Module 2 - GEARS

Lecture – 12 HELICAL GEARS-PROBLEMS

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12.1 HELICAL GEARS – PROBLEM 1

A 75 kW induction motor runs at 740 rpm in clock wise direction as shown in Fig.12.1. A 19 tooth helical pinion with 20^o normal pressure angle, 10 mm normal module and a helix angle of 23^o is keyed to the motor shaft. Draw a 3-dimensional sketch of the motor shaft and the pinion. Show the forces acting on the pinion and the bearing at A and B. The thrust should be taken out at A.

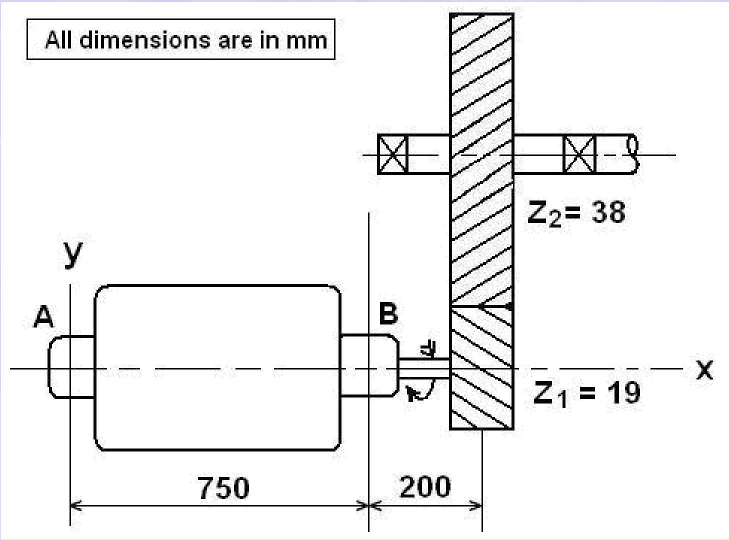

Fig.12.1 Helical gear layout diagram

<u>Data:</u> W=75kW, n_1=740rpm, Z_1 = 19, Z2 = 38, $Ø_n$=20^o, ψ = 23^o, m_n = 10 mm.

Question: Find reactions at A&B.

Solution: Transverse Pressure angle

$\tan \varnothing_n = \tan \varnothing \cos \psi$

$$\varphi = \tan^{-1}\left(\frac{\tan \varphi_n}{\cos \psi}\right)$$

$$= \tan^{-1}\left(\frac{\tan 20^o}{\cos 23^o}\right) = 21.57^o$$

$m = m_n / \cos \psi = 10 / \cos 23^o = 10.864$ mm

Pitch diameter of the pinion:

$d_1 = mZ_1 = 10.864 \times 19 = 206.4$ mm

Pitch line velocity:

$V = \pi d_1 n_1 /60 = \pi \times 206.4 \times 740 / 60000 = 8$ m/s

Tangential force on the pinion: F_t

$F_t = 1000W/V = 1000 \times 7\ 5 / 8 = 9375$ N

$F_r = F_t \tan \varnothing = 9375 \tan 21.57^o = 3706$ N

$F_a = F_t \tan \psi = 9375 \tan 23^o = 3980$ N

$F_n = F_t / \cos\varnothing_n \cos\psi = 9375 / \cos20^o \times \cos23^o = 10838$ N

3 forces, F_r in the –y direction, F_a in the x direction, and F_t in the +z direction are acting at the pitch point c of the pinion as shown in the sketch.

