Tutorial 8 Solution

Part-a

$\Sigma F_y = 0$:

$$\boxed{R_A + R_B = 6.2\ \text{kN}}$$

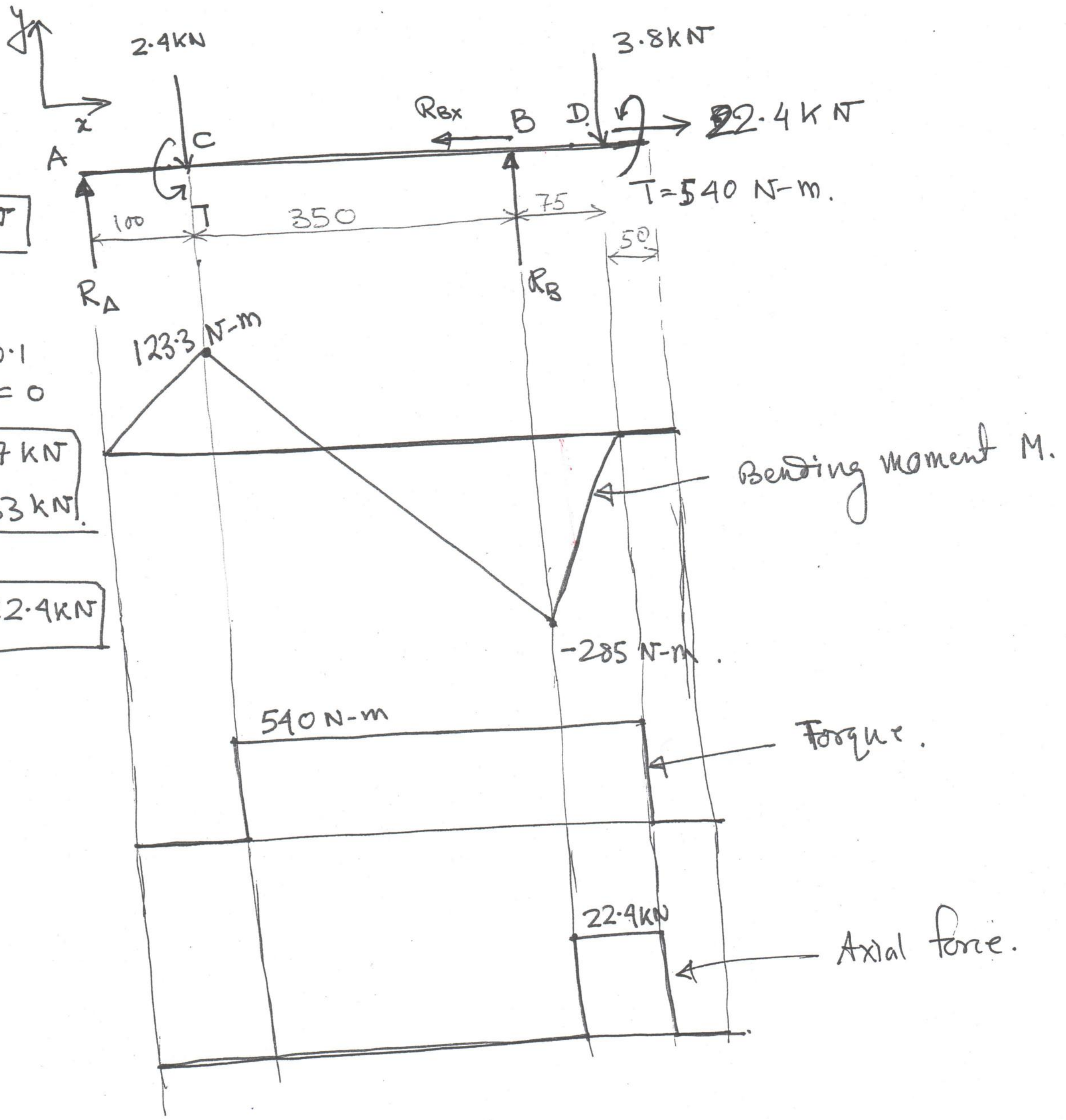
$\Sigma M_A = 0$:

$$+R_{Bx} \times 0.45 - 2.4 \times 0.1 - 3.8 \times 0.525 = 0$$

$$\Rightarrow \boxed{\begin{array}{l} R_B = 4.967\ \text{kN} \\ R_A = 1.233\ \text{kN} \end{array}}$$

$\Sigma F_x = 0$:

$$\boxed{R_{Bx} = 22.4\ \text{kN}}$$



Part-b

Most critical point: Just right of point B:

$$\boxed{\begin{array}{l} M = 285\ \text{N-m} \\ T = 540\ \text{N-m.} \\ F_{axial} = 22.4\ \text{kN} \end{array}}$$

$D/d = 1.2$ at B, $r/d = 0.05$ $\Rightarrow$

Static stress concentration factors.

$$\boxed{\begin{array}{lll} K_t = 2.0 & \text{(Axial)} \\ K_t = 1.95 & \text{(bending)} \\ K_{ts} = 1.6 & \text{(torsion)} \end{array}}$$

(A-15-7 — A-15-9)

Notch Sensitivity Factor: q depends on fillet radius which we do not know, yet.

