Introduction to Convective Transport

by

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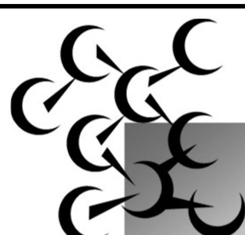

Molecular transport

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Transport phenomenon – holistic view

Net Species Transport = $\sum$ (Molecular Transport + Convective Transport)

- **Momentum Transport** = Molecular Momentum Transport + Convective Momentum Transport
- **Energy Transport** = Molecular Energy Transport + Convective Energy Transport
- **Mass Transport** = Molecular Mass Transport + Convective Mass Transport

Diffusion terms **Advection terms**

Rate equations

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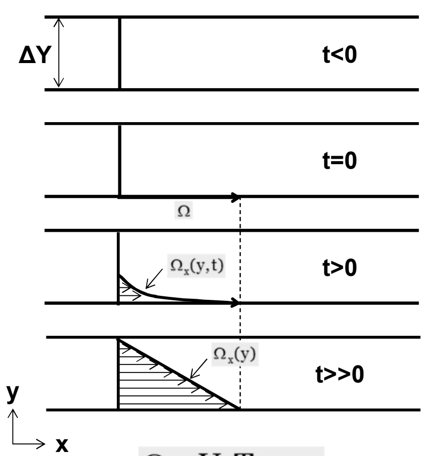
Molecular transport of species



$\Omega \sim V, T$ or ω

Momentum ►► **Newton' Law of viscosity**

Energy ►► **Fourier's Law of conductivity**

Mass ►► **Fick's Law of diffusion**

Basic forms of equation
(One dimensional/incompressible)

$$\frac{F}{A} = \tau = -(\mu)\frac{\Delta V}{\Delta Y}$$

$$\frac{Q}{A} = q = -(k)\frac{\Delta T}{\Delta Y}$$

$$\frac{N_{Ay}}{A} = j_{Ay} = -(\rho D_{AB})\frac{\Delta \omega_{Ay}}{\Delta Y}$$

Molecular transport of species

Momentum diffusivity = μ / ρ also called ν

Thermal diffusivity = $k / \rho C_p$ also called α

Mass diffusivity = D_{AB}

!! ALL HAVE THE SAME DIMENSIONS !!

(length)^2 / (time)

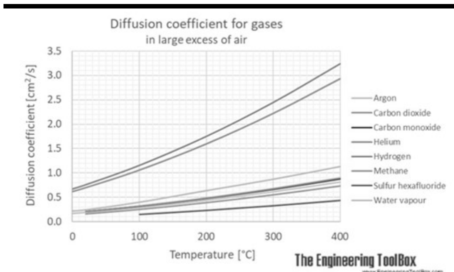
Taking any two at a time to form non-dimensional numbers

The Prandtl Number = $\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu C_p}{k}$

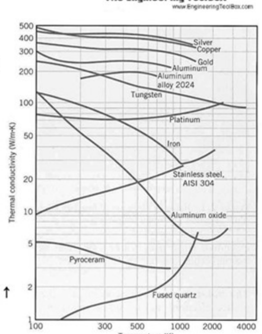
The Schmidt Number = $\mathrm{Sc} = \dfrac{\nu}{D_{AB}} = \dfrac{\mu}{\rho D_{AB}}$

The Lewis Number = $\mathrm{Le} = \dfrac{\alpha}{D_{AB}} = \dfrac{k}{\rho C_p D_{AB}}$

Variation with temperature



Temperature dependence of thermal conductivity of soilds



Solids

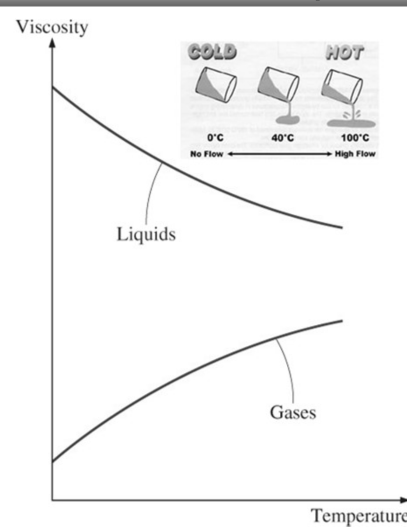
$k = k_e + k_l$

Pure metal: $k_e >> k_l$

Alloys: $k_e \sim k_l$

Non-metal : $k_l > k_e$

In general, $k \downarrow$ as $T \uparrow$



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Microscopic transport (Convection)

