

**UNIT - I**  
**CO2 - Partial differentiation**

1.  $f(x, y) = x^2 + xyz + z$  Find  $f_x$  at (1,1,1)  
a) 0      b) 1      c) 3      d) -1

2.  $f(x, y) = \sin(xy) + x^2 \ln(y)$  Find  $f_{yx}$  at  $(0, \pi/2)$   
a) 33      b) 0      c) 3      d) 1

3.  $f(x, y) = x^2 + y^3$ ;  $X = t^2 + t^3$ ;  $y = t^3 + t^9$  Find  $df/dt$  at  $t=1$ .  
a) 0      b) 1      c) -1      d) 164

4.  $f(x, y) = \sin(x) + \cos(y) + xy^2$ ;  $x = \cos(t)$ ;  $y = \sin(t)$  Find  $df/dt$  at  $t = \pi/2$   
a) 2      b) -2      c) 1      d) 0

5.  $f(x, y, z, t) = xy + zt + x^2 yzt$ ;  $x = k^3$ ;  $y = k^2$ ;  $z = k$ ;  $t = \sqrt{k}$   
Find  $df/dt$  at  $k = 1$   
a) 34      b) 16      c) 32      d) 61

6.  $f(x, y) = \sin(y + yx^2) / 1 + x^2$  Value of  $f_{xy}$  at (0,1) is  
a) 0      b) 1      c) 67      d) 90

7. Necessary condition of euler's theorem is \_\_\_\_\_  
a) z should be homogeneous and of order n  
b) z should not be homogeneous but of order n  
c) z should be implicit  
d) z should be the function of x and y only

8. If  $u = x^2 \tan^{-1}(y/x) - y^2 \tan^{-1}(x/y)$  then  $\frac{\partial^2 u}{\partial x \partial y}$  is  
a)  $\frac{x^2 + y^2}{x^2 - y^2}$       b)  $\frac{x^2 - y^2}{x^2 + y^2}$       c)  $\frac{x^2}{x^2 + y^2}$       d)  $\frac{y^2}{x^2 + y^2}$

9. If  $f(x, y)$  is a function satisfying euler's theorem then?

- a)  $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1)f$   
b)  $\frac{1}{x^2} \frac{\partial^2 f}{\partial x^2} + \frac{2}{xy} \frac{\partial^2 f}{\partial x \partial y} + \frac{1}{y^2} \frac{\partial^2 f}{\partial y^2} = n(n-1)f$   
c)  $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = nf$   
d)  $y^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + x^2 \frac{\partial^2 f}{\partial y^2} = n(n-1)f$

10. In euler theorem  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$ , here 'n' indicates?  
a) order of z      b) degree of z      c) neither order nor degree      d) constant of z

11. For homogeneous function with no saddle points we must have the minimum value as

- a) 90      b) 1      c) equal to degree      d) 0

12.  $f(x, y) = \sin(y/x)x^3 + x^2y$  find the value of  $f_x + f_y$  at  $(x,y)=(4,4)$ .

- a) 0      b) 78      c)  $4^2 \cdot 3(\sin(1) + 1)$       d) -12

13.  $f(x, y) = x^3 + xy^2 + 901$  satisfies the Euler's theorem.

- a) True      b) False

14. If  $z = x^n f(y/x)$  then?

- a)  $y \frac{\partial z}{\partial x} + x \frac{\partial z}{\partial y} = nz$       b)  $\frac{1}{y} \frac{\partial z}{\partial x} + \frac{1}{x} \frac{\partial z}{\partial y} = nz$   
c)  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz$       d)  $\frac{1}{x} \frac{\partial z}{\partial x} + \frac{1}{y} \frac{\partial z}{\partial y} = nz$

15. If  $z = e^{\frac{x^2+y^2}{x+y}}$  then,  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$  is?

- a) 0      b)  $z \ln(z)$       c)  $z^2 \ln(z)$       d)  $z$

16. Relative error in  $x$  is?

- a)  $\delta x$       b)  $\delta x/x$       c)  $\delta x/x * 100$       d) 0

19. The Jacobian of  $p, q, r$  w.r.t  $x, y, z$  given  $p = x + y + z, q = y + z, r = z$  is \_\_\_\_\_

- a) 0      b) 1      c) 2      d) -1

20. Given  $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$  then the value of  $\frac{\partial(u,v,w)}{\partial(x,y,z)}$  is \_\_\_\_\_

- a) 4      b) -4      c) 0      d) 1

21. If  $u = x + 3y^2 - z^3, v = 4x^2yz, w = 2z^2 - xy$  then  $\frac{\partial(u,v,w)}{\partial(x,y,z)}$  at  $(1,1,1)$ .

