

# Flow noise estimation models for axial flow past towed sonar arrays

**Rakesh Sekharipuram Sekar**

Department of Mechanical Engineering,  
Indian Institute of Technology Palakkad,  
Palakkad, Kerala, 678003, India  
Email: 132203001@smail.iitpkd.ac.in

**Senthil Rajan S**

Naval Physical Oceanographic Laboratory,  
Ernakulam, Kerala, 682021, India  
Email: senthilrajan.npol@gov.in

**Anoop Akkoorath Mana\***

Department of Mechanical Engineering,  
Indian Institute of Technology Palakkad,  
Palakkad, Kerala, 678003, India  
Email: akkoorath@iitpkd.ac.in

## ABSTRACT

*Towed sonar arrays house a series of pressure sensors inside a fluid-filled elastic tube. Towing of the sonar array in water generates a turbulent boundary layer on the exterior surface of the elastic tube. The pressure fluctuations in the turbulent boundary layer along with other ambient pressure fluctuations, excites the elastic tube and further generates pressure disturbances in the interior fluid. In this work, a new semi-empirical model of the turbulent pressure spectrum is presented. The new model predictions show a closer agreement with the available experimental results at all tow speeds. A three-dimensional vibroacoustic model of the fluid-filled elastic tube is also presented in this work. The vibroacoustic model is fully coupled and considers both breathing mode and first order variations in the elastic tube and the acoustic field variables. Further, the turbulent pressure spectrum semi-empirical model and the three-dimensional vibroacoustic model are used to compute*

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\*Address all correspondence for other issues to this author.

13 *the on-axis sound pressure level due to the external turbulent pressure excitation at differ-*  
14 *ent elastic tube diameters and tow speeds. At low frequencies, increasing tube diameter*  
15 *has little effect on flow noise, while at higher frequencies, flow noise decreases with larger*  
16 *diameters. Increasing tow speed raises flow noise across all frequencies.*

## 1 INTRODUCTION

17 Towed sonar arrays contain a series of pressure sensors enclosed within a fluid-filled elastic  
18 tube. As the sonar array is towed through the water, a thick layer of turbulent flow is generated over  
19 the exterior surface of the tube. The pressure fluctuations in this turbulent boundary layer (TBL),  
20 along with other ambient sea pressure variations, excite the elastic tube and subsequently produce  
21 acoustic pressure disturbances within the interior fluid. The hydrophones placed in the interior fluid  
22 picks these acoustic signals. The signals associated with the turbulent pressure fluctuations are  
23 called flow noise. Currently, the flow noise is measured either by towing the sonar array in open  
24 water using a dinghy or by allowing the hydrophone to free fall in water [1]. In the first case, noise  
25 from the boat and vibrations of the towline connections pollute the measured acoustic signals [2];  
26 whereas in the second case, the useful measurements can be made only at the terminal velocity  
27 of the hydrophone. This work aims at developing a fully-coupled vibroacoustic model for predicting  
28 the flow noise in towed sonar arrays which is useful over wide range of towing speeds and tube  
29 diameters.

30 A widely used model for predicting the turbulent pressure spectrum is that developed by Cor-  
31 cos [3] for the flow over a flat plate. In this model, the turbulent pressure is varying exponentially  
32 with respect to both the axes of the flat plate. Although this model is widely used in engineer-  
33 ing applications, it has a major shortcoming that it is overestimating the pressure level at low  
34 wavenumbers. Chase [4] presented a simpler turbulent pressure spectrum model for the flow over  
35 a flat plate. The model is based on experimental observations and uses direct dependence on  
36 the flow and dimensional parameters. Frendi and Zhang [5] analysed the Corcos [3] model and  
37 proposed a turbulent model for the flow over a flat plate based on large eddy simulation (LES) and  
38 direct numerical simulation (DNS) computational results. Frendi's model involves the use of an

auto spectrum which was derived by Goody [6]. The Frendi model predictions are found to match well with an earlier experimental result on flow over a flat plate. Some of the observations of the Chase and Frendi models are relevant to the present work and are discussed in section 2. Francis *et al.* [7] used LES and Reynolds averaged Navier Stokes (RANS) computational method to study the wavenumber frequency spectrum of the turbulent pressure field over a flat plate. This work presents an exhaustive discussion on similar problems in the literature.

Chase [4] developed a model for computing the turbulent pressure spectrum for an axial flow past a cylinder by modifying his earlier flat plate model. While modifying, Chase considered the radius of the cylinder as one of the parameters instead of the length of the flat plate. Chase derived azimuthal harmonic spectral density by integrating the turbulent pressure spectrum of the flat plate in the cross-flow direction. The details of this model are presented in section 2.1.

Carpenter and Kewley [8] conducted experiments for finding the flow noise inside a fluid-filled elastic tube while towed behind a ship and compared the results with that predicted by Chase [4]. The authors also proposed a tube transfer function for computing the flow noise inside the tube. Knight [9] performed similar analytical simulations as in [8] but with different types of hydrophones and compared the flow noise with that for an ideal hydrophone. The ideal hydrophone was assumed to have unit acoustic response and zero convective response. Knight also used an approximate tube transfer function to find the noise inside the fluid-filled elastic tube.

Unnikrishnan *et al.* [2] performed experiments to measure the turbulent pressure field outside the elastic tube by towing the sonar array in a quiet lake at different speeds. The work presents a comparison of the experimental results with the available semi-empirical model predictions. It was found that the semi-empirical model estimations match with the measurements only at high tow speeds. Karthik *et al.* [10] studied the turbulent pressure spectrum over a cylinder with the help of an LES computational model. The model predictions match well with the experimental results of Unnikrishnan *et al.* [2]. Karthik *et al.* also presented a non-dimensional turbulent flow noise spectrum for easy estimation of the spectrum at different tow speeds and tube diameters.

Both Carpenter and Kewley [8] and Knight [9] estimated the flow noise inside a fluid-filled elastic tube with the help of the Chase model for the turbulent pressure spectrum and an approximate

67 tube transfer function. Jineesh and Ebenezer [11] developed a better axisymmetric model of the  
68 fluid-filled elastic tube and used it to estimate the flow noise inside the tube. It was found that the  
69 earlier approximate transfer function model overestimated the flow noise inside the tube.

70 This paper develops a new semi-empirical model of the turbulent pressure spectrum for axial  
71 flow past a solid cylinder. It also presents a fully coupled three-dimensional vibroacoustic model  
72 of a fluid-filled elastic tube. Furthermore, these two models are used to compute the on-axis  
73 sound pressure level resulting from an external turbulent pressure excitation on the elastic tube.  
74 This paper is organized as follows: Section 2 discusses two existing semi-empirical models for  
75 estimating turbulent pressure spectrum for axial flow past a solid cylinder. Section 3 discusses the  
76 development of a new semi-empirical model for the turbulent pressure spectrum, which provides  
77 improved predictions compared to existing models. Section 4 discusses the development of a  
78 three-dimensional vibroacoustic model for estimating the on-axis flow noise inside a fluid-filled  
79 elastic tube. Further, Section 5 presents the results on interior acoustic pressure spectrum and  
80 on-axis flow noise inside a fluid-filled elastic tube and are compared with the available results in  
81 the literature.

## 2 REVIEW OF SEMI-EMPIRICAL MODELS OF TURBULENT PRESSURE SPECTRUM

82 One of the objective of this study is to predict the flow noise resulting from turbulent boundary  
83 layer excitation. To achieve this, a semi-empirical model that can estimate the turbulent pressure  
84 exerted by fluid flow on a cylindrical tube, is required. Two existing semi-empirical models for  
85 turbulent pressure fluctuation are discussed in this section. Further, flow noise at the outer surface  
86 of the tube is estimated using these models, showing its variation in comparison to available  
87 experimental results [2].

## 88 2.1 Chase model

89 Chase [4] proposed a semi-empirical model for predicting the frequency-wavenumber spec-  
90 trum of turbulent pressure field over a solid cylinder and is given by

$$\hat{p}_0(k_z, \omega) = C \rho^2 \nu_*^3 R^2 \left[ (k_z R)^2 + \frac{1}{12} \right] \left[ \frac{(\omega R - u_c k_z R)^2}{h^2 \nu_*^2} + (k_z R)^2 + b_1^{-2} \right]^{-2.5}. \quad (1)$$

91 The important parameters in the above equation are axial wavenumber  $k_z$ , frequency  $\omega$ , density  
92 of the fluid  $\rho$ , convective speed  $u_c$  ( $= 0.68u$ , where  $u$  is the tow speed),  $C = 0.063$ ,  $h = 3.7$ ,  
93  $\nu_* = 0.04U$  and tube radius  $R$  [4, 8, 9, 12, 13].

## 94 2.2 Frendi model

95 Frendi's model for the turbulent pressure spectrum for a flat plate is given by [5]

$$\hat{R}(k_z, k_2, \omega) = C_1 R^*(\omega) e^{-\hat{\alpha} r_k}. \quad (2)$$

96 In the above equation,  $C_1$  is given by

$$C_1 = \alpha^2 m \delta^2 \frac{1}{2\pi}, \quad (3)$$

97 where  $\alpha$  can be computed using

$$\alpha = \frac{a_1}{\pi} \frac{1}{\sqrt{1 + a_2 \left( \frac{\omega \delta}{u_t} - 50 \right)^2}}. \quad (4)$$

- 98 In the above equation,  $a_1 = 4.7$ ,  $a_2 = 3 \times 10^{-5}$  [5],  $u_t$  is the friction velocity ( $=0.04u$ , a small fraction  
99 of tow velocity  $u$ ) and  $\delta$  is the boundary layer thickness [14] given by

$$\delta = [48Re_a^{-1}Re_x^{(0.0226 \log Re_a + 0.2478)}]^{1/0.91}. \quad (5)$$

- 100 In the above equation,  $Re_a$  is the radius based Reynold's number ( $\frac{\rho u R}{\mu}$ ) and  $Re_x$  is length based  
101 Reynold's number ( $\frac{\rho u x}{\mu}$ ),  $x$  is distance of a point on the cylinder from the leading edge and  $\mu$  is  
102 the dynamic viscosity of the fluid medium. The constant  $m$  in Eq. (3) is a scaling factor which is  
103 approximately taken as  $1/7.7$  [5].  
104  $R^*(\omega)$  in Eq. (2) is the auto-spectrum given by [6]

$$R^*(\omega) = \frac{3\tau_w^2\omega^2(\frac{\delta}{u})^3}{[(\frac{\omega\delta}{u})^{0.75} + 0.5]^{3.7} + [1.1R_t^{-0.57}(\frac{\omega\delta}{u})]^7}. \quad (6)$$

- 105 In the above equation,  $\tau_w$  is the shear stress at the wall,  $R_t$  is the ratio of time scale [6] given by

$$R_t = \left(\frac{u_t}{u}\right) \left(\frac{u_t\delta}{\nu}\right), \quad (7)$$

- 106 where  $\nu$  is the kinematic viscosity.

- 107  $\hat{\alpha}$  in Eq. (2) is given by  $\hat{\alpha} = \alpha\delta$  and  $r_k$  depends on the axial and crossflow wavenumbers and  
108 is given by

$$|r_k|^2 = \left(k_z - \frac{\omega}{u_c}\right)^2 + (mk_2)^2. \quad (8)$$

- 109 In the above equation,  $k_2$  denotes the cross flow wavenumber and  $u_c$  is the convective speed ( $=$   
110  $0.68u$ , a large fraction of tow speed  $u$ ). Eq. (2) can be modified for estimating turbulent pressure

111 spectrum for an axial flow past a solid cylinder as

$$\hat{p}_0(k_z, \omega) = \int_{-1/2R}^{1/2R} \hat{R}(k_z, k_2, \omega) dk_2. \quad (9)$$

112 The estimation of flow noise using the models discussed in this section and its comparison with  
113 the findings of experiments are presented below.

