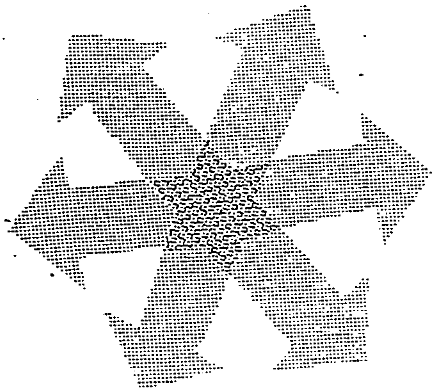
SOLUTIONS MANUAL



to accompany

CRANDALL & DAHL

An introduction to the mechanics of solids

SECOND EDITION

Prepared by

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서 문

본 책자는 Crandall & Dahl의 "An Introduction to the mechanics of Solids"의 연습문제를 풀이한 것이다.

위 책자는 역학을 공부하는 이들이 필히 익혀야 할 책으로서 PANGAL. N.NAYAK과 RANGNATH NAYAK에 의해서 제 7판에 대한 영자판 해답집이 이미 시중에 나온 바 있다.

제 2판이 출판된데 즈음하여 이에 대한 한글판 해답집을 엮어내기에 이르렀다.

위 책자는 역학을 공부하는데 있어 하나의 풀이예로서 도움을 줄 수도 있지만, 무의미하게 이용한다면 엄청난 손해를 야기할 수도 있다는 것을 말씀드린다. 다시 말해서 위 풀이들이 완전하다고 생각하고 비판없이 받아 들인다면, 스스로 생각하고 문제를 해결하는 능력을 마비시킬 수도 있다는 것을 꼭 명심하기 바란다.

따라서, 펴내는 이의 바램은 본 해답집이 보시는 이들로부터 Crandall 책의 문제를 풀이한 후에 심심풀이로 문제 해를 비교해 보시는 정도로만 이용되었으면 한다.

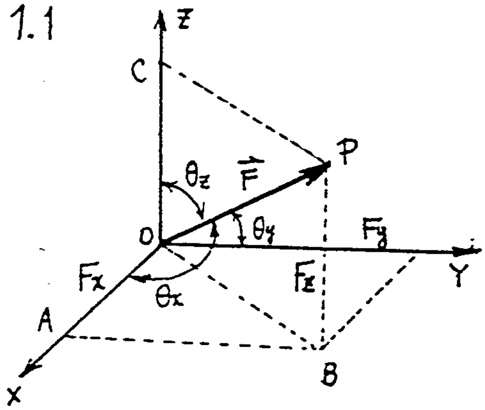
아무쪼록 이 풀이집이 보시는 이에게 조금이나마 도움이 되길 바랍니다.

－서울대학교 공과대학－

CONTENTS

CHAPTER 1.

1.1



$CP \parallel OB$, $OB \perp OC$ $\therefore CP \perp OC$

따라서, $\cos\theta_z = \dfrac{F_z}{|\vec{F}|}$

마찬가지로 $\cos\theta_y = \dfrac{F_y}{|\vec{F}|}$

$\cos\theta_z = \dfrac{F_x}{|\vec{F}|}$

피타고라스 정리에 의하면 $F_x^2 + F_y^2 + F_z^2 = |\vec{F}|^2$

$$|\vec{F}|^2\cos^2\theta_x + |\vec{F}|^2\cos^2\theta_y + |\vec{F}|^2\cos^2\theta_z = |\vec{F}|^2$$

$$\underline{\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1}$$

1.2 그림에서

$\bar{i} = \bar{a}\cos\theta - \bar{b}\sin\theta$

$\bar{j} = \bar{a}\sin\theta + \bar{b}\cos\theta$

또한 $\bar{i}\cdot\bar{a} = |\bar{i}||\bar{a}|\cos\theta = \cos\theta$

$\bar{i}\cdot\bar{b} = -\sin\theta$

$\bar{j}\cdot\bar{a} = \cos(\pi/2-\theta) = \sin\theta$

$\bar{j}\cdot\bar{b} = \cos\theta$

일반적으로 $\bar{i} = (\bar{i}\cdot\bar{a})\bar{a} + (\bar{i}\cdot\bar{b})\bar{b}$

$\bar{j} = (\bar{j}\cdot\bar{a})\bar{a} + (\bar{j}\cdot\bar{b})\bar{b}$

$$\bar{F} = F_x\bar{i} + F_y\bar{j} = F_x(\bar{a}\cos\theta - \bar{b}\sin\theta) + F_y(\bar{a}\sin\theta + \bar{b}\cos\theta)$$

$$= \bar{a}(F_x\cos\theta + F_y\sin\theta) + \bar{b}(F_y\cos\theta - F_x\sin\theta)$$

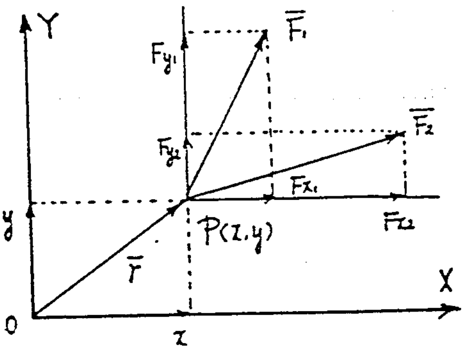
$\therefore$ $F_a = F_x\cos\theta + F_y\sin\theta \cdots ①$

$F_b = -F_x\sin\theta + F_y\cos\theta \cdots ②$

①과 ②로부터 $F_x = F_a\cos\theta - F_b\sin\theta$

$F_y = F_a\sin\theta + F_b\cos\theta$

1.3

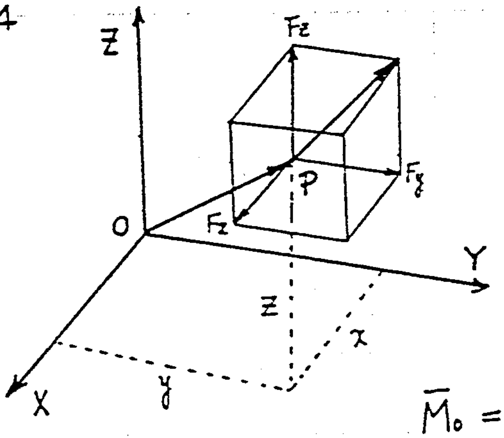


$$\overline{r} = \overline{i}x + \overline{j}y$$
$$\overline{F_1} = \overline{i}F_{x1} + \overline{j}F_{y1}$$
$$\overline{F_2} = \overline{i}F_{x2} + \overline{j}F_{y2}$$
$$\overline{M_o} = \overline{r} \times (\overline{F_1} + \overline{F_2})$$
$$= (\overline{i}x + \overline{j}y) \times [\overline{i}(F_{x1}+F_{x2}) + \overline{j}(F_{y1}+F_{y2})]$$
$$= \{x(F_{y1}+F_{y2}) - y(F_{x1}+F_{x2})\}\overline{k}$$

O점에 대한 $\overline{F_1}$과 $\overline{F_2}$의 moment를 각각 생각하면

$$\overline{M_{o1}} = \overline{r} \times \overline{F_1} = (xF_{y1} - yF_{x1})\overline{k}$$
$$\overline{M_{o2}} = \overline{r} \times \overline{F_2} = (xF_{y2} - yF_{x2})\overline{k}$$
$$\sum_{i=1}^{2} M_{oi} = \{x(F_{y1}+F_{y2}) - y(F_{x1}+F_{x2})\}\overline{k} = \overline{M_o}$$
$$\therefore \overline{r}\times\overline{F_1} + \overline{r}\times\overline{F_2} = \overline{r}\times(\overline{F_1}+\overline{F_2})$$

1.4



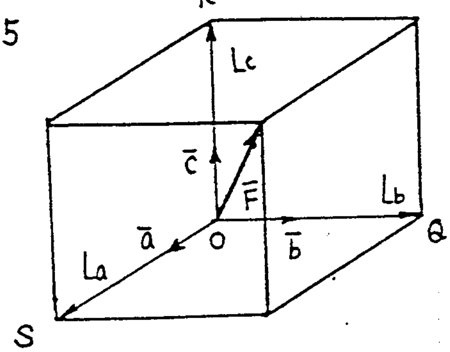
$\overline{M_o}$ (~~O점에 관한~~ Moment) $= \overline{r} \times \overline{F}$

$$\overline{M_o} = (x\overline{i} + y\overline{j} + z\overline{k}) \times (F_x\overline{i} + F_y\overline{j} + F_z\overline{k})$$
$$= \overline{i}(yF_z - zF_y) + \overline{j}(zF_x - xF_z)$$
$$+ \overline{k}(xF_y - yF_x)$$

따라서 $\overline{M_o}$는 다음과 같이 쓸수있다.

$$\overline{M_o} = \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

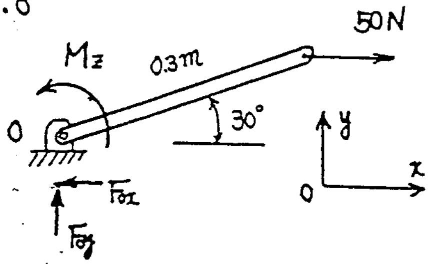
1.5



$$(\overline{F}\times\overline{b})\cdot\overline{c} = \{(L_a\overline{a} + L_b\overline{b} + L_c\overline{c})\times\overline{b}\}\cdot\overline{c}$$
$$= L_a(\overline{a}\times\overline{b})\cdot\overline{c} + L_b(\overline{b}\times\overline{b})\cdot\overline{c} + L_c(\overline{c}\times\overline{b})\cdot\overline{c}$$
$$(\overline{c}\times\overline{b})\cdot\overline{c} = 0 \quad (\because \overline{c}\times\overline{b} \perp \overline{c})$$
$$\overline{b}\times\overline{b} = 0$$
$$\therefore (\overline{F}\times\overline{b})\cdot\overline{c} = L_a(\overline{a}\times\overline{b})\cdot\overline{c}$$
$$\therefore L_a = \frac{(\overline{F}\times\overline{b})\cdot\overline{c}}{(\overline{a}\times\overline{b})\cdot\overline{c}}$$

같은 방법으로 $L_b = \dfrac{(\bar{F}\times\bar{c})\cdot\bar{a}}{(\bar{a}\times\bar{b})\cdot\bar{c}}$, $L_c = \dfrac{(\bar{F}\times\bar{a})\cdot\bar{b}}{(\bar{a}\times\bar{b})\cdot\bar{c}}$

1.6



평형조건으로부터

$\Sigma F_x = 0$ $\quad F_{ox} = 50\ N$

$\Sigma F_y = 0$ $\quad F_{oy} = 0$

$\Sigma M_{oz} = 0$ $\quad M_z = 50 \times 0.3 \sin 30^\circ$

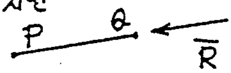
$= 75\ (N\cdot m)$

Ans. $F_{ox} = 50N$ $\quad M_z = 75(N\cdot m)$

1.7 (a) $\Sigma F_a = 0$, $\Sigma F_b = 0$ 이면 $\Sigma F = 0$ 를 만족하나 이 조건만으로는 x-y 면에서 Couple(짝힘)이 생길 수 있다.

임의의 점에 대하여 $\Sigma M_o = 0$ 가 되어야 Couple이 방지 될 수 있다.

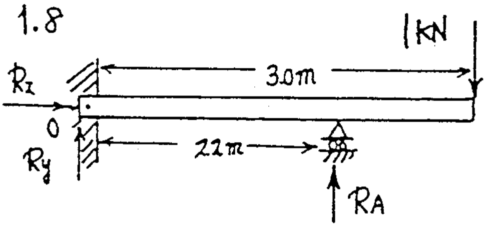
(b) $\Sigma M_p = 0$, $\Sigma M_Q = 0$ 이면 Couple은 발생하지 않지만 $\overline{PQ}$ 방향의 resultant force $\bar{R}$이 발생할 수 있다.



추가로 $\Sigma \bar{F}_{PQ} = 0$ 의 조건이 가해지면 resultant force가 없어진다

(c) $\Sigma M_o = 0$, $\Sigma M_p = 0$, $\Sigma M_Q = 0$ 라면 Couple과 resultant force가 각각 발생하지 않는다

1.8



$\Sigma F_x = 0$ $\quad \therefore\ R_x = 0$

$\Sigma F_y = 0$ $\quad R_y + R_A = 1000$ —①

$\Sigma M_o = 0$ $\quad 2.2 \times R_A = 3 \times 1000$

Ans) $\begin{cases} R_A = 1363.6\ N \\ R_y = -363.6\ N \end{cases}$ ①식에서부터

1.9

i) 위 실린더의 평형 조건

$\Sigma F_x = 0$ $\quad F_p \cos 45^\circ = F_c$

$\Sigma F_y = 0$ $\quad F_p \sin 45^\circ = 900\ N$

$\therefore F_p = 1272.8\ N$, $\quad F_c = 900\ N$

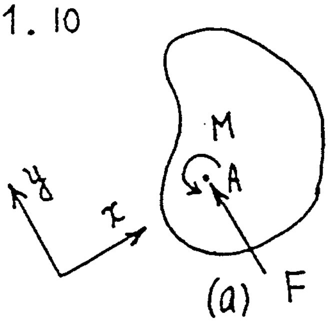
ii) 핀 실린더의 평형조건

$\Sigma F_x = 0 \quad F_A = F_P \cos 45° = 900\,N$

$\Sigma F_y = 0 \quad F_B = 900 + F_P \sin 45° = 1800\,N$

Ans. $\left(\begin{array}{l} F_A = 900\,N \\ F_B = 1800\,N \\ F_C = 900\,N \end{array} \right.$

1.10



i) 그림 (a) 에서의 평형조건

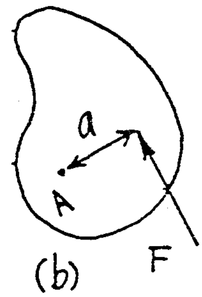
$\Sigma F_x = 0 , \quad \Sigma F_y = F , \quad \Sigma M_{Az} = M$

ii) 그림 (b) 에서의 평형 조건

$\Sigma F_x = 0 , \quad \Sigma F_y = F$

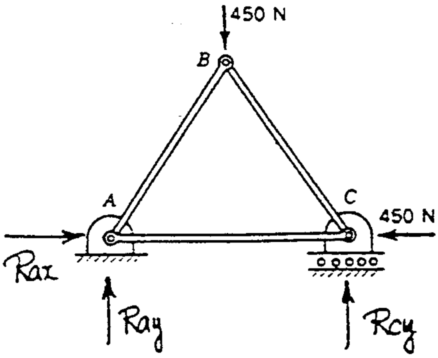
$\Sigma M_{Az} = a \cdot F = \frac{M}{F} \cdot F = M$

$a = \frac{|M|}{|F|}$



∴ 그림 (a)와 (b)로 부터 힘의 평형식이 같고 A점에 관한 moment 크기도 같다 그러므로 그림(a)와 같이 한점에 힘과 모우멘트가 작용하는 경우는 그림(b)와 같이 거리 $a = |M|/|F|$ 만큼 이동한 점에서 힘만을 가한 경우와 대등하다

1.11



i) A점에 관하여 ;

$\Sigma M_A = 0 \; ; \; -450 \times \frac{l}{2} + R_{cy} \cdot l = 0$

$\Sigma F_x = 0 \; ; \; R_{ax} = 450\,N$

$\Sigma F_y = 0 \; ; \; R_{ay} = 225\,N$] $R_{cy} = 225\,N$

ii) B점에 관하여 ;

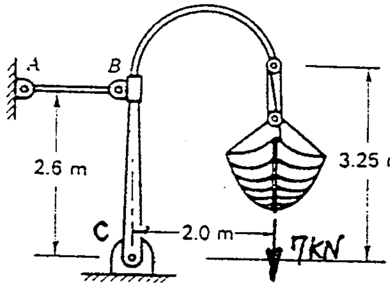
$\Sigma F_x = 0 \; ; \; \therefore F_{AB} = F_{BC}$ F_{AB} F_{BC}

iii) C점에 관하여 ; $\Sigma F_y = 0 \; ; \; F_{BC} \sin 60° = 225$

$\Sigma F_x = 0 \; ; \; F_{AC} - 450 + F_{BC} \cos 60° = 0 \quad \therefore F_{AC} = 320\,N , \; F_{BC} = 259.8\,N$

Ans.) $F_{AB} = F_{BC} = 259.8\,N , \quad F_{AC} = 320\,N$

1.12



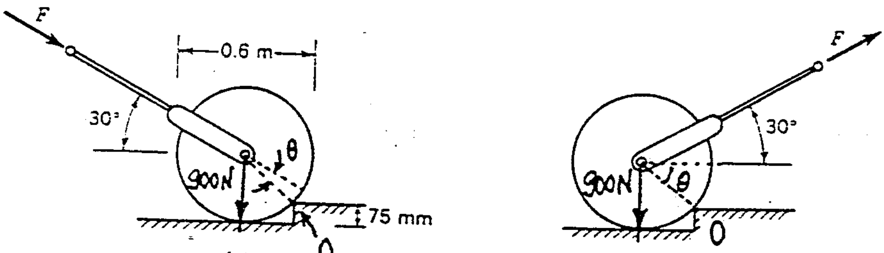
$\Sigma M_c = 0$ 로부터

$F_{AB} \times 2.6 - 7000 \times 2 = 0$

$\therefore\ F_{AB} = 5384.6\,N$

Ans.) $F_{AB} = 5384.6\ N$

1.13



(a) (b)

(a) $\theta = 60° - \cos^{-1}\dfrac{30-7.5}{30} = 18.59°$

$\Sigma M_o = 0$; $900\sqrt{0.3^2-(0.3-0.075)^2}$

$- F \times 0.3\sin 18.59° = 0$

$\therefore\ F = 1867\ N$

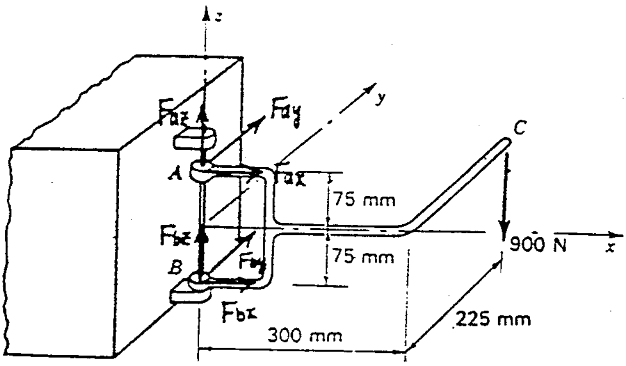
(b) $\theta = 30 + \sin^{-1}\dfrac{30-7.5}{30} = 78.59$

$\Sigma M_o = 0$; $900\sqrt{0.3^2-(0.3-0.075)^2}$

$- F \times 0.3\sin 78.59° = 0$

$\therefore\ F = 607\ N$

1.14



윗 구조물의 평형 조건으로부터

$\Sigma F_x = F_{Ax} + F_{Bx} = 0$

$\Sigma F_y = F_{Ay} + F_{By} = 0$

$\Sigma F_z = F_{Az} + F_{Bz} - 900 = 0$

$\Sigma M_x = 0$; $F_{By} \times 0.15 - 900 \times 0.225 = 0$

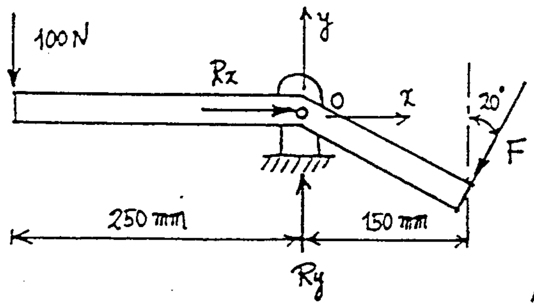
$\therefore\ F_{By} = -F_{Ay} = 1350\ N$

$\Sigma M_y = 0$; $F_{Ax} \times 0.15 + 900 \times 0.3 = 0$

$\therefore\ F_{Ax} = -F_{Bx} = -1800N$, $F_{Bz} = 900N$ ($\because F_{Az} = 0$)

Ans.) $F_{Ax} = -1800N$ $F_{Ay} = -1350N$, $F_{Az} = 0$, $F_{Bx} = 1800N$, $F_{By} = 1350N$, $F_{Bz} = 900N$

1.15



$\Sigma F_x = 0$; $-F\sin 20^\circ + R_x = 0$

$\therefore R_x = 53.6\,N$

$\Sigma F_y = 0$; $R_y - F\cos 20^\circ - 100 = 0$

$\therefore R_y = 247\,N$

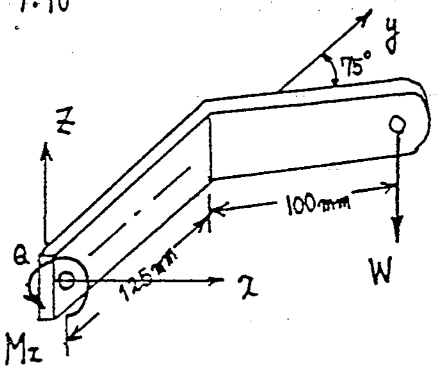
$\Sigma M_o = 0$;

$$100 \times 0.25 - F \times 0.15 \times \frac{1}{\cos 20^\circ} = 0$$

$\therefore F = 156.6\,N$

Ans.) $R_x = 53.6\,N$, $R_y = 247\,N$ $F = 156.6\,N$

1.16



$T_{max} = 100\ (N \cdot m)$

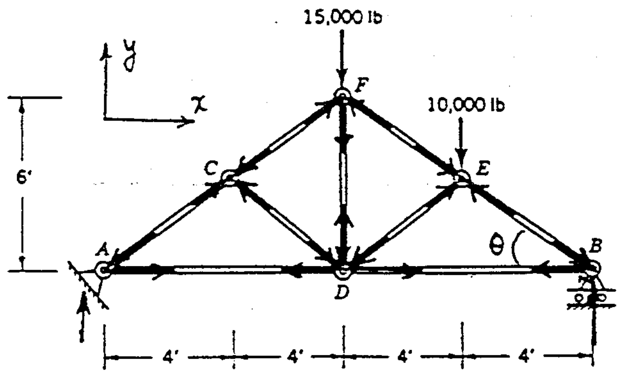
$\Sigma M_{ax} = 0$;

$M_x - W(0.125 + 0.1\cos 75^\circ) = 0$

$$\therefore W_{max} = \frac{100}{0.125 + 0.1\cos 75^\circ} = 662.8\,N$$

Ans.) $W_{max} = 662.8\,N$

1.17



Example 1-4로 부터

$F_{AC} = 16670$ (comp.)

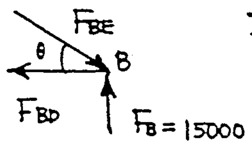
$F_{AD} = 13330$ (tens.)

$F_B = 15000$ (upward)

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

i) B점

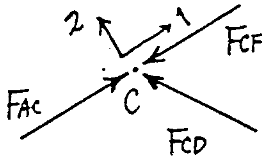


$\Sigma F_y = 0$; $F_{BE}\sin\theta = 15000$ $\therefore F_{BE} = 25000$

$\Sigma F_x = 0$; $F_{BD} = F_{BE}\cos\theta$ $\therefore F_{BD} = 20000$

(뒷장 계속)

ii) C점

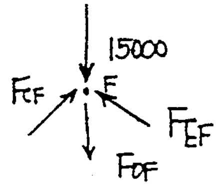


$\Sigma F_2 = 0$; $\therefore F_{CD} = 0$

$\Sigma F_1 = 0$; $F_{AC} = F_{CF}$

$= 16670$

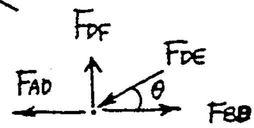
iii) F점



$\Sigma F_x = 0$;

$F_{EF} = F_{CF} = 16670$

iv) D점



$\Sigma F_x = 0$;

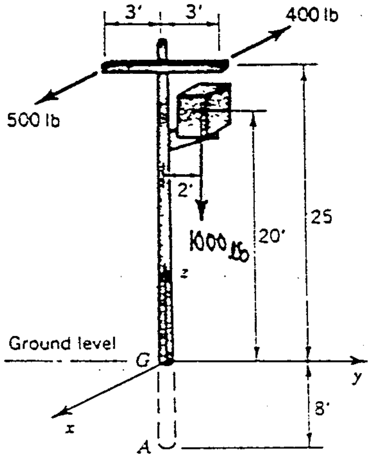
$F_{DE}\cos\theta = F_{BD} - F_{AD}$ $\quad \therefore F_{DE} = \frac{5}{4}(20000 - 13330) = 8337.5$

$\Sigma F_y = 0$; $\quad F_{DF} = F_{DE}\sin\theta = 8337.5 \cdot \frac{3}{5} = 5002.5$

Ans.) $F_{BD} = 20000$ lb (ten.) $\quad F_{EB} = 25000$ lb (comp.)

$F_{CD} = 0$, $F_{DE} = 8337.5$ lb (comp), $F_{CF} = F_{EF} = 16670$ lb (comp)

1.18



G에 작용하는 힘과 모우멘트는 전선과 변압기에 의한 양으로 나눌수 있다

i) 전선에 의한 힘과 모우먼트 성분

$F_{Gx} = 100\ \vec{i}$

$M_{Gy} = (500 \times 25 - 400 \times 25)\ \vec{j} = 2500\ \vec{j}$

$M_{Gz} = (3 \times 500 + 3 \times 400)\ \vec{k} = 2700\ \vec{k}$

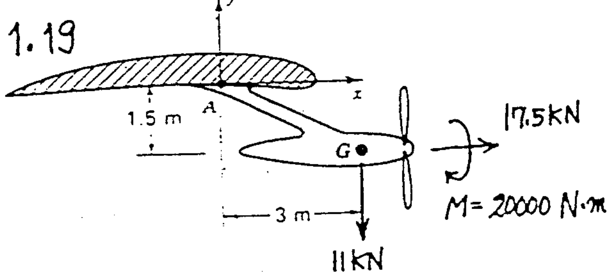
ii) 변압기에 의한 성분

$M_{Gz} = -1000\ \vec{k}$

$M_{Gz} = -2 \times 1000\ \vec{i} = -2000\ \vec{i}$

Ans.) G에서 작용하는 힘과 모우먼트 $\begin{cases} \vec{F} = 100\vec{i} - 1000\vec{k} \text{ (lb)} \\ \vec{M} = -2000\vec{i} + 2500\vec{j} + 2700\vec{k} \text{ (ft-lb)} \end{cases}$

1.19



$F_{Ax} = 17.5\,kN$; $\quad F_{Ay} = -11\,kN$

$M_x = 20000$ N·m

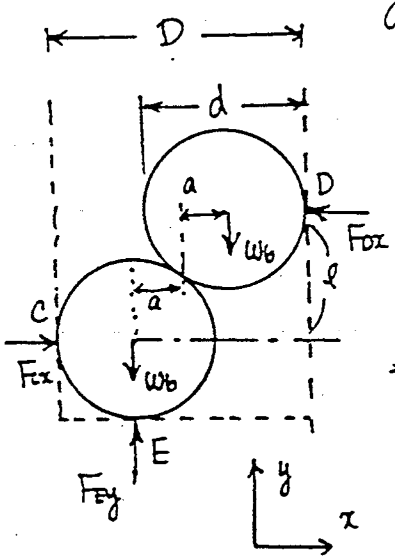
$M_z = 17.5 \times 1.5 - 11 \times 3 = -6.75$ (kN-m)

(뒷면에 계속)

$\vec{F} = 17500\,\vec{i} - 11000\,\vec{j}$ (N) $\vec{M} = 20000\,\vec{i} - 6750\,\vec{k}$ (N·m)

Ans.) $\vec{F} = 17500\vec{i} - 11000\vec{j}$ (N), $\vec{M} = 20000\vec{i} - 6750\vec{k}$ (N·m)

1.20



* 우선 구의 표면에 마찰력이 없다고 가정하면.

$a = \dfrac{D-d}{2}$, $\ell = 2\sqrt{\left(\frac{d}{2}\right)^2 - a^2} = \sqrt{D(2d-D)}$

평형 조건식 ; $\Sigma F_y = 0$ $F_{Ey} = 2W_b$

$\Sigma F_x = 0$ $F_{Cx} = F_{Dx}$

$\Sigma M_{Ez} = 0$; $(F_{Dx})\ell + (F_{Ey})\dfrac{d}{2} = W_b\dfrac{d}{2} + W_b\left(\dfrac{d}{2} + 2a\right)$

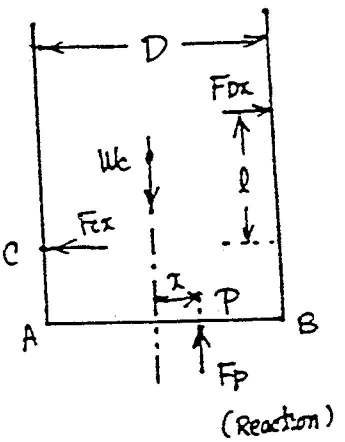
또는 $F_{Cx} = F_{Dx} = \dfrac{W_b(D-d)}{\ell} = \dfrac{W_b(D-d)}{\sqrt{D(2d-D)}}$

* AB 중심선으로부터 x만큼 떨어진 점에 작용하는 통자의 반력을 F_P라 하면 $\Sigma M_{Bz} = 0$;

$W_c \cdot \dfrac{D}{2} + F_{Cx} \cdot \dfrac{d}{2} = F_{Dx}\left(\dfrac{d}{2} + \ell\right) + F_P\left(\dfrac{D}{2} - x\right)$

$\Sigma F_y = 0$; $F_P = W_c$

위 두식으로 부터 $x = \dfrac{W_b}{W_c}(D-d)$



i) Orange juice can에 대하여 $D = 50$, $d = 45$, $W_b = 2.0$ $W_c = 0.57$

$x = \dfrac{2.0}{0.57}(50-45) = 17.5$

∴ $x < \dfrac{D}{2}$ 이므로 AB선상 내에 반력이 존재하며

따라서 "안정" 상태

ii) Beer can $D = 70$. $d = 45$ $W_b = 2.0$ $W_c = 1.0$

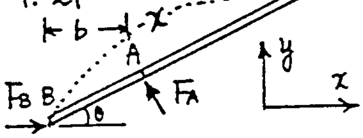
$x = \dfrac{2.0}{1.0}(70-45) = 50$. ∴ $x > \dfrac{D}{2}$

큰 반력이 AB선상 밖에 존재하므로 평형을 이룰수 없다

따라서 "불안정"

Ans.) Orange juice can ; 안정 , Beer can ; 불 안정

1. 21

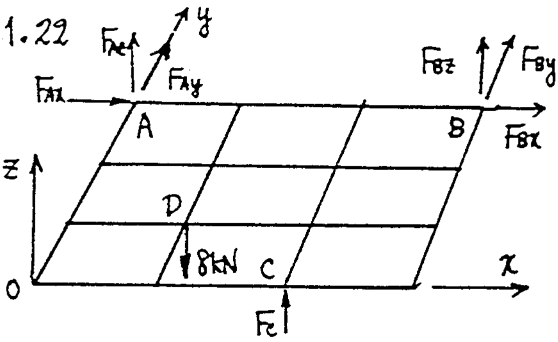


$\Sigma F_x = 0$; $F_B = F_A \sin\theta$

$\Sigma F_y = 0$; $F_A \cos\theta = W$

$\Sigma M_{Bz} = 0$; $F_A\, b/\cos\theta = W x \cos\theta$

1.22



가정

a) No friction at hinges

b) prop is rigid

c) No friction at prop

$\Sigma F_x = 0$; $F_{Ax} + F_{Bx} = 0$

~~부정정 문제에서~~ ~~적절한~~ 가정을 ~~취하면~~

$F_{Ax} = F_{Bx} = 0$.

$\Sigma F_y = 0$; $F_{Ay} + F_{By} = 0$, $\Sigma F_z = 0$; $F_{Az} + F_{Bz} + F_C = 8000$

$\Sigma M_{Bz} = 0$; $\therefore F_{Ay} = 0$, $F_{By} = 0$

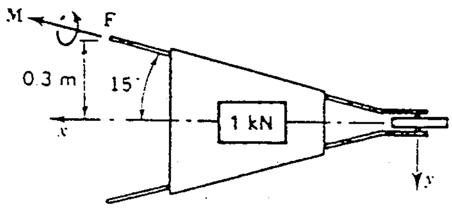
$\Sigma M_{Ay} = 0$; $F_{Bz} \times 3.9 + F_C \times 2.6 = 8000 \times 1.3$

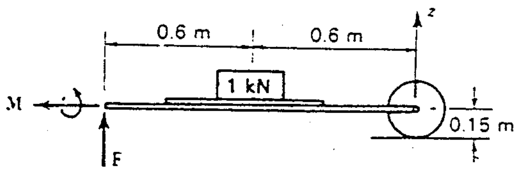
$\Sigma M_{Ax} = 0$; $8000 \times 2 = F_C \times 3$

~~위식을~~ 각각 ~~풀면~~ F를 ~~구할~~ ~~수~~ 있다.

Ans.) $\left[\begin{array}{l} F_{Bz} = -888.7\ N, \quad F_{Az} = 3555.7\ N, \quad F_C = 5333\ N \\ F_{Ax} = F_{Bx} = F_{Ay} = F_{By} = 0 \end{array} \right.$

1.23





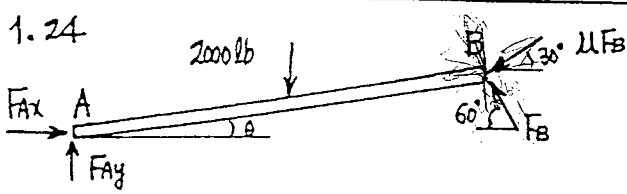
$\Sigma M_y = 0$; $0.6 \times 1000 = 1.2F + M \sin 15°$

$\Sigma M_x = 0$; $0.3F = M \cos 15°$ $\quad \therefore M(4\cos 15° + \sin 15°) = 600$

$\therefore M = 145.54$ (N·m), $F = 468.6$ (N)

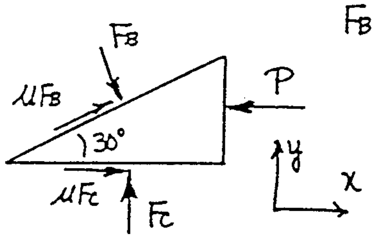
Ans.) $M = 145.54$ N·m , $F = 468.6$ N

1.24



a) $\Sigma M_A = 0$;

$2000 \times 2 = F_B \times 4 \sin 60° + \mu F_B 4 \sin 30°$



$F_B = 1396.6N$

쐐기의 평형 상태로부터

$\Sigma F_y = 0$; $F_C - F_B \cos 30 + \mu F_B \sin 30° = 0$

$\Sigma F_x = 0$; $-P + \mu F_C + (\mu \cos 30° + \sin 30°) F_B = 0$

구해진 F_B로부터 $F_C = 1000$ lb

$$P = 1000 \left[\mu + \frac{0.866\mu + 0.5}{0.866 - 0.5\mu} \right],$$ $\mu = 0.3$ 이라면 $P = 1364$ lb

b) 쐐기가 미끄러져 빠지는 것을 방지하려면 $P = 0$ 인 외력이 없는 상태에서 마찰력의 작용 방향은 (a)의 경우와 반대가 되므로, 등식을 세우면

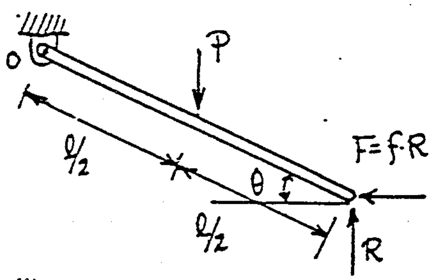
$$0 = 1000 \left[-\mu + \frac{0.5 - 0.866\mu}{0.866 + 0.5\mu} \right]$$ 가 된다.

이식을 풀면 $\mu = 0.27$

Ans.) a) $P = 1364$ lb b) $\mu = 0.27$

1.25

a)



$\Sigma M_O = 0$;

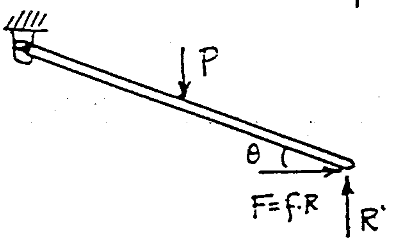
$P \cdot \frac{l}{2} \cos\theta + F \cdot l \sin\theta = R l \cos\theta$

$F = f \cdot R$ 이므로

$\frac{P}{2} \cos\theta = \frac{F}{f} \cos\theta - F \sin\theta$

$$\therefore F = \frac{Pf \cos\theta}{2(\cos\theta - f \sin\theta)} \quad \cdots ①$$

b)



$\Sigma M_O = 0$;

$P \frac{l}{2} \cos\theta = R \cdot l \cos\theta + f \cdot R\, l \sin\theta$

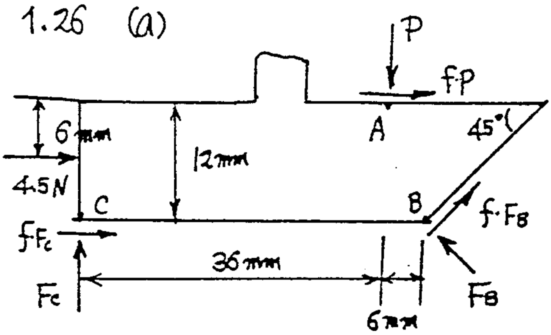
$\frac{P}{2} \cos\theta = \frac{F}{f} (\cos\theta + f \sin\theta) \quad (\because R = \frac{F}{f})$

$$\therefore F = \frac{Pf \cos\theta}{2(\cos\theta + f \sin\theta)} \quad \cdots ②$$

(a)의 경우의 마찰력이 (b)의 경우 보다 크며 (a)의 경우에 friction lock가 생길 수 있다.

Ans.) (a) $F = \frac{Pf \cos\theta}{2(\cos\theta - f \sin\theta)}$ (b) $F = \frac{Pf \cos\theta}{2(\cos\theta + f \sin\theta)}$

1.26 (a)



$\Sigma M_B = 0$;

$6P = 36F_C + 12f \cdot P + 4.5 \times 6$

$\Sigma F_x = 0$;

$4.5 + fP + fF_C + fF_B \cos 45° = F_B \cos 45°$

$\Sigma F_y = 0$;

$F_C + F_B \sin 45° + fF_B \cos 45° = P$

$\therefore P - 1.3\cos 45° F_B - F_C = 0$

연립하여 풀면 $P = 21.2$ N , $F_B = 22.34$ N , $F_C = 0.6633$ N

만약 문의 hinge 까지 거리를 L이라 하고 hinge로 부터 모우멘트를 생각하면 $\Sigma M_{hinge} = 0$;

$PL = F_B \sin 45° (L+6) + fF_B \sin 45° (L+6)$

$= F_B (1+f)(L+6) \sin 45°$

$L+6 \cong L \quad (\because L \gg 6mm)$

$\therefore P = F_B(1+f)\sin 45° = 22.34 \times 1.3 \times \sin 45° = 20.54$ (N)

(b) $f=0$ 인 경우에 대하여 같은 식을 유도하면

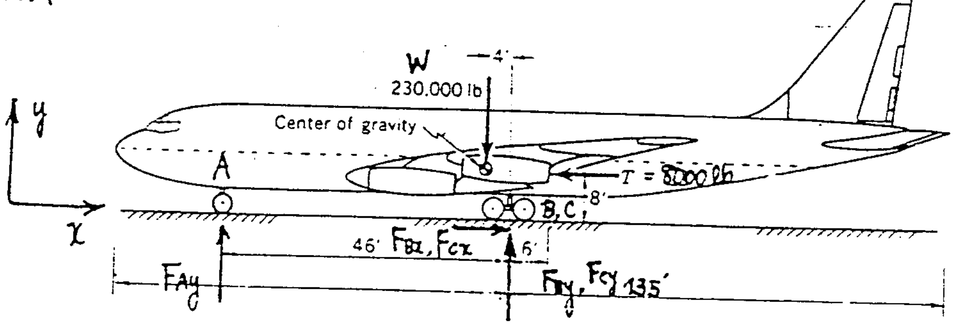
$$\left.\begin{aligned} P &= 6F_C + 4.5 \\ 4.5 &= F_B \cos 45° \\ P &= F_C + F_B \sin 45° \end{aligned}\right\} \quad \therefore F_B = 6.364\ (N)$$

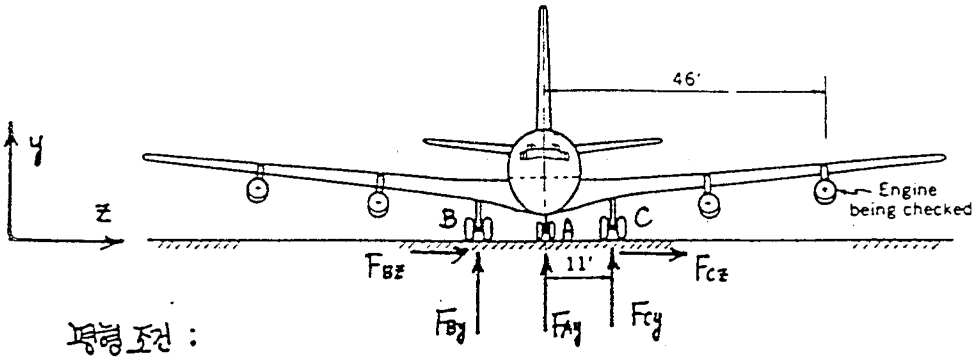
또한 $P = F_B \sin 45°$ 이므로

$P = 6.364 \times \sin 45° = 4.5$ (N)

Ans.) (a) $P = 20.54$ N (b) $P = 4.5$ N

1.27





평형조건 :

$\Sigma F_x = 0$; $F_{Bx} + F_{Cx} - 8000 = 0$ ········ (i)

$\Sigma F_y = 0$; $F_{Ay} + F_{By} + F_{Cy} - 230000 = 0$ ···· (ii)

$\Sigma F_z = 0$; $F_{Bz} + F_{Cz} = 0$ ······· (iii)

F_{Bz}, F_{Cz}는 부정정력 따라서, $F_{Bz} = F_{Cz} = 0$ 라 가정 할 수 있다.

B와 C의 중점에서의 모우멘트를 생각하면

$\Sigma M_x = 0$; $F_{By} \times 11 = F_{Cy} \times 11$ ······ (iv)

$\Sigma M_y = 0$; $F_{Cx} \times 11 \quad F_{Bx} \times 11 - 8000 \times 46 = 0$ ··· (v)

$\Sigma M_z = 0$; $-F_{Ay} \times 46 + 8000 \times 8 + 230000 \times 4 = 0$ ··· (vi)

(i)(ii)(iv)(v)와 (vi)으로 부터

$F_{Ay} = 213000\ lb \qquad F_{By} = F_{Cy} = 104300\ lb$

$F_{Bx} = -12700\ lb \qquad F_{Cx} = 20800\ lb$

$f_s \geq \dfrac{접선력}{수직력}$ 바퀴 A ; $f_s \geq \dfrac{12700}{104300} = 0.122$

바퀴 B ; $f_s \geq \dfrac{20800}{104300} = 0.2$ $\qquad \therefore f_s \geq 0.2$

Ans.) (a) $F_{Ay} = 213000\ lb \qquad F_{By} = F_{Cy} = 104300\ lb$

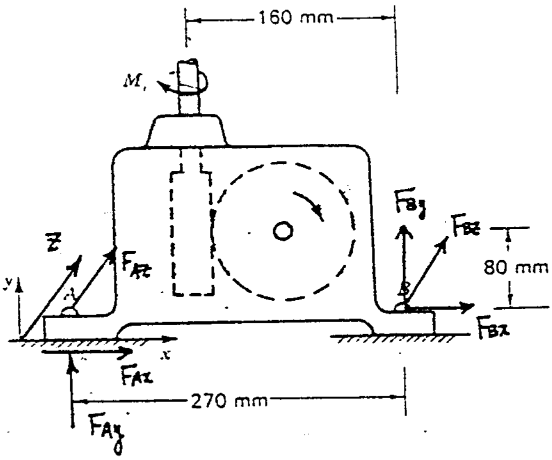
$F_{Bx} = -12700\ lb \qquad F_{Cx} = 20800\ lb \qquad F_{Bz} = F_{Cz} = 0$

(b) $f_s \geq 0.2$

1.28 $\Sigma M_y = 0$; $M_i = F_{Az} \times 0.11 - F_{Bz} \times 0.16$

$\Sigma M_z = 0$; $M_o = (F_{Ax} + F_{Bx}) \times 0.08 + F_{By} \times 0.16 - F_{Ay} \times 0.11$

$\Sigma F_x = 0$; $F_{Ax} + F_{Bx} = 0$, $\Rightarrow$ $F_{Ax} = F_{Bx} = 0$ 라 가정.



$\Sigma F_y = 0$; $F_{Ay} + F_{By} = 0$

$\Sigma F_z = 0$; $F_{Az} + F_{Bz} = 0$.

$\therefore F_{Az} = -F_{Bz} = \dfrac{M_i}{0.27} = \dfrac{15}{0.27}$

$= 55.56\ (N)$

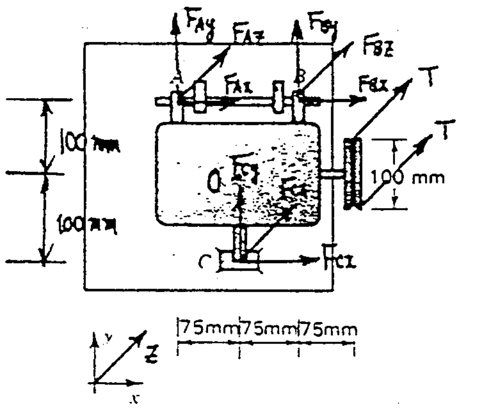
$F_{By} = -F_{Ay} = \dfrac{M_o}{0.27} = \dfrac{50 \times 15}{0.27}$

$= 2777.8\ (N)$

$(\because M_i = 15\ N\cdot m,\ M_o = 50 M_i)$

Ans.) $\underline{F_{Az} = -F_{Bz} = 55.56\ N}$, $\underline{F_{By} = -F_{Ay} = 2777.8\ N}$

1.29



$W = 80N$, $T = 125N$

$\Sigma F_x = 0$; $F_{Ax} + F_{Bx} + F_{Cx} = 0$

$\Sigma F_y = 0$; $F_{Ay} + F_{By} + F_{Cy} = 0$

$\Sigma F_z = 0$; $F_{Az} + F_{Bz} + F_{Cz} = 2T$

$\Sigma M_{xO} = 0$;

$(F_{Az} + F_{Bz}) \times 0.1 + W \times 0.1 = F_{Cz} \times 0.1 + 2T$

$\Sigma M_{yB} = 0$;

$2T \times 0.075 + F_{Cz} \times 0.075 + F_{Az} \times 0.15 = 0$

$\Sigma M_{zO} = 0$; $F_{Ay} \times 0.075 = F_{By} \times 0.075$

$F_{Ax} = F_{Bx} = F_{Cx} = 0$ (By assumption , F_{Ax}, F_{Bx}, F_{Cx} ; indeterminate)

$F_{Cy} = 0$ ($\because$ C점에서 y방향으로 고정되어 있지 않음)

$\Sigma F_y = 0$ 와 $\Sigma M_{zO} = 0$ 에서

$F_{Ay} + F_{By} = W$, $F_{Ay} = F_{By}$ $\quad \therefore F_{Ay} = F_{By} = \frac{W}{2} = 40\ N$

$\Sigma F_z = 0$, $\Sigma M_{xO} = 0$, $\Sigma M_{yB} = 0$ 에서

$F_{Az} + F_{Bz} + F_{Cz} = 2T$ $\quad \cdots ①$

$F_{Az} + F_{Cz} = -2T$ $\quad \cdots ②$

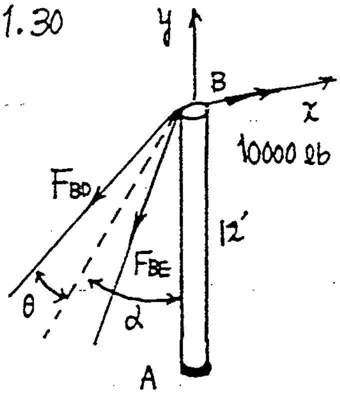
$F_{Az} + F_{Bz} - F_{Cz} = 2T - W$ $\quad \cdots ③$

①,②,③식으로 부터 $F_{Cz} = 151.5\ N$, $F_{Bz} = 299.25\ N$, $F_{Az} = -200.75\ N$

$\therefore\ \vec{F_A} = 40\vec{j} - 200.75\vec{k}$, $\vec{F_B} = 40\vec{j} + 299.25\vec{k}$, $\vec{F_C} = 151.5\vec{k}$

Ans.) $\vec{F_A} = 40\vec{j} - 200.75\vec{k}$ (N)
$\vec{F_B} = 40\vec{j} + 299.25\vec{k}$ (N), $\vec{F_C} = 151.5\vec{k}$ (N)

1.30



$\Sigma M_{Az} = 0$;

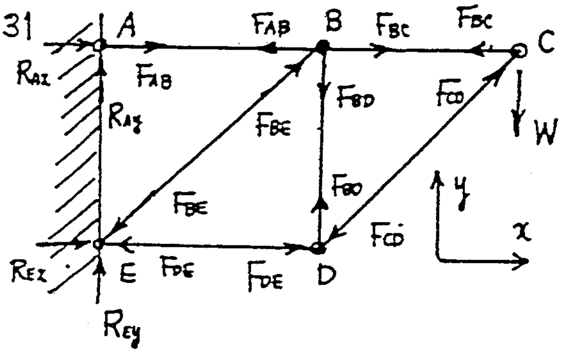
$2F_{BD}\cos\theta \cdot \sin\alpha \times 12 = 10000 \times 12$

$\sin\alpha = \dfrac{10}{\sqrt{10^2+12^2}}$ $\cos\theta = \dfrac{\sqrt{10^2+12^2}}{\sqrt{8^2+10^2+12^2}}$

$\therefore\ F_{BD} = F_{BE} = 8775\ N$

Ans.) $F_{BD} = F_{BE} = 8775\ N$

1.31



$\Sigma M_E = 0$; $2W = -R_{Ax}$

$\Sigma F_x = 0$; $R_{Ex} = -R_{Ax} = 2W$

$\Sigma F_y = 0$; $R_{Ay} + R_{Ey} = W$

C점 ; $\Sigma F_y = 0$;

$F_{CD}\cos 45° = W$

$\therefore\ F_{CD} = 1.41\,W$ (압축력)

$\Sigma F_x = 0$; $F_{BC} = F_{CD}\cos 45° = W$ (인장력)

D점, $\Sigma F_y = 0$; $F_{BD} = F_{CD}\sin 45° = W$ (인장력)

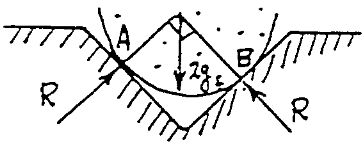
$\Sigma F_x = 0$; $F_{ED} = F_{CD}\cos 45° = W$ (압축력)

B점, $\Sigma F_y = 0$; $F_{EB}\cos 45° = F_{BD}$ $\therefore\ F_{BE} = 1.41\,W$ (압축력)

$\Sigma F_x = 0$; $F_{BA} = F_{BC} + F_{EB}\sin 45°$ $\therefore\ F_{AB} = 2W$ (인장력)

Ans.) $F_{AB} = 2W$, $F_{BD} = F_{BC} = W$ (인장력), $F_{BE} = F_{CD} = 1.41\,W$, $F_{DE} = W$ (압축력)

1.32



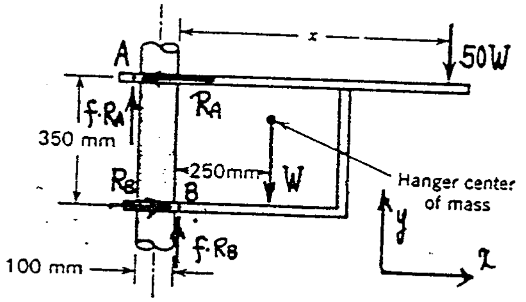
A.B점에서의 반력

$R = 2 \times 980 \times \cos 45° = 1386$ dyn

AB에서의 마찰력 ; $f = 0.2 \times R = 0.2 \times 1386 = 277\ (dyn)$

Ans.) $R = 1386\ dyn$, $f = 277\ dyn$

1.33



$\Sigma M_B = 0$;

$250\,W + 50W x + 100 f\cdot R_A = 350 R_A$

$\Sigma F_x = 0$; $R_A = R_B$

$\Sigma F_y = 0$; $f(R_A + R_B) = 51W$

두식을 연립하면

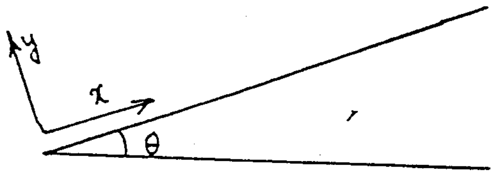
$R_A = R_B = \dfrac{51W}{2 \times 0.3} = 85W$

$(250 + 50x)\,W = (350 - 100 \times 0.3) R_A = 27200\,W$

$\therefore x = 539\ (mm)$

Ans.) 539 mm

1.34



우선, 차의 마력이 충분히 커서 차가 오를 수 있는 경사각은 마찰력에 좌우된다고 가정 하면,

$\Sigma M_A = 0$;

$L N_2 = 0.4L \cdot W\cos\theta - hW\sin\theta$

$\Sigma F_x = 0$; $f N_1 = W \sin\theta$

(∵ 뒷바퀴에만 동력이 걸림)

$\Sigma F_y = 0$; $N_1 + N_2 = W\cos\theta$

$\Sigma M_B = 0$; $N_1 = \left(0.6\cos\theta + \frac{h}{L}\sin\theta\right) W$

$\Sigma M_A = 0$; $N_2 = \left(0.4\cos\theta - \frac{h}{L}\sin\theta\right) W$

$\therefore\ f\left(0.6\cos\theta + \frac{h}{L}\sin\theta\right) = \sin\theta$

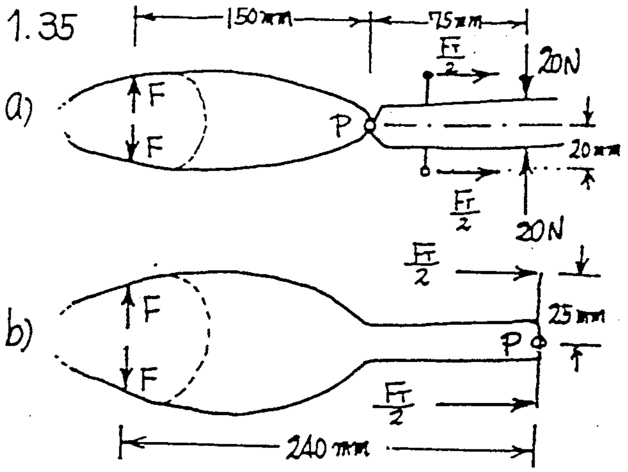
$\tan\theta = \dfrac{0.6\,L\cdot f}{L - f\cdot h} = \dfrac{0.6 \times 100 \times 0.7}{100 - 0.7 \times 2 \times 12} = 0.504$

$\therefore\ \theta = 26.8$

만약 마력이 충분치 못하다면 오를수 있는 경사각은 마력에 결정된다.

Ans.) 26

1.35



a) $\Sigma M_P = 0$;

$150 \times F = \frac{120}{2} \times 20 + 75 \times 20$

$\therefore F = 18\ (N)$

b) $\Sigma M_P = 0$

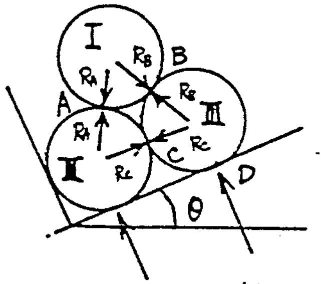
$\frac{F_T}{2} \times 25 + F \times 240 = 0$

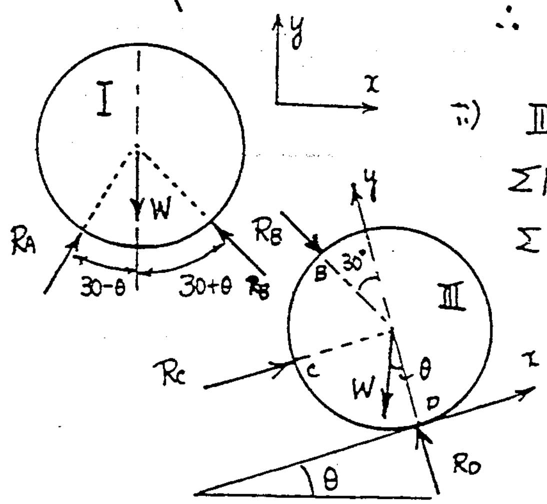
$\therefore F = -6.25 (N)$

(b)의 경우 forcep blade가 태아의 머리를 누르는 힘이 없으며, birth canal이 태아에 작용하는 힘과 서로 상쇄되는 결과가 되므로 태아에 좋은 forcep은 (b)의 Divergent-lever forcep이다.

Ans.) a) 18 N b) −6.25 N
Divergent-lever forcep이 좋다.

1.36





i) I 통나무에 대해서

$\Sigma F_x = 0$; $R_A \sin(30-\theta) = R_B \sin(30+\theta)$

$\Sigma F_y = 0$; $W = R_A \cos(30-\theta) + R_B \cos(30+\theta)$

$\therefore R_A = W \frac{\sin(30+\theta)}{\sin 60^\circ}$

$\therefore R_B = W \frac{\sin(30-\theta)}{\sin 60^\circ}$

ii) III 통나무에 대해서

$\Sigma F_x = 0$; $R_C + R_B \sin 30 = W \sin\theta$

$\Sigma F_y = 0$; $R_D = W\cos\theta + R_B \cos 30^\circ$

$R_C \geq 0$ 이어야 원상태 유지

$R_C = 0$ 이면

$R_B \sin 30^\circ = W \sin\theta$

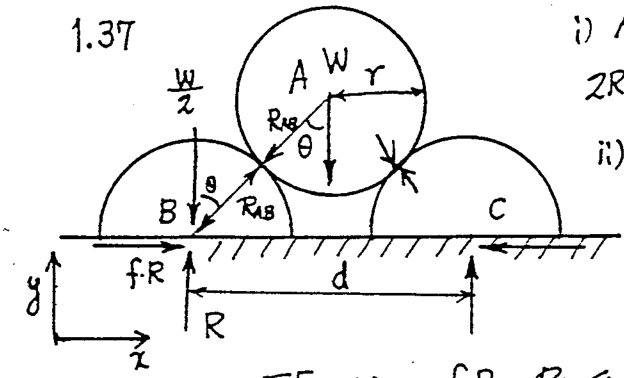
$\frac{\sin(30^\circ-\theta)}{\sin 60^\circ} = \frac{\sin\theta}{\sin 30^\circ}$

$\tan\theta = \left(\frac{1}{2} - \frac{\sqrt{3}}{2}\tan\theta\right)\frac{1}{\sqrt{3}}$ ∴ $\theta = \tan^{-1}\frac{1}{3\sqrt{3}}$

$\theta = 10.89°$

Ans.) 10.89°

1.37



i) A구에 대하여

$2R_{AB}$

ii) B구에 대하여

$\Sigma F_y = 0$; $R = \frac{W}{2} + R_{AB}\cos\theta$

$R_{AB} = \frac{W}{2\cos\theta}$ (∵ R=W)

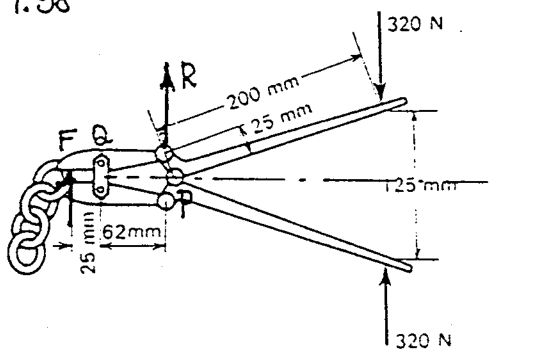
$\Sigma F_x = 0$; $f \cdot R = R_{AB}\sin\theta = \frac{W}{2}\tan\theta$

∴ $f = \frac{1}{2}\tan\theta = \frac{1}{2}\frac{\frac{d}{2}}{\sqrt{(2r)^2 - \left(\frac{d}{2}\right)^2}} = \frac{d}{2\sqrt{16r^2 - d^2}}$

d를 f와 r로 표현하면 $d = \frac{8fr}{\sqrt{1+4f^2}}$

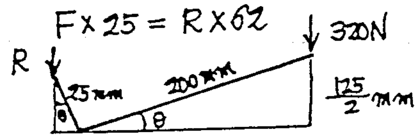
Ans.) $d_{max} = \frac{8fr}{\sqrt{1+4f^2}}$

1.38



Q점에 대한 모우먼트를 생각하면

$F \times 25 = R \times 62$



$R \times 25\sin\theta = \sqrt{200^2 - \left(\frac{125}{2}\right)^2} \times 320$

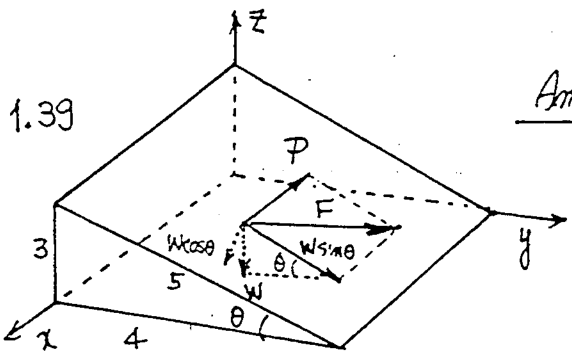
$\sin\theta = \frac{\frac{125}{2}}{200}$

∴ $R = 7781.7$ N

$F = \frac{62}{25} \times R = 19298.7$ N

Ans.) 19298.7 N

1.39



$F = \sqrt{P^2 + (W\sin\theta)^2}$

$= \sqrt{(0.4W)^2 + (0.6W)^2}$

$= 0.72W$

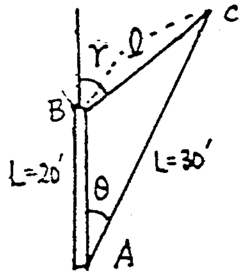
블록이 움직이기 위해서는 F가 마찰력 R_f 보다 커져야 한다

$R_f = f \cdot W\cos\theta$ $\qquad$ $0.72W \geq f \cdot W\cos\theta$

$f \leq 0.72 \times \frac{5}{4}$ $\qquad$ $\therefore\ f \leq 0.9$

Ans.) $f \leq 0.9$

1.40 가정; Post는 pin-joint이므로 굽힘 모우멘트를 받지 못한다



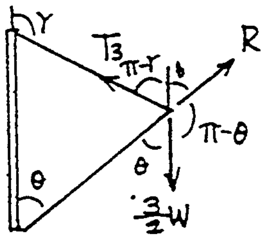
i) γ와 θ의 관계식

$l\sin\gamma = 30\sin\theta \quad \cdots ①$

$-l\cos\gamma + 30\cos\theta = 20 \quad \cdots ②$

①→② ; $-\cot\gamma\sin\theta + \cos\theta = \frac{2}{3} \quad \cdots ③$

ii) $\triangle ABC$를 포함하는 직면에 대하여



$\overline{DB}, \overline{BE}, \overline{BC}$에 걸리는 장력을 T_1, T_2, T_3 라 한다.

$\Sigma F_x = 0;\ T_3\sin(\pi-\gamma) = R\sin\theta \quad \cdots ④$

$\Sigma F_y = 0;\ T_3\cos(\pi-\gamma) + R\cos\theta = \frac{3}{2}W \quad \cdots ⑤$

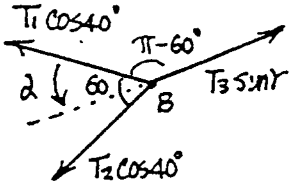
④⑤에서 $T_3 = \dfrac{1500}{(\cot\theta\sin\gamma - \cos\gamma)} \leq 1200 \quad \cdots ⑥$

$\therefore\ \cot\theta\sin\gamma - \cos\gamma \geq \frac{5}{4} \quad \cdots ⑦$

iii) B점을 포함하며 지면에 평행한 평면에 대하여

(참조; α의 범위는 T_1, T_2가 대칭이므로 양쪽으로 한계지어짐) (다음페이지 계속 ⇒)

$\theta^{(\circ)}$	$\frac{\cos\theta - \frac{2}{3}}{\sin\theta}$	γ°	$\cot\theta\sin\gamma - \cos\gamma$	α°(한계값)	α의 범위
5	3.781	14.8	1.953	임의값	$0 < \alpha < 60^{\circ}$
10	1.832	28.63	1.839	〃	$0 < \alpha < 60^{\circ}$
20	0.798	51.41	1.523	〃	〃
24	0.607	58.74	1.401	〃	〃
25	0.567	60.45	1.372	56.83	$3.17 < \alpha < 56.83$
27	0.494	63.70	1.316	51.18	$8.82 < \alpha < 51.18$
29	0.429	66.78	1.264	46.88	$13.12 < \alpha < 46.88$
30	0.399	68.25	1.238	⇒ T3가 한계 하중을 넘는다.	



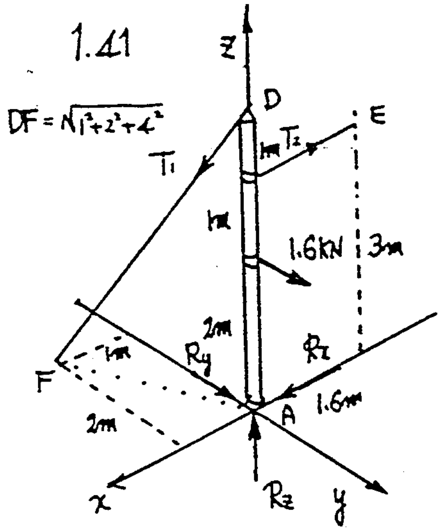
$$\frac{|T_2\cos 40°|}{\sin(\pi-\alpha)} = \frac{|T_3\sin\gamma|}{\sin 60°} \quad (\because \text{라미의 정리로부터})$$

$$T_2 = \frac{\sin\alpha\sin\gamma}{\cos 40°\sin 60°}T_3 \le 1200 \quad \cdots\cdots ⑧$$

③,⑦,⑧로 부터 θ, γ, α의 허용치를 구할수 있다.

즉, $\theta=30°$를 넣으면 T_3가 한계 하중을 초과하여 부적당하다.

1.41



$$\Sigma M_x = 0\ ; \quad 1600\times 2 = \frac{\sqrt5}{\sqrt{21}}\times\frac{2}{\sqrt5}\times T_1\times 4$$

$$\Sigma M_y = 0\ ; \quad T_2\times 3 = \frac{\sqrt5}{\sqrt{21}}\times\frac{1}{\sqrt5}\times T_1\times 4$$

위식을 연립하여 $T_1 = 1833N$

$T_2 = 533N$

$$\Sigma F_x = 0\ ; \quad R_x = T_2 - \frac{1}{\sqrt{21}}T_1 = 133N$$

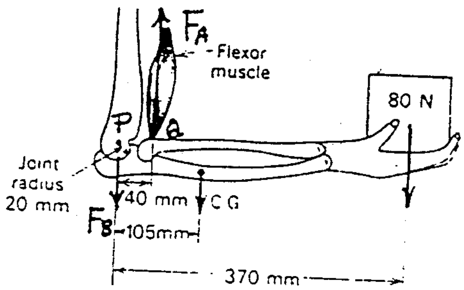
$$\Sigma F_y = 0\ ; \quad R_y = \frac{2}{\sqrt{21}}F_1 - R_z = -800N$$

$$\Sigma F_z = 0\ ; \quad R_z = T_1\cdot\frac{4}{\sqrt{21}} = 1600N$$

Ans.) $T_1 = 1833N$, $T_2 = 533N$

$R_x = 133N$, $R_y = -800N$ $R_z = 1600N$

1.42



i) 만약 골격사이 마찰이 없다면

$\Sigma M_P = 0$;

$F_A\times 40 = 16\times 105 + 80\times 370$

$\therefore F_A = 782N$

$\Sigma M_B = 0$:

$F_B\times 40 = 16\times 65 + 80\times 330$

$\therefore F_B = 686N$

ii) 마찰계수 $f = 0.015$ 이라면

(a) 하중을 들어 올릴때

$\Sigma M_P = 0$; $F_A\times 40 = 16\times 105 + 80\times 370 + 0.015\times F_B\times 20$

$\Sigma F_y = 0$; $F_A = F_B + 16 + 80$ $\therefore F_B = F_A - 96$

두식을 연립하면 $F_A \times 40 = 16\times105 + 80\times370 + 0.015\times(F_A - 96)\times 20$

$F_A = 787\ N \qquad \therefore F_B = 787 - 96 = 691\ N$

(b) 하중을 단순히 지지하고 있을때

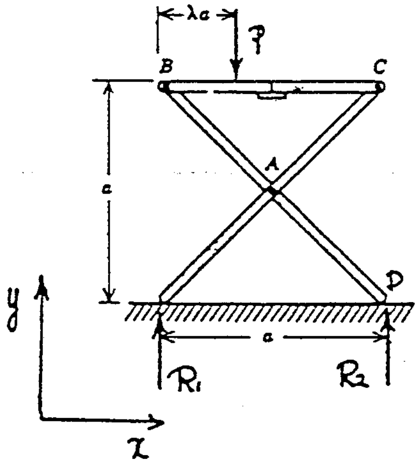
지지하는 상태에서는 마찰력이 작용하지 않으므로 $F_{friction} = 0$

$\therefore F_A = 782\ N$

Ans.) i) $F_A = 782N,\ F_B = 686N$

ii) (a) $F_A = 787N,\ F_B = 691N$, (b) $F_A = 782N,\ F_B = 686N$

1.43



a)

$\Sigma F_y = 0\ ;\quad R_1 + R_2 = P$

$\Sigma M_B = 0\ ;\quad aR_2 = \lambda a P$

$\therefore\ R_2 = \lambda P\ ,\ R_1 = (1-\lambda)P$

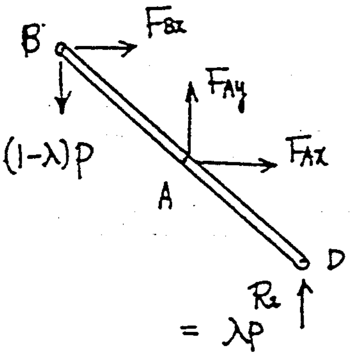
* Bar BD의 평형 조건

$\Sigma F_x = 0\ ;\quad F_{Ax} + F_{Bx} = 0$

$\Sigma F_y = 0\ ;\quad F_{Ay} = (1-\lambda)P - \lambda P = (1-2\lambda)P$

$\Sigma M_A = 0\ ;\quad F_{Bx}\cdot\frac{a}{2} - (1-\lambda)P\frac{a}{2} - \lambda P\cdot\frac{a}{2} = 0 \qquad \therefore F_{Bx} = P$

$\therefore \begin{cases} F_{Ax} = -P \\ F_{Ay} = (1-2\lambda)P \end{cases}$



$F_A = \sqrt{F_{Ax}^2 + F_{Ay}^2} = P\sqrt{2-4\lambda+4\lambda^2}$

b) λ에 대해서 최대인 값을 구하려면

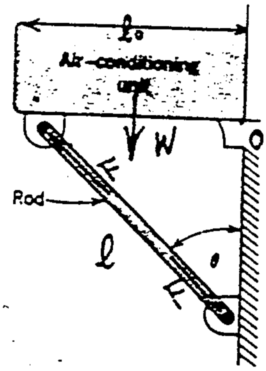
$$\frac{dF_A}{d\lambda} = P\frac{8\lambda-4}{2\sqrt{2-4\lambda+4\lambda^2}} = 0 \text{ 이므로}$$

$\lambda = \frac{1}{2}$ 에서 F_A는 최소, 최대는 $\lambda=0,\ \lambda=1$ 인 경계에서

$F_{max} = \sqrt{2}P$

Ans.) a) $F_A = \sqrt{2-4\lambda+4\lambda^2}\ P$, b) $F_{max} = \sqrt{2}P\ (\lambda=0, \lambda=1)$

1.44



Rod에 걸리는 압축력을 F라 하면,

$\Sigma M_o = 0$; $F\cos\theta = \frac{W}{2}$ $\therefore F = \frac{W}{2\cos\theta}$

비용이 가장 적게 들기 위해서는 재료가 적게 소모되어야 하므로 $\frac{dV}{d\theta} = 0$인 θ값을 찾으면 된다. ($l_o = l\sin\theta$)

$V = \pi r^2 l = \pi r^2 \frac{l_o}{\sin\theta}$ ....①

$F = A\cdot\sigma_R = \pi r^2 \sigma_R = \frac{W}{2\cos\theta}$ ....②

① ② 식으로부터 V와 θ에 대해서 정리하면

$V\sin\theta\cdot 2\cos\theta = \frac{W l_o}{\sigma_R} = C$ ..③ (∵ W, l_o은 주어진 값, σ_R은 임계 응력)

③식을 변수 V와 θ에 대해서 미분하면 $V\sin 2\theta = C$로 부터

$dV\sin 2\theta + 2V\cos 2\theta\, d\theta = 0$ $\frac{dV}{d\theta} = -2V\frac{\cos 2\theta}{\sin 2\theta} = 0$ $(0<\theta<\frac{\pi}{2})$

$\therefore \cos 2\theta = 0$ $\theta = \frac{\pi}{4}$

Ans.) $\theta = \frac{\pi}{4}$

1.45 i) 트렁크를 밀어 올리기 시작 할때 드는 힘 $(\theta = 30°)$

$F = W\sin 30° + fW\cos 30°$ ....①

ii) 트렁크가 밀려 내려오지 못하게 버티는 힘 $(\theta = 60°)$

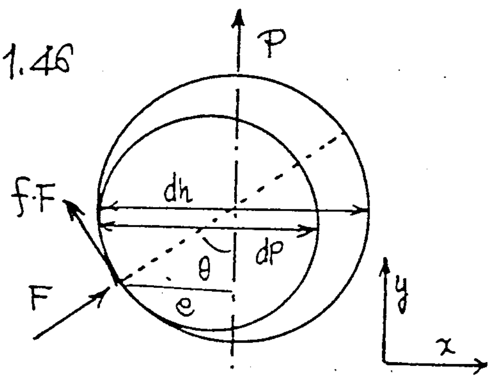
$F = W\sin 60° - fW\cos 60°$ ....②

① = ② 에서 $f(\cos 30° + \cos 60°) = \sin 60° - \sin 30°$

$\therefore f = 0.268$

Ans.) $f = 0.268$

1.46



$\Sigma F_x = 0$; $F\sin\theta = f\cdot F\cos\theta$

$\therefore f = \tan\theta$

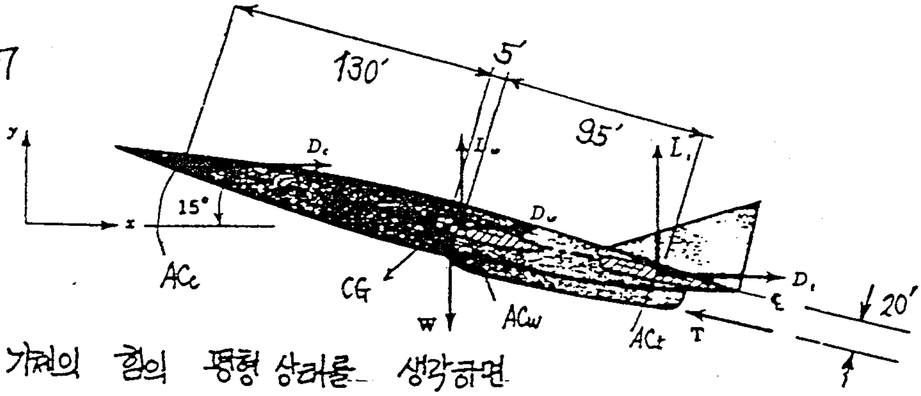
$\Sigma F_y = 0$; $F\cos\theta + f\cdot F\sin\theta = P$

$\therefore F = P\cos\theta$ $f = \tan\theta = 0.15$ $\therefore \theta = 8.53$

$e = \frac{dh}{2}\sin\theta = \frac{0.255}{2}\sin 8.53$

$= 0.0189$ (in)   Ans.) $e = 0.0189$(in)

1.47



가체의 힘의 평형 상태를 생각하면

$\Sigma F_x = 0$; $D_c + D_w + D_t = T\cos 15°$ ···· ①

$\Sigma F_y = 0$; $L_w + L_t + T\sin 15° = W$ ···· ②

$\Sigma M_{AC_w} = 0$; $D_c \times 135 \sin 15° - W \times 5 \cos 15 - L_t \times 95 \cos 15°$

$- D_t \times 95 \sin 15° + T \cdot 20 = 0$ ···③ 이다.

또한 주어진 값은 $(L/D)_t = 1.2$, $T = 160,000\ lb$

$W = 400,000\ lb$, $D_c = 100\ lb$ 이므로 위식에 대입.

$D_w + D_t = 154448$ ···· ①′

$L_w + L_t = 358589$ ···· ②′

$L_t \cos 15° + D_t \sin 15° = 13385.7$ ···③′

③′에 $L_t = 1.2 D_t$를 대입하면

$(1.2\cos 15° + \sin 15°)\ D_t = 13385.7$

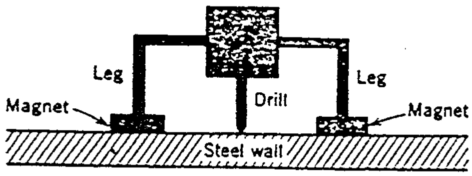
$\therefore\ D_t = 9440.3\ lb$, $D_w = 144507.7\ lb$

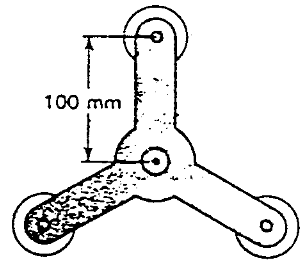
$L_t = 11328.4\ lb$, $L_w = 347260.6\ lb$

구하고자 하는 lift-to-drag ratio $(L/D)_w$는

$$(L/D)_w = \frac{347260.6}{144507.7} \cong 2.4$$

Ans.) $(L/D)_w = 2.4$

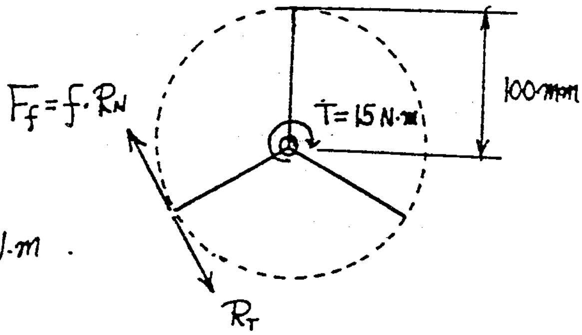




$T = 15\ N \cdot m$

수직력 $= 50\ N$

$f = 0.4$



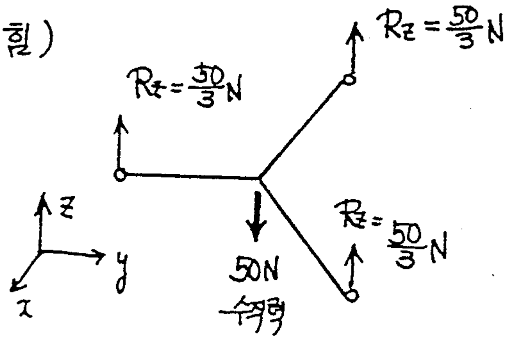
우선 드릴의 회전에 의한 반력 R_T에 대한 평형식

$T = 3 R_T \times 0.1 = 15\ N \cdot m$

$\therefore R_T = 50\ N$

마찰력 $F_f = f \cdot R_N$

(R_N ; 벽에 부착되는 힘)



그런데 드릴에 가해지는 수직력 50N으로 인한 leg에서의 반력 R_z는

$R_z = \frac{50}{3} N$

$\therefore R_N = F_M - R_z$

(단, F_M ; 자력에 의해 부착되는 힘)

결국 $F_f > R_T$ 이어야 하므로

$f \times (F_M - R_z) \geq R_T$ $\quad \therefore F_M \geqq \frac{50}{0.4} + \frac{50}{3}$

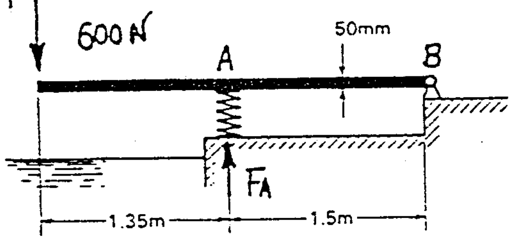
$F_M \geqq 141.67\ (N)$

따라서, 자력이 141.67 N 이상이어야 드릴이 가능하다

Ans.) 141.67 N

CHAPTER. 2.

2.1



[Given $k = 35000\ (N/m)$]

$\Sigma M_B = 0$;

$(1.35 + 1.5) \times 600 = 1.5 F_A$

$\therefore F_A = 1140 N$

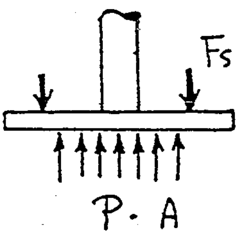
$F_A = k\,\delta_A$

$\therefore \delta_A = \dfrac{F_A}{k} = \dfrac{1140}{35000} = 0.03257$ (m)

널판의 강도가 강한 것과 약한 것을 비교하여 보면 강도가 약한 것은 굽힘에 의해서 moment-arm이 작아지기 때문에 실제로 처짐은 강한 것보다 작아지게 되나 그 영향은 diving board의 경우 무시 할 수 있다.

Ans.) $\delta = 3.257$ cm

2.2



압축된 스프링의 압축력

$F_s = 120{,}000 \times \dfrac{(250 - 207)}{1000}$

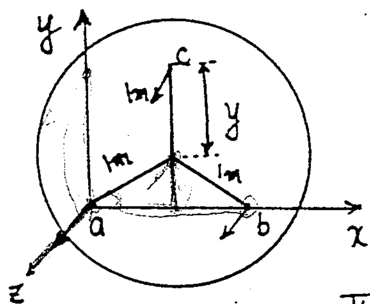
용기내 압력이 밸브를 미는 힘

$F_z = P \cdot A = P \cdot \dfrac{\pi}{4} (0.05)^2$

$\therefore P = \dfrac{120{,}000 \times \frac{43}{1000}}{\frac{\pi}{4}(0.05)^2} = 2627966$ (N/m^2)

Ans.) $P = 2.628\ MN/m^2$

2.3



$k_a = k_b = 14\ KN/m$

$k_c = 16\ KN/m$ $\quad W = 1.1 KN$

$\Sigma F_z = 0$; $\quad W = 2 k_a \delta_a + k_c \delta_c$

$\Sigma M_x = 0$;

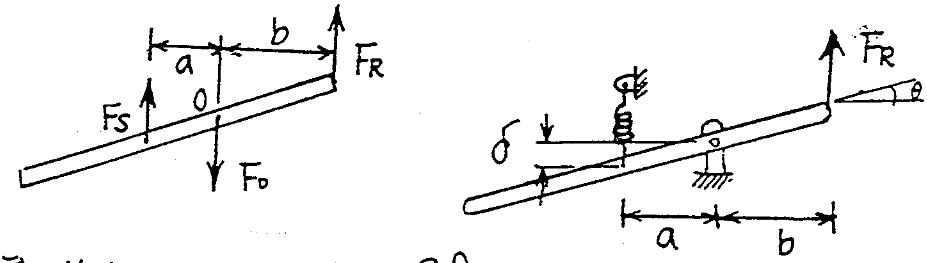
$W \sin 30° = k_c \delta_c (\sin 30° + y)$

$\delta_a = \delta_c$ 이므로 $\quad W\sin 30° = \frac{k_c W}{2k_a + k_c}(\sin 30° + y)$

$$\therefore y = \frac{2k_a}{k_c}\sin 30° = \frac{2\times 14}{16}\times\frac{1}{2} = 0.825\ (m)$$

Ans.) 0.825 m

2.4 만약 θ가 충분히 작다면 $\quad \delta \approx a\theta \quad (\because \theta \ll 1)$



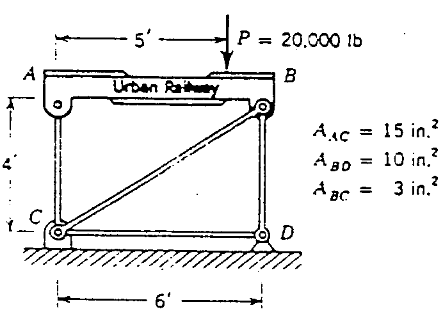
힘-변형 관계식은 $\quad F_s = k\delta$

모우멘트에 관한 평형을 생각하면 $\quad \Sigma M_o = 0$, 곧 $\quad aF_s = bF_R$

위 식에서 $\quad bF_R$이 상수이므로 aF_s도 역시 일정한 값 임을 알 수 있다. 따라서 가해지는 힘의 크기는 스프링의 탄성 계수나 그 위치에 따라 변하는 것이 아니며 작동자가 힘을 적게 들이려면 레버의 길이를 길게하여 0점으로부터 팔의 작용 위치를 멀게하여야 한다.

이때 가하여 지는 힘이 감소되어 잠은 하는 일이 감소되는 것을 의미하는 것이 아니고 다만 작동시 편리함을 덜어 줄수 있을 뿐이다.

2.5 탄성 계수 $E = 30\times 10^6$ psi 라 하자.



$$\begin{cases} F_{AC} = \frac{1}{6}P \\ F_{BD} = \frac{5}{6}P \\ F_{BC} = 0 \end{cases}$$

$\delta = \frac{FL}{EA}$ 로 부터

δ_{AC}와 δ_{BD}를 구하면

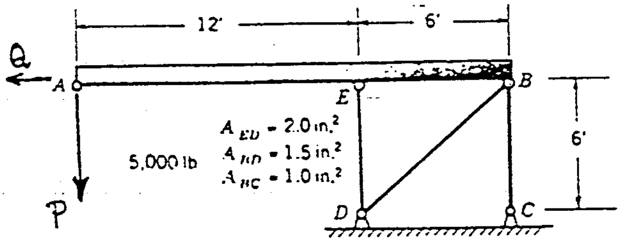
$$\delta_{AC} = \left(\frac{FL}{EA}\right)_{AC} = \frac{\frac{1}{6}PL}{15E} = \frac{20{,}000 \times 4 \times 12}{90 \times 30 \times 10^6} = 3.56 \times 10^{-4}\ (in)$$

$$\delta_{BD} = \left(\frac{FL}{EA}\right)_{BD} = \frac{PL}{12E} = \frac{20{,}000 \times 4 \times 12}{12 \times 30 \times 10^6} = 2.67 \times 10^{-3}\ (in)$$

$$\theta = \frac{\delta_{BD} - \delta_{AC}}{6 \times 12} = \frac{(26.7 - 3.56) \times 10^{-4}}{6 \times 12} = 3.2 \times 10^{-5}\ (rad)$$

Ans.) $\theta = 3.2 \times 10^{-5}$ rad.

2.6 a)



$\Sigma M_B = 0$;

$6F_{DE} = 18 \times 5000$

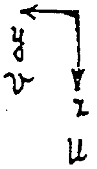
$\therefore F_{DE} = 15{,}000\ (lb)$

(압축력)

$\Sigma F_y = 0$;

$5000 = 15000 + F_{BC}$ $\therefore F_{BC} = -10000\ lb$

(인장력)



$\Sigma F_x = 0$; $\therefore F_{BD} = 0$

(∵ x방향의 수평성분 힘이 존재하지 않으므로)

b) A점에서의 수직 및 수평 변위를 알기 위해서는 각 부재의 변형에 의한 A점의 변위를 고려하여야 하므로 $\delta = \sum_{k=1}^{n} \frac{F_k L_k}{E A_k} \frac{\partial F_k}{\partial P}$ 로 표현 할 수 있다.

(단, F_k는 영향력을 받는 부재에 가해지는 외력, P는 구하고자 하는 변위 방향의 힘 성분)

AB 부재가 충분히 강하므로 굽힘에 의한 처짐은 무시하면 A점의 수직 및 수평 처짐은 다음 표와 같다.

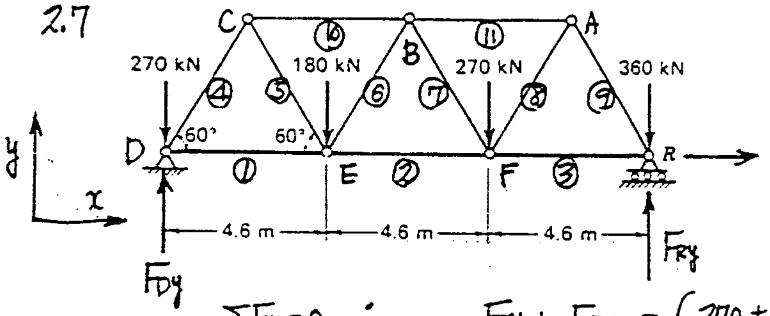
	F	L/EA	$\frac{\partial F}{\partial P}$	$\frac{\partial F}{\partial Q}$	$u = \frac{FL}{EA} \cdot \frac{\partial F}{\partial P}$	$v = \frac{FL}{EA} \frac{\partial F}{\partial Q}$
ED	-15000	1×10^{-7}	-3	0	4.5×10^{-3}	0
BC	10000	2×10^{-7}	2	1	4.0×10^{-3}	2×10^{-3}
BD	0	1.89×10^{-7}	0	$-\sqrt{2}$	0	0
	$E = 30 \times 10^6$ psi				8.5×10^{-3}	2.0×10^{-3}

따라서, 수직 변위 $\delta_V = u = 8.5\times10^{-3}$ ft $= 0.102$ in (↓)

수평 변위 $\delta_H = v = 2\times10^{-3}$ ft $= 0.024$ in (←)이다

Ans.) $\delta_V = 0.102$ in, $\delta_H = 0.024$ in

2.7



$A = 3250\ mm^2$

$E = 30\times10^{-6}$ psi

$\Sigma F_y = 0$; $F_{Dy} + F_{Ry} = (270 + 180 + 270 + 360)$

$\Sigma M_R = 0$;

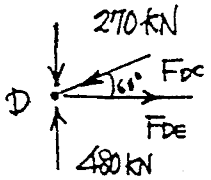
$$F_{Dy} \times (3\times4.6) = 270\times(4.6\times3) + 180\times(2\times4.6) + 270\times4.6$$

$\therefore F_{Dy} = 480\ kN$, $F_{Ry} = 600\ kN$

변형 후의 각 점을 ′로 표현하자. (D는 고정)

$$\delta_R = R'R \cong DR' - DR = (DE' - DE) + (E'F' - EF) + (F'R' - FR)$$

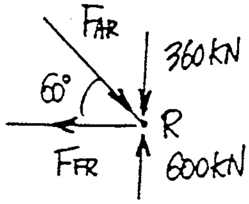
$$= \delta_{DE} + \delta_{EF} + \delta_{FR}$$



$\Sigma F_y = 0$; $480 = 270 + F_{DC}\sin60°$

$\Sigma F_x = 0$; $F_{DC}\cos60° = F_{DE}$

$\therefore F_{DE} = \frac{210}{\sin60°}\cdot\cos60° = 121.24\ kN$



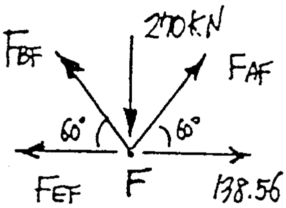
$\Sigma F_y = 0$; $600 = 360 + F_{AR}\sin60°$

$\Sigma F_x = 0$; $F_{AR}\cos60° = F_{FR}$

$\therefore F_{AR} = 277\ (kN)$, $F_{FR} = 138.56\ kN$

또한 F점에서 힘의 평형을 생각하면

$F_{AF} = F_{AR} = 277\ kN$ 이므로



$\Sigma F_y = 0$: $277\sin60° + F_{BF}\sin60° = 270$

$\Sigma F_x = 0$: $F_{EF} = 138.56 + 277\sin60° - F_{BF}\sin60°$

위 두식을 연립하면

$F_{BF} = 34.64\ kN$, $F_{EF} = 259.81\ kN$ 이다.

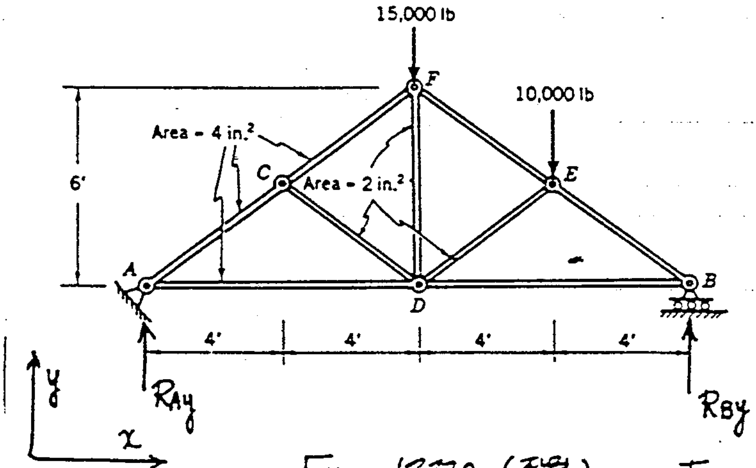
$\delta = FL/EA$ 이므로 수평 변위 δ_H는

$$\delta_H = 6.9 \times 10^{-6}\ (121.24 + 138.56 + 259.81)$$
$$= 3.588 \times 10^{-3}\ (m)$$

$\therefore\ \delta_H = 3.588\ mm$ (오른쪽으로)

Ans.) 오른쪽으로 3.588 mm 이동

2.8



$\Sigma F_y = 0$;

$R_{Ay} + R_{By} = 25000$

$\Sigma M_A = 0$;

$15000 \times 8 + 12 \times 10000$

$= 16 R_{By}$

$\therefore\ R_{By} = 15000$

$R_{Ay} = 10000$

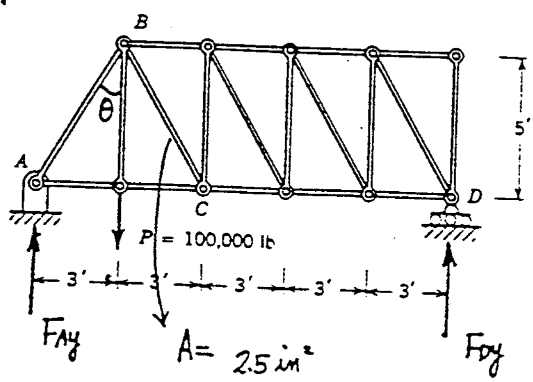
예제 풀이를 이용하면

$F_{AD} = 13330$ (장력) $\quad F_{BD} = 20000$ (장력)

$$\delta_H = \delta_{AD} + \delta_{BD}$$
$$= \frac{L}{EA}\ (F_{AD} + F_{BD}) = \frac{8}{4 \times 10 \times 10^6} \times (13330 + 20000)$$
$$= 6.666 \times 10^{-3}\ (ft) = 0.079\ in$$

Ans.) $\delta_H = 0.08\ in$

2.9



$\Sigma M_D = 0$; $F_{Ay} \times 15$

$= P \times 12$

$\therefore\ F_{Ay} = 80,000\ lb$

$\Sigma F_y = 0$:

$F_{Ay} + F_{Dy} = 100,000$

$\therefore\ F_{Dy} = 20,000\ lb$

점 B에서의 힘의 평형을 고려하면 $F_{Ay} + F_{BC}\cos\theta = P$

$$F_{BC} = \frac{20{,}000}{\cos\theta} = 23323.8\,(lb) \quad \left(\because \cos\theta = \frac{5}{\sqrt{3^2+5^2}}\right)$$

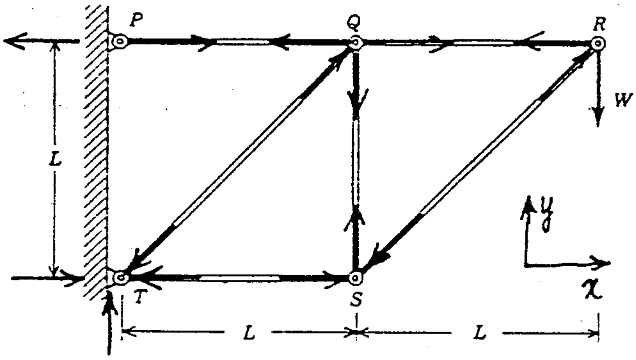
(압축력)

$$\delta_{BC} = \frac{F_{BC}L}{AE} = \frac{23323.8 \times \sqrt{5^2+3^2}}{2.5\times 30\times 10^6} = 0.02176\ (in)$$

수축.

