ME361 – Manufacturing Science Technology

Rolling

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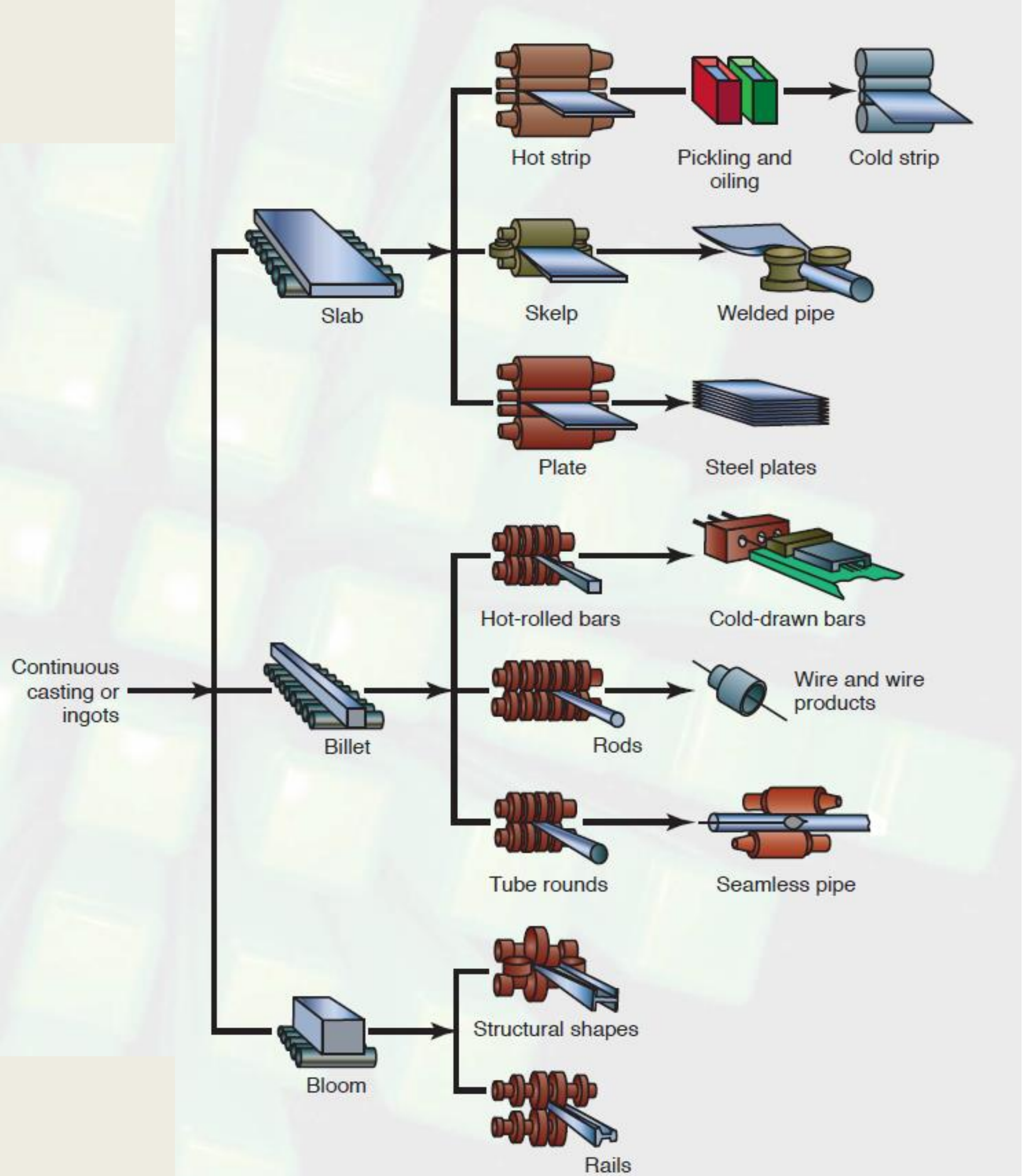
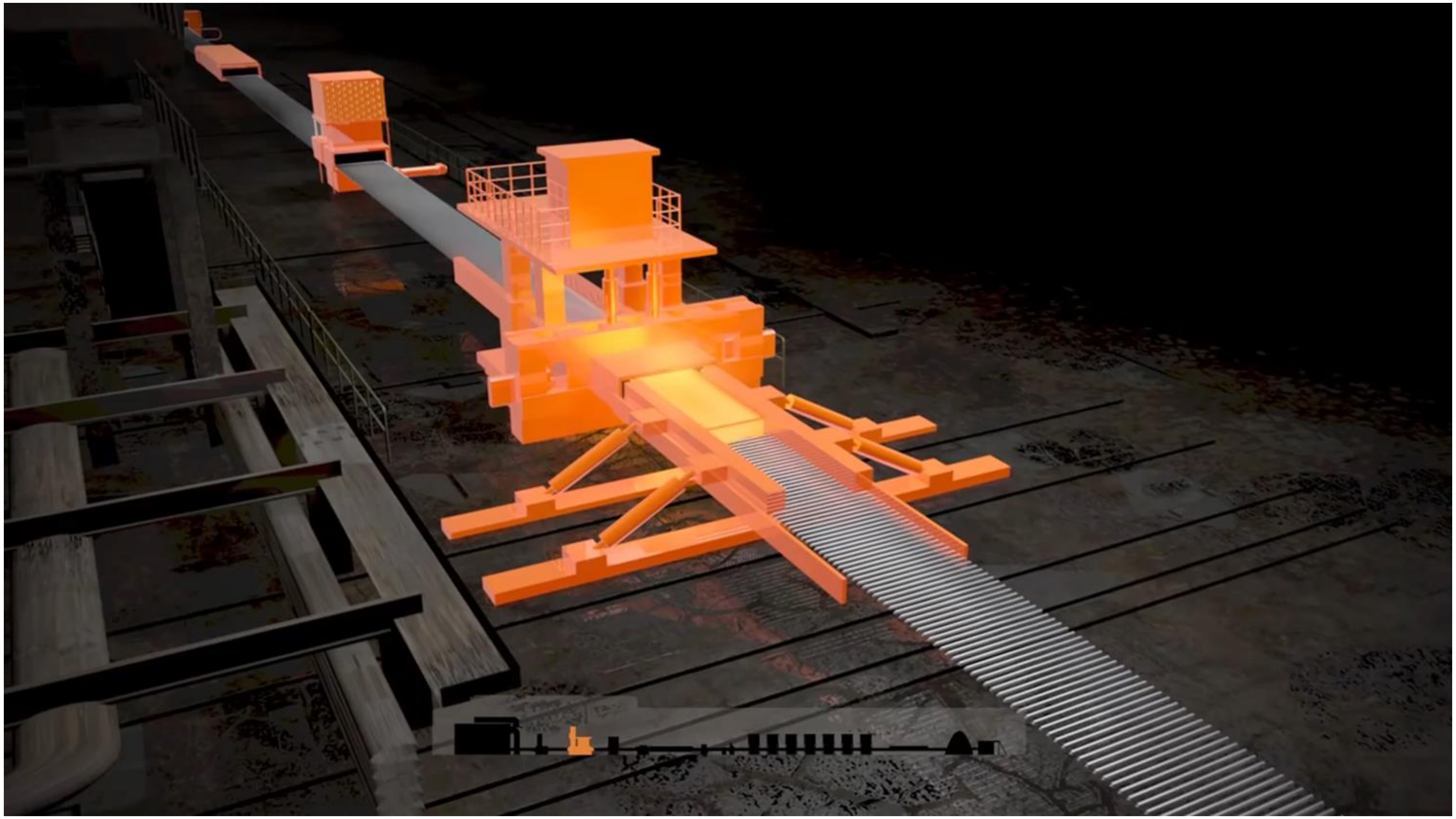
Rolling

- Accounts for ~90% of all metals produced by metal working processes
- Used to primarily make plates and sheets
  - Plates: thickness > 6 mm
  - Sheets: thickness < 6 mm



Source: Kalpakjian and Schmid

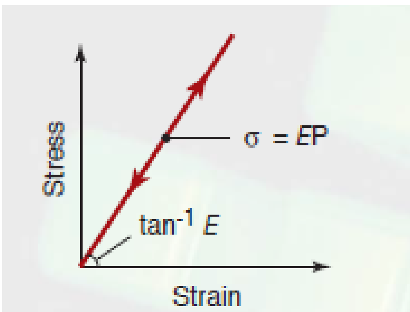
Rolling



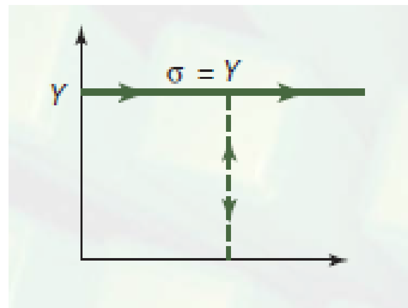
https://www.youtube.com/watch?v=LWM6b8P0r3E

Preliminaries. Stress-strain behavior.

