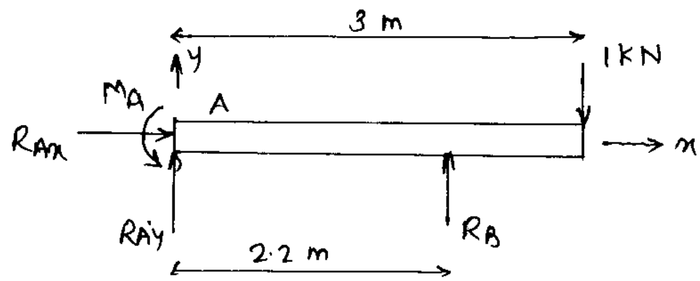
Solutions to Problems of Chapter 1

Problem 1.8



At B :

1. No longitudinal reaction because roller support is present.

Equilibrium Equations :

i) $\xrightarrow{+}$ $\Sigma F_x = 0$ $\Rightarrow$ $R_{Ax} = 0$

ii) $+\uparrow$ $\Sigma F_y = 0$ $\Rightarrow$ $R_{Ay} + R_B - 1 = 0.$

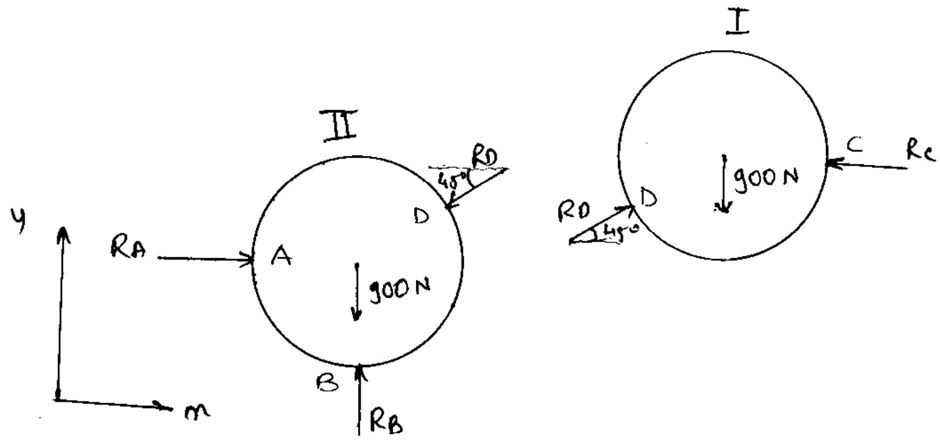
iii) $\circlearrowleft +$ $\Sigma M_A = 0$ $\Rightarrow$ $R_B \times 2.2 + M_A - 1 \times 3 = 0$.

Consider equations ii) and iii)

There are 2 equations but three unknown.

$\therefore$ Statically indeterminant.

Problem 1.9



No friction.

Equilibrium equations:→

1. Body I : $+\uparrow \sum F_y = 0 \Rightarrow R_D \sin 45^\circ - 900 = 0$

$$\Rightarrow R_D = \sqrt{2}\, 900 \text{ N}.$$

$\xrightarrow{+} \sum F_x = 0 \Rightarrow R_D \cos 45^\circ - R_c = 0$

$$\Rightarrow R_c = \sqrt{2} \times 900 \times \frac{1}{\sqrt{2}}$$
$$= 900 \text{ N}.$$

2. Body II: $\xrightarrow{+} \sum F_x = 0 \Rightarrow R_A - R_D \cos 45^\circ = 0$

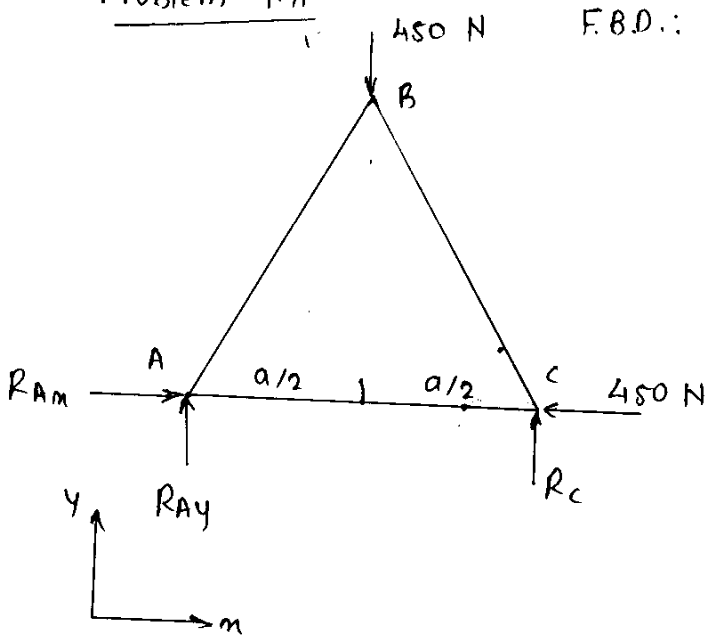
$$\Rightarrow R_A = \sqrt{2} \times 900 \times \frac{1}{\sqrt{2}}$$
$$= 900 \text{ N}.$$

$+\uparrow \sum F_y = 0 \Rightarrow R_B - 900 - R_D \sin 45 = 0.$

$$\Rightarrow R_B - 900 - \sqrt{2} \times 900 \times \frac{1}{\sqrt{2}} = 0.$$
$$\therefore R_B = 1800 \text{ N}.$$

Problem 1.11

F.B.D.:



B : Pinned joint

A : Hinged

C : Roller support

ABC : equilateral triangle.

