Chapter 4
Trees

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Chapter 4
Tree- part1

Preliminaries

Why Do We Need Trees?

- Lists, Stacks, and Queues are linear relationships
- Information often contains hierarchical relationships
  - File directories or folders
  - Moves in a game
  - Hierarchies in organizations
- Can build a tree to support fast searching

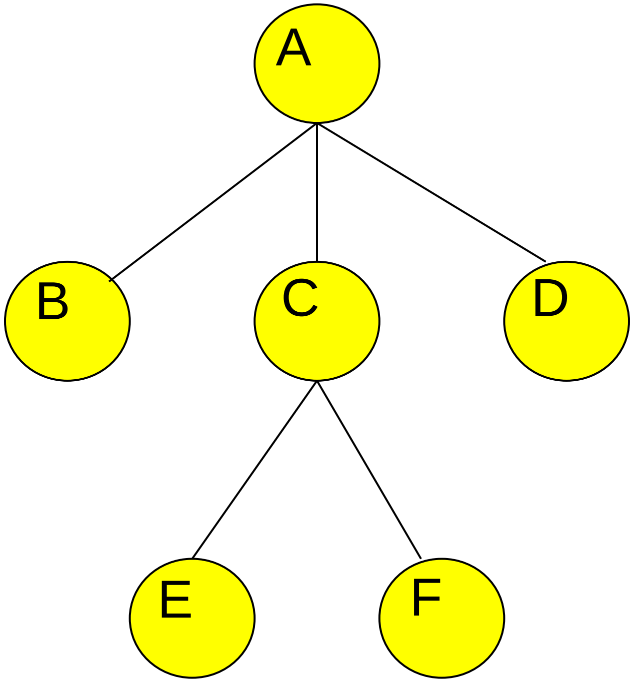
Tree Jargon

- root
- nodes and edges
- leaves

- parent, children, siblings
- ancestors, descendants

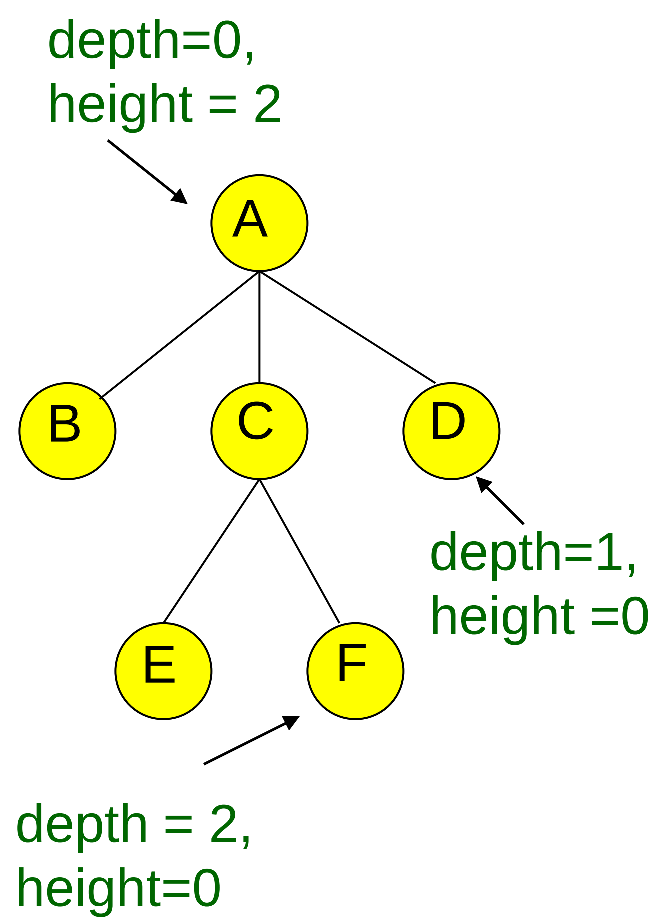
- subtrees

- path, path length
- height, depth



More Tree Jargon

- **Length** of a path = number of edges
- **Depth** of a node N = length of path from root to N
- **Height** of node N = length of longest path from N to a leaf
- **Depth of tree** = depth of deepest node
- **Height of tree** = height of root



Definition and Tree Trivia

- A tree is a set of nodes,i.e., either
  - it’s an empty set of nodes, or
  - it has one node called the root from which zero or more trees (subtrees) descend
- Two nodes in a tree have at most one path between them
- Can a non-zero path from node N reach node N again?

No. Trees can never have cycles (loops)

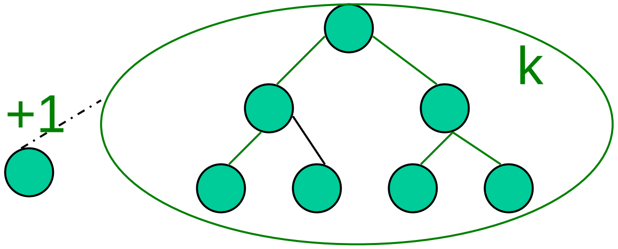
Paths

A tree with N nodes always has N-1 edges (prove it by induction)

Base Case: N=1 one node, zero edges

Inductive Hypothesis: Suppose that a tree with N=k nodes always has k-1 edges.

Induction: Suppose N=k+1…
The k+1st node must connect to the rest by 1 or more edges.

