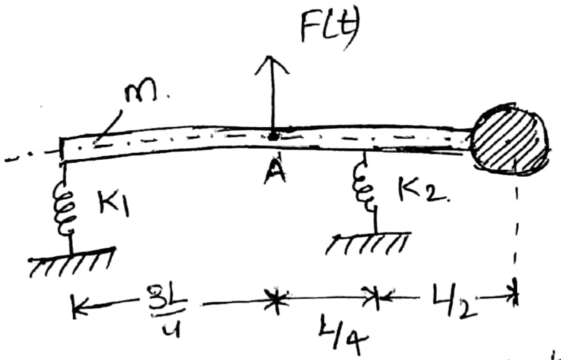
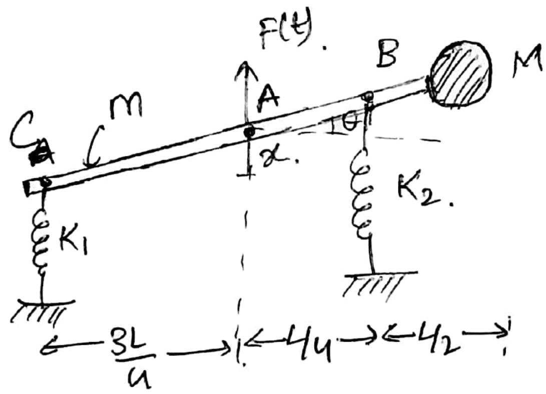
Q (3.3)



Let us consider at any instant the pt A to be ~~dis~~ at a distance x. and the beam is rotated by an angle θ



(for small θ)
Displacement of pt C $= x - \frac{3L}{4}\theta$.
Displacement of pt B $= x + \frac{L}{4}\theta$.

Applying LMB and AMB.

$$\Rightarrow \quad F(t) - K_1\left(x - \frac{3L}{4}\theta\right) - K_2\left(x + \frac{L}{4}\theta\right) = (m+M)\ddot{x} \qquad -(1)$$

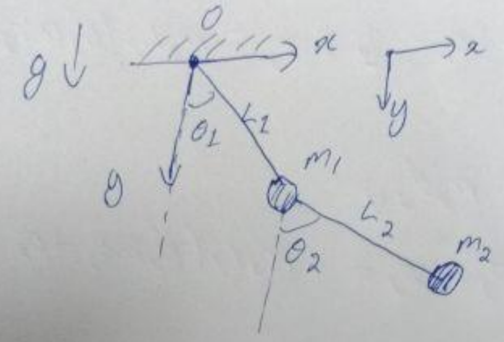
$$\Rightarrow \quad K_1\left(x - \frac{3L}{4}\theta\right)\frac{3L}{4} - K_2\left(x + \frac{L}{4}\theta\right)\frac{L}{4} = I\ddot{\theta} \qquad -(2)$$

where $I = I_{beam/A} + I_{M/A}$

$$= \frac{mL^2}{12} + M\left(\frac{3L}{4}\right)^2.$$

$$= \frac{mL^2}{12} + M\frac{9L^2}{16} = \frac{L^2}{4}\left[\frac{m}{3} + \frac{9M}{4}\right]$$

Q 3.4)



Position of m_1 wrt O is $\vec{r}_1 = L_1 \sin\theta_1 \hat{i} + L_1 \cos\theta_1 \hat{j}$

" " m_2 " O is $\vec{r}_2 = (L_1 \sin\theta_1 + L_2 \sin\theta_2)\hat{i} + (L_1\cos\theta_1 + L_2\cos\theta_2)\hat{j}$

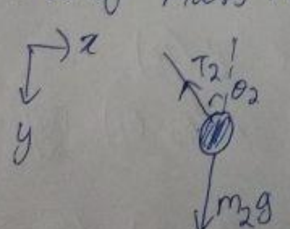
velocity of $m_1 = \dot{\vec{r}}_1 = \dot{\theta}_1 L_1 \cos\theta_1 \hat{i} - \dot{\theta}_1 L_1 \sin\theta_1 \hat{j}$

velocity of $m_2 = \dot{\vec{r}}_2 = (\dot{\theta}_1 L_1 \cos\theta_1 + \dot{\theta}_2 L_2 \cos\theta_2)\hat{i} + (-\dot{\theta}_1 L_1 \sin\theta_1 - \dot{\theta}_2 L_2 \sin\theta_2)\hat{j}$