Equations of Equilibrium:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow R_{Ax} - 450 = 0$

$\therefore R_{Ax} = 450 \text{ N}.$

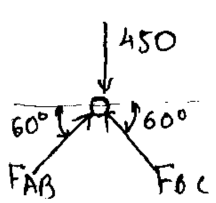
$+\circlearrowleft \Sigma M_A = 0 \Rightarrow a R_c - \frac{a}{2} 450 = 0$

$\therefore R_c = 225 \text{ N}.$

$+\uparrow \Sigma F_y = 0 \Rightarrow (R_A)_y + R_c - 450 = 0$

$\therefore R_{Ay} = 225 \text{ N}.$

Equilibrium of Pin B:



$\overset{+}{\rightarrow} \Sigma F_x = 0 \Rightarrow$

$F_{AB} \cos 60 - F_{BC} \cos 60 = 0$

$\Rightarrow F_{AB} = F_{BC}.$

Problem 1.11 (contd)

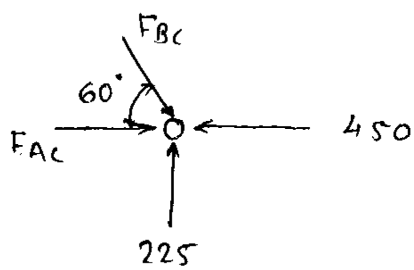
$+\uparrow \sum F_y = 0 \quad \Rightarrow \quad F_{AB} \sin 60 + F_{BC} \sin 60 - 450 = 0$

$\Rightarrow 2 F_{AB} \sin 60 = 450$

$\Rightarrow F_{AB} = 259.8$ N. (compression)

$\Rightarrow F_{BC} = 259.8$ N. (compression)

Equilibrium of joint C :



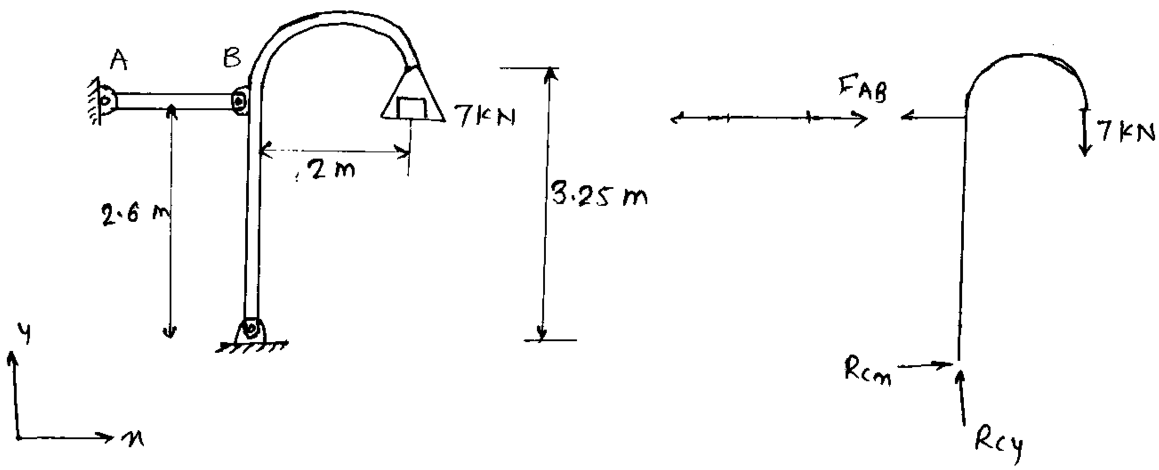
$\xrightarrow{+} \sum F_x = 0 \quad \Rightarrow$

$F_{AC} + F_{BC} \cos 60 - 450 = 0$

$F_{AC} = 450 - 259.8 \left(\frac{1}{2}\right)$

$= 320$ N. (compression)

Problem 1.12



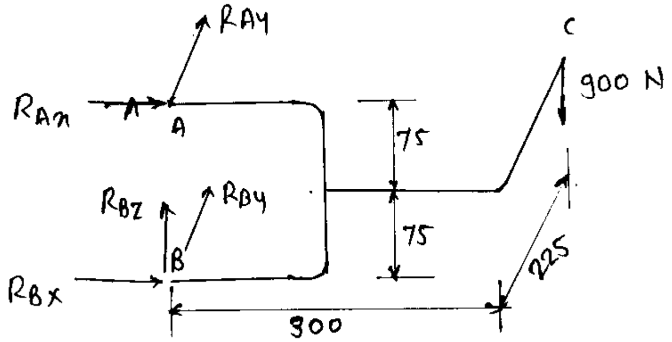
Equation of Equilibrium :

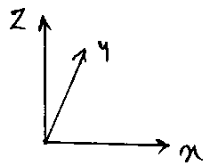
$\circlearrowleft + \sum M_C = 0 \Rightarrow -7 \times 2 + F_{AB} \times (2.6) = 0$

$\Rightarrow F_{AB} = \frac{14}{2.6}$

$= 5.39$ N.

