1. Solve the following relations.

a. $x(n) = x(n-1)+5$ or $x(1) = 0$

$x(n) = x(1)+5(n-1)$

let $x(1) = 0$:

$x(n) = 5(n-1)$

$\therefore x(n) = 5n-5$.

b. $x(n) = 3x(n-1)$ for $x\ n>1\ x(1)=4$

substituting

$x(n) = 3^{n-1} x(1)$

$x(1) = 4$

$x(n) = 4 \cdot 3^{n-1}$

$x(n) = 4.3^{n-1}$

c. $x(n) = x(n/2) + n$ for $n>1$, $x(1) = 1$ (solve for $n=2^k$)

$x(n) = n+n/2+n/4+\dots+1$

here $1+\frac{n}{2}+\frac{n}{4}+\dots+\frac{n}{2}\log n$ simplifies to $2n-1$

$\therefore x(n) = 2n-1$

d. $x(n) = x(n/3)+1$ or $n>1$ $x(1) = 1$ (solve for $n=3^k$)

$x(n) = 1+++1\dots$ (for $\log_3 n$ times)

$x(n) = \log_3 n$

$\therefore x(n) = \log_3 n$

2. Evaluate the following recurences completely

i) $T(n) = T(n/2)+1$, where $n=2K$ for all $k \geq 0$.

here

$T(n) = T(n/2)+1$

$T(n/2) = T(n/4)+1$

$T(n/4) = T(n/8)+1$

$\Rightarrow T(n) = 1+1+1 \ldots$ for $\log_2 n$ times

$\therefore T(n) = T(\frac{n}{2})+1$ for $n = 2^K$

$T(n) = \log_2 n$

$T(n) = O(\log n)$

ii $T(n) = T(n/3) + T(\frac{2n}{3}) + c_n$ where 'c' is a constant and 'n' is the input size.

$T(n) = aT(\frac{n}{b}) + f(n)$

$a=1$, $b=2$, $f(n) = cn$

i) calculate $n^{\log_b a}$

$n^{\log_3 1} = n^0 = 1$

ii) Compare $f(n)$ with $n^{\log_b a}$.

$f(n) = cn$

$f(n) = O(n^{0-1})$

iii) Apply case 3 for master theorm

if $f(n) = O(n^{\log_3 a} \log^x n)$ for some $x \geq 0$ than

$T(n) = \Theta(n^{\log_b a} \log^{K+1} n)$

since $f(n) = O(n)$

$T(n) = \Theta(\log n)$

$\therefore T(n) = T(n/3)+T(2n/3)$ then is: $T(n) = O(n\log n$

3. Consider the following recursion Algorithm

```
Min 1 (A[0 . . . . n-1])
  if n=1 return A[0]
  else temp = Min 1 [A [0 - - . n-2)]
    if temp<=A (n-1] return temp
  else
    reton A[n-1]
```

a. What does this algorithm compute?

⇒ This algorithm is designed to find the minimum element in an array 'A' of size 'n'

b. Set up recurrence relation for the algorithm basic operation count and solve it.

$T(n) = T(n-1) + 2$

* $T(1) = 1$
* $T(n) = T(n-1) + 2$

Expand $T(n) = T(n-2) + 2 + 2$

$T(n) = T(n-3) + 2 + 2 + 2$ [continue the pattern]

$T(n) = 1 + 2(n-1)$

$T(n) = 2n-1$

Best case.

4. Analyze the order of growth.

$F(n) = 2n^2+5$ and $g(n) = 7$

use the $\Omega(g(n))$ notation.

compute the limit

$$\lim_{n\to\infty} \frac{F(n)}{g(n)} = \lim_{n\to 8} \frac{2n^2+5}{7n}$$

Simplify the fraction

$$\lim_{n\to\infty} \frac{2n^2+5}{7n} = \lim_{n\to\infty}\left(\frac{2n^2}{7n} + \frac{5}{7n}\right) = \lim_{n\to\infty}\left(\frac{2}{7}n + \frac{5}{7n}\right)$$

Evaluate the limit

$$\lim_{n\to\infty}\left(\frac{2}{7}n + \frac{5}{7n}\right) = \infty$$

Conclusion

$$F(n) = \Omega(g(n))$$

$\therefore$ F(n) is asymtotically bounded below by g(n) meaning F(n) grows at least as fast a g(n). In simpler terms F(n) is a asymtotically quadratic.