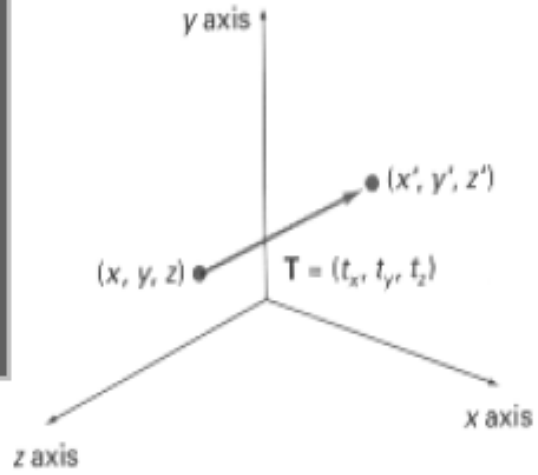
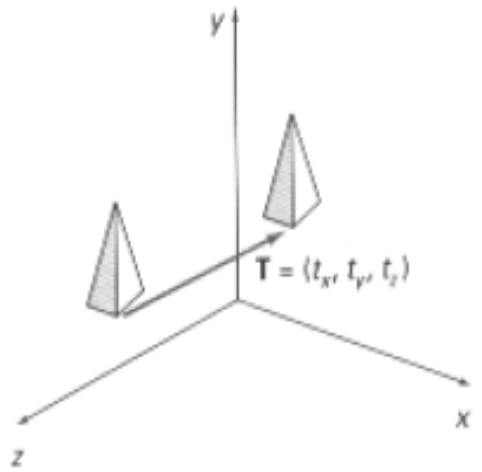
3D Transformation

Translation

In three-dimensional homogeneous coordinate representation, a point is transformed from position P = (x, y, z) to P′ = (x', y', z') This can be written as:

$$x' = x + t_x$$
$$y' = y + t_y$$
$$z' = z + t_z$$



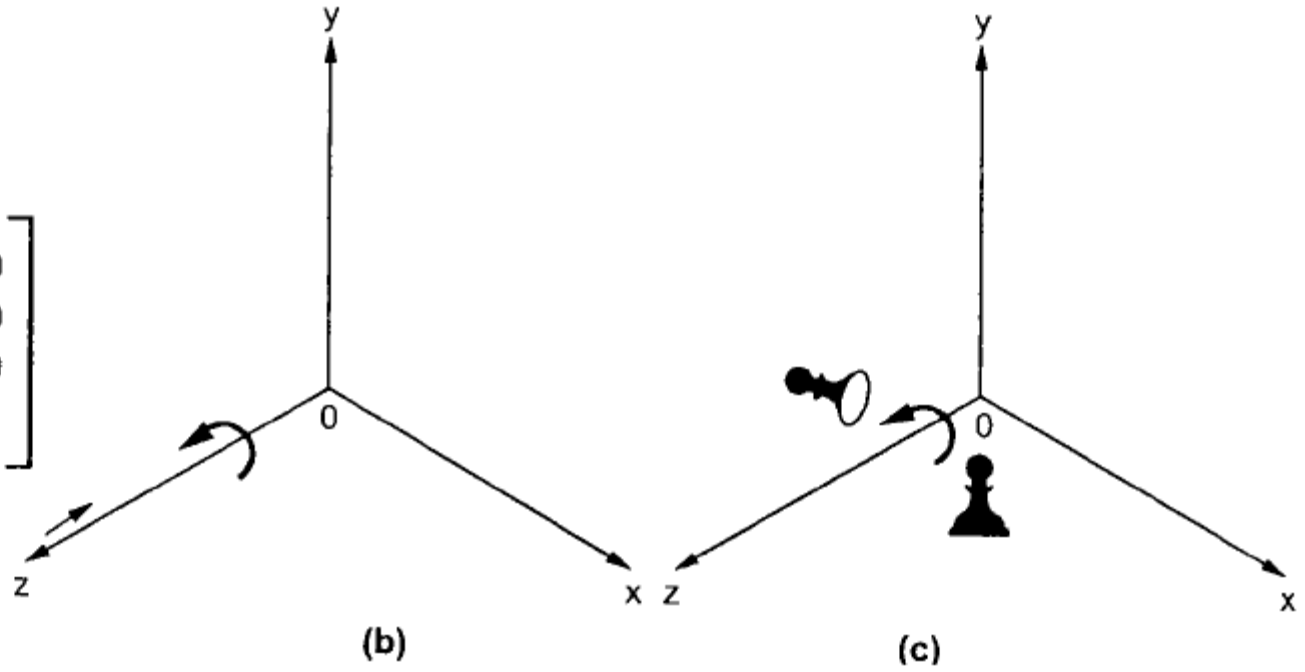


Translation

$$\begin{bmatrix} x' & y' & z' & 1 \end{bmatrix} = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ T_x & T_y & T_z & 1 \end{bmatrix}$$

