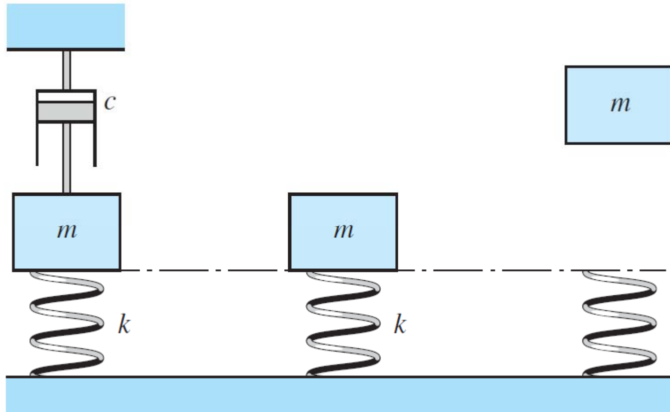


TABLE 7.1 Type of Loading

Load Type	Time Required to Apply Load (s)
Static loading	$t_{\text{applied loading}} > 3\tau$
"Gray area"	$\frac{1}{2}\tau < t_{\text{applied loading}} < 3\tau$
Dynamic loading	$t_{\text{applied loading}} < \frac{1}{2}\tau$

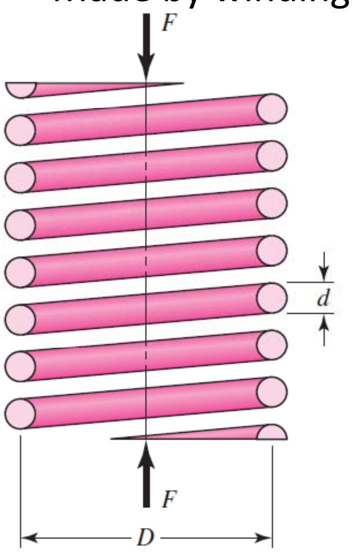
$$\tau = 2\pi\sqrt{\frac{m}{k}}$$

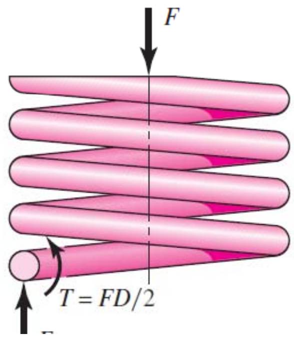
Mechanical Springs

- Controlled flexibility is required in many applications
  - Springs provide the solution
- Used for storing and releasing energy
- Soften impact loads (by absorbing energy)
- You will find springs in
  - Automotive suspensions
  - Stapler
  - Locks
  - IC engine valves
  - Pens
  - Door closers
  - Many other appliances

Helical springs

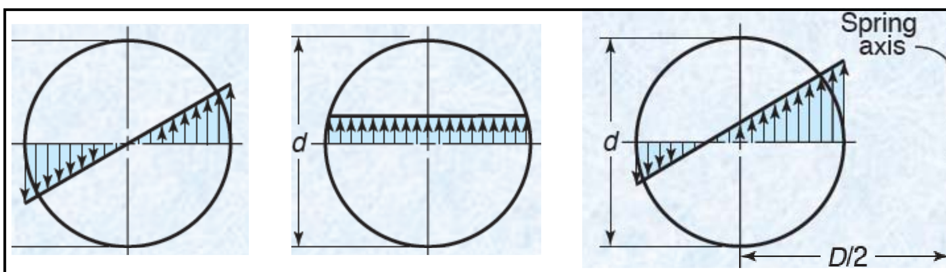
- Made by winding a spring wire over a mandrel