acceleration of $m_1 = \ddot{\vec{r}}_1 = (\ddot{\theta}_1 L_1 \cos\theta_1 - L_1 \sin\theta_1 \dot{\theta}_1^2)\hat{i}$
$- (\ddot{\theta}_1 L_1 \sin\theta_1 + L_1 \cos\theta \, \dot{\theta}_1^2)\hat{j}$

acceleration of $m_2 = \ddot{\vec{r}}_2 = (\ddot{\theta}_1 L_1 \cos\theta_1 - \dot{\theta}_1^2 L_1 \sin\theta_1 + \ddot{\theta}_2 L_2 \cos\theta_2 - \dot{\theta}_2^2 L_2 \sin\theta_2)\hat{i}$
$- (\ddot{\theta}_1 L_1 \sin\theta_1 + \dot{\theta}_1^2 L_1 \cos\theta_1 + \ddot{\theta}_2 L_2 \sin\theta_2 + \dot{\theta}_2^2 L_2 \cos\theta_2)\hat{j}$

FBD of mass m_2



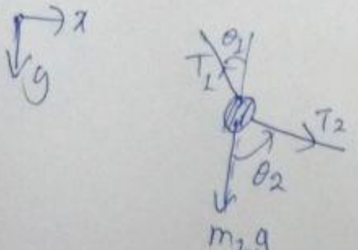
$\Sigma F_x = m_2 \ddot{r}_{2x}$

$\Rightarrow -T_2 \sin\theta_2 = m_2(\ddot{\theta}_1 L_1 \cos\theta_1 - \dot{\theta}_1^2 L_1 \sin\theta_1 + \ddot{\theta}_2 L_2 \cos\theta_2 - \dot{\theta}_2^2 L_2 \sin\theta_2)$ $\rightarrow (1)$

$\Sigma F_y = m_2 \ddot{r}_{2y}$

$\Rightarrow m_2 g - T_2 \cos\theta_2 = -m_2(\ddot{\theta}_1 L_1 \sin\theta_2 + \dot{\theta}_1^2 L_1 \cos\theta_1 + \ddot{\theta}_2 L_2 \sin\theta_2 + \dot{\theta}_2^2 L_2 \cos\theta_2)$ $\rightarrow (2)$

FBD of mass m_2



$\sum F_x = m_2 \ddot{r}_{2x}$

$\Rightarrow T_2 \sin\theta_2 - T_1 \sin\theta_1 = m_2 L_1(\ddot{\theta}_1 \cos\theta_1 - \dot{\theta}_1^2 \sin\theta_1)$ → (3)

$\sum F_y = m_2 \ddot{r}_{2y}$

$\Rightarrow m_2 g + T_2 \cos\theta_2 - T_1 \cos\theta_1 = -m_2 L_1(\ddot{\theta}_1 \sin\theta_1 + \dot{\theta}_1^2 \cos\theta_1)$ → (4)

on solving eqⁿ (1) to (4), Eqⁿ of motion are

$l_1 \ddot{\theta}_1 = (T_2/m_1) \sin(\theta_2 - \theta_1) - g \sin\theta_1$ → (7)

$l_1 \dot{\theta}_1^2 = (T_1/m_1) - (T_2/m_1)\cos(\theta_2 - \theta_1) - g\cos\theta_1$ → (8)

$l_2 \ddot{\theta}_2 = -(T_1/m_1) \sin(\theta_2 - \theta_1)$ → (9)

$l_2 \dot{\theta}_2^2 = (T_2/m_2) + (T_2/m_1) - (T_1/m_1)\cos(\theta_2 - \theta_1)$ → (10)

⇒ From (7) & (9)

$T_2 = m_1 \left(\dfrac{l_1 \ddot{\theta}_1 + g \sin\theta_1}{\sin(\theta_2 - \theta_1)} \right)$ → (11)

