Parser

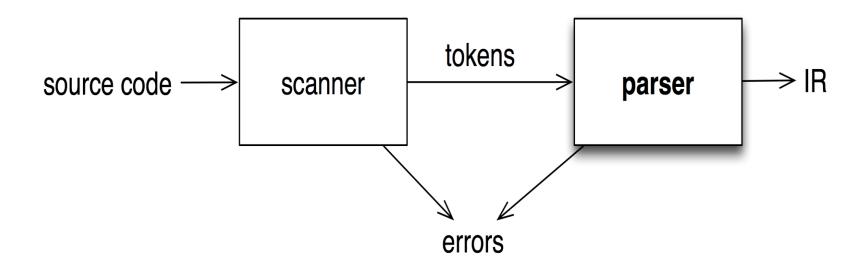
Second Phase of the compiler

- Parser Typically integrated with the lexical phase of the compiler
- Top Down Parser
- Bottom Up Parser

Functions of the Parser

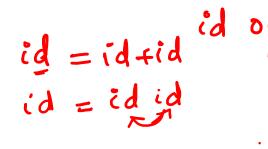
- Validate the syntax of the programming language
- Points out errors in the statements

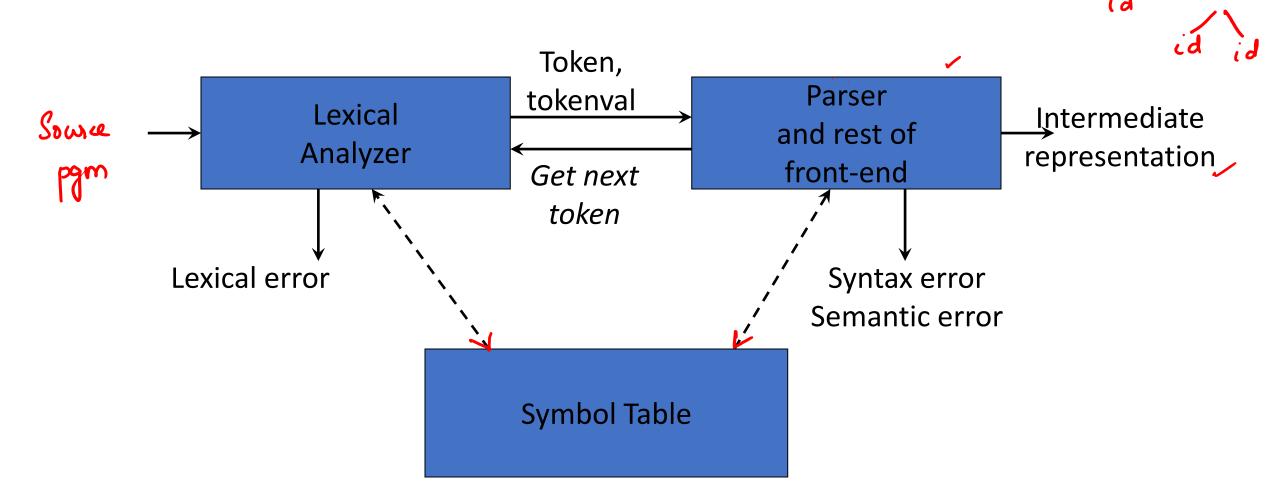
Role of the Parser



Role of the Parser

7= 4 2





General Types of Parsers

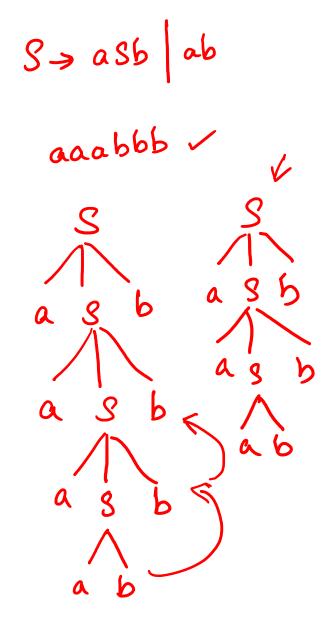
- Universal Parsers
 - Cocke- Younger-Kasami
 - Earley's Algorithm
- Top-Down ParsersBottom Up Parsers