Problem 1.14 :





- No friction
- No moment reactions at A & B
- No vertical reaction at A.
  At B, the reaction is provided by bracket.

Equilibrium equations:

$\xrightarrow{+}$ $\Sigma F_x = 0 \Rightarrow R_{Ax} + R_{Bx} = 0$ — i)

$\overset{+}{\nearrow}$ $\Sigma F_y = 0 \Rightarrow R_{Ay} + R_{By} = 0$ — ii)

$+\uparrow$ $\Sigma F_z = 0 \Rightarrow R_{Bz} - 900 = 0 \Rightarrow R_{Bz} = 900\ N$ — iii)

$\Sigma M_{Bx} = 0 \Rightarrow -R_{Ay} \times 150 - 900 \times 225 = 0$

$\Rightarrow R_{Ay} = -1350\ N$ — iv)

$\Sigma M_{By} = 0 \Rightarrow R_{Ax} \times 150 + 900 \times 300 = 0$

$\Rightarrow R_{Ax} = -1800\ N$. — v)

$\Sigma M_{Bz} = 0$ is identically satisfied.

Problem 1.14 (Contd)

Equations i) and v) gives

$$R_{Bx} = -R_{Ax} = 1800 \text{ N} \quad \text{— vi)}$$

Equations ii) and iv) gives

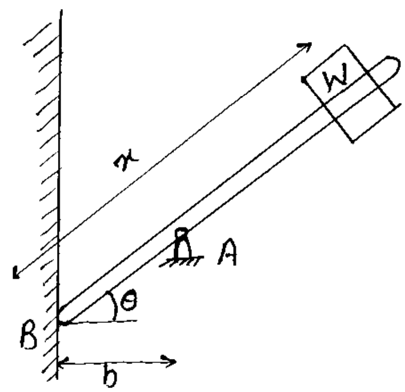
$$R_{By} = -R_{Ay} = 1350 \text{ N}. \quad \text{— vii)}$$

The reactions are

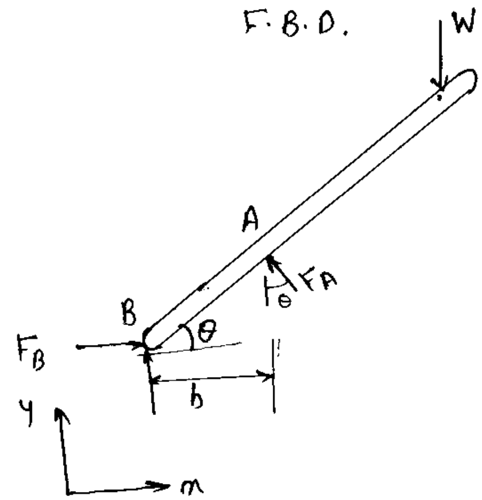
$$\underset{\sim}{R_A} = -1800\,\hat{i} - 1350\,\hat{j} \text{ N}$$

$$\underset{\sim}{R_B} = 1800\,\hat{i} + 1350\,\hat{j} + 900\,\hat{k} \text{ N},$$

Problem 1·21



F.B.D.



Assumptions :

i) inelastic contact

ii) No friction anywhere

iii) No deformation of rod

Assume equilibrium exists.

Equations of equilibrium :

$\xrightarrow{+} \Sigma F_x = 0 \quad \Rightarrow \quad F_B = F_A \sin\theta$ —— (1)

$+\uparrow \; \Sigma F_y = 0 \quad \Rightarrow \quad F_A \cos\theta - W = 0$

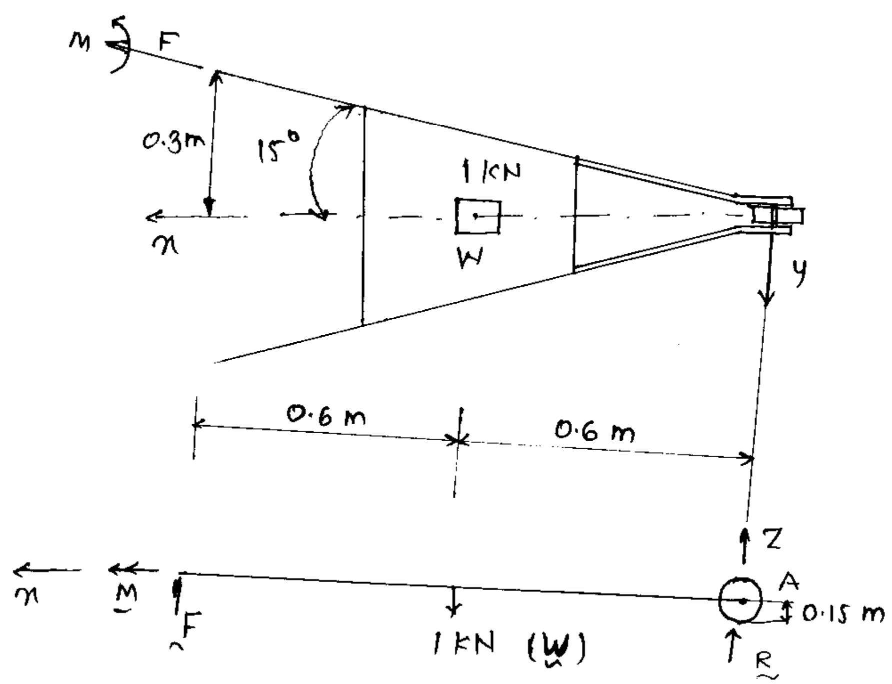
$\Rightarrow \quad F_A \cos\theta = W.$ —— (2)

