

## Unit - IV

Make up - January 2016

- a) With usual notation prove that  $\rho = \frac{(1+y'^2)^{3/2}}{y^2}$
- b) State Cauchy's Mean value theorem. Verify Cauchy's Mean value theorem for  $f(x) = \frac{1}{x^2}$ ,  $g(x) = \frac{1}{x} \ln[a, b]$ ,  $b > a > 0$
- c) Find the angle of intersection between the curves  $r = a \log \theta$  and  $r = \frac{a}{\log \theta}$
- a) State and Prove Lagrange's Mean value theorem.
- b) i) Find  $\frac{ds}{d\theta}$  for the curve  $r = a(1 - \cos \theta)$  ii) Find  $\frac{ds}{dx}$  for the curve  $ay^2 = x^3$
- c) Find the pedal equation for  $r^m = a^m \cos m\theta$

7 L2

7 L1

7 L3

6 L1

6 L3

7 L2

7 L1

7 L3

6 L1

6 L3

## Unit - V

- a) Obtain the reduction formula  $\int \cos^n x dx$  and hence evaluate  $\int_0^{\frac{\pi}{2}} \cos^n x dx$
- b) Find the area of the cardioid:  $r = a(1 - \cos \theta)$
- c) Evaluate
- i)  $\int_0^{\infty} \frac{t^5}{(1+t^2)^7} dt$
- ii)  $\int_0^{\frac{\pi}{6}} \cos^4 3\theta \sin^3 6\theta d\theta$
- a) Trace the curve  $r = a \sin 2\theta$
- b) Find the length of the arc of the parabola  $x^2 = 4ay$  measured from the vertex onto extremity of the latus-rectum.
- c) Find the volume of the solid obtained by revolving the cisoid  $y^2(2a - x) = x^3$  about its asymptote.

$$\int_0^{\frac{\pi}{2}} \cos^n x dx$$

7 L2

7 L4

7 L3

6 L5

7 L1

7 L4

7 L3

6 L3

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b) Test for the convergence of the series (i)  $\sum_{n=1}^{\infty} 3^n \cdot \left(\frac{n}{n+1}\right)^{n^2}$

(ii)  $2.3 + \frac{3.4}{2^2\sqrt{2}} + \frac{4.5}{3^2\sqrt{3}} + \dots \dots \dots \infty$

c) (i) State Leibnitz theorem.

(ii) If  $y = \tan^{-1}(x)$ , then show that  $(x^2+1)y_{n+2} + 2x(n+1)y_{n+1} + n(n+1)y_n = 0$

4. a) Obtain the Maclaurin's series expansion of the function  $f(x) = \log(1+e^x)$  upto terms containing  $x^4$

b) (i) State Cauchy's integral test.

(ii) Test for the convergence of the series  $\sum_{n=1}^{\infty} \frac{x^{(2n-2)}}{(n+1)\sqrt{n}}$

c) Test for the convergence of the series (i)  $\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \dots \dots \dots \infty$

(ii)  $1 + \frac{2!}{2^2} + \frac{3!}{3^3} + \dots \dots \dots \infty$

Unit - III

5. a) Find the volume of the greatest rectangular parallelopiped that can be inscribed in the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

b) The diameter and height of a right circular cone are measured as 4 cm and 6cm respectively, with a possible error of 0.1 cm. Find approximately the maximum area ( $=2\pi rh$ ).

c) Expand  $e^x \log(1+y)$  in powers of  $x$  and  $y$  upto third degree terms.

6. a) If  $u = x + y + z$ ,  $uv = y + z$  and  $uvw = z$  show that  $\frac{\partial(x, y, z)}{\partial(u, v, w)} = u^2 v$ .

b) If  $u$  is a homogeneous function of degree  $n$  in  $x$  and  $y$  then prove that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = nu$ . Hence prove that  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u$

c) Examine the function  $f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$  for extreme values.

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a) With usual r

b) State Cauc

$f(x) = \frac{1}{x^2}$

c) Find the ang

a) State and Pr

b) Find  $\frac{ds}{d\theta}$

c) Find the ped

a) Obtain the rec

b) Find the area

c) Evaluate

a) Trace the cu

b) Find the len

c) Find the volu

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Autonomous Institution affiliated to VTU, Belagavi)

First Semester B.Tech. (Autonomous Institution affiliated to VTU, Belagavi)

Make up Examinations – January 2016

Duration: 3 Hours

Max. Marks: 100

**Note: Answer Five full questions choosing One full question from each Unit.**

Marks BT.

