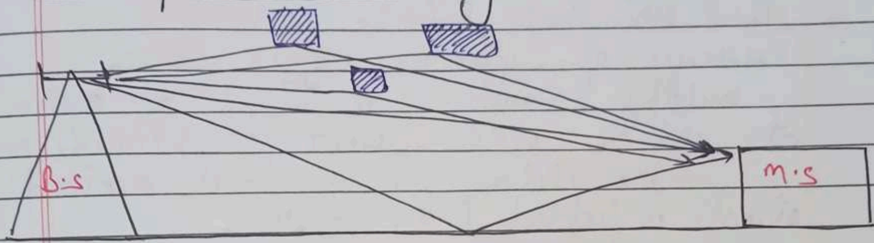
2|22

Multipath and Fading



wirelss channel is a multipath channel (their are serveral path). Multipath in the radio channel causes rapid fluxtuation in the signal amplitude called small scale fading.

fading is caused by distructive interference of two or more version of transmitted signal arriving at receiver at slightly different time.
Different amplitude with different phases due to different path length.
Resultant signal is time varying signal with random phase, amplitude, direction, angle of arrival and propogation deal
these multipath components combine vectorly at receivor antenna and cause total signal to fad and distor.
delayed ~~Dealed~~ signal are result of reflection, scattering, differaction from trees, hells, mountain etc.

Date: ____
Page: ____

Effect of fading :-

i) Multipath Propogation :-
Presence of reflecting object causes multiple version of signal to arrive at receiver with different amplitude and time delay, so total received signal is fadded or distorted.

ii) Speed of mobile :-
- It causes dopple shift at each multipath component.
- It causes random freq modulation. Doppler shift will be postive or negative depending upon whether mobile is moving forward or away from the base station.

iii) Speed of Surrounding objects :-
- If surrounding object move at greater rate than mobile this effect will dominate the small scale fading.
- If surrounding object moves at slower rate then mobile their effect can be egnored.
- the term chorence time determine how static the channel is
- Chorence Bandwidth is a measurment of maximum freq difference for which signal are strongly corelated in amplitude. channel Bandwidth can be contified by chorence Bandwidth

It tells about to what extend amplitude fluctuate and signal distored.

$f_1 \rightarrow$ faded

$f_1 + \frac{\Delta f}{B_C}$ $\rightarrow$ out of fade

* Doppler Effect :-

when a wave source and/or a receiver is/are moving the freq of received signal will not be same as ~~trans~~ transmitted signal.

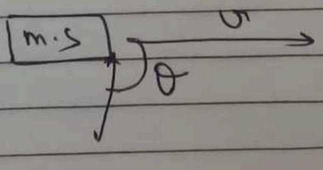
when their are moving each other the freq of received is greater then the freq of transmitted signal.

when their are moving away from each the freq of received signal is less then transmitted signal.

Received freq = $f_R = f_c \pm f_D$

If signal arrive at mobile station at 'θ' angle and mobile station is movin with velocity v. and $\lambda = c/f$.

$$\boxed{f_D = \frac{v}{\lambda} \cos\theta}$$



Question.

A vehicle is travelling at 60 km/h towards the base station of height 30 m mobile station is 1 km away from Base station. freq of operation is 900 MHz. What is received freq at mobile station

$$\tan\theta = \frac{30}{1000}$$

$$\theta = \tan^{-1}\left(\frac{30}{1000}\right)$$

$$\theta = 1.72^\circ$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = 0.33\times10^{-6}$$

$$v = \frac{60\times1000}{3600} = 16.66$$

$$f_D = \frac{v}{\lambda}\cos\theta = \frac{16.66}{0.33\times10^{-6}}\cos(1.72^\circ)$$

$$f_D = 50.49\ Hz.$$

$$f_R = 900\ MHz + 50.49\ Hz$$

Date:
Page:

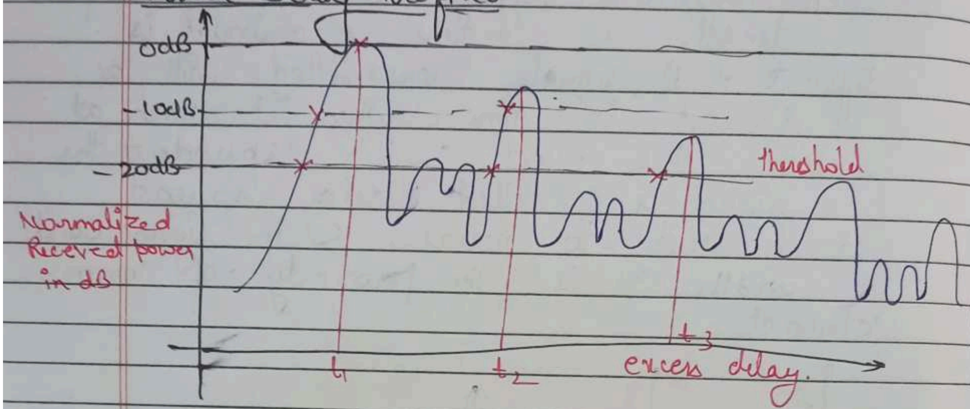
* Delay Spread :-

Each multipath signal travells different path length so time of arrival is different. A singale transmitted will be spread in time when it reach at receiver. this effect which spreadout the signal is called delay spread.
It lead to increase in signal Band width. It is the property of comm. channel.

Date:
Page:

* Time Dispersion Parameters :-

- Power Delay Profile :-



i) Level Crossing rate :- Average no. of time/sec that the signal envelope crosses the level in positive going direction.

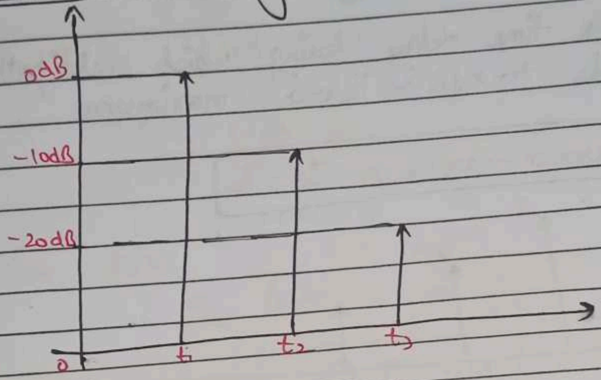
ii) Fading Rate :- No. of times signal envelop crosses the middle value is positive going direction for unit time (direction/time)

iii) Depth of fading :- Ratio of mean square value and minimum value of fading signal.

iv) Fading duration :- Time for which signal is below the given threshold.

v) Quality of Serive (QoS) :- It depends upon all four above parameters.

- **Mean Excess Delay:-**



$$\boxed{\bar{\tau} = \frac{\sum a_k^2 \tau_k}{\sum a_k^2}}$$

$$\boxed{\bar{\tau} = \frac{\sum P_k \tau_k}{\sum P_k}}$$

It is the first moment of power delay profile which is given by P_k, τ_k.

- **RMS Delay Spread:-** (σ_τ)

It is the square of second control moment. (with respect to origin)

$$\boxed{\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}}$$ (second central moment)

$$\boxed{\overline{\tau^2} = \frac{\sum a_k^2 \tau_k^2}{\sum a_k^2}}$$

$$\boxed{\overline{\tau^2} = \frac{\sum P_k \tau_k^2}{\sum P_k}}$$

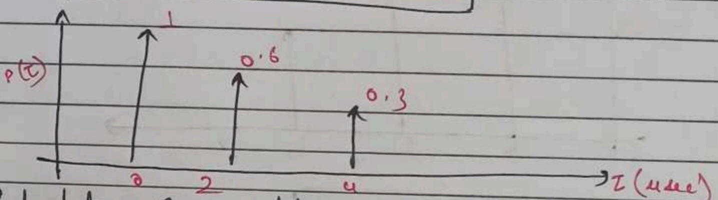
Date:
Page:

- Maximum Excess Delay :-

It is the time delay during which multipath energy falls to x dB below maximum

$\boxed{\text{below maximum} = \tau_x - \tau_0}$

Question



Calculate RMS delay spread?

$$\bar{\tau} = \frac{\sum P_k \tau_k}{\sum P_k}$$

$$= \frac{0\times1 + 2\times0.6 + 4\times0.3}{1+0.6+0.3} = \frac{2.4}{1.9}$$

$$\bar{\tau} = 1.26 \ \mu sec$$

$$\overline{\tau^2} = \frac{\sum P_k \tau_k^2}{\sum P_k}$$

$$= \frac{0\times1 + 4\times0.6 + 16\times0.3}{1+0.6+0.3} = \frac{2.4+4.8}{1.9}$$

$$\overline{\tau^2} = 3.79 \ \mu sec$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$= \sqrt{3.76 - (1.26)^2}$$

$$\boxed{\sigma_\tau = 1.48 \ \mu sec}$$

If BPSK is used what is maximum bite rate without use of any equilizer.

for No ISI σ_τ / T_s must be less then 0.1

$\frac{\sigma_\tau}{T_s} \leq 0.1$ for DBPSK & BFSK | $\frac{\sigma_\tau}{2T_s} \leq 0.1$ for QPSK & m-array

$$\frac{\sigma_\tau}{T_s} \leq 0.1$$

$$\frac{1.48}{T_s} \leq 0.1$$

$$\frac{1.48}{0.1} \leq T_s$$

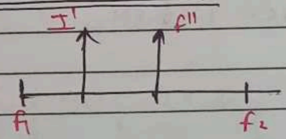
$$14.8 \leq T_s$$

for BPSK

$$R_s = \frac{1}{T_s} = \frac{1}{14.8\,\mu sec}$$

$$\boxed{R_s = 67.56 \text{ Kbps.}}$$ (Data rate)

* Coherence Bandwidth:-



$$\boxed{f' + BC = f'''}$$

A statical measure the range of freq over which ~~rate~~ the channel can be considel as ~~p~~ flat means channel pass all spectral components with equal gain and linear phase.

It ~~repeat~~ represent corelation between two fading signal ~~envo~~ envelopes at f_1 & f_2.

It is the function of delay spread two freq that are larger than coherence bandwidth faded ~~ins inpaden~~ ~~independently~~. independently.

It is useful in diversity technic multiple copies of same signal are send using different freq those are separated by coherence

Coherence B.W & delay spread
B_c & σ_τ
Coherence ~~time & Doppler~~ spread
T_c & B_D

Bandwidth.

If we define Coherence bandwidth as a range of freq over which corelation is above 90%

$$\boxed{B_c = \frac{1}{50\sigma_\tau}}$$

for 50% Corelation

$$\boxed{B_c = \frac{1}{5\sigma_\tau}}$$

Question: For a certain multipath channel if $\sigma_\tau = 1.37$ usec then for 50% corelation what will be coherence Bandwidth?

$$B_c = \frac{1}{5 \times 1.37}$$

$$B_c = 146 \text{ kHz}$$

⇒ GSM required 200 kHz Bandwith as a Coherence bandwidth.
AMPS (analog mobile phone serives) can be used without equilzer.
AMPS required 30 kHz.

* Coherence time and Doppler Spread :-

Doppler spread is maximum freq shift (doppler shift)

$$\boxed{B_D = f_{m\,max}}$$

Doppler spread tells freq dispersiveness of channel. It is obtained from doppler spectrum. If ~~base~~ bandwidth of base band signal is much

Date:
Page:

greater then B_0 then effect of doppler spread is negligible $\boxed{B_s >> B_0}$

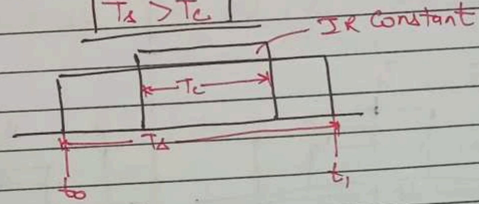
$\boxed{B_s = \frac{1}{T_s}}$ (from symbol rate)

this is called slow fading.