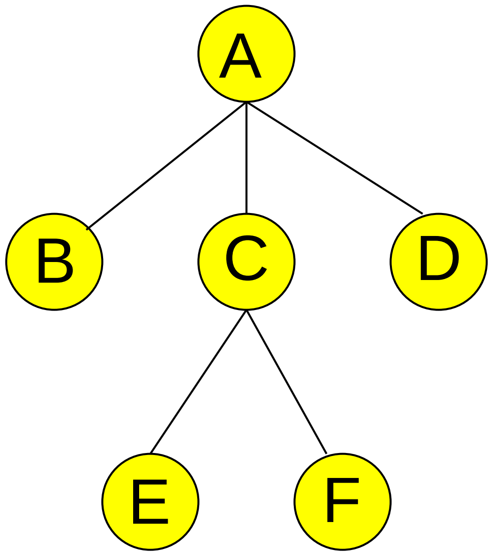


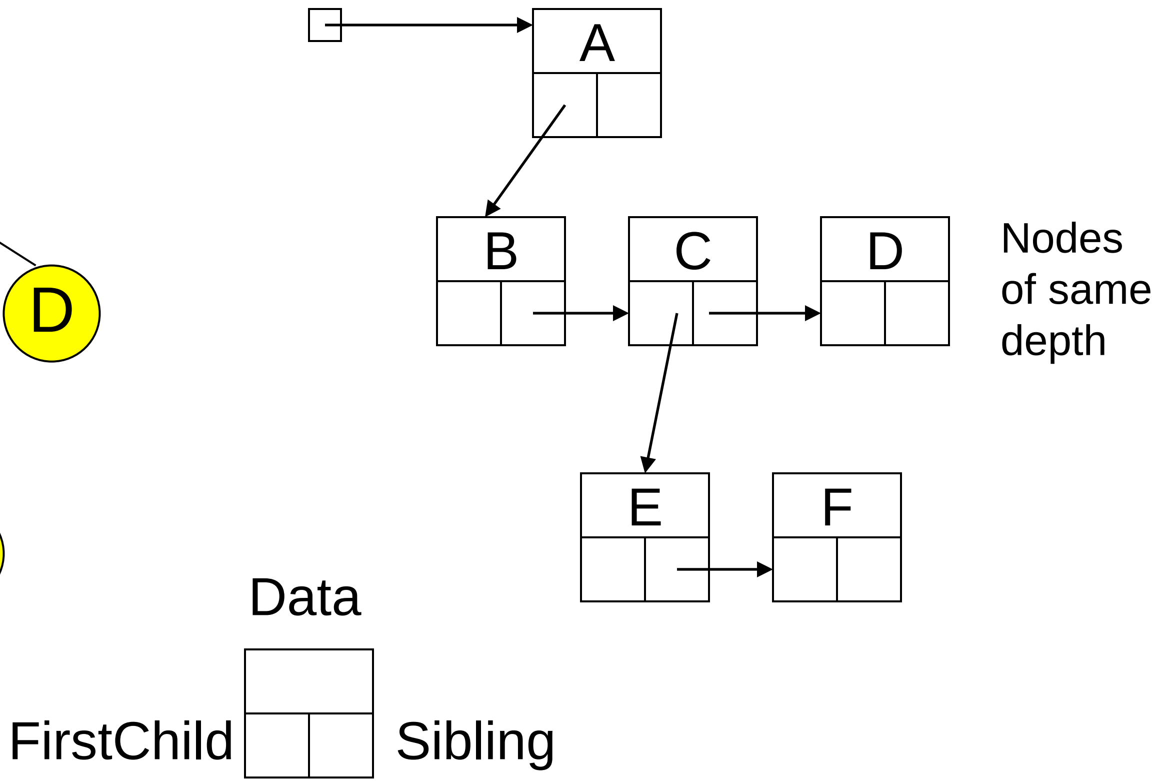
If more, we get a cycle. So it connects by just 1 more edge

Implementation of Trees

- One possible pointer-based Implementation
  - tree nodes with value and a pointer to each child
  - but how many pointers should we allocate space for?
- A more flexible pointer-based implementation
  - 1st Child / Next Sibling List Representation
  - Each node has 2 pointers: one to its first child and one to next sibling
  - Can handle arbitrary number of children

Arbitrary Branching





Tree Traversals

```
// Print using a preorder traversal
void printhelp(GTNode<E>* root) {
if (root->isLeaf()) cout << "Leaf: ";
else cout << "Internal: ";
cout << root->value() << "\n";
// Now process the children of "root"
for (GTNode<E>* temp = root->leftmostChild();
temp != NULL; temp = temp->rightSibling())
printhelp(temp);
}
```

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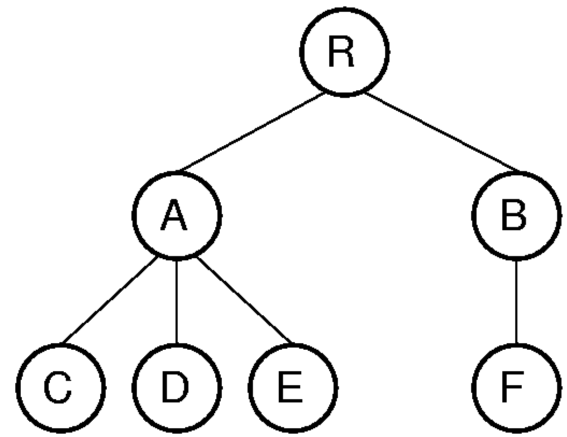
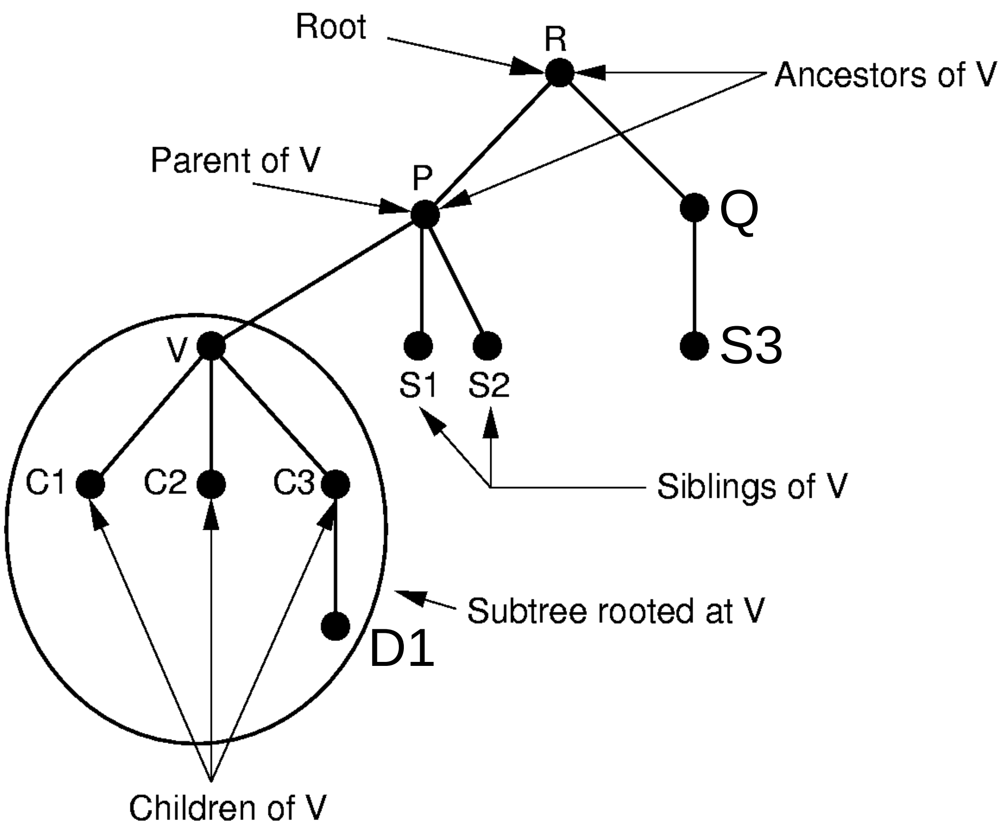