Bearing at A is made to take the Axial reaction $R_AX = 3980$ N

Taking moments about the z axis

$-F_r (950) + F_a (206.4/2) + R_B^y (750) = 0$, i.e.,

$-3706 \times 950 + 3980 \times 103.2 + R_B^y (750) = 0$

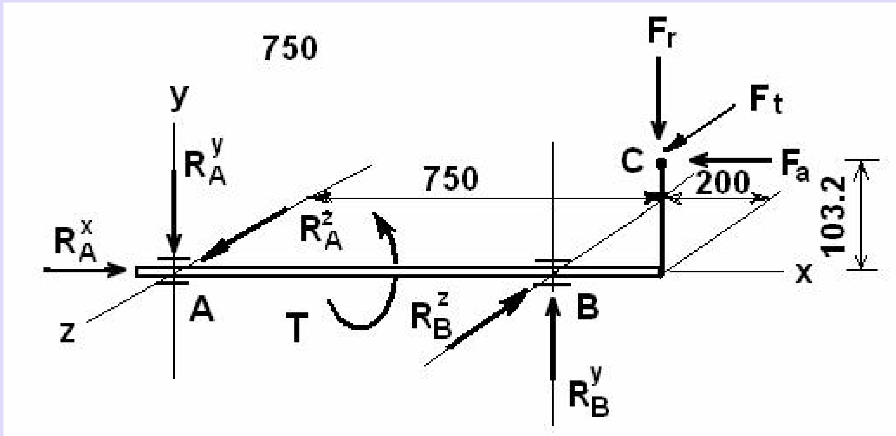

Fig. 12.2 Reaction the shaft bearings due to forces at the pinion pitch point

R_B^y = 4146.7 N ↑

$\Sigma F^Y = 0$, from which R_a^y = 440.7 N↓

Taking moment about y axis,

$R_B^z (750) - F_t(950) = 0$

i.e, $750\ R_B^z - 9375 x\ 950 = 0 \longrightarrow\ R_B^z = 11875$ N

$\Sigma F^z = 0$, from which R_A^z = 2500 N

$T = F_t\ (206.4/2) = 9375 \times (103.2) = 96750$ Nmm = 96.75 Nm

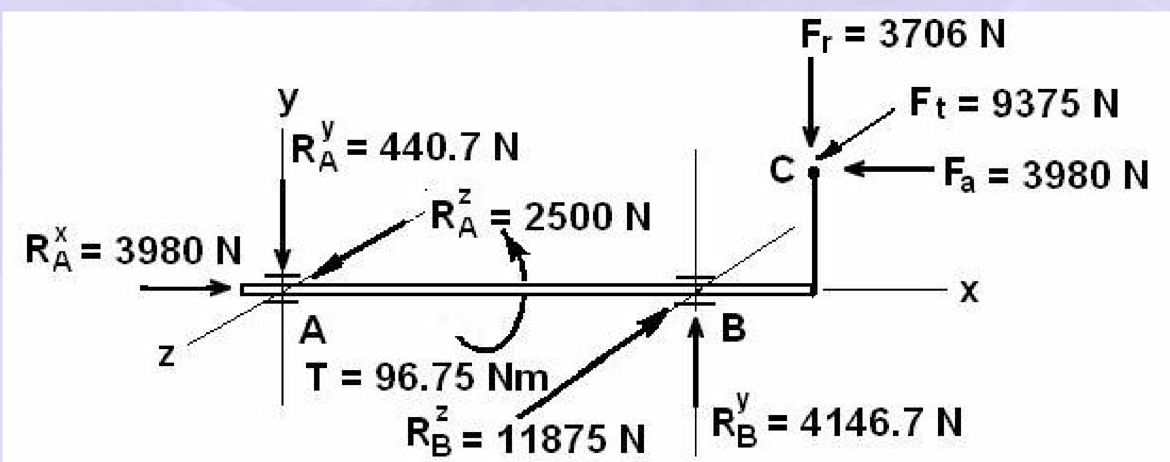

Fig.12.3 Reaction on shaft bearings due to forces at the pinion pitch point from calculation

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12.2 HELICAL GEARS - PROBLEM 2

A helical gear drive shown in Fig.12.4 transmits 20 kW power at 1440 rpm to a machine input shaft running at 360rpm. The motor shaft pinion has 18 teeth, 20^{O} normal pressure angle and a normal module of 4mm and 30^{o} right hand helix. Determine all dimensions of the gear and the pinion. b=1.2 p_a. Comment the chosen gears.

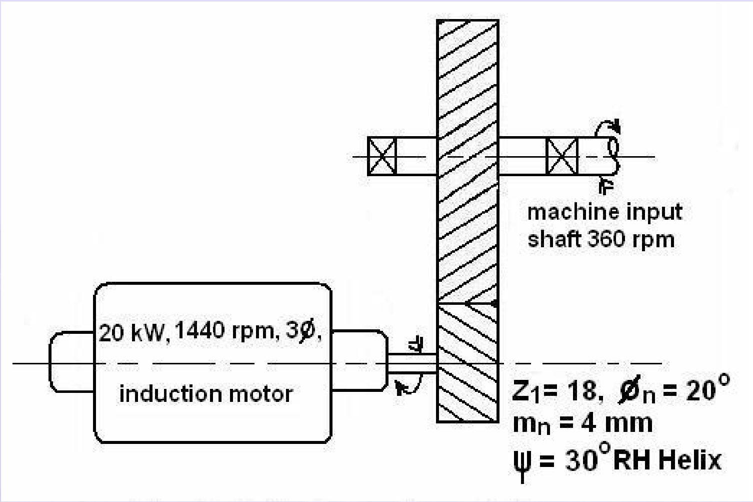


Fig.12.4 Helical gear layout diagram

The pinion material is made of C45 steel with hardness 380 Bhn and tensile strength σ_{ut} = 1240 MPa. The gear is made of ductile iron grade 120/90/02 of hardness 331 Bhn and tensile strength σ_{ut}= 974 MPa. Both gears are hobbed, HT and OQ&T and ground.

Given data:

W=20 kW, n_1 =1440 rpm, Z_1= 18, m_n= 4mm, $\O_n$= 20^{o}, b=1.2 p_a, n_2= 360 rpm, $\Psi = 30^{o}$ RH Helix

The following assumptions are made:

(a) Tooth profiles are std. involutes.

(b) Gears mesh along their pitch circles

(c) All loads are transmitted at the pitch point and mid planes of the gears.

(d) All power losses are neglected.

Solution:

$\tan \varnothing_n = \tan \varnothing . \cos \psi$

1. Transverse pressure angle $\varnothing = \tan^{-1}(\tan \varnothing n / \cos \psi) = \tan^{-1}(\tan 20^o/\cos 30^o) = 22.8^o$