Coherence time is the ~~statical~~ statistical measurment of time over which channel impulse reponse is time invarient system.

If symbol period is much greater time coherence time then channel will change during transmission of signal hence their will be distortion at receiver

$\boxed{T_s > T_c}$



$$T_c \propto \frac{1}{f_{max}}$$

Coherence times is inversly propostional to the maximum freq and the constant is 0.423.

$$\boxed{T_c = \frac{0.423}{f_{max}}}$$

Date:
Page:

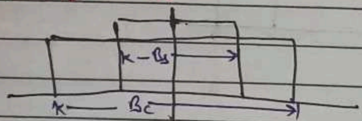
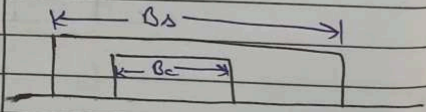
Two signal arriving with time ~~de~~ separation greater then Coherence time are effected differently by the channel.

Larger the coherence time channel changes slowly

$T_s > T_c \longrightarrow$ slow

* Type of Small scale fading based on multipath time delay spread

σ_τ (delay spread)

Flat fading	freq selective fading
i) $B_s << B_c \Rightarrow \frac{1}{T_s} << \frac{1}{\sigma_\tau}$ $\Rightarrow \sigma_\tau << T_s$	$B_s >> B_c \Rightarrow \sigma_\tau >> T_s$
	
ii) all Spectral Component of freq get preserved	all spectral Component of freq are not preserved
iii) Rayleigh or Ricean distributed	Multipath components resolved
It occurs due to fluxeation in gain of multipath channel which lead to change in amplitud of reactied signal with time	

Continuous wave → narrow band
discreate wave → wide band

Date:
Page:

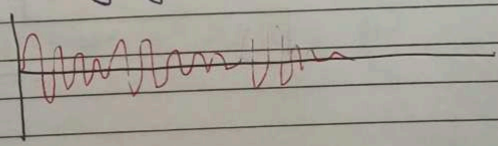
$\sigma_\tau << T_s$. It means bandwidth of symbol is narrow. the fadding distribution is statistical characteristic of varation of received signal over time.

* Flat fading:-

- It occure due to fluxcation in gain of multipath channel which lead to change in amplitude of received signal with time.
- It occure when symbol period is much greater than delay spread. $(T_s >> \sigma_\tau)$. It means bandwidth of symbol is narrow
- the fading distribution is statistical characteristic of varation of received signal over time.
- Two most common fading distribution are :-

i) Rayleigh
ii) Ricean

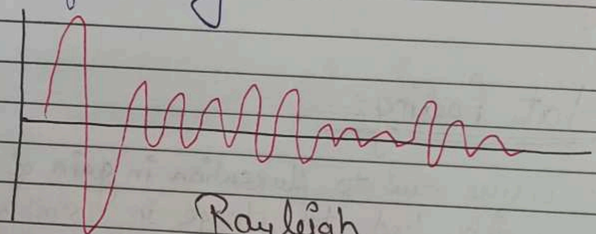
If all multipath component have aproximately same amplitude then envolop of received signal is Rayleigh.



Date:
Page:

their is no dominant signal (LOS).
It occure when mobile station is far away from base station.

If LOS is exist then distribution will be Ricean. Ricean convert to Rayleigh when dominant Component fad away.