Rotation

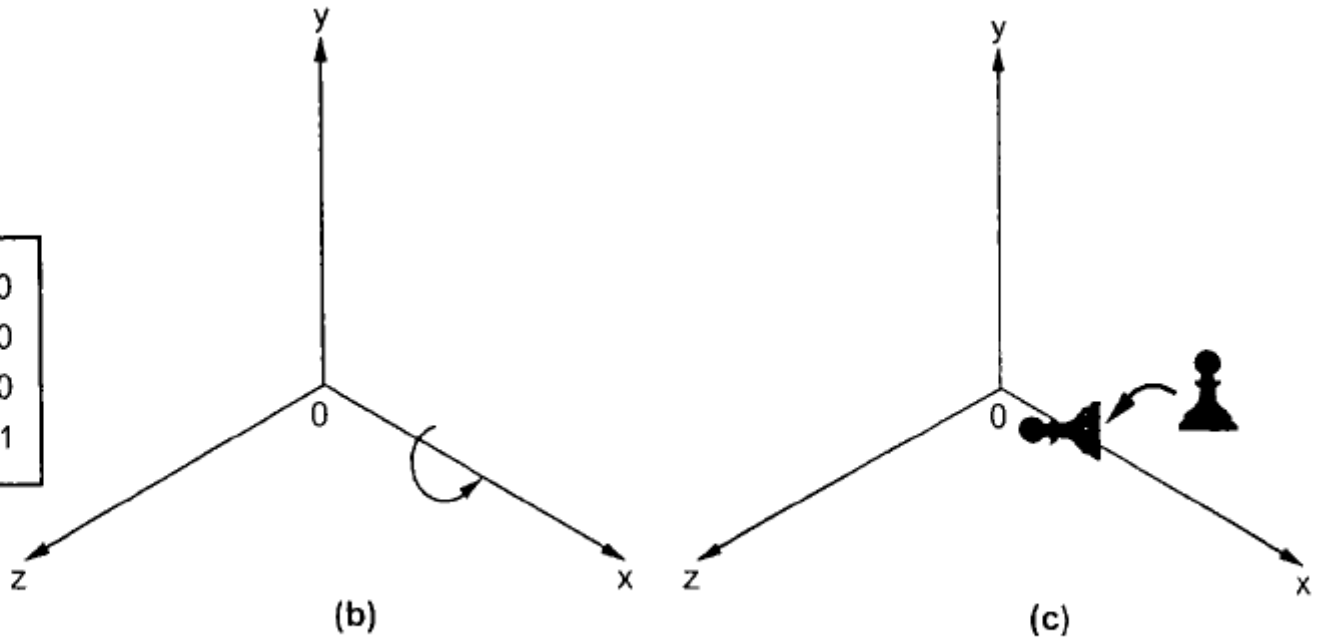
$$R_z = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation about z axis