- a) Define the Rank of a Matrix. Find the rank of the matrix

3	4	5	6	7
4	5	6	7	8
5	6	7	8	9
10	11	12	13	14
15	16	17	18	19

using elementary row transformations.

- b) Check whether the system of equations

$$x_1 + x_2 - x_3 = 0$$

$$2x_1 - x_2 + x_3 = 3$$

$4x_1 + 2x_2 - 2x_3 = 2$  is consistent. Hence solve it by using Gauss elimination method.

- c) Check whether the linear transformation

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

is orthogonal.

- a) Using Rayleigh's power method, obtain the largest eigen value and the corresponding eigen vector of the matrix  $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ , select  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

as the initial eigen vector and carry out five iterations.

- b) If  $Y=AX$  is an orthogonal transformation with  $A = \begin{bmatrix} 1 & 2 & a \\ \frac{1}{3} & 2 & b \\ 2 & -2 & c \end{bmatrix}$  find a, b, c and

Find the spectral and modal matrix of  $A = \begin{bmatrix} 5 & 3 \\ 3 & 5 \end{bmatrix}$

- c) Find the spectral and modal matrix of  $A =$

Unit - II

- a) (i) State Cauchy's root test.  
(ii) Test for the convergence of the series  $\frac{2}{4} + \frac{2.4}{4.7} + \frac{2.4.6}{4.7.10} + \dots \infty$

6. a) If  $p$  be the radius of curvature at any point  $P$  on the parabola  $y^2 = 4ax$  and  $S$  be its focus then show that  $p^2$  varies as  $(SP)^3$
- b) State and prove Lagrange's mean value theorem.
- c) Show that following curves intersect each other orthogonally  $r^n = a^n \cos n\theta$

7 L3  
7 L2  
6 L3

## Unit - V

9. a) Obtain the reduction formula for  $\int \sin^n x \, dx$ . Hence evaluate  $\int_0^{\pi/2} \sin^n x \, dx$

7 L4

- b) Evaluate (i)  $\int_0^{2a} \frac{x^3}{\sqrt{2ax - x^2}} \, dx$

$$(ii) \int_0^{\infty} \frac{dx}{(1+x^2)^6}$$

7 L5  
6 L3

- c) Find the entire length of the cardioid  $x = a(1 + \cos \theta)$ .

7 L4

10. a) Trace the curve  $x = a \cos^3 t$ ,  $y = a \sin^3 t$ .

- b) Find the area enclosed between one arch of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$  and its base.

7 L3

- c) Find the surface area generated by the revolution of the portion of the parabola  $y^2 = 4ax$  bounded between the vertex and the upper end of the latus rectum, about the  $x$ -axis.

6 L3

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- c) (i) State Leibnitz theorem.  
(ii) if  $y = (x^2 - 1)^n$ , then show that  $(x^2 - 1)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$

4. a) (i) State D'Alembert's ratio test.

(ii) Test for the convergence of the series  $\sum_{n=1}^{\infty} \left(\frac{n+2}{n+3}\right)^n x^n, x > 0$ .

- b) Obtain Taylor's series expansion of  $y = \tan^{-1}(x)$  at  $x=1$  upto terms containing  $(x-1)^3$

- c) State Cauchy's integral test. Using Cauchy's integral test, test for convergence of the series  $\sum_{n=1}^{\infty} \frac{5e^n}{e^{2n} + 16}$

### Unit - III

5. a) If  $u = 2xy, v = x^2 - y^2$  where  $x = r \cos \theta, y = r \sin \theta$  find  $\frac{\partial(u, v)}{\partial(r, \theta)}$ .

- b) State and prove Euler's theorem. If  $\log u = \frac{x^3 + y^3}{3x + 4y}$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u \log u$

- c) Expand  $f(x, y) = (1 + x - y)^{-1}$  in powers of  $(x-1)$  and  $(y-1)$  upto second degree terms.

6. a) Find the possible error in surface area  $(= 4\pi r^2)$  and volume  $(= \frac{4}{3}\pi r^3)$  of a sphere of radius  $r$ , if  $r$  is measured as 18.5 inches with a possible error of 0.1 inch.

- b) If  $u = f(r, s, t)$  where  $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$  then show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 0$ .