Ans.) $\delta_{BC} = 0.02176$ in 감소

2.10



a) R점에서

$\Sigma F_y = 0$;

$F_{RS}\sin 45 = W$

$\therefore F_{RS} = \sqrt{2}\,W$

$\Sigma F_x = 0$;

$F_{RQ} = \cos 45\, F_{RS}$

$= W$

Q점에서 $\Sigma F_x = 0$; $F_{QR} + F_{QT}\cos 45 = F_{PQ}$ ⋯ ①

$\Sigma F_y = 0$; $F_{QT}\sin 45 = F_{QS}$ ⋯ ②

S점에서 $\Sigma F_x = 0$; $F_{ST} = F_{SR}\cos 45° = W$ ⋯ ③

$\Sigma F_y = 0$; $F_{QS} = F_{SR}\sin 45° = W$ ⋯ ④

위 비식을 각각 풀면 $F_{PQ} = 2W\ (T)$ $F_{QR} = W(T)$

$F_{TQ} = \sqrt{2}\,W\ (C)$, $F_{QS} = W\ (T)$, $F_{TS} = W\ (C)$ $F_{SR} = \sqrt{2}\,W(C)$

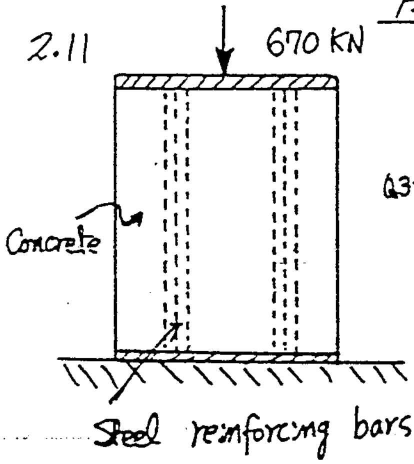
b)

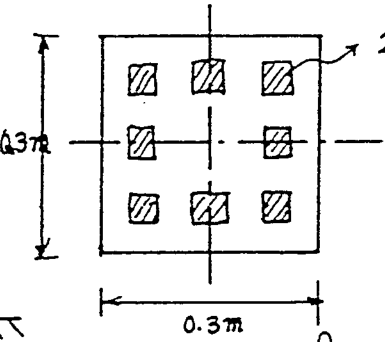
	F	L/EA	$\frac{\partial F}{\partial W}$	$\frac{FL}{EA}\frac{\partial F}{\partial W}$
PQ	$2W$	L/EA	2	$4\frac{WL}{EA}$
QR	W	〃	1	$\frac{WL}{EA}$
QT	$-\sqrt{2}W$	$\sqrt{2}L/EA$	$-\sqrt{2}$	$2\sqrt{2}\frac{WL}{EA}$
QS	W	L/EA	1	$\frac{WL}{EA}$
ST	$-W$	L/EA	-1	$\frac{WL}{EA}$
RS	$-\sqrt{2}W$	$\sqrt{2}L/EA$	$-\sqrt{2}$	$2\sqrt{2}\frac{WL}{EA}$

$$\delta_v = \sum_{k=1}^{n} \frac{F_k L_k}{EA_k}\frac{\partial F_k}{\partial W} = 12.657\frac{WL}{EA}$$

Ans.) $\delta_v = 12.657\frac{WL}{EA}$

2.11





25×25 mm

$E_c = 17\ GN/m^2$ for Concrete

$\delta_c = \left(\frac{F}{EA}\right)_c L$

$\delta_s = \left(\frac{F}{EA}\right)_s L$

$\delta_c = \delta_s$ (∵ 서로 밀착되어 있음)

$A_c = 0.085\ m^2$, $A_s = \frac{0.005}{8}\ m^2$

$F_c + 8F_s = 670\ KN$ ···· ①

$$\frac{F_c \times L}{17\times10^9\times0.085} = \frac{F_s \times L}{200\times10^9\times\frac{0.005}{8}} \cdots ② \quad \therefore F_c = 11.56\,F_s. \quad (\because E_s = 200\ GN/m^2)$$

①식으로부터 $(11.56 + 8)F_s = 670\ KN$ $\therefore F_s = 34.25\ KN$

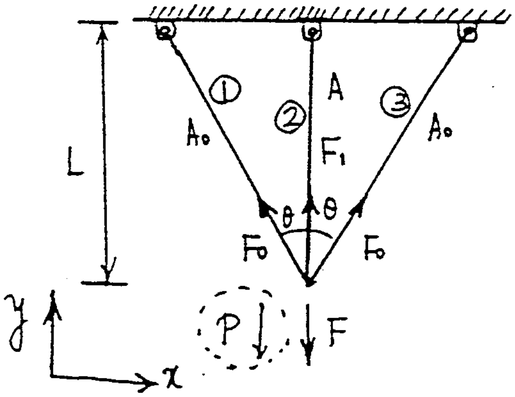
②에 대입하면 . $F_c = 3960\ KN$

$\sigma_s = F_s/A_s = 34.25/\frac{0.005}{8} = 54800\ KN/m^2$

$\sigma_c = F_c/A_c = \ \ /0.085 = 4658.8\ KN/m^2$

Ans.) $\sigma_s = 54.8\ MN/m^2$, $\sigma_c = 4.66\ MN/m^2$

2.12



$\Sigma F_y = 0$;

$F_1 + 2F_0\cos\theta = F$ ···①

bar ①, ③ 만이 F를 받는다면

$$\delta_1 = \frac{\frac{F}{2\cos\theta}\times L}{EA_0\cos\theta}\ \frac{1}{2\cos\theta}\times 2$$

$$\left(\because \delta = \frac{FL}{EA}\frac{\partial F}{\partial P}\right)$$

bar ②가 작용하므로 이에 작용하는 힘 F_1과 대응하는 처짐 δ_2를 구하여 보면 $\delta_2 = \dfrac{F_1 \cdot L}{2EA_0 \cos^3\theta}$

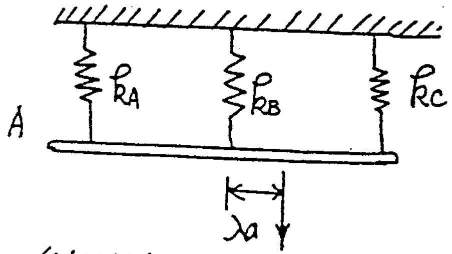
$$\delta_1 - \delta_2 = \frac{F_1 \cdot L}{EA} \Rightarrow \frac{F \cdot L}{2EA_0\cos^3\theta} = \frac{F_1 \times L}{EA}$$

$$\therefore \; AF = (A + 2A_0\cos^3\theta)F_1 \quad \cdots ②$$

①, ②에서 $F_0 = \dfrac{A \cdot F}{A + 2A_0\cos^3\theta}$, $F_1 = \dfrac{FA_0\cos^2\theta}{A + 2A_0\cos^3\theta}$

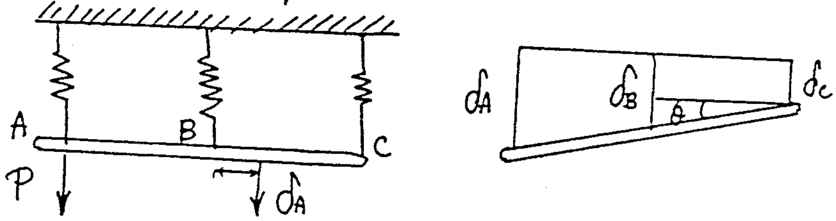
Ans.) $F_0 = \dfrac{AF}{A + 2A_0\cos^3\theta}$, $F_1 = \dfrac{FA_0\cos^2\theta}{A + 2A_0\cos^3\theta}$

2.13 a)



$$\delta_A = \frac{P[2k_C - \lambda(k_B + 2k_C)]}{k_Ak_B + 4k_Ak_C + k_Bk_C}$$

b)



$$k_A\delta_A + k_B\delta_B + k_C\delta_C = P, \qquad \delta_A + \delta_C = 2\delta_B$$

$$\Sigma M_C = 0 \; ; \qquad 2k_A\delta_A + k_B\delta_B = 2P$$

$$\begin{bmatrix} k_A & k_B & k_C \\ 1 & -2 & 1 \\ 2k_A & k_B & 0 \end{bmatrix} \begin{bmatrix} \delta_A \\ \delta_B \\ \delta_C \end{bmatrix} = \begin{bmatrix} P \\ 0 \\ 2P \end{bmatrix}$$

$$\det. A = \begin{vmatrix} k_A & k_B & k_C \\ 1 & -2 & 1 \\ 2k_A & k_B & 0 \end{vmatrix} = k_Ak_B + k_Bk_C + 4k_Ak_C$$

$$\delta_A = \frac{1}{\det.A}\begin{vmatrix} P & k_B & k_C \\ 0 & -2 & 1 \\ 2P & k_B & 0 \end{vmatrix} = \frac{1}{\det A}(2k_BP + 4k_CP - k_BP)$$

$$\therefore\ \delta_A = \frac{2k_BP + 4k_CP - k_BP}{k_Ak_B + k_Bk_C + 4k_Ak_C} = \frac{(k_B + 4k_C)P}{k_Ak_B + k_Bk_C + 4k_Ak_C}$$

$$\delta_c = \frac{1}{\det.A}\begin{vmatrix} k_A & k_B & P \\ 1 & -2 & 0 \\ 2k_A & k_B & 2P \end{vmatrix} = \frac{1}{\det A}(-4k_AP + k_BP + 4k_AP - 2k_BP)$$

$$= \frac{-k_B \cdot P}{k_Ak_B + k_Bk_C + 4k_Ak_C}$$

$$\theta = \frac{\delta_A - \delta_c}{2a} \qquad \therefore\ \delta = \delta_c + (1-\lambda)a\theta$$

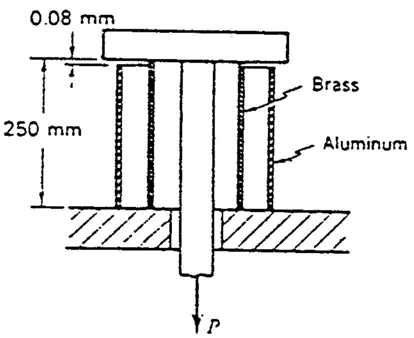
$$\therefore\ \delta = \frac{1-\lambda}{2}\delta_A + \frac{1+\lambda}{2}\delta_c$$

$$= \frac{P}{k_Ak_B + k_Bk_C + 4k_Ak_C}\left[-k_B + \frac{1-\lambda}{2}(4k_C + 2k_B)\right]$$

$$= \frac{P}{k_Ak_B + k_Bk_C + 4k_Ak_C}\left[2k_C - \lambda(k_B + 2k_C)\right]$$

— Q.E.D —

2.14



$$A = \pi r^2 - \pi(r-t)^2$$
$$= \pi(2r-t)t$$
$$= \pi(D-t)t$$

r: 반경, D: 직경, t: 두께.

$$A_{Al} = \pi(0.25 - 0.00625) \times 0.00625$$
$$= 4.79 \times 10^{-3} \quad (m^2)$$

$$A_{Br.} = \pi(0.15 - 0.00625) \times 0.00625 = 2.82 \times 10^{-3} \quad (m^2)$$

$E_{Al} = 69 \times 10^9\ N/m^2$, $E_{Br} = 103 \times 10^9\ N/m^2$ 라고 놓으면 하중-처짐 곡선은 다음과 같이 구하여진다.

i) $0 \leq \delta \leq 8 \times 10^{-5} (m)$

$$\delta = \frac{PL_B}{E_B A_B} = \frac{P \times 0.25}{(103 \times 10^9) \times (2.82 \times 10^{-3})} = 8.61 \times 10^{-10} P$$

$$\therefore P = 1.16 \times 10^9 \delta$$

ii) $\delta \geq 8 \times 10^{-5} (m)$

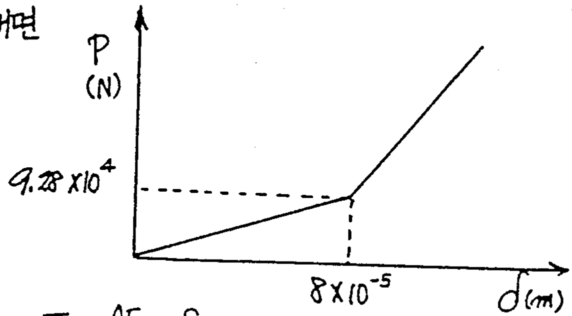
$$P = \frac{E_B A_B}{L_B} \delta + \frac{E_A A_A}{L_A} (\delta - 8 \times 10^{-5})$$

$$= \frac{(103 \times 10^9)(2.82 \times 10^{-3})}{0.25} \delta + \frac{(69 \times 10^9)(4.79 \times 10^{-3})}{(0.25 - 8 \times 10^{-5})} (\delta - 8 \times 10^{-5})$$

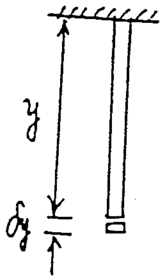
$$= 2.48 \times 10^9 \delta - 105797.1$$

그래프로 그려 나타내면
오른쪽 그림과 같다.

Ans.)



2.15



$$\delta = \frac{FL}{AE} \qquad F = \frac{AE}{L} \cdot \delta$$

$$\therefore k = \frac{AE}{L}.$$

$$\left[\begin{array}{l} k = 10^6 \ lb/in \\ w = 1.6 \ lb/ft \end{array} \right.$$

$d\delta = \frac{wy}{EA} dy$ 이므로 $\quad \delta = \int_0^L \frac{wy}{EA} dy = \frac{wL^2}{2EA}$

$AE = kL$ 이므로 $\quad \therefore \delta = \frac{wL}{2k}$ 이다.

여기서 k라 함은 5980 ft rope 전체의 Spring Const 이므로

$$k = \frac{10^6}{5980} \ (lb/in) \qquad \therefore \delta = \frac{(1.6 \times \frac{1}{12}) \times (5980 \times 12)}{2 \times (10^6/5980)}$$

$$= 28.6 \ in = 2.384 \ ft$$

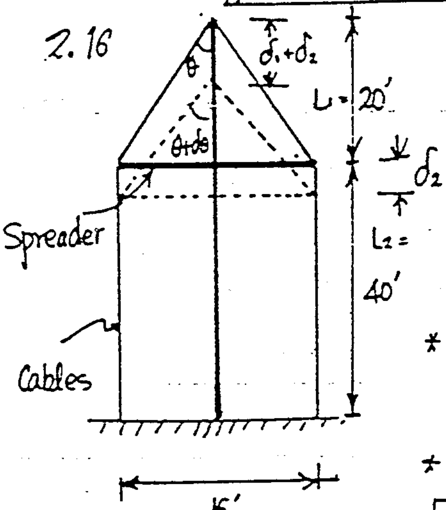
$$6000 - (5980 + 2.384) = 17.616 \ ft.$$

6'의 사람이 매달린다면 약 12 ft 가량이 모자란다 이를 rope 끝에 W의 무게를 달아 줄을 늘여 뜨린다면

$$W = k \cdot \delta' = \frac{10^6}{5980} \times 12 \times 12 = 24080\ (lb)$$

Ans.) 24080 lb 무게의 추를 로프 끝에 매달아 놓는다.

2.16



* 가정

i) Spreader는 견고히 mast에 부착된다

ii) Cable과 spreader 사이에는 마찰이 없다.

iii) 감판은 견고하다.

iv) turnbuckle의 길이는 무시할 수 있다.

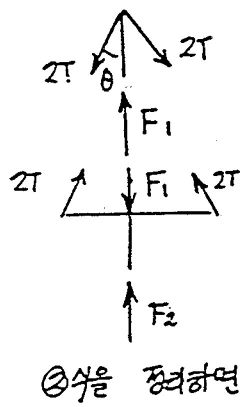
* 변형 전

$$\sqrt{L_1^2+8^2} + L_2 = L_3 \quad \cdots ①$$

* 변형 후

$$\sqrt{(L_1-\delta_1)^2+8^2} + (L_2-\delta_2) = L_3-\delta_3 \quad \cdots ②$$

또한, $\delta_3 = \delta_3' - \delta_3''$ 이다. (δ_3' ; turnbuckle의 조임에 의한 길이 감소, δ_3'' ; 장력에 의한 길이 증가)



Mast 위에서 평형식

$$F_1 = 2T\cos\theta \times 2 = 4T\cos\theta \quad \cdots ③$$

Mast 아래에서의 평형식

$$\Sigma F_y = 0 ; \quad 4T\cos\theta - 4T - F_1 + F_2 = 0 \quad \cdots ④$$

③, ④로부터 $T = F_2/4$, $F_1 = F_2\cos\theta$

$(1+\epsilon)^n \cong 1+n\epsilon$ (단, $\epsilon \ll 1$)를 이용하여

②식을 정리하면 $\sqrt{L_1^2+8^2} - \dfrac{\delta_1 L_1}{\sqrt{L_1^2+8^2}} + L_2 + \delta_2 = L_3 - \delta_3$

①식을 사용하면 $-\dfrac{\delta_1 L_1}{\sqrt{L_1^2+8^2}} = \delta_2 - \delta_3$ or $-0.93\,\delta_1 = \delta_2 - \delta_3 \quad \cdots ⑤$

$$\delta_1 = \frac{F_1 L_1}{A_1 E_1} = \frac{F_2\cos\theta \times 20}{20 \times 1.6 \times 10^6} = 5.8 \times 10^{-7}\, F_2 \quad (ft)$$

마찬가지로 $\delta_2 = 12.5 \times 10^{-7}\ F_2\ (ft)$

$$\delta_3' = \frac{15}{20} \times \frac{1}{12} = 0.0625\ ft.$$

$$\delta_3'' = \frac{T L_3}{12 \times 140{,}000} = 3.67 \times 10^{-5}\ T\ (ft) = 9.18 \times 10^{-6} F_2\ (ft)$$

$$\delta_3 = \delta_3' - \delta_3'' = 0.0625 - 9.18 \times 10^{-6}\ F_2$$

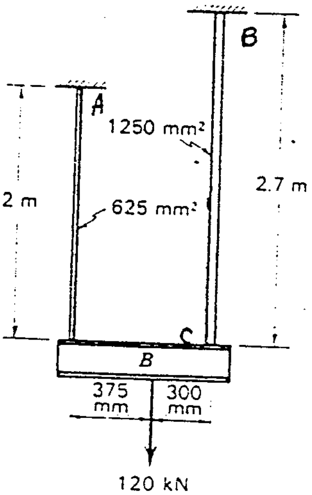
④식에 $\delta_1, \delta_2, \delta_3$를 각각 대입하면

$$-(0.93)(5.8 \times 10^{-7}) F_2 = 12.5 \times 10^{-7} F_2 - 0.0625 + 9.18 \times 10^{-6} F_2$$

$$\therefore\ F_2 = 5700\ lb$$

Ans.) $F_2 = 5700\ lb$

2.17



$E_s = 200\ GN/m^2$

$\Sigma M_C = 0$; $F_A \times 675 = 120{,}000 \times 300$

$\therefore F_A = 53333 N$, $F_B = 66667 N$

$$\delta_A = \frac{53333 \times 2}{(200 \times 10^9)(625 \times 10^{-6})} = 8.122 \times 10^{-4} (m)$$

$$\delta_C = \frac{66667 \times 2.7}{(200 \times 10^9)(1250 \times 10^{-6})} = 6.853 \times 10^{-4} (m)$$

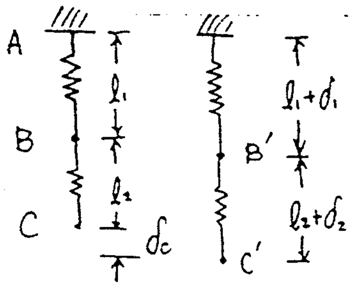
$$\therefore \delta_B = \delta_C + \frac{\delta_A - \delta_C}{L_{AC}} L_{BC} = 6.853 \times 10^{-4}$$

$$+ \frac{300}{675}(8.122 - 6.853) \times 10^{-4}$$

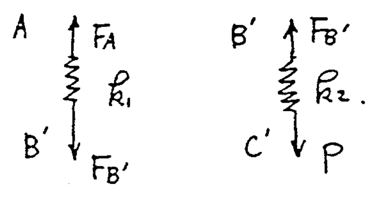
$$= 7.417 \times 10^{-4}\ (m)$$

Ans.) $\delta_B = 7.417 \times 10^{-4}\ m.$

2.18.



AB'의 스프링. B'C'의 스프링.



평형식 $B'C'$; $F_{B'} = P$, AB' ; $F_A = F_{B'} = P$

힘-처짐식 $\delta_1 = \frac{P}{k_1}$, $\delta_2 = \frac{P}{k_2}$

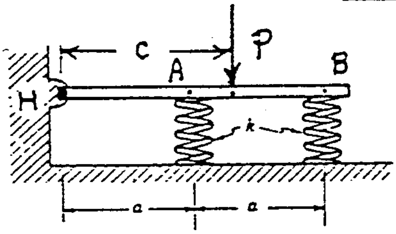
기하학적 적합조건식 ; C의 변위 $= (l_1+\delta_1)+(l_2+\delta_2) - (l_1+l_2)$

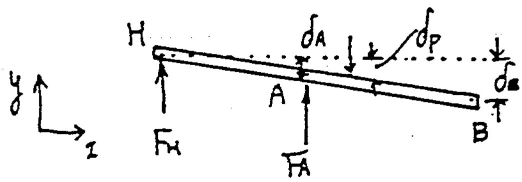
$= \delta_1 + \delta_2 = P\left(\frac{1}{k_1}+\frac{1}{k_2}\right)$

전체 스프링 constant $= \frac{P}{\delta_c} = \frac{k_1 k_2}{k_1+k_2}$

Ans.) $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$

2.19





기하학적 조건으로 부터

$\delta_A = \frac{1}{2}\delta_B$, $\delta_P = \frac{c}{2a}\delta_B$.

힘-변형 관계 ;

$F_A = k\delta_A = \frac{k}{2}\delta_B$

$F_B = k\delta_B$

$\Sigma M_{Hz} = 0$;

$P\cdot c = F_A\cdot a + F_B\cdot 2a = \frac{5}{2}k\,a\,\delta_B$

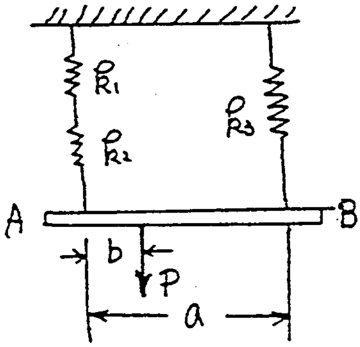
$\therefore\ P = \frac{5}{2}k\,\delta_B\frac{a}{c}$

$\frac{P}{\delta_P} = \frac{\frac{5}{2}k\delta_B\frac{a}{c}}{\frac{c}{2a}\delta_B} = 5\frac{a^2}{c^2}k$, $\frac{P}{\delta_P} = \frac{20}{9}k$ 이라면

$5\frac{a^2}{c^2}k = \frac{20}{9}k$ $\therefore\ c = 1.5a$.

Ans.) $c = 1.5a$.

2.20.



문제(18)로 부터 k_1, k_2의 합성 스프링상수 $k_4 = \frac{k_1 k_2}{k_1+k_2}$

A.B에 주어지는 힘을 각각 F_A, F_B라 할때

$F_A = k_4\delta$ ....①

$F_B = k_3\delta$ ....②

평형식 ; $F_A + F_B = P$, $F_A\cdot a = P(a-b)$

$\therefore\ F_A = P\left(1-\frac{b}{a}\right)$, $F_B = P\frac{b}{a}$

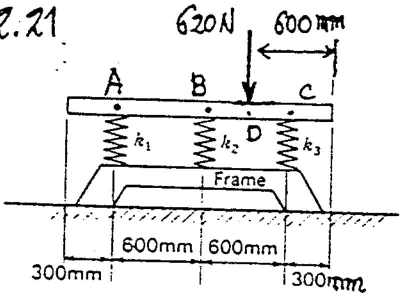
①, ②로 부터 $\frac{F_A}{k_4} = \frac{F_B}{k_3}$ or $\frac{(b-a)}{a k_4} = \frac{b}{a k_3}$

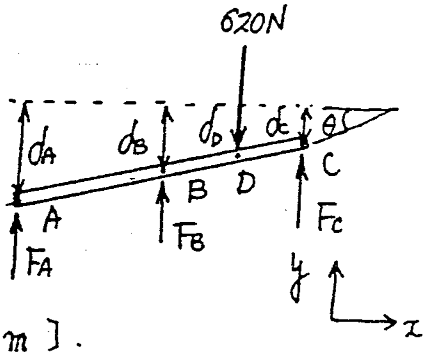
곧, $\frac{b}{a} = \frac{k_3}{k_3+k_4} = \frac{k_3(k_1+k_2)}{k_1k_2+k_3(k_1+k_2)}$ 이며

$$\delta = \frac{P(k_1+k_2)}{k_1k_2+k_2k_3+k_3k_1}$$

Ans.) $b = \frac{(k_1+k_2)k_3\ a}{k_1k_2+k_2k_3+k_3k_1}$, $\delta = \frac{(k_1+k_2)P}{k_1k_2+k_2k_3+k_3k_1}$

2.21





$\delta_B = \delta_A - 0.6\,\theta$

$\delta_D = \delta_A - 0.9\,\theta$

$\delta_C = \delta_A - 1.2\,\theta$

$\left\{ \begin{array}{l} \delta : [m]. \\ \theta : [radian] \end{array} \right.$

$k_1 = k_3 = 17\,kN/m, \quad k_2 = 21\,kN/m$

힘-처짐 관계식 $F_A = k_1\delta_A$, $F_B = k_2\delta_B$. $F_C = k_3\delta_C$ 을

위식에 대입 $F_A = 17\times10^3\,\delta_A$, $\cdots$①

$F_B = 21\times10^3\,(\delta_A - 0.6\theta)$ $\cdots$②

$F_C = 17\times10^3\,(\delta_A - 1.2\theta)$ $\cdots$③

$\Sigma F_y = 0$; $F_A + F_B + F_C = 620\ N$

위식에 ①.②.③식 대입 $55\,\delta_A - 33\theta = 0.62$ $\cdots$④

$\Sigma M_{Dz} = 0$; $0.9F_A + 0.3F_B = 0.3F_C$

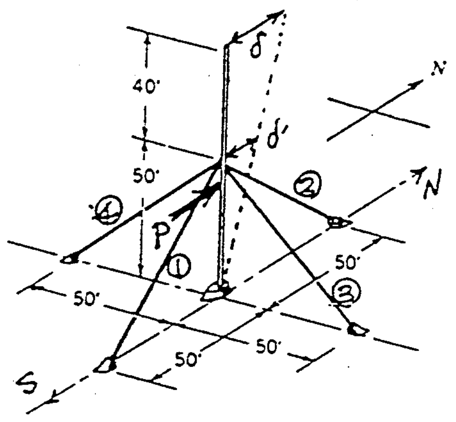
or $55\,\delta_A + 7.8\theta = 0$, $\cdots$⑤

$40.8\theta = -0.62$ $\therefore\ \theta = -0.0152$, $\delta_A = 2.155\times10^{-3}$

$F_A = 36.59\ N$ $F_B = 236.78\ N$, $F_C = 346.72\ N$

Ans.) $\theta = 0.0152$ rad (시계 방향)

2.22



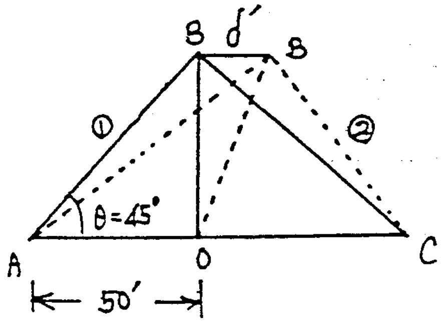
• 가정 ;

i) Ball joint는 마찰이 없고 모우먼트를 전달 받지 못한다.

ii) 깃대 는 견고하다.

iii) 깃대의 처짐은 S-N plane 내에 있다.

iv) 처짐이 상당히 작아 이에 따른 각 변화는 무시 할 수 있다.



로프 ①과 ②의 변형을 생각하면

$AB' \approx AB + BB'\cos(\angle BB'A)$

$\approx AB + BB'\cos 45° = AB + \delta'\cos 45°$

$\therefore \delta_1 =$ elongation of rope ①

$= \delta' \cos 45°$

$= \frac{5}{9} \times 0.707\,\delta$

② 로우프의 응력은 압축력이 될 수 없으므로 (∵ wire는 압축력을 받을 수 없다) ②에 걸리는 힘은 zero, 또한 δ'의 변형은 S-N plane에서 일어나므로 ③,④ 로우프의 신장도 무시할 수 있다.

힘-변위 관계식 $T_1 = A_1E_1\left(\frac{\delta_1}{L_1}\right) = \frac{\pi}{4}\left(\frac{1}{4}\right)^2 \times 30 \times 10^6 \times \frac{\frac{5}{9} \times 0.707\delta}{1.414 \times 50}$

$\therefore T_1 = 8230\,\delta$

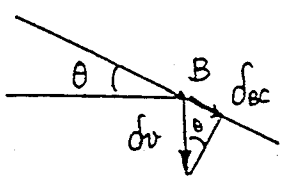
깃대의 평형식 ; $T_1\left(\frac{1}{\sqrt{2}}\right) \times 50 = 900 \times 45$

$\therefore T_1 = 1140$ lb

$8230\,\delta = 1140$ $\quad\therefore \delta = 0.139$ ft $= 1.68$ in

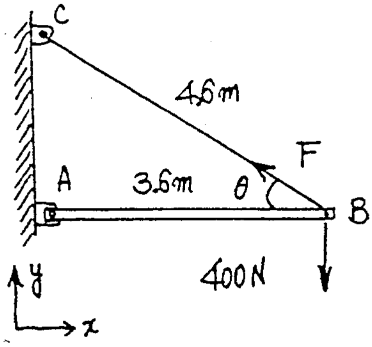
Ans.) $\delta = 1.68$ in

2.23



$\delta = \frac{WL}{EA} = \frac{400 \times 4.6}{E \cdot A} = 23 \times 10^{-3}$

$\therefore EA = 80{,}000$



$\Sigma F_y = 0$; $F \sin\theta = 400$

$\therefore F = \dfrac{4.6}{\sqrt{4.6^2 - 3.6^2}} \times 400$

$= 642.6\,N$

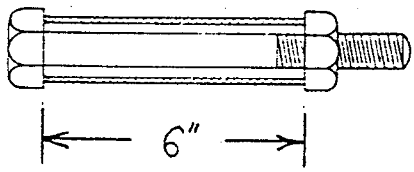
$\delta_{BC} = \dfrac{FL}{EA} = \dfrac{642.6 \times 4.6}{80{,}000}$

$= 0.0369$ (in)

$\therefore \delta_v = \dfrac{\delta_{BC}}{\sin\theta} = 0.05935$ m.

Ans.) $\delta_v = 59.35$ mm

2.24. Bolt area = 1.00 in², Sleeve area = 0.60 in²



$\delta_b = \delta_b' - \delta_b''$

$\begin{cases} \delta_b' ; \ 90° \text{ 회전에 의한 길이변화} \\ \delta_b'' ; \ \text{인장에 의한 〃} \end{cases}$

$\delta_b' = \dfrac{1}{16} \times \dfrac{1}{12} \times \dfrac{1}{4} = \dfrac{1}{768}$ (ft), $\delta_b'' = \dfrac{T \cdot L}{EA_b}$

볼트의 인장력과 Sleeve의 압축력이 같고 $\delta_b = \delta_s$ 이므로

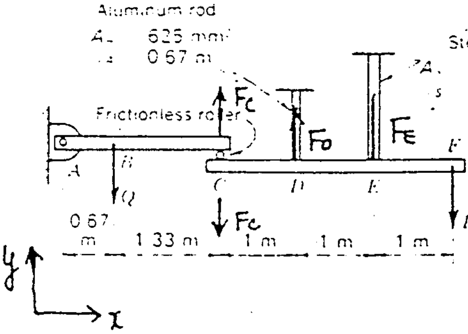
$\delta_s = \dfrac{TL}{EA_s} = \delta_b = \dfrac{1}{768} - \dfrac{TL}{EA_b}$

$\dfrac{TL}{E}\left(\dfrac{1}{A_s} + \dfrac{1}{A_b}\right) = \dfrac{1}{768}$ $\therefore T = \dfrac{E}{768L} \dfrac{A_s A_b}{A_s + A_b}$

$\therefore T = 2441.406$ lb

Ans.) $T = 2441.406$ lb

2.25



* 알루미늄 rod AC에서

$\Sigma M_A = 0$; $0.67Q = 2F_C$

$\therefore F_C = 0.335Q$

* Steel rod 에서

$\Sigma F_y = 0$; $F_C + P = F_D + F_E$

$\therefore 0.335Q = F_D + F_E - P \cdots ①$

$3P = F_D + 2F_E \cdots\cdots ②$

$$\delta_D = \frac{F_D L_A}{E_A A_A}, \quad \delta_E = \frac{F_E L_E}{E_S A_S} \qquad \delta_D = \delta_E \quad \cdots\cdots ③$$

$$\therefore \frac{F_D \times 0.67}{69 \times 10^9 \times 625} = \frac{F_E \times 1.0}{200 \times 10^9 \times 1250}$$

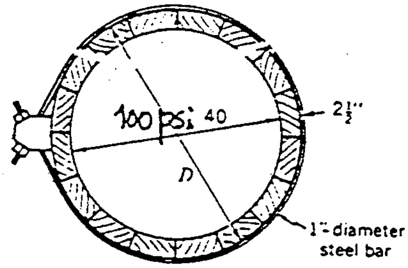
$$F_E = 3.884\ F_D \quad \cdots\cdots ④$$

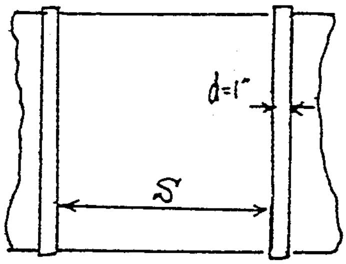
②④로부터 $F_D = 0.342\,P$, $F_E = 1.329\,P$,

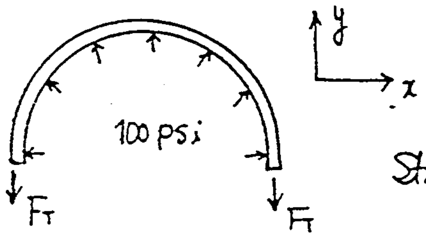
①로부터 $Q = \dfrac{(0.342 + 1.329 - 1)\,P}{0.335} = 2.003\,P$

Ans.) $Q = 2.003\,P$

2.26







$\Sigma F_y = 0$; $F_T \times 2 = 40 \times l \times 100$

$\therefore F_T = 2000\,S$ (lb)

$$\text{Strain (tangential)} = \frac{2000S}{\frac{\pi}{4}d^2 E} = \frac{2000S}{\frac{\pi}{4}(1)^2 \times 30 \times 10^6} = \frac{dL}{L}$$

$L = 2\pi R \quad \therefore dL = 2\pi\, dR \qquad \dfrac{dL}{L} = \dfrac{dR}{R} = \dfrac{2000\,l}{\frac{\pi}{4} 30 \times 10^6}$

$dR \le \left(\dfrac{0.03}{2}\right)''$ 　　한계 값 $dR = \left(\dfrac{0.03}{2}\right)''$

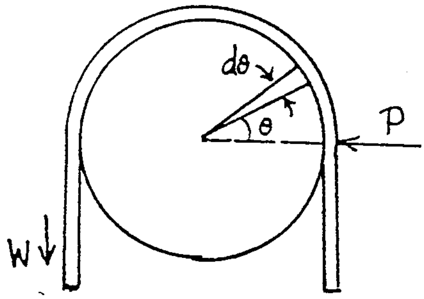
$$\therefore l = \frac{\pi}{4}\,\frac{30 \times 10^6}{2000} \times \frac{0.03/2}{22.5} = 7.65''$$

Ans.) 7.65 in

2.27. 예제 2.7로부터 $\dfrac{dT}{d\theta} = f \cdot T$

적분하면 $T = Ce^{f\theta}$ ($C = \text{const}$)

$\theta = \pi$ 이면, $T = W = Ce^{\pi f} \qquad \therefore C = We^{-\pi f}$



또한, $T = We^{f(\theta-\pi)}$ 이며

$\theta = 0$ 일때 $\quad T_0 = We^{-f\pi}$

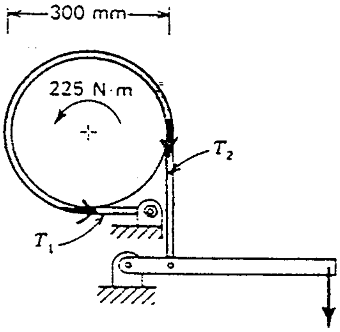
이러한 경우가 가능하려면

$f \cdot P = T_0$

$\therefore\ P = \frac{W}{f} e^{-f\pi}$

Ans.) $P = \frac{W}{f} e^{-\pi f}$

2.28



$(T_2 - T_1) \times 0.15 = 225$

$T_2 = T_1 e^{\frac{3}{3}\pi f}$

$\therefore\ T_2 = T_1 + 1500$

$T_2 = T_1 e^{0.6\pi}$

위 값을 구하면 $\quad T_1 = 268.5\,lb$

$T_2 = 1768.5\,lb$.

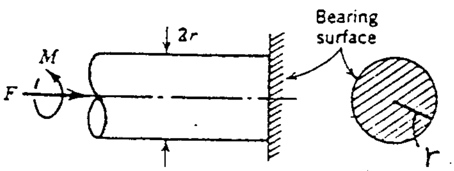
Ans.) $T_1 = 268.5\,lb \quad T_2 = 1768.5\,lb$

2.29. $\quad T = T_0 e^{f\theta} \quad (\theta = 8\pi,\ f = 0.3,\ T_0 = 200N)$

$\therefore\ T = 200\, e^{8\pi \times 0.3} = 376300\ (N)$

Ans.) $\quad T = 376.3\ kN$

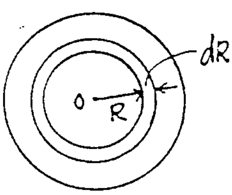
2.30



$\sigma = \frac{F}{A} = \frac{F}{\pi r^2}$

$\tau = f \cdot \sigma = \frac{F \cdot f}{\pi r^2}$

(단위 면적당 작용하는 마찰력)

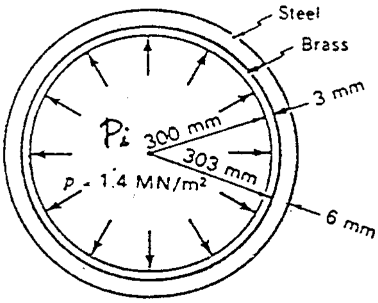


$\therefore\ dM = \tau \cdot R\, 2\pi R\, dR = \frac{2F \cdot f}{r^2} R^2 dR$

$M = \frac{2Ff}{r^2} \int_0^r R^2 dR = \frac{2f \cdot r \cdot F}{3}$

Ans.) $M = \frac{2frF}{3}$

2.31



$$P_i d_s = 2(F_B + F_S)$$

$$\therefore F_B + F_S = \frac{1}{2} P_i d_s$$

$$= \frac{1}{2} \times 1.4\times10^6 \times 0.6 \times 0.006 = 2520\,N \quad \cdots ①$$

$$\delta_B = \left(\frac{FL}{EA}\right)_B$$

$$\Delta D_B = \frac{\delta_B}{\pi} = \frac{F_B \times 0.603}{(103\times10^9)(0.003\times0.006)}$$

$$= 6.505\times10^{-7}\ \frac{F_B}{2}$$

$E_B = 103\ GN/m^2$

$E_S = 205\ GN/m^2$

$$\delta_S = \left(\frac{FL}{EA}\right)_S$$

$$\Delta D_S = \frac{\delta_S}{\pi} = \frac{F_S \times 0.612}{(205\times10^9)(0.006\times0.006)} = 1.659\times10^{-7}\ \frac{F_S}{2}$$

$\Delta D_B = \Delta D_S$; $F_B = 0.255\, F_S \quad \cdots ②$

①②로부터 $F_S = 2008\,N$, $F_B = 512\,N$

Ans.) $F_S = 2008\,N$, $F_B = 512\,N$

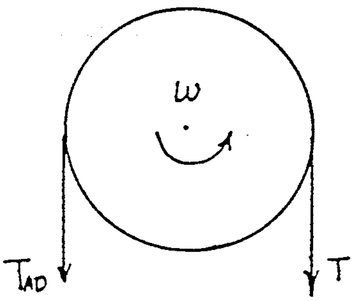
2.32 $$\Delta D_b = 6.505\times10^{-7}\ \frac{F_B}{2} = 3.33\times10^{-4}\times\frac{1}{2}\ (m)$$

$$= 0.1665\ (mm)$$

$$\therefore \Delta R_b = \frac{\Delta D_b}{2} = 0.08326\ mm$$

Ans.) $\Delta R_b = 0.08326\ mm$

2.33



$$\Delta N - T\Delta\theta = 0.$$

$$\Delta T - f\Delta N = 0$$

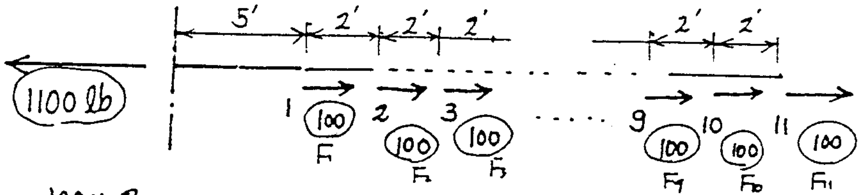
$$\therefore T = T_{AD}\, e^{f\theta}$$

$$\frac{dN}{R\,d\theta} = \frac{T\,d\theta}{R\,d\theta} = \frac{T}{R} = \frac{T_{AD}e^{f\theta}}{R}$$

$$\therefore \frac{dN}{R\,d\theta} = \frac{T}{R} = \frac{T_{AD}e^{f\theta}}{R}$$

— Q.E.D —

2.34



$$\delta_{11} = \frac{100 \times 2}{EA}$$

$$\delta_{10} = \frac{200 \times 2}{EA}$$

$$\vdots$$

$$\delta_2 = \frac{1000 \times 2}{EA}$$

$$\delta_1 = \frac{1100 \times 5}{EA}$$

~~EA를 구하기 위해서~~ $k = \frac{EA}{L}$

$$EA = kL = 29400\ lb/in \times 1\ ft$$

$$= 29400 \times 12\ \ lb$$

$$\therefore\ EA = 352800\ \ lb$$

$$\delta = \frac{1}{EA}\left[1100 \times 5 + 200(1 + \cancel{2} \cdots + 10)\right]$$

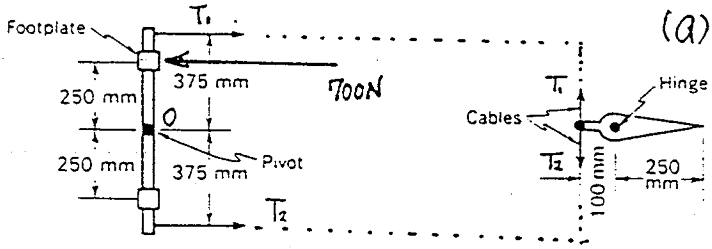
$$= 0.0468\ [ft]$$

$$\delta_{total} = 2\delta = 0.0936\ (ft)$$

$$L = 50 + 2 \times 4 + 0.0936 = 58.094\ \ ft.$$

<u>Ans.) $L = 58.094\ ft$</u>.

2.35



(a) $\Sigma M_o = 0$;

$$375T_1 = 375T_2 + 250 \times 700$$

$$\therefore\ T_1 - T_2 = \frac{2}{3} \times 700$$

$$T_0 = 1.4kN,\quad EA = kL = 60 \times 10^6 \times 0.025\ (N)$$

$$\therefore\ \delta_1 = \frac{(T_1 - T_0) \times 6}{EA},\quad \delta_2 = \frac{(T_0 - T_2) \times 6}{EA}\quad (단,\ T_2 > 0)$$

$T_2 > 0$ 면 $\delta_1 = \delta_2$ 이어야 한다

$$\therefore\ T_1 - T_0 = T_0 - T_2 \qquad \therefore\ T_1 + T_2 = 2T_0 = 2800$$

~~위 두식을~~ 연립하면 $T_1 = 1633.6\ N$

$$T_2 = 1166.7\ N$$

$$\delta = \frac{(1633.3 - 1400) \times 6}{60 \times 10^6 \times 0.025} = 9.33 \times 10^{-4} \ (m)$$

$$\theta = \frac{\delta}{0.1} = 0.00933 \ (rad) = 9.33 \times 10^{-3} \ (rad) = 0.535^\circ$$

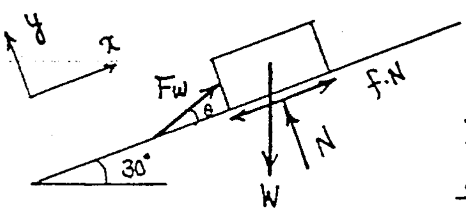
(b) $T_2 = 0$ 라면 $\quad 375\, T_1 = 250 \times 700$

$$\therefore \ T_1 = \frac{2}{3} \times 700 \qquad \delta = \frac{\frac{2}{3} \times 700 \times 6}{60 \times 10^6 \times 0.025} = 1.867 \times 10^{-3} \ (m)$$

$$\theta_1 = \frac{\delta_1}{0.1} = 1.867 \times 10^{-2} \ (rad) = 1.07^\circ$$

Ans.) (a) 0.535° (b) 1.07°

2.36 $\quad W = 10$ tons $\quad f = 0.2 \quad \tan\theta = \frac{2}{5}$



i) 미끄러져 내려올때

$\Sigma F_x = 0$; $W \sin 30^\circ = f \cdot N + F_w \cos\theta$

$\Sigma F_y = 0$; $W \cos 30^\circ = N + F_w \sin\theta$

위 두식을 연립하면

$$F_w = \frac{W(\sin 30^\circ - f \cos 30^\circ)}{\cos\theta - f \sin\theta} = 7651.5 \ (lb)$$

$$\therefore \ \delta_w = \frac{F_w L}{EA} = \frac{7651.5 \times \sqrt{2^2 + 5^2}}{(1.6 \times 10^6) \times (2 \times 2)} = 6.438 \times 10^{-3} \ (ft) = 0.07726 \ (in)$$

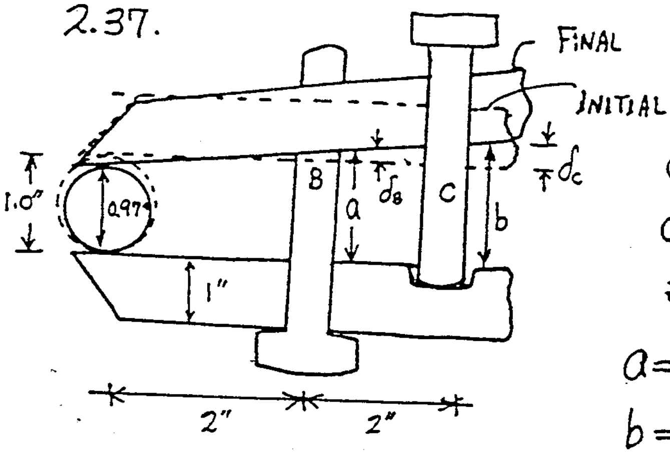
ii) 끌어 올릴때

$$F_w = \frac{W(\sin 30^\circ + f \cos 30^\circ)}{\cos\theta + f \sin\theta} = 13427.1 \ (lb)$$

$$\therefore \ \delta_w = \frac{F_w L}{AE} = \frac{13427.1 \times \sqrt{29}}{(1.6 \times 10^6) \times (2 \times 2)} = 0.011298 \ (ft) = 0.1356 \ (in)$$

Ans.) i) 미끄러져 내려올때 : $\delta_w = 0.07726$ in
ii) 끌어 올릴때 ; $\delta_w = 0.1356$ in

2.37.



C볼트를 N번 회전 시켰을때

δ_B = 볼트 B의 신장량

δ_c = 볼트 C의 수축량

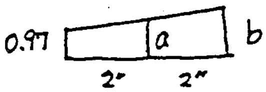
라 하자.

$$a = 1 + \delta_B \quad \cdots ①$$

$$b = 1 + \frac{N}{20} - \delta_c \quad \cdots ②$$

Jaw가 견고하다면 변형 후 기하학적 조건으로 부터

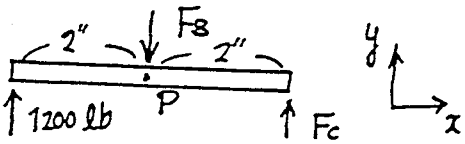
$$2a = 0.97 + b \quad \cdots ③$$



①.②를 ③에 대입

$$2\delta_B + \delta_c = \frac{N}{20} - 0.03 \quad \cdots\cdots ④$$

Equilibrium



$\Sigma F_y = 0$; $1200 + F_c = F_B$

$\Sigma M_{pz} = 0$; $1200 \times 2 = F_c \times 2$

$\therefore$ $F_B = 2400$ lb, $F_c = 1200$ lb

힘 - 변형 관계식 : $\delta_B = \frac{F_B L_B}{A_B E_B} = \frac{2400 \times 1}{0.1 \times 30 \times 10^6} = 0.0008$ in

$$\delta_c = \frac{F_c L_c}{A_c E_c} = \frac{1200 \times 1}{0.1 \times 30 \times 10^6} = 0.0004 \text{ in}$$

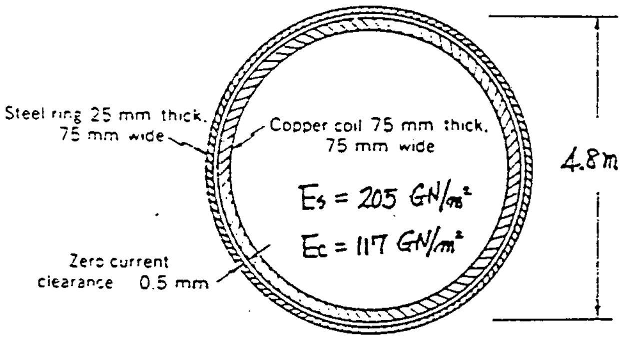
$N = 0.64$ ($\because$ ④로 부터)

Ans.) $N = 0.64$ (회전)

2.38 $\delta_{RS} = \delta_{RC} - 0.5 \quad \cdots\cdots ①$

축과 구리사이에 작용하는 압력 = P_i

$$P_o = \frac{70{,}000}{0.075} = 933333.3 \ N/m^2$$



$$\delta_R = \frac{\Delta P \cdot r^2}{Et}$$

$$\delta_{RS} = \frac{P_i\, r^2}{E_s t_s}$$

$$\delta_{RC} = \frac{(P_o - P_i)\, r^2}{E_c t_c}$$

$\delta_{RS} = \delta_{RC} - 0.5$ 이므로

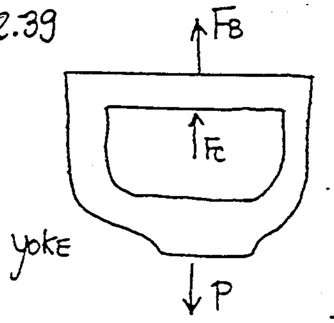
$$\frac{P_i (2.4)^2 \times 1000}{(205 \times 10^9) \times 0.025} = \frac{(P_o - P_i) \times (2.4)^2 \times 1000}{(117 \times 10^9) \times 0.075} - 0.5$$

$\therefore P_i = 63275\ (N/m^2)$, $\quad F_c = (P_o - P_i)\, r\, b$ 로 부터

$$F_c = (933333.3 - 63275) \times 2.4 \times 0.075 = 156610.50\ (N)$$

Ans.) $F_c = 156610.5\ N$

2.39



yoke의 평형조건

$P = F_B + F_c \quad \cdots\cdots ①$

$(P = 325\ lb)$

$\frac{\delta_B}{L_B} = f_B\left(\frac{F_B}{A_B}\right), \quad \frac{\delta_c}{L_c} = f_c\left(\frac{F_c}{A_c}\right)$

f_B, f_c는 응력 - 변형도 곡선의 기울기에 해당.

$\delta_B = \delta_c = \delta$ 를 이용. $\quad L_B f_B\left(\frac{F_B}{A_B}\right) = L_c f_c\left(\frac{F_c}{A_c}\right) \quad \cdots\cdots ②$

<풀이과정> $\quad \left(\frac{F_B}{A_B}\right) A_B + \left(\frac{F_c}{A_c}\right) A_c = \left(\frac{P}{A}\right)_B \times 0.01 + \left(\frac{P}{A}\right)_c \times 0.02 = P$

또한, $\quad \left(\frac{\delta}{L}\right)_B L_B = \left(\frac{\delta}{L}\right)_c L_c \quad \therefore \left(\frac{\delta}{L}\right)_c \Big/ \left(\frac{\delta}{L}\right)_B = L_B / L_c = \frac{3}{2}$

* 도표

TRIAL	$(\delta/L)_B$	$(\delta/L)_c$	$(P/A)_B$	$(P/A)_c$	F_B	F_c	P
1	.003	.0045	7,500	13,400	75	268	343
2	.0028	.0042	7,000	13,900	70	260	330
3	.0026	.0039	6,500	12,800	65	256	321

< 앞면으로부터 계속 >

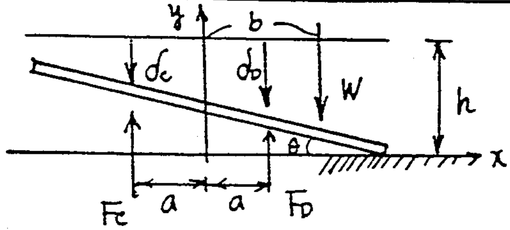
4	.0027	.0041	6750	12963	67.5	259.0	326

* 근사해 ⇒ $P_B = 67\,(lb)$, $P_c = 258\,(lb)$

이때 $\left(\frac{\delta}{L}\right)_B = 0.0027$ $\therefore\ \delta = 1.8 \times 0.0027 = 4.86 \times 10^{-3}$ in

Ans.) $\delta = 4.86 \times 10^{-3}$ in

2.40.



$W = F_c + F_D$

$F_D = \frac{a+b}{2a} W$

$F_c = \frac{-(b-a)}{2a} W$

$$\theta = \frac{\delta_D - \delta_c}{2a} = \frac{h - \frac{1}{2}(\delta_c + \delta_B)}{L} \quad \cdots\cdots\cdots ①$$

$$F_D = \frac{a+b}{2a} W = k_D \delta_D \ , \quad F_c = \frac{a-b}{2a} W = k_c \delta_c \ \cdots ②$$

①식을 정리 $L(\delta_D - \delta_c) = 2ah - a(\delta_c + \delta_D) \ \cdots ①'$

② → ①'

$$L\left(\frac{a+b}{2a}\frac{W}{k_D} - \frac{a-b}{2a}\frac{W}{k_c}\right) + a\left(\frac{a-b}{2a}\frac{W}{k_c} + \frac{a+b}{2a}\frac{W}{k_D}\right) = 2ah$$

$k_c/k_D = \nu$, $a/L = s$ 라 하면

$$\left[\nu\left(s + \frac{b}{L}\right) - \left(s - \frac{b}{L}\right)\right] + s\left[\left(s - \frac{b}{L}\right) + \nu\left(s + \frac{b}{L}\right)\right] = \frac{4s^2 k_c}{W} h$$

$$\therefore (\nu + 1 - s + \nu s)\frac{b}{L} = \frac{4s^2 k_c h}{W} + s - \nu s - s^2 - \nu s^2$$

$$= s^2\left(\frac{4k_c h}{W} + \frac{1}{s} - \frac{\nu}{s} - 1 - \nu\right)$$

그런데 $\nu + 1 - s + \nu s = 2 + \nu - 1 + \nu s - s$

$$= 2 + (\nu - 1)(s + 1) = 2 - (1 - \nu)(1 + s)$$

$$\frac{1}{s} - \frac{\nu}{s} - 1 - \nu = -2 + \frac{1}{s} - \frac{\nu}{s} + 1 - \nu$$

$= -2 + (1 - \nu)\left(1 + \frac{1}{s}\right)$ 이므로

b/L 를 정리하면

$$\therefore \frac{b}{L} = \frac{S^2}{2-(1-\nu)(1+S)}\left[\frac{4k_c h}{W} - 2 + (1-\nu)\left(1+\frac{1}{S}\right)\right]$$

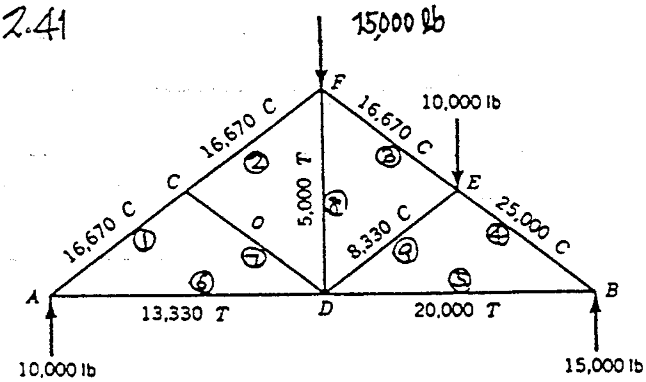
$$= \frac{S^2}{1-\frac{1}{2}(1-\nu)(1+S)}\left[\frac{2k_c h}{W} - 1 + \frac{1}{2}(1-\nu)\left(1+\frac{1}{S}\right)\right]$$

$$\left(단,\ \nu = k_c/k_o,\ S = a/L\right)$$

즉, $$\frac{b}{L} = \frac{(a/L)^2}{1-\frac{1}{2}(1-\nu)(1+a/L)}\left[\frac{2k_c h}{W} - 1 + \frac{1}{2}(1-\nu)\left(1+\frac{L}{a}\right)\right]$$

- Q.E.D -

2.41



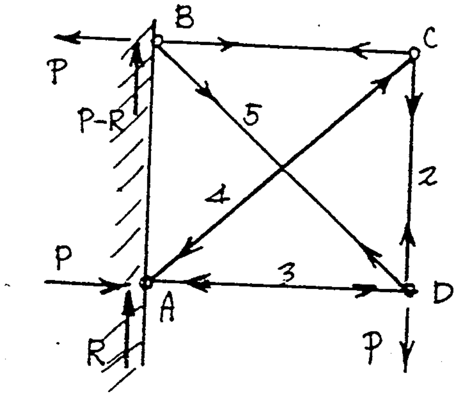
에너지 방법을 이용한다

$$\delta_p = \sum_{i=1}^{n} F_i \frac{L_i}{A_i E_i}\frac{\partial F_i}{\partial P}$$

	F	$\frac{L}{EA}$	$\frac{\partial F}{\partial P}$	$\frac{FL}{EA}\cdot\frac{\partial F}{\partial P}$
1	-16670	$\frac{5}{4\times(10\times10^6)}$	$-\frac{2.5}{3}$	$208375\ \frac{1}{10\times10^6\times12}$
2	-16670	$\frac{5}{4\times(10\times10^6)}$	$-\frac{2.5}{3}$	〃
3	-16670	〃	$-\frac{2.5}{3}$	〃
4	-25000	〃	$-\frac{2.5}{3}$	$312500\ \frac{1}{10\times10^6\times12}$
5	20000	〃	$\frac{2}{3}$	$200000\ \frac{1}{10\times10^6\times12}$
6	13,3 0	〃	$\frac{2}{3}$	$133000\ \frac{1}{10\times10^6\times12}$
7	0	$\frac{5}{2\times(10\times10^6)}$	0	0
8	5000	$\frac{6}{2\times(10\times10^6)}$	0	0
9	-8330	$\frac{5}{2\times(10\times10^6)}$	0	0
Σ				0.011 (ft)

Ans.) $\delta_{VH} = 0.011$ ft

2.42



A점 ; $F_4 = \sqrt{2}R$

$F_3 = P - F_4\cos 45° = P - R$

B점 ; $F_5 = \sqrt{2}(P-R)$

$F_1 = P - F_5\cos 45° = R$

C점 ; $F_2 = F_4\cos 45° = R$

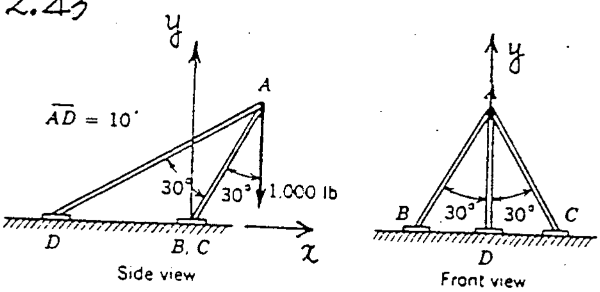
	F	$\frac{L}{EA}$	$\frac{\partial F}{\partial P}$	$\frac{FL}{EA}\cdot\frac{\partial F}{\partial P}$
1	R	$\frac{L}{EA}$	1	$\frac{RL}{EA}$
2	R	〃	1	〃
3	$-(P-R)$	〃	1	$\frac{(R-P)L}{EA}$
4	$-\sqrt{2}R$	$\sqrt{2}\frac{L}{EA}$	$-\sqrt{2}$	$\frac{2\sqrt{2}RL}{EA}$
5	$\sqrt{2}(P-R)$	〃	$-\sqrt{2}$	$\frac{2\sqrt{2}(R-P)L}{EA}$
Σ				$(3+4\sqrt{2})R-(1+2\sqrt{2})P/_{EA}$

$\delta_{AV} = 0$ 이므로 $(3+4\sqrt{2})R = (1+2\sqrt{2})P$

$\therefore\ R = \frac{1+2\sqrt{2}}{3+4\sqrt{2}}P = 0.442P$ $\quad\therefore\ F_{CD} = 0.442P$ (tension)

Ans.) $F_{CD} = 0.442P$ (tension)

2.43



$\overline{AD} = 10'$, $A_{AD} = 1\ in^2$

$A_{AB} = A_{AC} = 2.1\ in^2$

$E_{AD} = E_{AC} = 30\times10^6$ psi

* 가정 ; ~~부정정인 구조물을~~ 정정 ~~구조물로~~

보기 위하여 base와 member 사이의 bolt joint를 pin joint로 가정하고 마찰이 없다고 본다.

$$\overline{AB} = \overline{AC} = \sqrt{(AD\sin 30^\circ \tan 30^\circ)^2 + (AD\sin 30^\circ \frac{1}{\cos 30^\circ})^2} = 6.455 \text{ (ft)}$$

$\Sigma M_z = 0$; $\quad F_{AD}(5\tan 30^\circ) = 1000 \times (5\tan 30^\circ)$

$$\therefore F_{AD} = 1000 \text{ lb}$$

$$\vec{F_{AB}} = F_{AB} \frac{1}{6.455}(5\tan 30^\circ \vec{i} + 5\vec{j} - 5\tan 30^\circ \vec{k})$$

$\Sigma F_x = 0$; $\quad 2\left(\frac{F_{AB}\, 5\tan 30^\circ}{6.455}\right) = 1000 \cos 30^\circ \quad (\because A_{AB} = A_{AC})$

$$\therefore F_{AB} = 968.25 \text{ (lb)}$$

① 수직 이동

	F (lb)	$\frac{L}{EA}$	$\frac{\partial F}{\partial P}$	$\frac{FL}{EA}\cdot\frac{\partial F}{\partial P}$
AD	1000	10^{-6}	1	10^{-3}
AB	−968.25	1.0246×10^{-7}	−0.96825	9.6056×10^{-5}
AC	〃	〃	〃	〃
Σ				1.192×10^{-3} (ft)

$\therefore \delta_{AV} = 1.192 \times 10^{-3} \text{ (ft)} = 0.0143 \text{ (in)}$ (아래방향으로)

② 수평 이동

$\Sigma M_z = 0$; $\quad 5 = F'_{AD}\, 5\tan 30^\circ \qquad \therefore F_{AD} = 1.732$

$$F\,\vec{F_{AB}}' = F_{AB}\frac{1}{6.455}(5\tan 30^\circ \vec{i} + 5\vec{j} - 5\tan' \vec{k})$$

$\Sigma F_x = 0$; $\quad F'_{AB}\frac{1}{6.455}\, 5\tan^{-1} 30^\circ = F'_{AD} - 1$

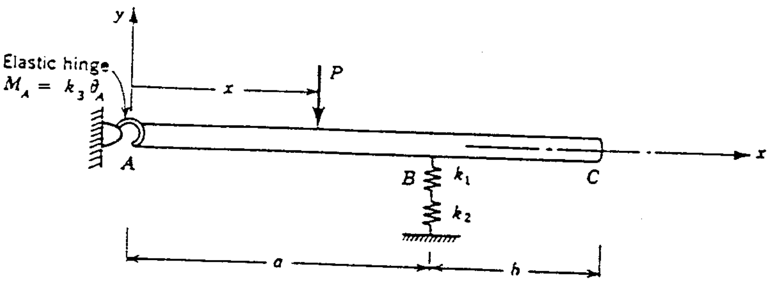
$$\therefore F'_{AB} = 0.559.$$

	F (lb)	$\frac{L}{EA}$	$\frac{\partial F}{\partial P}$	$\frac{FL}{EA}\cdot\frac{\partial F}{\partial P}$
AD	1000	10^{-6}	1.732	1.732×10^{-3}
AB	−968.25	1.0246×10^{-7}	−0.559	5.546×10^{-5}
AC	〃	〃	〃	〃
Σ				1.84×10^{-3} (ft)

$\therefore \delta_{AH} = 1.84 \times 10^{-3} \text{ ft} = 0.0221 \text{ in}$ (right)

Ans.) $\delta_{AH} = 0.0221$ in , $\delta_{AV} = 0.0143$ in

2.44



(a) $\sum M_{zA} = 0$; $Px = k_3\theta_A + ka\theta_A \cdot a$ ····①

$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} \rightarrow k = \frac{k_1k_2}{k_1+k_2}$ ······②

$F_B = ka\theta_A \quad \therefore \theta_A = F_B/ka$ ······③

③을 ①에 대입 $Px = \frac{k_3F_B}{ka} + \frac{ka^2F_B}{ka}$

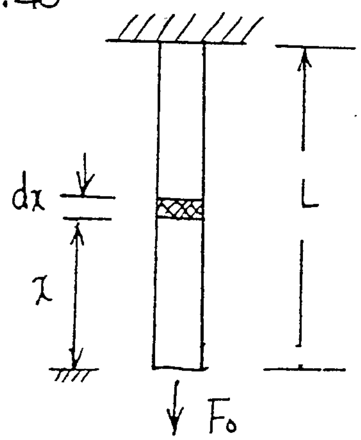
$\therefore F_B = \frac{kaPx}{k_3 + ka^2} = \frac{k_1k_2aPx}{(k_1+k_2)k_3 + k_1k_2a^2}$

b) $\theta_A = \frac{F_B}{ka} = \frac{(k_1+k_2)Px}{(k_1+k_2)k^3 + k_1k_2a^2)}$

Ans.) $F_B = \frac{k_1k_2aPx}{(k_1+k_2)k^3 + k_1k_2a^2}$,

$\theta_A = \frac{(k_1+k_2)Px}{(k_1+k_2)k_3 + k_1k_2a^2}$

2.45



(a) 막대기의 자중을 생각하면

$F_x = wx + F_0$, $dU = \frac{F_x^2}{2EA}dx$

$\therefore U = \int_0^L \frac{L(wx+F_0)^2}{2EA}dx$

$= \frac{1}{2EA}\left[\frac{w^2}{3}x^3 + wF_0x^2 + F_0^2x\right]_0^L$

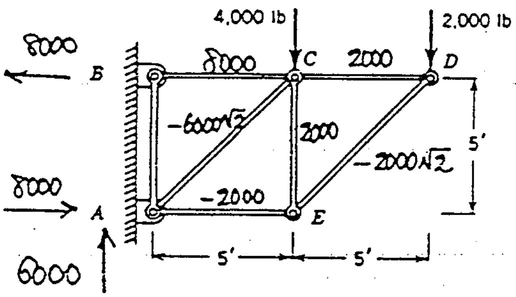
$= \frac{1}{6EA}\left(\frac{W_0}{L^2} + 3WF_0L + 3F_0^2\right)$

(b) 막대기 자중 없이 생각하면

$U = \frac{F_0^2L}{2EA}$

Ans.) $\begin{cases} (a)\ U = \dfrac{L}{6EA}(w_0L^2 + 3wF_0L + 3F_0^2) \\ (b)\ U = \dfrac{F_0^2 L}{2EA} \end{cases}$

2.46.



(a) 경우 $(P_c)_{max} = 6000\sqrt{2}$
$= 8485\ lb$

$(P_t)_{max} = 8000\ lb$

(1) $(\sigma_{max})_t = \dfrac{8000}{2.17}$

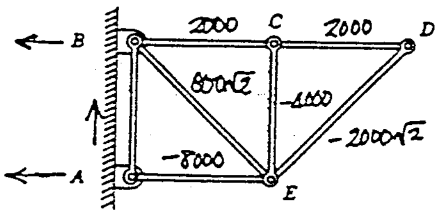
$= 3686.6\ psi < 20000\ (o)$

$$\frac{10^8 I}{L^2} = \frac{10^8 \times 0.59}{(5\sqrt{2} \times 12)^2} = 8194.41\ (lb) > 2000\sqrt{2} \quad (o)$$

그러나 $8194.4 < (P_c)_{max} = 8485\ lb$ 이므로

(1)의 재료를 사용 할 수 없다. (2) 재료는 나중에 생각.

(b) 경우



$(P_t)_{max} = 6000\sqrt{2}\ lb$
$(P_c)_{max} = 8000\ lb$

(1) $(\sigma_{max})_t = \dfrac{6000\sqrt{2}}{2.17}$

$= 3910.3\ psi < 20000 \quad (o)$

$$\frac{10^8 I}{L^2} = \frac{10^8 \times 0.59}{(5 \times 12)^2} = 16388.9\ lb > 8000\ lb \quad (o)$$

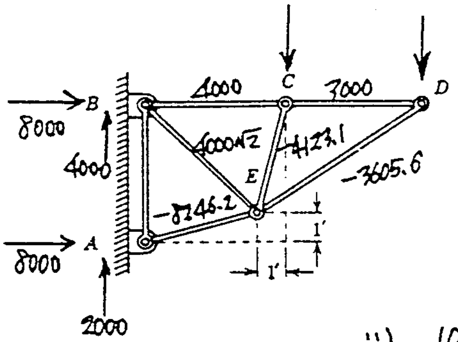
총중량 $= 293.566\ lb$

(1)의 재료가 합당하므로 (2)의 경우는 고려하지 않아도 된다.

(C) 경우

$(P_t)_{max} = 4000\sqrt{2}\ lb$

$(P_c)_{max} = 8246.2\ lb$



(1) $(\sigma_{max})_t = \dfrac{4000\sqrt{2}}{2.17}$

$= 2606.8\ Psi < 20{,}000\ Psi$ (O)

i) $\dfrac{10^8 I}{L^2} = \dfrac{10^8 \times 0.59}{(\sqrt{17}\times 2)^2}$

$= 24101.3 > 8246$ (O)

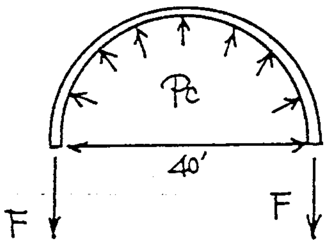
ii) $\dfrac{10^8 I}{L^2} = \dfrac{10^8 \times 0.59}{(\sqrt{36+16}\times 12)^2} = 7879.3 > 3605.6$

(O)

총중량 $= (5\times3 + 4\sqrt{2} + 2\sqrt{17} + \sqrt{52}) \times 7.5 = 270.9$ (lb)

∴ (C)의 경우로 설계하고 (1)의 재료를 사용하면 주어진 형하중 최소무게 (270.9 lb)로 설계 할 수 있다.

2.47

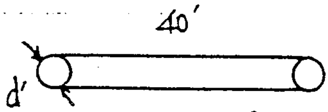


$2F = P_c \cdot d' \times (40\times12)$

$= P_c \frac{1}{4}\sqrt{170}$

∴ $F = 782.3\ P_c$ (lb)

(단, P_c 단위는 Psi)



$A = 170 \times \frac{\pi}{4}(\frac{1}{4})^2$

$d' = \sqrt{\frac{4}{\pi}A} = \frac{1}{4}\sqrt{170}$

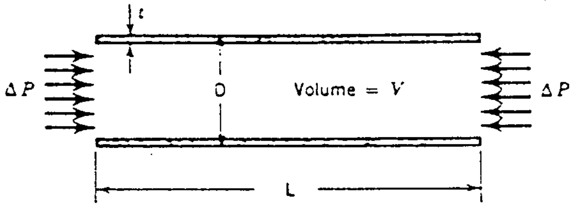
$P_i \times (31\times12)\left(\frac{75\times12}{310}\right)$

$= 2FA = 2\times 782.3\ P_c\{170\ \frac{\pi}{4}(\frac{1}{4})^2\}$

∴ $P_i = 12.1\ P_c$

Ans.) $F = 782.3\ P_c$ lb , $P_i = 12.1\ P_c$ Psi

2.48



$\delta_R = \dfrac{\Delta P\, r^2}{Et}\left(1+\dfrac{t}{2r}\right)$

$V = \dfrac{\pi D^2}{4} L$

$$\Delta V = \pi\left(\frac{D}{2}+\delta_R\right)^2 L - \frac{\pi}{4}D^2 L = \pi L \delta_R (D+\delta_R)$$

$$k = \frac{\Delta V}{V}\frac{1}{\Delta p} = \frac{1}{V}\cdot \pi L \delta_R (D+\delta_R)\frac{1}{\Delta p}$$

$$= \frac{1}{V}\pi L D \delta_R \frac{1}{\Delta p} \qquad (\because\ \delta_R^2 \ll 1)$$

$$= \frac{1}{V}\pi L D \frac{\cancel{\Delta p}\, r^2}{Et}\left(1+\frac{t}{2r}\right)\frac{1}{\cancel{\Delta p}} = \frac{\pi L r^2}{V}\frac{1}{Et}(D+t)$$

$$= \frac{1}{Et}(D+t) = \frac{11.25\times10^{-3}}{(205\times10^{9})(1.25\times10^{-3})}$$

$$= 4.39\times10^{-11}\ (m^2/N) = 4.39\times10^{-5}\ (mm^2/N)$$

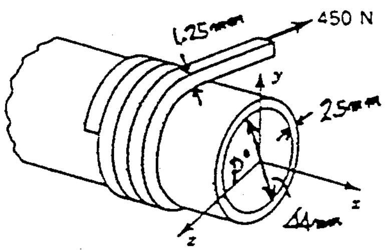
Ans.) $k = 4.39\times10^{-5}\ (mm^2/N)$

2.49. $F = \pi r^2 p = \pi \times (0.05)^2 \times \left(\frac{60}{760}\times 101325\right)$

$= 62.8\ (N)$

Ans.) $F = 62.8\ N$

2.50



(a)

$2\times450 = P_0\times(49\times10^{-3})(1.25\times10^{-3})$

$\therefore P_0 = 14.6938\ (MN/m^2)$

$$\delta_R = \frac{Pr^2}{Et}\left(1-\frac{t}{2r}\right)$$

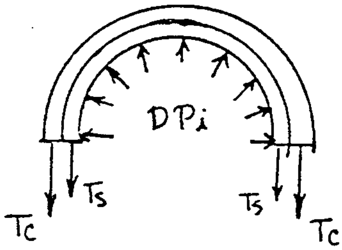
$$= \frac{14.6938\times10^{6}\times(24.5\times10^{-3})^2\times\left(1-\frac{2.5}{49}\right)}{(140\times10^{9})(2.5\times10^{-3})}$$

$$= 2.391\times10^{-5}\ (m)$$

$\therefore\ \delta_R = 0.02391\ (mm)$

(b) $P_i D b = 2(T_s + T_c)$ ····· ①

$$\delta_{Rs} = \frac{T_s L_s}{2\pi E_s A_s}, \quad \delta_{Rc} = \frac{T_c L_c}{2\pi E_c A_c}$$



$$\delta_{RS} = \delta_{RC}$$

$$\frac{T_c \cdot \pi (44+2.5)\times 10^{-3}}{(140\times 10^9)(2.5\times 10^{-3})\times(1.25\times 10^{-3})} = \frac{T_s\, \pi (49+1.25)\times 10^{-3}}{(205\times 10^9)(1.25\times 10^{-3})^2}$$

$$\therefore T_c = 1.476\, T_s \quad \cdots ②$$

①② 식으로 부터

$$P_i (44\times 10^{-3})(1.25\times 10^{-3}) = 2(1.476+1)\, T_s$$

$$\therefore P_i = 90036.478\ T_s = 61000.194\ T_c$$

$$(T_c)_{max} = (210\times 10^6)\times(2.5\times 10^{-3})\times(1.25\times 10^{-3}) = 656.25\ N$$

$$(T_s)_{max} = (1.0\times 10^9)\times(1.25\times 10^{-3})^2 = 1562.5\ N$$

또한 외부에서 450 N이 작용하므로

$$\left.\begin{aligned}(T_s)'_{max} &= (T_s)_{max} - 450 = 1112.5\ (N)\\ (T_c)'_{max} &= (T_c)_{max} + 450 = 1106.25\ (N)\end{aligned}\right\} \cdots ③$$

②, ③ 으로부터 Tc가 먼저 극한값에 도달 한다는 것을 알 수 있다.

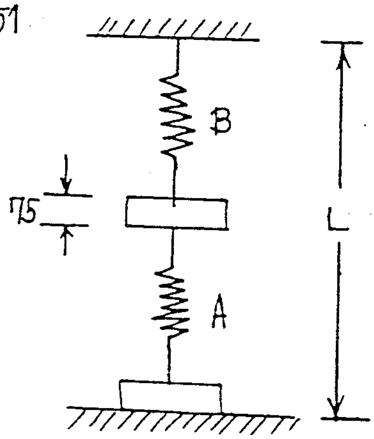
$$\therefore P_i \leqq 61000.194 \times 1106.25\ (N/m^2)$$

$$\leqq 67.481\ (MN/m^2)$$

Ans.) (a) $\delta_R = 2.391\times 10^{-5}\, m$

(b) $P_i \leqq 67.481\ MN/m^2$

2.51



Spring	free length	Spring Constant
A	250 mm	1.4 MN/m
B	200 mm	0.875 MN/m

$$\delta_A + \delta_B = L - (600-75) \cdots ①$$

$$k_A \delta_A = k_B \delta_B \quad \cdots ②$$

①②로부터 $\delta_A = \frac{k_B}{k_A+k_B}(L-525)$, $\delta_B = \frac{k_A}{k_A+k_B}(L-525)$

하중 W로 δ 만큼 처짐이 생길때 최종 늘어난 Spring 길이

$\delta_B' = \delta_B + \delta$, $\delta_A' = \delta_A - \delta$

$\therefore F_B = k_B(\delta_B + \delta)$, $F_A = k_A(\delta_A - \delta)$

$\Sigma F_y = 0$, $W = F_B - F_A$

$$W = (k_A+k_B)\delta + k_B\delta_B - k_A\delta_A$$

$$= (k_A+k_B)\delta + \frac{k_A k_B}{k_A+k_B}(L-525) - \frac{k_A k_B}{k_A+k_B}(L-525)$$

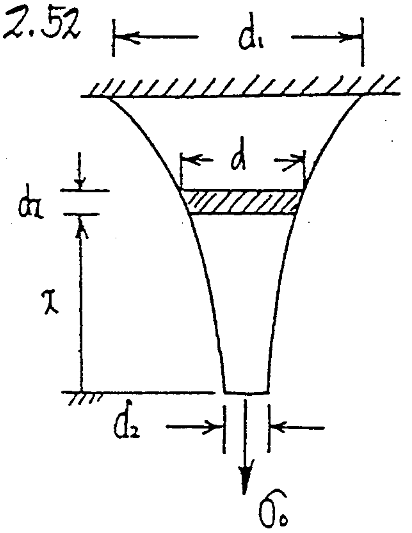
$$\therefore \frac{W}{\delta} = k_A + k_B$$

$$\text{Sensibility} = \frac{\delta}{W} = \frac{1}{k_A+k_B} = \frac{1}{1.4+0.875}$$

$$= 0.43956 \ \text{mm/kN}$$

결국, Sensibility는 k_A 및 k_B에만 관계 될 뿐 길이 L에는 무관하다.

2.52



$$\frac{\pi d^2}{4}\sigma_0 = \int_0^x \gamma \frac{\pi d^2}{4} dx + \frac{\pi d_2^2}{4}\sigma_0$$

양변을 x에 관해 미분

$$\frac{\pi}{4}\sigma_0 \frac{d}{dx}(d^2) = \gamma \frac{\pi}{4} d^2$$

$$\frac{\gamma}{\sigma_0} dx = \frac{d(d^2)}{d^2} \quad \text{양변 적분}$$

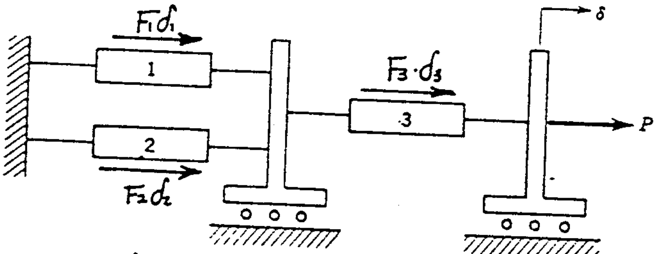
$$\frac{\gamma}{\sigma_0} x = \ln(d^2) + C$$

$$\therefore d^2 = A e^{\frac{\gamma}{\sigma_0}x}$$

$$d^2 = d_2^2 e^{\frac{\gamma}{\sigma_0}x} \qquad \therefore d = d_2 e^{\frac{\gamma}{2\sigma_0}x}$$

Ans.) $d = d_2 e^{\frac{\gamma}{2\sigma_0}x}$

2.53



$$\int_0^{\delta} p\, d\delta = \sum_{i=1}^{3} \int_0^{e_i} F_i\, de_i$$

$$U = \int_0^{e_1} F_1\, de_1 + \int_0^{e_2} F_2\, de_2 + \int_0^{e_3} F_3\, de_3$$

$\delta_1 = \delta_2$ 이며 $\quad F_1 + F_2 = F_3 = P$ 이므로

$$U = \int_0^{e_1} (F_1 + F_2)\, de_1 + \int_0^{e_3} F_3\, de_3$$

$$= \int_0^{e_1} p\, de_1 + \int_0^{e_3} F_3\, de_3 \qquad (\because F_1 + F_2 = F_3 = P)$$

또한, $\delta_1 + \delta_3 = e_1 + e_3 = \delta p = const$ 이므로

$$U = \int_0^{e_1+e_3} p\, d(e_1+e_3) = \int_0^{\delta} p\, d\delta$$

$$\therefore \quad U = \int_0^{\delta} p\, d\delta = \sum_{i=1}^{3} \int_0^{e_i} F_i\, de_i$$

— Q.E.D —

2.54. $$U^* = \int_0^{F_1} e_1\, dF_1 + \int_0^{F_2} d_2\, dF_2 + \int_0^{F_3} e_3\, dF_3$$

$$= \int_0^{F_1+F_2} e_1\, d(F_1+F_2) + \int_0^{F_3} e_3\, d(F_3)$$

$$(\because e_1 = e_2 \;,\; e_1 = const)$$

$$= \int_0^{P} e_1\, dp + \int_0^{P} e_3\, dp \qquad (\because F_1 + F_2 = F_3 = P)$$

$$= \int_0^{P} \delta\, dp \qquad (\because e_1 + e_3 = \delta)$$

$$\therefore U^* = \int_0^{P} \delta\, dp = \sum_{i=1}^{3} \int_0^{F_i} e_i\, dF_i$$

— Q.E.D —

2.55 $$\sum_i \int_0^{S_i} \vec{P_i} \cdot d\vec{S_i} = U$$

S_i외의 다른 변위는 없다고 본 상태에서

S_i에 비해 대단히 작은 ΔS_i가 발생 했다고 가정

변위는 평형을 유지하며 서서히 변하고 potential work의 증분은 potential Energy의 증분 ΔU와 같을 것이다.

$$\vec{P_i}\cdot\Delta\vec{S_i} = \Delta U$$

$$\vec{P_i}\cdot\Delta\vec{S_i} = P_i \Delta S_i \cos\theta = (P_i \cos\theta)\Delta S_i$$

$$= f_i \Delta S_i$$

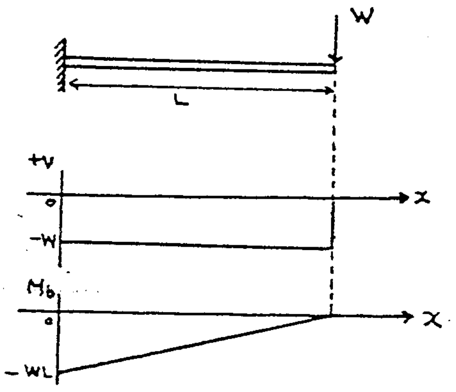
$$\therefore \quad f_i \Delta S_i = \Delta U$$

따라서, ΔS_i 값이 매우 작다면 $f_i = \dfrac{\partial U}{\partial S_i}$

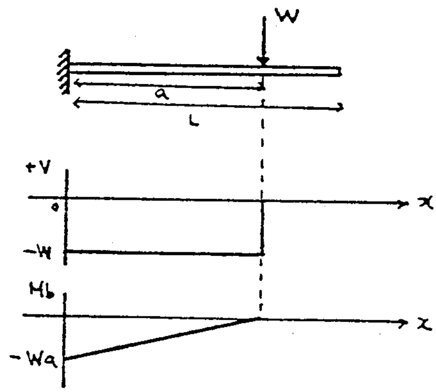
— Q.E.D —

CHAPTER 3

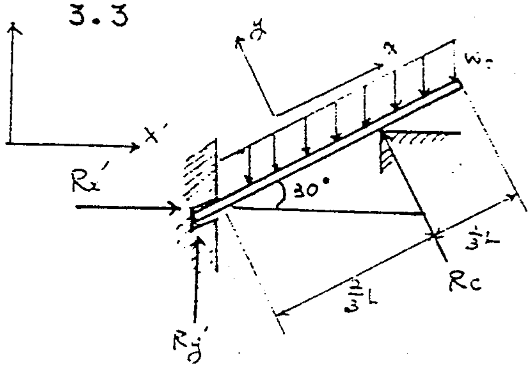
3.1



3.2



3.3



$\Sigma F_x' = 0 \qquad R_x' = R_c \sin 30°$

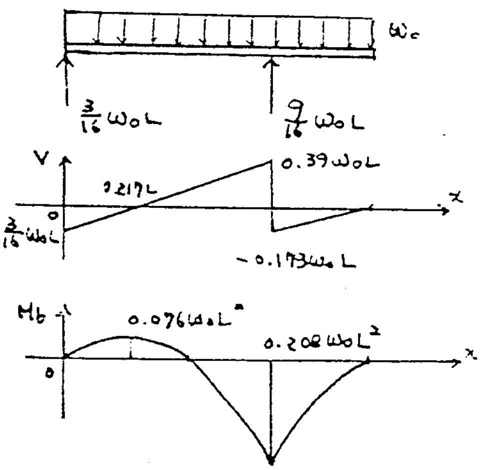
$\Sigma F_y' = 0 \qquad R_y' + R_c \cos 30° = w_0 L \cos 30°$

$\Sigma M_c = 0 \qquad \frac{2}{3} R_c L = \frac{1}{2} w_0 (L \cos 30°)^2$

$\therefore\ R_c = \frac{9}{16} w_0 L$

$R_x' = \frac{9}{32} w_0 L$

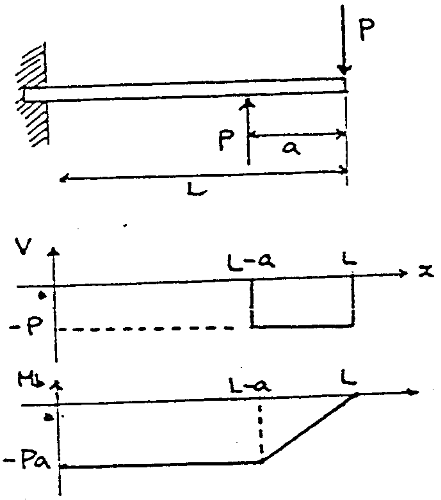
$R_y' = \frac{7\sqrt{3}}{32} w_0 L$



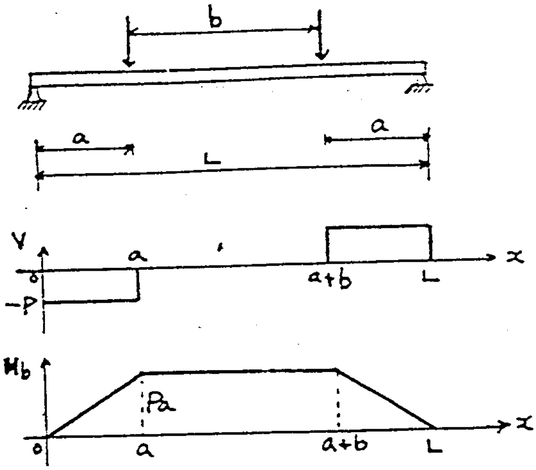
$R_y = R_y' \cos 30° - R_x' \sin 30°$

$= \frac{3}{16} w_0 L$

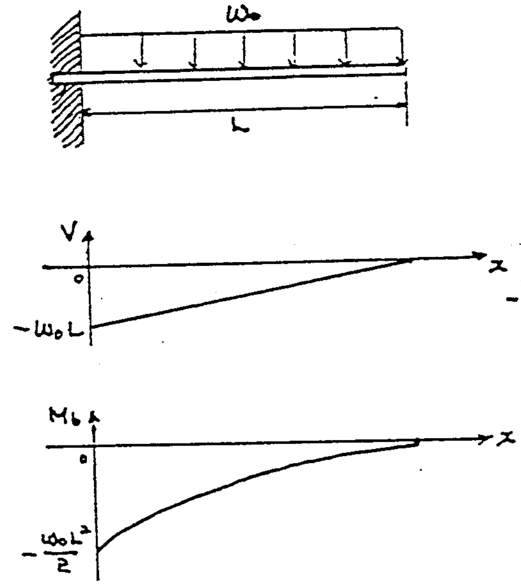
3.4



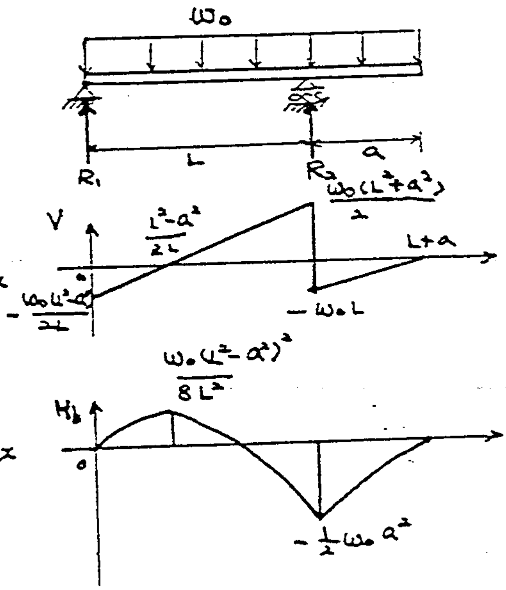
3.5



3.6

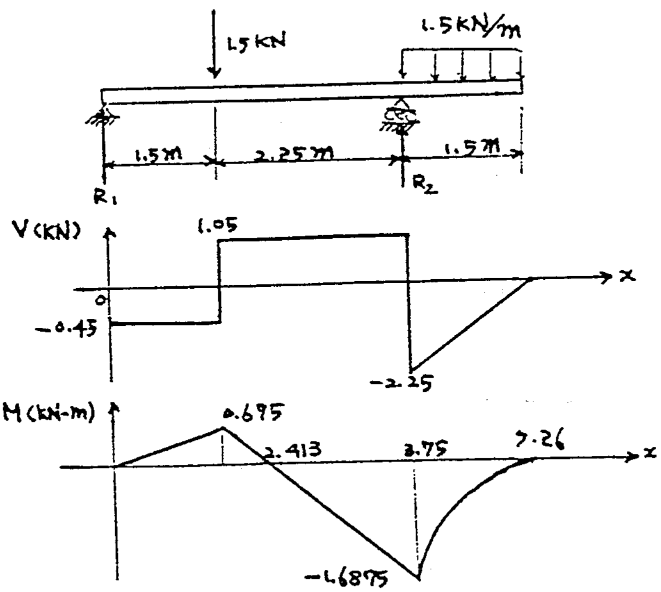


3.7



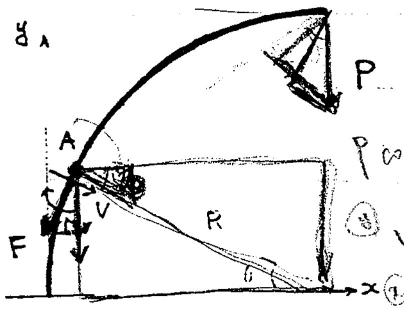
$$R_1 = \frac{w_0(L^2-a^2)}{2L}, \quad R_2 = \frac{w_0(L+a)^2}{2L}$$

3.8



$R_1 = 0.45 \text{ KN}$ $\quad$ $R_2 = 3.3 \text{ KN}$

3.9



F = Normal (axial) force

V = Transverse (shear) force

$P\cos(\frac{\pi}{2}-\theta) + F = 0$

$V\sin\theta + P + F\cos\theta = 0$

$F\sin\theta = V\cos\theta$ $\quad$ $V = \frac{\sin\theta}{\cos\theta}F$

$M - P(R\cos\theta) = 0$

$F + P\cos\theta = 0$

$PR\cos\theta + M = 0$ $\quad$ $M = -PR\cos\theta$

$V + P\sin\theta = 0$

$V\sin\theta$ $\quad$ $\frac{V + P\sin\theta}{F}$

$\therefore \quad M = -PR\cos\theta$

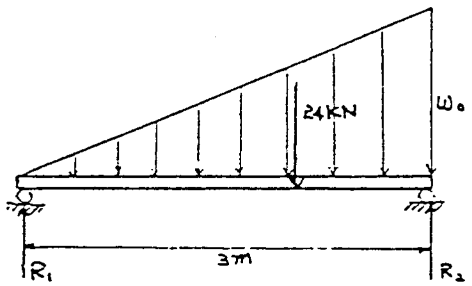
$\sin\theta = \frac{V}{P}$

$F = -P\cos\theta$ (compression)

$V = P\sin\theta$

$V = -P\sin\theta$

3.10



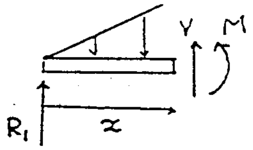
$R_1 = 8000$ N

$R_2 = 16000$ N

$$\int_0^3 w_0 \frac{x}{L} dx = 24{,}000$$

$$= \frac{1}{2} w_0 \frac{x^2}{L} \Big]_0^3$$

$\therefore \; w_0 = 16000 \; (N/m)$



$$V = -R_1 + \frac{1}{2} x \left(w_0 \frac{x}{L}\right)$$

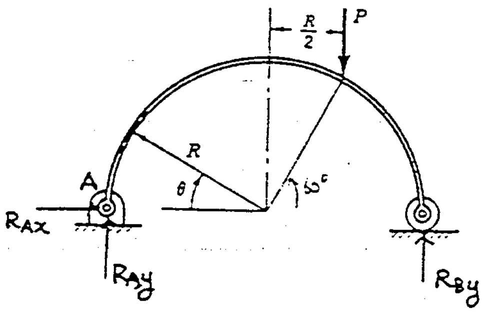
$$= -8000 + \frac{8000}{3} x^2$$

$$M = R_1 x - \frac{1}{2}\left(w_0 \frac{x}{L}\right) \cdot x \cdot \left(\frac{x}{3}\right)$$

$$= R_1 x - \frac{w_0 x^2}{6L}$$

$$= 8000\,x - \frac{8000}{9} x^3$$

3.11



$\Sigma (M_z)_B = 0 \; : \; R_{By}(2R) - P(\frac{3}{2}R) = 0$

$\Sigma F_x = 0 \; : \; R_{Ax} = 0$

$\Sigma F_y = 0 \; : \; R_{Ay} + R_{By} - P = 0$

$\therefore \; R_{By} = \frac{3}{4}P \qquad R_{Ay} = \frac{P}{4}$

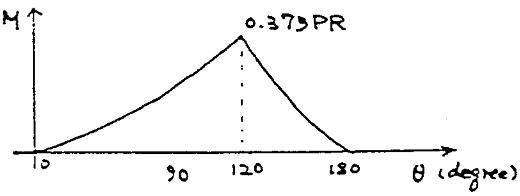
i) $\theta < 120°$

$$M - \frac{P}{4}\left(R(1-\cos\theta)\right) = 0$$

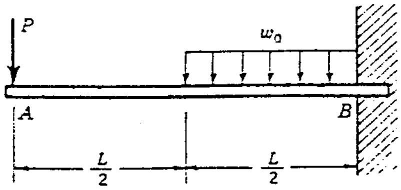
$$M = \frac{PR}{4}(1-\cos\theta)$$

ii) $\theta > 120°$

$$M = \frac{3PR}{4}(1+\cos\theta)$$



3.12

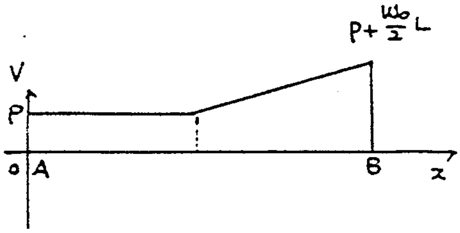


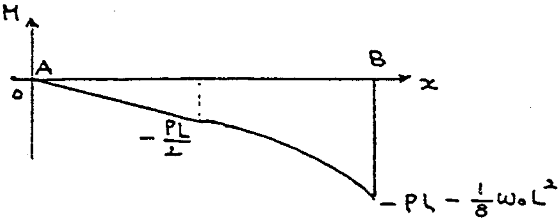
$$q(x) = -P\langle x\rangle_{-1} - w_0\langle x-\tfrac{L}{2}\rangle^0$$

$$V = -\int_{-\infty}^{x} q\,dx = \{P\langle x\rangle^0 + w_0\langle x-\tfrac{L}{2}\rangle^1\}$$

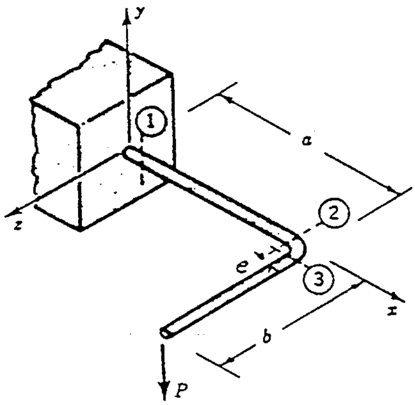
$$M = -\int_{-\infty}^{x} V\,dx = \{-P\langle x\rangle^1 - \tfrac{w_0}{2}\langle x-\tfrac{L}{2}\rangle^2\}$$

$(x<L)$





3.13



Section 3 $\quad V_{y3} = -P$

$M_{x3} = -Pb$

Section 2.

e<<b 라고 가정하고 무시하면

$V_{y2} = -P$

$M_{tx2} = Pb$

Section 3 $\quad V_{y1} = -P$

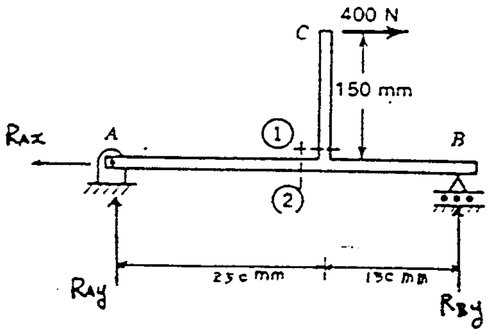
$M_{bz1} = -Pa$

$M_{tx1} = Pb$

Ans) $V_{y1} = -P,\ M_{bz} = -Pa,\ M_{tx1} = Pb$

$V_{y2} = -P,\ M_{tx2} = Pb,\ V_{y3} = -P,\ M_{bx3} = -Pb$

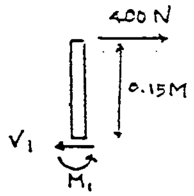
3.14



$R_{Ay} = -R_{By} = -150\ N$

$R_{Ax} = 400\ N$

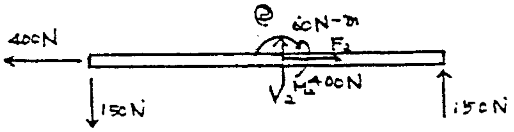
Section 1.



$V_1 = 400N$

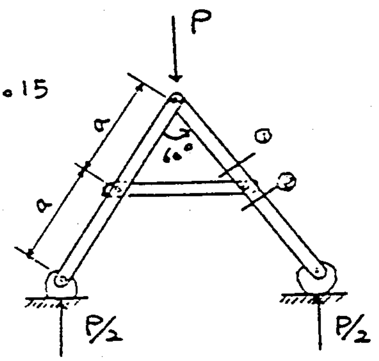
$M_1 = -60\ N\text{-}m$

Section 2.

