6. a) Prove that $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\,d\theta \cdot \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$.

b) i) Define the beta function $\beta(m,n)$ and show that $\beta(m,n) = \beta(n,m)$.

ii) Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta$ in terms of Gamma function.

c) Show that $\beta(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}}\,dx$.

Unit – IV

7. a) Find the Laplace transform of (i) $\frac{e^{-at} - e^{-bt}}{t}$, (ii) $t\sin^2 t$

b) Rewrite the function $f(t) = \begin{cases} t^2 & 0<t<2 \\ 4 & 2\le t<4 \\ 0 & t\ge 4 \end{cases}$ using unit step function and find its transform.

c) If f(t) is a periodic function with period T so that f(t+ T)= f(t) for all values of t, P

L(f(t)) = $\frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt$.

8. a) Find the inverse Laplace transform of (i) $\frac{s^2}{(s+2)^3}$ (ii) $\frac{s+4}{(s^2+2s+2)}$

b) Solve $x''(t) + 4x(t) = 2t - 8\,;\, x(0) = 1,\quad x'(0) = 0$ by the method of transform.

c) Find the inverse Laplace transform of $\frac{4s+5}{(s+2)^2(s-1)^2}$ using partial fractions.

Unit – V

9. a) Construct a partial differential equation by eliminating the function F, from the $F(x+y+z, xy+z^2) = 0$.

b) Solve $(y+z)p - (z+x)q = x - y$ by Lagrange's method

c) Use the method of separation of variables to solve $\frac{\partial z}{\partial x} = 2\frac{\partial z}{\partial y} + z$, $z(x,0) = 6e^{-3x}$.

10. a) Construct a partial differential equation by eliminating the constants a, b from the equations $i)\, z = ax^2 + by^2$ $\quad ii)\, z = ax + by + a^2 + b^2$

b) Solve $x^2\frac{\partial u}{\partial x} + y^2\frac{\partial u}{\partial y} = 0$ by the method of separation of variables.

c) Derive one dimensional heat flow equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

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NMAM INSTITUTE OF TECHNOLOGY, NITTE

(An Autonomous Institution affiliated to VTU, Belagavi)

Second Semester B.E. (Credit System) Degree Examinations

April - May 2015

14MA201 - ENGINEERING MATHEMATICS - II

Duration: 3 Hours | Max. Marks: 100

*Note: Answer **Five full** questions choosing **One full** question from **each Unit**.*

Unit – I

1. a) Solve $p^2 + 2py\cot x = y^2$ — 6

   b) Solve $(x^2y^2 + xy + 1)ydx + (x^2y^2 - xy + 1)xdy = 0$ — 7

   c) In a single closed electric circuit ,the current I at time t is governed by the differential equation $E - RI - L\dfrac{dI}{dt} = 0$. Show that the current increases with time and that it approaches $\dfrac{E}{R}$ as a limit, given that a constant e.m.f E is impressed at time t=0,no current having flowed previously. — 7

2. a) Solve $x^2 + p^2x = yp$ — 6

   b) Solve $xdy + ydx - y^2 \log x dx = 0$ — 7

   c) Find the orthogonal trajectories of the family $r = 4a\sec\theta\tan\theta$ — 7

Unit – II

3. a) Solve the differential equation $(D^3 + D^2 + D + 1)y = \cos 2x$ — 6

   b) Solve the differential equation $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} - 12y = x^3\log x$ — 7

   c) Solve $(D^2 - 2D + 3)y = x^2 + \cos x$ using the method of undetermined coefficients. — 7

4. a) Solve the differential equation $(D-7)^2 y = 2e^{7x} - 5$. — 6

   b) Solve the differential equation $(D^3 - D)y = 2x + 1 + 4\cos x + 2e^x$ — 7

   c) A spring is such that 1.96kg weight stretches it 19.6cms,an impressed force $\dfrac{1}{2}\cos 8t$ is acting on the spring. If the weight is started from the equilibrium point with an imparted upward velocity of 14.7 cms. per sec, determine the position of the weight as a function of time. — 7

Unit – III

5. a) Evaluate $\int_0^1\int_0^2\int_1^2 x^2yz\,dxdydz$. — 6

   b) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar co-ordinates. — 7

   c) Change the order of integration and hence evaluate $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dydx$ — 7