### 114 2.3 Flow noise

115 Here, the estimation of flow noise, as measured by a series of hydrophones placed at the  
116 outer surface of a solid cylinder, is discussed and compared with the experimental results [2]. The  
117 flow noise associated with the turbulent pressure spectrum  $p_0(k_z, \omega)$  as registered by an array of  
118 hydrophones is [2]

$$Q(\omega) = \int_{-\infty}^{\infty} \hat{p}_0(k_z, \omega) H(k_z) dk_z. \quad (10)$$

119 In the above equation,  $H(k_z)$  is the hydrophone response function. The hydrophone array is a set  
120 of large number of similar elements with specific length arranged at a fixed distance apart. This  
121 array acts as noise filter and its response is given by [2]

$$H(k_z) = \frac{\sin(k_z d N / 2)}{N \sin(k_z d / 2)} \frac{\sin(l k_z / 2)}{l k_z / 2}, \quad (11)$$

122 where  $N$  is the number of hydrophone elements in the array,  $d$  is the distance between two hy-  
123 drophones and  $l$  is the length of individual hydrophones.

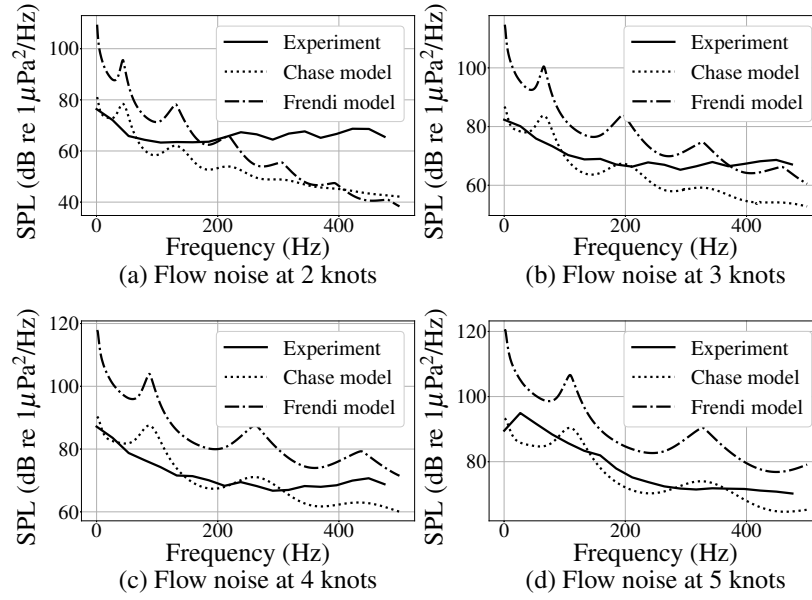


Fig. 1. Comparison of flow noise predicted by Chase [4] and Frendi [5] models with the experimental results [2] at different tow speeds.

**124** The sound pressure level (SPL) associated with the flow noise  $Q(\omega)$  is given by

$$SPL = 10 \log_{10} \left( \frac{Q(\omega)}{p_{ref}^2} \right), \quad (12)$$

**125** where  $p_{ref} = 1 \times 10^{-6}$  Pa is the reference acoustic pressure in water.

**126** Figure 1 compares the flow noise estimated by the Chase [4] and Frendi [5] models with the  
**127** experimental results [2] at various tow speeds for a solid cylinder having a diameter of 0.01 m.  
**128** The sonar array consists of 66 hydrophones, each of length 8 mm, placed at the outer surface of  
**129** the cylinder with an interval of 16 mm. It is evident from Fig. 1 that the Frendi model consistently  
**130** overestimates the flow noise at almost all frequencies and tow speeds, whereas the Chase model  
**131** aligns well with experiments at high tow speeds. However, at low speeds, there is a significant dif-  
**132** ference between the Chase model and the experiment, especially at high frequencies. Regardless  
**133** of tow speed, both models predict a significant reduction in flow noise with frequency compared  
**134** to the experimental results. To address these differences, a new model of the turbulent pressure



135 spectrum is proposed in this work to better match the experimental data, particularly at low tow  
136 speeds. The development of this new model is discussed in the following section.

### 3 A NEW SEMI-EMPIRICAL MODEL OF THE TURBULENT PRESSURE SPECTRUM

137 It is shown in the previous sections that the Chase and Frendi models show a significant  
138 deviation from the experimental results [2] at low tow speeds and at high frequencies. In this  
139 section, a new semi-empirical model of the turbulent pressure spectrum is developed that closely  
140 aligns with the experimental results [2]. This new model is derived using the insights from both the  
141 Chase and Frendi models and is referred to as the *hybrid model*.

#### 142 3.1 The hybrid model

143 In the *hybrid model*, the pressure spectrum of the Chase model [4] (Section 2.1) is used in  
144 conjunction with the exponential decay function present in the Frendi model [5] (Section 2.2).  
145 Accordingly, the turbulent pressure spectrum is given by

$$\hat{p}(k_z, k_2, \omega) = C_3 \bar{P}(\omega) e^{-\hat{\alpha} r_k}. \quad (13)$$

146 In the above equation, the autospectrum  $\bar{P}(\omega)$  is given by

$$\bar{P}(\omega) = \int_{-\infty}^{\infty} \hat{p}_0(\omega, k_z) dk_z, \quad (14)$$

147 where  $\hat{p}_0(\omega, k_z)$  is the same as that used in the Chase model (Eq. (1)). In this new model, the  
148 wavenumber dependency is included in the form of an exponential function  $e^{-\hat{\alpha} r_k}$ , where  $\hat{\alpha} = \alpha \delta$   
149 with

$$\alpha = \frac{a_1}{\pi} \frac{1}{\sqrt{1 + a_2 \left( \frac{\omega \delta}{u_t} - 50 \right)^2}}, \quad (15)$$

$$\delta = [48Re_a^{-1}Re_x^{(0.0226 \log Re_a + 0.2478)}]^{1/0.91} \quad (16)$$

150 and

$$|r_k|^2 = \left(k_z - \frac{\omega}{u_c}\right)^2 + (mk_2)^2. \quad (17)$$

151 In Eq. (15),  $a_1$  and  $a_2$  determine the behavior of the spectrum at low and high frequencies, respec-  
 152 tively. It has been observed in Fig. 1 that the predictions of the Frendi model deviate more at higher  
 153 frequency ranges. Therefore, the value of  $a_2$  is decreased from  $3 \times 10^{-5}$  to  $3 \times 10^{-6}$ . Different  
 154 values of  $a_1$  and  $C_3$  were tested to match the experimental results given in Fig. 1. A better match  
 155 is found with the experimental data when  $a_1 = 1$  and  $C_3 = 1 \times 10^{-4}$ . Furthermore, the turbulent  
 156 pressure spectrum given in Eq. (13) is integrated over the cross-flow wavenumber  $k_2$  from  $-1/2R$   
 157 to  $1/2R$  to obtain the pressure spectrum  $\hat{p}_0(k_z, \omega)$  for the axial flow past a solid cylinder [4]. Thus,

$$\hat{p}_0(k_z, \omega) = \int_{-1/2R}^{1/2R} \hat{p}(k_z, k_2, \omega) dk_2. \quad (18)$$

158 The hybrid model is used to compute the flow noise for axial flow past a solid cylinder. The  
 159 results of the new model and their comparison with the existing models and Unnikrishnan's exper-  
 160 imental results [2] are presented in the next subsection.

### 161 3.2 Flow noise

162 The flow noise can be computed using Eq. (10). Here, the turbulent pressure spectrum  
 163  $\hat{p}_0(k_z, \omega)$  for the new hybrid model is given by Eq. (18) and the hydrophone response function  
 164  $H(k_z)$  is given by Eq. (11). A comparison of flow noise measured in SPL (see Eq. (12)) computed  
 165 using the new hybrid model, Chase model [4] and Unnikrishnan's experiment [2] are shown in  
 166 Fig. 2. It can be seen that the predictions of the new hybrid model are consistent with the mea-

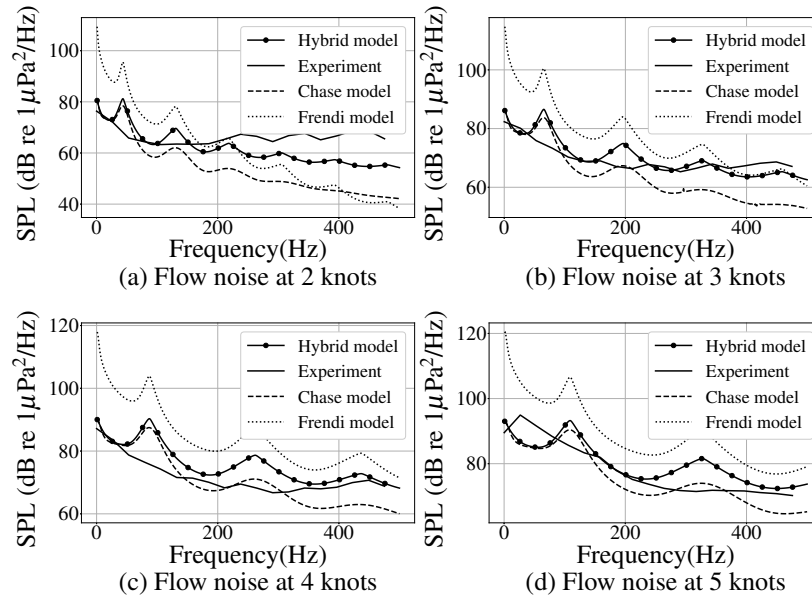


Fig. 2. A comparison of flow noise predicted by the hybrid model, the Chase model [4] and Frendi model [5] with that measured from experiments [2] at different tow speeds.

sured values [2] at all frequencies and towing speeds. Although the hybrid model underpredicts noise at high frequencies for the 2 knots case, the predictions are better than that by the existing Chase and Frendi models.

