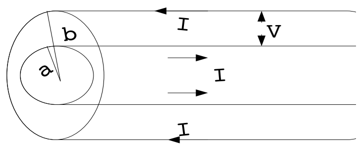
IITK/ PHY103, **Problem set-13** **Date: April 5, 2015** [DpC/TKG]

1. A long coaxial cable carries current I(the current flows down the surface of inner cylinder of radius a and back along the outer cylinder of radius b. If the conductors are held at a potentil difference V, calculate the power transported down the cable.



2. Consider a solenoid of length l and radius R ($l >> R$) and a time dependent surface current density $\vec{K} = K(t)\hat{\phi}$ is flowing on the surface. Show that the power flow through the surface is the same as the rate of change of the magnetic energy stored inside the solenoid.

3. Find the nature of polarization for of the EM waves whose electric field components are given by:
(a) $\vec{E} = \hat{z}E_0 \sin(kx - \omega t) + \hat{y}E_0 \sin(kx - \omega t)$
(b) $\vec{E} = \hat{z}E_0 \sin(kx - \omega t) + \hat{y}E_0 \cos(kx - \omega t)$
(c) $\vec{E} = Re[E_0(\hat{x} + (1+i)\hat{y})e^{-i(kz-\omega t)}]$.

4. The magnetic field in vacuum is given by $\vec{B} = (2 \times 10^{-6}T)\ \cos(\beta x)\cos[(2.4 \times 10^{12}\ s^{-1})t]\ \hat{z}$.
(a) Show that it satisfies the differential wave equation.
(b) Find the value of β.
(c) Find the electric field.
(d) Calculate the Poynting vector ($\vec{S}$) and the direction of energy flow at a given time at any arbitrary point x. Find the average of $\vec{S}$ over a full time period.

Practice Problems

1. A long straight conductor carries a steady current I. Show that the energy dissipated by Joules heating is replenished by the inflow of energy through the Poynting vector.

2. Find the amplitude, angular frequency, phase constant, wave length, nature of polarization and direction of propagation for each of the EM waves represented by the

following fields. Find the corresponding magnetic fields.

(a) $\vec{E} = E_0 \sin[(2 \times 10^4 m^{-1})x - (2.5 \times 10^{12} s^{-1})t]\hat{z} + E_0 \sin[(2 \times 10^4 m^{-1})x - (2.5 \times 10^{12} s^{-1})t]\hat{y}$;

(b) $\vec{E} = E_0 \cos[(10^7 m^{-1})y - (3 \times 10^{15} s^{-1})t]\hat{z} - E_0 \sin[(10^7 m^{-1})y - (3 \times 10^{15} s^{-1})t]\hat{x}$;

(c) $\vec{E} = (5V/m)\cos[(10^7 m^{-1})z - (3 \times 10^{15} s^{-1})t]\hat{x} + (7V/m)\cos[(10^7 m^{-1})z - (3 \times 10^{15} s^{-1})t - \pi/6]\hat{y}$.

3. Write down a plane-wave solution to Maxwell's equations corresponding to a right circularly polarized wave with an electric field amplitude of 10 V/m, wavelength of 3 cm and propagating along the z direction in free space. Write the corresponding magnetic field. Evaluate the Poynting vector.

4. The electric field in vacuum is written as $\vec{E} = \hat{z}A[\sin(ax - \omega t) + \sin(bx + 2\omega t)]$. For what condition on a and b, can this be a valid expression for EM wave?