Контрольна робота No2

Диференціальне числення функції однієї змінної

Контрольна робота №2

Варіант 2

№1.2

$y \cdot \sin x - \cos(x-y) = 0$

$y' \cdot \sin x + y \cdot (\sin x)' - (\cos(x-y))' = 0$

$y' \cdot \sin x + y \cdot \cos x + \sin(x-y) \cdot (1-y') = 0$

$y' \cdot \sin x + y \cdot \cos x + \sin(x-y) - y' \sin(x-y) = 0$

$y'(\sin x - \sin(x-y)) = -y\cos x - \sin(x-y)$

$y' = -\dfrac{y\cos x + \sin(x-y)}{\sin x - \sin(x-y)}$

№2.2

$\begin{cases} x = e^t \sin t \\ y = e^t \cos t \end{cases} \qquad \begin{cases} x' = e^t \cdot \sin t + e^t \cdot \cos t \\ y' = e^t \cos t - e^t \sin t \end{cases}$

$y'(x) = \dfrac{e^t \cos t - e^t \sin t}{e^t \sin t + e^t \cdot \cos t} = \dfrac{\cos t - \sin t}{\sin t + \cos t}$

N3.2

$$y=(x^2+1)^{\sin x}$$

$$\ln y=\ln(x^2+1)^{\sin x}$$

$$\ln y=\sin x\cdot\ln(x^2+1)$$

$$y'\cdot\frac{1}{y}=\cos x\ln(x^2+1)+\sin x\cdot\frac{1}{x^2+1}\cdot 2x$$

$$y'=\cancel{y}(\cos x\ln(x^2+1)+\frac{\cancel{x}\cdot 2x\sin x}{x^2+1})\cdot(x^2+1)^{\sin x}$$

N4.2

$$y=\frac{2\sin^2 x}{\cos 2x}$$

$$y'=\frac{(2\sin^2 x)'\cdot\cos 2x-2\sin^2 x\cdot(\cos 2x)'}{\cos^2 2x}$$

$$y'=\frac{4\sin x\cdot\cos x\cdot\cos 2x-2\sin^2 x\cdot(-2\sin 2x)}{\cos^2 2x}$$

$$y'=\frac{4\sin x(\cos x\cdot\cos 2x+\sin x\cdot\sin 2x)}{\cos^2 2x}$$

$$y'=\frac{4\sin x\cdot\cos(-x)}{\cos^2 2x}$$

$$y'=\frac{2\sin 2x}{\cos^2 2x}$$

$y' = \frac{2tg2x}{\cos 2x}$

$dy = \frac{2tg2x}{\cos 2x}dx$

N 5.2

$y = (x-2)^2$

$y = 4x - x^2 + 4$

$tg\gamma = tg(\alpha - \beta) = \frac{tg\alpha - tg\beta}{1 + tg\alpha\, tg\beta}$

$(x-2)^2 = 4x - x^2 + 4$

$x^2 - 4x + 4 = 4x - x^2 + 4$

$2x^2 - 8x = 0$

$2x(x-4) = 0$

$x_1 = 0$

$x_2 = 4$

$A(0;4) = B(0;4)$

$C(4;4) = D(4;4)$

$tg\alpha = y' = 2x - 4$

$tg\beta = y' = 4 - 2x$

$A: tg\alpha = 2\cdot(0) - 4 = -4$

$tg\beta = 4 - 2\cdot 0 = 4$

$A: tg\gamma = \left|\frac{-4-4}{1-4\cdot 4}\right| = \left|\frac{-8}{-15}\right| = \frac{8}{15}$

$C = tg\alpha = 2 \cdot 4 - 4 = 4$ $\quad$ $tg\beta = 4 - 2 \cdot 4 = -4$

$C = tg\gamma = \left|\frac{4+4}{1-4\cdot 4}\right| = \left|\frac{8}{-15}\right| = \frac{8}{15}$

$\gamma = arctg\frac{8}{15}$

№5.2

$\lim\limits_{x\to\frac{\pi}{2}} \frac{\ln(\sin x)}{\pi - 2x} = \frac{\frac{1}{\sin x}\cdot\cos x}{-2} = \frac{ctg\, x}{-2} \stackrel{!}{=} \cancel{\frac{ctg\frac{\pi}{2}}{-2}} = \cancel{0} = \frac{ctg\frac{\pi}{2}}{-2} = \frac{\infty}{-2} = \infty$