Unit - 2

Electromagnetic Waves

krishnavel

Electric field Intensity ($\vec{E}$):

It is defined as the ratio of electrostatic force to the electric charge

$$\boxed{\vec{E} = \frac{\vec{F}}{q}}$$

Unit : newton / coulomb

Electric Displacement vector ($\vec{D}$):

It is defined as the electric flux (Q) per unit area. It is also known as electric flux density.

$$\vec{D} = \frac{Q}{4\pi r^2} \quad \rightarrow (1)$$

W.K.t Electric field intensity,

$$\vec{E} = \frac{Q}{4\pi \varepsilon r^2} \quad \rightarrow (2)$$

comparing (1) & (2) $\Rightarrow$ $\boxed{\vec{D} = \varepsilon \vec{E}}$

Electrical permittivity (ε):

It is defined as the ratio of displacement current ($\vec{D}$) to the electric field Intensity ($\vec{E}$).

$$\boxed{\varepsilon = \frac{\vec{D}}{\vec{E}}}$$

Unit : $C^2 N^{-1} m^{-2}$

Dielectric constant (or) Relative permittivity (ε_r) :

It is defined the ratio of permittivity of the medium (ε) to the permittivity of free space (or) vacuum (ε_0).

$$\boxed{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}} \quad (\text{or}) \quad \boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

where $\varepsilon_0 \rightarrow$ permittivity in free space or vacuum.

$$\varepsilon_0 = 8.854 \times 10^{-12} \, C^2 N^{-1} m^{-2}$$

Magnetic flux density ($\vec{B}$) :

It is defined as the number of magnetic flux lines of force (ϕ_m) passing normally through unit area of cross section at that point.

$$\boxed{\vec{B} = \frac{\phi_m}{A}}$$

Magnetic field Intensity ($\vec{H}$) :

It is defined as the force experienced by a unit north pole placed at the given point in a magnetic field.

$$\boxed{\vec{H} = \frac{\vec{F}}{m}}$$

Magnetic permeability (μ) :

It is defined as ratio of magnetic flux intensity ($\vec{B}$) to the magnetic field intensity ($\vec{H}$)

$$\boxed{\mu = \mu_0 \mu_r = \frac{\vec{B}}{\vec{H}}}$$

Unit : $NS^2 C^{-2}$

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Relative permeability (μ_r):

It is defined as the ratio of permeability of the medium (μ) to the permeability of free space (or) vaccum (μ_0).

$$\boxed{\mu_r = \frac{\mu}{\mu_0}} \quad \text{(or)} \quad \boxed{\mu = \mu_0 \mu_r}$$

where $\mu_0 \rightarrow$ Permeability in free space or vaccum

$\mu_0 = 4\pi \times 10^{-7} \ NS^2C^{-2}$

Gauss law for electric field:

It states that, the total flux through any closed surface is equal to $\frac{1}{\varepsilon_0}$ times of the total charge (Q) enclosed in the surface.

$$\boxed{\oint_S \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}}$$

Faraday's law:

It states that, the induced electromotive force (ε) in a coil is equal to the rate of change of the magnetic flux (ϕ) linking the coil.

$$\boxed{\varepsilon = -\frac{d\phi}{dt}}$$

The negative sign implies the decrease in magnetic flux.

Ampere's circuit law:

It states that, the line integral of magnetic field ($\vec{B}$)

surrounding any closed path is equal to μ times of net current (I) passing through that path.

$$\oint_l \vec{B} \cdot dl = \mu_0 I$$

$$\oint_l \frac{\vec{B}}{\mu_0} dl = I$$

$$\because \vec{H} = \frac{\vec{B}}{\mu_0}$$

$$\boxed{\oint_l \vec{H} \cdot dl = I}$$

Gauss law for magnetic field :

Gauss law for magnetic field states that the magnetic flux ($\vec{B}$) passing through the closed surface is equal to zero.

$$\boxed{\oint_s \vec{B} \cdot \vec{ds} = 0}$$

Derivations of Maxwell's equations

16m **Maxwell's first equation from electric gauss law:**

S → dielectric medium of surface

V → volume

Q → Total charge

ρ → charge density

According to gauss law, for electric field,

$$\boxed{\oint_S \vec{E}\cdot ds = \frac{Q}{\varepsilon_0}}$$

$$\oint_S \varepsilon_0 \vec{E}\, ds = Q \rightarrow ①$$

W.K.T ⇒ $\boxed{\vec{D} = \varepsilon\vec{E}}$

Since, $\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$

$$\Rightarrow \vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

Since, $\boxed{\varepsilon_r = 1}$ (For Air)

$$\boxed{\vec{D} = \varepsilon_0 \vec{E}} \rightarrow ②$$

Sub ② in ① ⇒

$$\oint_S \vec{D}\cdot ds = Q \rightarrow ③$$

Since, $\boxed{Q = \oint_V \rho\, dv} \rightarrow ④$

comparing ③ & ④ ⇒

$$\boxed{\oint_S \vec{D} \cdot ds = \oint_V \rho \, dv} \rightarrow ⑤$$

Eqn ⑤ is called Maxwell's first equation in Integral form.