2. Transverse module: $m = m_n /\cos \psi$

   i.e., $m = 4 / \cos 30^o = 4.62$ mm

3. Pinion pitch dia.:$d_1 = Z_1$ m = 18 x 4.62 = 83.2 mm

4. Gear, no. of teeth: $Z_2 = Z_1 (n_1/n_2)$ = 18(1440/360) = 72

5. Gear dia.: $d_2 = Z_2$ m = 72 x4.62 = 335.7mm

6. $p = \pi m = \pi x 4.62 = 14.51$ mm

7. $p_a = p / \tan \psi = 14.51 / \tan 30^o = 25.13$ mm

8. $b = 1.2 p_a = 1.2 \times 25.13 = 30.16$ mm

9. $V = \pi d_1\ n_1 /60000 = \pi x 83.2 x 1440 /60000 = 6.27$ m/s

10. $d_{b1} = d_1 \cos\varnothing = 83.2 \cos 22.8^o = 76.7$ mm

    $d_{b2} = d_2 \cos\varnothing = 335.7 \cos 22.8^o = 309.5$ mm

11. Addendum: h_a or $a = 1 m_n = 4.0$ mm

12. Dedendum: $h_f = 1.25\ m_n = 1.25 \times 4.0 = 5.00$ mm

13. $F_t = 1000\ W / V = 1000 \times 20/6.27 = 3190$ N

14. $F_r = F_t \tan \varnothing = 3190 x \tan 22.8^o = 1341$ N

15. $F_a = F_t \tan \psi = 3190 x \tan 30^o = 1842$ N

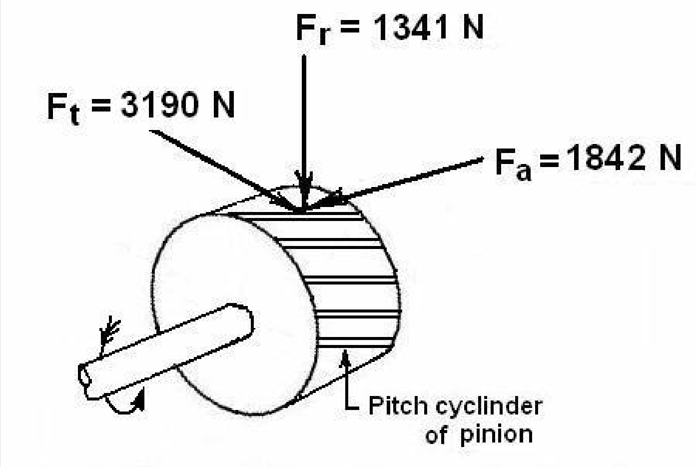


Fig. 12.5 View of the forces acting on pitch cylinder of the helical drive pinion

Bending stress on the pinion:

$$\sigma_{b1} = \frac{F_t}{b\,m_n\,J} K_v\,K_o\,(0.93 K_m)$$

J= 0.45 for $Z_{V1}=Z_1 / \cos^3 \psi = 18 / \cos^3 30^o = 27.7$ or 28 and $\psi = 30^o$ from Fig.12.6

J-multiplication factor from Fig.12.7 = 1.013 from Fig.12.7

$Z_{v2} = Z_2 / \cos^3 \psi = 72/ \cos^3 30^o = 110.9$ or 111 teeth mating gear.

J = 0.45 x 1.013 = 0.4559

HELICAL GEAR - TOOTH BENDING STRESS

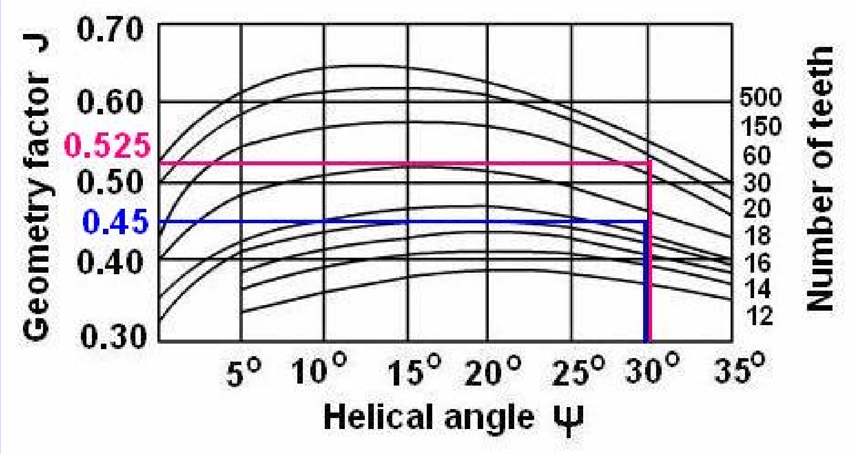


Fig.12.6 Geometry factor J for helical gear with $\varphi_n = 20^o$ and mating with 75 tooth gear

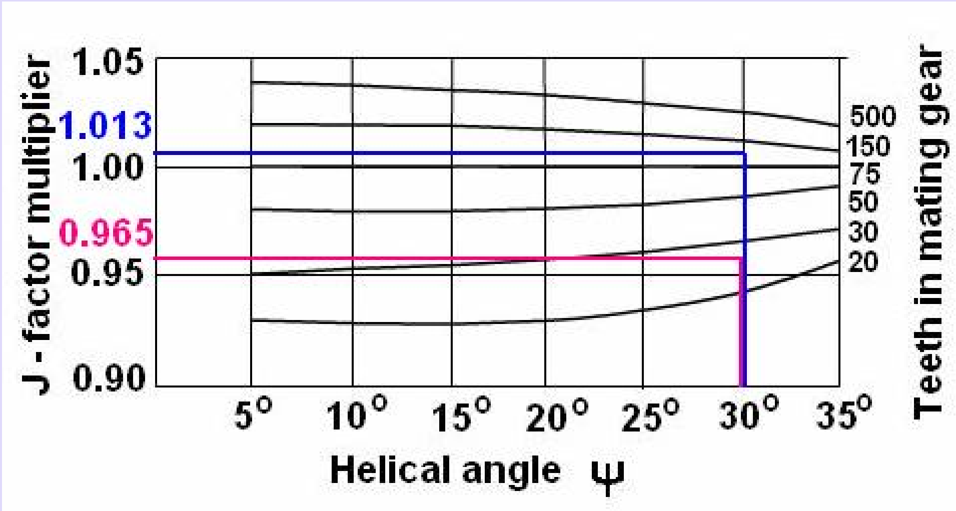


Fig.12.7 J- factor multiplier when the mating gear has tooth other than 75

$$K_v = \left[\frac{78 + (200V)^{0.5}}{78} \right]^{0.5} = \left[\frac{78 + (200x\,6.27)^{0.5}}{78} \right]^{0.5} = 1.21$$

K_o = 1.25 assuming uniform source of power and moderate shock from driven machinery, Table 12.1

K_m= 1.5 for b=30.16 mm & less rigid mountings, less accurate gears, contact across full face, Table 12.2

HELICAL GEAR –TOOTH BENDING STRESS (AGMA)

Table 12.1 -Overload factor K_o

	Driven Machinery		
Source of power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 12. 2 Load distribution factor K_m

	Face width (mm)			
Characteristics of Support	0 - 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.2	1.3	1.4	1.7
Less rigid mountings, less accurate gears, contact across the full face	1.5	1.6	1.7	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0	Over 2.0	Over 2.0	Over 2.0

