ME-231A-Practice Problem Set 4

P1. Air enters a nozzle steadily at 2.21 kg/m^3 and 30 m/s and leaves at 0.762 kg/m^3 and 180 m/s. If the inlet area of the nozzle is 80 cm^2, determine (*a*) the mass flow rate through the nozzle, and (*b*) the exit area of the nozzle.

P2. A gas-filled pneumatic strut in an automobile suspension system behaves like a piston-cylinder apparatus. At one instant when the piston is L=0.15m away from the closed end of the cylinder, the gas density is uniform at ρ=18 kg/m^3 and the piston begins to move away from the closed end at V=12 m/s. Assume as a simple model that the gas velocity is one-dimensional and proportional to distance from the closed end; it varies linearly from zero at the end to u=V at the piston. Find the rate of change of gas density at this instant. Obtain an expression for the average density as a function of time.

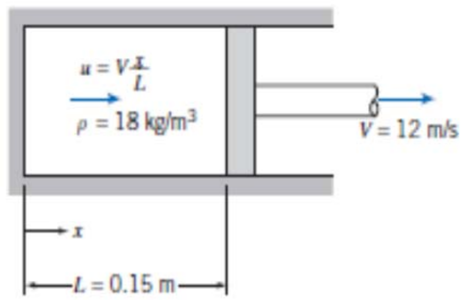


Fig.P2

P3. The components of a mass flow vector **ρu** are $\rho u = 4x^2y$, $\rho v = xyz$, $\rho w = yz^2$. Compute the net outflow through the closed surface formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.
(a) Integrate over the closed surface.
(b) Integrate over the volume bounded by that surface.

P4. A viscous liquid is sheared between two parallel disks of radius R, one of which rotates while the other is fixed. The velocity field is purely tangential, and the velocity varies linearly with z from V_θ=0 at z=0 (the fixed disk) to the velocity of the rotating disk at its surface (z=h). Derive an expression for the velocity field between the disks.

P5. The velocity field for a free vortex in the $r\theta$ plane is $\vec{V} = \hat{e}_\theta C / r$. Find the stream function for this flow. Evaluate the volume flow rate per unit depth between r_1=0.20 m and r_2=0.24 m, if C=0.3 m^2/s. Check the flow rate calculated from the stream function by integrating the velocity profile along this line.

P6. Let a one-dimensional velocity field be $u = u(x, t)$, with $v = 0$ and $w = 0$. The density varies as $\rho = \rho_o(2 - cos\omega t)$. Find an expression for $u(x, t)$ if $u(0, t)$ = U.

P7. Deduce the following relationship between V and the stream function ψ, for a 2-D steady compressible flow.

$$V_x = \frac{\rho_o}{\rho}\frac{\partial \psi}{\partial y}$$

$$V_y = -\frac{\rho_o}{\rho}\frac{\partial \psi}{\partial x}$$

where ρ_o is the density of fluid at some reference condition. Show that this satisfies the differential form of the continuity equation.

P8. Find V_y and ψ when

(a) $V_x = e^{-x}\cosh y + 1$ (b) $V_x = V_o\left(\frac{2y}{ax} - \frac{y^2}{a^2x^2}\right)$

Use V_y=0 at y=0.

P9. Consider a steady, two-dimensional, incompressible flow of a Newtonian fluid with the velocity field $u = -2xy$, $v = y^2 - x^2$, and $w = 0$. (a) Does this flow satisfy conservation of mass? (b) Find the pressure field $p(x, y)$ if the pressure at point ($x = 0, y = 0$) is equal to p_a.

P10. In spherical polar coordinates, the flow is called axisymmetric if $v_\theta = 0$ and $\partial/\partial\theta = 0$, so that $v_r = v_r(r, \theta)$ and $v_\theta = v_\theta(r, \theta)$. Show that a stream function $\psi(r, \theta)$ exists for this case and is given by

$$v_r = \frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}, v_\theta = \frac{1}{r\sin\theta}\frac{\partial \psi}{\partial r}$$

P11. A viscous liquid of constant density and viscosity falls due to gravity between two parallel plates a distance 2h apart, as in the figure. The flow is fully developed, that is, $w = w(x)$ only. There are no pressure gradients, only gravity. Set up and solve the Navier-Stokes equation for the velocity profile $w(x)$.

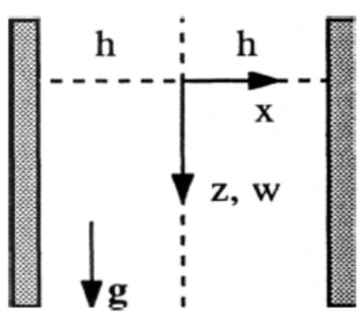


Fig.P11

P12. Consider the steady, laminar, incompressible flow between two large parallel plates as shown. The upper plate moves with velocity V to the right and the lower plate is stationary. The pressure gradient in the flow direction is zero. The lower region between the plates is filled with fluid A, and the upper region filled with fluid B. Assume fully developed laminar flow to obtain velocity profile.

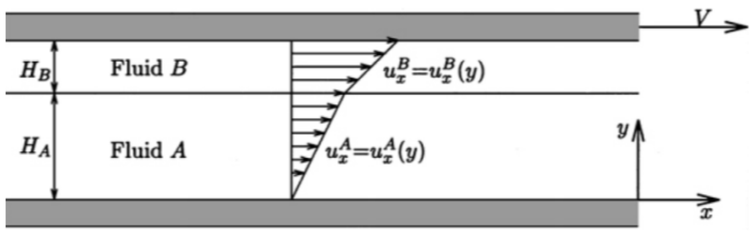


Fig. P12

P13. A constant-thickness film of viscous liquid flows in laminar motion down a plate inclined at angle θ, as in Fig. P13. The velocity profile is $u = Cy(2h - y)$, $v = w = 0$ Find the constant C in terms of the specific weight and viscosity and the angle θ. Find the volume flux Q per unit width in terms of these parameters.

