

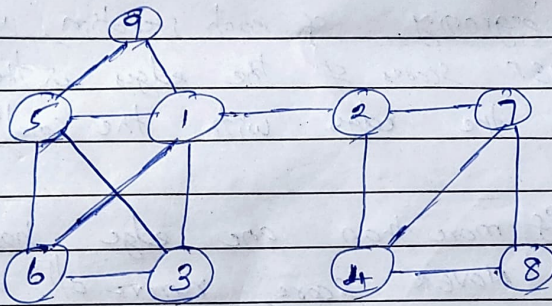
# Neighborhood overlap based approach (NOVER)

Neighborhood overlap = number of nodes who are neighbors of both A & B

number of nodes who are neighbors of at least one of A or B.

Note one should not count neither A nor B as part of the neighbors in the denominator & each node should be counted only once.

Example



1 → {2, 3, 5, 6, 9}

6 → {1, 3, 5}

2 → {1, 4, 7}

7 → {2, 8}

3 → {1, 5, 6}

8 → {4, 7}

4 → {2, 7, 8}

9 → {1, 5}

5 → {1, 3, 6, 9}

Edge

Union of Neighb.

Intersec.

NOVER

1-2

{3, 5, 6, 9, 4, 7}

{}

0/6 = 0.0

1-3

{2, 5, 6, 9}

{5, 6}

2/4 = 0.5

1-5

{2, 3, 6, 9}

{3, 6, 9}

3/4 = 0.75

1-6

{2, 3, 5, 9}

{3, 5}

2/4 = 0.5

1-9

{2, 3, 5, 6}

{5}

1/4 = 0.25

2-4

{1, 7, 8}

{7}

1/3 = 0.33

2-7

{1, 4, 8}

{4}

1/3 = 0.33



Edge	Union of Neighb.	Intersec.	NOVER
2-5	{1, 6, 9}	{1, 6}	$2/3 = 0.67$
3-6	{1, 5}	{1, 5}	$2/2 = 1.0$
4-7	{2, 8}	{2, 8}	$2/2 = 1.0$
4-8	{2, 7}	{7}	$1/2 = 0.5$
5-6	{1, 3, 9}	{1, 3}	$2/3 = 0.67$
5-9	{1, 3, 6}	{1, 3}	$1/3 = 0.33$
7-8	{2, 4}	{4}	$1/2 = 0.5$

### NOVER based GN algorithm

\* At the beginning of each iteration, we compute the NOVER scores of the edges in the graph & remove the edges with the smallest NOVER score.

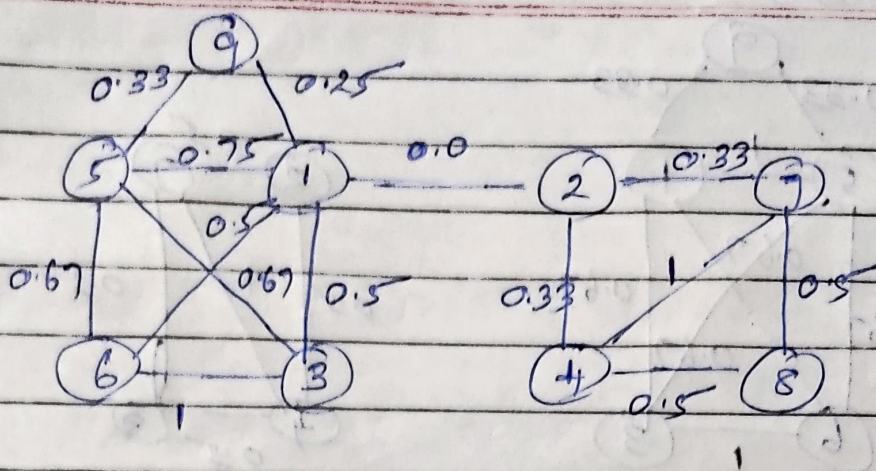
- If more than one edge has the smallest NOVER score, remove all such competing edges at the same time.

- If the graph gets disconnected to two or more communities (components), we compute the total modularity score of the resulting communities.

\* Repeat the iterations until there are no more edges.

\* The partition (set of communities) with the total modularity score is the optimal partition.