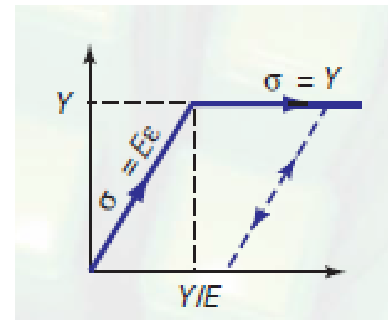
(a) Perfectly elastic



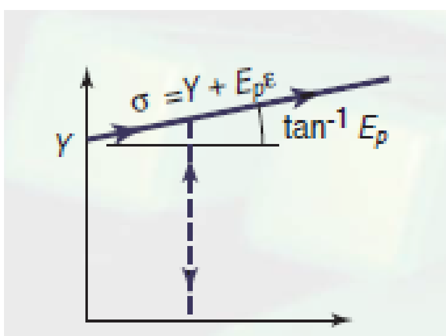
(b) Rigid, perfectly plastic



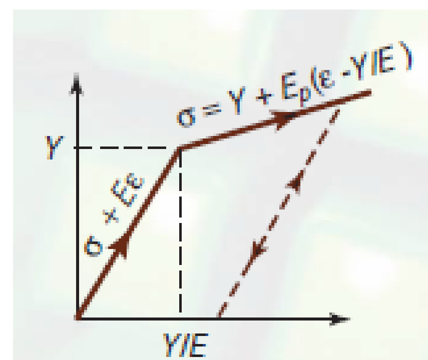
(c) Elastic, perfectly plastic



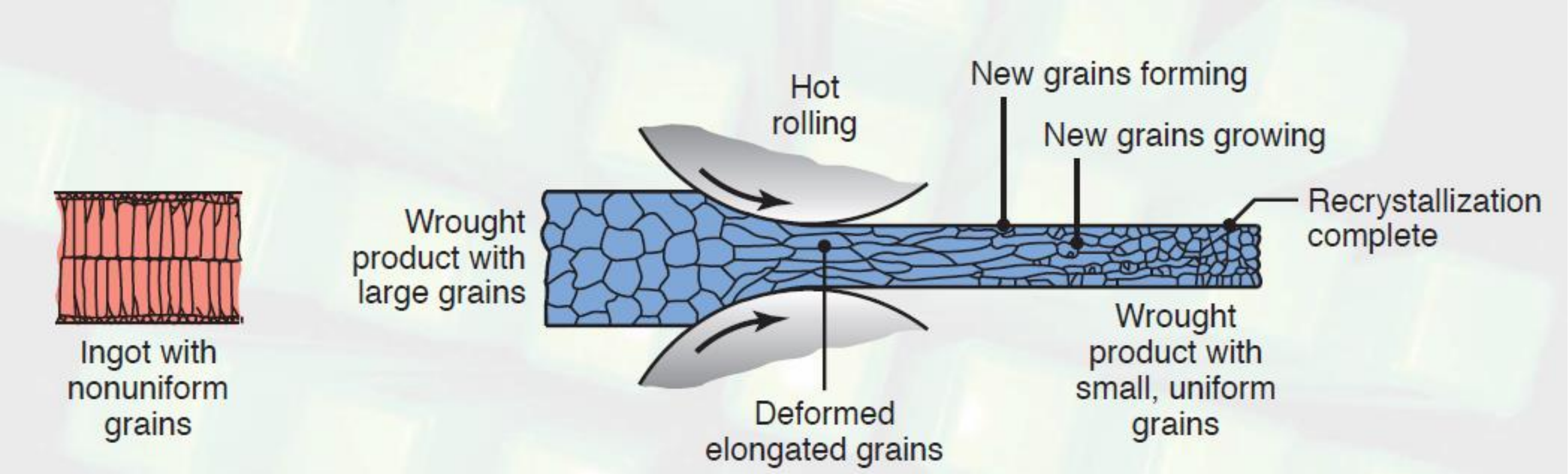
(d) Rigid, linearly strain hardening



(e) Elastic, linearly strain hardening

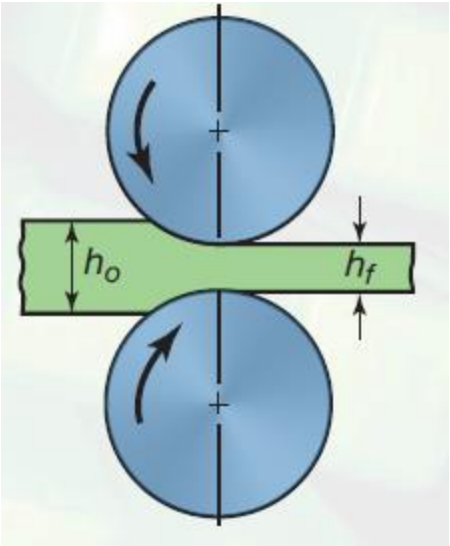


Grain structure in rolling



Source: Kalpakjian and Schmid

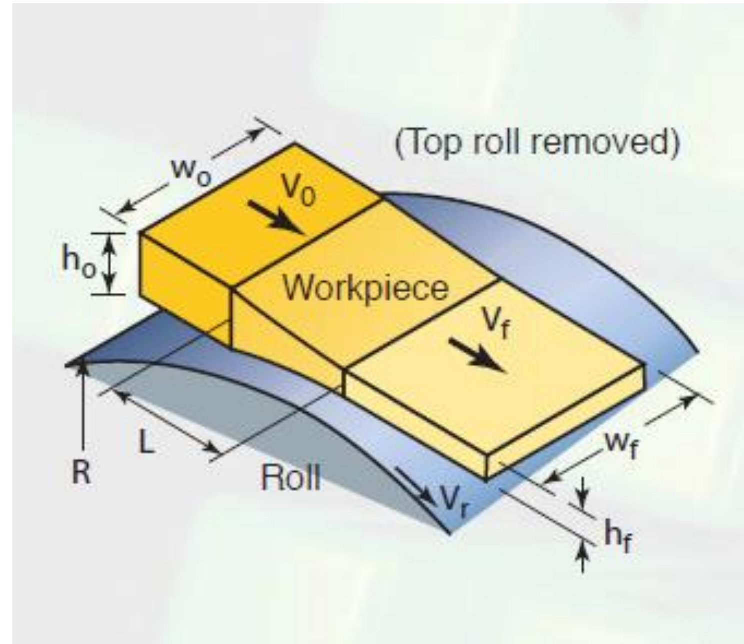
Rolling analysis: objectives and assumptions



- Objectives:
  - To determine roll separating forces
  - To determine torque and power required to drive the rolls
- Assumptions:
  - Rolls are straight and rigid cylinders
  - Width is larger than thickness and no significant widening takes place, i.e. problem is of plane strain type
  - The coefficient of friction is low and constant over the entire roll-workpiece interface