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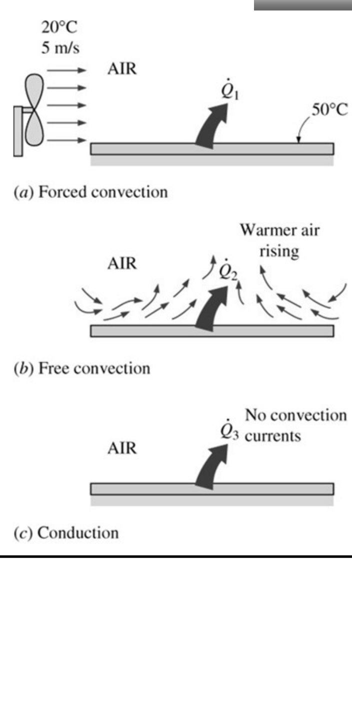
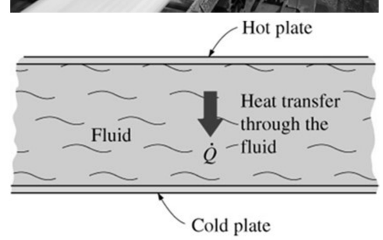
The convection problem





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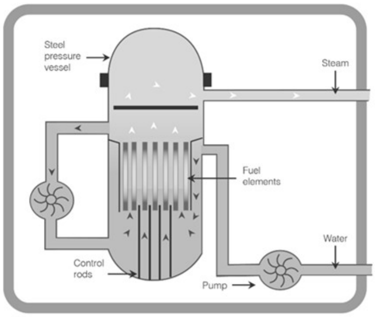
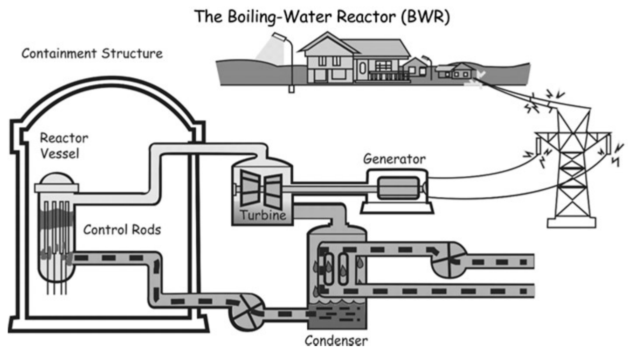
The nuclear power plant (BWR)





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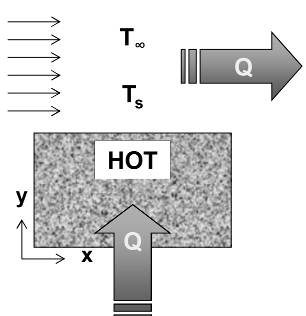
The Convection problem for interface transport

Typically we encounter solid-fluid boundaries

Net transport is linearly dependent on difference of driving potential at the interface



Newton's law of cooling

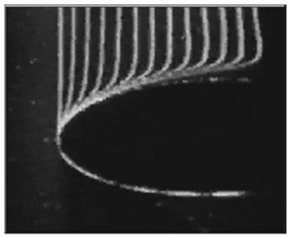
$$q'' = \frac{Q}{A_{cs}} = h \cdot \left(T_s - T_\infty\right) = -k_f \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

Similarly for mass transfer

$$N''_A = \frac{N}{A_{cs}} = h_m \cdot \left(C_{A,s} - C_{A,\infty}\right) = -D_{AB} \left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

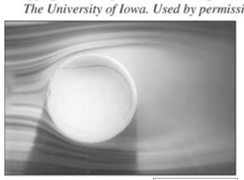
Heat/Mass transfer coefficients are 'fudge' factors

The velocity boundary layer (external flow)



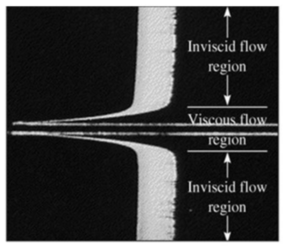
The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.

"Hunter Rouse: Laminar and Turbulent Flow Film." Copyright IIHR-Hydroscience & Engineering, The University of Iowa. Used by permission.



External flow over a tennis ball, and the turbulent wake region behind.

Courtesy NASA and Cislunar Aerospace, Inc.



The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).

Fundamentals of Boundary Layers, National Committee from Fluid Mechanics Films, © Education Development Center.

