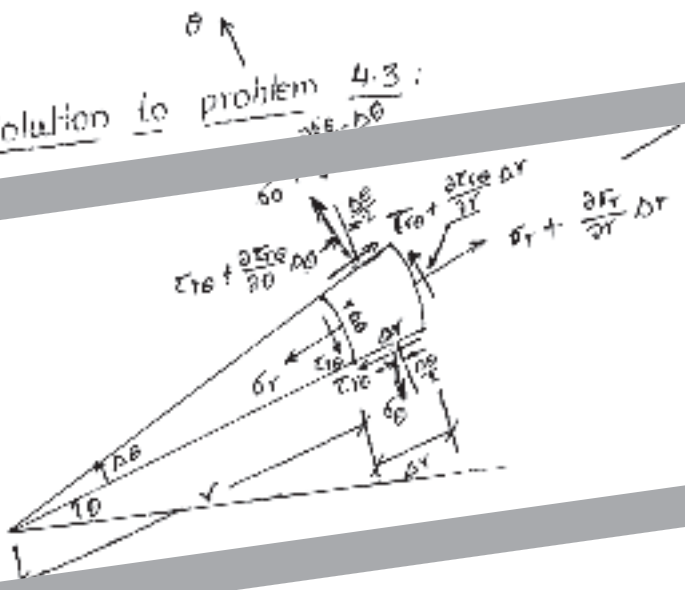
Solutions to HW Problems

Chapter 4

Solution to problem 4.3:



i) $\tau_{\theta r} = \tau_{r\theta}$

ii) unit thickness in z-direction

iii) $\sin\frac{\Delta\theta}{2} \cong \frac{\Delta\theta}{2}$; $\cos\frac{\Delta\theta}{2} \cong 1$.

$$\Sigma F_r = 0 \Rightarrow -\sigma_r(r\Delta\theta) + \left(\sigma_r + \Delta r\frac{\partial \sigma_r}{\partial r}\right)(r+\Delta r)\Delta\theta - \sigma_\theta \sin\frac{\Delta\theta}{2}(\Delta r)$$

$$\Rightarrow \sigma_r \Delta r \Delta\theta + \frac{\partial \sigma_r}{\partial r} r \Delta r \Delta\theta + \frac{\partial \sigma_r}{\partial r}(\Delta r)^2 \Delta\theta - 2\sigma_\theta \frac{\Delta\theta \Delta r}{2}$$

$$- \frac{\partial \sigma_\theta}{\partial \theta}\frac{(\Delta\theta)^2 \cdot \Delta r}{2} + \frac{\partial \tau_{r\theta}}{\partial \theta}\Delta\theta\Delta r = 0.$$

$$\Rightarrow \frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} + \frac{1}{r}\left(\frac{\partial \sigma_r}{\partial r}\Delta r - \frac{\partial \sigma_\theta}{\partial \theta}\frac{\Delta\theta}{2}\right) = 0.$$

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0 \qquad \text{as } \Delta r \to 0,\ \Delta\theta \to 0$$