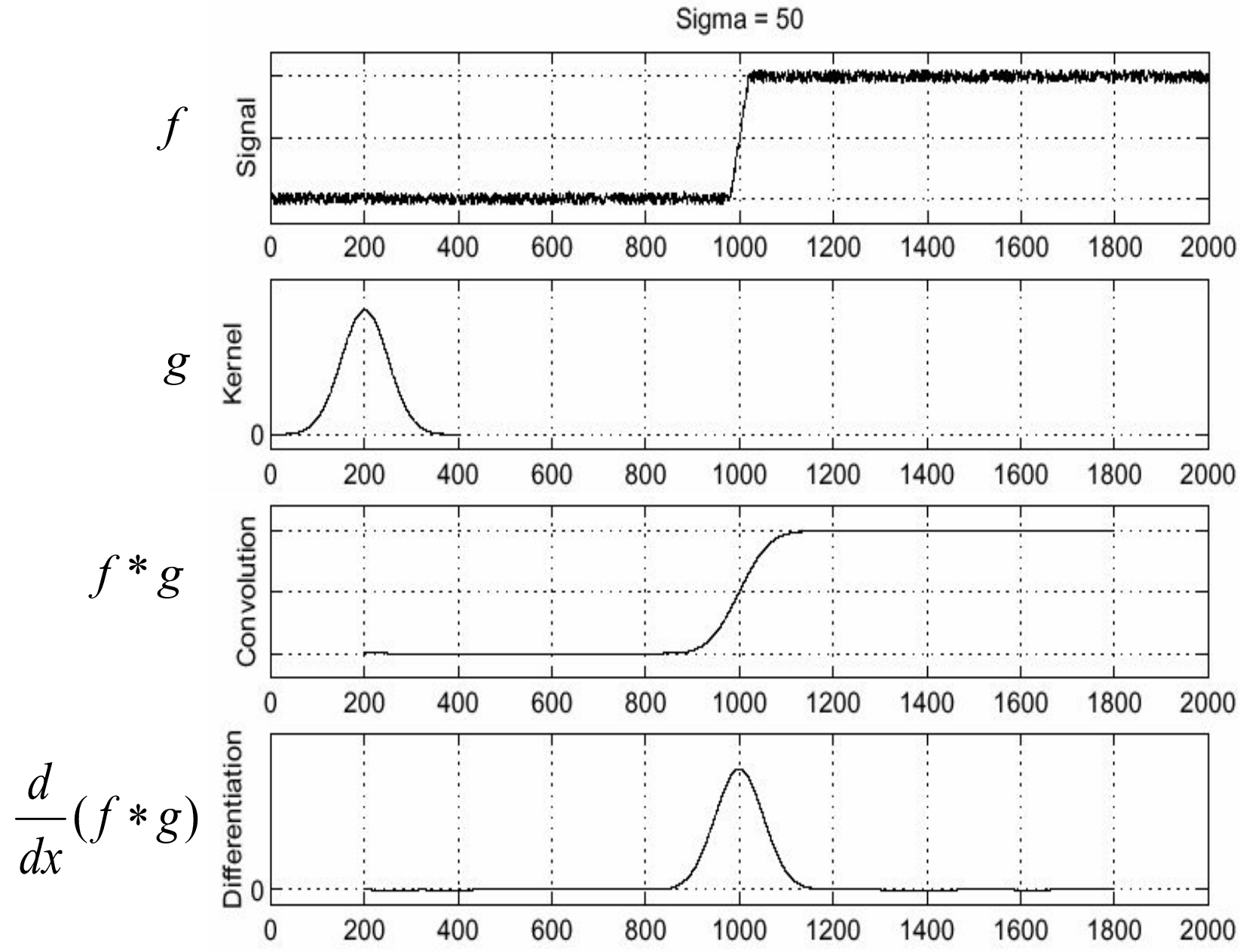
Solution: smooth first



Source: S. Seitz

Solution: smooth first

To find edges, look for peaks in $\frac{d}{dx}(f * g)$



Source: S. Seitz

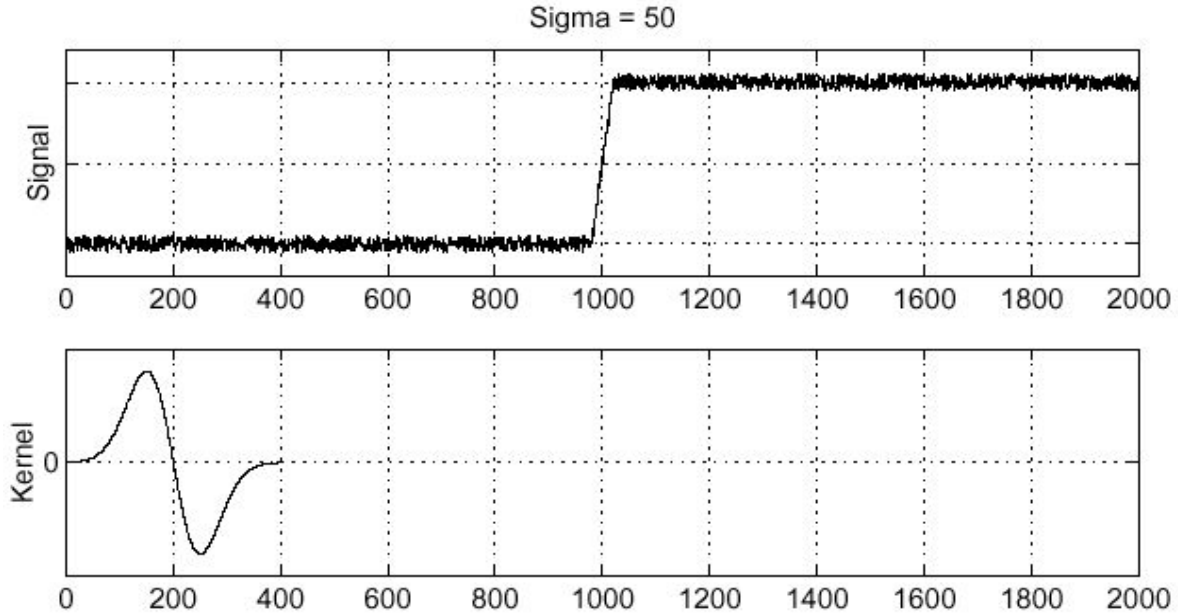