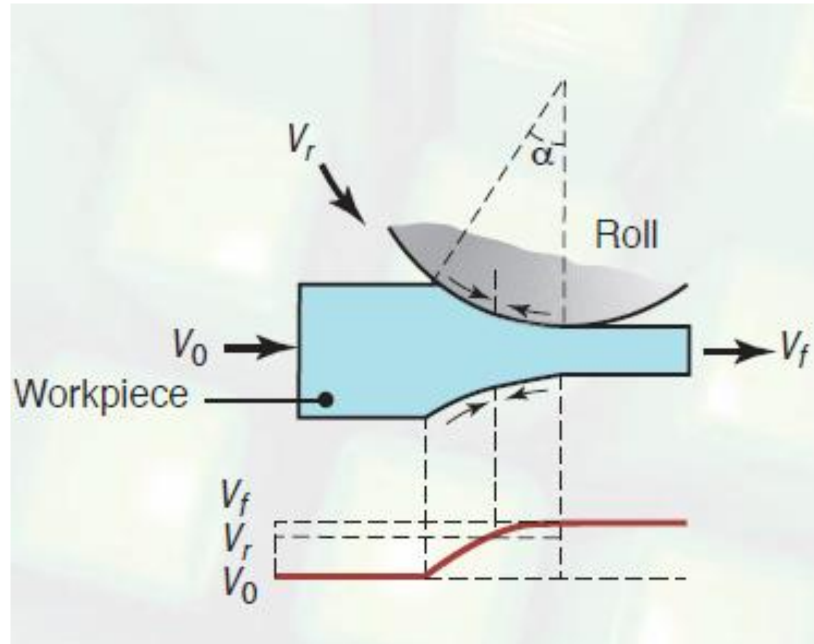
Mechanics of flat rolling

- Strip of thickness h_0 enters the roll gap and is reduced to thickness of h_f
- Surface speed of roll is V_r
- Entry speed of strip is V_o and exit speed is V_f
- Since volume rate is constant, velocity of strip must increase as it moves through the roll gap (similar to fluid flow through a converging channel).



Mechanics of flat rolling

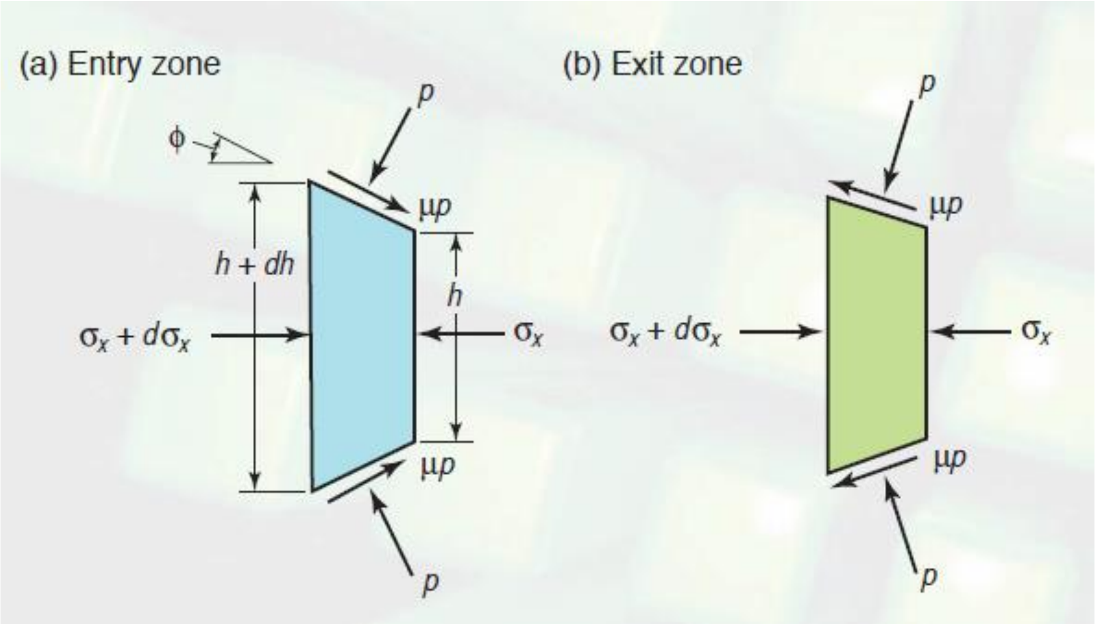
- In general, $V_f > V_r > V_o$
- However, because V_r is constant along the roll gap, sliding occurs between the roll and the strip
- At a point along the roll gap, $V_r = V_o = V_f$; this point is known as the neutral point, or no-slip point
- To the left of the neutral point, roll moves faster than the workpiece, and to the right the workpiece moves faster than the roll
- Frictional forces oppose each other at the neutral point, and these forces are greater on the left of the neutral point, than on the right, which pulls the strip into the gap



$$Forward\ slip = \frac{V_f - V_r}{V_r}$$

Roll pressure distribution

- Slab method of analysis for plane strain



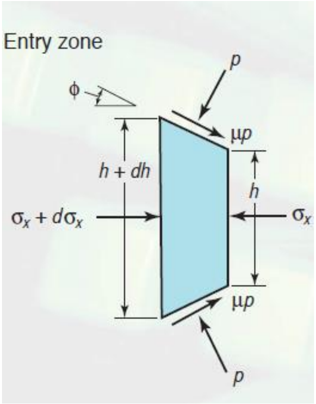
- Balance horizontal forces in this element:

$$\boxed{(\sigma_x + d\sigma_x)(h + dh) - 2pRd\phi \, sin \, \phi - \sigma_x h \pm 2\mu pRd\phi \, cos \, \phi = 0}$$

Roll pressure distribution

Balance horizontal forces in this element:

$$\boxed{(\sigma_x + d\sigma_x)(h + dh) - 2pRd\phi \sin\phi - \sigma_x h \pm 2\mu pRd\phi \cos\phi = 0}$$



Simplifying and neglecting 2nd order terms:

$$\boxed{\frac{d(\sigma_x h)}{d\phi} = 2pR(\sin\phi \mp \mu\cos\phi)}$$

Since α is usually only a few degrees, take: $\sin\phi = \phi$ and $\cos\phi = 1$

$$\boxed{\frac{d(\sigma_x h)}{d\phi} = 2pR(\phi \mp \mu)} \qquad (1)$$

Digression for plane strain

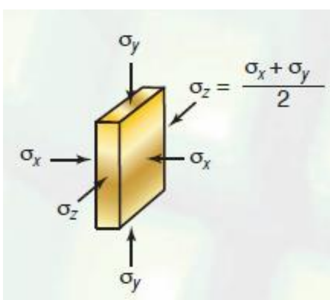
Recalling Hooke's law equations:

$$\boxed{\begin{aligned} \varepsilon_x &= \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \\ \varepsilon_y &= \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + \sigma_z)\right] \\ \varepsilon_z &= \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right] \end{aligned}}$$

For plane strain, $\varepsilon_z = 0$;

$\sigma_z = \frac{\sigma_x + \sigma_z}{2}$;

since $\nu = \frac{1}{2}$ in plastic deformation



Use a yield criterion (Distortion energy/ von-Mises):

$$\boxed{(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 = 2Y^2}$$

$\sigma_x, \sigma_y, \sigma_z$ – principle stresses;

Y – the uniaxial yield stress

$$\boxed{\sigma_y - \sigma_x = \frac{2}{\sqrt{3}} Y = Y'}^{*} \quad \text{or} \quad \boxed{d\sigma_y = d\sigma_x} \qquad (2)$$

*σ_x and σ_y are assumed to be principle stresses, even though there is a shear component of $\mu\sigma_y$. However, since μ is usually small, this is reasonable

Roll pressure distribution

From the plane strain criterion:

$$\boxed{\sigma_y - \sigma_x = \frac{2}{\sqrt{3}} Y = Y'} \xrightarrow[\text{rolling}]{\text{Specifically for}} \boxed{p - \sigma_x = Y'_f} \qquad (2)$$