- c) Determine the point in the plane  $3x - 4y + 5z = 50$  nearest to the origin.

7. a)

### Unit - IV

- b) State Rolle's theorem. Verify Rolle's theorem for the function  $f(x) = |x-a|^m |x-b|^n$  in  $[a, b]$ ,  $b > a$  and  $m, n > 1$

- c) Find the pedal equation of the curve  $a^2 = r^2 \cos 2\theta$

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- a) If  $p$  be the radius of curvature of the curve  $r^n = b^n \sin m\theta$  show that  $r^n = b^n \sin m\theta$

- b) State and prove the theorem of L'Hopital.

where  $n$  is a positive integer

- c) Evaluate  $\int_0^{\pi/2} \sin x \cos x dx$

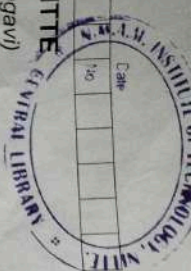
(ii)

- d) Find the area of the region bounded by the curve  $y = x^2$  and the line  $y = 4$

- e) Trace the curve  $y^2 = 4ax$  and find the area of the region bounded by the curve and the x-axis.

- f) Find the area of the region bounded by the curve  $y = x^2$  and the line  $y = 4$

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Time: 3 Hours

Note: Answer Five full questions choosing One full question from each Unit.

Max. Marks: 100

**Unit - I**

Marks BT\*

Using Rayleigh's power method, obtain the largest eigen value and the corresponding eigen vector of the matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}, \text{ select } \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

as the initial eigen vector and carry out five iterations.

7 L\*3

Test for consistency and solve the system of equations

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + 9z &= 16 \end{aligned}$$

using Gauss elimination method.

7 L3

If A=

$$\begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$$

and B=

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$$

Find rank of A, rank of B and rank

6 L3

of A+B using elementary row transformations.

1) Show that the equations

$$\begin{aligned} y_1 &= x_1 + 2x_2 + 5x_3 \\ y_2 &= 2x_1 + 4x_2 + 11x_3 \\ y_3 &= -x_2 + 2x_3 \end{aligned}$$

represent a regular linear transformation. Find the inverse of this transformation.

7 L5

2) Find the spectral and modal matrix of A=

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Reduce the quadratic form  $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$  into canonical form.

7 L3

**Unit - II**

1) Use the ratio test.

Convergence of the series  $\frac{1}{1^2} + \frac{1+2}{1^2+2^2} + \frac{1+2+3}{1^2+2^2+3^2} + \dots \infty$

6 L3

2) Use the ratio test.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

7 L3

$$(ii) \frac{3}{4} + \frac{3.5}{4.8} + \frac{3.6.5}{4.8.12} + \dots \infty$$



- a) Obtain the reduction formula  $\int \sin^n x \, dx$ . Hence evaluate  $\int_0^{\frac{\pi}{2}} \sin^n x \, dx$  where  $n$  is a positive integer. 7 L3
- b) Trace the curve  $r = a \sin 3\theta$ . 7 L3
- c) Evaluate (i)  $\int_0^1 x^2(1-x^2)^2 \, dx$  (ii)  $\int_0^2 \frac{x^4}{\sqrt{4-x^2}} \, dx$ . 6 L3
- a) Determine the area bounded by the cissoid  $y^2(2a-x) = x^3$ ,  $a > 0$  with explanation and asymptote. 7 L5
- b) Find the length of the parabola  $y^2 = 8x$  cut off by the line  $3y = 8x$ . 7 L3
- c) Obtain the volume of the solid generated by the revolution of the cycloid  $x = a(\theta - \sin \theta)$ ,  $y = a(1 - \cos \theta)$ ,  $a > 0$  about its base. 6 L3

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4. a) Find the  $n^{\text{th}}$  derivative of  $y = (ax + b)^m$  where  $m > n$  and hence find the  $n^{\text{th}}$  derivative of  $\frac{1}{ax + b}$ .

- b) Obtain the Taylor's series of  $\sin x$  in powers of  $\left(x - \frac{\pi}{2}\right)$  up to terms containing

$$\left(x - \frac{\pi}{2}\right)^4.$$

- c) Test the convergence of the following series:

$$\frac{x}{1 \cdot 2} + \frac{x^2}{3 \cdot 4} + \frac{x^3}{5 \cdot 6} + \dots \infty \quad (x > 0)$$

## Unit - III

5. a) Expand the function  $f(x, y) = x^2 + xy + y^2$  at  $(3, 4)$  up to third degree terms.