We will take guess values of K_f and K_{fs} to be same as the static stress concentration factors.

From A-20: AISI 1020 CD Steel:

$$S_{ut} = 470\ \text{MPa}, \quad S_y = 390\ \text{MPa}.$$

(Eq 6-19) $\longrightarrow$ $$K_a = a\, S_{ut}^{\,b} = 4.51(470)^{-0.251}$$

$$= 0.883$$

$$K_b = 0.9 \quad (\text{guess}).$$

$$K_c = K_d = K_e = 1.$$

$$\Rightarrow \quad S_e = (0.883)(0.9)(1)(1)(1)(0.5)(470)$$

$$= \underline{186\ \text{MPa}}$$

Bending moment is completely reversing.
Axial load and Torque are constants.

$$\Rightarrow \quad M_a = 285\ \text{N-m}, \quad T_a = 0, \quad F_a = 0$$

$$M_m = 0, \quad T_m = 546\ \text{N-m}, \quad F_m = 22.4\ \text{kN}$$

Ignore axial load first:

Von-Mises Stresses. (eq 7-5, 7-6)

$$\sigma_a' = \left\{ \left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2 \right\}^{1/2}$$

$$\sigma_m' = \left\{ \left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2 \right\}^{1/2}.$$

$K_f = K_t = 1.95$?

K_f ~~0.62~~ 1.95

$K_{fs} = K_{ts}$

$K_{fs} = 1.6$

$$\Rightarrow \quad \sigma_a' = \frac{32 K_f M_a}{\pi d^3} = \frac{5661}{d^3}$$

$$\sigma_m' = \sqrt{3}\cdot\frac{16 K_{fs} T_m}{\pi d^3} = \frac{7621.57}{d^3}.$$

Modified Goodman criteria: $$\frac{1}{n_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{5661}{186\times10^6\, d^3} + \frac{7621.57}{470\times10^6\, d^3} \quad \Rightarrow \quad \boxed{d = 45.4\ \text{mm}}$$

Since we started in a conservative manner,
we can choose the standard size to be
Table A-17 $\boxed{d = 45\text{ mm}}$ instead of next value $d = 50\text{ mm}$

Now we do a through calculation with this starting dia
$d = 45\text{ mm}$. (at right of b).

For the fillet: $r = 0.05 \times 45\text{ mm} = 2.25\text{ mm}$.

$\Rightarrow q \simeq 0.78$, $q_{shear} \simeq 0.81$ (Fig 6-20, 6-21)

(Eq 6-32)
$$\Rightarrow K_{f\,axial} = 1 + q(K_{t\,axial} - 1) = 1 + 0.78(2-1) = 1.78$$
$$K_{f\,bending} = 1 + 0.78(1.95-1) = 1.741$$
$$K_{f\,shear} = 1 + 0.81(1.6-1) = 1.486.$$

$\Rightarrow$ k_a remains the same $\Rightarrow$ $k_a = 0.883$

(Eq 6-20) $\leftarrow$ $k_b = \left(\frac{d}{7.62}\right)^{-0.107} = \left(\frac{45}{7.62}\right)^{-0.107} = 0.827.$

$k_c = k_d = k_e = 1.$

$$\Rightarrow \boxed{S_e = 171.6\text{ MPa}}$$

$\sigma_a\big|_{bending} = \frac{32 K_f M_a}{\pi d^3} = 55.5\text{ MPa}$

$\sigma_a\big|_{axial} = \tau_a = 0$

$\sigma_m{}_{bending} = 0$, $\sigma_m\big|_{axial} = K_{fs} \cdot \frac{22.4 \times 10^3}{\pi d^2/4} = 25.07\text{ MPa}$

$\tau_m = K_{fs} \cdot \frac{16 T_m}{\pi d^3} = 44.9\text{ MPa}$

$\Rightarrow \boxed{\sigma_a' = 55.5\text{ MPa}}$ $\sigma_m' = \left(\left(\sigma_{m\,axial}\right)^2 + 3\tau_m^2\right)^{1/2} \simeq 83.17\text{ MPa}$ May be less. 81.2

Eq. 6-55, 6-56.

$\Rightarrow \boxed{n_f = \dfrac{1}{\sigma_a'/S_e + \sigma_m'/S_{ut}} = 2.}$ (okay).

Static yield: $\sigma'_{max} = \sigma_a' + \sigma_m' = 138.7$ MPa.

$\Rightarrow n_y = S_y / \sigma'_{max} = 2.812$ (okay).

$\Rightarrow$ $\boxed{d = 45 \text{ mm at the right of B is okay}}$

At the left of B: $\dfrac{D}{d} = 1.2 \Rightarrow \boxed{D = 54\text{mm}}$

Again this is not a standard size. Since major part of the shaft will be of diameter D. We should choose standard value for this

Let $\boxed{D = 60 \text{ mm}, \Rightarrow d = 50\text{mm}.}$ (Table A-17).