Universal Parsers

- Can parse any Grammar
- Use in NLP
- But too inefficient in Compilers

Top Down Parsers

- Build the parse trees from the top to the bottom
- Recursive Descent parsers requires backtracking
- LL Parsers No Backtracking



Example

Consider the Grammar

$$S \rightarrow c A d$$

 $A \rightarrow ab \mid a$

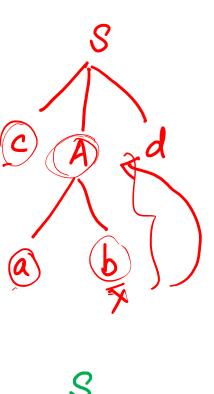
Let the input be "cad"

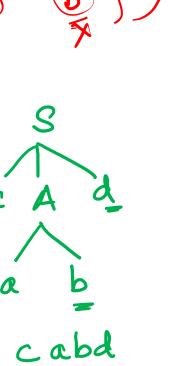
$$S \rightarrow cAd \mid dC$$

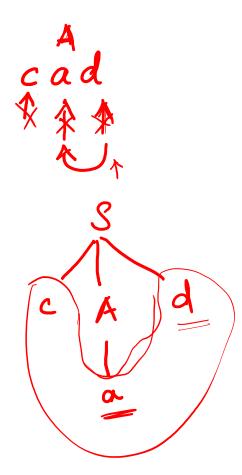
$$A \rightarrow ab \mid C$$

$$C \rightarrow c \mid E$$



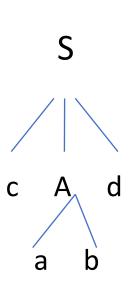


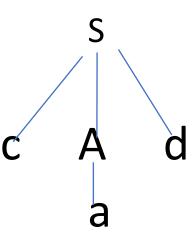




Parsing (Recursive Descent)

Expand A using the first alternative $A \rightarrow ab$

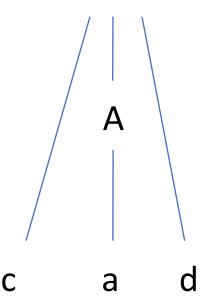




Bottom up Parsers

- Start from the bottom and work up to the root for parsing a string
- LR parsers are bottom up parsers

S



Parsing

- Both Top Down and Bottom up parsers parse the string based on a viable-prefix property
- This property states that before the string the is fully processed, if there is an error, the parser will identify it and recovers

Context Free Grammars - CFG

- Programming language constructs are defined using context free grammar
- For example

```
E \rightarrow E + E \mid E * E \mid (E) \mid id
```

Expression grammar involving the operators, +, *, ()

Context Free Grammars

- Defined formally as (V, T, P, S)
 - V Variables / Non-terminals
 - T Terminals that constitute the string
 - P Set of Productions that has a LHS and RHS
 - S Special Symbol, subset of V

NT
$$\rightarrow \alpha$$

$$A \rightarrow A\alpha$$

$$A \rightarrow B \beta$$

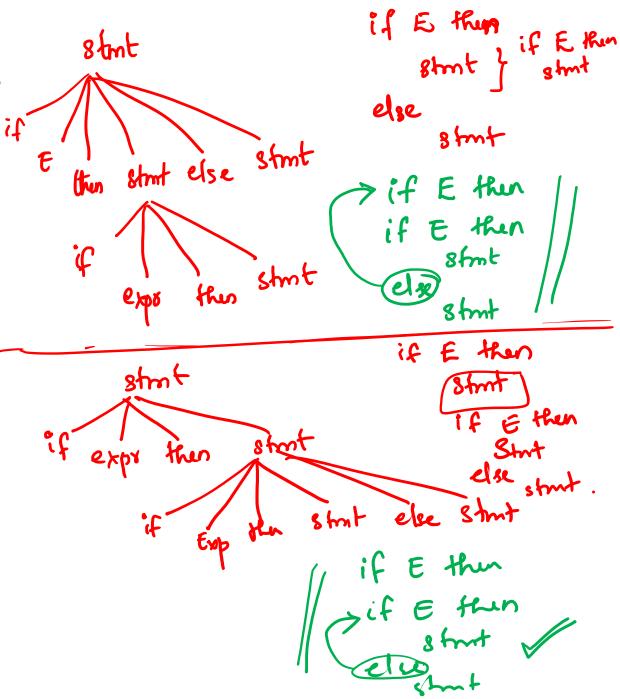
$$A \rightarrow A\alpha \beta \beta \beta$$