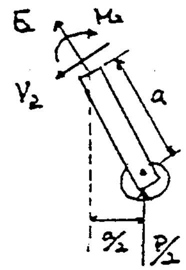


$F_2 = 400\ N$ $\quad V_2 = 150N$ $\quad M_2 = -375\ N\text{-}m$

3.15



Section 2

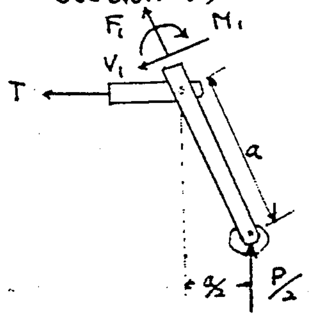


$M_2 = \frac{Pa}{4}$

$F_2 = -\frac{P}{2}\sin 30° = -\frac{P}{4}$

$V_2 = \frac{P}{2}\cos 30° = \frac{\sqrt{3}}{4}P$

Section 1 ;



$$\Sigma M = 0 \qquad Ta \sin 60^\circ = \frac{P}{2}(2a)\cos 60^\circ$$

$$\therefore T = \frac{Pa}{4}$$

$$M_1 = \frac{Pa}{4}$$

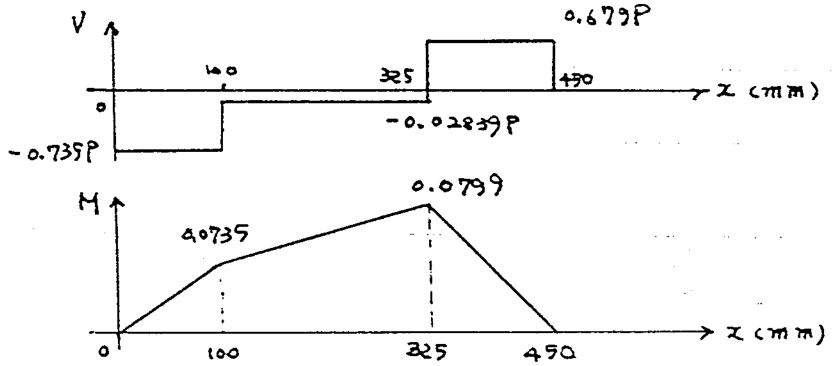
$$V_1 = \frac{P}{2}\sin 30^\circ - T\sin 60^\circ = -\frac{P}{4}$$

$$F_1 = -\frac{P}{2}\cos 30^\circ - T\cos 60^\circ = -\frac{5\sqrt{3}}{12}P$$

3.16

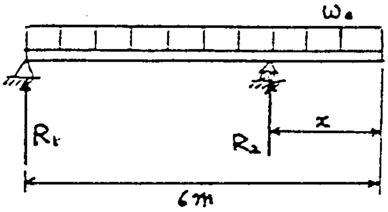
$$\Sigma M_B = 0 \ ; \quad R_A \cdot 450 = (P\cos 45^\circ \cdot 350) + P(\cos 45^\circ \cdot (3)$$
$$+ \ (P\sin 45^\circ \cdot 125) - (P \sin 45^\circ \cdot 20)$$

$$\therefore R_A = 0.735P \qquad \therefore R_D = \sqrt{2}P - R_A = 0.679P$$



B와 C 에 붙어있는 근육은 중간 부분의 Bending Moment 를 분산시켜 즉 max. Bending Moment 의 값을 낮게 해주는 효과.

3.17



$$R_2(6-x) = \frac{1}{2} w_0 (6)^2$$

$$\therefore R_2 = \frac{18 w_0}{6-x}$$

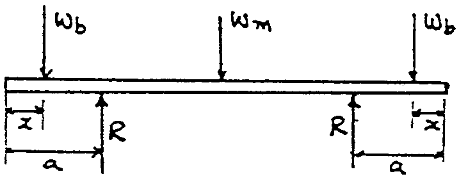
$$R_1 = 6w_0 - \frac{18 w_0}{6-x} = \frac{18w_0 - 6w_0 x}{6-x}$$

$R_1 \times 4.2 = \frac{w_0 \times 4.2^2}{2}$ 여기에 R_1 을 대입

$$\frac{18w_0 - 6w_0 x}{6-x} \cdot 4.2 = \frac{w_0 \times 4.2^2}{2}$$

$$\therefore x = 1.385 \ (m)$$

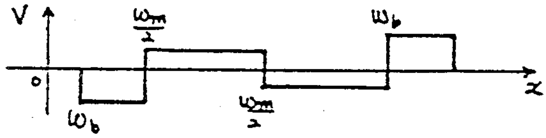
3.18



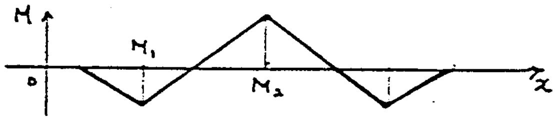
$$R = W_b + \frac{W_m}{2}$$

$$|M_1| = W_b \ (a-x)$$

$$|M_2| = \left(W_b + \frac{W_m}{2}\right)\left(\frac{L}{2} - a\right) - W_b\left(\frac{L}{2} - x\right)$$



$|M_1| = |M_2|$ 인 경우에 max bending moment 가 최소로 될것이므로

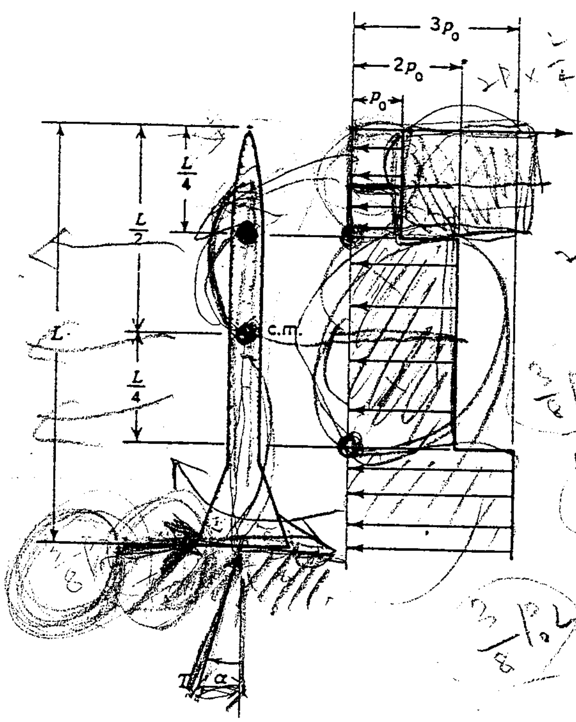


$|M_1| = |M_2|$ 에서 x를 구하면

$$x = a - \frac{W_m}{8W_b}(L-2a)$$

즉 W_b를 $a - \frac{W_m}{8W_b}(L-2a)$ 되는 지점에 놓아야 bending moment 의 최대 값이 최소가 되므로 목수의 생각은 틀렸다.

3.19



$2P_0 \times \frac{L}{4} \times \frac{3L}{8}$

$\frac{3}{8}P_0L - \frac{P_0L}{4}$

$\frac{P_0L}{4}$

$\frac{3}{8}P_0L = 2P_0 \times \frac{L}{2}$

P_0L

(a) $\Sigma M_{c.m.} = 0$

$(T\sin\alpha)\frac{L}{2} = (2P_0\cdot\frac{L}{4})(\frac{3}{8}L)$

$\therefore T\sin\alpha = \frac{3}{8}P_0L$

(b) i) $\frac{L}{4}$ 점

$V = \frac{P_0L}{4}$

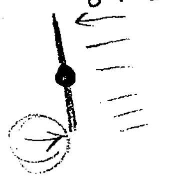
$M_b = -\frac{P_0}{2}(\frac{L}{4})^2$

$= -\frac{P_0L^2}{32}$

ii) $\frac{3}{4}L$ 점

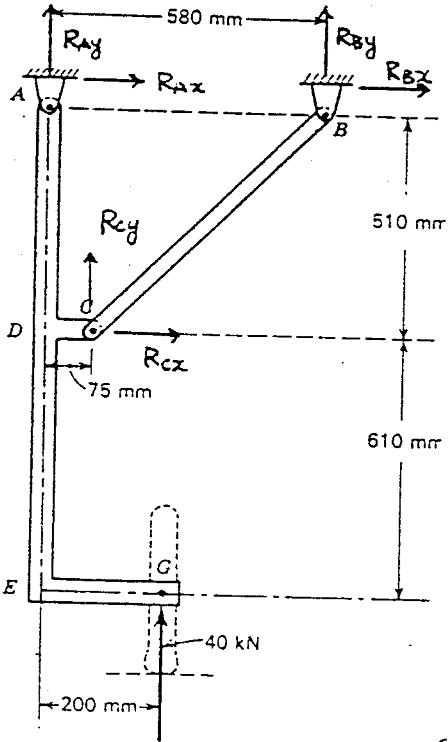
$V = T\sin\alpha - \frac{3}{4}P_0L$

$= -\frac{3}{8}P_0L$



$M_b = (T\sin\alpha)\frac{L}{4} - \frac{3P_0}{2}(\frac{L}{4})^2 = 0$

3.20



a)

$\Sigma M_B = 0;$

$580 R_{Ay} + 40000 \times 200 = 0$

$\therefore R_{Ay} = -13793.1N$

$R_{By} = -26206.9N$

CD 가 없는 왼편 부재만의 free Body로 생각하면

$R_{Ay} + R_{cy} + 40000 = 0$

$\therefore R_{cy} = -26206.9\ N$

$\Sigma M_A = 0$; $75 R_{cy} + 510 R_{cx} + 40000 \times 200 = 0$

$\therefore \quad R_{cx} = -11832.3\,N$

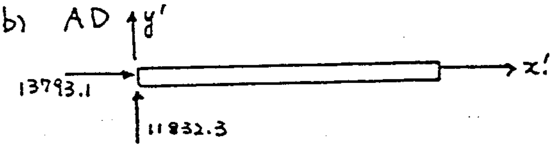
$R_{Ax} + R_{cx} = 0$ 에서

$\therefore \; R_{Ax} = 11832.3\,N$

$R_{Ax} + R_{Bx} = 0$ 에서

$R_{Bx} = -11832.3\,N$

b) AD

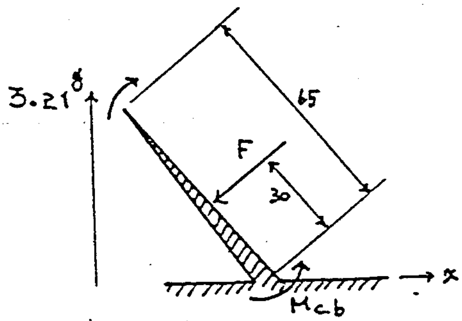


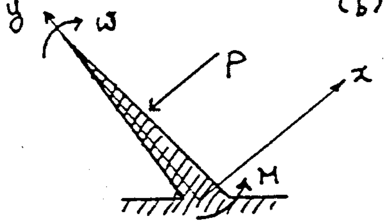
Axial Force $= 13793.1$ (N)(comp)

$V = -11832.3$ (N)

$M_b = 11832.3\,x$ (N-m)

3.21





(a) $M_{c.b} = -30\mu m \times 2.2\,nN$

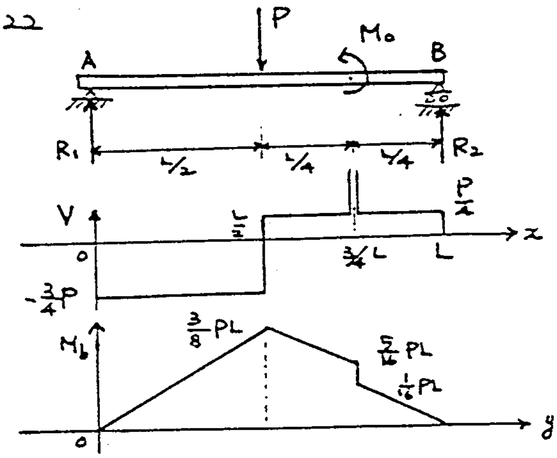
$= -6.6 \times 10^{-14}$ (N-m)

(b) $M = -\int_0^L P y\, dy = c\omega\mu f(\theta) \cdot \frac{L^3}{3}$

$= -(9.154 \times 10^{-14})\, c\mu\omega f(\theta)$

(c ; 상수)

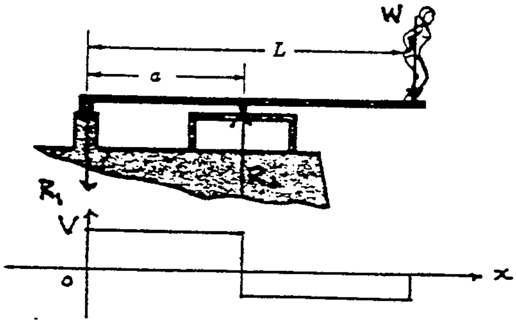
3.22



$R_1 = \frac{3}{4}P$

$R_2 = \frac{1}{4}P$

3.24

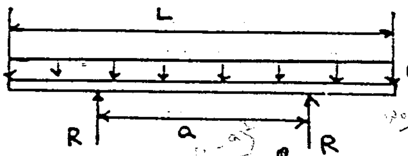


$R_2 = \frac{L}{a}W \quad R_1 = \frac{L-a}{a}W$

$x=a$ 에서 Max. bending moment 가 발생한다는 것을 알수있다
($\because \frac{\partial M_b}{\partial x} = 0$ 는 $V=0$)

$\therefore M_a = -R_1 a = \frac{-(L-a)W}{a}\cdot a = -(L-a)W = const$

3.25



$R = \frac{L w_0}{2}$

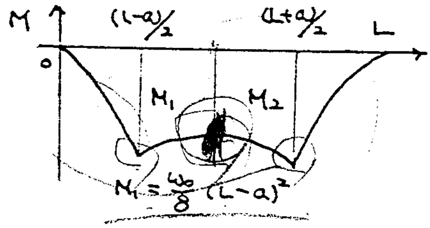
$$q(x) = -w_0 \langle x\rangle^0 + \frac{1}{2}w_0 L\langle x - \frac{L-a}{2}\rangle_{-1} + \frac{w_0 L}{2}\langle x - \frac{L+a}{2}\rangle_{-1}$$

$$M_b(x) = -\frac{w_0}{2}\langle x\rangle^2 + \frac{1}{2}w_0 L\langle x - \frac{L-a}{2}\rangle^1 + \frac{w_0 L}{2}\langle x - \frac{L+a}{2}\rangle^1$$

$M_b(\frac{L}{2}) = -\frac{w_0}{8}L^2 + a\frac{w_0 L}{4} = 0$ 에서 $L = 2a$

$\therefore$ $a \le \frac{L}{2}$ 과 $a > \frac{L}{2}$ 인 구간을 나누어서 생각한다.

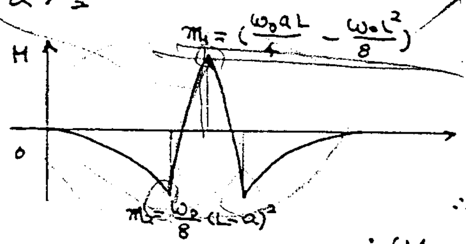
CASE I] $a \le \frac{L}{2}$



$(M_1)_{min}$ 는 $M_2 = 0$ 즉 $a = \frac{L}{2}$일때

$(M_{max})_{CASE I} = \frac{w_0 L^2}{32}$

CASE II] $a > \frac{L}{2}$



$m_1 = m_2$ 때 $(M_{b\,max})_{min}$ 이된다

즉 $(\frac{w_0 aL}{4} - \frac{w_0 L^2}{8}) = \frac{w_0}{8}(L-a)^2$

전개하면 $2aL - L^2 = L^2 - 2aL + a^2$

$\therefore a^2 - 4aL + 2L^2 = 0$

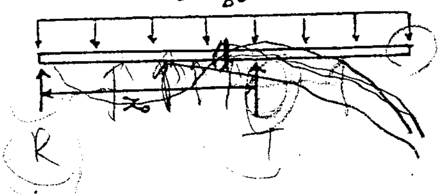
$\therefore a = (2-\sqrt{2})L = 0.5859L$

$\therefore (M_{max})_{CASE II} = \frac{w_0}{8}(1-0.5859)^2 L^2$
$= \frac{w_0 L^2}{46.65}$

$(M_{max})_{CASE II} < (M_{max})_{CASE I}$; $\therefore$ $a = 0.5859L$.

8.26

$\omega_0 = g_0 \cos\theta$



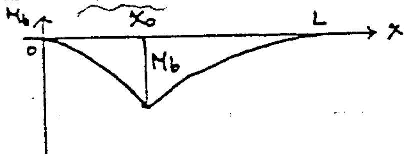
$R_1 = g_0 L\cos\theta \left(1 - \frac{L}{2x_0}\right)$

$T = \frac{g_0 L^2 \cos\theta}{2x_0}$

$$g = -g_0 \cos\theta \langle x\rangle^0 + R_1\langle x\rangle_{-1} + T\langle x - x_0\rangle_{-1}$$

$$M_b = -\frac{g_0\cos\theta}{2}\langle x\rangle^2 + R_1\langle x\rangle^1 + T\langle x - x_0\rangle^1$$

CASE I> $x_0 \le \frac{L}{2}$

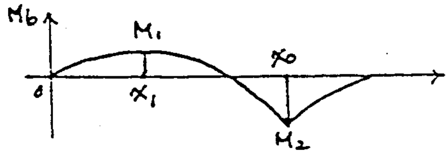


$$M_b = g_0 L\cos\theta\left(x_0 - \frac{L}{2}\right) - \frac{g_0\cos\theta}{2}x_0^2$$
$$= -\frac{g_0\cos\theta}{2}(x_0^2 - 2Lx_0 + L^2)$$
$$= -\frac{g_0\cos\theta}{2}(x_0 - L)^2$$

$\therefore |M_{b\,max}|_{min}$ 은 $x_0 = \frac{L}{2}$ 일때

$\cdot \ (M_{b\,max})_{CASE\,I} = \frac{g_0 L^2\cos\theta}{8}$

CASE II> $x_0 > \frac{L}{2}$



$V=0$; $0 = -g_0\cos\theta\, x_1 + g_0\cos\theta\left(1 - \frac{L}{2x_0}\right)$

$\therefore x_1 = L\left(1 - \frac{L}{2x_0}\right)$

$$\therefore M_1 = -\frac{g_0\cos\theta}{2}L^2\left(1 - \frac{L}{2x_0}\right)^2 + g_0 L\cos\theta\left(1 - \frac{L}{2x_0}\right)^2 L$$

$$\therefore M_1 = \frac{1}{2}g_0 L^2\left(1 - \frac{L}{2x_0}\right)^2\cos\theta$$

$$M_2 = \frac{1}{2}g_0(L - x_0)^2\cos\theta$$

$|M_1| = |M_2|$ 일때 $|M_{max}|_{min}$ $\quad \frac{1}{2}g_0 L^2\left(1 - \frac{L}{2x_0}\right)^2\cos\theta = \frac{1}{2}g_0(L - x_0)^2\cos\theta$

$L^2\left(1 - \frac{L}{x_0} + \frac{L^2}{4x_0^2}\right) = L^2 - 2x_0 L + x_0^2 \rightarrow 4\left(\frac{x_0}{L}\right)^4 - 8\left(\frac{x_0}{L}\right)^3 + 4\left(\frac{x_0}{L}\right) - 1 = 0$

let $\frac{x}{L} = t \qquad 4t^4 - 8t^3 + 4t - 1 = 0 \quad \therefore (2t^2 - 1)(2t^2 - 4t + 1) = 0$

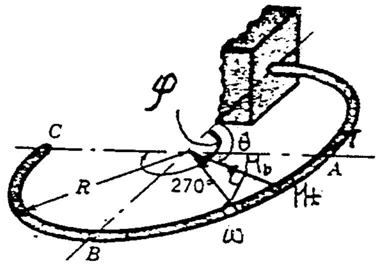
$2t^2 - 4t + 1 > 0$ 이므로 $\qquad t = \frac{\sqrt{2}}{2} \quad \therefore x_0 = 0.707L$ 일때

$(M_{b\,max})_{CASE\,II} = \frac{1}{2}g_0 L^2\left(1 - \frac{\sqrt{2}}{2}\right)^2\cos\theta = 0.043\, g_0 L^3\cos\theta$

$\therefore (M_{b\,max})_{CASE\,II} < (M_{b\,max})_{CASE\,I} \quad \therefore x_0 = 0.707L = 53\ (ft)$

$\therefore x_0 = 53\,ft$

3.27



$M_b = WR\sin(\varphi-\theta)$

$M_t = WR\{1-\cos(\varphi-\theta)\}$

Load의 위치가 다음과 같을 때

1) A점 : $\varphi = 90°$

$M_t = WR(1-\sin\theta) \rightarrow M_t)_{max} = WR$ at $\theta=0$

$M_b = WR\cos\theta \rightarrow M_b)_{max} = WR$ at $\theta=0$

2) B점 : $\varphi = 180°$

$M_t = WR(1+\cos\theta) \rightarrow M_t)_{max} = 2WR$ at $\theta=0$

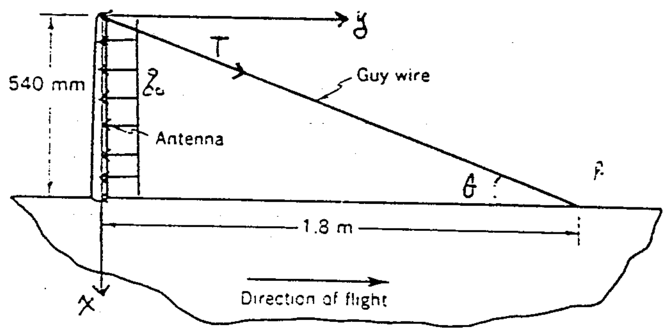
$M_b = WR\sin\theta \rightarrow M_b)_{max} = WR$ at $\theta=90°$

3) C점 : $\varphi = 270°$

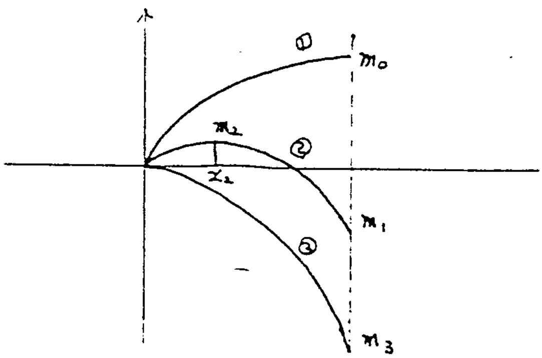
$M_t = WR(1+\sin\theta) \rightarrow M_t)_{max} = 2WR$ at $\theta=90°$

$M_b = -WR\cos\theta \rightarrow M_b)_{max} = WR$ at $\theta=180°, 0°$

3.28



$0.54 q_o = D$



$q = T\cos\theta \langle x\rangle_{-1} - q_o\langle x\rangle^0$

$-V = T\cos \langle x\rangle^0 - q_o\langle x\rangle^1$

$M_b = T\cos\theta\langle x\rangle^1 - \frac{1}{2}q_o\langle x\rangle^2$

① $T\cos\theta - q_0 L = 0 \quad (\because V = 0 \ \text{at} \ x = L)$

$M_b = T\cos\theta \, L - \frac{1}{2} q_0 L^2 = \frac{1}{2} q_0 L^2 = 0.27D$

② $T\cos\theta - q_0 x_2 = 0 \quad (\because V = 0 \ \text{at} \ x = x_2) \quad \therefore x_2 = \frac{T}{q_0}\cos\theta$

$M_b)_2 = T\cos\theta \, \frac{T\cos\theta}{q_0} - \frac{1}{2} q_0 \left(\frac{T\cos\theta}{q_0}\right)^2 = \frac{T^2\cos^2\theta}{2q_0}$

$M_b)_1 = \left| T\cos\theta \cdot L - \frac{1}{2} q_0 L^2 \right|$

$M_b)_1 = M_b)_2$ 일 때 $|M_{b\,max}|_{min}$

$\therefore \frac{T^2\cos^2\theta}{2q_0} = \frac{1}{2} q_0 L^2 - TL\cos\theta, \quad \cos^2\theta \, T^2 + 2D\cos\theta \, T - D^2 = 0$

$\therefore T = \frac{-2D\cos\theta + \sqrt{4D^2\cos^2\theta + 4D^2\cos^2\theta}}{2\cos^2\theta} = \frac{(\sqrt{2}-1)D}{\cos\theta} = 0.432D$

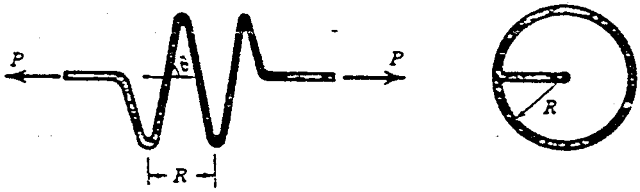
$\therefore M_b)_{max} = \frac{T^2\cos^2\theta}{2q_0} = \frac{(T\cos\theta)^2}{2D} \cdot L = 0.0463D$

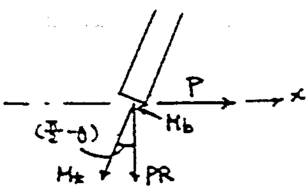
③ $M_b)_{max} = \frac{1}{2} q_0 L^2 = 0.27D$

①,②,③의 경우를 비교하면 ②의 경우가 $M_b)_{max}$가 최소

$\therefore \underline{T = 0.432D}$

3.29





$M_t = PR\cos(90° - \theta) = PR\sin\theta$

$M_b = RP\sin(90° - \theta) = PR\cos\theta$

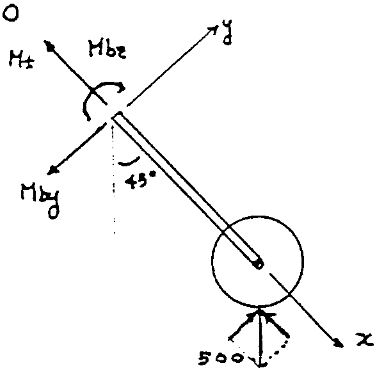
$2\pi R\cos\theta = R$

$\cos\theta = \frac{1}{2\pi} \quad \therefore \theta = 80.9°$

$\therefore M_b)_{max} = PR\cos 80.9° = 0.158PR$

$M_t)_{max} = PR\sin 80.9° = 0.985PR$

3.30



I) $M_t = -\frac{500}{\sqrt{2}} \times 6 = -2121.3$ (lb - in)

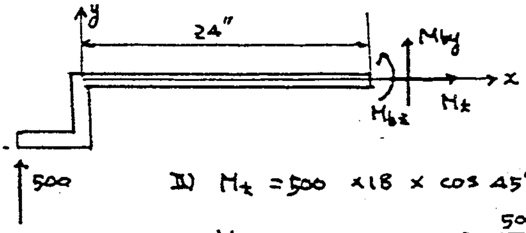
$M_{bz} = \frac{500}{\sqrt{2}} \times 18 = 6364.0$ (lb - in)

$M_{by} = -\frac{500}{\sqrt{2}} \times 6 = -2121.3$ (lb - in)

$\therefore M_t)_{max} = 2121.3$ (lb - in)

$M_b)_{max} = \sqrt{(6364)^2 + (2121.3)^2}$

$= 6708.2$ (lb - in)



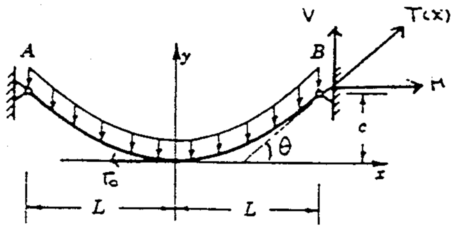
II) $M_t = 500 \times 18 \times \cos 45° = 6364$ (lb - in)

$M_{bz} = 500 \times 24 + 2 \cdot \frac{500}{\sqrt{2}} \times 6 \times \cos 45° = 15000$ (lb - in)

$\therefore M_t)_{max} = 6364$ (lb - in)

$M_b)_{max} = 15000$ (lb - in)

3.31



$\Sigma F_x = 0 \; ; \; H = T_0 = \text{const.}$

$\Sigma F_y = 0 \; ; \; V = H \tan\theta = \omega_0 x$

$\therefore \tan\theta = \frac{\omega_0 x}{H}$

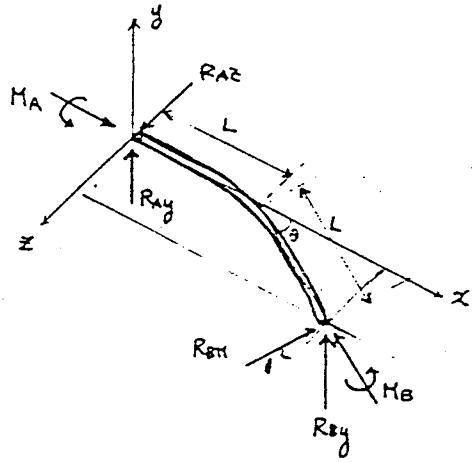
즉 $\frac{dy}{dx} = \frac{\omega_0 x}{H}$ $\qquad \therefore \; y = \frac{\omega_0 x^2}{H} + c$

$x = 0 \; ; \; y = 0 \qquad \therefore \; c = 0$

$\therefore \; y = \frac{\omega_0 x^2}{2H}$

$x = L \; : \; y = c \qquad \therefore \; \underline{H = \frac{\omega_0 L^2}{2c}}$

3.32



$\Sigma F_x = 0$; $R_{Bx} \sin\theta = 0$ ∴ $\underline{R_{Bx} = 0}$

$\Sigma F_y = 0$; $R_{Ay} + R_{By} = 0$ — (1)

$\Sigma F_z = 0$; $R_{Az} - R_{By} \cos\theta = 0$

∴ $\underline{R_{Az} = 0}$

$\Sigma (M_x)_A = 0$;

$M_A - M_B\cos\theta - R_{By} L \sin\theta = 0$ — (2)

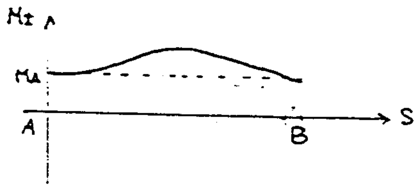
$\Sigma (M_y)_A = 0$; identically satisfied

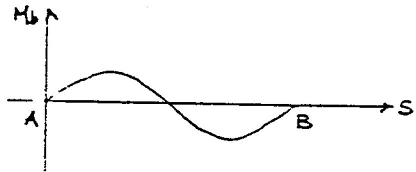
$\Sigma (M_z)_A = 0$; $-M_B \sin\theta + R_{By} L (1+\cos\theta) = 0$

— (3)

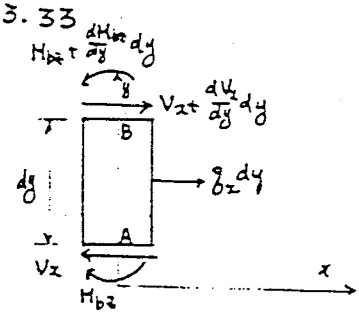
(1),(2),(3)을 풀면 $\underline{R_{Ay} = -\frac{M_A}{L}\tan\left(\frac{\theta}{2}\right) = -R_{By}}$

$\underline{M_B = M_A}$





3.33



dy 에 대한 Free body diagram을 그리면

$\Sigma F_x = 0$; $V_x + \frac{dV_x}{dy} dy - V_x + q_x dy = 0$

$$\therefore \underline{\frac{dV_x}{dy} + q_x = 0}$$

$\Sigma (M_z)_B = 0$;

$$M_{bz} + \frac{dM_{bz}}{dy} dy - V_x dy + \overset{\to 0}{q_x dy \frac{dy}{2}} - M_{bz} = 0$$

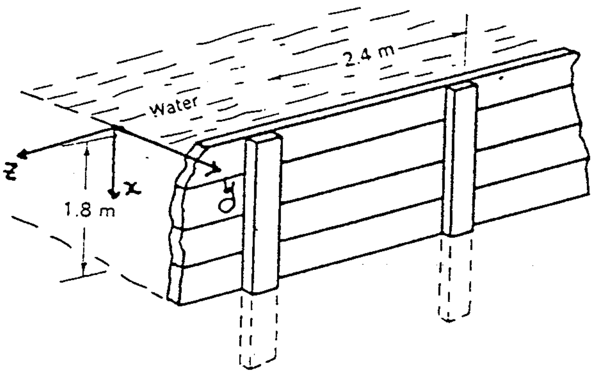
$$\therefore \underline{\frac{dM_{bz}}{dy} - V_x = 0}$$

direction triplet = (direction of member)(direction of loading)(direction of moment vector) 라고 하면,

이 식이 right hand rule 에 일치하면 (a)식, 그렇지 않으면 (b) 식이 된다.

direction of member	loading	triplet	equation type
x	z	xzy	(b)
y	z	yzx	(a)
z	x	zxy	(a)
z	y	zyx	(b)

3.34



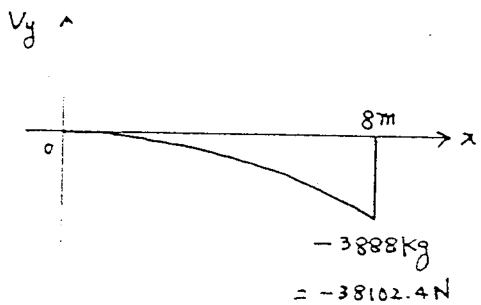
1개의 upright는 2.4m 폭의 수압을 받게 되므로

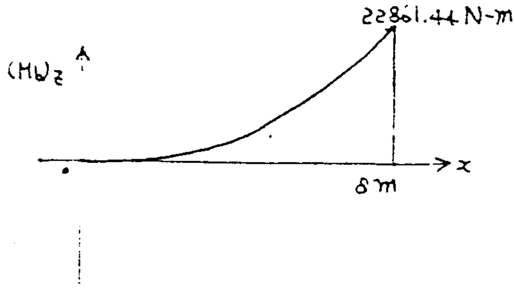
$$dF = \rho g b y\, dy$$

$$\therefore F = \frac{1}{2}\rho g b y^2 = -V_x$$

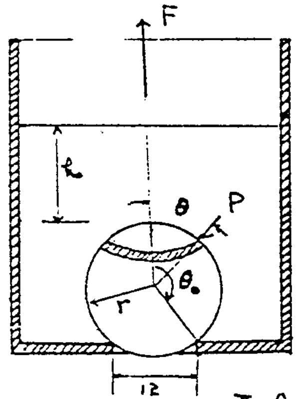
$$M_b = -\int_0^y V_x\, dy$$

$$= \frac{1}{6}\rho g b y^3$$





3.35



$$F = W + \int_0^{\theta_0} 2\pi r \sin\theta \rho g \{h_o + r(1-\cos\theta)\} r\cos\theta d\theta$$

$$= W + 2\pi r^2 \rho g \int_0^{\theta_0} \sin\theta \cos\theta (h_o + r - r\cos\theta) d\theta$$

let $\cos\theta = t$

$$\pi - \theta_0 = \tan^{-1} \tfrac{3}{4}$$

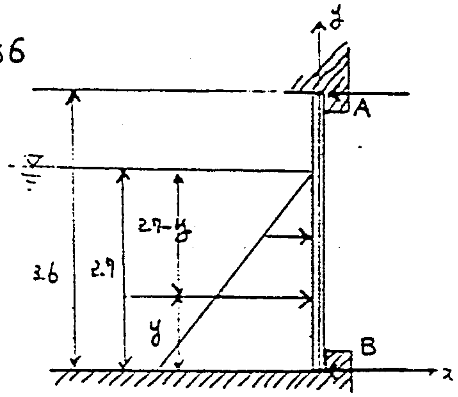
$$F = W + 2\pi r^2 \rho g \int_1^{\cos\theta_0} t(h_o + r - rt)(-dt)$$

$$= W + 2\pi r^2 \rho g \left[\frac{r}{3}(\cos^3\theta_0 - 1) + \frac{1}{2}(h_o + r)\sin^2\theta_0 \right]$$

$$= 160 + 2\pi \left(\frac{10}{12}\right)^2 62.4 \left[\frac{10}{3\times 12}(-0.8^3 - 1) + \frac{1}{2}\left(\frac{18+10}{12}\right) 0.6^2 \right]$$

$$= 160 \text{ (lb)}$$

3.36



$\Sigma M_{Bz} = 0$;

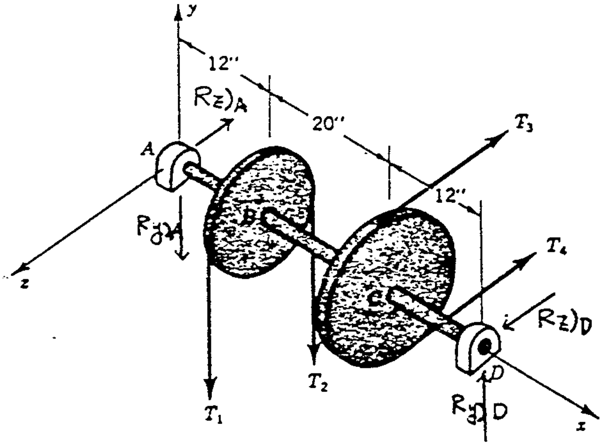
$$3.6 \times R_A = \int_0^{2.7} \rho g (2.7 - y) y \cdot b dy$$

$$= 23520 \left[\frac{2.7}{2} y^2 - \frac{y^3}{3} \right]_0^{2.7}$$

$$= 77157.36$$

$\therefore R_A = 21432.6$ (N)

3.37



$$\frac{T_1}{T_2} = \frac{T_3}{T_4} = 3$$

$2\pi \times (T_1 - T_2) \times 1750 \times \frac{1}{3} = 25 \times 33000$; rotational power

$\therefore T_1 - T_2 = 225.09 \qquad \therefore T_1 = 337.636\ \text{lb}, \quad T_2 = 112.545\ \text{lb}$

$2\pi \times (T_3 - T_4) \times \frac{1}{2} \times 1750 = 25 \times 33000$

$\therefore T_3 - T_4 = 150.06 \qquad \therefore T_3 = 225.09\ \text{lb}, \quad T_4 = 75.03\ \text{lb}$

$\Sigma F_y = 0; \quad P_y = T_1 + T_2 + R_y$ — ①

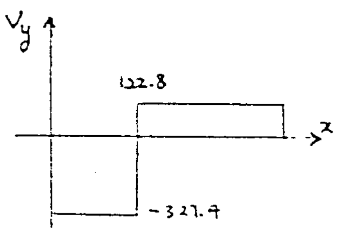
$\Sigma M_{Az} = 0; \ (T_1 + T_2) \times 12 = 44 P_y$ — ②

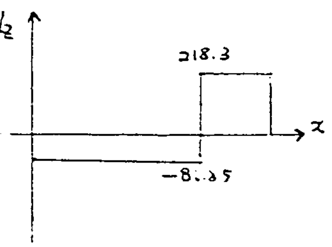
①, ② 에서, $P_y = 122.777\ \text{lb}, \quad R_y = -327.4\ \text{lb}$

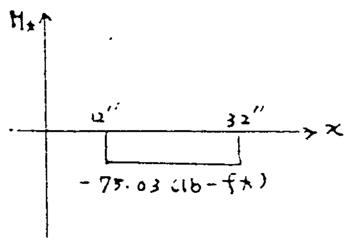
$\Sigma F_z = 0; \quad P_z = (T_3 + T_4) + R_z$ — ③

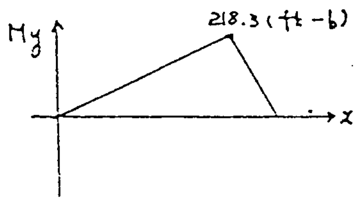
$\Sigma M_{Ay} = 0; \ 32(T_3 + T_4) = 44 P_z$ — ④

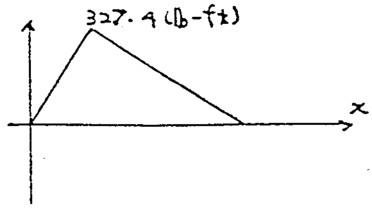
③, ④ 에서, $P_z = 218.27\ \text{lb} \qquad R_z = -81.85\ \text{lb}$

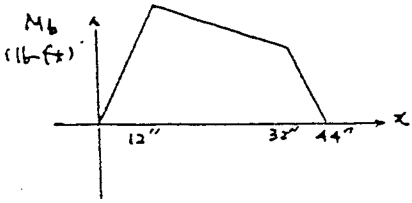


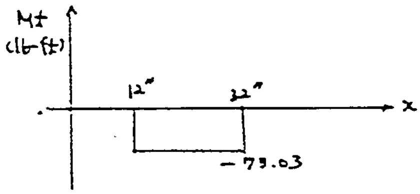




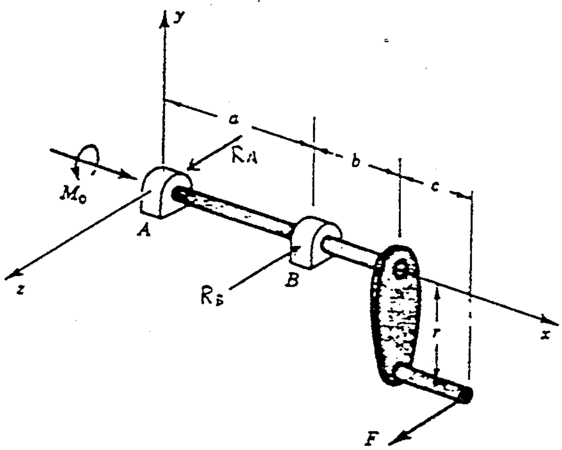








3.38

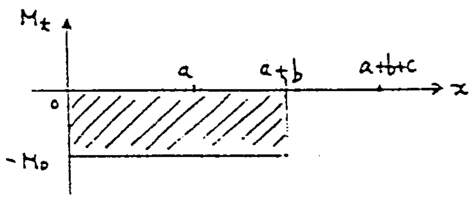


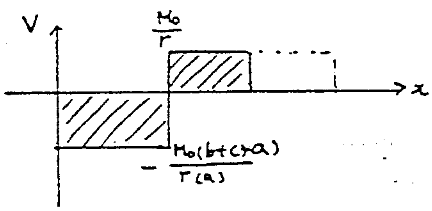
$$\Sigma M_x = 0 \; ; \; F = \frac{M_o}{r}$$

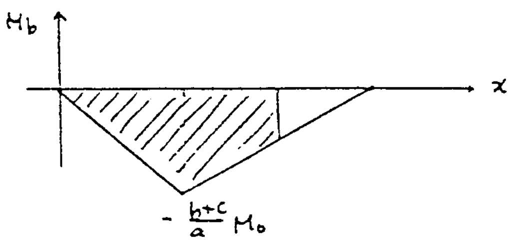
$$\Sigma (M_y)_A = 0 \; ; R_B = \frac{M_o}{r} \frac{(a+b+c)}{a}$$

$$\Sigma (M_y)_B = 0 \; ; \; R_A = \frac{M_o}{r} \frac{(b+c)}{a}$$

$\Sigma F_z = 0$; 위의 식으로
자동 만족

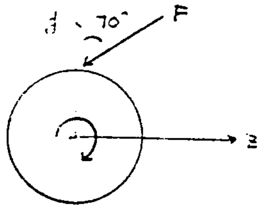






3.39

I)



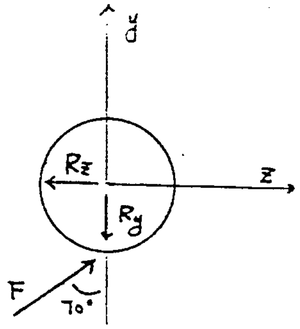
아래 기어

$\sum M_z = 0$

$130 = F \sin 70 \ \overline{2\cdot 3}$

$\therefore F = 6640.5\ N$

II)



$\sum F_y = 0 : R_y = F\cos 70° = 2271.2\ (N)$

$\sum F_z = 0 : R_z = F \sin 70° = 6240\ (N)$

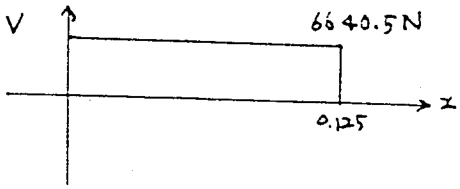
$M_z = -F \sin 70° \ \frac{0.125}{2}$

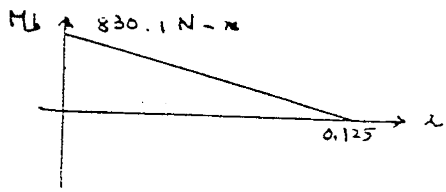
$= -390 \quad (N-m)$

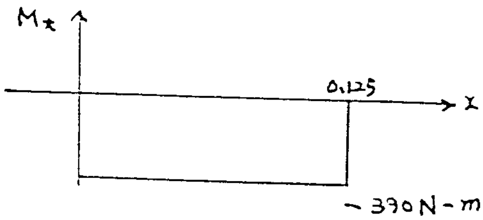
$M_b)_y = -(0.125 - x)\ F \sin 70° = 6240\ (0.125 - x)$

$M_b)_z = (0.125 - x)\ F\cos 70° = 2271.2\ (0.125 - x)$

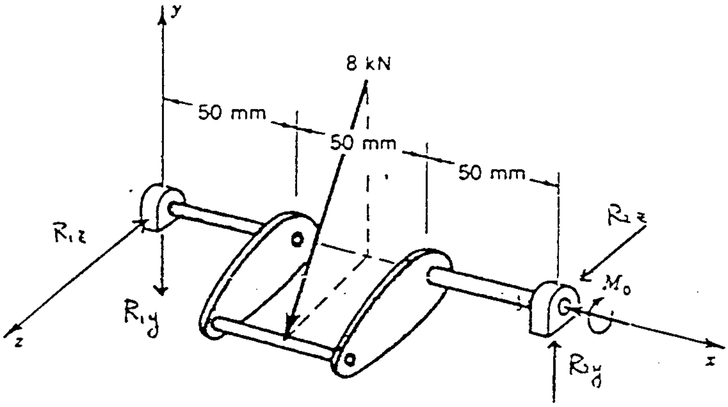
$\therefore\ M_b = 6640.5\ (0.125 - x) \quad (N-m)$







3.40



$2l = 0.075$

$\therefore l = 0.0375\ m$

$\cos\theta = \frac{37.5}{125}$

$\therefore \theta = 72.54^\circ$

$\Sigma F_z = 0;\ R_{2z} - R_{1z} = -8000\cos\theta$ $\quad R_{1z} = 1200\ N$

$\Sigma M_y = 0;\ 150 \cdot R_{2z} + 8000\cos\theta\ 75 = 0$ $\quad R_{2z} = -1200\ N$

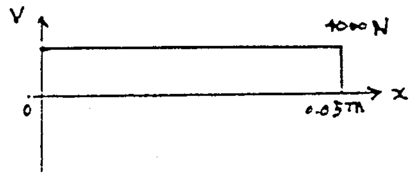
$\Sigma F_y = 0;\ 8000\sin\theta + R_{1y} = R_{2y}$ $\quad R_{1y} = -3815.8\ N$

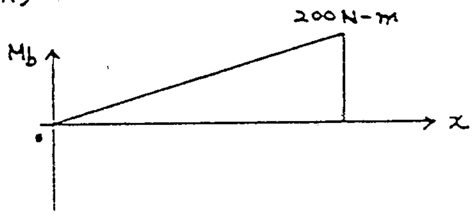
$\Sigma M_z = 0;\ 150 R_{2y} = 8000\sin\theta\ 75$ $\quad R_{2y} = 3815.8\ N$

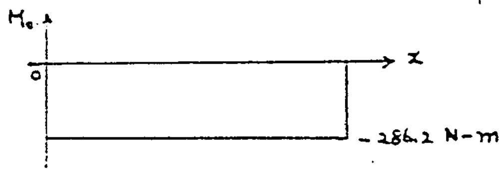
$M_0 = -8000\sin\theta \cdot 0.0375 = -286.2\ (N\text{-}m)$

$V = \sqrt{R_{1z}^2 + R_{1y}^2} = \sqrt{R_{2z}^2 + R_{2y}^2} = 4000\ (N)$

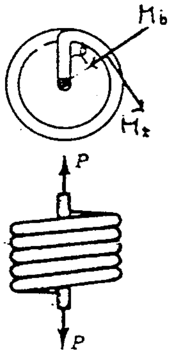
$M_b = Vx = 4000x\ (N\text{-}m)$







3.41

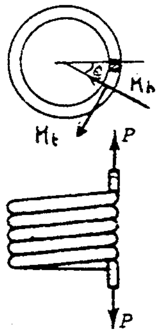


(a)

$M_b = 0$

$M_t = PR$

$V = P$



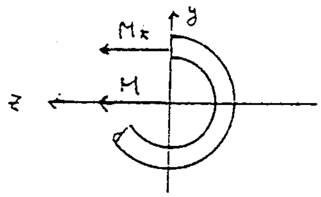
(b)

$M_b = PR \sin\theta$

$M_t = PR(1-\cos\theta)$

$V = P$

3.42 I) A점

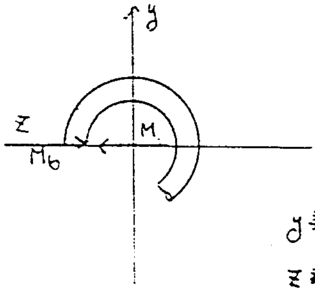


$M_t = M$

$M_b = 0$

$V = 0$

II) B 점



$M_b = M$

$M_t = 0$

$V = 0$

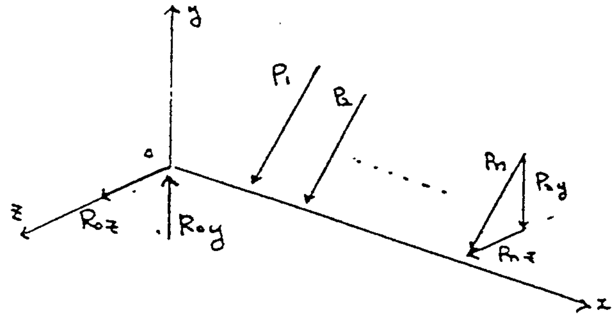
y축상; Pure twisting moment

z축상; Pure bending moment

그외 모든점 : Bending + Twisting

Shear 는 아무데도 없다.

3.43



자중에 의한 bending-moment 는 무시한다.

$x_n < x < x_{n+1}$ 의 구간에서

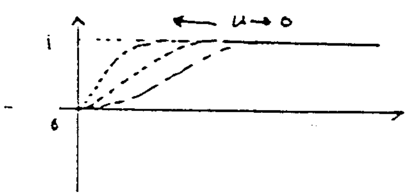
$$M_{By} = \sum_{i=1}^{n} P_{iz}(x-x_i) + R_{oz}\cdot x = \left(\sum_{i=1}^{n} P_{iz} + R_{oz}\right)x - \sum_{i=1}^{n} P_{iy} x_i$$
$$= Ax + B$$
$$M_{Bz} = \sum_{i=1}^{n} P_{iy}(x-x_i) + R_{oy}\cdot x = \left(\sum_{i=1}^{n} P_{iy} + R_{oy}\right)x - \sum_{i=1}^{n} P_{iy} x_i$$
$$= Cx + D$$

A, B, C, D 상수 ∴ M_{by}, M_{bz} 는 linear하고 max는 x_n or x_{n+1} 에서 일어날 것이다.

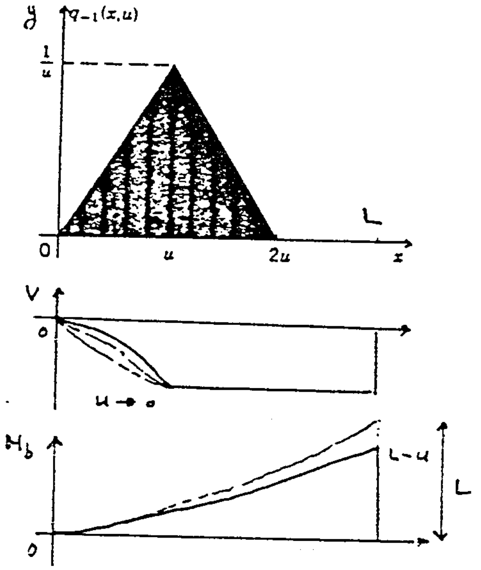
$$M_b = \sqrt{(M_{by})^2 + (M_{bz})^2} = \sqrt{(A^2+C^2)x^2 + 2(AB+CD)x + (B^2+C^2)}$$
$$= \sqrt{ax^2+bx+c} \geq 0$$
$$\frac{d^2 M_b}{dx^2} = \frac{4ac-b^2}{4(ax^2+bx+c)^{\frac{3}{2}}} \qquad \text{또한} \left(\begin{array}{ll} ax^2+bx+c \geq 0 & a \geq 0 \\ b^2-4ac \leq 0 & \end{array}\right)$$
$$\therefore \frac{d^2 M_b}{dx^2} \geq 0 \qquad \text{-Q.E.D.-}$$

3.44

$$p = \int_0^{2u} \delta_{-1}(x,u)\,dx = \int_0^{u} \frac{x}{u^2}\,dx + \int_u^{2u}\left(\frac{2}{u} - \frac{x}{u^2}\right)dx = 1$$



∴ $u \to 0$ 일수록 $\delta_{-1}(x,u)$ 는 unit concentrated force



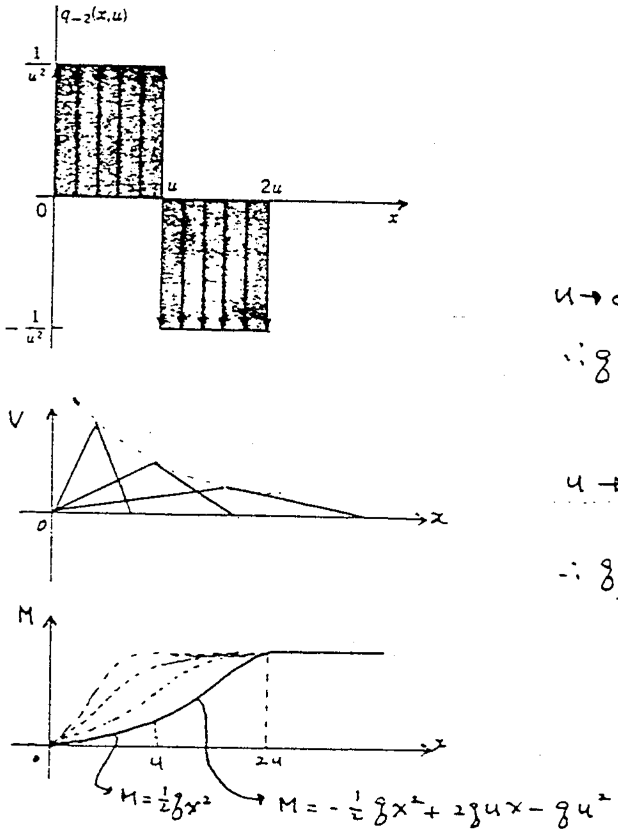
$u \to 0$ 때 $V(x) = -\langle x \rangle^0$

$$q = -\frac{dV_x}{dx} = \langle x \rangle_{-1}$$

$u \to 0$ 때 $M(x) = \langle x \rangle^1$

$$q = \frac{d^2M}{dx^2} = \langle x \rangle_{-1}$$

3.45



$u \to 0$; $V_x = -\langle x \rangle_{-1}$

$$\therefore q = -\frac{dV}{dx} = \langle x \rangle_{-2}$$

$u \to 0$; $M_x = \langle x \rangle^0$

$$\therefore q = \frac{d^2M}{dx^2} = \langle x \rangle_{-2}$$

3.46

i) $0 < x < u$

$$\int_{-\infty}^{x} g_{-2}(x,u)\,dx = \int_0^x \frac{1}{u^2}\,dx = \frac{x}{u^2} = g_{-1}(x,u)$$

ii) $u < x < 2u$

$$\int_{-\infty}^{x} g_{-2}(x,u)\,dx = \int_u^x \left(-\frac{1}{u^2}\right)dx + \int_0^u \frac{1}{u^2}\,dx$$

$$= \frac{2}{u} - \frac{x}{u^2} = g_{-1}(x,u)$$

$$\therefore \int_{-\infty}^{x} g_{-2}(x,u)\,dx = g_{-1}(x,u)$$

한편 $\lim_{u\to 0} g_{-2}(x,u) = \langle x \rangle_{-2}$, $\lim_{x\to u} g_{-1}(x,u) = \langle x \rangle_{-1}$ 이므로

$$\lim_{x\to u} \int_{-\infty}^{x} g_{-1}(x,u)\,dx = \int_{-\infty}^{x} \left\{ \lim_{x\to u} g_{-1}(x,u) \right\} dx$$

$$= \int_{-\infty}^{x} \langle x \rangle_{-1}\,dx$$

$$\lim_{u\to 0} \int_{-\infty}^{x} g_{-2}(x,u)\,dx = \lim_{u\to 0} g_{-1}(x,u) = \langle x \rangle_{-1}$$

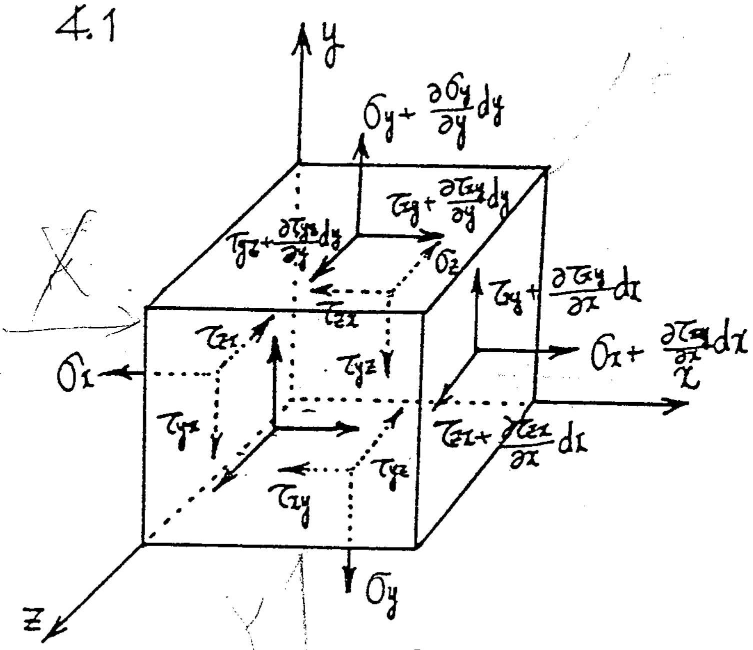
$$\therefore \int_{-\infty}^{x} \langle x \rangle_{-2}\,dx = \langle x \rangle_{-1}$$

$$\therefore \int_{-\infty}^{x} g_{-2}(x-a,u)\,dx = g_{-1}(x-a,u)$$

— Q.E.D. —

CHAPTER. 4

4.1



$$\Sigma F_x = 0 ; \quad \left(\sigma_x + \frac{\partial \sigma_x}{\partial x} dx\right) dy dz - \sigma_x dy dz + \left(\tau_{xy} + \frac{\partial \tau_{xy}}{\partial y} dy\right) dx dz$$
$$- \tau_{xy} dx dz + \left(\tau_{zx} + \frac{\partial \tau_{zx}}{\partial z} dz\right) dx dy - \tau_{zx} dx dz = 0$$
$$\therefore \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}\right) dx dy dz = 0$$

따라서, $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} = 0$

같은 방법으로,

$\Sigma F_y = 0 ; \quad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} = 0$

$\Sigma F_z = 0 ; \quad \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} = 0$

\- Q.E.D -

4.2 문제 4.1과 관련하여 힘의 평형식에 body force 항이 더 첨가 되어진다

$$\therefore \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + X = 0$$

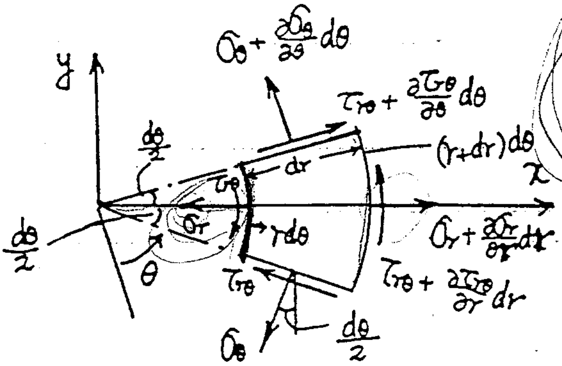
(단, X는 단위 체적당 body force를 나타냄)

같은 방법으로

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0$$

$$\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + Z = 0$$

4.3



$\sin \frac{d\theta}{2} \cong \frac{d\theta}{2}$

$\cos \frac{d\theta}{2} \cong 1$

$(\because \frac{d\theta}{2} \ll 1)$

$\Sigma F_x = 0$;

$$(\sigma_r + \frac{\partial \sigma_r}{\partial r} dr)(r+dr)d\theta - \sigma_r r d\theta - \tau_{r\theta} dr - \sigma_\theta \frac{dr d\theta}{2}$$
$$+ (\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta) dr - (\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta) \frac{dr d\theta}{2} = 0$$

Third term 이상을 무시하면

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\Sigma F_y = 0 ; \ \{\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta}(d\theta) - \sigma_\theta\} dr \cos\frac{d\theta}{2} + (\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial r} dr)$$
$$(r+dr)d\theta - \tau_{r\theta} r d\theta + (\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta + \tau_{r\theta}) dr \sin\frac{d\theta}{2} = 0$$

역시 third term 이상을 무시하여 정리하면

$$\frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{r\theta}}{\partial r} + 2\frac{\tau_{r\theta}}{r} = 0$$

– Q.E.D –

4.4 1) r과 θ의 관계

$$F_x = [(\sigma_r + \frac{\partial \sigma_r}{\partial r} dr)(r+dr)d\theta - \sigma_r r d\theta - \tau_{r\theta} dr - \sigma_\theta dr \frac{d\theta}{2}$$
$$+ (\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta} d\theta) dr - (\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta} d\theta) dr \frac{d\theta}{2}] dz$$

$$F_x \cong \left(r\frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta + \frac{\partial \tau_{r\theta}}{\partial \theta}\right)(dr\,d\theta\,dz)$$

(고차 미분항은 생략하였음)

$$F_y = \Big[\left(\sigma_\theta + \frac{\partial \sigma_\theta}{\partial \theta}d\theta\right)dr + \left(\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial \theta}d\theta\right)dr\frac{d\theta}{2} - \sigma_\theta dr + \tau_{r\theta}dr\frac{d\theta}{2}$$

$$+ \left(\tau_{r\theta} + \frac{\partial \tau_{r\theta}}{\partial r}dr\right)(r+dr)\,d\theta - \tau_{r\theta}(r\,d\theta)\Big]dz$$

$$\cong \left(\frac{\partial \sigma_\theta}{\partial \theta} + \tau_{r\theta} + r\frac{\partial \tau_{r\theta}}{\partial r} + \tau_{r\theta}\right)(dr\,d\theta\,dz)$$

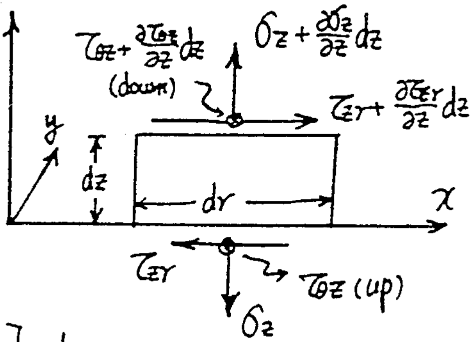
$$F_z = \Big[\left(\tau_{zr} + \frac{\partial \tau_{zr}}{\partial r}dr\right)(r+dr)\,d\theta - \tau_{zr}\,r\,d\theta + \left(\tau_{\theta z} + \frac{\partial \tau_{\theta z}}{\partial \theta}d\theta\right)dr$$

$$- \tau_{\theta z}dr\Big]dz \cong \left(r\frac{\partial \tau_{zr}}{\partial r} + \tau_{zr} + \frac{\partial \tau_{\theta z}}{\partial \theta}\right)(dr\,d\theta\,dz)$$

2) z 성분에 대하여

$$F_x \fallingdotseq \left[\left(\tau_{zr} + \frac{\partial \tau_{zr}}{\partial z}dz\right)dr - \tau_{zr}dr\right]r\,d\theta$$

$$= \left(r\frac{\partial \tau_{zr}}{\partial z}\right)dr\,d\theta\,dz$$



$$F_y = \left[\left(\tau_{\theta z} + \frac{\partial \tau_{\theta z}}{\partial z}dz\right)dr - \tau_{\theta z}dr\right]r\,d\theta$$

$$= \left(r\frac{\partial \tau_{\theta z}}{\partial z}\right)dr\,d\theta\,dz$$

$$F_z = \left[\left(\sigma_z + \frac{\partial \sigma_z}{\partial z}dz\right)dr - \sigma_z dr\right]r\,d\theta = \left(r\frac{\partial \sigma_z}{\partial z}\right)dr\,d\theta\,dz$$

$\Sigma F_x = 0$;

$$\left(r\frac{\partial \sigma_r}{\partial r} + \sigma_r - \sigma_\theta + \frac{\partial \tau_{r\theta}}{\partial \theta} + r\frac{\partial \tau_{zr}}{\partial r}\right)dr\,d\theta\,dz = 0$$

$\Sigma F_y = 0$;

$$\left(\frac{\partial \sigma_\theta}{\partial \theta} + \tau_{r\theta} + r\frac{\partial \tau_{r\theta}}{\partial r} + \tau_{r\theta} + r\frac{\partial \tau_{\theta z}}{\partial z}\right)dr\,d\theta\,dz = 0$$

$\Sigma F_z = 0$;

$$\left(r\frac{\partial \tau_{zr}}{\partial r} + \tau_{zr} + \frac{\partial \tau_{\theta z}}{\partial \theta} + r\frac{\partial \sigma_z}{\partial z}\right)dr\,d\theta\,dz = 0$$

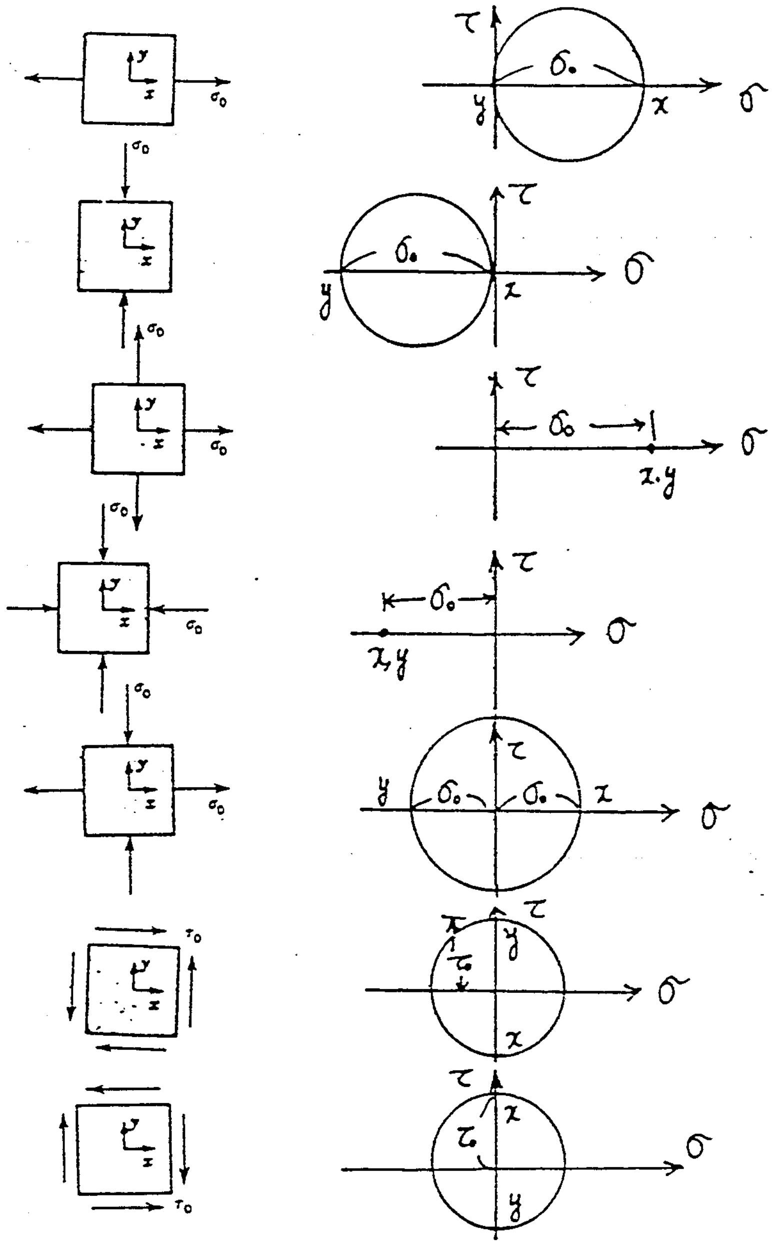
정리하여 나타내면 다음과 같다

$$* \begin{cases} \dfrac{\partial \sigma_r}{\partial r} + \dfrac{1}{r}\dfrac{\partial \tau_{r\theta}}{\partial \theta} + \dfrac{\partial \tau_{zr}}{\partial r} + \dfrac{(\sigma_r - \sigma_\theta)}{r} = 0 \\ \dfrac{\partial \tau_{r\theta}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \sigma_\theta}{\partial \theta} + \dfrac{\partial \tau_{\theta z}}{\partial z} + \dfrac{2\tau_{r\theta}}{r} = 0 \\ \dfrac{\partial \tau_{zr}}{\partial r} + \dfrac{1}{r}\dfrac{\partial \tau_{\theta z}}{\partial \theta} + \dfrac{\partial \sigma_z}{\partial z} + \dfrac{\tau_{zr}}{r} = 0 \end{cases}$$

4.5 a축의 방향 전환은 physical plane에서 a축이 180° 회전한 것과 같다. 이는 Mohr's circle plane에서 $180° \times 2 = 360°$를 회전한 것과 같다.

따라서, 원래의 위치와 마찬가지가 되며 응력 성분의 방향과 크기도 같을 수 밖에 없다.

4.6



4.7 $\sigma_x' = \dfrac{\sigma_x+\sigma_y}{2} + \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

$\sigma_y' = \dfrac{\sigma_x+\sigma_y}{2} - \dfrac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

$\tau_{xy}' = -\dfrac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$

~~주축은~~ $\tau_{xy}'=0$ 일때, 즉 $\tan 2\theta_p = \dfrac{2\tau_{xy}}{\sigma_x-\sigma_y}$

(a) $\theta_p = \frac{1}{2}\tan^{-1}\dfrac{2\times 80}{40-0} = 37.98°$

$\sigma_{1,2} = \dfrac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2} = 20 \pm \sqrt{20^2+80^2}$

$\therefore \sigma_1 = 102.462\ MN/m^2,\quad \sigma_2 = -62.462\ MN/m^2,\quad \theta_p = 37.98°$

(b) $\theta_p = \frac{1}{2}\tan^{-1}\dfrac{2\times(-60)}{140-20} = -22.5°$

$\sigma_{1,2} = \dfrac{140+20}{2} \pm \sqrt{\left(\dfrac{140-20}{2}\right)^2+(-60)^2}$

$\therefore \sigma_1 = 164.85\ MN/m^2,\quad \sigma_2 = -4.85\ MN/m^2,\quad \theta_p = 22.5°$

(c) $\theta_p = \frac{1}{2}\tan^{-1}\left(\dfrac{2\times 100}{-120-150}\right) = -21.82°$

$\sigma_{1,2} = -35 \pm \sqrt{85^2+100^2}$

$\therefore \sigma_1 = 96.24\ MN/m^2,\quad \sigma_2 = -166.24\ MN/m^2,\quad \theta_p = -21.82°$

(d) $\theta_p = \frac{1}{2}\tan^{-1}\dfrac{2\times 8000}{10000+4000} = 24.41°$

$\sigma_{1,2} = 3000 \pm \sqrt{7000^2+8000^2}$

$\therefore \sigma_1 = 13630\ psi,\quad \sigma_2 = -7630\ psi,\quad \theta_p = 24.41°$

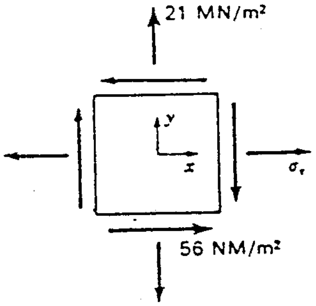
(e) $\theta_p = \frac{1}{2}\tan^{-1}\dfrac{2\times(-6000)}{-10000-20000} = -79.10°$

$\sigma_{1,2} = 5000 \pm \sqrt{15000^2+6000^2}$

$\therefore \sigma_1 = 21155.5\ psi,\quad \sigma_2 = -11155.5\ psi$

$\theta_p = -79.10°$

4.8



minimum principal stress

$$\sigma_2 = -7\ MN/m^2.$$

$$\sigma_2 = \frac{\sigma_x + 21}{2} - \sqrt{\left(\frac{\sigma_x - 21}{2}\right)^2 + 56^2} = -7$$

$$\left(\frac{\sigma_x - 21}{2}\right)^2 + 56^2 = \left(\frac{\sigma_x + 35}{2}\right)^2$$

$$112\sigma_x = 112^2 + 21^2 - 35^2 \qquad \therefore \sigma_x = 105$$

$$\sigma_x = 105\ MN/m^2, \quad \theta_p = \frac{1}{2}\tan^{-1}\frac{2\times(-56)}{105-21} = -26.565°$$

Ans.) $\sigma_x = 105\ MN/m^2$, $\theta_p = -26.565°$

4.9 $\sigma_x = -120\ MN/m^2$, $\sigma_y = 50\ MN/m^2$, $\tau_{xy} = 100\ MN/m^2$

$\theta = 30°$

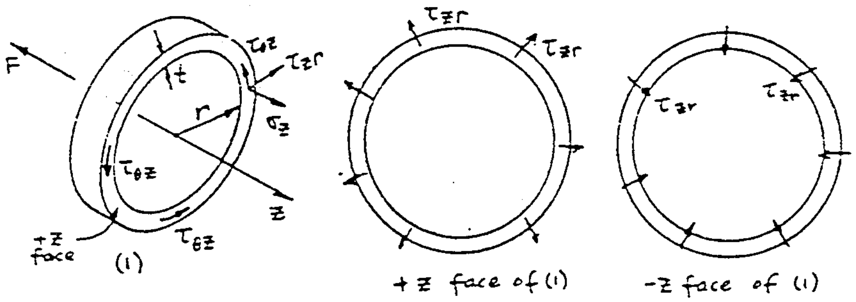
$$\sigma_x' = \frac{-120+50}{2} + \frac{-120-50}{2}\cos 60° + 100\sin 60° = 9.1025\ (MN/m^2)$$

$$\sigma_y' = -35 + 85\cos 60° - 100\sin 60° = -79.1025\ (MN/m^2)$$

$$\tau_{xy}' = 85\sin 30° + 100\cos 60° = 123.6122\ (MN/m^2)$$

Ans.) $\sigma_x' = 9.1025\ MN/m^2$, $\sigma_y' = -79.1025\ MN/m^2$.
$\tau_{xy}' = 123.6122\ MN/m^2$

4.10



i) +z 축에 대하여 축 대칭이므로 응력은 θ값에 무관하다

+z 면에서 τ_{zr}이 상수 값을 가지며 방향이 outward라 하자

F의 작용점에서 먼거리에 있는 점의 응력은 St. Venant's

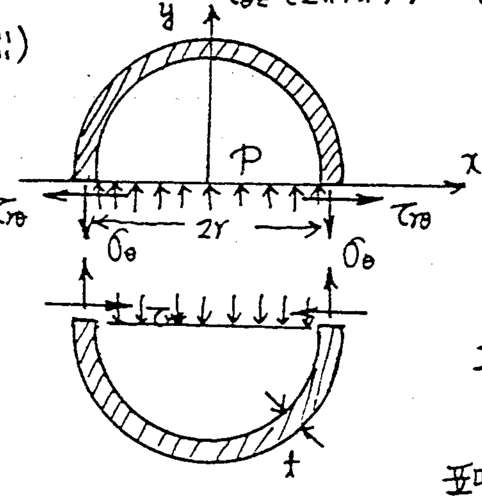
Principle에 의해서 z와는 무관하다. 또한 $-z$ 면에서의 τ_{zr}은 $+z$면의 응력과 크기는 같고 방향이 반대가 될 것이다 (작용과 반작용의 원리). 그런데 ⊕⊖축의 선택은 임의적이므로 같은 요소에 대한 같은 물리적 응력 상태를 가져야 하므로, 결국 $\tau_{zr}=0$ 일수 밖에 없다.

평형식 ; $\Sigma F_z = 0$; $F = \sigma_z 2\pi r t$ $\therefore \sigma_z = \dfrac{F}{2\pi r t}$

$\Sigma M_z = 0$;

$\tau_{\theta z}(2\pi r t)\, r = 0$ $\therefore \tau_{\theta z} = 0$

ii)



i)과 마찬가지로 circular symmetry가 있고 x, y축 선택이 임의이므로. $\tau_{r\theta}=0$

Equilibrium ;

$\Sigma F_y = 0$; $2[\sigma_\theta (t)(1)] = p(2r)(1)$

$\therefore \sigma_\theta = \dfrac{pr}{t}$

표면 안쪽 응력 $\sigma_r = -p$

표면 바깥쪽 응력 $\sigma_r = 0$ 이므로

벽 내부의 응력은 $-p \leqq \sigma_r \leqq 0$ 이다.

$(r/t) \gg 1$ 이면 $\sigma_\theta \gg \sigma_r$, $\sigma_z \gg \sigma_r$ 이고

σ_r은 무시될 수 있다 $\therefore \sigma_r \fallingdotseq 0$

* 요약 ; $\left[\begin{array}{l} \tau_{r\theta} = \tau_{\theta z} = \tau_{zr} = 0 \quad \therefore r, \theta, z = \text{주축} \\ \sigma_r \fallingdotseq 0 \ ; \quad \sigma_\theta = \dfrac{pr}{2t}, \quad \sigma_z = \dfrac{F}{2\pi r t} \end{array} \right.$

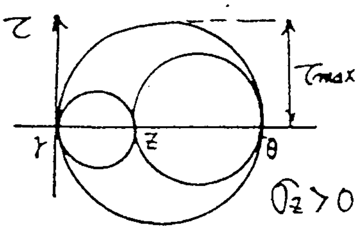
4.11 문제 4.10 으로 부터

$\sigma_r \fallingdotseq 0$

$\sigma_\theta = \dfrac{pr}{2t}$

$\sigma_z = \dfrac{F}{2\pi r t}$

Shear stress $= 0$



$\tau_{max} = \frac{1}{2} | \sigma_{max} - \sigma_{min} | =$ radius of largest circle

a) i) 만약 $F>0$ 이면 $\sigma_z > 0$ 이다

$\tau_{max} = \frac{1}{2}\sigma_z$ or $\frac{1}{2}\sigma_\theta$, $\sigma_{max} = \sigma_z$ or σ_θ

$\therefore$ $\tau_{max} \neq \sigma_{max}$ (앞페이지 그림 참조)

ii) $F<0$ 이면 $\sigma_z<0$

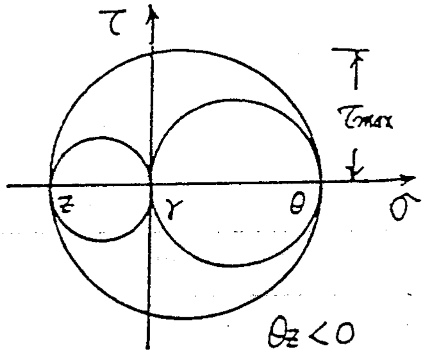
$\tau_{max} = \frac{1}{2}(\sigma_\theta - \sigma_z)$

$= \frac{1}{2}(\frac{pr}{t} + \frac{F}{2\pi rt})$

$\sigma_{max} = \sigma_\theta = \frac{pr}{t}$

$\therefore \frac{1}{2}(\frac{pr}{t} + \frac{F}{2\pi rt}) = \frac{pr}{t}$

$\therefore$ $F = 2\pi r^2 p$



b) $F>0$ 이면

$\tau_{max} = \frac{1}{2}\sigma_{max}$ (항상 성립)

$F \le 0$ 이면 $\sigma_z \le 0$ $\frac{1}{2}(\frac{pr}{t} + \frac{F}{2\pi rt}) = \frac{1}{2}(\frac{pr}{t})$

$\therefore$ $F=0$. $\therefore$ $F \geq 0$ 일때 성립

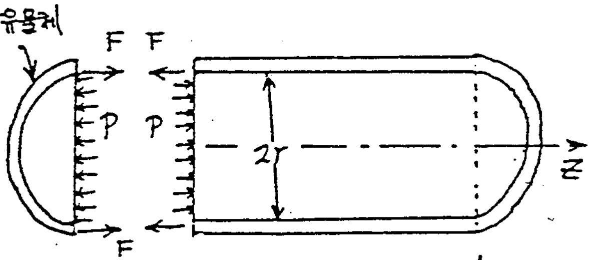
Ans.) (a) $F = 2\pi r^2 p$ (b) $F \geq 0$

4.12

왼편 자유물체의 평형식;

$\Sigma F_z = 0$:

$F = p\pi r^2$



* Circular symmetry 와 +z축의 선정이 임의적이므로.

$\tau_{zr} = 0$ (Pro 4.10 참조)

$\tau_{\theta r} = 0$

$\Sigma M_z = 0$; $\tau_{\theta z} = 0$ $\therefore$ r, θ, z는 주축.

σ_z, σ_θ를 표현하면

$\Sigma F_z = 0$; $\quad 2\pi r t\, \sigma_z = \pi r^2 p \qquad \therefore\ \sigma_z = \frac{rp}{2t}$

$\Sigma F_t = 0$; $\quad b r p = \sigma_\theta b t \qquad \therefore\ \sigma_\theta = \frac{pr}{t}$

$-p \leqq \sigma_r \leqq 0 \qquad r/t \gg 1$ 이면 $\quad \sigma_\theta \gg \sigma_r$

$\therefore\ \sigma_r \cong 0$.

Ans.) $\sigma_r \cong 0, \quad \sigma_\theta = \frac{pr}{t}, \quad \sigma_z = \frac{rp}{2t}$

4.13 $\quad \sigma_r = 0, \quad \sigma_\theta = \frac{pr}{t} \quad \sigma_z = \frac{pr}{2t}$

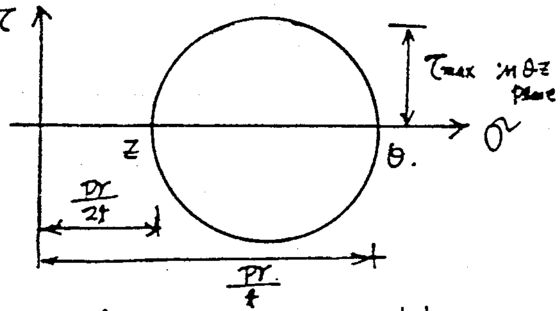
θ-z plane에서 Mohr 원을 그리면. $(\tau_{r\theta} = \tau_{zr} = 0)$

$(\tau_{max})_{\theta z} = \frac{1}{2}(\sigma_\theta - \sigma_z)$

$= \frac{pr}{4t}$

$(\sigma_{max})_{\theta z} = \sigma_\theta = \frac{pr}{t}$

$\therefore\ (\tau_{max})_{\theta z} = \frac{1}{4}(\sigma_{max})_{\theta z}$



< 주의; $(\tau_{max})_{\theta z}$ 는 θ-z plane에서 최대치 이지만 최대 전단 응력은 아님에 주의 할 것 >

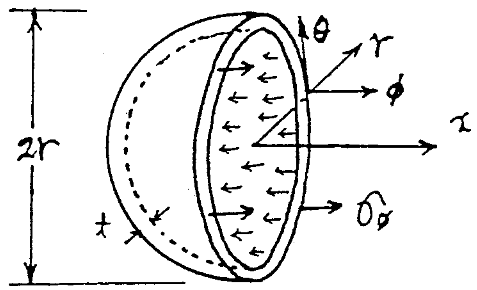
4.14

<문제 4.10>을 이용하여 r, ϕ, θ 가 주축임을 안다.

평형식 ; $\Sigma F_x = 0$;

$-p\pi r^2 + \sigma_\phi (2\pi r t) = 0$

$\therefore\ \sigma_\phi = \frac{pr}{2t}$



마찬가지 방법으로 $\quad \sigma_\theta = \frac{pr}{2t}$,

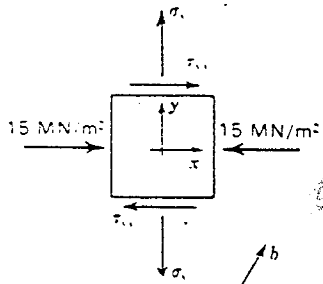
표면 안쪽 $\quad \sigma_r = -p$, $\quad$ 표면 바깥쪽 $\quad \sigma_r = 0$

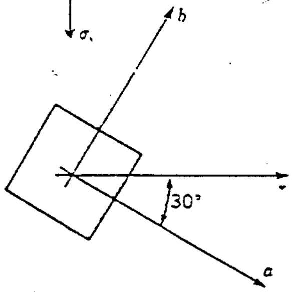
따라서 표면내에서 $\quad -p \leqq \sigma_r \leqq 0$ 이고 $\quad r/t \gg 1$ 이면

$\sigma_r \ll \sigma_\theta, \sigma_\phi \qquad \therefore\ \sigma_r \cong 0$

Ans.) $\quad \sigma_r = 0, \quad \sigma_\phi = \sigma_\theta = \frac{pr}{2t}$

4.15





Shear stress = 0. $\tau_{xy}=0$

$$\sigma_y = 75\ MN/m^2, \quad \sigma_x = -15\ MN/m^2$$

$$\sigma_a = \frac{-15+75}{2} + \frac{-15-75}{2}\cos(-60^\circ)$$

$$= 7.5\ (MN/m^2)$$

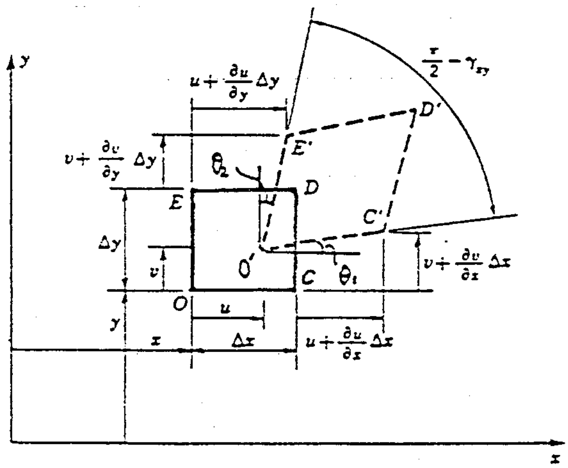
$$\sigma_b = 30 + 45\cos(-60^\circ) = 52.5\ (MN/m^2)$$

$$\tau_{ab} = 45\sin(-60^\circ)$$

$$= -38.97\ (MN/m^2)$$

Ans.)
$$\begin{cases} \sigma_a = 7.5\ MN/m^2 \\ \sigma_b = 52.5\ MN/m^2 \\ \tau_{ab} = -38.97\ MN/m^2 \end{cases}$$

4.16



$$\epsilon_x = \frac{O'C'-OC}{OC} \cong \frac{(\Delta x + u + \frac{\partial u}{\partial x}\Delta x - u) - \Delta x}{\Delta x} = \frac{\partial u}{\partial x}$$

$$\epsilon_y = \frac{O'E'-OE}{OE} \cong \frac{(\Delta y + v + \frac{\partial v}{\partial y}\Delta y - v) - \Delta y}{\Delta y} = \frac{\partial v}{\partial y}$$

$$\gamma_{xy} = \theta_1 + \theta_2 \cong \frac{(v + \frac{\partial v}{\partial x}\Delta x) - v}{\Delta x} + \frac{(u + \frac{\partial u}{\partial y}\Delta y) - u}{\Delta y} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}$$

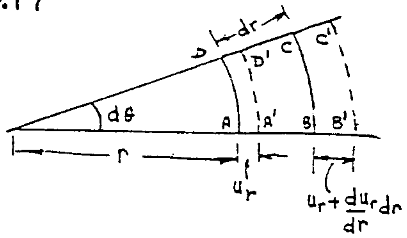
< 참고; z 방향 처짐 w를 고려하면 A'B'는 x-y 평면과 작은 각을 이루나, 이에 대한 고려는 무시하여도 무방하다)

같은 방법으로 다른 항도 구하면 다음과 같다.

$$\begin{cases} \epsilon_x = \frac{\partial u}{\partial x} \\ \epsilon_y = \frac{\partial v}{\partial y} \\ \epsilon_z = \frac{\partial w}{\partial z} \end{cases} \qquad \begin{cases} \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \\ \gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \\ \gamma_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{cases}$$

– Q.E.D –

4.17



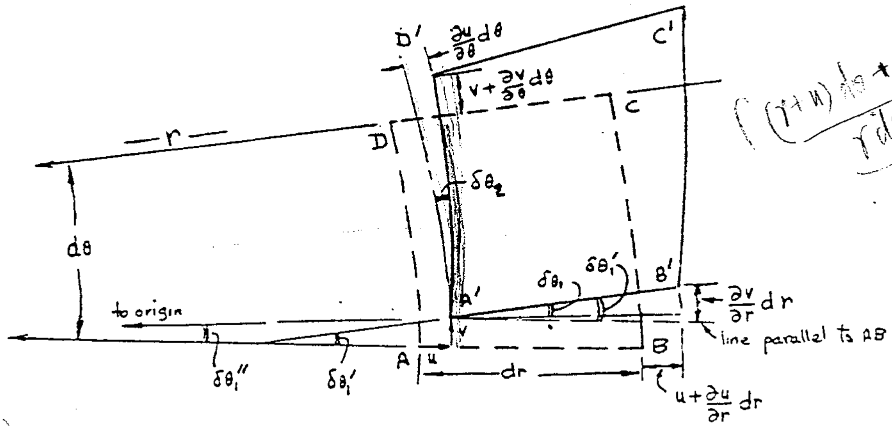
$$\epsilon_r = \frac{A'B' - AB}{AB} = \frac{\left(dr + u_r + \frac{du_r}{dr}dr - u_r\right) - dr}{dr} = \frac{du_r}{dr}$$

$$\epsilon_\theta = \frac{A'D' - AD}{AD} = \frac{(r+u_r)\,d\theta - r\,d\theta}{r\,d\theta} = \frac{u_r}{r}$$

$\gamma_{r\theta} =$ ∠DAB 의 각변화 $= 0$

— Q.E.D —

4.18



$\frac{(r+u)\,d\theta + v \ldots}{r\,d\theta}$

$$\epsilon_r = \frac{A'B' - AB}{AB} = \frac{\left[\left(dr + u + \frac{\partial u}{\partial r}dr\right) - u\right] - [dr]}{dr} = \frac{\partial u}{\partial r}$$

$$\epsilon_\theta = \frac{A'D' - AD}{AD} = \frac{\left[(r+u)\,d\theta + v + \frac{\partial v}{\partial \theta}d\theta - v\right] - [r\,d\theta]}{r\,d\theta} = \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}$$

$$\gamma_{r\theta} = \delta\theta_1 + \delta\theta_2 = (\delta\theta_1' - \delta\theta_1'') + \delta\theta_2 = \left(\frac{\frac{\partial v}{\partial r}dr}{dr} - \frac{v}{r}\right) + \frac{\frac{\partial u}{\partial \theta}d\theta}{r\,d\theta}$$

$$= \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta}$$

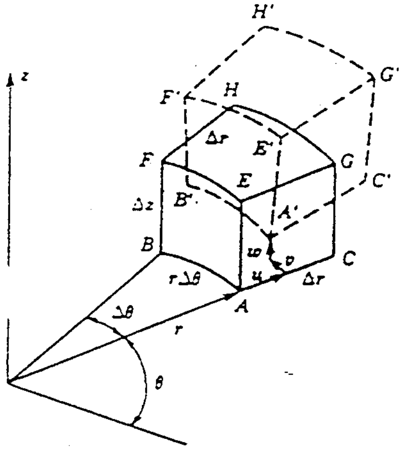
— Q.E.D —

4.19 그림의 B'F'E'A'를 r-θ 평면에 투영하고

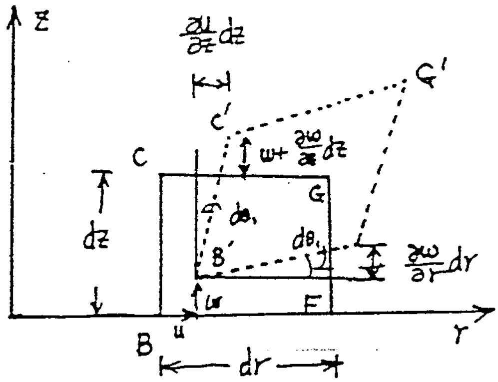
〈문제 4.18〉의 결과를 이용한다

$$\epsilon_r = \frac{\partial u}{\partial r}, \qquad \epsilon_\theta = \frac{1}{r}\frac{\partial v}{\partial \theta} - \frac{u}{r}$$

$$\gamma_{r\theta} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial u}{\partial \theta} - \frac{v}{r}$$



B'F'G'C'를 BFGC에 평행한 평면에 투영시키고 이들이 이루는 각이 매우 작으므로 이각을 무시한다.



$$\epsilon_z = \frac{B'C' - BC}{BC}$$

$$= \frac{\left[dz + w + \frac{\partial w}{\partial z}dz - w\right] - dz}{dz} = \frac{\partial w}{\partial z}$$

$$\gamma_{zr} = d\theta_1 + d\theta_2 = \frac{\frac{\partial w}{\partial r}dr}{dr} + \frac{\frac{\partial u}{\partial z}dz}{dz} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}$$

마찬가지로, A'B'C'D'를 ABCD를 포함하는 평면에 투영하여 문제를 풀면 $\gamma_{\theta z} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta}$ 를 얻을수 있다.

4.20 $\epsilon_x = 800 \times 10^{-6}$, $\epsilon_y = 100 \times 10^{-6}$, $\gamma_{xy} = -800 \times 10^{-6}$

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\epsilon_1 = 981.5 \times 10^{-6}, \qquad \epsilon_2 = -81.5 \times 10^{-6}$$

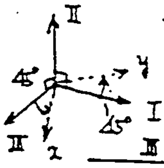
$$\theta_p = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x - \epsilon_y} = -24.4^\circ$$

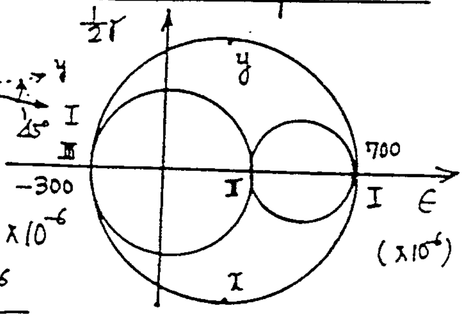
Ans.) $\epsilon_1 = 981.5 \times 10^{-6}$, $\epsilon_2 = -81.5 \times 10^{-6}$, $\theta_p = -24.4^\circ$

4.21 $\epsilon_{I} = 700 \times 10^{-6}$

$\epsilon_{II} = 300 \times 10^{-6}$

$\epsilon_{III} = -300 \times 10^{-6}$

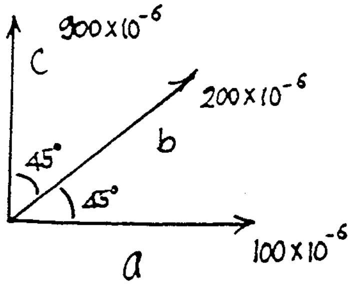


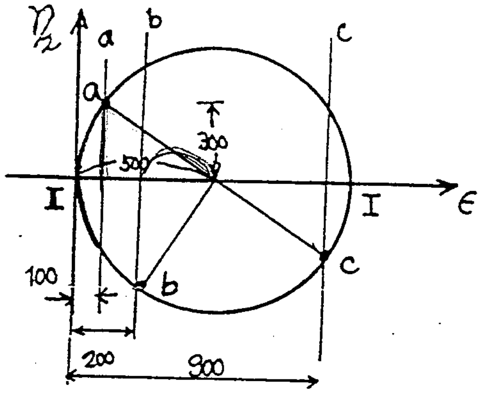


$$\gamma_{max} = 2\left(\frac{1}{2}\left|\epsilon_{I} - \epsilon_{III}\right|\right) = 1000 \times 10^{-6}$$

Ans.) $\gamma_{max} = 1000 \times 10^{-6}$

4.22 (a) $\epsilon_a = 100 \times 10^{-6}$, $\epsilon_b = 200 \times 10^{-6}$, $\epsilon_c = 900 \times 10^{-6}$

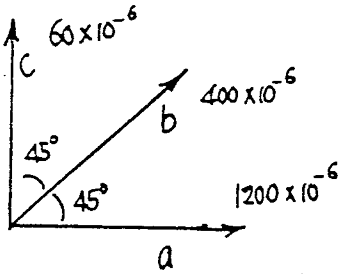


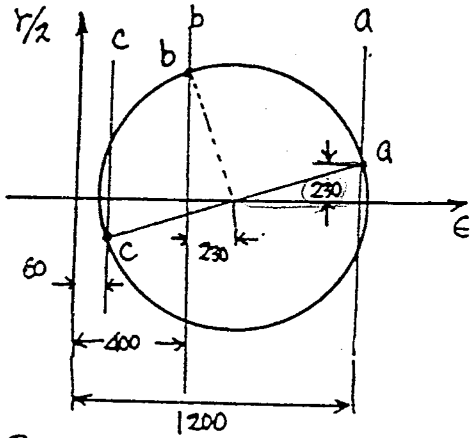


(b) $\epsilon_a = 1200 \times 10^{-6}$

$\epsilon_b = 400 \times 10^{-6}$

$\epsilon_c = 60 \times 10^{-6}$

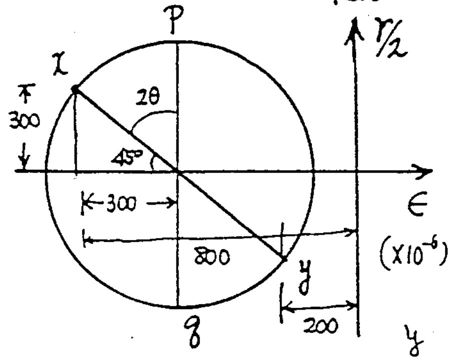


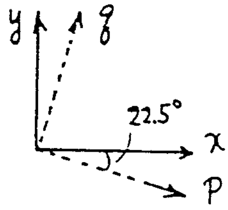


4.23 $\epsilon_x = -800 \times 10^{-6}$

$\epsilon_y = -200 \times 10^{-6}$

$\gamma_{xy} = -600 \times 10^{-6}$



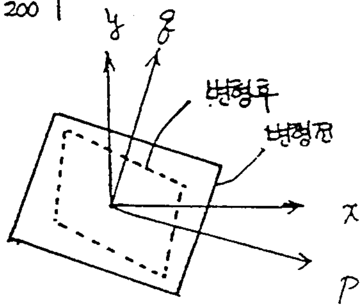


$2\theta = 45°$

$\therefore \theta = 22.5°$

p.q ; max shear axes

[γ_{pq} ; negative]



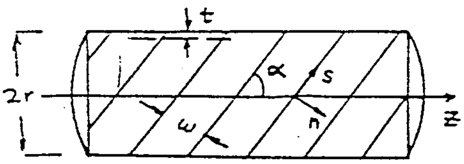
4.24 $u = w = 0$. $v = Cr$

<문제 4.19 >.에서의 변형도 성분을 이용하여 풀면

$$\begin{cases} \epsilon_r = \dfrac{\partial u}{\partial r} = 0 \\ \epsilon_\theta = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta} + \dfrac{u}{r} = 0 \\ \epsilon_z = \dfrac{\partial w}{\partial z} = 0 \end{cases} \qquad \begin{cases} \gamma_{r\theta} = \dfrac{\partial v}{\partial r} + \dfrac{1}{r}\dfrac{\partial u}{\partial \theta} - \dfrac{v}{r} = C - C = 0 \\ \gamma_{\theta z} = \dfrac{1}{r}\dfrac{\partial w}{\partial \theta} + \dfrac{\partial v}{\partial z} = 0 \\ \gamma_{zr} = \dfrac{\partial u}{\partial z} + \dfrac{\partial w}{\partial r} = 0 \end{cases}$$

변위 $v = Cr$로 부터 유발되는 Strain은 없고 이 운동은 z축에 관한 회전 운동이다.