Y'_f: flow stress for strain-hardened case

Recalling force balance equation:

$$\boxed{\frac{d(\sigma_x h)}{d\phi} = 2pR(\phi \mp \mu)} \quad (1) \xrightarrow[\text{into (1)}]{\text{Substituting (2)}} \boxed{\frac{d\left[\left(p - Y'_f\right)h\right]}{d\phi} = 2pR(\phi \mp \mu)}$$

$$\boxed{\frac{d}{d\phi}\left[Y'_f\left(\frac{p}{Y'_f} - 1\right)h\right] = 2pR(\phi \mp \mu)} \qquad (3)$$

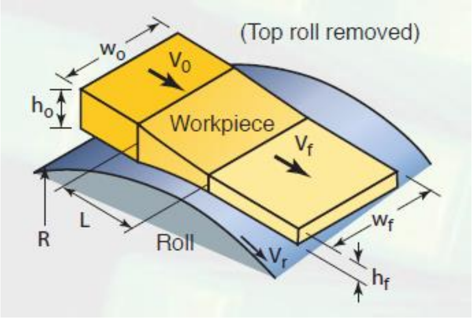
Differentiating this…

Roll pressure distribution

$$Y'_f h \frac{d}{d\phi}\left(\frac{p}{Y'_f}\right) + \left(\frac{p}{Y'_f} - 1\right)\frac{d}{d\phi}(Y'_f h) = 2pR(\phi \mp \mu) \qquad (4)$$

Second term is usually very small, because as h decreases, Y'_f increases, thus making the product nearly a constant, and its derivate thus becomes zero, thus (4) becomes:

$$\frac{\frac{d}{d\phi}\left(\frac{p}{Y'_f}\right)}{\frac{p}{Y'_f}} = \frac{2R}{h}(\phi \mp \mu) \qquad (5)$$



If h_f is the final thickness:

$$h = h_f + 2R(1 - \cos\phi) \xrightarrow{\approx} h = h_f + R\phi^2 \qquad (6)$$

Now, substitute (6) into (5)

Roll pressure distribution

Recalling again:

$$\frac{\frac{d}{d\phi}\left(\frac{p}{Y'_f}\right)}{\frac{p}{Y'_f}} = \frac{2R}{h}(\phi \mp \mu) \qquad (5)$$

$$h = h_f + R\phi^2 \qquad (6)$$

Now, substitute (6) into (5), and integrating:

$$ln\frac{p}{Y'_f} = ln\frac{h}{R} \mp 2\mu\sqrt{\frac{R}{h_f}}tan^{-1}\sqrt{\frac{R}{h_f}}\phi + ln\,C$$

$$p = CY'_f\frac{h}{R}e^{\mp\mu H}$$

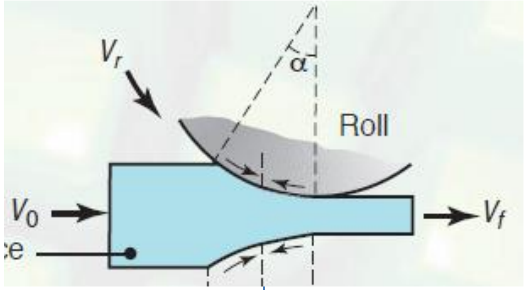
wherein

$$H = 2\sqrt{\frac{R}{h_f}}tan^{-1}\sqrt{\frac{R}{h_f}}\phi \qquad (7)$$

- $p = f(h, \phi)$
- p increases with increasing material strength, increasing μ, and increasing R/h_f ratio

Roll pressure distribution

Roll pressure: $p = CY'_f \frac{h}{R} e^{\mp \mu H}$ wherein $H = 2\sqrt{\frac{R}{h_f}} tan^{-1} \sqrt{\frac{R}{h_f}} \phi$ (7)



At entry:

$$\phi = \alpha; p = Y'_f; H = H_o$$

$$C = \frac{R}{h_0} e^{\mu H_0}$$

$$p = Y'_f \frac{h}{h_0} e^{\mu(H_o - H)} \quad (8)$$

At exit:

$$\phi = 0; p = Y'_f; H = H_f = 0$$

$$C = \frac{R}{h_f}$$

$$p = Y'_f \frac{h}{h_f} e^{\mu H} \quad (9)$$

Determination of the neutral point

- All velocities at the neutral point are the same, i.e. $V_r = V_o = V_f$
- Pressures at the neutral point are also the same, hence the neutral point can be obtained by simply equating the pressure at the entry to that of the exit:

Pressure at entry $\boxed{p = Y'_f \frac{h}{h_o} e^{\mu(H_o - H)}} = \boxed{p = Y'_f \frac{h}{h_f} e^{\mu H}}$ Pressure at exit

$$\boxed{\frac{h_0}{h_f} = \frac{e^{\mu H_o}}{e^{2\mu H_n}} = e^{\mu(H_o - 2H_n)}} \longrightarrow \boxed{H_n = \frac{1}{2}\left(H_0 - \frac{1}{\mu} ln \frac{h_0}{h_f}\right)} \quad (10)$$

Substituting (10) into (7)

$$\boxed{H = 2\sqrt{\frac{R}{h_f}} tan^{-1} \sqrt{\frac{R}{h_f}} \phi} \quad (7) \longrightarrow \boxed{\phi_n = \sqrt{\frac{h_f}{R}} tan\left(\sqrt{\frac{h_f}{R}} \frac{H_n}{2}\right)}$$

Roll pressure distribution: influence of friction

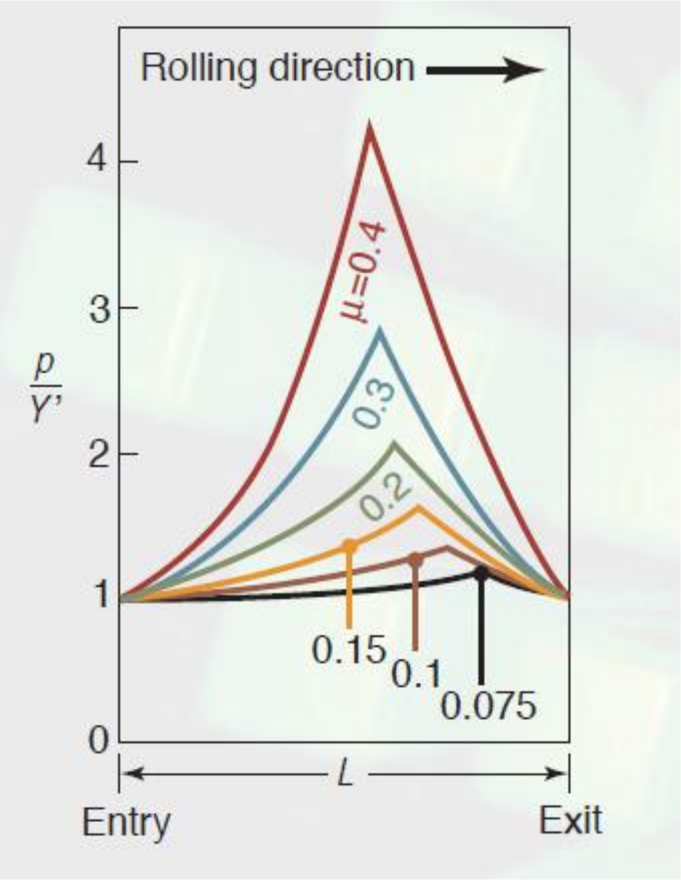
Recalling roll pressure:

$$\boxed{p = CY'_f \frac{h}{R} e^{\mp \mu H}}$$

$$\boxed{H = 2\sqrt{\frac{R}{h_f}} tan^{-1} \sqrt{\frac{R}{h_f}} \phi}$$

- $p = f(h, \phi)$
- p increases with increasing material strength, increasing μ, and increasing R/h_f ratio
- Neutral point shifts to the exit as friction decreases, and without friction, the rolls slip, and the neutral point shifts completely to the exit