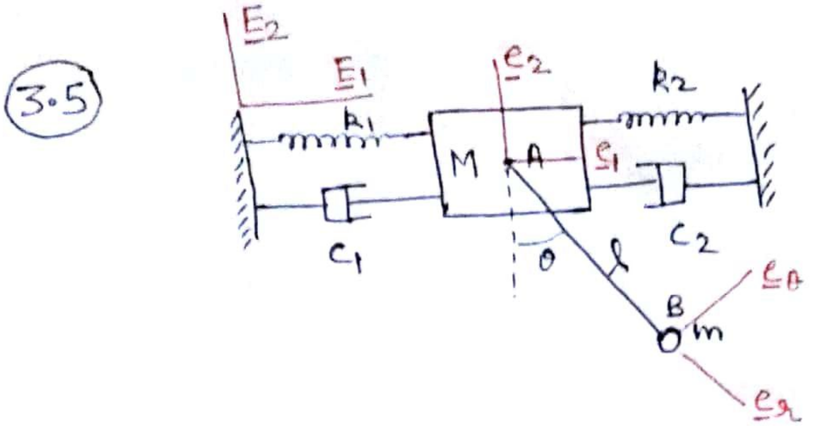
$T_1 = -m_1 \left(\dfrac{l_2 \ddot{\theta}_2}{\sin(\theta_2 - \theta_1)} \right)$ → (12)

Substituting eqⁿ (11) & (12) in eqⁿ (8) & eqⁿ (10)

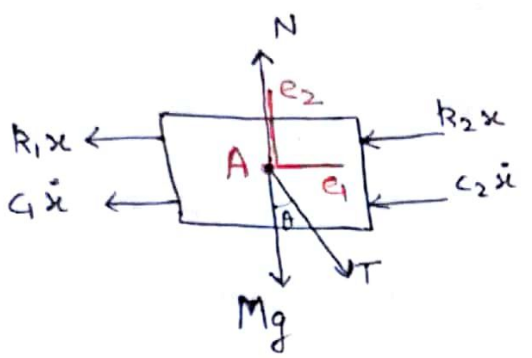
$-l_1 \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) = l_2 \ddot{\theta}_2 + l_1 \ddot{\theta}_1 \cos(\theta_2 - \theta_1) + g \sin\theta_2$

$(m_1 + m_2) l_1 \ddot{\theta}_1 + m_2 l_2 \ddot{\theta}_2 \cos(\theta_2 - \theta_1) = m_2 l_2 \dot{\theta}_2^2 \sin(\theta_2 - \theta_1) - (m_1 + m_2) g \sin\theta_1$

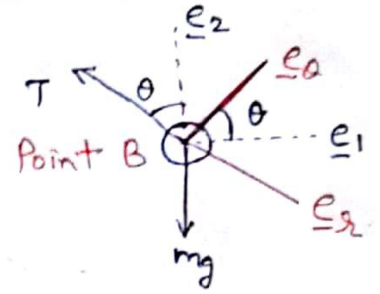
(3.5)



FBD of Mass 'M'



FBD of Mass 'm'



$$\underline{a}_A = \ddot{x}\,\underline{E}_1 = \ddot{x}\,\underline{e}_1$$

$$\underline{a}_A = \underline{a}_B + \underline{a}_{A|B} \quad \text{or} \quad \underline{a}_B = \underline{a}_A + \underline{a}_{B|A}$$

$$\underline{a}_{B|A} = l\ddot{\theta}\,\underline{e}_\theta - l\dot{\theta}^2\,\underline{e}_r$$

$$\underline{e}_\theta = \cos\theta\,\underline{e}_1 + \sin\theta\,\underline{e}_2$$

$$\underline{e}_r = \sin\theta\,\underline{e}_1 - \cos\theta\,\underline{e}_2$$

$$\therefore\ \underline{a}_B = (\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta)\underline{e}_1 + (l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta)\underline{e}_2$$

EOM for Mass 'M'

$$T\sin\theta - (k_1+k_2)x - (c_1+c_2)\dot{x} = M\ddot{x} \quad \text{--- (1)}$$

$$N - Mg - T\cos\theta = 0 \quad \text{--- (2)}$$

EOM for Mass 'm'

$$T\cos\theta - mg = m(l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta) \quad \text{--- (3)}$$

$$-T\sin\theta = m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) \quad \text{--- (4)}$$

Eliminate $T\sin\theta$ from eq^n (1) & (4)

$$-m(\ddot{x} + l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) = M\ddot{x} + (c_1+c_2)\dot{x} + (k_1+k_2)x$$

$$(M+m)\ddot{x} + (c_1+c_2)\dot{x} + (k_1+k_2)x + m(l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta) = 0$$

--- (5)