4.25



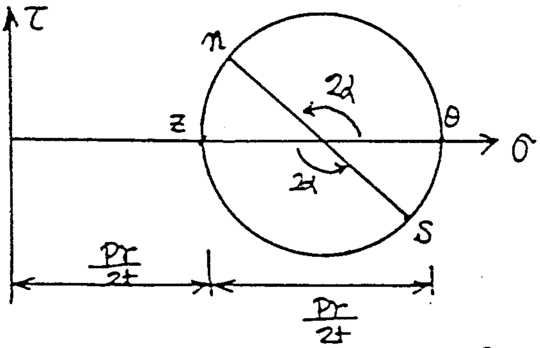
기하학적 조건 ; $2\pi r \cos\alpha = W$

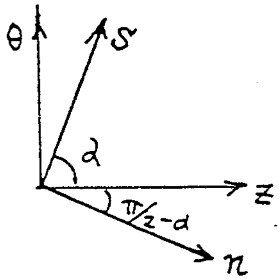
<문제 4.12 >로 부터

$\sigma_r \cong 0$, Shear stress$=0$

$\sigma_\theta = \dfrac{Pr}{t}$, $\sigma_z = \dfrac{Pr}{2t}$

θ-z 평면을 n-s 평면으로 바꾸기 위하여 Mohr 원을 이용하면





$$\sigma_n = \frac{\sigma_\theta + \sigma_z}{2} - \frac{\sigma_\theta - \sigma_z}{2}\cos(\pi - 2\alpha)$$

$$= \frac{3Pr}{4t} + \frac{Pr}{4t}\cos 2\alpha$$

$\sigma_{max} = \sigma_\theta = \dfrac{Pr}{t}$, 만약 $\sigma_n = 0.8\,\sigma_{max}$ 이라면

$$\sigma_n = 0.8\frac{Pr}{t} = \frac{3Pr}{4t} + \frac{Pr}{4t}\cos 2\alpha$$

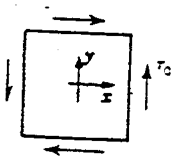
$\therefore \cos 2\alpha = 0.2 \quad , \quad \alpha = 39.23 \quad , \quad \cos\alpha = 0.775$

$$W = 2\pi r \cot\alpha \cdot \sin\alpha = 2\pi r \cos\alpha$$

$$\therefore \frac{W}{r} = 2\pi \cos 39.23° \cong 4.86$$

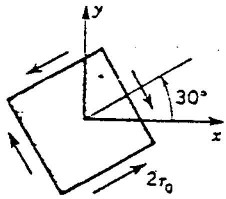
Ans.) $\frac{W}{r} \leq 4.86$

4.26 (a)



(i)

\+



(ii)

Mohr 원을 이용하여 (ii)를 x-y 축의 응력으로 변환시키면

$\sigma_x = -2\tau_0 \sin(-60) = \sqrt{3}\tau_0$

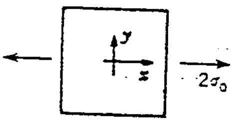
$\sigma_y = 2\tau_0 \sin(-60) = -\sqrt{3}\tau_0$

$\tau_{xy} = -2\tau_0 \cos(-60°) = -\tau_0$

이를 (i)과 합하면 $\sigma_x = \sqrt{3}\tau_0$, $\sigma_y = -\sqrt{3}\tau_0$, $\tau_{xy} = \tau_0 - \tau_0 = 0$

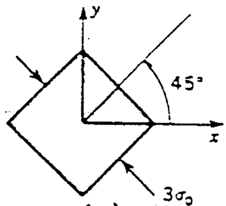
따라서, 합한 결과는 x, y축이 주축이다

(b)



(i)

\+



(ii)

마찬가지로 (ii)를 x.y축의 응력으로 나타내면

$\sigma_x = \frac{-3\sigma_0}{2} + \frac{-3\sigma_0}{2}\cos(-90°)$

$= -1.5\sigma_0$

$\sigma_y = -\frac{3}{2}\sigma_0 - \frac{-3\sigma_0}{2}\cos(-90°)$

$= -1.5\sigma_0$

$\tau_{xy} = \left(-\frac{3\sigma_0}{2}\right) = 1.5\sigma_0$

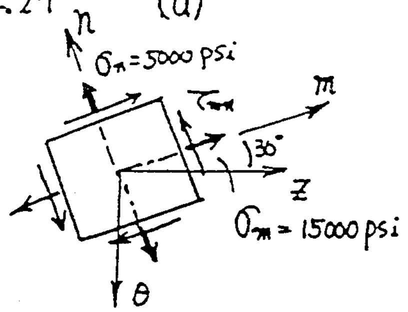
(i).(ii)를 합하면 $\sigma_x = 2\sigma_0 - 1.5\sigma_0 = 0.5\sigma_0$, $\sigma_y = -1.5\sigma_0$

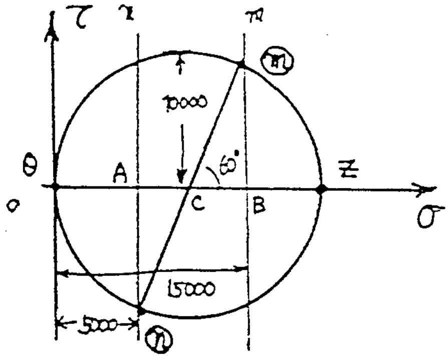
$\tau_{xy} = 1.5\sigma_0$ 따라서 주축 방향은 다음과 같다.

$$\theta_p = \frac{1}{2}\tan^{-1}\frac{2\times1.5}{0.5-(-1.5)} = 28.15°$$

Ans.) (a) $\theta_p = 0$. (b) $\theta_p = 28.15$

4.27 (a)





<문제 4.10>의 결과를 이용하여 $\sigma_z = \frac{F}{2\pi r t}$, $\sigma_\theta = \frac{pr}{t}$,

주축 θ, z를 30° 회전시키면 σ_m, σ_n, τ_{mn}을 구할수있다.

$$\sigma_m = \frac{\sigma_\theta + \sigma_z}{2} + \frac{-\sigma_\theta + \sigma_z}{2}\cos 60^\circ = \frac{3F}{8\pi} + 25P = 15000$$

$$\sigma_n = \frac{\sigma_\theta + \sigma_z}{2} - \frac{-\sigma_\theta + \sigma_z}{2}\cos 60^\circ = \frac{F}{8\pi} + 75P = 5000$$

위 두식을 연립하여 풀면 $P = 0$, $F = 40000\pi$ (lb)

$$\tau_{mn} = \frac{\sigma_z - \sigma_\theta}{2}\sin 60^\circ = -8670 \text{ psi}$$

(b) $\sigma_m = 15{,}000$, $\sigma_n = 15{,}000$

$$\left.\begin{aligned} \frac{3F}{8\pi} + 25P &= 15000 \\ \frac{F}{8\pi} + 75P &= 15000 \end{aligned}\right) \quad \begin{aligned} P &= 150 \text{ (psi)} \\ F &= 30000\pi \text{ (lb)} \end{aligned}$$

$$\tau_{mn} = -\frac{\sigma_z - \sigma_\theta}{2}\sin 60 = 0$$ //

Ans.) [(a) $P = 0$, $F = 40000\pi$ lb , $\tau_{mn} = -8670$ psi

(b) $p = 150$ psi , $F = 30000\pi$ lb , $\tau_{mn} = 0$

4.28 $\sigma_1 = 20$ MN/m², $\sigma_2 = -45$ MN/m²

$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 20^2} = 20 \quad \cdots ①$$

$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 20^2} = -45 \quad \cdots ②$$

①+② ; $\sigma_x + \sigma_y = -25 \quad \cdots ③$

①−② : $65 = 2\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + 20^\circ}$ $\therefore (\sigma_x - \sigma_y)^2 = 2625$

$\therefore\ \sigma_x - \sigma_y = \pm 51.235 \cdots ④$

③④로 부터 2개의 해를 구한다.

i) $\begin{cases} \sigma_x = 13.118\ MN/m^2 \\ \sigma_y = -38.118\ MN/m^2 \end{cases}$ ii) $\begin{cases} \sigma_x = -38.118\ MN/m^2 \\ \sigma_y = 13.118\ MN/m^2 \end{cases}$

Ans.) $\left[\begin{array}{l} \text{i) } \sigma_x = 13.118\ MN/m^2,\ \sigma_y = -38.118\ MN/m^2 \\ \text{ii) } \sigma_x = -38.118\ MN/m^2,\ \sigma_y = 13.118\ MN/m^2 \end{array} \right.$

4.29 $u(x,y) = (\cos\beta - 1)x - \sin\beta\, y$

$v(x,y) = \sin\beta\, x + (\cos\beta - 1)y$

$\epsilon_x = \frac{\partial u}{\partial x} = \cos\beta - 1, \quad \epsilon_y = \frac{\partial v}{\partial y} = \cos\beta - 1$

$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -\sin\beta + \sin\beta = 0$

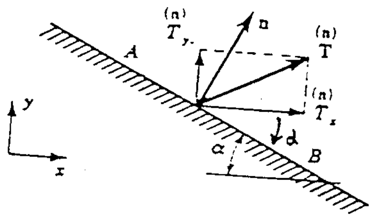
$W_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}(\sin\beta + \sin\beta) = \sin\beta$

$$\begin{cases} \epsilon_x = \cos\beta - 1 \\ \epsilon_y = \cos\beta - 1 \\ \gamma_{xy} = 0 \\ W_z = \sin\beta \end{cases} \underset{\beta \to 0}{\Longrightarrow} \begin{cases} \epsilon_x = 0 \\ \epsilon_y = 0 \\ \gamma_{xy} = 0 \\ W_z = \beta \end{cases}$$

$\beta \to 0$ 일때 $(\because \sin\beta \cong \beta)$

Ans.) $\epsilon_x = \epsilon_y = \gamma_{xy} = 0, \quad W_z = \beta$

4.30



$\sigma_x' = \frac{T_x + T_y}{2} + \frac{T_x - T_y}{2}\cos(-2\alpha)$

$\sigma_y' = \frac{T_x + T_y}{2} - \frac{T_x - T_y}{2}\cos(-2\alpha)$

$\tau_{xy}' = \frac{T_x - T_y}{2}\sin(-2\alpha)$

위식의 σ_x', σ_y'가 동시에 Zero가 되려면 $T_x + T_y = 0$ 이고 $\cos 2\alpha = 0$ 인 $\alpha = 45°$ 이어야 함

이때, $\tau_{xy}' = T_x$ 이다

Ans.) $T_x + T_y = 0, \quad \alpha = 45°$

4.31 $\sigma_x' = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$

극값을 찾기위해서 $\frac{\partial \sigma_x'}{\partial \theta}=0$ 인 θ를 구하면, 다음과 같다.

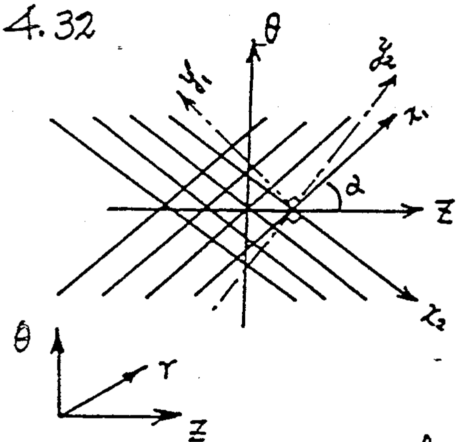
$$\frac{\partial \sigma_x'}{\partial \theta} = -(\sigma_x-\sigma_y)\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\therefore \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

위 값은 $\tau_{xy}'=0$ 로 만드는 θ 값에 해당한다.

즉, σ_x'의 극값은 $\tau_{xy}'=0$ 로 되는 각에서 갖으며 그 각은 주축의 방향각과 일치한다 -Q.E.D-

4.32



Filament는 축방향 장력만 견딤

축 방향 응력을 σ라 하고

filament 응력을 θ-z 축으로 변환한다.

i) x_1 방향 filament

$$\sigma_{x_1}' = \frac{\sigma}{2} + \frac{\sigma}{2}\cos(-2\alpha)$$

$$\sigma_{y_1}' = \frac{\sigma}{2} - \frac{\sigma}{2}\cos(-2\alpha)$$

$$\tau_{xy_1}' = -\frac{\sigma}{2}\sin(-2\alpha)$$

ii) x_2 방향 filament.

$$\sigma_{x_2}' = \frac{\sigma}{2} + \frac{\sigma}{2}\cos 2\alpha \qquad \sigma_{y_2}' = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\alpha$$

$$\tau_{xy_2}' = -\frac{\sigma}{2}\sin 2\alpha.$$

위 두 응력 성분을 합하면,

$$\sigma_z = \sigma_{x_1}' + \sigma_{x_2}' = \sigma(1+\cos 2\alpha)$$

$$\sigma_\theta = \sigma_{y_1}' + \sigma_{y_2}' = \sigma(1-\cos 2\alpha), \quad \tau_{\theta z} = 0 \text{ 이다.}$$

연립하면 $\frac{1+\cos 2\alpha}{1-\cos 2\alpha} = \frac{1}{2}$ $\quad \therefore 3\cos 2\alpha = -1.$

$\cos 2\alpha = -\frac{1}{3}$ $\quad \therefore \alpha = 54.7°$

Ans.) $\alpha = 54.7$

4.33 $V = \int_{-h/2}^{h/2} \tau_{xy}\, dy$, $M = -\int_{-h/2}^{h/2} y\, \sigma_x\, dy$

$$q = \sigma_{y\,(y=\frac{h}{2})} - \sigma_{y\,(y=-\frac{h}{2})}$$

(a) $\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0$ 를 이용하자.

$$\int_{-h/2}^{h/2} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} \right) dy = \frac{\partial}{\partial x} \int_{-h/2}^{h/2} \tau_{xy}\, dy + \int_{-h/2}^{h/2} \frac{\partial \sigma_y}{\partial y}\, dy$$

$$d\sigma_y = \frac{\partial \sigma_y}{\partial x} dx + \frac{\partial \sigma_y}{\partial y} dy$$

그런데, 적분을 보의 두께 방향으로 수행하므로 $dx = 0$

$$\therefore \ d\sigma_y = \frac{\partial \sigma_y}{\partial y} dy$$

$$\int_{-h/2}^{h/2} \left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} \right) dy = \frac{\partial V}{\partial x} + \int_{-h/2}^{h/2} d\sigma_y$$

$$= \frac{\partial V}{\partial x} + \left\{ \sigma_{y\,(y=\frac{h}{2})} - \sigma_{y\,(h=-\frac{h}{2})} \right\} = \frac{\partial V}{\partial x} + q = 0$$

$$\underline{\frac{\partial V}{\partial x} + q = 0} \ {}_{\prime\prime}$$

(b) $\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0$ 를 이용한다.

$$\int_{-h/2}^{h/2} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) y\, dy = \frac{\partial}{\partial x} \int_{-h/2}^{h/2} \sigma_x y\, dy + \int_{-h/2}^{h/2} \frac{\partial \tau_{xy}}{\partial y} y\, dy$$

$$d\tau_{xy} = \frac{\partial \tau_{xy}}{\partial x} dx + \frac{\partial \tau_{xy}}{\partial y} dy$$

위와 마찬가지로 $dx = 0$ $\quad \therefore \ d\tau_{xy} = \frac{\partial \tau_{xy}}{\partial y} dy$

$$\int_{-h/2}^{h/2} \left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \right) y\, dy = -\frac{\partial M}{\partial x} + \int_{-h/2}^{h/2} y\, d(\tau_{xy})$$

$$= -\frac{\partial M}{\partial x} + \int_{-h/2}^{h/2} d(y\tau_{xy}) - \int_{-h/2}^{h/2} \tau_{xy}\, dy$$

(∵ 둘째 항을 부분 적분 수행)

$$= -\frac{\partial M}{\partial x} + \frac{h}{2}(\tau_{xy})_{y=\frac{h}{2}} - \frac{h}{2}(\tau_{xy})_{y=-\frac{h}{2}} - V$$

$$(\tau_{xy})_{y=\frac{h}{2}} = (\tau_{xy})_{y=-h/2} = 0 \quad \text{이므로}$$

$$\int_{-h/2}^{h/2}\left(\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) y\,dy = -\frac{\partial M}{\partial x} - V = 0$$

$$\therefore \quad \frac{\partial M}{\partial x} + V = 0$$

- Q.E.D -

4.34 $\quad \sigma_\theta = \sigma_\varphi = \frac{pr}{2t}$

$$V = \frac{4}{3}\pi r^3 = 35 \times 10^{-5} \times 10^{-3} \; (m^3)$$

$$r^3 = \frac{3 \times 35}{4\pi} \times 10^{-2} \times 10^{-12} \qquad \therefore \; r = 0.4372 \times 10^{-3} \; (m)$$

$$\sigma_\theta = \sigma_\varphi = \frac{150 \times \frac{101325}{760}}{2 \times (1 \times 10^{-6})} \times 0.4372 \times 10^{-3}$$

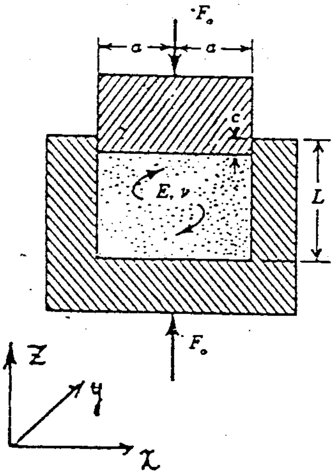
$$= 4371640 \;\; N/m^2 = 4.37 \;\; MN/m^2$$

Ans.) $\sigma_\theta = \sigma_\varphi = 4.37 \;\; MN/m^2$

CHAPTER 5.

$\sigma = \epsilon E$

5.1



응력 - 변형도 관계식으로부터

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y+\sigma_z)]$$
$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z+\sigma_x)]$$
$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)]$$

그런데 x, y 방향 변형이 억제되었으므로

$$\epsilon_x = \epsilon_y = 0$$

따라서 $\sigma_x = \nu(\sigma_y + \sigma_z)$

$\sigma_y = \nu(\sigma_x + \sigma_z)$

$\sigma_x = \sigma_y$ 이고

위 식을 연립하면 $\sigma_x = \sigma_y = \frac{\nu}{1-\nu}\sigma_z$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x+\sigma_y)] = \frac{1}{E}\left[\sigma_z - \frac{2\nu^2}{1-\nu}\sigma_z\right] = \frac{c}{L}$$

$\therefore \quad \sigma_z = \frac{(1-\nu)}{(1-\nu-2\nu^2)}\frac{Ec}{L}$ $\qquad F_o = 4a^2 \times \sigma_z$

$$= \frac{4a^2Ec}{L}\frac{(1-\nu)}{(1-\nu-2\nu^2)}$$

Ans.) $F_o = \frac{4a^2Ec}{L}\frac{(1-\nu)}{(1-\nu-2\nu^2)}$

5.2 $\epsilon = \frac{\Delta V}{V} = \frac{V'-V}{V} = \frac{V'}{V} - 1 = (1+\epsilon_x)(1+\epsilon_y)(1+\epsilon_z) - 1$

$= \epsilon_x + \epsilon_y + \epsilon_z + \epsilon_x\epsilon_y + \epsilon_y\epsilon_z + \epsilon_z\epsilon_x + \epsilon_x\epsilon_y\epsilon_z$

ϵ 값은 상당히 작으므로 ϵ의 2차항 이상을 무시하여도 무방

$\therefore \quad \epsilon_v \cong \epsilon_x + \epsilon_y + \epsilon_z$

5.3 (a) 5.2의 식을 이용하면

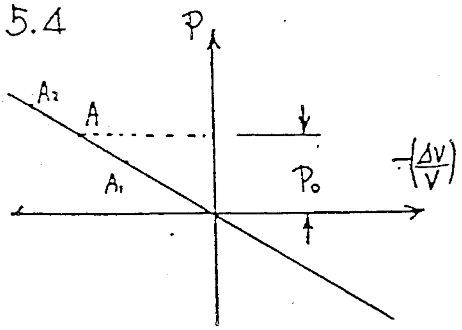
$$\frac{\Delta V}{V} = \epsilon_x + \epsilon_y + \epsilon_z = \frac{1}{E}[\sigma_x + \sigma_y + \sigma_z - 2\nu(\sigma_x+\sigma_y+\sigma_z)]$$
$$= \frac{1}{E}(\sigma_x+\sigma_y+\sigma_z)(1-2\nu)$$

(b) $B = \frac{p}{(-\Delta V/V)} = \frac{-\frac{1}{3}(\sigma_x+\sigma_y+\sigma_z)}{-\frac{1}{E}(\sigma_x+\sigma_y+\sigma_z)(1-2\nu)}$ $\quad$ ($\because p = -\frac{1}{3}(\sigma_x+\sigma_y+\sigma_z)$ 또한 5.3(a)로 부터)

$$= \frac{E}{3(1-2\nu)}$$

** isotropic ; 재료의 거동이 어느 방향에 대해서도 일정한 성질 (등방성)

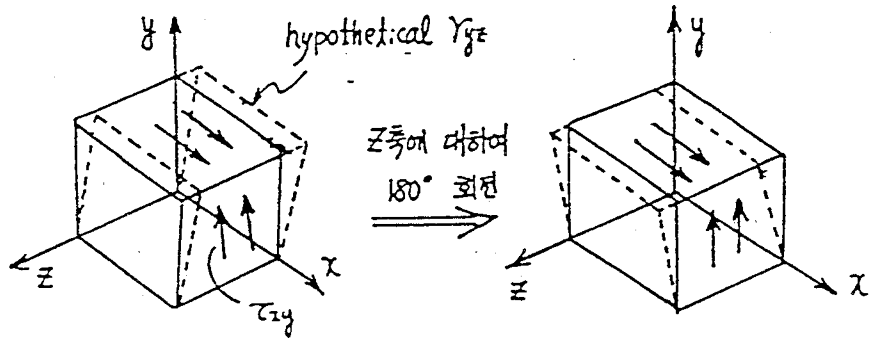
5.4



Poisson's ratio $\nu > \frac{1}{2}$이면 $B<0$이고 압력 P와 체적 감소율 $-\left(\frac{\Delta V}{V}\right)$와의 관계는 왼편 그림과 같다.
물체가 P_0의 압력에서 평형상태를 이루고 있다고 하면 물체의 상태는 A로 표시된다.

만약 열동으로 인한 약간의 상태변화로 물체의 체적이 증가하였다면 물체 상태는 A_2로 변하고 따라서 압력이 증가하며, 체적이 증가하는 방향으로 변화를 가속시키는 압력차가 존재하게 된다.
또한, 체적이 약간 감소하게 되게 될때 물체의 압력이 감소하며 체적 감소가 가속될 것이며 결국 zero volume 상태가 될 것이다.
따라서, 이러한 물질은 불안정하다.
실제로, 현존 물체는 $B>0$ 이며 따라서 $\nu<\frac{1}{2}$인 범위에 있다.

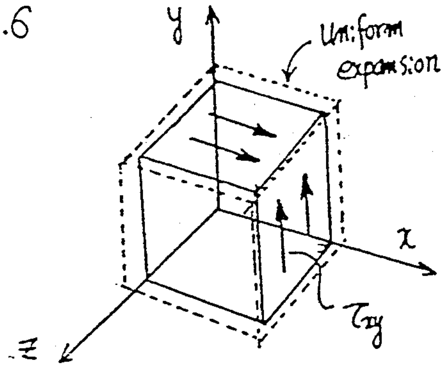
5.5



z축에 대하여 180° 회전한 후 τ_{xy}는 변하지 않은 반면 γ_{yz}는 방향이 바뀌었다. 이 물질은 등방성이므로 좌표계에 무관하게 shear-strain behavior가 동일하여야 하므로 .

$\gamma_{yz} = 0$ 이다.
즉, 전단응력성분 τ_{xy}로 인한 전단 변형도 성분 γ_{yz}를 유발시키지는 못한다.

5.6



y축에 관하여 180° 회전을 시키면 Uniform expansion은 불변이지만 τ_{xy}의 방향이 반대가 된다. expansion과 전단 응력이 선형 관계라고 가정한다면, (expansion) = $c(\tau_{xy})$ 이다. 위 식에서 같은 팽창 현상에 대해서 c가 양,음 부호를 전부 성립시킬 수 없으므로 $c=0$ 이어야 또한 expansion 역시 zero 이어야 한다.

따라서, 등방성인 물질에서는 전단 응력에 의하여 발생하는 균일 팽창 및 contraction은 존재치 않는다.

5.7

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha(T - T_0) \quad \cdots\cdots ①$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_z + \sigma_x)] + \alpha(T - T_0) \quad \cdots\cdots ②$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha(T - T_0) \quad \cdots\cdots ③$$

②식을 σ_y에 대하여 표현하면

$$\sigma_y = [\epsilon_y - \alpha(T - T_0)]E + \nu(\sigma_x + \sigma_z) \quad \cdots\cdots ④$$

④ → ①

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu\{\sigma_z + \nu(\sigma_x + \sigma_z) + \{\epsilon_y - \alpha(T - T_0)\}E\}] + \alpha(T - T_0)$$

$$= \frac{1}{E}(1-\nu^2)\sigma_x - \frac{1}{E}\nu(\nu+1)\sigma_z - \nu\epsilon_y + \alpha(1+\nu)(T - T_0) \quad \cdots ⑤$$

④ → ③

$$\epsilon_z = \frac{1}{E}(1-\nu^2)\sigma_z - \frac{1}{E}\nu(\nu+1)\sigma_x - \nu\epsilon_y + \alpha(1+\nu)(T - T_0) \quad \cdots ⑥$$

⑤,⑥ 으로부터

$$\epsilon_x(1-\nu) + \nu\epsilon_z = \frac{1}{E}(1+\nu)(1-2\nu)\sigma_x - \nu\epsilon_y + \alpha(1+\nu)(T - T_0)$$

$$\therefore \sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\epsilon_x + \nu(\epsilon_y + \epsilon_z) - \alpha(1+\nu)(T - T_0)]$$

$$= \frac{E}{1+\nu}\epsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_x + \epsilon_y + \epsilon_z) - \frac{\alpha E(T - T_0)}{1-2\nu}$$

마찬가지로 σ_y, σ_z를 변형도 성분과 T로 표현할 수 있다

$$\sigma_y = \frac{E}{1+\nu}\epsilon_y + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_x+\epsilon_y+\epsilon_z) - \frac{\alpha E(T-T_0)}{1-2\nu}$$

$$\sigma_z = \frac{E}{1+\nu}\epsilon_z + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_x+\epsilon_y+\epsilon_z) - \frac{\alpha E(T-T_0)}{1-2\nu}$$

Ans.) $\sigma_i = \frac{E}{1+\nu}\epsilon_i + \frac{\nu E}{(1+\nu)(1-2\nu)}(\epsilon_x+\epsilon_y+\epsilon_z) - \frac{\alpha E(T-T_0)}{1-2\nu} \quad (i=x,y,z)$

5.8 $\sigma_x = 130\,MN/m^2$, $\sigma_y = -70\,MN/m^2$, $\tau_{xy} = 80\,MN/m^2$

($\nu = 0.3$, $E = 205\,GN/m^2$, $G = 78.8\,GN/m^2$ 이라 하자)

$$\epsilon_x = \frac{1}{E}(130+70\nu) = 0.737\times10^{-3}$$

$$\epsilon_y = \frac{1}{E}(-70-130\nu) = -0.532\times10^{-6}$$

$$\epsilon_z = \frac{1}{E}(-130\nu-70\nu) = -0.0878\times10^{-3}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{80}{78.8\times10^3} = 1.015\times10^{-3}$$

$$\theta = \frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = \frac{1}{2}\tan^{-1}\frac{1.015}{0.737+0.001} = 19.3^\circ$$

$$\epsilon_{I,II} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \frac{\gamma_{xy}^2}{4}} \begin{cases} = 1.447\times10^{-3} \\ = -0.178\times10^{-3} \end{cases}$$

Ans.) $\epsilon_I = 1.447\times10^{-3}$, $\epsilon_{II} = -0.178\times10^{-3}$, $\epsilon_{III} = \epsilon_z = -0.0878\times10^{-3}$

$\theta = 19.3^\circ$

5.9 $\sigma_x = 145\,MN/m^2$, $\tau_{xy} = 42\,MN/m^2$, $\epsilon_z = -3.6\times10^{-4}$

$\epsilon_z = \frac{1}{E}[-\nu(\sigma_x+\sigma_y)]$ 이므로

$$\sigma_y = -\left(\frac{E}{\nu}\epsilon_z + \sigma_x\right) = -\left[\frac{200\times10^9}{0.3}\times(-3.6\times10^{-4}) + 145\times10^6\right] = 95\,MN/m^2$$

Ans.) $\sigma_y = 95\,MN/m^2$

5.10 $\epsilon_1 = 3.2\times10^{-4}$, $\epsilon_2 = -5.4\times10^{-4}$

$$\epsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y)$$

$$\epsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$\frac{1}{2}\tan^{-1}\frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = 60°$, $(\epsilon_x-\epsilon_y)\tan 120° = \gamma_{xy}$ ····· ①

$\frac{\epsilon_x+\epsilon_y}{2} + \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2} = 3.2\times10^{-4}$ ······ ②

$\frac{\epsilon_x+\epsilon_y}{2} - \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2+\left(\frac{\gamma_{xy}}{2}\right)^2} = -5.4\times10^{-4}$ ····· ③

②-③ $\sqrt{(\epsilon_x-\epsilon_y)^2+\gamma_{xy}^2} = 8.6\times10^{-4} = \gamma_{xy}\sqrt{\left(\frac{1}{\tan 120°}\right)^2+1}$

$\therefore \gamma_{xy} = 7.448\times10^{-4}$

②+③ ; $\epsilon_x+\epsilon_y = -2.2\times10^{-4}$

① ; $\epsilon_x-\epsilon_y = -4.3\times10^{-4}$ $\Rightarrow \epsilon_x = -3.25\times10^{-4}$, $\epsilon_y = 1.05\times10^{-4}$

알루미늄 $\begin{cases} E = 69\ GN/m^2 \\ \nu = 0.33 \end{cases}$, $G = 26\ GN/m^2$

$\nu\epsilon_x + \epsilon_y = \frac{1-\nu^2}{E}\sigma_y$ 에 대입

$\sigma_y = \frac{E}{1-\nu^2}(\epsilon_y+\nu\epsilon_x) = \frac{69\times10^9}{1-0.33^2}(1.05\times10^{-4}-0.33\times3.25\times10^{-4})$

$= -0.1742\ (MN/m^2)$

$\sigma_x = \frac{69\times10^9}{1-0.33^2}(1.05\times10^{-4}\times0.33-3.25\times10^{-4})$

$= -22.48\ (MN/m^2)$

$\tau_{xy} = G\gamma_{xy} = 26\times10^9\times7.448\times10^{-4} = 19.36\ (MN/m^2)$

Ans.) $\sigma_x = -22.48\ MN/m^2$, $\sigma_y = -0.1742\ MN/m^2$

$\tau_{xy} = 19.36\ MN/m^2$

5.11 $\epsilon_x = \ln\frac{L_f}{L_0}$

$AL_f = A_0L_0 \rightarrow \epsilon_x = \ln\frac{A_0}{A}$ $\therefore \frac{A_0}{A} = e^{\epsilon_x}$

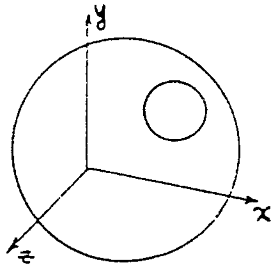
$\Delta A = \frac{A_0-A}{A_0} = 1-\frac{A}{A_0} = 1-e^{-\epsilon_x}$ - Q.E.D -

5.12 $\sigma_\theta = \frac{Pr}{t}$, $\sigma_z = \frac{Pr}{2t}$, $\sigma_r = 0$

$\epsilon_0 = \frac{1}{E}(\sigma_z-\nu\sigma_\theta) = \frac{Pr}{2Et}(1-2\nu)$ $\therefore P = \frac{2Et\epsilon_0}{r(1-2\nu)}$

Ans.) $P = \frac{2Et\epsilon_0}{r(1-2\nu)}$

5·13 열이 가해진 후에 plate는 일정한 strain을 갖는 stress free의 상태로 될 것이다.



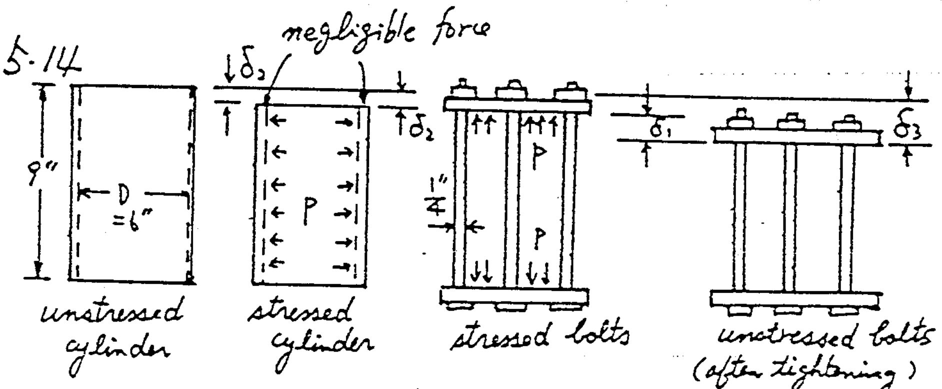
$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha(T-T_0)$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$$

(이러한 해는 equilibrium. compatibility stress-strain-temperature relation, boundary condition을 모두 만족한다.)

따라서 변위는 크기만 일정하게 변한다. 그러므로 원은 원의 형태를 그대로 유지하면서 크기만 변한다.

5·14



$$\delta_1 + \delta_2 = \delta_3 = \frac{1}{2}\left(\frac{1}{20}\right) = 0.025'' \quad \text{—(1)}$$

평형 조건; Cylinder; $\sigma_\theta = p \cdot \dfrac{D}{2t} = p \cdot \dfrac{6}{2(\frac{1}{16})} = 48p$ psi

$$\sigma_r = \sigma_z = 0$$

Bolts; Force $= p \cdot \frac{\pi}{4}(D^2) = p \cdot \frac{\pi}{4}(6)^2 = 28.3p$ lb

stress $\sigma_z = \dfrac{28.3p}{3} \cdot \dfrac{1}{\frac{\pi}{4}(\frac{1}{4})^2} = 192p$ psi

힘과 변형과의 관계;

Cylinder; $\varepsilon_z = \dfrac{1}{E_B}[\sigma_z - \nu_B(\sigma_\theta + \sigma_r)] = -\dfrac{\nu_B \sigma_\theta}{E_B}$

$$\text{Bolts} \ ; \ \varepsilon_z = \frac{\sigma_z}{E_s}$$

따라서 $\delta_2 = (-\varepsilon_z)_{cylinder} \cdot (L)_{cyl.} = -\left(-\frac{\nu_B}{E_B} 48P\right) \cdot 9$

$$\delta_1 = (\varepsilon_z)_{bolt} \cdot (L)_{bolt} = \left(\frac{192P}{E_s}\right) \cdot 10$$

(1)식에서 $\frac{(9)(48)\nu_B P}{E_B} + \frac{1920P}{E_s} = 0.025$

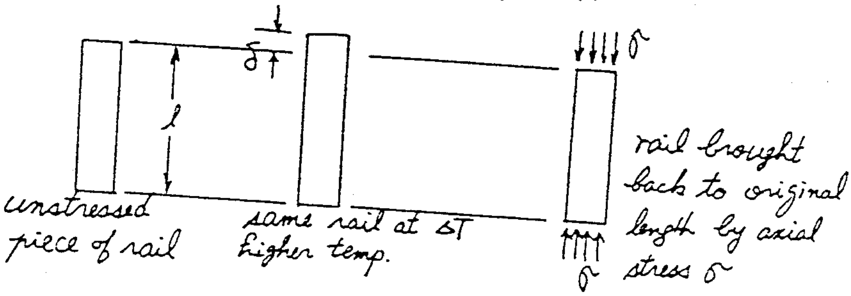
한편, $\nu_B = \nu_{BRASS} = 0.34$; $E_B = 15.2 \times 10^6$ psi

$E_s = E_{steel} = 30 \times 10^6$ psi

그러므로 $\frac{(9)(48)(0.34)(P)}{15.2 \times 10^6} + \frac{1920P}{30 \times 10^6} = 0.025$

$\therefore P = 338$ psi   Ans $P = 338$ psi

5.15 rail의 양단이 움직일 수 없다면, 열이 가해질 때 thermal stress가 발생되게 될 것이다.

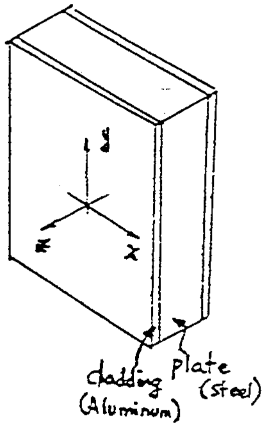


$$\delta = \alpha \cdot l \cdot \Delta T \qquad \therefore \sigma = E \cdot \frac{\delta}{l} = E\alpha(\Delta T)$$

따라서, $E\alpha\Delta T$ 만큼의 stress가 rail의 양단에서 발생할 것이다. rail이 점점 더 길어지면, 양단에서의 힘이 점점 커지므로 rail은 buckling을 일으키려 할 것이다. buckling을 방지하기 위해서는 lateral constraints가

필요하며, lateral support의 간격은 rail의 단면과 탄성계수에 의해 정해진다. rail이 curve를 이루지 않는 한, rail 양단에서의 힘이 인장 (ΔT가 negative)인 경우에는 buckling 문제는 일어나지 않는다. rail이 curve인 경우에는 오히려 펴지려 할 것이다.

5.16 서로 다른 열팽창에 의해서 응력이 발생한다. cladding의 두께가 plate의 두께에 비해서 상당히 작다면, plate에서의 응력은 작으므로 무시한다.



대칭조건; shear stress = 0 (양단 제외)

Geometry; $\left.\begin{array}{l}(\varepsilon_x)_{clad} = (\varepsilon_x)_{plate} \\ (\varepsilon_y)_{clad} = (\varepsilon_y)_{plate}\end{array}\right) \text{—(1)}$

Stress-strain-temp. relation;

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha \Delta T \cdots$$

평형조건; $\sigma_z = 0$

(notation; subscript p for plate; c for cladding)

plate; 모든 응력 $\simeq 0$. $\therefore (\varepsilon_x)_p = \alpha_p \cdot \Delta T = (\varepsilon_y)_p$

cladding; (1) 식으로 부터

$$(\varepsilon_x)_c = \frac{1}{E_c}[(\sigma_x)_c - \nu_c(\sigma_y)_c] + \alpha_c \Delta T = \alpha_p \cdot \Delta T$$

$$(\varepsilon_y)_c = \frac{1}{E_c}[(\sigma_y)_c - \nu_c(\sigma_x)_c] + \alpha_c \Delta T = \alpha_p \cdot \Delta T$$

이 식을 풀면 $(\sigma_x)_c = (\sigma_y)_c = \dfrac{E_c(\alpha_p - \alpha_c)}{1-\nu_c} \cdot \Delta T$

이것이 두개의 주응력이며, 세 번째 주응력은 $\sigma_z = 0$.

Tresca yield criterion을 이용하면,

$$\tau_{max} = \frac{1}{2}Y$$

cladding ; $\tau_{max} = \frac{1}{2}\left|(\sigma_{max} - \sigma_{min})\right| = \frac{1}{2}\left[\frac{E_c(\alpha_p - \alpha_c)\Delta T}{1-\nu_c} - 0\right]$

material property ; $\alpha_c = 12\times10^{-6}\ °F^{-1}$, $\alpha_p = 6\times10^{-6}\ °F^{-1}$

$\nu_c = 0.33$; $E_c = 11\times10^{6}$ psi $Y_c = 5000$ psi

$$\therefore \frac{5000}{2} = \frac{11\times10^{6}(12-6)(10^{-6})(\Delta T)}{2(1-0.33)} \qquad \therefore \Delta T = 51°F$$

Ans $\Delta T = 57°F$ (Von Mises yield criterion gives the same answer)

5.17 Pulley를 가열하는 것이 더 쉽다해도, pulley를 가열하거나 shaft를 냉각시키거나 같은 효과를 가져올 것이다. 그러나, 실제로는 pulley가 shaft end에서 어느 정도 떨어져 고정되어 있다면 pulley를 가열하는 것이 더 편하다.

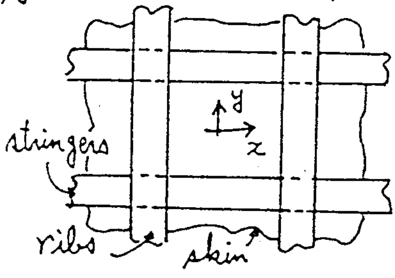
pulley hole의 직경이 shaft의 직경보다 0.05mm 작고, assembling 하는 동안에 clearance 0.025mm를 필요로 하므로, 열 팽창은 다음과 같아야 한다.

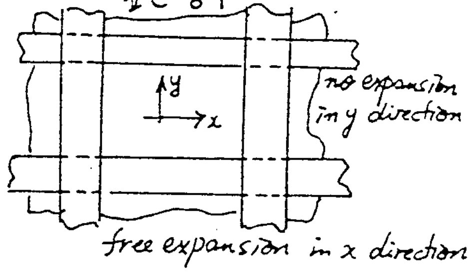
열 팽창 = 0.05 + 0.025 = 0.075 (mm), $\alpha = 11\times10^{-6}\ °C^{-1}$

$$\therefore \varepsilon = \frac{0.075}{25} = \alpha\Delta T \qquad \therefore \Delta T = \frac{0.075}{25\times11\times10^{-6}} = 272.7\ °C$$

Ans $\Delta T = 272.7°C$

5.18 uniform temp. skin이 ribs보다 50°F 높은경우





i) Rib에 의해서 lateral strain은 없다.

Geometry ; $\varepsilon_y = 0$

평형조건 ; $\sigma_x = \sigma_z =$ shear stress $= 0.$

Stress - strain - temp. relation ;

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] + \alpha\Delta T = 0$$

$$\therefore \sigma_y = -E\alpha\Delta T = -(11\times10^{6})(12\times10^{-6})(50) = -6600 \text{ psi}$$

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] + \alpha\Delta T = -\frac{(0.33)(-6600)}{11\times10^{6}}$$

$$+(12\times10^{-6})(50) = 800\times10^{-6}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] + \alpha\Delta T = 800\times10^{-6}$$

ii) Rib에 의한 lateral restraint가 없다면

all stresses $= 0$. $\varepsilon_x = \varepsilon_y = \varepsilon_z = \alpha\Delta T = 600\times10^{-6}$

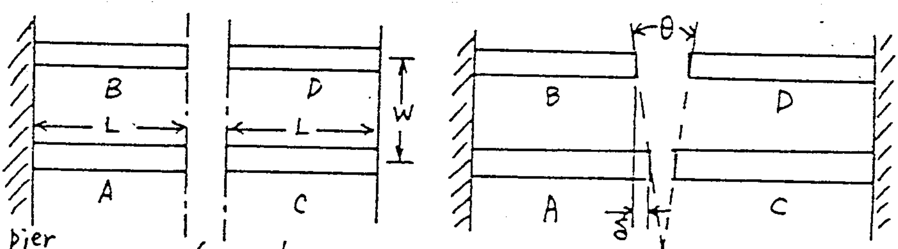
Ans. i) $\sigma_x = \sigma_z = 0.$ $\sigma_y = -6600$ psi

$\varepsilon_x = \varepsilon_z = 800\times10^{-6}$, $\varepsilon_y = 0$

ii) $\sigma_x = \sigma_y = \sigma_z = 0$, $\varepsilon_x = \varepsilon_y = \varepsilon_z = 600\times10^{-6}$

5.19 2개의 member로서 truss를 표시할 수 있다.

그리고 태양은 한쪽 member에만 열을 주게 될 것이다.



a) uniform temp. b) A & C heated

A 와 C의 팽창 $= (\alpha\Delta T)\cdot L = \delta$

2 span의 양끝은 θ 만큼의 각 차이를 갖게될 것이다.

$$\theta \simeq \frac{2\delta}{W} = 2(\alpha\Delta T)\left(\frac{L}{W}\right)$$

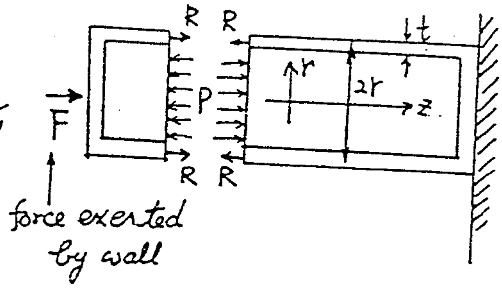
Estimate ; $L = 200ft$; $w = 30ft$; $\Delta T = 50°F$

$\alpha = 6\times10^{-6}\ °F^{-1}$ (for steel)

$\therefore \theta = 2(6\times10^{-6})(50)\left(\frac{200}{30}\right) = 0.004\ (rad) = 0.2\ (degree)$

Ans $\theta = 0.2$ degree

5·20



Geometry ; $\varepsilon_z = 0$ in cylinder wall

평형 조건 ; (그림에서)

$F + R = p\pi r^2$

$\therefore R = p\pi r^2 - F$

$\therefore$ Axial stress in cylinder $= \dfrac{R}{2\pi rt} = \dfrac{p\pi r^2 - F}{2\pi rt} = \sigma_z$

Circumferential stress $= \sigma_\theta = \dfrac{pr}{t}$, $\sigma_r \simeq 0$

Stress-strain relation 에서

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_\theta)\right] = 0$$

$\therefore \sigma_z = \nu(\sigma_r + \sigma_\theta)$ 즉, $\dfrac{p\pi r^2 - F}{2\pi rt} = \nu \cdot \dfrac{pr}{t}$

따라서 $F = p\pi r^2(1-2\nu)$ Ans. $F = p\pi r^2(1-2\nu)$

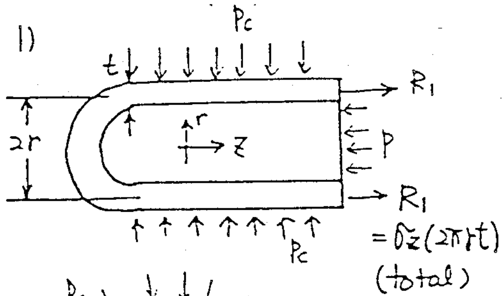
5·21 tank 와 cavity 의 벽 사이에는 마찰력이 없다고 가정하고, p_c = contact pressure between tank and cavity 로 놓자.

Geometry ; $\varepsilon_\theta = 0$ (tank 의 벽에서)

평형 조건 ;

$\Sigma F_z = 0$; $p\pi r^2 = R_1 = \sigma_z (2\pi r t)$

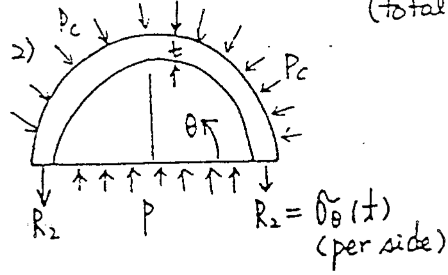
$$\therefore \sigma_z = \frac{p \cdot r}{2t}$$



$\Sigma F_{vertical} = 0$;

$$2\sigma_\theta t = (p - p_c)(2r)$$

$$\therefore \sigma_\theta = (p - p_c)\frac{r}{t}$$



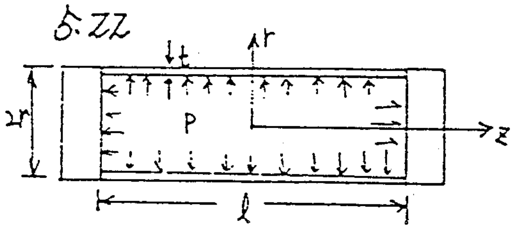
그리고 σ_r 은 $-p$ 와 $-p_c$ 사이에 있다.
p_c 가 p 와 같은 order 라고 생각하면
즉, $\left(\frac{t}{r}\right) \ll 1$ 이라면 σ_r 은 무시할 수 있다.

Stress-strain relation ; $\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] = 0$

$$\therefore \sigma_\theta = \nu(\sigma_r + \sigma_z) = \nu(\sigma_z) = \frac{\nu p r}{2t}$$

Ans $\sigma_\theta = \nu p r / 2t$

5.22



Geometry ; Rigid end 는 tangential strain (ε_θ) 를 방지한다. 그러나 (l/r) 이 상당히 크다면 이러한 end effect 는 무시할 수 있다.

평형 조건 ; $\sigma_z = \frac{p \cdot r}{2t}$; $\sigma_\theta = \frac{p \cdot r}{t}$; $\sigma_r \simeq 0$

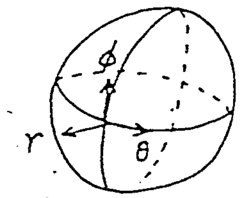
Stress-strain relation ; $\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_\theta + \sigma_r)] = \frac{pr}{2Et}(1 - 2\nu)$

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] = \frac{pr}{2Et}(2 - \nu)$$

$$\delta_D = D \cdot \varepsilon_\theta = 2r \cdot \varepsilon_\theta , \quad \delta_l = l\, \varepsilon_z$$

$$\therefore \frac{\delta_l}{\delta_D} = \frac{l}{2r}\frac{(1-2\nu)}{(2-\nu)} \qquad \text{Ans. } \left(\delta_l/\delta_D\right) = \frac{l}{2r}\frac{(1-2\nu)}{(2-\nu)}$$

5.23



internal pressure $= p$
radius $= r$

평형조건 ; $\sigma_\theta = \sigma_\phi = \dfrac{pr}{2t}$, $\sigma_r \simeq 0$

Stress - strain relation ;

$$\varepsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\sigma_\phi + \sigma_r)\right] = \frac{pr}{2Et}(1-\nu)$$

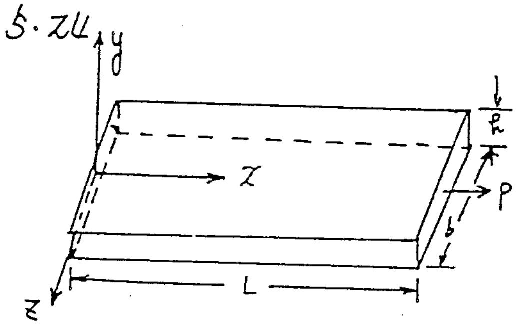
$$\delta_D = (D)\cdot\varepsilon_\theta = 2r\varepsilon_\theta = \frac{pr^2}{Et}(1-\nu)$$

문제 (5.22) 에서 $(\delta_D)_{cylinder} = \dfrac{pr^2}{Et}(2-\nu)$

$$\therefore \frac{(\delta_D)_{cylinder}}{(\delta_D)_{sphere}} = \left(\frac{pr^2}{Et}(2-\nu)\right)\Big/\left(\frac{pr^2}{Et}(1-\nu)\right) = \frac{2-\nu}{1-\nu}$$

Ans $(\delta_D)_{cylinder}/(\delta_D)_{sphere} = (2-\nu)/(1-\nu)$

5.24



Bounding surface 상의 다음 응력들은 전체적인 평형조건을 만족한다.

$$\sigma_x = \frac{P}{bh} ; \ \sigma_y = \sigma_z = 0$$

$$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$$

Stress - strain relation ;

$$\left.\begin{aligned} &\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] = \frac{P}{Ebh}, \quad \varepsilon_y = \varepsilon_z = \frac{-\nu P}{Ebh} \\ &\text{All shear strains} = 0 \end{aligned}\right\} -(1)$$

따라서 $\dfrac{\partial u}{\partial x} = \dfrac{P}{Ebh}$; $\dfrac{\partial v}{\partial y} = -\dfrac{\nu P}{Ebh} = \dfrac{\partial w}{\partial z}$; $\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right)$, etc $= 0$

$u = v = \omega = 0$ 인 원점에 대하여 적분해 주면,

$$u = \frac{Px}{Ebh} \; ; \quad v = \frac{-\nu Py}{Ebh} \; ; \quad \omega = -\frac{\nu Pz}{Ebh}$$

i) Equilibrium ; stress의 derivatives가 모두 "0" 이므로 만족.

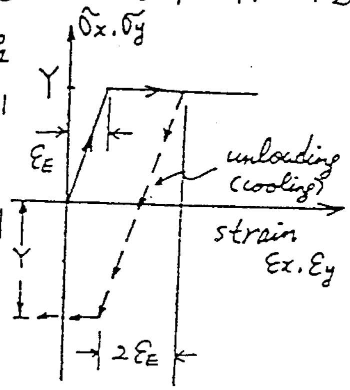
ii) Geometric compatibility ; 변위가 모두 single value 이며 연속이므로 만족.

iii) Stress-strain relation ; (1)식에서 만족

따라서 위의 해는 모든 principle of elasticity를 만족함.

$$\text{Ans.} \quad u = \frac{Px}{Ebh}, \quad v = -\frac{\nu Py}{Ebh}, \quad \omega = -\frac{\nu Pz}{Ebh}$$

5.25 처음 yielding이 일어나는 온도 이상으로 100°F 까지 가열했을 때, cladding은 $\sigma_x = \sigma_y = Y$로 소성이 될 것이다. 온도가 다시 낮아짐에 따라서 cladding은 탄성적으로 하중이 줄어들게 된다. 냉각하는 도중에 strain이 $2\varepsilon_E$ (그림참조)를 넘게되면 다시 소성이 될 것이다. 약 50°F의 온도는 ε_E 만큼의 strain을 발생시킨다. (문제(5.16)).



plate가 주변보다 150°F 높은 온도에서부터 냉각될 때, strain은 $2\varepsilon_E$ 보다 큰 $3\varepsilon_E$이 될 것이다. 그러므로, cladding 은 $\sigma_x = \sigma_y = -Y$로 주변온도에서 소성이 될 것이다.

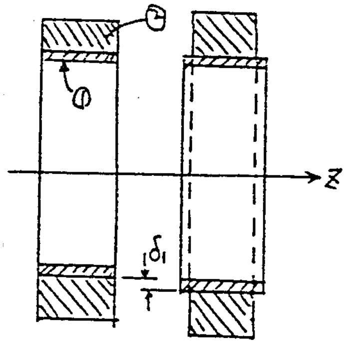
5.26 서로 다른 열 팽창은 다음과 같은 일을 야기시킬 것이다.

i) 두 개의 hoop의 지름의 차이, 접촉면에서의 contact pressure p

ii) 서로 다른 lateral expansion.

만일 hoop 사이에 마찰력이 크거나, 2개의 hoop가 서로 접착되어 있다면, lateral expansion은 axial stress나 shear stress로 나타날 것이다. 따라서 문제의 편의상 hoop는 마찰력이 없고 자유롭다고 가정한다. 그러면 lateral expansion은 구속되지 않을 것이다.

각각의 hoop에서 (t/R)은 작기 때문에, 2개의 hoop에 대하여 같은 반경을 사용한다. $(=300\,mm)$. 가열한 후에 반경은 δ_1 만큼 증가한다.



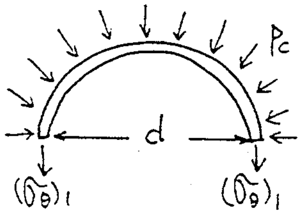
Geometry ; tangential strain ε_θ in both hoops $= \dfrac{\delta_1}{300\times10^{-3}}$

Symmetry ; 접촉면에서 $\tau_{r\theta}=0$

평형 조건 ; $\sigma_z=0$,

$\sigma_r=0$ 으로 가정.

i) inner hoop ; $\sum F_{vertical}=0$;



$$-P_c(0.6)(0.06)-(\sigma_\theta)_1(0.06)(0.003)(2)=0$$

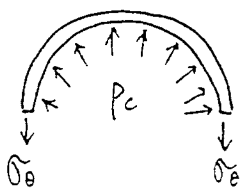
$$\therefore (\sigma_\theta)_1=-100P_c$$

ii) outer hoop ;

$\sum F_{vertical}=0$;

$$P_c(0.6)(0.06)-(\sigma_\theta)_2(0.06)(0.006)(2)=0$$

$$\therefore (\sigma_\theta)_2=50P_c$$



·Stress-strain relation ;

$$\varepsilon_\theta=\frac{1}{E}\left[\sigma_\theta-\nu(\sigma_z+\sigma_r)\right]+\alpha\Delta T=\frac{\sigma_\theta}{E}+\alpha\Delta T$$

따라서 inner hoop ; $(\varepsilon_\theta)_1=-\dfrac{100P_c}{E_1}+\alpha_1\Delta T=\delta_1/300\times10^{-3}$

outer hoop; $(\varepsilon_\theta)_2 = \frac{50P_c}{E_2} \times \alpha_2 \Delta T = \delta_1/300\times10^{-3}$

$$\therefore -\frac{100P_c}{E_1} + \alpha_1 \Delta T = \frac{50P_c}{E_2} + \alpha_2 \Delta T \quad -①$$

material property (① (brass); $E_1 = 100\times10^9$, $\alpha_1 = 20.4\times10^{-6}$
② (steel); $E_2 = 200\times10^9$; $\alpha_2 = 11.4\times10^{-6}$

따라서 ①식에 material property를 대입하면

$$P_c\left[\frac{100}{100\times10^9} + \frac{50}{200\times10^9}\right] = (20.4 - 11.4)\times10^{-6}\times85$$

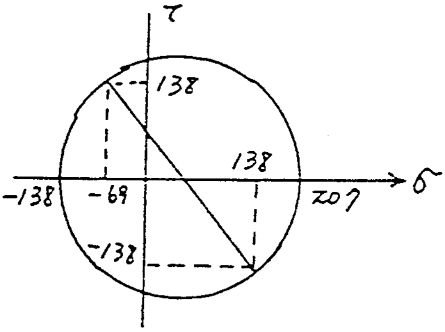
$$\therefore P_c = 6.12\times10^5 (N/m^2)$$

Brass hoop (inner); $\sigma_\theta = -100P_c = -61.2 (MN/m^2)$

Steel hoop (outer); $\sigma_\theta = 50P_c = 30.6 (MN/m^2)$

Ans. $(\sigma_\theta)_{brass} = -61.2\ MN/m^2$ $(\sigma_\theta)_{steel} = 30.6\ MN/m^2$

5.27 우선 x-y 평면에서 Mohr's circle을 이용하여 principal stress를 求한다.



principal stress;

$\sigma_1 = 207\ MN/m^2$

$\sigma_2 = -138\ MN/m^2$

$\sigma_3 = \sigma_z = 0$

a) Mises yield criterion

$$Y = \bar{\sigma} = \sqrt{\frac{1}{2}\{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\}}$$

$$= 300.764 (MN/m^2) < 330\ MN/m^2$$

따라서 yielding이 발생하지 않는다.

b) Max. shear stress criterion ; $|\sigma_{max} - \sigma_{min}| = Y$

$\sigma_{max} = \sigma_1 = 207\ MN/m^2$, $\sigma_{min} = \sigma_2 = -138\ MN/m^2$

$\therefore Y = |207 - (-138)| = 345 \ (MN/m^2) > 330 \ MN/m^2$

따라서 yielding이 발생한다.

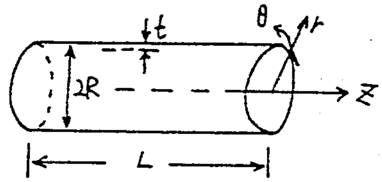
Ans. a) Mises yield criterion 으로는 yielding이 발생하지 않음.

b) Max. shear stress criterion 으로는 yielding이 생김.

5.28 p = internal pressure closed ends

평형조건; $\sigma_\theta = \dfrac{PR}{t}$; $\sigma_z = \dfrac{PR}{2t} = \dfrac{1}{2}\sigma_\theta$

$\sigma_r \simeq 0$



Yield criterion (Max. shear stress criterion)

$|\sigma_{max} - \sigma_{min}| = Y$; $\sigma_{max} = \sigma_\theta$; $\sigma_{min} = 0$

1020 HR steel에 대한 항복 응력 $Y = 250 \ MN/m^2$

$\therefore \sigma_\theta = Y = 250 \times 10^6 \ N/m^2$, $\sigma_z = \dfrac{1}{2}\sigma_\theta = 125 \times 10^6 \ N/m^2$

elastic 에서 stress-strain relation;

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] = \frac{1}{200 \times 10^9}(250 \times 10^6 - 0.3 \times 125 \times 10^6)$$

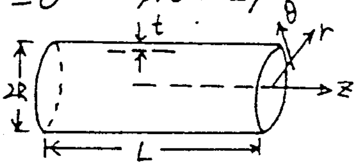
$$= 1.0625 \times 10^{-3}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)] = \frac{(125 - 0.3 \times 250) \times 10^6}{200 \times 10^9} = 0.25 \times 10^{-3}$$

Ans. length (ε_z) ; 0.025%, circumferential (ε_θ) ; 0.106 %.

5.29 평형조건; $\sigma_z = \dfrac{PR}{2t}$; $\sigma_\theta = \dfrac{PR}{t}$; $\sigma_r \simeq 0$ closed ends, internal pressure: p

plastic 에서의 stress-strain relation;



$$d\varepsilon_z^P = \frac{d\bar{\varepsilon}}{\bar{\sigma}}[\sigma_z - \frac{1}{2}\sigma_\theta] = 0$$

$$d\varepsilon_\theta^p = \frac{d\bar{\varepsilon}}{\bar{\sigma}}[\sigma_\theta - \tfrac{1}{2}\sigma_z] = \left(\frac{3}{4}\right)\left(\frac{PR}{t}\right)\frac{d\bar{\varepsilon}}{\bar{\sigma}}$$

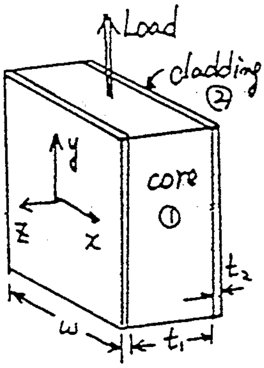
따라서 $\varepsilon_z^p = \int d\varepsilon_z^p = 0\ ;\ \varepsilon_\theta^p = \int d\varepsilon_\theta^p = \int\left(\frac{3}{4}\right)\left(\frac{PR}{t}\right)\frac{d\bar{\varepsilon}}{\bar{\sigma}} \neq 0 = \varepsilon_0.$

elastic strain을 무시하면 $\varepsilon_z = \varepsilon_z^p\ .\ \varepsilon_\theta = \varepsilon_\theta^p$

$$\therefore \frac{\text{axial strain}}{\text{tangential strain}} = \frac{\varepsilon_z}{\varepsilon_\theta} = \frac{0}{\varepsilon_0} = 0$$

Ans (axial strain / tangential strain) = 0

5.30 실제 test에 있어서 시편의 폭 w는, 두께가 작기 때문에 두께 보다 상당히 클것이다. 이러한 경우 해석은 가능하나, stress σ_z가 무시될 수 없기 때문에 상당히 복잡해 진다. 이러한 경우는 일반화된 plane strain의 경우이다. 문제를 단순하게 하기 위해서 w가 t_1에 비하여 상당히 작다는 비현실적인 가정을 한다. 이러한 가정하에서는 stress σ_x. τ_{xy} . τ_{xz}는 edge를 따라서 모두 "0"이다. 그러므로 그러한 응력들은 전체적으로 0 으로 취할 수있다.



Symmetry ; $\tau_{yz} = 0$, 평형 조건 ; $\sigma_z = 0$

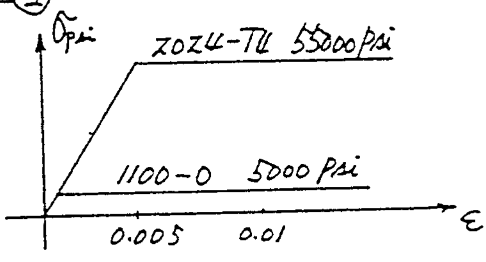
따라서 stress는 σ_y만이 남는다.

하중을 제거한 후에 y 방향의 순수한 힘은 0 이다.

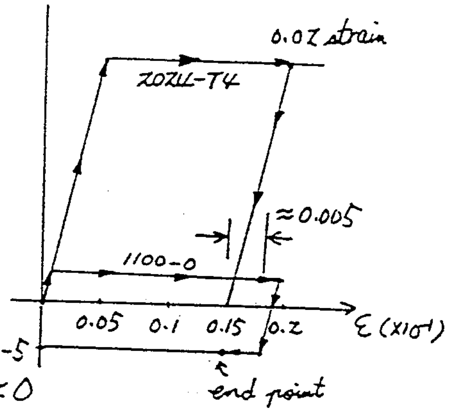
$\Sigma F_y = 0\ ;\ (\sigma_y)_1 t_1 + 2(\sigma_y)_2 t_2 = 0 \quad \therefore (\sigma_y)_1 = -2\left(\frac{t_2}{t_1}\right)(\sigma_y)_2$ — ①

Geometry ; $(\varepsilon_y)_1 = (\varepsilon_y)_2$ — ②

stress - strain data ;
elastic perfectly plastic model은 오른편 그림과 같은 형태를 이룬다.



a) ①식에서 second term을 무시하면, $(\sigma_y)_1 = 0$. 이것은 core에 대한 stress-strain history의 end point를 결정해 준다. 또, ②식을 이용하여 cladding에 대한 end point를 결정할 수 있다. 이것은 오른편처럼 graph로 나타낼 수 있다.



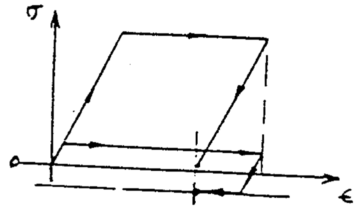
따라서 residual stress $\left(\begin{array}{l} \text{core} \simeq 0 \\ \text{cladding} = -5000 \text{ psi} \end{array} \right.$

b) $t_1 = 0.041''$, $t_2 = 0.005''$ 인 경우 ①식은 다음과 같이 된다.

$(\sigma_y)_1 = -0.244\,(\sigma_y)_2$, 이 경우에 양 end point는 strain axis 상의 오른쪽으로 이동해 갈 것이다. 이 이동이 약 0.005보다 작다고 가정하면 $(\sigma_y)_2 = -5000$ psi

$(\sigma_y)_1 = -0.244(-5000) = 1220$ psi

[end point의 이동 $\simeq \frac{1220}{E} < 0.005$]



따라서 residual stress는 $\left(\begin{array}{l} \text{core} \quad 1220 \text{ psi} \\ \text{cladding} \quad -5000 \text{ psi} \end{array} \right.$

Ans. a) $(\sigma_y)_1 = 0$, $(\sigma_y)_2 = -5000$ psi b) $(\sigma_y)_1 = 1220$ psi $(\sigma_y)_2 = -5000$ psi

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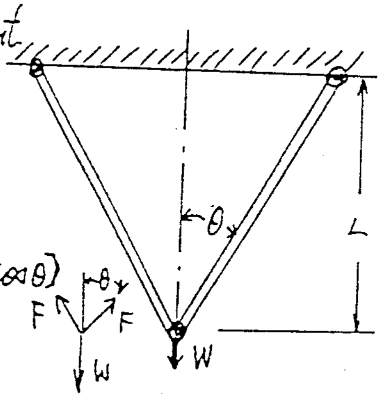
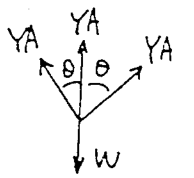
5.31 a) 미소 변화로 가정하면 θ는 constant.



평형조건; $2F\cos\theta = W$ $\therefore F = \frac{W}{2\cos\theta}$

yielding; $F = Y \cdot A$ $\therefore W = 2YA\cos\theta$

b)

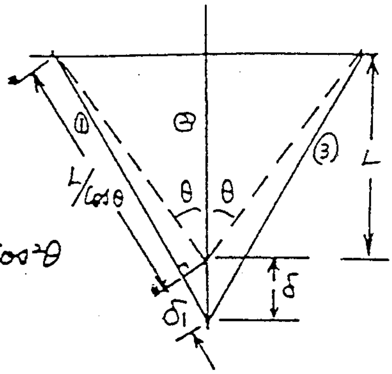


평형조건; $W = YA[1 + 2\cos\theta]$

$\therefore$ increase $= YA$

c) Geometry ; $\delta_1 = \delta_3 = \delta\cos\theta$

$\delta_2 = \delta$



rod에 발생되는 strain은;

$$\varepsilon_1 = \frac{\delta_1}{L/\cos\theta} = \frac{\delta\cos\theta}{L/\cos\theta} = \frac{\delta}{L}\cos^2\theta$$

$$\varepsilon_2 = \frac{\delta}{L}$$

stress - strain relation (elastic) $F = A(E\varepsilon)$

$$F_1 = F_3 = EA\frac{\delta}{L}\cos^2\theta \quad ; \quad F_2 = EA\frac{\delta}{L}$$

평형조건; $W = 2F_1\cos\theta + F_2 = EA\frac{\delta}{L}[1 + 2\cos^3\theta]$

$$\therefore \ \delta = \frac{WL/EA}{1 + 2\cos^3\theta}$$

d) c)에서 하중 W에 의해 생기는 stress는 다음과 같다.

$$\sigma_1 = \sigma_3 = \frac{F_1}{A} = E\frac{\delta}{L}\cos^2\theta = \frac{(W/A)\cos^2\theta}{1+2\cos^3\theta} \ ; \ \sigma_2 = \frac{(W/A)}{1+2\cos^3\theta}$$

b)에서 소성하중 $= W_p = YA[1 + 2\cos\theta]$,

elastic unloading을 가정하고, $-W_p$에 의한 stress를 구하여 yield stress에 더하라. 그러면 $W = -W_p$인 경우에

$$\sigma_1 = \sigma_3 = -\frac{Y(1+2\cos\theta)\cos^2\theta}{1+2\cos^3\theta} \quad ; \ \sigma_2 = -\frac{Y(1+2\cos\theta)}{1+2\cos^3\theta}$$

Residual stress ;

rod ①.③ ; $Y - \frac{Y(1+2\cos\theta)\cos^2\theta}{1+2\cos^3\theta} = Y\left(\frac{1-\cos^2\theta}{1+2\cos^3\theta}\right)$

rod ② ; $Y - \frac{Y(1+2\cos\theta)}{1+2\cos^3\theta} = -\frac{2Y\sin^2\theta\cos\theta}{1+2\cos^3\theta}$

∴ rod ①②③ 의 residual stress는 모두 yield stress

Y보다 작다. 따라서 elastic unloading 이라는 가정은 옳다.

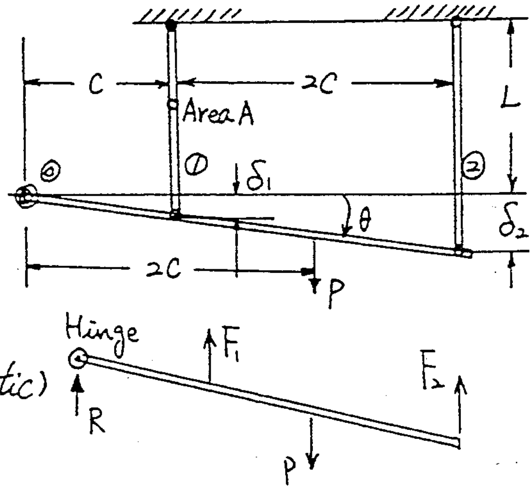
* $\theta=0$ 인 경우; 3개의 rod에 uniaxial tension 이 걸린다.
따라서 residual stress는 0. 윗 식은 이것을 만족한다.
$\theta=\pi/2$ 인 경우; ①③은 무관하고 rod ②에 단순한 tension만이 작용한다. 따라서 residual stress는 0. rod ②에 대한 구한 식이 이것을 만족한다.

Ans. a) $W=2YA\cos\theta$ b) 하중의 증분 $=YA$

c) $\delta=\dfrac{WL/EA}{1+2\cos^3\theta}$ d) rod ①③ ; $Y\left(\dfrac{1-\cos^2\theta}{1+2\cos^3\theta}\right)$

rod ② ; $-\dfrac{2Y\sin^2\theta\cos\theta}{1+2\cos^3\theta}$

5.32 모든 joint는 pin joint이고 θ를 미소각으로 이상화 하자.



Geometry ; $\delta_1=c\theta$; $\delta_2=3c\theta$

$\therefore \varepsilon_1=\dfrac{c\theta}{L}$, $\varepsilon_2=\dfrac{3c\theta}{L}$

평형조건 ; $\sum M_{⓪}=0$

$F_1(c)+F_2(3c)=P(2c)$

$\therefore F_1+3F_2=2P$

stress - strain relation (elastic)

$F=\sigma A=E\varepsilon A$

$F_1=\dfrac{EAc\theta}{L}$; $F_2=\dfrac{3EAc\theta}{L}$

$\therefore \left(\dfrac{EAc\theta}{L}\right)+3\left(\dfrac{3EAc\theta}{L}\right)=2P$ $\therefore F_1=\dfrac{P}{5}$; $F_2=\dfrac{3P}{5}$

$\sigma_1=\dfrac{P}{5A}$; $\sigma_2=\dfrac{3P}{5A}$

rod ①이 소성이 되면, 다른 rod들도 소성이 된다.

$F_1=F_2=YA$, $P=(F_1+3F_2)/2=2YA$

reverse plastic flow가 생기지 않는 elastic unloading을 가정하면, $-2YA$에 의한 elastic stress는;

$$\sigma_1 = -\frac{2}{5}Y \quad ; \quad \sigma_2 = -\frac{6}{5}Y$$

Residual stress = plastic stress + change in stress on unloading

∴ Residual stress;

rod ① ; $Y - \frac{2}{5}Y = \frac{3}{5}Y$

rod ② ; $Y - \frac{6}{5}Y = -\frac{1}{5}Y$

모두 Y 보다 작은 값이다. 따라서 elastic unloading 이라는 가정은 옳다.

rod ①이 막 소성이 되었을 때;

$$\sigma_1 = E\left(\frac{c\theta}{L}\right) = Y \quad \therefore \theta = \frac{YL}{EC}$$

elastic unloading을 할 때;

$$\frac{EAC\theta}{L} = \frac{P}{5}, \quad P = -2YA \quad \therefore \theta = -\frac{2}{5}\frac{YL}{EC}$$

$$\therefore \text{residual angle} = \frac{YL}{EC} - \frac{2}{5}\frac{YL}{EC} = \frac{3}{5}\left(\frac{YL}{EC}\right)$$

Ans. rod① $= \frac{3}{5}Y$, rod② $= -\frac{1}{5}Y$ residual angle $= \frac{3}{5}\left(\frac{YL}{EC}\right)$

5.33 1020 CR max. uniform strain $\simeq 0.01$

1020 HR max. uniform strain $\simeq 0.20$

$$\therefore \frac{(\Delta l)_{CR}}{(\Delta l)_{HR}} \simeq \frac{L(0.01)}{L(0.20)} = 0.05$$

fracture 때의 true strain ; 1020 CR $\varepsilon_f = 0.70$

1020 HR $\varepsilon_f = 0.90$

$$\therefore \frac{(\varepsilon_f)_{CR}}{(\varepsilon_f)_{HR}} = \frac{0.7}{0.9} = 0.78$$

fracture에 대한 average strain의 비로서 두개의 material의 relative ductility D_R을 측정하자.

대단히 긴 시편에서는 average strain $\simeq$ uniform strain

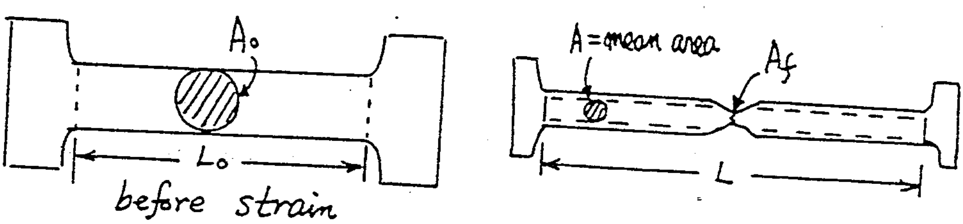
$$D_R = \frac{1020\,CR\ ductility}{1020\,HR\ ductility} = 0.05$$

상당히 짧은 시편에서는 average strain ≃ fracture strain

$D_R = 0.78$

Ans. $(\varepsilon_f)_{CR}/(\varepsilon_f)_{HR} = 0.78$

5.34



elastic strain은 무시한다.

% elongation $= PE = 100\left(\frac{L-L_0}{L_0}\right)$; % reduction in Area $= RA$

$$= 100\left(\frac{A_0 - A_f}{A_0}\right)$$

volume constancy (plastic flow) $A_0 L_0 = AL$

그림에서 necking이 없을 때; $A_f = A$, necking의 경우; $A_f < A$

$\therefore A_f \leq A$

따라서 $A_0 L_0 \geq A_f \cdot L$; $L = L_0\left[1 + \frac{PE}{100}\right]$; $A_f = A_0\left[1 - \frac{RA}{100}\right]$

$\therefore A_0 L_0 \geq A_0\left[1 - \frac{RA}{100}\right] L_0\left[1 + \frac{PE}{100}\right]$ 즉 $\left[1 - \frac{RA}{100}\right]\left[1 + \frac{PE}{100}\right] \leq 1$

만일, $\left[1 - \frac{RA}{100}\right]\left[1 + \frac{PE}{100}\right]$가 1보다 작다면, fracture 전에 necking이 생기고, 1이라면 necking 없이 fracture가 발생한다.

5.35 σ_x를 제외한 다른 stress는 0.

$\bar{\sigma} = \sigma_x$, $\quad d\varepsilon_x^p = \frac{d\bar{\varepsilon}^p}{\bar{\sigma}}\left[\sigma_x - 0\right] = \frac{\partial \bar{\varepsilon}^p \sigma_x}{\bar{\sigma}}$

$\therefore \bar{\varepsilon}^p = \int d\bar{\varepsilon}^p = \int d\varepsilon_x^p = \varepsilon_x^p$ $\quad$ − Q.E.D −

5.36 $u = Ar + \frac{B}{r}$

Equilibrium ; $\frac{d\sigma_r}{dr} + \frac{\sigma_r - \sigma_\theta}{r} = 0$ — ①

Strain - displacement ;

$$\varepsilon_r = \frac{du}{dr} = A - \frac{B}{r^2} ; \ \varepsilon_\theta = \frac{u}{r} = A + \frac{B}{r^2} ; \ \varepsilon_z = \frac{dw}{dz} \quad — ②$$

Stress - strain ;

$$\left.\begin{aligned} &\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)], \quad \varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_z + \sigma_r)] \\ &\varepsilon_r - \varepsilon_\theta = \frac{1}{E}[(\sigma_r - \sigma_\theta)(1+\nu)] = \left(A - \frac{B}{r^2}\right) - \left(A + \frac{B}{r^2}\right) = -\frac{2B}{r^2} \end{aligned}\right\} ③$$

$\therefore \sigma_r - \sigma_\theta = -\frac{E}{1+\nu} \cdot \frac{2B}{r^2}$ ①식에 대입

$$\frac{d\sigma_r}{dr} = \frac{E}{1+\nu} \cdot \frac{2B}{r^3} \qquad \therefore \sigma_r = -\frac{EB}{1+\nu} \cdot \frac{1}{r^2} + C$$

$$\left.\begin{aligned} &\text{at } r = r_i ; \sigma_r = -P_i \\ &\text{at } r = r_o ; \sigma_r = -P_o \end{aligned}\right) \therefore \left(\begin{aligned} -P_i &= -\frac{EB}{1+\nu} \cdot \frac{1}{r_i^2} + C \\ -P_o &= -\frac{EB}{1+\nu} \cdot \frac{1}{r_o^2} + C \end{aligned} \right.$$

$$\therefore P_o - P_i = \frac{EB}{1+\nu} \cdot \frac{r_i^2 - r_o^2}{r_i^2 r_o^2}$$

따라서 $B = \frac{(1+\nu)(P_o - P_i) r_i^2 r_o^2}{E(r_i^2 - r_o^2)}$

②③에서 $A - \frac{B}{r^2} = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)]$, $A + \frac{B}{r^2} = \frac{1}{E}[\sigma_\theta - \nu(\sigma_z + \sigma_r)]$

$\therefore A - \frac{B}{r_i^2} = \frac{1}{E}[-P_i - \nu(\sigma_\theta + \sigma_z)]$, $A + \frac{B}{r_i^2} = \frac{1}{E}[\sigma_\theta - \nu(-P_i + \sigma_z)]$

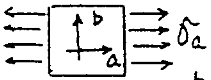
위 두식에서 σ_θ를 소거하면 $(1+\nu)\left(A - \frac{B}{r_i^2}\right) = \frac{1}{E}[(\nu^2 - 1)P_i - \nu^2 \sigma_z]$

$\therefore A = \frac{1}{E(1+\nu)}[(\nu^2 - 1)P_i - \nu^2 \sigma_z] + \frac{B}{r_i^2}$ 여기에 B를 대입하면

A를 얻게 된다,

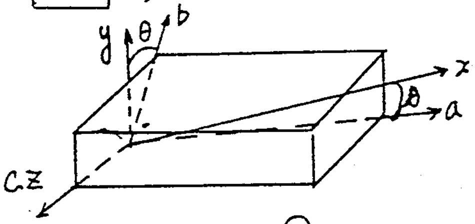
5.37 문제 (5.36)의 결과에 $P_o = 0$. $\sigma_z = 0$을 대입하여 얻는다.

5.38

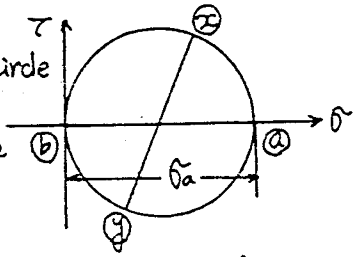


state of stress

other stress = 0



Mohr's circle in the a-b plane



$$\sigma_x = \frac{\sigma_a}{2}(1+\cos 2\theta)$$

$$\sigma_y = \frac{\sigma_a}{2}(1-\cos 2\theta)$$

$$\tau_{xy} = -\frac{\sigma_a}{2}\sin 2\theta$$

나머지 stress는 0

stress - strain ;

$$\varepsilon_x = S_{11}\sigma_x + S_{12}(\sigma_y+\sigma_z)$$

$$= \frac{a}{2}\left[(S_{11}+S_{12})+(S_{11}-S_{12})\cos 2\theta\right]$$

$$\varepsilon_y = S_{11}\sigma_y + S_{12}(\sigma_x+\sigma_z)$$

$$= \frac{a}{2}\left[(S_{11}+S_{12})-(S_{11}-S_{12})\cos 2\theta\right]$$

$$\gamma_{xy} = S_{44}\tau_{xy} = -\frac{\sigma_a}{2}\left[S_{44}\sin 2\theta\right]$$

이러한 strain을 (a. b. c) coordinate로 다시 바꿔주기 위해 Mohr's circle을 이용하면

$$\gamma_{ab} = (\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$= \frac{\sigma_a}{2}\left[2(S_{11}-S_{12})\cos 2\theta\right]\sin 2\theta - \frac{\sigma_a}{2}\left[S_{44}\sin 2\theta\right]\cos 2\theta$$

$$\therefore \frac{\gamma_{ab}}{\sigma_a} = \left[S_{11} - S_{12} - \frac{1}{2}S_{44}\right]\sin 2\theta\cdot\cos 2\theta \quad \text{— Q.E.D. —}$$

한 걸음 더 나아가서 ;

$$\frac{\gamma_{ab}}{\varepsilon_a} = \frac{\left[S_{11}-S_{12}-\frac{1}{2}S_{44}\right]\sin 2\theta\cos 2\theta}{(S_{11}+S_{12})+(S_{11}-S_{12})\cos^2 2\theta + \frac{1}{2}S_{44}\sin^2 2\theta}$$

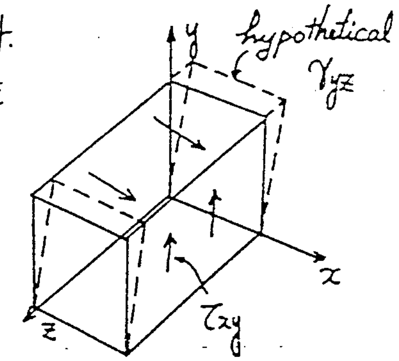
(Table 5.7) 에서 $S_{11}-S_{12}-\frac{1}{2}S_{44} = 0.419\times10^{-7}\ in^2/lb$

$$\frac{\gamma_{ab}}{\sigma_a} = 0.419\times10^{-7}\cdot\frac{1}{2}\sin 4\theta$$

$$\therefore \left(\frac{\gamma_{ab}}{\sigma_a}\right)_{max} = 0.2095\times10^{-7}\ in^2/lb$$

Ans. $(\gamma_{ab}/\sigma_a)_{max} = 0.2095\times10^{-7}\ in^2/lb$

5.39 τ_{xy}가 γ_{yz}를 발생시킨다고 하자. z축에 관해 180° 회전 시키면 γ_{yz}는 반대가 되지만, τ_{xy}는 그렇지 않다. x.y.z 축은 회전후에도 structural axis 이므로, 회전에 의해서 stress-strain 의 관계는 변화가 없다.



τ와 γ와의 관계는 선형적이어야 한다. 헌데 $\gamma_{yz} = c\,\tau_{yz}$ 에서 c는 위의 모순에 의해 $c=0$이다. 따라서 τ_{xy}에 의한 shear strain γ_{yz}는 없다. γ_{zx}도 같은 요령.

5.40 pipe가 충분히 길다고 생각하면 중간 부분에서는 lateral restraint가 없으므로 $\sigma_r = \sigma_\theta = 0$, z 방향은 구속되었으므로 $\varepsilon_z = 0$ 으로 놓을 수 있다.

stress - strain relation;

$$\varepsilon_z = \alpha\Delta T + \frac{1}{E}[\sigma_z - \nu(\sigma_\theta + \sigma_r)] = 0 \qquad \therefore \sigma_z = -E\alpha\Delta T$$

$$\sigma_z = -(200\times10^9)(12\times10^{-6})(-35^\circ) = 84\times10^6\ (N/m^2)$$

$$\varepsilon_\theta = \alpha\Delta T + \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)]$$

$$\varepsilon_r = \alpha\Delta T + \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)]$$

$$\therefore \varepsilon_\theta = \varepsilon_r = \alpha\Delta T - \frac{\nu}{E}\sigma_z = \alpha\Delta T + \nu\alpha\Delta T$$

$$= \alpha\Delta T(1+\nu) = (12\times10^{-6})(-35)(1+0.3)$$

$$= -5.46\times10^{-4}$$

Ans. $\sigma_r = \sigma_\theta = 0$, $\sigma_z = 84\times10^6\ N/m^2$; $\varepsilon_\theta = \varepsilon_r = -5.46\times10^{-4}$. $\varepsilon_z = 0$

5.41 Bolt의 displacement = turn(δ_1) + 온도증가(δ_2)
+ Al의 팽창에 의한 displacement(δ_3) = δ_b

$\delta_1 = \frac{1}{4} \times \frac{1}{16} = \frac{1}{64}$ (압축), $\delta_2 = \alpha_S \cdot L \cdot \Delta T$ (인장)

$\delta_3 = \frac{F \cdot L}{E_S A_S}$ (인장) (F: Al의 팽창에 의한 인장력)

Sleeve 의 displacement δ_s = 온도증가 (δ_2') + Al의 팽창에 의한 (즉. Al와 Fe 간의 팽창계수의 차이에 의해 생기는 힘) displacement (δ_3')

$\delta_2' = \alpha_a \cdot L \cdot \Delta T$ (인장), $\delta_3' = \frac{F \cdot L}{E_a A_a}$ (압축)

Geometry; $\delta_b = \delta_s$

$$\therefore -\frac{1}{64} + \alpha_S \cdot L \cdot \Delta T + \frac{F \cdot L}{E_S A_S} = \alpha_a \cdot L \cdot \Delta T - \frac{FL}{E_a A_a}$$

$$F = \left\{ (\alpha_a - \alpha_s) L \cdot \Delta T + \frac{1}{64} \right\} \Big/ \left(\frac{L}{E_S A_S} + \frac{L}{E_a A_a} \right)$$

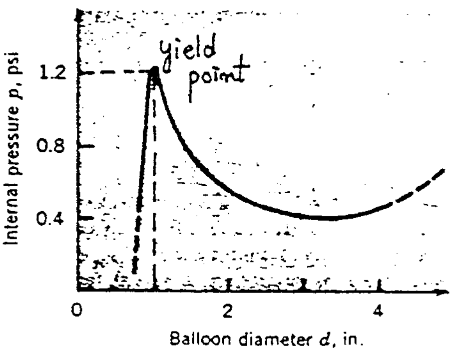
$$= \left\{ (12-6) \times 10^{-6} \times 4 \times 40 + \frac{1}{64} \right\} \Big/ \left(\frac{4}{(30 \times 10^6)(\frac{\pi}{4})(\frac{1}{2})^2} + \frac{4}{(10 \times 10^6)(\frac{\pi}{4})(\frac{5}{16})} \right)$$

$$= 7183.4 \text{ (lb)}$$

$\therefore \sigma_S = F/A_S = 36580$ psi (tension)

$\sigma_A = F/A_A = 29270$ psi (compression)) Ans.

5.42



rubber는 비압축성

$\therefore 4\pi r^2 t = const = C'$

$r^2 t = C''$

$$\sigma_\theta = \sigma_\phi = \frac{p \cdot r}{2t} = \frac{p \cdot r}{2 \cdot \frac{C}{r^2}}$$

$$= C p d^3$$

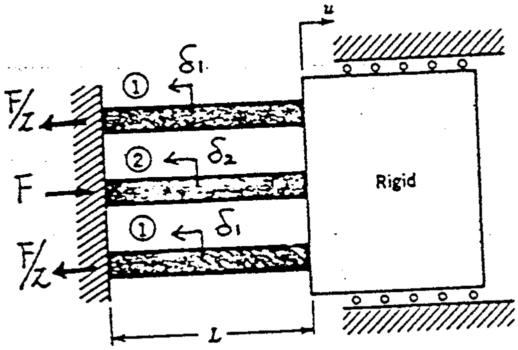
그림에서 대략 $p = 4d - 1$

따라서 $\sigma_\theta = \sigma_\phi = c\,(4d^4 - d^3)$
그림에서 대략 직경이 $1\ in$ 압력이 $1.2\ psi$ 에서 항복이
일어남을 알수 있다.

Ans. $\sigma_\theta = \sigma_\phi = c\,(4d^4 - d^3)$, $c = const$

5.43 (a) 변화된 온도 ; T'
Geometry 에 의해서 그림과 같이 힘이 작용하게 된다.

$$\therefore \delta_1 = \alpha_1 L T' + \frac{(F/2)\cdot L}{E_1 A}$$



$$= \alpha L T' + \frac{F\cdot L}{2EA}$$

$$\delta_2 = \alpha_2 L T' - \frac{F\cdot L}{E_2 A}$$

$$= 2\alpha L T' - \frac{FL}{2EA}$$

$\delta_1 = \delta_2$ 이어야 하므로 $\alpha L T' + \frac{F\cdot L}{2EA} = 2\alpha L T' - \frac{FL}{2EA}$

$$\therefore F = EA\alpha T'$$

material ① ; $\sigma_1 = (F/2)/A = \frac{1}{2} E\alpha T' \Rightarrow \sigma_y = \frac{1}{2} E\alpha T_y$

$$\therefore T_y = \frac{2\sigma_y}{E\alpha} \quad (\sigma_y \text{ ; yield stress})$$

(b) 실제 변형 $\delta = \alpha L T' + \frac{F\cdot L}{2EA} = \frac{3}{2}\alpha L T'$

$$\therefore u = \alpha L T_y + \frac{\alpha L T_y}{2} = \frac{3}{2}\alpha L T_y = \frac{3L\sigma_y}{E}$$

(c) $F = 2\sigma_y AE$, $T' = 2T_y$

$$\therefore u = \alpha L \cdot 2T_y + \frac{2EA\sigma_y \cdot L}{2EA} = \frac{7L\sigma_y}{E}$$

(d) $U_{plastic} = U_{total} - U_{elastic}$

$$= \frac{7}{2}\alpha L T_y - \frac{3}{2}\alpha L T_y = 2\alpha L T_y = \frac{4L\sigma_y^2}{E}$$

Ans. a) $T_y = \frac{2\sigma_y}{E\alpha}$ b) $u = \frac{3L\sigma_y^2}{E}$ c) $u = \frac{7L\sigma_y^2}{E}$ d) $U_p = \frac{4L\sigma_y^2}{E}$

5.44 문제(5.41)과 같은 요령. (압축(-). 인장(+))

i) Bolt ; δ_b = axial strain + 온도변화 + 인장력

$$= -(0.0005)(150+1.5) + (12\times10^{-6})(151.5)(95) + \frac{P(151.5)}{(200\times10^9)(310\times10^{-6})} = 2.44\times10^{-6}P + 0.09696$$

ii) washer ; δ_w = 온도변화 + 압축력

$$= (12\times10^{-6})(1.5)(95) + \frac{(-P)(1.5)}{(200\times10^9)(625\times10^{-6})} = 1.2(-P)\times10^{-8} + 1.71\times10^{-3}$$

iii) sleeve ; δ_s = 온도변화 + 압축력

$$= (24\times10^{-6})(150)(95) + \frac{-P(150)}{(70\times10^9)(625\times10^{-6})} = -3.43\times10^{-6}P + 0.342$$

Geometry ; $\delta_b = \delta_w + \delta_s$ i) ii) iii) 을 대입 $\therefore P = 41353.7N$

$\sigma_b = P/A_b = 1.33\times10^8 (N/m^2)$, $\sigma_w = P/A_w = 6.62\times10^7 (N/m^2)$

Ans $\sigma_b = 1.33\times10^8\ N/m^2$ (인장) ; $\sigma_w = 6.62\times10^7\ N/m^2$ (압축)

5.45 대기압 P_0 라 놓으면

Equilibrium ; $kA\sigma = (A - kA)P - P_0 A$

$$\therefore \sigma = \frac{1-k}{k}P - \frac{1}{k}P_0$$

yielding ; $280\times10^6 = \left(\frac{1-k}{k}\right)P_y - \frac{P_0}{k}$ $\therefore P_y = \frac{P_0 + 280\times10^6 k}{1-k}$

lateral 방향의 stress는 없다고 가정하면, uniaxial loading이 된다. $\sigma_1 = \sigma,\quad \sigma_2 = \sigma_3 = 0$ — ①

i) Mises yielding criterion ;

$$\bar{\sigma} = \sqrt{\frac{1}{2}\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\}}$$

①을 대입하면 $\bar{\sigma} = \sigma = Y$

ii) Max. shear stress criterion ;

$$\bar{\tau} = \frac{1}{2}|\sigma_{max} - \sigma_{min}| = \frac{1}{2}|\sigma - 0| = \frac{1}{2}Y$$

$\therefore Y = \sigma$

Ans. $\sigma = \dfrac{1-k}{k}p - \dfrac{P_o}{k}$, $P_y = \dfrac{P_o + 280\times10^6 k}{1-k}$

---

i) ii) 에서 서로 같다.

5.46 (a) δ_s 가 δ_c 보다 residual stress에 의한 displacement 만큼 더 커야한다. ↳ copper

따라서 $\alpha_s L \Delta T + \dfrac{L}{E_c}(\sigma_{resi.})_c = \alpha_c \cdot L \cdot \Delta T$

$$\therefore \Delta T = \frac{(\sigma_{resi})_c}{(\alpha_c - \alpha_s)E_c} = \frac{21\times10^6}{(16.2-10.8)(10^{-6})(118\times10^9)} = 33\,^\circ C$$

(b) copper 의 residual stress 에 의해서 신장되는 量만큼을 tensile stress를 가해서 늘려놓고, electroplating을 하면 copper 에는 residual stress가 남지 않는다.

$$\delta = \frac{L}{E_c}(\sigma_{resi.})_c = \frac{L}{E_s A_s} = \left(\frac{P}{A_s}\right)\cdot\frac{L}{E_s} = \sigma_s \cdot \frac{L}{E_s}$$

$$\therefore \sigma_s = \frac{A_s}{E_s}(\sigma_{resi.})_c = \frac{200\times10^9}{118\times10^9}\times(21\times10^6)$$

$$= 35.6\times10^6\,(N/m^2)$$

(a)의 열을 가해주는 방법 보다는 (b)의 방법이 우월하다.

Ans. a) $\Delta T = 33°C$ b) $35.6\times10^6\ N/m^2$

5.47 $P_o = 0$, $P_i = p$

Thin walled tube ; $t/r_i \ll 1$ 이면 σ_r은 σ_z, σ_θ에 비하여 미소하다. 따라서 $\sigma_r \simeq 0$. —①

Eq. (5.9) 에서 $P_o = 0$ 인 경우이므로.

$$\sigma_r = -\frac{p[(r_o/r)^2-1]}{(r_o/r_i)^2-1} = -\frac{p[(r_o/r)^2-1]}{[(r_i+t)/r_i]^2-1}$$

$$\simeq -p\cdot\frac{r_i}{2t}[(r_o/r)^2-1] \quad (\because \frac{t}{r_i} \ll 1 \text{ 이면 } (\frac{t}{r_i})^2 = 0)$$

윗 식에서 $r_i < r < r_o$ 이면 $-p \leq \sigma_r \leq 0$ 따라서 적합조건 만족.

$\lim_{(t/r_i)=0} \sigma_r \approx 0$ 따라서 ①과 일치한다.

Thin walled tube 에서 $\sigma_\theta \simeq \frac{pr}{t}$ —②

Eq. (5.10) 에서 $P_o = 0$. $P_i = p$ 이므로

$$\sigma_\theta = p\frac{(r_o/r)^2+1}{(r_o/r_i)^2-1} \approx \frac{p\cdot r_i}{2t}[(r_o/r)^2+1]$$

윗식에서 t가 아주 작으면 $r_i \approx r_o \simeq r$ ∴ $\sigma_\theta = \frac{pr}{t}$

따라서 ②식과 일치한다.

$\frac{t}{r_i} = 0.05$ 인 경우. 윗식에서

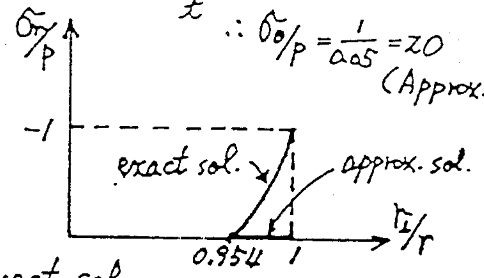
i) $\sigma_r = -\frac{p[(r_o/r)^2-1]}{(1.05)^2-1} = -9.756\,p[(\frac{1.05r_i}{r})^2-1]$; exact sol.

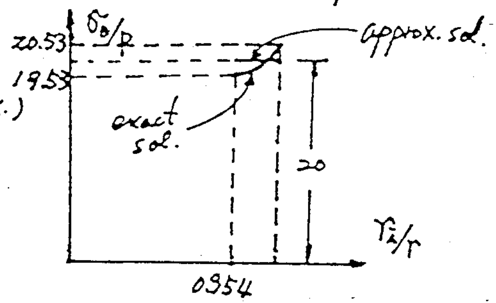
$\sigma_r \approx 0$; approximate sol.

ii) $\sigma_\theta = p\frac{(r_o/r)^2+1}{(r_o/r_i)^2-1} = 9.756\,p[(1.05\frac{r_i}{r})^2+1]$; exact sol.