A comparison of the turbulent pressure spectrum  $\hat{p}_0(k_z, \omega)$  predicted by the hybrid model (Eqs. (13)-(18)), the Chase model (Eq. (1)) and the Frendi model (Eq. (9)) for different frequencies at 2 knots is shown in Fig. 3. Here, the diameter of the cylinder is chosen as 10 mm, density of the fluid is  $1000 \text{ kg/m}^3$  and the SPL is calculated at 11 m from the leading edge of the solid cylinder. It can be seen from Fig. 3 that for a given frequency, at low wavenumbers, the turbulent pressure spectrum increases at a slow rate. It peaks at convective wavenumber  $k_c (= \omega/u_c)$  forming a convective ridge. It can be seen that while all the models predict a ‘flat’ spectrum at lower wavenumbers and a ridge at convective wavenumber, their predictions differ significantly at large wavenumbers. The presence of an exponential function results in an exponential decrease in the spectrum at large wavenumbers for the hybrid and Frendi models. Chase model predicts a higher spectrum with a smaller slope at large wavenumbers. It can be seen from Fig. 3 that the predictions by the three models are closer at lower frequencies. However, at high frequencies,

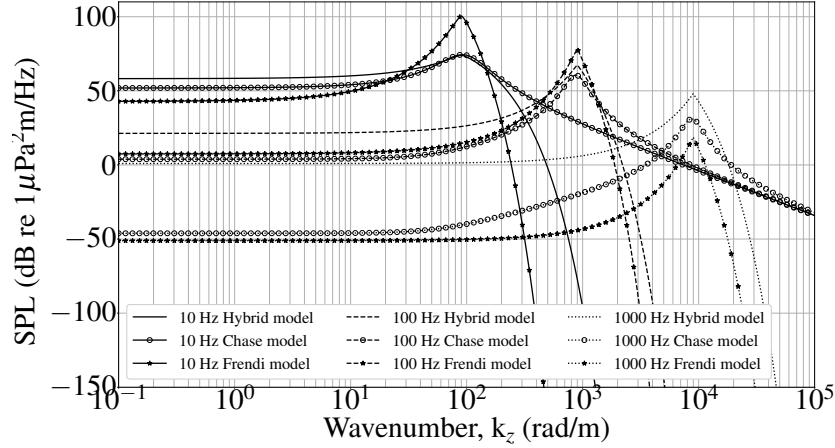


Fig. 3. A comparison of the turbulent pressure spectrum  $\hat{p}_0(k_z, \omega)$  given by the hybrid model (Eq. (18)), Chase (Eq. (1)) and Frendi (Eq. (9)) model at 2 knots.

the hybrid model predicts a spectrum that is higher than the rest. This difference in the spectrum predicted by the hybrid model helps to achieve closer agreement with the measured flow noise, as shown in Fig. 2

### 3.3 Non-dimensional power spectral density

The non-dimensional power spectral density,  $Q_{ND}$ , of the flow noise is defined as

$$Q_{ND} = \frac{Q(\omega)}{\rho^2 D U^3}, \quad (19)$$

where  $Q(\omega)$  is the flow noise given by Eq. (10) and  $D$  is the cylinder diameter. The non-dimensional power spectral density calculated using the hybrid model (Eqs. (13)-(18)) at different tow speeds are shown in Fig. 4.

It can be seen that, the non-dimensional power spectral density for different tow speeds collapse to a single curve against the non-dimensional frequency  $\omega D/u$ . One can therefore obtain the power spectral density at different tow speeds and cylinder diameters using this “single” non-dimensional curve.

While developing a new model of turbulent pressure field for axial flow past a solid cylinder, it is assumed that the cylinder is rigid and therefore the pressure field is not altered by the cylinder

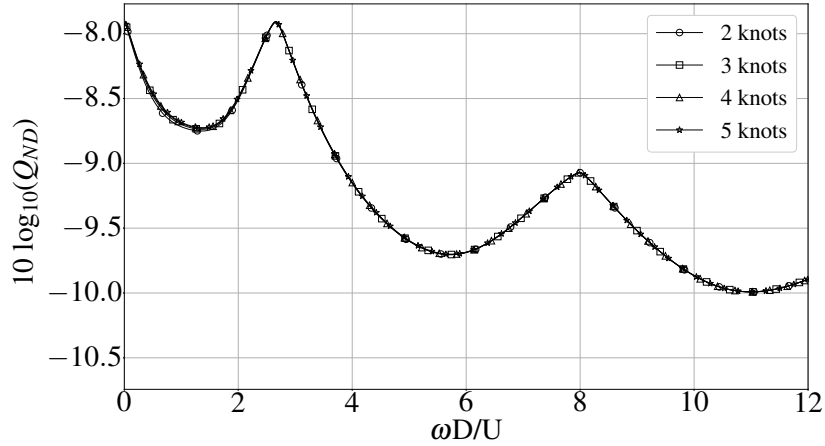


Fig. 4. Non-dimensional power spectral density for different tow speeds using the new hybrid model.

196 displacement field. However, the cylindrical tube in towed sonar arrays is not rigid and can be  
 197 assumed to be elastic. The turbulent pressure fluctuation outside the elastic tube, creates vibration  
 198 inside the tube and in turn generates acoustic waves in the fluid inside the tube. The following  
 199 section presents a three-dimensional vibroacoustic model of a fluid-filled elastic tube, which is  
 200 further used with the new hybrid model of the external turbulent pressure excitation to estimate  
 201 on-axis flow noise.

#### 4 THREE-DIMENSIONAL VIBROACOUSTIC (3D-VA) MODEL OF A FLUID-FILLED ELASTIC TUBE

202 This section develops a fully-coupled three-dimensional vibroacoustic (3D-VA) model of the  
 203 fluid-filled elastic tube. A schematic of the fluid-filled tube is shown in Fig. 5. First, the displacement  
 204 field of the elastic tube is derived from the Navier-Lame equilibrium equation (see Section 4.1) and  
 205 then the acoustic pressure field inside the tube is derived from the acoustic wave equation (see  
 206 Section 4.2). The structure (elastic tube) and the fluid (interior fluid) are then coupled with the  
 207 help of stress and displacement boundary conditions at the interface (see Section 4.3). External  
 208 pressure excitation is also taken into account in the form of a stress boundary condition on the  
 209 outer surface of the tube. The boundary conditions, when expressed in terms of the unknown  
 210 displacement and pressure fields, form a system of linear algebraic equations. The unknown  
 211 displacement and pressure fields are then calculated by solving this system of equations (see

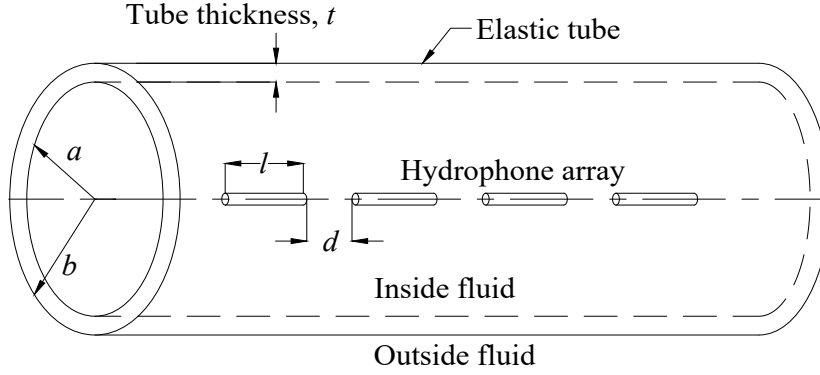


Fig. 5. Fluid filled elastic tube.

212 Section 4.4).

#### 213 4.1 The elastic tube displacement and stress fields

214 This section involves modeling of an elastic tube using the Navier-Lame equilibrium equation in  
 215 three-dimensional cylindrical coordinates. The Navier-Lame equilibrium equation is given by [15]

$$\mu \nabla^2 \mathbf{U}(r, \theta, z, t) + (\lambda + \mu) \nabla \nabla \cdot \mathbf{U}(r, \theta, z, t) = \rho_s \ddot{\mathbf{U}}(r, \theta, z, t), \quad (20)$$

216 where  $\mathbf{U}$  is the displacement vector ( $= \{W_e, \Theta_e, U_e\}^T$ ,  $W_e$  represents the radial,  $\Theta_e$  represents  
 217 the azimuthal and  $U_e$ , the axial displacement fields),  $\lambda$  and  $\mu$  are the Lamé's coefficients,  $\rho_s$  is the  
 218 density of the tube and  $\nabla$  is the gradient operator in three dimension given by

$$\nabla = \frac{\partial}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial}{\partial \theta} \mathbf{e}_\theta + \frac{\partial}{\partial z} \mathbf{e}_z. \quad (21)$$

219 The displacement vector  $\mathbf{U}$  may be represented using the Helmholtz decomposition method as

$$\mathbf{U} = \nabla \phi + \nabla \times \boldsymbol{\psi}, \quad (22)$$

220 where  $\phi$  is a scalar potential and  $\psi$  is a vector potential. The scalar and vector potential functions  
 221 satisfy the Navier-Lame equation for  $n = 0$  and for all positive integer values of  $n$ . A complete  
 222 solution to Navier-Lame equation can be obtained as

$$\phi(r, \theta, z, t) = \sum_{n=0}^{\infty} [A_1 J_n(\beta_1 r) + A_2 Y_n(\beta_1 r)] [A_3 \cos(n\theta) + A_4 \sin(n\theta)] e^{i(k_z z - \omega t)}, \quad (23)$$

223

$$\psi_r(r, \theta, z, t) = \sum_{n=0}^{\infty} [C_1 J_{n+1}(\beta_2 r) + C_2 Y_{n+1}(\beta_2 r)] \sin(n\theta) e^{i(k_z z - \omega t)}, \quad (24)$$

224

$$\psi_\theta(r, \theta, z, t) = \sum_{n=0}^{\infty} -[C_1 J_{n+1}(\beta_2 r) + C_2 Y_{n+1}(\beta_2 r)] \cos(n\theta) e^{i(k_z z - \omega t)} \quad (25)$$

225 and

$$\psi_z(r, \theta, z, t) = \sum_{n=0}^{\infty} [B_1 J_n(\beta_2 r) + B_2 Y_n(\beta_2 r)] [B_3 \cos(n\theta) + B_4 \sin(n\theta)] e^{i(k_z z - \omega t)} \quad (26)$$

226 In this work, the above expressions are truncated to only  $n = 0$  and  $n = 1$  terms and further used  
 227 to compute the elastic tube displacement and stress fields. A detailed derivation of the potential  
 228 functions are given in Section S1 of the Supplemental material.

**229** 4.1.1 Elastic tube displacement components

**230** In this subsection, the displacement components of the elastic tube in radial ( $W_e$ ), azimuthal  
**231** ( $\Theta_e$ ) and axial ( $U_e$ ) directions, are derived. The displacement components are given by Eq. (22).

$$W_e(r, \theta, z, t) = \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \psi_z}{\partial \theta} - \frac{\partial \psi_\theta}{\partial z}, \quad (27)$$

**232**

$$\Theta_e(r, \theta, z, t) = \frac{1}{r} \frac{\partial \phi}{\partial \theta} + \frac{\partial \psi_r}{\partial z} - \frac{\partial \psi_z}{\partial r} \quad (28)$$

**233** and

$$U_e(r, \theta, z, t) = \frac{\partial \phi}{\partial r} + \frac{1}{r} \frac{\partial \psi_z}{\partial \theta} - \frac{\partial \psi_\theta}{\partial z} \quad (29)$$

**234** Substituting for the scalar (Eq. (23)) and the vector potential (Eqs. (24) - (26)) functions, the radial  
**235** displacement field is

$$\begin{aligned} W_e(r, \theta, z, t) = & \mathbf{e}^{i(k_z z - \omega t)} \left\{ \frac{1}{r} [r\beta_1 J_0(r\beta_1) \cos(\theta) - J_1(r\beta_1)(\cos(\theta) + r\beta_1)] E_1 + \frac{1}{r} \sin(\theta) [r\beta_1 J_0(r\beta_1) \right. \\ & - J_1(r\beta_1)] E_2 + \frac{1}{r} [r\beta_1 Y_0(r\beta_1) \cos(\theta) - Y_1(r\beta_1)(\cos(\theta) + r\beta_1)] F_1 + \frac{1}{r} \sin(\theta) [r\beta_1 Y_0(r\beta_1) \\ & - Y_1(r\beta_1)] F_2 + ik_z [J_1(r\beta_2) + J_2(r\beta_2) \cos(\theta)] G_1 + ik_z [J_1(r\beta_2) + J_2(r\beta_2) \cos(\theta)] G_2 \\ & \left. + [\frac{1}{r} J_1(r\beta_2) \cos(\theta)] H_1 + [\frac{1}{r} Y_1(r\beta_2) \cos(\theta)] H_2 - [\frac{1}{r} J_1(r\beta_2) \sin(\theta)] I_1 - [\frac{1}{r} Y_1(r\beta_2) \sin(\theta)] I_2 \right\}, \quad (30) \end{aligned}$$