Eliminating T from eqⁿ (3) & (4), we get

$$-m\frac{(\ddot{x} + l\ddot{\theta}\cos\theta + l\dot{\theta}^2\sin\theta)\cos\theta - mg}{\sin\theta} = m(l\ddot{\theta}\sin\theta - l\dot{\theta}^2\cos\theta)$$

$$-\cos\theta\,\ddot{x} - l\ddot{\theta}\cos^2\theta \mp \cancel{l\dot{\theta}^2\cos\theta\sin\theta} - g\sin\theta = l\ddot{\theta}\sin^2\theta - \cancel{l\dot{\theta}^2\cos\theta\sin\theta}$$

$$\ddot{x}\cos\theta + g\sin\theta + l\ddot{\theta} = 0 \quad -(6)$$

For small oscillations, $\theta \approx$ small

$\therefore \cos\theta \approx 1$, $\sin\theta \approx \theta$

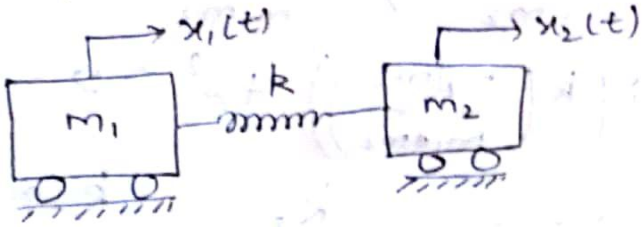
Eqⁿ (5) & (6) become

$$(M+m)\ddot{x} + (c_1+c_2)\dot{x} + (k_1+k_2)x + m(l\ddot{\theta} - l\dot{\theta}^2\theta) = 0 \quad -(7)$$

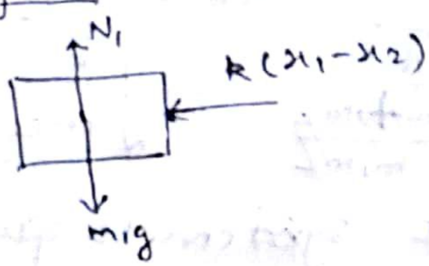
$$\ddot{x} + l\ddot{\theta} + g\theta = 0 \quad -(8)$$

Two coupled linear eqⁿs. (Differential eqⁿs)

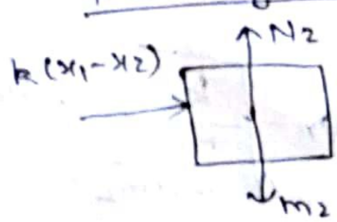
(36)



FBD of m_1



FBD of m_2



EOM for m_1

$$-k(x_1-x_2) = m_1\ddot{x}_1 \quad -(1)$$

EOM for m_2

$$k(x_1-x_2) = m_2\ddot{x}_2 \quad -(2)$$

$$m_1\ddot{x}_1 + k(x_1-x_2) = 0$$

$$m_2\ddot{x}_2 + k(x_2-x_1) = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}^{-1}}_{[M]}\underbrace{\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}}_{[K]}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$[D] = [M]^{-1}[K]$$

$$[D] = k\begin{bmatrix} \frac{1}{m_1} & 0 \\ 0 & \frac{1}{m_2} \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{k}{m_1} & -\frac{k}{m_1} \\ -\frac{k}{m_2} & \frac{k}{m_2} \end{bmatrix}$$

Finding eigen values of $[D]$ matrix

$$\begin{vmatrix} \frac{k}{m_1}-\lambda & -\frac{k}{m_1} \\ -\frac{k}{m_2} & \frac{k}{m_2}-\lambda \end{vmatrix} = 0$$

$$\left(\frac{k}{m_1}-\lambda\right)\left(\frac{k}{m_2}-\lambda\right) - \frac{k^2}{m_1 m_2} = 0$$

$$-\lambda\left(k\left(\frac{m_1+m_2}{m_1 m_2}\right)\right) + \lambda^2 = 0$$

$$\lambda = 0, \quad \cancel{\frac{k}{m_1 m_2}} \quad \frac{k(m_1+m_2)}{m_1 m_2}$$

