Problem sheet 6 – for the tutorial class on 09/03/2021

I. What language over $\{a, b\}$ does the CFG with productions

$$S \to aaS \mid bbS \mid Saa \mid Sbb \mid abSab \mid abSba \mid baSba \mid baSab \mid \Lambda$$

generate? Prove your answer.

Is this grammar ambiguous?

II. In each case below, find a context-free grammar with no Λ-productions that generates the same language, except possibly for Λ, as the given CFG.

a. $S \to AB \mid \Lambda \qquad A \to aASb \mid a \qquad B \to bS$

b. $S \to AB \mid ABC$
$A \to BA \mid BC \mid \Lambda \mid a$
$B \to AC \mid CB \mid \Lambda \mid b$
$C \to BC \mid AB \mid A \mid c$

III. In each case, given the context-free grammar G, find a CFG G' with no Λ-productions and no unit productions that generates the language $L(G) - \{\Lambda\}$.

a. G has productions

$$S \to ABA \qquad A \to aA \mid \Lambda \qquad B \to bB \mid \Lambda$$

b. G has productions

$$S \to aSa \mid bSb \mid \Lambda \qquad A \to aBb \mid bBa \qquad B \to aB \mid bB \mid \Lambda$$

c. G has productions

$$S \to A \mid B \mid C \qquad A \to aAa \mid B \qquad B \to bB \mid bb$$
$$C \to aCaa \mid D \qquad D \to baD \mid abD \mid aa$$

IV. In each case, given the context-free grammar G, find an equivalent CFG with no useless variables.

i. G has productions

$$S \to ABC \mid BaB \qquad A \to aA \mid BaC \mid aaa$$
$$B \to bBb \mid a \qquad C \to CA \mid AC$$

ii. G has productions

$$S \to AB \mid AC \quad A \to aAb \mid bAa \mid a \quad B \to bbA \mid aaB \mid AB$$
$$C \to abCa \mid aDb \qquad D \to bD \mid aC$$

V. In each case below, given the context-free grammar G, find a CFG G_1 in Chomsky normal form generating $L(G) - \{\Lambda\}$.

a. G has productions $S \to SS \mid (S) \mid \Lambda$

b. G has productions $S \to S(S) \mid \Lambda$

c. G has productions

$$S \to AaA \mid CA \mid BaB \quad A \to aaBa \mid CDA \mid aa \mid DC$$
$$B \to bB \mid bAB \mid bb \mid aS \quad C \to Ca \mid bC \mid D \quad D \to bD \mid \Lambda$$

VI Find a Greibach normal-form grammar equivalent to the following CFG:

$$S \to AA \mid 0$$

$$A \to SS \mid 1$$