Вариант 2

№1.2

$$\int_0^1 \frac{x\,dx}{1+\sqrt{x}}$$

$t = 1+\sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\,dx$

$$\int_0^1 \frac{2x\sqrt{x}}{t}\,dt = \int_0^1 \frac{2\sqrt{x}^2\cdot\sqrt{x}}{t}\,dt = \int_0^1 \frac{2(t-1)^2\cdot(t-1)}{t}\,dt =$$

$$= \int_0^1 \frac{2(t-1)^3}{t}\,dt = \int_0^1 \frac{2(t^3-3t^2+3t-1)}{t}\,dt = \frac{2t^3}{3} - 3t^2 + 6t - 2\ln t\Big|_0^1 =$$

$$= \frac{2(1+\sqrt{x})^3}{3} - 3(1+\sqrt{x})^2 + 6(1+\sqrt{x}) - 2\ln(1+\sqrt{x})\Big|_0^1 =$$

$$= \frac{16}{3} - 12 + 12 - 2\ln 2 - \frac{2}{3} + 3 - 6 = \frac{14}{3} - 3 - 2\ln 2 = \frac{5}{3} - 2\ln 2$$

№2.2

$$\int_0^{\frac{\pi}{3}} \sin x\cos^2 x\,dx$$

$t = \cos x$

$dt = -\sin x\,dx$

$$\int_0^{\frac{\pi}{3}} -t^2\,dt = -\frac{t^3}{3}\Big|_0^{\frac{\pi}{3}} = -\frac{\cos^3 x}{3}\Big|_0^{\frac{\pi}{3}} = -\frac{\cos^3\frac{\pi}{3}}{3} + \frac{\cos^3 0}{3} =$$

$$= -\frac{\frac{1}{8}}{3} + \frac{1}{3} = -\frac{1}{24} + \frac{1}{3} = \frac{7}{24}$$

№3.2

$$\int_1^2 x \ln x \, dx$$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x} dx \qquad v = \frac{x^2}{2}$

$$\ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2x} dx = \ln x \cdot \frac{x^2}{2} - \frac{1}{2} \cdot \frac{x^2}{2} = \frac{\ln 1 \cdot x^2}{2} - \frac{x^2}{4} \Big|_1^2 =$$

$$= \frac{\ln 2 \cdot 2^2}{2} - \frac{2^2}{4} - \frac{\ln 1 \cdot 1}{2} + \frac{1}{4} = 2\ln 2 - \frac{3}{4}$$

№4.2

$$\int_0^3 |1-x| \, dx = \int_0^1 1-x \, dx + \int_1^3 -(1-x) \, dx = \left(x - \frac{x^2}{2}\right)\Big|_0^1 +$$

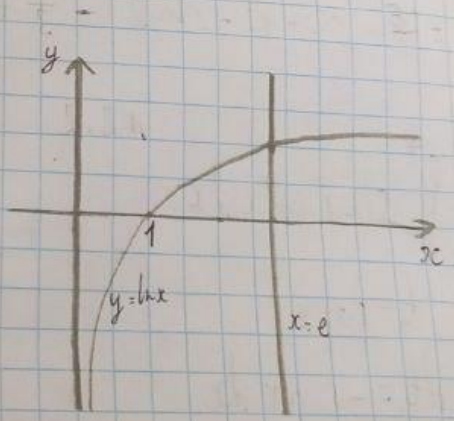
$$+ \left(-x + \frac{x^2}{2}\right)\Big|_1^3 = \frac{1}{2} + 2 = \frac{5}{2}$$

№5.2

$y = \ln x \qquad x = e, \; y = 0$

$y = \ln 0$

$y = 1$



$$\int \ln x \, dx = x \ln x - x$$

$$x \ln x - x \Big|_1^e = e \cdot \ln e - e - \ln 1 + 1 = e - e + 1 = 1$$

~~№6.2~~

№7.2

$$V = \pi \cdot \int_0^{\pi} \sin^2 x \, dx$$

$$\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx = \frac{x}{2} - \frac{\sin 2x}{4} \Big|_0^{\pi} = \frac{\pi}{2} - \frac{\sin 2\pi}{4} = \frac{\pi}{2}$$

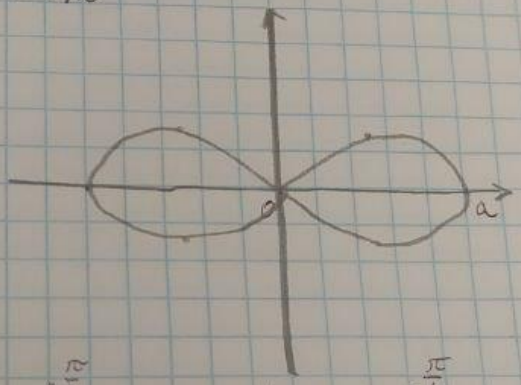
$$V = \pi \cdot \frac{\pi}{2} = \frac{\pi^2}{2}$$

№6.2

$r^2 = a^2 \cos 2\varphi$

$\varphi_1 = 0$

$\varphi_2 = \frac{\pi}{4}$



$$S = \frac{1}{2} \int_{\varphi_1}^{\varphi_2} r^2 d\varphi = 4 \cdot \frac{1}{2} \int_0^{\frac{\pi}{4}} a^2 \cdot \cos 2\varphi \, d\varphi = 2a^2 \int_0^{\frac{\pi}{4}} \cos 2\varphi \, d\varphi =$$

$$= a^2 \sin 2\varphi \Big|_0^{\frac{\pi}{4}} = a^2 \sin \frac{\pi}{2} - a^2 \sin 0 = a^2 - a^2 \cdot 0 = a^2$$