

Introduction to Arrays

Mar 28, 2022

AGENDA:

- = Little more on Space Complexity
- = Arrays / List Data structure
 - Access / Retrieve
 - Update
 - Iteration over an array
- = 3-4 interesting problems

xx Space Complexity

Def:

How does $\frac{\text{space taken up by program}}{\text{Input}}$ vary as $N \rightarrow \infty$?

V.E-

$$\left\{ \begin{array}{l} \text{Space taken up by program} \\ = \text{Space taken up by Input} \\ + \text{Auxiliary / Extra space.} \end{array} \right.$$

if the input is array
 $O(n)$

$O(n)$

$O(1)$

Only this is considered while calculating space complexity.



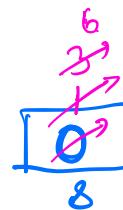
1. Space complexity by below code :

$$\rightarrow s=0$$

for i in range(0,10):

$$s=s+i.$$

$O(1)$



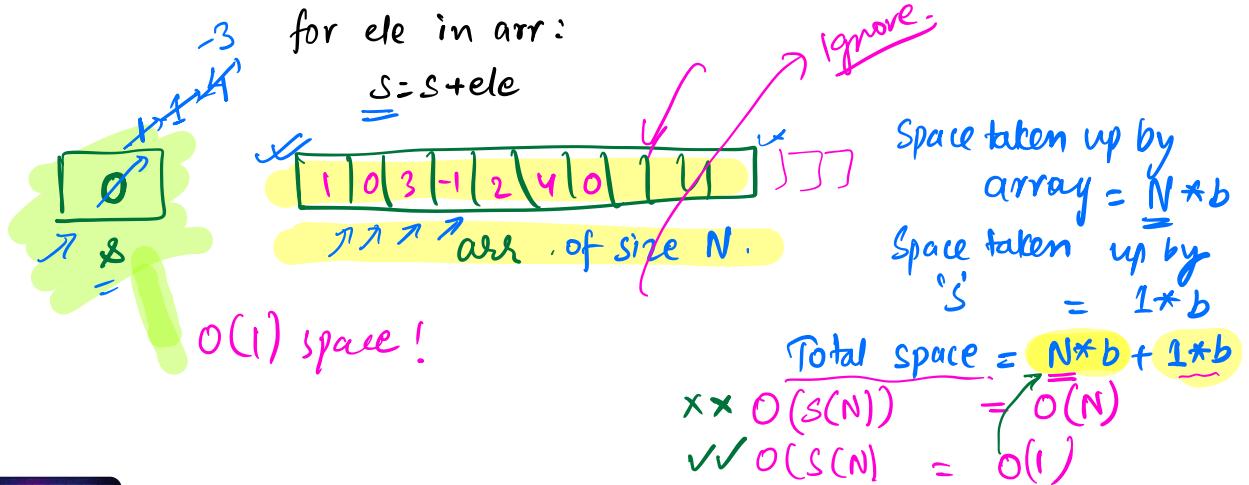
$$S(N) = 2 * b$$
$$O(S(N)) = \underline{O(1)}$$



2. Space complexity by below code :

// arr of size N is passed as input.

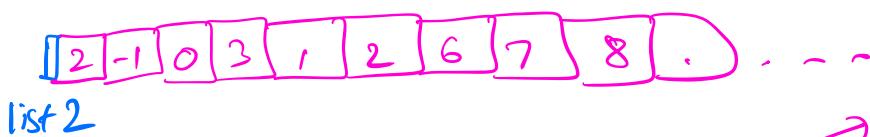
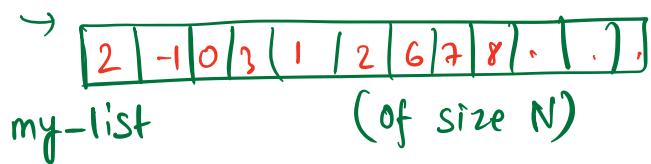
$$\checkmark s=0$$



3. Space complexity by below code :

```
def copy_list(my-list):
    list2 = []
    for x in my-list:
        list2.append(x)
    return list2
```

O(N) space



Total space = Input space

$N \times b$
(my-list)

Ignored!

(list-2)

\downarrow
 $N \times b$. bytes

$O(s(n)) = O(N)$

ARRAYS / LIST.