$$\tau_{\max} = \frac{Tr}{J} + \frac{F}{A}$$

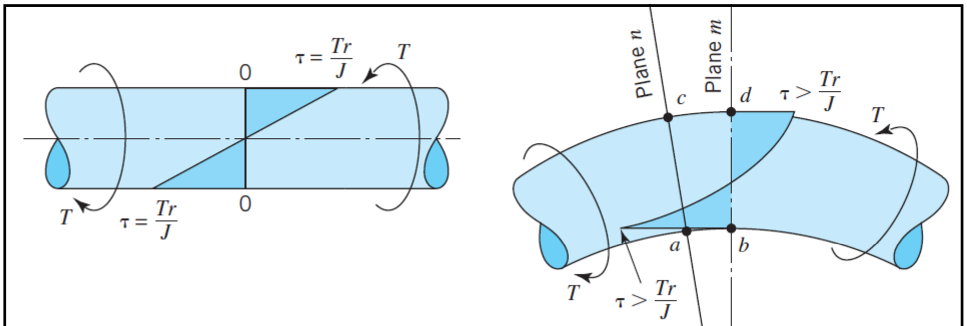
$C = D/d$; Spring index



$$\tau = \frac{Tr}{J} + \frac{F}{A} = \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$\tau = \frac{8FD}{\pi d^3}\left(1 + \frac{1}{2C}\right) = \frac{8FD}{\pi d^3} K_s$$

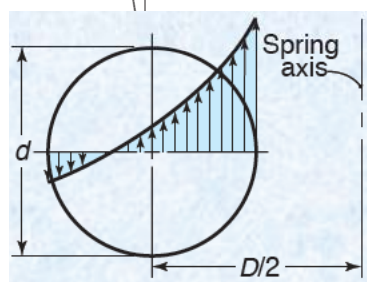
K_s = Shear stress correction factor



$$\tau = \frac{8FD}{\pi d^3} K_W, \; K_W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

$$\tau = \frac{8FD}{\pi d^3} K_B, \; K_B = \frac{4C+2}{4C-3}$$

K_B = Bergstraesser factor



Deflection of springs

$$\text{Strain Energy } U = \frac{T^2 l}{2GJ} + \frac{F^2 l}{2GA}; \; T = \frac{FD}{2}$$