Bending stress in the pinion is

$$\sigma_{b1} = \frac{F_t}{b\,m_n\,J} K_v\,K_o\,(0.93\,K_m)$$

$$= \frac{3190}{30.2 \times 4.00 \times 0.4559} \times 1.21 \times 1.25\,(0.93 \times 1.5)$$

$$= 122.2 \text{ MPa}$$

- For the gear J = 0.525, for Z_{V2}= 111 & $\psi=30^o$ from Fig. 12.6
- J-factor multiplier = 0.965 for Zv_1=28 & $\psi=30^o$ from Fig.12.7

For the gear, J = 0.525 x 0.965 = 0.5066

Bending stress for the gear is

$$\sigma_{b2} = \frac{F_t}{b\,m_n\,J} K_v\,K_o\,(0.93\,K_m)$$

$$= \frac{3190}{30.2 \times 4.0 \times 0.5066} \times 1.21 \times 1.25\,(0.93 \times 1.5)$$

$$= 110 \text{ MPa}$$

Corrected bending fatigue strength of the pinion:

$\sigma_e = \sigma_{e'} \, k_L \, k_v \, k_s \, k_r \, k_T \, k_f \, k_m$

$\sigma_{e'} = 0.5\sigma_{ut}$ =.0.5x1240 =620 MPa

k_L = 1.0 for bending

k_V = 1.0 for bending for m ≤ 5 module,

k_s = 0.645 for σ_{ut} = 1240 MPa from Fig.12.8

k_r = 0.897 for 90% reliability from the Table 12.3

k_T =1.0 with Temp. < 120°C,

k_f = 1.0

k_m = 1.33 for σ_{ut} = 1240 MPa from the Fig.12.9

σ_e = 620x1x1x0.645x1x1x0.897x1.33 = 477 MPa

SPUR GEAR – PERMISSIBLE TOOTH BENDING STRESS (AGMA)

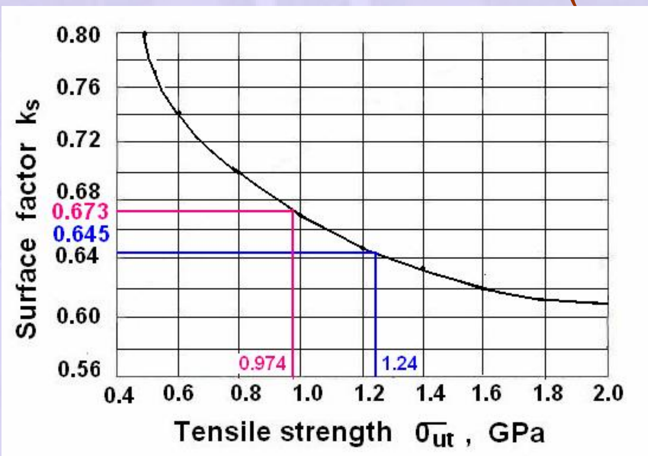


Fig. 12.8 Surface factor k_s

Table 12.3 Reliability factor k_r

Reliability factor R	0.50	0.90	0.95	0.99	0.999	0.9999
Factor k_r	1.000	$\boxed{0.897}$	0.868	0.814	0.753	0.702

k_f = fatigue stress concentration factor. As this factor is included in J factor, kf =1 is taken.

k_m = Factor for miscellaneous effects. For idler gears subjected to two way bending, = 1. For other gears subjected to one way bending, the value is taken from the Fig.12.9. Use k_m = 1.33 for σ_{ut} less than 1.4 GPa.

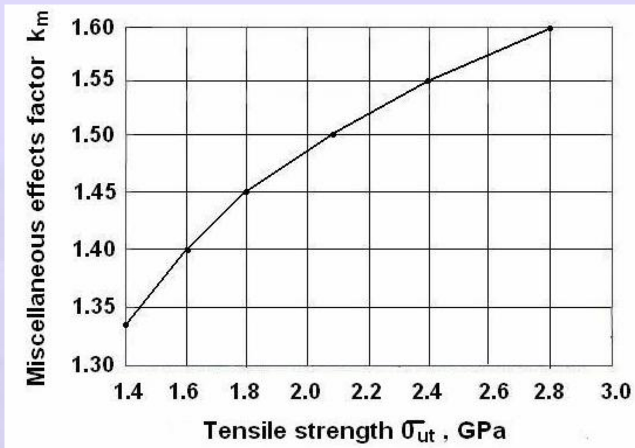


Fig.12.9 Miscellaneous effects factor, k_m

Corrected fatigue strength of the gear:

$\sigma_e = \sigma_{e'}\, k_L\, k_v\, k_s\, k_r\, k_T\, k_f\, k_m$

$\sigma_e' = 0.35\sigma_{ut}$ =.0.35x974 =340.9 MPa

k_L = 1.0 for bending

k_V = 1.0 for bending for m ≤ 5 module,

k_s = 0.673 for σ_{ut} = 974 MPa from Fig.12.8

k_r = 0.897 for 90% reliability from the Table 12.3

k_T = 1.0 with Temp. < 120^{o}C,

k_f = 1.0

k_m = 1.33 for σ_{ut} = 974 MPa from Fig.12.9

σ_e = 340.9x1x1x0.673x0.897x1x1x1.33 = 273.7MPa

Factor of safety for the pinion on bending:

$s_{b1} = \sigma e / \sigma_{b1} = 477 / 122.2 = 3.9$

Factor of safety for the gear on bending:

$s_{b2} = \sigma_e / \sigma_{b2} = 273.7/110 = 2.49$

Table 12.4 Guidance on the necessary safety factor

Factor of safety against	Long life gearing	Finite life gearing
Tooth breakage $S_B \geq$.	1,8 ⋯ 4	1,5 ⋯ 2
Pitting S_G	1,3 ⋯ 2,5	0,4 ⋯ 1
Scoring S_F	3 ⋯ 5	3 ⋯ 5

As per Niemen Table 12.4, the minimum factor of safety for infinite life in bending fatigue is 1.8. Since both the case the factor of safety exceeds this value, the gears will have infinite life.

