

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$$

- b) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 4$
- c) Find the extreme values of the function $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$

6. a) If $u = f(x-y, y-z, z-x)$ then find the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$

- b) If $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$. Evaluate $\frac{\partial(u, v, w)}{\partial(x, y, z)}$ at $(1, -1, 0)$.

- c) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction.

Unit – IV

7. a) Find angle between the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$.

- b) Find $\frac{ds}{dx}$ and $\frac{ds}{dy}$ for the curve $x^{2/3} + y^{2/3} = a^{2/3}$

- c) Find the radius of curvature at any point on the curve $x = a \cos^3 \theta$, $y = a \sin^3 \theta$,

8. a) State and prove Lagrange's mean value theorem.

- b) Verify Rolle's theorem for the function $(x-a)^m(x-b)^n$ in (a, b) , where m and n are positive integers.

- c) State Cauchy's mean value theorem. Verify Cauchy's mean-value theorem for the functions e^x and e^{-x} in the interval (a, b)

Unit – V

9. a) Evaluate $\int_0^\infty \frac{x^2}{(1+x^2)^{7/2}} dx$

- b) Obtain the reduction formula for $\int \sin^n x dx$ and hence evaluate $\int_0^{\pi/2} \sin^n x dx$

- c) Trace the curve $y^2(a-x) = x^2(a+x)$

10. a) Evaluate $\int_0^{\pi} \frac{\sqrt{1 - \cos x}}{1 + \cos x} \sin^2 x dx$

- b) Find the surface area of the solid formed by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

- c) Find the area included between the curve $y^2(2a-x) = x^3$ and its asymptote.

BT* Bloom's Taxonomy, L* Level

NMAM INSTITUTE OF TECHNOLOGY, NITTE
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First Semester B.E. (Credit System) Degree Examinations
Supplementary Examinations – July 2018

17MA101 – ENGINEERING MATHEMATICS - I

Duration: 3 Hours

Note: Answer Five full questions choosing One full question from each Unit.

Max. Marks: 100

Marks BT*

Unit - I

1. a) Find the rank of the matrix

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

by reducing it to echelon form.

6 L*2

- b) Solve the following system of equations by Gauss-Seidel method.

$$10x + y + z = 12, \quad x + y + 10z = 12, \quad x + 10y + z = 12$$

Take $x^{(0)} = y^{(0)} = z^{(0)} = 0$ and carry out four iterations.

7 L3

- c) Using the Rayleigh's power method, find the largest eigen value and the

corresponding eigen vector with the given initial vector. $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and given

vector is $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$. Carryout 5 iterations.

7 L3

- a) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2yz + 2xz - 2xy$ to canonical form.

6 L2

- b) Show that the transformation, $y_1 = 2x_1 + x_2 + x_3, y_2 = x_1 + x_2 + 2x_3, y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation.

7 L3

- c) Diagonalize the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

7 L3

Unit - II

- a) Test for the convergence of the series,

$$\frac{1}{2\sqrt{1}} + \frac{x^2}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} + \frac{x^6}{5\sqrt{4}} + \dots \infty$$

6 L3

- b) If $x = \sin t, y = \sin pt$ prove that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} + (p^2 - n^2)y_n = 0$

7 L3

- c) Find the Maclaurin series expression of $f(x) = \sqrt{1 + \sin 2x}$ up to the term containing x^4 .

7 L3

- a) Test the convergence of the series.

$$\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^2} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^3} - \frac{4}{3}\right)^{-3} + \dots$$

6 L4

- b) Obtain the n^{th} derivative of $\log(ax+b)$

7 L3

- c) Expand $\log x$ in powers of $(x-1)$ up to 3^{rd} degree terms.

7 L3

Unit - III

- a) Show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$ where $\log u = \frac{x^3 + y^3}{3x + 4y}$.

6 L3

P.T.O.

9. a) Find the surface area of the solid generated by the revolution of the cardioid $r = a(1 - \cos \theta)$ about the initial line.

6 L1 5 2

- b) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ with usual notations.

7 L1 5 1

- c) Obtain the reduction formula for $\int \cos^n x dx$. Hence

$$\text{evaluate } \int_0^{\frac{\pi}{2}} \cos^n x dx.$$

7 L1 5 1

2. a) Evaluate i) $\int_0^{\infty} x^6 e^{-2x} dx$ using gamma function.

ii) $\int_0^{\frac{\pi}{2}} \sin^7 \theta \cos^9 \theta d\theta$ using beta & gamma function.

6 L1 5 2

- b) Find the volume generated by revolving one arch of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about the x-axis.