$\sigma_\theta = \dfrac{P r_i}{t}$; approximate sol. $\quad r \geq r_i \;\therefore\; \dfrac{r_i}{r} \leq 1$

$\therefore \sigma_\theta / p = \dfrac{1}{0.05} = 20$ (Approx.)

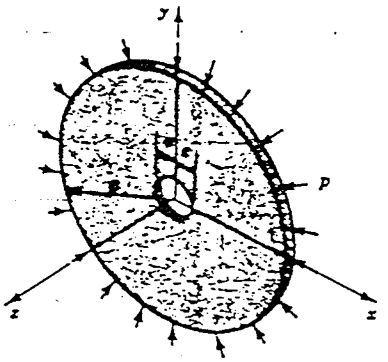




exact sol.

$(\sigma_r/p) = -9.756\left[1.05^2\left(\frac{r_i}{r}\right)^2 - 1\right]$, $(\sigma_\theta/p) = 9.756\left[1.05^2\left(\frac{r_i}{r}\right)^2 + 1\right]$

5. 48 $\qquad$ (a) $r_i = c$. $r_o = a$. $P_o = P$. $P_i = 0$



axial force는 없으므로

$\sigma_z = 0$, shear stresses $= 0$

$$\sigma_r = -\frac{P\left[(a/c)^2 - (a/r)^2\right]}{(a/c)^2 - 1}$$

$$= -P\left[\frac{1 - (c/r)^2}{1 - (c/a)^2}\right] \quad \text{—①}$$

또, $$\sigma_\theta = -p\left[\frac{1 + (c/r)^2}{1 - (c/a)^2}\right] \quad \text{—②}$$

$$\sigma_{1,2} = \frac{\sigma_r + \sigma_\theta}{2} \pm \sqrt{\left(\frac{\sigma_r - \sigma_\theta}{2}\right)^2}$$

$c/a \to 0$ 이면 $\quad \sigma_r = -p\left[1 - (c/r)^2\right]$, $\sigma_\theta = -p\left[1 + (c/r)^2\right]$

$\therefore \sigma_{1,2} = -P \pm p\left(\frac{c}{r}\right)^2$ 따라서 $\sigma_{max} = \left|-p \pm p\left(\frac{c}{r}\right)^2\right|_{max}$

$$= p + p\left(\frac{c}{r}\right)^2 = p\left(1 + \left(\frac{c}{r}\right)^2\right)$$

최대값은 $r = c$ 에서 생긴다. $\sigma_{max} = 2p$

(b) $c = 0$ 이면 ①②에서 $\sigma_r = \sigma_\theta = -p \quad \therefore \sigma_{max} = p$

따라서 hole에 의한 응력 집중계수 ; $\dfrac{2p}{p} = 2$

Ans. (a) $\sigma_{max} = 2P$. (b) 응력집중 계수 $= 2$

5.49 (Tensor Analysis)

$$\sigma_{ij,i} = \frac{\partial \sigma_{ij}}{\partial i}, \quad u_{i,j} = \frac{\partial u_i}{\partial j} \quad u_{j,i} = \frac{\partial u_j}{\partial i}$$

Kronecker delta $\delta_{ij} \begin{cases} = 0 & (i \neq j) \\ = 1 & (i = j) \end{cases}$

위의 관계를 이용하여 i, j에 xyz를 순차적으로 대입하면 된다.

$\sigma_{ij,i} + X_i = 0$; Equilibrium condition

$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$; strain-displacement relation

$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\delta_{ij}\theta + \alpha\delta_{ij}\Delta T$; stress-strain relation

〈tensor에 관한 수학책 참조〉

5.50 Strain energy ; $U = \frac{1}{2}\int(\sigma_{ij}\varepsilon_{ij})\,dV$

i에 $x.y.z$, j에 xyz를 순차적으로 대입.

$$U = \frac{1}{2}\int(\sigma_{xx}\varepsilon_{xx} + \sigma_{xy}\varepsilon_{xy} + \sigma_{xz}\varepsilon_{xz} + \sigma_{yx}\varepsilon_{yx} + \sigma_{yy}\varepsilon_{yy} + \sigma_{yz}\varepsilon_{yz} + \sigma_{zx}\varepsilon_{zx} + \sigma_{zy}\varepsilon_{zy} + \sigma_{zz}\varepsilon_{zz})\,dV$$

여기에서 $\sigma_{xx} = \sigma_x$, $\sigma_{yy} = \sigma_y$, $\sigma_{zz} = \sigma_z$. $\sigma_{xy} = \sigma_{yx}$, $\sigma_{yz} = \sigma_{zy}$

$\sigma_{zx} = \sigma_{xz}$ 로 바꿔준다. (ε도 마찬가지)

또 $\gamma_{xy} = 2\varepsilon_{xy}$. $\gamma_{yz} = 2\varepsilon_{yz}$ $\gamma_{zx} = 2\varepsilon_{zx}$

또 전단응력 σ_{xy}. σ_{yz} σ_{zx} 는 τ_{xy}. τ_{yz} τ_{zx}

이라 같이 우리가 상용하던 notation으로 바꿔주면.

$$U = \frac{1}{2}\int(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})\,dV$$

— Q.E.D. —

5.51 $\Pi = \frac{1}{2}\int(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{zx}\gamma_{zx})\,dV$

이 식을 모두 stress의 term으로 바꿔주기 위해 (5.2)식의 stress strain의 관계식을 이용하면

$$\Pi = \frac{1}{2}\int\Big[\frac{1}{E}(\sigma_x^2+\sigma_y^2+\sigma_z^2) - \frac{2\nu}{E}(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x)$$

$$+\frac{1}{G}(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2)\Big]\,dV \qquad -\text{Q.E.D}-$$

윗 식을 모두 strain으로 바꿔 주기 위해 문제(5.7)의 결과를 이용하면, $\gamma = \frac{\tau}{G}$ 와

$$\sigma_i = \frac{E}{1+\nu}\varepsilon_i + \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x+\varepsilon_y+\varepsilon_z) - \cancel{\frac{\alpha E(T-T_0)}{1-2\nu}}\rightarrow 0 \ (\because T=T_0),\quad (i=x.y.z)$$

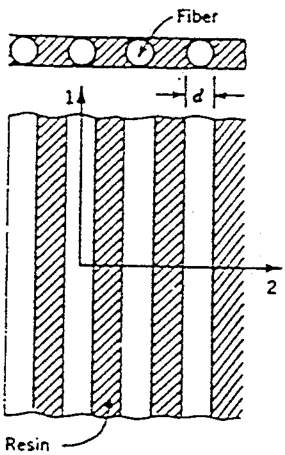
$$\Pi = \int\Big[\frac{E(1-\nu)}{2(1+\nu)(1-2\nu)}(\varepsilon_x+\varepsilon_y+\varepsilon_z)^2 - \frac{E}{(1+\nu)}\Big\{\varepsilon_x\varepsilon_y + \varepsilon_y\varepsilon_z + \varepsilon_z\varepsilon_x$$

$$-\frac{1}{4}(\gamma_{xy}^2+\gamma_{yz}^2+\gamma_{zx}^2)\Big\}\Big]\,dV \quad \left(\because G = \frac{E}{2(1+\nu)}\right)$$

$-$ Q.E.D. $-$

5.52 (a) ① 방향의 길이의 변화

$$\frac{PL}{E_1 A} = \frac{P_f\cdot L}{E_f\cdot A_f} = \frac{P_r\cdot L}{E_r\cdot A_r} \ —①, \quad P = P_f + P_r$$



따라서 $P = \left(1+\frac{E_r A_r}{E_f A_f}\right)P_f$

①식에 대입.

$$\frac{L}{E_1 A}\left(1+\frac{E_r A_r}{E_f A_f}\right)P_f = \frac{P_f\cdot L}{E_f A_f}$$

$$\therefore \frac{1}{E_1 A} = 1/(E_f A_f + E_r A_r)$$

$$\therefore E_1 = \frac{1}{A}(E_f A_f + E_r A_r)$$

$$A_f/A = 40\% = 0.4 \quad \therefore A_r/A = 0.6$$

따라서 $E_1 = 0.4E_f + 0.6E_r = (0.4)(5\times10^7) + (0.6)(5\times10^5)$

$= 2.03 \times 10^7 \ (psi)$

② 방향의 길이의 변화;

$$\frac{PL}{E_2 A} = \frac{PL_f}{E_f \cdot A} + \frac{PL_r}{E_r \cdot A} \text{ 따라서 } \frac{1}{E_2} = \frac{0.4}{E_f} + \frac{0.6}{E_r}$$

$$\therefore E_2 = 1 \Big/ \left(\frac{0.4}{E_f} + \frac{0.6}{E_r} \right) = \frac{E_r \cdot E_f}{0.4E_r + 0.6E_f}$$

$$= (5 \times 10^5) \cdot (5 \times 10^7) \big/ (0.4)(5 \times 10^5) + (0.6)(5 \times 10^7)$$

$$= 8.278 \times 10^5 \ (psi)$$

(b) 기하학적으로 대칭이므로 $E_1' = E_2' = E$, $P = P_1 + P_2$

first layer에 걸리는 힘 ; P_1 : second layer ; P_2

늘어난 길이 (①② 방향 모두 같다)

$$\frac{PL}{E(2A)} = \frac{P_1 L}{E_1 A} = \frac{P_2 L}{E_2 A} \text{ — ① } P_1 + P_2 = P \text{ 이므로}$$

$$P_1 \left(1 + \frac{E_2}{E_1}\right) = P$$

$$\therefore \frac{L}{L(2A)} \cdot \frac{E_1 + E_2}{E} P_1 = \frac{P_1 L}{E_1 A} \quad \text{따라서 } E = (E_1 + E_2)/2$$

$$\therefore E = (2.03 \times 10^7 + 8.278 \times 10^5)/2 = 1.0564 \times 10^7 \ (psi)$$

(c) ①② 방향이 서로 같은 material property를 갖고, 기하학적으로 대칭이므로 isotropy이다.

Ans. (a) $E_1 = 2.03 \times 10^7$ psi, $E_2 = 8.278 \times 10^5$ psi (b) $E = 1.0564 \times 10^7$

5.53 $\varepsilon_{ij} = S_{ji} \tau_i \qquad S_{ij} = S_{ji} = 1 \cdots 6$

where $\varepsilon_j = \varepsilon_x \cdots \gamma_{zx}$, $\tau_i = \sigma_x \cdots \tau_{zx}$

$ij = x$, $S_{ij} = S_{ji} = 1 \cdots 6$, $\tau_i = \sigma_x \cdots \tau_{zx}$로 변화시키면

$$\varepsilon_x = S_{11}\sigma_x + S_{12}\sigma_y + S_{13}\sigma_z + S_{14}\tau_{xy} + S_{15}\tau_{yz} + S_{16}\tau_{zx}$$

나머지도 같은 요령 —Q.E.D—

5.54 $\sigma_{mean} = 30.000 psi$, $\sigma_{ultimate} = 130.000 psi$
endurance limit를 60.000 psi로 가정하고, reduction factor K_f를 3으로 놓으면,

$K_f = \frac{60.000}{\sigma_o} = 3 \quad \therefore \sigma_o = 20.000$

비례관계; $130.000 : 20.000 = 10.000 : \sigma_a$

$\therefore \sigma_a = \frac{10}{13} \times 20.000 = 15384.6$ psi

Ans 15384.6 psi

5.55 endurance limit; $\sigma_e = 40.000$ psi

$\sigma_a = 0.15 \sigma_m$ —① 안정계수 3 . $K_f = 3$

load line. curve ; $\sigma_a = 0.15 \sigma_m$ 과 안정계수를 3으로 하고 fatigue-strength reduction factor K_f를 더해준 σ_a curve 와의 만나는 점이 σ_m이 된다.

위의 설명에서 $\sigma_a = 4.444 - \frac{4.444}{33.333} \sigma_m$ —②

①② 에서 $\sigma_m = 15685$ psi Ans 15685 psi

5.56 HMZ1A-T8 (mean stress ; 12000 psi
alternating stress ; 17.000 psi

Fig 5-42b 에서 (endurance limit ; 약 10.000 psi
ultimate strength ; 약 50.000 psi

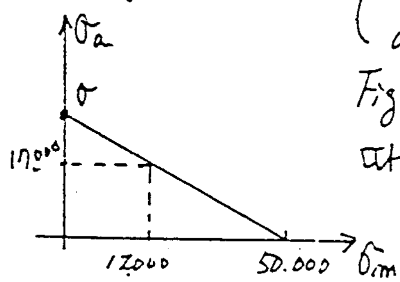


Fig 5-42 b는 $\sigma_{mean} = 0$ 인 경우이다

따라서 $\sigma \doteqdot 22368$ psi

Fig 5.42b 에서 대략 Cycle $\doteqdot$ 14000 (cycles)

Ans 약 14000 cycle

CHAPTER 6

6.1 $\tau_{max} = \frac{M_t \cdot r_o}{I_z} = \frac{1100 \times \frac{5\times10^{-2}}{2}}{\frac{\pi}{32}(5\times10^{-2})^4} = 4.482\times10^7 \, (N/m^2)$

$\phi = \frac{M_t \cdot L}{G \cdot I_z} = \frac{1100\times1.5}{(80\times10^9)\cdot\frac{\pi}{32}(5\times10^{-2})^4} = 0.0336 \, (rad) = 1.926°$

Ans. $\tau_{max} = 4.482\times10^7 \, N/m^2$ $\phi = 1.926°$

6.2 $\tau_{max} = \frac{M_t \cdot r_o}{I_z} = 25000 \times \frac{d_o/2}{\frac{\pi}{32}(d_o^4 - d_i^4)} = 82\times10^6$

따라서 $d_o = 644(d_o^4 - d_i^4)$

$\phi = \frac{M_t \cdot L}{G \cdot J} = \frac{25000\times2.5}{(80\times10^9)\cdot\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{\pi}{90} \, (rad)$

따라서 $d_o^4 - d_i^4 = 2.2797\times10^{-4}$

②를 ①에 대입하면 $d_o = 0.1468 \, m$ $d_i = 0.124 \, m$

Ans. $d_i = 124 \, mm$ $d_o = 146.8 \, mm$

6.3 $\sigma_r = \sigma_\theta = \sigma_z = \tau_{r\theta} = \tau_{rz} = 0$

$\tau_{\theta z} = G_1 r \frac{d\phi}{dz} \quad (0 < r < r_i)$, $\tau_{\theta z} = G_2 r \frac{d\phi}{dz} \quad (r_o > r > r_i)$

한편, twisting moment $M_t ; \int_A r \tau_{\theta z} \, dA$

$= \frac{d\phi}{dz}\left\{2\pi G_1 \int_0^{r_i} r^3 dr + 2\pi G_2 \int_{r_i}^{r_o} r^3 dr\right\}$

$= \frac{\pi}{2}\frac{d\phi}{dz}\left\{(G_1 - G_2) r_i^4 + G_2 r_o^4\right\}$

이 식은 다음과 같이 쓸 수 있다.

$$M_t = \frac{d\phi}{dz}\{G_1 I_1 + G_2 I_2\} \quad \text{where} \left(\begin{array}{l} I_1 = \frac{\pi}{2} r_i^4 \\ I_2 = \frac{\pi}{2}(r_o^4 - r_i^4) \end{array} \right.$$

∴ 비틀림각의 총합 ; $\phi = \dfrac{M_t \cdot L}{G_1 I_1 + G_2 I_2}$

$$\tau_{\theta z} = G_1 \cdot r \frac{d\phi}{dz} = \frac{G_1 M_t \cdot r}{G_1 I_1 + G_2 I_2}, \quad 0 < r < r_i$$

$$\tau_{\theta z} = G_2 \cdot r \frac{d\phi}{dz} = \frac{G_2 M_t \cdot r}{G_1 I_1 + G_2 I_2}, \quad r_i < r < r_o$$

6.4 4130 HT steel ; $G = 11.9 \times 10^6$ psi , 항복응력 $Y = 200.000$ psi

$$\tau_{\theta z} = \frac{1}{2} Y = 100.000 \text{ psi}$$

$$\phi_Y = \frac{M_t \cdot L}{G \cdot J} = \left(\frac{M_t \cdot r_o}{J}\right) \cdot \left(\frac{L}{G r_o}\right) = \tau_{\theta z} \cdot \frac{L}{G r_o}$$

$$= 100.000 \times \left(\frac{2}{11.9 \times 10^6 \times \frac{5}{4} \times \frac{1}{12} \times \frac{1}{2}}\right) = 0.3227 \text{ (rad)}$$
$$= 18.49°$$

Ans. $\phi_Y = 18.49°$

6.5 1020 CR steel ; $G = 11.5 \times 10^6$ psi $\quad Y = 86000$ psi

$\tau_{\theta z} = \frac{1}{2} Y = 43000$ psi $\qquad d_o = 2''$

(a) $M_t = 2500$ ft-lb

$\tau_{max} = \dfrac{M_t \cdot r_o}{J}$ 에서 $43000 = \dfrac{2500 \times 12 \times 1}{\frac{\pi}{32}(d_o^4 - d_i^4)}$ $\quad \therefore d_i = 1.727''$

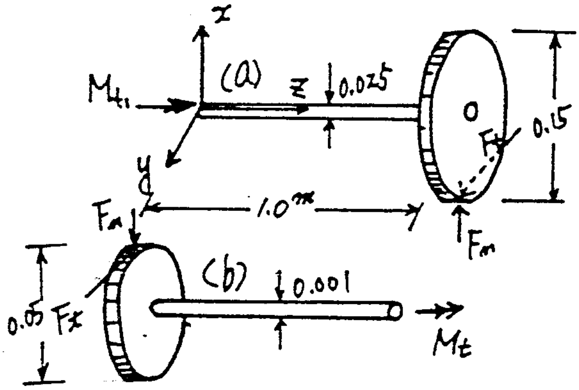
(b) $M_t = 5000$ ft-lb

(a)와 마찬가지로 $43000 = \dfrac{5000 \times 12 \times 1}{\frac{\pi}{32}(d_o^4 - d_i^4)}$ $\quad \therefore d_i = 1.1562''$

Ans. (a) $d_i = 1.727$ in $\quad$ (b) $d_i = 1.1562$ in

6.6 $M_t = 10(N\text{-}m),\ \Sigma M_z = 0,\ M_{t1} = 0.075 F_t$

$M_t = 0.025 F_t$



따라서 $M_{t1} = 3M_t = 30(N\text{-}m)$

부재 ⓐ의 비틂각

$$\phi_1 = \frac{M_{t1} \cdot L_1}{G_1 J_1} = \frac{30 \times 1}{(80\times10^9)\frac{\pi}{32}(0.025)^4}$$

$= 9.7785 \times 10^{-3}$ (rad)

$= 0.56027°$

따라서 부재 ⓑ의 비틂각 $\phi_1' = \frac{0.015}{0.05}\phi_1 \quad \therefore \phi_1' = 1.68081°$

외력에 의한 부재 ⓑ의 비틂각

$$\phi_2 = \frac{M_t L_2}{G_2 J_2} = \frac{10 \times 0.67}{(80\times10^9)\frac{\pi}{32}(0.01)^4} = 0.085307 \text{ (rad)}$$

$= 4.88773°$

$\therefore$ total torsion angle $\phi = \phi_1' + \phi_2 = 6.57°$

Ans. $\phi = 6.57°$

6.7 $M_{t1} = 3M_t$

$$\tau_1 = \frac{M_{t1} \cdot r_1}{J_1} = \frac{3 \times 0.0125}{\frac{\pi}{32}(0.025)^4} M_t = 977848 M_t$$

$$\tau_2 = \frac{M_t \cdot r_2}{J_2} = \frac{0.005}{\frac{\pi}{32}(0.01)^4} M_t = 5092958 M_t \qquad \therefore \tau_2 > \tau_1$$

$\therefore 275 \times 10^6 = 5092958\, M_t$

Ans. $M_t = 53.996$ (N-m)

6.8 아랫쪽 gear의 비틂각을 ϕ라 하면, 기하학적 적합조건에 의해서 윗쪽 gear의 비틂각은 $\frac{3}{5}\phi$ 이다.

따라서 아랫쪽 gear의 최종 비틂각은 $(\frac{3\pi}{180} - \phi)$ rad.

F_A를 gear가 접하는 점의 tangential force라 놓으면

lower shaft ; $M_{tL} = F_A \times 125 = \dfrac{(\frac{3\pi}{180} - \phi) G \cdot I_L}{L} = \dfrac{(\frac{3\pi}{180} - \phi) G (\frac{125}{2})^4}{L}$ —①

upper shaft ; $M_{tu} = F_A \times 75 = \dfrac{\phi \cdot G \cdot I_u}{L} = \dfrac{\phi \cdot G \cdot \frac{\pi}{2} (\frac{75}{2})^4}{L}$ —②

$G = 80 \times 10^9$

①② 식에서 $\theta = 0.0252$ (rads)

$(\tau_{\theta z})_{max.upper} = \dfrac{2 M_{tu}}{\pi (r_{ou})^3}$ $\quad (\tau_{\theta z})_{max.lower} = \dfrac{2 M_{tL}}{\pi (r_{oL})^3}$

①② 식을 이용하면

Ans. $(\tau_{\theta z})_{max.upper} = 45.9$ MN/m²

$(\tau_{\theta z})_{max, lower} = 33.6$ MN/m²

6.9 gear의 접촉점 A.B에서의 tangential force를 F_{AT}. F_{BT}라 놓으면

upper shaft ; $F_{AT} \times 50 + F_{BT} \times (75/2) = 0$ —①

lower shaft ; $F_{AT} \times 50 + F_{BT} \times (125/2) = 0$ —②

①② 식에서 $F_{AT} = F_{BT} = 0$

Ans. shear stress $= 0$

6.10 평형 조건 $\Sigma M_x = 0$; $-M_A - M_c + M_o = 0$ —①

기하학적 적합조건 ; $\phi_{AB} = \phi_{BC}$ —②

$M_A = \dfrac{\phi_{AB}}{L_1} G_1 I_{AB}$ $\quad I_{AB} = \dfrac{\pi}{2} \left(\dfrac{d_1}{2}\right)^4$

$M_B = \dfrac{\phi_{BC}}{L_2} G_2 I_{BC}$ $\quad I_{BC} = \dfrac{\pi}{2} \left(\dfrac{d_2}{2}\right)^4$ $\quad G_1 = G_2$

① ② 에서

Ans. $M_A = \dfrac{M_0}{1+\dfrac{L_1}{L_2}\left(\dfrac{d_2}{d_1}\right)^4}$ $M_C = \dfrac{M_0}{1+\dfrac{L_2}{L_1}\left(\dfrac{d_1}{d_2}\right)^4}$

6.11 문제 6.10과 마찬가지로

$$\phi_{AB} = \frac{2M_A}{\pi G_1}\left(\frac{L_1}{r_1^4}\right) \qquad \phi_{BC} = \frac{2M_C}{\pi G_2}\left(\frac{L_2}{r_2^4}\right)$$

$$\phi_{AB} = \phi_{BC}, \quad M_A = M_C$$

Ans. $\left(\dfrac{G_2}{G_1}\right)\left(\dfrac{L_1}{L_2}\right)\left(\dfrac{d_2}{d_1}\right)^4 = 1$

6.12 $\tau = \dfrac{M_t \cdot r}{J}$ —①

$$M_t = 12\times5000 + 10P = 60.000 + 10P$$

6개의 bolt를 1개의 bolt로 치환하기 위해

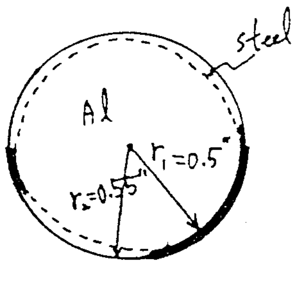
$$J = \Sigma\left(\int r^2 dA + Al^2\right) = 6\times\frac{\pi}{32}\times0.75^4 + \frac{\pi}{4}\times\left(\frac{3}{4}\right)^2(4\times25+2\times9)$$

$$= 87.66\ (in^4) \text{ —②}$$

$$\therefore \tau_{max} = \frac{60000+10P}{87.66}\times\sqrt{45} = 10{,}000 \qquad \therefore P = 7068\,(lb)$$

Ans. $P = 7068$ lb

6.13



Steel; $G_{st} = 7.8\times10^{10}\ N/m^2$

$\tau_{st} = 1360\ MN/m^2$

Al ; $G_{Al} = 26\times10^9\ N/m^2$

$\tau_{Al} = 100\ MN/m^2$

(a) $\dfrac{(\tau_{\theta z})_{r_2}}{(\tau_{\theta z})_{r_1}} = \dfrac{G_{st}\cdot r_2}{G_{Al}\cdot r_1}$ $\dfrac{(\tau_{\theta z})_{r_2}/\tau_{st}}{(\tau_{\theta z})_{r_1}/\tau_{Al}} = \dfrac{G_{st}}{G_{Al}}\cdot\dfrac{\tau_{Al}}{\tau_{st}}\cdot\dfrac{r_2}{r_1}$

$$\frac{(\tau_{\theta z})_{r_2}/Y_{st}}{(\tau_{\theta z})_{r_1}/Y_{Al}} = \frac{78}{26} \times \frac{100}{1360} \times \frac{27.5}{25} =$$

$r = r_1$ 에서 Al이 먼저 항복을 일으킨다.

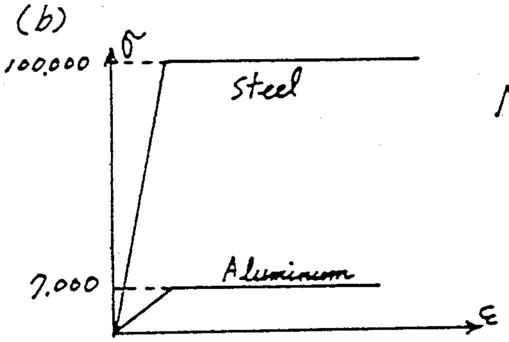
composite shaft에 대하여

$$(\tau_{\theta z})_{r_1} = \frac{2 \cdot M_t \cdot G_{Al} \cdot r_1}{\pi\{G_{Al} \cdot r_1^4 + G_{st}(r_2^4 - r_1^4)\}} = \frac{1}{2} Y_{Al} = 50 \times 10^6$$

$$\therefore M_t = \frac{\pi \times 0.5 \times 100 \times \{26 \times 10^3 \times (0.025/2)^4 + 78 \times 10^3 \times ((0.0275/2)^4 - (0.025/2)^4)\}}{26 \times 10^3 \times (0.025/2)}$$

$$\cong 367 \ (N \cdot m)$$

(b)



shaft가 완전히 항복되었다고 하면

$$M_t = 50 \int_0^{r_1} 2\pi r^2 dr + 680 \int_{r_1}^{r_2} 2\pi r^2 dr = 1155.2 \ (N \cdot m)$$

$$\frac{d\phi}{dz} = \frac{\tau_{\theta z}}{G \cdot r} = \frac{50 \times 10^6}{26 \times 10^9 \times (0.025/2)} = 0.23 \ (rad) = 13.22^\circ$$

Ans. (a) 366.9 N·m (b) 1155.2 N·m, 13.22°

6.14 고정단에서의 거리를 x라 잡으면 $M_t = \int_0^L \frac{q_0}{L} x \, dx$

$$\therefore M_{tx} = M_x - \int_0^x \frac{q_0}{L} x \, dx = M_x - \frac{q_0}{2L} x^2$$

$$d\phi = \frac{M_{tx}}{GJ} dx. \quad \therefore \phi = \int_0^L \frac{1}{GJ}\left(M_t - \frac{q_0}{2L} x^2\right) dx$$

$$= \frac{1}{GJ}\left(M_t \cdot L - \frac{q_0}{6L} L^3\right) \qquad M_t = \frac{q_0 \cdot L}{2}$$

$$\therefore \phi = \frac{2 M_t \cdot L}{3GJ} \ ; \text{Ans.}$$

6.15 구속되어 있으므로 회전각은 "0"이다. 구속되지 않을 때의 회전각 ; ϕ

$\phi = \dfrac{2M_t \cdot L}{3GJ}$ 구속에 의한 회전각은 방향이 반대이므로 $\phi' = -\dfrac{M_{t_2} \cdot L}{GJ}$

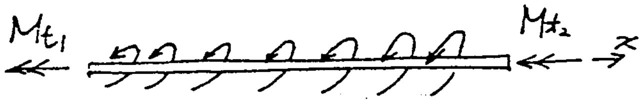
$$\phi + \phi' = \frac{2M_t \cdot L}{3GJ} + \left(-\frac{M_{t_2} \cdot L}{GJ}\right) = 0 \quad \text{—①}$$

평형조건 ; $\sum M_x = 0$; $M_{t_1} + M_{t_2} = M_t$ —②

①② 에서

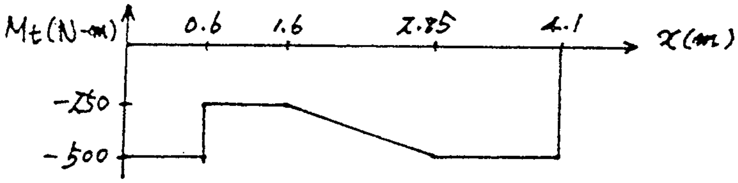
$M_{t_1} = \frac{1}{3} M_t$

$M_{t_2} = \frac{2}{3} M_t$



Ans. $M_{t_1} = \frac{1}{3} M_t$, $M_{t_2} = \frac{2}{3} M_t$

6.16 beam 에서의 B.M.D 과 같은 요령이다.



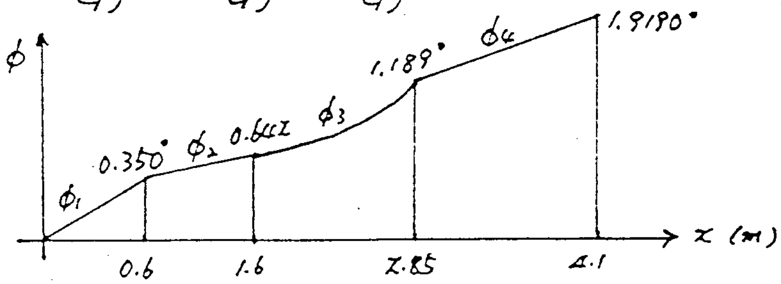
$$\phi = \int \frac{M_t}{GJ} dx$$

$$\phi_1 = \int \frac{500}{GJ} dx = \frac{500}{GJ} x \quad (0 \le x \le 0.6)$$

$$\phi_2 = \int \frac{250}{GJ} dx = \frac{250}{GJ}(x - 0.6) + \frac{500 \times 0.6}{GJ} \quad (0.6 \le x \le 1.6)$$

$$\phi_3 = \frac{100}{GJ} x^2 + \frac{570}{GJ} x + \frac{500 \times 0.6}{GJ} + \frac{250 \times 1}{GJ} - \frac{570 \times 1.6}{GJ} \quad (1.6 \le x \le 2.85)$$

$$\phi_4 = \frac{500}{GJ} x + \frac{1018.17}{GJ} + \frac{500 \times (-2.85)}{GJ} \quad (2.85 \le x \le 4.1)$$



6.17 $\tau = \frac{M_t \cdot r_o}{J}$ $J = \frac{\pi}{32}\{d_o^4 - d_i^4\} = \frac{\pi}{32}\{d_o^4 - (\frac{d_o}{2})^4\} = \frac{15\pi}{512} d_o^4$

$$M_t = \frac{\tau_{max} \cdot J}{r_o} = \frac{15\pi}{256} d_o^3 \cdot \tau_{max}$$

$$HP = \frac{M_t \omega}{76} = \frac{1}{76} \cdot \frac{15\pi}{256} d_o^3 \cdot \tau_{max} \cdot \frac{2\pi \cdot rpm}{60} = (2.54\times10^{-4})(rpm)(\tau_{max})(d$$

Ans. $HP = 2.54\times10^{-4} (rpm)(\tau_{max})(d_o^3)$

6.18 두께를 미소하다고 가정하면 shear flow는 항상 일정하다.

q = shear flow, minimum thickness $= t_{min} = 0.15''$

Tresca의 yield criteria를 적용하면

max. shear stress $= \frac{q}{t_{min}} = \frac{Y}{2} = \frac{35000}{2}$

$$\therefore q = 0.15 \times \frac{35000}{2}$$

$$(M_t)_{max} \fallingdotseq 2qA = 2\times 0.15 \times \frac{35000}{2} \times \frac{\pi}{4}(5^2 - 4.5)^2$$

$$= 19586 \text{ lb-in}$$

Ans. $(M_t)_{max.} = 19586$ lb-in

6.19 일반적으로 thin-walled tube 에서 $I_p \simeq 2\pi r^3 \cdot t$

$$\tau = \frac{M_t \cdot r}{I_p}$$

(a) outer tube; $M_{t_o} = \frac{\tau_y \cdot I_p}{2r} = \frac{\tau_y \cdot 2\pi(2r)^3 \cdot t}{2r} = 8\pi r^2 t \tau_y$

inner tube; $\tau = \frac{1}{2}\tau_y$ $M_{t_i} = \frac{\frac{1}{2}\tau_y \cdot 2\pi r^3 \cdot t}{r} = \pi r^2 t \tau_y$

$$\therefore T_y = M_{t_o} + M_{t_i} = 9\pi r^2 t \tau_y$$

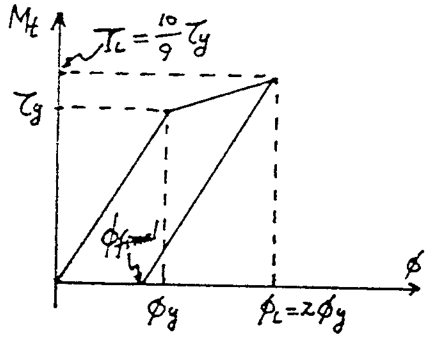
$$\phi_y = \frac{M_t \cdot L}{G I_p} = \frac{\tau_y \cdot L}{2Gr}$$

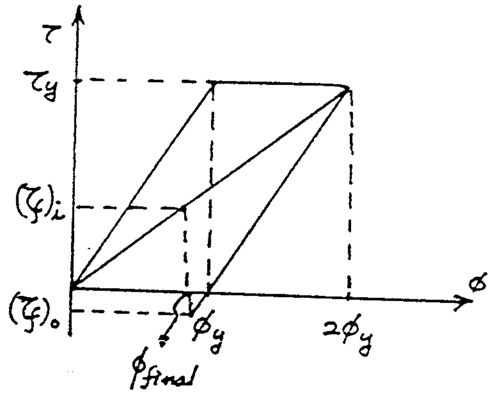
(b) outer와 inner tube 에서의 stress는 τ_y

$M_{t_o} = 8\pi r^2 \tau_y$ $M_{t_i} = 2\pi r^2 t \tau_y$ $\therefore T_L = 10\pi r^2 t \tau_y$

inner tube 에서 stress를 비교하면 $\phi_L = 2\phi_y = \frac{\tau_y \cdot L}{Gr}$

(c)





$(M_t)_{outer} = \tau_{outer} \times 4\pi r^2 t$

$(M_t)_{inner} = \tau_{inner} \times \pi r^2 t \qquad (M_t)_{outer} + (M_t)_{inner} = 0$

∴ 최종 stress에 대해서 $\tau_{outer} = -\frac{1}{4}\tau_{inner}$

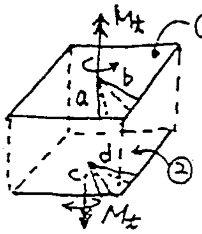
그림으로 부터 $(\tau_{outer})_{final} = -\frac{1}{9}\tau_y \quad (\tau_{inner})_{final} = \frac{4}{9}\tau_y$

Ans. (a) $T_y = 9\pi r^2 t \tau_y \quad \phi_y = \tau_y \cdot L/2Gr$ (b) $T_L = 10\pi r^2 t \tau_y$, $\phi_L = \tau_y L/Gr$

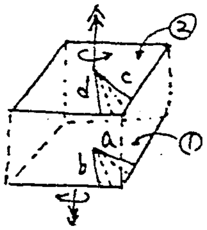
(c) $\phi_f = 4\tau_y L/9Gr$, $\tau_{of} = -\tau_y/9 \quad \tau_{if} = 4\tau_y/9$

6.20

(a)

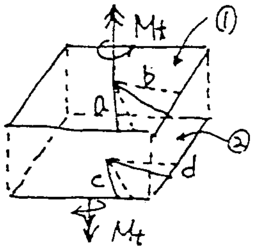


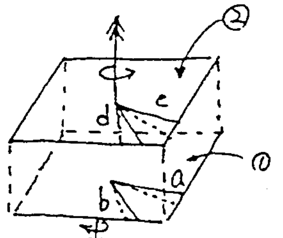
inverting



위의 두 그림 같은 면을 비교하면, 변형이 대각선에 관하여 대칭이며 양쪽 대각선에 대하여 비슷한 위치에 있는 요소는 같은 변형을 한다. 따라서 기하학적으로 가능한 변형 형태이다.

(b)



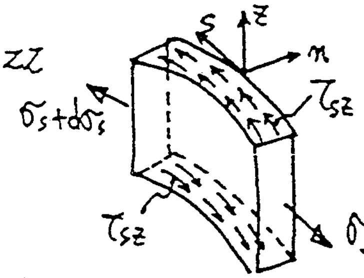


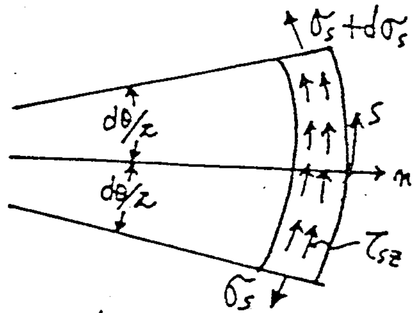
대칭축에 대하여 대칭을 이루지 않으므로 이러한 변형 형태는 기하학적으로 불가능하다.

6.21 (a) 대각선에 관해서 대칭이 아니므로, 이것은 기하학적으로 불가능한 변형 형태이다.

(b) 대각선에 관해 대칭이므로, 가능한 변형 형태이다.

6.22





표면에 작용하는 하중은 없다. 이러한 shaft에 대해서는, 위·아랫면에서의 τ_{sz}는 s-coordinate의 주어진 값과 같아야만 한다.

평형조건 ; $\Sigma F_n=0$; $\left[\sigma_s \frac{d\theta}{2} + (\sigma_s+d\sigma_s)\frac{d\theta}{2}\right] dn\, dz = 0$

$$\therefore \sigma_s = 0$$

6.23 승객이 타기 전 ; shaft의 torque $M_{t_i} = 3000$ lb-ft

승객이 탄 후 ; shaft의 torque $=(3000-3\overline{W})$ lb-ft

$\therefore$ Torque의 감소량 $= 3\overline{W}$ lb-ft

따라서 $\overline{W}$에 의한 각 변화 $\phi = \frac{M_t \cdot L}{GI_z} = \frac{3\overline{W}\times 12\times 60}{12\times 10^6 \times \frac{\pi}{32}\times 256} = 7.15\overline{W}\times 10^{-6}$ (rad)

$\therefore \delta_1 = 7.15\overline{W}\times 10^{-6}\times 36$ (in)

$\overline{W}$에 의한 tension에 의한 cable의 길이의 증가량

$$\delta_2 = \frac{F\cdot L}{AE} = \frac{\overline{W}}{(1/2)} \times \frac{200\times 12}{30\times 10^6} = 13.3\overline{W}\times 10^{-6}\times 12 \text{ (in)}$$

따라서 총 변위 $\delta = \delta_1 + \delta_2 = 420\overline{W}\times 10^{-6}$ in $= 0.2$ (in)

$\therefore \overline{W} = 475$ lb   Ans. 475 lb

6.24 switch에서 l만큼 떨어진 점의 torsion moment (torque)

$$(M_t)_l = M_0 + \int_0^l dM_t = M_0 + \frac{dM_t}{dl}\int_0^l dl = M_0 + l\frac{dM_t}{dl} \quad \left(M_0 : \text{switch의 torque}\right)$$

$\therefore (M_t)_l = 0.33 + 0.45l$

$$(\tau_{\theta z})_{max} = \frac{T \cdot r}{J} = \frac{(0.33+0.45l)\times 0.0015}{\frac{\pi}{32}\times(0.003)^4} = 280\times10^6$$

$\therefore l = 2.565$ (m)

$$\frac{d\phi}{dl} = \frac{(M_t)_l}{GI_p} \; ; \quad \phi_L = \frac{(M_t)_l \cdot L}{GI_p} = \int_0^{2.565} \frac{(0.33+0.45l)\,dl}{(80\times10^9)\cdot\frac{\pi}{32}(0.003)^4} = 3.657 \text{ (rad)}$$

$\therefore \phi_{total} = 2\phi_L = 7.315$ (rad) $= 1.164$ (revolution)

Ans. $l = 2.565$ (m) total play $= 1.164$ (revolutions)

6.25 w = foil 위의 물의 무게 $= 2\times(2\times3)\times10^{-8}\times1$ (grams)

$= 12\times10^{-8}$ grams

축 AB에 생기는 moment $gw = 2w = 24\times10^{-8}$ gram-cm

M_B = DE를 수평으로 되돌리는데 필요한 moment

M_A = A를 지지함으로써 A단에 생기는 moment $=0$ ($\because \phi_{AC}=0$)

평형을 이루기 위해 $M_B = gw = 24\times10^{-8}$ gram-cm

$$\therefore \phi_B = \frac{M_t \cdot L}{GI_p} = \frac{24\times10^{-8}\times4}{2.7\times10^{8}\,\frac{\pi}{32}(0.001)^4} = 0.036 \text{(rads)} = 2.06^\circ$$

Ans. $\phi_B = 2.06^\circ$

6.26 shear flow는 일정하므로. 2024-T4 Al ; $Y = 400\,MN/m^2$

$\tau_{AB} = \tau_{DC} = \frac{1}{2}\tau_{AD} = \frac{1}{2}\tau_{BC}$

$M_t = 2Aq = 2A\times(t\tau)_{BC}$ —①

$$\begin{cases} A = (52-2.6)\times(39-13)\times10^{-6} = 1.86238\times10^{-3}\,(m^2) \\ q = (t\tau)_{BC} = \frac{Y}{2}\cdot t_{BC} = (200\times10^{-6})\cdot(1.3\times10^{-3}) = 260000\,N/m^2 \end{cases}$$

윗 식을 ①에 대입

$\therefore M_t = 2\times1.862\times10^{-3}\times260\,000 = 968.4$ (N-m)

Ans. $M_t = 968.4$ (N-m)

6.27 (a) shear flow는 일정하기 때문에

$$1.3\tau_y = (2.6-x)\tau_y - x\tau_y \qquad \underline{\therefore x = 0.6 \ mm}$$

(b) $(M_t)_L = 2Aq$

$$= 2\cdot(52-2.6)(39-1.3)\times10^{-6}\cdot(1.3\times10^{-3}\tau_y)$$

$$\tau_y = \frac{1}{2}Y = 200\times10^{6}\ N/m^2$$

$$\underline{\therefore (M_t)_L = 968.4 \ (N\text{-}m)}$$

Thin wall의 모든 부분에서 shear flow가 앞의 문제와 같기 때문에 $(M_t)_L$도 같아진다. 실제로는 $(M_t)_L$이 $(M_t)_Y$보다 약간 큰데 그 이유는, 항복이 생기는 단면에서의 shear stress가 일정하다는 가정이 옳지 않기 때문이다.

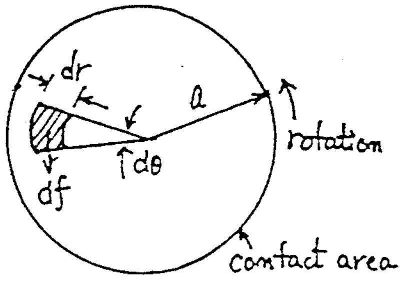
6.28

contact area $= \pi a^2$

pressure $= 30$ psi

Normal load $= 1000(lb) = 30\pi a^2$

$$\therefore a = 3.25''$$



contact area의 면적 요소에 생기는 friction force $df = 0.6pdA = 0.6p\times rd\theta\cdot dr$

$$\text{Torque} = df\times r = 0.6pr^2 d\theta dr$$

$$\text{Total torque} = \iint_A 0.6pr^2 d\theta dr = \int_{r=0}^{a}\int_{\theta=0}^{2\pi} 0.6pr^2 d\theta dr = 0.6p\cdot 2\pi\cdot\frac{a^3}{3}$$

$$= \frac{0.6\times30\times2\pi\times(3.25)^3}{3} = 1300\ (lb\text{-}in)$$

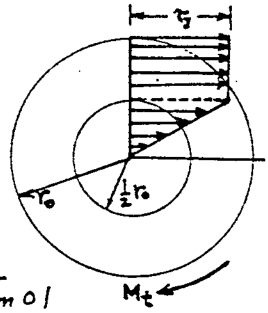
$\therefore$ 두개의 tire에 의한 torque $= 2600$ lb-in

Torque in steering column (due to 20:1 reduction gear)

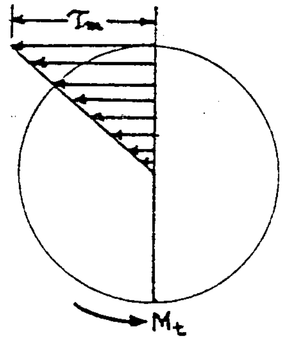
$$= \frac{2600}{20} = 130\ (lb\text{-}in),\quad \tau_{max} = \frac{T\cdot r}{I_P} = \frac{2T}{\pi r^3} = \frac{2\times130}{\pi\times(\frac{3}{8})^3} = 1570\ (psi)$$

$$\underline{\text{Ans. } \tau_{max} = 1570\ psi}$$

6.29 중첩이론을 적용하자.
하중에 의해서 생기는 stress의 분포는 우측의 그림과 같다. 여기에 linear reserved stress를 중첩시킨다.
이때의 torque는 $-M_t$ 이다.



maximum 중첩응력(superposed stress) τ_m이 $2\tau_y$ 보다 작다면, linearity를 가정하는 것이 가능하다. 왜냐하면 최대 잔류응력(max residual stress) $\tau_r = \tau_m - \tau_y \le \tau_y$
중첩된 응력은 우편과 같다.



최초의 torque ; $\phi_i = 2\phi_y$

$$M_t = \frac{4}{3}T_y\left[1 - \frac{1}{4}\left(\frac{1}{3}\right)^2\right] = 1.29T_y$$

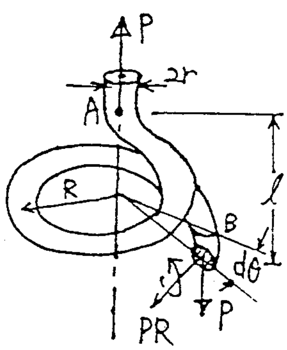
elastic moment T_y는 비틀각 ϕ_y를 발생시킨다.
중첩된 elastic moment는 $1.29\,T_y$
따라서 중첩된 springback의 변형각은 $1.29\,\phi_y$

$$\therefore \phi_2 = \phi_i - 1.29\,\phi_y = 0.71\,\phi_y = 0.355\,\phi_i$$

[note ; $\tau_m < 2\tau_y$, 왜냐하면 Torque $= 1.29\,T_y < 2T_y$]

Ans. $\phi_{residual} = 0.355\,\phi_i$

6.30 spring의 한 점을 wire에 수직하게 자르고 pitch가 작다고 가정하면 equilibrium condition.



$$\Sigma M = 0 \quad \therefore M_t = PR \;,\; \Sigma F = 0 \;\therefore P = V$$

$$L = R\,d\theta$$

$$\phi = \int \frac{M_t \cdot L}{GJ} = \int_0^{2n\pi} \frac{PR \cdot R\,d\theta}{GJ} = \frac{2n\pi \cdot PR^2}{GJ}$$

$$\delta = R\phi = \frac{2n\pi\, PR^3}{G\left(\frac{\pi}{2}\right)r^4} = \frac{4nPR^3}{Gr^4}$$

6.31 Estimate ; Twisting Torque = 5 N-m Compressive Force = 15N

Compressive stress $= \frac{F}{A} = \frac{15}{\frac{\pi}{4}\cdot(0.006)^2} = 530516$ (N/m^2)

Shear stress $= \frac{M_t \cdot \frac{d}{2}}{J} = \frac{16 M_t}{\pi d^3} = \frac{16 \times 5}{\pi \times (0.006)^3} = 117892550$ (N-m)

$\frac{\text{Compressive stress}}{\text{Shear stress}} = 4.5 \times 10^{-3}$

Ans. 4.5×10^{-3} (σ_c/τ)

6.32 (a) loading을 완전한 elastic 이라고 가정하면, 1100-0 과 2024-T4는 같은 elastic modulus를 갖는다.

$$\tau = \frac{M_t \cdot r}{I_z} = \frac{120 \times 0.0067}{\frac{\pi}{32}(11.3^4 - 6.7^4) \times (10^{-3})^4} = 5.731 \times 10^8 \ N/m^2$$

$$\tau_{max} = \frac{\sigma_y}{2} = 17.5 \times 10^6 \ N/m^2$$

$\tau > \tau_{max}$ 따라서 elastic loading은 아니다.

(b) cladding은 plastic, core는 elastic 이라 가정하면

$$\text{moment due to cladding} = M_c = \sum \frac{J}{r} \tau_y$$

$$= \sum 2\pi r^2 t \tau_y = 2\pi\{(6.7+0.4)(10^{-3})\}^2(0.8 \times 10^{-3})(17.5 \times 10^6)$$
$$+ 2\pi\{(11.3-0.4)(10^{-3})\}^2(0.8 \times 10^{-3})(17.5 \times 10^6) = 14.885 \ (N\text{-}m)$$

applied moment $= 120$

$\therefore$ moment due to core $= M_A = 120 - 14.885 = 105.115$ (N-m)

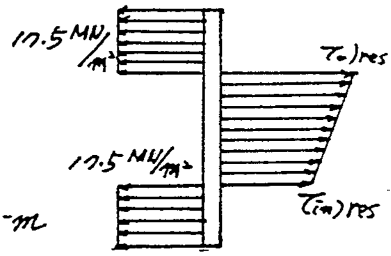
$$J_{core} = \frac{\pi}{2}[(10.5)^4 - (7.5)^4](10^{-3})^4 = 1.4123 \times 10^{-8} \ (m^4)$$

$$\therefore \tau = \frac{M_A \cdot r}{J_{co}} = \frac{105.115 \times r}{1.4123 \times 10^{-8}} = 7.443 \times 10^9 r$$

$\therefore (\tau_i)_{core} = 55.821 \ MN/m^2$ $(\tau_o)_{core} = 78.149 \ MN/m^2$

(c) 잔류응력을 구하기 위해, 역 방향으로 120 N-m 의 moment에 기인한 stress 와 앞의 stress 와 중첩해야 한다.

앞의 계산으로 부터 역 방향의 moment를 가할 때, cladding에서는 역방향의 yielding stress에 도달한다. cladding에 의한 moment는 $-14.885\ N\text{-}m$



따라서 core 의 moment $= 14.885\ N\text{-}m$

$$\therefore \tau = \frac{M_A \cdot r}{J_{co}} = \frac{14.885 \times r}{1.4123 \times 10^{-8}} = 1.054 \times 10^{9}\, r$$

$$\therefore (\tau_i)_{residual} = 7.905\ MN/m^2,\quad (\tau_o)_{residual} = 11.066\ MN/m^2$$

Ans. pure aluminum ; $\tau_{residual} = -14.885\ N\text{-}m$

aluminum alloy ; $(\tau_i)_{res.} = 7.905\ MN/m^2$

$(\tau_o)_{res.} = 11.066\ MN/m^2$

6.33 hole 의 깊이 $= L\ (ft)$

물의 부력과 무게는 무시한다.

$$\text{weight of metal supported} \simeq 12L \times \frac{\pi}{4}(3^2-2^2)\cdot\rho \quad \left(\rho = \frac{lb}{in^3}\right)$$

$$= 12L \times \frac{\pi}{4} \times 5 \times 0.28 = 13.2L\ lb$$

단면 전체에 걸쳐 uniform tensile stress를 가정하면

$$\text{tensile stress} \quad \sigma_z = \frac{13.2L}{\frac{\pi}{4}(3^2-2^2)} = 3.36L\ psi$$

$$\text{shear stress} \quad \tau_{\theta z} = \frac{M_t \cdot r_o}{I_z} = \frac{2000 \times 12 \times \frac{3}{2}}{\frac{\pi}{32}(3^4-2^4)} = 5650\ psi$$

Mohr's circle에 依해서

$$\tau_{max} = \sqrt{\left(\frac{\sigma_z}{2}\right)^2 + \tau_{\theta z}^2} = \sqrt{5650^2 + \left(\frac{3.36L}{2}\right)^2} = \frac{50000}{2}$$

$$\left(\because \tau_{max} = \frac{Y}{2}\right)$$

$$\therefore L = 14500\ ft$$

Ans. $L = 14500\ ft$

6.34 spring constant $= 125\ \frac{lb}{in}$

$$\text{Torque} = P\cdot x \qquad \phi = \frac{(Px)L}{GI} = \frac{Px \cdot L}{G \cdot \frac{\pi}{32} d^4} \quad (d;\ \text{wire diameter})$$

load에 의한 deflection $= \phi x$

spring constant $k = \frac{P}{\phi \cdot x} = \frac{G\pi d^4}{32x^2 L} = \frac{12\times10^6\pi}{32}\frac{d^4}{x^2 L}$

$$\frac{d^4}{x^2 L} = \frac{32\times125}{12\times10^6\pi} = \frac{1}{3000\pi} \quad \text{—①}$$

static load $1000\ lb$

dynamic load $\pm 6''\times125\ \frac{lb}{in} = \pm750\ lb$

$\therefore$ maximum load $= 1000+750 = 1750\ lb$

$(M_t)_{max} = 1750\ lb\text{-}in$

$$\tau = \frac{(1750x)\frac{d}{2}}{\frac{\pi}{32}d^4} = 8900\frac{x}{d^3}$$

따라서 $\frac{x}{d^3}\cdot8900 \le 50.000$ or $\frac{x}{d^3} \le 5.62$ —②

$x_{max} = 30''$ $\quad L_{max} = 120''$

①식으로 부터 $d^4 = \frac{900\times120}{3000\pi} = 11.4 \quad \therefore d = 1.84''$

②식에 대입하면 $\frac{x}{d^3} = \frac{30}{612} = 4.85 < 5.62$ ②식을 만족한다.

$$\tau = \frac{4.85}{5.62}\times50.000 = 43.000\ psi$$

①식은 다음과 같이 쓸 수 있다. $Ld^2 = \frac{3000\pi}{x^2/d^6}$

②식에 대입하면 $Ld^2 \geqslant \frac{3000\pi}{(5.62)^2}$

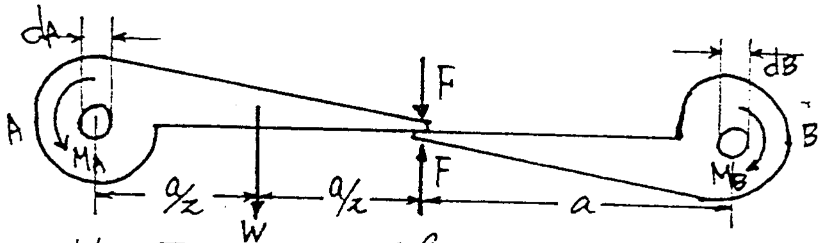
Ld^2은 부피에 비례하고, cost를 최소되게 design 해야 한다.

$Ld^2 = \frac{3000\pi}{(5.62)^2}$ $\left(\begin{array}{l} L=120'' \text{ 이면 } d=1.57'' \quad x=21.4'' \\ x=30'' \text{ 이면 } d=1.75'' \quad L=98.0'' \end{array} \right.$

두 경우 모두 $\tau = 50000\ psi$ 이다.

Ans. $L=120''$; $d=1.57''$, $x=30''$; $d=1.75''$

6.35 $\quad \phi_A = \frac{M_A\cdot L}{G J_A} \quad \phi_B = \frac{M_B\cdot L}{G J_B} \quad$ —①



그림에서 $M_B = Fa$ $\quad M_A = W \cdot \frac{a}{2} - Fa$

기하학적 적합조건에서 $\phi_A = \phi_B$, $\quad \frac{M_A}{M_B} = \left(\frac{d_A}{d_B}\right)^4 = \frac{W}{2F} - 1$

$$\therefore F = \frac{W}{2\left[1 + \left(\frac{d_A}{d_B}\right)^4\right]}$$

① 식에 대입하면

$$\phi_A = \phi_B = \frac{1}{G \cdot \frac{\pi}{32} \cdot d_B^4} \cdot \frac{W \cdot a \cdot L}{\left[1 + \left(\frac{d_A}{d_B}\right)^4\right]} = \frac{16 W \cdot a L}{\pi G (d_A^4 + d_B^4)}$$

Ans. $F = W / 2\left[1 + \left(\frac{d_A}{d_B}\right)^4\right]$, $\phi_A = \phi_B = 16 W \cdot a \cdot L / \pi G (d_A^4 + d_B^4)$

6.36 $\varepsilon_r = \frac{\partial u}{\partial r} = 0$, $\quad \varepsilon_\theta = \frac{\partial v}{r \partial \theta} + \frac{u}{r} = 0$, $\quad \varepsilon_z = \frac{\partial w}{\partial z} = 0$.

$$\gamma_{\theta r} = \frac{\partial v}{\partial r} + \frac{1}{r}\frac{\partial r}{\partial \theta} - \frac{v}{r} = \phi - \phi = 0, \quad \gamma_{zr} = \frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} = 0$$

$$\gamma_{\theta z} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \theta} = r\frac{d\phi}{dz}$$

$\therefore u = 0 \quad v = r\phi \quad w = 0$ 일 때 $\varepsilon_r = \varepsilon_\theta = \varepsilon_z = \gamma_{r\theta} = \gamma_{zr} = 0$

$\gamma_{\theta z} = r\frac{d\phi}{dz}$ 를 만족한다.

i) Hooke's law 에 대입.

$$\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_\theta + \sigma_z)] = 0, \quad \gamma_{r\theta} = \frac{1}{G}\tau_{r\theta} = 0$$

$$\varepsilon_\theta = \frac{1}{E}[\sigma_\theta - \nu(\sigma_r + \sigma_z)] = 0, \quad \gamma_{rz} = \frac{1}{G}\tau_{rz} = 0$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_r + \sigma_\theta)] = 0, \quad \gamma_{\theta z} = \frac{1}{G}\tau_{\theta z} = r\frac{d\phi}{dz}$$

$$\therefore \tau_{\theta z} = Gr\frac{d\phi}{dz}$$

ii) 평형 조건식에 대입.

$$\frac{\partial \sigma_r}{\partial r} + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = 0$$

$$\frac{\partial \tau_\theta}{\partial r} + \frac{1}{r}\frac{\partial \sigma_\theta}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{\partial \tau_{r\theta}}{r} = \frac{\partial \tau_{\theta z}}{\partial z} = Gr\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}\right)$$

$$= Gr\frac{\partial}{\partial z}\left(\frac{M_t}{GI_z}\right) = 0$$

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} = G\frac{\partial}{\partial \theta}\left(\frac{d\phi}{dz}\right)$$

$$= G\frac{\partial}{\partial \theta}\left(\frac{M_t}{GI_z}\right) = 0$$

따라서 $\sigma_r = \sigma_\theta = \sigma_z = \tau_{r\theta} = \tau_{rz} = 0$, $\tau_{\theta z} = \frac{T \cdot r}{J}$ 의 해는 elastic theory 의 범위 내에서 exact solution 이 된다.

6.37 대칭 조건 ; $\gamma_{r\theta} = \gamma_{rz} = 0$. $\frac{\partial}{\partial \theta} = 0$. $\gamma_{\theta z} = r\frac{d\phi}{dz}$

$\varepsilon_r = \varepsilon_\theta = 0$ 으로 가정하자.

i) Hooke's law 에서 다음 식을 얻을 수 있다.

$$\sigma_r = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + \frac{E}{1+\nu}\varepsilon_r$$

$$\sigma_\theta = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + \frac{E}{1+\nu}\varepsilon_\theta$$

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_r + \varepsilon_\theta + \varepsilon_z) + \frac{E}{1+\nu}\varepsilon_z$$

$$\therefore \sigma_r = \sigma_\theta = \frac{\nu E \varepsilon_z}{(1+\nu)(1-2\nu)}$$

$$\sigma_z = \left[\frac{\nu E}{(1+\nu)(1-2\nu)} + \frac{E}{1+\nu}\right]\varepsilon_z = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\varepsilon_z$$

$$\tau_{r\theta} = \tau_{rz} = 0 \quad , \quad \tau_{\theta z} = Gr\frac{d\phi}{dz}$$

ii) 평형 조건

$$\frac{\partial \sigma_r}{\partial r} + \frac{\partial \tau_{r\theta}}{r\partial \theta} + \frac{\partial \tau_{zr}}{\partial z} + \frac{\sigma_r - \sigma_\theta}{r} = \frac{\nu E}{(1+\nu)(1-2\nu)} \cdot \frac{\partial \varepsilon_z}{\partial r} = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{\partial \sigma_\theta}{r\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z} + \frac{2\tau_{r\theta}}{r} = \frac{\nu E}{(1+\nu)(1-2\nu)} \cdot \frac{1}{r}\frac{\partial \varepsilon_z}{\partial \theta} = 0$$

$$\frac{\partial \tau_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta} + \frac{\partial \sigma_z}{\partial z} + \frac{\tau_{zr}}{r} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \frac{\partial \varepsilon_z}{\partial z} = 0$$

$$\frac{\partial \varepsilon_z}{\partial r} = \frac{1}{r}\frac{\partial \varepsilon_z}{\partial \theta} = \frac{\partial \varepsilon_z}{\partial z} = 0 \qquad \therefore \varepsilon_z = \text{constant}$$

$$\therefore \sigma_z = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)} \varepsilon_z = \text{constant.}$$

surface loading 이 없으므로. $\sigma_z = 0$. $\therefore \varepsilon_z = 0$.

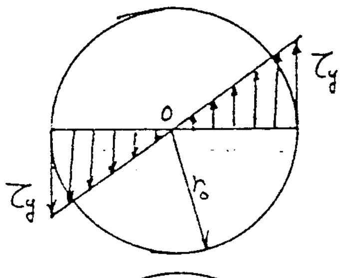
6.38 $\tau_i = \dfrac{T_i \cdot r}{J} = \dfrac{T_i \left(\frac{1.9}{2} \times 10^{-3}\right)}{\frac{\pi}{32}(1.9\times10^{-3})^4}$

$$\bar{\tau} = \frac{1}{4}\sum_i \tau_i = \frac{1}{4}\left[\frac{(0.95\times10^{-3})}{\frac{\pi}{32}(1.9\times10^{-3})^4}\right] \times (0.1 + 0.125 + 0.1 + 0.11)$$

$$= 80749264 \ N/m^2 = 80.75 \ MN/m^2$$

Ans. $\bar{\tau} = 80.75 \ MN/m^2$

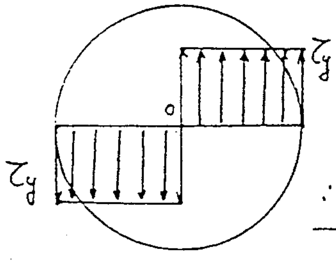
6.39 Efficiency of spring $= U_y / U_L$



$$M_t = \int_0^r \left(\frac{\tau_y}{r_0} \cdot r\right) r (2\pi r dr)$$

$$= \frac{1}{2}\pi r_0^3 \tau_y$$

$$\therefore U_y = \frac{M_t^2 L}{2GJ} = \left(\frac{\pi}{2} r_0^3 \tau_y\right)^2 \frac{L}{2GJ}$$



$$T_L = \int_0^{r_0} \tau_y \cdot r(2\pi r) dr = \frac{2}{3}\pi r_0^3 \tau_y$$

$$\therefore U_L = \frac{T_L^2 L}{2GJ} = \left(\frac{2}{3}\pi r_0^3 \tau_y\right)^2 \frac{L}{2GJ}$$

$$\therefore \text{Efficiency} = U_y / U_L = 9/16 = 56.25\%$$

6.40 $\gamma = k\tau^2 \qquad \gamma_{\theta z} = r\dfrac{d\phi}{dz} \qquad \therefore k\tau^2 = r\dfrac{d\phi}{dz}$ (from Eq.(6.3))

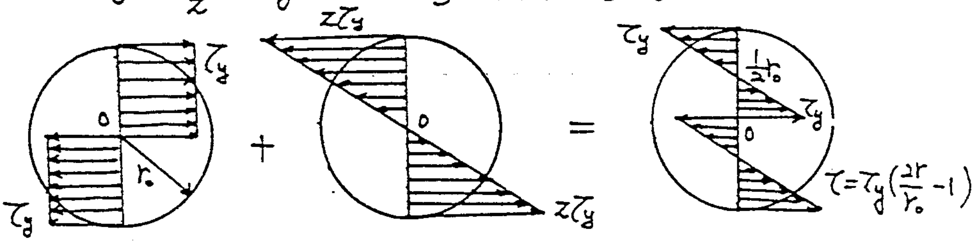
$$\therefore M = \int_A r\tau\, dA = \int r\cdot 2\pi r\left(\frac{r}{k}\right)^{\frac{1}{2}}\left(\frac{d\phi}{dz}\right)^{\frac{1}{2}} dr$$

$$= \int_0^{r_o}\left(\frac{1}{k}\cdot\frac{d\phi}{dz}\right)^{\frac{1}{2}}\left(2\pi r^{\frac{5}{2}}\right) dr = 2\pi\cdot\left(\frac{1}{k}\frac{d\phi}{dz}\right)^{\frac{1}{2}}\cdot\left(\frac{2}{7}r_o^{\frac{7}{2}}\right)$$

$$\therefore \frac{d\phi}{dz} = \frac{49kM_t^2}{16\pi^2 r^7}, \quad \tau_{max} = \left(\frac{r_o}{k}\cdot\frac{d\phi}{dz}\right)^{\frac{1}{2}} = \frac{7M_t}{4\pi r_o^3}$$

Ans. $\dfrac{d\phi}{dz} = \dfrac{49kM_t^2}{16\pi^2 r^7}, \quad \tau_{max} = \dfrac{7M_t}{4\pi r_o^3}$

6.41 $T_y = \dfrac{\pi}{2}r_o^3\tau_y, \quad T_L = \dfrac{2}{3}\pi r_o^3\tau_y = \dfrac{4}{3}T_y$



Elastic theory에서 어느 한쪽 방향으로 완전 소성으로 pretwisting 된 상태에서 구속을 풀면, 그리고 반대 방향으로 moment를 증가시켜 가면 반대 방향의 항복 응력에 도달할 때 까지는 선형적으로 변한다. 그러나 일단 반대 방향의 항복 응력에 도달하게 되면, plastic하게 되어 선형적인 변화를 하지 않는다. 선형을 벗어날 때의 moment를 구해보면, 그 때의 shear stress는 $\tau = \left(\frac{2r}{r_o}-1\right)\tau_y$ 이다.

$$\therefore (M_t)'_y = \int_0^r\left(\frac{2r}{r_o}-1\right)\tau_y\cdot r(2\pi r)dr = \left[2\pi\tau_y\left(\frac{r_o^3}{2}-\frac{r_o^3}{3}\right)\right]$$

$$= \frac{1}{3}\pi r_o^3\tau_y = \frac{2}{3}T_y$$

따라서 M_t가 $\frac{2}{3}T_y$ 의 값보다 크게될 때부터 선형이 아니다.

— Q.E.D. —

즉, 반대 방향으로 moment를 증가시켜 갈때는, $M_t = T_y \dfrac{\phi}{\phi_y}$ 의 관계를 유지하며 선형적으로 변한다. Ans. $M_t = T_y(\phi/\phi_y)$

6.42 Fig (6.28) (6.29)에서

$$U = \frac{1}{2G}\iiint \tau_{sz}^2 \, dn\, ds\, dz \qquad M_t = 2\tau_{sz} \cdot t \cdot A \text{ 를 대입.}$$

$$U = \frac{1}{2G}\iiint \frac{M_t^2}{4t^2A^2} \, dn\, ds\, dz$$ 여기에 Castigliano's theorem 적용

$$\therefore \phi = \frac{\partial U}{\partial M_t} = \frac{1}{2G}\iiint \frac{M_t}{2t^2A^2} \, dn\, ds\, dz$$

$$= \frac{M_t}{4A^2G}\iiint \frac{1}{t^2} \, dn\, ds\, dz = \frac{M_t}{4A^2G}\iint \left[\frac{n}{t^2}\right]_{-\frac{t}{2}}^{\frac{t}{2}} ds\, dz$$

$$= \frac{M_t}{4A^2G}\iint \frac{1}{t} \, dz\, ds = \frac{M_t \cdot L}{4A^2G}\int \frac{ds}{t}$$ — Q.E.D. —

6.43 (a) $$\phi_1 = \frac{M_t \cdot L_i}{GJ_1} = \frac{M_t \cdot L_i}{G \cdot \frac{\pi}{32}(2R_i)^4} = \frac{2M_t L_i}{\pi R_i^4 G}$$

$$\phi_2 = \frac{M_t L_o}{GJ_2} = \frac{M_t L_o}{G \cdot \frac{\pi}{2}(R_o^4 - R_i^4)} = \frac{2M_t L_o}{\pi(R_o^4 - R_i^4)G}$$

중첩의 원리; $$\phi = \phi_1 + \phi_2 = \frac{2M_t}{\pi G}\left(\frac{L_i}{R_i^4} + \frac{L_o}{R_o^4 - R_i^4}\right)$$

$$\therefore k_t = \frac{M_t}{\phi} = \frac{\pi G}{2}\left[\frac{1}{L_i/R_i^4 + L_o/(R_o^4 - R_i^4)}\right]$$

(b) $$\tau_i = \frac{M_t \cdot r_i}{J_i} = \frac{M_t \cdot R_i}{\frac{\pi}{32}(2R_i)^4} = \frac{2M_t}{\pi R_i^3}$$

$$\tau_o = \frac{M_t \cdot R_o}{\frac{\pi}{2}(R_o^4 - R_i^4)} = \frac{2M_t R_o}{\pi(R_o^4 - R_i^4)}$$

$$\tau_i = \tau_o; \quad 1 = (1/R_i^3)/(R_o/(R_o^4 - R_i^4))$$

$$\therefore \left(\frac{R_o}{R_i}\right)^4 - 1 = \left(\frac{R_o}{R_i}\right)$$

trial error method에 의해서 $\frac{R_o}{R_i}$의 값을 구하면

$$\frac{R_o}{R_i} \simeq 1.22$$

Ans. (a) $k_t = \dfrac{\pi G}{2[L_i/R_i^4 + L_o/(R_o^4 - R_i^4)]}$,

(b) $\left(\frac{R_o}{R_i}\right)^4 - 1 = \left(\frac{R_o}{R_i}\right)$, $\frac{R_o}{R_i} = 1.22$

6.44 $M_A = \overline{W}\cdot b + M_B - M_D$

$$\phi_{AB} = \frac{M_A \cdot a}{GJ} = \frac{a}{GJ}(\overline{W}\cdot b + M_B - M_D),\ \phi_{BC} = \frac{a}{GJ}(\overline{W}\cdot b - M_D)$$

중첩의 원리; $\phi_{AC} = \phi_{AB} + \phi_{BC} = \frac{a}{GJ}(2\overline{W}\cdot b + M_B - 2M_D) = 0$

따라서 $2M_D - M_B = 2\overline{W}\cdot b$ —①

$$\phi_{BD} = \frac{2a}{GJ}(\overline{W}\cdot b - M_D)\ ..\quad \phi_{DE} = \frac{2a}{GJ}\overline{W}\cdot b$$

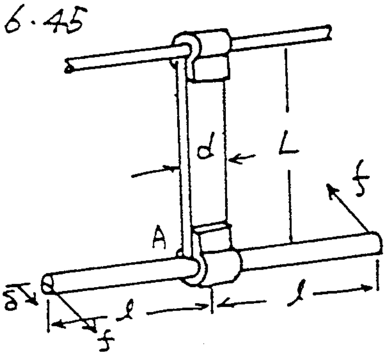
중첩의 원리 ; $\phi_{AE} = \phi_{AB} + \phi_{BD} + \phi_{DE} = \frac{a}{GJ}(5\overline{W}\cdot b + M_B - 3M_D) = 0$

따라서 $3M_D - M_B = 5\overline{W}\cdot b$ —②

①② 에서 $M_D = 3\overline{W}\cdot b$, $M_B = 4\overline{W}\cdot b$

Ans. $M_B = 4\overline{W}\cdot b$ $M_D = 3\overline{W}\cdot b$

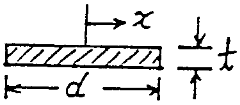
6.45



Al 봉은 bending이 일어나지 않는다고 가정한다.

$f = G_o \frac{Mm}{r^2}$, $M_t = G_o \frac{Mm}{r^2}\cdot 2l$ —①

where G_o : 두 질점간의 만유 인력
M : Bucket 의 무게
m ; Bucket의 갯수



Mylar tape의 A부분을 보면, tape의 양측면에서 전단 변형이 max. 이 된다.
($\because \tau = \frac{VQ}{bI}$ 에서 $Q_{max.}$ 이 되기 때문)

$$\tau = G\frac{x\phi}{L} \quad \therefore M_t = 2\int_0^{\frac{d}{2}} \tau \cdot x\,dx \cdot t = 2\int_0^{\frac{d}{2}} G\cdot\frac{x\phi}{L}\cdot x\cdot t\,dx$$

$$= \frac{2G\phi t}{L}\int_0^{\frac{d}{2}} x^2\,dx = \frac{G\phi d^3 t}{12L}$$

따라서 $\phi = \frac{12\cdot M_t\cdot L}{G d^3 t}$

$$\delta = \phi\cdot l = \frac{12M_t\cdot L\cdot l}{Gd^3t} = \frac{24G_0 M m L l^2}{Gd^3t\cdot r^2}$$

($\because$ ①에서 $M_t = G_0\frac{M\cdot m}{r^2}$)

$$= \frac{24\times(6.67\times10^{-11})(100)(5)(10)(1)^2}{\frac{1.4\times10^9}{2(1+0.4)}\times(25.4\times10^{-3})^3(0.254\times10^{-3})(0.4)^2} = 2.4\times10^{-5}(m)$$

($\because G = \frac{E}{2(1+\nu)}$)

Ans. $\delta = 2.4\times10^{-5}\ m$

CHAPTER 7

7.1 $\bar{y} = \dfrac{\int y\,dA}{\int dA} = \dfrac{\int_1 y\,dA + \int_2 y\,dA + \int_3 y\,dA}{\int_1 dA + \int_2 dA + \int_3 dA}$, $\int_1 y\,dA = \bar{y}_1 A_1 \cdots\cdots$

$\therefore \bar{y} = \dfrac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3} = \dfrac{\sum_i \bar{y}_i A_i}{\sum_i A_i}$ 같은 요령으로. $\bar{z} = \dfrac{\sum_i \bar{z}_i A_i}{\sum_i A_i}$

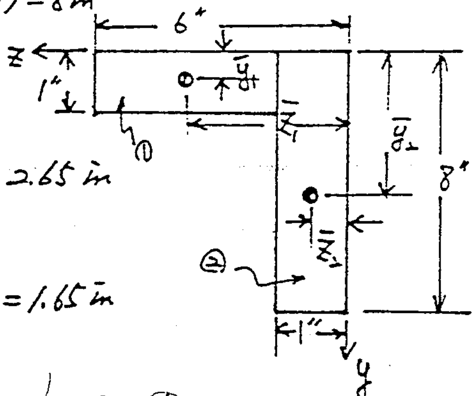
7.2 $A_1 = (6-1)(1) = 5\,in^2$, $A_2 = (8)(1) = 8\,in^2$

$\bar{y}_1 = 0.5\,in$. $\bar{z}_1 = 3.5\,in$

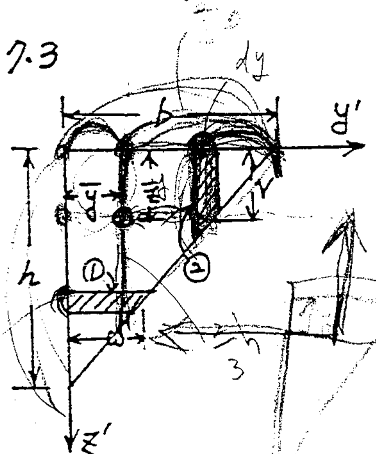
$\bar{y}_2 = 4\,in$, $\bar{z}_2 = 0.5\,in$

$\bar{y} = \dfrac{\sum_i \bar{y}_i A_i}{\sum_i A_i} = \dfrac{(0.5)(5)+(4)(8)}{13} = 2.65\,in$

$\bar{z} = \dfrac{\sum_i \bar{z}_i A_i}{\sum_i A_i} = \dfrac{(3.5)(5)+(0.5)(8)}{13} = 1.65\,in$



7.3



For the area ①

$I_{yy} = \int y^2 dA = \int_{-2h/3}^{h/3} y^2\left(\frac{2b}{3} + \frac{b}{h}y\right)dy = \frac{bh^3}{36}$

For the area ②

$I_{zz} = \int z^2 dA = \int_{-2b/3}^{b/3} z^2\left(\frac{2h}{3} + \frac{h}{b}z\right)dz = \frac{b^3 h}{36}$

For the area ③ $dA = dy \cdot dz$

$I_{yz} = \int yz\,dA = \int dy \int dz\,(yz)$

$= \int_{-\frac{2}{3}h}^{\frac{h}{3}} dy \int_{-\left(\frac{b}{3}+\frac{b}{h}y\right)}^{\frac{b}{3}} dz\,(yz) = \int_{-\frac{2}{3}h}^{\frac{h}{3}} y \cdot \frac{1}{2}\left[\left(\frac{b}{3}\right)^3 - \left(\frac{b}{3}+\frac{b}{h}y\right)^2\right]dy$

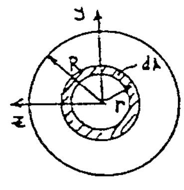
$= \int_{-\frac{2}{3}h}^{\frac{h}{3}} \left(-\frac{1}{2}\right)\left(\frac{b}{h}y^2\right)\left(\frac{2}{3}b + \frac{b}{h}y\right)dy = -\frac{1}{2}\left(\frac{b}{h}\right)\left[\frac{2}{9}by^3 + \frac{b}{h}\cdot\frac{y^4}{4}\right]_{-\frac{2}{3}h}^{\frac{h}{3}}$

$= -\dfrac{b^2h^2}{72}$ — Q.E.D —

7.4 $I_x = \int r^2 dA = \int (y^2+z^2)\, dA = \int y^2 dA + \int z^2 dA = I_{yy} + I_{zz}$

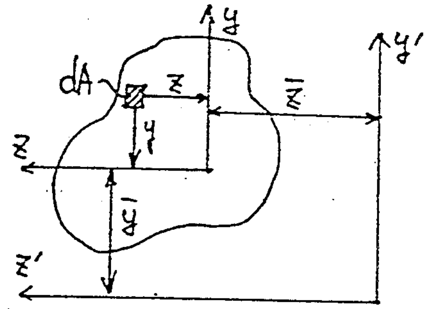
symmetry ; $I_{yy} = I_{zz}$

$\therefore I_{yy} = I_{zz} = \frac{1}{2} I_x$



$I_x = \int r^2 dA = \int_0^R r^2 (2\pi r dr) = \frac{\pi}{2} R^4$

$\therefore I_{yy} = I_{zz} = \frac{\pi}{4} R^4$ — Q.E.D —



7.5 $I_{y'y'} = \int y'^2 dA = \int (y+\bar{y})^2 dA$

$= \int y^2 dA + \int 2y\bar{y}\, dA + \int \bar{y}^2 dA$

$= I_{yy} + 2\bar{y} \int y\, dA + \bar{y}^2 A$

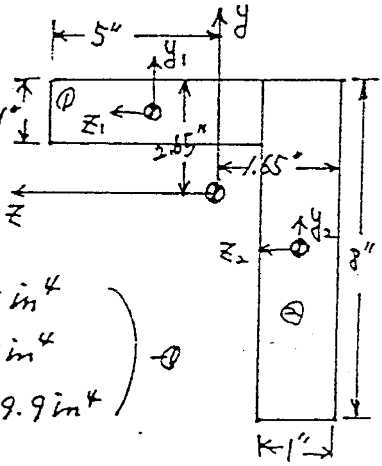
$\int y\, dA = 0$ ($\because$ y-z축이 centroid를 통하기 때문)

$\therefore I_{y'y'} = I_{yy} + \bar{z}^2 A$ 같은 방법으로 $I_{z'z'} = I_{zz} + \bar{y}^2 A$

$I_{y'z'} = \int y'z'\, dA = \int (y+\bar{y})(z+\bar{z})\, dA = \int yz\, dA + \int \bar{y} z\, dA + \int \bar{y}\bar{z}\, dA + \int y\bar{z}\, dA$

$\int y\, dA = \int z\, dA = 0$

$\therefore I_{y'z'} = I_{yz} + \bar{y}\bar{z} A$



7.6 Area ① $I_{y_1y_1} = \frac{1}{12}(5)(1)^3 = 0.417\ in^4$

$I_{z_1z_1} = \frac{1}{12}(1)(5)^3 = 10.417\ in^4$

$I_{y_1z_1} = 0. \quad A = 5\ in^2$

$\therefore (I_{yy})_1 = 0.417 + (2.15)^2(5) = 23.5\ in^4$

$(I_{zz})_1 = 10.417 + (1.85)^2(5) = 27.5\ in^4$ — ①

$(I_{yz})_1 = 0 + (2.15)(1.85)(5) = 19.9\ in^4$

Area ② 같은 방법,

$(I_{yy})_2 = 57.3\ in^4, (I_{zz})_2 = 11.3\ in^4 \quad (I_{yz})_2 = 12.4\ in^4$ — ②

① + ② $I_{yy} = 80.8\ in^4 \quad I_{zz} = 38.8\ in^4 \quad I_{yz} = 32.3\ in^4$

Ans. $I_{yy} = 80.8\ in^4 \quad I_{zz} = 38.8\ in^4 \quad I_{yz} = 32.3\ in^4$

7.7 $\bar{y} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2}{A} = \frac{-(0)(at) - (a)(2at)}{3at} = -\frac{2}{3}a$

$\bar{z} = \frac{-(\frac{a}{2})(at) - (0)(2at)}{3at} = -\frac{a}{6}$

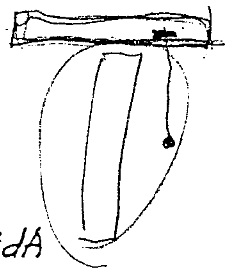
평행축 정리 이용.

$I_{yy} = [\frac{1}{12}at^3 + (\frac{4a^2}{9})at] + [\frac{1}{12}8a^3t + (\frac{a^2}{9})2at] = \frac{4}{3}a^3t$

$I_{zz} = [\frac{1}{12}ta^3 + (\frac{a^2}{9})at] + [\frac{1}{12}t^3 2a + (\frac{a^2}{9})2at] = \frac{1}{4}a^3t$

$I_{yz} = [0 + (\frac{2a}{3})(-\frac{a}{3})at] + [0 + (-\frac{a}{3})(\frac{a}{6})2at] = -\frac{1}{3}a^3t$

7.8 $I_{zz} = \frac{1}{12}(5.75)(200 - 2\times 7.75)^3 + 2[(130)(7.75)(100 - \frac{7.75}{2})^2 + \frac{1}{12}(130)(7.75)^3] = 21638087.83 = 2.164\times10^7 (mm^4)$



Ans. $I_{zz} = 2.164\times10^7$ mm

7.9 $n = z\cos\theta - y\sin\theta$

$m = z\sin\theta + y\cos\theta$

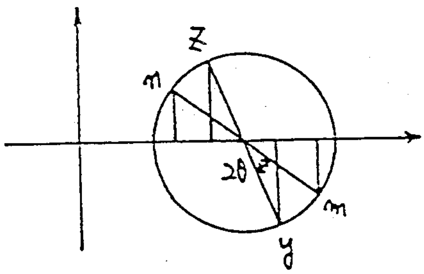
$I_{mm} = \int m^2 dA = \int (z\sin\theta + y\cos\theta)^2 dA$

$= \int z^2\sin^2\theta\, dA + \int y^2\cos^2\theta\, dA + \int 2yz\sin\theta\cos\theta\, dA = I_{zz}\sin^2\theta + I_{yy}\cos^2\theta + 2I_{yz}\sin\theta\cos\theta$

I_{nn}, I_{nm} 도 같은 방법으로 얻어진다.

$\cos^2\theta = \frac{1}{2}(1+\cos2\theta)$, $\sin^2\theta = \frac{1}{2}(1-\cos2\theta)$, $2\sin\theta\cos\theta = \sin2\theta$

를 이용하면 (4.23)(4.24) 식이 얻어진다.



$I_{mn} = 0$ 에서 $\tan2\theta = \frac{-2I_{yz}}{I_{yy} - I_{zz}}$

$I_{1,2} = \frac{I_{yy}+I_{zz}}{2} \pm \sqrt{\left(\frac{I_{yy}-I_{yz}}{2}\right)^2 + I_{yz}^2}$

– Q.E.D –

7.10 $I_{yy} = 80.8\ in^4$, $I_{zz} = 38.8\ in^4$, $I_{yz} = 32.3\ in^4$

$\tan2\theta = \frac{32.3}{\frac{1}{2}(80.8 - 38.8)} = 1.54$ $\therefore \theta = 28.5°$

$$I_{11} = \frac{1}{2}(38.8+80.8) + \sqrt{\left(\frac{38.8-80.8}{2}\right)^2 + (32.3)^2} = 98.33\ (in^4)$$

$$I_{22} = \frac{1}{2}(38.8+80.8) - \sqrt{\left(\frac{38.8-80.8}{2}\right)^2 + (32.3)^2} = 21.27\ (in^4)$$

7.11 문제 7.3에서 $I_{yy} = bh^3/36$, $I_{zz} = b^3h/36$, $I_{yz} = -b^2h^2/72$

$$\theta = \frac{1}{2}\tan^{-1}\left(\frac{b^2h^2/72}{\frac{1}{2}\left[\frac{bh^3}{36} - \frac{b^3h}{36}\right]}\right) = \frac{1}{2}\tan^{-1}\left(\frac{bh}{h^2-b^2}\right)$$

$$I_{11} = \frac{1}{2}\left(\frac{bh^3}{36} + \frac{b^3h}{36}\right) + \sqrt{\frac{1}{4}\left(\frac{bh^3}{36} - \frac{b^3h}{36}\right)^2 + \left(\frac{b^2h^2}{72}\right)^2}$$

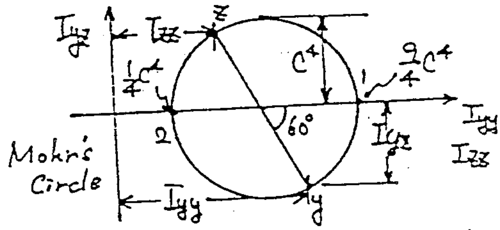
$$= \frac{bh}{72}\left[(h^2+b^2) + \sqrt{(h^2-b^2)^2 + h^2b^2}\right]$$

$$I_{22} = \frac{bh}{72}\left[(h^2+b^2) - \sqrt{(h^2-b^2)^2 + h^2b^2}\right] \qquad \text{- Q.E.D -}$$

7.12 $I_{11} = \frac{1}{12}(c)(3c)^3 = \frac{9}{4}c^4$

$I_{22} = \frac{1}{12}(3c)(c)^3 = \frac{1}{4}c^4$

$I_{12} = 0$



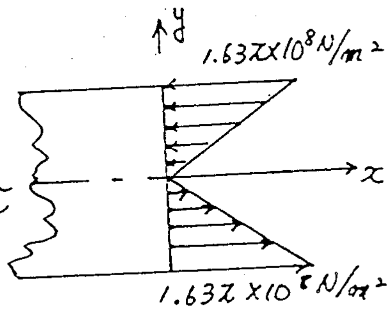
$$I_{yy} = \frac{1}{2}\left(\frac{C^4}{4} + \frac{9C^4}{4}\right) + \frac{1}{2}\left(\frac{9C^4}{4} - \frac{C^4}{4}\right)\cos 60° = 1.75C^4$$

$$I_{zz} = \frac{1}{2}\left(\frac{C^4}{4} + \frac{9C^4}{4}\right) - \frac{1}{2}\left(\frac{9C^4}{4} - \frac{C^4}{4}\right)\cos 60° = 0.75C^4$$

$$I_{yz} = (C^4)\sin 60° = 0.866C^4$$

7.13 $I_{zz} = \frac{1}{12}bh^3 = \frac{1}{12}\times 0.025\times(0.05)^3 = 2.60417\times10^{-7}\ (m^4)$

$$\sigma_{max} = \frac{M y_{max}}{I_{zz}} = \frac{(1.7\times10^3)(0.025)}{2.60417\times10^{-7}} = 1.632\times10^8\ (N/m^2) < \sigma_Y$$



∴ σ_Y 보다 작으므로 linear하다.

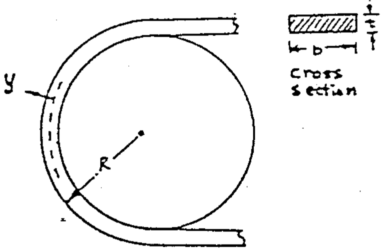
7.14 $I_{ZZ} = 2.164 \times 10^{7} mm^2 = 2.164 \times 10^{-5} m^4$

$M_{max} = 5000 \times 6 = 30000$ (N-m)

$\therefore \sigma_{max} = \dfrac{M_{max} \cdot y_{max}}{I_{ZZ}} = \dfrac{30{,}000 \times 0.1}{2.164 \times 10^{-5}} = 138632 \times 10^{5}\ N/m^2$

Ans. $\sigma_{max} = 138.632\ MN/m^2$

7.15



$\text{max. strain} = \dfrac{y_{max}}{R} = \dfrac{t/2}{R}$

$\text{max. stress} = E\left(\dfrac{y_{max}}{R}\right) = \dfrac{E \cdot t}{2R}$

$\dfrac{E \cdot t}{2R} = 280 \times 10^{6}$

$\therefore t = \dfrac{2 \cdot \frac{1}{2}(0.3+t) \times 280 \times 10^{6}}{200 \times 10^{9}}$

$= 4.206 \times 10^{-4}\ m$

t가 $\frac{t}{2}$로 되면 $\sigma'_{max} = 280 \times \frac{1}{2} = 140\ MN/m^2$

Ans. $t = 4.206 \times 10^{-4}\ m$, $\sigma'_{max} = 140\ MN/m^2$

7.16 $I_{ZZ} = \frac{1}{12}(12)(300)^3 + 2\left[\frac{1}{12}(210)(25)^3 + (210)(25)(162.5)^2\right]$

$= 3.048 \times 10^{8}\ (mm^4) = 3.048 \times 10^{-4}\ (m^4)$

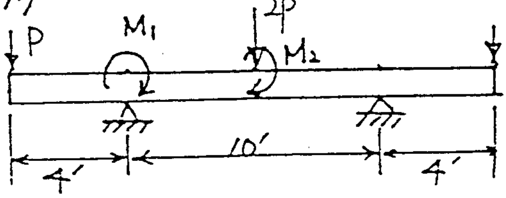
$M_b = \frac{1}{2} qLx - \frac{1}{2} q x^2$

$(M_b)_{max} = \frac{q}{2}\left(\frac{L^2}{4}\right) = \frac{1}{8} qL^2 = 203906.25$ (N-m)

$\therefore \sigma_{max} = \dfrac{(M_b)_{max} \cdot y_{max}}{I_{ZZ}} = \dfrac{203906.25 \times (0.175)}{3.048 \times 10^{-4}} = 117.1 \times 10^{6}\ (N/m^2)$

Ans. $\sigma_{max} = 117.1\ MN/m^2$

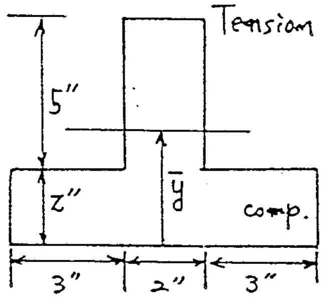
7.17



평형조건; $\Sigma M_{1,2} = 0$ $M_1 = 4P$

$M_2 = 9P - 10P = -P$

$\therefore M_{max} = 4P$



$$\bar{y} = \frac{(5\times2)\times4.5 + (8\times2)\times1}{26} = 2.346\,(in)$$

$$I_{zz} = \frac{1}{12}(2)(5)^3 + (2\times5)(4.5-2.346)^2 + \frac{1}{12}(8)(2)^3 + (2\times8)(2.346-1)^2 = 101.6\,(in^4)$$

$$\sigma_t = \frac{4P_t}{I_{zz}}(7-2.346) = 5000 \quad \therefore P_t = 27280\,(lb)$$

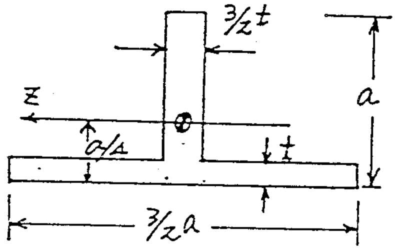
$$\sigma_c = \frac{4P_c}{I_{zz}}(2.346) = 20.000 \quad \therefore P_c = 216400\,(lb)$$

$\therefore P_{max} = P_t = 27280\,lb$ (Tension 으로 파괴된다.)

Ans. $P_{max} = 27280\ lb$

7.18

$$\bar{y} = \frac{(a)(\frac{3}{2}t)(\frac{a}{2}) + (\alpha a)(t)(0)}{(a)(\frac{3}{2}t) + (\alpha a)(t)} = \frac{3a}{4(\alpha+\frac{3}{2})} \quad -①$$



($\because t \ll a$ 이므로 t/a는 무시)

$$(\sigma_t)_{max} = \frac{(M_b)_{max}\cdot y_t}{I_{zz}}, \quad (\sigma_c)_{max} = \frac{(M_b)_{max}\cdot y_c}{I_{zz}}$$

$$3(\sigma_t)_{failure} = (\sigma_c)_{failure}$$

$$\therefore \frac{(\sigma_t)_{max}}{(\sigma_c)_{max}} = \frac{1}{3} = \frac{y_t}{y_c} \qquad y_t = \bar{y} = \frac{3a}{4(\alpha+\frac{3}{2})}$$

$$y_c = a - \bar{y} = \frac{a(4\alpha+3)}{4(\alpha+\frac{3}{2})}$$

따라서 $\frac{1}{3} = \left(\frac{3a}{4(\alpha+\frac{3}{2})}\right) \Big/ \left(\frac{a(4\alpha+3)}{4(\alpha+\frac{3}{2})}\right) \quad \therefore \alpha = \frac{3}{2}$

$$I_{zz} = \frac{1}{12}\left(\frac{3}{2}t\right)(a)^3 + \left(\frac{3at}{2}\right)\left(\frac{a}{4}\right)^2 + \frac{1}{12}\left(\frac{3a}{2}\right)t^3 + \left(\frac{3at}{2}\right)\left(\frac{a}{4}\right)^2$$
$$= \frac{5}{16}a^3t$$

$\alpha = \frac{3}{2}$ 을 ①에 대입하면 $\quad y_t = \bar{y} = \frac{a}{4}$

$$\therefore (\sigma_t)_{max} = \frac{(M_b)_{max} \cdot \frac{a}{4}}{\frac{5}{16} a^3 t} = \frac{4}{5} \cdot \frac{(M_b)_{max}}{a^2 t}$$

한편 $(M_b)_{max} = \frac{1}{8} w_0 \cdot l^2 = \frac{1}{8} w_0 (10a)^2 = \frac{25}{2} w_0 a^2$

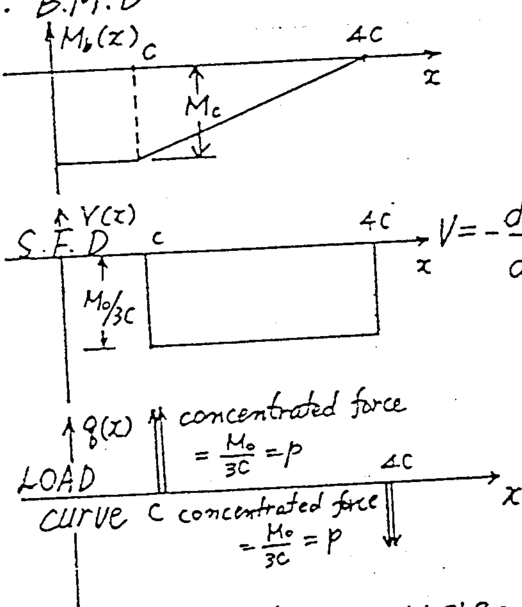
$$\therefore (\sigma_t)_{max} = \frac{4}{5a^2 t}\left(\frac{25}{2} w_0 a^2\right) = 10 \cdot \frac{w_0}{t} \qquad \therefore w_0 = \frac{t}{10} \sigma_t$$

Ans. $\alpha = \frac{3}{2}$, $w_0 = \frac{t}{10} \sigma_t$

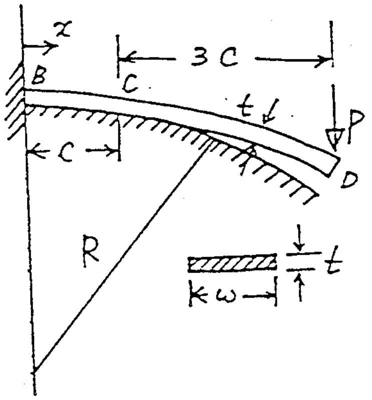
7.19 구간 BC에서 strip의 곡률은 일정하다. 따라서 BC구간에서의 Bending moment는 일정하다.

$$M_0 = -(EI/R) = -\frac{1}{12}(Ewt^3/R) = -P(3C)$$

∴ B.M.D



$$V = -\frac{dM_b}{dx}$$



$$q = -\frac{dV}{dx}$$

Strip과 block 사이의 반작용은 집중 하중이다.

$$P = \frac{M_0}{3C} = \left(\frac{1}{3C}\right)\frac{1}{12}\frac{Ewt^3}{R} = \frac{Ewt^3}{36CR}$$

Ans. $P = Ewt^3/36CR$

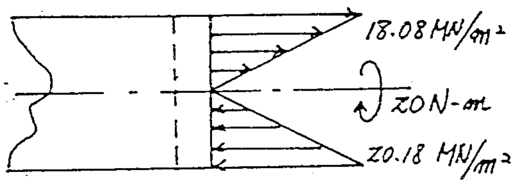
7.20 $M_b = 400 \times 0.05 = 20$ (N·m)

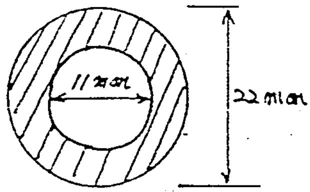
(a) $I_{zz} = \dfrac{\pi d^4}{64} = \dfrac{\pi}{64}(22\times10^{-3})^4 = 1.150\times10^{-8}\ (m^4)$

Bending stress at B $(\sigma_{max})_B = \dfrac{20\times0.011}{1.15\times10^{-8}} = 19.13\ (MN/m^2)$

Compressive stress $\sigma_c = \dfrac{-P}{\frac{\pi}{4}d^2} = \dfrac{-400}{\frac{\pi}{4}(22\times10^{-3})^2} = -1.052 (MN/m^2)$

$\sigma = \sigma_B + \sigma_c = 18.08\ MN/m^2$ (tension). $20.18\ MN/m^2$ (compression).