<u>Differential form</u>:

From eqn ⑤, Applying gauss divergence theorem on L.H.S ⇒

$$\oint_S \vec{D} \cdot ds = \oint_V \vec{\nabla} \cdot \vec{D} \cdot dv \rightarrow ⑥$$

From ⑤ & ⑥, we can write,

$$\oint_V \vec{\nabla} \cdot \vec{D} \, dv = \oint_V \rho \, dv \rightarrow ⑦$$

$$\boxed{\vec{\nabla} \cdot \vec{D} = \rho} \rightarrow ⑧$$

Eqn ⑧ ⇒ Maxwell's first equation in differential form.

<u>Maxwell's ~~first~~ IInd equation from magnetic gauss law</u>:

According to gauss law for magnetic field,

$$\boxed{\phi = 0} \rightarrow ⑨$$

W.K.T ⇒ $$\boxed{\phi = \oint_S \vec{B} \cdot ds} \rightarrow ⑩$$

comparing ⑨ and ⑩ ⇒

$$\oint_S \vec{B} \cdot ds = 0 \rightarrow ⑪$$

Eqn ⑪ ⇒ Maxwell's second equation in integral form

Differential form :

From eqn (11), using gauss divergence theorem.

$$\oint_S \vec{B}\cdot ds = \oint_V \vec{\nabla}\cdot\vec{B}\,dv = 0 \rightarrow (12)$$

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0} \rightarrow (13)$$

Eqn (13) represents Maxwell's 2^{nd} equation in differential form

Maxwell's third equation from Faraday's law :

According to faraday's law,

$$\boxed{\varepsilon = -\frac{d\phi}{dt}} \rightarrow (14)$$

$\varepsilon \rightarrow$ Electromotive force

$\phi \rightarrow$ Magnetic flux

W.K.T $\Rightarrow$ $$\boxed{\varepsilon = \oint_L \vec{E}\cdot dl} \rightarrow (15)$$

$$\boxed{\phi = \oint_S \vec{B}\cdot ds} \rightarrow (16)$$

Sub (15) & (16) in (14) $\Rightarrow$

$$\oint_l \vec{E}\cdot dl = \frac{-d\left[\oint_S \vec{B}\cdot ds\right]}{dt}$$

$$\oint_l \vec{E}\cdot dl = -\oint_S \frac{d\vec{B}}{dt}\cdot ds \rightarrow (17)$$

Eqn (17) represents Maxwells 3^{rd} eqn in integral form.

Differential form :

From eqn (17), Applying stoke's theorem on L.H.S,

$$\oint_l \vec{E}.dl = -\oint_S \vec{\nabla} \times \vec{E}.dS \rightarrow (18)$$

Comparing (17) & (18) $\Rightarrow$

$$-\oint_S \vec{\nabla} \times \vec{E}.dS = \oint_S \frac{\partial \vec{B}}{\partial t} ds \rightarrow (19)$$

Eqn (19) becomes,

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \rightarrow (20)$$

Eqn (20), represents Maxwells 3rd eqn in differential form

Maxwell's Fourth equation from Ampere's law :

From ampere's law,

$$\boxed{\oint_l \vec{H}.\vec{dl} = I} \rightarrow (21)$$

W.K.T $\Rightarrow$ $I = \oint_S \vec{J}.\vec{ds} \rightarrow (22)$

Sub (22) in (21) $\Rightarrow$

$$\oint_l \vec{H}.\vec{dl} = \oint_S \vec{J}.\vec{ds} \rightarrow (23)$$

From stoke's theorem,

$$\oint_l \vec{H}.\vec{dl} = \oint_S \vec{\nabla} \times \vec{H}.\vec{ds} \rightarrow (24)$$

comparing (23) & (24) $\Rightarrow$

$$\oint_S \vec{\nabla} \times \vec{H} \cdot d\vec{s} = \oint_S \vec{J} \cdot d\vec{s} \rightarrow (25)$$

Eqn (25) becomes,

$$\vec{\nabla} \times \vec{H} = \vec{J} \rightarrow (26)$$

Applying gauss divergence theorem on both sides,

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} \rightarrow (27)$$

From vector identity, $\boxed{\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0}$

Eqn (27) becomes,

$$\vec{\nabla} \cdot \vec{J} = 0 \rightarrow (28)$$

According to eqn of continuity,

$$\boxed{\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0}$$

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$\vec{\nabla} \cdot \vec{J} = 0$ only if $\frac{\partial \rho}{\partial t} = 0 \rightarrow (29)$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \vec{J}_d} \rightarrow (30)$$

Taking divergence on both sides ⇒

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot (\vec{J} + \vec{J}_d) \rightarrow (31)$$

Using vector identity, $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = 0$

Eqn (31) becomes, $\vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J}_d = 0$

$$\vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{J_d} = 0$$

Since, $\boxed{\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}}$

$$-\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J_d} = 0$$

$$\vec{\nabla} \cdot \vec{J_d} = \frac{\partial \rho}{\partial t}$$

From Maxwell 1st eqn, w.k.t $\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$

$$\Rightarrow \vec{\nabla} \cdot \vec{J_d} = \frac{\partial (\vec{\nabla} \cdot \vec{D})}{\partial t}$$

$$\vec{\nabla} \cdot J_d = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\vec{J_d} = \frac{\partial \vec{D}}{\partial t} \rightarrow (32)$$

Sub eqn (32) in (30) $\Rightarrow$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}} \rightarrow (33)$$

Eqn (33) $\Rightarrow$ Maxwell's 4th eqn in differential form