$\Sigma M_{Bz} = 0 \quad \Rightarrow \quad F_A \cdot \dfrac{b}{\cos\theta} = W \cdot x \cos\theta$ —— (3)

From above equations (2 and 3)

$$x = \frac{b}{\cos^3\theta} .$$

Problem 1·23



Forces in vector form:

$$\underset{\sim}{M} = M\cos 15\,\hat{i} - M\sin 15\,\hat{j}$$

$$\underset{\sim}{F} = F\,\hat{k}$$

$$\underset{\sim}{W} = -1\,\hat{k}\ \text{kN}$$

$$\underset{\sim}{r}_F = 1.2\,\hat{i} - 0.3\,\hat{j}\ \text{m}$$

$$\underset{\sim}{r}_w = 0.6\,\hat{i}$$

Equilibrium equations

Σ Moment about A $= \underset{\sim}{0} \Rightarrow$

$$\underset{\sim}{M} + \underset{\sim}{r}_F \times \underset{\sim}{F} + \underset{\sim}{r}_w \times \underset{\sim}{W} = 0$$

$$(M\cos 15\,\hat{i} - M\sin 15\,\hat{j}) + (1.2\,\hat{i} - 0.3\,\hat{j}) \times F\hat{k} + 0.6\,\hat{i} \times (-1\,\hat{k}) = 0$$

$$M\cos 15\,\hat{i} - M\sin 15\,\hat{j} + (-1.2F\hat{j}) - 0.3F\hat{i} + 0.6\,\hat{j} = 0$$

$$\therefore\quad M\cos 15 - 0.3F = 0 \qquad \text{— i)}$$

$$M\sin 15 + 1.2F = 0.6 \qquad \text{— ii)}$$

Problem 1.23 (Contd)

Solving equations i) and ii) simultaneously

$$M(4\cos 15 + \sin 15) = 0.6$$

$$\Rightarrow \quad M = 0.145 \text{ kN·m}$$

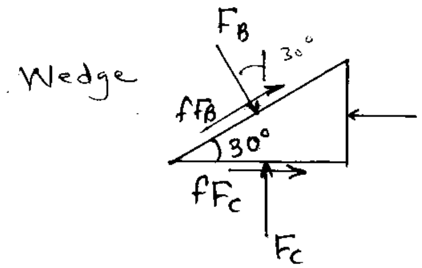
$$F = \frac{M\cos 15}{0.3} = \frac{0.145 \times \cos 15}{0.3}$$

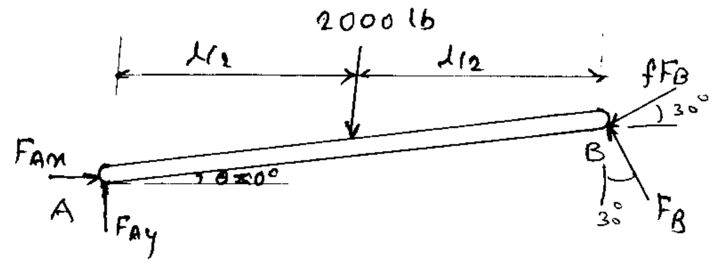
$$= 0.467 \text{ kN.}$$

Problem 1.24 :

Assuming general friction coefficient f, when the wedge is moving, tangential force at contact =

$= f \times$ normal force.





a) Equilibrium of AB :

$$\Sigma M_{Az} = 0 \Rightarrow -2000 \times \frac{l}{2} + F_B \cos 30° \times l - f F_B \sin 30° \times l = 0$$

$$\therefore \quad F_B = \frac{1000}{0.866 - \frac{1}{2} f}.$$

Equilibrium of Wedge :

$$\Sigma F_y = 0 \Rightarrow F_C - F_B \cos 30° + f F_B \sin 30° = 0$$

$$\Sigma F_x = 0 \Rightarrow -P + fF_C + (f \cos 30° + \sin 30°) F_B = 0.$$

Solving above system gives :

$$F_C = 1000 \text{ lb}$$

$$P = 1000 \left(f + \frac{0.866 f + 0.5}{0.866 - 0.5 f} \right)$$

for $f = 0.3$ $\quad P = 1000 \left(0.3 + \frac{0.866 \times 0.3 + 0.5}{0.866 - 0.5 \times 0.3} \right)$

$$= 1364 \text{ lb}.$$

b) If f is very small, the wedge will slip out when force p is removed. The borderline occurs when the wedge is just prevented from slipping out by the the friction forces.

We go through the previous analysis, replacing f by $-f$ everywhere, because the tendancy we are now investigating is 'slip in opposite direction.

