Варіант 2

№1.2

$$\cos x \cos y\, dy + \sin x \sin y\, dx = 0$$

$$\cos x \cos y\, dy = -\sin x \sin y\, dx$$

$$\frac{\cos y\, dy}{\sin y} = -\frac{\sin x\, dx}{\cos x}$$

$$\int \frac{\cos y}{\sin y}\, dy = \int \frac{\sin x}{\cos x}\, dx$$

$$\ln \sin y = \ln \cos x + C$$

$$\sin y = e^{c} \cos x$$

$$y = \arcsin(C \cos x), \quad x = \frac{\pi}{2}.$$

№2.2

$$xy' = \sqrt{x^2 - y^2} + y$$

$$\frac{x\,dy}{dx} = \sqrt{x^2 - y^2} + y$$

$$x\, dy = (\sqrt{x^2 - y^2} + y)\, dx$$

$$x(u\, dx + x\, du) = (\sqrt{1-u^2} + u)\, x\, dx$$

$$u x\, dx + x^2 du = \sqrt{1-u^2}\, x\, dx + u x\, dx$$

$$x^2 du = \sqrt{1-u^2}\, x\, dx$$

$$\frac{du}{\sqrt{1-u^2}}=\frac{dx}{x}$$

$$\int\frac{1}{\sqrt{1-u^2}}du=\int\frac{1}{x}dx$$

$$\arcsin u=\ln x+C$$

$$y=x\sin(\ln x+C),\ x=y,\ x=-y$$

№3.2

$$y'-4y=e^{2x}$$

$$\cancel{y'}\ u'v+u'v-4uv=e^{2x}$$

$$u'v+u(v'-4v)=e^{2x}$$

$$v'-4v=0$$

$$v'=4v$$

$$\frac{dv}{dx}=4v$$

$$dv=4v\,dx$$

$$\frac{dv}{v}=4dx$$

$$\int\frac{1}{v}dv=\int 4dx$$

$\ln|v| = 4x$

$v = e^{4x}$

$v = e^{4x}$

$u' e^{4x} = e^{2x}$

$u' e^{2x} = 1$

$u' = \frac{1}{e^{2x}}$

$\frac{du}{dx} = \frac{1}{e^{2x}}$

$du = \frac{dx}{e^{2x}}$

$\int 1 du = \int \frac{1}{e^{2x}} dx$

$u = C - \frac{1}{2e^{2x}}$

$u = C - \frac{1}{2e^{2x}}$

$y = \frac{e^{2x}(2Ce^{2x} - 1)}{2}$

$y = Ce^{4x} - \frac{e^{2x}}{2}$

№4.2

$$\frac{2x\,dx}{y^3}+\frac{y^2-3x^2}{y^4}dy=0$$

$$(y^2-3x^2)dy+2xy\,dx=0$$

$$(y^2-3x^2)dy=-2xy\,dx$$

$$(u^2-3)x^2(u\,dx+x\,du)=-2ux^2dx$$

$$(u^2-3)(u\,dx+x\,du)=-2u\,dx$$

$$(u^2x-3x)du=(u-u^3)dx$$

$$(u^2-3)x\,du=(u-u^3)dx$$

$$\left(\frac{3}{(u-1)u(u+1)}-\frac{u}{(u-1)(u+1)}\right)du=\frac{dx}{x}$$

$$\int\frac{3}{(u-1)u(u+1)}-\frac{u}{(u-1)(u+1)}du=\int\frac{1}{x}dx$$

$$-\frac{\ln(u^2-1)}{2}+\frac{3\ln(u+1)}{2}-3\ln(u)+\frac{3\ln(u-1)}{2}=\ln(x)+C$$

$$\frac{(u-1)^{\frac{3}{2}}(u+1)^{\frac{3}{2}}}{u^3\sqrt{u^2-1}}=e^cx$$

$$\frac{x^3\left(\frac{y}{x}-1\right)\left(\frac{y}{x}+1\right)}{y^3}=cx$$

$$\frac{x}{y}-\frac{x^3}{y^3}=cx$$

№5.2

$(xy^2+y)dx - dy = 0$

$-dy = (-xy^2 - y)dx$

$-y' = -xy^2 - y$

$y' = xy^2 + y$

$y' - y = xy^2$

$\frac{y'}{y^2} - \frac{1}{y} = x$

$-u' - u = x$

$-u' = x + u$

$1 - v' = v$

$-v' = v - 1$

$-\frac{dv}{dx} = v - 1$

$-dv = (v - 1)dx$

$\frac{dv}{v-1} = -dx$

$\int \frac{1}{v-1} dv = \int -1 dx$

$\ln|v - 1| = C - x$

$$V-1=e^{C-x}$$

$$x+y-1=\frac{C}{e^x}$$

$$\frac{1}{y}+x-1=\frac{C}{e^x}$$

$$y=-\frac{e^x}{(x-1)e^x+C}$$

№6.2

$$y''+10y'+26y=0$$

$$\lambda^2+10\lambda+26=0$$

$$\lambda^2+10\lambda+26 \rightarrow \lambda_{1,2}=\pm i-5 \quad k=1 \quad T: \frac{C_1\sin(x)}{e^{5x}}+\frac{C\cos(x)}{e^{5x}}$$

$$y=\sum P_{k-1}(x)e^{\alpha x}\sin\beta x+Q_{k-1}(x)e^{\alpha x}\cos\beta x$$

$$\lambda=\alpha\pm\beta i \quad P_{k-1}(x),\ Q_{k-1}(x)\rightarrow C_1+\ldots+C_k x^{k-1}$$

$$y=\frac{C_1\sin(x)}{e^{5x}}+\frac{C\cos x}{e^{5x}}$$

$$y=\frac{C_1\sin x+C\cos x}{e}$$

N 7.2

$y'' + 8y' + 15y = 30x$

$e^{\alpha x}(P_m(x)\cos\beta x + Q_n(x)\sin\beta x)$

$y_1 = x^s e^{\alpha x}(R_n(x)\cos\beta x + T_n(x)\sin\beta x)$

$\alpha + \beta i = 0 \rightarrow s = 0$

$y_0 = Ax + B$

$y_0' = A$

$y_0'' = 0$

$15Ax + 15B + 8A = 30x$

$\begin{cases} 15A = 30 \\ 15B + 8A = 0 \end{cases} \quad \begin{cases} A = 2 \\ B = -\frac{16}{15} \end{cases}$

$y_0 = 2x - \frac{16}{15}$

$y = \frac{C}{e^{3x}} + \frac{C_1}{e^{5x}} + 2x - \frac{16}{15}$

$\frac{\cos(x)}{e^{5x}}$