Derivative of sigmoid activation function?

$$f(x) = \frac{1}{1+e^{-x}}$$

$$f'(x) = \frac{\partial}{\partial x}\left[\frac{1}{1+e^{-x}}\right] \qquad = \frac{\partial}{\partial x}\left[f(x)\right]$$

$$= \frac{\partial}{\partial x}\left[\left(1+e^{-x}\right)^{-1}\right]$$

$$= \frac{\left(1+e^{-x}\right)\cdot 0 - 1 * \left(-e^{-x}\right)}{\left(1+e^{-x}\right)^2}$$

$$= \frac{e^{-x}}{\left(1+e^{-x}\right)^2}$$

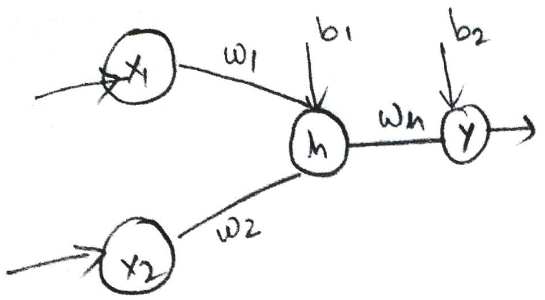
$$= \frac{1 - 1 + e^{-x}}{\left(1+e^{-x}\right)^2}$$

$$= \frac{1+e^{-x}}{\left(1+e^{-x}\right)^2} - \frac{1}{\left(1+e^{-x}\right)^2}$$

$$= \frac{1}{\left(1+e^{-x}\right)} - \frac{1}{\left(1-e^{-x}\right)^2}$$

$$= \frac{1}{\left(1+e^{-x}\right)}\left(1 - \frac{1}{1+e^{-x}}\right)$$

$$\boxed{f'(x) = f(x) * \left(1 - f(x)\right)}$$



Given; $x_1 = 0.5$
$x_2 = 1.1$
$w_1 = 0.25$, $w_2 = 0.3$
$w_h = 0.33$
$b_1 = -0.1$
$b_2 = 0.3$
$y = \text{target} = 0.8$
$\eta = 0.25$

$h_{in} = w_1 x_1 + w_2 x_2 + b_1$
$= 0.5 * 0.25 + 0.3 * 1.1 + (-0.1)$
$= 0.125 + 0.33 - 0.1$
$= 0.355$

$h_{out} = Act(h_{in})$
$= \text{Sigmoid}(0.355)$
$= \dfrac{1}{1+e^{-0.355}}$

$= 0.587$

$y_{in} = h_{out} * w_h + b_2$
$= 0.58 * 0.33 + 0.3$
$= 0.491$

$y_{out} = Act(y_{in})$
$= \text{Sigmoid}(0.491)$
$= 0.62$

Error (E) $= \dfrac{1}{2}\left[(y - y_{out})^2\right]$
$= \dfrac{1}{2}\left[(0.8 - 0.62)^2\right]$
$= \dfrac{0.0324}{2} = 0.0162$

Now to find new value of w_h, w_1, w_2.

$w_{h\,new} = w_{h\,old} - \eta \dfrac{\partial E}{\partial w_{h\,old}}$

$\dfrac{\partial E}{\partial w_{h\,old}} = \dfrac{\partial\left[\frac{1}{2}(y - y_{out})^2\right]}{\partial w_{h\,old}}$ here we can not differentiate

to calculate derivate wrt $w_{h\,old}$ then we will expand y_{out} and the sequence is $E \to y_{out} \to y_{in} \to w_h$

$$\frac{\partial E}{\partial w_{h\,old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial w_{h\,old}} \quad \text{— (eq}^n\text{ 1)}$$

$$\frac{\partial y_{in}}{\partial w_{h old}} = \frac{\partial(h_{out} * w_h + b_1)}{\partial w_{h old}}$$ $w_h = w_{h old}$

$$= h_{out} = 0.587$$

$$\frac{\partial y_{out}}{\partial y_{in}} = y_{out}(1 - y_{out})$$

$$= 0.62 * (1 - 0.62)$$

$$= 0.235$$

$$\frac{\partial E}{\partial y_{out}} = \frac{1}{2}((y - y_{out})^2)$$

$$= \cancel{2} * \frac{1}{\cancel{2}}(y - y_{out}) * -1$$

$$= y_{out} - y$$

$$= 0.62 - 0.8$$

$$= -0.18$$

$$\frac{\partial E}{\partial w_{hold}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial w_{hold}}$$

