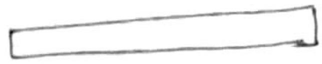
Quiz - 3 Solution



$\longrightarrow x$ Free Free beam

E, A, ρ

Governing PDE

$$\boxed{u_{,xx} = \frac{1}{c^2} u_{,tt}} \qquad \text{where } c = \sqrt{\frac{E}{\rho}}$$

Obtain solution through separation of variable approach

$$u(x,t) = g(x)\,h(t)$$

we get

$$g(x) = A\cos\frac{\omega}{c}x + B\sin\frac{\omega}{c}x$$

$$h(t) = C\cos\omega t + D\sin\omega t$$

<u>Objective</u>: To find frequencies and mode shapes

Free Free Condition at both ends of beam

$\therefore$ At $x=0$ and $x=l$ traction $\underline{t}=0$

$$\therefore \boxed{E\frac{\partial u}{\partial x}\bigg|_{x=0,l} = 0} \quad \text{Condition to be met}$$

$$\therefore g'(0) = 0 \quad \text{and} \quad g'(l) = 0$$

$$g'(x) = \frac{\omega}{c}\left[-A\sin\frac{\omega}{c}x + B\cos\frac{\omega}{c}x\right]$$

$$g'(0) = \frac{\omega}{c}B = 0 \Rightarrow B = 0$$

$$g'(l) = -\frac{A\omega}{c}\sin\frac{\omega l}{c} = 0$$

$$\Rightarrow \sin\frac{\omega l}{c} = 0$$

$$\sin\frac{\omega l}{c} = \sin n\pi$$

$$\omega_n = \frac{n\pi c}{l} = \frac{n\pi}{l}\sqrt{\frac{E}{\rho}}$$

For $n=0$, $\omega_n = 0 \Rightarrow$ Rigid body motion of rod.

$$\therefore g(x) = A\cos\frac{\omega}{c}x = A\cos\frac{n\pi c}{c\,l}x = A\cos\frac{n\pi x}{l}$$

for $n=0$; $g(x) = A$ (Rigid body mode shape)

$n = 0, 1, 2, 3 \ldots\ldots$

$g_n(x) = A_n \cos\frac{n\pi x}{l}$ $\equiv$ Eigen func. | mode shapes

<u>Mass normalization of eigen functions</u>

$$\int_0^l \rho A \left(A_n \cos\frac{n\pi x}{l}\, A_n \cos\frac{n\pi x}{l}\right) dx = 1$$

$$A_n^2 = \frac{2}{\rho l A}$$

$$A_n = \sqrt{\frac{2}{\rho A l}}$$

Normalized Eigen Vectors OR mode shapes

$$\boxed{g_n(x) = \sqrt{\frac{2}{\rho A l}} \cos\frac{n\pi x}{l}}$$ $n = 0, 1, 2, 3 \ldots$