- a) -184      b) -90      c) 20      d) 40

22. If  $x = r \cos \theta, y = r \sin \theta$  then the value of  $\frac{\partial(x,y)}{\partial(r,\theta)}$  is \_\_\_\_\_

- a) 1      b) 0      c)  $r$       d)  $\frac{1}{r}$

23. If  $u + v = e^x \cos y$  and  $u - v = e^x \sin y$  the value of  $J\left(\frac{u,v}{x,y}\right)$  is \_\_\_\_\_

- a)  $e^{2x}$       b)  $\frac{e^{2x}}{2}$       c)  $\frac{-e^{2x}}{2}$       d) 0

24. Which among the following is the definition of Jacobian of  $u$  and  $v$  w.r.t  $x$  and  $y$ ?

- a)  $J\left(\frac{x,y}{u,v}\right)$       b)  $J\left(\frac{u,v}{x,y}\right)$       c)  $\frac{\partial(x,y)}{\partial(u,v)}$       d)  $\frac{\partial(u,v)}{\partial(x,y)}$

25. Given  $f(x,y) = e^x \cos y$ , what is the value of the fifth term in Taylor's series near  $(1, \frac{\pi}{4})$  where it is expanded in increasing order of degree & by following algebraic identity rule?

- a)  $\frac{-e(x-1)(y-\frac{\pi}{4})}{\sqrt{2}}$       b)  $-\sqrt{2}e(x-1)\left(y-\frac{\pi}{4}\right)$   
c)  $\frac{e(x-1)^2}{\sqrt{2}}$       d)  $\frac{e(y-\frac{\pi}{4})^2}{\sqrt{2}}$

26. Consider the  $f(x, y) = x^2 + y^2 - a$ . For what values of  $a$  do we have critical points for the function.

- a) independent of  $a$                       b) for any real number except zero  
c)  $a \in (0, +\infty)$                       d)  $a \in (-1, 1)$

27.  $f(x, y) = \sin(x) \cdot \cos(y)$  Which of the following is a critical point?

- a)  $\left(\frac{\pi}{4}, \frac{\pi}{4}\right)$                       b)  $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$                       c)  $\left(0, \frac{\pi}{4}\right)$                       d)  $(0, 0)$

28. The point  $(0,0)$  in the domain of  $f(x, y) = \sin(xy)$  is a point of \_\_\_\_\_

- a) Saddle                      b) Minima                      c) Maxima                      d) Constant

29. Maximize the function  $x + y - z = 1$  with respect to the constraint  $xy=36$ .

- a) 0                      b) -8                      c) 8                      d) No Maxima exists

30. A partial differential equation requires

- a) exactly one independent variable                      b) more than one dependent variable  
c) two or more independent variables                      d) equal number of dependent and independent variables

31. If  $u = e^x(x \cos y - y \sin y)$ , then  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \dots$

- a) 0                      b)  $u$                       c)  $e^u$                       d) none

32. If  $x = uv$ ,  $y = u/v$  then  $\frac{\partial(x,y)}{\partial(u,v)}$  is,

- a)  $-2u/v$                       b)  $-2v/u$                       c) 0                      d) 1

33. If  $J_1 = \frac{\partial(u,v)}{\partial(x,y)}$ ,  $J_2 = \frac{\partial(x,y)}{\partial(u,v)}$  then  $J_1 J_2$  is

- a) 2                      b) 0                      c) 1                      d) none

34. If  $u = x^y$ , then  $\frac{\partial u}{\partial x}$  is

- a) 0                      b)  $yx^{y-1}$                       c)  $x^y \log x$                       d) none