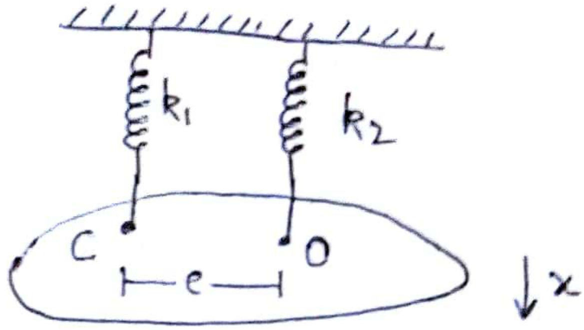
$$\therefore \omega_n = \sqrt{\frac{k}{\frac{m_1 m_2}{m_1+m_2}}} \quad \cancel{\therefore \omega_n = \sqrt{\frac{k}{\frac{m_1+m_2}{m_1 m_2}}}} = \sqrt{\frac{k}{m_{eq}}}$$

$\therefore$ This 2-DOF system is equivalent to this SDOF system.

where $m_{eq} = \dfrac{m_1 m_2}{m_1+m_2}$

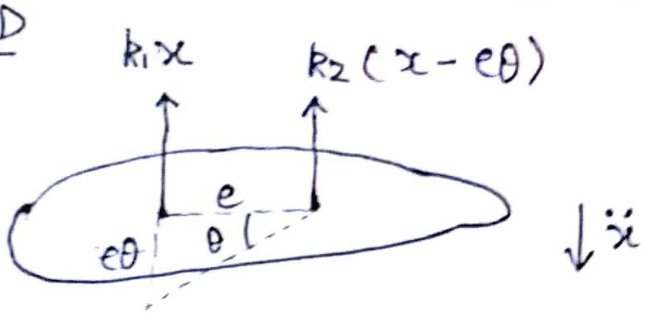
$$\ddot{x} + \frac{k}{m_{eq}}x = 0 \Rightarrow \boxed{\ddot{x} + \omega_n^2 x = 0}$$

(3.7)



Moment of Inertia about mass centre 'c' $= I_c$

FBD



EOM for mass 'm'

$$-k_1 x - k_2(x - e\theta) = m\ddot{x}$$

$$m\ddot{x} + k_1 x + k_2(x - e\theta) = 0$$

$$m\ddot{x} + (k_1 + k_2)x - k_2 e\theta = 0 \quad \text{—(1)}$$

Torque balance about C

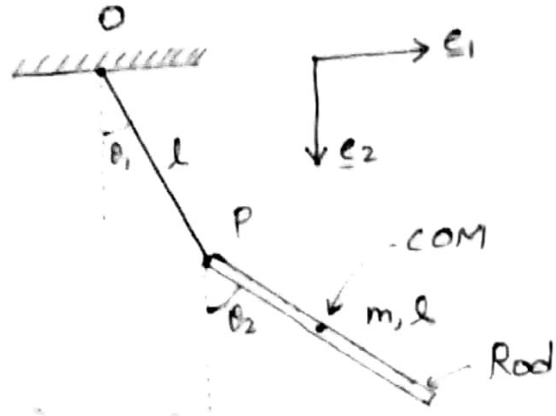
$$I_c\ddot{\theta} = k_2(x - e\theta)e$$

$$I_c\ddot{\theta} - k_2 e x + k_2 e^2\theta = 0 \quad \text{—(2)}$$

Matrix form

$$\begin{bmatrix} m & 0 \\ 0 & I_c \end{bmatrix}\begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 e \\ -k_2 e & k_2 e^2 \end{bmatrix}\begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

3.8

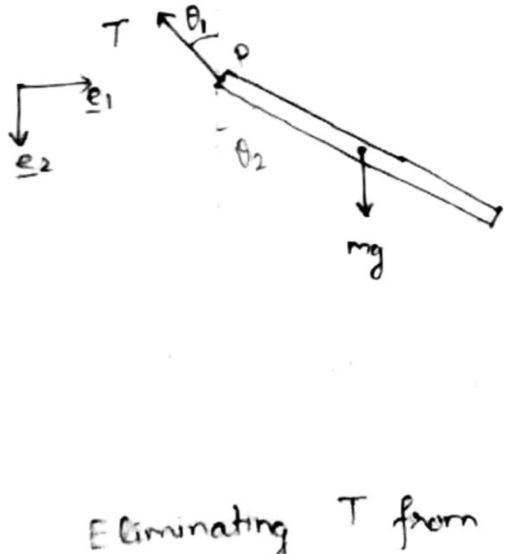


$$r_{CM/O} = l\left[\sin\theta_1 + \frac{\sin\theta_2}{2}\right]e_1 + l\left[\cos\theta_1 + \frac{\cos\theta_2}{2}\right]e_2$$