$$U = \frac{4F^2 D^2}{G\pi d^4} \pi D N_a + \frac{2F^2}{G\pi d^2} \pi D N_a$$

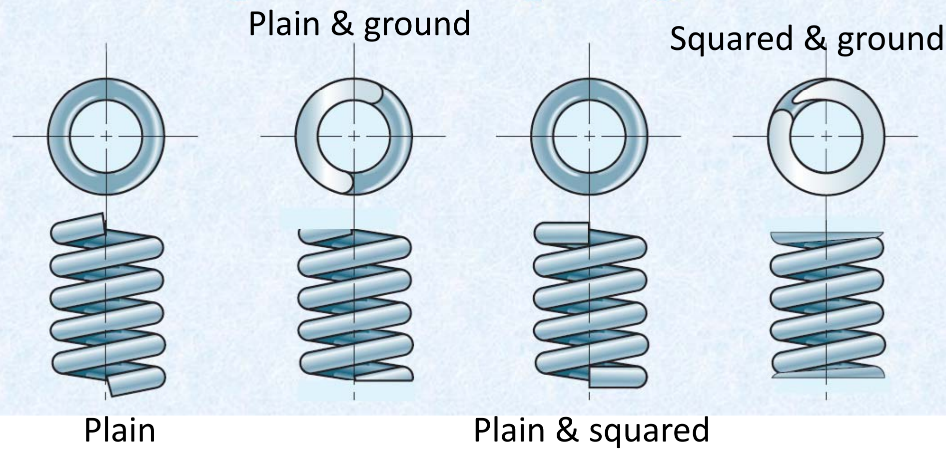
N_a - Number of active coils

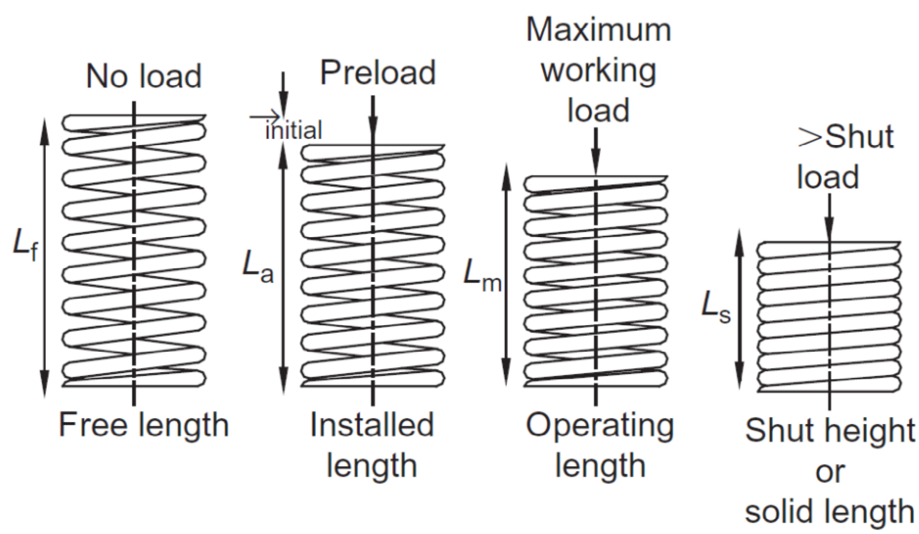
G- Shear modulus

$$U = \frac{4F^2 D^3}{G d^4} N_a \left(1 + \frac{1}{2C^2}\right) \sim \frac{4F^2 D^3}{G d^4} N_a$$

$$y = \frac{dU}{dF} = \frac{8FD^3}{G d^4} N_a; \; k = \frac{F}{y} = \frac{G d^4}{8 D^3 N_a}$$

Compression Spring Ends





Term	Plain	Plain and Ground	Squared or Closed	Squared and Ground
	Type of Spring Ends			
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, L_0	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, L_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
Pitch, p	$(L_0 - d)/N_a$	$L_0/(N_a + 1)$	$(L_0 - 3d)/N_a$	$(L_0 - 2d)/N_a$

Clash allowance

- The force-deflection response of springs should be ideally linear till it reaches solid length
- But it is not so as the some coils starts touching each other before solid length is reached due to manufacturing inaccuracies

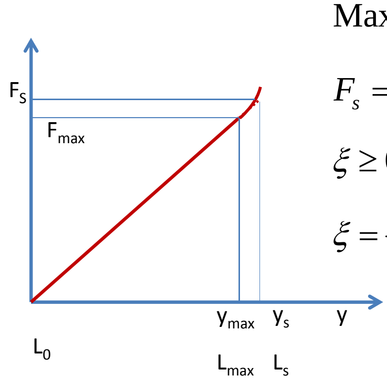
Force at solid length: $F_s = ky_s = k(L_0 - L_s)$

Maximum force: $F_{\max} = ky_{\max} = k(L_0 - L_{\max})$

$$F_s = (1+\xi)F_{\max}, \; F_{\max} \le \frac{7}{8}F_s$$

$$\xi \ge 0.15$$

$$\xi = \frac{y_s}{y_{\max}} - 1 = \frac{L_0 - L_s}{L_0 - L_{\max}} - 1$$



Stability

- Like columns, compression springs can buckle when their deflection exceeds a limit y_{cr}.

$$y_{cr} = L_0 C_1^{'} \left\{ 1 - \left[1 - \frac{C_2^{'}}{\lambda_{eff}^2} \right]^{1/2} \right\}; \quad \lambda_{eff} = \frac{\alpha L_0}{D}$$

L_0 – Free length, α – End condition costant

$$L_0 < \frac{\pi D}{\alpha} \left\{ \frac{2(E-G)}{2G+E} \right\}^{1/2} \text{ for absolute stability,}$$

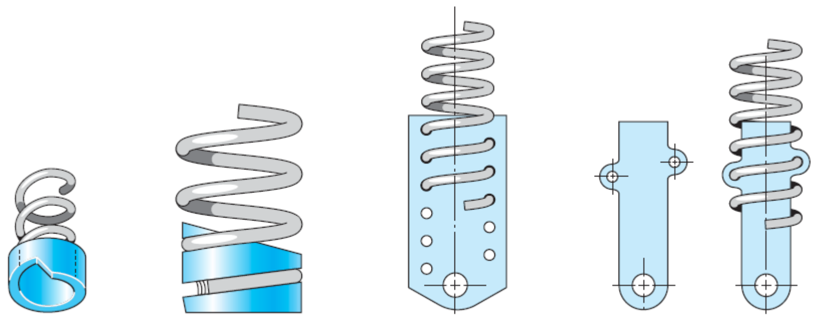
E-Young's modulus, G-Shear modulus

For steel springs, $L_0 < 2.63 \dfrac{D}{\alpha}$ for absolute stability

Stability

End Condition	Constant α
Spring supported between flat parallel surfaces (fixed ends)	0.5
One end supported by flat surface perpendicular to spring axis (fixed); other end pivoted (hinged)	0.707
Both ends pivoted (hinged)	1
One end clamped; other end free	2

*Ends supported by flat surfaces must be squared and ground.