Derivative theorem of convolution

- This theorem gives us a very useful property:

$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

f

$\frac{d}{dx} g$



Source: S. Seitz

Derivative theorem of convolution

- This theorem gives us a very useful property:

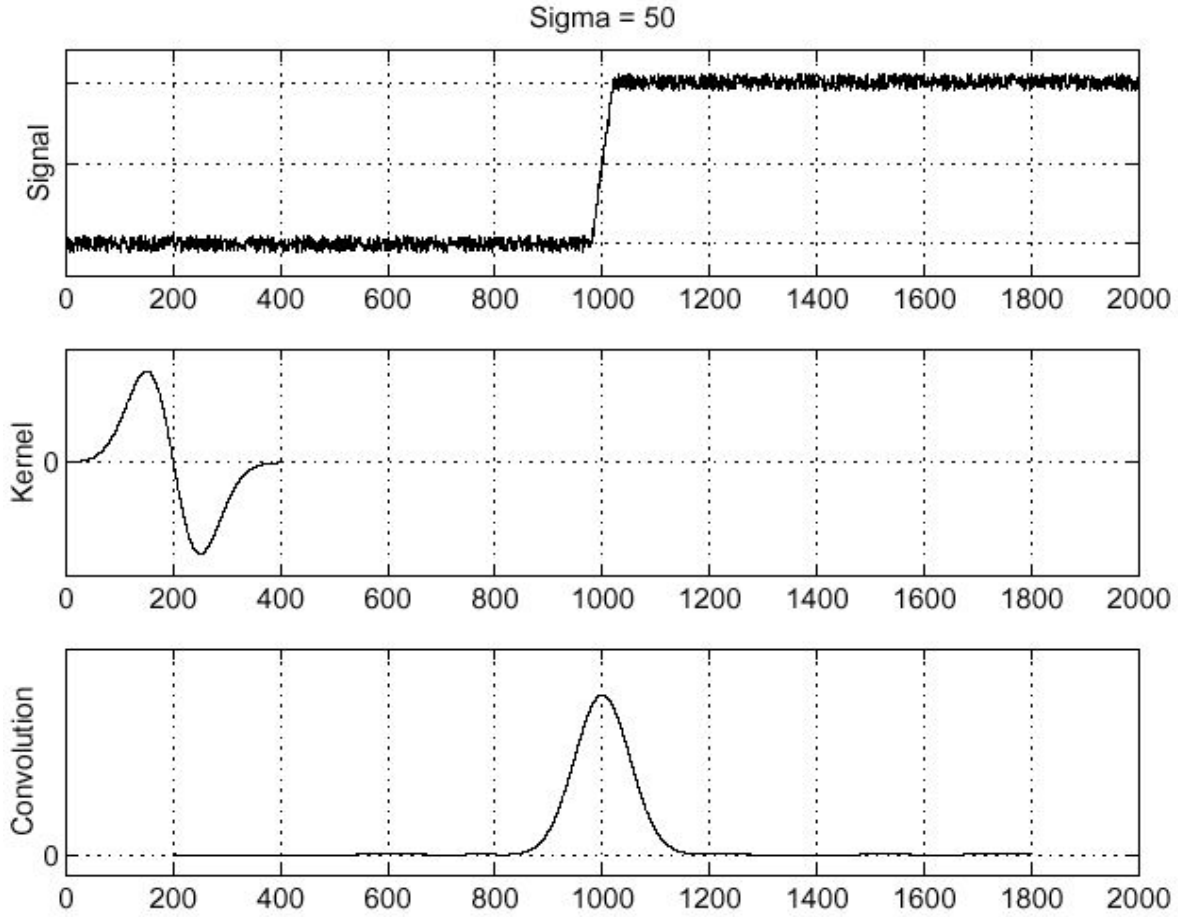
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- This saves us one operation:

We can precompute: $\frac{d}{dx} g$

f

$f * \frac{d}{dx} g$



Source: S. Seitz

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$\frac{d}{dx} g$

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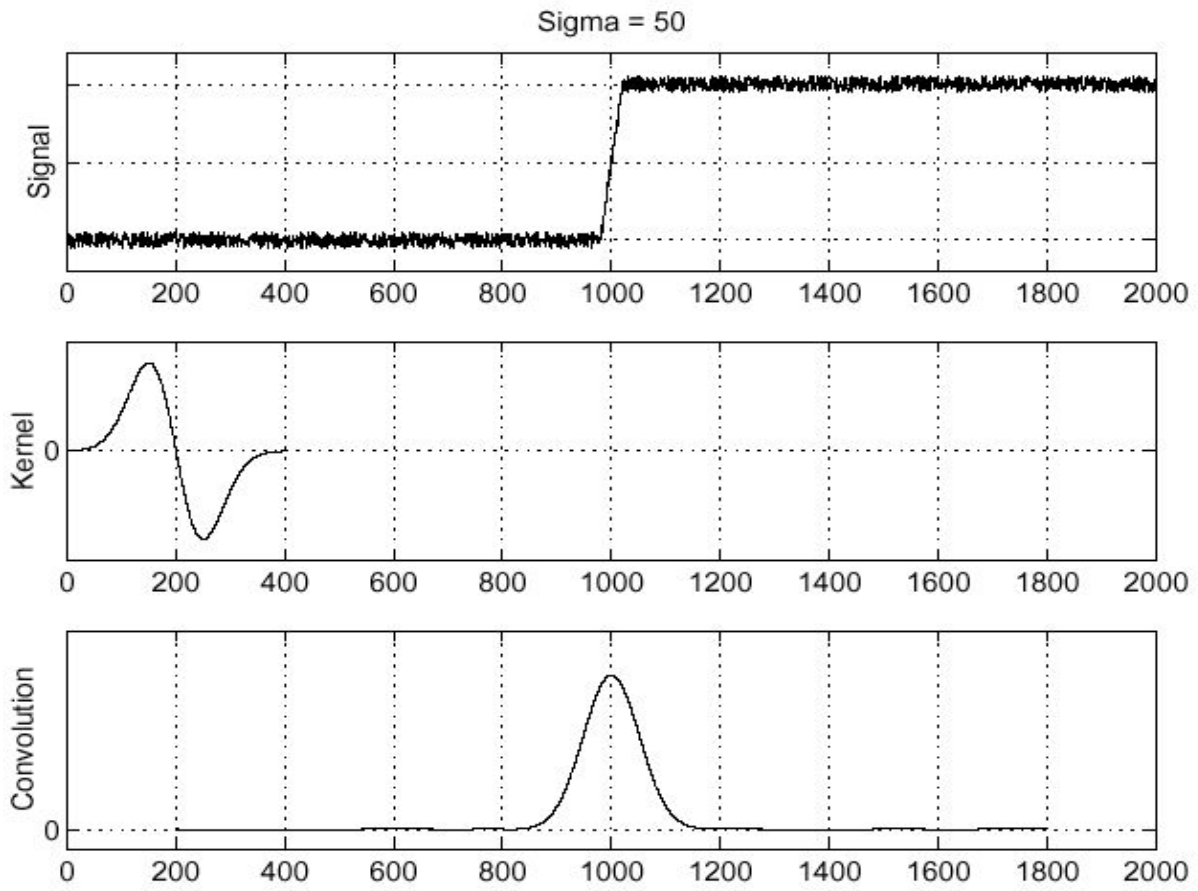
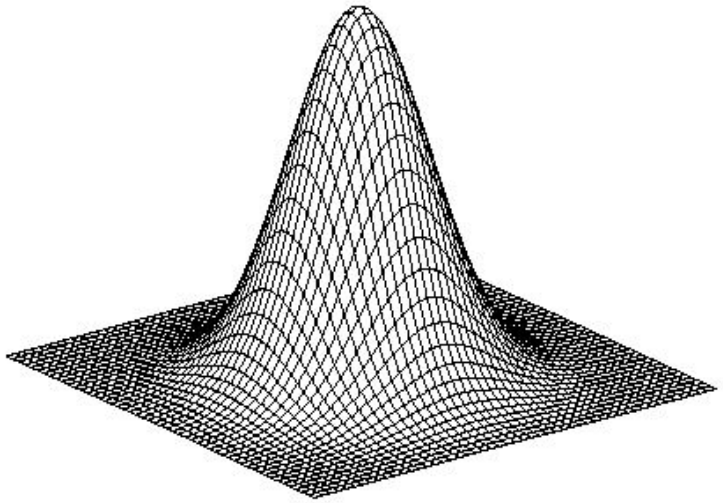
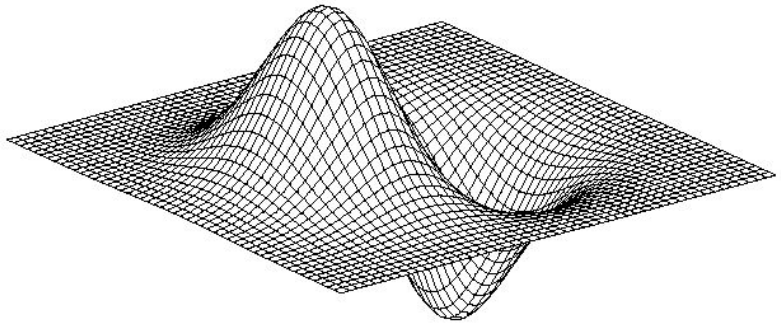


Source: S. Seitz

Derivative of Gaussian filter (central derivative)



