Question

Construct npda's that accept the following languages on $\Sigma = \{a, b, c\}$.

(a) $L = \{a^n b^{2n} : n \geq 0\}$.

(b) $L = \{wcw^R : w \in \{a, b\}^*\}$.

(c) $L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$.

(d) $L = \{a^n b^{n+m} c^m : n \geq 0, m \geq 1\}$.

(e) $L = \{a^3 b^n c^n : n \geq 0\}$.

(f) $L = \{a^n b^m : n \leq m \leq 3n\}$.

(g) $L = \{w : n_a(w) = n_b(w) + 1\}$.

(h) $L = \{w : n_a(w) = 2n_b(w)\}$.

(i) $L = \{w : n_a(w) + n_b(w) = n_c(w)\}$.

(j) $L = \{w : 2n_a(w) \leq n_b(w) \leq 3n_c(w)\}$.

(k) $L = \{w : n_a(w) < n_b(w)\}$.

#introduction to formal languages and automata

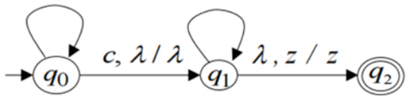
Answer

(a)The NPDA for language {a^n b^2n}

The initial stack symbol is $.
Start with S →aSbb | abb
convert to GNF: S→aSBB | aBB; B→b.
Derive the canonical three state npda then eliminate the q1 state by using a special stack symbol, Y, to mark it
(q0, z, q2) → a
(q0, A, q3) (q3, z, q2) → a a
(q3, z, q2) → a a
(q0, A, q3) (q3, z, q2) → a a a
(q3, z, q2) → a a a
(q0, A, q1) (q1, z, q2) → a a a b
(q1, z, q2) → a a a b

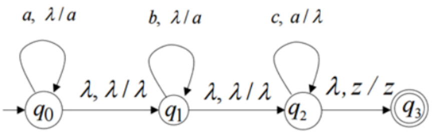
(b)L={wcw^r :w∈{a,b}*}

M = ({q0 , q1, q2}, Σ, {a, b, z}, δ, q0, z, {q2})
Instantaneous description (ID): (current state, remaining input, stack)
δ(q0 , a, a) = {(q0 , aa)}
δ(q0, b, a) = {(q0, ba)}
, δ(q0, a, b) = {(q0 , ab)}
, δ(0 b b) {(0 bb)}
δ(q0 , b, b) = {(q0, bb)},
δ(q0, a, z) = {(q0 , az)},
δ(q0, b, z) = {(q0, bz)},
δ(q0, c, z) = {(q1, z)},
δ(q0, c, a) = {(q1, a)},
δ(q0, c, b) = {(q1 , b)},
δ(q1, a, a) = {(q1, λ)},
δ(q1 b b) = {(q1 λ)}
δ(q1, λ, z) = {(q2 , z)}



(c) L={a^n b^m c^n+m :n>0,m>0}

Machine definition: M = ({q 0, q 1, q 2, q 3 , q 4}, Σ, {a, z}, δ, q 0, z, {q 0, q 4 })
δ(q 0 , λ, z) = {(q 1, z)},
δ(q 1 , a, z) {(q 1 , az)}
δ(q 1 , a, a) = {(q 1 , aa)},
δ(q 1, λ, a) = {(q 2, a)},
δ(q 1, λ, z) = {(q 2 , z)},
δ(q 2, b, z) = {(q 2, az)},
δ(q 2 , b, a) = {(q 2, aa)},
δ(q 2, c, a) = {(q 3, λ)},
δ(q 3, c, a) = {(q 3, λ)},
δ(q 3, λ, z) = {(q 4, λ)}



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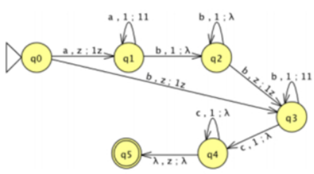
(d)L={a^n b^n+m c^m}

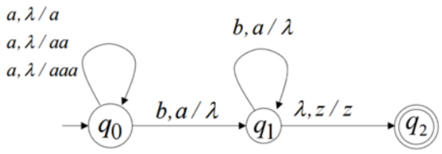
An NPDA that accepts L is M = (Q, Σ, Γ, δ, q0, z, F), where Q = {q0, q1, . . . , q5},

Σ = {a, b, c}, Γ = {0, 1, z}, F = {q5}, and the transition function δ is represented as the following graph



(f) L={a^n b^m :n<=m<=3n}

Machine definition: M = ({q 0, q 1, q 2 }, Σ, {a, b, z}, δ, q 0 , z, {q 2 })
Instantaneous description (ID): (current state remaining input stack)
δ(q 0, a, z) = {(q 0, az)}
, δ(q 0 , a, z) = {(q 0, aaz)},
δ(q 0 a z) = {(q 0 aaaz)}
, δ(q 0 , a, a) = {(q 0, aa)},
δ(q 0, a, a) = {(q 0 , aaa)},
δ(q 0, a, a) = {(q 0, aaaa)}
δ(q 1 , b, a) = {(q 1, λ)},
δ(q 1, λ, z) = {(q 2 , z)}

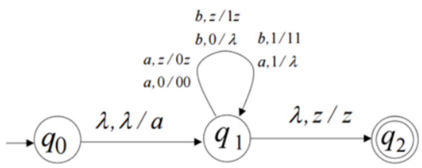


(g)L={w:na(w)=nb(w)+1}

Instantaneous description (ID): (current state, remaining input, stack)
δ(q 0, λ, z) = {(q 1 , az)},
δ(q 1 , a, z) = {(q 1, az)},
δ(q 1 , a, a) = {(q 1 , aa)},
δ(q 1 , a, b) = {(q 1, λ)},
δ(q 1 b) {(q 1 b)}
δ(q 1, b, z) = {(q 1, b z)},
δ(q 1, b, a) = {(q 1 , λ)}
, δ(q 1, b, b) = {(q 1, bb)},
δ(q 1, λ, z) = {(q 2, λ)},
δ(q 1, c, z) = {(q 1 , c)},
δ(q 1, c, a) = {(qa , a)},
δ(q 1, c, b) = {(q 1, b)}

δ(q 0, a, a) = {(q 0, aaaa)} δ(q 1, c, b) = {(q 1, b)}.

δ(q 0 , b, a) = {(q 1, λ)},



(h) L={w:na(w)=2nb(w)}

Instantaneous description (ID): (current state, remaining input, stack)

δ(q 0, λ, z) = {(q 1 , az)},

δ(q 1 , a, z) = {(q 1, az)},

δ(q 1 , a, a) = {(q 1 , aa)},

δ(q 1 , a, b) = {(q 1, λ)},

δ(q 1 b) {(q 1 b)}

δ(q 1, b, z) = {(q 1, b z)},

δ(q 1, b, a) = {(q 1 , λ)}

, δ(q 1, b, b) = {(q 1, bb)},

, δ(q 2, b, b) = {(q 2, bb)},

δ(q 2, λ, z) = {(q 3, λ)},

δ(q 2, c, z) = {(q 2 , c)},

δ(q 2, c, a) = {(qa , a)},

δ(q 2, c, b) = {(q 2, b)}.