

1. Given an array of $\{4, -2, 5, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11\}$ integers and find the maximum and minimum product that can be obtained by multiplying two integers from the array

Solution:

Array is $[4, -2, 3, 10, -5, 2, 8, -3, 6, 7, -4, 1, 9, -1, 0, -6, -8, 11, 9]$
we need to consider the largest and smallest product that can be formed by selecting two numbers from the array

1. Sort the array

Sorted array

$[-9, -8, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]$

2. Identify possible candidates for maximum product

3. Identify possible candidates for minimum product.

Calculating maximum products

The two longest positive numbers are 10 and 11

$$10 \times 11 = 110$$

• The two smallest negative numbers are -9 and -8

$$-9 \times -8 = 72$$

The maximum product is 110

Calculating minimum product:

The largest positive and negative number is 11 and -9

$$11 \times -9 = -99$$

The smallest positive and negative

$$-9 \times -8 = -72$$

-99 is smaller than 72 so

maximum product = 110, and minimum product = -99

2. Demonstrate the binary search method to search for the key = 23 from the array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

Sol: Given key = 23 and
array = {2, 5, 8, 12, 16, 23, 38, 56, 72, 91}

1. Initialize pointers

low = 0 and high = 9

calculate $\text{mid} = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor = \left\lfloor \frac{0 + 9}{2} \right\rfloor = 4$

compare $\text{arr}[\text{mid}]$ with key:

$\text{arr}[4] = 16$

Since $16 < 23$ update $\text{low} = \text{mid} + 1 = 5$

calculate $\text{mid} = \left\lfloor \frac{\text{low} + \text{high}}{2} \right\rfloor = \left\lfloor \frac{5 + 9}{2} \right\rfloor = 7$

compute $\text{arr}[\text{mid}]$ with key

$\text{arr}[7] = 56$

Since $56 > 23$ update $\text{high} = \text{mid} - 1 = 6$

$\text{mid} = \left\lfloor \frac{5 + 6}{2} \right\rfloor = 5$

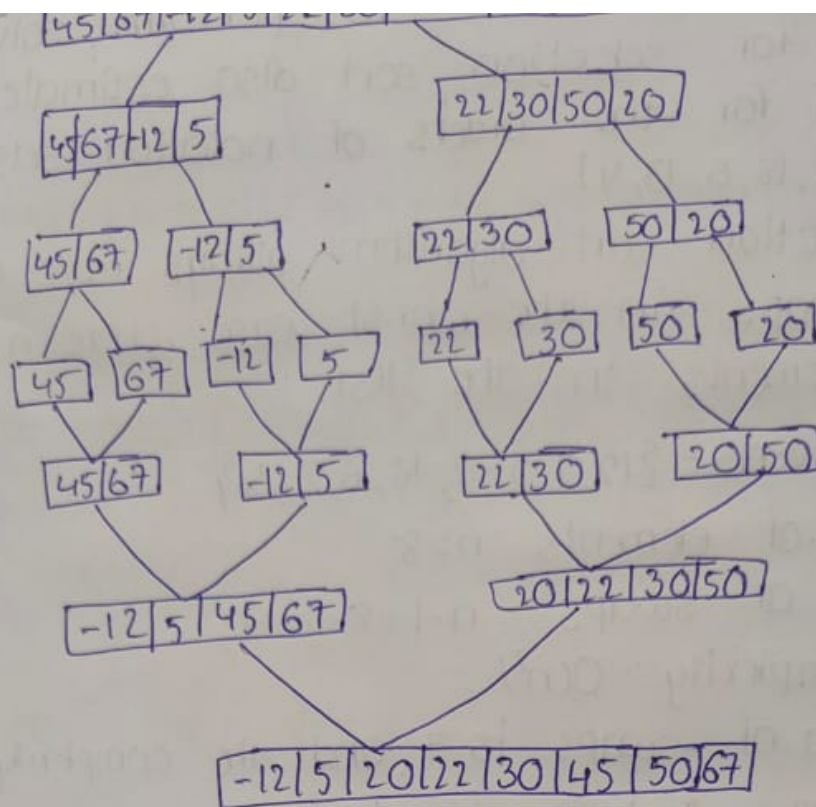
$\text{arr}[\text{mid}] = \text{arr}[5] = 23$

$23 = 23$ The key is found at index 5

\therefore key = 23 is found at index 5.

3. Apply merge sort and other list of 8 elements
data $d = [45, 67, -12, 5, 22, 30, 50, 20]$. Set up recurrence
relation for the numbers of key comparison
made by merge sort.

sol:



Sorted list = $\{-12, 5, 20, 22, 30, 45, 50, 67\}$

4. Find the no. of times to perform supplying for selection sort estimate the time.

Recurrence relation for comparisons:

$$T(n) = 2T(n/2) + O(n)$$

if $n=1$, $T(1)=0$

→ At each level of recursion most $n-1$ comparisons to merge two size n so it becomes

$$T(n) = 2T(n/2) + (n-1)$$

Solving recurrence relation we get

$$T(n) = n \log_2(n) = n+1$$

$$\therefore T(n) = O(n \log n)$$

The recurrence relation is $T(n) = 2T(n/2) + O(n)$

$$T(n) = n \log_2(n) - n + 1$$

ii. Find the no. of times to perform solving applying for selection sort also estimate time complexity for the orders of notation sets (12, 7, 5, -2, 18, 6, 13, 4)

The selection sort algorithm always makes exactly $n-1$ swaps in the worst case, where n is the no. of elements in the list

$$\text{givens} = \{12, 7, 5, -2, 18, 6, 13, 4\}$$

$$\text{No. of elements } n = 8$$

$$\text{no. of swaps} = n - 1 = 8 - 1 = 7$$

$$\text{Time complexity :- } O(n^2)$$

The no. of swaps is 7, and the complexity is $O(n^2)$

5. Find the index of the target value 10 using binary search for following of value = 10

$$\text{low} = 0 \quad \text{and} \quad \text{high} = 9$$

$$\text{mid} = \frac{\text{low} + \text{high}}{2} = \frac{0 + 9}{2} = 4$$

$$\text{list}[4] = \text{mid } 10 \quad \text{Mid} = \text{value}$$

$$\text{Since } 10 == 10 \quad \text{Mid} = 10 \quad \text{Mid} = \text{value}$$

Since $10 == 10$ the target is found at index 4

\therefore The target value = 10 is found at index 4