INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

Classification of PDE and Method of Characteristics

MSO 203B

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1. Classify the following PDE's in terms of Linearity:
   a) $u_t + u_x + uu_x = u_{xxt}$
   b) $u_x^2 + u_t^2 = 1$
   c) $u_t + u^2 u_x = 0$
   d) $u_t + x^2 u_x = \sin(x)$
2. Solve the following equations using Method of characteristics
   a) $yu_x - xu_y = 0;\ u(0,y) = 2y^2$ for $y > 0$.
   b) $u_t + (x+t)u_x = t;\ u(x,0) = \exp(x)$.
   c) $u_t + xu_x + u = 3x;\ u(x,0) = \tan^{-1} x$.
   d) $x(y^2 - u^2)u_x - y(u^2 + x^2)u_y = (x^2 + y^2)u$.
3. Prove that the problem $u_x + u_y = 1$ subject to $u(x,x) = 1$ has no solution.
4. Prove that the problem $u_x + u_y = 1$ subject to $u(x,x) = x$ has infinitely many solutions.
5. Prove that the Cauchy Problem

$$u_x - u_y = 1;\ \ u(x,0) = x^2$$

admits an unique solution.