(*a*) Contoured and plate (*b*) (*c*) (*d*)

Spring surge

- When a helical compression spring is subjected to excitation close to its natural frequency, coils can start clashing each other.
- The forcing frequency should be well separated from the natural frequency of the spring to avoid this.
- Forcing frequency should be much smaller than natural frequency

Frequency in Hz, $f = \frac{1}{2}\sqrt{\frac{kg}{W}}$ when ends are always in contact with plates

k-spring stiffness, g-accleration due to gravity

$W = \frac{\pi^2 d^2 D N_a \gamma}{4}$; γ is specific weight

Spring Materials

- Springs are manufactured either by hot or cold working
- Winding the spring wire over a mandrel: this process induces residual stresses due to bending
- Pre-hardened wire should not be used for C < 4 or d > 6 mm
- Materials
  - Plain carbon steels
  - Alloy steels
  - Nickel alloys
  - Spring brass

Tensile Strength

$$S_{ut} = \frac{A}{d^m}$$

- **Annealing**
- **Set Removal or Pre-setting**

Table 10–4

Constants A and m of $S_{ut} = A/d^m$ for Estimating Minimum Tensile Strength of Common Spring Wires

Source: From *Design Handbook*, 1987, p. 19. Courtesy of Associated Spring.

Material	ASTM No.	Exponent m	Diameter, in	A, kpsi · in^m	Diameter, mm	A, MPa · mm^m	Relative Cost of wire
Music wire*	A228	0.145	0.004–0.256	201	0.10–6.5	2211	2.6
OQ&T wire†	A229	0.187	0.020–0.500	147	0.5–12.7	1855	1.3
Hard-drawn wire‡	A227	0.190	0.028–0.500	140	0.7–12.7	1783	1.0
Chrome-vanadium wire§	A232	0.168	0.032–0.437	169	0.8–11.1	2005	3.1
Chrome-silicon wire‖	A401	0.108	0.063–0.375	202	1.6–9.5	1974	4.0
302 Stainless wire#	A313	0.146	0.013–0.10	169	0.3–2.5	1867	7.6–11
		0.263	0.10–0.20	128	2.5–5	2065	
		0.478	0.20–0.40	90	5–10	2911	
Phosphor-bronze wire**	B159	0	0.004–0.022	145	0.1–0.6	1000	8.0
		0.028	0.022–0.075	121	0.6–2	913	
		0.064	0.075–0.30	110	2–7.5	932	

Table 10-5

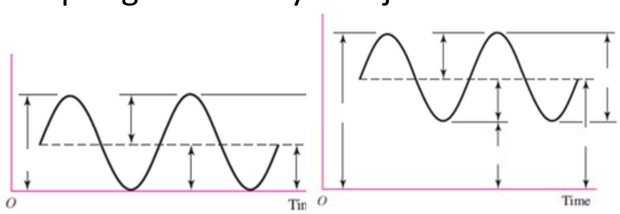
Mechanical Properties of Some Spring Wires

Material	Elastic Limit, Percent of S_{ut} Tension	Torsion	Diameter d, in	E Mpsi	E GPa	G Mpsi	G GPa
Music wire A228	65–75	45–60	<0.032	29.5	203.4	12.0	82.7
			0.033–0.063	29.0	200	11.85	81.7
			0.064–0.125	28.5	196.5	11.75	81.0
			>0.125	28.0	193	11.6	80.0
HD spring A227	60–70	45–55	<0.032	28.8	198.6	11.7	80.7
			0.033–0.063	28.7	197.9	11.6	80.0
			0.064–0.125	28.6	197.2	11.5	79.3
			[illegible]	[illegible]	[illegible]	11.4	78.6
Oil tempered A239	[illegible]	[illegible]		[illegible]	[illegible]	11.2	77.2
Valve spring A230	[illegible]	[illegible]		[illegible]	[illegible]	11.2	77.2
Chrome-vanadium A231	88–93	65–75		29.5	203.4	11.2	77.2
A232	88–93			29.5	203.4	11.2	77.2
Chrome-silicon A401	85–93	65–75		29.5	203.4	11.2	77.2
Stainless steel							
A313*	65–75	45–55		28	193	10	69.0
17-7PH	75–80	55–60		29.5	208.4	11	75.8
414	65–70	42–55		29	200	11.2	77.2
420	65–75	45–55		29	200	11.2	77.2
431	72–76	50–55		30	206	11.5	79.3
Phosphor-bronze B159	75–80	45–50		15	103.4	6	41.4
Beryllium-copper B197	70	50		17	117.2	6.5	44.8
	75	50–55		19	131	7.3	50.3
Inconel alloy X-750	65–70	40–45		31	213.7	11.2	77.2