What is an array?

** Collection of elements

same-type.

{ C++,
Java
etc.

→ different-types { Python.

array → [1, 2, 5, 6, -3, 0]

→ [1, 2, "scalar", true, 5.0]

why do we need array?

a=1
b=2
c=5
d=6
e=-3
f=0

100 students in the class.

Store their marks.

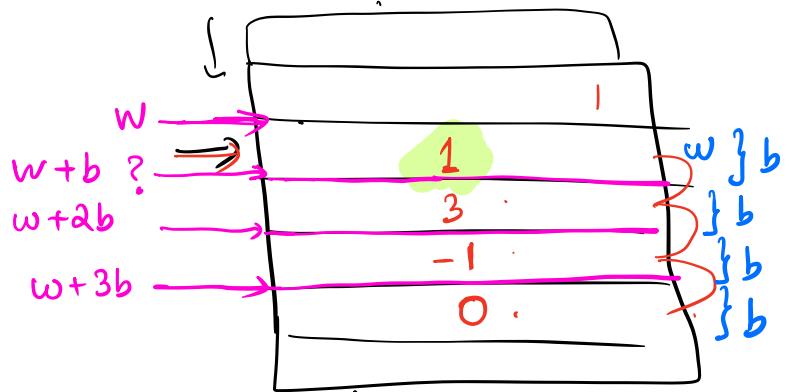
{ a1 = 97 - }
 a2 = 83 - }
 a3 = 76 - }
 a4 = 100 - }
 .
 a100 = ? = }

* what is the max marks?

* what is the marks of
76th student?

** Array is a contiguous block of memory.

How is array stored in memory?



array of size 4.

$[1, 3, -1, 0]$

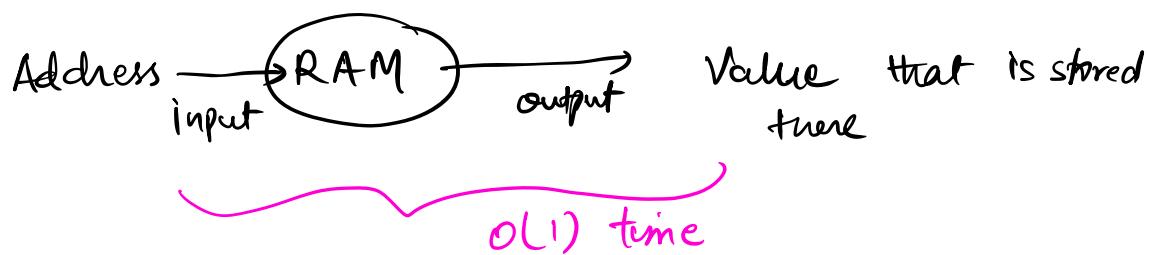
$w \rightarrow$ Starting address of array.
3rd element $\rightarrow ? (-1)$

$w+2b$

w, n th element by $\{w+(n-1)b\}$

** What is the time complexity of accessing an array element?

RAM \rightarrow Random Access Memory.



When you declare an array,
you know the starting address.

$$\text{ar} = \boxed{w} \rightarrow [1, 3, -1, 0, 2, 6, 4]$$

ar[3] = ?

$w + 3b$

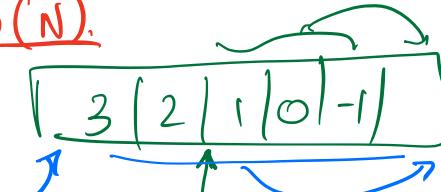
AM

O
in constant time
w: starting
address of the array.

Recap.

1) Accessing the element / Retrieve $\rightarrow O(1)$

2) Insert. $\rightarrow O(N)$.



3) Iterate over an array?



R

O. . .

Q:

:-

in constant time

Given an array, find sum of array elements.

$$\text{arr} = [7, 17, 1, 2, 5, 4, 3]$$

* How do you access an array element?