Hence we get

$$P = 1000\left[-f + \frac{0.5 - 0.866f}{0.866 + 0.5f}\right]$$

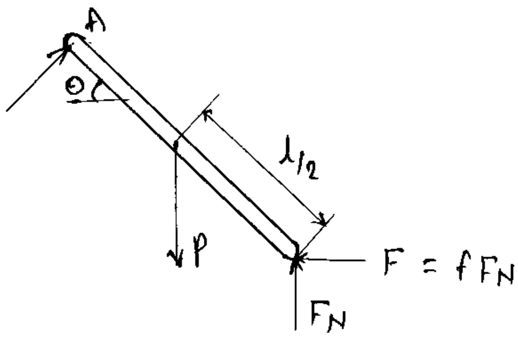
We require $P = 0$ or

$$\frac{0.5 - 0.866f}{0.866 + 0.5f} - f = 0.$$

$\therefore f = 0.27.$

Problem 1.25

Free Body diagram of link for counterclockwise rotation.



Friction force

$F = f F_N$

Equilibrium equations:

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow$

$$-F(l)\sin\theta - P\frac{l}{2}\cos\theta + F_N l\cos\theta = 0$$

$$\Rightarrow F\left(\frac{1}{f} - \tan\theta\right) = P/2.$$

i.e. $F = \dfrac{Pf}{2(1 - f\tan\theta)}$.

For c.w. rotation F is in the opposite direction.

$+\circlearrowleft \Sigma M_A = 0$

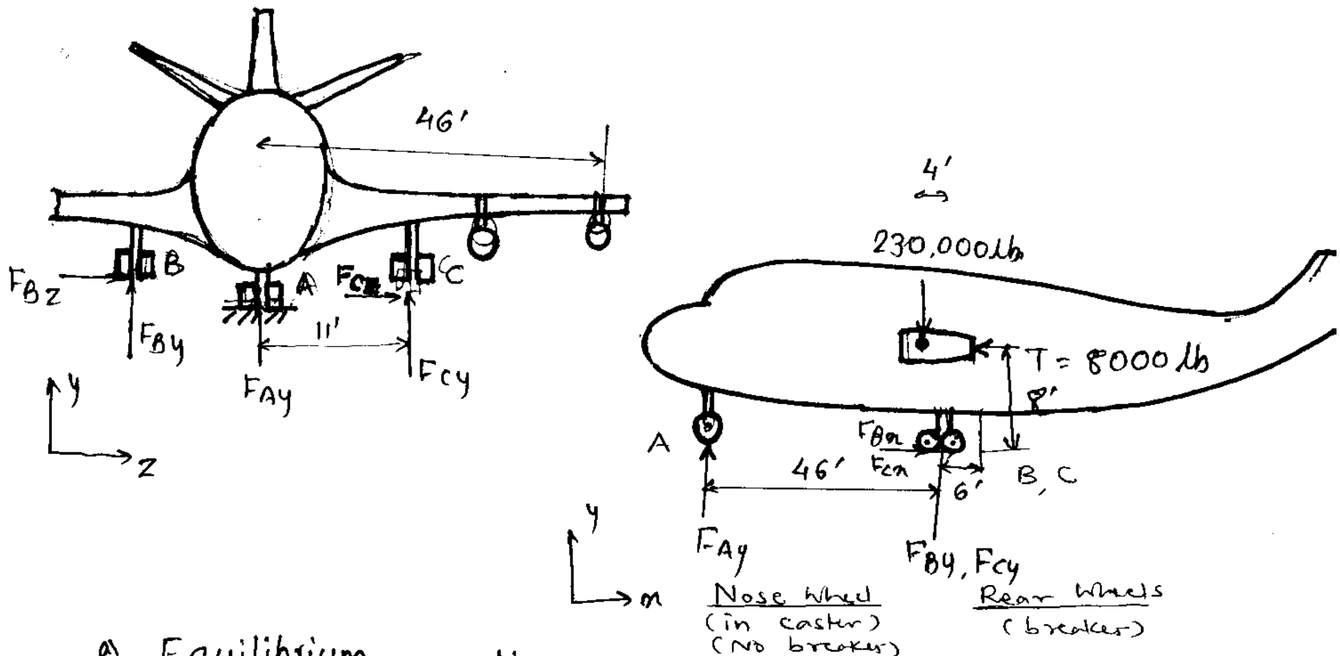
$$F(l)\sin\theta - P\frac{l}{2}\cos\theta + F_N l\cos\theta = 0.$$

$$F = \frac{Pf}{2(1+f\tan\theta)}.$$

The mechanism becomes friction lock for c.c.w. dirn.

As $F \Rightarrow \infty$ for $f = \dfrac{1}{\tan\theta}$.

Problem 1.27.



a) Equilibrium equations:

$\Sigma F_x = 0 \Rightarrow F_{Bx} + F_{Cx} - 8000 = 0$ —— i)

$\Sigma F_y = 0 \Rightarrow F_{Ay} + F_{By} + F_{Cy} - 230{,}000 = 0$ —— ii)

$\Sigma F_z = 0 \Rightarrow F_{Bz} + F_{Cz} = 0$ —— iii)

F_{Bz}, F_{Cz} are indeterminate. A plausible assumption is

$F_{Bz} = F_{Cz} = 0$.