$$= -0.18 * 0.235 * 0.587$$

$$= -0.024$$

$$(w_h)_{new} = w_{hold} - \eta \cdot \frac{\partial E}{\partial w_{hold}}$$

$$= 0.33 - (0.25 * -0.024)$$

$$= 0.33 - (-0.06)$$

$$= 0.336$$

Similarly for w_1 :- $(w_1)_{new} = (w_1)_{old} - \eta \frac{\partial E}{\partial w_{old}}$

$$\frac{\partial E}{\partial (w_1)_{old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial h_{out}} * \frac{\partial h_{out}}{\partial h_{in}} * \frac{\partial h_{in}}{\partial w_{1 old}} \longleftarrow eq^n (2)$$

$$\frac{\partial y_{in}}{\partial h_{out}} = \frac{\partial(h_{out} * w_h + b_1)}{\partial h_{out}}$$

$$= w_h = 0.33$$

$$\frac{\partial h_{out}}{\partial h_{in}} = h_{out} * (1 - h_{out})$$

$$= 0.587 * 0.41$$

$$= 0.242$$

$$\frac{\partial h_{in}}{\partial w_{old}} = \frac{\partial(w_1 x_1 + w_2 x_2 + b_1)}{\partial w_{1 old}}$$ $w_1 = w_{p old}$

$$= x_1 = 0.5$$

Now Putting the values in eq^n (2)

$$\frac{\partial E}{\partial w_{1old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial h_{out}} * \frac{\partial h_{out}}{\partial h_{in}} * \underline{\frac{\partial h_{in}}{\partial w_{1old}}}$$

$$= -0.18 * 0.235 * 0.33 * 0.242 * \underline{0.5}$$

$$= -0.0016$$

$$w_{1new} = w_{1old} - \eta \frac{\partial E}{\partial w_{1old}}$$

$$= 0.25 - (0.25 * -0.0016)$$

$$= 0.25 + 0.00042$$

$$= 0.25042.$$

Similarly we will calculate for w_2.

$$(w_2)_{new} = (w_2)_{old} - \eta \frac{\partial E}{\partial w_{2old}}$$

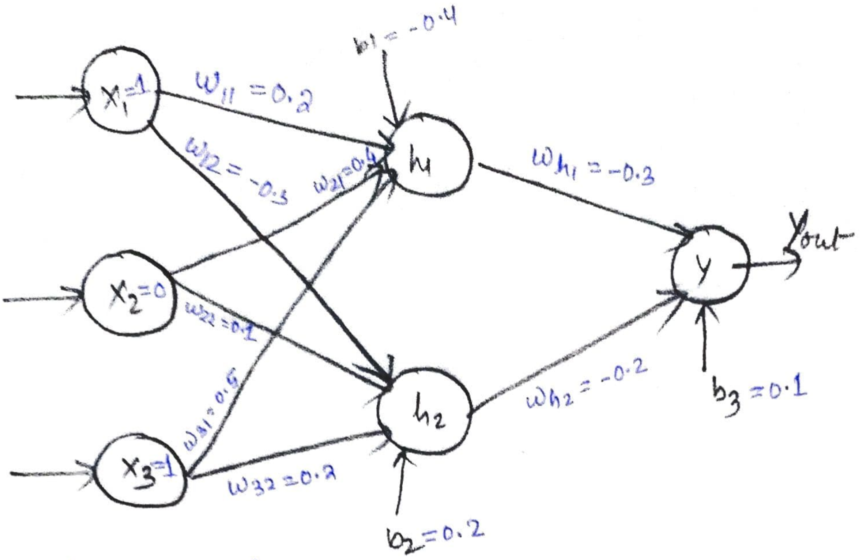
$$\frac{\partial E}{\partial w_{2old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial h_{out}} * \frac{\partial h_{out}}{\partial h_{in}} * \underline{\frac{\partial h_{in}}{\partial w_{2old}}}$$