Ans: The gear is weaker among the two in bending fatigue as its factor of safety is lower.

Contact stress on helical gears is given by:

$$\sigma_H = C_p \sqrt{\frac{F_t}{bdI}\left(\frac{\cos\psi}{0.95CR}\right) K_v\, K_o\, (0.93K_m)}$$

Cp = 166 (MPa)$^{0.5}$ for steel pinion vs cast iron gear from Table 12.5.

$$I = \frac{\sin\varphi\cos\varphi}{2}\ \frac{i}{i+1} = \frac{\sin 22.8^o \cos 22.8^o}{2}\ \frac{4}{4+1} = 0.143$$

Table 12.5 Elastic coefficient Cp for spur gears, in $\sqrt{MPa}$

Pinion Material (μ = 0.3 in all cases)	Gear Material			
	Steel	Cast Iron	Al Bronze	Tin Bronze
Steel, E = 207 GPa	191	$\boxed{166}$	162	158
Cast Iron, E = 131 GPa	166	149	149	145
Al Bronze, E=121 GPa	162	149	145	141
Tin Bronze, E=110 GPa	158	145	141	137

Contact ratio is given by:

$$CR_t = \left(\frac{\sqrt{(r_1 + a)^2 - r_{b1}{}^2} + \sqrt{(r_2 + a)^2 - r_{b2}{}^2} - (r_1 + r_2)\sin\varphi}{\pi m \cos\varphi} \right)$$

Using standard tooth system with a= $1m_n$, CR_t:

$$CR_t = \left(\frac{\sqrt{(41.6 + 4.0)^2 - 38.35^2}}{\pi \, x \, 4.62 \cos 22.8^o} \right) + \left(\frac{\sqrt{(167.85 + 4.0)^2 - 154.75^2}}{\pi \, x \, 4.62 \cos 22.8^o} \right) - \left(\frac{(41.6 + 167.85)\sin 22.8^o}{\pi \, x \, 4.62 \cos 22.8^o} \right) = 1.365$$

K_v =1.21, K_o = 1.25, K_m =1.5

$$\sigma_H = C_p \sqrt{\frac{F_t}{b\,d\,I}\left(\frac{\cos\psi}{0.95\ CR}\right) K_v\ K_o\ (0.93 K_m)}$$

$$= 166 \sqrt{\frac{3190}{30.2 \, x \, 83.2 \, x \, 0.143}\left(\frac{\cos 30^o}{0.95 \, x \, 1.365}\right) 1.21 x 1.25 (0.93 \, x \, 1.5)}$$

$$= 587 MPa$$

Surface fatigue strength of pinion is:

$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T$

σ_{sf}' = surface fatigue strength of the material

= 2.8 (Bhn) – 69 from Table 12.6

= 2.8x 380 -69

= 995 MPa

HELICAL GEAR – SURFACE FATIGUE STRENGTH

$K_L = 0.9$ for 10^8 cycles from Fig.12.10

$K_H = 1.005$ for K = 380/331 = 1.14 & i= 4 from Fig.12.11

$K_R = 1.0$ for 99% reliability from Table 12.7

$K_T = 1.0$ assuming temp. < 120^0C

For the pinion material,

$\sigma_{sf1} = \sigma_{sf}' K_L K_H K_R K_T = 995 \times 0.9 \times 1 \times 1.005 \times 1 = 900$ MPa

Table 12.6 Surface fatigue strength σ_{sf}' (MPa) for metallic spur gears

(10^7 cycle life, 99% reliability and temperature <120^oC)

Material	σ_{sf}' (MPa)
Steel	2.8(Bhn) — 69 MPa
Nodular Iron	0.95[2.8(Bhn) – 69] MPa
Cast Iron , grade 20	379
Cast Iron , grade 30	482
Cast Iron , grade 40	551
Tin Bronze, AGMA 2C (11% Sn)	207
Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)	448

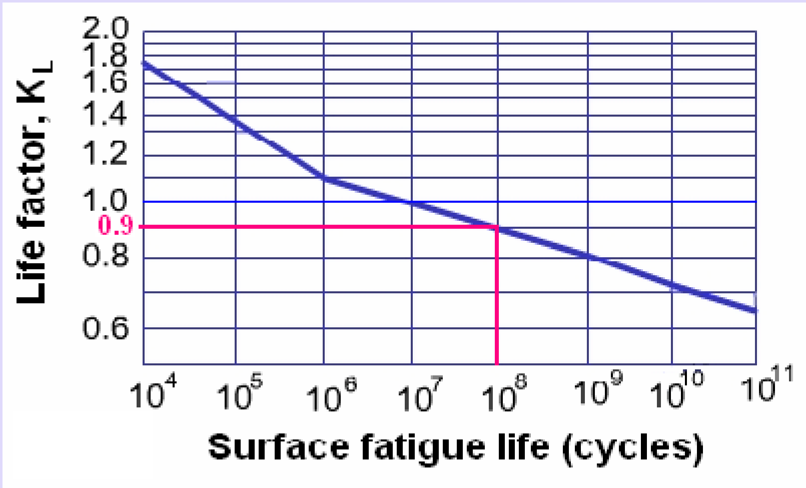


Fig.12.10 Life Factor K_L

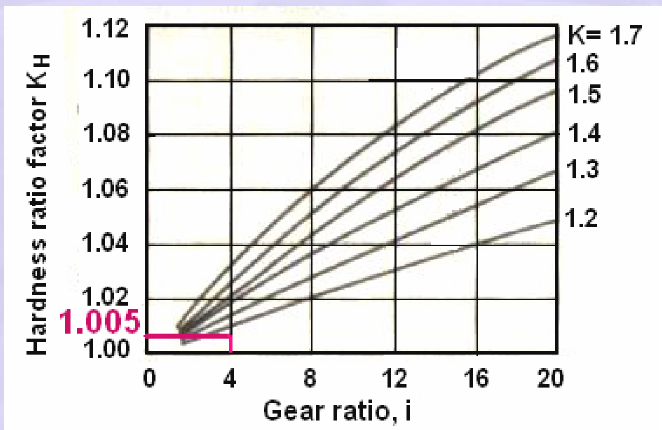


Fig. 12. 11 Hardness ratio factor, K_H
K = Brinell hardness ratio of pinion and gear, K_H = 1.0 for values of K below 1.2

Table 12.7 Reliability factor K_R

Reliability (%)	K_R
50	1.25
99	1.00
99.9	0.80

K_T (Temperature factor) = 1 for T≤ 120°C based on Lubricant temperature.