Context Free Grammar

Example
stmt → if E then stmt else stmt
stmt → if E then stmt
stmt → a

 $E \rightarrow b$

Here, stmt, E are Non-terminals, if, then, else, a, b are all terminals



Grammar - notations

- Terminals $a,b,c,... \in T$
 - specific terminals: 0, 1, id, +
- Non-terminals *A,B,C,...* ∈
 - specific non-terminals: expr, term, stmt
- Grammar symbols $X,Y,Z \in (N \cup T)$
- Strings of terminals $u, v, w, x, y, z \in T^*$
- Strings of grammar symbols $\alpha, \beta, \gamma \in (N \cup T)^*$

Derivation

- The *one-step derivation* is defined by α A $\beta \Rightarrow \alpha$ γ β where $A \rightarrow \gamma$ is a production in the grammar
- In addition, we define
 - \Rightarrow is *leftmost* \Rightarrow_{lm} if α does not contain a nonterminal
 - \Rightarrow is $rightmost \Rightarrow_{rm}$ if β does not contain a nonterminal
 - Transitive closure \Rightarrow^* (zero or more steps)
 - Positive closure ⇒⁺ (one or more steps)

Derivation

• The language generated by G is defined by $L(G) = \{w \mid S \Rightarrow^+ w\}$

Derivation

$$E \rightarrow E + E$$

 $E \rightarrow E * E$
 $E \rightarrow (E)$
 $E \rightarrow - E$
 $E \rightarrow \text{id}$

$$E \Rightarrow -E \Rightarrow -id$$

$$E \Rightarrow_{rm} E + E \Rightarrow_{rm} E + id \Rightarrow_{rm} id + id$$

$$E \Rightarrow^{*} E^{*} E \Rightarrow^{*} id * id + id$$

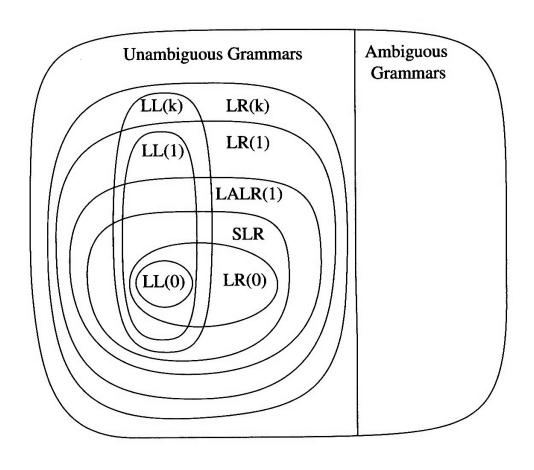
$$E \Rightarrow E + E \Rightarrow id + E$$

$$\Rightarrow id + id \rightarrow$$

Parsers

- Context Free grammars are already defined for all programming constructs
- All strings that are part of the programming language will be based on this construct
- Hence, parsers are designed keeping in mind the CFG

Hierarchy of Grammar Classes



Hierarchy

- LL(*k*):
- Left-to-right, Leftmost derivation, k tokens lookahead
- LR(*k*):
- Left-to-right, Rightmost derivation, k tokens lookahead
- SLR:
- Simple LR (uses "follow sets")
- LALR:
- LookAhead LR (uses "lookahead sets")

Top Down Parsers

 LL methods (Left-to-right, Leftmost derivation) and recursive-descent parsing

$$E \rightarrow T + T$$
 $T \rightarrow (E)$
 $T \rightarrow -E$
 $T \rightarrow id$