**236** where,  $E_1, E_2, F_1, F_2, G_1, G_2, H_1, H_2, I_1$  and  $I_2$  are unknown constants with  $E_1 = A_1 A_3, E_2 =$   
**237**  $A_1 A_4, F_1 = A_2 A_3, F_2 = A_2 A_4, G_1 = C_1, G_2 = C_2, H_1 = B_1 B_4, H_2 = B_2 B_4, I_1 = B_1 B_3$  and



**238**  $I_2 = B_2 B_3$ . The azimuthal displacement field is

$$\begin{aligned} \Theta_e(r, \theta, z, t) = & \mathbf{e}^{i(k_z z - \omega t)} \left\{ \frac{-1}{r} [J_1(r\beta_1) \sin(\theta)] E_1 + \frac{1}{r} [J_1(r\beta_1) \cos(\theta)] E_2 + \frac{-1}{r} [Y_1(r\beta_1) \sin(\theta)] F_1 \right. \\ & + \frac{1}{r} [Y_1(r\beta_1) \cos(\theta)] F_2 + [ik_z J_2(r\beta_2) \sin(\theta)] G_1 + [ik_z Y_2(r\beta_2) \sin(\theta)] G_2 \\ & - \frac{\beta_2 \sin(\theta)}{2} [J_0(r\beta_2) - J_2(r\beta_2)] H_1 - \frac{\beta_2 \sin(\theta)}{2} [Y_0(r\beta_2) - Y_2(r\beta_2)] H_2 \\ & + \frac{1}{r} \{-r\beta_2 J_0(r\beta_2) \cos(\theta) + J_1(r\beta_2) [r\beta_2 + \cos(\theta)]\} I_1 + \frac{1}{r} \{-r\beta_2 Y_0(r\beta_2) \cos(\theta) \\ & \left. + Y_1(r\beta_2) [r\beta_2 + \cos(\theta)]\} I_2 \right\}. \quad (31) \end{aligned}$$

**239** The axial displacement field is

$$\begin{aligned} U_e(r, \theta, z, t) = & \mathbf{e}^{i(k_z z - \omega t)} \left\{ ik_z [J_0(r\beta_1) + J_1(r\beta_1) \cos(\theta)] E_1 + [ik_z J_1(r\beta_1) \sin(\theta)] E_2 \right. \\ & + ik_z [Y_0(r\beta_1) + Y_1(r\beta_1) \cos(\theta)] F_1 + [ik_z Y_1(r\beta_1) \sin(\theta)] F_2 \\ & \left. - \beta_2 [J_0(r\beta_2) + J_1(r\beta_2) \cos(\theta)] G_1 - \beta_2 [Y_0(r\beta_2) + Y_1(r\beta_2) \cos(\theta)] G_2 \right\}. \quad (32) \end{aligned}$$

**240** The spatio-temporal  $(r, \theta, z, t)$  displacement field is then transformed to the wavenumber-frequency

**241**  $(r, \theta, k_z, \omega)$  domain using the Fourier transform pairs,

$$\hat{G}(r, \theta, k_z, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(r, \theta, z, t) e^{-i(k_z z - \omega t)} dz dt, \quad (33)$$

and

$$g(r, \theta, z, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{G}(r, \theta, k_z, \omega) \mathbf{e}^{i(k_z z - \omega t)} dk_z d\omega. \quad (34)$$

**242** The transformed displacement components are given below.

**243** (a) Radial displacement:

$$\begin{aligned} \hat{W}_e(r, \theta, k_z, \omega) = & \left\{ \frac{-\chi_1}{2} \{2J_1(r\beta_1) + [J_2(r\beta_1) - J_0(r\beta_1)] \cos(\theta)\} \right\} \hat{P}_1(k_z, \omega) \\ & + \left\{ \frac{\chi_1}{2} [J_0(r\beta_1) - J_2(r\beta_1)] \sin(\theta) \right\} \hat{P}_2(k_z, \omega) + \left\{ \frac{-\chi_1}{2} \{2Y_1(r\beta_1) \right. \\ & + [Y_2(r\beta_1) - Y_0(r\beta_1)] \cos(\theta)\} \right\} \hat{Q}_1(k_z, \omega) + \left\{ \frac{\chi_1}{2} [Y_0(r\beta_1) - Y_2(r\beta_1)] \sin(\theta) \right\} \hat{Q}_2(k_z, \omega) \\ & - \left\{ \chi_2 [J_1(r\beta_2) + J_2(r\beta_2) \cos(\theta)] \right\} \hat{R}_1(k_z, \omega) - \left\{ \chi_2 [Y_1(r\beta_2) + Y_2(r\beta_2) \cos(\theta)] \right\} \hat{R}_2(k_z, \omega) \\ & + \left\{ \frac{1}{r} [J_1(r\beta_2) \cos(\theta)] \right\} \hat{S}_1(k_z, \omega) + \left\{ \frac{1}{r} [Y_1(r\beta_2) \cos(\theta)] \right\} \hat{S}_2(k_z, \omega) \\ & + \left\{ \frac{1}{r} [J_1(r\beta_2) \sin(\theta)] \right\} \hat{T}_1(k_z, \omega) + \left\{ \frac{1}{r} [Y_1(r\beta_2) \sin(\theta)] \right\} \hat{T}_2(k_z, \omega), \quad (35) \end{aligned}$$

**244** where  $\hat{P}_1(k_z, \omega)$ ,  $\hat{P}_2(k_z, \omega)$ ,  $\hat{Q}_1(k_z, \omega)$ ,  $\hat{Q}_2(k_z, \omega)$ ,  $\hat{R}_1(k_z, \omega)$ ,  $\hat{R}_2(k_z, \omega)$ ,  $\hat{S}_1(k_z, \omega)$ ,  $\hat{S}_2(k_z, \omega)$ ,  $\hat{T}_1(k_z, \omega)$

**245** and  $\hat{T}_2(k_z, \omega)$  are the unknown variables and  $\chi_1 = \frac{\beta_1}{jk_z}$  and  $\chi_2 = \frac{jk_z}{\beta_2}$ .

**246** (b) Azimuthal displacement:

$$\begin{aligned} \hat{\Theta}_e(r, \theta, k_z, \omega) = & \left\{ \frac{-\chi_1}{r\beta_1} [J_1(r\beta_1) \sin(\theta)] \right\} \hat{P}_1(k_z, \omega) + \left\{ \frac{\chi_1}{r\beta_1} [J_1(r\beta_1) \cos(\theta)] \right\} \hat{P}_2(k_z, \omega) \\ & + \left\{ \frac{-\chi_1}{r\beta_1} [Y_1(r\beta_1) \sin(\theta)] \right\} \hat{Q}_1(k_z, \omega) + \left\{ \frac{\chi_1}{r\beta_1} [Y_1(r\beta_1) \cos(\theta)] \right\} \hat{Q}_2(k_z, \omega) \\ & - \left\{ \chi_2 J_2(r\beta_2) \sin(\theta) \right\} \hat{R}_1(k_z, \omega) - \left\{ \chi_2 Y_2(r\beta_2) \sin(\theta) \right\} \hat{R}_2(k_z, \omega) \\ & + \left\{ \frac{\sin(\theta)}{r} [J_1(r\beta_2) - r\beta_2 J_0(r\beta_2)] \right\} \hat{S}_1(k_z, \omega) + \left\{ \frac{\sin(\theta)}{r} [Y_1(r\beta_2) - r\beta_2 Y_0(r\beta_2)] \right\} \hat{S}_2(k_z, \omega) \\ & + \left\{ \beta_2 J_0(r\beta_2) \cos(\theta) - \frac{1}{r} \{J_1(r\beta_2) * [r\beta_2 + \cos(\theta)]\} \right\} \hat{T}_1(k_z, \omega) \end{aligned}$$

$$+ \left\{ \beta_2 Y_0(r\beta_2) \cos(\theta) - \frac{1}{r} \{Y_1(r\beta_2) * [r\beta_2 + \cos(\theta)]\} \right\} \hat{T}_2(k_z, \omega) \quad (36)$$

**247** (c) Axial displacement:

$$\begin{aligned} \hat{U}_e(r, \theta, k_z, \omega) = & \left\{ J_0(r\beta_1) + J_1(r\beta_1) \cos(\theta) \right\} \hat{P}_1(k_z, \omega) + \left\{ J_1(r\beta_1) \sin(\theta) \right\} \hat{P}_2(k_z, \omega) \\ & + \left\{ Y_0(r\beta_1) + Y_1(r\beta_1) \cos(\theta) \right\} \hat{Q}_1(k_z, \omega) + \left\{ Y_1(r\beta_1) \sin(\theta) \right\} \hat{Q}_2(k_z, \omega) \\ & + \left\{ J_0(r\beta_2) + J_1(r\beta_2) \cos(\theta) \right\} \hat{R}_1(k_z, \omega) + \left\{ Y_0(r\beta_2) + Y_1(r\beta_2) \cos(\theta) \right\} \hat{R}_2(k_z, \omega). \end{aligned} \quad (37)$$

**248** The unknown variables in the displacement components, are related to the unknown constants

**249**  $A_1, A_2, A_3, A_4, B_1, B_2, B_3, B_4, C_1$  and  $C_2$  in the potential functions as,

$$\hat{P}_1(k_z, \omega) = 2\pi i A_1 A_3 k_z \delta(k + k_z) \delta(\omega - \omega_0), \quad (38)$$

$$\hat{P}_2(k_z, \omega) = 2\pi i A_2 A_3 k_z \delta(k + k_z) \delta(\omega - \omega_0), \quad (39)$$

$$\hat{Q}_1(k_z, \omega) = 2\pi i A_1 A_4 k_z \delta(k + k_z) \delta(\omega - \omega_0), \quad (40)$$

$$\hat{Q}_2(k_z, \omega) = 2\pi i A_2 A_4 k_z \delta(k + k_z) \delta(\omega - \omega_0), \quad (41)$$

$$\hat{R}_1(k_z, \omega) = -2\pi C_1 \beta_1 \delta(k + k_z) \delta(\omega - \omega_0), \quad (42)$$

$$\hat{R}_2(k_z, \omega) = -2\pi C_2 \beta_1 \delta(k + k_z) \delta(\omega - \omega_0), \quad (43)$$

$$\hat{S}_1(k_z, \omega) = 2\pi B_1 B_4 \delta(k + k_z) \delta(\omega - \omega_0), \quad (44)$$

$$\hat{S}_2(k_z, \omega) = 2\pi B_2 B_4 \delta(k + k_z) \delta(\omega - \omega_0), \quad (45)$$

$$\hat{T}_1(k_z, \omega) = -2\pi B_1 B_3 \delta(k + k_z) \delta(\omega - \omega_0), \quad (46)$$

250 and

$$\hat{T}_2(k_z, \omega) = -2\pi B_2 B_3 \delta(k + k_z) \delta(\omega - \omega_0), \quad (47)$$

251 where  $\delta()$  is the dirac delta function. In the above equations (Eqs. (38)-(47)), two unknown vari-  
252 ables can be expressed in terms of other variables as,

$$\hat{Q}_2(k_z, \omega) = \frac{\hat{P}_2(k_z, \omega) \hat{Q}_1(k_z, \omega)}{\hat{P}_1(k_z, \omega)} \text{ and } \hat{T}_2(k_z, \omega) = \frac{\hat{S}_2(k_z, \omega) \hat{T}_1(k_z, \omega)}{\hat{S}_1(k_z, \omega)}. \quad (48)$$

253 Thus, of the ten unknown variables (Eqs. (38)-(47)), present in the displacement fields (Eqs. (35)  
254 - (37)), only eight are independent.