$\Sigma M = 0$ at a point midwaway between B & C,

$\Sigma M_x = 0 \Rightarrow F_{By} \times 11 = F_{Cy} \times 11$ —— iv)

$\Sigma M_y = 0 \Rightarrow F_{Cx} \times 11 - F_{Bx} \times 11 - 8000 \times 46 = 0$ — v)

$\Sigma M_z = 0 \Longrightarrow -F_{Ay} \times 46 + 8000 \times 8 + 230{,}000 \times 4 = 0$ — vi)

Solving above set of equations: (i, ii, iv, v, vi)

$F_{Ay} = 21{,}300$ lb , $F_{By} = F_{Cy} = 104{,}300$ lb

$F_{Bx} = -12{,}700$ lb, $F_{Cx} = 20{,}800$ lb.

b)

Problem 1.27 (contd)

b) Coefficient of friction :

$$f_s \geq \frac{\text{tangential force}}{\text{Normal force}}$$

at wheel B :

$$f_s \geq \frac{12\,700}{104{,}300}$$

$$\geq 0.122$$

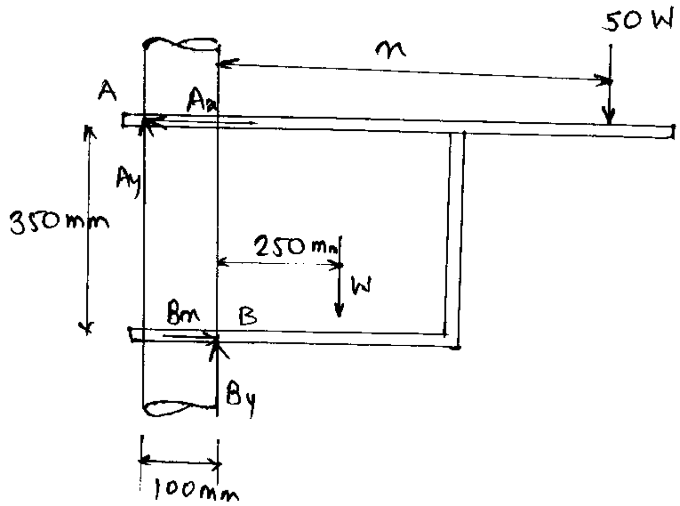
at wheel C :

$$f_s \geq \frac{20800}{104{,}300}$$

$$\geq 0.2$$

$$\therefore f = 0.2$$

Problem 1.33



Because of the load, the hanger tilts so as to have the contact at points A and B of the post

coefficient of friction between post and support
$f = 0.3$

To Find: x so as to have no slip.

Equilibrium equations:

$\xrightarrow{+} \Sigma F_x = 0 \Rightarrow B_x - A_x = 0$ — i)

$+\uparrow \Sigma F_y = 0 \Rightarrow A_y + B_y - 50W - W = 0$ — ii)

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow B_x \times 0.35 + 0.1 \times B_y - 0.35W - (x+0.1)\,50W = 0$ — iii)

$A_y = f \cdot \times A_x$ — iv)

$B_y = f \times B_x$ — v)

5 equations and 5 unknowns.

i) $\Rightarrow A_x = B_x$ Then

iv) & iv) $\Rightarrow A_y + B_y = 2fB_x$

ii) $\Rightarrow 51W = 2fB_x$

$\therefore B_x = \dfrac{51W}{2f}$

Problem 1.33 (Contd)

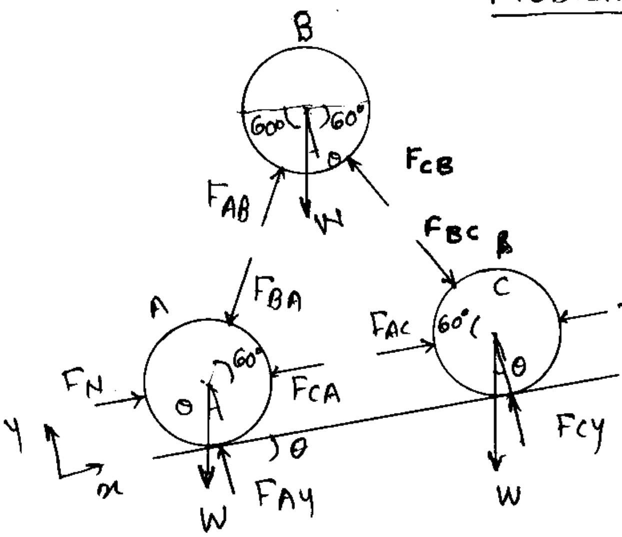
$$B_y = \frac{51W}{2f} \times f = \frac{51W}{2}.$$

iii) $\Rightarrow$ $$\frac{51W}{2f} \times 0.35 + \frac{51W}{2} \times 0.1 - W \times 0.35 - 50W(x+0.1) = 0$$

$$\frac{51 \times 0.35}{0.6} + \frac{51 \times 0.1}{2} - 0.35 - 50x - 5 = 0.$$

$$x = 0.539 \text{ m}.$$

Problem 1.36



Assume all logs to be of equal diameter.

When c is about to move $f = 0$ and $F_{AC} = 0$.