Rotation

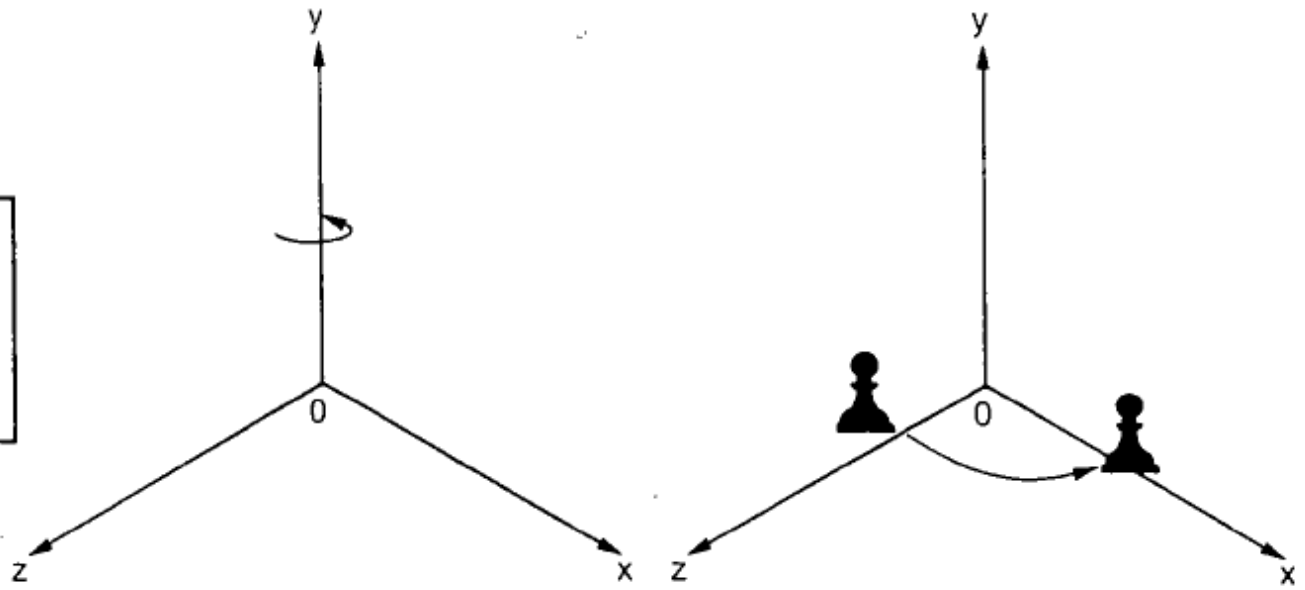
$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Rotation about x axis

Rotation

$$R_y = \begin{bmatrix} \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



(a) (b) (c)