#### 255 4.1.2 Elastic stress components

256 The elastic tube and the acoustic fluid inside the tube are coupled through displacement and  
257 stress boundary conditions. Of all the stress components, only  $\tau_{rr}$ ,  $\tau_{r\theta}$ ,  $\tau_{rz}$ , and  $\tau_{z\theta}$  are of interest  
258 to us. These components may be computed using the constitutive relations [15]. They are,

$$\tau_{rr}(r, \theta, z, t) = (\lambda + 2\mu) \frac{\partial W_e}{\partial r} + \frac{\lambda}{r} \left( W_e + \frac{\partial \Theta_e}{\partial \theta} \right) + \lambda \frac{\partial U_e}{\partial z}, \quad (49)$$

259

$$\tau_{rz}(r, \theta, z, t) = \mu \left( \frac{\partial W_e}{\partial z} + \frac{\partial U_e}{\partial r} \right), \quad (50)$$

260

$$\tau_{r\theta}(r, \theta, z, t) = \mu \left( \frac{1}{r} \frac{\partial W_e}{\partial \theta} + \frac{\partial \Theta_e}{\partial r} - \frac{\Theta_e}{r} \right), \quad (51)$$

261 and

$$\tau_{z\theta}(r, \theta, z, t) = \mu \left( \frac{\partial \Theta_e}{\partial z} + \frac{1}{r} \frac{\partial U_e}{\partial \theta} \right). \quad (52)$$

262 The constitutive relations above are transformed into the frequency - wavenumber ( $\omega - k_z$ ) domain  
 263 using the Fourier transform pair (Eqs. (33) and (34)). Furthermore, displacement components  
 264 derived in the previous subsection are substituted to obtain closed form expressions for these  
 265 stress components (see Eqs. (S.40), (S.44), (S.45) and (S.46) in the supplemental material). A  
 266 detailed derivation of the stress components using the constitutive relations are given in Section S2  
 267 in supplemental material.

#### 268 4.2 The interior fluid acoustic pressure and displacement fields

269 The interior fluid is assumed to be confined inside an infinitely long elastic tube. The acoustic  
 270 wave propagation in the fluid is governed by

$$\nabla^2 p_f(r, \theta, z, t) = \frac{1}{c_a^2} \frac{\partial^2 p_f(r, \theta, z, t)}{\partial t^2}, \quad (53)$$

271 where  $p_f$  is the acoustic pressure,  $c_a$  is the speed of sound in the fluid inside the tube and  $\nabla^2$  is  
 272 the Laplacian. In cylindrical coordinates,

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}. \quad (54)$$

273 Assuming a plane wave propagation in the  $z$  direction and using a variable separable form for  $p_f$ ,

$$p_f(r, \theta, z, t) = R(r)\Theta(\theta)e^{j(k_z z - \omega t)}. \quad (55)$$

**274** Substituting Eq. (55) in Eq. (53), and rearranging gives

$$\frac{r^2}{R} \frac{\partial^2 R(r)}{\partial r^2} + \frac{r}{R} \frac{\partial R(r)}{\partial r} + \alpha^2 r^2 = -\frac{1}{\Theta} \frac{\partial^2 \Theta}{\partial \theta^2}, \quad (56)$$

**275** where  $\alpha^2 = \frac{\omega^2}{c_a^2} - k_z^2$ . For the above equation, only those solutions are valid for which the left hand

**276** side and the right hand sides are equal to a positive constant ( $n^2$ ). Therefore,

$$\frac{r^2}{R} \frac{\partial^2 R(r)}{\partial r^2} + \frac{r}{R} \frac{\partial R(r)}{\partial r} + \alpha^2 r^2 = -\frac{1}{\Theta} \frac{\partial^2 \Theta}{\partial \theta^2} = n^2. \quad (57)$$

**277** From the above equation,  $\Theta(\theta)$  can be obtained by solving

$$\frac{\partial^2 \Theta}{\partial \theta^2} + n^2 \Theta = 0. \quad (58)$$

**278** A general solution to Eq. (58) is

$$\Theta(\theta) = P_{f01} \cos(n\theta) + P_{f02} \sin(n\theta), \quad (59)$$

**279** where  $P_{f01}$  and  $P_{f02}$  are unknowns. Similarly from Eq. (57),  $R(r)$  is governed by,

$$\frac{\partial^2 R}{\partial r^2} + \frac{1}{r} \frac{\partial R}{\partial r} + \left( \alpha^2 - \frac{n^2}{r^2} \right) R = 0. \quad (60)$$

**280** A general solution to Eq. (60) is

$$R(r) = P_{f03} J_n(\alpha r) + P_{f04} Y_n(\alpha r), \quad (61)$$

281 where  $P_{f03}$  and  $P_{f04}$  are unknowns. As  $r \rightarrow 0$ ,  $Y_n(\alpha r) \rightarrow -\infty$ , the second term on the right hand  
 282 side must vanish for all valid pressure fields inside a cylindrical tube. Therefore,  $P_{f04} = 0$ . Thus,  
 283 using Eqs. (55), (59) and (61),

$$p_f(r, \theta, z, t) = P_{f03} J_n(\alpha r) [P_{f01} \cos(n\theta) + P_{f02} \sin(n\theta)] e^{j(k_z z - \omega t)} \quad (62)$$

284 The above equation is valid for  $n = 0, 1, 2, \dots$  etc. A complete solution to the acoustic pressure field  
 285 may be written as,

$$p_f(r, \theta, z, t) = \sum_{n=0}^{\infty} P_{f03} J_n(\alpha r) [P_{f01} \cos(n\theta) + P_{f02} \sin(n\theta)] e^{j(k_z z - \omega t)} \quad (63)$$

286 Only  $n = 0$  and  $n = 1$  terms in Eq. (63) are considered in this work. Further, Eq. (63) is transformed  
 287 to the frequency-wavenumber  $(\omega - k_z)$  domain using Eq. (33) and is given by

$$\hat{p}_f(r, \theta, k_z, \omega) = \hat{P}_{f1}(k_z, \omega) [J_0(\alpha r) + J_1(\alpha r) \cos(\theta)] + \hat{P}_{f2}(k_z, \omega) J_1(\alpha r) \sin(\theta), \quad (64)$$

288 where  $\hat{P}_{f1}(k_z, \omega)$  and  $\hat{P}_{f2}(k_z, \omega)$  are two unknowns, which are function of the constants  $P_{f01}$ ,  $P_{f02}$   
 289 and  $P_{f03}$ . They are related as,

$$\hat{P}_{f1}(k_z, \omega) = 2\pi P_{f03} P_{f01} \delta(k + k_z) \delta(\omega - \omega_0) \quad (65)$$

290 and

$$\hat{P}_{f2}(k_z, \omega) = 2\pi P_{f03} P_{f02} \delta(k + k_z) \delta(\omega - \omega_0). \quad (66)$$

291 The acoustic particle velocity in the radial, azimuthal and axial directions can be obtained with the  
292 help of Euler equation,

$$\nabla p_f(r, \theta, z, t) = -\rho \frac{\partial \mathbf{u}_f(r, \theta, z, t)}{\partial t}, \quad (67)$$

293 where  $\nabla$  is the gradient operator and  $\mathbf{u}_f$  is the acoustic particle velocity. The acoustic fluid particle  
294 velocity  $\mathbf{u}_f$  may be represented as,

$$\mathbf{u}_f(r, \theta, z, t) = u_{fr}(r, \theta, z, t)\mathbf{e}_r + u_{f\theta}(r, \theta, z, t)\mathbf{e}_\theta + u_{fz}(r, \theta, z, t)\mathbf{e}_z, \quad (68)$$

295 where  $u_{fr}$ ,  $u_{f\theta}$  and  $u_{fz}$  are the radial, azimuthal and axial components of the fluid particle velocity,  
296 respectively. In the above equation, as for the acoustic pressure, a harmonic variation in the form  
297 of  $e^{j(k_z z - \omega t)}$  is assumed for the particle velocity. Substituting for  $\mathbf{u}_f$  (Eq. (68)) and transforming to  
298 the frequency-wavenumber domain, Eq. (67) results

$$\begin{aligned} \frac{\partial \hat{p}_f(r, \theta, k_z, \omega)}{\partial r} \mathbf{e}_r + \frac{1}{r} \frac{\partial \hat{p}_f(r, \theta, k_z, \omega)}{\partial \theta} \mathbf{e}_\theta + \frac{\partial \hat{p}_f(r, \theta, k_z, \omega)}{\partial z} \mathbf{e}_z \\ = j\rho\omega \hat{u}_{fr}(r, \theta, k_z, \omega) \mathbf{e}_r + j\rho\omega \hat{u}_{f\theta}(r, \theta, k_z, \omega) \mathbf{e}_\theta + j\rho\omega \hat{u}_{fz}(r, \theta, k_z, \omega) \mathbf{e}_z. \end{aligned} \quad (69)$$

#### 299 4.2.1 Radial component of the fluid particle displacement

300 Comparing and equating the radial components on the left and right hand sides of Eq. (69)  
301 result in

$$\frac{\partial \hat{p}_f(r, \theta, k_z, \omega)}{\partial r} = j\rho\omega \hat{u}_{fr}(r, \theta, k_z, \omega) \quad (70)$$



**302** Substituting for the acoustic pressure  $\hat{p}_f(r, \theta, k_z, \omega)$  from Eq. (64) and simplifying yields

$$\begin{aligned} \hat{u}_{fr}(r, \theta, k_z, \omega) = \frac{1}{j\rho\omega} \left\{ \alpha J_0(r\alpha) \cos(\theta) - \frac{J_1(r\alpha)}{r} [r\alpha + \cos(\theta)] \right\} \hat{P}_{f1} \\ + \frac{1}{j\rho\omega} \left\{ \frac{\alpha \sin(\theta)}{2} [J_0(r\alpha) - J_2(r\alpha)] \right\} \hat{P}_{f2}. \end{aligned} \quad (71)$$

**303** Let  $U_{fr}$  be the radial displacement of the fluid particle, defined by

$$U_{fr}(r, \theta, z, t) = \tilde{U}_{fr}(r, \theta) e^{j(k_z z - \omega t)}. \quad (72)$$

**304** In the frequency-wavenumber domain, the fluid particle displacement and velocity in the radial

**305** direction are related as

$$\hat{U}_{fr}(r, \theta, k_z, \omega) = \frac{j}{\omega} \hat{u}_{fr}(r, \theta, k_z, \omega). \quad (73)$$

**306** Substituting for  $\hat{u}_{fr}(r, \theta, k_z, \omega)$  (Eq. (71)) in the above equation, the radial component of the fluid