$$= -0.18 * 0.235 * 0.33 * 0.242 * \underline{1.1}$$

$$= \cancel{0.0004225}$$

$$(w_2)_{new} = w_{2old} - \eta \frac{\partial E}{\partial w_{2old}}$$

$$= 0.25 - 0.25 - 0.$$



Activation function = Sigmoid , η = learning rate = 0.9, target(y) = 1

$(h_1)_{in} = w_{11}x_1 + w_{21}x_2 + w_{31}x_3 + b_1$

$=$

$(h_1)_{out} = Act(h_1)_{in} = \text{Sigmoid}(\quad) = 0.332$

$h_{2in} = w_{12}x_1 + w_{22}x_2 + w_{32}x_3 + b_2$

$h_{2out} = Act(h_2)_{in} = \text{Sigmoid}(\quad) = 0.525$

$Y_{in} = h_{1out} * w_{h_1} + h_{2out} * w_{h_2} + b_3$

$Y_{out} = Act(Y_{in}) = 0.474$

Error $(E) = \frac{1}{2}[(y - Y_{out})^2]$

Now to reduce the error, we need to backpropagate.

weight updation :-

$$(w_{h_1})_{new} = (w_{h_1})_{old} - \eta \frac{\partial E}{\partial (w_{h_1})_{old}}$$

$$\frac{\partial E}{\partial (w_{h_1})_{old}} = \frac{\partial E}{\partial Y_{out}} * \frac{\partial Y_{out}}{\partial Y_{in}} * \frac{\partial Y_{in}}{\partial (w_{h_1})_{old}}$$

$$\frac{\partial Y_{in}}{\partial (w_{h_1})_{old}} = \frac{\partial(h_{1out} * w_{h_1} + \cancel{h_{2out} * w_{h_2}} + b_3)}{\partial w_{h_1 old}}$$

$$= (h_1)_{out} = 0.332$$

$$\frac{\partial Y_{out}}{\partial Y_{in}} = Y_{out} * (1 - Y_{out})$$

$$= 0.474 * (1 - 0.474)$$

$$= 0.249$$

$$\frac{\partial E}{\partial Y_{out}} = \frac{1}{\cancel{2}} \times \cancel{2}\,[y - Y_{out}] * -1$$

$$= Y_{out} - y = 0.474 - 1$$

$$= -0.526$$

$$W_{11\,new} = W_{11\,old} - \frac{\partial E}{\partial W_{11\,old}}$$

$$\frac{\partial E}{\partial W_{11\,old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial h_{1out}} * \frac{\partial h_{1\,out}}{\partial h_{1\,in}} * \frac{\partial h_{1\,in}}{\partial W_{11\,old}}$$

$$\frac{\partial y_{in}}{\partial h_{1out}} = \frac{\partial(h_{1out} * W_{h1} + \cancel{h_{2out} * W_{h2}}^{\,0} + b_3)}{\partial h_{1out}}$$

$$= W_{h1_{out}}$$

$$\frac{\partial h_{1out}}{\partial h_{1\,in}} = \frac{\partial\, Act(h_{1in})}{\partial h_{1in}} = h_{1out} * (1 - h_{1out})$$

$$= 0.332 * (1 - 0.332)$$

$$\frac{\partial h_{1in}}{\partial W_{11\,old}} = \frac{\partial(W_{11}x_1 + W_{21}x_2 + W_{31}x_3 + b_1)}{\partial W_{11\,old}}$$

$$= x_1 = 1$$

So Putting all these values;

$$\frac{\partial E}{\partial W_{11\,old}} = \frac{\partial E}{\partial y_{out}} * \frac{\partial y_{out}}{\partial y_{in}} * \frac{\partial y_{in}}{\partial h_{1\,out}} * \frac{\partial h_{1\,out}}{\partial h_{1in}} * \frac{\partial h_{1\,in}}{\partial W_{11\,old}}$$

$$= -0.526 * 0.249 * 0.332 * 0.221 * 1$$

$$= (\quad)$$

$$\frac{\partial E}{\partial W_{11\,new}} = W_{11\,old} - \eta \frac{\partial E}{\partial W_{11\,old}}$$

$$= (\quad) \checkmark$$

Similarly Calculate for $W_{12}, W_{21}, W_{22}, W_{31}, W_{32}$