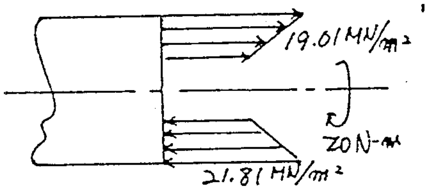




(b) $\sigma_c = \dfrac{-400}{\frac{\pi}{4}\{(22\times10^{-3})^2-(11\times10^{-3})^2\}} = -1.403\ (MN/m^2)$

$I_{zz} = \dfrac{\pi}{64}\{(22\times10^{-3})^4-(11\times10^{-3})^4\} = 1.078\times10^{-8}\ (m^4)$

$\therefore (\sigma_B)_{max} = \dfrac{20\times0.011}{1.078\times10^{-4}} = 20.41\ (MN/m^2)$



$\sigma = \sigma_c + \sigma_B = \begin{pmatrix} 19.01 \text{ (tension)} \\ 21.81 \text{ (compression)} \end{pmatrix}$

(c) tension ; $\left(\dfrac{19.01-18.08}{18.08}\right)\times100 = 5.144\ (\%)$

compression ; $\left(\dfrac{21.81-20.18}{20.18}\right)\times100 = 8.077\ (\%)$

Ans. (c) tension 5.144 %, compression 8.077 %

7.21 $(M_B)_{max} = 1.8\times26000 = 46800\ (N\text{-}m)$, $I_{zz} = \dfrac{b^4}{12}$

$\tau_u = \dfrac{P}{A} = \dfrac{2600}{b_s^2} = 10\times10^6 \quad \therefore b_s = 0.016\ m$

$\sigma_u = \dfrac{M\cdot y}{I} = \dfrac{12}{b_t^4}(46800)\cdot\dfrac{b_t}{2} = 52\times10^6 \quad \therefore b_t = 0.1754\ m$

b_s 와 b_t 중에서 큰것을 b로 택한다. $\therefore b = 0.1754\ m$

Ans. $b = 0.1754\ m$

7.22 Bending stress $\sigma_b = -\dfrac{M \cdot y}{I_{zz}}$ $I_{zz} = \pi r^3 t$, $y = r\sin\theta$

$$\therefore \sigma_b = \frac{M(r\sin\theta)}{\pi r^3 t} = -\frac{M}{\pi r^2 t}\sin\theta$$

Ans. $\sigma_b = -\dfrac{M}{\pi r^2 t}\sin\theta$

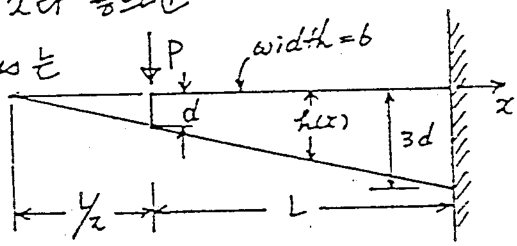
7.23 유체의 비중 ; γ $\quad g = \pi r^2 \cdot \gamma$; 단위 길이당 하중

$$(M_b)_{max} = \frac{1}{8} g L^2 = \frac{1}{8}\pi r^2 \cdot \gamma \cdot L^2$$

$$\therefore \sigma_{max} = \frac{\frac{1}{8}\pi r^2 L^2 \cdot \gamma}{\pi r^3 t} \cdot r = \frac{\gamma L^2}{8t}$$; tank의 반경에 무관하다.

— Q. E. D —

7.24 자유단에서 부터의 거리를 x라 놓으면 x점에서의 maximum stress는 다음과 같다. $\dfrac{M_b(x)}{I_{yy}(x)} \cdot \dfrac{h(x)}{2}$



$$M_b(x) = p \cdot x$$

$$I_{yy}(x) = \frac{1}{12} b h^3 = \frac{1}{12} b d^3 \left[1 + \frac{2x}{L}\right]^3$$

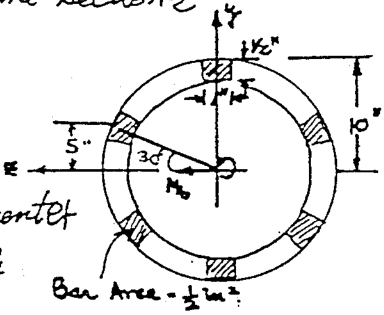
$$(\sigma_x) = \frac{P \cdot x \frac{1}{2} d\left(1 + \frac{2x}{L}\right)}{\frac{1}{12} b d^3 \left(1 + \frac{2x}{L}\right)^3} = \frac{6Px}{bd^2\left(1 + \frac{2x}{L}\right)^2}$$

최대가 되기 위해서 $\dfrac{d}{dx}\left(\dfrac{x}{\left(1 + \frac{2x}{L}\right)^2}\right) = 0 \quad \therefore x = \dfrac{L}{2}$

$$\therefore (\sigma_x)_{max} = \frac{6P \cdot \frac{L}{2}}{bd^2 (2)^2} = \frac{3PL}{4bd^2}$$

Ans. $x = \dfrac{L}{2}$ $\quad \sigma_{max} = \dfrac{3PL}{4bd^2}$

7.25 plane section은 변형후에도 plane section을 유지한다고 가정하자.



그러면, $\sigma_{max} = \dfrac{M_b \cdot y_{max}}{I_{yy}}$

I_{yy}의 계산 ; shell 자체의 2차 moment와 bar 자체의 moment는 미소하므로 무시한다. $\therefore I_{yy} = A\bar{y}^2$

$$I_{yy} = \left(\frac{1}{2}\right)[10^2+10^2+5^2+5^2+5^2+5^2] = 150 \text{ in}^4$$

$$\therefore \sigma_{max} = \frac{100.000 \times 10}{150} = 6667 \text{ psi}$$

Ans. $\sigma_{max} = 6667$ psi

7.26 $\rho = 6\,\mu m$, $M_b = 5\times10^{-7}$ N-m, $(I_{yy})_{total} = 4\times10^{-8}\ mm^4$

$$E = \frac{M\rho}{I} = \frac{(5\times10^{-7})(6\times10^{-6})}{(4\times10^{-8}\times10^{-12})} = 75\ (MN/m^2) \quad (\rho\,;\text{curvature})$$

Ans. $E = 75\ MN/m^2$

7.27 $\tau_{yz}(y_1) = \dfrac{4P}{3\pi r^4}(r^2-y_1^2)$. 두께가 1이므로

$$U_t = \frac{1}{2G}\int_V \left(\frac{Ty}{J}\right)^2 dV = \frac{1}{2G}\int_A \left(\frac{Ty}{J}\right)^2 2\pi y\,dy. \quad (T=PR,\ J=\frac{\pi}{2}r^4)$$

$$\therefore U_t = \frac{1}{2G}\cdot\frac{1}{\pi^2 r^8}P^2R^2\int_0^r y^2\, 2\pi y\,dy = \frac{P^2R^2}{\pi G r^4}$$

$U_s = \int_V \dfrac{\tau^2}{2G}dV$ 에서 두께가 1이므로, $U_s = \int_A \dfrac{\tau^2}{2G}dA$

$$U_s = \frac{1}{2G}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \tau_{yz}^2 (2r\cos\theta)(r\cos\theta)\,d\theta$$

$$\tau_{yz} = \frac{4P}{3\pi r^4}(r^2-y^2) = \frac{4P}{3\pi r^4}(r\cos\theta)^2 = \frac{4P}{3\pi r^2}\cos^2\theta$$

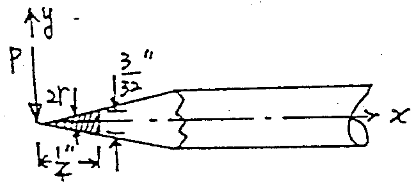
$$\therefore U_s = \frac{r^2}{G}\left(\frac{4P}{3\pi r^2}\right)^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^6\theta \, d\theta = \frac{5P^2}{9\pi G r^2}$$

따라서 $\dfrac{U_s}{U_t} = \dfrac{5}{9}\left(\dfrac{r}{R}\right)^2$

Ans. $U_t = \dfrac{P^2R^2}{\pi G r^4}$, $U_s = \dfrac{5P^2}{9\pi G r^2}$

7.28 Trial I $P \simeq 1$(lb) ; with perfectly sharp point

$\sigma = \dfrac{M \cdot y}{I_{yy}}$; $M = P \cdot x$



$r = \dfrac{3}{64}\left(\dfrac{x}{1/4}\right) = \dfrac{3}{16}x$

$y = r$

$I_{yy} = \dfrac{\pi r^4}{4} = \dfrac{\pi}{4}\left(\dfrac{3}{16}\right)^4 x^4$

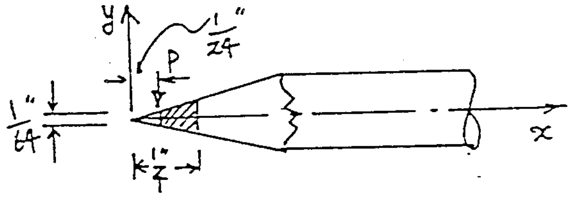
$$\therefore \sigma(x) = \frac{(P \cdot x)\left(\frac{3}{16}x\right)}{\frac{\pi}{4}\left(\frac{3}{16}\right)^4 \cdot x^4} = 192\frac{P}{x^2}$$

연필심이 날카로울수록 $x \to 0$ 가 되어 $\sigma \to \infty$ 로 된다.
따라서 파괴되려는 경향이 농후해 진다.

Trial II

연필 끝은 1/64"가 편평하고, 하중1(lb)라 가정하자.

$M = P\left(x - \dfrac{1}{24}\right)$



r. y는 I에서와 같다.

$$\sigma = \frac{P\left(x-\frac{1}{24}\right)\left(\frac{3}{16}x\right)}{\frac{\pi}{4}\left(\frac{3}{16}\right)^4 \cdot x^4}$$

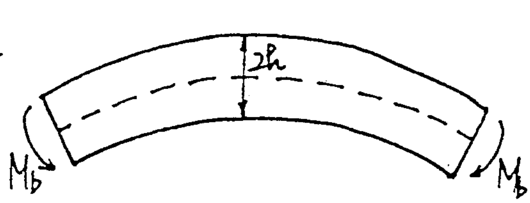
$$= \frac{192P\left(x-\frac{1}{24}\right)}{x^3} \quad \frac{1}{24}" \le x \le \frac{1}{4}"$$

$x = 3/48$" 에서 $\sigma_{max} = 16,300$ psi

7.29 2개의 bar를 soldering 하였으므로 높이가 2h인 하나의 bar가 된다.

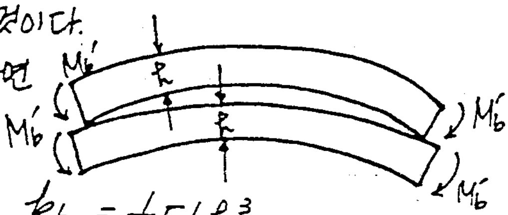
따라서 $\frac{d\phi}{dS} = \frac{M_b}{EI_{yy}} = \frac{12}{8}\frac{M_b}{Ebh^3}$

$$\frac{M_b}{d\phi/dS} = K_{b1} = \frac{2}{3}Ebh^3$$



seperate 되어 있는 경우에는 각각의 curvature $d\phi/ds$로 구부러지고 양단에서만 접촉할 것이다.

각각의 moment를 M_b'이라 하면



$$M_b' = E\left(\frac{1}{12}bh^3\right)\frac{d\phi}{dS}$$

$$M_b = 2M_b' = \frac{1}{6}Ebh^3\frac{d\phi}{dS} \qquad \therefore\ k_{b2} = \frac{1}{6}Ebh^3$$

Ratio of stiffness $\left(\frac{\text{soldered}}{\text{separate}}\right) = \frac{k_{b1}}{k_{b2}} = 4.0$

max. bending stress의 비는 max. bending strain의 비와 같다. 각각의 경우 max. strain은

$$\text{max. strain} = \frac{d\phi}{dS}\left(\frac{1}{2}\text{ beam height}\right)$$

soldered beam; $\frac{d\phi}{dS} = \frac{M_b}{k_{b1}}$; $\varepsilon_{max} = \frac{2h}{2}\left(\frac{M_b}{k_{b1}}\right) = \frac{hM_b}{k_{b1}}$

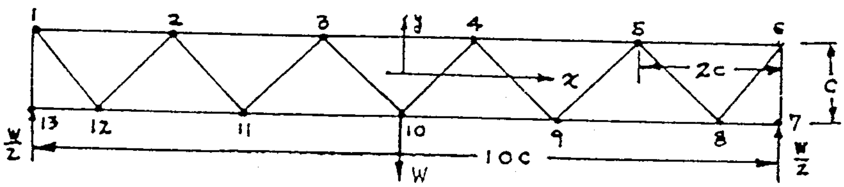
seperate beam; $\frac{d\phi}{dS} = \frac{M_b}{k_{b2}}$; $\varepsilon_{max.} = \frac{h}{2}\left(\frac{M_b}{k_{b2}}\right) = \frac{hM_b}{2k_{b2}}$

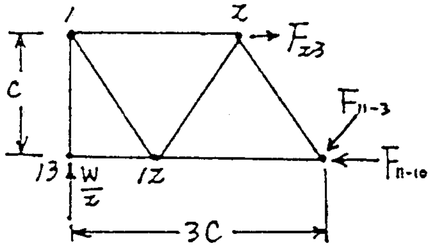
$\therefore$ Ratio of stresses; $\left(\frac{\text{soldered}}{\text{seperate}}\right) = \left(hM_b/k_{b1}\right)/\left(hM_b/2k_{b2}\right)$

$$= \frac{1}{2}$$

Ans. $\left(\frac{\text{Soldered}}{\text{Seperate}}\right)_{\text{stiffness}} = 4.$ $\left(\frac{\text{Soldered}}{\text{Seperate}}\right)_{\text{stress}} = \frac{1}{2}$

7.30





Equilibrium ;

$\Sigma M_{11} = 0$ (pin-joint transfers no moment)

$\therefore \frac{W}{2}(3C) + F_{23}(C) = 0$

$F_{23} = -\frac{3}{2}\overline{W}$

같은 방법으로, $F_{12} = -\frac{\overline{W}}{2}$; $F_{23} = -\frac{3}{2}\overline{W}$; $F_{34} = -\frac{5\overline{W}}{2}$, $F_{45} = -\frac{3}{2}\overline{W}$

$F_{56} = -\frac{\overline{W}}{2}$; $F_{78} = 0$; $F_{89} = \frac{2}{2}\overline{W}$; $F_{910} = \frac{4\overline{W}}{2}$; $F_{1011}\ \frac{4\overline{W}}{2}$; $F_{1112} = \frac{2}{2}\overline{W}$;

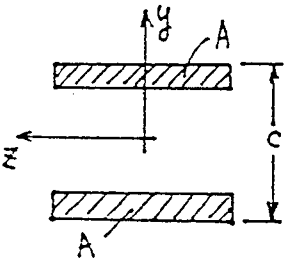
$F_{1213} = 0$

따라서 stress ($\sigma = F/A$)는 다음과 같다.

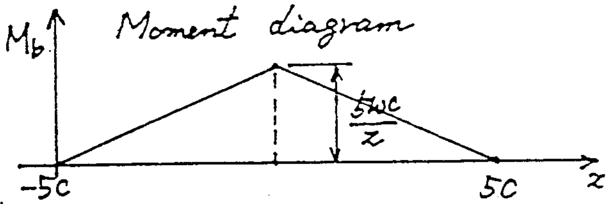
$\sigma_{12} = -\frac{\overline{W}}{2A}$; $\sigma_{23} = -\frac{3\overline{W}}{2A}$; $\sigma_{34} = -\frac{5\overline{W}}{2A}$; $\sigma_{45} = -\frac{3\overline{W}}{2A}$; $\sigma_{56} = -\frac{\overline{W}}{2A}$

$\sigma_{78} = 0$; $\sigma_{89} = \frac{\overline{W}}{A}$; $\sigma_{910} = \frac{2\overline{W}}{A}$; $\sigma_{1011} = \frac{2\overline{W}}{A}$; $\sigma_{1112} = \frac{\overline{W}}{A}$; $\sigma_{1213} = 0$

<u>Bending Theory</u> ; 단순 bending으로 가정하고, 대각선 부재는 무시.

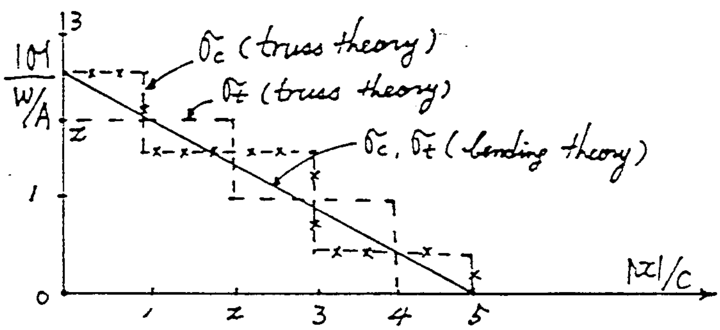


$I_{yy} \simeq 2\left[A\left(\frac{C}{2}\right)^2\right] = \frac{AC^2}{2}$



$M_b = \frac{5\overline{W}C}{2}\left[1 - \frac{|x|}{5C}\right]$

$\sigma_T = |\sigma_C| = \frac{M_b\left(\frac{C}{2}\right)}{I_{yy}} = \frac{5\overline{W}}{2A}\left[1 - \frac{|x|}{5C}\right]$



대각선 부재는 shear force를 전달하고, beam의 곡률에 의해 생기는 y 방향의 작은 힘들을 전달한다.

7.31 bending 이므로 strain $\varepsilon = \frac{y}{\rho}$ $\therefore \sigma = c|\varepsilon|^n = \frac{c}{\rho^n}|y|^n$

$$M_b = 2\int_0^{\frac{h}{2}} \sigma b \cdot y\, dy = \frac{2bc}{\rho^n}\int_0^{\frac{h}{2}} y^{n+1} dy = \frac{2bc}{(n+2)\rho^n}\left(\frac{h}{2}\right)^{n+2}$$

$$\therefore \frac{c}{\rho^n} = \frac{(n+2)M_b}{2b\left(\frac{h}{2}\right)^{n+2}}$$

$$\sigma_{max} = \frac{c}{\rho^n}\left|\frac{h}{2}\right|^n = \frac{(n+2)M_b}{2b\left(\frac{h}{2}\right)^{n+2}}\left(\frac{h}{2}\right)^n = \frac{2(n+2)}{bh^2}M_b$$

Ans. $\sigma_{max} = \frac{2(n+2)}{bh^2}M_b$

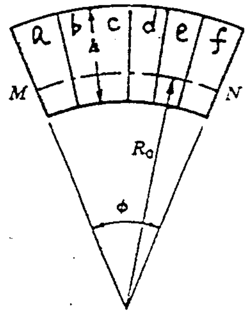
7.32 $u = -\frac{xy}{\rho}$; $v = \frac{x^2+\nu(y^2-z^2)}{2\rho}$; $\omega = \frac{\nu yz}{\rho}$

$$\varepsilon_x = \frac{\partial u}{\partial x} = -\frac{y}{\rho}\ ;\ \varepsilon_y = \frac{\partial v}{\partial y} = \nu\frac{y}{\rho}\ ;\ \varepsilon_z = \frac{\partial u}{\partial z} = \frac{\nu y}{\rho}$$

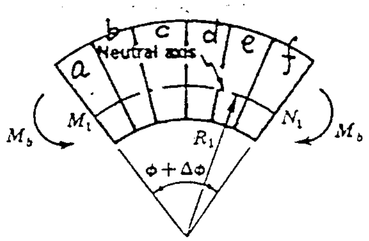
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -\frac{x}{\rho} + \frac{x}{\rho} = 0$$

$$\gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial \omega}{\partial y} = -\frac{\nu z}{\rho} + \frac{\nu z}{\rho} = 0$$

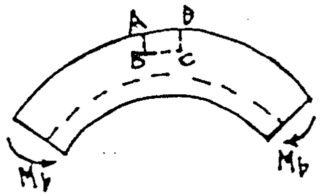
$$\gamma_{zx} = \frac{\partial \omega}{\partial x} + \frac{\partial u}{\partial z} = 0 + 0 = 0.$$

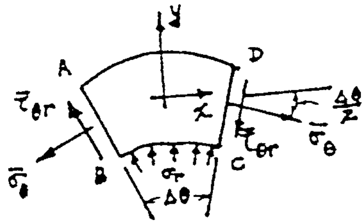


7.33 변형하기 전에 circular segments a.b.c.d…는 같은 하중을 받으므로 같은 형태로 변형할 것입니다. y축에 대하여 회전을 해도, 회전하기 전과 변화가 없을 것이다.
∴ 이것은 curve 면에 수직한 flat end를 갖는 segment만이 이것을 만족한다.
radial plane section은 변형 후에도 plane을 유지한다. 또 longitudinal strain은 없으므로 중립축은 변화가 없다.



$R_0\phi = R_1(\phi+\Delta\phi)$, $\frac{1}{R_1} = \frac{\phi+\Delta\phi}{R_0\phi} = \frac{1}{R_0} + \frac{\Delta\phi}{R_0\phi}$ —①





Free-body diagram of ABCD

$\Sigma F_y = 0$, AB.CD 에서의 shear stress $\tau_{\theta r}$의 y성분은 서로 상쇄된다. Normal stress σ_θ는 y방향의 성분이 "0"이 아니다. 이것과 평형을 이루기 위해서 σ_r이 존재한다.

Tangential strain distribution;

중립축으로 부터 y만큼 떨어진 circular arc를 생각해 보면.

Length before deformation $= (R_0+y)\phi$

Length after deformation $= (R_1+y)(\phi+\Delta\phi)$

Tangential strain $\varepsilon_\theta = \dfrac{(R_1+y)(\phi+\Delta\phi)-(R_0+y)\phi}{(R_0+y)\phi}$

①식을 사용하면

$$\varepsilon_\theta = \frac{R_0-R_1}{R_1}\left(\frac{y}{R_0+y}\right)$$

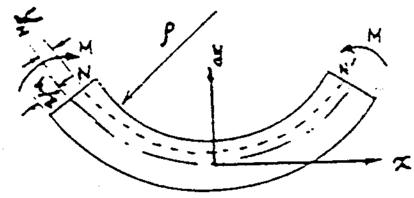
$R_0 \gg y$ 가 아니면 non-linear 이다.

7.34 geometrical results;

$$\varepsilon_x = -\frac{1}{\rho}(y-y_N) \quad —①$$

$$\varepsilon_z = \text{constant} = \varepsilon_0 \quad —②$$

equilibrium ; $\sigma_y = 0$ (h/ρ small)



<Neutral Axis는 centerline으로 부터 y_N 만큼의 거리에 NN'이다>

$$\Sigma F_x = 0 \;;\; \int_{-h/2}^{h/2} \sigma_x \, dy = 0 \quad —③$$

$$\Sigma F_z = 0 \;;\; \int_{-h/2}^{h/2} \sigma_z \, dy = 0 \quad —④$$

$$\Sigma M_z = 0 \;;\; -\int_{-h/2}^{h/2} y \cdot \sigma_x \, dy \quad —⑤$$

stress - strain $\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z)$ — ⑥

$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y)$ — ⑦

①,②,⑥,⑦ ; $\sigma_x = \frac{E}{1-\nu^2}\left[-\frac{(y-y_N)}{\rho} + \nu\varepsilon_o\right]$, $\sigma_z = \frac{E}{1-\nu^2}\left[\varepsilon_o - \frac{\nu(y-y_N)}{\rho}\right]$

$\Sigma F_x = 0$. $\Sigma F_z = 0$. $\left(\frac{y_N}{\rho} + \nu\varepsilon_o\right)h = 0$, $\left(\varepsilon_o + \frac{\nu y_N}{\rho}\right)h = 0$

$\therefore y_N = \varepsilon_o = 0$

$\therefore \sigma_x = -\frac{E}{1-\nu^2}\cdot\frac{y}{\rho}$

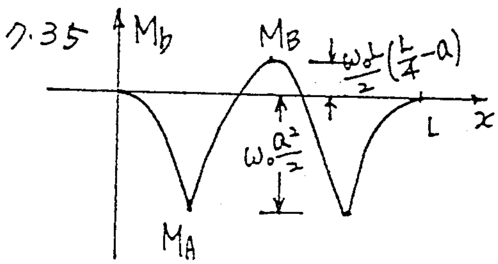
$\Sigma M_z = 0$: $M = -\int_{-h/2}^{h/2} y\left[-\frac{E}{1-\nu^2}\cdot\frac{y}{\rho}\right]dy = \frac{Eh^3}{12(1-\nu^2)\cdot\rho}$

$\therefore \frac{d\phi}{ds} = \frac{1}{\rho} = \frac{12(1-\nu^2)\cdot M}{Eh^3}$

$\sigma_x = -\frac{Ey}{1-\nu^2}\left(\frac{1}{\rho}\right) = \frac{12My}{h^3}$

Ans. $\sigma_x = \frac{12My}{h^3}$

7.35



$M_A = M_B$;

$\frac{w_o \cdot L}{2}\left(\frac{L}{4} - a\right) = w_o\frac{a^2}{2}$

$\therefore 4a^2 + 4La - L^2 = 0$

$\therefore a = \frac{-2 \pm \sqrt{4+4}}{4}L$ 에서

$a = 0.2071L = 178.1\ mm$ $(a > 0)$

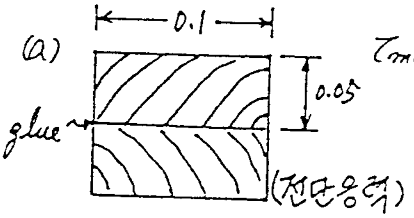
$\therefore M = \frac{1}{2}w_o a^2 = 0.01586\, w_o$

$\sigma_{max} = \frac{My}{I} = \frac{(0.01586 w_o)(3\times10^{-3})}{\frac{1}{12}(190\times10^{-3})(6\times10^{-3})^3} = 7\times10^6$

$\therefore w_o = 503.1\ N/m$

Ans. $w_o = 503.1\ N/m$

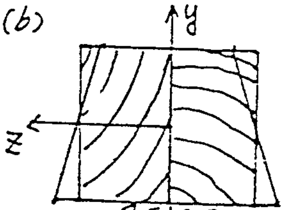
7.36 (a)



$$\tau_{max} = \frac{VQ}{bI} = \frac{P\times\frac{1}{2}\times(0.1)\times(0.05)^2}{(0.1)\times\frac{1}{12}\times(0.1)^4} = 150P$$

$$\therefore \tau_{max} = 150P \ (N/m^2)$$

(b)



(굽힘응력)

$$\varepsilon_x = \frac{M\cdot y}{E\cdot I_{zz}} = \frac{3P\times(0.05)}{E\times\frac{1}{12}\times(0.1)^4} = 18000\frac{P}{E}$$

$$|\varepsilon_z| = |\nu\varepsilon_x| = 18000\frac{\nu P}{E}$$

$$\sigma_z = E|\varepsilon_z| = 18000\nu P$$

$\therefore \nu = 0.4$로 놓으면 $\quad \sigma_z = 7200P \ (N/m^2)$

Ans. (a) $\tau = 150P \ (N/m^2)$ (b) $\sigma_z = 7200P \ (N/m^2)$

7.37 (a) just above the solder joint

$$Q = 2\times(\tfrac{1}{4})(4+1) = 2.5 \ (in^3) \quad \therefore \frac{q}{b} = \frac{VQ}{I_{zz}} = \frac{6000\times2.5}{75} = 200 \ (lb/in)$$

(b) just below solder joint

$$Q = 2.5 + 4.7 = 7.2 \ (in^3) \quad \therefore \frac{q}{b} = \frac{6000\times7.2}{75} = 575 \ (lb/in)$$

(c) at the neutral surface

$$Q = 6\times(\tfrac{1}{4})\times3 + 4.7 = 9.2 \ (in^3) \quad \therefore \frac{q}{b} = \frac{6000\times9.2}{75} = 740 \ (lb/in)$$

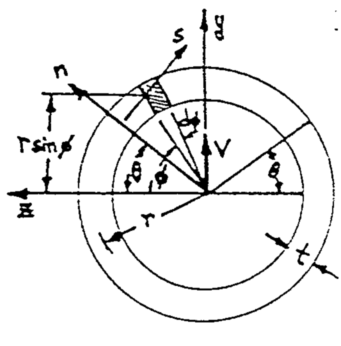
Ans a) 200 lb/in b) 575 lb/in c) 740 lb/in

7.38 $I_{yy} = \pi r^3 t \quad (t/r \ll 1)$

Q = 빗금친 부분의 1차 moment

$$= 2\int_{\theta}^{\pi/2}(r\sin\phi)(t\,r\,d\phi)$$

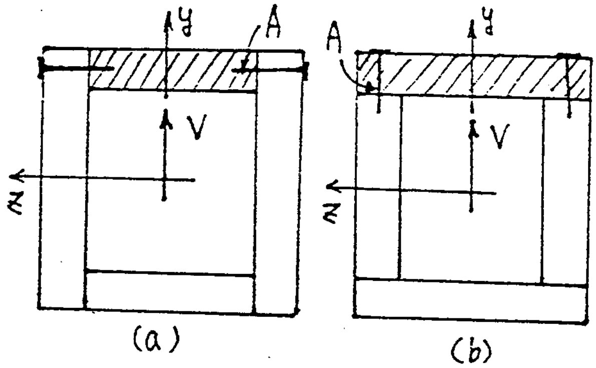
$$= 2r^2t\cos\theta$$



$$\tau_{sx} = \tau_{xs} = \frac{1}{2}\left(\frac{VQ}{I_{yy}}\right) = \frac{V}{\pi r}\cos\theta$$

└→(빗금친 부분이 양쪽 면을 갖으므로) Ans. $\tau_{sx} = \frac{V}{\pi r}\cos\theta$

7.39 못이 접촉면 A에서 shear flow를 전달한다.
(a),(b) 에서 다른 모든 조건은 같기 때문에 전달하는 shear flow가 작을수록 좋다.



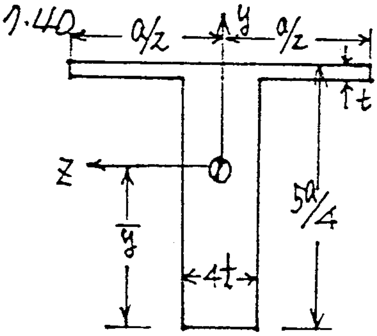
$$\text{shear flow} = \frac{VQ}{I_{yy}}$$

Q ; 빗금친 면의 1차 moment (a)가 (b)보다 작다.

I_{yy} ; (a)(b) 서로 같다.

Ans. (a)쪽의 설계가 (b)보다 좋다.

7.40



$$\bar{y} = \frac{(at)(\frac{5}{4}a) + (4t)(\frac{5}{4}a)(\frac{5}{8}a)}{(at) + (4t)(\frac{5}{4}a)} = 0.73a$$

$$(\text{stem } \sigma_x)_{max} = \frac{M}{I_{yy}}(y_{max})_{stem} = \frac{M}{I_{yy}}(0.73a)$$

$$(\text{flange } \sigma_x)_{max} = \frac{M}{I_{yy}}(y_{max})_{flange} = \frac{M}{I_{yy}}(0.52a)$$

$$\therefore \frac{(\text{stem } \sigma_x)_m}{(\text{flange } \sigma_x)_m} = \frac{0.73a}{0.52a} = 1.40$$

$(\text{stem } \tau_{xy})_{max}$ 은 $y=0$ 에서

$$Q = \frac{1}{2}(4t)(0.73a)\left(\frac{0.73a}{2}\right) = 1.06\,a^2t$$

$(\text{flange } \tau_{zx})_{max}$ 은 flange와 stem 의 연결점에서

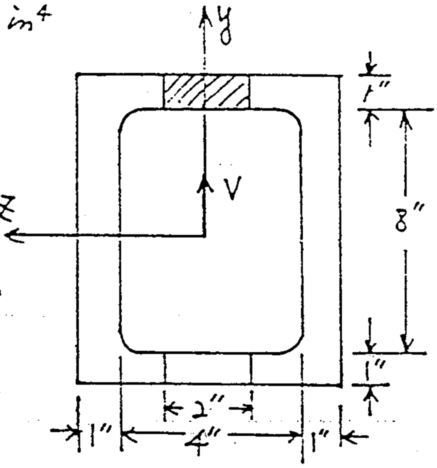
$$Q(\text{half flange}) = \left(\frac{a}{2}\right)(t)(0.52a) = 0.26\,a^2t$$

$$\therefore \frac{(\text{stem } \tau_{xy})_m}{(\text{flange } \tau_{zx})_m} = \left(\frac{V}{I_{yy}}\,\frac{1.06\,a^2t}{4t}\right) \Big/ \left(\frac{V}{I_{yy}}\cdot\frac{0.26\,a^2t}{t}\right) = 1.02$$

Ans. $(\sigma_{xs}/\sigma_{xf})_m = 1.4$ $\quad (\tau_{xys}/\tau_{zxf})_m = 1.02$

7.41 Bolt는 빗금친 부분의 양단에서 shear flow를 전달한다.

$I_{yy} = \frac{1}{12}\left[(6)(10)^3 - (4)(8)^3\right] = 330 \text{ in}^4$



빗금친 부분의 1차 moment ; Q

$Q = (1)(2)(4.5) = 9 \text{ in}^3$

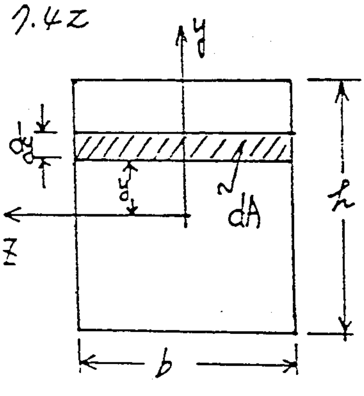
q_{zx}를 각 끝에서의 shear flow라 하면, $2q_{zx} = \frac{VQ}{I_{yy}} = \frac{(10{,}000)(9)}{330} = 272 \text{ lb/in}$

$\therefore q_{zx} = 136 \text{ lb/in}$

Total shear flow (길이 "S"에 대한)는 1개의 bolt에 의해서 전달된다 $\therefore 136 \times S = 400 \quad S = 2.95''$

Ans. $S = 2.95''$

7.42



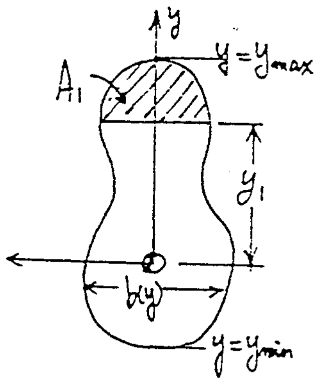
$$\tau_{xy} = \frac{V}{2I_{yy}}\left[\left(\frac{h}{2}\right)^2 - y^2\right] \quad \left(\tau = \frac{VQ}{bI}\right)$$

따라서 $\text{Shear force} = \int_A \tau_{xy}\, dA = \int_{y=-h/2}^{h/2} \tau_{xy} \cdot b\, dy$

$$= \frac{Vb}{2I_{yy}} \int_{-h/2}^{h/2} \left[\left(\frac{h}{2}\right)^2 - y^2\right] dy = \frac{Vb}{2I_{yy}}\left[h\left(\frac{h}{2}\right)^2 - \frac{1}{3}\left(\frac{h}{2}\right)^3\right]$$

$$= \frac{V}{I_{yy}}\left(\frac{1}{12} b h^3\right) = \frac{V}{I_{yy}} \cdot I_{yy} = V \quad \text{- Q.E.D -}$$

7.43 $Q = \int_{A_1} y\, dA = \int_{y_1}^{y_{max}} by\, dy = -\int_{y_{max}}^{y_1} by\, dy$



$\therefore \frac{dQ}{dy_1} = -by_1$

$$\int_A \tau_{xy}\, dA = \int_A \frac{VQ}{bI_{yy}}\, dA = \frac{V}{I_{yy}} \int \frac{Q}{b}(b\, dy)$$

$(b = b(y)\ ;\ \text{a function of } y)$

$$= \frac{V}{I_{yy}} \int_{y_{min}}^{y_{max}} Q\, dy = \frac{V}{I_{yy}} \left[Qy - \int y \frac{dQ}{dy} dy \right]_{y_{min}}^{y_{max}}$$

$$= \frac{V}{I_{yy}} \left[Qy + \int by^2 dy \right]_{y_{min}}^{y_{max}}$$ 괄호안의 첫 항은 $Q=0$ 이므로 (at $y=y_{max}$, $y=y_{min}$)

$$\therefore \int_A \tau_{xy}\, dA = \frac{V}{I_{yy}} \int_{y_{min}}^{y_{max}} y^2 (b\, dy) = \frac{V}{I_{yy}} \int_A y^2 dA = \frac{V}{I_{yy}} (I_{yy}) = V$$

7.44 $$\sigma_{max} = \frac{M_b}{I_{zz}} y_{max} = \frac{M_b}{I_{zz}} \left(\frac{h}{2}\right) = \left(\frac{1}{8} w_o L^2 \cdot h\right) \Big/ \left(\frac{bh^3}{12} \cdot \frac{1}{2}\right) = \frac{3}{4} \cdot \frac{w_o L}{bh^2}$$

$$(\tau_{yz})_{max} = \frac{V \cdot Q_{max}}{I_{zz} \cdot b_{min}} = \left(\frac{1}{2} w_o L \cdot \frac{h^2 b}{8} \cdot \frac{1}{b}\right) \Big/ \left(\frac{bh^3}{12}\right) = \frac{3}{4} \cdot \frac{w_o L}{bh}$$

$$\therefore (\sigma / \tau_{yz})_{max} = L/h$$

Ans. $(\sigma / \tau_{yz})_{max} = L/h$

7.45 $$Q_{max} = (0.75h)(2t)\left(\frac{h}{2}\right) + (0.5h)(t)(0.25h)$$

$$= 0.875 t h^2$$

$$I_{zz} = \frac{t}{12} h^3 + 2\left(\frac{3}{4}h\right)(2t)\left(\frac{h}{2}\right)^2 = \frac{5}{6} t h^3$$

($\because$ flange 의 자체 2차 moment 는 미소하므로 무시)

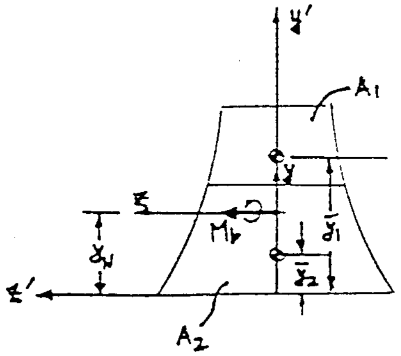
$$(M_b)_{max} = \frac{1}{4} PL \qquad V = \frac{1}{2} P$$

$$\sigma_{max} = \frac{(M_b)_{max} \cdot y_{max}}{I_{zz}} = \frac{\frac{1}{4} PL \cdot \frac{h}{2}}{\frac{5}{6} t h^3} = \frac{3PL}{20 h^2 t}$$

$$\tau_{max} = \frac{V \cdot Q_{max}}{b_{min} I_{zz}} = \frac{\frac{1}{2} P \cdot 0.875 t h^2}{t \cdot \frac{5}{6} t h^3} = \frac{21}{40} \cdot \frac{P}{h \cdot t}$$

$$\therefore (\sigma / \tau)_{max} = \frac{2}{7} \cdot \frac{L}{h}$$ Ans. $(\sigma / \tau)_{max} = \frac{2}{7} \frac{L}{h}$

7.46 (a) $\varepsilon_x = -\frac{d\phi}{ds} y$



기하학적 적합조건 ; $y = y' - y_N$

평형조건 ; $\sigma_y = 0$, $\sigma_z = 0$

$\sum F_x = 0$; $-\int_A \sigma_x dA = 0$

$\sum M_z = 0$; $-M_b + \int_A \sigma_x y\, dA = 0$

stress-strain relation ;

$(\sigma_x)_i = E_i (\varepsilon_x)_i = -E_i\left(\frac{d\phi}{ds}\right) y \quad (i = 1, 2)$

$\int_A \sigma_x dA = -E_1\left(\frac{d\phi}{ds}\right)\int_{A_1}(y' - y_N)\, dA - E_2\left(\frac{d\phi}{ds}\right)\int_{A_2}(y' - y_N)\, dA = 0$

한편 $\int_{A_1} y' dA = \bar{y}_1 A_1$, $\int_{A_2} y' dA = \bar{y}_2 A_2$

$\therefore -E_1\left(\frac{d\phi}{ds}\right)(\bar{y}_1 - y_N)A_1 - E_2\left(\frac{d\phi}{ds}\right)(\bar{y}_2 - y_N)A_2 = 0$

$\therefore y_N = \frac{E_1 \bar{y}_1 A_1 + E_2 \bar{y}_2 A_2}{E_1 A_1 + E_2 A_2}$ — Q.E.D —

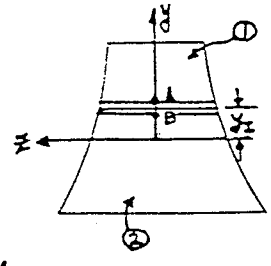
(b) $M_b = -\int_A \sigma_x \cdot y\, dA = E_1\left(\frac{d\phi}{ds}\right)\int_{A_1}(y' - y_N)^2 dA + E_2\left(\frac{d\phi}{ds}\right)\int_{A_2}(y' - y_N)^2 dA$

$= E_1\left(\frac{d\phi}{ds}\right)(I_{yy})_1 + E_2\left(\frac{d\phi}{ds}\right)(I_{yy})_2$

$\therefore \frac{d\phi}{ds} = \frac{1}{\rho} = \frac{M_b}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2}$ — Q.E.D —

(c) $(\sigma_x)_i = -E_i\left(\frac{d\phi}{ds}\right) y = -E_i \frac{M_b \cdot y}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2}$ — Q.E.D —

7.47 두 재료의 접촉면 근처에 점 A, B를 그림처럼 잡고, lateral strain ε_z를 생각하자. σ_x를 제외한 모든 응력은 "0"으로 가정.

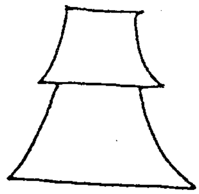


$(\sigma_x)_A = -\lambda E_1 y_I$; $(\sigma_x)_B = -\lambda E_2 y_I$ $\left(\lambda = \dfrac{M_b}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2} \right)$

$\therefore (\varepsilon_z)_A = \dfrac{1}{E}(\sigma_z - \nu\sigma_x - \nu\sigma_y) = \lambda \nu_1 E_1 y_I$

$(\varepsilon_z)_B = \lambda \nu_2 E_2 y_I$

따라서 일반적으로 lateral strain은 서로 다르다. 따라서 접촉면에 저항이 없다면 우측 그림과 같이 변형할 것이다. 그러나. 실제로는 기하학적 적합조건을 만족시켜야 한다. 따라서 σ_z가 존재할 것이고 그러므로 ε_z. τ_{yz}도 존재한다. 만일 $E_1\nu_1 = E_2\nu_2$ 이면 σ_x만 존재한다.



7.48 $I_a = 23.70 \times 10^6 mm^4 = 23.70 \times 10^{-6} m^4$

$I_s = 2\left[(0.00625)(100\times10^{-3})(0.1+0.00315)^2 + \frac{1}{12}(0.1)(0.00625)^3\right]$

$= 13.7 \times 10^{-6} m^4$

Material Property ; $E_a = 75\, GN/m^2$ $E_s = 200\, GN/m^2$

$\left(\dfrac{d\phi}{ds}\right)_1 = \dfrac{M_b}{E_a I_a}$ $\left(\dfrac{d\phi}{ds}\right)_2 = \dfrac{M_b}{E_a I_a + E_s I_s}$

Stiffness ; $(k_b)_1 = E_a I_a$ $(k_b)_2 = E_a I_a + E_s I_s$

$\Delta k_b = \dfrac{(k_b)_2 - (k_b)_1}{(k_b)_1} = \dfrac{E_s I_s}{E_a I_a} = 1.54$

$\sigma_x = -E \cdot y \cdot \left(\dfrac{d\phi}{ds}\right)$

따라서 $\sigma_a = -E_a y_a \left(\dfrac{d\phi}{ds}\right)_2$, $\sigma_s = -E_s y_s \left(\dfrac{d\phi}{ds}\right)_2$

$\therefore (\sigma_s/\sigma_a)_{max} = \dfrac{E_s (y_s)_{max}}{E_a (y_a)_{max}} = 2.833$

Ans. Ratio of stiffness ; 2.54 $(\sigma_s/\sigma_a)_{max} = 2.833$

7.49 material property ; $E_a = 75\, GN/m^2$ $E_s = 200\, GN/m^2$

$(\sigma_Y)_a = 109.1\, MN/m^2$ $(\sigma_Y)_s = 254.5\, MN/m^2$

$$I_s = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(0.048^4 - 0.042^4) = 1078 \times 10^{-7}\ (m^4)$$

$$I_a = \frac{\pi}{64}(0.042^4 - 0.03^4) = 1.13 \times 10^{-7}\ (m^4)$$

$$k = \frac{M_b}{E_a I_a + E_s I_s} = \frac{M_b}{30035}$$

$$\therefore (\sigma_y)_s = k E_s (y_s)_{max} = \frac{(M_b)_s}{30035}(200 \times 10^9)(0.024) = 2545 \times 10$$

$$\therefore (M_b)_s = 1592\ N\text{-}m$$

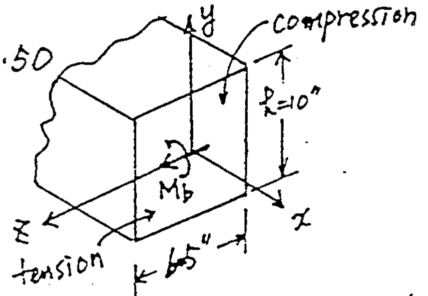
$$(\sigma_y)_a = k E_a (y_a)_{max} = \frac{(M_b)_a}{30035}(75 \times 10^9)(0.021) = 109.1 \times 10^6$$

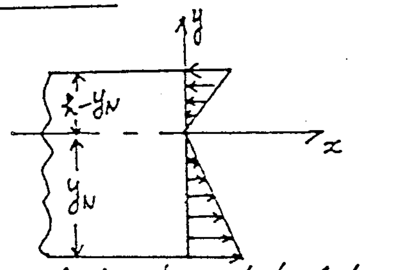
$$\therefore (M_b)_a = 2081\ N\text{-}m$$

따라서 M_b는 $(M_b)_s$ 와 $(M_b)_a$ 중에서 작은 값이 된다.

Ans. $M_b = 1592\ N\text{-}m$

7.50





Elastic stress distribution

$E_T = E_{tension} = 7.5 \times 10^6\ psi$; $Y_T = 10.000\ psi$

$E_C = E_{compressive} = 30 \times 10^6\ psi$; $Y_C = 20.000\ psi$

y_N = height of neutral axis ; ρ = radius of curvature

Geometry ; $\varepsilon_x = -y/\rho$

Equilibrium ; $\sigma_y = \sigma_z = 0$; $\Sigma F_x = \int_{-y_N}^{h-y_N} b\,\sigma_x\,dy = 0$

$$\Sigma M_z = M_b + \int_{-y_N}^{h-y_N} b\,\sigma_x\,y\,dy = 0$$

Stress-Strain ; Compression ; $\sigma_x = E_C \varepsilon_x$

Tension ; $\sigma_x = E_T \varepsilon_x$

Yield Criterion ; Compression ; $|\sigma_x| = Y_C$

Tension ; $|\sigma_x| = Y_T$

따라서 $\Sigma F_x = -\frac{E_c}{\rho}\int_0^{h-y_N} by\,dy - \frac{E_T}{\rho}\int_{-y_N}^{0} by\,dy = 0$

즉, $-\frac{E_c \cdot b}{\rho}\frac{1}{2}(h-y_N)^2 + \frac{E_T \cdot b}{\rho}\frac{1}{2}y_N^2 = 0$

$\therefore y_N = \frac{2}{3}h$

따라서 $\Sigma M_z = M_b - \frac{E_c}{\rho}\int_0^{h/3} by^2 dy + \frac{E_T}{\rho}\int_{-2h/3}^{0} by^2 dy = 0$

$\therefore M_b = \frac{bh^3}{81\rho}[E_c + 8E_T]$

Max. Compressive stress $= \sigma_c = \frac{E_c(h/3)}{\rho} = E_c\left(\frac{h}{3}\right)\cdot\frac{81M_b}{bh^3[E_c+8E_T]}$

Max. tensile stress $= \sigma_T = \frac{E_T(2h/3)}{\rho} = E_T\left(\frac{2h}{3}\right)\frac{81M_b}{bh^3[E_c+8E_T]}$

따라서 $M_b = \frac{bh^2}{27}\sigma_c\left[1+8\frac{E_T}{E_c}\right] = \frac{bh^2}{27}\frac{\sigma_T}{2}\left[\frac{E_c}{E_T}+8\right]$

$\frac{bh^2}{27} = \frac{(5)(100)}{27} = 18.5$; $\left[1+8\frac{E_T}{E_c}\right] = 1+8\cdot\frac{7.5}{30} = 3$

$\left[\frac{E_c}{E_T}+8\right] = \frac{30}{7.5}+8 = 12$

따라서, $|\sigma_c| = Y_c$; $M_b = (18.5)(20000)(3) = 1.11\times10^6$ lb-in

$\sigma_T = Y_T$; $M_b = (18.5)\left(\frac{10000}{2}\right)(12) = 1.11\times10^6$ lb-in

Ans. $(M_b)_{max} = 1.11\times10^6$ lb-in

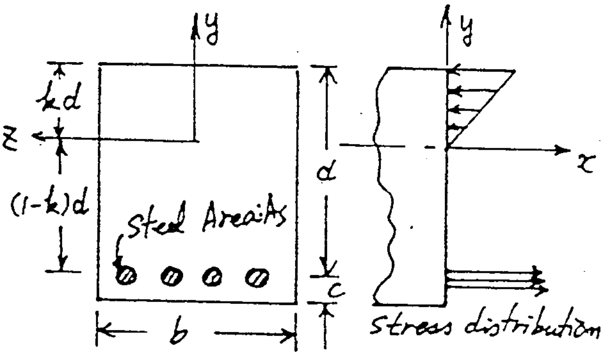
7.51 Geometry ; $\varepsilon_x = -\frac{y}{\rho}$

Equilibrium ;

$\sigma_y = \sigma_z = 0$

$\Sigma F_x = \int \sigma_x dA = 0$

$\Sigma M_z = M_b + \int \sigma_x y\,dA = 0$



Stress-Strain ; Concrete ; Compression $\sigma_x = E_c\varepsilon_x$, Steel ;

Tension $\sigma_x = 0$ $\qquad$ $\sigma_x = E_s\varepsilon_x$

steel에 대해서 strain에 대한 평균치를 구하면.

$$\Sigma F_x = -\int_0^{kd} \frac{E_c \cdot y}{\rho} b\,dy + \frac{E_s(1-k)d}{\rho} A_s = 0$$

$$E_s A_s (d - kd) - E_c b \frac{(kd)^2}{2} = 0 \quad — ①$$

$$\Sigma M_z = M_b - \int_0^{kd} \frac{E_c y}{\rho} b y\,dy - E_s \frac{(1-k)d}{\rho} A_s (1-k)d = 0$$

giving $\dfrac{1}{\rho} = \dfrac{M_b}{\frac{1}{3}E_c b k^3 d^3 + E_s A_s (1-k)^2 d^2}$

$$\sigma_{steel} = \sigma_s = \frac{E_s(1-k)d}{\rho} = \frac{M_b E_s (1-k)d}{\frac{1}{3}E_c b k^3 d^3 + E_s A_s (1-k)^2 d^2} = \frac{M_b}{A_s d\left[\frac{1}{3}\frac{E_c b k^3 d}{A_s E_s (1-k)} + (1-k)\right]}$$

①식으로 부터

$$\frac{E_c b d}{E_s A_s} = \frac{2(1-k)}{k^2} \;;\; \therefore \frac{1}{3}\frac{E_c b k^3 d}{A_s E_s (1-k)} = \frac{2}{3}k$$

$$\sigma_s = \frac{M_b}{A_s d\left[\frac{2}{3}k + 1 - k\right]} = \frac{M_b}{A_s d\left[1 - \frac{k}{3}\right]} \quad \text{- Q.E.D -}$$

$$(\sigma_{concrete})_m = (\sigma_c)_m = E_c \frac{kd}{\rho} = \sigma_s \frac{kd}{(1-k)d}\frac{E_c}{E_s} = \sigma_s \frac{E_c}{E_s}\left(\frac{k}{1-k}\right)$$

①식으로 부터

$$\frac{E_c}{E_s} = \frac{2A_s(1-k)}{b d k^2} \;;\; \therefore (\sigma_c)_m = \frac{M_b}{A_s d\left[1-\frac{k}{3}\right]} \cdot \frac{2A_s(1-k)}{b d k^2} \cdot \frac{k}{1-k} = \frac{2M_b}{b d^2 k\left[1 - \frac{k}{3}\right]}$$

- Q.E.D -

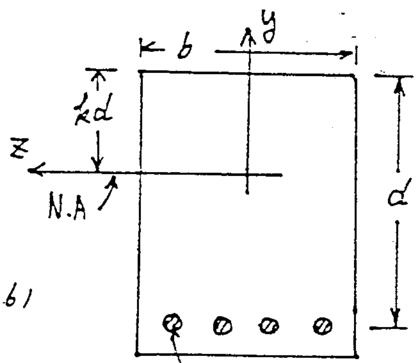
7.52 문제 (7.51) 참조.

$$E_s A_s (1-k) - \frac{1}{2}E_c b k^2 d = 0$$

또는 $k^2 + \dfrac{2E_s A_s}{E_c b d} k - \dfrac{2E_s A_s}{E_c b d} = 0$

$$\frac{2E_s A_s}{E_c b d} = \frac{(2)(30\times10^6)(2.2)}{(1.5\times10^6)(9)(16)} = 0.61$$

$\therefore k^2 + 0.61k - 0.61 = 0$ 따라서 $k = 0.475$



For $\sigma_s = 20000$ psi $\quad M_b = 20000 A_s d(1-\frac{k}{3})$

$= 20000(2.2)(16)(0.842) = 592000$ lb-in

$(\sigma_c)_{max} = 1350$ psi $\quad M_b = 1350 \cdot \frac{1}{2} bd^2 k(1-\frac{k}{3})$

$= 1350 \cdot (\frac{1}{2})(9)(256)(0.475)(0.842) = 625000$ lb-in

$\therefore (M_b)_{max} = 592000$ lb-in

Ans. $(M_b)_{max} = 592000$ lb-in

7.53 그림은 문제 (7.51) 참조.

$\sigma_s = \dfrac{E_s(1-k)d}{\rho}$; $(\sigma_c)_{max} = \dfrac{E_c kd}{\rho}$ $\quad \therefore \dfrac{\sigma_s}{(\sigma_c)_m} = \dfrac{1-k}{k}\left(\dfrac{E_s}{E_c}\right)$

$\dfrac{20000}{1350} = \dfrac{1-k}{k}\left(\dfrac{30\times10^6}{1.5\times10^6}\right) \quad \therefore k = 0.575$

문제 (7.51)의 ①로부터 ; $A_s = \frac{1}{2}\frac{E_c}{E_s}(bd)\cdot\frac{k^2}{1-k}$

$= \frac{1}{2}\left(\frac{1.5\times10^6}{30\times10^6}\right)(9)(16)\frac{(0.332)}{(0.425)} = 2.8$ in^2

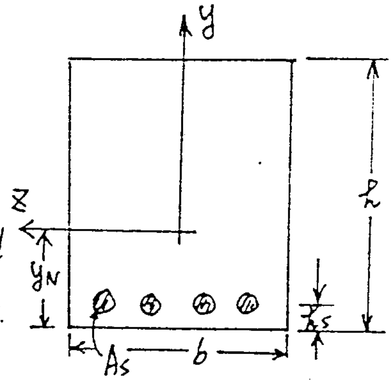
즉 $(5)\frac{\pi}{4}d_s^2 = 2.8$ $\quad$ (d_s ; diameter of steel bars)

$\therefore d_s = 0.84$ in

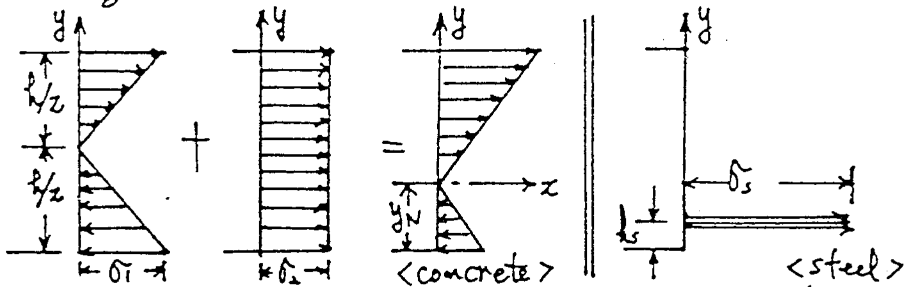
$M_b = \sigma_s A_s d(1-\frac{k}{3}) = 20000(2.8)(16)(1-\frac{0.575}{3}) = 728000$ lb-in

Ans. $d_s = 0.84$ in $\quad M_b = 728000$ lb-in

7.54 (a) Prestressing wire가 release해 지면 beam은 곡률 반경 ρ_1으로 굽게된다.
strain과 stress의 분포는 높이가 y_N인 중립축을 중심으로 직선적으로 될 것이다.
concrete에서의 응력 분포는 symmetric



bending stress 와 uniform stress 로 분해될 수 있다.



centerline 주변의 bending stress 와 uniform stress 에 의한 moment 는 "0" 이다.

Equilibrium ; $\Sigma F_x = \sigma_2 (bh) + \sigma_s A_s = 0$ —— ①

$$\Sigma M_z = -2\left[\left(\frac{\sigma_1}{2}\right)\left(\frac{h}{2}\right)(b)\right]\left[\frac{2}{3}\left(\frac{h}{2}\right)\right] + \sigma_s A_s \left(\frac{h}{2} - h_s\right) = 0 \quad \text{—— ②}$$

①식에서 $\sigma_2 = -\dfrac{\sigma_s A_s}{bh}$

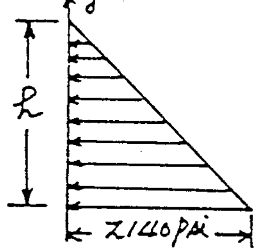
②식에서 $\sigma_1 = \dfrac{6\sigma_s A_s}{bh^2}\left(\dfrac{h}{2} - h_s\right)$

$A_s = 76\,\frac{\pi}{4}(0.15)^2 = 1.34\ in^2$; $b = 9''$; $h = 18''$; $h_s = 6''$

$\sigma_s = 130\,000\ psi$

$\therefore\ \sigma_2 = \dfrac{-(130000)(1.34)}{(9)(18)} = -1070\ psi$, $\sigma_1 = \dfrac{6(130000)(1.34)(9-6)}{(9)(18)(18)} = 1070\ psi$

결과적으로 concrete 에 걸리는 stress 의 분포는 ;



[$\frac{2h}{3} \geq h_s \geq \frac{h}{3}$ 일 때는 tensile stress 는 없다.]

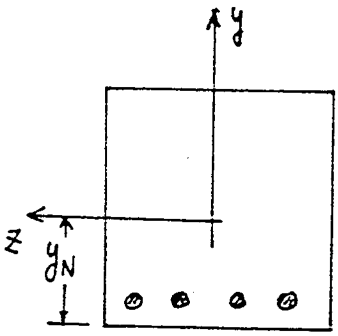
(b) Bending

Bending 에 의해서 발생되는 stress 를 중첩시킨다. concrete 에 tensile stress 가 발생한다고 가정하나, 결국에는 compressive prestresses 에 의해서 상쇄된다.

문제 (7.46) 에서

$$y_N = \frac{E_c \bar{y}_c A_c + E_s \bar{y}_s A_s}{E_c A_c + E_s A_s}$$

$$\frac{1}{\rho_b} = \frac{M_b}{E_c (I_{yy})_c + E_s (I_{yy})_s}$$



($\frac{1}{\rho_b}$; moment M_b 에 의해 생기는 곡률의 변화량)

$\bar{y}_c = 9''$; $A_c = (9)(18) = 162 \text{ in}^2$

$\bar{y}_s = 6''$; $A_s = 1.34 \text{ in}^2$

$E_c = 1.5 \times 10^6$ psi ; $E_s = 30 \times 10^6$ psi

$$\therefore y_N = \frac{(1.5\times10^6)(9)(162) + (30\times10^6)(6)(1.34)}{(1.5\times10^6)(162) + (30\times10^6)(1.34)} = 8.5''$$

$$(I_{yy})_c = \left[\frac{1}{12}(9)(18)^3 + (9)(18)(9-8.5)^2\right] = 4170 \text{ in}^4$$

$$(I_{yy})_s = \left[\ 0 \quad + (1.34)(8.5-6)^2\right] = 8.4 \text{ in}^4$$

$$E_c(I_{yy})_c + E_s(I_{yy})_s = 10^6\left[(1.5)(4170) + (30)(8.4)\right] = 6512\times10^6 \text{ lb-in}^2$$

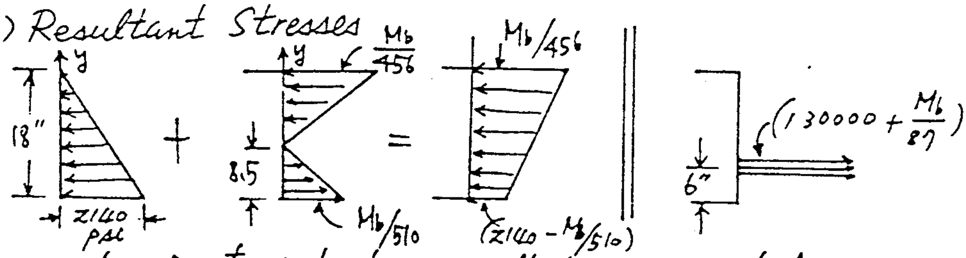
$$(\sigma_x)_{concrete} = -\frac{M_b(1.5\times10^6)y}{6512\times10^6}$$ (y: 중립축에서 부터 측정한 거리)

$$\text{Max. Compressive stress} = \frac{M_b(1.5\times10^6)(9.5)}{6512\times10^6} = \frac{M_b}{456}$$

$$\text{Max. Tensile stress} = \frac{M_b(1.5\times10^6)(8.5)}{6512\times10^6} = \frac{M_b}{510}$$

$$\text{Bending Stress in Steel} = \frac{M_b(30\times10^6)(8.5-6)}{6512\times10^6} = \frac{M_b}{87}$$

(c) Resultant Stresses



concrete ; prestress + bending = resultant steel

Requirments

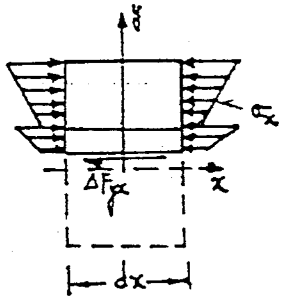
1) No tension in concrete ; $2140 - \frac{M_b}{510} > 0 \quad \therefore M_b < 1090000$ lb-in

2) Concrete compressive stress < 2250 psi ; $\frac{M_b}{450} < 2250 \quad \therefore M_b < 1030000$ lb-in

3) Steel tensile stress < 150 000 psi ; $130000 + \frac{M_b}{87} < 150000 \quad \therefore M_b < 1740000$ lb-in

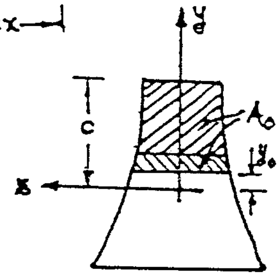
$(M_b)_{max} = 1030000$ lb-in

7.55 문제 (7.46) 으로 부터



$$\sigma_x = \frac{-E_i M_b \cdot y}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2}$$

$$\Delta F_{yx} = \left[\int_{A_0} \sigma_x \, dA\right]_{x+dA} - \left[\int_{A_0} \sigma_x \, dA\right]_x$$



또는 $\frac{dF_{yx}}{dx} = \frac{d}{dx} \int_{A_0} \sigma_x \, dA$

$$= \frac{d}{dx}\left[-\frac{M_b}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2} \int_{A_0} E y \, dA\right]$$

$$= -\left(\frac{dM_b}{dx}\right)\left(\frac{1}{E_1 (I_{yy})_1 + E_2 (I_{yy})_2}\right)\left(\int_{A_0} E y \, dA\right)$$

$-\frac{dM_b}{dx} = V$ 또 $\tau_{xy} = \tau_{yx} = \frac{q_{yx}}{b} = \frac{1}{b}\frac{dF_{yx}}{dx}$

$$\therefore \tau_{xy} = \tau_{yx} = \frac{V}{b[E_1 (I_{yy})_1 + E_2 (I_{yy})_2]} \int_{A_0} E y \, dA \qquad \text{- Q.E.D -}$$

7.56 문제 (7.48) 에서

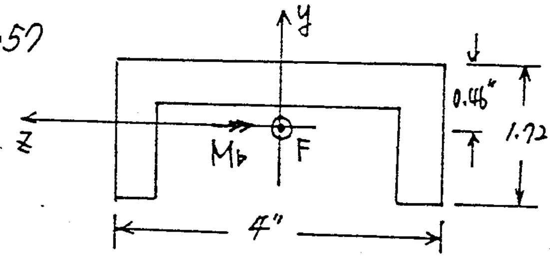
$$q_{yx} = \frac{V}{E_1 I_1 + E_2 I_2} \int_{A_0} E_1 y \, dA = \frac{V}{E_1 I_1 + E_2 I_2} E_1 \left[\frac{y^2}{2}\right]_{0.1}^{0.10625}$$

$$= \frac{(27000)(200 \times 10^9)(\frac{1}{2})[(0.10625)^2 - (0.1)^2]}{(75 \times 10^9)(23.7 \times 10^{-6}) + (200 \times 10^9)(13.3 \times 10^{-6})} = 784300 \, (N/m)$$

$$\therefore S = \frac{F \times 2}{q_{yx}} = \frac{2 \times 22000}{784300} = 0.0561\ (m)$$

Ans. $S = 0.0561\ m$

7.57



$A = 2.12\ in^2$

$I_{zz} = 0.44\ in^4$

$F = 3000\ lb$

$M_b = Fl = 3000 \times 1.71 = 5130\ lb\text{-}in$

Direct Stress ; $(\sigma_t)_1 = \frac{F}{A} = \frac{3000}{2.12} = 1415\ psi$

Bending Stress ; $(\sigma_t)_2 = \frac{M_b \cdot y}{I_{zz}} = \frac{5130 \times 0.46}{0.44} = 5363\ psi$

$$(\sigma_c)_2 = \frac{5130}{0.44}(0.46 - 1.72) = -14{,}690\ psi$$

$\therefore (\sigma_c)_{max} = -14{,}690 + 1415 = -13{,}275\ psi$

$(\sigma_t)_{max} = 5360 + 1415 = 6775\ psi \quad \therefore \sigma_{max} = 13275\ psi$

Ans. $\sigma_{max} = 13275\ psi$

7.58 문제(7. 23) 참조 $\sigma_{max} = \frac{\gamma \cdot L^2}{8t}$

$2\pi r t \cdot \sigma_t = \pi r^2 p \qquad \therefore \sigma_t = \frac{p \cdot r}{2t} \qquad \sigma_{max} = \sigma_t$ 에서

따라서 $\frac{\gamma L^2}{8t} = \frac{p \cdot r}{2t} \qquad \therefore p = \frac{\gamma L^2}{4r}$

Ans. $\sigma_{max} = \frac{\gamma L^2}{8t}, \quad p = \frac{\gamma L^2}{4r}$

7.59 cutting force를 mid-depth of cut에 작용하는 point load로 치환하자. 그러면 collet에 작용하는 moment와 torques는

a) due to 1600 N force ; $M_t = -(1600)(0.005) = -8\ N \cdot m$

$M_{bz} = (1600)(0.075) = 120\ N\text{-}m$

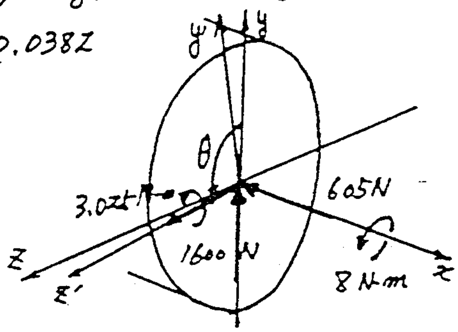
b) due to 605N force ; $M_{by} = -(605)(0.005) = -3.025$ N-m

M_{by}와 M_{bz}는 하나의 moment $M_b = (M_{by}^2 + M_{bz}^2)^{1/2}$로 합성할 수 있다. y축에 대한 $\theta = \tan^{-1}(M_{bz}/M_{by})$ 각으로 합성된다.

$\therefore M_b = \sqrt{(3.025)^2 + (120)^2} = 120.0382$

$\theta = \tan^{-1}(120/3.025)$

따라서 결과적인 moment, force, torque는 우편의 그림과 같다.



새로운 좌표축 y', z'을 그림과 같이 잡으면 resultant bending moment는 z'을 중심으로 발생된다. M_b는 관성의 주축을 중심으로 작용한다.

$I_{yy'} = \frac{\pi}{4}\left(\frac{0.0125}{2}\right)^4$; $I_x = 2(I_{yy'})$

$A = \pi\left(\frac{0.0125}{2}\right)^2$

Stresses;

1) 605 N ; $\sigma_x = -\frac{F}{A} = -4.93$ MN/m²

2) 1600N ; $\tau_{xy'} = \frac{V \cdot Q}{b I_{yy'}}$ 에서 Q는 $y' = \pm(0.0125/2)$ 에서 0

$y = 0$ 에서 $Q_{max} = \frac{2}{3} r^3$

$$\therefore (\tau_{xy})_{max} = \frac{(1600) \times \left(\frac{2}{3}\right)\left(\frac{0.0125}{2}\right)^3}{(0.0125)(0.0125)^4 \frac{\pi}{64}} = 17.38 \text{ MN/m}^2$$

3) M_b ; $\sigma_x = \frac{M_b \cdot y'}{I_{y'y'}}$; $y' = \pm(0.0125/2)$ 에서 max.

$y = 0$ 에서 "0"

$$\therefore (\sigma_x)_{max} = \frac{(120.0382)(0.0125/2)}{\frac{\pi}{64}(0.0125)^4} = 626 \text{ MN/m}^2$$

4) M_t ; $\tau = \frac{M_t(r)}{I_x}$; $r = \pm(0.0125/2)$ 에서 max

$r = 0$ 에서 "0"

$$\therefore \tau_{max} = \frac{(8) \cdot (0.0125/2)}{2 \cdot \frac{\pi}{64}(0.0125)^4} = 20.86 \text{ MN/m}^2$$

Resulting Stresses; linear superposition 을 하기 위해 문제를 단순화 시키자. 즉, bending stress 가 다른 stress 보다 훨씬 크기 때문에 max. shear stress 의 위치는 max. bending stress 의 위치와 일치하게 된다고 볼 수 있다.

따라서 그 위치는 ; $y' = (0.0125/2)$, $z' = 0$ 이다.

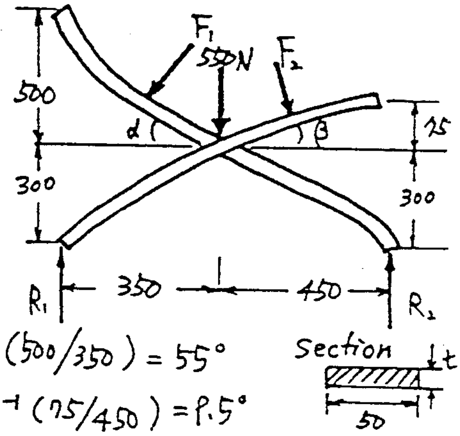
$$\sigma_x = -626 - 4.93 = -630.92 \quad MN/m^2$$

$$\tau_{xz} = 20.86 \quad MN/m^2$$

$$\therefore \tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2} = 316.15 \ MN/m^2$$

Ans. $\tau_{max} = 316.15 \ MN/m^2$

7.60 one side frame 에 대해 하중 상태는 우측 그림과 같다고 생각하자.



$\therefore$ 평형조건; $R_1 = 310\,N$

$R_2 = 240\,N$

$$\begin{cases} F_1 \sin\alpha = F_2 \sin\beta \\ F_1 \cos\alpha = 550 - F_2 \cos\beta \end{cases} \quad \begin{cases} \alpha = \tan^{-1}(500/350) = 55° \\ \beta = \tan^{-1}(75/450) = 9.5° \end{cases}$$

$\therefore F_1 = 0.2 F_2 = 100\,N$

Bending Moment

$$M_{12} = 100 \times \sqrt{(0.5)^2 + (0.35)^2} \times \frac{1}{2} = 31 \ (N\text{-}m)$$

$$M_{23} = 500 \times \sqrt{(0.075)^2 + (0.45)^2} \times \frac{1}{2} = 114 \ (N\text{-}m)$$

$$M_{24} = 310 \times 0.35 = 108 \ (N\text{-}m)$$

$$M_{25} = 240 \times 0.45 = 108 \ (N\text{-}m)$$

$\therefore (M_b)_{max} = 114 \ (N\text{-}m) \qquad \sigma_Y = 600 \ MN/m^2$

$$\sigma_{max} = \frac{(M_b)_{max} \cdot y_{max}}{I_{zz}} = \frac{\sigma_Y}{2}$$

$$\frac{114 \times 0.5t}{\frac{0.05}{12} \times t^3} = 300 \times 10^6 \qquad \therefore t = 6.75 \times 10^{-3}\ (m)$$

Ans. $t = 6.75 \times 10^{-3}$ m

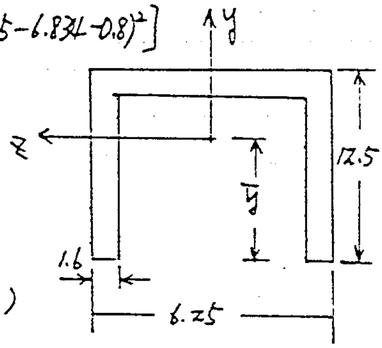
7.61 1) Compressive Stress due to direct load

2) Bending Stress

$$\bar{y} = \frac{\frac{1}{2}(12.5)^2(6.25) - \frac{1}{2}(10.9)^2(3.05)}{12.5 \times 6.25 - 10.9 \times 3.05} = 6.834\ (mm).$$

$$I_{yy} = 2\left[\frac{1}{12}(1.6)(12.5)^3 + (1.6)(12.5)(6.25-6.834)^2 + \frac{1}{12}(3.05)(1.6)^3 + (3.05)(1.6)(12.5-6.834-0.8)^2\right]$$

$$= 651.4\ (mm^4)$$



$$A = [12.5 + 12.5 + (6.25 - 3.2)](1.6)$$

$$= 44.88\ (mm^2)$$

$$M_b = -300 \times 100 = -30000\ (N\text{-}mm)$$

Compressive Stress ; $\sigma_F = \frac{F}{A} = \frac{-300}{44.88} = -6.684\ (N/mm^2)$

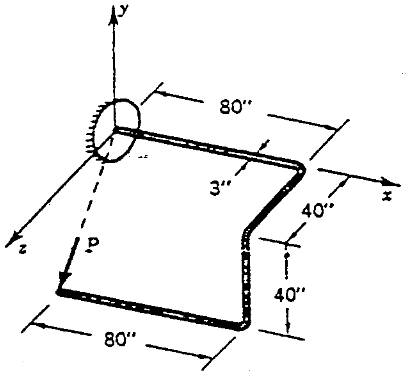
Bending Stress ; $\sigma_M = \frac{M_b \cdot y_{max}}{I_{yy}} = \frac{-30000\,(6.843)}{651.1} = -315.3\ (N/mm^2)$

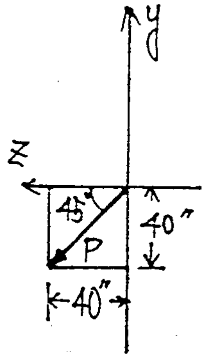
$$\therefore \sigma_{max} = -6.684 - 315.3 = -322\ MN/mm^2$$

이값은 yielding stress의 약 54%에 해당한다. 따라서 이 값은 상당히 커서 각압증에 additional stress가 발생한다. 따라서 300 N은 너무 큰 하중이다.

7.62 Rod에는 Bending moment M_b와 twisting moment M_t가 동시에 작용한다. $\sigma_b = \frac{M_b \cdot d}{2I_{yy}}$, $\tau = \frac{M_t \cdot d}{4I_{yy}} = \frac{M_t \cdot d}{2I_x}$

Von Mises의 yielding criterion에 의하면 yielding은 다음과 같은 상태에서 발생한다.





$$\bar{\sigma} = \sqrt{\frac{1}{2}(\sigma_b^2+\sigma_b^2)+3(\tau)^2} = Y$$

(Y; yield strength in tension)

$$\bar{\sigma} = \sqrt{\left(\frac{M_b\cdot d}{2I_{yy}}\right)^2+3\left(\frac{M_t\cdot d}{4I_{yy}}\right)^2}$$

$$= \frac{M_b\cdot d}{2I_{yy}}\left[1+\frac{3}{4}\left(\frac{M_t}{M_b}\right)^2\right]^{1/2}$$

B. C. D 에 $\bar{\sigma}$를 계산한다. 그중에서 maximum이 있을 것이다.

1) just before B ; $M_b = 80P,\ M_t = 0$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)80P$

2) just after B ; $M_b = \frac{80P}{\sqrt{2}},\ M_t = \frac{80P}{\sqrt{2}}$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)106P$

3) just before C ; $M_b = 63.2P,\ M_t = 56.8P$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)87P$

4) just after C ; $M_b = 63.2P\ \ M_t = 56.8P$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)87P$

5) just before D ; $M_b = \frac{80}{\sqrt{2}}P,\ M_t = \frac{80}{\sqrt{2}}P$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)106P$

6) just after D ; $M_b = 80P,\ M_t = 0$ $\quad \bar{\sigma} = \left(\frac{d}{2I_{yy}}\right)80P$

$$(\bar{\sigma})_{max} = 106P\left(\frac{d}{2I_{yy}}\right)$$

3" diameter rod에 대해 ; $I_{yy} = \frac{3.14}{4}\left(\frac{3}{2}\right)^4 = 3.98\ in^4$

4130 HT에 대해 ; $Y = 190000\ psi$

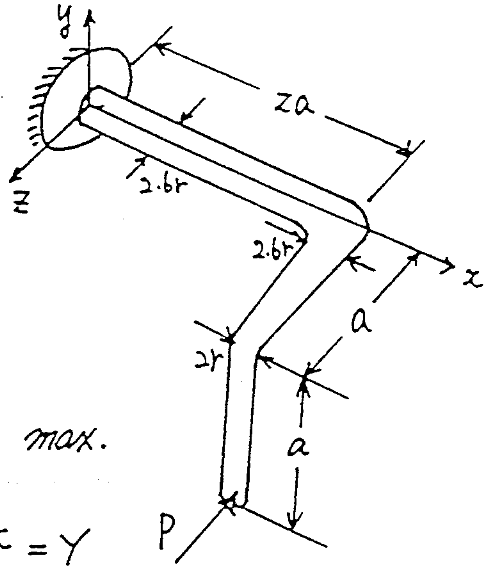
$\therefore \frac{(106)(P)(3)}{(2)(3.98)} = 190000 \qquad \therefore P = 4750\ lb$

Ans. $P = 4750\ lb$

7.63 Bending과 Torsion의 합성에 의해서 Stress가 발생한다.
(Direct Stress는 무시한다.)

AB; Bending에 의한 B점에서의 max. stress

BC; Twisting moment=0;
Bending moment=const.
∴ Max. stress는 minimum section에서 생긴다. 즉 B점에서 생긴다. 이 Max. stress는 AB에 대한 stress와 같다.



CD; Twisting moment = const.
Bending moment는 D점에서 max.
∴ max. stresses at D

Von Mises Yield criterion; $\bar{\sigma} = Y$

문제(7.62)에서 $\bar{\sigma} = \dfrac{M_b \cdot d}{2 \cdot I_{yy}}\left[1+\dfrac{3}{4}\left(\dfrac{M_t}{M_b}\right)\right]^{1/2}$

$\underline{B}$; $M_b = Pa$; $M_t = 0$; $I_{yy} = \frac{\pi}{4}r^4$; $\therefore \bar{\sigma} = \sigma_b = \dfrac{4Pa}{\pi r^3} = 1.26\dfrac{Pa}{r^3}$

$\underline{D}$; $M_b = 2Pa$; $M_t = Pa$; $I_{yy} = \frac{\pi}{4}(1.3r)^4$

$$\therefore \bar{\sigma} = \frac{(2Pa)(2.6r)}{2(\frac{\pi}{4})(1.3r)^4}\left[1+\frac{3}{4}\left(\frac{1}{2}\right)\right]^{1/2} = 1.36\frac{Pa}{r^3}$$

$$\therefore (\bar{\sigma})_{max} = 1.36\frac{Pa}{r^3} = Y \quad (\text{for yielding})$$

따라서 yielding이 일어날 조건; $P = 0.735\dfrac{r^3 Y}{a}$

만일, Tresca (max. shear stress) yield criterion을 사용하면 $P = 0.77\dfrac{r^3 Y}{a}$

Ans. Mises; $P = 0.735\, r^3Y/a$ Tresca; $P = 0.77\, r^3Y/a$

7.64 Total stress는 Bending, Torsion, Internal, shear stress

등을 중첩함으로써 얻어진다.

Equilibrium ; 1) Piston : $F = p\pi\left(\frac{R}{\sqrt{6}}\right)^2$ 즉 $p = \frac{6F}{\pi R^2}$

2) Tube at built in end ; $M_{bz} = -5FR$

$M_t = -6FR$

Geometry ; $I_{yy} = \pi R^3 t$; $I_x = 2\pi R^3 t$

Stresses ; 1) Shear ; stress = 0. ($\because Q = \int y\,dA = 0$ for N)

2) Bending ; $\sigma_x = -\frac{M_{bz}\cdot y}{I_{yy}} = -\frac{(-5FR)R}{\pi R^3 t} = \frac{5F}{\pi R t}$

3) Torsion ; $\tau_{xz} = M_t \cdot r / I_x = -(6FR)(R)/2\pi R^3 t$

$= -3F/\pi R t$.

4) Internal Pressure ;

Circumferential Stress ; $\sigma_z = \frac{PR}{t} = \frac{6F}{\pi R t}$

Axial Stress ; $\sigma_x = \frac{PR}{2t} = \frac{3F}{\pi R t}$

Total Stresses ; $\sigma_x = \frac{5F}{\pi R t} + \frac{3F}{\pi R t} = \frac{8F}{\pi R t}$

$\sigma_z = \frac{6F}{\pi R t}$ $\qquad \tau_{xz} = \frac{-3F}{\pi R t}$

$(t/R) \ll 1$ 이므로 outer stress는 무시한다.

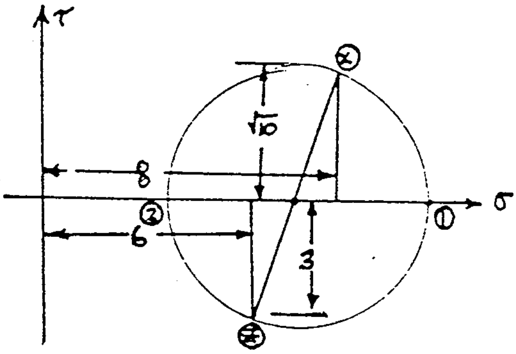
Mohr's 원; x-z plane에 stress/$(F/\pi R t)$로 그린다.

$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}$

$\sigma_{max} = \sigma_1 = \frac{F}{\pi R t}(7+\sqrt{10})$

$\sigma_{min} = \sigma_y = 0$

$\therefore \tau_{max} = \frac{F}{2\pi R t}(7+\sqrt{10})$

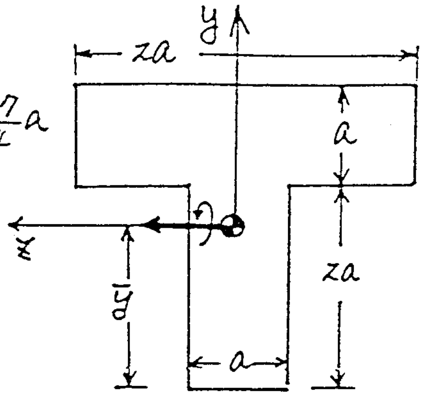


항복에 대해서 $\tau_{max} = Y/2$

$\therefore F = \frac{\pi R t Y}{7+\sqrt{10}} = 0.31RtY$ $\qquad$ Ans. $F = 0.31RtY$

7.65 $(\sigma_x)_{max} = Y$ 일때 $M_Y = M_b$

$$\bar{y} = \frac{(2a)(a)(a)+(2a)(a)(\frac{5a}{2})}{(2a)(a)+(2a)(a)} = \frac{7a^3}{4a^2} = \frac{7}{4}a$$



$$I_{yy} = \left[\frac{1}{12}(a)(2a)^3 + (2a)(a)(\tfrac{3}{4}a)^2\right] + \left[\frac{1}{12}(2a)(a)^3 + (2a)(a)(\tfrac{3a}{4})^2\right] = \frac{37}{12}a^4$$

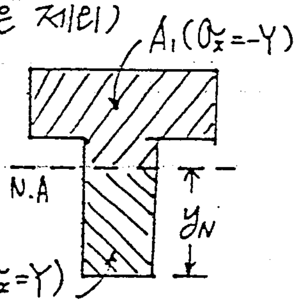
$$y_{max} = \frac{7}{4}a$$

$$(\sigma_x)_{max} = \frac{(M_b)(\frac{7a}{4})}{\frac{37}{12}a^4} = Y \quad \therefore M_b = M_Y = (\tfrac{37}{12})(\tfrac{4}{7})Ya^3 = \frac{37}{21}Ya^3$$

Fully Plastic ; $\sigma_x = \pm Y$ ($\sigma_x = 0$ 인 중립축은 제외)

Equilibrium ; $\Sigma F_x = 0$; $-YA_1 + YA_2 = 0$

$\Sigma M_z = 0$; $M_b + \int \sigma_x y\, dA = 0$

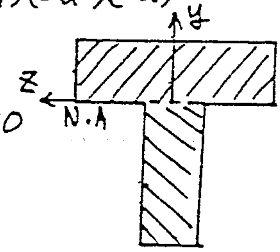


첫번째 방정식에서 $y_N = 2a$

두번째 방정식에서 $M_b = M_L$

따라서 $M_b = -\int \sigma_x y\, dA = -(-Y)(2a^2)(\frac{a}{2}) - (Y)(2a^2)(-a)$

$= 3Ya^3 = M_L$



$\therefore K = M_L/M_Y = (3Ya^3)/(\frac{37}{21}Ya^3) = 1.70$

Ans. $K = 1.70$

7.66 $M_L = -\int_A \sigma_x y\, dA = Y_1 \int_1 y\, dA + Y_2 \int_2 y\, dA$

$$\int y\, dA = 2\int_0^{\pi} (F d\theta)(t)(F \sin\theta) = 4F^2 t$$

$$\therefore M_L = (109.1\times10^6)(4)\left(\frac{0.021+0.015}{2}\right)^2(0.006)$$

$$+ (254.5\times10^6)(4)\left(\frac{0.024+0.021}{2}\right)^2(0.003) = 2394\ (N\text{-}m)$$

Ans. $M_L = 2394$ N-m

7.67 2024-T4 Aluminum ; $Y = 55000$ psi

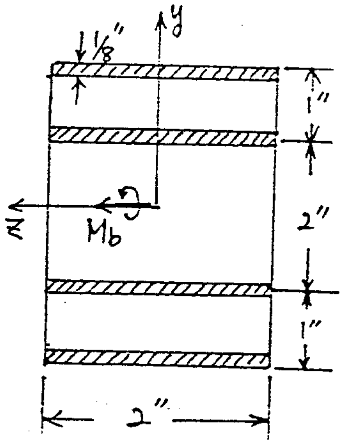
$\epsilon_y = 55000/11\times10^6 = 0.005$: yield strain

strain ϵ_x 는 중립축으로 부터의 거리에 비례한다. ④에서의 strain $= 0.016$

①에서의 strain $= -0.016$, ②에서의 strain $= -0.008$

③에서의 strain $= 0.008$. 이것은 모두 ϵ_y보다 크다 따라서, 모든 strip은 plastic 상태이다.

$\therefore$ Moment M_b는 limit moment M_L이 된다.



Stresses ; $(\sigma_x)_1 = (\sigma_x)_2 = -55000$ psi

$(\sigma_x)_3 = (\sigma_x)_4 = 55000$ psi

$\Sigma M_z = 0$; $M_b = -\int \sigma_x y\,dA = -\sum_i (\sigma_x)_i A_i \bar{y}_i$

$= (2)(55000)[(2)(\frac{1}{8})(2) + (2)(\frac{1}{8})(1)] = 82500$ lb-in

Unloading ; Unloading은 elastic으로 가정한다. 그리고 residual stress를 구하기 위해, moment $-M_b$에 의해서 발생되는 elastic stress를 위에서 구한 loading stress에 더해 준다.

$$I_{yy} = 2[(2)(\frac{1}{8})(2)^2 + (2)(\frac{1}{8})(1)^2] = 2.5 \text{ in}^4$$

Moment $-M_b$에 의한 Elastic stress

$$(\sigma_x)_i = -\frac{(-M_b)y_i}{I_{yy}} = \frac{M_b \cdot y_i}{I_{yy}}$$

따라서 $(\sigma_x)_1 = (82500)(2)/(2.5) = 66000$ psi $= -(\sigma_x)_4$

$(\sigma_x)_2 = (82500)(1)/(2.5) = 33000$ psi $= -(\sigma_x)_3$

$\therefore$ Residual Stress

$(\sigma_x)_1 = -55{,}000 + 66{,}000 = 11{,}000$ psi

$(\sigma_x)_2 = -55000 + 33{,}000 = -22{,}000$ psi

$(\sigma_x)_3 = 55{,}000 - 33{,}000 = 22{,}000$ psi

$(\sigma_x)_4 = 55{,}000 - 66{,}000 = -11{,}000$ psi

Ans.

7.68 Wire diameter ; $d = 0.036''$

From postage scale test ; $M_L = 0.61$ lb-in

초기 항복에 대한 탄성해 ; $M_b = M_Y = \left(\frac{\pi d^4}{64}\right)\left(\frac{2}{d}\right)Y = \frac{\pi d^3}{32}Y$

<Table 7.1> 에서 ; $\frac{M_L}{M_Y} = 1.3$

$\therefore M_L = 1.3 M_Y = \frac{(1.3)\pi d^3 Y}{32} = 0.61$ lb-in

즉 $Y = \frac{(0.61)(32)}{(1.3)(\pi)(0.036)^3} = 102{,}000$ psi

여기에 적당한 재료는 1020 CR steel 이다.

Ans. $Y = 102{,}000$ psi material ; 1020 CR steel

7.69 (a) Butt-brazed joint

450 Newton 에 의한 tensile stress ; $\sigma_t = \frac{450}{0.001 \times 0.02} = 18\ MN/m^2$

$(\sigma_b)_p = KEy = \frac{1}{0.3}(110\times10^9)(0.0005) = 183.3\ MN/m^2$

$(\sigma_b)_s = KEy = \frac{1}{0.3}(78\times10^9)(0.0005) = 130\ MN/m^2$

$\therefore \sigma_p = \sigma_t + (\sigma_b)_p = 201.3\ MN/m^2 <$ Fatigue limit, Tensile strength

$\sigma_s = \sigma_t + (\sigma_b)_s = 148\ MN/m^2 >$ Fatigue limit

$\therefore$ Silver-brazing alloy 가 파괴 되었다.

$t = 1.2\,mm$ 인 경우.

$\sigma_t = \frac{450}{0.0012 \times 0.025} = 15\ MN/m^2$

$(\sigma_b)_p = KEy = \frac{1}{0.3}(110\times10^9)(0.0006) = 220\ MN/m^2$

$(\sigma_b)_s = KEy = \frac{1}{0.3}(78\times10^9)(0.0006) = 156\ MN/m^2$

$\therefore\ \sigma_p = \sigma_t + (\sigma_b)_p = 235\ MN/m^2$

$\sigma_s = \sigma_t + (\sigma_b)_s = 171\ MN/m^2$

따라서 Tensile Strength $> \sigma_p >$ Fatigue Limit

Tensile Strength $> \sigma_s >$ Fatigue Limit

$\therefore$ 불안정하다. ($\because \sigma_p . \sigma_s$ 가 Fatigue Limit 보다 크다)

(b) $t = 0.3\,mm$ 일 때;

$$\sigma_t = \frac{450}{0.0003 \times 0.025} = 60\ MN/m^2$$

$$(\sigma_b)_p = \frac{1}{0.3}(110\times10^9)(0.00015) = 55\ MN/m^2$$

$$(\sigma_b)_s = \frac{1}{0.3}(78\times10^9)(0.00015) = 39\ MN/m^2$$

$\sigma_p = \sigma_t + (\sigma_b)_p = 115\ MN/m^2 <$ Fatigue Limit

$\sigma_s = \sigma_t + (\sigma_b)_s = 99\ MN/m^2 <$ Fatigue Limit

$\therefore$ 적당하다. ($\because \sigma_p . \sigma_s$ 가 Fatigue Limit 보다 작다)

(c) Lap-brazed joint;

$(\sigma_b)p = \frac{1}{0.3}(110\times10^9)(0.001) = 367\ MN/m^2 >$ Fatigue Limit

$\therefore$ 불안정 하다.

(d) Modified lap-brazed joint;

(a)의 경우와 같다.

(e) Butt-brazed joint

$$\sigma_t = \frac{450}{0.001\times0.075} \fallingdotseq 6\ MN/m^2$$

또 (a)에서 $(\sigma_b)_p = 183.3\ MN/m^2 \quad (\sigma_b)_s = 130\ MN/m^2$

따라서 $\sigma_p = \sigma_t + (\sigma_b)_p = 189.3\ MN/m^2 <$ Fatigue Limit

$\sigma_s = \sigma_t + (\sigma_b)_s = 136\ MN/m^2 >$ Fatigue Limit

∴ Silve - brazing alloy가 불안정하다.

7.70 (a) ⓓ의 방법이 $Q_d = Q_{max}$ 이 되어서 τ가 가장 크기 때문에 가장 나쁘다. ($\because \tau = VQ/bI$)

(b) ⓑ의 방법이 $Q_d = Q_{min}$이 되어서 τ가 가장 작기 때문에 가장 좋다.

ⓓ의 경우; Reaction at both ends $= \frac{1}{2} \times 18 \times 10^3 (N)$

$$I = \frac{1}{12} bh^3 \Rightarrow \frac{1}{12} \times 75 \times (150^3 - 145^3) = 3.31 \times 10^6 \ (mm^4)$$

$$Q_{max} = 75^2 \left(\frac{75}{2}\right) - (72.5)(70)\left(\frac{72.5}{2}\right) = 2.697 \times 10^4 (mm^3)$$

$$\therefore \tau = \frac{VQ}{bI} = \frac{(9 \times 10^3)(2.697 \times 10^{-5})}{(0.005)(3.31 \times 10^{-6})} = 14.67 \ MN/m^2$$

Ans $\tau = 14.67 \ MN/m^2$

7.71 문제에서 $E_w = 0.06 E_m$.

$$(M_b)_{max} = \frac{PL}{4} \ (\text{at loading point})$$

$$I_w = \frac{1}{12}(0.15)(20t)^3 = 100 t^3$$

$$I_m = 2\left[\frac{1}{12}(0.15)t^3 + (0.15) \cdot t \, (10.5t)^2\right] = 33.1 t^3$$

$$k = \frac{1}{\rho} = \frac{M_b}{E_m I_m + E_w I_w} = \frac{\frac{1}{4}PL}{100t^3 \cdot E_w + 33.1 t^3 E_m} = \frac{PL}{156.4 E_m t^3}$$

$$(\because E_w = 0.06 E_m)$$

$\sigma = kEy$ 에서 $\sigma_{max} = kE y_{max}$

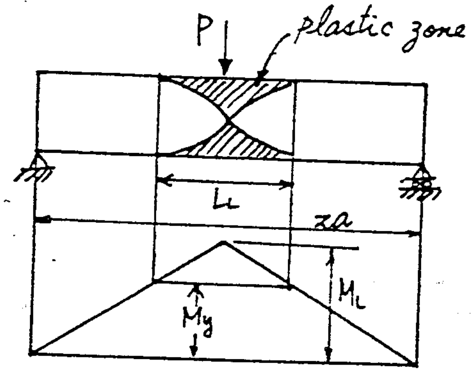
$$\therefore (\sigma_m)_{max.} = \frac{PL}{156.4 E_m t^3} E_m \cdot 11t = 0.07033 \frac{PL}{t^2}$$

$$(\sigma_w)_{max.} = \frac{PL}{156.4 E_m t^3} E_w 10t = \frac{PL}{156.4 E_m t^3}(0.06 E_m) 10t$$

$$= 0.00384 (PL/t^2)$$

$\therefore\ \sigma_{max} = 0.07033 \frac{PL}{t^2}$ Ans. $\sigma_{max} = 0.07033 \frac{PL}{t^2}$

7.72 (참조. text p.456~)



Elastic region 에서는 curvature 가 linear 하게 증가하다가 plastic region에 들어서면 급격히 증가하게 된다. P가 점점 더 커져서 plastic hinge를 이루게 되면 plastic한 중간점을 중심으로 길이가 a인 두개의 beam이 hinged된 것처럼 작용을 하게 될 것이다. 즉, moment는 전달하지 못하고, Force만을 전달한다. 따라서 plastic zone의 양 끝에서의 moment는 M_y가 될것이고, shear force p도 양쪽 hinge에 각각 P/2 씩 전달하게 된다.