Leftmost derivation:

$$E \Rightarrow_{lm} T + T$$

$$\Rightarrow_{lm} id + T$$

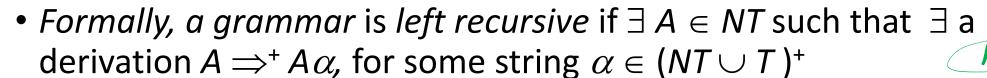
$$\Rightarrow_{lm} id + id$$

Top Down Parsers – LL (1) Parsers

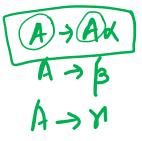
- LL parsers cannot handle
 - Left Recursive Grammar
 - Left Factoring

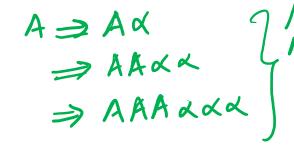


Left Recursive Grammar



• $A \rightarrow A \alpha / \beta | \gamma$

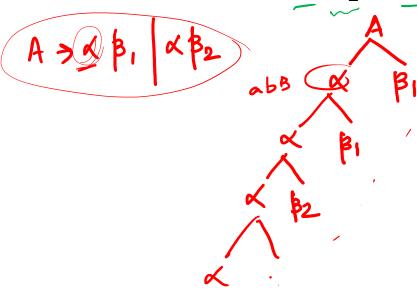


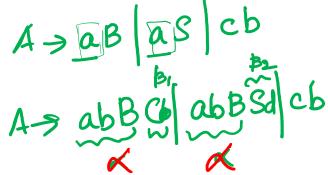


Left Factor

 When a nonterminal has two or more productions whose right-hand sides start with the same grammar symbols the grammar is said to have left-factor property

• Example $A \rightarrow \alpha \beta_1 / \alpha \beta_2 / ... | \alpha \beta_n | \gamma$

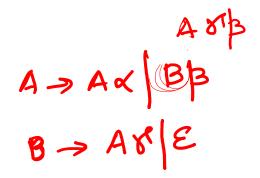






Pre-requisites for Top-Down Parser

- Eliminate Left Recursion
- Left Factor the grammar



Eliminating Left Recursion

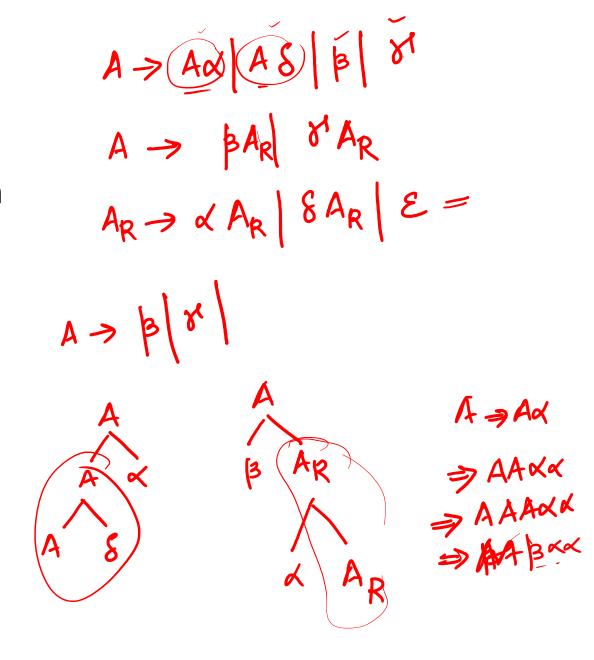
```
Arrange the nonterminals in some order A_1, A_2, ..., A_n
  for i = 1, ..., n do
         for j = 1, ..., i-1 do
                    replace each
                            A_i \rightarrow A_i \gamma
                    with
                             e^{A_i \to \delta_1 \gamma | \delta_2 \gamma | \dots | \delta_k \gamma}
                    where
                             A_i \rightarrow \delta_1 \mid \delta_2 \mid \dots \mid \delta_k
          end
       \rightarrow eliminate the immediate left recursion in A_i \checkmark
  end
```