Q. Print sum of 1st and 5th element.

$$A = [5, -4, 8, \underset{\nearrow}{9}, 10]$$

1st elem. $A[0]$

5th element $= A[4]$.
 $A[0] + A[4]$

** Accessing each element of the array to get total sum:

Approach: Iterate over the array, sum each element.

```
s=0  
N = len(arr)  
for i in range(0, N):  
    s=s+arr[i]  
return s
```

Python

```
s=0  
for element in arr:  
    s=s+element  
return s
```

No. of iterations = N

TC: O(N)

SC: O(1)

Problem Solving

Q. Given N array elements, find no. of elements having atleast 1 element greater than itself.

e.g.

$$[2, 5, 1, 4, \cancel{8}, \cancel{0}, 8, 1, 3, \cancel{8}]$$



Ans = ? $\textcircled{7}$ ✓
 $\underline{10 - 3 = 7}$

How many 8's.
✓ 3

e.g.

$$[1, 2, 3, 4, \cancel{5}, \cancel{5}, 6, \cancel{7}, \cancel{7}, \cancel{7}]$$

$\textcircled{7}$ ✓

$10 - 3 = 7$

Approach :

1. Find total no. of elements = N
2. Find Max no. of list = maxim
3. Count how many max we have. = count
4. Ans = $N - \text{count}$

Pseudo-code :

```
{  
    N = len(arr)  
    #maxi = max(arr) ← Don't use this  
    maxi = arr[0]  
    for i in range(1, N):  
        maxi = max(maxi, arr[i])  
        if arr[i] > maxi:  
            maxi = arr[i]  
}  
Iterate and compare.
```

```
{  
    # Count how many maxi are there.  
    count = 0  
    for element in arr:  
        if element == maxi:  
            count += 1  
}  
Iterating and compare.
```

return $N - count$.

Time complexity → $O(N)$
Space → $O(1)$

Break till : 10:10 pm.

Q. Reverse the given array.

Input = [5, 2, 7, -1, 0] ←

Output = [0, -1, 7, 2, 5] ↴

Naive approach:

#inp = [5, 2, 7, -1, 0]

Create a new array.

outp = [] [0, -1, 7, 2, 5] moving back.

for i in range (n-1, -1, -1):
 outp.append (inp[i]) [n-1, 0]

→ range (start, end) → [start, end - 1]

$\rightarrow \text{range}(\text{start}, \text{end}, \text{step}) \rightarrow$ $\left[\begin{matrix} \text{start}, \text{start}-1, \dots, \text{end}+1 \\ \text{start}-2, \dots \end{matrix} \right]$



Time complexity: $O(N)$

Space complexity: $O(N)$

You created
a new
array.

9{:::-1]}

Can we optimise?

↳ Time
↳ Space.

$O(N) \rightarrow$ ✓ Best!

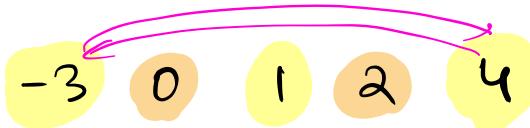
Can we do it in $O(1)$?

inp =

-3	0	1	2	4
----	---	---	---	---

Modify this array itself.

Inplace algorithm



4 2 1 0 -3

inp:

1	2	3	4	5
---	---	---	---	---

out: 5 4 3 2

Approach.

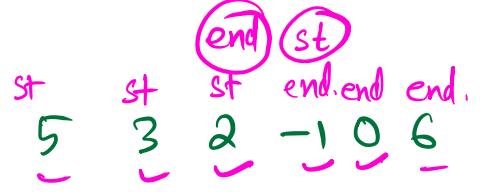
Start $\rightarrow 0$
end $\rightarrow N-1$

Pseudo-code.

```

def reverse(arr):
    N = len(arr)
    start = 0
    end = N - 1
    while (start < end):
        swap(arr[start], arr[end])
        start += 1
        end -= 1

```



fun swap (a, b) :

a, b = b, a. // In python

X {
 a = b
 b = a } X

{
 temp = a
 a = b
 b = temp }.

Time: O(N)

No. of iterations = N/2
O(N)

Space:

O(1)

Inplace.

Q.