**307** particle displacement is given by

$$\begin{aligned} \hat{U}_{fr}(r, \theta, k_z, \omega) = \left\{ \frac{1}{r\rho\omega^2} \{ r\alpha J_0(r\alpha) \cos(\theta) - J_1(r\alpha) [r\alpha + \cos(\theta)] \} \right\} \hat{P}_{f1} \\ + \left\{ \frac{\sin(\theta)}{r\rho\omega^2} [r\alpha J_0(r\alpha) - J_1(r\alpha)] \right\} \hat{P}_{f2}. \end{aligned} \quad (74)$$

**308** 4.2.2 Azimuthal component of the fluid particle displacement

**309** Comparing and equating the azimuthal components on the left and right hand sides of Eq. (69)

**310** result in

$$\frac{1}{r} \frac{\partial \hat{p}_f(r, \theta, k_z, \omega)}{\partial \theta} = j \rho \omega \hat{u}_{f\theta}(r, \theta, k_z, \omega) \quad (75)$$

**311** Substituting for the acoustic pressure  $\hat{p}_f(r, \theta, k_z, \omega)$  from Eq. (64) and simplifying yields

$$\hat{u}_{f\theta}(r, \theta, k_z, \omega) = \left\{ \frac{1}{r \rho \omega} [j \sin(\theta) J_1(r\alpha)] \right\} \hat{P}_{f1} + \left\{ \frac{-1}{r \rho \omega} [j \cos(\theta) J_1(r\alpha)] \right\} \hat{P}_{f2}. \quad (76)$$

**312** Let  $U_{f\theta}$  be the azimuthal displacement of the fluid particle defined by

$$U_{f\theta}(r, \theta, z, t) = \tilde{U}_{f\theta}(r, \theta) e^{j(k_z z - \omega t)}. \quad (77)$$

**313** In the frequency-wavenumber domain, the fluid particle displacement and velocity in the azimuthal

**314** directions are related as

$$\hat{U}_{f\theta}(r, \theta, k_z, \omega) = \frac{j}{\omega} \hat{u}_{f\theta}(r, \theta, k_z, \omega). \quad (78)$$

**315** Substituting for  $\hat{u}_{f\theta}(r, \theta, k_z, \omega)$  (Eq. (76)) in the above equation, the azimuthal component of the

**316** fluid particle displacement is given by

$$\hat{U}_{f\theta}(r, \theta, k_z, \omega) = \left\{ \frac{-1}{r \rho \omega^2} [J_1(r\alpha) \sin(\theta)] \right\} \hat{P}_{f1} + \left\{ \frac{1}{r \rho \omega^2} [J_1(r\alpha) \cos(\theta)] \right\} \hat{P}_{f2}. \quad (79)$$

### 317 4.3 Boundary conditions

318 The field variables derived in the previous subsections involve unknown variables. It is shown  
 319 in Section 4.1 that the elastic tube displacement (Eqs. (35), (36) and (37)) and stress (Eqs. (49),  
 320 (50), (51) and (52)) fields have eight unknowns:  $\hat{P}_1$ ,  $\hat{P}_2$ ,  $\hat{Q}_1$ ,  $\hat{R}_1$ ,  $\hat{R}_2$ ,  $\hat{S}_1$ ,  $\hat{S}_2$  and  $\hat{T}_1$ . Section 4.2  
 321 shows that the interior fluid pressure (Eq. (64)) and the fluid particle displacement fields (Eqs. (74)  
 322 and (79)) have two unknowns:  $\hat{P}_{f1}$  and  $\hat{P}_{f2}$ . These ten unknown variables can be calculated with  
 323 the help of boundary conditions on the inner ( $r = a$ ) and outer ( $r = b$ ) surfaces of the elastic tube.  
 324 This subsection discusses the ten boundary conditions that are used to compute the ten unknown  
 325 variables. Note that in the previous subsection, all field variables are expressed in the  $r - \theta - k_z - \omega$   
 326 domain. These closed form expressions are transformed numerically into the  $r - k_\theta - k_z - \omega$  domain  
 327 using the discrete Fourier transform pair given below before use in the boundary conditions.

$$\hat{G}(r, k_\theta, k_z, \omega) = \sum_{\theta=0}^{N-1} g(r, \theta, k_z, \omega) e^{-j2\pi k_\theta \theta / N} \quad (80)$$

$$g(r, \theta, k_z, \omega) = \frac{1}{N} \sum_{k_\theta=0}^{N-1} \hat{G}(r, k_\theta, k_z, \omega) e^{j2\pi k_\theta \theta / N} \quad (81)$$

328 The ten boundary conditions are given below.

- 329 1. On the interior surface of the elastic tube ( $r = a$ ), the radial component of the normal stress  
 330  $\tau_{rr}$  is equal to the negative of the acoustic pressure  $p_f$ . In the  $k_\theta - k_z - \omega$  domain, this can be  
 331 written as

$$\hat{\tau}_{rr}(a, k_\theta, k_z, \omega) = -\hat{p}_f(a, k_\theta, k_z, \omega) \quad (82)$$

- 332 2. On the exterior surface of the elastic tube ( $r = b$ ), the radial component of the normal stress  
 333  $\tau_{rr}$  is equal to the negative of the external turbulent pressure  $p_0$ . In the  $k_\theta - k_z - \omega$  domain,

334 this may be written as

$$\hat{\tau}_{rr}(b, k_\theta, k_z, \omega) = -\hat{p}_0(k_z, \omega) \quad (83)$$

335 where  $\hat{p}_0(k_z, \omega)$  is assumed to be a function of  $k_z$  and  $\omega$  alone and is given by Eq. (18) for the  
 336 hybrid model. For cases where the external pressure varies with  $\theta$ , an appropriate pressure  
 337 spectrum  $\hat{p}_0(k_\theta, k_z, \omega)$  must be used.

338 3. The exterior and interior surfaces of the elastic tube are assumed to be shear-free. Therefore  
 339  $\tau_{rz}|_{r=a,b} = 0$ ,  $\tau_{r\theta}|_{r=a,b} = 0$  and  $\tau_{z\theta}|_{r=a,b} = 0$ . In the  $k_\theta - k_z - \omega$  domain, this may be written as

$$\hat{\tau}_{rz}(r = a, k_\theta, k_z, \omega) = 0, \quad (84)$$

$$\hat{\tau}_{rz}(r = b, k_\theta, k_z, \omega) = 0, \quad (85)$$

$$\hat{\tau}_{r\theta}(r = a, k_\theta, k_z, \omega) = 0, \quad (86)$$

$$\hat{\tau}_{r\theta}(r = b, k_\theta, k_z, \omega) = 0, \quad (87)$$

$$\hat{\tau}_{z\theta}(r = a, k_\theta, k_z, \omega) = 0, \quad (88)$$

and

$$\hat{\tau}_{z\theta}(r = b, k_\theta, k_z, \omega) = 0. \quad (89)$$

340 4. The radial ( $W_e$ ) and azimuthal ( $\Theta_e$ ) components of the elastic tube displacements on the inte-  
 341 rior surface ( $r = a$ ) must be equal to the respective fluid particle displacement ( $U_{fr}$  and  $U_{f\theta}$ )  
 342 at  $r = a$ . In the  $k_\theta - k_z - \omega$  domain, this may be written as

$$\hat{U}_{fr}(a, k_\theta, k_z, \omega) = \hat{W}_e(a, k_\theta, k_z, \omega) \quad (90)$$

and

$$\hat{U}_{f\theta}(a, k_\theta, k_z, \omega) = \hat{\Theta}_e(a, k_\theta, k_z, \omega). \quad (91)$$

The expressions for the acoustic pressure, the fluid and the elastic tube displacement components, and the elastic tube stress components derived in the previous subsections are substituted in the above boundary conditions. The resulting equations are given in detail in Section S3 in the supplemental material.

#### 4.4 Solution methodology

Previous section discussed the boundary conditions associated with the elastic tube displacements and the acoustic pressure variations inside and outside the elastic tube. These boundary conditions result in ten equations involving twelve unknown variables (see section S3 in the supplemental material). Of the twelve unknown variables, only ten are independent. A further simplification of the boundary condition to include only the ten independent unknowns are presented in section S4 in supplemental material. The resulting system of algebraic equations may be represented in a matrix form,

$$\mathbf{A}(r, k_\theta, k_z, \omega) \mathbf{x} = \mathbf{b}(r, k_\theta, k_z, \omega), \quad (92)$$

where  $\mathbf{A}$  is the coefficient matrix of order  $10 \times 10$ ,  $\mathbf{x}$  is the unknown variable vector of order  $10 \times 1$  and  $\mathbf{b}$  is the constant vector of order  $10 \times 1$ . A detailed representation of  $\mathbf{A}$ ,  $\mathbf{x}$  and  $\mathbf{b}$  are given in section S4 in the supplemental material. Eq. (92) is solved numerically for the unknown variable vector  $\mathbf{x}$  and the solution is used to calculate the interior acoustic pressure field  $\hat{p}_f(r, k_\theta, k_z, \omega)$ . The acoustic pressure field  $\hat{p}_f(r, k_\theta, k_z, \omega)$  is further used to compute (a) azimuthal variation in the acoustic pressure field inside the elastic tube, (b) on-axis flow noise spectrum, and (c) on-axis flow noise.

#### 362 4.4.1 Azimuthal variation in the interior acoustic pressure field

363 The interior acoustic pressure field  $\hat{p}_f(r, k_\theta, k_z, \omega)$  is first multiplied with the square of the hy-  
 364 drophone response function  $H(k_z)$  (Eq. (11)) and then integrated over the entire axial wavenumber  
 365 domain to get

$$Q(r = a, k_\theta, f) = 4\pi \int_{-\infty}^{\infty} \hat{p}_f(r = a, \theta, k_z, \omega) |H(k_z)|^2 dk_z. \quad (93)$$

366 The factor  $4\pi$  is used to account for the negative frequency and radian frequency measure [8,  
 367 9, 12]. The azimuthal variation in  $Q(r = a, \theta, f)$  may be computed using the inverse Fourier  
 368 transform,

$$Q(r = a, \theta, f) = \frac{1}{N} \sum_{k_\theta=0}^{N-1} Q(r = a, k_\theta, f) e^{j2\pi k_\theta \theta / N}. \quad (94)$$

369 The azimuthal variation in acoustic pressure level at the tube inner surface is computed using

$$SPL(r = a, \theta, f) = 10 \log_{10} \left( \frac{|Q(a, \theta, f)|}{p_{ref}^2} \right), \quad (95)$$

370 where  $p_{ref} = 1\mu Pa$  is the reference acoustic pressure in water.