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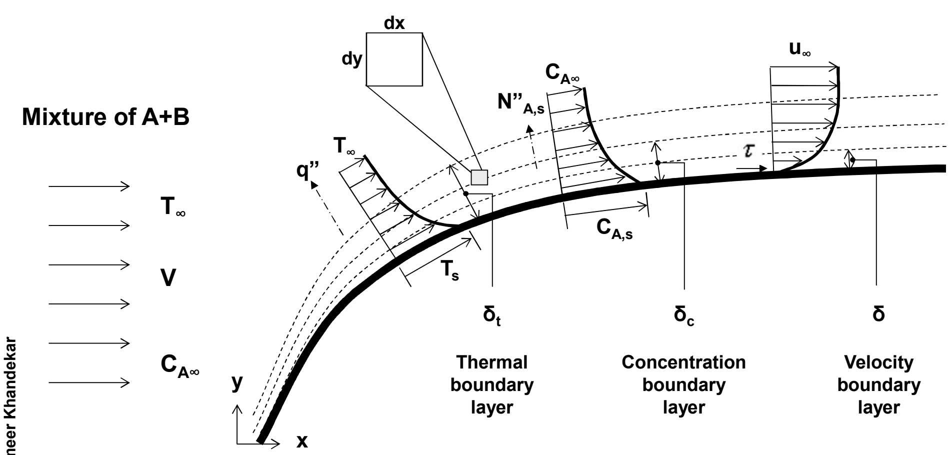
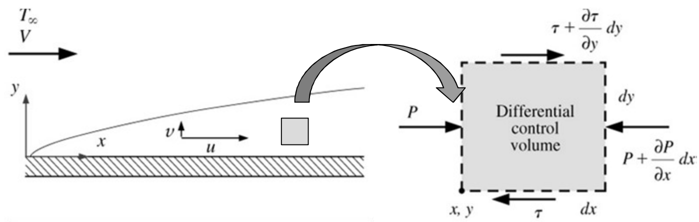
The concept of boundary layer



Boundary layer approximations



1) Velocity components:
$v \ll u$

2) Velocity grandients:
$\frac{\partial v}{\partial x} \ll 0, \frac{\partial v}{\partial y} \ll 0$
$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$

3) Temperature gradients:
$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

The governing equations for steady two dimensional incompressible fluid flow with negligible viscous dissipation:

continuity $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

x – momentum $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

y – momentum $$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

energy $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

Scaling Laws

Connecting the molecular level to microscopic level

$$\frac{\delta}{\delta_t} \approx \mathrm{Pr}^n \approx \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

If Pr~1 ►►δ~δ_t (gases)

If Pr>>1 ►► δ_t <<δ (Oils)

If Pr<<1 ►► δ_t >>δ (liquid metals)

Similarly

$$\frac{\delta}{\delta_c} \approx Sc^n \approx \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}} \qquad \frac{\delta}{\delta_t} \approx Le^n \approx \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}}$$

Implications of the boundary layers

Velocity boundary layer ► **velocity gradients** ►**shear stress**

Thermal boundary layer ►**temperature gradients** ►**heat flux**

Concentration boundary layer ►**concentration gradients** ►**molar flux**

For practical engineering design and applications the direct implications are:

Momentum transfer ► ► Friction Factor (non-dimensional shear stress)

Energy transfer ► ► Nusselt Number (heat transfer coefficient)

Mass Transfer ► ► Sherwood Number (mass transfer coefficient)

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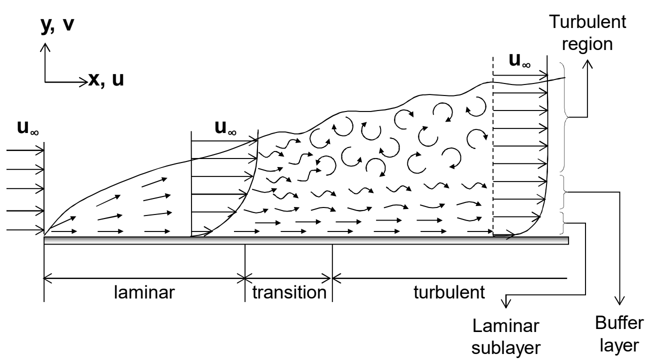
Microscopic transport parameters



Reynolds Number

Inertia force
Viscous force

$$\mathrm{Re} = \frac{(\rho \cdot V^2)}{(\mu \cdot V / L)} = \frac{\rho \cdot V \cdot L}{\mu}$$

Criterion for flow transition

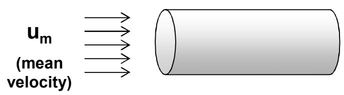
LAMINAR ►TURBULENT

Laminar region ►Highly ordered flow ►streamlines

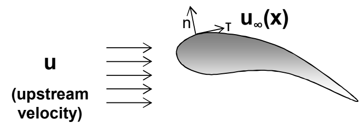
Turbulent region ► highly irregular, 3D ►enhanced species transport ► increased boundary layer thickness ►mixing►flatter velocity profile

Microscopic transport parameters

In engineering practice we usually encounter two types of flow



Internal flows:
flow confined in a conduit, pipe, duct, etc. of constant cross section



External flows:
Flow past a flat plate, submerged bodies aerofoils, bluff bodies, etc.