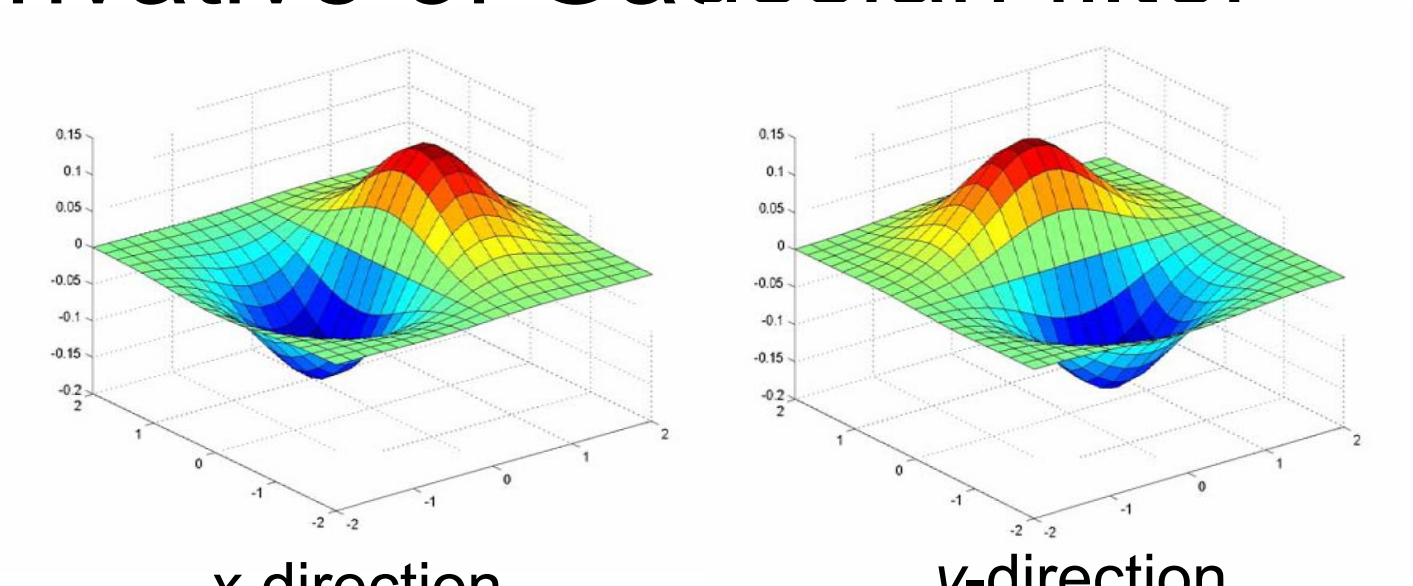
* [1 0 -1] =



2D-gaussian

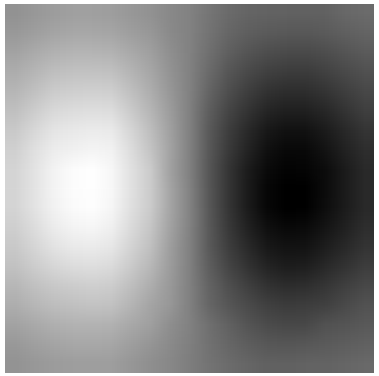
x - derivative

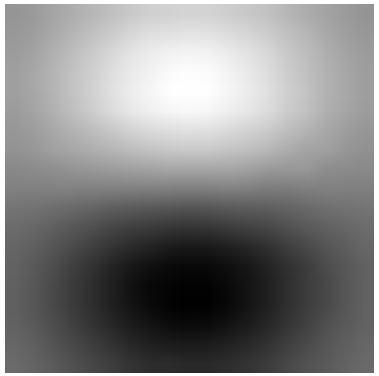
Derivative of Gaussian filter



x-direction

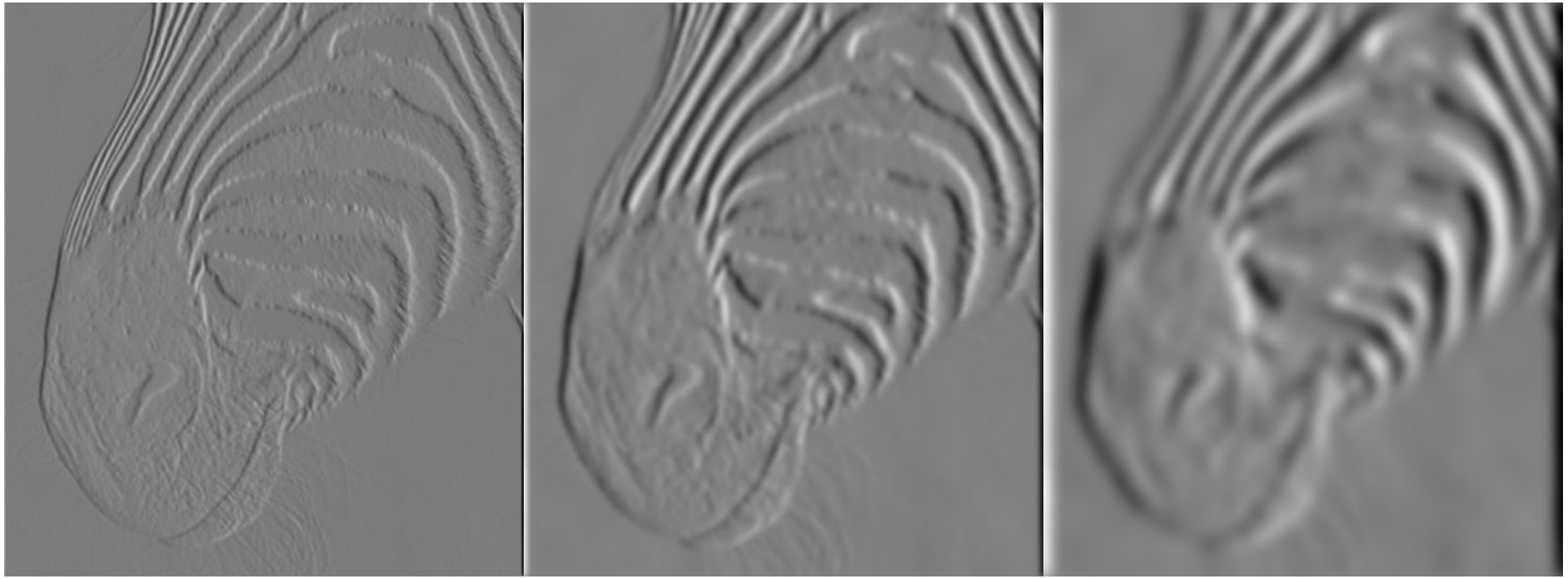
y-direction





Derivative of Gaussian filter

Tradeoff between smoothing at different scales

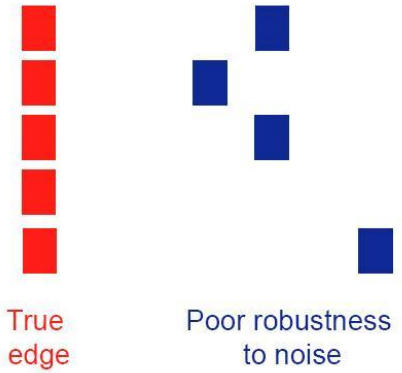


1 pixel 3 pixels 7 pixels

Smoothed derivative removes noise, but blurs edge.
Also finds edges at different “scales”.

Designing an edge detector

- Criteria for an “optimal” edge detector:
  - **Good detection:** the optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges)