Reverse part of the given array.

e.g. → Input: [4, 3, 5, 1, 0, 7, 2, 8]
→ start = 3
→ end = 6 {→ 0-indexed}

Output: [4 3 5 2 7 0 1 8]

def reverse(arr, start, end):

N = len(arr)
~~start = 0
end = n - 1~~
{ while (start < end) :
 Swap (arr[start], arr[end])
 start += 1
 end -= 1

Time complexity → Best case → ? $O(1)$

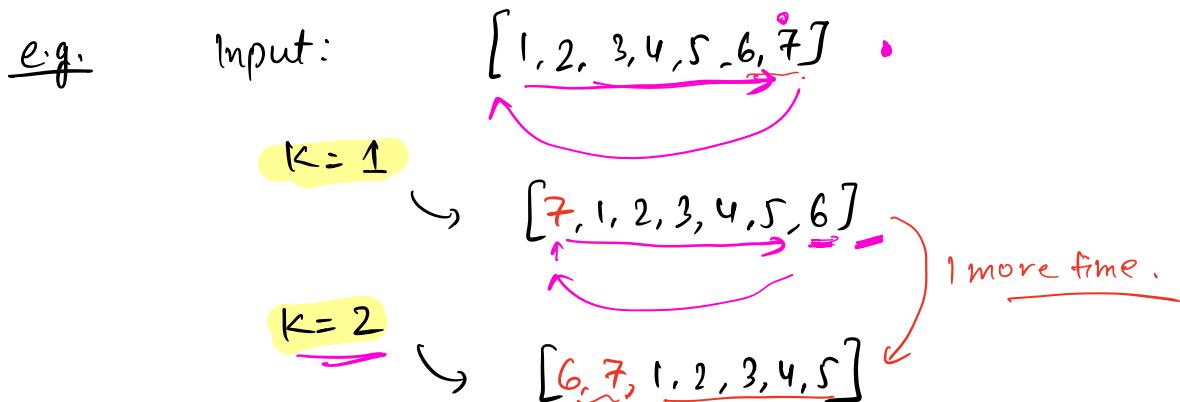
↓
Worst case → $O(N)$ { start = 0
 end = $N-1$

Time complexity: $O(N)$.

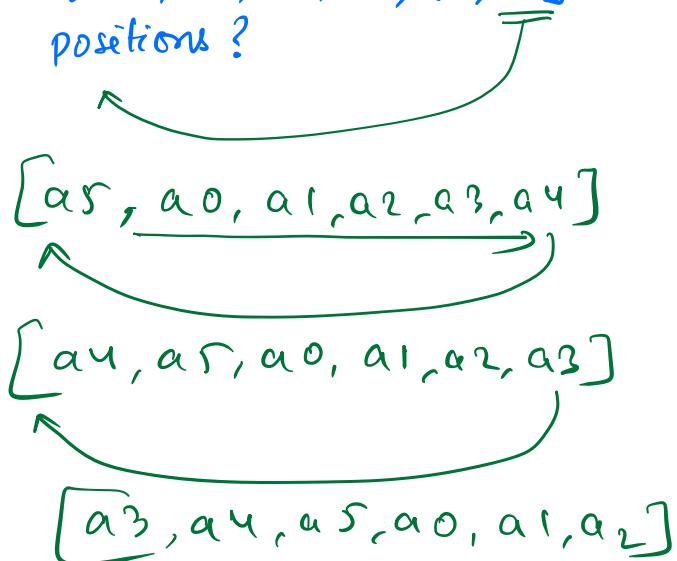
Space complexity: $O(1)$

Q. ** Very interesting Interview Problem: "Google/Amazon"

Given an input array, rotate the given array 'K' times from left to right.



Rotate the array
[a₀, a₁, a₂, a₃, a₄, a₅]
by 3 positions?



** What happens if $k > n$? **

Array has 5 elements.

rotate the array k times.

arr =	[1, 2, 3, 4, 5]	←
{ k = 1 :	5, 1, 2, 3, 4	
k = 2 :	4, 5, 1, 2, 3	
k = 3 :	3, 4, 5, 1, 2	
k = 4 :	2, 3, 4, 5, 1	
<u>k = 5</u> :	1, 2, 3, 4, 5	← k = 0 5%5
k = 6 :	5, 1, 2, 3, 4	← k = 1 6%5
k = 7 :	4, 5, 1, 2, 3	← k = 2 7%5

Conclusion:

**

if $k > N$:
 $k = k \% N$

Brute Force approach.

* $\left\{ \begin{array}{l} \text{Pick up the last element} \\ \text{Insert it at the first.} \end{array} \right.$
 $\underbrace{\hspace{1cm}}_{O(N)}$

How many times you need to perform this?