평형 조건 ; $M_y = \frac{P}{2}\left(a - \frac{L_L}{2}\right)$, $P = \frac{2M_L}{a}$

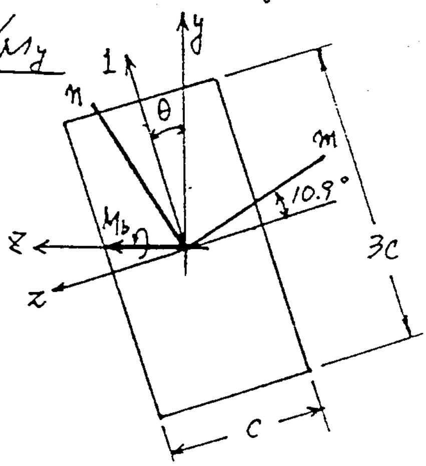
$\therefore M_y = \frac{M_L}{a}\left(a - \frac{L_L}{2}\right)$ $\quad \therefore L_L = 2a\left(1 - \frac{M_y}{M_L}\right) = 2a\left(1 - \frac{1}{f}\right)$

단 $f = M_L / M_y$

Ans. $L_L = 2a\left(1 - \frac{1}{f}\right)$, $f = M_L/M_y$

7.73.

$$\begin{cases} \dfrac{d\alpha}{dS_1} = 0.3849 \dfrac{M_{bz}}{EC^4} \\ \dfrac{d\beta}{dS_2} = -2 \dfrac{M_{bz}}{EC^4} \end{cases}$$



$$\frac{d\alpha}{dS_1} = \frac{d\phi}{dS}\cos\theta \quad , \quad \frac{d\beta}{dS_2} = \frac{d\phi}{dS}\sin\theta$$

$$\therefore \left(\frac{d\phi}{dS}\right)^2 = \left(\frac{d\alpha}{dS_1}\right)^2 + \left(\frac{d\beta}{dS_2}\right)^2$$

$$\therefore \left(\frac{d\phi}{dS}\right) = \sqrt{(0.3849)^2 + (-2)^2} \cdot \frac{M_{bz}}{EC^4} = 2.037\frac{M_{bz}}{EC^4}$$

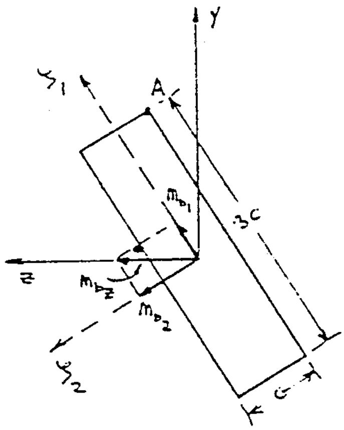
$$\alpha = \tan^{-1}\frac{(-2)}{0.3849} = -79.6^\circ$$

$$(\sigma_x)_A = -E \cdot m\left(\frac{d\phi}{dS}\right) = -E\,(0.7746C)\left(2.037\frac{M_{bz}}{EC^4}\right)$$

$$\simeq -1.6\left(M_{bz}/C^3\right) \quad (\because m = 0.7746C)$$

Ans. $\frac{d\phi}{dS} = 2.037\frac{M_{bz}}{EC^4}$

7.74



임의의 축 ξ_1, ξ_2 에 대하여

$$\sigma_x = -\left[\frac{(\xi_1 I_{22} - \xi_2 I_{12})M_{b_2} + (\xi_1 I_{12} - \xi_2 I_{11})M_{b_1}}{I_{11}I_{22} - I_{12}^2}\right]$$

만일, ξ_1 ξ_2 가 principal axis 라면,

$I_{12} = 0$ 따라 σ_x는 다음과 같이 된다.

$$\sigma_x = -\left[\frac{\xi_1}{I_{11}}M_{b_2} - \frac{\xi_2}{I_{22}}M_{b_1}\right] \quad \text{—①}$$

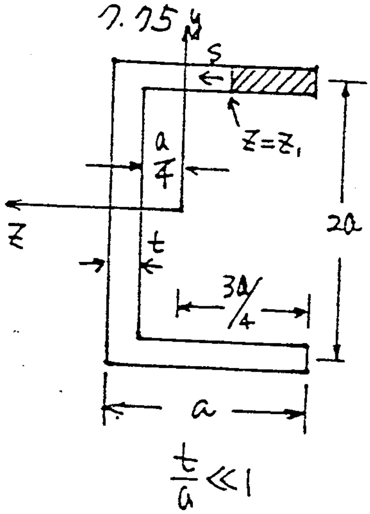
A점의 좌표 ; $\xi_1 = 1.5C$, $\xi_2 = -0.5C$

$$\therefore I_{11} = \frac{1}{12}(C)(3C)^3 = \frac{9}{4}C^4 , \quad I_{22} = \frac{1}{12}(3C)(C)^3 = \frac{1}{4}C^4$$

$$M_{b1} = M_{bz}\cos 60^\circ = 0.5M_{bz} , \quad M_{b2} = M_{bz}\sin 60^\circ = 0.866M_{bz}$$

$\therefore \sigma_x = -1.6\frac{M_{bz}}{C^3}$ Ans. $\sigma_x = -1.6(M_{bz}/C^3)$

7.75



shear center는 z축상에 있고, $I_{yz}=0$.
y축 방향의 힘 V에 의한 shear flow
$q_{sx} = q_{xs} = q$.

i) upper leg (z 방향이 +s 방향);
first moment of area : Q

$$Q = (z_1 + \tfrac{3}{4}a)(t)(a)$$

$$\therefore q = -\frac{VQ}{I_{yy}} = -\frac{V}{I_{yy}} at(z_1 + \tfrac{3}{4}a)$$

$$(-\tfrac{3}{4}a \le z_1 \le \tfrac{a}{4} \ ; \ y=a)$$

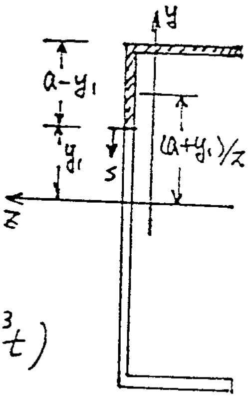
ii) lower leg (−z 방향이 +s 방향);

$$q = -\frac{V}{I_{yy}} at(z_1 + \tfrac{3}{4}a) \quad \left(-\tfrac{3}{4}a \le z_1 \le \tfrac{a}{4} \ ; \ y=-a\right)$$

iii) 오른편 그림의 빗금친 부분;

$$Q = (a^2 t) + (a-y_1)(t)\frac{(a+y_1)}{2}$$

$$= \frac{3a^2}{2}t - \frac{t}{2}y_1^2 \ ; \quad q = -\frac{VQ}{I_{yy}}$$



Resultant z-direction force

$$\text{upper}; \ F_{1z} = \int_{-\frac{3}{4}a}^{a/4} q \, dz = -\frac{V}{I_{yy}}\left(\frac{a^3}{2}t\right)$$

$$\text{lower}; \ F_{2z} = \int_{-\frac{3}{4}a}^{\frac{a}{4}} (-q) \, dz = \frac{V}{I_{yy}}\left(\frac{a^3 t}{2}\right)$$

Resultant y-direction force

$$F_{3y} = \int_{-a}^{a} (-q)\, dy = \frac{V}{I_{yy}} \int_{-a}^{a} \left(\frac{3a^2 t}{2} - \frac{t}{2}y^2\right) dy = \frac{V}{I_{yy}}\left(\frac{8}{3}a^3 t\right)$$

horizontal force에 의한 clockwise의 moment

$$\frac{V}{I_{yy}}\left(\frac{a^3t}{2}\right)(2a) = \frac{V}{I_{yy}}(a^4t) \quad -①$$

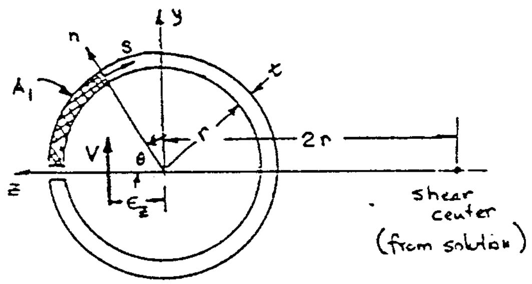
vertical force에 의한 counterclockwise의 moment

$$\frac{V}{I_{yy}}\left(\frac{8a^3t}{3}\right)\cdot d \quad -②$$

①=②에서 $d = \frac{3}{8}a$

Ans. 무게 중심에서 z 방향으로 $\left(\frac{3}{8}a+\frac{a}{4}\right)$

7.76



대칭성에 의해 shear center는 z축상에 있고, $I_{yz}=0$. shear center가 원점으로부터 z의 양의 방향으로 ε_z 만큼 떨어져 있다고 가정하면,

빗금친 부분 A_1 ; $Q=\int y\,dA = \int_{\phi=0}^{\phi=\theta}(r\sin\phi)(r\,d\phi) = r^2t(1-\cos\theta)$

$$\therefore q_{xs} = -\frac{V}{I_{yy}}r^2t(1-\cos\theta)$$

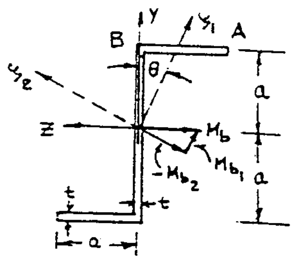
원점에 관한 q_{xs}의 moment; $-\int_{\theta=0}^{\theta=2\pi} q_{xs}(r\,d\theta)(r)$

$$= \frac{V}{I_{yy}}(2\pi r^4 t) \quad -①$$

원점에 관한 shear force의 moment; $-V\varepsilon_z$ -②

①=②에서 $\varepsilon_z = -\frac{2\pi r^4 t}{\pi r^3 t} = -2r$ Ans. $\varepsilon_z = -2r$

7.77

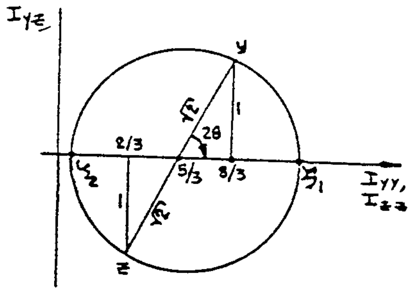


$I_{yy} = \frac{8}{3}a^3t$

$I_{zz} = \frac{2}{3}a^3t$

$I_{yz} = -a^3t$

Mohr's circle : (I/a^3t is plotted)



Mohr's circle 에서 $I_{11} = 3.08\,a^3t$

$I_{22} = 0.26\,a^3t$; $\theta = 22.5°$

문제 (7.74) 에서 $\sigma_x = \frac{M_{b1}}{I_{22}}\xi_2 - \frac{M_{b2}}{I_{11}}\xi_1$

$M_{b1} = M_b \sin 22.5° = 0.384\,M_b$

$M_{b2} = -M_b \cos 22.5° = -0.923\,M_b$

max. stress 는 A. B 에서 생긴다.

A ; $\xi_1 = 1.307\,a$ B ; $\xi_1 = 0.923\,a$

$\xi_2 = -0.539\,a$ $\xi_2 = 0.384\,a$

$$(\sigma_x)_A = \frac{(0.384M_b)(-0.539a)}{(0.26\,a^3t)} - \frac{(-0.923M_b)(1.307a)}{(3.08\,a^3t)} = -0.406\frac{M_b}{a^2t}$$

$$(\sigma_x)_B = 0.844\frac{M_b}{a^2t} \qquad \therefore (\sigma_x)_{max} = (\sigma_x)_B = 0.844\frac{(M_b)_{max}}{a^2t}$$

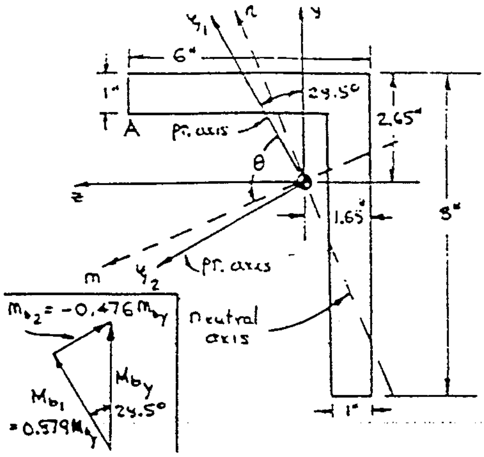
$$(M_b)_{max} = PL \qquad \therefore (\sigma_x)_{max} = \frac{0.844PL}{a^2t}$$

Ans. $(\sigma_x)_{max} = 0.844PL/a^2t$

7.78

$I_{11} = 98.2\ in^4$, $I_{22} = 21.4\ in^4$

y축을 1축으로, z축을 2로
바꾸어 주면, 1.2축은
principal axis 이므로
$I_{12}=0$.



θ를 1축으로 부터 m 까지의
반시계 방향의 각도로 놓으면

$$\frac{d\phi}{ds}\cos\theta=\frac{M_{b2}}{EI_{11}}$$

$$\frac{d\phi}{ds}\sin\theta=-\frac{M_{b1}}{EI_{22}}$$

$$\therefore \tan\theta=-\frac{M_{b1}I_{11}}{M_{b2}I_{22}}=-\frac{(0.879M_{by})(98.2)}{(-0.476M_{by})(21.4)}=8.5$$

$\therefore \theta=83.3°$ 따라서 m. n 의 위치는 그림과 같다.
6개의 모서리에 대해 모두 σ_x를 구하여 가장 큰 것이
σ_{max}이 된다.
max. m coordinate를 갖는 점은 A ; $\xi_1=3.53''$
$\xi_2=3.03''$

$$\therefore (\sigma_x)_A=(\sigma_x)_{max}=\frac{M_{b1}}{I_{22}}\xi_2-\frac{M_{b2}}{I_{11}}\xi_1=\frac{(0.879M_{by})(3.03)}{21.4}-\frac{(-0.476M_{by})(3.53)}{98.2}$$

$$=0.141M_{by}=0.141\,(100.000)$$

$$=14100\ (psi)$$ <u>Ans 14100 psi</u>

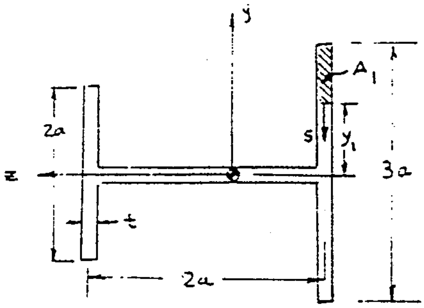
7.79
대칭성에 의해서 shear center는 z축상에 있고,
$I_{yz}=0$.

vertical force V_y에 대해

$$q_{xs} = q_{sx} = -\frac{V_y}{I_{yy}}\int y\,dA$$

horizontal member는

$\int y\,dA \simeq 0$ 이므로 $q_{xs}=0$.



i) 오른쪽 vertical leg ;

$$\int_{A_1} y\,dA = \int_{y_1}^{3a/2} y(t\,dy)$$

$$= \frac{t}{2}\left(\frac{9a^2}{4} - y_1^2\right)$$

$$\therefore q_{xs} = -\frac{V_y \cdot t}{I_{yy}}\left(\frac{9a^2}{4} - y_1^2\right)$$

total force ; $F_1 = \dfrac{V_y \cdot t}{2I_{yy}}\displaystyle\int_{-3a/2}^{3a/2}\left(\frac{9a^2}{4} - y^2\right)dy$

$$= \frac{V_y \cdot t}{2I_{yy}}\left(\frac{9}{2}a^3\right)$$

ii) 왼쪽 vertical leg ;

$$\int y\,dA = \frac{t}{2}(a^2 - y_1^2)$$

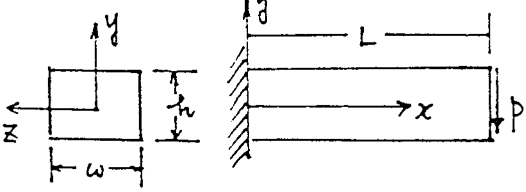
$$F_2 = \frac{V_y \cdot t}{2I_{yy}}\int_{-a}^{a}(a^2 - y^2)\,dy = \frac{V_y \cdot t}{2I_{yy}}\left(\frac{4}{3}a^3\right)$$

i) ii)에서 평형조건 $\sum M_{shear\ center} = 0$

$$\frac{V_y \cdot t}{2I_{yy}}\left(\frac{9}{2}a^3\right)(\delta) = \frac{V_y \cdot t}{2I_{yy}}\left(\frac{4}{3}a^3\right)(2a - \delta)$$

$\therefore \delta = \dfrac{16}{35}a$ Ans $\delta = \dfrac{16}{35}a$

7.80



(w/h)가 작고, P를 적당히 작용시키면 x-y plane 에서 plane stress로 고려할 수 있다.

$y = h/2$ 에서 ; σ_x = tension ; $\sigma_y = 0$; $\tau_{xy} = 0$

∴ x는 principal tension direction

y는 〃 compression 〃

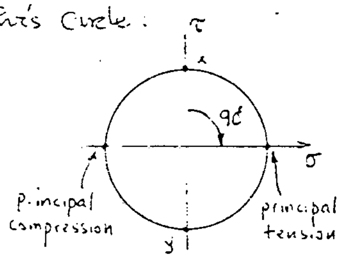
$y = -h/2$ 에서 ; σ_x = compression ; $\sigma_y = 0$; $\tau_{xy} = 0$

∴ x는 principal compression direction

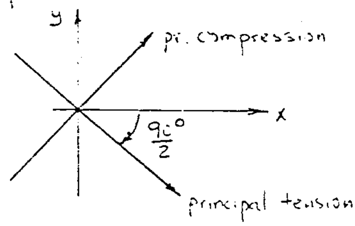
y는 〃 tension 〃

$y = 0$ 에서 ; $\sigma_x = 0$: $\sigma_y = 0$; τ_{xy} = negative

Mohr's Circle :

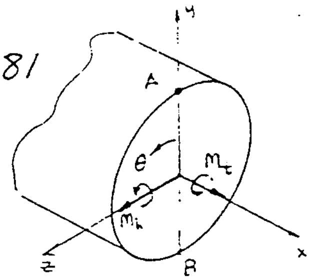


Principal directions —



따라서 trajectories는 옳다.

7.81



Bending은 normal stress σ_x. Torsion은 shear stress $\tau_{x\theta}$를 일으킬 것이다. (θ는 접선 방향의 각도)

이와 같은 stress는 A. B에서 최대가 될 것이고
max. shear stress도 이와 같은 점에서 생길 것이다.

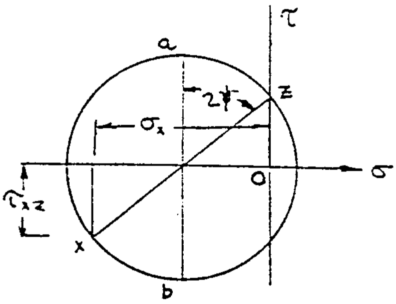
A점에서의 Stress ;

$$\sigma_x = -\frac{(M_b)(r)}{I_{yy}}$$

$$\tau_{x\theta} = \tau_{xz} = \frac{(M_t)(r)}{I_x} = \frac{M_t \cdot r}{2I_{yy}}$$

$$\left(\because I_{xx} = \frac{\pi}{2}r^4 = 2\left(\frac{\pi}{4}r^4\right) = 2I_{yy}\right)$$

다른 응력은 "0" 이다



< Mohr's circle >

Mohr's circle 에서

$$\tau_{max} = \left[\left(\frac{\sigma_x}{2}\right)^2 + (\tau_{xz})^2\right]^{1/2} = \frac{r}{2I_{yy}}\sqrt{(M_b)^2+(M_t)^2}$$

$$\tan 2\psi = \frac{-\sigma_x/2}{\tau_{xz}} = \frac{M_b \cdot r}{2I_{yy}} \cdot \frac{2I_{yy}}{M_t \cdot r} = \frac{M_b}{M_t}$$

$$\therefore \psi = \frac{1}{2}\tan^{-1}\left(\frac{M_b}{M_t}\right)$$

Hollow shaft 에서도, solid shaft 와 마찬가지로
$I_x = 2I_{yy}$ 이고, max. shear stress도 r_{max}
에서 생기므로 성립된다.

CHAPTER 8

8.1 (a) $EIy'' = -PL + Px$

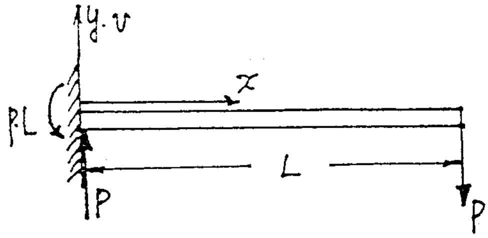
$EIy' = -PLx + \frac{1}{2}Px^2 + C_1$

$y'(0) = 0 \quad \therefore C_1 = 0$

$EIy = \frac{1}{6}Px^3 - \frac{1}{2}PLx^2 + C_2$

$y(0) = 0 \quad \therefore C_2 = 0$

$\therefore y = \frac{P}{6EI}(x^3 - 3Lx^2)$; Ans.



(b) $EIy'' = Px - Pa - P\langle x-a\rangle^1$

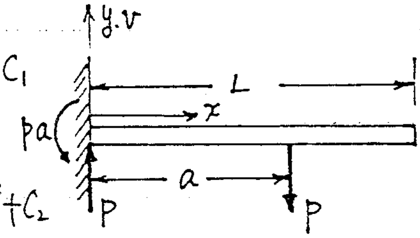
$EIy' = \frac{1}{2}Px^2 - Pax - \frac{1}{2}P\langle x-a\rangle^2 + C_1$

$y'(0) = 0 \quad \therefore C_1 = 0$

$EIy = \frac{1}{6}Px^3 - \frac{Pa}{2}x^2 - \frac{1}{6}P\langle x-a\rangle^3 + C_2$

$y(0) = 0 \quad \therefore C_2 = 0$

$\therefore y = \frac{P}{6EI}(x^3 - 3ax^2 - \langle x-a\rangle^3)$: Ans.



(c) $EIy'' = \frac{3}{16}\omega_0 Lx - \frac{\sqrt{3}}{4}\omega_0 x^2 + \frac{9}{16}\omega_0 L\langle x - \frac{2}{3}L\rangle^1$

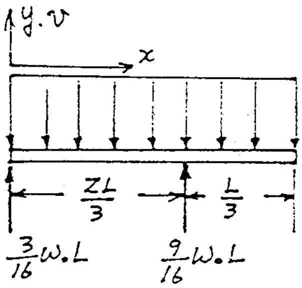
$EIy' = \frac{3}{32}\omega_0 Lx^2 - \frac{\sqrt{3}}{12}\omega_0 x^3 + \frac{9}{32}\omega_0 L\langle x - \frac{2}{3}L\rangle^2 + C_1$

$EIy = \frac{1}{32}\omega_0 Lx^3 - \frac{\sqrt{3}}{48}\omega_0 x^4 + \frac{3}{32}\omega_0 L\langle x - \frac{2}{3}L\rangle^3$

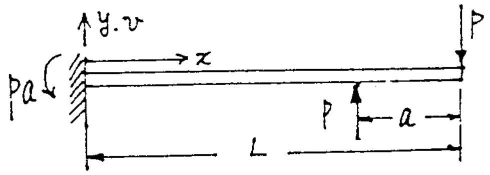
$+ C_1 x + C_2$

$y(0) = 0 \quad \therefore C_2 = 0, \quad y(\frac{2}{3}L) = 0 \quad \therefore C_1 = -0.0032\omega_0 L^3$

$\therefore y = \frac{1}{EI}\left[\frac{1}{32}\omega_0 Lx^3 - \frac{\sqrt{3}}{48}\omega_0 x^4 + \frac{3}{32}\omega_0 L\langle x - \frac{2}{3}L\rangle^3 - 0.0032\omega_0 L^3 x\right]$; Ans.



(d)



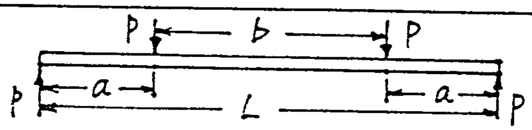
$$EIy'' = -Pa + P\langle x-L+a\rangle^1$$

$$EIy' = -Pax + \frac{1}{2}P\langle x-L+a\rangle^2 + C_1 \;;\; y'(0)=0 \quad \therefore C_1 = 0$$

$$EIy = -\frac{1}{2}Pax^2 + \frac{1}{6}P\langle x-L+a\rangle^3 + C_2 \;;\; y(0)=0 \quad \therefore C_2 = 0$$

$$\therefore y = \frac{P}{6EI}\left[-3ax^2 + \langle x-L+a\rangle^3\right] \;;\; \text{Ans.}$$

(e)



$$EIy'' = Px - P\langle x-a\rangle^1 - P\langle x-a-b\rangle^1$$

$$EIy' = \frac{1}{2}Px^2 - \frac{1}{2}P\langle x-a\rangle^2 - \frac{1}{2}P\langle x-a-b\rangle^2 + C_1$$

$$EIy = \frac{1}{6}Px^3 - \frac{1}{6}P\langle x-a\rangle^3 - \frac{1}{6}P\langle x-a-b\rangle^3 + C_1x + C_2$$

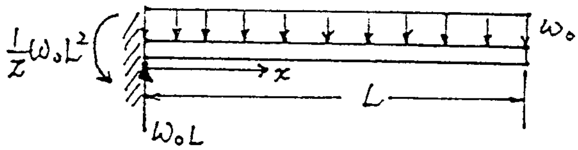
$$y(0)=0 \;:\; \therefore C_2 = 0$$

$$y(2a+b)=0 \;;\; \frac{1}{6}P(2a+b)^3 - \frac{1}{6}P(a+b)^3 - \frac{1}{6}Pa^3 + C_1(2a+b) = 0$$

$$\therefore C_1 = -\frac{1}{2}Pa(a+b)$$

$$\therefore y = \frac{P}{6EI}\left[x^3 - \langle x-a\rangle^3 - \langle x-a-b\rangle^3 - 3a(a+b)x\right] \;;\; \text{Ans.}$$

(f)



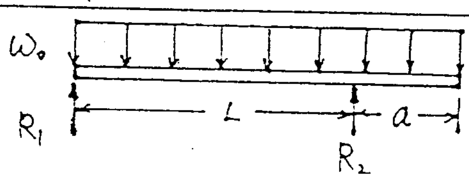
$$EIy'' = \omega_0 Lx - \frac{1}{2}\omega_0 L^2 - \frac{1}{2}\omega_0 x^2$$

$$EIy' = \frac{1}{2}\omega_0 Lx^2 - \frac{1}{2}\omega_0 L^2 x - \frac{1}{6}\omega_0 x^3 + C_1 \;;\; y'(0)=0 \quad \therefore C_1 = 0$$

$$EIy = \frac{1}{6}\omega_0 Lx^3 - \frac{1}{4}\omega_0 L^2 x^2 - \frac{1}{24}\omega_0 x^4 + C_2 \;;\; y(0)=0 \quad \therefore C_2 = 0$$

$$\therefore y = \frac{\omega_0}{24EI}\left(4Lx^3 - 6L^2x^2 - x^4\right) \;;\; \text{Ans.}$$

(g)



$$EIy'' = \frac{w_0(L^2-a^2)}{2L}x - \frac{w_0(L+a)^2}{2L}\langle x-L\rangle^1 - \frac{1}{2}w_0x^2$$

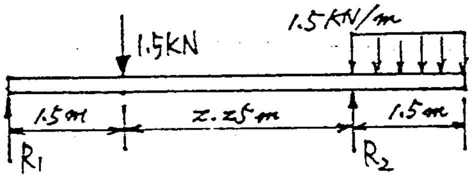
$$EIy' = \frac{w_0(L^2-a^2)}{4L}x^2 - \frac{w_0(L+a)^2}{4L}\langle x-L\rangle^2 - \frac{1}{6}w_0x^3 + C_1$$

$$EIy = \frac{w_0(L^2-a^2)}{12L}x^3 - \frac{w_0(L+a)^2}{12L}\langle x-L\rangle^3 - \frac{1}{24}w_0x^4 + C_1x + C_2$$

$$y(0)=0 \quad \therefore C_2=0, \quad y(L)=0 \quad \therefore C_1 = \frac{1}{24}w_0L(2a^2-L^2)$$

$$\therefore y = \frac{w_0}{EI}\left[\frac{(L^2-a^2)}{12L}x^3 - \frac{(L+a)^2}{12L}\langle x-L\rangle^3 - \frac{1}{24}x^4 - \frac{1}{24}L(L^2-2a^2)x\right]$$

(h)



$R_1 = 0.45\,kN$

$R_2 = 3.3\,kN$

$$EIy'' = R_1x - p\langle x-a\rangle^1 + R_2\langle x-a-b\rangle^1 - \frac{1}{2}w_0\langle x-a-b\rangle^2$$

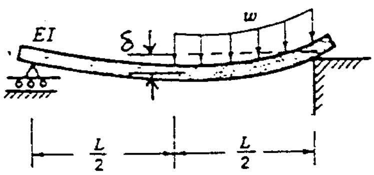
$$EIy' = \frac{1}{2}R_1x^2 - \frac{P}{2}\langle x-a\rangle^2 + \frac{R_2}{2}\langle x-a-b\rangle^2 - \frac{1}{6}w_0\langle x-a-b\rangle^3 + C_1$$

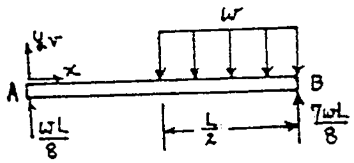
$$EIy = \frac{1}{6}R_1x^3 - \frac{P}{6}\langle x-a\rangle^3 + \frac{R_2}{6}\langle x-a-b\rangle^3 - \frac{1}{24}w_0\langle x-a-b\rangle^4 + C_1x + C_2$$

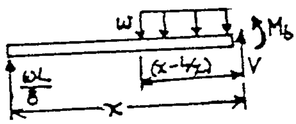
$$y(0)=0 \quad \therefore C_2=0, \quad y(a+b)=0 \quad \therefore C_1 = 0.2953$$

$$\therefore y = \frac{1}{EI}[0.075x^3 - 0.25\langle x-1.5\rangle^3 + 0.55\langle x-3.75\rangle^3$$
$$- 0.0625\langle x-3.75\rangle^4 + 0.2953x] \text{ : Ans.}$$

8.2







Bending Moment

$$EIy'' = M_b = \frac{wL}{8}x - \frac{w}{2}\langle x-\frac{L}{2}\rangle^2$$

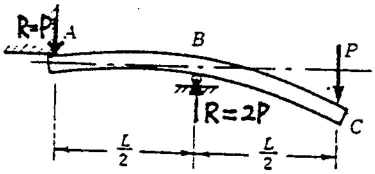
적분하면 $EIy = \frac{wLx^3}{48} - \frac{w}{24}\langle x-\frac{L}{2}\rangle^4 + C_1x + C_2$

$$y(0)=0 \quad \therefore C_2=0, \quad y(L)=0 \quad \therefore C_1 = -\frac{7wL^3}{384}$$

$$\therefore EIy = \frac{wLx}{48}\left(x^2 - \frac{7}{8}L^2\right) - \frac{w}{24}\left\langle x - \frac{L}{2}\right\rangle^4$$

$$\delta = (-y)_{x=\frac{L}{2}} = \frac{5wL^4}{768EI}$$

Ans. $(5wL^4)/(768EI)$



8.3 Bending Moment

$$EIy'' = -Px + 2P\left\langle x - \frac{L}{2}\right\rangle^1$$

적분하면

$$EIy = -\frac{Px^3}{6} + \frac{P}{3}\left\langle x - \frac{L}{2}\right\rangle^3 + C_1 x + C_2$$

Boundary Condition $\quad y(0) = 0, \quad \therefore C_2 = 0$

$$y\left(\frac{L}{2}\right) = 0, \quad \therefore C_1 = \frac{PL^2}{24}$$

$$\therefore EIy = \frac{Px}{6}\left(\frac{L^2}{4} - x^2\right) + \frac{P}{3}\left\langle x - \frac{L}{2}\right\rangle^3$$

$$\delta = (-y)_{x=L} = \frac{PL^3}{12EI}$$

Ans. $\dfrac{PL^3}{12EI}$

8·4 문제 (8·1(e)) 에서 $b = L - 2a$를 대입하면

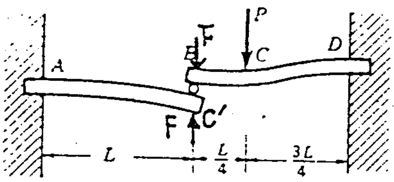
$$y = \frac{P}{6EI}\left[x^3 - \langle x-a\rangle^3 - \langle x-L+a\rangle^3 - 3a(L-a)x\right]$$

$$\therefore \delta = y_{x=\frac{L}{2}} = \frac{P}{6EI}\left[\left(\frac{L}{2}\right)^3 - \left(\frac{L}{2}-a\right)^3 - 3a(L-a)\frac{L}{2}\right] = \frac{Pa}{24EI}[4a^2 - 3L^2]$$

Ans. $\dfrac{Pa}{24EI}(4a^2 - 3L^2)$

8.5 $\quad v_B = v_{C'} \quad$ — ①

중첩의 원리를 이용하여 displacement를 구하면,

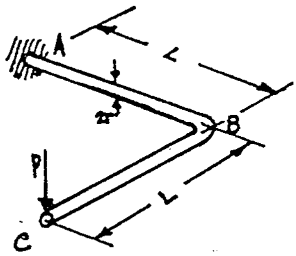


Beam ① ; $v_{C'} = -\dfrac{FL^3}{3EI}$

Beam ② ; $v_B = -\left[\dfrac{P\left(\frac{3}{4}L\right)^3}{3EI} + \dfrac{L}{4}\cdot\dfrac{P\cdot\left(\frac{3}{4}L\right)^2}{2EI} - \dfrac{FL^3}{3EI}\right]$ $\quad$ 이 두식을 ①에 대입하면.

$F = \frac{81}{256} P$ Ans. $\frac{81}{256} P$

8.6



중첩 이론.

$\delta_c = (\delta_c)_{bending} + (\delta_c)_{torsion}$

$(\delta_c)_{bending} = (\delta_{BA}) + (\delta_{CB})$

$(\delta_c)_{torsion} = L \times (\phi_{BA})_{torsion}$

δ_{BA} ; P에 의해서 생기는 Bending Moment로 인한 A에 대한 B의 변위

δ_{CB} ; 〃 〃 〃 〃 B에 대한 C의 상대 변위

$\delta_{BA} = \frac{PL^3}{3EI}$; $I = \frac{\pi}{4} r^4$ $\therefore \delta_{BA} = \frac{4PL^3}{3\pi E r^4}$

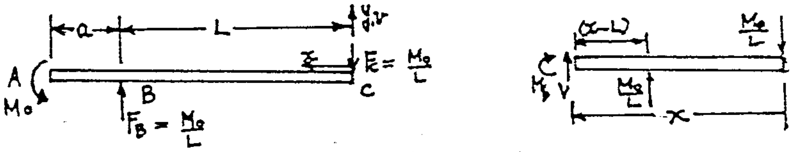
$\delta_{CB} = \frac{PL^3}{3EI} = \frac{4PL^3}{3\pi E r^4}$ $\phi_{BA} = \frac{M_t \cdot L^2}{G I_z} = \frac{PL \cdot L^2}{G \cdot \frac{\pi}{2} r^4}$ 모두를 조합하면

$\delta_c = \frac{8PL^3}{3\pi E r^4} + \frac{2PL^3}{\pi G r^4} = \frac{4PL^3}{3\pi E r^4}(5+3\nu)$ $\left(\because G = \frac{E}{2(1+\nu)}\right)$

ν : Poisson's Ratio

Ans. $\delta = \frac{4PL^3}{3\pi E r^4}(5+3\nu)$

8.7



Bending Moments ; $EIy'' = -\frac{M_o}{L}x + \frac{M_o}{L}\langle x-L \rangle^1$, $EIy' = \frac{-M_o}{2L}x^2 + \frac{M_o}{2L}\langle x-L \rangle^2 + C_1$

적분하면 $EIy = -\frac{M_o}{6L}x^3 + \frac{M_o}{6L}\langle x-L \rangle^3 + C_1 x + C_2$

Boundary Condition $y(0) = 0$, $y(L) = 0$

대입하면, $C_2 = 0$. $C_1 = \frac{M_o L}{6}$

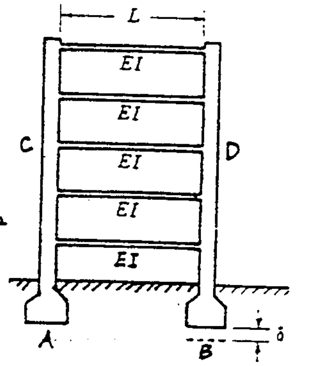
$\therefore y' = \frac{1}{EI}\left(-\frac{M_o}{2L}x^2 + \frac{M_o}{2L}\langle x-L \rangle^2 + \frac{M_o L}{6}\right)$

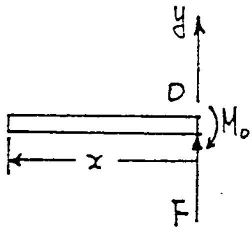
$\phi = (-y')_{x=L+a} = \frac{1}{EI}\left(\frac{M_o}{2L}(L+a)^2 - \frac{M_o}{2L}a^2 - \frac{M_o L}{6}\right) = \frac{M_o(L+3a)}{3EI}$

Ans. $\phi = M_o(L+3a)/(3EI)$

8.8 Beam C.D 만을 취하자. 다른 모든 부재도 CD와 같은 형태로 변형할 것이다.

D 에서의 반력은 힘 F와 moment M_0 라 하자.





Bending Moment

$EI\ddot{y} = M_b = Fx - M_0$

$EI\dot{y} = F\frac{x^2}{2} - M_0 x + C_1$

$EIy = F\frac{x^3}{6} - M_0\frac{x^2}{2} + C_1 x + C_2$

B.C $y(0) = 0,\quad y'(0) = 0$

$y(L) = \delta,\quad y'(L) = 0$

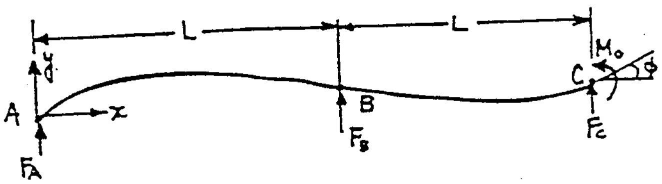
$\therefore C_1 = C_2 = 0,\quad F = \frac{12EI\delta}{L^3},\quad M_0 = \frac{6EI\delta}{L^2}$

$\therefore M_b = \frac{12EI\delta}{L^3}x - \frac{6EI\delta}{L^2}$

$y(L) = -\delta,\quad \delta = -\frac{1}{EI}\left[-\frac{1}{6}M_0 L^2\right] \quad \therefore M_0 = \frac{6EI\delta}{L^2}$

Ans. $M_0 = \frac{6EI\delta}{L^2}$

8.9



Equilibrium ; $\sum F_y = 0$: $F_A + F_B + F_C = 0$ —①

$\sum M_{Bz} = 0$: $F_A - F_C = \frac{M_0}{L}$ —②

Bending Moment ; $EIy'' = M_b = F_A \cdot x + F_B \langle x - L \rangle^1$

$EIy' = F_A\frac{x^2}{2} + \frac{F_B}{2}\langle x-L\rangle^2 + C_1$

$EIy = F_A\frac{x^3}{6} + \frac{F_B}{6}\langle x-L\rangle^3 + C_1 x + C_2$

B.C $y(0) = 0,\ y(L) = 0,\ y(2L) = 0$

$\therefore C_2 = 0,\quad C_1 = -\frac{F_A}{6}L^2$

$\frac{8F_A}{6} + \frac{F_B}{6} - \frac{2F_A}{6} = 0$ —③

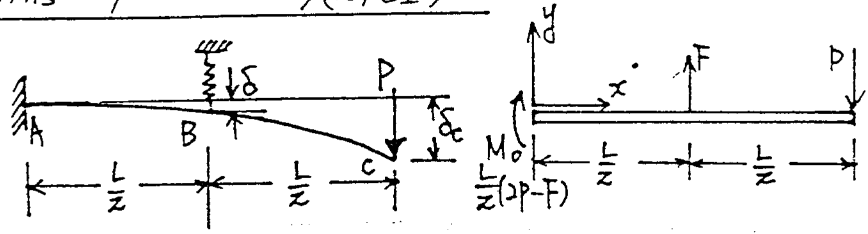
①②③ 에서

$$F_A = -\frac{M_0}{4L} \;;\; F_B = \frac{3M_0}{2L} \;;\; F_c = -\frac{5M_0}{4L}$$

$$\therefore \phi \simeq (y')_{x=2L} = \frac{7M_0L}{24EI}$$

Ans $\phi = (7M_0L)/(24EI)$

8.10



Bending Moment ; $EIy'' = M_b = (P-F)x + \frac{L}{2}(2P-F) + F\langle x-\frac{L}{2}\rangle^1$

적분하면 $EIy' = (P-F)\frac{x^2}{2} + \frac{L}{2}(2P-F)x + \frac{F}{2}\langle x-\frac{L}{2}\rangle^2 + C_1$

$$EIy = (P-F)\frac{x^3}{6} + \frac{L}{4}(2P-F)x^2 + \frac{F}{6}\langle x-\frac{L}{2}\rangle^3 + C_1x + C_2$$

Bending Moment를 적분한 식에 Boundary Condition을 대입.

B.C. $y(0)=0$. $y'(0)=0$ $\quad\therefore C_1 = C_2 = 0$

Spring ; $F = -kv_B = -k(y)_{x=\frac{L}{2}}$

$$= -\frac{k}{EI}\left(\frac{7PL^3}{48} - \frac{FL^3}{12}\right) \quad \therefore F = \frac{7P}{4}\cdot\frac{x}{x-1} \;\left(x \equiv \frac{kL^3}{12EI}\right)$$

$$\delta = -v_B = \frac{F}{k} = \frac{7P}{4k}\left(\frac{kL^3/12EI}{kL^3/12EI - 1}\right)$$

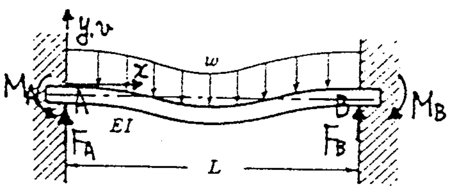
$$\delta_c = (-y)_{x=L} = \frac{PL^3}{3EI}\left[1 - \frac{25}{32\left(\frac{24EI}{kL^3}+1\right)}\right]$$

Ans. $\delta_c = \frac{PL^3}{3EI}\left[1 - \frac{25}{32\left(\frac{24EI}{kL^3}+1\right)}\right]$

8.11 $\Sigma F_y = 0$; $F_A + F_B = wL$

대칭 조건 : $M_A = M_B$, $F_A = F_B = \frac{wL}{2}$

따라서 $\Sigma M_{Az} = 0$을 만족한다.



Bending Moment ; $EIy'' = M_b = \omega L \cdot \frac{x}{2} - \omega \frac{x^2}{2} - M_A$

적분하면 $EIy' = \omega L \frac{x^2}{4} - \omega \frac{x^3}{6} - M_A x + C_1$

$EIy = \omega L \frac{x^3}{12} - \omega \frac{x^4}{24} - M_A \frac{x^2}{2} + C_1 x + C_2$

Boundary Condition $y(0) = 0 \quad y'(0) = 0$ —①

$y(L) = 0 \quad y'(L) = 0$ —②

① 에서 $C_1 = C_2 = 0$

② 에서 $M_A = \frac{\omega L^2}{12}$ —③

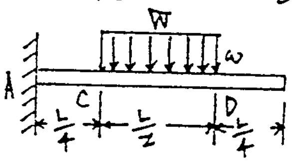
$\therefore M_b = \omega L \frac{x}{2} - \frac{\omega L^2}{12} - \frac{\omega x^2}{2} \qquad \frac{dM_b}{dx} = 0 \quad at \quad x = \frac{L}{2}$

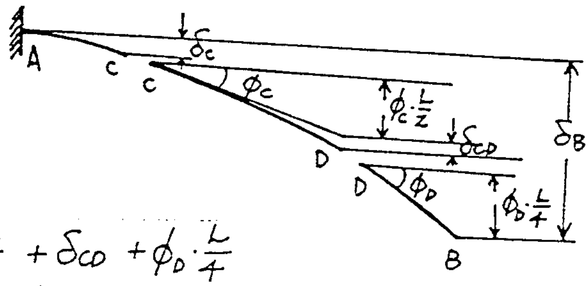
$\therefore |M_b|_{max} = \frac{\omega L^2}{24}$ —④ ③.④ 에서 $M_{max} = \frac{\omega L^2}{12}$

$|v|_{max} = y\left(\frac{L}{2}\right) = \frac{\omega L^4}{384EI}$

Ans. $M_{max} = \omega L^2/12 \qquad V_{max} = \omega L^4/384EI$

8.12 중첩의 원리를 적용한다. 즉, beam이 3개의 cantilever로 구성되어 있다고 가정하자.





그림에서 $\delta_B = \delta_C + \phi_C \cdot \frac{L}{2} + \delta_{CD} + \phi_D \cdot \frac{L}{4}$

$\delta_C = \frac{(\omega L/2)(L/4)^3}{3EI} + \frac{(\omega L^2/8)(L/4)^2}{2EI} = \frac{5\omega L^4}{768EI}$

$\phi_C = \frac{(\omega L/2)(L/4)^2}{2EI} + \frac{(\omega L^2/8)(L/4)}{EI} = \frac{3\omega L^3}{64EI}$

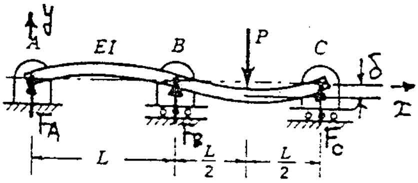
$\delta_{CD} = \frac{\omega(L/2)^4}{8EI} = \frac{\omega L^4}{128EI} \quad ; \quad \phi_{CD} = \frac{\omega(L/2)^3}{6EI} = \frac{\omega L^3}{48EI}$

$\therefore \phi_D = \phi_C + \phi_{CD} = (13\omega L^3)/(192EI)$

$\therefore \delta_B = \frac{5\omega L^4}{768EI} + \frac{3\omega L^3}{64EI} \cdot \frac{L}{2} + \frac{\omega L^4}{128EI} + \frac{13\omega L^3}{192EI} \cdot \frac{L}{4} = \frac{7\omega L^4}{128EI} = \frac{7}{64}\frac{\overline{W} L^3}{EI}$ -Q.E.D-

8·13 $\Sigma F_y = 0 \;:\; F_A + F_B + F_C = P$ -①

$\Sigma M_{Bz} = 0 \;;\; F_C - F_A = \frac{P}{2}$ -②



Bending Moment;

$EIy'' = M_b = F_A x + F_B\langle x-L\rangle' - P\langle x-\frac{3}{2}L\rangle$

적분하면 $EIy = F_A\frac{x^3}{6} + \frac{F_B}{6}\langle x-L\rangle^3 - \frac{P}{6}\langle x-\frac{3}{2}L\rangle^3 + C_1 x + C_2$

Boundary Condition ; $y(0) = y(L) = y(2L) = 0$

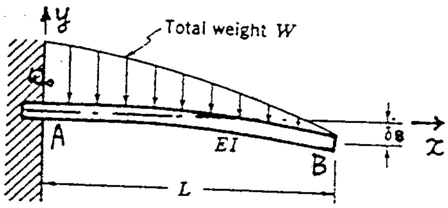
①②와 Boundary Condition 으로부터

$C_2 = 0 \;;\; C_1 = \frac{PL^2}{64} \;;\; F_A = -\frac{3}{32}P \;;\; F_B = \frac{22}{32}P \;;\; F_C = \frac{13}{32}P$

$\therefore y = \frac{1}{EI}\left(-\frac{P}{64}x^3 + \frac{11P}{96}\langle x-L\rangle^3 - \frac{P}{6}\langle x-\frac{3}{2}L\rangle^3 + \frac{PL^2}{64}x\right)$

$\delta = (-y)_{x=\frac{3}{2}L} = \frac{23PL^3}{1536EI}$ Ans. $\delta = (23PL^3)/(1536EI)$

8·14 $\omega_0 \cdot \frac{1}{2}\cdot L = \overline{W}$ $\therefore \omega_0 = \frac{2\overline{W}}{L}$

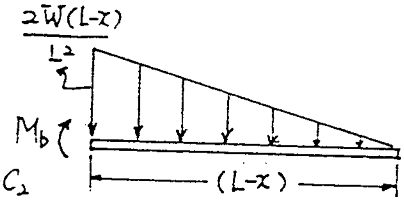


x점에서의 하중;

$\frac{L-x}{L}\omega_0 = \frac{2\overline{W}(L-x)}{L^2}$

x점에서의 Bending Moment ; $M_b = -\frac{2\overline{W}(L-x)}{L^2}\cdot\frac{(L-x)}{2}\cdot\frac{(L-x)}{3} = -\frac{\overline{W}(L-x)^3}{3L^2}$

$\therefore EIy'' = M_b = -\frac{\overline{W}(L-x)^3}{3L^2}$



적분하면 $EIy' = \frac{\overline{W}(L-x)^4}{12L^2} + C_1$

$EIy = -\frac{\overline{W}(L-x)^5}{60L^2} + C_1 x + C_2$

Boundary Condition ; $y(0) = y'(0) = 0$ $\therefore C_1 = -\frac{\overline{W}L^2}{12}$

$C_2 = \frac{\overline{W}L^3}{60}$

$\therefore y = \frac{1}{EI}\left[-\frac{\overline{W}(L-x)^5}{60L^2} - \frac{\overline{W}L^2 x}{12} + \frac{\overline{W}L^3}{60}\right]$

$\delta_B = -(y)_{x=L} = \frac{\overline{W}L^3}{15EI}$. Ans. $\delta_B = \overline{W}L^3/15EI$

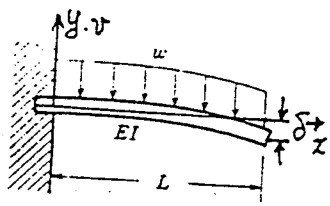
8.15 Max. bending moment는 고정단에서 생긴다. $M_{max} = \frac{1}{2}\omega L^2$

(a) $y = \frac{\omega}{EI}\left(\frac{1}{6}Lx^3 - \frac{1}{4}L^2x^2 - \frac{1}{24}x^4\right)$ 문제 (8.1 (f)) 참조

$\therefore v_{max} = |y(L)| = \frac{\omega L^4}{8EI}$

(b) $\delta_B \doteq 0$ — ①

(a)



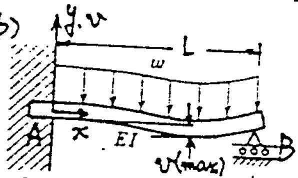
R에 의한 B점의 처짐 $= \frac{RL^3}{3EI}$

ω에 의한 B점의 처짐 $= -\frac{\omega L^4}{8EI}$

①에서 $\frac{RL^3}{3EI} = \frac{\omega L^4}{8EI}$ $\quad \therefore R = \frac{3}{8}\omega L$

Bending Moment; $M_b = EIy'' = \frac{3}{8}\omega L(L-x) - \frac{\omega}{2}(L-x)^2$

(b)



$EIy' = -\frac{3}{16}\omega L(L-x)^2 + \frac{\omega}{6}(L-x)^3 + C_1$

$EIy = \frac{3}{48}\omega L(L-x)^3 - \frac{\omega}{24}(L-x)^4 + C_1x + C_2$ — ②

B.C. $\quad y(0) = y'(0) = 0$

C_1, C_2를 구하여 ②에 대입하면

$$y = \frac{1}{EI}\left[\frac{1}{16}\omega L(L-x)^3 - \frac{\omega(L-x)^4}{24} + \frac{\omega L^3 x}{48} - \frac{\omega L^4}{48}\right]$$

max. deflection은 $y' = 0$을 만족하는 $x \simeq 0.58L$에서 갖게 된다.

$v_{max} = |y(0.58L)| = 0.42\,\frac{\omega L^4}{48EI}$

$\frac{dM_b}{dx} = 0$ 을 만족하는 점 $x_A = \frac{5}{8}L$

$(M_b)_{x=\frac{5}{8}L} = \frac{9}{128}\omega L^2 \qquad (M_b)_{x=0} = \frac{\omega L^2}{8}$

$\therefore (M_b)_{max} = \frac{\omega L^2}{8}$

Ans. (a) $M_{max} = \omega L^2/2 \quad v_{max} = \omega L^4/8EI$

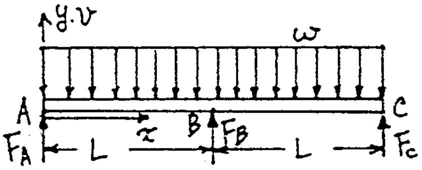
(b) $M_{max} = \omega L^2/8 \quad v_{max} = (0.42)(\omega L^4/48EI)$

8·16 (a) 문제 (8·1 (g))에서 $a=0$인 경우.

$$y = \frac{1}{EI}\left[\frac{\omega L x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega L^3 x}{24}\right], \quad M_b = +EIy'' = \left[\frac{\omega L}{2}x - \frac{\omega}{2}x^2\right]$$

$$v_{max} = y\left(\frac{L}{2}\right) = \frac{5\omega L^4}{384EI}, \quad (M_b)_{max} = |M_b|_{x=\frac{L}{2}} = \frac{\omega L^2}{8}$$

(b) $F_A = F_c$ —①



$F_A + F_B + F_c = 2\omega L$ —②

Bending Moment ;

$$EIy'' = M_b = F_A \cdot x + F_B\langle x-L\rangle^1 - \frac{\omega x^2}{2}$$

$$\therefore EIy = \frac{F_A}{6}x^3 + \frac{F_B}{6}\langle x-L\rangle^3 - \frac{\omega x^4}{24} + C_1 x + C_2$$

Boundary Condition ; $y(0) = y(L) = y(2L) = 0$ 에서

$$C_2 = 0, \quad C_1 = -\frac{\omega L^3}{48}$$

$$F_A = F_c = \frac{3\omega}{8}L, \quad F_B = \frac{5}{4}\omega L$$

$$\therefore y = \frac{1}{EI}\left[\frac{3L}{48}x^3 + \frac{10L}{48}\langle x-L\rangle^3 - \frac{1}{24}x^4 - \frac{L^3}{48}x\right]$$

$\frac{dy}{dx}=0$을 만족하는 $x \simeq 0.45L$ 에서 v_{max} 이 생긴다.

$$|v|_{max} = \frac{\omega L^4}{200EI} = 1.91\frac{\omega L^4}{384EI}$$

구간 AB에서 $M_b = \frac{3\omega L x}{8} - \frac{\omega x^2}{2}$, $x = \frac{3}{8}L$ 에서 max.

$$(M_b)_{x=\frac{3}{8}L} = \frac{9\omega L^2}{128}, \quad (M_b)_{x=L} = \frac{\omega L^2}{8}$$

$$\therefore |M_b|_{max} = \frac{\omega L^2}{8}$$

Ans. (a) $R = \omega L$, $M_{max} = \frac{1}{8}\omega L^2$, $\delta_{max} = \frac{5\omega L^4}{384EI}$

(b) $R = \frac{3}{8}\omega L$, $M_{max} = \frac{1}{8}\omega L^2$, $\delta_{max} = 1.91\frac{\omega L^4}{384EI}$

8·17 (a) $EIy'' = -M_0 + \frac{M_0}{a}\langle x+a-L\rangle^1$

$$EIy' = -M_0 x + \frac{M_0}{2a}\langle x+a-L\rangle^2 + C_1 \;;\; y'(0)=0 \quad \therefore C_1 = 0$$

$$EIy = -\frac{1}{2}M_0 x^2 + \frac{M_0}{6a}\langle x+a-L\rangle^3 + C_2 \;;\; y(0)=0 \quad \therefore C_2 = 0$$

$$\therefore\ y = \frac{M_0}{6EI}\left[-3x^2 + \frac{1}{a}\langle x-a-L\rangle^3\right]$$

$$y(L) = \frac{M_0}{6EI}\left[-3L^2 + a^2\right] \qquad \lim_{a/L \to 0} = \frac{M_0 L^2}{6EI}\left(-3 + \left(\frac{a}{L}\right)^2\right) = -\frac{M_0 L^2}{2EI}$$

(b) $EIy'' = -M_0 \qquad EIy' = -M_0 x + C_1\ ;\ y'(0) = 0 \quad \therefore C_1 = 0$

$\qquad EIy = -\frac{1}{2}M_0 x^2 + C_2\ ;\ y(0) = 0 \quad \therefore C_2 = 0$

$$\therefore\ y = -\frac{M_0}{2EI}x^2$$

$$y(L) = -\frac{M_0 L^2}{2EI}$$

Ans. (a) $y(L) = \frac{M_0}{6EI}\left[-3L^2 + a^2\right]$ (b) $y(L) = -\frac{M_0 L^2}{2EI}$

$(a/L) \to 0$일 경우에는 두 경우의 값이 서로 같다.

8.18 $g_n(x) = \langle a-x\rangle^n$

즉, $g_n(x) = \begin{cases} (a-x)^n & x \le a \\ 0 & x > a \end{cases}$

$$\int_x^\infty \langle a-x\rangle^n dx = \int_x^a \langle a-x\rangle^n dx + \int_a^\infty \langle a-x\rangle^n dx$$

정의로 부터 $\int_a^\infty \langle a-x\rangle^n dx = 0$

또 $\int_x^a \langle a-x\rangle^n dx = \begin{cases} \frac{(a-x)^{n+1}}{n+1} & x \le a \\ 0 & x > a \end{cases}$

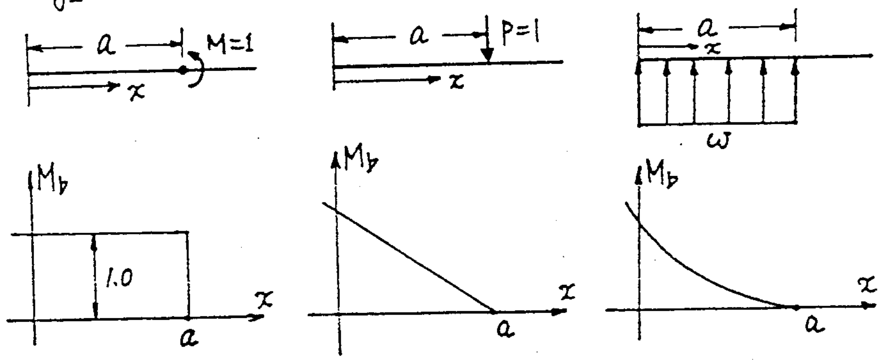
$$\therefore \int_x^\infty \langle a-x\rangle^n dx = \frac{\langle a-x\rangle^{n+1}}{(n+1)}$$

$x_0 < a \quad \int_{x_0}^x \langle a-x\rangle^n dx = \begin{cases} \frac{(a-x_0)^{n+1}}{n+1} - \frac{(a-x)^{n+1}}{n+1} = C - \frac{(a-x)^{n+1}}{n+1} & x \le a \\ \frac{(a-x_0)^{n+1}}{n+1} = C & x > a. \end{cases}$

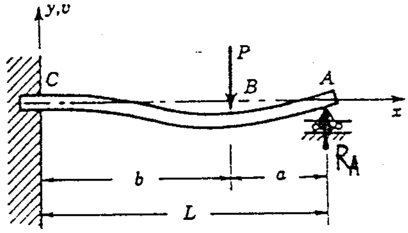
$$\therefore \int \langle a-x\rangle^n dx \equiv \int_{x_0}^x \langle a-x\rangle^n dx = -\frac{\langle a-x\rangle^{n+1}}{n+1} + C. \qquad -Q.E.D.-$$

Beam에 걸리는 하중과 bending moment는 beam을 따라서 우측에서 좌측으로 나아간다.

$g_0(x)$; unit couple에 의한 moment
$g_1(x)$; unit point load에 의한 moment
$g_2(x)$; unit continuous load에 의한 moment.



8.19 $M_b = \sum_{j=1}^{m} P_j \langle a_j - x \rangle^1$



$m=2$를 취하면

$P_1 = -P$, $a_1 = b$

$P_2 = R_A$, $a_2 = L$

또한, $a_2 > x$ 이라면, $\langle a_2 - x \rangle^1 = (a_2 - x) = (L - x)$

$\therefore M_b = R_A(L-x) - P\langle b-x \rangle^1 = EIy''$

$EIy' = -\frac{R_A}{2}(L-x)^2 + \frac{P}{2}\langle b-x \rangle^2 + C_1$

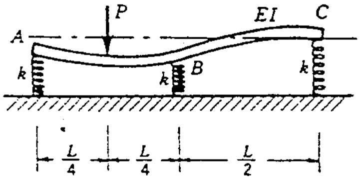
$EIy = \frac{R_A}{6}(L-x)^3 - \frac{P}{6}\langle b-x \rangle^3 + C_1 x + C_2$

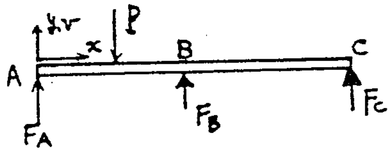
여기에 B.C. $y(0) = y'(0) = 0$, $y(L) = 0$ 를 대입하면

$C_1 = \frac{1}{2}(R_A L^2 - Pb^2)$ $\qquad C_2 = -\frac{1}{6}(R_A L^3 - Pb^3)$

$\therefore R_A = \frac{P}{2} \cdot \frac{b^2}{L^3}(3L - b)$ $\qquad$ Ans. $R_A = \frac{Pb^2}{2L^3}(3L-b)$

8.20





변형이 일어난 후에 A점을 기준으로 잡으면

δ = displacement of A, $\delta - v_B$ = displacement of B

$\delta - v_C$ = displacement of C

Force - deflection ; $F_A = k\delta$

$$F_B = k(\delta - v_B) = F_A - kv_B$$

$$F_C = k(\delta - v_C) = F_A - kv_C$$

Equilibrium; $\Sigma F_y = 0$; $F_A + F_B + F_C = P$ 또는, $3F_A - k(v_B + v_C) = P$ —①

$\Sigma M_{Az} = 0$; $\frac{PL}{4} = F_B \cdot \frac{L}{2} + F_C \cdot L = L\left[\frac{3}{2}F_A - k\left(\frac{v_B}{2} + \frac{v_C}{2}\right)\right]$

또는 $6F_A - 2k(v_B + v_C) = P$ —②

Bending Moment ;

$$EIv'' = M_b = F_A \cdot x - P\langle x - \tfrac{L}{4}\rangle' + F_B\langle x - \tfrac{L}{2}\rangle'$$

$$= F_A \cdot x - P\langle x - \tfrac{L}{4}\rangle' + (F_A - kv_B)\langle x - \tfrac{L}{2}\rangle'$$

적분하면; $EIv = \frac{F_A}{6}x^3 - \frac{P}{6}\langle x - \frac{L}{4}\rangle^3 + \frac{(F_A - kv_B)}{6}\langle x - \frac{L}{2}\rangle^3 + C_1 x + C_2$

Boundary Condition;

$v(0) = 0$, $v(L/2) = v_B$, $v(L) = v_C$

$\therefore C_2 = 0$

$$EIv_B = \frac{F_A \cdot L^3}{48} - \frac{PL^3}{384} + C_1\frac{L}{2} \quad —③$$

$$EIv_C = \frac{F_A \cdot L^3}{6} - \frac{27PL^3}{384} + \frac{(F_A - kv_B) \cdot L^3}{48} + C_1 L \quad —④$$

① ② ③ ④로 부터

$F_A = \dfrac{P(\frac{7}{2} + \frac{13}{384}N)}{(6 + \frac{N}{12})}$, $F_B = \dfrac{P(2 + \frac{22}{384}N)}{(6 + \frac{N}{12})}$, $F_C = \dfrac{P(\frac{1}{2} - \frac{3}{384}N)}{(6 + \frac{N}{12})}$, $N \equiv \dfrac{kL^3}{EI}$

Ans. $F_A = P(\frac{7}{2} + \frac{13}{384}N)/(6 + \frac{N}{12})$, $F_B = P(2 + \frac{22}{384}N)/(6 + \frac{N}{12})$

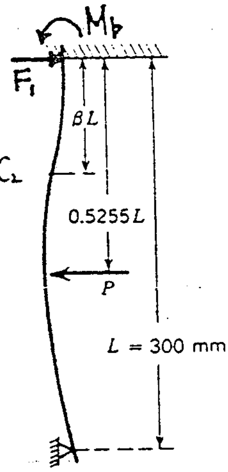
$F_C = P(\frac{1}{2} - \frac{3}{384}N)/(6 + \frac{N}{12})$ where $N \equiv \frac{kL^3}{EI}$

8·21 (a) Bending Moment ;

$$EIy'' = -M_0 + F_1 x - P\langle x-0.5255L\rangle^1$$

$$EIy' = -M_0 x + \frac{F_1}{2}x^2 - \frac{1}{2}P\langle x-0.5255L\rangle^2 + C_1$$

$$EIy = -\frac{1}{2}M_0 x^2 + \frac{F_1}{6}x^3 - \frac{1}{6}P\langle x-0.5255L\rangle^3 + C_1 x + C_2$$



Boundary Condition ; $y'(0)=0$. $y(0)=0$

$$\therefore C_1 = C_2 = 0$$

$$y(L) = y''(L) = 0$$

$$\therefore -\frac{1}{2}M_0 L^2 + \frac{1}{6}F_1 L^3 + \frac{1}{6}P(0.4745L)^3 = 0 \quad -①$$

$$-M_0 + F_1 L - 0.4745PL = 0 \quad -②$$

①② 에서 $F_1 = 0.7385P$, $M_0 = 0.2640PL$

i) $\beta < 0.5255$ 인 경우 $y''=0$ 에서 $\therefore x = \frac{M_0}{F_1} = 0.3575L$

ii) $\beta \geq 0.5255$ 인 경우 $y''=0$ 에서 $\therefore x = L$

(b) $\phi_1 = y'(0.3575L) = -0.04719\frac{PL^2}{EI}$

$$\phi_2 = y'(L) = 0.1053\frac{PL^2}{EI} \quad \therefore \beta = 1 \quad \phi_{max} = 0.1053\frac{PL^2}{EI}$$

(c) $k = \frac{S}{\ } = \frac{2\phi_{max}\times 1.5}{\ } = 0.3159\frac{L^2}{EI}$

Ans. (a)(b)(c) 에서 $\beta=1$. $\phi_{max} = 0.1053\frac{PL^2}{EI}$ $k = 0.3159\frac{L^2}{EI}$

8·22 Castigliano's theorem ; $\delta = \frac{\partial U}{\partial P}$ (2구간으로 나누어 생각)

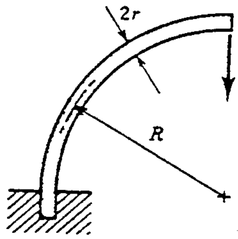
$$U = \frac{1}{2}\int \frac{M_b^2}{EI}dx = \frac{1}{4EI}\int_0^{\frac{L}{2}} P^2(L-x)^2 dx + \frac{1}{2EI}\int_{\frac{L}{2}}^{L} P^2(L-x)^2 dx$$

$$= \frac{P^2}{4EI}\left[-\frac{(L-x)^3}{3}\right]_0^{\frac{L}{2}} + \frac{P^2}{2EI}\left[-\frac{(L-x)^3}{3}\right]_{\frac{L}{2}}^{L} = \frac{3P^2L^3}{32EI}$$

$$\therefore \delta = \frac{\partial U}{\partial P} = \frac{3PL^3}{16EI}$$

Ans. $\delta = 3PL^3/16EI$

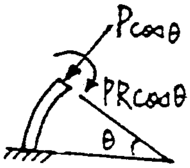
8·23 Castigliano's theorem을 적용하면.



$$U=\int\frac{F^2}{2EA}ds+\int\frac{M^2}{2EI}ds$$

$$=\int_0^{\frac{\pi}{2}}\frac{P^2\cos^2\theta}{2EA}(Rd\theta)+\int_0^{\frac{\pi}{2}}\frac{P^2R^2\cos^2\theta}{2EI}(R\,d\theta)$$

$$=\frac{P^2R}{8r^2E}+\frac{P^2R^3}{2r^4E}$$

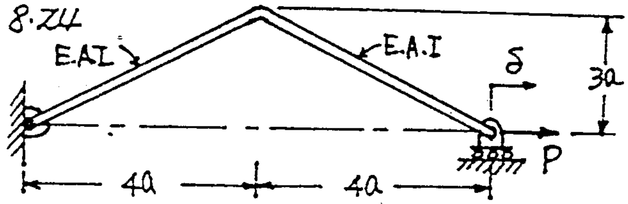


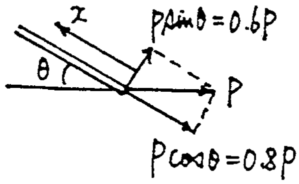
$$\therefore\delta=\frac{\partial U}{\partial P}=\underbrace{\frac{PR}{4r^2E}}_{=\text{axial}}+\underbrace{\frac{PR^3}{r^4E}}_{=\text{bending}}\qquad\therefore\frac{\delta_a}{\delta_b}=\frac{r^2}{4R^2}$$

—Q.E.D—

Ans. $\delta=\frac{P\cdot R}{4r^2E}+\frac{PR^3}{r^4E}$

8·24





Castigliano's theorem을 적용하면.

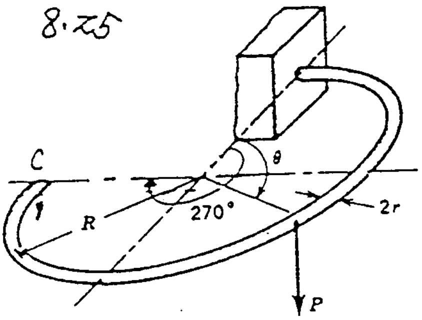
$$U=\int\frac{F^2}{2EA}dx+\int\frac{M^2}{2EI}dx$$

$$=2\left[\int_0^{5a}\frac{(0.8P)^2}{2EA}dx+\int_0^{5a}\frac{(0.6P\cdot x)^2}{2EI}dx\right]=\frac{16P^2a}{5EA}+5\frac{P^2a^3}{EI}$$

$$\therefore\delta=\frac{\partial U}{\partial P}=\frac{32Pa}{5EA}+\frac{10Pa^3}{EI}$$

Ans. $\delta=\frac{32Pa}{5EA}+\frac{10Pa^3}{EI}$

8·25



(1) Vertical deflection (임의의 각 ; α)

$$(M_b)_\alpha=-PR\sin(\theta-\alpha)$$

$$m_b=-R\sin\left(\alpha-\frac{\pi}{2}\right)=-R\cos\alpha$$

$$(M_t)_\alpha=PR(1-\cos(\theta-\alpha))$$

$$m_t=R\left(1+\cos\left(\alpha-\frac{\pi}{2}\right)\right)=R(1+\sin\alpha)$$

$$\therefore \delta_v = \int \frac{M_b \cdot m_b}{EI} ds + \int \frac{M_t \cdot m_t}{G \cdot J} ds$$

($\alpha > \theta$ 때 $M_b = M_t = 0$ 이므로 $\alpha = \theta$까지만 적분해 주면 된다.)

$$= \int_0^\theta \frac{PR^2 \sin(\theta-\alpha)\cos\alpha}{EI} R d\theta + \int_0^\theta \frac{PR^2\{1-\cos(\theta-\alpha)\}(1+\sin\alpha)}{GJ} R d\theta$$

$$= \frac{PR^3}{EI}\left[\sin\theta \cdot \left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right) - \cos\theta \cdot \frac{\sin^2\theta}{2}\right] + \frac{PR^3}{GJ}\left[\theta - \cos\theta\sin\theta\right.$$

$$\left. + \sin\theta\cos\theta - \sin\theta - \cos\theta + 1 - \cos\theta \cdot \frac{\sin^2\theta}{2} + \frac{1}{4}\sin\theta\sin 2\theta - \frac{1}{2}\theta\sin\theta\right]$$

$$\therefore \delta_v = \frac{PR^3\theta}{2EI}\sin\theta + \frac{PR^3}{GJ}\left(1 + \theta - \frac{1}{2}\theta\sin\theta - \sin\theta - \cos\theta\right)$$

(2) Horizontal deflection

C점에 Horizontal load H가 있다고 가정을 하고, 임의점에서의 접선력을 F_A라 하면,

$$F_A = \sin\left(\frac{3}{2}\pi - \alpha\right) \times H = -H\cos\alpha$$

$$(M_b)_H = HR\sin\left(\frac{3}{2}\pi - \alpha\right) = -HR\cos\alpha$$

$$M_t = PR(1 - \cos\langle\theta - R\rangle)$$

$$(M_b)_v = PR\sin\langle\theta - R\rangle$$

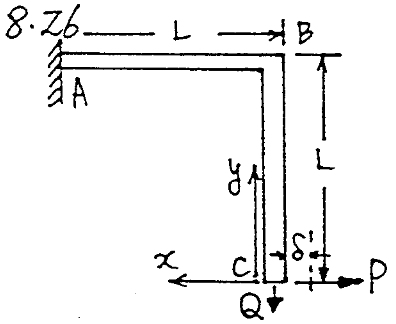
Castigliano's theorem 이용.

$$U = \int_0^{\frac{3}{2}\pi} \frac{H^2R}{2EA}\cos^2\alpha\, d\alpha + \int_0^{\frac{3}{2}\pi} \frac{R^3}{2EI}\left[H^2\cos^2\alpha + P^2\sin^2\langle\theta-\alpha\rangle\right] d\alpha$$

$$\int_0^{\frac{3}{2}\pi} \frac{R^3}{2GJ} \cdot P^2 \cdot (1 - \cos\langle\theta-\alpha\rangle)^2 d\alpha, \quad \left(\frac{\partial U}{\partial H}\right)_{H=0} = 0 \quad \therefore \delta_H = 0$$

Ans. $\delta_v = \frac{PR^3\theta}{2EI}\sin\theta + \frac{PR^3}{GJ}\left(1 + \theta - \frac{1}{2}\theta\sin\theta - \sin\theta - \cos\theta\right), \ \delta_H = 0$

8.26



Castigliano's theorem을 적용.

구간 AB ; $\begin{cases} M_b = PL - Qx \\ F_A = P \end{cases}$

구간 BC ; $\begin{cases} M_b = P \cdot y \\ F = Q \end{cases}$

Strain energy ; $\overline{U} = \int_0^L \frac{(PL-Qx)^2}{2EI}dx + \int_0^L \frac{P^2}{2EA}dx + \int_0^L \frac{P^2y^2}{2EI}dy + \int_0^L \frac{Q^2}{2EA}dy$

$\frac{\partial \overline{U}}{\partial Q} = \int_0^L \frac{2(PL-Qx)}{2EI}(-x)\,dx + \int_0^L \frac{2Q}{2EA}dy$

$= \frac{QL^3}{3EI} - \frac{PL^3}{2EI} + \frac{QL}{EA} = 0 \quad \therefore Q = \frac{3L^2A}{2L^2A+6I}P$ —①

$\delta = \frac{\partial \overline{U}}{\partial P} = \int_0^L \frac{(PL-Qx)}{EI}L\,dx + \frac{PL}{EA} + \frac{PL^3}{3EI}$

$= \frac{PL^3}{EI} - \frac{QL^3}{2EI} + \frac{PL}{EA} + \frac{PL^3}{3EI} = \left[\frac{4L^3}{3EI} + \frac{L}{EA} - \frac{L^3}{2EI}\left(\frac{3L^2A}{2L^2A+6I}\right)\right]P$

$= 0.025$

$I = \frac{\pi}{64}(0.013)^4, \quad A = \frac{\pi}{4}(0.013)^2, \quad L = 1.25m$

$E = 2\times10^{11}$ 을 대입하면 $\quad P = 6.15\,N$ ①에 대입

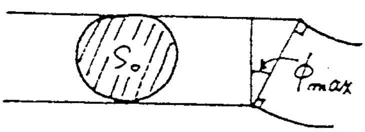
$\therefore Q = 9.23\,N$

i) 구간 AB $\quad \sigma_{max} = \frac{P}{A} + \frac{M_b}{I}\cdot\frac{d}{2} = 35.70\ MN/m^2$

ii) 구간 BC $\quad \sigma_{max} = \frac{Q}{A} + \frac{M_b}{I}\frac{d}{2} = 35.7\ MN/m^2$

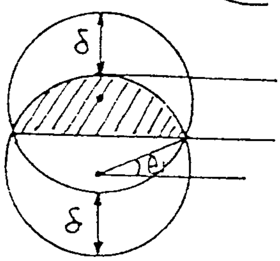
Ans. $\sigma_{max} = 35.7\ MN/m^2$

8.27 문제 (8.1(g)) 에서 $\delta_{max} = \frac{5wL^4}{384EI}$, $\quad \phi_{max} = \frac{wL^3}{24EI}$



$S_0 = \pi(0.072)^2\cos\phi_{max}$

$\simeq \pi R_i^2 \quad (\because \phi_{max} \simeq 0)$



그림에서 $\sin\theta_i = \frac{\delta}{2R_i}$

θ_i 이 infinitesimal 이라고 가정하면

$\sin\theta_i \simeq \theta_i = \frac{\delta}{2R_i}$

$$S_1 = 2\int_{\theta_1}^{\frac{\pi}{2}} 2R_i\cos\theta\,(R_i\cos\theta)\,d\theta = 2R_i^2\left(\frac{\pi}{2}-\theta_1-\sin\theta_1\cos\theta_1\right)$$

$$= \pi R_i^2 - 2R_i\frac{\delta}{R_i} = \frac{1}{2}\pi R_i^2 = \frac{1}{2}S_0 = 8.486\times10^{-3}\ (m^2)$$

$\therefore \dfrac{\delta}{R_i} = \dfrac{1}{4}\cdot\dfrac{1}{\pi}$ 따라서 위의 가정이 맞다.

$$\left(\begin{aligned} &w = (7.79\times10^3)(9.8)\times\frac{\pi}{4}[0.15^2-0.144^2] = 105.8\ N/m \\ &I = \pi R^3 t = 3.744\times10^{-6}\ m^4 \end{aligned}\right.$$ δ에 대입하면

$$\therefore \frac{5\cdot(105.8)\cdot L^4}{384\cdot(200\times10^9)(3.744\times10^{-6})} = \frac{1}{4\pi}(0.072)$$

$L = 7.47\ m$

(2) $w = (7.79\times10^3)\times\frac{\pi}{4}(0.15^2-0.126^2) = 397.2\ N/m$

$I = \frac{\pi}{64}(0.15^4-0.126^4) = 1.248\times10^{-5}\ m^4$

$$\delta_{max} = \frac{5\times(397.2)(7.47)^4}{384\times(200\times10^9)\times1.248\times10^{-5}} = 6.452\times10^{-3}$$

if $\theta_1 \ll 1$, $\sin\theta_1 \simeq \theta_1 = \dfrac{\delta}{2R_i} = \dfrac{6.452\times10^{-3}}{0.126} = 0.0512\ (rad)$

$\therefore S_1 = 2\times(0.063)^2\left[\frac{\pi}{2}-0.0152-0.0512\times1\right] = 0.01166\ (m^2)$

$\therefore \Delta S = 0.01166 - 8.486\times10^{-3} = 3.17\times10^{-3}\ (m^2)$

따라서 $\dfrac{\Delta S}{S_1}\times100 = \dfrac{3.17\times10^{-3}}{8.486\times10^{-3}}\times100 = 37.36(\%)$ 증가.

Ans. $L = 7.47\ m$, 37.36%

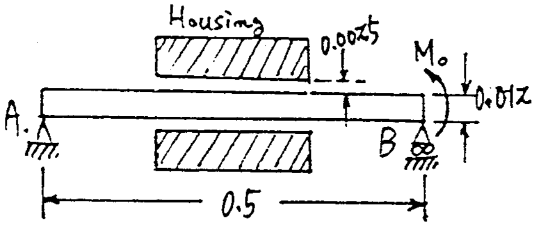
8.28 문제 (8.1 (g))에서

$$\delta_{max} = \frac{5wL^4}{384EI} = \frac{1}{360}L$$

$$\therefore I_{min} = \frac{5\times360\,wL^3}{384E} = \frac{5\times360\times15000\times3.6^3}{384\times7\times10^9} = 4.686\times10^{-4}(m^4)$$

Ans. $I_{min} = 4.686\times10^{-4}\ m^4$

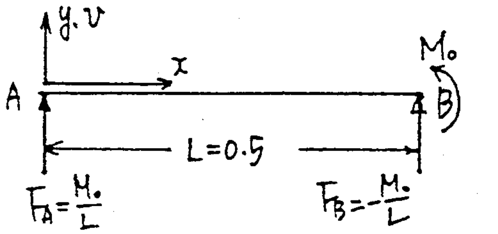
8.29 Bending Moment;



$$EIv'' = M_b = \frac{M_0}{L}x$$

적분하면 $EIv = \frac{M_0}{6L}x^3 + C_1 x + C_2$

Boundary Condition;



$$v(0) = v(L) = 0$$

$$\therefore C_2 = 0, \quad C_1 = -\frac{M_0 L}{6}$$

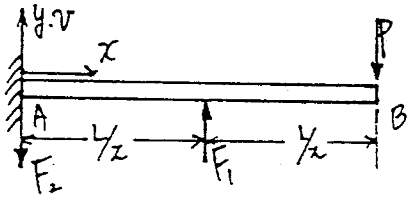
$$\therefore v = \frac{1}{EI}\left\{\frac{M_0}{6}\left(\frac{x^3}{L} - xL\right)\right\}$$

$|v|_{max}$ 은 $\frac{dv}{dx}=0$ 을 만족하는 $x = \frac{L}{\sqrt{3}}$ 에서 생긴다. $|v|_{max} = \frac{M_0}{6EI}L^2\left(\frac{\sqrt{3}}{9} - \frac{\sqrt{3}}{3}\right) = -\frac{\sqrt{3}M_0L^2}{27EI}$

$$\therefore \frac{\sqrt{3}M_0\,(0.5)^2}{27\times(200\times10^9)(\frac{\pi}{64}\times0.02^4)} = 1.25\times10^{-3} \quad \therefore M_0 = 15.87\ \text{N-m}$$

Ans. $M_0 = 15.87$ N-m

8.30 Beam AB



Bending Moment

$$EIv'' = -M_0 - F_2x + F_1\langle x-\tfrac{L}{2}\rangle$$

$$EIv' = -M_0x - \tfrac{1}{2}F_2x^2 + \tfrac{1}{2}F_1\langle x-\tfrac{L}{2}\rangle^2 + C_1$$

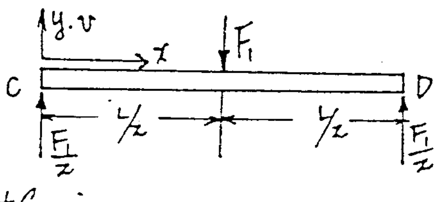
$$EIv = -\tfrac{1}{2}M_0x^2 - \tfrac{1}{6}F_2x^3 + \tfrac{1}{6}F_1\langle x-\tfrac{L}{2}\rangle^3 + C_1x + C_2$$

Boundary Condition $v'(0) = v(0) = 0$. $\therefore C_1 = C_2 = 0$

$$\therefore EIv = -\tfrac{1}{2}M_0x^2 - \tfrac{1}{6}F_2x^3 + \tfrac{1}{6}F_1\langle x-\tfrac{L}{2}\rangle^3 \quad \text{—①}$$

Equilibrium; $\Sigma M_0 = 0$. $M_0 + F_2L = F_1(\frac{L}{2})$; $F_1L = 2F_2L + 2M_0$ —②

Beam CD



$$EIv'' = \tfrac{1}{2}F_1x + (-F_1)\langle x-\tfrac{L}{2}\rangle^1$$

$$EIv' = \tfrac{1}{4}F_1x^2 - \tfrac{1}{2}F_1\langle x-\tfrac{L}{2}\rangle^2 + C_1$$

$$EIv = \tfrac{1}{12}F_1x^3 - \tfrac{1}{6}F_1\langle x-\tfrac{L}{2}\rangle^3 + C_1x + C_2\ ;$$

$v(0) = v(L) = 0 \quad \therefore C_2 = 0. \quad C_1 = \frac{-1}{16} F_1 L^2$

$$\therefore v = \frac{F_1}{EI}\left[\frac{1}{12}x^3 - \frac{1}{6}\langle x - \frac{L}{2}\rangle^3 - \frac{1}{16}L^2 x\right] \quad \text{— ③}$$

①③ 식에서 Geometry를 적용. $v_{AB}(\frac{L}{2}) = v_{CD}(\frac{L}{2})$

$$-\frac{1}{8}M_0 L - \frac{F_2}{48}L^3 = -\frac{1}{48}F_1 L^3$$

$\therefore F_1 L = F_2 L + 6M_0$ — ④ $\quad \Sigma F_y = 0\,;\ F_1 = F_2 + P$ — ⑤

②④⑤ 에서, $F_1 = \frac{5}{3}P\,,\ F_2 = \frac{2}{3}P\,.\ M_0 = \frac{1}{6}PL$

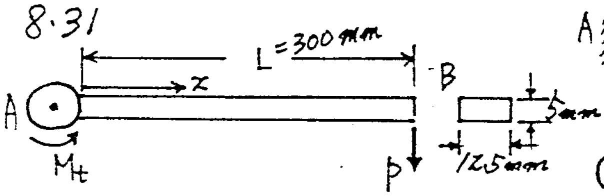
따라서 $v(\frac{L}{2}) = -\frac{F_1 L^3}{48EI} = -\frac{5PL^3}{144EI} \qquad \therefore \delta_E = \frac{3PL^3}{144EI}$

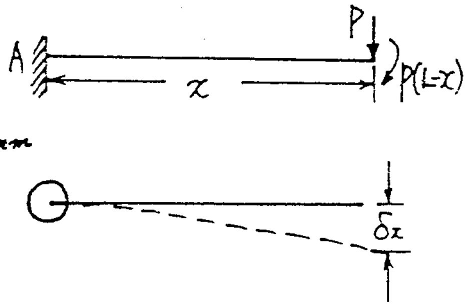
$$v(L) = \frac{1}{EI}\left(-2M_0 L^2 - \frac{1}{6}F_2 L^3 + \frac{F_1}{48}L^3\right)$$

$$= \frac{1}{EI}\left(-\frac{PL^3}{3} - \frac{PL^3}{9} + \frac{PL^3}{144}\right) = -\frac{63PL^3}{144EI}$$

Ans. $\delta_P = \frac{63PL^3}{144EI}$

8·31





Equilibrium ; $P = M_t/L$

중첩의 원리에 의한 x점에서의 deflection;

$$\delta_x = \frac{Px^3}{3EI} + \frac{P(L-x)x^2}{2EI} = \frac{P}{EI}\left[\frac{x^3}{3} + \frac{x^2(L-x)}{2}\right]$$

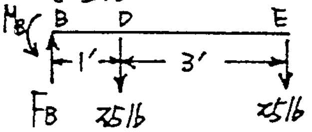
$x = 0.3\,m$ 에서 $\delta_x = \frac{P}{EI}\left[\frac{(0.3)^3}{3} + \frac{(0.3)^2(0.38-0.3)}{2}\right]$

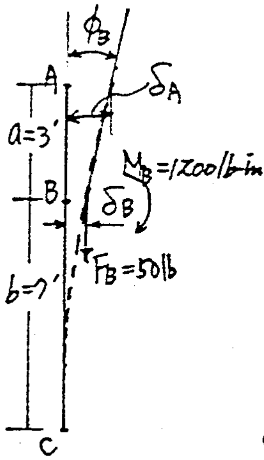
$$= \frac{M_t}{(0.38)(200\times10^9)(1.302\times10^{-10})}\left[\frac{(0.3)^3}{3} + \frac{(0.3)^2(0.08)}{2}\right]$$

$$= 1.2733\times10^{-3} M_t$$

Ans. $\delta = 1.2733 \times 10^{-3} M_t \cdot m$ M_t는 N-m.

8.32





평형조건; $F_B = 50lb$

$M_B = 1200\ lb\text{-}in$

Signpost의 자중과 Axial force F_B는 Signpost에 bending-moment를 야기시키지만, 작은 값이므로 무시한다.

중첩의 원리; $\delta_A = \delta_B + AB \times \phi_B$

$$= \delta_B + a\phi_B$$

$$\delta_B = \frac{M_B \cdot b^2}{2EI}, \quad \phi_B = \frac{M_B \cdot b}{EI} \qquad a = 36'', \ b = 84''$$

따라서 $\delta_A = \dfrac{M_B \cdot b}{EI}\left(\dfrac{b}{2} + a\right) = \dfrac{84 \times 1200}{\frac{\pi}{64}(256 - 150) \times 30 \times 10^6} \times 78 = 0.0503''$

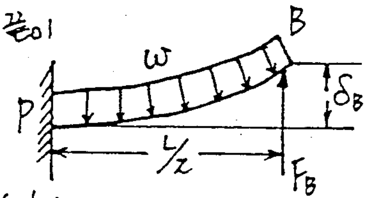
Ans. $\delta_A = 0.0503''$

8.33 Pontoon이 δ_P만큼 잠아 앉는다면, 상방으로 작용하는 힘은;

$$F_P = \gamma A \delta_P$$

대칭성에 의해서, 양단에서의 반력은 서로 같다.

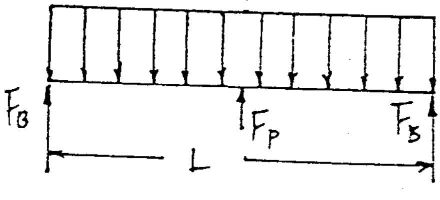
그리고, bridge는 중앙점(P)에서의 기울기는 "0"이다. 따라서 bridge의 반은 한쪽끝이 고정된 cantilever로 생각할 수 있다.



그런후에 중첩의 원리를 적용하자.

$$\delta_B = \frac{F_B \cdot (\frac{L}{2})^3}{3EI} - \frac{\omega (\frac{L}{2})^4}{8EI} = \frac{L^3}{8EI}\left(\frac{F_B}{3} - \frac{\omega L}{16}\right)$$

이것은 B점이 고정되어 있다고 생각하면 δ_P도 될 것이다.



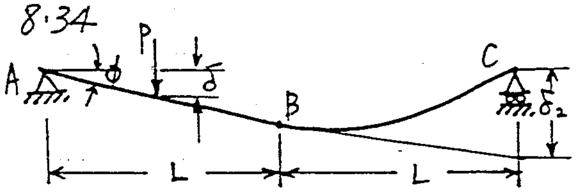
$$\therefore F_P = \gamma A \delta_P = \frac{\gamma A L^3}{8EI}\left(\frac{F_B}{3} - \frac{\omega L}{16}\right)$$

전체 bridge의 평형조건에서 $2F_B + F_P = \omega L$

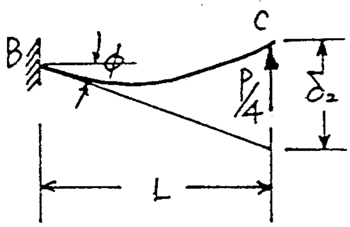
∴ $F_B = \dfrac{\left(1 + \frac{\gamma A L^3}{128EI}\right)}{\left(2 + \frac{\gamma A L^3}{128EI}\right)}$. ∴ $\delta_P = \dfrac{5\omega L^4}{384EI\left(1+\frac{\gamma A L^3}{48EI}\right)}$

Ans. $\delta = \dfrac{5\omega L^4}{384EI\left(1+\frac{\gamma A L^3}{48EI}\right)}$

8·34



중첩의 원리.
Rigid beam의 기울기를 ϕ로 놓으면, 그림에서 $\delta \simeq \phi L$
$\delta_2 \simeq 2\phi L$



Slope ϕ로, B점에 고정된 Cantilever beam으로 BC를 고려하면.

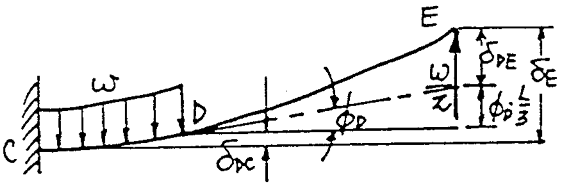
$$\delta_2 = \frac{P}{4}\cdot\frac{(L)^3}{3EI} = 2\phi L$$

$$\therefore \phi = \frac{PL^2}{24EI}$$

따라서 $(\delta)_{L/2} = \phi\cdot\frac{L}{2} = \frac{PL^3}{48EI}$; 하중 상태에서의 처짐.

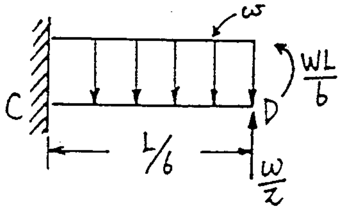
Ans. $\delta = \dfrac{PL^3}{48EI}$

8·35 분포하중 $\omega = 3\overline{W}/L$. 대칭에 의해서 양단에서의 반력은 $\overline{W}/2$이다. 또 대칭성에 의해서 중앙점의 기울기는 "0"이다. 따라서 중앙점이 "0" 기울기로 고정되어 있는 cantilever로 고려할 수 있다.



$$\delta_E = \delta_{DC} + \phi_D\cdot\frac{L}{3} + \delta_{DE}$$

Beam CD ;



$$\delta_{DC} = -\frac{\omega(\frac{L}{6})^4}{8EI} + \frac{\overline{W}}{2}\frac{(\frac{L}{6})^3}{3EI} + \frac{\overline{W}L}{6}\cdot\frac{(\frac{L}{6})^2}{2EI}$$

$$= \frac{29\overline{W}L^3}{10368EI}$$

$$\phi_D = -\frac{\omega(\frac{L}{6})^3}{6EI} + \frac{\overline{W}}{2}\frac{(\frac{L}{6})^2}{2EI} + \frac{\overline{W}L}{6}\frac{(\frac{L}{6})}{EI}$$

$$= \frac{42\overline{W}L^2}{1296EI}$$

$$\delta_{DE} = \frac{\frac{\overline{W}}{2}(\frac{L}{3})^3}{3EI} = \frac{\overline{W}L^3}{162EI}$$

$$\therefore \delta_E = \frac{\overline{W}L^3}{EI}\left(\frac{1}{162} + \frac{29}{10368} + \frac{14}{1296}\right) = \frac{205\overline{W}L^3}{10368EI}$$

이것은 고정점을 E로 보면 $\delta_C = \frac{205\overline{W}L^3}{10368EI}$

exact solution ; $\delta_{exact} = \frac{\overline{W}L^3}{48EI}$

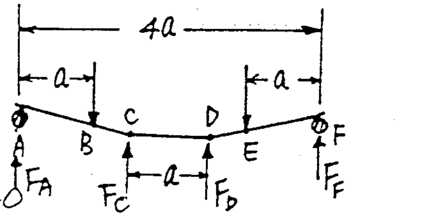
$$\therefore \frac{\delta_{approx}}{\delta_{exact}} = \frac{10368}{48\times205} \simeq 1.05 \quad (\text{약 } 5\% \text{ 의 error})$$

Ans. $\delta = (205\overline{W}L^3)/(10368EI)$ $\delta_{approx}/\delta_{exact} = 1.05$

8·36 i) CD에서 $EIv'' = M_b = 0$

ii) CD에서 $\frac{dM_b}{dx} = V_y = 0$

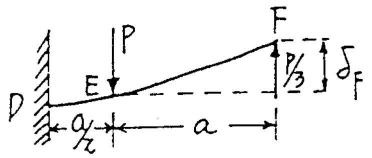
iii) CD에서의 임의점에서의 하중 = 0



iv) C·D점에서만 point force가 작용한다. 그렇지 않으면 $M_b \neq 0$.

평형조건 ; $F_A + F_C = F_D + F_F = P$

$M_b = 0$ at C and D $\therefore F_A = F_F = P/3$, $F_D = F_C = 2P/3$



DF를 D점이 고정된 cantilever로 생각하자. D점에서의 기울기는 "0". 그리고, 중첩의 원리를 사용하면

$$\text{Deflection at } E = \frac{\frac{Pa}{3}\left(\frac{a}{2}\right)^2}{2EI} - \frac{\left(\frac{2P}{3}\right)\left(\frac{a}{2}\right)^3}{3EI}$$

$$\text{Deflection of } EF = \frac{\left(\frac{P}{3}\right)(a)^3}{3EI}$$

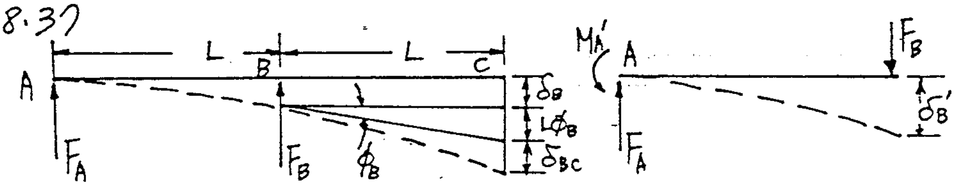
$$\text{Slope at } E = \frac{\left(\frac{Pa}{3}\right)\left(\frac{a}{2}\right)}{EI} - \frac{\left(\frac{2P}{3}\right)\left(\frac{a}{2}\right)^2}{2EI}$$

$$\therefore \delta_F = (\text{deflection at } E) + a \times (\text{slope at } E) + (\text{deflection of } EF)$$

$$= \frac{5Pa^3}{24EI} \qquad \therefore P = \frac{24EId}{5a^3}$$

Ans. $P = (24EId)/(5a^3)$

8.37



$$\text{Upper ; } \delta_B = \frac{(P-F_B)L^3}{3EI} + \frac{PL \cdot L^2}{2EI} \qquad \delta_B' = \frac{F_B L^3}{3EI}$$

$$= \frac{L^3}{6EI}(5P - 2F_B)$$

기하학적 적합조건 ; $\delta_B = \delta_B'$ $\quad \therefore \dfrac{F_B L^3}{3EI} = \dfrac{L^3}{6EI}(5P - 2F_B)$

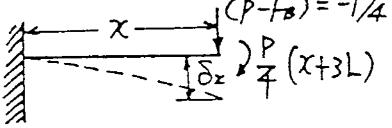
즉 $F_B = \frac{5}{4}P$

따라서, $\delta_C = \delta_B + L\phi_B + \delta_{BC}$

$$= \frac{5}{12}\frac{PL^3}{EI} + L \cdot \left[\frac{PL \cdot L}{EI} + \frac{\left(-\frac{P}{4}\right) L^2}{2EI}\right] + \frac{PL^3}{EI} = \frac{13PL^3}{8EI}$$

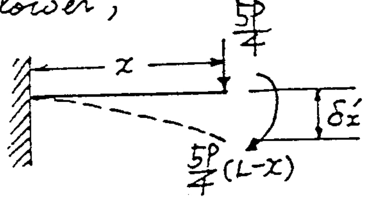
Deflection curves ; Method of superposition

upper ; $\quad x \le L \qquad \delta_x = \dfrac{P}{4} \cdot \dfrac{(x+3L)x^2}{2EI} - \dfrac{P}{4} \cdot \dfrac{x^3}{3EI}$



$$= \frac{P}{4EI}\left\{\frac{x^3}{6} + \frac{3}{2}x^2 L\right\} \quad x \le L$$

lower;



$$\delta'_x = \frac{(\frac{5P}{4})\cdot x^3}{3EI} + \frac{(\frac{5P}{4})(L-x)x^2}{2EI}$$

$$= \frac{P}{4EI}\left\{\frac{5}{2}Lx^2 - \frac{5}{6}x^3\right\} \quad 0\le x\le L$$

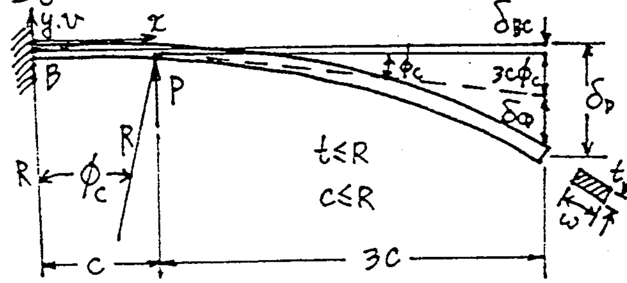
$$\delta'_x - \delta_x = \frac{P}{4EI}(Lx^2 - x^3) > 0 \quad \text{for } 0<x<L$$

$$= 0 \quad \text{for } x=L$$

∴ 가정은 정확하다.

Ans. $\delta = (13PL^3)/(8EI)$, The assumption is correct.