$$v_{CM/O} = \dot{r}_{CM/O} = l\left[\dot\theta_1\cos\theta_1 + \frac{\dot\theta_2\cos\theta_2}{2}\right]e_1 - l\left[\dot\theta_1\sin\theta_1 + \frac{\dot\theta_2\sin\theta_2}{2}\right]e_2$$

$$a_{CM/O} = \ddot{r}_{CM/O} = l\left[\ddot\theta_1\cos\theta_1 - \dot\theta_1^2\sin\theta_1 + \frac{\ddot\theta_2\cos\theta_2 - \dot\theta_2^2\sin\theta_2}{2}\right]e_1$$
$$- l\left[\ddot\theta_1\sin\theta_1 + \dot\theta_1^2\cos\theta_1 + \frac{\ddot\theta_2\sin\theta_2 + \dot\theta_2^2\cos\theta_2}{2}\right]e_2$$

FBD of rod



$$\Sigma F_x = m\,a_{CM_x} \quad — (1)$$

$$\Rightarrow -T\sin\theta_1 = ml\left[\ddot\theta_1\cos\theta_1 - \dot\theta_1^2\sin\theta_1 + \frac{\ddot\theta_2\cos\theta_2 - \dot\theta_2^2\sin\theta_2}{2}\right]$$

$$\Sigma F_y = m\,a_{CM_y} \quad — (2)$$

$$\Rightarrow mg - T\cos\theta_1 = -ml\left[\ddot\theta_1\sin\theta_1 + \dot\theta_1^2\cos\theta_1 + \frac{\ddot\theta_2\sin\theta_2 + \dot\theta_2^2\cos\theta_2}{2}\right]$$

Eliminating T from eq (1), eq (2) we get I eq^n of motion.
2 DOF system, 2 independent variables θ_1, θ_2
Three unknowns at this stage θ_1, θ_2, T.
So, Applying Torque / moment balance equation about P

$$\Sigma T_{/P} = r_{CM/P} \times m a_{cm} + \underline{\underline{I}}_{cm}\cdot\alpha + \omega \times \underline{\underline{I}}_{cm}\cdot\omega \quad — (3)$$

$$\Rightarrow r_{CM/P} \times mg(e_2) = r_{CM/P} \times m a_{cm} + \underline{\underline{I}}_{cm}\cdot\alpha + \underbrace{\omega \times \underline{\underline{I}}_{cm}\cdot\omega}_{=0}$$

Reason: Direction of $\underline{\underline{I}}_{cm}\cdot\omega$ is same as ω direction i.e. $\hat{k}$ dir.

$$I_{CM} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}, \quad \underline{\alpha} = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_2 \end{bmatrix}, \quad \underline{\omega} = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\underline{r}_{CM/P} = \begin{bmatrix} \frac{l}{2}\sin\theta_2 \\ \frac{l}{2}\cos\theta_2 \\ 0 \end{bmatrix}$$ Also, $I_{CM}\cdot\underline{\alpha} = I_{zz}\ddot{\theta}_2 = \frac{ml^2}{12}\ddot{\theta}_2\hat{k}$

$$\underline{r}_{CM/P} \times \underline{a}_{CM} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{l}{2}\sin\theta_2 & \frac{l}{2}\cos\theta_2 & 0 \\ a_{cmx} & a_{cmy} & 0 \end{vmatrix}$$

$$= \hat{k}\left(\frac{l}{2}\left\{\sin\theta_2\, a_{cmy} - \cos\theta_2\, a_{cmx}\right\}\right)$$

$$\frac{l}{2}\hat{k}\left[-\sin\theta_2\left\{\ddot{\theta}_1\sin\theta_1 + \dot{\theta}_1^2\cos\theta_1 + \frac{\ddot{\theta}_2\sin\theta_2 + \dot{\theta}_2^2\cos\theta_2}{2}\right\} - \cos\theta_2\left\{\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1 + \frac{\ddot{\theta}_2\cos\theta_2 - \dot{\theta}_2^2\sin\theta_2}{2}\right\}\right]$$

Substituting this expression in eq (3) we get our IInd eq^n of differential motion.