Next critical location is the spur gear, because of key way.

D = 60mm.

Load: $M_a = 1233$ N-m., $M_m = 0$ $\boxed{K_f = 5.0}$

$T_m = 540$ N-m, $T_a = 0$, $\boxed{K_{fs} = 3.0}$

No axial load.

$\Rightarrow \sigma_a' = K_f . \dfrac{32 M_a}{\pi D^3} = 29.07$ MPa

$\sigma_m' = \sqrt{3} . K_{fs} . \dfrac{T_m \times 16}{\pi D^3} = 66.16$ MPa.

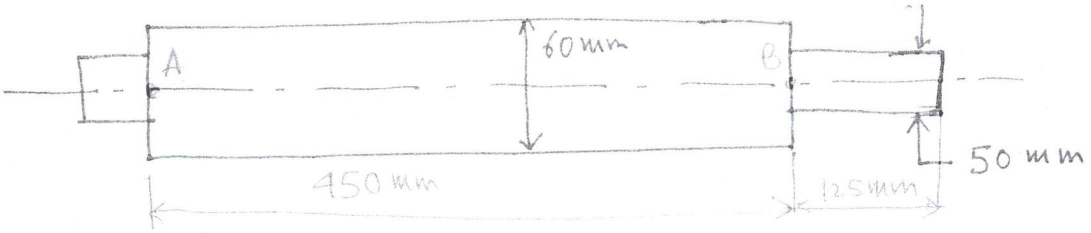
$\Rightarrow n_f = \left(29.07/171.6 + \dfrac{66.16}{470}\right)^{-1} = 3.22$ (okay)

$\boxed{\text{So, the sizes mentioned above are okay.}}$

Shaft size: Between bearings A and B: $\boxed{D = 60 \text{ mm.}}$

For bearing A and Bearing B and the remaining: $\boxed{d = 50 \text{ mm}}$

ME
637



Part-C : Design of rectangular key.

Material — AISI 1006 HR steel. $\Rightarrow$ $\boxed{S_y = 170\,MPa \quad S_{sy} = 0.577 S_y = 98.1\,MPa}$

Key for spur gear :

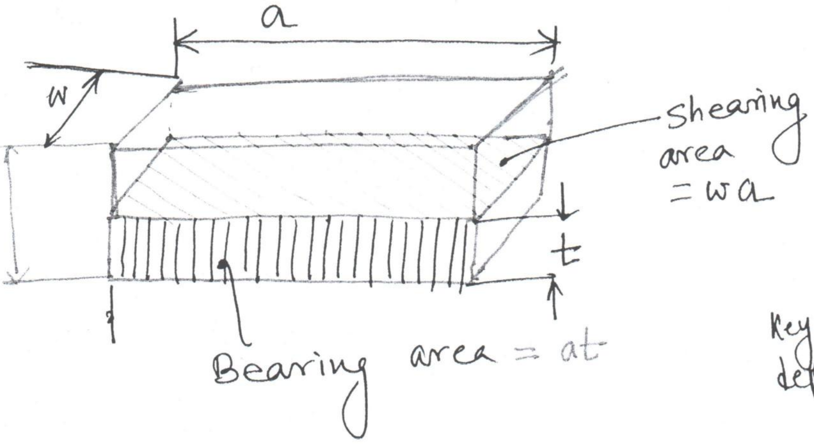
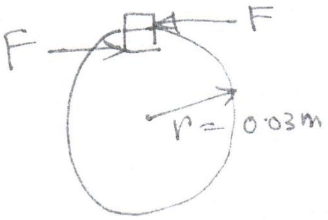


Table 7-6 : Key size for shaft (dia = 60 mm)

w = 16 mm
h = 12 mm
t = 5.5 mm

Key way depth.

Failure due to shearing : T = 540 N-m $\Rightarrow$ F = 540/0.03 = 18 kN



$$\frac{S_{sy}}{n} = \frac{F}{wa} \Rightarrow \frac{98.1\times10^6}{1.8} = \frac{18\times10^3}{16\times10^{-3}\times a}$$

$$\Rightarrow \boxed{a = 20.64\,mm}$$

Failure due to bearing: $\frac{S_y}{n} = \frac{F}{at} \Rightarrow \frac{170\times10^6}{1.8} = \frac{18\times10^3}{5.5\times a}$

$$\Rightarrow \boxed{a = 34.65\,mm}$$

$\Rightarrow$ Key length should be larger than 34.65 mm.

Key for worm : Table 7-6 : Key size (shaft dia = 50mm)

$\boxed{w = 12\,mm, \quad h = 10\,mm, \quad t = 5\,mm}$

Shearing : $a = \frac{20.64}{(50/60)} \times \frac{16}{12}\,mm = 27.52\,mm.$

Bearing: $a = 34.65 \times \frac{60}{50} \times \frac{5.5}{5}\,mm = 45.74\,mm.$

$\Rightarrow$ Key length $a \geq 45.74\,mm$ (for worm).

Since both keys should have $a \simeq 100\,mm$ (width of worm & gear hub) — both are safe. are safe.