Eliminate Left Recursion

- Rewrite every left-recursive production $A \rightarrow A \alpha / \beta \mid \gamma \mid A \delta$
- into a right-recursive production:

$$A \rightarrow \beta A_R / \gamma A_R$$

 $A_R \rightarrow \alpha A_R / \delta A_R / \epsilon$



•
$$A \rightarrow B C \mid \mathbf{a}$$

 $B \rightarrow C A \mid A \mathbf{b}$
 $C \rightarrow A B \mid C C \mid \mathbf{a}$

(1)
$$A \rightarrow BC | a$$
 $B \rightarrow CA | Ab$
 $B \rightarrow CA | BCb | ab$

(2) $B \rightarrow CAB_R | abB_R$
 $B_R \rightarrow CbB_R | E$

• i = 1: nothing to do • i = 2, j = 1: $B \rightarrow CA \mid \underline{A} \mathbf{b}$ $\Rightarrow B \rightarrow CA \mid \underline{B} C \mathbf{b} \mid \underline{\mathbf{a}} \mathbf{b}$ $\Rightarrow_{(imm)} B \rightarrow CA B_R \mid \mathbf{a} \mathbf{b} B_R$ • $B_R \rightarrow C \mathbf{b} B_R \mid \varepsilon$ • i = 3, j = 1: $C \rightarrow \underline{A} B \mid CC \mid \mathbf{a}$ $\Rightarrow C \rightarrow \underline{B} C B \mid \underline{\mathbf{a}} B \mid CC \mid \mathbf{a}$

•
$$i = 3, j = 2$$
: $C \rightarrow \underline{B} C B \mid \mathbf{a} B \mid C C \mid \mathbf{a}$
 $\Rightarrow C \rightarrow \underline{C} \underline{A} \underline{B}_{\underline{R}} C B \mid \underline{\mathbf{a}} \underline{\mathbf{b}} \underline{B}_{\underline{R}} C B \mid \mathbf{a} B \mid C C \mid \mathbf{a}$
 $\Rightarrow_{(imm)} C \rightarrow \mathbf{a} \mathbf{b} B_{R} C B C_{R} \mid \mathbf{a} B C_{R} \mid \mathbf{a} C_{R}$
 $C_{R} \rightarrow A B_{R} C B C_{R} \mid C C_{R} \mid \varepsilon$

Example - Expression Grammar

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

$$E \rightarrow TE'$$

$$E' \rightarrow +TE' \mid E$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT' \mid E$$

$$F \rightarrow (E) \mid id$$

Modified Grammar

G

- $E \rightarrow TE'$
- E' \rightarrow +TE' | ε
- $T \rightarrow FT$
- T' \rightarrow *FT' | ε
- $F \rightarrow (E) \mid id$

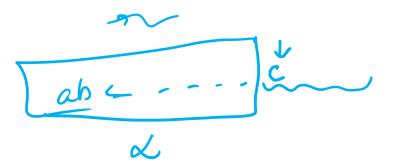
Left Factoring

Replace productions

$$A \rightarrow \alpha \; \beta_1 \; / \; \alpha \; \beta_2 \; / \; ... \; | \; \alpha \; \beta_n \; | \; \gamma$$
 with

$$A \to \alpha A_R \mid \gamma$$

$$A_R \to \beta_1 / \beta_2 / \dots / \beta_n$$



Left Factoring

METHOD: For each nonterminal A, find the longest prefix α common to two or more of its alternatives. If $\alpha \neq \epsilon$ — i.e., there is a nontrivial common prefix — replace all of the A-productions $A \to \alpha \beta_1 \mid \alpha \beta_2 \mid \cdots \mid \alpha \beta_n \mid \gamma$, where γ represents all alternatives that do not begin with α , by

$$A \to \alpha A' \mid \gamma$$

 $A' \to \beta_1 \mid \beta_2 \mid \cdots \mid \beta_n$

Here A' is a new nonterminal. Repeatedly apply this transformation until no two alternatives for a nonterminal have a common prefix. \square

Left Factoring - Example

$$S \rightarrow \underline{iCtS} \mid \underline{iCtS}eS \mid a$$
 $C \rightarrow b$

$$S \rightarrow eS \mid \epsilon$$

Left Factoring

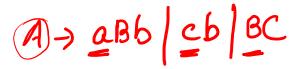
- S \rightarrow iCTSS' | a
- S' \rightarrow eS | ϵ
- $\cdot C \rightarrow b$