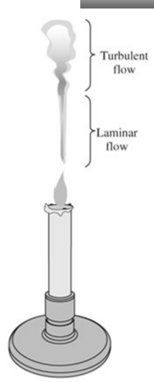
Force exerted by fluid on the object

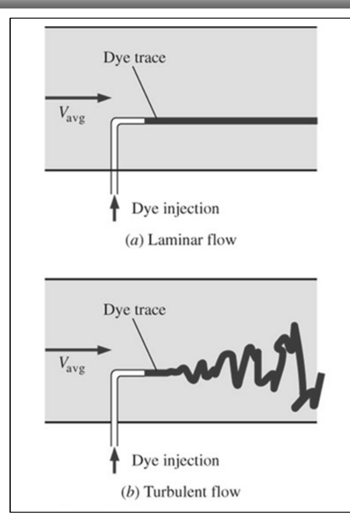
(a) Forces present even if the fluid is stationary (Buoyancy)

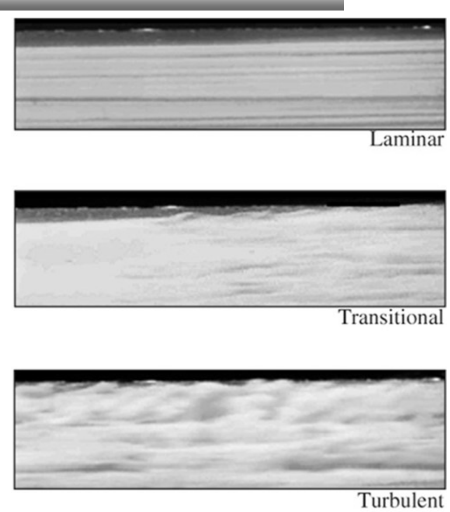
(b) Forces due to fluid motion (frictional drag and form drag)

The motion of fluid is due to kinetic energy and we are interested to know how much of this kinetic energy is manifested as pressure drop/shear stress

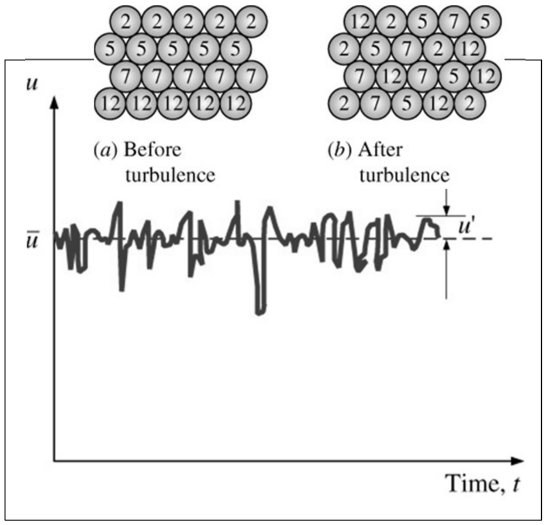
Laminar Transitional and turbulent flows

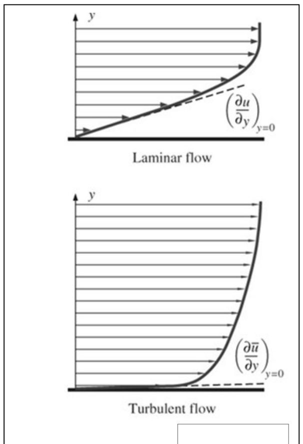






Turbulent flow characteristics

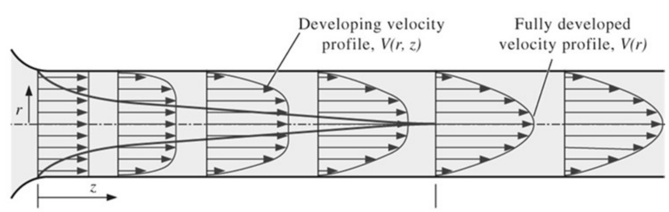


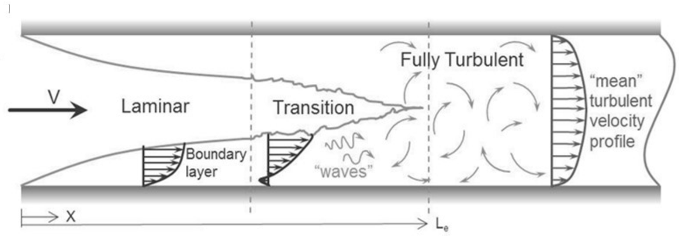


The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream velocity.