35. If  $u = x^y$ , then  $\frac{\partial u}{\partial y}$  is

- a) 0                      b)  $yx^{y-1}$                       c)  $x^y \log x$                       d) none

36. If  $u = x^3 + y^3$ , then  $\frac{\partial^2 u}{\partial x \partial y}$  is equal to

- a) -3                      b) 3                      c) 0                      d)  $3x+3y$

37. If  $u = x^2 + 2xy + y^2 + x + y$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to

- a)  $2u$                       b)  $u$                       c) 0                      d) none

38. If  $u = \log \frac{x^2}{y}$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to

- a)  $2u$                       b)  $3u$                       c)  $u$                       d) 1

39. If  $A = f_{xx}(a, b)$ ,  $B = f_{xy}(a, b)$ ,  $C = f_{yy}(a, b)$  then  $f(x, y)$  will have a maximum at  $(a, b)$  if

- a)  $f_x = 0, f_y = 0, AC < B^2$  and  $A < 0$                       b)  $f_x = 0, f_y = 0, AC = B^2$  and  $A > 0$   
c)  $f_x = 0, f_y = 0, AC > B^2$  and  $A > 0$                       d)  $f_x = 0, f_y = 0, AC > B^2$  and  $A < 0$

40. If  $z = \sin^{-1} \frac{\sqrt{x^2+y^2}}{x+y}$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to

- a) 0                      b) 1/2                      c) 1                      d) 2

41. If  $u = \sin^{-1}(x/y) + \tan^{-1}(y/x)$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is equal to

- a) u                      b) 2u                      c) 3u                      d) 0

42. If an error of 1% is made in measuring its length and breadth, the percentage error in the area of a rectangle is

- a) 0.2%                      b) 0.02%                      c) 2%                      d) 1%

43.  $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$  is a homogeneous function of degree.....

- a) 1                      b) 2                      c) 0                      d) 1/2

44. If u and v are functions of r, s where r, s are functions of x, y then  $\frac{\partial(u,v)}{\partial(r,s)} \cdot \frac{\partial(r,s)}{\partial(x,y)} = \dots$

- a)  $\frac{\partial(u,v)}{\partial(x,y)}$                       b)  $\frac{\partial(u,v)}{\partial(r,s)}$                       c)  $\frac{\partial(r,s)}{\partial(x,y)}$                       d) none

45. The necessary conditions for a function  $f(x, y)$  to have an extreme at (a, b) are.....

- a)  $f_x > 0, f_y > 0$                       b)  $f_x < 0, f_y > 0$                       c)  $f_x = 0, f_y = 0$                       d)  $f_x < 0, f_y < 0$

46. If  $u = (x - y)^4 + (y - z)^4 + (z - x)^4$ , then  $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$  is

- a) 1                      b) u                      c) 4u                      d) 0

47. If  $u = \cos^{-1}(x/y) + \tan^{-1}(y/x)$  then  $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$  is

- a) u                      b) 2u                      c) 0                      d) 1

48. If  $u = f(x + ay) + g(x - ay)$  then  $\frac{\partial^2 u}{\partial y^2}$  equals

- a)  $\frac{\partial^2 u}{\partial x^2}$                       b)  $a \frac{\partial^2 u}{\partial x^2}$                       c)  $a^2 \frac{\partial^2 u}{\partial x^2}$                       d)  $\frac{\partial^2 u}{\partial x \partial y}$

49. If  $u = x^4 + y^4 + 3x^2 y^2$ , then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$  is

- a) u                      b) 3u                      c) 4u                      d) 2u

50. If  $u = f(y/x)$  then

- a)  $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0$                       b)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$   
c)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$                       d)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$

**Answers: 1)c 2)d 3.)d 4)b 5)b 6)a 7)a 8.)b 9)a 10)a 11)d 12)c 13)b 14)c 15)b 16)b 17)a 18)a 19)b 20)a 21)a 22)c 23)c 24)b 25)a 26)a 27)c 28)d 29)d 30) 31)a 32)b 33)c 34)b 35)c 36)c 37)d 38) 39)d 40)a 41)a 42)d 43)c 44)a 45)c 46)d 47)c 48)c 49.)c 50)b**