Equilibrium equations:

For log C : $\Sigma F_x = 0$

$$F_{BC} \cos 60 - W \sin\theta = 0.$$

$$\sin\theta = \frac{F_{BC} \cos 60}{W}. \quad \text{——— (1)}$$

For log B :

$$\Sigma F_x = 0 \Rightarrow -F_{CB} \cos 60 + F_{AB} \cos 60 - W \sin\theta = 0$$

$$\Sigma F_y = 0 \Rightarrow F_{CB} \sin 60 + F_{AB} \sin 60 - W \cos\theta = 0.$$

Solving for $F_{CB} = F_{BC} = \dfrac{W(\sin\theta \sin 60 - \cos\theta \cos 60)}{-2 \sin 60 \cos 60}$

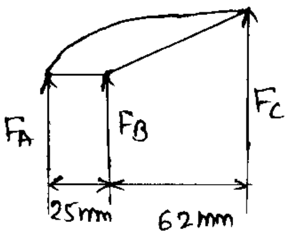
$$= \frac{W \cos(\theta + 60)}{\cos 30}. \quad \text{—— (2)}$$

(1) & (2) $\Rightarrow \sin\theta = \dfrac{\cancel{W} \cos(\theta + 60) \cos 60}{\cos 30}$

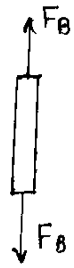
$$\tan\theta = \frac{2}{3} \frac{\cos^2 60}{\cos 30}$$

$$\theta = 10.8°.$$

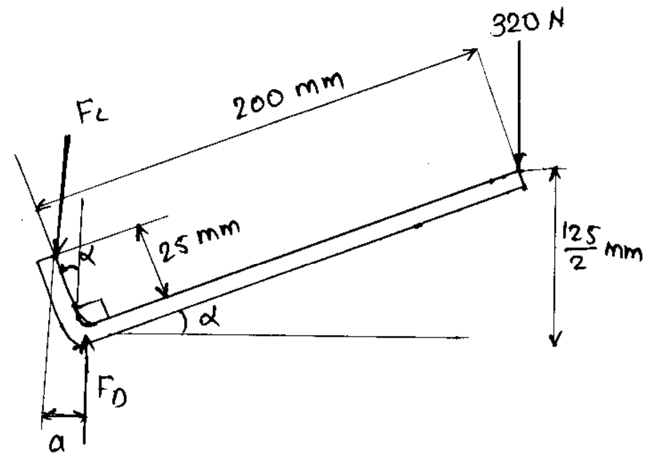
Problem 1.38



Cutter



2 force member



Handle.

1. To calculate α, a :

$$\alpha = \sin^{-1} \frac{125/2}{200} = 18.2^\circ$$

$$a = 25 \sin\alpha = 7.8 \text{ mm}.$$

Equilibrium of handle:

At points B, C there will be force in y-direction only due to the presence of 2 force member.

$$\therefore +\circlearrowleft \Sigma M_D = 0 \Rightarrow F_C \times a - 320 \times 200 \cos\alpha = 0.$$

$$\therefore F_C = \frac{320 \times 200 \cos 18.2^\circ}{7.8}$$

$$= 7794.9 \text{ N}$$

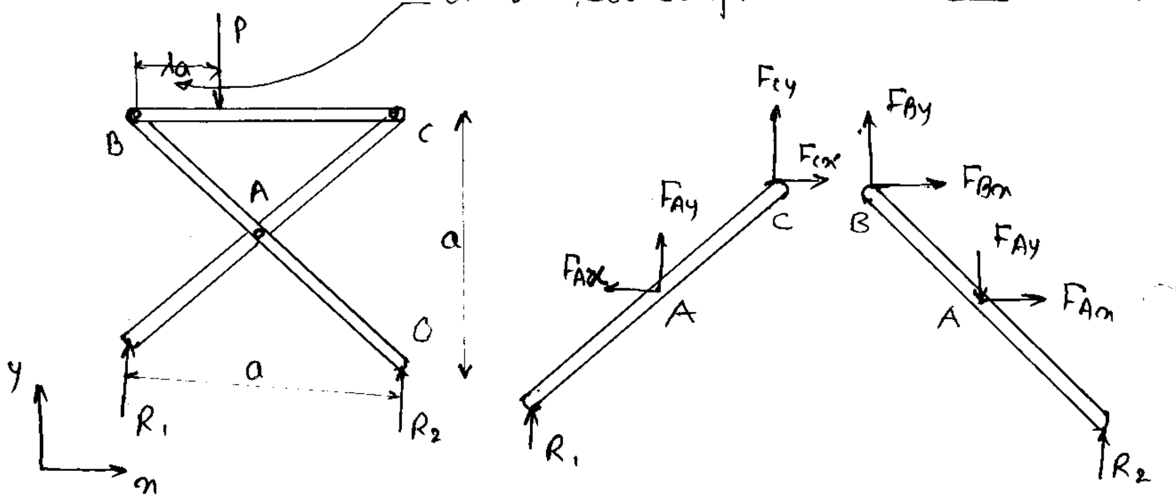
Equilibrium of cutter:

$$+\circlearrowleft \Sigma M_B = 0 \Rightarrow -F_A \times 25 + 7794.9 \times 62 = 0.$$

$$\therefore F_A = \frac{F_C \times 62}{25} = \frac{7794.9 \times 62}{25}$$

$$F_A = 19.331 \text{ kN}$$

Problem 1.43. — a is subscript (It is not $\lambda \times a$)