R, A, C, D, E, B, F

Exercise



R, P, V, C1, C2, C3, D1, S1, S2, Q, S3

Binary Trees

Binary Trees

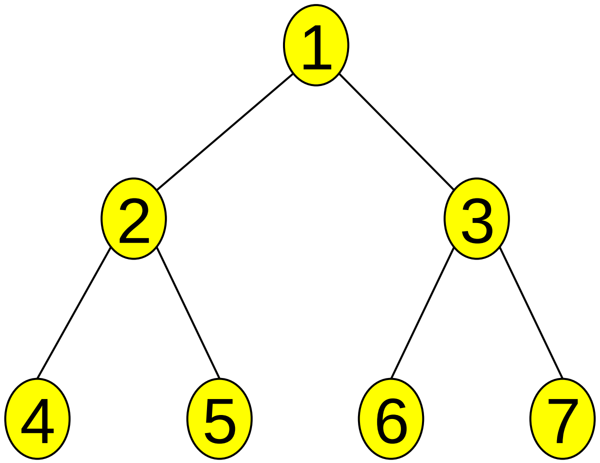
- Every node has at most two children
  - Most popular tree in computer science
- Given N nodes, what is the minimum depth of a binary tree? (This means all levels but the last are full!)
  - At depth d, you can have $N = 2^d$ to $N = 2^{d+1}-1$ nodes

$$2^d \le N \le 2^{d+1}-1 \ \text{ implies } \ d_{min} = \lfloor \log_2 N \rfloor$$

Minimum depth vs node count

- At depth d, you can have $N = 2^d$ to $2^{d+1}-1$ nodes
- minimum depth d is $\Theta(\log N)$

$T(n) = \Theta(f(n))$ means
$T(n) = O(f(n))$ and $f(n) = O(T(n))$,
i.e. T(n) and f(n) have the same growth rate



d=2

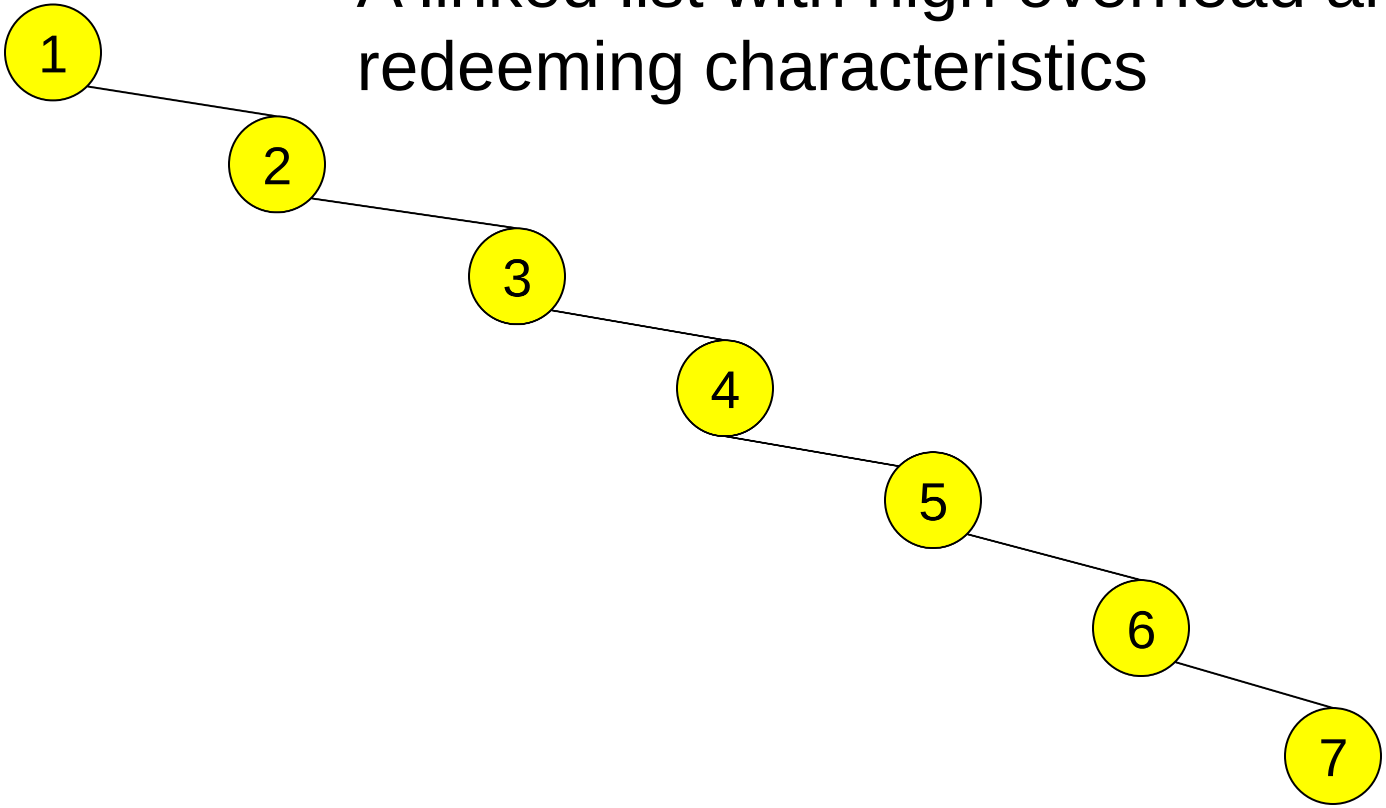
$N=2^2$ to 2^3-1 (i.e, 4 to 7 nodes)

Maximum depth vs node count

- What is the maximum depth of a binary tree?
  - Degenerate case: Tree is a linked list!
  - Maximum depth = N-1
- Goal: Would like to keep depth at around log N to get better performance than linked list for operations like Find