- b) If  $\tan u = \left(\frac{x^3 + y^3}{x - y}\right)$ , then show that (i)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

$$(ii) \quad x^2 \frac{\partial^2 u}{\partial x^2} + y^2 \frac{\partial^2 u}{\partial y^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} = 2 \cos 3u \sin u.$$

- c) If  $y_1 = \frac{x_2 x_3}{x_1}, y_2 = \frac{x_3 x_1}{x_2}$  and  $y_3 = \frac{x_1 x_2}{x_3}$ , find  $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)}$ .

6. a) Find the possible percentage error in computing the resistance  $r$  from the formula  $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$ , if both  $r_1$  and  $r_2$  are in errors by 2%.

- b) A rectangular box open at the top is to have volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

- c) If  $u = x + \frac{y^2}{x}, v = \frac{y^2}{x}$ , then prove that  $JJ' = 1$ .

7.

## Unit - IV

- a) Find  $\frac{ds}{d\theta}$  for the curve  $r^2 = a^2 \cos 2\theta$ .

- b) If  $\rho$  is the radius of curvature at any point  $P$  on the parabola  $y^2 = 4ax$  and  $S$  be its focus, then show that  $\rho^2$  varies as  $(SP)^3$

- c) State and prove Lagrange's mean value theorem.

8.

- a) Find the pedal equation of the curve  $\frac{2a}{r} = 1 - \cos \theta$ .

- b) For a polar curve  $r = f(\theta)$  prove that the radius of curvature

$$\rho = \frac{(r^2 + r_1^2)^{\frac{3}{2}}}{r^2 + 2r_1 r_2 - r_2^2} \text{ where } r_1 = f'(\theta), r_2 = f''(\theta)$$

- c) Show that the tangents drawn at the extremities of any chord of the cardioid  $r = a(1 + \cos \theta)$  which passes through the pole are perpendicular to each other.

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- a) Obtain the

- b) Trace the

- c) Evaluate (

- a) Determine

- b) Find the le

- c) Obtain the

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**NMAM INSTITUTE OF TECHNOLOGY, NITTE**  
(An Autonomous Institution affiliated to VTU, Belagavi)  
**First Semester B.E. (Credit System) Degree Examinations**

November - December 2016

**16MA101 - ENGINEERING MATHEMATICS - I**

Duration: 3 Hours

**Note: Answer Five full questions choosing One full question from each Unit.**

**Unit - I**

**Marks BT\***

- a) Find the modal and spectral matrix of  $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$  6 L\*3

- b) Using the Gauss elimination method solve

$$\begin{aligned} x + 2y + 3z &= 9 \\ 2x + y - 2z &= -1 \\ 3x - y - 3z &= -4 \end{aligned}$$

7 L2

- c) Prove that the linear transformation  $y_1 = 3x_1 - 3x_2 + 4x_3$ ,  $y_2 = 2x_1 - 3x_2 + 4x_3$  and  $y_3 = -x_2 + x_3$  is a regular linear transformation. Also find the inverse of this transformation. 7 L3

- a) Determine the rank of the following matrix

$$\begin{bmatrix} 1 & -1 & 0 & 2 \\ 2 & -1 & 2 & 5 \\ 3 & -4 & -2 & 6 \\ -1 & 0 & -1 & -3 \end{bmatrix}$$

6 L3

- b) Reduce the quadratic form,  $5x^2 + 2y^2 + 2z^2 + 2zy$ , to canonical form. Also specify the matrix of the transformation.

7 L2

- c) Using the Rayleigh's power method, find the dominant eigen value and the corresponding eigen vector of the matrix  $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ . Take the initial approximation to the eigen vector as  $[1, 0, 0]^T$  7 L3

**Unit - II**

- a) Using MacLaurin's series obtain the expansion of  $e^{\cos x}$  as far as terms containing  $x^4$ . 6 L3

- b) State Cauchy's integral test and use it to show that  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converges for  $p > 1$  and diverges for  $p \leq 1$ . 7 L5

- c) State Cauchy's root test and examine the nature of the series:  $\sum \left( \frac{n+2}{n+3} \right)^n x^n$  where  $x > 0$ . 7 L3