8.38



i) $v'' \simeq -\frac{1}{R} = const. = \frac{M_b}{EI}$

ii) $\frac{dM_b}{dx} = V_y = 0$

iii) Intensity of loading은 없다.

iv) C점에서 위로 힘 P가 작용한다. 따라서 $M_b = const.$

$x \le C$ 인 경우 $M_b = -P(4C-x) + P(C-x) = -3PC$

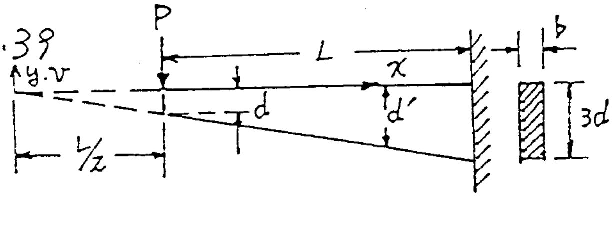
$$\therefore P = -\frac{M_b}{3C} = \frac{EI}{3CR}$$

중첩의 원리; $\delta_D = \delta_{BC} + 3C\cdot\phi_c + \delta_{CD}$

$$= \frac{1}{2}\frac{C^2}{R} + 3C\cdot\frac{C}{R} + \frac{P(3C)^3}{3EI} = \frac{13C^2}{2R}$$

Ans. $\delta_D = (13C^2)/(2R)$

8.39



임의의 점 x에서

$d'(depth) = 2dx/L$

$$\therefore I_x = \frac{bd'^3}{12} = \frac{2bd^3x^2}{3L^3}$$

Bending Moments $EI_x \cdot v'' = M_b = -P(x-\frac{L}{2}),\ \frac{L}{2} \le x \le \frac{3}{2}L$

I_x에 대해 적분; $\frac{2Ebd^3}{3L^3} v'' = -\frac{P}{x^3}(x-\frac{L}{2})$

적분하면; $\frac{2Ebd^3}{3L^2} v' = \frac{P}{x} - \frac{PL}{4x^2} + C_1$

$$\frac{2Ebd^3}{3L^2} v = P\ln x + \frac{PL}{4x} + C_1 x + C_2$$

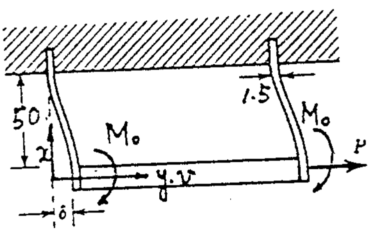
Boundary Condition ; $v(\frac{3}{2}L) = v'(\frac{3}{2}L) = 0$

$$\therefore v = \frac{3PL^3}{2Ebd^3}\left\{\ln\frac{2x}{3L} + \frac{2}{3} + \frac{L}{4x} - \frac{5}{9}\frac{x}{L}\right\}$$

$x = \frac{L}{2}$ 에서 $v = \frac{3PL^3}{2Ebd^3}(\frac{8}{9} - \log_e 3)$

$$\delta \equiv |v|_{L/2} = \frac{3PL^3}{2Ebd^3}(\log_e 3 - \frac{8}{9})\ ;\ \text{Ans.}$$

8.40



(대칭이므로 한쪽의 vertical rod 만을 고려한다.)

(a) Bending moment

$$EIv'' = M_0 - \frac{1}{2}Px$$

$$EIv' = M_0 x - \frac{1}{4}Px^2 + C_1$$

$$EIv = \frac{1}{2}M_0 x^2 - \frac{1}{12}Px^3 + C_1 x + C_2$$

Boundary Condition

$v'(0) = v'(L) = 0\ ,\ v(L) = 0$

따라서 $C_1 = 0\ ,\ M_0 = \frac{1}{4}PL$

$$C_2 = \frac{PL^3}{12} - \frac{1}{2}M_0 L^2 = -\frac{1}{24}PL^3$$

$$\therefore EIv = \frac{1}{8}PLx^2 - \frac{1}{12}Px^3 - \frac{1}{24}PL^3$$

$$\delta = -v(0) = \frac{PL^3}{24EI}$$

$$I = \frac{1}{12}(6\times10^{-3})(1.5\times10^{-3})^3 = 1.6875\times10^{-12}\ (m^4)$$

$E = 100\ GN/m^2$ 윗 식에 대입하면

$$\delta = \frac{(50\times10^{-3})^3 \cdot P}{24\cdot(100\times10^9)(1.6875\times10^{-12})} \quad \therefore P = 32400\delta \ (N)$$

δ의 단위는 m.

(b) $EIv'' = M_0 - \frac{P}{2}x = \frac{P}{4}L - \frac{P}{2}x$

따라서 $(M_b)_{max} = \frac{1}{4}PL$

$$\sigma_y = \frac{(M_b)_{max}}{I}\left(\frac{t}{2}\right) = \frac{\frac{1}{4}(32400\delta)}{(1.6875\times10^{-12})}(50\times10^{-3})\cdot\frac{(1.5\times10^{-3})}{2}$$

$= 1.8\times10^{11}\delta = 350\times10^6 \ (N/m^2)$ ($\because$ yielding load와 같아야 하기 때문)

$\therefore \delta = 1.94 \ mm$

Ans. $P = 32400\delta \ (N)$, $\delta = 1.94 \ (mm)$

8.41 $\dfrac{dv_s}{dx} = \dfrac{(\tau_{zy})_{max}}{G} = \dfrac{3P}{2Gbh}$

$(\tau_{zy})_{max} = \dfrac{3P}{2bh}$ for rectangular sections

$$v_s(x) = \int_{x=0}^{x}\frac{3P}{2Gbh}dx + v_s(0) = \frac{3Px}{2Gbh} + 0$$

$v_b(x) = \dfrac{Px^3}{3EI} + \dfrac{P(L-x)x^2}{2EI}$; 중첩의 원리 적용.

$$\frac{v_s(L)}{v_b(L)} = \frac{3PL/2Gbh}{PL^3/3EI} = \frac{9EI}{2GbhL^2} \ ; I = \frac{1}{12}bh^3 ; \frac{E}{G} = 2(1+\nu)$$

$\therefore \left(\dfrac{v_s}{v_b}\right)_{x=L} = \dfrac{3(1+\nu)}{4}\dfrac{h^2}{L^2}$; $\nu \leq 0.5$ (Thermodynamics 에서)

$\dfrac{h}{L} \leq 10^{-1}$ 이면 $\left(\dfrac{v_s}{v_b}\right)_{x=L} \leq \dfrac{3\times1.5}{4}\times10^{-2} = 1.025\times10^{-2}$

$\therefore$ Shear deflection은 약 1% 정도

$$\left(\frac{v_s}{v_b}\right)_{x=L} = \frac{3(1+\nu)}{4}\cdot\frac{h^2}{L^2} \quad \text{— Q.E.D. —}$$

8.42 Castigliano's Theorem을 사용하자.

$$\tau_{xy} = \frac{V}{2I}\left[\left(\frac{h}{2}\right)^2 - y^2\right]$$

$$U_s = \int \frac{\tau_{xy}^2}{2G}\,dx\,dy\,dz = \frac{1}{2G}\cdot\frac{V^2}{4I^2}\int\left(\frac{h^2}{4} - y^2\right)dx\,dy\,dz$$

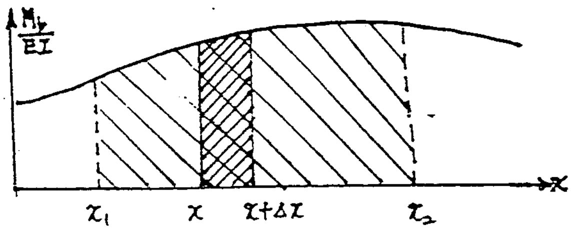
$$= \frac{V^2}{8GI^2}\cdot b\cdot L\cdot\left[\frac{h^4}{16}\cdot\frac{h}{2} - \frac{h^2}{2}\cdot\frac{1}{3}\left(\frac{h}{2}\right)^3 + \frac{1}{5}\left(\frac{h}{2}\right)^5\right]\times 2$$

$$= \frac{2V^2Lh^2}{480GI^2}\cdot bh^3 = \frac{2V^2h^2L}{40GI} = \frac{1+\nu}{10EI}\cdot V^2h^2L$$

Castigliano's theorem; $\frac{\partial U_s}{\partial V} = \frac{(1+\nu)Vh^2L}{5EI} = v_s$

$$\therefore \left(\frac{v_s}{v_b}\right)_{x=L} = \frac{3(1+\nu)}{5}\cdot\frac{h^2}{L^2}$$ — Q. E. D. —

8.43



(a) 곡률이 매우 크다면, Δx는 중립축을 따라서 측정되어야 한다. 구간 dx에서 곡률 반경이 일정하다고 가정하자.

그러면 구간 dx의 양끝에서의 접선 사이의 각도 $d\phi$는;

$$d\phi = \Delta x / R$$

$\frac{1}{R} \simeq \frac{d^2v}{dx^2} = \frac{M_b}{EI}$ $\therefore d\phi = \frac{M_b\cdot\Delta x}{EI} = \frac{M_b}{EI}$ 그림표에서 빗금친 부분

적분하면, $\phi_{12} = \int_{x_1}^{x_2}\frac{M_b}{EI}dx$; $\frac{M_b}{EI}$ 그림표에서 x_1과 x_2 사이의 면적

(b) 기하학적인 형태에서 Δx에 의한 ε의 증가량 $d\varepsilon$은

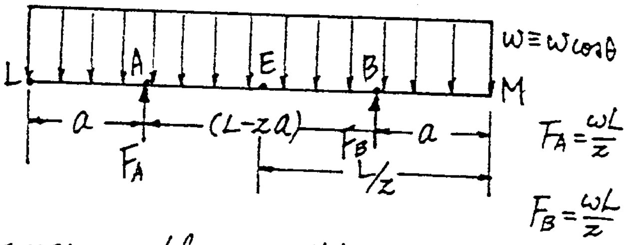
$d\varepsilon = (x_2 - x)\,d\phi = (x_2 - x)\cdot\frac{M_b}{EI}\cdot\Delta x$; x_2에 대한 빗금 부분의 1차 moment

적분하면 $\varepsilon_{12} = \int_{x_1}^{x_2} d\varepsilon = \int_{x_1}^{x_2}(x_2 - x)\frac{M_b}{EI}dx$; x_2에 대한 x_1과 x_2 사이의 M_b/EI 그림표의 1차 moment

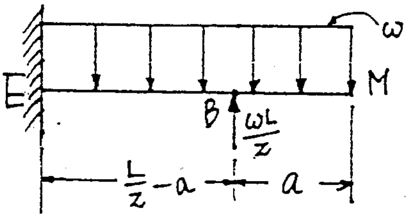
8·44 임의점 에서의 하중

i) axial load ; $\frac{W}{L}\sin\theta$

ii) normal load ; $\frac{W}{L}\cos\theta$



EM을 E점에 기울기 "0"으로 고정된 cantilever로 생각한다.



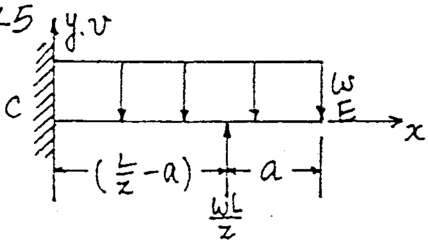
Load M이 작용하는 단면들은 E점에서의 단면에 평행하여야 한다. 왜냐하면 EL과 EM의 처짐은 E점에 대하여 대칭이기 때문이다.

중첩 이론에 의해서 M점에서의 기울기 ϕ_M

$$\phi_M = -\frac{w\left(\frac{L}{2}\right)^3}{6EI} + \frac{wL}{2}\cdot\frac{\left(\frac{L}{2}-a\right)^2}{2EI} = 0$$

$$\therefore\ a = L\left(\frac{\sqrt{3}-1}{2\sqrt{3}}\right) = 0.211L \qquad \text{Ans. } a = 0.211L$$

8·45



8·44와 같은 요령으로 E점이 기울기 "0"으로 고정되어 있는 cantilever로 CE를 생각하자.

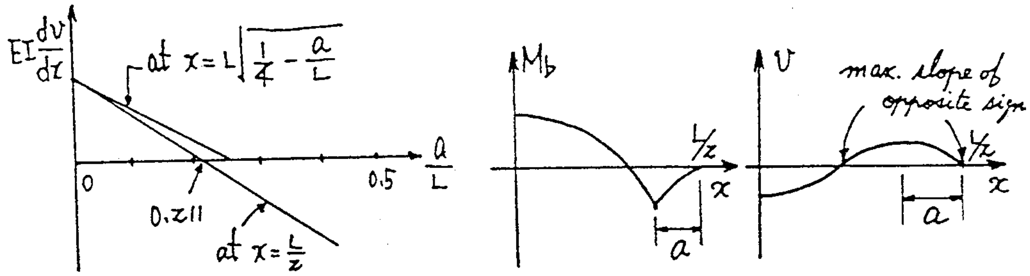
$$EIv'' = M_b = \frac{wL}{2}\left\langle \frac{L}{2}-a-x\right\rangle^1 - \frac{w}{2}\left(\frac{L}{2}-x\right)^2$$

Boundary Condition $\quad v(0) = v'(0) = 0$

$$v' = \frac{1}{EI}\left\{-\frac{wL}{4}\left\langle\frac{L}{2}-a-x\right\rangle^2 + \frac{w}{6}\left(\frac{L}{2}-x\right)^3 + \frac{3(L-2a)^3-L^2}{48}wL\right\}$$

$(v')_{max}$는 $M_b = 0$ or $\begin{cases} x = \sqrt{\frac{1}{4}-\frac{a}{L}}\cdot L & \text{for } \frac{a}{L} < \frac{1}{4} \\ x = \frac{L}{2} & \text{for } \frac{a}{L} > \frac{1}{4} \end{cases}$

$\frac{a}{L}$ 과 v' 과의 관계를 그림표로 그리면 ;



$\frac{dv}{dx}$ 가 절대값은 같고, 부호가 반대인 2개의 극값을 갖도록 (a/L)의 값을 취하면 $0.211 < \frac{a}{L} < 0.25$

(a/L) 값이 이 값의 범위에서 변하면, $\left|\frac{dv}{dx}\right|$의 값이 증가한다.

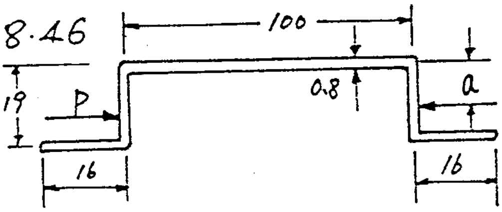
따라서 (a/L)의 선택은 $\min\cdot\left[\left|\frac{dv}{dx}\right|_{max}\right]$을 만족해야 한다.

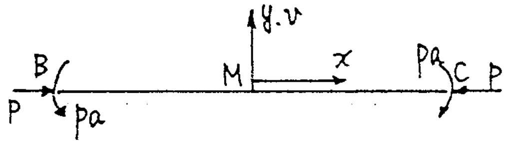
Tirial-Error method에 의해서 $(a/L) \simeq 0.22$

$$\text{이때} \left(\frac{dv}{dx}\right)_{x=\sqrt{\frac{1}{4}-\frac{a}{L}}\cdot L} = -\left(\frac{dv}{dx}\right)_{x=\frac{L}{2}}$$

Ans. $(a/L) = 0.22$

8.46





Axial force의 bending 효과는 무시한다. $(\simeq Pv)$

따라서 $M_b = EIv'' = -Pa = -\frac{3}{8}P$

Boundary Condition ; $v(0) = v'(0) = 0$

$\therefore\ v = -\frac{P}{EI}\cdot\frac{3}{16}x^2$; M점이 극지점인 포물선이 된다.

$E = 200\times10^9\ N/m^2$, $I = \frac{bh^3}{12} = \frac{1}{12}(0.013)(0.0008)^3 = 5.547\times10^{-13}\ m^4$

$\therefore\ v \cong -0.005Px^2 \qquad |v|_{max} = 0.02P$

$|v|_{max} = 1.25\ mm$ 라 두면 $\quad P = 11.68\ N$

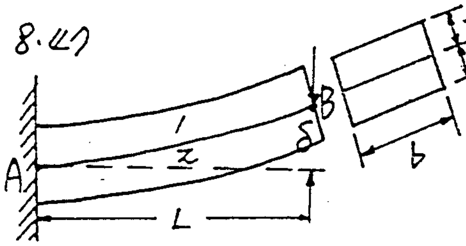
Stress ; $(\sigma_x)_{max} = \frac{M\cdot y}{I_{yy}} = \frac{(11.68)(0.0095)}{5.547\times10^{-13}} \times \frac{0.0008}{2} = 80.03\ MN/m^2$

Steel Data ; $\left(\begin{array}{lll} 1020\ HR\ ; & Y \simeq 255\ MN/m^2 & \text{안정계수} ;\ 6 \\ 1020\ CR\ ; & Y \simeq 600\ MN/m^2 & \text{안정계수} ;\ 7.5 \\ 4130\ HT\ ; & Y = 1880\ MN/m^2 & \text{안정계수} ;\ 17.3 \end{array}\right)$

∴ 1020 HR 이 가장 적당하며, plastic yielding 은 $P=37.38N$ 에서 생긴다.

Ans. $v = -\frac{Pax^2}{2EI}$; parabola , 1020HR , $P_Y = 37.38\ N$

8.47

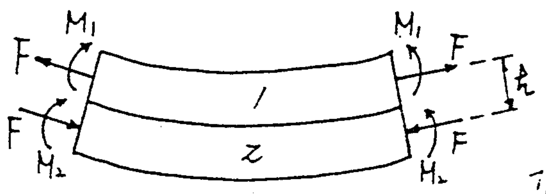


i) 일반 beam theory에서 plane section은 변형후에도 plane을 유지한다.

ii) 어떤 net force나 moment는 어떤 단면을 통해서도 전달될 수 없다.

A·B are areas of stress concentration

따라서, stress와 strain은 beam 중에서 변화가 없다.
beam과 beam 사이에는 크기는 같고 방향이 반대인 힘이 전달된다.



평형조건 ; 단면중에서 tensile stress가 일정하다고 가정하면,

$M_1 + M_2 = F\cdot h$

기하학적 적합조건; 접촉면에서의 strain과 곡률반경은 두 beam에서 서로 같아야 한다.

strain ; $\varepsilon_1 = \frac{F}{E_1 bh} + \frac{M_1\cdot\frac{h}{2}}{E_1 I} + \alpha_1 T$ —①

$\varepsilon_2 = \frac{F}{E_2 bh} + \frac{M_2\cdot\frac{h}{2}}{E_2 I} + \alpha_2 T$ —②

곡률 ; $\frac{1}{R_1} \simeq \frac{M_1}{E_1 I}$, $\frac{1}{R_2} \simeq \frac{M_2}{E_2 I}$ where $I = \frac{bh^3}{12}$

$R_1 = R_2$, $M_1 = \frac{E_1}{E_2}M_2$ 또 $M_1+M_2 = M_2(1+\frac{E_1}{E_2}) = F\cdot h$

$$\therefore M_2 = \frac{F \cdot h}{1+\frac{E_1}{E_2}} \quad , \quad M_1 = \frac{F \cdot h}{1+\frac{E_2}{E_1}}$$

①②에 대입하고, $\varepsilon_1 = \varepsilon_2$로 놓으면

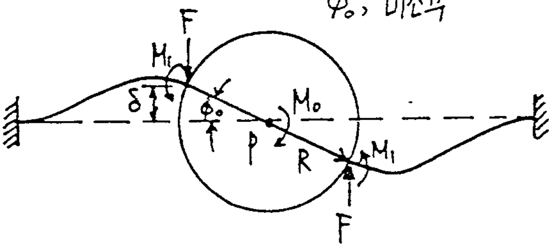
$$M_2 = \frac{(\alpha_2-\alpha_1)T}{\frac{1+\frac{E_1}{E_2}}{E_1 b h^2} + \frac{1+\frac{E_1}{E_2}}{E_2 b h^2} + \frac{6}{bh^2}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

원의 기하학에서; $\delta = \frac{L^2}{2R} = \frac{M_2 L^2}{\alpha E_2 I} = \frac{6M_2 L^2}{E_2 b h^3}$

따라서 $\delta = \frac{(\alpha_2-\alpha_1)TL^2}{h} \cdot \frac{6}{14+\frac{E_1}{E_2}+\frac{E_2}{E_1}}$

Ans. $\delta = 6(\alpha_2-\alpha_1)TL^2 E_1 E_2 / h(14E_1E_2+E_1^2+E_2^2)$

8·48



ϕ_0; 미소각

기하학적 적합조건 ; $\delta \simeq R\phi_0$

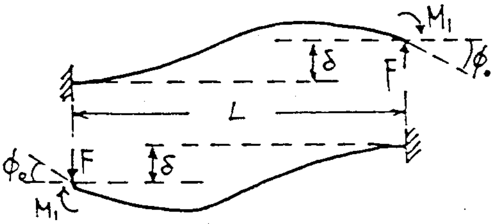
Strip과 disc에 작용하는 힘;

Disc의 평형조건;

$\sum M_P = 0$; $M_0 = 2FR + 2M_1$ ——①

Strip의 변형;

$$\delta = \frac{FL^3}{3EI} - \frac{M_1 L^2}{2EI}$$



$$\phi_0 = \frac{M_1 L}{EI} - \frac{FL^2}{2EI} \qquad \delta = R\phi_0$$

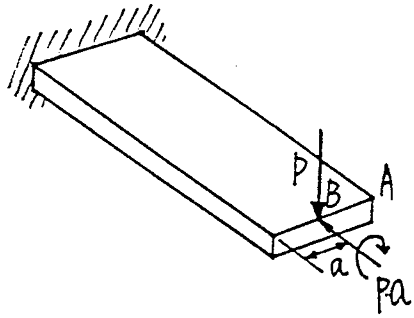
위의 3식과 ①식에서

$$F = \frac{M_0}{2R+\frac{2}{3}L\times\left(\frac{2L+3R}{L+2R}\right)}$$

$$\phi_0 = -\frac{FL^2}{2EI} + \frac{FL^2}{3EI}\cdot\frac{2L+3R}{L+2R} \qquad \therefore\ K \equiv \frac{M_0}{\phi_0} = \frac{EI}{L^2}\cdot\frac{2R+\frac{2}{3}L\times\left(\frac{2L+3R}{L+2R}\right)}{-\frac{1}{2}+\frac{1}{3}\cdot\frac{2L+3R}{L+2R}}$$

$$K = \frac{8EI}{L^3}(3R^2+3RL+L^2) \qquad \text{Ans. } K = \frac{8EI}{L^3}(3R^2+3RL+L^2)$$

8.49 문제는 오른편 그림과 같이 치환할 수 있다.



A점에서의 bending deflection은 B점에서의 bending deflection과 같다.

$$(\delta_A)_{bending} = \frac{P(10a)^3}{3E\cdot\frac{1}{2}\cdot 2a\cdot a^3} = \frac{2000P}{Ea}$$

Torsion에 의한 deflection $(\delta)_{torsion} = a\phi$

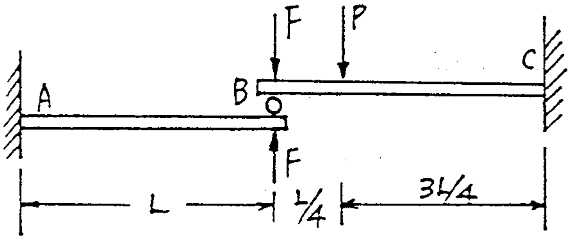
$$\frac{M_t}{\phi} = C_2\cdot\frac{Ga'b'^3}{L} \quad \text{where} \quad C_2 = 0.229, \ (\text{by table } (6.1)), \quad a' = 2a,\ b' = a,\ M_t = Pa$$

$$\therefore \phi = \frac{Pa\times 10a}{0.229\times 2a\times a^3} = \frac{5P}{0.229Ga^2}$$

$$\therefore \delta_A = (\delta)_{bending} + (\delta)_{torsion} = \frac{P}{a}\left\{\frac{2000}{E} + \frac{5}{0.229G}\right\} = \frac{P}{Ea}\left\{2000 + 56.2(1+\nu)\right\}$$

Ans. $\delta = \frac{P}{Ea}\left\{2000 + 56.2(1+\nu)\right\}$

8.50



Limiting value P를 구하는 것이므로, $M_A = M_C = M_L$,

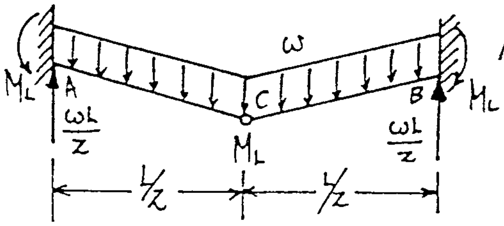
$FL = M_L$,

$-FL + \frac{3}{4}PL = M_L$

$\therefore P = \frac{8}{3L}M_L$

Ans. $P = \frac{8}{3L}M_L$

8.51 문제 (8.11)에서 $M_A = M_B = M_L$ 평형조건 $\Sigma M_A = 0$;

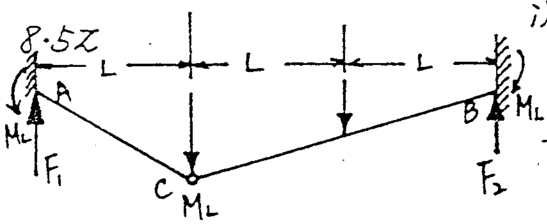


$$M_L = -M_L + \frac{1}{2}\omega L\left(\frac{L}{2}\right) - \frac{1}{2}\omega\left(\frac{L}{2}\right)^2$$

$$\therefore \omega = \frac{16M_L}{L^2}$$

Ans. $\omega = 16M_L/L^2$

8.52



i) $\frac{a}{L} \leq \frac{1}{2}$ $\quad \Sigma M_B = \Sigma M_c = 0$;

$-M_L + F_1 L = M_L$

$-M_L + 3F_1 L - 2(1-\frac{a}{L})PL - Pa = -M_L$

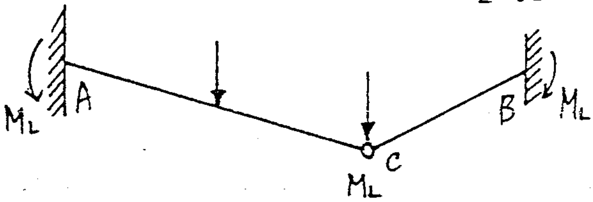
$\therefore 6M_L = (2L-a)P \quad \therefore P = \frac{6M_L}{2L-a}$

ii) $\frac{a}{L} \geq \frac{1}{2}$ $\quad \Sigma M_B = \Sigma M_c = 0$;

$-M_L + F_1 2L - P(L-a) = M_L$,

$-M_L + 3F_1 L - 2P(L-a) - Pa = -M_L$

$\therefore P = \frac{6M_L}{L+a}$



Ans. $a \geq \frac{L}{2}$; $P = \frac{6M_L}{L+a}$ $\qquad a \leq \frac{L}{2}$; $P = \frac{6M_L}{2L-a}$

8.53 material 1 ; $(\sigma_y)_1 = Y$, material 2 ; $(\sigma_y)_2 = 0.5Y$

i) material 1 에서 붕괴가 발생되면.

$M_L = \frac{3}{4}P \cdot \frac{1}{4}L = \frac{3}{16}PL$

$M_L = \frac{3}{16}PL = (\sigma_y)_1 \cdot \frac{b^2}{2} \cdot \frac{b}{4} \times 2 \qquad \therefore P = \frac{4}{3}\frac{b^3 Y}{L}$

ii) material 2 에서 붕괴가 발생되면.

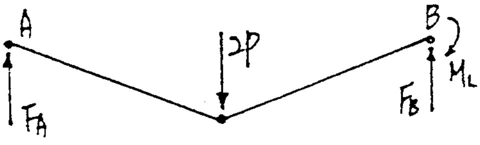
$M_L = \frac{1}{4}P \cdot \frac{L}{2} = \frac{1}{8}P \cdot L$

$M_L = \frac{1}{8}P \cdot L = \frac{1}{2}Y \cdot \frac{b^2}{2} \cdot \frac{b}{4} \cdot 2 \qquad \therefore P = \frac{b^3 Y}{L}$

i) ii) 에서 ii)의 $P = \frac{b^3 Y}{L}$ 가 더 작은 값이다. 따라서 $P = \frac{b^3 Y}{L}$ 에서 material 2가 먼저 붕괴한다.

Ans. $P = b^3 Y / L$

8.54

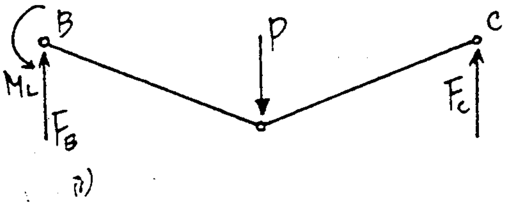


i)

i) $-M_L + F_A(\frac{L}{4}) = M_L$

$-M_L + \frac{1}{2}F_A L - 2P(\frac{L}{4}) = -M_L$

$\therefore P = \frac{8}{L}M_L$



ii) $-M_L + F_C\left(\frac{L}{4}\right) = M_L$

$-M_L + F_C\left(\frac{L}{2}\right) - P\left(\frac{L}{4}\right) = -M_L$

$\therefore P = \frac{16}{L}M_L$

i) ii) 에서 작은값 $P = \frac{8M_L}{L}$ 에서 i)의 형태로 붕괴된다.

Ans. $P = 8M_L/L$

8.55 Bending moment;

$EIv'' = -M_A + F_A x - P\langle x - \frac{L}{3}\rangle^1$

$EIv' = -M_A x + \frac{1}{2}F_A x^2 - \frac{1}{2}P\langle x - \frac{L}{3}\rangle^2 + C_1$

$EIv = -\frac{1}{2}M_A x^2 + \frac{1}{6}F_A x^3 - \frac{1}{6}P\langle x - \frac{L}{3}\rangle^3 + C_1 x + C_2$

Boundary Condition;

$v'(0) = 0 \quad \therefore C_1 = 0 \quad , \quad v(0) = 0 \quad \therefore C_2 = 0$

$v'(L) = 0 \ ; -M_A L + \frac{1}{2}F_A L^2 - \frac{2}{9}PL^2 = 0 \quad -①$

$v(L) = 0 \ ; -\frac{1}{2}M_A L^2 + \frac{1}{6}F_A L^3 - \frac{4}{81}PL^3 = 0 \quad -②$

①② 에서 $F_A = \frac{20}{27}P , \quad M_A = \frac{4}{27}PL$

따라서, B점에서의 moment ; $M_B = EIv''\left(\frac{L}{3}\right) = -\frac{4}{27}PL + \frac{20}{27}P\left(\frac{L}{3}\right) = \frac{8}{81}PL$

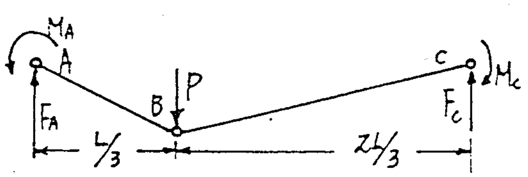
C점에서의 moment ; $M_C = EIv''(L) = -\frac{4}{27}PL + \frac{20}{27}PL - \frac{2}{3}PL = -\frac{2}{27}PL$

$\therefore M_{max} = M_A = \frac{4}{27}PL \quad \therefore P_y = \frac{27}{4}\frac{M_y}{L} \ (\because M_{max} = M_{yielding})$

$-M_L + F_A \cdot \frac{L}{3} = M_L \ ; \ F_A = \frac{6M_L}{L}$

$-M_L + F_A \cdot L - \frac{2}{3}PL = -M_L \ ; \ P = \frac{3}{2}F_A$

$\therefore P_L = \frac{9M_L}{L}$



$\therefore \frac{P_L}{P_y} = \frac{\frac{9}{L}\cdot M_L}{\frac{27}{4}\cdot\frac{M_y}{L}} = \frac{4}{3}\frac{M_L}{M_y} = \frac{4}{3}k$ — Q. E. D. —

8.56 (8.55)와 같은 요령으로.

$$EIv'' = -M_0 + Fx - P\langle x-\tfrac{L}{2}\rangle^1 , \quad EIv' = -M_0x + \tfrac{1}{2}Fx^2 - \tfrac{P}{2}\langle x-\tfrac{L}{2}\rangle^2 + C_1$$

$$EIv = -\frac{M_0}{2}x^2 + \frac{1}{6}Fx^3 - \frac{P}{6}\langle x-\frac{L}{2}\rangle^3 + C_1x + C_2$$

B.C $v'(0)=0 \ \therefore C_1=0$, $v(0)=0 \ \therefore C_2=0$

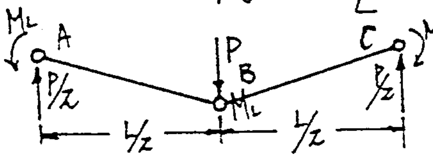
$v'(L)=0 \ ; \ -M_0L + \frac{1}{2}FL^2 - \frac{1}{8}PL^2 = 0$ — ①

$v(L)=0 \ ; \ -\frac{1}{2}M_0L + \frac{1}{6}FL^3 - \frac{P}{48}L^3 = 0$ — ②

①② 에서 $F_A = \frac{P}{2}$, $M_0 = \frac{1}{8}PL$

또, $M_B = EIv''(\frac{L}{2}) = -\frac{1}{8}PL + \frac{1}{4}PL = \frac{1}{8}PL \quad \therefore M_{max} = \frac{1}{8}PL$

따라서 $P_y = 8\cdot\frac{M_y}{L}$



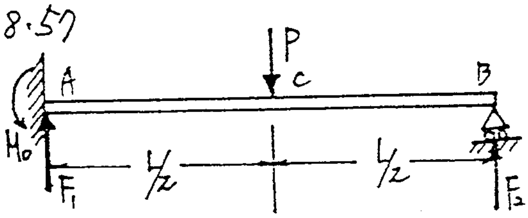
평형 조건 $\Sigma M_c = 0$;

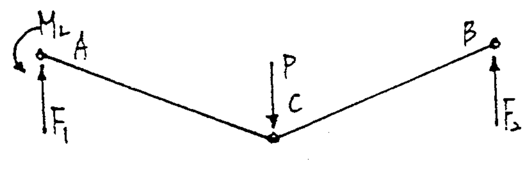
$-M_L + \frac{P}{2}\cdot\frac{L}{2} = M_L$

$\therefore P_L = 8\cdot\frac{M_L}{L}$

$\therefore P_L/P_y = (8\cdot M_L/L)/(8\cdot\frac{M_y}{L}) = \frac{M_L}{M_y} = k$ — Q. E. D. —

8.57





문제(8.19)에서

$F_1 = \frac{11}{16}P$, $F_2 = \frac{1}{2}P\cdot\frac{1}{L^3}\cdot$

$\times\frac{L^2}{4}(3L - \frac{L}{2})$

$= \frac{5}{16}P$

$\therefore M_0 = \frac{1}{2}PL - \frac{5}{16}PL = \frac{3}{16}PL$

$M_B = \frac{1}{2}F_2L = \frac{5}{32}PL$

따라서 $M_{max} = \frac{3}{16}PL$

$\therefore P_y = \frac{16}{3}\cdot\frac{M_y}{L}$

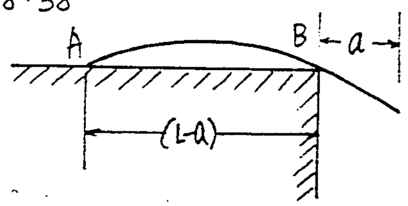
평형조건 $\Sigma M_A = \Sigma M_c = 0$;

$$\left.\begin{aligned} &-M_L + F_2\cdot\frac{L}{2} = M_L \\ &-M_L + F_2\cdot L - \frac{1}{2}PL = 0 \end{aligned}\right) \therefore P = \frac{6M_L}{L}$$

$\therefore P_L = 6\cdot\frac{M_L}{L}$

$$\therefore \frac{P_L}{P_y} = \frac{\frac{6}{L}M_L}{\frac{16}{3L}M_y} = \frac{9}{8}\frac{M_L}{M_y} = \frac{9}{8}K \quad - Q.E.D. -$$

8·58



$F_B(L-a) = \frac{1}{2}\omega L^2 \quad \therefore F_B = \frac{\omega L^2}{2(L-a)}$

$F_A = \omega L - F_B = \frac{\omega L(L-2a)}{2(L-a)}$

Bending Moment;

$$EIv'' = \frac{\omega L(L-2a)}{2(L-a)}x - \frac{1}{2}\omega x^2 + \frac{\omega L^2}{2(L-a)}\langle x+a-L\rangle^1$$

$$EIv' = \frac{\omega L(L-2a)}{4(L-a)}x^2 - \frac{1}{6}\omega x^3 + \frac{\omega L^2}{4(L-a)}\langle x+a-L\rangle^2 + C_1$$

$$EIv = \frac{\omega L(L-2a)}{12(L-a)}x^3 - \frac{1}{24}\omega x^4 + \frac{\omega L^2}{12(L-a)}\langle x+a-L\rangle^3 + C_1x + C_2$$

B.C. $v(0) = v(L-a) = 0 \quad \therefore C_2 = 0$

$$C_1 = -\frac{\omega}{24}(L^3 - 3aL + a^2L + a^3)$$

따라서 $$v = \frac{1}{EI}\left[\frac{\omega L(L-2a)}{12(L-a)}x^3 + \frac{\omega L^2}{12(L-a)}\langle x+a-L\rangle^3 - \frac{1}{24}\omega x^4 - \frac{\omega}{24}(L^3 - 3aL + a^2L + a^3)x\right]$$

8·59 문제에서 $\frac{d^2v}{dx^2} = \frac{1}{E}\frac{I_{yz}\cdot M_{by} + I_{yy}M_{bz}}{I_{yy}\cdot I_{zz} - I_{yz}^2} \quad \frac{d^2w}{dx^2} = -\frac{1}{E}\frac{I_{zz}\cdot M_{by} + I_{yz}M_{bz}}{I_{yy}\cdot I_{zz} - I_{yz}^2}$

$I_{zz} = \frac{8}{3}ta^3, \quad I_{yy} = \frac{2}{3}ta^3. \quad I_{yz} = -ta^3$

$M_{bz} = -P(L-x), \quad M_{by} = 0$

$$\therefore \frac{d^2v}{dx^2} = \frac{-\frac{2}{3}P(L-x)}{Ea^3t\left(\frac{2}{3}\cdot\frac{8}{3} - 1\right)} = -\frac{6P}{7E}(L-x)\cdot\frac{1}{a^3t}$$

적분하여 B.C $v'(0) = v(0) = 0$를 대입하면,

$$v = -\frac{8P}{7Ea^3t}\left(\frac{1}{2}x^2 - \frac{1}{6}x^3\right) \quad \therefore v(L) = -\frac{2PL^3}{7Ea^3t}$$

$$\frac{d^2w}{dx^2} = \frac{1}{E \cdot a^3 t} \cdot \frac{-P(L-x)}{\left(\frac{2}{3}\cdot\frac{8}{3} - 1\right)} = \frac{-9P}{7Ea^3t}(L-x)$$

적분하여. B.C $w'(0) = w(0) = 0$을 대입하면

$$w = -\frac{9P}{7Ea^3t}\left(\frac{L}{2}x^2 - \frac{1}{6}x^3\right) \quad \therefore w(L) = -\frac{3PL^3}{7Ea^3t}$$

Ans. $v(L) = (-2PL^3)/(7Ea^3t)$, $w(L) = (-3PL^3)/(7Ea^3t)$

8.60 문제 (7.2) (7.78)에서

$$\left(\begin{array}{lll} I_{yz} = 32.3 \text{ in}^4 & I_{yy} = 38.8 \text{ in}^4 & I_{zz} = 80.8 \text{ in}^4 \\ E = 30\times10^6 \text{ psi} & M_{bz} = 100.000 \text{ lb-in} & \end{array}\right)$$

문제 (8.59) 식을 이용하면,

$$\frac{d^2v}{dx^2} = \frac{1}{E}\cdot\frac{I_{yz}M_{by} + I_{yy}M_{bz}}{I_{yy}\cdot I_{zz} - I_{yz}^2} = \frac{(32.3)(10^5)}{(30\times10^6)\{(38.8)(80.8) - (32.3)^2\}}$$

$$= 5.147\times10^{-5}$$

B.C $v'(0) = v(0) = 0$을 이용하면

$$v = 2.574\times10^{-5}x^2 \quad \therefore v(120) = 0.3706 \text{ in}$$

$$\frac{d^2w}{dx^2} = -\frac{(80.8)(10^5)}{(30\times10^6)\{(38.8)(80.8) - (32.3)^2\}} = -1.284\times10^{-4}$$

B.C $w'(0) = w(0) = 0$을 이용하면

$$w = (-6.418\times10^{-5})x^2 \quad \therefore w(120) = -0.9242 \text{ in}$$

Ans vertical = 0.3706 in, horizontal = −0.9242 in

8.61 $\delta = \frac{L^2}{2R} = \frac{L^2}{2\rho}$, $M_0 = \frac{1}{\rho}EI$ $\quad\therefore \delta = \frac{M_0L^2}{2EI}$

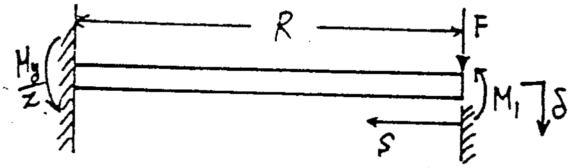
ϕ를 미소각으로 가정 : $\phi \simeq \frac{L}{\rho} = \frac{M_0 L}{EI}$ — Q.E.D. —

8.62 (a) $\phi_x = \dfrac{M_x \cdot L}{2GJ} = \dfrac{2(1+\nu)}{2E} \cdot \dfrac{M_x}{\frac{\pi}{2} r^4} \cdot R = \dfrac{2(1+\nu)}{\pi r^4 E} R M_x$

$$\left(\because G = \frac{E}{2(1+\nu)} \right)$$

$$\therefore k_x = \frac{M_x}{\phi_x} = \frac{\pi r^4 E}{2(1+\nu) \cdot R}$$

(b)



평형조건 $\Sigma M = 0$;

$\frac{1}{2} M_y = F \cdot R - M_1$

$$\delta = \frac{FR^3}{3EI} - \frac{M_1 R^2}{2EI}$$

$$\phi_y = \frac{FR^2}{2EI} - \frac{M_1 R}{EI}$$

기하학적 적합 조건;

$\delta = R\phi_y$; $\frac{1}{3} FR - \frac{1}{2} M_1 = \frac{1}{2} F - M_1$

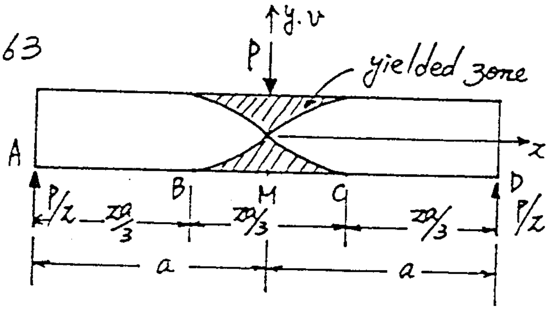
$$\therefore M_1 = \frac{1}{3} FR = \frac{2EI\phi_y}{R} \quad \therefore F = \frac{3M_y}{4R}$$

따라서 $\phi_y = \dfrac{FR^2}{2EI} - \dfrac{FR^2}{3EI} = \dfrac{FR^2}{6EI} = \dfrac{M_y \cdot R}{8EI}$

$$\therefore k_y = \frac{M_y}{\phi_y} = \frac{8EI}{R} = \frac{2\pi r^4 E}{R} \qquad \left(\frac{k_y}{k_x} = 4(1+\nu) \leq 6 \right)$$

Ans. $k_x = \pi r^4 E / 2(1+\nu) \cdot R \qquad k_y = 2\pi r^4 E / R$

8.63



moment - curvature ;

$$M_b = \frac{3}{2} M_y \left\{ 1 - \frac{1}{3} \left[\frac{(1/\rho)_y}{(1/\rho)} \right]^2 \right\} \text{ -①}$$

$$|x| \leq \frac{a}{3}$$

$$M_b = \frac{EI_{yy}}{\rho}, \quad \frac{a}{3} < |x| \leq a \text{ -②}$$

ρ : radius of curvature

또, $M_y = \dfrac{bh^2}{6} Y$, $\dfrac{1}{\rho} \simeq \dfrac{d^2 v}{dx^2}$, $\left(\dfrac{1}{\rho}\right)_y = \dfrac{\varepsilon_y}{h/2} = \dfrac{2Y}{Eh}$

그림에서 $M_b = \dfrac{P(a-x)}{2} \qquad 0 < x < a$

① ② 식에 대입하면,

$$\frac{P(a-x)}{2} = \frac{3}{2}M_y\left\{1-\frac{1}{3}\left[\frac{2Y}{Eh}\Big/\frac{d^2v}{dx^2}\right]^2\right\}, \quad |x| \le \frac{a}{3} \quad — ③$$

$$\frac{P(a-x)}{2} = EI\frac{d^2v}{dx^2} \qquad \frac{a}{3} \le |x| \le a \quad — ④$$

또, table 에서 $(M_b)_{x=0} = 1.5M_y = M_L$

$$\therefore \frac{Pa}{2} = 1.5M_y \quad \text{or} \quad p = \frac{3M_y}{a}$$

따라서 ③④식은 다음과 같이 된다.

$$\sqrt{\frac{1}{3}}\left[\left(\frac{2Y}{Eh}\right)\Big/\left(\frac{d^2v}{dx^2}\right)\right] = \left(\frac{x}{a}\right)^{1/2} \quad |x| \le \frac{a}{3} \quad — ⑤$$

$$EI\frac{d^2v}{dx^2} = \frac{3M_y}{2a}(a-x) \quad \frac{a}{3} < |x| < a \quad — ⑥$$

B.C $v'(0) = 0$, $v(a) = 0$,

⑤식에서의 $v(\pm\frac{a}{3})$ = ⑥식에서의 $v(\pm\frac{a}{3})$

⑤식에서의 $v'(\pm\frac{a}{3})$ = ⑥식에서의 $v'(\pm\frac{a}{3})$

적분하여 B.C를 이용하면

$$v = -\frac{20}{27}\cdot\frac{a^2M_y}{EI} + \frac{2\sqrt{a}\,Y}{\sqrt{3}\,Eh}\cdot\frac{4}{3}x^{3/2} \quad |x| \le \frac{a}{3}$$

$$v = \frac{1}{EI}\left\{(a-x)M_y\cdot a\left[\frac{1}{4}\left(1-\frac{x}{a}\right)^2 - 1\right]\right\} \quad \frac{a}{3} \le |x| \le a$$

$Y = \frac{6M_y}{bh^2}$, $I = \frac{bh^3}{12}$ 을 대입하면

$$v = -\frac{a^2M_y}{EI}\left[\frac{20}{27} - \frac{4}{3\sqrt{3}}\left(\frac{x}{a}\right)^{3/2}\right] \quad |x| \le \frac{a}{3}$$

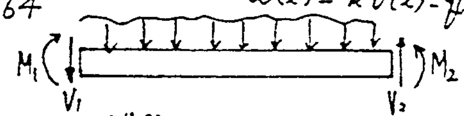
$$v = -\frac{M_y\cdot a^2}{4EI}\left(3-\frac{x}{a}\right)\left[1-\left(\frac{x}{a}\right)^2\right] \quad \frac{a}{3} \le |x| \le a \quad \text{— Q.E.D. —}$$

$x = 0$ 에서 $v_p = -\frac{20M_y\cdot a^2}{27EI}$

문제 (8.4)에서 $v(\frac{L}{2}) = -\frac{PL^3}{48EI} \Rightarrow v_c = -\frac{M_y\cdot a^2}{3EI}\left(\because \begin{matrix} L = 2a \\ M_y = \frac{1}{2}Pa\end{matrix}\right)$

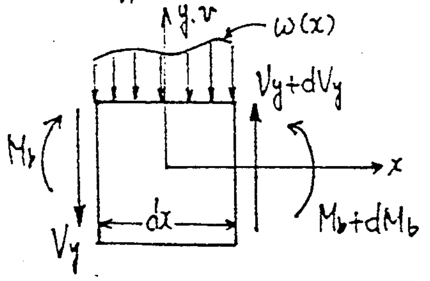
$$\therefore \frac{v_p}{v_c} = \frac{\left(-\frac{20}{27}\right)}{\left(-\frac{1}{3}\right)} = \frac{20}{9} \quad \text{— Q.E.D. —}$$

8.64 $\omega(x) = k v(x) - q(x)$ 평형조건 ;



$\Sigma F_y = 0$; $dV_y = \omega(x) dx$

$= k v(x) dx - q(x) dx$



$\Sigma M_z = 0$; $V_y dx = -dM_b$

$$\frac{dM_b}{dx} = -V_y, \quad \frac{d^2M_b}{dx^2} = -\frac{dV_y}{dx}$$

$$= q(x) - k v(x)$$

Beam bending 에서 $M_b = EI \dfrac{d^2 v}{dx^2}$

$$\therefore EI \frac{d^4 v}{dx^4} + k v(x) = q(x) \quad -①$$

$q(x) = 0$ 인 경우에는 $v = e^{sx}$ 가 ①식의 해가 된다.

s를 구하기 위해 $v = e^{sx}$를 ①식에 대입하면

$(EIs^4 + k) v = 0 \quad \therefore s^4 = -\dfrac{k}{EI}$, $\beta^4 \equiv -\dfrac{s^4}{4} = \dfrac{k}{4EI}$로 놓으면

$$s = \pm\beta \pm i\beta$$

$$\therefore v = e^{(\beta+i\beta)x}, \quad v = e^{(\beta-i\beta)x}, \quad v = e^{(-\beta+i\beta)x}, \quad v = e^{(-\beta-i\beta)x}$$

①식에서 $q(x) = 0$ 이면, linear 이므로, 위의 4개의 해의 combination도 해가 된다.

$$v = Ae^{(\beta+i\beta)x} + Be^{(\beta-i\beta)x} + Ce^{(-\beta+i\beta)x} + De^{(-\beta-i\beta)x}$$

$$= e^{\beta x}[(A+B)\cos\beta x + (A-C)\sin\beta x]$$

$$+ e^{-\beta x}[(C+D)\cos\beta x + (C-D)\sin\beta x]$$

$$= e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + e^{\beta x}(C_3 \cos\beta x + C_4 \sin\beta x)$$

$x=0$ 에 point load가 작용하고, beam이 매우 길다면

$x \to \infty$ 이면 $v = v' = 0$

$\therefore x > 0$ 이면 $C_3 = C_4 = 0 \quad \therefore v = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x)$

또, Boundary Condition ; $v'(0) = 0 \quad -②$

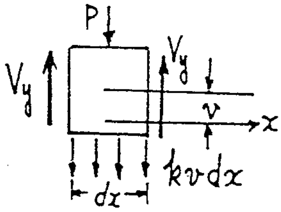
또 다른 B.C.가 $x=0$ 근처의 점에서의 shear stress에 의해 생긴다.

$$2V_y + kv\,dx = P$$

$dx \to 0$ 이면 $\quad V_y = \frac{P}{2} = -EIv''' \quad$ —②

①②를 $\quad v = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x)$ 에 대입하여,

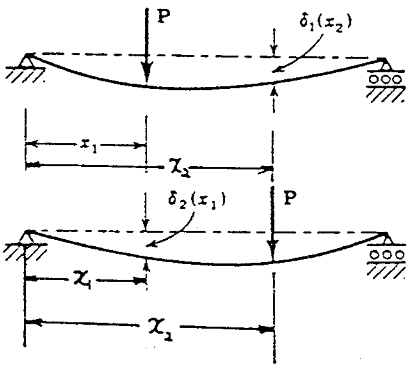
$$C_1 = C_2 = -\frac{P\beta}{2k}$$



$$\delta \equiv -v(0) = -C_1$$

$$= \frac{P\beta}{2k} = \frac{P}{2k}\left(\frac{k}{4EI}\right)^{1/4} = \frac{P}{(64EIk^3)^{1/4}} \qquad \text{— Q. E. D. —}$$

8.65 Table (8·1) 에서,



(a) $\delta(x_2) = \frac{P(L-x_1)}{6LEI}\left[\frac{L}{(L-x_1)}(x_2-x_1)^3 - x_2^3 + \{L^2-(L-x_1)^2\}x_2\right]$

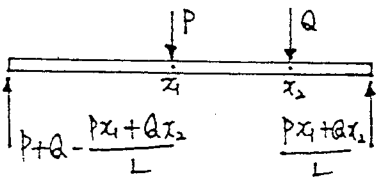
$$= \frac{P}{6EIL}x_1(L-x_2)(2Lx_2 - x_1^2 - x_2^2) \quad \text{—①}$$

$\delta(x_1) = \frac{P(L-x_2)}{6EIL}\left[\frac{L}{(L-x_2)}(x_1-x_2)^3 - x_1^3 + \{L^2-(L-x_2)^2\}x_1^2\right]$

$$= \frac{P}{6EIL}x_1(L-x_2)(2Lx_2^2 - x_1^2 - x_2^2) \quad \text{—②}$$

∴ ①②에서 $\delta(x_1) = \delta(x_2)$ — Q. E. D. —

(b) Castigliano's theorem.

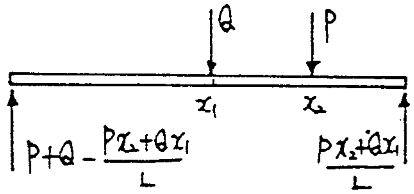


$$M_b = \left(P+Q-\frac{Px_1+Qx_2}{L}\right)x - P\langle x-x_1\rangle^1 - Q\langle x-x_2\rangle^1$$

$$U = \int \frac{M_b^2}{2EI}\,dx$$

$$\therefore\ \delta(x_2) = \left(\frac{\partial U}{\partial Q}\right)_{Q=0} = \int \frac{M_b \cdot m}{EI}\,dx = \frac{1}{EI}\int M_b\left[\left(1-\frac{x_2}{L}\right)x - \langle x-x_2\rangle^1\right]dx$$

$$= \frac{P}{EI}\int\left[\left(1-\frac{x_1}{L}\right)x - \langle x-x_1\rangle^1\right]\left[\left(1-\frac{x_2}{L}\right)x - \langle x-x_2\rangle^1\right]dx \quad \text{—①}$$



같은 요령으로

$$M_b=\left(P+Q-\frac{Px_2+Qx_1}{L}\right)x-Q\langle x-x_1\rangle^1-P\langle x-x_2\rangle^1$$

$$\therefore\ \delta(x_1)=\left(\frac{\partial U}{\partial Q}\right)_{Q=0}=\int\frac{M_b\cdot m}{EI}dx$$

$$=\frac{P}{EI}\int\left[\left(1-\frac{x_2}{L}\right)x-\langle x-x_2\rangle^1\right]\left[\left(1-\frac{x_1}{L}\right)x-\langle x-x_1\rangle^1\right]dx \quad -②$$

①② 에서 $\delta(x_1)=\delta(x_2)$ — Q. E. D. —

8·66 material property (copper) $\left(\begin{array}{l}\rho_0=8.95\times10^3\ kg/m^3\\ E=117\ GN/m^2\end{array}\right.$

$$I=\frac{\pi}{64}d^4=\frac{\pi}{64}(0.025)^4=1.917\times10^{-8}(m^4)$$

mirror 의 위치는 rod 의 중앙이므로.

$$M_b=\frac{1}{8}\omega L^2$$

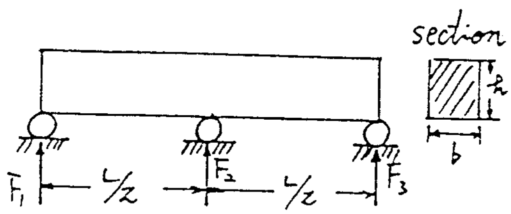
또 mirror 는 rod가 처져 생기는 곡률과 일치하는 곡률을 갖게될 것이다.

$$\frac{d\phi}{ds}=\frac{M_b}{EI}=\frac{\frac{1}{8}(8.95\times10^3)(\frac{\pi}{4})(0.025)^2(9.8)(2.5)^2}{(117\times10^9)(1.917\times10^{-8})}$$

$$=\frac{1}{0.015}(m) \qquad \therefore\ \rho=66.68\ m$$

Ans. $\rho=66.68\ m$

8·67



Bending moment ;

$$EIv''=F_1x-\frac{1}{2}\omega x^2+F_2\langle x-\frac{L}{2}\rangle^1$$

$$EIv'=\frac{1}{2}F_1x^2-\frac{1}{6}\omega x^3+\frac{1}{2}F_2\langle x-\frac{L}{2}\rangle^2+C_1$$

$$EIv = \frac{1}{6}F_1 x^3 - \frac{1}{24}\omega x^4 + \frac{1}{6}F_2\langle x - \frac{L}{2}\rangle^3 + C_1 x + C_2$$

Boundary Condition ;

$v(0) = 0 \quad \therefore C_2 = 0$

$v'(\frac{L}{2}) = 0 \ ; \ \frac{1}{8}F_1 L^2 - \frac{1}{48}\omega L^3 + C_1 = 0 \quad \therefore C_1 = -\frac{1}{8}F_1 L^2 + \frac{1}{48}\omega L^3$

$v(\frac{L}{2}) = 0 \ ; \ \frac{1}{128}\omega L^4 - \frac{1}{24}F_1 L^3 = 0$

$\therefore F_1 = \frac{3}{16}\omega L \ , \quad F_2 = \frac{5}{8}\omega L$

max. bending moment는 $v''' = 0$을 만족하는 곳에서 발생.

i) $x = \frac{3}{16}L \ ; \ (M_b)_{\frac{3}{16}L} = \frac{9}{512}\omega L^2$

ii) $x = \frac{1}{2}L \ ; (M_b)_{\frac{1}{2}L} = -\frac{1}{32}\omega L^2 \qquad \therefore (M_b)_{max} = \frac{\omega}{32}L^2$

따라서 가운데 바퀴에서 파괴가 일어난다.

8.68 문제 (8.1 (g)) 에서

(a) $(M_b)_{max} = \frac{1}{8}\omega L^2 , \quad \sigma_{max} = \frac{(M_b)_{max}}{I}\cdot\frac{h}{2} = \frac{3\omega L^2}{4bh^2}$

$\omega L = \overline{W}$ 이므로 $\quad \overline{W} = \frac{4bh^2}{3L}(\sigma_{max}) \quad , \quad \overline{W_B} = \gamma bhL$

$$\therefore \frac{\overline{W}}{\overline{W_B}} = \frac{4h}{3L^2}\left(\frac{\sigma_{max}}{\gamma}\right)$$

(b) $v = \frac{\omega}{EI}\left[\frac{L}{12}x^3 - \frac{1}{24}x^4 - \frac{1}{24}L^3 x\right]$

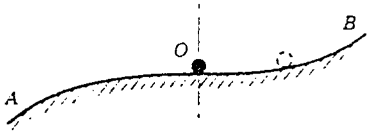
$\therefore \delta_{max} = \frac{\omega}{EI}\cdot\frac{5L^4}{384} = \frac{5\overline{W}L^3}{384EI} \quad (\because \overline{W} = \omega L)$

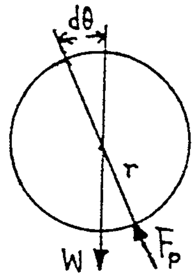
$\left.\begin{aligned} \therefore \overline{W} &= \frac{32Ebh^3\delta_{max}}{5L^3} \\ \overline{W_B} &= \gamma bhL \end{aligned}\right) \quad \therefore \frac{\overline{W}}{\overline{W_B}} = \frac{32h^2\delta_{max}}{5L^4}\left(\frac{E}{\gamma}\right)$

— Q. E. D. —

CHAPTER 9

9.1





$\Sigma F = 0$ 이 아니다. 따라서 평형은 있을 수 없다. 또, 불균형 moment $M_P = Wr\delta\theta$ (G)는 구를 O점에서 부터 구르게 할 것이다. 따라서 평형은 불안정하다.

9.2 A점에서의 tension $= P$

$\Sigma M_{Bz} = 0$;

$(kx + P\sin\theta)\sqrt{L^2+x^2} = P\cos\theta \cdot x$

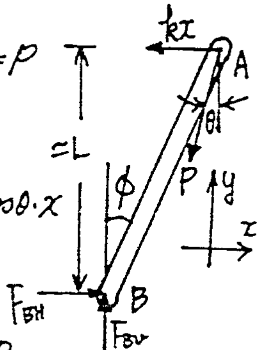
$\theta \ll 1$ $\quad \sin\theta \simeq x/2L$

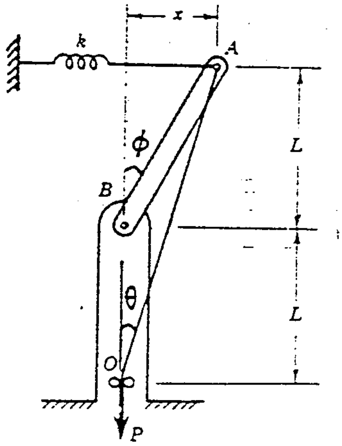
$\sqrt{L^2+x^2} \simeq L$

$\cos\theta = 1 \quad \therefore \dfrac{Px}{L} = x\left(k + \dfrac{P}{2L}\right)$

$\therefore P = 2kL$

Ans. $P_{cr} = 2kL$





9.3(a) B.C는 아무런 moment도 전달할 수 없기 때문에, A.D점의 반력은 AB. DC에만 전해진다.

$\Sigma M_D = 0$; $(F_{AV} + 2F_{AH})L = M + kxL$ —①

$\Sigma F_y = 0$; $F_{AV} = F_{DV}$ —②

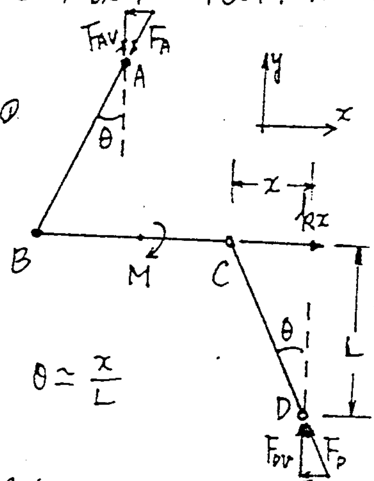
$\Sigma F_x = 0$; $F_{AH} + F_{DH} = kx$ —③

Geometry ; $F_{AH} \simeq F_{AV} \cdot \dfrac{x}{L}$ —④

$F_{DH} \simeq F_{DV} \cdot \dfrac{x}{L}$ —⑤

$\theta \simeq \dfrac{x}{L}$

①~⑤에서 $M = \dfrac{kL^2}{2}$; Ans.



(b) M이 반대 방향으로 작용하면, stability 문제는 없다. AD. CD가 tension으로 붕괴되거나, BC가 bending에 의해 붕괴 될때까지 M이 작용.

9.4 평형조건 $\sum M_{Bz}=0$; $P(L\sin\theta+\varepsilon)=k\theta$

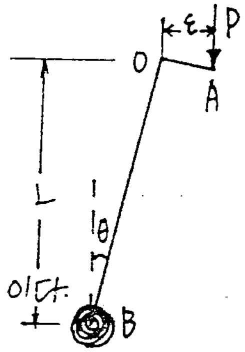
θ가 미소각이면, $\sin\theta\simeq\theta$

$$\therefore \theta=\frac{P\varepsilon}{k-PL}$$

$P\to\frac{k}{L}$ 이면 $\theta\to\infty$ 이다.

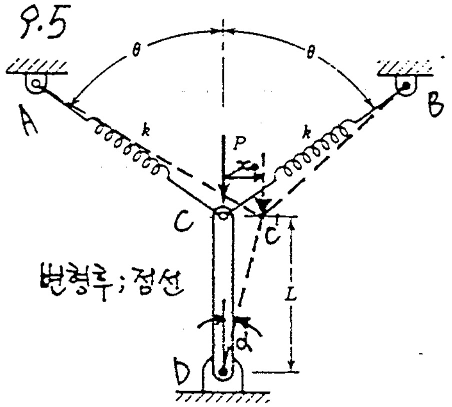
따라서, $P=\frac{k}{L}$ 는 한계 하중 (limiting load)이다.

$\therefore$ $P<\frac{k}{L}$ 이면 평형은 안정이다.



Ans. $\theta=(P\varepsilon)/(k-PL)$, $P<k/L$

9.5



그림에서 α를 미소각으로 가정하면

$\delta_{AC}\simeq x\cos\theta$, $\delta_{BC}=-x\sin\theta$

평형조건; $\sum M_{Dz}=0$;

$Px_0=F_{AC}L\sin\theta-F_{BC}L\sin\theta$

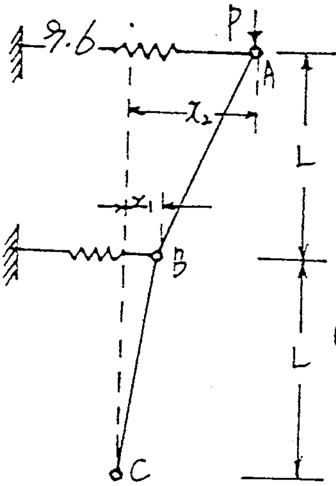
$=(kx\cos\theta+kx\sin\theta)L\sin\theta$

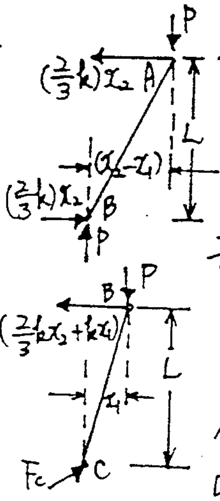
$=kxL\sin\theta(\sin\theta+\cos\theta)$

$\therefore P_{cr}=kL\sin\theta(\sin\theta+\cos\theta)$

Ans. $P_{cr}=kL\sin\theta(\sin\theta+\cos\theta)$

9.6





구간 AB의 평형조건;

$\sum M_B=0$; $P(x_2-x_1)=\frac{2}{3}kx_2L$ —①

구간 BC의 평형조건;

$\sum M_C=0$;

$(P)(x_1)=k(x_1+\frac{2}{3}x_2)L$ —②

식①②가 해를 갖아야만 평형을 만족하게 된다. ①②에서 x_1을 제거하면

$$\left[\left(2-\frac{kL}{P}\right)\left(1-\frac{2}{3}\frac{kL}{P}\right)-1\right]x_2=0.$$

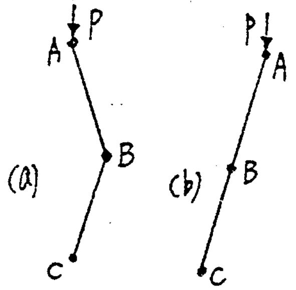
만일 $x_2=0$ 이면 $x_1=0$ 인 trivial solution을 얻게된다. 따라서 non-trivial solution을 얻으려면,

$$\left[\left(2-\frac{kL}{P}\right)\left(1-\frac{2}{3}\frac{kL}{P}\right)-1\right]=0 \qquad \therefore P=\frac{kL}{3} \text{ or } P=2kL$$

stability

$x_2=-x_1$ 이면 $P=\frac{kL}{3}$로 neutral equilibrium을 유지한다. (a)

$x_2=\frac{3}{2}x_1$ 이면 $P=2kL$로 neutral equilibrium을 유지한다. (b)



(a)는 $P<\frac{kL}{3}$일 경우 안정

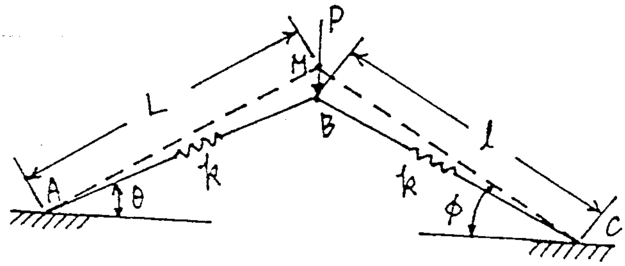
$P>\frac{kL}{3}$일 경우 불안정

(b)는 $P<2kL$일 경우 안정

$P>2kL$일 경우 불안정

구조물은 모든 가능한 상황에서 안정되어야 하므로 $P<\frac{kL}{3}$을 만족해야 한다. $\underline{\text{Ans. } P<\frac{kL}{3}}$

9.7 그림에서



$2k(AM-AB)\sin\theta=P$

$AM\cos\phi=AB\cos\theta$

θ와 ϕ를 미소각으로 보면

$\sin\theta\simeq\theta$

$\cos\phi\simeq1-\frac{\phi^2}{2}$, $\cos\theta\simeq1-\frac{\theta^2}{2}$ $\quad\therefore P=kL\theta(\phi^2-\theta^2)$ —①

snap through는 $\frac{dP}{d\theta}=kL(-\phi^2+3\theta^2)=0$ 이어야 한다.

$\therefore\ \theta=\pm\frac{\phi}{\sqrt{3}}$ ①에 대입하면 $P=\pm\frac{2kL\phi^3}{3\sqrt{3}}$; snap through

stability ; $d\theta=\varepsilon$ 이고 $P=\text{const.}$ 이면

restoring force ; $P_{rest.}=kL(\theta+\varepsilon)[-\phi^2+(\theta+\varepsilon)^2]-P\simeq kL\varepsilon(-\phi^2+3\theta^2)$

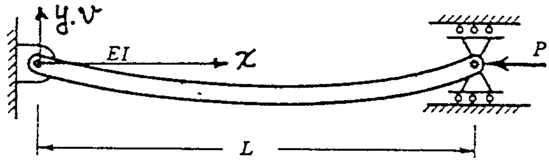
즉. $\phi^2<3\theta^2$일때 안정. $|\theta|>\frac{\phi}{\sqrt{3}}$; 안정 $\quad|\theta|\leq\frac{\phi}{\sqrt{3}}$; 불안정.

Ans. $P = kL\theta(\phi^2 - \theta^2)$, snap through : $P = \pm \frac{2kL\phi^3}{3\sqrt{3}}$

$|\theta| < \frac{\phi}{\sqrt{3}}$ 일때 불안정, $|\theta| > \frac{\phi}{\sqrt{3}}$ 일때 안정.

9.8

$EIv'' = M_b = -Pv$ —①



$\therefore EIv'' + Pv = 0$

$v'' + \frac{P}{EI}v = v'' + k^2 v = 0$

$(k^2 = \frac{P}{EI})$

이 방정식의 일반해는 : $v = A\cos kx + B\sin kx$

Boundary Condition ; $v(0) = v(L) = 0$

$\therefore A = 0.$ $\quad B\sin kL = 0$

Non-trivial sol. 은 $\sin kL = 0$ $\quad \therefore kL = n\pi = \sqrt{\frac{P}{EI}}L$

따라서 $P_{cr} = \frac{n^2\pi^2 EI}{L^2}$ 여기에서 좌굴하중은 $n=1$ (최소) 일 때만 의미가 있으므로

$P_{cr} = \frac{\pi^2 EI}{L^2}$ $\qquad$ Ans. $P_{cr} = \frac{\pi^2 EI}{L^2}$

9.9 Eq.(9.10) 식으로 부터

$v = C_1 + C_2 x + C_3 \sin\sqrt{\frac{P}{EI}}x + C_4\cos\sqrt{\frac{P}{EI}}x$ —①

Boundary Condition $v(0) = v(L) = v'(0) = v'(L) = 0$ ①에 대입.

$\therefore C_1 + C_4 = 0, \quad C_2 + C_3\cdot\sqrt{\frac{P}{EI}} = 0$

$C_1 + C_2 L + C_3\sin\sqrt{\frac{P}{EI}}\cdot L + C_4\cos\sqrt{\frac{P}{EI}}\cdot L = 0$

$C_2 + C_3\sqrt{\frac{P}{EI}}\cos L\sqrt{\frac{P}{EI}} - C_4\sqrt{\frac{P}{EI}}\sin\sqrt{\frac{P}{EI}}\cdot L = 0$

위의 4식에서 C_2, C_4를 소거하면

$C_1(1 - \cos\sqrt{\frac{P}{EI}}\cdot L) + C_3(\sin\sqrt{\frac{P}{EI}}L - \sqrt{\frac{P}{EI}}\cdot L) = 0$ —②

$C_1\sqrt{\frac{P}{EI}}\sin\sqrt{\frac{P}{EI}}\cdot L + C_3\sqrt{\frac{P}{EI}}(\cos\sqrt{\frac{P}{EI}}\cdot L - 1) = 0$ —③

②③ 에서 non-trivial sol.은 계수 C_1, C_3 에 대한 determinant=0

이어야 한다.

$$\therefore (1-\cos\beta L)(\cos\beta L-1)-\sin^2\beta L+\beta L\sin\beta L=0\ , \quad \beta\equiv\sqrt{\frac{P}{EI}}$$

$$\sin\frac{\beta L}{2}\left(\frac{\beta L}{2}\cos\frac{\beta L}{2}-4\sin\frac{\beta L}{2}\right)=0$$

i) $\sin\frac{\beta L}{2}=0$ 일 경우 $\frac{\beta L}{2}=n\pi \quad \therefore P_{cr}=\frac{(2n)^2\pi^2EI}{L^2}$

$$v_n=C_1\left(1-\cos\frac{2n\pi x}{L}\right)$$

좌굴하중은 최소($n=1$) 일 때만 의미를 갖는다.

$$\therefore P_{cr}=(4\pi^2EI)/L^2\ , \quad v=C_1\left(1-\cos\frac{2\pi x}{L}\right)$$

ii) $\tan\frac{\beta L}{2}=\frac{\beta L}{8}$ 인 경우 i)보다 큰 좌굴 하중이 계산된다.

i) ii) 에서 $P_{cr}=(4\pi^2EI)/L^2$: Ans.

9.10 Eq (9.9) 에서

$$EI\frac{d^4v}{dx^4}+P\frac{d^2v}{dx^2}=0$$

방정식의 해는 ; $v=C_1+C_2x+C_3\sin\sqrt{\frac{P}{EI}}x+C_4\cos\sqrt{\frac{P}{EI}}x$

Boundary Condition ; $v(0)=v'(0)=0$.

L 에서 shear force $=EI\frac{d^3v}{dx^3}=0$, moment $=M_b=EI\frac{d^2v}{dx^2}=0$

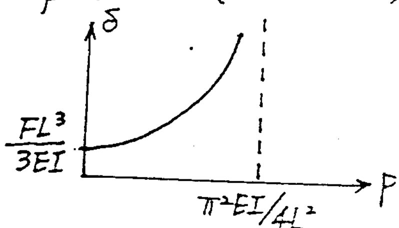
따라서 $C_1=-C_4=\delta+\frac{F}{P}L$

$$C_3=-C_2\sqrt{\frac{EI}{P}}=\left(\delta+\frac{F}{P}L\right)\cot kL \qquad k\equiv\sqrt{\frac{P}{EI}}$$

$$\therefore v=\left(\delta+\frac{F}{P}L\right)(1-\cos kx)-\left(\delta+\frac{F}{P}L\right)\cot kL(kx-\sin kx)$$

$$\delta=(v)_{x=L}=\left(\delta+\frac{F}{P}L\right)(1-kL\cot kL)$$

$$\therefore \delta=\frac{F}{P}L\frac{(\sin kL-kL\cos kL)}{\cos kL}$$



Ans. $\delta=\left(\frac{F}{P}L\right)(\sin kL-kL\cos kL)/(\cos kL)$

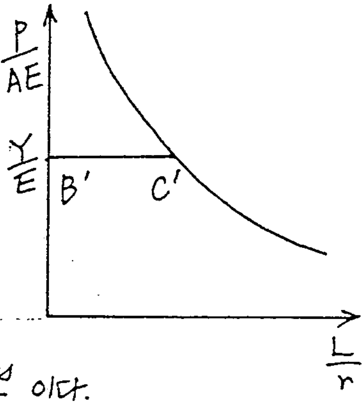
9.11 Eq.(9.21)은 다음과 같이 쓸 수 있다.

$$\frac{P}{AE} \geq \frac{Y}{E} \quad \text{for plastic flow} \quad —①$$

Eq.(9.22)은 다음과 같이 쓸 수 있다.

$$\frac{P}{AE} < \frac{\pi^2}{4}\cdot\frac{I}{AL^2} = \frac{\pi^2}{4}\cdot\frac{r^2}{L^2} \quad \text{for stability}$$

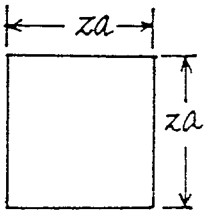
또는 $\frac{P}{AE} \geq \frac{\pi^2}{4}\left(\frac{r}{L}\right)^2$ for instability —②



②식은 cantilever column에 대한 instability lucus를 나타낸다. column의 재료나 기하학적인 형상에 무관하게 $\left(\frac{P}{AE}\right)$ 対 $\left(\frac{L}{r}\right)^2$의 graph는 고정된 곡선을 형성한다.

BC에 대응되는 좌표는 $\frac{Y}{E} = \frac{\text{yield stress}}{\text{modulus}}$ 이다.

9.12



i) Yielding ; $P = AY = 4a^2 Y$

1020 HR steel ; $Y = 255\ MN/m^2$

문제에서 주어진 하중 ; $P = 50\ KN$

$$\therefore a = \sqrt{\frac{50\times10^3}{4\times255\times10^6}} = 0.007\ (m)$$

ii) Buckling ; $P = \frac{\pi^2}{4}\cdot\frac{EI}{L^2}$: $I = \frac{bh^3}{12} = \frac{4}{3}a^4$

$$\therefore P = \frac{\pi^2}{3}\cdot\frac{Ea^4}{L^2} \qquad \therefore a^4 = \frac{3L^2P}{\pi^2 E}$$

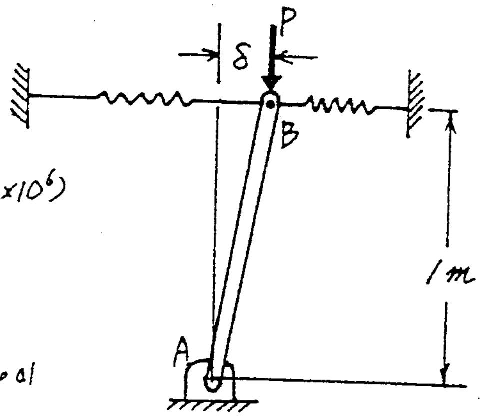
$$\therefore a = \sqrt[4]{\frac{3\times2^2\times50\times10^3}{\pi^2\times200\times10^9}} = 0.02348\ (m)$$

i) ii)에서 큰값 0.02348 (m) ∴한변의 길이 $2a = 0.04696$

Ans 0.04696 (m) ; 한 변의 길이

9.13 i) A점에 관한 moment 에서

$P_{cr} = 2kL = 2\times 80\times 10^3 = 160.000\ N$



ii) Yielding

$P_{cr} = YA = \frac{\pi}{4}d^2 Y = \frac{\pi}{4}\times(25\times 10^{-3})^2(400\times 10^6)$

$= 196.350\ N$

iii) B점이 옆으로 이동하지 않는다면,

양단이 hinged 된 beam 으로 buckling이 가능하다.

$P_{cr} = \frac{\pi^2 EI}{L^2} = \pi^2\times(76\times 10^9)\cdot\frac{\pi}{64}(0.025)^4 = 14.383N$

i) ii) iii) 에서 buckling이 가장 먼저 일어나며, $P_{cr} = 14383N$

Ans. Buckling이 가장 먼저 일어난다. $P_{cr} = 14383\ N$

9.14 (Eq. 9.17) 에서 $\delta = \frac{M_0}{P}\left(\frac{1-\cos L\sqrt{\frac{P}{EI}}}{\cos L\sqrt{\frac{P}{EI}}}\right)$

미소각 θ에 대해서 $\theta \equiv L\sqrt{\frac{P}{EI}}$, $1-\cos\theta \simeq \frac{\theta^2}{2}$, $\cos\theta \simeq 1.0$

$\therefore \delta \simeq \frac{M_0}{P}\cdot\frac{\theta^2}{2} = \frac{M_0 L^2}{2EI}$ — Q. E. D. —

9.15 (a) Yielding ; $(P_{cr})_Y = YA$ (Y : Yielding Stress)

Buckling ; $(P_{cr})_B = \frac{\pi^2 EI}{L^2}$

위의 2식에서 작은 Pcr 값이 critical value 이다.

(b) 평형조건 ; $(L\cos\theta)(kL\sin\theta) = PL\sin\theta$ $\therefore P = kL\cos\theta$.

(c) Stable이 될려면 restoring force가 (+) 이어야 한다.

$P_{res.} = kL\cos(\theta+\varepsilon) - kL\cos\theta = -kL\varepsilon\sin\theta > 0$

$(\because \varepsilon \ll 1 \quad \sin\varepsilon \simeq \varepsilon,\ \cos\varepsilon \simeq 1)$

$\therefore \sin\theta < 0$ 에서 stable 하다 $\therefore \pi < \theta < 2\pi$

(d) $\varepsilon = \theta$를 미소각으로 가정하면,

(b)에서 $P = kL\cos\theta = kL\left(1-\frac{1}{2}\theta^2\right)$ $\quad \therefore P = \frac{1}{2}kL(2-\theta^2)$

(e) stable하려면 restoring force는 (+)이어야 한다.

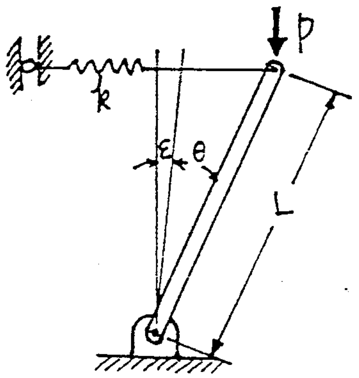
$$P_{res.} = \frac{1}{2}kL\{2-(\theta+\varepsilon')^2\} - \frac{1}{2}kL(2-\theta^2)$$
$$= \frac{1}{2}kL(-\varepsilon')(2\theta+\varepsilon') < 0$$

$P_{res.} < 0$ 이므로 θ에 관계없이 항상 unstable하다.

(f) $\frac{dP}{d\theta} = -kL\theta = 0$ 인 점에서 P가 maximum value를 갖는다.

$\therefore P_{max} = kL$ at $\theta = 0$.

(g) 평형조건 $\Sigma M_A = 0$; $PL\sin\theta = kL\cos\theta\cdot(L\sin\theta - L\sin\varepsilon)$



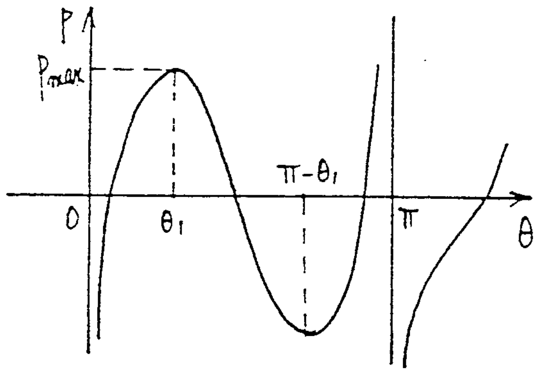
$$\therefore P = kL\cdot\cos\theta\cdot\left(1-\frac{\sin\varepsilon}{\sin\theta}\right)$$

$$\frac{dP}{d\theta} = kL\left(\frac{\sin\varepsilon\cdot\cos\theta}{\sin^2\theta}\right)\cos\theta - kL\sin\theta\left(1-\frac{\sin\varepsilon}{\sin\theta}\right)$$

$$= kL\frac{\sin\varepsilon\cdot\cos^2\theta - \sin^3\theta + \sin\varepsilon\sin^2\theta}{\sin^2\theta}$$

$$= kL\frac{\sin\varepsilon - \sin^3\theta}{\sin^2\theta}$$

$\frac{dP}{d\theta} = 0$; $\sin^3\theta_1 = \sin\varepsilon$ 인 $(0<\theta_1<\frac{\pi}{2})$ θ_1, $\pi-\theta_1$ 에서 P의 극값을 갖는다. P-θ의 graph를 그리면



$0<\theta<\theta_1$; Stable

$\theta_1<\theta<\pi-\theta_1$; unstable

$\pi-\theta_1<\theta<\pi$; stable

θ는 안정된 상태를 유지하려 한다. $\therefore$ $\theta=\theta_1$에서 $\theta=\pi-\theta_1$으로 jump가 발생하게 된다.

(h) (d)에서 $p = kL\left(1-\frac{\theta^2}{2}\right) = kL\left(1+\beta\cdot\frac{x^2}{L^2}\right)$ $\therefore \beta = -\frac{1}{2}$

9.16 $\bar{\sigma} = \left\{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\right\}^{1/2}$

Biaxial tension을 받는 spherical shell 에서는 $\sigma_1 = \sigma_2$, $\sigma_3 = 0$

평형 조건 ; $p = \frac{2\sigma_1 t_0}{r_0}$ (p = pressure) —①

위에서 $\bar{\sigma} = \sigma_1 = \sigma_2$ 이므로 $p = \frac{2\bar{\sigma} t_0}{r_0}$ —②

$$\bar{\varepsilon} = \int \left\{\frac{2}{9}\left[(d\varepsilon_I - d\varepsilon_{III})^2 + (d\varepsilon_I - d\varepsilon_{II})^2 + (d\varepsilon_{II} - d\varepsilon_{III})^2\right]\right\}^{1/2}$$

체적의 미소 요소에 있어서의 체적 증가 ; $d(\Delta V)$

$$d(\Delta V) = (d\varepsilon_I + d\varepsilon_{II} + d\varepsilon_{III})\cdot\Delta V$$

체적의 증가는 없으므로 $d\varepsilon_I + d\varepsilon_{II} + d\varepsilon_{III} = 0$.

sperical shell 에서, $d\varepsilon_I = d\varepsilon_{II} = \frac{dr_0}{r_0}$ $\therefore d\varepsilon_{III} = -2\frac{dr_0}{r_0}$

즉 $\frac{dt_0}{t_0} = -2\frac{dr_0}{r_0}$ $\therefore \bar{\varepsilon} = \int \frac{2}{3}\frac{dr_0}{r_0}$ or $d\bar{\varepsilon} = 2\frac{dr_0}{r_0}$

②식을 log를 취하여 미분하면

$$\frac{dp}{p} = \frac{d\bar{\sigma}}{\bar{\sigma}} + \frac{dt_0}{t_0} - \frac{dr_0}{r_0} = \frac{d\bar{\sigma}}{\bar{\sigma}} - \frac{3}{2}d\bar{\varepsilon} \quad —③$$

초기에 $\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = E$ 이므로, $\frac{d\bar{\sigma}}{\bar{\sigma}} - \frac{3}{2}d\bar{\varepsilon} > 0$ 즉 $\frac{d\bar{\sigma}}{d\bar{\varepsilon}} > \frac{3}{2}\bar{\sigma}$

따라서 초기에는 $dp > 0$ 또는 p가 증가한다.

또, $\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{3}{2}\bar{\sigma}$ 일때 $dp = 0$ $\frac{d\bar{\sigma}}{d\bar{\varepsilon}} < \frac{3}{2}\bar{\sigma}$ 일때 $dp < 0$
p는 감소

$\therefore$ Max. pressure는 $\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{3}{2}\bar{\sigma}$ 일때 생긴다. —Q.E.D.—

③식을 적분 ; $\ln p = \ln\bar{\sigma} - \frac{3}{2}\bar{\varepsilon} + \ln C_1$

또는 $p = C_1\bar{\sigma}\exp\left(-\frac{3}{2}\bar{\varepsilon}\right)$

$\bar{\varepsilon} \ll 1$ 이면 $\exp(-\frac{3}{2}\bar{\varepsilon}) \simeq 1$

$\therefore\ p \simeq C_1 \bar{\sigma} = \frac{2\bar{\sigma} t_0}{r_0} \qquad \therefore\ C_1 = \frac{2t_0}{r_0}$

따라서, $p = \frac{2t_0\bar{\sigma}}{r_0} \exp(-\frac{3}{2}\bar{\varepsilon})$

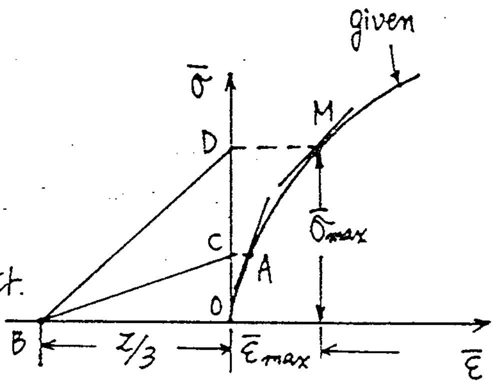
Max. pressure 는 $\bar{\sigma} = \bar{\sigma}_{max}$, $\bar{\varepsilon} = \bar{\varepsilon}_{max}$ 에서 생긴다.

$$p_{max} = \frac{2t_0\bar{\sigma}_{max}}{r_0} \exp\left(-\frac{3}{2}\bar{\varepsilon}_{max}\right)$$

$\bar{\sigma}_{max}$ 과 $\bar{\varepsilon}_{max}$ 의 결정.

임의의 점 A에서의 접선의 기울기와 BC의 기울기가 같아지는 점에서 $\bar{\sigma}_{max}$. $\bar{\varepsilon}_{max}$ 을 나타낸다.

M에서 근사적으로 다음 식이 성립한다.



$$\frac{d\bar{\sigma}}{d\bar{\varepsilon}} = \frac{3}{z}\bar{\sigma}\left(= \frac{OD}{OB}\right)$$

$$\text{Ans. } p_{max} = \frac{2t_0\bar{\sigma}_{max}}{r_0} \exp\left(-\frac{3}{z}\bar{\varepsilon}_{max}\right)$$

9.17 $F = A\sigma$

$$A = \frac{A_0 \cdot L_0}{L_0 + \delta} = \frac{A_0}{1 + \frac{\delta_0}{L_0}} = \frac{A_0}{1+\varepsilon} \qquad \therefore F = \frac{A_0\sigma}{1+\varepsilon},\ \frac{F}{A_0} = \frac{\sigma}{1+\varepsilon}$$

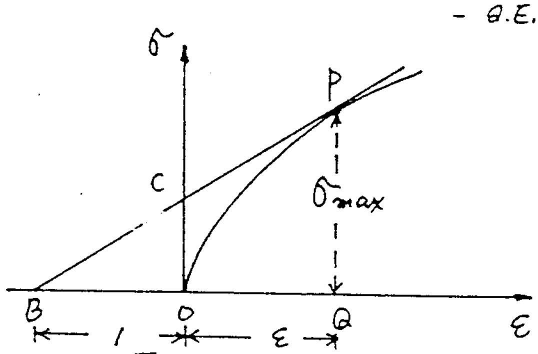
\- Q.E.D -

오른편 graph 에서

$\sigma_{max} = PQ$

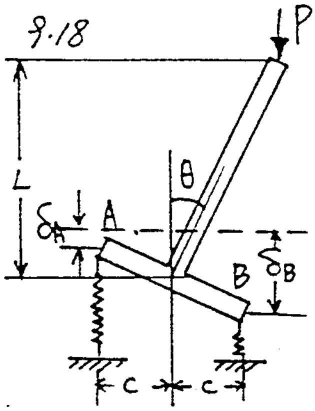
$\frac{F_{max}}{A_0} = \frac{\sigma_{max}}{1+\varepsilon}$

또, $\frac{OC}{QP} = \frac{1}{1+\varepsilon}$



$\therefore\ OC = \frac{QP}{1+\varepsilon} = \frac{\sigma_{max}}{1+\varepsilon} \qquad \therefore\ \frac{F_{max}}{A_0} = OC$

9.18



$F_A = \frac{P}{2}(1-\frac{L\theta}{c})$ —①

$F_B = \frac{P}{2}(1+\frac{L\theta}{c})$ —②

$\delta_B - \delta_A = 2c\theta$ —③

force-deflection relation;

$F_A = k_e\delta_A$. $F_B = k_e\delta_B$ —④

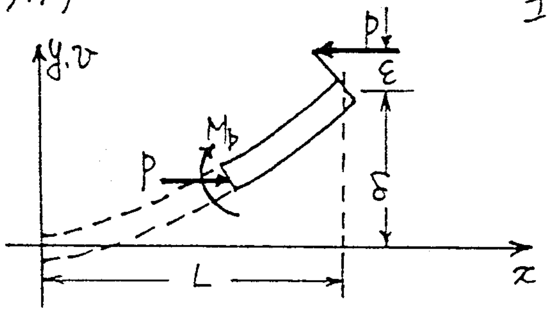
③④에서 $F_B - F_A = 2k_e c\theta$ —⑤

①②를 ⑤에 대입하면 $\theta(2k_e c - \frac{PL}{c}) = 0$

∴ non-trivial sol. $P_{cr} = \frac{2k_e c^2}{L}$ (임의의 θ에 대해서)

Ans $P_{cr} = 2k_e c^2/L$

9.19



그림에서

$M_b = P(\delta+\varepsilon-v) = EIv''$

$\therefore EIv'' = P(\delta+\varepsilon-v)$ —①

$\phi = -(\delta+\varepsilon-v)$라 놓으면 —②

①식은 다음과 같이 된다.

$EI\frac{d^2\phi}{dx^2} + \frac{P}{EI}\phi = 0$ —③

③식의 일반해;

$\phi = C_1 \sin\sqrt{\frac{P}{EI}}\cdot x + C_2\cos\sqrt{\frac{P}{EI}}x$

②식에서 $v = \delta+\varepsilon + C_1\sin\sqrt{\frac{P}{EI}}\cdot x + C_2\cos\sqrt{\frac{P}{EI}}\cdot x$

Boundary Condition;

$v(0) = v'(0) = 0$, $v(L) = \delta$를 대입하면

$C_1 = 0$ $C_2 = -(\delta+\varepsilon)$ $\therefore v = (\delta+\varepsilon)[1-\cos\sqrt{\frac{P}{EI}}\cdot x]$

$\therefore v(L) = \delta = (\delta+\varepsilon)[1-\cos\sqrt{\frac{P}{EI}}\cdot L]$

따라서 $\delta = \frac{\varepsilon(1-\cos\sqrt{\frac{P}{EI}}L)}{\cos\sqrt{\frac{P}{EI}}\cdot L} = \varepsilon(\sec\sqrt{\frac{P}{EI}}\cdot L - 1)$ — Q.E.D. —

9.20 평형조건: $p(x+\varepsilon) = 2kLx\left(1+\beta\frac{x^2}{L^2}\right)$ —①

$\frac{dp}{dx}=0$ 에서 max. value ; $\frac{dp}{dx} = (x+\varepsilon)\left(1+3\beta\frac{x^2}{L^2}\right) - x\left(1+\beta\frac{x^2}{L^2}\right) = 0$ —②

$$\therefore x+\varepsilon = \frac{x\left(1+\beta\frac{x^2}{L^2}\right)}{1+3\beta\frac{x^2}{L^2}}$$

위의 $(x+\varepsilon)$을 ①에 대입하면,

$$p_{max} = \frac{2kLx\left(1+\beta\frac{x^2}{L^2}\right)}{x+\varepsilon} \simeq 2kL\left(1+3\beta\frac{x^2}{L^2}\right)$$

$p_{cr} = 2kL$ $\therefore \frac{P_{cr}-P_{max}}{P_{cr}} = 1-\frac{P_{max}}{P_{cr}} = -3\beta\frac{x^2}{L^2}$ — Q.E.D. —

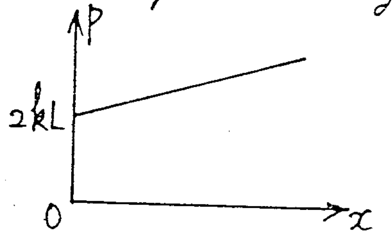
②식에서 $\beta\frac{x^2}{L^2} = -\frac{1}{2}\left(\varepsilon+3\beta\varepsilon\frac{x^2}{L^2}\right) = -\frac{1}{2}\varepsilon\frac{P_{max}}{P_{cr}}$

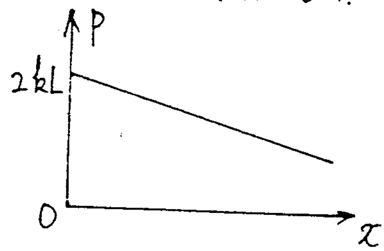
$$\therefore \left(1-\frac{P_{max}}{P_{cr}}\right)^{\frac{3}{2}} = \frac{3\sqrt{-3\beta}}{2}\cdot\frac{\varepsilon}{L}\cdot\frac{P_{max}}{P_{cr}}$$ — Q.E.D. —

9.21 $f = kx(1+\alpha x/L)$, $px = 2kx(1+\alpha x/L)L$

$$\therefore p = 2kL(1+\alpha x/L)$$

따라서 postbuckling curve는 다음과 같다.





$p(x+\varepsilon) = 2kx(1+\alpha x/L)L$ —①

$\frac{dp}{dx}=0$ 에서 max. value ;

$$\frac{dp}{dx} = (1+2\alpha x/L)(x+\varepsilon) - x(1+\alpha x/L) = 0$$ —②

①②에서 $P_{max} = \frac{2kx(1+\alpha x/L)L}{x+\varepsilon} = 2kL(1+2\alpha x/L)$

따라서 $\dfrac{P_{cr}-P_{max}}{P_{cr}} = 1 - \dfrac{P_{max}}{P_{cr}} = -2\alpha x/L$ — Q.E.D. —

또, $\left(1-\dfrac{P_{max}}{P_{cr}}\right)^2 = -4\alpha\cdot(-\alpha)\dfrac{x^2}{L^2}$

②식에서 $\varepsilon + \alpha\dfrac{x^2}{L} + 2\alpha\varepsilon\dfrac{x}{L} = 0$ 이므로 $-\alpha\dfrac{x^2}{L^2} = \dfrac{\varepsilon}{L}(1+2\alpha x/L)$

따라서 $\left(1-\dfrac{P_{max}}{P_{cr}}\right)^2 = -4\alpha\cdot\dfrac{\varepsilon}{L}(1+2\alpha x/L) = -4\alpha\cdot\dfrac{\varepsilon}{L}\dfrac{P_{max}}{P_{cr}}$

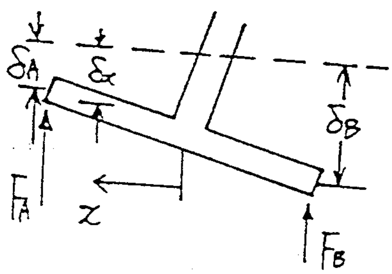
— Q.E.D. —

9.22 $P_e - 2k_e c\theta = P_t + 2k_t c\theta$ —①

$P_t = \dfrac{2c^2}{L}k_t$, $P_e = \dfrac{2c^2}{L}k_e$ 이므로 ①식에 대입하면

$\theta = \dfrac{P_e - P_t}{2(k_e+k_t)}\cdot\dfrac{1}{c} = \left(\dfrac{k_e-k_t}{k_e+k_t}\right)\left(\dfrac{c}{L}\right)$ — Q.E.D. —

9.23



평형 조건; $P(L\theta+\varepsilon) = (F_B - F_A)c$ —①

$P = F_A + F_B$

윈편 그림에서

$\delta_A - \delta_x = -(c-x)\theta$

$\delta_B - \delta_x = (c+x)\cdot\theta$

$\therefore F_A = \frac{1}{2}P_o - k_e(c-x)\theta$, $F_B = \frac{1}{2}P_o + k_t(c+x)\cdot\theta$ —②

②를 ①에 대입; $P(L\theta+\varepsilon) = [(k_e+k_t)c - (k_e-k_t)x]\,\theta\cdot c(1+\beta\theta^2)$

$P_A = P_{x=c}$ 이므로 $P(L\theta+\varepsilon) = 2k_t c^2\theta\,(1+\beta\theta^2)$ —③

$\dfrac{dP}{d\theta} = 0$ 에서 max. value ; $\dfrac{dP}{d\theta} = (1+3\beta\theta^2)(\varepsilon+L\theta) - L\theta(1+\beta\theta^2) = 0$

$\therefore \dfrac{1+\beta\theta^2}{L\theta+\varepsilon} = \dfrac{1+3\beta\theta^2}{L\theta}$ —④

③④에서 $(P_A)_{max} = \dfrac{2k_t c^2}{L}(1+3\beta^2)$

그런데 $\beta < 0$ 이므로,

$(P_A)_{max}$ 은 $P_t \left(= \frac{2k_t C^2}{L}\right)$ 보다 작다. $\therefore P_t > P_r$

— Q.E.D. —