$$S_{yt} = 0.75 S_{ut};\ S_{sy} = 0.577 S_{yt}$$

Helical Compression Springs: Fatigue design

- Springs are always subjected to fluctuating loads



$$F_m = \frac{F_{\min} + F_{\max}}{2}$$

$$F_a = \frac{F_{\max} - F_{\min}}{2}$$

- Calculate τ_a and τ_m

Approximate Strength Ratios of Some Common Spring Materials

Material	S_{ys}/S_u	S'_{es}/S_u
Hard–drawn wire	0.42	0.21
Music wire	0.40	0.23
Oil–tempered wire	0.45	0.22
Chrome–vanadium wire	0.52	0.20
Chrome–silicon wire	0.52	0.20

Helical Compression Springs: Fatigue design

Zimmerli's approach

- Observed that size, material and tensile strength has no effect on the endurance limit when d < 10 mm
- Springs are many times subjected to shot peening for improving fatigue life
- For R=0, the value of S_{se} (S_a) is as follows for infinite life
  - S_{se} = 310 MPa for unpeened springs
  - S_{se} = 465 MPa for peened springs
- The above is not for fully reversed loading as $\tau_a = \tau_m$

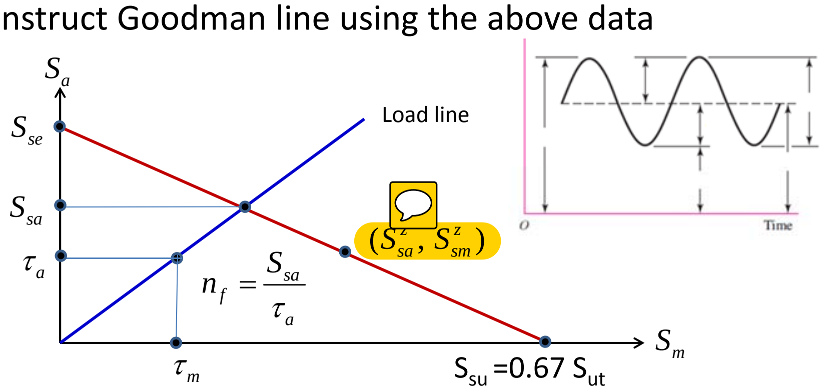
(F. P. Zimmerli, "Human Failures in Spring Applications," *The Mainspring, no. 17, Associated Spring* Corporation, Bristol, Conn., August–September 1957)

Helical Compression Springs: Fatigue design

- Calculate τ_m, τ_a from load history
- Use Zimmerly data for **infinite life** given below

  For unpeened springs $S_{sa}^{z} = 241\ \text{MPa},\ S_{sm}^{z} = 379\ \text{MPa}$

  For peened springs $S_{sa}^{z} = 398\ \text{MPa},\ S_{sm}^{z} = 534\ \text{MPa}$
- Construct Goodman line using the above data