Total. TC: $O(N * K)$

\int
Quadratic

\downarrow
 K times.

K is an input.

$$\begin{aligned} N &\rightarrow \cancel{K \text{ max}} 10^5 \\ K &\rightarrow 10^5 \\ &\underline{10^{10}} \end{aligned}$$

Space complexity:
Time complexity:

** Let's try to optimise on Time.

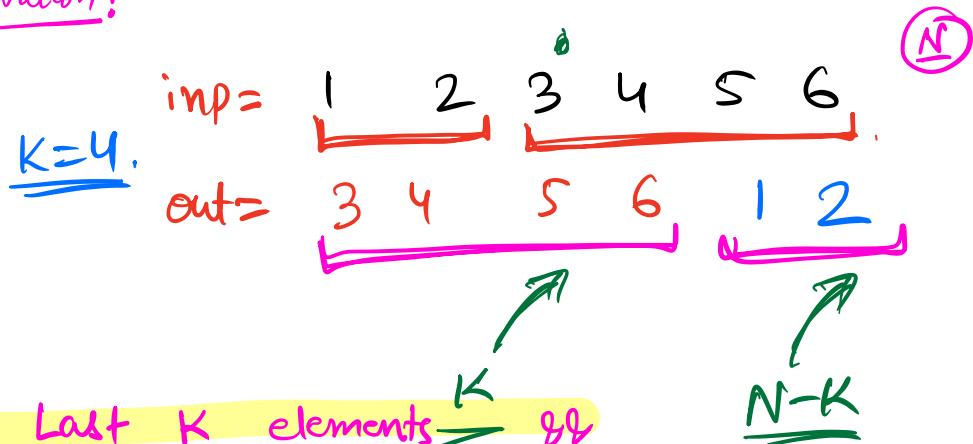


$K < N$ $K = K \% N$
Original array : $[1, 2, 3, 4, 5]$
 $K=3$; New array : $[3, 4, 5, 1, 2]$

** We will be using a new array.

How can we directly translate original values to new array?

Observation!

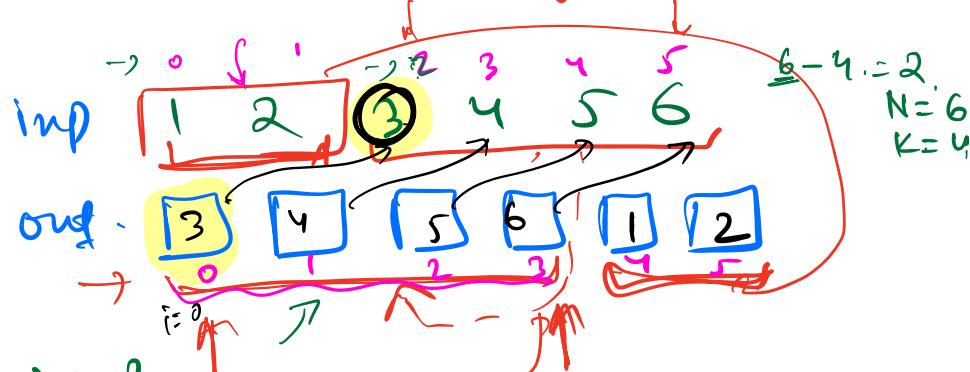


* Obs. → Last K elements \xrightarrow{K} go

come at start.

1st $n-K$ elements go at end.

* Pick up values from your input array.
Start filling values in your output array.



Translation: ?

For first 4 values,

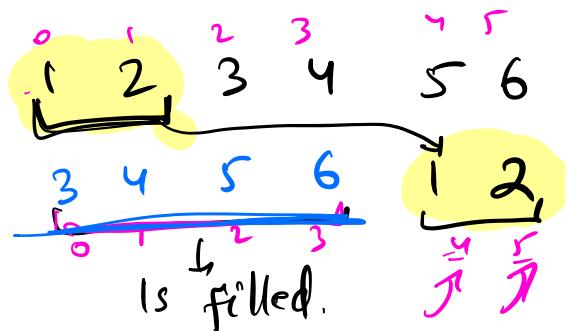
$$i = 0, 1, 2, 3$$

$$\begin{aligned} \text{out}[0] &= \text{inp}[2] \\ \text{out}[1] &= \text{inp}[3] \\ \text{out}[2] &= \text{inp}[4] \\ \text{out}[3] &= \text{inp}[5] \end{aligned}$$