Dimensionless pressure distribution



Source: Kalpakjian and Schmid

Roll pressure distribution: influence of reduction

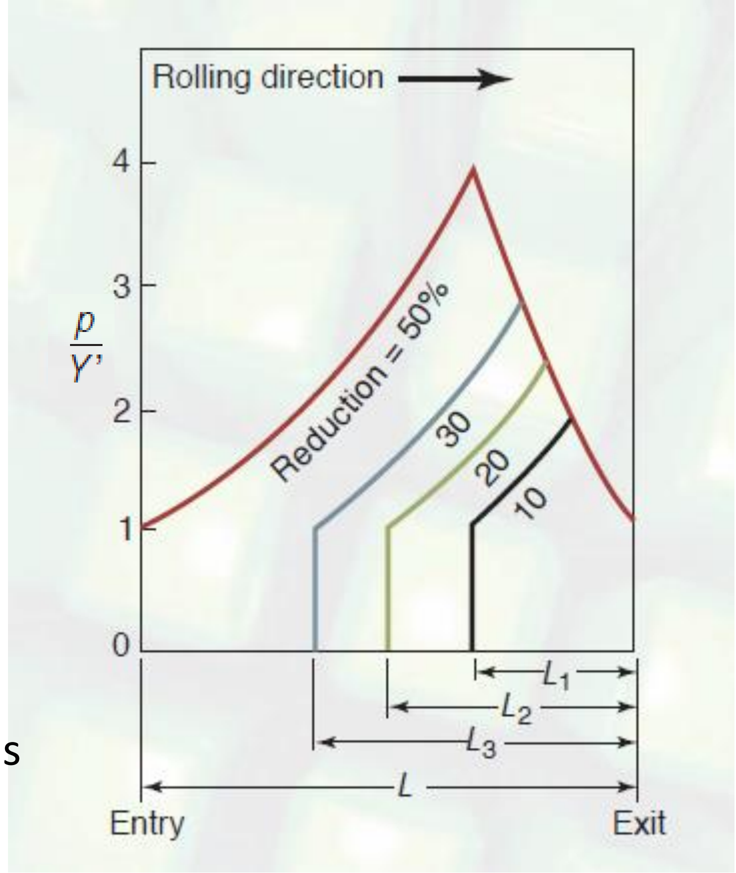
Recalling roll pressure:

$$p = CY'_f \frac{h}{R} e^{\mp \mu H}$$

$$H = 2\sqrt{\frac{R}{h_f}} tan^{-1} \sqrt{\frac{R}{h_f}} \phi$$

- $p = f(h, \phi)$
- p increases with increasing material strength, increasing μ, and increasing R/h_f ratio
- As thickness reduction increases, the length of contact in the roll gap increases, which increases the peak pressure

Dimensionless pressure distribution



Source: Kalpakjian and Schmid

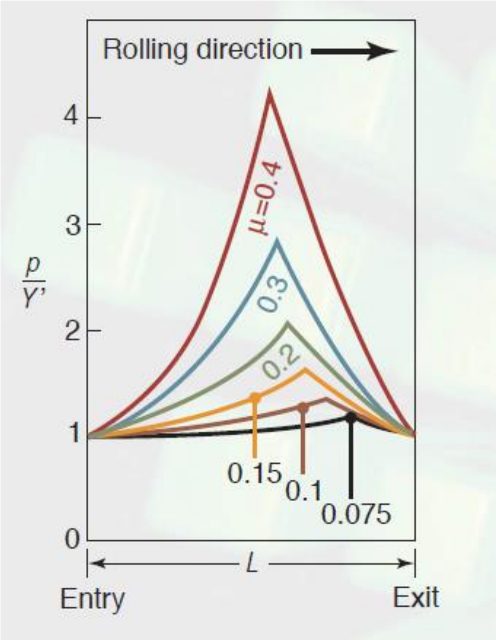
Rolling forces

- Given the pr. vs. contact-length curve, forces can be calculated from the area under the curve multiplied by the strip width, w
- Alternatively, roll-separating force is:

$$\boxed{F = \int_0^{\phi_n} w\,pRd\phi + \int_{\phi_n}^{\alpha} w\,pRd\phi}$$

- Simpler method yet:

$$\boxed{F = Lwp_{av}}$$



Wherein the arc of contact: $\boxed{L = \sqrt{R\Delta h};}$ $\boxed{\Delta h = h_o - h_f}$

And the average pressure: $\boxed{p_{av} = \overline{Y'}\left(1 + \frac{\mu L}{2h_{av}}\right)}$ wherein $\overline{Y'}$ is the average flow stress in plane strain in the roll gap

Roll torque and power

- Roll force: $$\boxed{F = \int_0^{\phi_n} w\, pR d\phi + \int_{\phi_n}^{\alpha} w\, pR d\phi}$$

- Roll torque, T, for each roll can be calculated as:

$$\boxed{T = \int_{\phi_n}^{\alpha} w\, \mu p R^2 d\phi - \int_0^{\phi_n} w\, \mu p R^2 d\phi}$$

Minus sign indicates a change in direction of friction force at the neutral point.
If frictional forces are equal, the torque will be zero

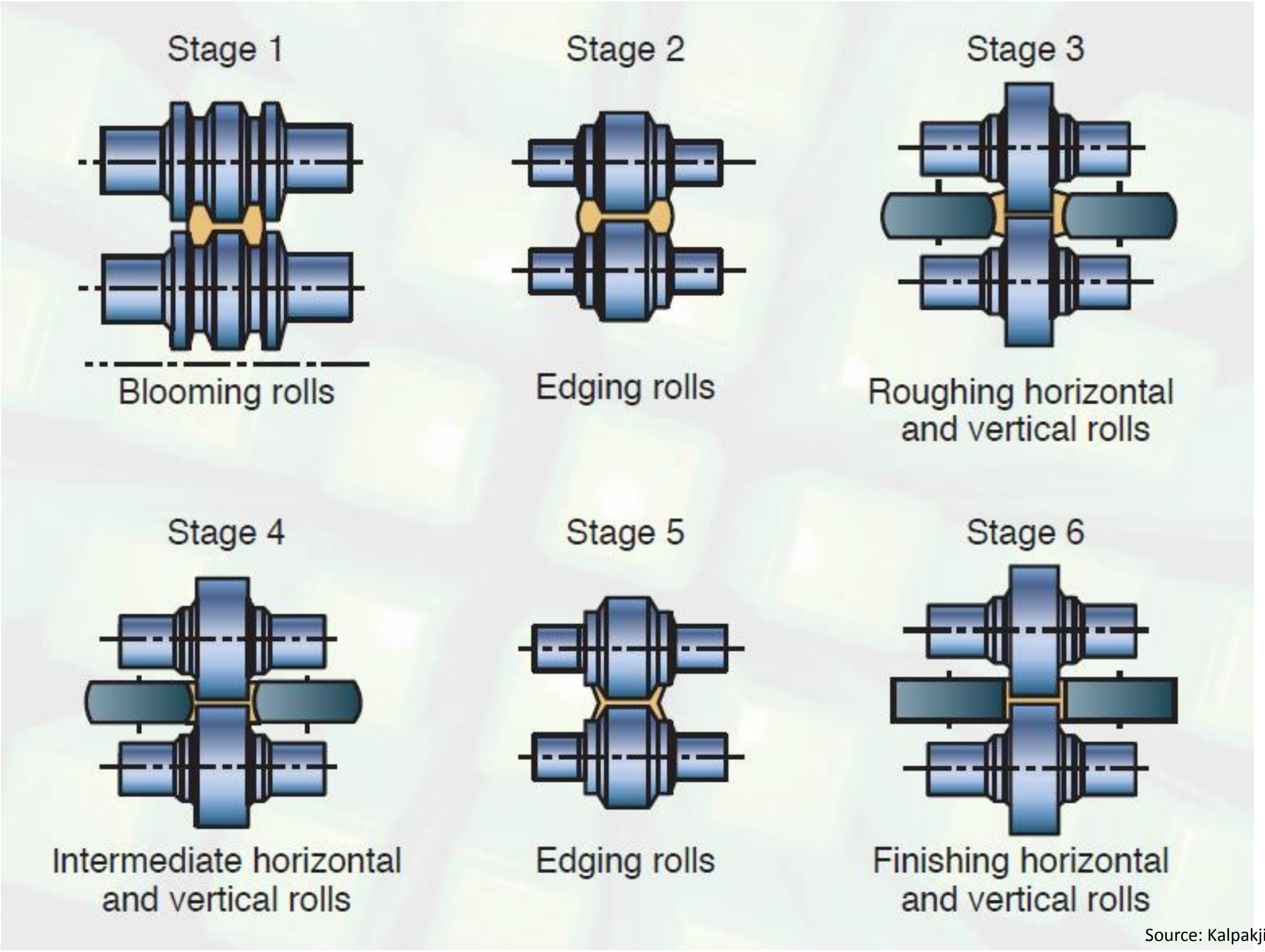
- Roll torque, T, can also be estimated by assuming roll force acts in the middle of the arc of contact, i.e. a moment arm of 0.5L, and that F is ⊥ to the plane of the strip:

$$\boxed{T = \frac{FL}{2}} \longrightarrow \boxed{Power = T\omega} \xrightarrow{\omega = 2\pi N} \boxed{Power = \frac{\pi F L N}{60000}\,\mathrm{kW}}$$

Other rolling operations:

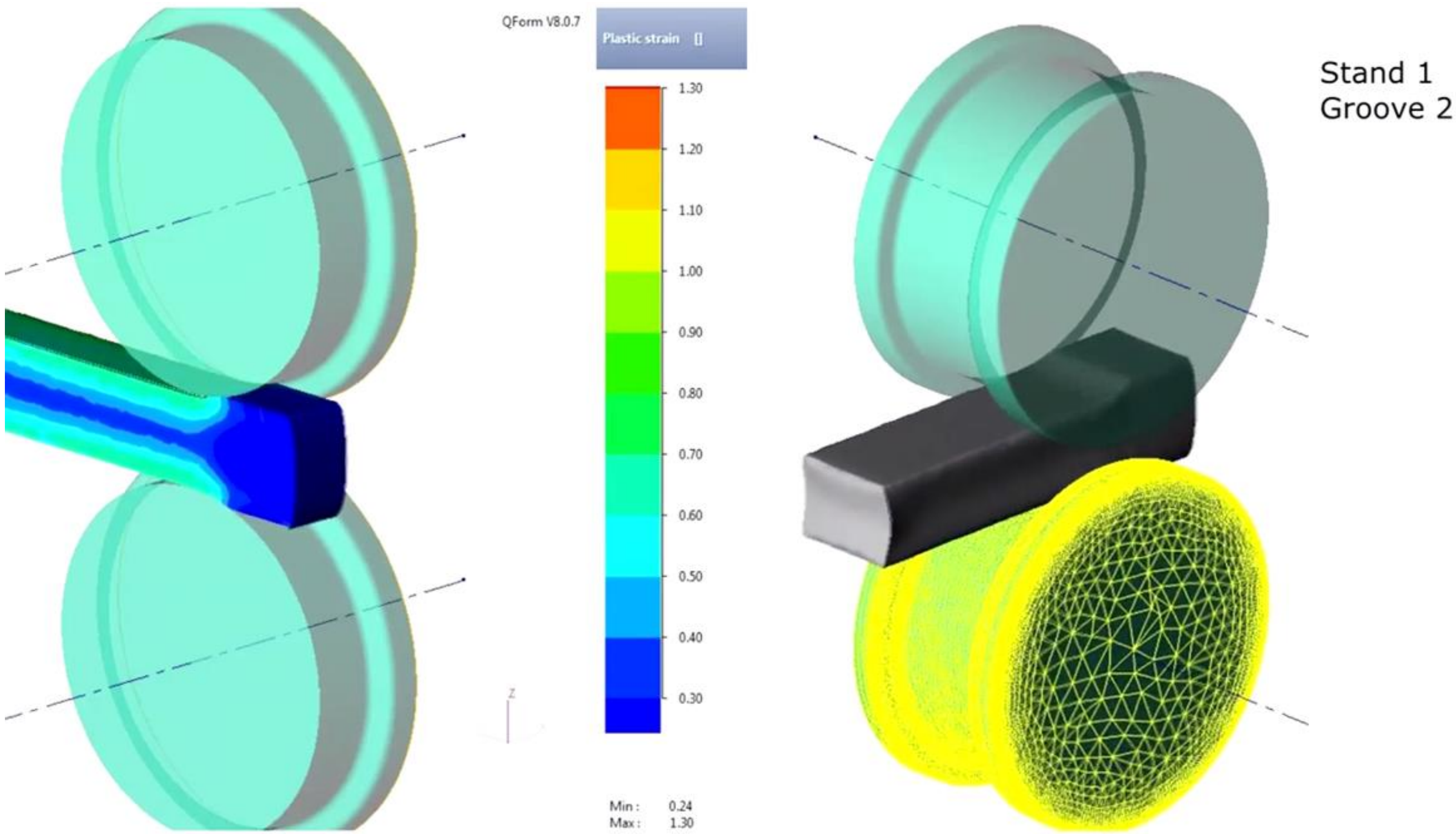
- Rolling of special sections and shapes
- Thread rolling
- Skew rolling
- …