7 L1 5 2

- c) Using reduction formula evaluate i) $\int_0^{\frac{\pi}{6}} \sin^3 6x \cos^4 3x dx$

ii) $\int_0^{\frac{\pi}{2}} \frac{\sin^4 \theta}{(1 + \cos \theta)^2} d\theta$

7 L1 5 2

Bloom's Taxonomy, L* Level; CO* Course Outcome; PO* Program Outcome

4. a) State Rolle's theorem. Verify Rolle's theorem for $(x+2)^3(x-3)^4$ in $[-2, 3]$
- b) Obtain Taylor's series expansion of $\sin x$ in powers of $(x - \frac{\pi}{2})$ up to

terms containing $\left(x - \frac{\pi}{2}\right)^4$.

- c) Discuss the convergence of the series

i) $1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \dots, x > 0$.

ii) $\sum_{n=1}^{\infty} \sqrt{\frac{3^n - 1}{2^n + 1}}$

Unit - III

5. a) If $u = e^x \sin(yz)$ where $x = t^2, y = t - 1, z = \frac{1}{t}$ then find $\frac{du}{dt}$ at $t = 1$.
- b) If $\tan u = \frac{x^3 + y^3}{x - y}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$.
- c) If the perimeter of a triangle is constant then prove that the area of this triangle is maximum when the triangle is equilateral.
6. a) Find the unit vector normal to the surface $xyz^3z^2 = 4$ at $(-1, -1, 2)$.
- b) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 3$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.
- c) Find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + 3xyz)$.

Unit - IV

7. a) Evaluate $\iint_D xy \, dx \, dy$ over the first quadrant of the circle $x^2 + y^2 = a^2$.
- b) Using double integral find the area of the cardioid $r = a(1 + \cos \theta)$.
- c) Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

8. a) Evaluate $\int_0^1 \int_0^{x+y} \int_0^x (x+y+z) \, dz \, dy \, dx$.

- b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ by changing to polar coordinates.

- c) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$.

4. a) State Rolle's theorem. Verify Rolle's theorem for $(x+2)^3(x-3)^4$ in $[-2, 3]$
- b) Obtain Taylor's series expansion of $\sin x$ in powers of $(x - \frac{\pi}{2})$ up to

terms containing $\left(x - \frac{\pi}{2}\right)^4$.

- c) Discuss the convergence of the series

i) $1 + \frac{x}{2} + \frac{x^2}{3^2} + \frac{x^3}{4^3} + \dots, x > 0$.

ii) $\sum_{n=1}^{\infty} \sqrt{\frac{3^n - 1}{2^n + 1}}$

Unit - III

5. a) If $u = e^x \sin(yz)$ where $x = t^2, y = t - 1, z = \frac{1}{t}$ then find $\frac{du}{dt}$ at $t = 1$.
- b) If $\tan u = \frac{x^3 + y^3}{x - y}$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$ and $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u$.
- c) If the perimeter of a triangle is constant then prove that the area of this triangle is maximum when the triangle is equilateral.
6. a) Find the unit vector normal to the surface $xyz^3z^2 = 4$ at $(-1, -1, 2)$.
- b) A particle moves along the curve $x = t^3 + 1, y = t^2, z = 2t + 3$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.
- c) Find $\text{div} \vec{F}$ and $\text{curl} \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + 3xyz)$.

Unit - IV

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8. a) Evaluate $\int_0^1 \int_0^{x+y} \int_0^x (x+y+z) \, dz \, dy \, dx$.

- b) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$ by changing to polar coordinates.

- c) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16a^2}{3}$.

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First Semester B.E. (Credit System) Degree Examinations
November - December 2018

18MA101 – ENGINEERING MATHEMATICS - I

Max. Marks: 100

Duration: 3 Hours

Note: Answer Five full questions choosing One full question from each Unit.

Marks BT* CO* PO*

Unit – I

a) Find the rank of the following matrix using elementary row transformation.

$$\begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$$

6 L*1 1 2

b) Test for consistency and solve the system of equations by Gauss elimination method.

$$\begin{aligned} 2x - 7y + 4z &= 9 \\ -3x + 8y + 5z &= 6 \\ x + 9y - 6z &= 1 \end{aligned}$$

7 L3 1 2

c) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

7 L1 1 2

a) Find the spectral and modal matrix for $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

6 L1 1 2

b) Find the inverse of the following matrix by using elementary row operations.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

7 L3 1 2

c) Show that the equations $y_1 = 2x_1 + 3x_2 + 4x_3$, $y_2 = 4x_1 + 3x_2 + x_3$, $y_3 = x_1 + 2x_2 + 4x_3$ represent a regular linear transformation. Find the inverse of this transformation.

7 L1 1 2

Unit – II

a) (i) State D'Alembert's ratio test.

(ii) Test for the convergence of the series $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \dots$

State and prove Cauchy's mean value theorem.

6 L3 2 2

7 L1 2 1

c) Obtain the Maclaurin's expansion of $e^x \cos x$ up to third degree terms.

7 L1 2 2