Rotation about y axis

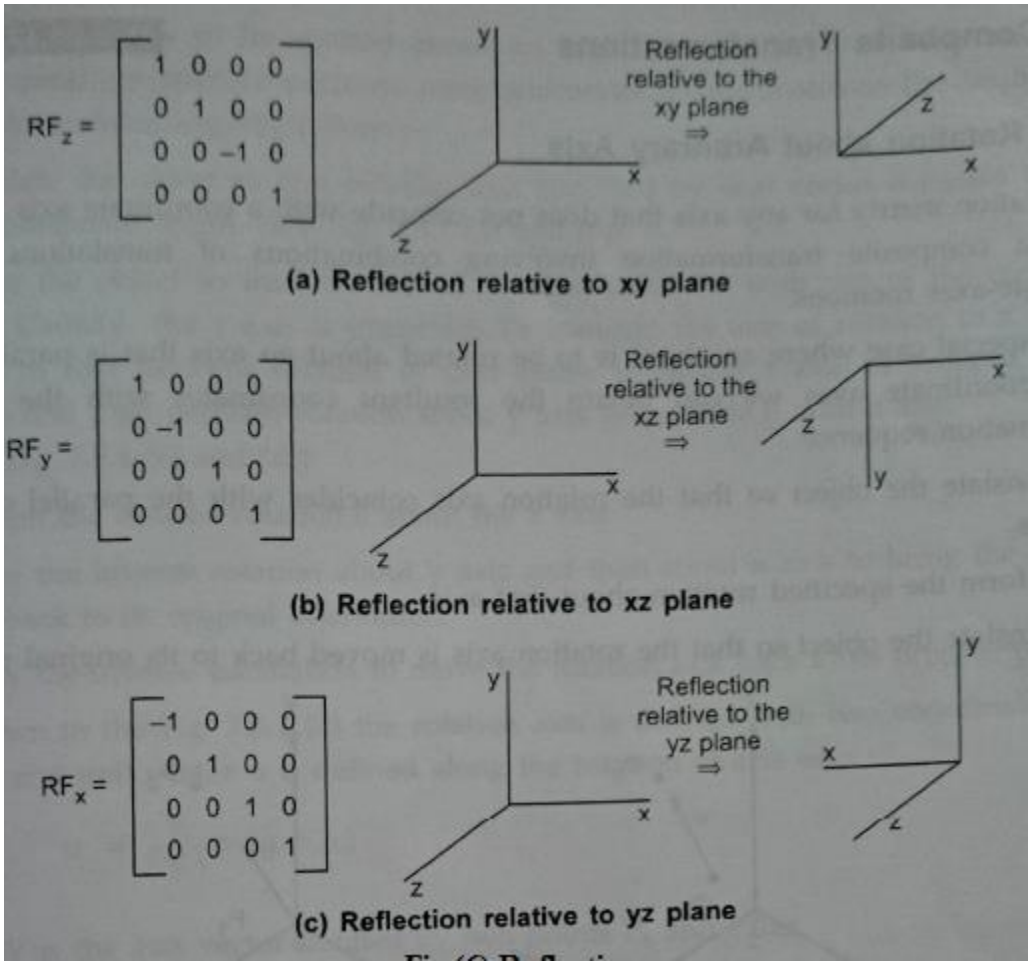
Scaling

- The matrix expression for the scaling transformation of a position P = (x_i, y_i, z_i) relative to coordinate origin can be written as:

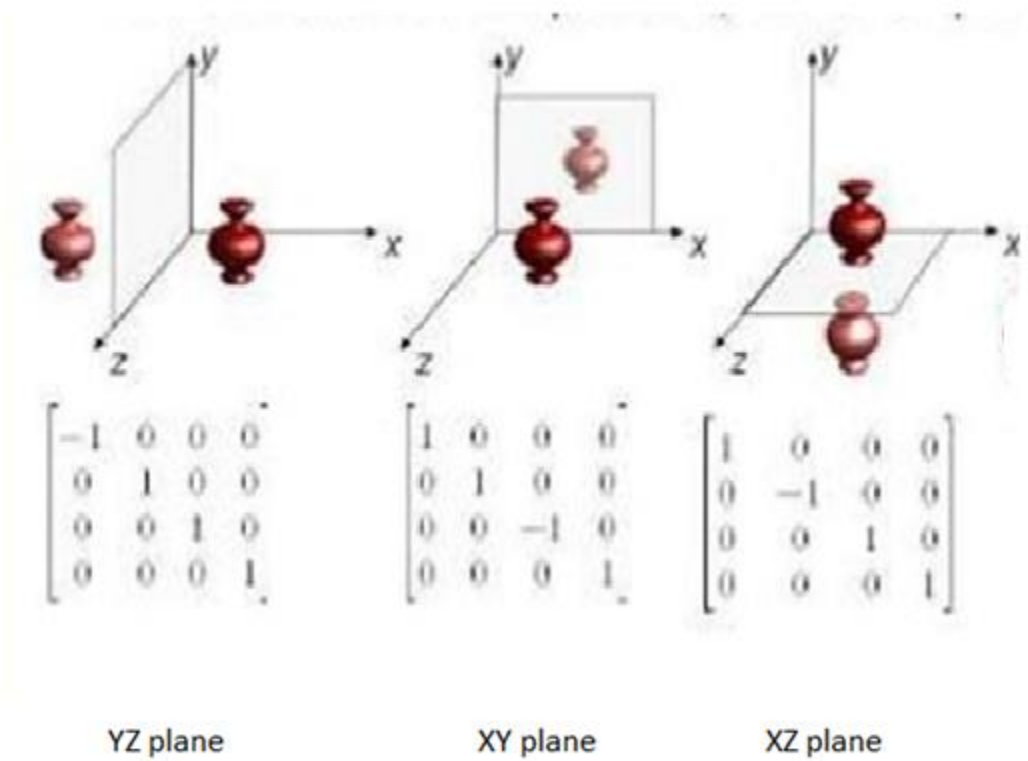
$$\begin{bmatrix} x_i & y_i & z_i & w \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} S_x x_i & S_y y_i & S_z z_i & w \end{bmatrix}$$

Reflection

Types of Reflection
- Reflection Relative to XY Plane
- Reflection Relative to YZ Plane
- Reflection Relative to XZ Plane



Reflection



$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

YZ plane XY plane XZ plane

Shearing

f Shearing → Shearing in X axis
f Shearing → Shearing in Y axis
f Shearing → Shearing in Z axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ shy & 1 & 0 & 0 \\ shz & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & Shx & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & Shz & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & Shx & 0 \\ 0 & 1 & Shy & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Shearing about X axis- X coordinate will change but no change in Y and Z axis coordinate.
- Shearing about Y axis- Y coordinate will change and changes are made only in X and Z axis.
- Shearing about Z axis- Z coordinate will change but no change in X and Y axis.