Rayleigh

$$\text{Pdf of Rayleigh} = \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{\sigma^2}\right) \quad 0 < r < \infty$$
$$= 0 \quad r < \sigma$$

σ is rms value of received signal before detection. the Probability that the ~~envol~~ envelop of received signal does not exceed a specific value of r is given by CDF

$$\text{CDF} \quad P(R) = P_r(r \le R)$$
$$= \int_0^R P_r \, dr$$
$$= \int_0^R \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{\sigma}\right) dr$$
$$= \cancel{1 - e^{R^2}}$$
$$\boxed{\text{CDF} = 1 - e^{R^2/2\sigma^2}}$$

Date:
Page:

$\Rightarrow$ average value $E(r) = \int_0^{\infty} r\,p(r)\,dr$

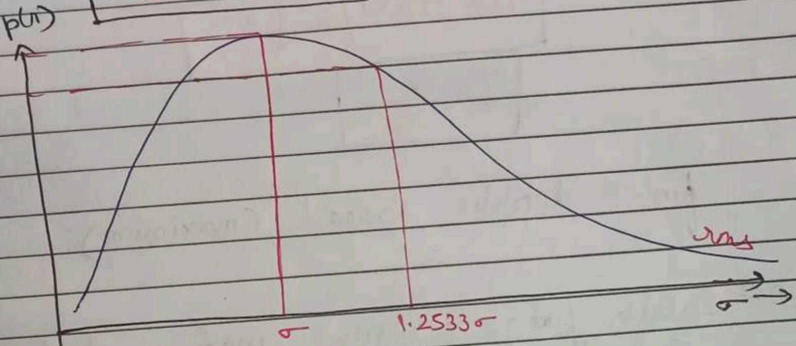
$= \sigma\sqrt{\pi/2}$

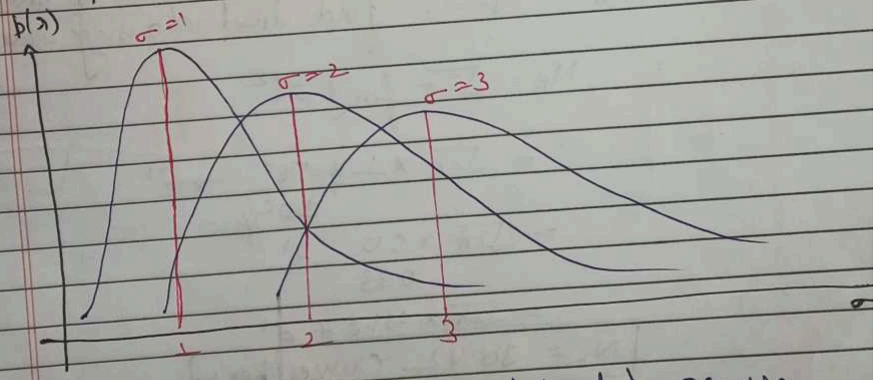
$$\boxed{E(r) = 1.2533\sigma}$$

$\Rightarrow$ rms value $= \sqrt{\int_0^{\infty} r^2 p(r)\,dr}$

$= \sqrt{2}\,\sigma$

$$\boxed{rms = 1.414\sigma}$$





In pratically when $\sigma \geq 3$ deep fad occure

Date:
Page:

• Ricean :-

$$\boxed{P(r) = \frac{r}{\sigma^2} \exp\left[-\frac{(r^2+A^2)}{2\sigma^2}\right] I_0 \left[\frac{Ar}{\sigma^2}\right]}$$

where I_0 = vessel function of first ~~zero~~ kind of zero order

A = peak value of dominant signal

→ No of level crossing per second.

$$\boxed{N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2}}$$

$$\boxed{\rho = \frac{R}{r_{rms}}}$$

f_m = doppler spread (maximum)

Question Carrier freq 900MHz maxi speed of mobile 50 Km/h $\rho = 1$ find level crossing rate