A degenerate tree

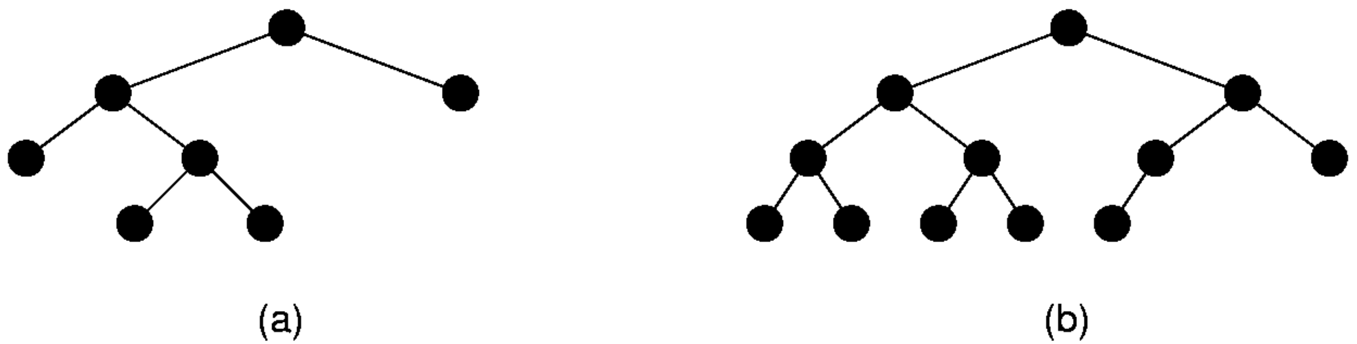
A linked list with high overhead and few redeeming characteristics



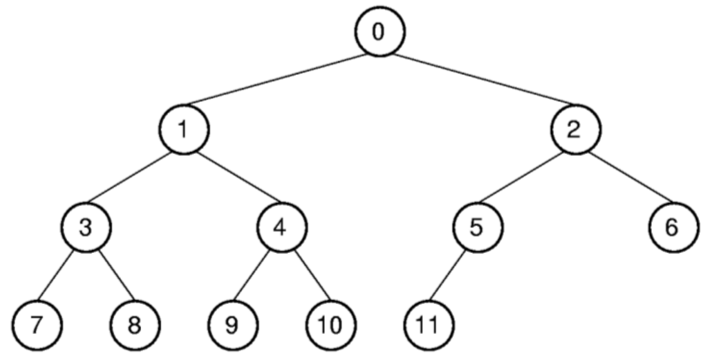
Full and Complete Binary Trees

Full binary tree: Each node is either a leaf or internal node with exactly two non-empty children.

Complete binary tree: If the height of the tree is *d*, then all levels except possibly level *d-1* are completely full. The bottom level has all nodes to the left side.



Array Implementation



Position	0	1	2	3	4	5	6	7	8	9	10	11
Parent	--	0	0	1	1	2	2	3	3	4	4	5
Left Child	1	3	5	7	9	11	--	--	--	--	--	--
Right Child	2	4	6	8	10	--	--	--	--	--	--	--
Left Sibling	--	--	1	--	3	--	5	--	7	--	9	--
Right Sibling	--	2	--	4	--	6	--	8	--	10	--	--

Array Implementation

Parent (r) = $\lfloor (r - 1) / 2 \rfloor$ if $r \neq 0$ and $r < n$.

Leftchild(r) = $2r + 1$ if $2r+1 < n$.

Rightchild(r) = $2r + 2$ if $2r + 2 < n$.

Leftsibling(r) = $r - 1$ if r is even, $r > 0$ and $r < n$.

Rightsibling(r) = $r + 1$ if r is odd, $r + 1 < n$.

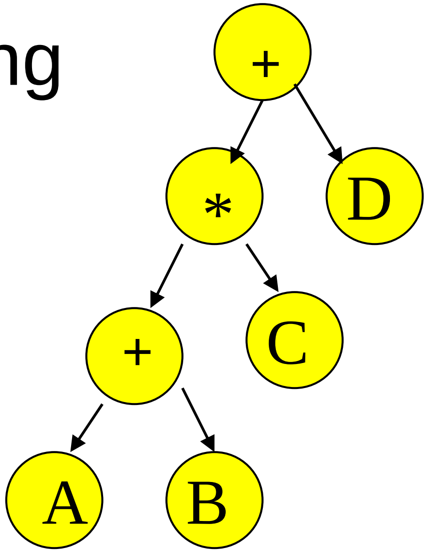
Traversing Binary Trees

- The definitions of the traversals are recursive definitions. For example:
  - Visit the root
  - Visit the left subtree (i.e., visit the tree whose root is the left child) and do this recursively
  - Visit the right subtree (i.e., visit the tree whose root is the right child) and do this recursively
- Traversal definitions can be extended to general (non-binary) trees

Traversing Binary Trees

- Preorder: Node, then Children (starting with the left) recursively + * + A B C D
- Inorder: Left child recursively, Node, Right child recursively A + B * C + D
- Postorder: Children recursively, then Node

  A B + C * D +



Exercise

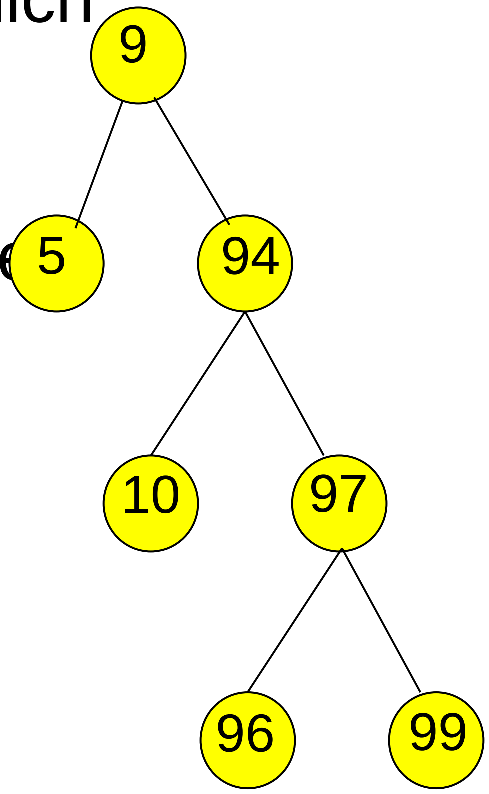
A certain binary tree has the preorder enumeration as BEFCGDH and the inorder enumeration as FEBGCHD . Try to draw the binary tree and give the postorder enumeration.

