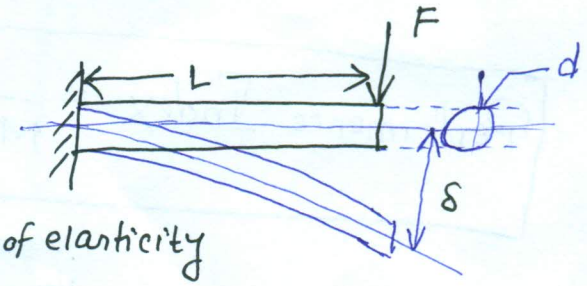
Problem Statement :- Cost-effective material for circular c/s cantilever beam load at end having high stiffness & light weight.

Sol:

Free variable — Diameter 'd' & material choice

Constraint — Length of beam & tip deflection



From basic Mech. of solids,

Tip deflection, $\delta = \frac{FL^3}{3EI}$ — (1)

E = Modulus of elasticity

I = Area Moment of Inertia

$\therefore I = \frac{\pi d^4}{64}$ — (2)

$\therefore F = \left(\frac{3EI}{L^3}\right)\delta$ — (3)

↳ equivalent Stiffness

Mass of beam, m = density × volume

$= \rho \times \frac{\pi}{4} d^2 L$ — (4)

From (2) & (3)

$$d^2 = \left(\frac{64}{3}\frac{FL^3}{E\delta\pi}\right)^{1/2} \quad - (5)$$

Put eq. (5) in (4), we get

$$m = \frac{\pi}{4} \rho L \left(\frac{64}{3} \frac{F L^3}{E \pi \delta} \right)^{1/2}$$

$$= \underbrace{\left(\frac{\pi}{4} \times \frac{8}{\sqrt{3}\pi} \right)}_{\text{Constant}} \times \underbrace{(L \times L^{3/2})}_{\text{Geometric}} \times \underbrace{\left(\frac{F}{\delta} \right)^{1/2}}_{\text{Functional parameter}} \times \underbrace{\left(\frac{\rho}{\sqrt{E}} \right)}_{\text{Material property Index (M.P.I)}}$$

$$\boxed{\text{Performance Index} = \frac{1}{\text{M.P.I}} = \frac{\sqrt{E}}{\rho}}$$ Maximize it

Limiting case :- $\frac{E^{1/2}}{\rho} = C$

Taking log both sides

$\log E = 2 \log \rho + 2 \log C$

(Straight line equation)

Slope, $m = 2$

$\tan \theta = 2$

$\therefore \boxed{\theta = 63.4^\circ}$ (Fixed)