Rolling of different sections and shapes



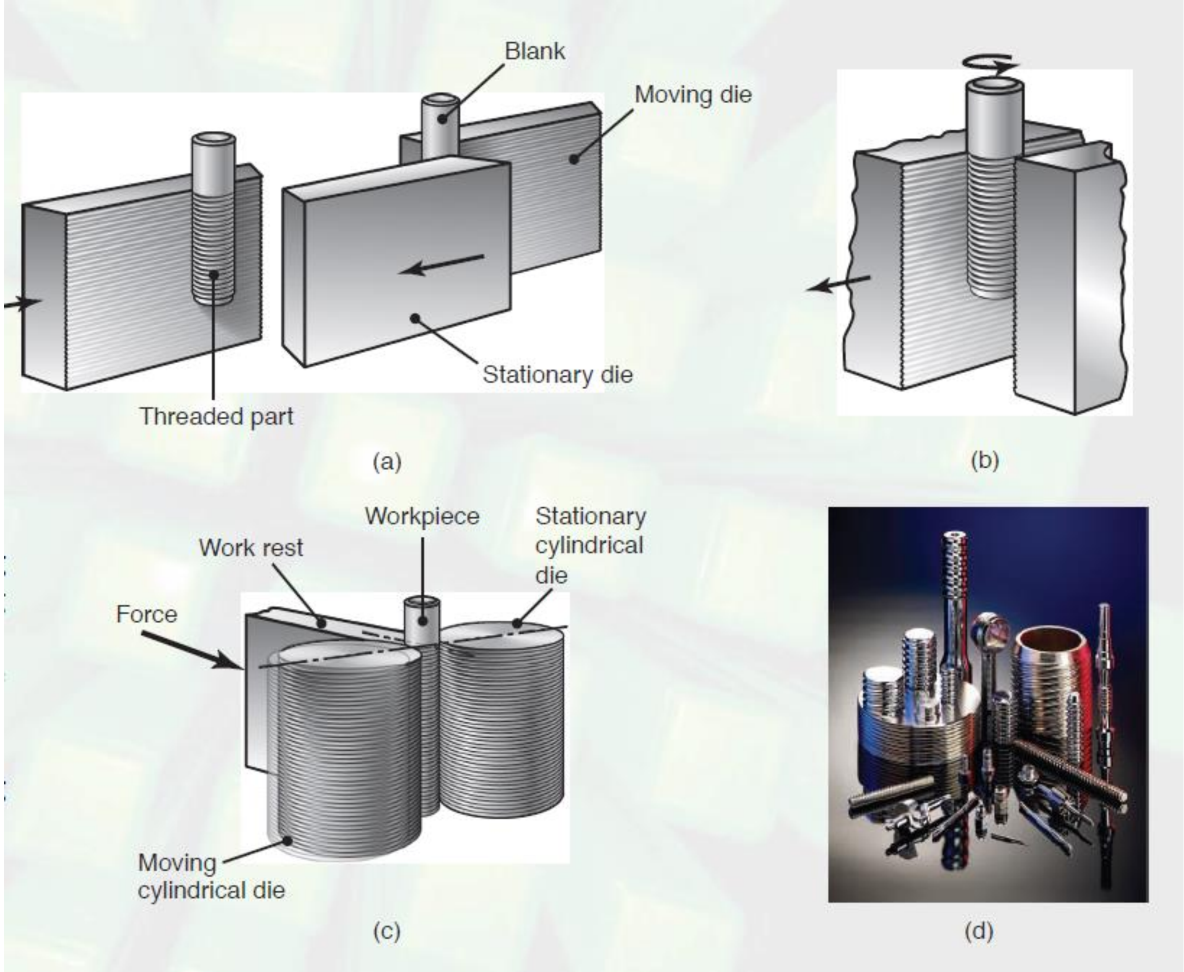
Source: Kalpakjian and Schmid

Rolling of sections: Numerical analysis



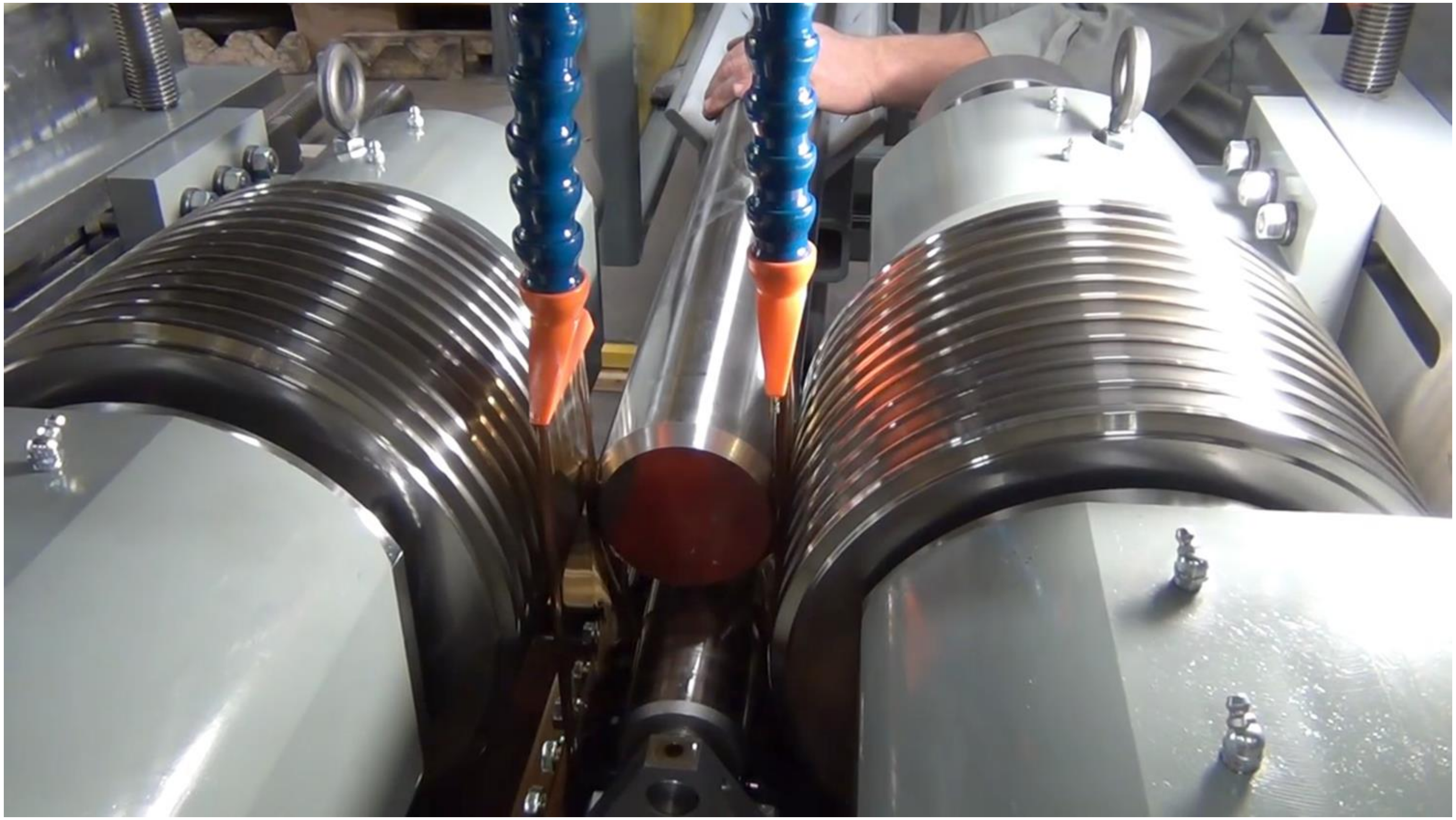
https://www.youtube.com/watch?v=aJnJjUSgmwI&t=13s