Postorder enumberation: FEGHDCB

Binary Search Trees

Binary Search Trees

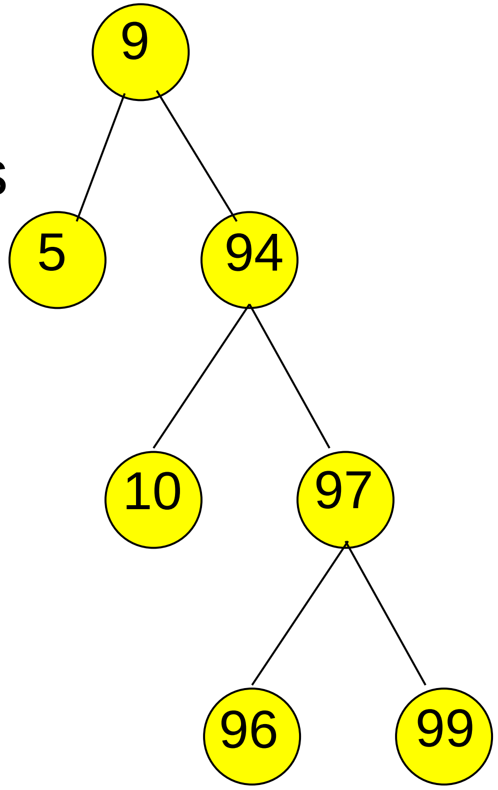
- Binary search trees are binary trees in which
  - all values in the node's left subtree are less than node value
  - all values in the node's right subtree are greater than node value
- Operations:
  - Find, FindMin, FindMax, Insert, Delete



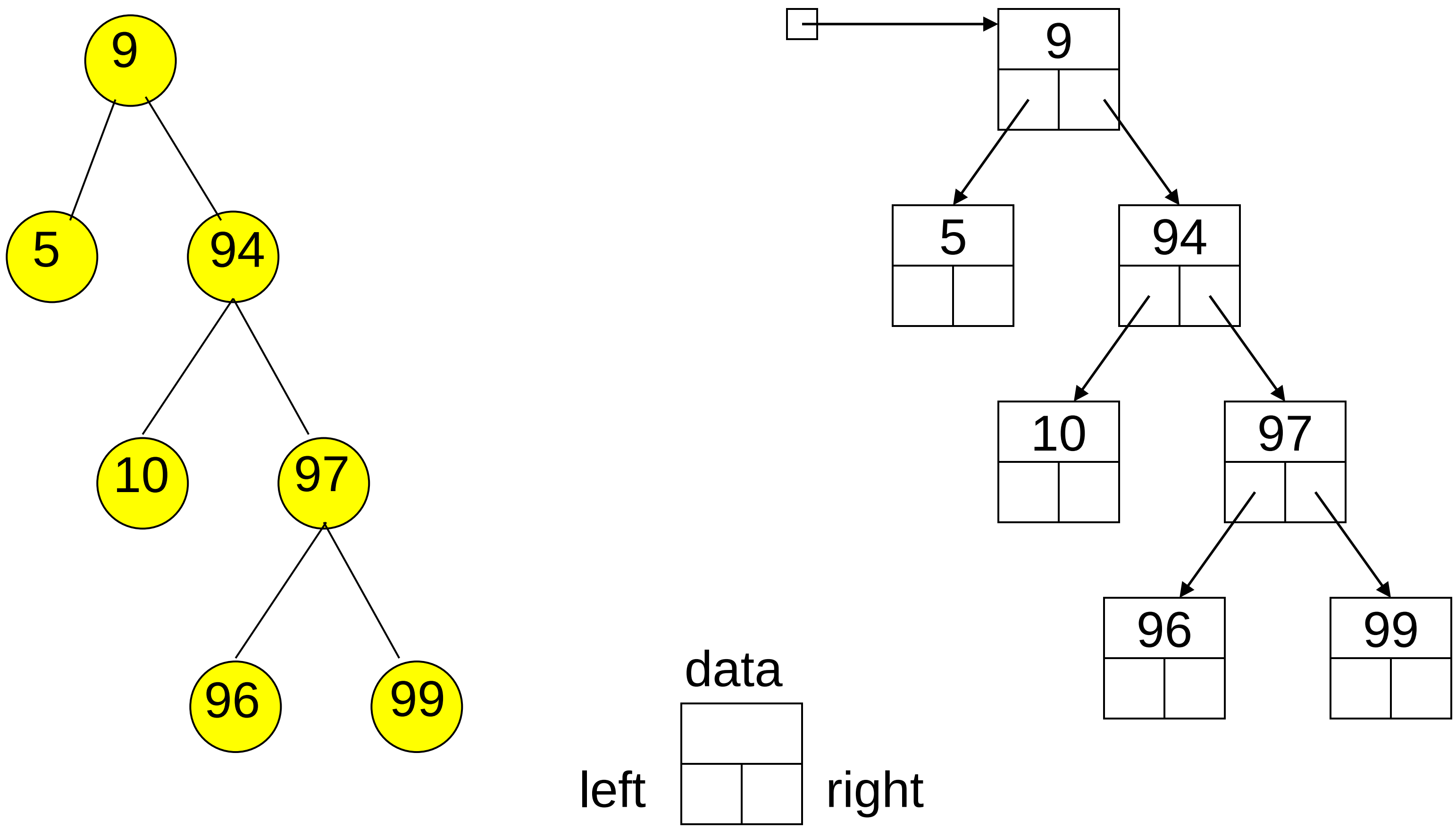
What happens when we traverse the tree in inorder?

Operations on Binary Search Trees

- How would you implement these?
  - Recursive definition of binary search trees allows recursive routines
  - Call by reference helps too
- FindMin
- FindMax
- Find
- Insert
- Delete



Binary SearchTree



Find

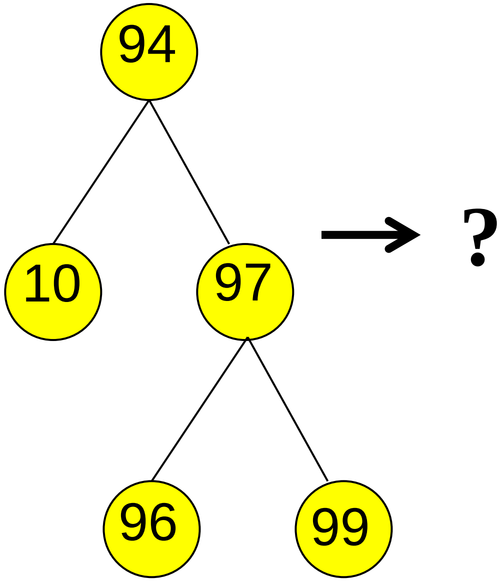
```
Find(T : tree pointer, x : element): tree pointer
{
case {
  T = null : return null;
  T.data = x : return T;
  T.data > x : return Find(T.left,x);
  T.data < x : return Find(T.right,x)
}
}
```