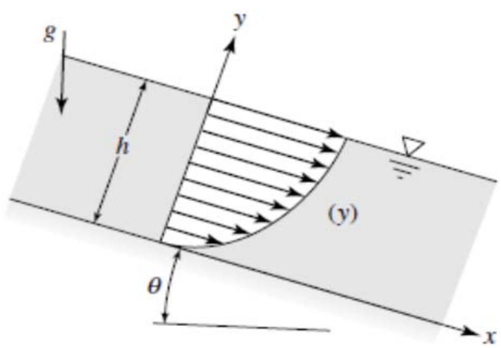


Fig.P13

P14. Assume the liquid film in Fig.P13 is horizontal (i.e., θ=0°) and that the flow is driven by a constant shear stress on the top surface (y=h), τ_{yx}=C. Assume that the liquid film is thin enough and flat and that the flow is fully developed with zero net flow rate (flow rate Q=0). Determine the velocity profile u(y) and the pressure gradient dp/dx.

P15. Water and oil flow down a vertical plane. The flow is steady, laminar and fully developed. Simplify the Navier stokes equation separately for water and oil and write the relevant boundary conditions. Obtain the two velocity profiles. Sketch these qualitatively, taking care near the oil-water interface.

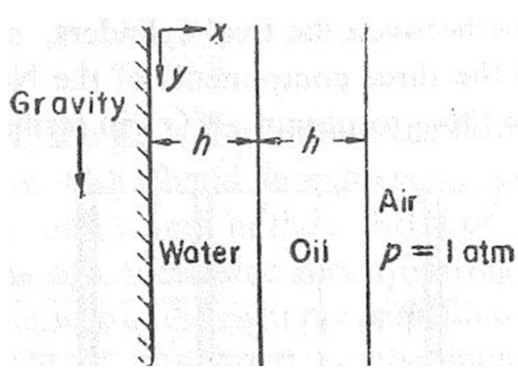


Fig.P15

P16. A wetted wall column is used to measure mass transfer coefficients. A liquid of density ρ and viscosity μ flows down inside of a tube of radius R shown. After an initial region, the flow becomes fully developed and the thickness of fluid layer is constant at h. simplify the Navier-stokes and continuity equation to obtain $V_z(r)$ for laminar conditions.

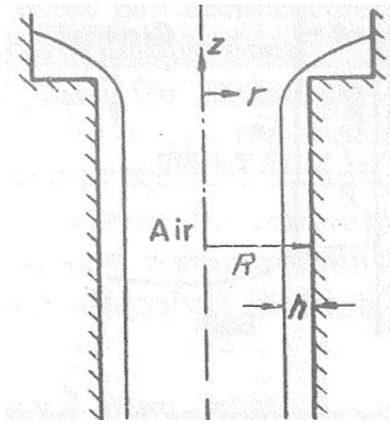
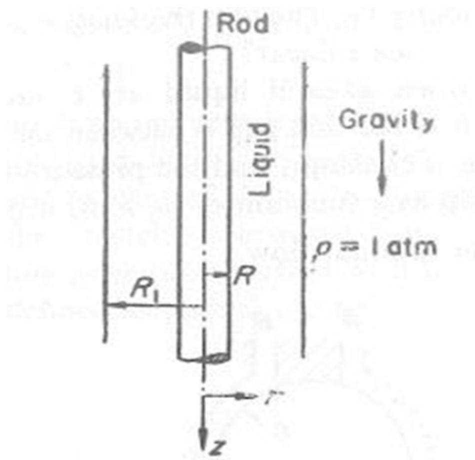


Fig.P16

P17. A viscous incompressible liquid flows down a long, cylindrical rod of radius R, as a thin, fully developed laminar film. Simplify the continuity and the Navier-stokes equation to obtain $V_z(r)$.



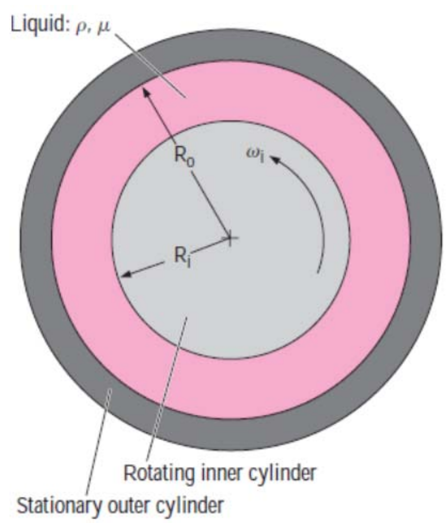


Fig.P17 **Fig.P18**

P18. An incompressible Newtonian liquid is confined between two concentric circular cylinders of infinite length a solid inner cylinder of radius R_i and a hollow, stationary outer cylinder of radius R_o (Fig. P18, the z-axis is out of the page). The inner cylinder rotates at angular velocity ω_i. The flow is steady, laminar, and two-dimensional in the r-θ plane. The flow is also *rotationally symmetric*, meaning that nothing is a function of coordinate u (u_θ and P are functions of radius r only). The flow is also circular, meaning that velocity component $u_r = 0$ everywhere. Generate an exact expression for velocity component u_θ as a function of radius r and the other parameters in the problem. You may ignore gravity.