Department of Physics, IIT-Kanpur

Instructors: TKG & DpC PHY103 (2014-15, II-SEM) Answers

Final Answers of the Exercise problems of Assignment 1:

Ex. 3c):

$$
\begin{aligned}
[\mathbf{r}\times\boldsymbol{\nabla}]_x &= (y\frac{\partial}{\partial z} - x\frac{\partial}{\partial y}) \\
[\mathbf{r}\times\boldsymbol{\nabla}]_y &= (z\frac{\partial}{\partial x} - z\frac{\partial}{\partial x}) \\
[\mathbf{r}\times\boldsymbol{\nabla}]_z &= (x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}). \qquad (1)
\end{aligned}
$$

Final Answers of the Exercise problems of Assignment 2:

Ex.4:

a)

$$\boldsymbol{\nabla}\times(\hat{\phi}/s) = 2\pi\delta^2(\mathbf{s}).$$

b) $J = 2\pi$.

Final Answers of the Exercise problems of Assignment 3:

Ex.1: The Laplacian operator in spherical coordinates is given in Griffith's book.

Ex. 2:

For spherical shell

$$\rho(r) = Q\frac{\delta(r-R)}{4\pi r^2}.$$

For the line charge density on the ring:

$$
\begin{aligned}
\lambda &= Q\frac{\delta(s-R)\delta(z)}{2\pi s} \text{ cylindrical coordinates} \\
&= Q\frac{\delta(r-R)\delta(\theta-\pi/2)}{2\pi r^2\sin\theta} \text{ spherical coordinates.}
\end{aligned}
$$

Ex.3: Yes, $g(x)$ is the Gaussian representation of Dirac delta function.

Ex.4: $f(x) = 1 - \theta(x-b) = \theta(b-x)$.

Ex. 6:

The electric field in the xy plane:

$$\mathbf{E}(x,y) = \frac{q}{4\pi\epsilon_0}\sum_{k=1}^{6}\frac{(x-a\cos(k\pi/3))\hat{\imath} + (y-a\sin(k\pi/3))\hat{\jmath}}{[(x-a\cos(k\pi/3))^2 + (y-a\sin(k\pi/3))^2]^{3/2}}.$$

$$
\begin{aligned}
E_x(x,0) &= \frac{q}{4\pi\epsilon_0}\sum_{k=1}^{6}\frac{(x-a\cos(k\pi/3))}{(x^2-2ax\cos(k\pi/3)+a^2)^{3/2}} \\
E_y(x,0) &= \frac{-qa}{4\pi\epsilon_0}\sum_{k=1}^{6}\frac{\sin(k\pi/3)}{(x^2-2ax\cos(k\pi/3)+a^2)^{3/2}} = 0.
\end{aligned}
$$

The field at the origin is zero.
For $x \gg a$, the electric field is

$$
E_x(x,0) \simeq \frac{1}{4\pi\epsilon_0}\Big[\frac{6q}{x^2}+\frac{9qa^2}{2x^4}\Big].
$$

The leading term is same as if the charges are concentrated at the origin.

The field due to other five charges at (0,0) is

$$
\mathbf{E}_{5\text{ charges}}(0,0) = +\frac{q}{4\pi\epsilon_0 a^2}\hat{i}.
$$

Ex. 7:
i) Spherical shell:

$$
\mathbf{E}_{in} = 0, \qquad \mathbf{E}_{out} = \frac{\sigma}{\epsilon_0}\frac{R^2}{r^2}\hat{r}
$$

ii) Cylindrical :

$$
\mathbf{E}_{in} = \frac{\rho s}{2\epsilon_0}\hat{s}, \qquad \mathbf{E}_{out} = \frac{\rho}{2\epsilon_0}\frac{R^2}{s}\hat{s}.
$$

iii) Infinite slab:

$$
\mathbf{E}_{in} = \frac{\rho z}{\epsilon_0}\hat{z}, \qquad \mathbf{E}_{out} = \frac{\rho\, d}{2\epsilon_0}\frac{z}{|z|}\hat{z}.
$$

Ex.8:
The potential at $z = z_0$ on the z axis is

$$
V(0,0,z_0) = \frac{\sigma R}{2\epsilon_0}\ln\Big[\frac{(z_0+L/2)+\sqrt{(z_0+L/2)^2+R^2}}{(z_0-L/2)+\sqrt{(z_0-L/2)^2+R^2}}\Big].
$$