$$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2}$$

$$= \sqrt{2\pi} \times 1 \times \frac{50}{\cancel{900}\, c/f} \times e^{-1}$$

$$= \frac{\sqrt{2\pi} \times 50}{0.33} e^{-1}$$

$$= \sqrt{2\pi} \times 41.67\, e^{-1}$$

$$\boxed{N_R = 38.42 \text{ crosses/per sec}}$$

average fad duration :-

$$\boxed{\bar{\tau} = N_R \, P_r(u \le R)}$$

$$\boxed{\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}}$$

$$\bar{\tau} = \frac{e^{1^2} - 1}{1 \times 41.67 \times \sqrt{2\pi}}$$

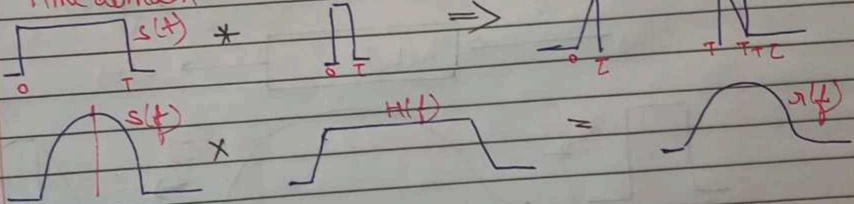
$$\boxed{\bar{\tau} = 16.5 \, \text{msec}}$$

* Flat Fading Channel Model :- $[T_s > \sigma_\tau]$

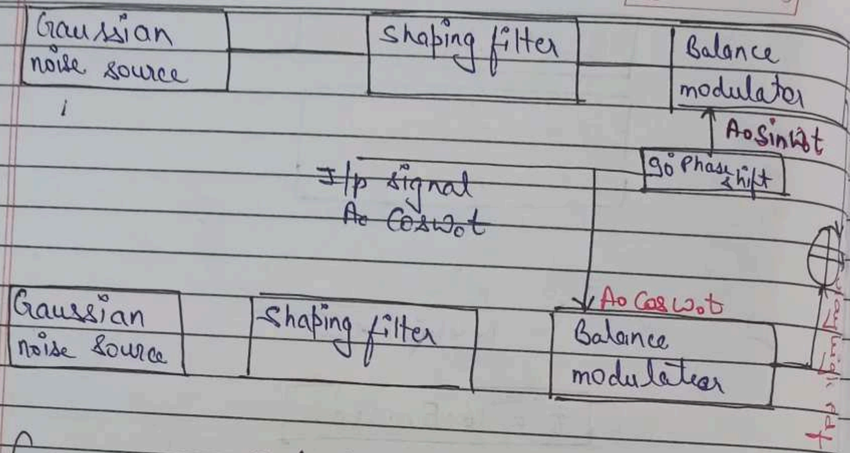
$$s(t) \rightarrow \boxed{h(t,\tau)} \rightarrow r(t)$$

$$r(t) = s(t) * h(t,\tau)$$

Time domain

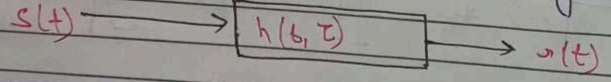


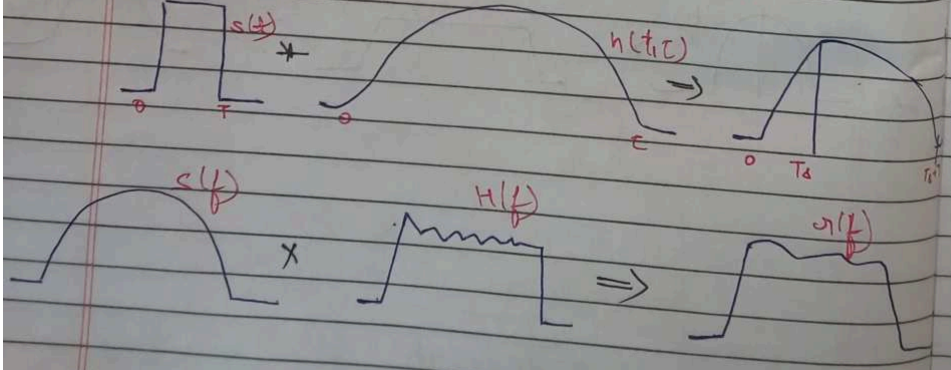
In GSM carrier is authogonal.
In CDMA noise is authogonal.
ISI → Intersymbol Interference



* Frequency Selective :- ($\sigma_\tau > T_s$)

for freq selective ($\sigma_\tau > T_s$) means at receiver copies of 2nd symbol will be arrive before the completion of copies 1st symbols. so ISI induces which causes distortion of receiv baseband signal.





