Assume the utility function $U(x_1, x_2) = x_1^{0.1} x_2^{0.9}$ *for a consumer where* x_1 *is gasoline and* x_2 *is a numeraire good (all other goods clubbed together and it has unit price). He has exogenous income* I = $2000. *Due to disturbances in the supplying countries, the price of gasoline goes up from $2 to $4 per gallon.*

1. *Derive the optimal consumption of gasoline as a function of* p_1 *(the price of gasoline) and* p_2 *(the price of other goods). Then, get the exact optimal consumption bundle.*

Max $x_1^{0.1}x_2^{0.9}$ subject to $p_1x_1 + p_2x_2 = 2000$ → Set up Lagrange → From F.O.C. derive $x_1 = 200/p_1$ and $x_2 = 1800/p_2$.

Prior to the price increase, $p1 = 2$. Thus $x1 = 200/2 = 100$. We can similarly calculate that $x2 = 1800/1 = 1800$ (since $p2 = 1$). The optimal consumption bundle is therefore $A = (100,1800)$. After the price increase, the person consumes 200/4=50 gallons of gasoline.

2. *Calculate the Hicks substitution effect from this price change. Assume* $p_2 = \$1$.

To calculate the substitution effect, we first have to know how much utility the consumer gets before the price increase. We already calculated that $x1 = 100$, and $x2 = 1800$. The original bundle is $A = (100,1800)$ — which gives utility $u_A = u(100,1800) = 100^{0.1}1800^{0.9} \approx 1348$.

Then we ask what the least is that we could spend and reach this utility level again after the price increase; i.e. we solve *Min* $4x_1 + x_2$ *subject to* $x_1^{0.1} x_2^{0.9} = 1348$. There are different ways to get the solution. You may express one variable in terms of the other (using the constraint) and express the objective function in terms of a single variable only and then proceed to get F.O.C. You may set up Lagrange as well. Solution: $x_1 = 53.59$ and $x_2 = 1929.19$.

Change due to substitution effect ('pure' price effect) = 53.59 – 100 = - 46.41 gallons
Change due to total price effect = 50 – 100 = - 50 gallons
Change due to income effect → - 46.41 + INC = -50 → - 3.59 gallons
Compensating variation (CV) = (4*53.59+1929.19) – 2000 = $143.55