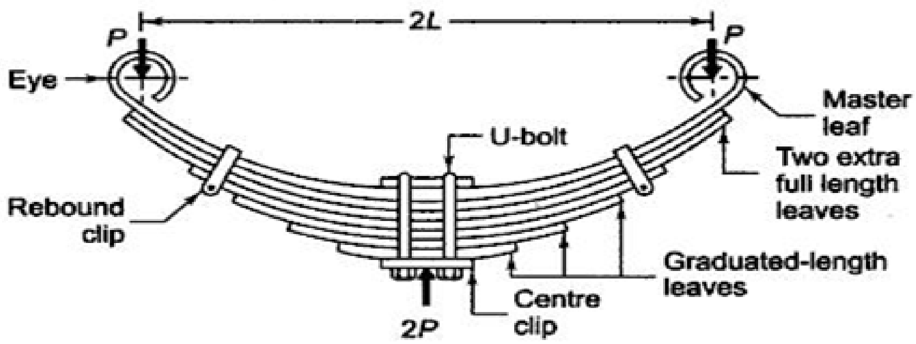


Spring design

- Spring index: $4 \le C \le 12$, Choose C
- If stiffness k is specified then
  - From F_{max} calculate y_{max}
  - Using the condition $\xi \ge 0.15$ determine F_s or y_s
  - **The factor of safety at solid length (n_s) should be at least 1.2: Using this fix d. Choose standard size**
  - You will have to choose the material at this point to get S_{ut} and S_{sy}
  - Check if factor of safety at F_{max} is sufficient
- Now using $k = \frac{d^4 G}{8 D^3 N_a}$ fix N_a ; ($4 \le N_a \le 12$) then get L_0
- Check for fatigue, buckling and surge
- Iterate till all conditions are satisfied

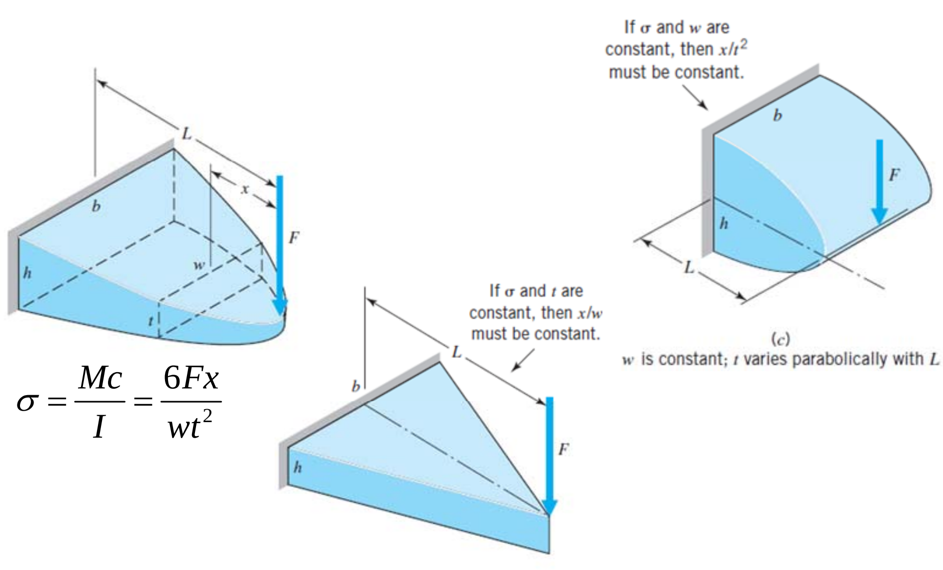
Multi-leaf springs

- Used in the suspension of trucks, railway wagons and SUVs
- Consists of a series of plates, usually having semi-elliptical shape
- Leaf at the top has maximum length- master leaf
- The leaves are held together by two U bolts and a center clip
- Re-bound clips are provided to keep the leaves in alignment



- The leaves share the load only when applied in direction shown
- The leaves tend to separate when load is reversed