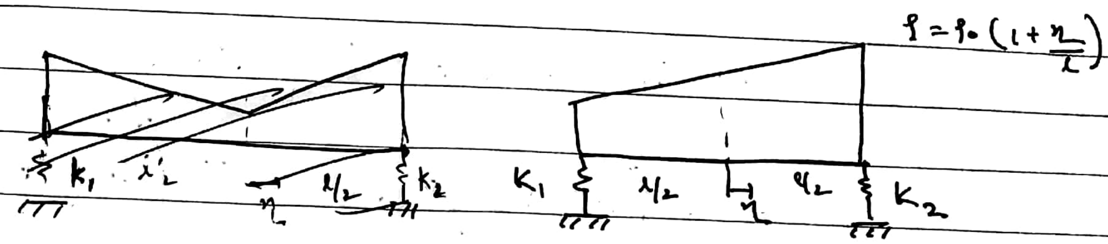
$$\Rightarrow \underline{r}_{cm/P} \times g(\underline{e}_2) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{l}{2}\sin\theta_2 & \frac{l}{2}\cos\theta_2 & 0 \\ 0 & g & 0 \end{vmatrix} = \frac{gl}{2}\sin\frac{\theta}{2}\hat{k}$$

$$-\frac{ml^2}{2}\left[\sin\theta_2\left\{\ddot{\theta}_1\sin\theta_1 + \dot{\theta}_1^2\cos\theta_1 + \frac{\ddot{\theta}_2\sin\theta_2 + \dot{\theta}_2^2\cos\theta_2}{2}\right\} + \cos\theta_2\left\{\ddot{\theta}_1\cos\theta_1 - \dot{\theta}_1^2\sin\theta_1 + \frac{\ddot{\theta}_2\cos\theta_2 - \dot{\theta}_2^2\sin\theta_2}{2}\right\} + \frac{ml^2}{12}\ddot{\theta}_2\right] = mg\frac{l}{2}\sin\frac{\theta}{2} \quad \text{— (4)}$$

This is the required II equation of motion

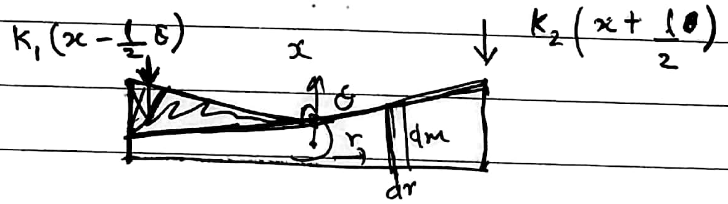
I eq^n of motion we get by eliminating T from eq (1), eq (2).

3.9



$$\rho(\eta) = \rho_0 \left(1 + \frac{\eta}{L}\right)$$

The two degrees of freedom are translation (x) as well as rotation (θ) of the center.



{Taking everything from eqⁿ position hence ignoring g}

Force balance :

$$-K_1\left(x - \frac{l}{2}\theta\right) - K_2\left(x + \frac{l}{2}\theta\right) = \int (\ddot{x} + r\ddot{\theta})\, dm$$

$$= \ddot{x}\int dm + \ddot{\theta}\int r\, dm$$

$$= \ddot{x}\int \rho\, dr + \ddot{\theta}\int r\rho\, dr$$

$$= \ddot{x}\int_{-l/2}^{l/2} \rho_0\left(1 + \frac{r}{L}\right) dr + \ddot{\theta}\int_{-l/2}^{l/2} r\rho_0\left(1 + \frac{r}{l}\right) dr$$

$$= \ddot{x}\left(\rho_0\, r^2\right)\Big|_{-l/2}^{l/2} + \ddot{\theta}\rho_0\left(\frac{r^3}{3l}\right)\Big|_{-l/2}^{l/2}$$

classmate

Date

Page

$$\boxed{-k_1\left(x-\frac{\theta l}{2}\right)-k_2\left(x+\frac{\theta l}{2}\right)=\ddot{x}\rho_0 l+\ddot{\theta}\frac{\rho_0 l^2}{12}} \quad — (1)$$