Above 120^oC, it is less than 1 to be taken from AGMA standards.

For gear: $\sigma_{sf}' = 0.95[2.8(Bhn)-69] = 0.95[2.8x331-69] = 815$ MPa

$K_L = 0.9$ for 10^8 cycles from Fig.12.10

$K_H = 1.005$ for K = 380/331 = 1.14 & i=4 from Fig.12.11

$K_R = 1.0$ for 99% reliability from Table 12.7

$K_T = 1.0$ assuming temp. < 120^0C

$\sigma_{sf\ 2} = \sigma_{sf}' K_L K_H K_R K_T = 815 \times 0.9 \times 1.005 \times 1 \times 1 = 795$ MPa

HELICAL GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)

Allowable surface fatigue stress for design is given by

$[\sigma_H] = \sigma_{Sf} / s_H$

Factor of safety s_H = 1.1 to 1.5

Design equation is: $\sigma_H \le [\sigma_H]$

Factor of safety for the pinion against pitting:

$s_{H1} = \sigma_{Sf1} / \sigma_H = 900 /587 = 1.53$

Factor of safety for gear against pitting:

$s_{H2} = \sigma_{Sf2} / \sigma_H = 795 /587 = 1.35$

In both case the factor of safety is more than 1.3 against pitting (Table 12.4) and the design is adequate. Among these, gear is slightly weaker than pinion and is likely to fail first.

The factor of safety in surface fatigue is proportional to square root of load and that in bending fatigue is directly proportional to load. Hence, the equivalent bending factor of safety for corresponding surface fatigue $(s_{H2})^2 = 1.35^2 = 1.81$ is compared with (S_{b2}) and is <2.49. So the gears are likely to fail due to surface fatigue and not due to bending fatigue.

12.3 HELICAL GEARS - PROBLEM 3

In a crossed helical gear drive, the shaft angle is 90^o and the gear ratio is 1:1 with the helix angle $\psi_1 = \psi_2 = 45^o$. The normal module is 4 mm and the number of teeth in the gears are $Z_1 = Z_2 = 50$. The above identical gears are to be so changed that the driven gear has a pitch diameter of around 200 mm in the new arrangement.

Data: $\Sigma = \psi_1 + \psi_2 = 90^o$; $\psi_1 = \psi_2 = 45^o$; $m_n = 4$ mm;
$Z_1 = Z_2 = 50$ and $d_2 \sim 200$ mm.

Solution:

$$d_1 = \frac{m_n z_1}{\cos\psi_1} = \frac{m_n z_1}{\sin\psi_2} \text{ and } d_2 = \frac{m_n z_2}{\cos\psi_2}$$

Centre distance: $C = 0.5\,(d_1 + d_2) = 0.5 m_n\,(Z_1 + Z_2)/\cos\psi$

$= 0.5 \text{ x } 4 \text{ x } (2\text{x}50) / \cos 45^o$

$= 282.84$ mm

$$C = \frac{1}{2}(d_1 + d_2) = \frac{1}{2}\left(\frac{m_n Z_1}{\sin\psi_2} + \frac{m_n Z_2}{\cos\psi_2}\right) = \frac{m_n Z}{2}\left(\frac{\sin\psi_2 + \cos\psi_2}{\sin\psi_2 \cos\psi_2}\right)$$

Also $$Z = \frac{d_2 \cos\psi_2}{m_n}$$

Therefore $$C = \frac{m_n}{2} \text{ x } \frac{d_2 \cos\psi_2}{m_n} \text{ x } \left(\frac{\sin\psi_2 + \cos\psi_2}{\sin\psi_2 \cos\psi_2}\right) = \frac{d_2}{2}(1 + \cot\psi_2)$$

Or $$\cot\psi_2 = \frac{2C}{d_2} - 1 = \frac{2 \text{ x } 282.84}{200} - 1 = 1.828, \quad \psi_2 = 28.675^o$$

Hence, $$Z_2 = \frac{d_2 \cos\psi_2}{m_n} = \frac{200 \text{ x } \cos 28.675^o}{4} = 43.86$$

Taking an integral value for Z = 44 and substituting

$$\frac{2C}{m_n Z} = \frac{\sin\psi_2 + \cos\psi_2}{\sin\psi_2 \cos\psi_2} \qquad \text{or}$$

$$\frac{C}{m_n Z} = \frac{282.84}{4x44} = \frac{\sin\psi_2 + \cos\psi_2}{2\sin\psi_2 \cos\psi_2}$$

Squaring: $2.5826 = \dfrac{1+\sin 2\psi_2}{\sin^2 2\psi_2}$

Solving we get $\psi_2 = 28.9^o$

Final values $d_1 = 4 \times 44 / \sin 28.85^o = 364.75$ mm

$d_2 = 4 \text{ X } 44 / \cos 28.85^o = 200.94$ mm which is near to 200 mm

$C = 0.5\,(d_1 + d_2) = (364.75 + 200.94) = 282.84$ mm equal to original centre distance.