Uniform strength cantilever

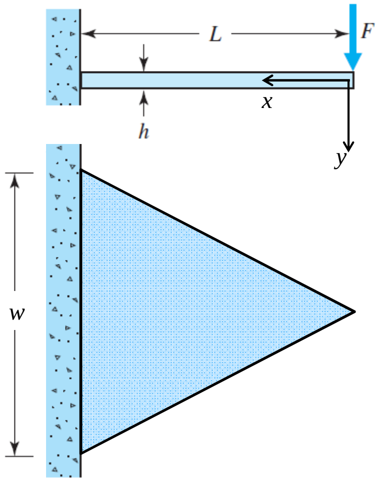


$$\sigma = \frac{Mc}{I} = \frac{6Fx}{wt^2}$$

(b)
t is constant; w varies linearly with L

(c)
w is constant; t varies parabolically with L

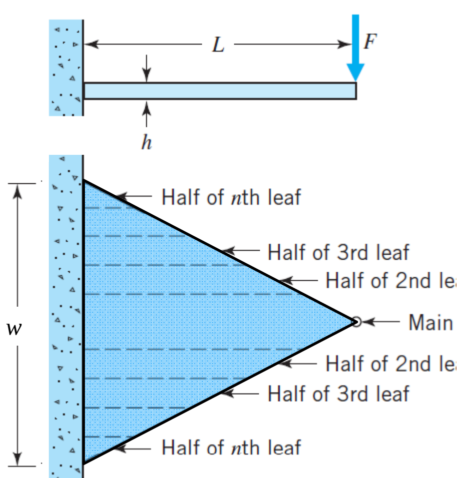
Uniform strength cantilever

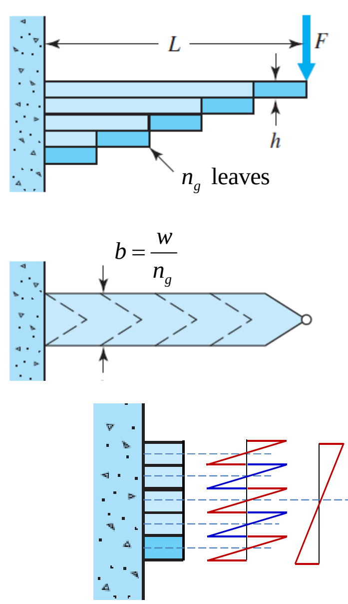


$$\frac{d^2y}{dx^2} = -\frac{M}{EI(x)} = -\frac{12Fx}{Eb(x/L)h^3} = -\frac{12FL}{Ewh^3}$$

$$y_{tip} = \frac{6FL^3}{Ewh^3}$$

$\sigma = \dfrac{6FL}{wh^2}$, is same at all sections of the beam along its span

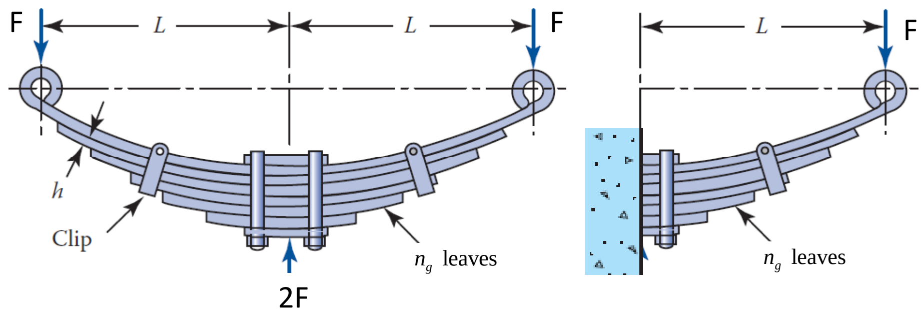




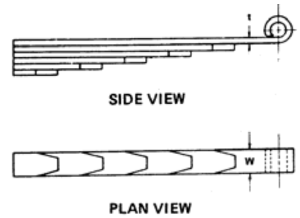
$$\sigma = \frac{6FL}{n_g bh^2}, \quad y_{tip} = \frac{6FL^3}{En_g bh^3}$$

- **Friction between leaves is assumed to be zero**
- **Perfect contact assumed between leaves**

Leaf spring



$$\sigma = \frac{6FL}{n_g b h^2}, \quad y_{tip} = \frac{6FL^3}{E n_g b h^3}$$



- **This analysis neglects effects of curvature and is valid for small deflections only**
- **The sharp corners at the ends of the leaves are made flat**

Leaf springs with additional full length leaves

- Leaf springs also have to withstand additional loads
  - Axial thrust (due to acceleration and deceleration)
  - Lateral loads during turning of vehicle (cornering)
  - Torque reactions about the axis of the shaft (axle)
- Additional full length leaves are added
- The graduated set and the additional leaves act like parallel springs: Deflection is same for both
- No friction between leaves and perfect contact assumed
  - Total load shared by the n_f additional leaves :– F_f
  - Total load shared by the graduated leaves :– F_g
  - Force on the spring $2F=(2F_f + 2F_g)$

n_f : number of additional full length leaves (not including master)
n_g : number of graduated length leaves (includes master)

$$y = \frac{4F_f L^3}{E n_f b h^3} = \frac{6F_g L^3}{E n_g b h^3}; \quad F_f = \frac{3n_f}{2n_g} F_g$$

$$F = F_f + F_g = F_g + \frac{3n_f}{2n_g} F_g \rightarrow F_g = \frac{2n_g}{2n_g + 3n_f} F; \; F_f = \frac{3n_f}{2n_g + 3n_f} F$$