LL (1) Parser – Predictive parser

- L input is scanned from left to right
- L left derivation
- (1) looking at 1 input symbol

Predictive Parser LL (1)

- Eliminate left recursion from grammar
- Left factor the grammar
- Compute FIRST and FOLLOW
- Two variants:
 - Recursive (recursive calls)
 - Non-recursive (table-driven)





- Recursive-descent parsing is a top-down method of syntax analysis in which a set of recursive procedures is used to process the input
- One procedure is associated with each nonterminal of a grammar
- A simple form of recursive-descent parsing, called predictive parsing, in which the lookahead symbol unambiguously determines the flow of control through the procedure body for each nonterminal
- The sequence of procedure calls during the analysis of an input string implicitly defines a parse tree for the input, and can be used to build an explicit parse tree, if desired.

```
void SC) }
                               call A()
                               if 'd' match ('d')
void A() {
       Choose an A-production, A \to X_1 X_2 \cdots X_k;
      (for ( i = 1 \text{ to } k ) {
              if (X_i is a nonterminal)
                     call procedure X_i();
              else if (X_i equals the current input symbol a)
                     advance the input to the next symbol;
              else /* an error has occurred */;
```

```
⇒もけて
```

```
stmt → expr;
| if (expr) stmt
| for (optexpr; optexpr) stmt
| other

optexpr → ε ✓
| expr ✓

for (; i←f)
```

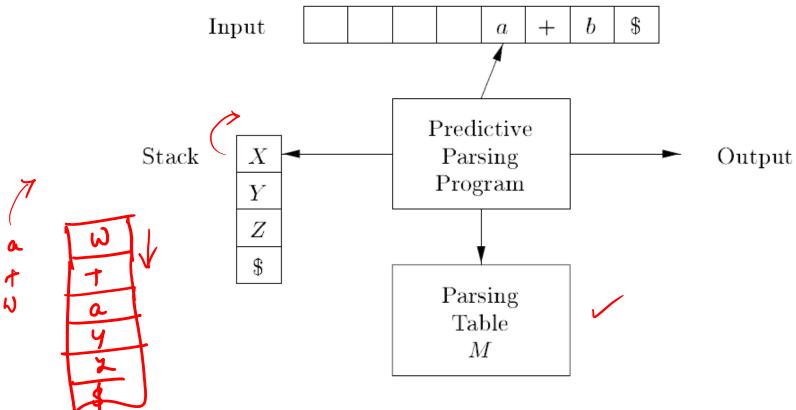
```
void stmt() {
       switch ( lookahead ) {
       case expr:
               match(\mathbf{expr}); match(';'); break;
       case if:
               match(\mathbf{if}); \ match('('); \ match(\mathbf{expr}); \ match(')'); \ stmt();
               break:
       case for:
               match(\mathbf{for}); \ match('(');
               optexpr(); match(';'); optexpr(); match(';'); optexpr();
               match(')'; stmt(); break;
       case other;
               match(\mathbf{other}); break;
       default:
               report("syntax error");
```

```
void optexpr() {
     if ( lookahead == expr ) match(expr);
}

void match(terminal t) {
     if ( lookahead == t ) lookahead = nextTerminal;
     else report("syntax error");
}
```

Non-Recursive Predictive Parsing





FIRST ()

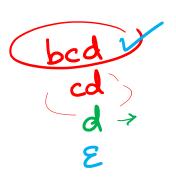
- FIRST function is computed for all terminals and non-terminals
- FIRST(α) = the set of terminals that begin all strings derived from α

```
FIRST (a)
FIRST (A)
FIRST (aAB)
```

FIRST ()

```
if a \in T
• FIRST(a) = \{a\}
  FIRST(\varepsilon) = \{\varepsilon\}
  FIRST(A) = \bigcup_{A \to \alpha} FIRST(\alpha)
                                         for A \rightarrow \alpha \in P
                                   A \rightarrow d_1 \left| d_2 \right| d_3 \right|
                         FIRST(A) = FIRST(X1) U FIRST(X2) U FIRST(X3)
```