Equilibrium equations:

$+\circlearrowleft \ \Sigma M_O = 0 \Rightarrow P(a-\lambda_a) - R_1 a = 0.$

$$R_1 = P\left(1 - \frac{\lambda_a}{a}\right) \quad — (1)$$

$\Sigma F_y = 0 \Rightarrow$

$$R_1 + R_2 - P = 0.$$

$$R_2 = P - R_1 = \frac{\lambda_a}{a} P. \quad — (2)$$

For the whole strut

$+\circlearrowleft \ \Sigma M_C = 0 \Rightarrow -R_1 a - F_{Ax}\frac{a}{2} - F_{Ay}\frac{a}{2} = 0.$ (bar AC) — (3)

$+\circlearrowleft \ \Sigma M_B = 0 \Rightarrow R_2 a + F_{Ax}\frac{a}{2} - F_{Ay}\frac{a}{2} = 0,$ (bar AB) — (4)

(1) - (4) $\Rightarrow$

$$F_{Ax} = -P$$

$$F_{Ay} = -P\left(1 - \frac{\lambda_a}{a}\right) + P\frac{\lambda_a}{a}.$$

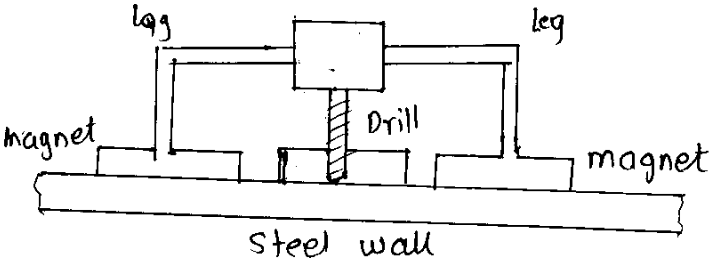
$$= -P\left(1 - \frac{2\lambda_a}{a}\right).$$

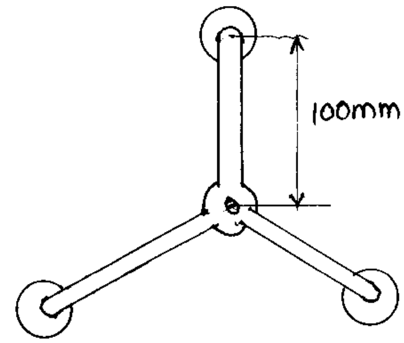
Magnitude of the shear $= (F_{Ax}^2 + F_{Ay}^2)^{1/2}$

$$= P\left[1 + \left(1 - \frac{2\lambda_a}{a}\right)^2\right]^{1/2}.$$

Maximum shear $= \sqrt{2}P$ when $\lambda_a = 0, a$

Problem 1.48

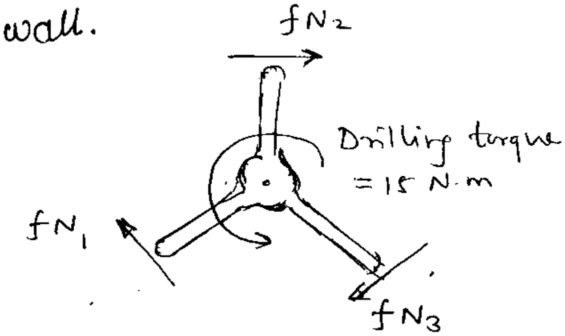




1. In space lab, it is not possible to apply either force or torque to hold the drill in equilibrium.

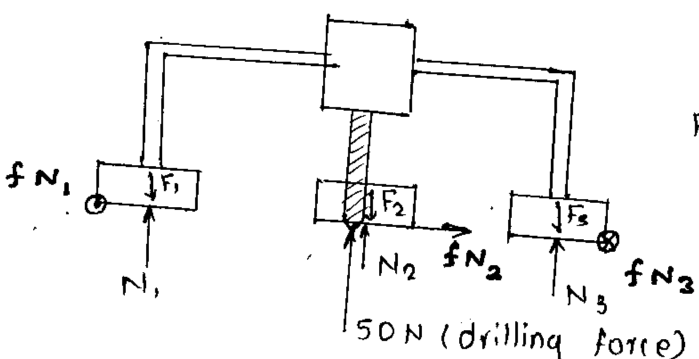
2. Legs of drill holder are provided with magnet.

3. Drilling torque = 15 N.m, Drilling force = 15 N
   $f \cancel{\mu} = 0.4$ between magnet and wall.



To find, minimum holding force.

F.B.D.



F: magnetic force.

drilling torque = 15.

Due to symmetry $F_1 = F_2 = F_3 = F$ say
$N_1 = N_2 = N_3 = N$, say

$$+\uparrow \Sigma F_y = 0 \Rightarrow 3N + 50 = 3F \quad \text{— i)}$$

$$+\circlearrowleft \Sigma M_y = 0 \Rightarrow 3 \times \cancel{\mu} f N \times 0.1 = 15 \quad \text{— ii)}$$

$$\Rightarrow N = \frac{15}{0.1 \times 3 \times 0.4} = 125\ N.$$

$$\text{i)} \Rightarrow \quad F = N + \frac{50}{3} = 125 + 16.67$$

$$= 141.67\ N.$$