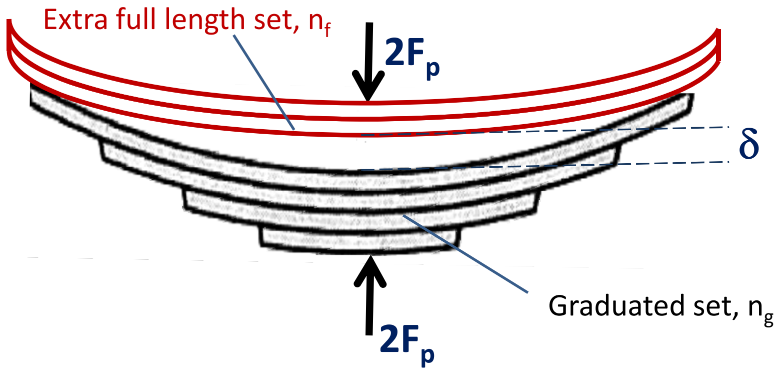
$$\sigma_f = \frac{6F_f L}{n_f b h^2} = \frac{18FL}{(2n_g + 3n_f) b h^2}$$

This is the maximum stress and varies along the span

$$\sigma_g = \frac{6F_g L}{n_g b h^2} = \frac{12FL}{(2n_g + 3n_f) b h^2}; \quad y_{\max} = \frac{12FL^3}{(2n_g + 3n_f) E b h^3}$$

- Maximum stress in additional leaves is 50% more than that in graduated leaves

Pre-loading or Nipping



- The gap of δ is closed by tightening the center U bolt
- The graduated set and the extra full length sets make full contact after this
- The gap δ has to be chosen such a way that when the service load (2F) is applied, the stress in both sets will be the same

- During closing the gap, both sets have the same force $2F_p$
- This force causes
  - Compressive stress at the top fibers of the full length set
  - Tensile stresses in the top fibers of graduated set

$$\sigma_f = -\frac{6F_pL}{n_f bh^2}; \quad \sigma_g = \frac{6F_pL}{n_g bh^2}$$

- When the external force 2F acts, it causes tensile stress on top fibers of both sets

$$\sigma_f = \frac{18FL}{(2n_f + 3n_g)bh^2}; \quad \sigma_g = \frac{12FL}{(2n_f + 3n_g)bh^2}$$

- Add the two stresses and then equate to get F_p so that the stresses during service in both sets are equal

$$\sigma_f = \frac{18FL}{(2n_f + 3n_g)bh^2} - \frac{6F_pL}{n_f bh^2} = \sigma_g = \frac{12FL}{(2n_f + 3n_g)bh^2} + \frac{6F_pL}{n_g bh^2}$$

$$F_p = \frac{Fn_f n_g}{(2n_f + 3n_g)n}; \; n = n_f + n_g;$$

Putting back in previous equation

$$\sigma_f = \frac{6FL}{nbh^2} = \sigma_g$$

$$\delta = \frac{6F_pL^3}{n_g Ebh^3} + \frac{4F_pL^3}{n_f Ebh^3} = \frac{2FL^3}{nEbh^3}$$

Material

- Made of hardened steel

G92600 (SAE 9260)	G86600(SAE 8660)
G40680 (SAE 4068)	G51600(SAE 5160)
G41610 (SAE 4161)	G51601(SAE 51B60)
G61500 (SAE 6150)	H51600(SAE 5160H)
	G50601(SAE 50B60)

- Typical properties

Hardness:	Bhn 388-461 (3000 kg mass) Brinell indentation diameter 3.10–2.85 mm Rockwell C 42-49
Tensile strength:	1300–1700 MPa
Yield strength (0.2% offset):	1170–1550 MPa
Reduction of area:	25% min
Elongation:	7% min

Source: SAE Spring design manual