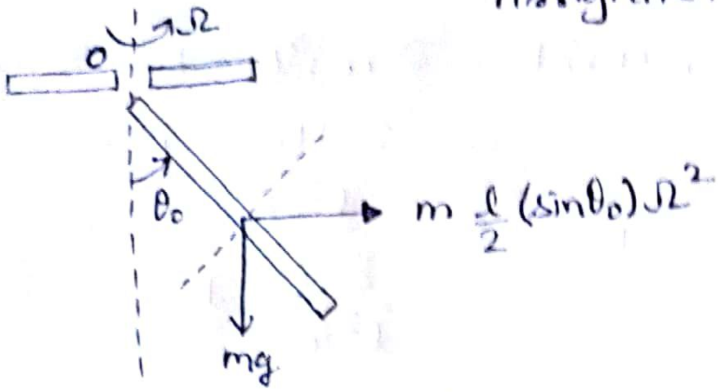
(9)



a) For equilibrium position

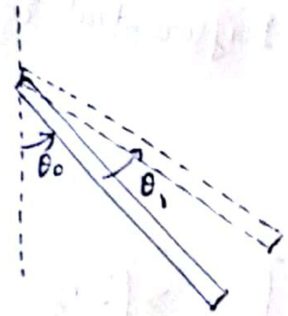
Net force on the system is 0.

$$\therefore \quad m\frac{l}{2}\cos\theta_0\Omega^2\sin\theta_0 = mg\sin\theta_0$$

$$\cos\theta_0 = \frac{2g}{l\Omega^2} \Rightarrow \boxed{\theta_0 = \cos^{-1}\left(\frac{2g}{l\Omega^2}\right)}$$

b) Taking moment about point O

$$\Sigma T_{/O} = J\ddot{\theta}$$



θ_1 is small

System is displaced from equilibrium pos. by small angle θ_1

$$m\frac{l}{2}\sin(\theta_0+\theta_1)\Omega^2\frac{l}{2}\cos(\theta_0+\theta_1) - mg\frac{l}{2}\sin(\theta_0+\theta_1) = \frac{ml^2}{3}\ddot{\theta}_1$$

$$\Rightarrow \frac{l\Omega^2}{4}\sin(\theta_0+\theta_1)\cos(\theta_0+\theta_1) - \frac{g}{2}\sin(\theta_0+\theta_1) = \frac{l\ddot{\theta}_1}{3}$$

$$\Rightarrow \quad \ddot{\theta}_1 + \frac{3g}{2l}\sin(\theta_0+\theta_1) - \frac{3\Omega^2}{4}\sin(\theta_0+\theta_1)\cos(\theta_0+\theta_1) = 0$$

$$\Rightarrow \quad \ddot{\theta}_1 + \frac{3g}{2l}\sin(\theta_0+\theta_1) - \frac{3\Omega^2}{8}\sin 2(\theta_0+\theta_1) = 0$$

$$\Rightarrow \quad \ddot{\theta}_1 + \frac{3g}{2l}[\sin\theta_0\cos\theta_1 + \cos\theta_0\sin\theta_1] - \frac{3\Omega^2}{8}\sin 2\theta_0\cos 2\theta_1 + \sin 2\theta_1 \cdot \sin 2\theta_0 = 0$$

for small θ_1, $\cos\theta_1 \simeq 1$, $\sin\theta_1 \simeq \theta_1$

$$\Rightarrow \ddot{\theta}_1 + \frac{3g}{2l}[\sin\theta_0 + \theta_1\cos\theta_0] - \frac{3\Omega^2}{8}[\sin 2\theta_0 + 2\theta_1\cos 2\theta_0] = 0$$

Since, $\cos\theta_0 = \frac{2g}{l\Omega^2}$, $\cos 2\theta_0 = 2\cos^2\theta_0 - 1$

$= 2 \cdot \frac{4g^2}{l^2\Omega^4} - 1$

$\sin\theta_0 = \frac{\sqrt{l^2\Omega^4 - 4g^2}}{l\Omega^2}$, $= \frac{8g^2}{l^2\Omega^4} - 1$

$$\ddot{\theta}_1 + \frac{3g}{2l}\left[\theta_1 \frac{2g}{l\Omega^2}\right] + \frac{3g}{2l}\sin\theta_0 - \frac{3\Omega^2}{8} 2\theta_1\left[\frac{8g^2}{l^2\Omega^4} - 1\right] - \frac{3l\Omega^2}{8} 2\sin\theta_0 \frac{2g}{l\Omega^2} = 0$$

$$\ddot{\theta}_1 + \frac{3g^2}{l^2\Omega^2}\theta_1 + \frac{3g}{2l}\sin\theta_0 - \frac{6g^2\theta_1}{l^2\Omega^2} + \frac{3}{4}\Omega^2\theta_1 - \frac{3g}{2l}\sin\theta_0 = 0$$

$$\ddot{\theta}_1 - \frac{3g^2}{l^2\Omega^2}\theta_1 + \frac{3}{4}\Omega^2\theta_1 = 0$$

$$\boxed{\ddot{\theta}_1 + \left(\frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2}\right)\theta_1 = 0}$$ Required Differential Eqn.

(c) For motion θ_1 to be harmonic.

$$\frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2} > 0$$

$$\Omega^4 > \frac{4g^2}{l^2} \Rightarrow \Omega^2 > \frac{2g}{l} \Rightarrow \boxed{\Omega > \sqrt{\frac{2g}{l}}}$$

(d) $$\omega_n^2 = \frac{3\Omega^2}{4} - \frac{3g^2}{l^2\Omega^2}$$

$$\boxed{\omega_n = \sqrt{\frac{3\Omega^2}{4} - \frac{3g^2}{l^2\Omega^2}}}$$

(e) For very large Ω, $\frac{3}{4}\Omega^2 >> \frac{3g^2}{l^2\Omega^2}$

$$\therefore \omega_n = \sqrt{\frac{3}{4}\Omega^2} = \frac{\sqrt{3}}{2}\Omega$$

Hence, ω_n does not depend on g.