FIRST () – Algorithm



```
A \Rightarrow B \subset D_{\varepsilon}
B \Rightarrow b \mid \varepsilon
C \Rightarrow C \mid \varepsilon
D \Rightarrow d \mid \varepsilon
```

• FIRST $(X_1X_2...X_k) =$ if for all $j = 1, ..., i-1 : \varepsilon \in \text{FIRST}(X_j)$ then

add non- ε in FIRST (X_i) to FIRST $(X_1X_2...X_k)$ if for all $j = 1, ..., k : \varepsilon \in \text{FIRST}(X_j)$ then

add ε to FIRST $(X_1X_2...X_k)$ CLEST(E) = \S b. $\varepsilon \S$

FIRST (A) = FIRST (BCD)
b, Eg =
$$\{b, c, d, 2\}$$

$$FIRST(C) = \{c, \xi\}$$

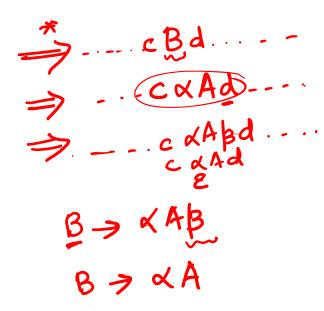
$$FIRST(D) = \{d\}$$

FOLLOW

• FOLLOW(A) = the set of terminals that can immediately follow non-terminal A

FOLLOW - Algorithm

```
• FOLLOW(A) =
        if A is the start symbol S then
                 add $ to FOLLOW(A)
        for all (B \rightarrow \alpha A \beta) \in P do
                 add FIRST(\beta)\{\epsilon} to FOLLOW(A) \checkmark
        for all (B \rightarrow \alpha A \beta) \in P and \varepsilon \in FIRST(\beta) do
                 add FOLLOW(B) to FOLLOW(A)
        for all (B \rightarrow \alpha A) \in P do
                 add FOLLOW(B) to FOLLOW(A)
```



Example

$$FI(+) = \frac{2}{3} + \frac{2}{3}$$
 $FI(+) = \frac{2}{3} + \frac{2}{3}$ $FI(-) = \frac{2}{3} + \frac{2}{3}$

 $FI(E) = FI(T) = \{(id\}$

 $FI(T) = FI(F) = {(,id)}$

FI(F) = { C, id }

FI(E1) = 3 + , E3

• E'
$$\rightarrow$$
 \pm TE' | ϵ

• T
$$\rightarrow$$
 FT'

• T'
$$\rightarrow$$
 *FT' | ϵ

•
$$F \rightarrow (E) \mid id$$

• F
$$\rightarrow$$
 (E) | id

FO(E) = { }, \$ }

FO(E) = { }, \$ }

FO(T) = FI(E) U FO(E) = { }, \$ }

FO(F) = FI(T) U FO(T) = { *, +,), \$ }

FO(E) = FO(E) = { }, \$ }

FO(E) = FO(E) = { }, \$ }

FIRST

- FIRST (E) = FIRST (T) = FIRST(F) = {(, id}
- FIRST (E') = $\{+, \epsilon\}$
- FIRST (T') = $\{ *, \epsilon \}$

FOLLOW

```
    FOLLOW(E) = FIRST(')')
        ={$, )}
        FOLLOW(T) = FIRST(E') U FOLLOW(E)
        ={+, $, )}
        FOLLOW(F) = { *, +, $, )}
        FIRST(T') U FOLLOW(T)
```

```
    FOLLOW (E') = FOLLOW (E)
    = {$, )}
    FOLLOW (T') = FOLLOW (T)
```

= {\$, +,)}

Another Example

• Ambiguous grammar $S \rightarrow \mathbf{i} C \mathbf{t} S S' \mid \mathbf{a}$ $S' \rightarrow \mathbf{e} S \mid \varepsilon$ $C \rightarrow \mathbf{b}$