} in terms of N, i, K

$6-4=2$
 $N-K$ ✓
 $N-K+1$
 $N-K+2$
 $N-K+3$

{ for $i = 0, 1, 2, 3$
 → $out[i] = input[N-K+i]$ ✗ ✗
 (1st part)



↗ $out[4] = inp[0]$ $K+1$
 ↗ $out[5] = inp[1]$ $K+2$

for $i = 0, 1 :$

→ $output[K+i] = input[i]$ ✗
 ↓ ↓
 ↗ ↗

Pseudo-code .

$K = K \% N$ $0, 1, 2, 3$
 $out = [0] * N$ ←
 filled up } for i in range($0, K$): ← K
 1st part. } $out[i] = inp[N-K+i]$
 for i in range($0, N-K$): $N-K$
 $out[K+i] = inp[i]$
 ↓ ↓

Time complexity : $O(N)$ // ← No
Space " : $O(N)$ // ← Yes.

↳ You are creating
a new array.

* Can we optimize on space ?



IDEA

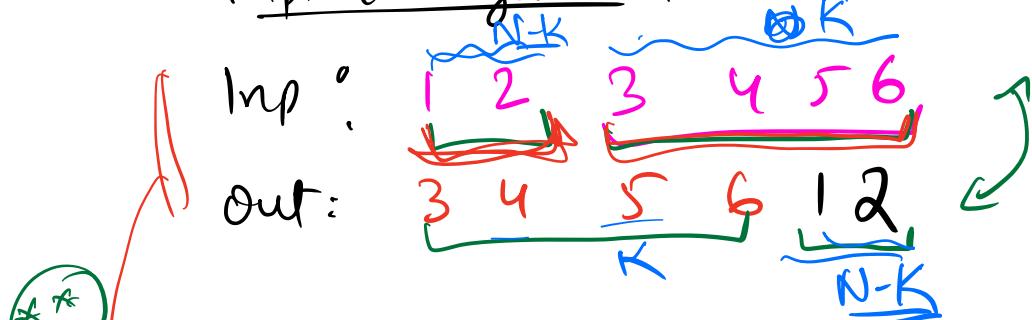
Imp: [1, 2, 3, 4, 5, 6]

K=4; Output: [3, 4, 5, 6, 1, 2]

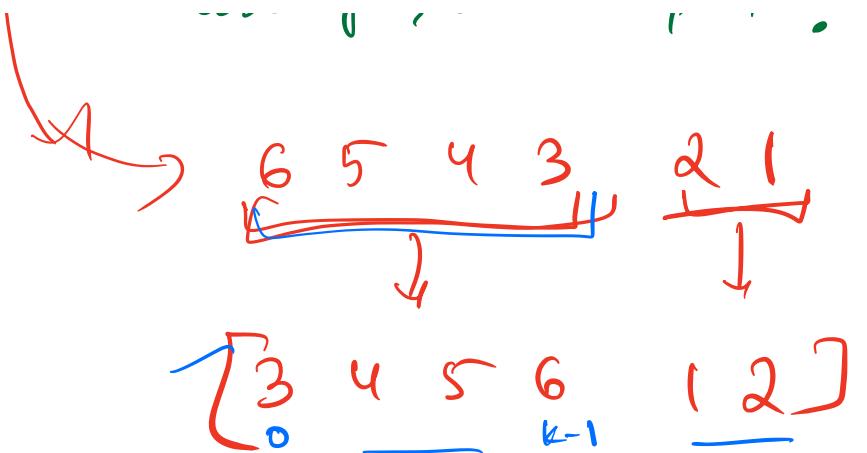
* observation!

Trying to do in $O(1)$ space complexity.

* Inplace algorithm.



Think on how can reverse the array or its parts!



- ① Reverse the whole array . TC: $O(N)$ –
 SC: $O(1)$
- ② Reverse [array , 0, k-1] TC: $O(N)$ –
 SC: $O(1)$
- ③ Reverse [array , k, N-1] TC: $O(N)$ –
 SC: $O(1)$

~~Post~~ .