Designing an edge detector

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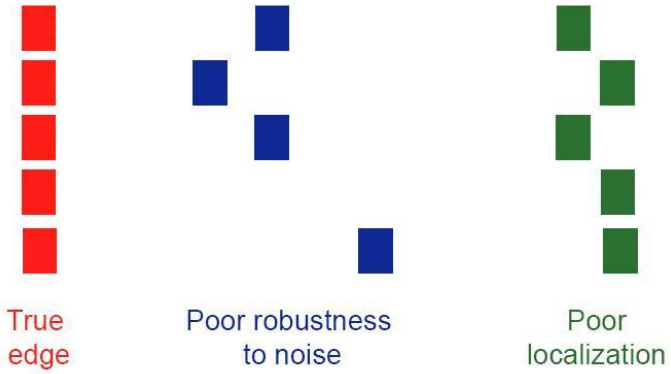
  - **Good localization:** the edges detected must be as close as possible to the true edges



Designing an edge detector

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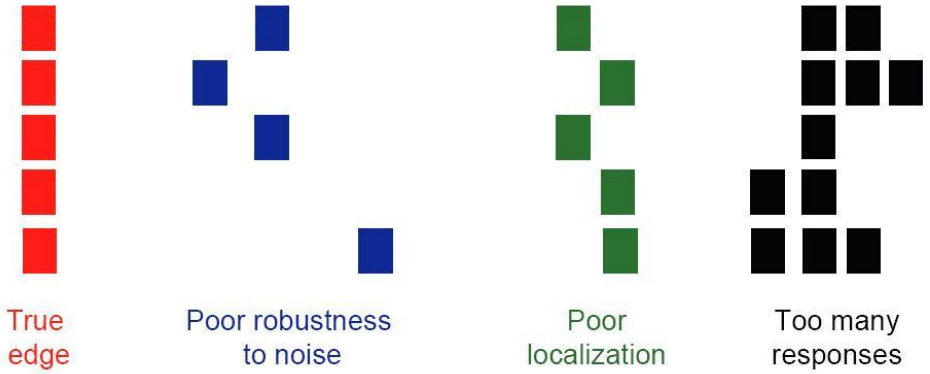
  - **Good localization:** the edges detected must be as close as possible to the true edges
  - **Single response:** the detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge



Summary

- Convolutions and Cross-Correlation
- Edge detection
- Image Gradients
- A simple edge detector

Next time: Detecting lines