Modularity (1, 3, 5, 6, 9)

$$\text{Mod}(1, 3) = 1 - (5 \times 3) / (2 \times 14) = 0.46$$

$$\text{Mod}(1, 5) = 1 - (5 \times 4) / (2 \times 14) = 0.28$$

$$\text{Mod}(1, 6) = 1 - (5 \times 3) / (2 \times 14) = 0.46$$

$$\text{Mod}(1, 9) = 1 - (5 \times 2) / (2 \times 14) = 0.64$$

$$\text{Mod}(3, 5) = 1 - (3 \times 4) / (2 \times 14) = 0.57$$

$$\text{Mod}(3, 6) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

$$\text{Mod}(3, 9) = 0 - (3 \times 2) / (2 \times 14) = -0.21$$

$$\text{Mod}(5, 6) = 1 - (4 \times 3) / (2 \times 14) = 0.57$$

$$\text{Mod}(5, 9) = 1 - (4 \times 2) / (2 \times 14) = 0.71$$

$$\text{Mod}(6, 9) = 0 - (3 \times 2) / (2 \times 14) = -0.21$$

$$\text{Modularity}(1, 3, 5, 6, 9) = 3.95$$

Modularity (2, 4, 7, 8)

$$\text{Mod}(2, 4) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

$$\text{Mod}(2, 7) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

$$\text{Mod}(2, 8) = 0 - (3 \times 2) / (2 \times 14) = -0.21$$

$$\text{Mod}(4, 7) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

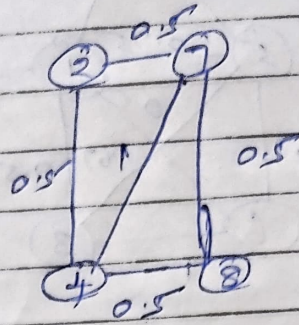
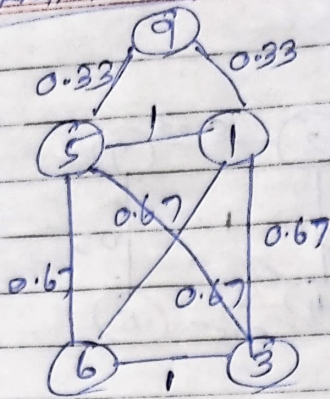
$$\text{Mod}(4, 8) = 1 - (3 \times 2) / (2 \times 14) = 0.78$$

$$\text{Mod}(7, 8) = 1 - (3 \times 2) / (2 \times 14) = 0.78$$

$$\text{Modularity}(2, 4, 7, 8) = 3.39$$

$$\text{Total Modularity Score} = 7.34$$



Iteration-1

1-3

Union  
 $\{5, 6, 9\}$ Intersection  
 $\{5, 6\}$ Never  
 $2/3 = 0.67$ 

1-5

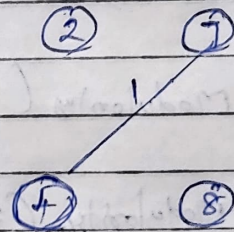
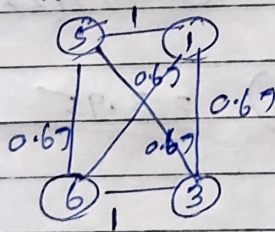
1-6

1-9

⋮

1 ←  $\{3, 5, 6, 9\}$ 3 ←  $\{1, 5, 6\}$ Iteration-2

⑨

modularity (1, 3, 5, 6)

$$\text{Mod}(1, 3) = 1 - (5 \times 3) / (2 \times 14) = 0.46$$

$$\text{Mod}(1, 5) = 1 - (5 \times 4) / (2 \times 14) = 0.28$$

$$\text{Mod}(1, 6) = 1 - (5 \times 3) / (2 \times 14) = 0.46$$

$$\text{Mod}(3, 5) = 1 - (3 \times 4) / (2 \times 14) = 0.57$$

$$\text{Mod}(3, 6) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

$$\text{Mod}(5, 6) = 1 - (4 \times 3) / (2 \times 14) = 0.57$$



$$\text{Modularity } (1, 3, 5, 6) = 3.02$$

$$\text{Modularity } (4, 7)$$

$$\text{Modularity } (4, 7) = 1 - (3 \times 3) / (2 \times 14) = 0.68$$

$$\text{Modularity } (4, 7) = 0.68$$

$$\text{Total Modularity Score} = 3.70$$