~~TC:~~ $O(N)$

~~SC:~~ $O(1)$

Doubt Session.

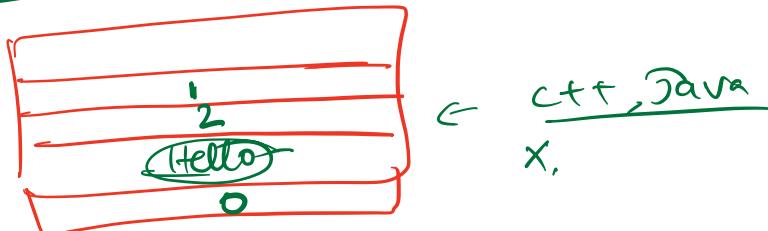
q insert(my-list, 0, num) =

my-list.insert(element, location)

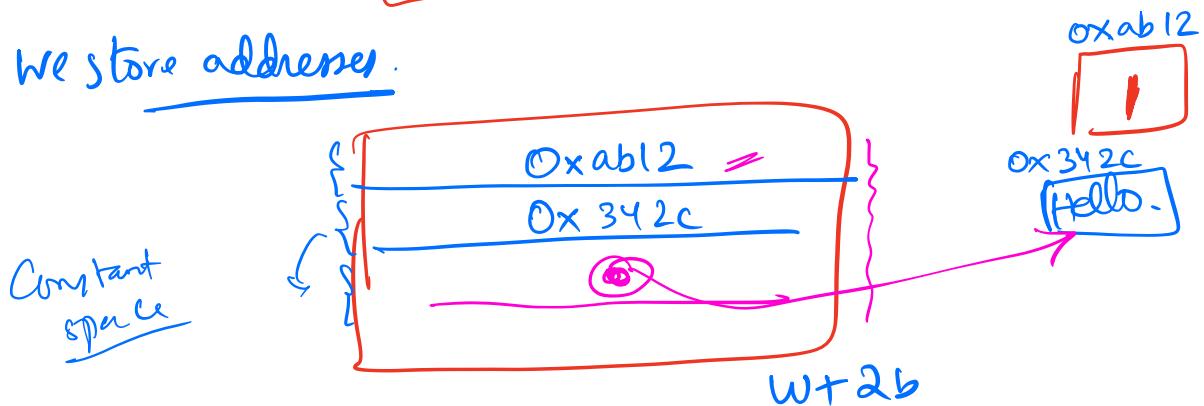
[1, 2, "Hello", 3, true]

3b bytes 2b bytes b ytes

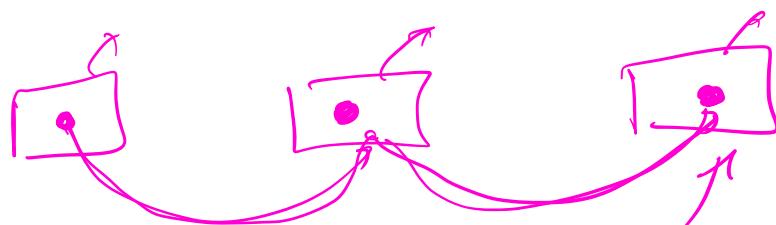
$$\boxed{w+3b}$$



We store addresses.



$$w + (n-1)b.$$



```
i = 1
while ( i < N ) :
    x = i
    while ( x > 0 ) : 7
```

$x = 2$ }

$i + 1$

$i = 1$

1

2

3

4

⋮

⋮

N

inner loop iter.

$\frac{1}{2}$ —

$\frac{2}{2}$ —

$\frac{3}{2}$ —

$\frac{4}{2}$ —

⋮ —

$\frac{N}{2}$ —

1

2

3

⋮

⋮

2^N

2^N

$1 + 2 + 3 + 4 + \dots + N$

$\frac{2^N(2^N + 1)}{2}$

$= \frac{N(N+1)}{2}$

$O(N^2)$

$1 < N$

$2^N * 2^N$

~~$O(2^{N+1})$~~

2^N

4^N

$(2 \times 2 \times \dots) \times \underbrace{(2 \times 2 \times \dots)}_{N \text{ times}}$

$= \underbrace{(4 \times 4 \times 4 \times \dots)}_{N \text{ times}} = 4^N$

$O(4^N)$