FindMin

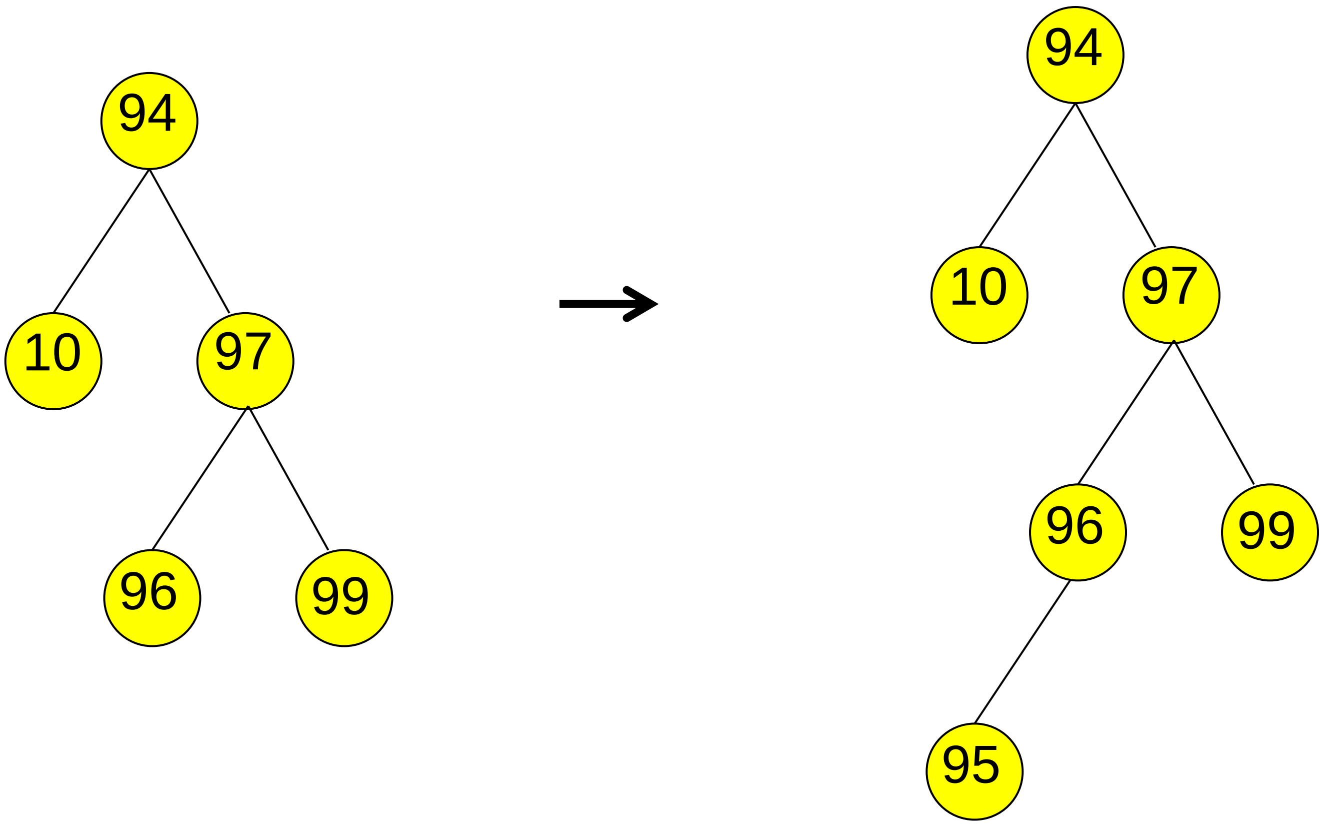
- Design recursive FindMin operation that returns the smallest element in a binary search tree.
  - ```
    FindMin(T : tree pointer) : tree
    pointer {
    // precondition: T is not null
    //
    ???
    }
    ```
    ```

Insert Operation

- `Insert(T: tree, X: element)`
  - Do a “Find” operation for X
  - If X is found → update (no need to insert)
  - Else, “Find” stops at a NULL pointer
  - Insert Node with X there
- Example: Insert 95

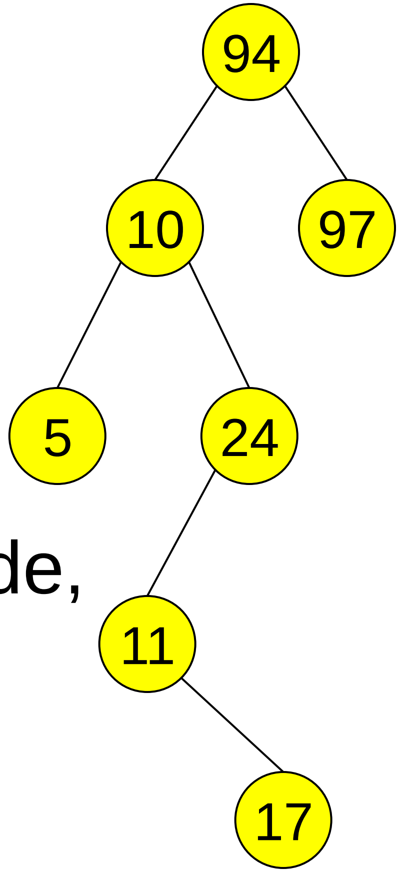


Insert 95



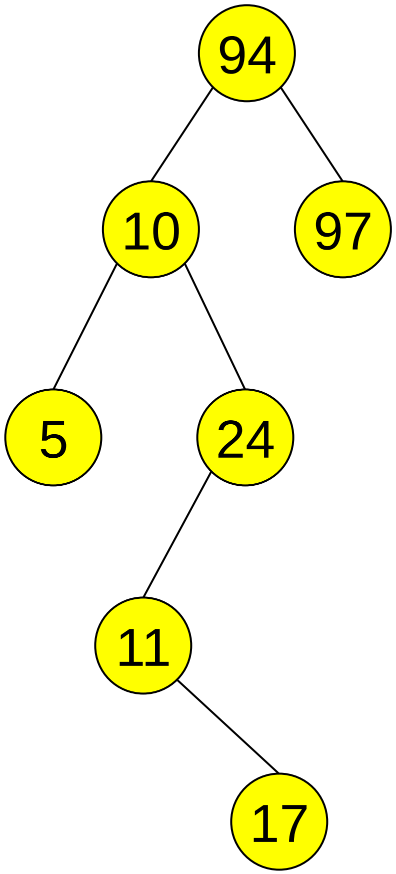
Delete Operation

- Delete is a bit trickier…Why?
- Suppose you want to delete 10
- Strategy:
  - › Find 10
  - › Delete the node containing 10
- Problem: When you delete a node, what do you replace it by?



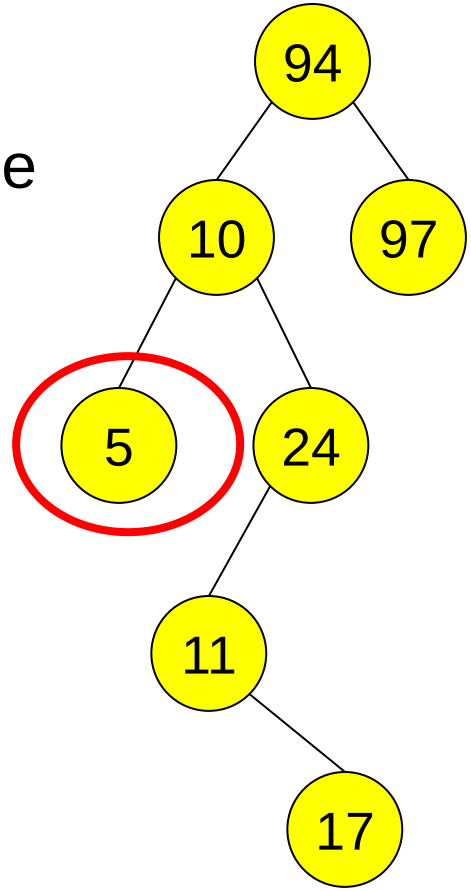
Delete Operation

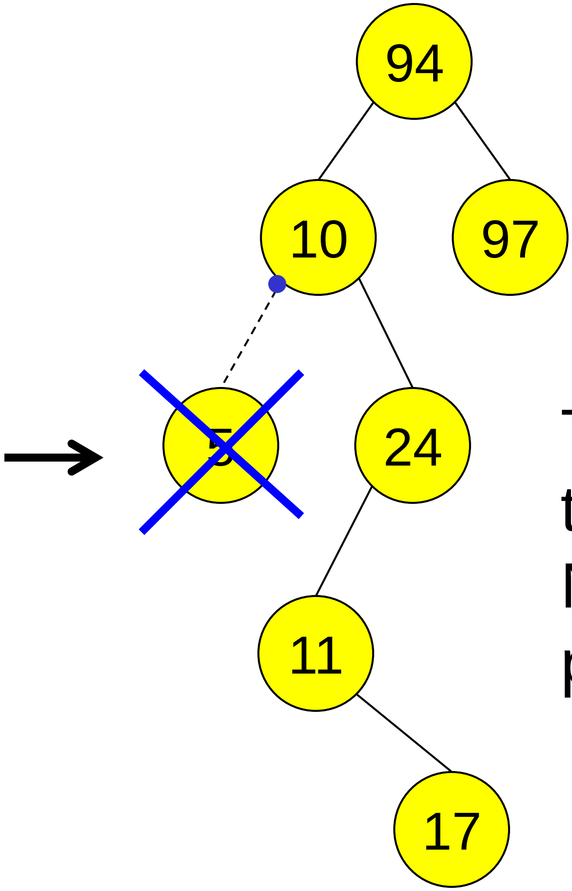