Fo(s) =
$$\{4, e\}$$

Fo(s') = $\{4, e\}$
Fo(c) = $\{4, e\}$

$$FI(C) = \{b\}$$

 $FI(8') = \{e, E\}$
 $FI(3) = \{i, a\}$

$$R \rightarrow (\tau) | c$$

$$T \rightarrow T, R | R$$

$$R \rightarrow (\tau) | c$$

$$T \rightarrow RT' | -$$

$$T' \rightarrow RT$$

- First (S) = {i,a}
- First (S') = {e, ε }
- First (C) = {b}
- Follow (S) ={\$, e}
- Follow(S') = {\$, e}

Predictive Parsing Table

- Row for each non-terminal
- Column for each terminal symbol
 Table[NT, symbol] = Production that matches the [NT, symbol] if First(NT) has ε, then add production
 NT → ε in all [NT, a] for all 'a' in FOLLOW(NT)

Parsing action

```
push($)
 push(S)
 a := lookahead
```

repeat

```
X := pop()
     if X is a terminal or X = $ then
             match(X) // move to next token, a := lookahead
     else if M[X,a] = X \rightarrow Y_1 Y_2 ... Y_k then
             push(Y_k, Y_{k-1}, ..., Y_2, Y_1) // such that Y_1 is on top
             produce output and/or invoke actions
     else
             error()
     endif
until X = $
```

E-17E

Parsing action Example

Stack	Input String	Action		
\$E	id + id* id \$	[E, id]		
\$E'T	id + id *id \$	[T, id] モラフモ		
\$E'T'F	id + id *id \$	[F, id] $T > FT^{r}$		
\$E'T'id	(id + id *id \$	id, id -> pop stack and move input		
\$E'T'	<u>+ id *id\$</u>	[T', +] -> replace with ε		
\$E'	+ id *id\$	[E', +]		
\$ <u>E</u> 'T+	± id *id\$	+, + → pop stack and move		
\$E'T	id * id \$	[T, id]		
\$E' <u>T'F</u>	id *id\$	[F, id]		

Stack	Input String	Action		
\$E'T'id	id *id\$	id, id -> pop		
\$E'T'	*id \$	[T', *]		
\$E'T <u>'F*</u>	*id \$	*,* -> pop, and move		
\$E'T'F	id\$	[F, id]		
\$E'T'id	<u>id</u> \$	id, id → pop		
\$E'T'	\$	T', \$ -> replace with ε		
\$E'	\$	E', \$ -> replace with ε		
\$	\$	Accept		

Error Recovery in LL (1) parser

- Panic mode
 - Discard input until a token in a set of designated synchronizing tokens is found
- Phrase-level recovery
 - Perform local correction on the input to repair the error
- Error productions
 - Augment grammar with productions for erroneous constructs
- Global correction
 - Choose a minimal sequence of changes to obtain a global least-cost correction

Error Recovery

- Panic Mode
 - Add synchronizing actions to undefined entries based on FOLLOW
- Phrase Mode
 - Change input stream by inserting missing +, *, (, or)
 For example: id id is changed into id * id or id + id

Error Recovery

- Error Production
 - Add productions that will take care of incorrect input combinations

Error Recovery

	id	+	*	()	\$
Е	$E \rightarrow TE'$			$E \rightarrow TE'$	synch	synch
E'		E'→ +TE'			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
Т	$T \rightarrow FT'$	synch		$T \rightarrow FT'$	synch	synch
T'		$T' \rightarrow \epsilon$	T' → *FT'		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
F	$F \rightarrow id$	synch	synch	$F \rightarrow (E)$	synch	synch

LL (1)

• A grammar G is LL(1) if for each collections of productions $A \to \alpha_1 \mid \alpha_2 \mid ... \mid \alpha_n$ for nonterminal A the following holds:

- 1. FIRST(α_i) \cap FIRST(α_j) = \emptyset for all $i \neq j$
- 2. if $\alpha_i \Rightarrow^* \varepsilon$ then
 - 2.a. $\alpha_i \Rightarrow^* \varepsilon$ for all $i \neq j$
 - 2.b. $\mathsf{FIRST}(\alpha_j) \cap \mathsf{FOLLOW}(A) = \emptyset$ for all $i \neq j$

If then Grammar

- The if then grammar has multiple entries in the parsing table.
- So, confusion on which production to apply
- Ambiguous grammar hence not LL (1)