#### 371 4.4.2 On-axis flow noise spectrum level

372 The acoustic pressure field  $\hat{p}_f(r, k_\theta, k_z, \omega)$  is integrated over the azimuthal wavenumbers to  
 373 obtain the on-axis flow noise spectrum as a function of frequency ( $\omega$ ) and axial wavenumber ( $k_z$ ).

$$\hat{p}_f(r = 0, k_z, \omega) = \int_{-\infty}^{\infty} \hat{p}_f(r = 0, k_\theta, k_z, \omega) dk_\theta. \quad (96)$$

**374** The on-axis flow noise spectrum level can be calculated by

$$SPL(r = 0, k_z, \omega) = 10 \log_{10} \left( \frac{|\hat{p}_f(r = 0, k_z, \omega)|}{p_{ref}^2} \right). \quad (97)$$

**375** 4.4.3 On-axis flow noise

**376** First, the on-axis flow noise spectrum is computed using Eq. (96). It is further multiplied with  
**377** the square of the hydrophone response function  $H(k_z)$  (Eq. (11)) and integrated over the axial  
**378** wavenumbers  $k_z$  to obtain on-axis flow noise  $Q(r = 0, f)$ . Thus,

$$Q(r = 0, f) = 4\pi \int_{-\infty}^{\infty} \hat{p}_f(r = 0, k_z, \omega) |H(k_z)|^2 dk_z. \quad (98)$$

**379** The on-axis flow noise level can be computed using

$$SPL(r = 0, f) = 10 \log_{10} \left( \frac{|Q(r = 0, f)|}{p_{ref}^2} \right). \quad (99)$$

## 5 RESULTS AND DISCUSSIONS

**380** Previous sections discussed the development of a fully-coupled three-dimensional vibroacous-  
**381** tic (3D-VA) model of a fluid-filled elastic tube under external pressure excitations. In this section,  
**382** the 3D-VA model is used to estimate the interior acoustic pressure field and flow noise in towed  
**383** sonar arrays. Section 5.1 presents the interior acoustic pressure field for azimuthally varying ex-  
**384** ternal pressure excitation over the fluid-filled elastic tube. Section 5.2 discusses the on-axis flow  
**385** noise spectrum due to an external turbulent pressure excitation. External turbulent pressure exci-  
**386** tation is computed using the hybrid model developed in this work (see Section 3). The results are  
**387** then compared with those obtained using the tube transfer function [9] and the axisymmetric vi-  
**388** broacoustic [11, 16] models available in the literature. Section 5.3 presents the on-axis flow noise  
**389** computed using the 3D-VA model and further compares the results with those predicted using

390 tube transfer function [9] and the axisymmetric [11] models. Further, Section 5.4 discusses the  
 391 flow noise variation for various elastic tube diameters at different tow speeds.

### 392 5.1 Interior acoustic pressure field for azimuthally varying external excitation

393 The developed 3D-VA model of the fluid-filled elastic tube is initially tested with an exterior har-  
 394 monic pressure excitation that has a known azimuthal variation. Results for two different external  
 395 pressure excitation are presented here : (a)  $\hat{p}_0(\theta, k_z) = \sin(\theta)$  (see Fig. 6) and (b)  $\hat{p}_0(\theta, k_z) = \cos(\theta)$   
 396 (see Fig. 7).

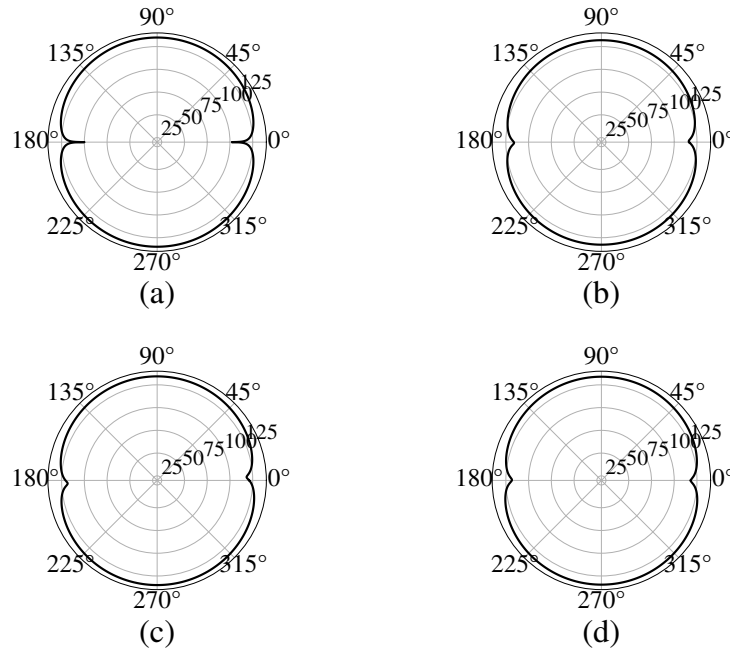


Fig. 6. (a): Azimuthal variation in the exterior pressure field  $\hat{p}_0(\theta, k_z) = \sin(\theta)$ . (b), (c) and (d): Azimuthal variation in the interior pressure field ( $r = a$ ) at 10 Hz, 100 Hz and 1000 Hz, respectively, for  $\hat{p}_0(\theta, k_z) = \sin(\theta)$ .

397 Fig. 6(a) shows the azimuthal variation in the acoustic pressure (see Eqs. (93) - (95) with  $\hat{p}_f$   
 398 being replaced with  $\sin(\theta)$ ) at the outer surface of the elastic tube. Figs. 6(b) - 6(d) shows the  
 399 resulting azimuthal variation in the acoustic pressure at the inner surface ( $r=a$ ) of the elastic tube  
 400 for (b): 10 Hz, (c) 100 Hz and (d) 1000 Hz. Fig. 7 shows similar results for  $\hat{p}_0(\theta, k_z) = \cos(\theta)$ . The  
 401 Figs. 6 and 7 confirms that the 3D-VA model accurately captures the azimuthal variation (restricted



**402** to  $n = 0$  and  $n = 1$ ) in the external pressure excitation.

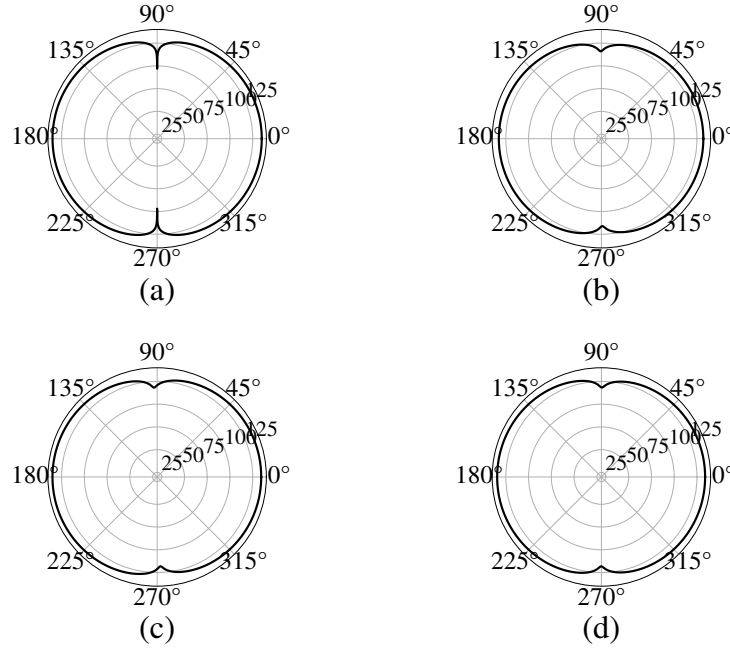


Fig. 7. (a): Azimuthal variation in the exterior pressure field  $\hat{p}_0(\theta, k_z) = \cos(\theta)$ . (b), (c) and (d): Azimuthal variation in the interior pressure field ( $r = a$ ) at 10 Hz, 100 Hz and 1000 Hz, respectively, for  $\hat{p}_0(\theta, k_z) = \cos(\theta)$ .

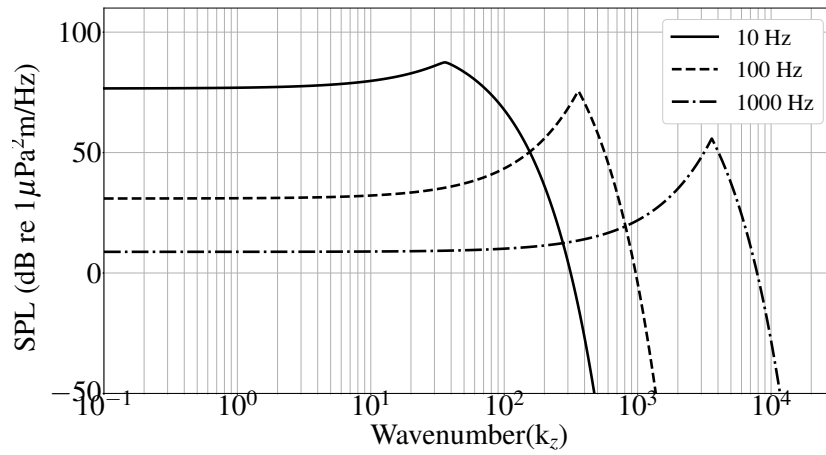


Fig. 8. The external turbulent pressure spectrum level computed using the hybrid model (Eq. (18)).

Table 1. The elastic tube, interior fluid and the hydrophone array parameters used for estimation of flow noise inside the cylinder

Property	Values
Tube diameter (m)	0.04
Tube thickness (m)	0.005
Tow speed/Flow velocity (knots)	5
Number of hydrophones	50
Length of hydrophone (m)	0.05
Hydrophone spacing (m)	0.25
Exterior fluid density ( $\text{kg/m}^3$ )	1000
Interior fluid density ( $\text{kg/m}^3$ )	800
Reference pressure ( $\mu\text{ Pa}$ )	1

## 5.2 On-axis flow noise spectrum level for external turbulent pressure excitation

This section presents on-axis flow noise spectrum when the elastic tube is excited by an external turbulent pressure field. The external turbulent pressure field (see Fig. 8) is computed using the hybrid model (Eq. (18)). Further, the 3D-VA model is used to calculate the on-axis flow noise spectrum (see Eqs. (96) and (97)). This flow noise spectrum is compared with that computed using the tube transfer function model [9] (see Fig. 9) and the axisymmetric model [11] (see Fig. 10). The elastic tube, inside fluid, and hydrophone array parameters used for computation are listed in Table 1. It can be seen from Figs. 9 and 10 that the on-axis acoustic pressure computed using the 3D-VA model follows the external turbulent pressure excitation given in Fig. 8 - gradually increasing up to and peaking at the convective wavenumber ( $u_c = \omega/k_c$ ), and decreasing exponentially beyond. Thus, the flow noise inside the tube is dominated by the contribution from wavenumbers less than the convective wavenumber.

In the tube transfer function model (Fig. 9) and the axisymmetric model (Fig. 10) predictions, the peak occurs at a lower wavenumber than the convective wavenumber. For example, at 10 Hz, the peak occurs at 1.58 rad/m in the tube transfer function model and at 1.56 rad/m in the axisymmetric model. Note that the convective wavenumber at 10 Hz for 5 knots is  $k_c = 35.93$  rad/m. This

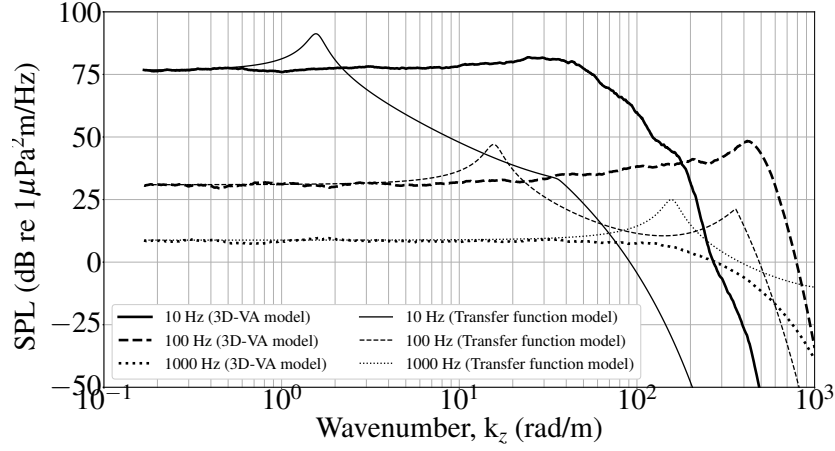


Fig. 9. Comparison of on-axis flow noise spectrum level due to a turbulent pressure excitation computed using the 3D-VA model and the tube transfer function model [9].