12.4 HELICAL GEARS - PROBLEM 4

In a turbine drive 300 kW power is transmitted using a pair of double helical gear. The pinion speed is 2950 rpm and that of the gear is about 816.5 rpm. There are no space constraints on the gear drive. Selecting suitable materials, design the pinion and the gear to last for 10^8 cycles.

Data: W = 300kW; n_1 = 2950rpm; n_2 = 816.5 rpm; Life 10^8 cycles.

Solution: Since there are no constraints for the drive design, the number of teeth on the pinion is assumed as Z_1 = 29. Helix angle of 35^o and normal pressure angle $\varphi_n = 20^o$ are taken for the gears and b = 1.2 p_a is assumed.

$$\omega_1 = \frac{2\pi n_1}{60} = \frac{2\pi \times 2950}{60} = 308.77 \text{ rad/s}$$

$i = n_1 / n_2$ = 2950 / 816.5 = 3.612

$Z_2 = i Z_1$ = 3.612 x 29 = 104.8 rounded to 105

Torque:

$$T_1' = \frac{1000W}{\omega} = \frac{1000x300}{308.77} = 971.6Nm$$

The double helical gear is considered as two single helical gears coupled together sharing the torque equally. Torque on each half is T_1 =971.6/2 =485.8 Nm=485800 Nmm.

The AGMA bending stress equation:

$$\sigma_b = \frac{F_t}{b\, m_n\, J} K_v\, K_o\, (0.93 K_m)$$

$p = \pi m = \pi m_n / \cos\psi = \pi m_n / \cos 35^o = 3.833 m_n$

$p_a = p / \tan\psi$.

Assuming $b = 1.2p_a = 1.2\, p / \tan\psi = 1.2 \times 3.833 m_n / \tan 35^o = 6.569 m_n$

$F_t = 2T_1 / d_1 = 2T_1 / mZ_1 = 2T_1 \cos\psi / m_n Z_1 = 2 \times 485800 \times \cos 35^o / m_n \times 29$

$= 27444 / m_n$ N

J for the pinion with teeth $Z_{v1} = Z_1 / \cos^3\psi = 29 / \cos^3 35^o = 82$, $\psi = 35^o$ is: J=0.47 from Fig. 12.6

J multiplier for mating with Zv2 = $Z_2/\cos^3\psi = 105/\cos^3 45^o$ =297, is =1.015 from Fig. 12.7

For pinion J = 0.47 x 1.015 = 0.4771

HELICAL GEAR - TOOTH BENDING STRESS

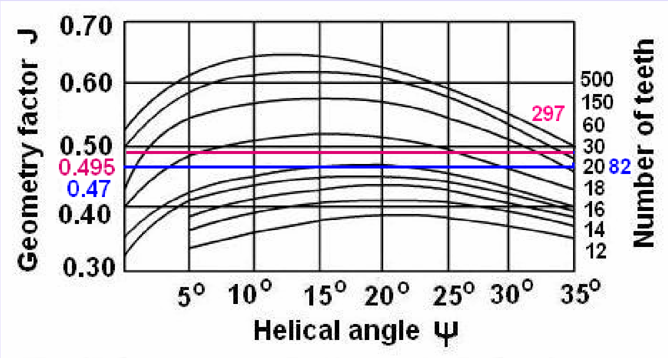


Fig.12.6 Geometry factor J for helical gear with $\varphi_n = 20^o$ and mating with 75 tooth gear.

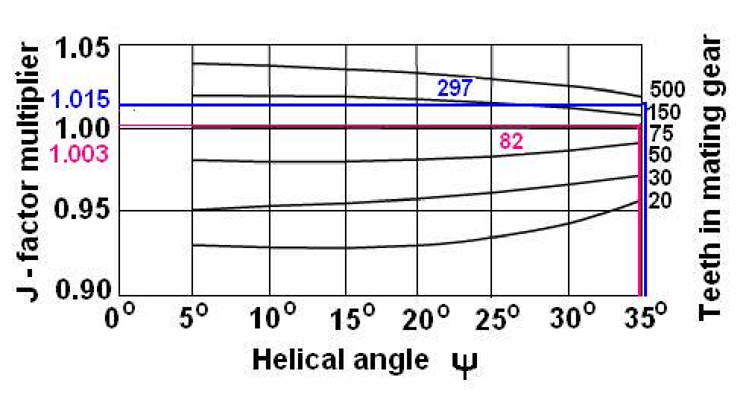


Fig.12.7 J- factor multiplier when the mating gear has tooth other than 75

J factor for the gear with teeth Z_{v2} = 297 and $\psi = 35^o$ is J =0.495 from Fig. 12.6

J multiplier for mating with Z_{v1}= 82 is = 1.003 from Fig. 12.7

For gear J = 0.495 x 1.003 = 0.4965

$$K_v = \left[\frac{78 + (200V)^{0.5}}{78}\right]^{0.5} = 1.25 \quad \text{assumed since V is not known.}$$

K_o = 1.25 assuming uniform source of power and moderate shock from driven machinery, Table 12.1.

K_m= 1.3 expecting b=150 mm Accurate mountings, small bearing clearances, minimum deflection, precision gears, Table 12.2.

Helical Gear –Tooth Bending Stress (AGMA)

Table 12.1 -Overload factor K_o

	Driven Machinery		
Source of power	Uniform	Moderate Shock	Heavy Shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

Table 12.2 Load distribution factor K_m

	Face width (mm)			
Characteristics of Support	0 - 50	150	225	400 up
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.2	1.3	1.4	1.7
Less rigid mountings, less accurate gears, contact across the full face	1.5	1.6	1.7	2.0
Accuracy and mounting such that less than full-face contact exists	Over 2.0	Over 2.0	Over 2.0	Over 2.0

For the pinion:

$$\sigma_{b1} = \frac{F_t}{b m_n J} K_v K_o (0.93 K_m)$$

$$= \frac{27444}{6.569 m_n^3 \times 0.4771} \times 1.25 \times 1.25 \times (0.93 \times 1.3) = \frac{16542}{m_n^3}$$

For the gear:

$$\sigma_{b2} = \frac{F_t}{b\,m_n\,J} K_v K_o (0.93 K_m)$$

$$= \frac{27444}{6.569 m_n^3 \times 0.4965} \times 1.25 \times 1.25 \times (0.93 \times 1.3)$$

$$= \frac{15895}{m_n^3}$$

The pinion material is made from C45 steel with hardness 380 Bhn and tensile strength σ_{ut}= 1240 MPa. The gear is made from ductile iron grade 120/90/02 of hardness 331 Bhn and tensile strength σ_{ut}= 974 MPa. Both gears are hobbed, HT and OQ&T and ground.