- Problem: When you delete a node, what do you replace it by?
- Solution:
  - If it has no children, by NULL
  - If it has 1 child, by that child
  - If it has 2 children, by the node with the smallest value in its right subtree (the successor of the node)



Delete “5” - No children

Find 5 node

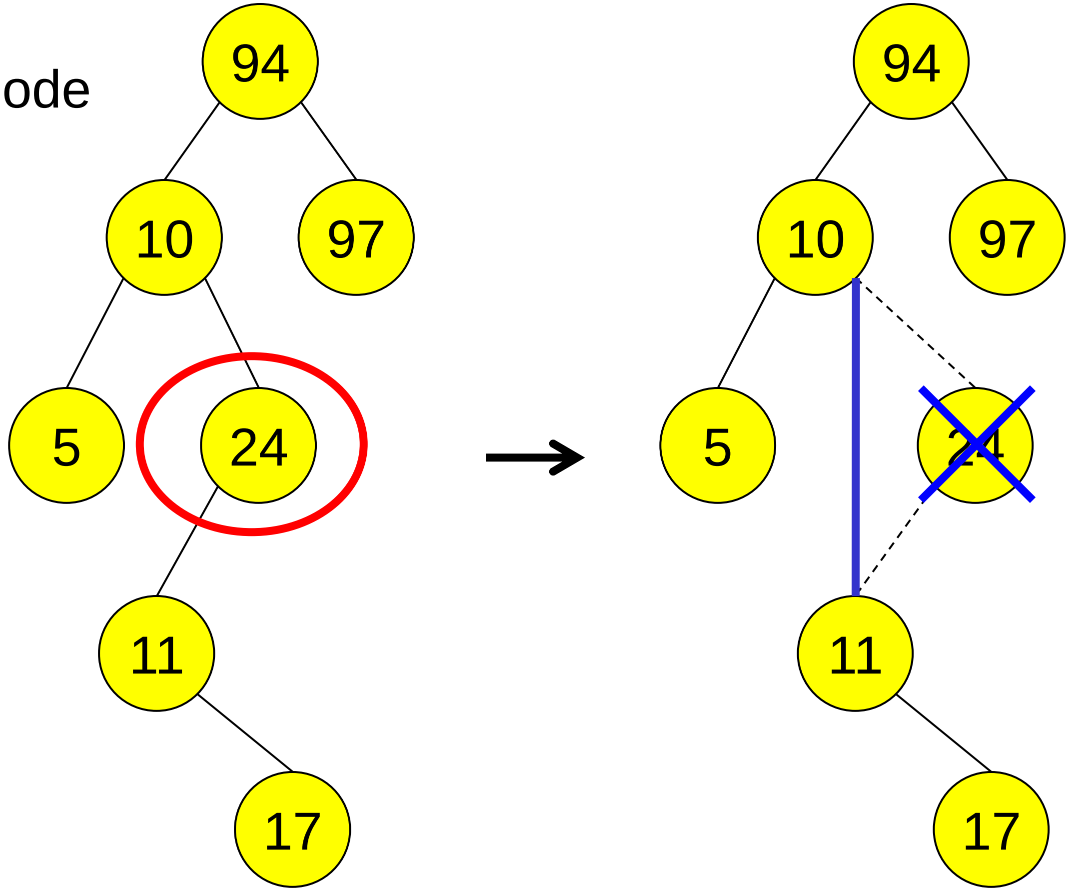




Then Free the 5 node and NULL the pointer to it

Delete “24” - One child

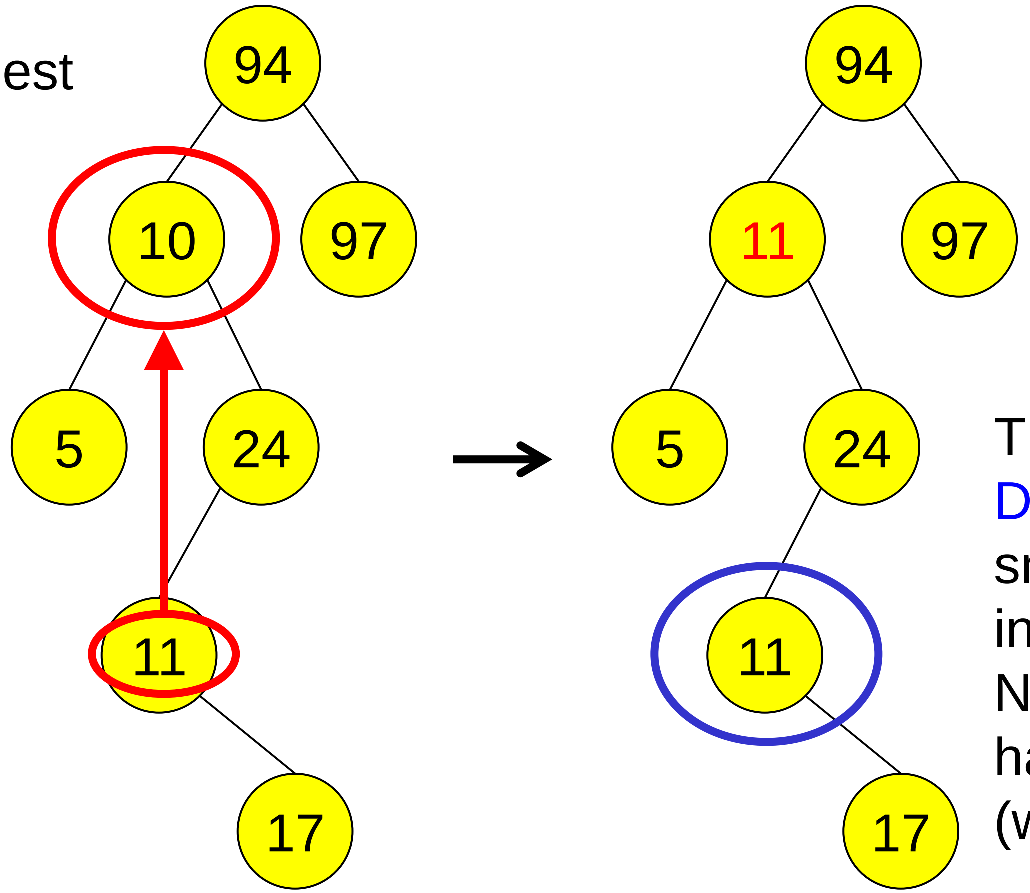
Find 24 node



Then Free
the 24 node and
replace the
pointer to it with
a pointer to its
child

Delete "10" - two children

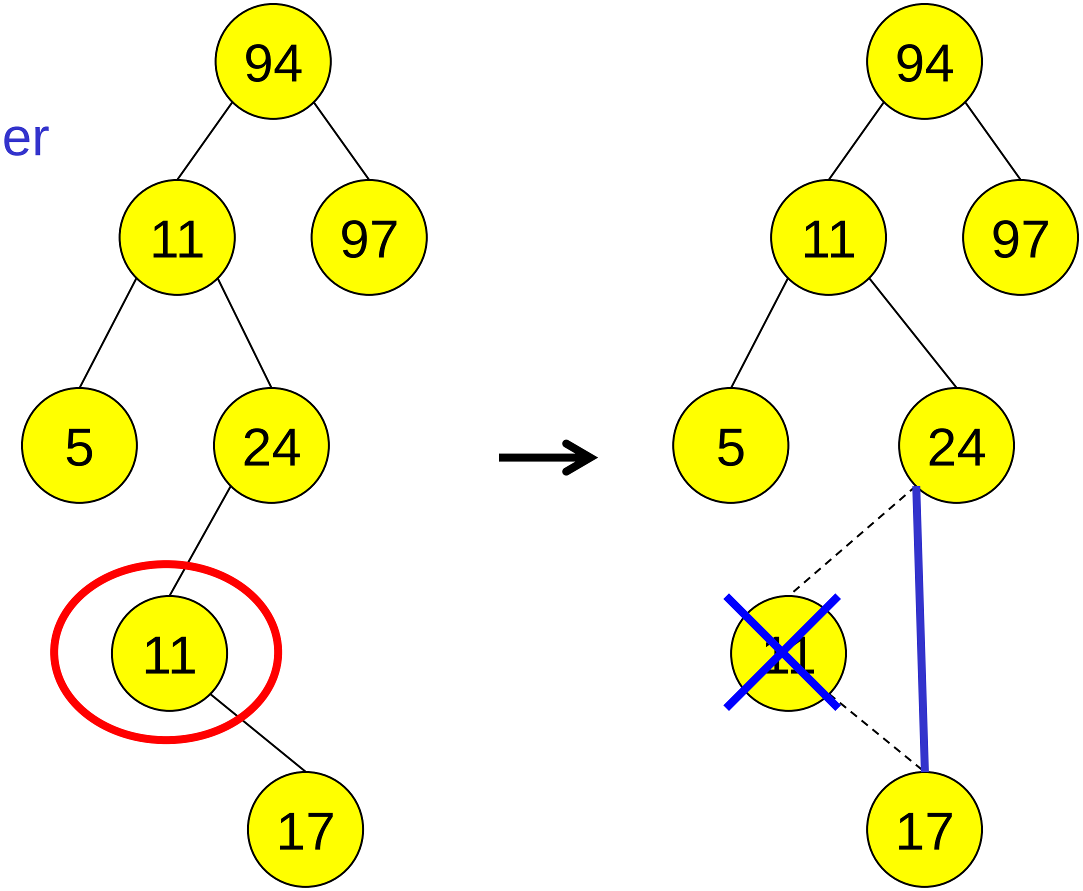
Find 10,
Copy the smallest value in right subtree into the node



Then (recursively) Delete node with smallest value in right subtree
Note: it can not have two children (why?)

Then Delete “11” - One child

Remember
11 node



Then Free
the 11 node and
replace the
pointer to it with
a pointer to its
child

Remove from Text

```
private BinaryNode remove( Comparable x, BinaryNode t) {
if ( t == null) return t;                              // not found
if ( x.compareTo( t.element ) < 0 )
    t.left = remove( x, t.left );                      // search left
else if ( x.compareTo( t.element) > 0 )
    t.right = remove(x, t.right );                     // search right
else if  (t.left != null && t.right != null)           // found it; two children
    {   t.element = findMin (t.right ).element;        // find the min, replace,
        t.right = remove( t.element, t.right);  }         and remove it
else t = (t.left != null ) ? t.left : t.right;         // found it; one child
return t;   }
```

FindMin Solution

```
FindMin(T : tree pointer) : tree pointer {
// precondition: T is not null //
if T.left = null return T
else return FindMin(T.left)
}
```

Note: Look at the “remove” method in the book.