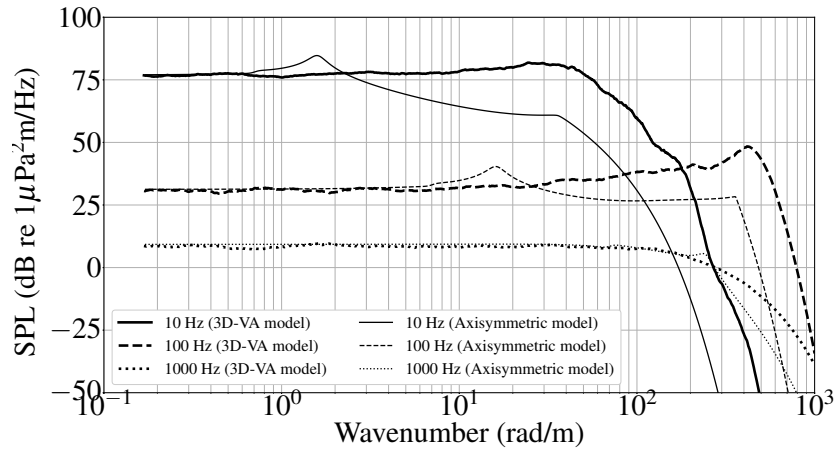


Fig. 10. Comparison of on-axis flow noise spectrum level due to a turbulent pressure excitation computed using the 3D-VA model and the axisymmetric model [11].

419 smaller wavenumber where the first peak occurs in the transfer function and the axisymmetric  
 420 models corresponds to the breathing mode wavenumber,  $k_b$ , of the elastic tube given by [9]

$$k_b^2 = 2\rho_0\omega^2 R/Et. \quad (100)$$

421 In Eq. (100),  $\rho_0$  is the density of the inside fluid,  $R$  is the outer radius of the elastic tube,  $E$  is  
 422 the Young's modulus of the tube and  $t$  is the thickness of the tube. The tube transfer function  
 423 model and the axisymmetric model consider only the breathing mode ( $n = 0$ ) variations while

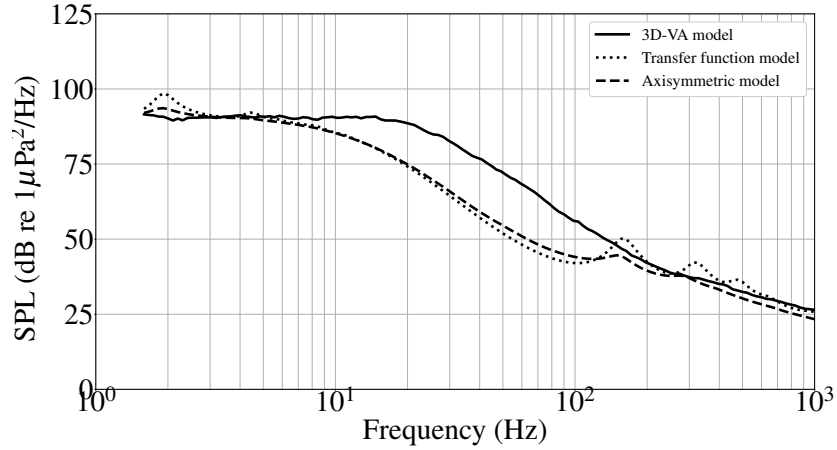


Fig. 11. The on-axis flow noise due to the turbulent pressure excitation computed using the 3D-VA model, the tube transfer function model [9] and the axisymmetric model [11].

424 modeling the fluid-filled elastic tube. The present 3D-VA model considers both  $n = 0$  (breathing)  
 425 and  $n = 1$  (first order) variations in the solid and fluid displacement fields. The absence of peaks at  
 426 the breathing wavenumber in the present 3D-VA model indeed demonstrates a cumulative effect  
 427 of including both  $n = 0$  and  $n = 1$  order terms in the fully-coupled vibroacoustic formulation. The  
 428 same is the reason for the difference in the flow noise spectrum between the 3D-VA model and  
 429 the other models beyond the breathing wavenumber.

### 430 5.3 On-axis flow noise for external turbulent pressure excitation

431 This section presents the flow noise as heard by the hydrophones placed inside the fluid-filled  
 432 elastic tube. The flow noise is computed using Eqs. (98) and (99). Note that flow noise at a given  
 433 frequency can be obtained by integrating the corresponding flow noise spectrum (Figs. 9 and 10)  
 434 over the wavenumber. Fig. 11 shows the variation in flow noise with frequency, computed using  
 435 the 3D-VA model. It also depicts the flow noise predicted by the tube transfer function [9] and  
 436 the axisymmetric [11] models. Note that in all cases, the external turbulent pressure excitation is  
 437 given by Eq. (18) (the hybrid model). The elastic tube, interior fluid and hydrophone parameters  
 438 are given in Table 1. It can be seen from Fig. 11 that the flow noise decreases with frequency. This  
 439 decrease is attributed to the reduction in the external turbulent pressure excitation with frequency  
 440 as shown in Fig. 8. As mentioned earlier, the 3D-VA model considers both  $n = 0$  (breathing) and  
 441  $n = 1$  variation in the acoustic pressure field. This results in a better flow noise prediction than the

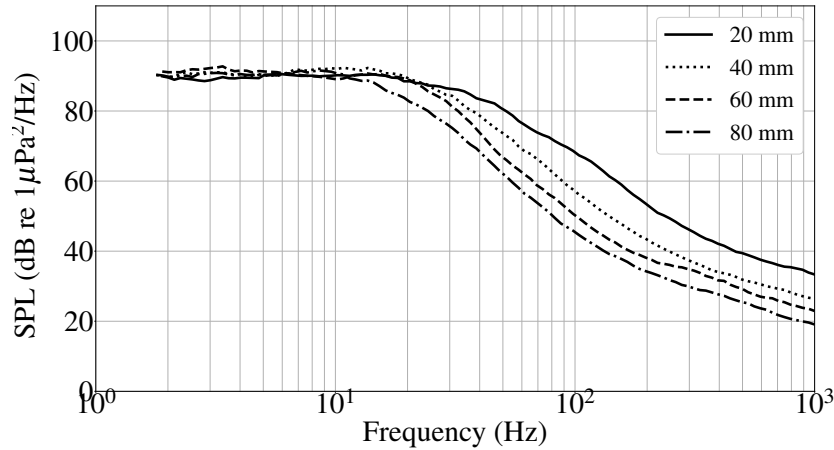


Fig. 12. Comparison of on-axis flow noise estimated using 3D-VA model, due to a turbulent pressure excitation at 5 knots over an elastic tube for different diameters.

transfer function and axisymmetric models, where only the  $n = 0$  or the breathing wavenumber is considered. The transfer function and the axisymmetric models underpredict the flow noise in the mid frequency range (10 Hz - 200 Hz). It is evident from Figs. 9 and 10 that the difference between the 3D-VA and other models is not significant as the frequency increases. For that reason, the flow noise predictions (Fig. 11) by the three models are quite close to each other at high frequencies (beyond  $\sim 200$  Hz).

#### 5.4 On-axis flow noise for different tube diameters and tow speeds

In this section, the on-axis flow noise (Eqs. (98) and (99)) is computed for different elastic tube diameters at different tow speeds. Fig. 12 shows the comparison of on-axis flow noise estimated for different elastic tube diameters at 5 knots and Fig. 13 shows the variation in the flow noise for a tube of 40 mm diameter at different tow speeds. The variation in flow noise is attributed to the changes in external turbulent pressure excitation with tube diameters and tow speeds, as shown in Fig. 4 (the non-dimensional plot). It was shown in Fig. 4 that an increase in diameter or a decrease in tow speed results in a reduction in the non-dimensional power spectral density. This leads to a decrease in on-axis flow noise inside the fluid-filled elastic tube with increasing tube diameter (Fig. 12) or decreasing tow speeds (Fig. 13).

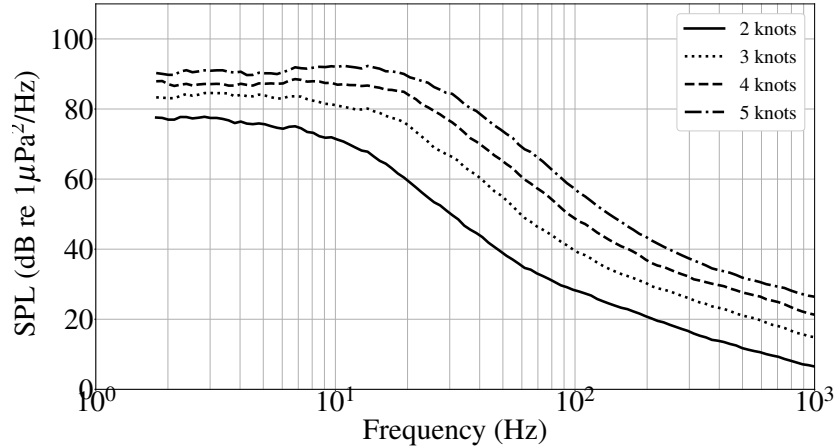


Fig. 13. Comparison of on-axis flow noise estimated using 3D-VA model, due to a turbulent pressure excitation over an elastic tube of 40 mm diameter at different tow speeds.

## 6 CONCLUSIONS

A new semi-empirical (hybrid) model is developed for estimating the wavenumber-frequency spectrum of turbulent pressure for an axial flow past a solid cylinder. The hybrid model is derived using insights from different turbulent pressure semi-empirical models (Chase [4] and Frendi *et al.* [5]) and the experimental results of Unnikrishnan *et al.* [2]. The hybrid model predictions are found to be superior to the existing semi-empirical models and compares reasonably well with available experimental results.

A fully-coupled three-dimensional vibroacoustic model (3D-VA model) is developed for computing the pressure field inside the fluid-filled elastic tube due to external turbulent pressure excitations. In this 3D-VA model, the structure (elastic tube) is modeled using the Navier-Lame equilibrium equations, and the fluid inside the tube is modeled using the acoustic wave equation. The 3D-VA model is first tested for an exterior harmonic pressure excitation having a known azimuthal variation. The interior pressure field is found to follow the same azimuthal variation as that of the external excitation.

Next, the 3D-VA model is used in conjunction with the hybrid model of the turbulent pressure spectrum to find the on-axis flow noise. The results are then compared with the on-axis flow noise estimated using an existing transfer function model [9] and an axisymmetric vibroacoustic model. The transfer function and the axisymmetric models consider only the breathing mode ( $n = 0$ ) of

the elastic tube, but the 3D-VA model considers both  $n = 0$  (breathing) and  $n = 1$  (first order) variations in modeling the elastic tube and the fluid inside the tube. Consequently, it is observed that the two other models underpredict the flow noise compared to the 3D-VA model.

The on-axis flow noise is then estimated for different elastic tube diameters and tow speeds. At low frequencies, an increase in the tube diameter causes negligible variation in flow noise, but at higher frequencies, the flow noise decreases with increase in the tube diameter. When the tow speed is increased, the flow noise is found to increase at all frequencies.

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