Thread rolling



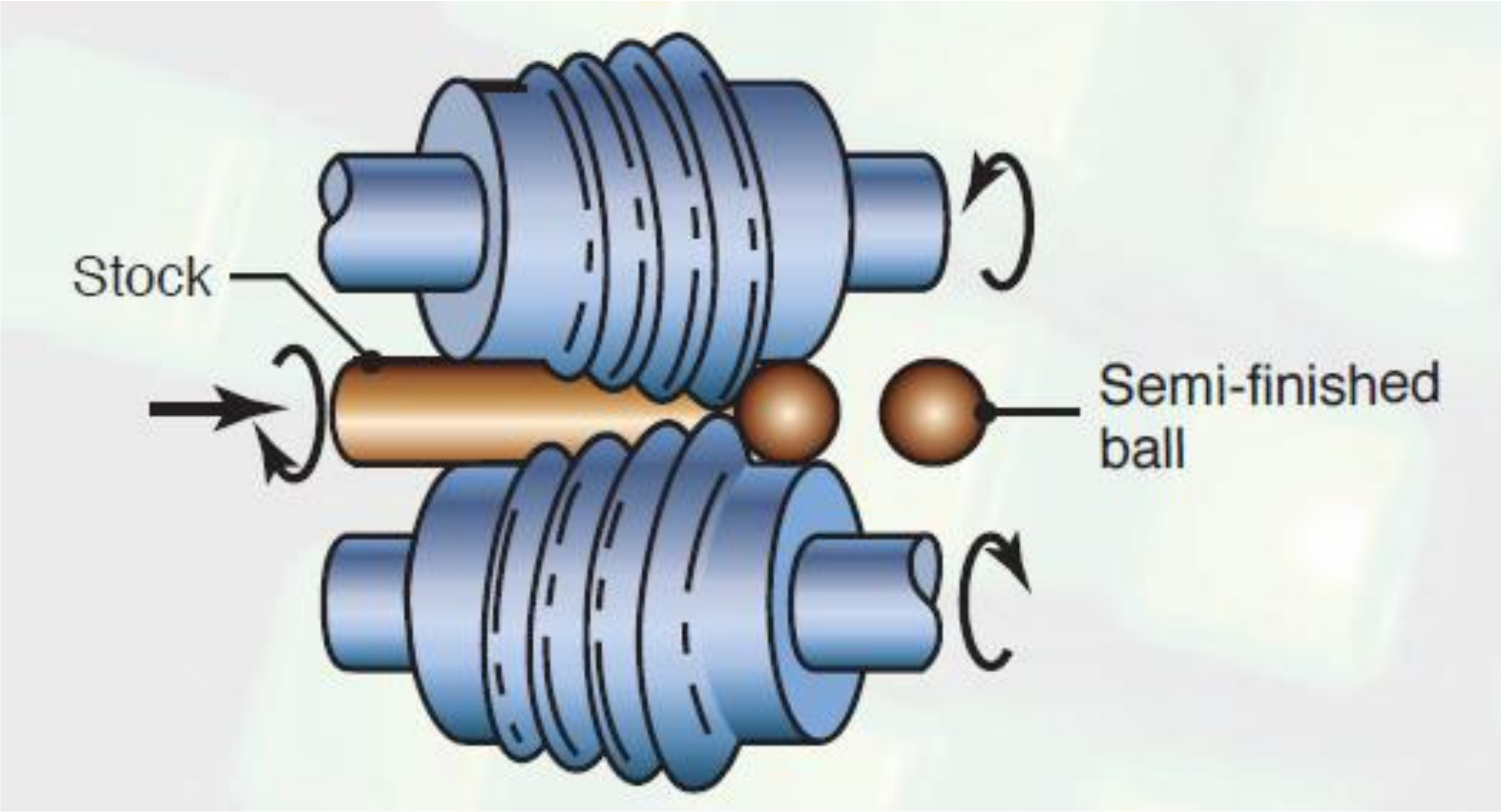
Source: Kalpakjian and Schmid

Thread rolling



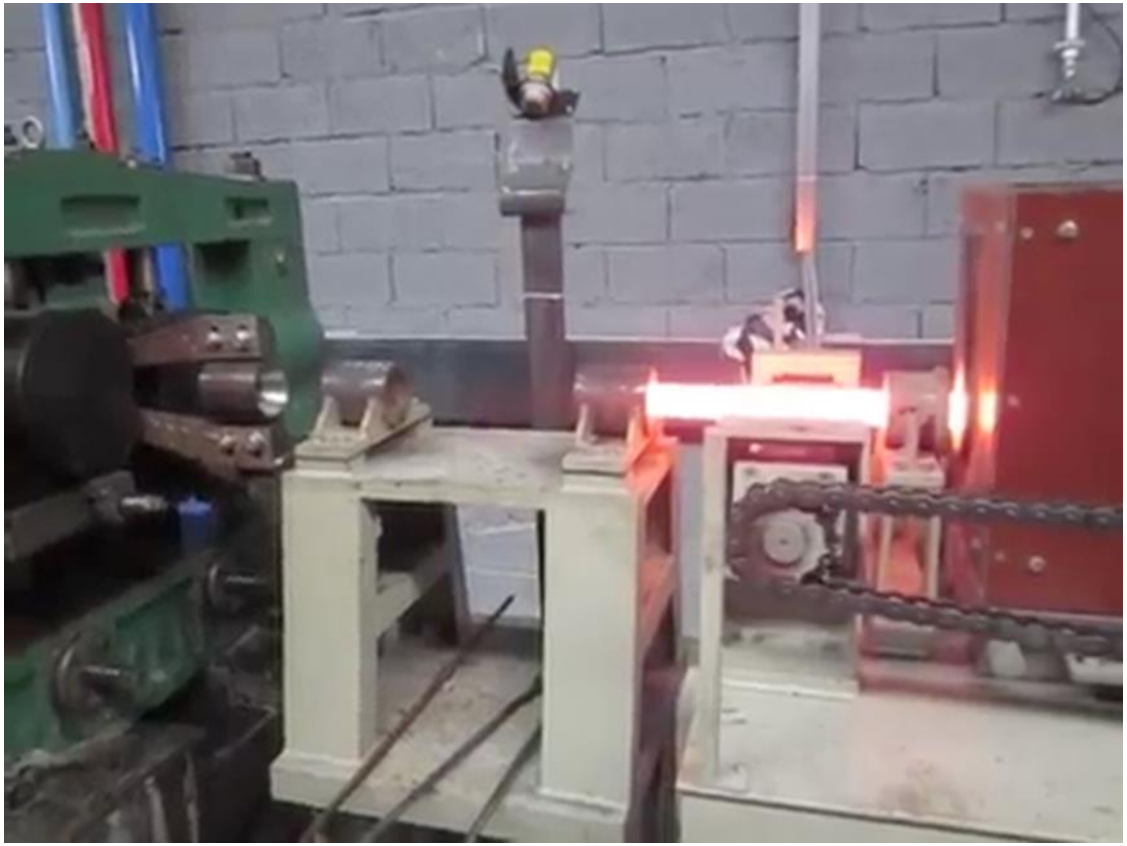
https://www.youtube.com/watch?v=bclnb_Cp4sE

Skew rolling



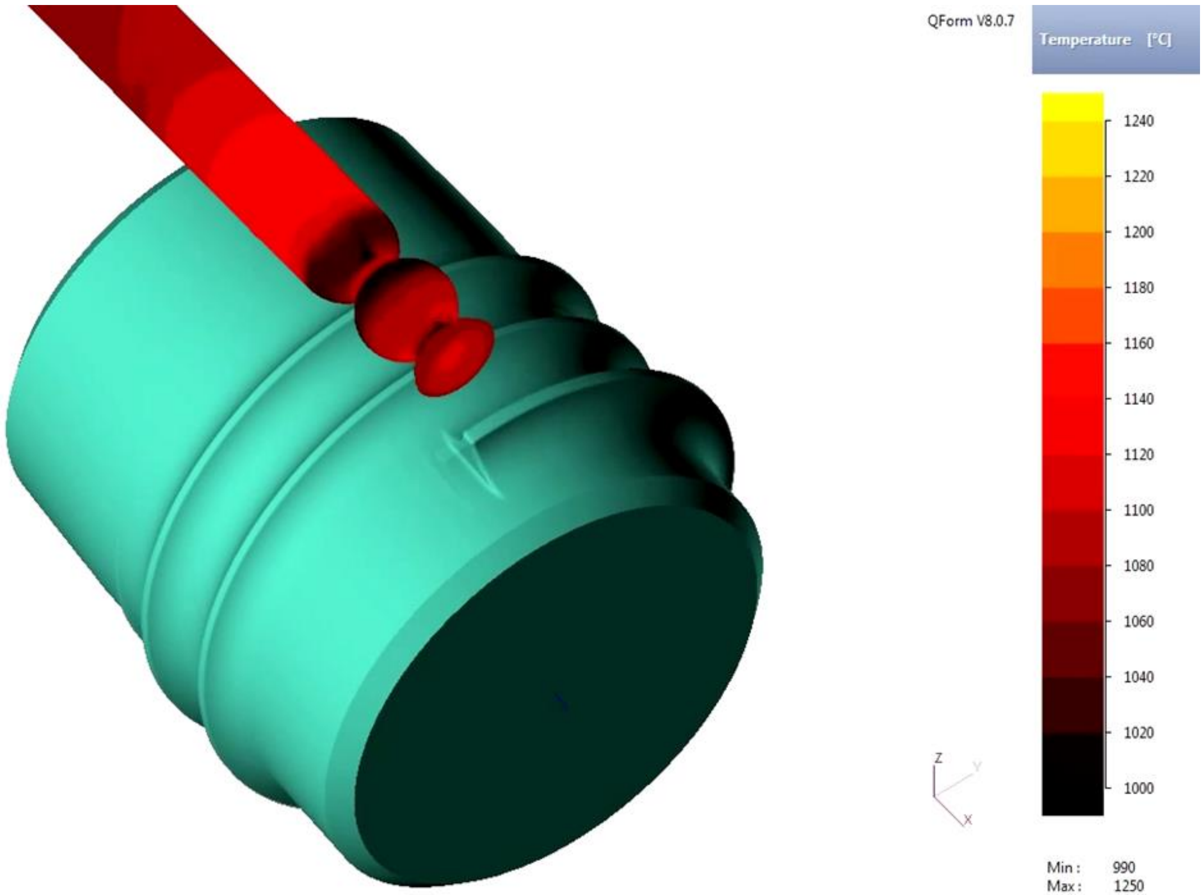
Source: Kalpakjian and Schmid

Skew rolling



https://www.youtube.com/watch?v=BazQnUg0k2Q

Skew rolling: numerical analysis



https://www.youtube.com/watch?v=67F_GrfZTzM