Development of BL in internal flows (pipe)

FIGURE 6–11
The development of the velocity profile in a circular pipe. $V = V(r, z)$ and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in the flow direction, $V = V(r)$.





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Microscopic transport parameters

Internal flows = Darcy friction factor $f = \dfrac{-(dp/dx)D}{\frac{1}{2}\rho \cdot u_m^2}$

External flows = Coefficient of friction $C_f = \dfrac{\tau_s}{\frac{1}{2}\rho \cdot u_\infty^2} = \dfrac{(\text{Drag force per unit area})}{\frac{1}{2}\rho \cdot u^2}$

(note the difference between basis velocities for kinetic energy scaling)

We can see that the functional form of these equations are in the form:

$$f \text{ or } C_f = \frac{2}{\text{Re}} \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0} \approx \text{F}^{\underline{\text{n}}}(\text{Re})$$

!! The significance of this result should not be overlooked !!

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Microscopic transport parameters

Now coming back to the definition of heat and mass transfer coefficients

$$h \cdot (T_s - T_\infty) = -k_f \cdot \left. \frac{\partial T}{\partial y} \right|_{y=0}$$

$$h_m \cdot (C_{A,s} - C_{A,\infty}) = -D_{AB} \left. \frac{\partial C_A}{\partial y} \right|_{y=0}$$

Again, it can be shown that the functional form of these equations are in the form:

$$h = \frac{k_f}{L} \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0} \qquad OR \qquad Nu = \frac{hL}{k_f} = \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0}$$

$$h_m = \frac{D_{AB}}{L} \left. \frac{\partial C_A^*}{\partial y^*} \right|_{y^*=0} \qquad OR \qquad Sh = \frac{h_m L}{D_{AB}} = \left. \frac{\partial C_{AB}^*}{\partial y^*} \right|_{y^*=0}$$

Microscopic transport parameters

FINAL PICTURE WHICH EMERGES

$$C_f \approx \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$Nu \approx \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$Sh \approx \left.\frac{\partial C^*_{AB}}{\partial y^*}\right|_{y^*=0}$$

The Nusselt Number is to the thermal boundary layer what the friction factor is to the velocity boundary layer

The Sherwood Number is to the concentration boundary layer what the Nusselt Number is to the thermal boundary layer

All three quantities manifest the non-dimensional gradients at the interface of the particular specie they respectively represent

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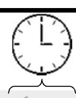
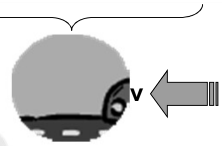
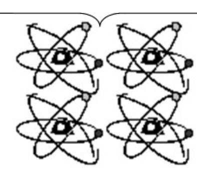


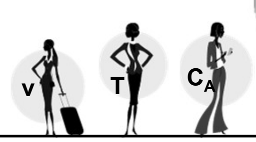
$$\rho\left(\frac{\partial v_i}{\partial t} + v_x\frac{\partial v_i}{\partial x} + v_y\frac{\partial v_i}{\partial y} + v_z\frac{\partial v_i}{\partial z}\right) = \mu\left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} + \frac{\partial^2 v_i}{\partial z^2}\right) - \frac{\partial p}{\partial x} \qquad \Big| i = x,y,z$$

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial C_A}{\partial t} + v_x\frac{\partial C_A}{\partial x} + v_y\frac{\partial C_A}{\partial y} + v_z\frac{\partial C_A}{\partial z}\right) = \rho D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2}\right)$$







Summary and Conclusions

- **A holistic view of transport phenomena was provided**
- **“Thermodynamics” and “Transport Phenomena” are complimentary**
- **Momentum, energy and mass transport are guided by very similar laws**
- **Conservation of species is applicable in all cases**
- **Information at all levels: molecular, microscopic and macroscopic is needed**
- **Understanding can be improved by a unified study of specie transport**
- **Thermal/fluid management and design of thermal/fluid systems is indeed vital for the success of critical technologies and systems**

End of Lecture