Date:
Page:

* Thumb Rule :-

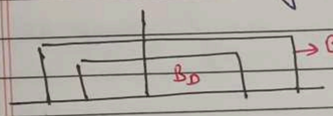
$T_s > 10\,\sigma_\tau \Rightarrow$ Flat fading
$T_s < 10\,\sigma_\tau \Rightarrow$ freq selective

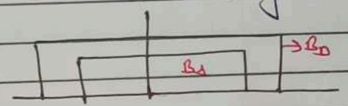
* Type of Small scale fading based on Doppler spread :-

slow | fast

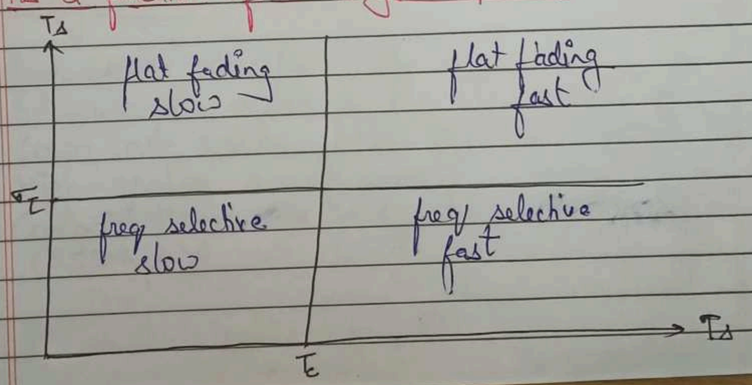
slow	fast
i) $T_c >> T_s$	$T_c << T_s$
ii) low doppler spread	High doppler spread
channel variation is smaller then baseband signal	channel variation is faster then baseband signal.

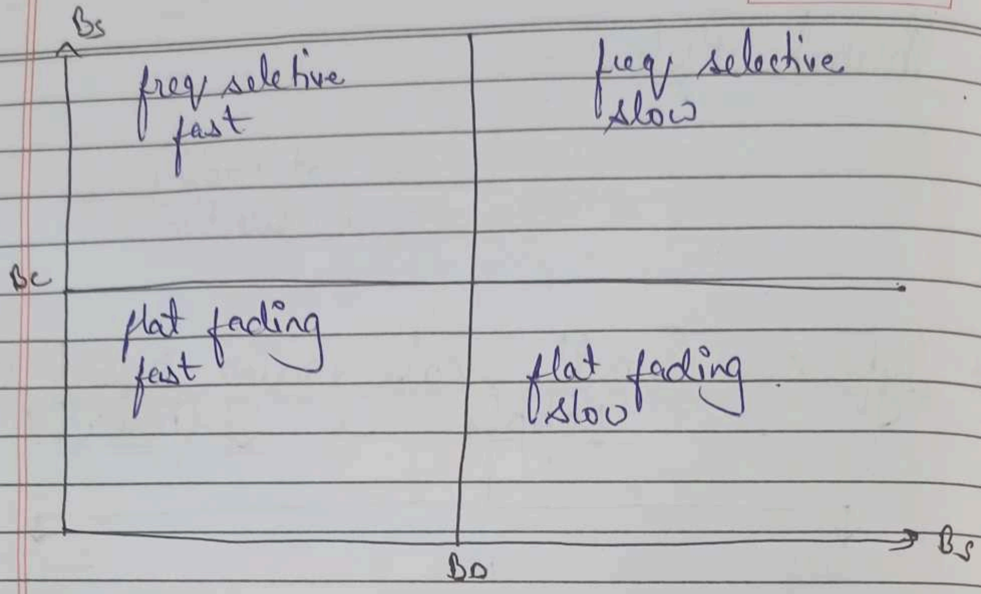
iii)





As a function of T_s (symbol period)





As a function of BD (signal bandwidth)