6. Big Omega notation P.T $g(n) = n^3 + 2n^2 + 4n$ is $\Omega(n^3)$

sol. $g(n) \geq c \cdot n^3$

$g(n) = n^3 + 2n^2 + 4n$

find constant and n_0

$n^3 + 2n^2 + 4n \geq c \cdot n^3$

Divide the both sides with n^3

$1 + \frac{2n^2}{n^3} + \frac{4n}{n^3} \geq c.$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq c$

here $\frac{2}{n}$ and $\frac{4}{n}$ approaches 0

$1 + \frac{2}{n} + \frac{4}{n^2}$

Example $c = \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq \frac{1}{2}$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1$ $\quad (1 \geq \frac{1}{2}, n \geq 1)$

$1 + \frac{2}{n} + \frac{4}{n^2} \geq 1/2$ $\quad (n \geq 1)\ n_0 = 1$

Thus, $g(n) = n^3 + 2n^2 + 4n$ is indeeded $\Omega(n^3)$

7. Big theta notation: Determine wheather $h(n) = 4n^2 + 3n$ is $\Theta(n^2)$ or not.

$c_{12} n^2 \leq h(n) \leq c_2 n^2$

In upper bound $h(n)$ is $O(n^2)$

In lower bound $h(n)$ is $\Omega(n^2)$

$h(n) = 4n^2 + 3n$

$h(n) \leq c_1 n^2$

$4n^2 + 3n \leq c_2 n^2$

$4n^2 + 3n \leq 5n^2$

Let's $c_2 = 5$

Divide both sides by n^2

$4 + \frac{3}{n} \le 5$

$4(n) = 4n^2 + 3n$ is $O(n^2)$ $(c_2 = 5, n_0 = 1)$

lower bound:

$h(n) = 4n^2 + 3n$

$h(n) \ge c_1 n^2$

$4n^2 + 3n \ge 4n^2 + 3n \ge 4n^2$

Divide both sides with n^2

$4 + \frac{3}{n} \ge 4$

$4(n) = 4n^2 + 3n$

$h(n) = 4n^2 + 3n$

8. Let's $f(n) = n^3 - 2n^2 + n$ and $g(n) = n$ show wheather $f(n) = \Omega(g(n))$ is true or false and justify your answer

$f(n) \ge c \cdot g(n)$

substituting $f(n)$ and $g(n)$ into this inequality we get

find c and n_0 hols $n \ge n_0$

$n^3 - 2n^2 + n \ge -cn^2$

$n^3 - 2n^2 + n + cn^2 \ge 0$

$n^3 + (c-2)n^2 + n \ge 0$ $\quad (n^3 \ge 0)$

$n^3 + (1-2)n^3 + n \ge 0$

$n^3 + (1-2)n^3 + n = n^3 - n^2 + n \ge 0$ $\quad (c = 2$

$f(n) = n^3 - 2n^2 + n$ is $\Omega(g(n)) = \Omega(-n^2)$

$\therefore$ statement $f(n) = \Omega(g(n)$ is True.

9. Determine wheather $h(n) = n\log n$ is in $\Theta(n\log n)$ prove a rigorous proof your conclusion

$c_1 n\log n < h(n) \leq c_2 n\log n$

upper bound:

$h(n) \leq c_2 n\log n$

$h(n) = n\log n + n$

$n\log n + n \leq c_2 n\log n$

Divide both sides by $\log n$

$$1 + \frac{n}{n\log n} \leq 2$$

$$1 + \frac{1}{\log n} \leq 2$$

$1 + \frac{1}{\log n} \leq 2$ (simplify then $h(n)$ is $\Theta(n\log n)$
($c_2 = 2$, $n_0 = 2$)

lower bound:

$h(n) \geq c_1 n\log n$

$h(n) = n\log n + n$

$n\log n + n \geq c_1 n\log n$

Divide both sides by $n\log n$

$$1 + \frac{n}{n\log n} \geq c_1$$

$1 + \frac{1}{n\log n} \geq c_1$ (simplify)

$1 + \frac{1}{\log n} \geq 1$ ($c_1 = 1$)

$\frac{1}{\log n} \geq 0$ for all $n > 1$

$h(n)$ is $\Omega(n\log n)$ ($c_1 = 1$, $n_0 = 1$)

$h(n) = n\log n + n$ is $\Theta(n\log n)$

10. solve the following recurrence relations and find the order of growth for solutions $T(n) = 4T(n/2) + n^2$

Sol $T(1) = 1$

$T(n) = 4T(n/2) + n^2$, $T(1) = 1$

$T(n) = aT(n/b) + f(n)$

$a = 4$, $b = 2$, $f(n) = n^2$

Apply master's theorm

$T(n) = aT(n/b) + f(n)$

$f(n) = O(n^{\log_b a - 1})$ $\qquad$ $f > 0$

$T(n) = \Theta(n^{\log_b a})$

$f(n) = \Theta(n^{\log_b a})$ $\qquad$ $T(n) = \Theta(n^{\log_b a} \log n)$

$f(n) = \Omega(n^{\log_b a + 1})$, then $f(n) = f(n)$

Calculating $\log_b a$:

$\log_b a = \log_2 4 = 2$

$f(n) = n^2 = \Theta(n^2)$

$f(n) = \Theta(n^2) = \Theta(n^{\log_b a})$

$T(n) = 4T(n/2) + n^2$

$T(n) = \Theta(n^{\log_b a} \log n) = \Theta(n^2 \log n)$

order of growth

$T(n) = 4T(n/2) + n^2$ with $T(1) = 1$ is $\Theta(n^2 \log n)$