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14MA201

6. a) Evaluate $\int_0^1 \frac{x^\alpha - 1}{\log x} dx \;\; \alpha \geq 0$ using differentiation under the integral sign.

b) With the usual notation , prove that $\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$

c) Evaluate $\int_0^2 x(8-x^3)^{\frac{1}{3}} dx$ interms of Gamma function

Unit – IV

7. a) Find the Laplace transform of (i) $\int_0^t \frac{1-e^t}{t} dt$, (ii) $e^{5t}\cos^3 t$

b) If f(t) is a periodic function with period T so that f(t+ T)= f(t) for all values of t,

$$L(f(t)) = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t) dt.$$

c) Rewrite the following function using unit step function and find its Laplace transform

$$f(t) = \begin{cases} t-1 & 0 \leq t < 2 \\ 3-t & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases}$$

8. a) Find the inverse Laplace transform of (i) $\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}$ (ii) $\log(\frac{s+1}{s+2})$

b) Solve the following by the method of Laplace transform

$$x''(t) + x(t) = 6\cos 2t \;\; ; x(0) = 3, \quad x'(0) = 1$$

c) State and prove convolution theorem.

Unit – V

9. a) Construct partial differential equations by eliminating the constants a, b from the equations: $i)\; 2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\quad ii)\; z = (x-a)^2 + (y-b)^2 + 1$.

b) Derive one dimensional heat flow equation in the form $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.

c) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ by Lagrange's method.

10. a) $Solve\; \frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0, z = e^y \; and \; \frac{\partial z}{\partial x} = 1$ by direct integration

b) Derive one dimensional wave equation in the form $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

c) Solve $x^2 \frac{\partial u}{\partial x} + y^2 \frac{\partial u}{\partial y} = 0$ by the method of separation of variables.

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NMAM INSTITUTE OF TECHNOLOGY, NITTE

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Second Semester B.E. (Credit System) Degree Examinations

Make up / Supplementary Examinations - July 2015

14MA201 – ENGINEERING MATHEMATICS - II

Duration: 3 Hours — Max. Marks: 100

Note: Answer ***Five full*** *questions choosing* ***One full*** *question from* ***each Unit.***

Unit – I

a) Solve $p(p+y)=x(x+y)$ — 6

b) Solve $(xy^2-e^{1/x^3})dx-x^2ydy=0$ — 7

c) When a resistance R ohms is connected in series with an inductance L henries with an e.m.f E volts, the current i amperes at time t is given by $L\frac{di}{dt}+Ri=E$. If $E=10\sin t$ volts and $i=0$, when $t=0$, find i as a function of t. — 7

a) Solve $y=-px+x^4p^2$ — 6

b) Show that the system of parabolas $y^2=4a(x+a)$ is self orthogonal. — 7

c) Solve $\left(5x^4+6x^2y^2-8xy^3\right)dx+\left(4x^3y-12x^2y^2-5y^4\right)dy=0$ — 7

Unit – II

a) Solve the differential equation $(D^3-6D^2+11D-6)y=e^{-2x}+e^{-3x}$ — 6

b) Solve the differential equation $(D^2-D-2)y=36xe^{2x}$ — 7

c) A spring is such that it would be stretched by 19.6 cms by a 4.9 kg weight. Let the weight be attached to the spring and pulled down 15 cms below the equilibrium position. If the weight is started with an upward velocity of 9.8 cms per second describe the motion. No damping or impressed force is present. — 7

a) Solve the differential equation $(D^2+36)y=4\cos 6x$ — 6

b) Solve the differential equation $\frac{d^4y}{dx^4}-y=\cos x\cosh x$ — 7

c) Solve $(D^2-2D+1)y=e^{2x}\left(e^x+1\right)^{-2}$ using the method of variation of parameters. — 7

Unit – III

a) Change the order of integration and hence evaluate $\int_0^\infty\int_x^\infty\frac{e^{-y}}{y}dydx$ — 6

b) Evaluate $\int_0^a\int_0^x\int_0^{x+y}e^{x+y+z}\,dzdydx$. — 7

c) Using double integrals find the area bounded by the cardioid $r=a(1+\cos\theta)$ — 7

P.T.O.