Moment balance

$$k_1\left(x-\frac{l\theta}{2}\right)\frac{l}{2}-k_2\left(x+\frac{l\theta}{2}\right)\frac{l}{2}=\int(\ddot{x}+r\ddot{\theta})\,r\,dm$$

$$=\ddot{x}\int r\,dm+\ddot{\theta}\int r^2 dm$$

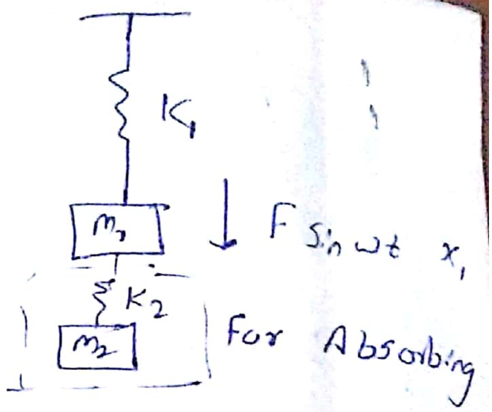
$$=\ddot{x}\int_{-l/2}^{l/2}\rho_0\left(1+\frac{r}{L}\right)r\,dr+\ddot{\theta}\int_{-l/2}^{l/2}\rho_0\left(1+\frac{r}{L}\right)r^2 dr$$

$$=\ddot{x}\rho_0\left(\frac{r^3}{3L}\right)\Big|_{-l/2}^{l/2}+\ddot{\theta}\rho_0\left(\frac{r^3}{3}\right)\Big|_{-l/2}^{l/2}$$

$$=\ddot{x}\rho_0\frac{l^2}{12}+\ddot{\theta}\rho_0\frac{l^3}{12}$$

Scanned by CamScanner

Q3.29



Piece of Machinery → 4800 lb $(2.1352\times10^4 N)$

deflection (static) = 1.2 in $(3.05\times10^{-2} m)$

- Initial stiffness:

$$F = k_1 x_1$$

$$k_1 = \frac{2.1352\times10^4}{3.05\times10^{-2}} = 700\ kN/m$$

$$\omega_n = \sqrt{\frac{K_1}{m_1}} \Rightarrow \sqrt{\frac{700\times10^3}{2.1352\times10^4}\times g} \Rightarrow 17.93\ rad$$

- Force applied.

$$F_1 = 444.8 \sin\omega_n t \quad \text{[Resonance, so } \omega = \omega_n]$$
$$= F_0 \sin\omega_n t$$

- Now Applying absorber

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_1 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} F_0 \sin\omega_n t \\ 0 \end{bmatrix}$$

Assume soln $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \sin\omega_n t$. $(\omega_n = \omega)$

Problem changes to

$$\begin{pmatrix} k_1 + k_2 - m_1\omega^2 & -k_2 \\ -k_2 & k_2 - m_2\omega^2 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$

Using Cramer's rule, we may write

$$X_1 = \frac{(K_2 - m_2 \omega^2) F}{|Z(\omega)|} \quad (1)$$

$$X_2 = \frac{-K_2 F}{|Z(\omega)|} \quad (2)$$

Where

$$|Z(\omega)| = \begin{vmatrix} K_1 + K_2 - m_1\omega^2 & -K_2 \\ -K_2 & K_2 - m_2\omega^2 \end{vmatrix}$$

$$\Rightarrow K_1K_2 - m_1K_2\omega^2 - K_1m_2\omega^2 - K_2m_2\omega^2 + m_1m_2\omega^4$$

- Now, it's clear from eq (1) that $X_1 = 0$ when

$$\omega = \sqrt{\frac{K_2}{m_2}} = \omega_n. \quad \text{(Declared before)}$$

So, for new system

$$\sqrt{\frac{K_2}{m_2}} = 17.93 \text{ rad/s}. \quad -(3)$$

if we simplify eq (2) by using $\omega = \sqrt{\frac{K_2}{m_2}}$, the same simplifies to

$$X_2 = \frac{-K_2 F}{-K_2^2} = \frac{F}{K_2}$$

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For our case

$X_2 = 0.1\ in \Rightarrow 2.54 \times 10^{-3}\ m$

So, $k_2 = \frac{448.8}{2.54 \times 10^{-3}} \Rightarrow 175.1 kN/m$

$\omega_n = \sqrt{\frac{k_2}{m_2}} \Rightarrow m_2 = \frac{k_2}{\omega_n^2} \Rightarrow 544.7 kg$.

$\rightarrow \quad \mu = \frac{m_2}{m_1} \Rightarrow 0.2502.$