Corrected bending fatigue strength of the pinion:

$\sigma_e = \sigma_e' \, k_L \, k_v \, k_s \, k_r \, k_T \, k_f \, k_m$

$\sigma_e' = 0.5\sigma_{ut}$ =.0.5x1240 =620 MPa

k_L = 1.0 for bending

k_V = 1.0 for bending for $m \le 5$ module,

k_s = 0.645 for σ_{ut} = 1240 MPa from Fig.12.8

k_r = 0.897 for 90% reliability from the Table 12.3

k_T = 1.0 with Temp. < 120^{o}C, k_f = 1.0

k_m = 1.33 for σ_{ut} = 1240 MPa from the Fig.12.9

σ_e = 620x1x1x0.645x1x1x0.897x1.33 = 477 MPa

Corrected bending fatigue strength of the gear:

$\sigma_e = \sigma_e' \, k_L \, k_v \, k_s \, k_r \, k_T \, k_f \, k_m$

$\sigma_e' = 0.35\sigma_{ut}$ =.0.35x974 =340.9 MPa

k_L = 1.0 for bending

k_V = 1.0 for bending for $m \le 5$ module,

k_s = 0.673 for σ_{ut} = 974 MPa from Fig.12.8

k_r = 0.897 for 90% reliability from the Table 12.3

k_T = 1.0 with Temp. < 120^{o}C, k_f = 1.0

k_m = 1.33 for σ_{ut} = 974 MPa from Fig.12.95

σ_e = 340.9x1x1x0.673x0.897x1x1x1.33 = 273.7MPa

Permissible stress for the pinion in bending fatigue with factor of safety 1.6 for finite life gearing from Table 12.4:

$[\sigma_b]_1 = \sigma_e / s_b$ = 477/1.6 = 298 MPa

Permissible stress for the pinion in bending fatigue with factor of safety 1.6,

$[\sigma_b]_2 = \sigma_e / s_b$ = 273.7/1.6 = 171 MPa

For the pinion,

$$\sigma_{b2} = \frac{16542}{m_n^3} = [\sigma]_2 = 298$$

m_n = 3.81 mm

For the gear,

$$\sigma_{b2} = \frac{15895}{m_n^3} = [\sigma]_2 = 171$$

m_n = 4.53 mm

Take a standard value of 5 mm as given in Table 12.8.

Table 12.8 Standard modules in mm

0.3	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**1.0**
1.25	**1.5**	**1.75**	**2.0**	**2.25**	**2.5**	**3**
3.5	**4**	**4.5**	$\boxed{\mathbf{5}}$	**5.5**	**6**	**6.5**
7	**8**	**9**	**10**	**11**	**12**	**13**
14	**15**	**16**	**18**	**20**	**22**	**24**
26	**28**	**30**	**33**	**36**	**39**	**42**
45	**50**	**Further increase is in terms of 5 mm**				

$m = m_n / \cos 35^o = 5 / \cos 35^o = 6.104$ mm

$d_1 = mZ_1 = 6.104 \times 29 = 177.01$ mm

$d_2 = mZ_2 = 6.104 \times 105 = 640.92$ mm

$p = 3.833 m_n = 3.833 \times 5 = 19.165$ mm

$p_a = p / \tan \psi. = 19.165 / \tan 35^o = 27.37$ mm

$b = 1.2 p_a = 1.2 \times 27.37 = 32.84$ mm, take 35 mm

$d_{a1} = d_1 + 2m_n = 177.01 + 2 \times 5 = 187.01$ mm

$d_{a2} = d_2 + 2m_n = 640.92 + 2 \times 5 = 650.92$ mm

Transverse pressure angle: $\tan Ø_n = \tan Ø \cos \psi$

$$\varphi = \tan^{-1}\left(\frac{\tan \varphi_n}{\cos \psi}\right) = \tan^{-1}\left(\frac{\tan 20^o}{\cos 35^o}\right) = 23.96^o$$

$d_{b1} = d_1 \cos Ø = 177.01 \cos 23.96^o = 161.76$ mm

$d_{b2} = d_2 \cos Ø = 640.92 \cos 23.96^o = 585.69$ mm

$C = 0.5(d_1 + d_2) = 0.5(177.01 + 640.92) = 408.97$mm

$V = 0.5 \omega d_1 = 0.5 \times 308.77 \times 177.01 \times 10^{-3} = 27.33$ m/s

$F_t = 2T_1/d_1 = 2 \times 485800 / 177.01 = 5489$N

Contact stress on the gears is given by:

$$\sigma_H = C_p \sqrt{\frac{F_t}{bdI}\left(\frac{\cos \psi}{0.95CR}\right) K_v K_o (0.93 K_m)}$$

$C_p = 166\ (\text{MPa})^{0.5}$ for steel pinion vs cast iron gear from Table 12.5.

$$I = \frac{\sin \varphi \cos \varphi}{2} \frac{i}{i+1}$$

$$= \frac{\sin 23.96^o \cos 23.96^o}{2} \frac{3.621}{3.621+1} = 0.1454$$

Table 12.5 Elastic coefficient Cp for spur gears, in $\sqrt{MPa}$

Pinion Material (μ = 0.3 in all cases)	Gear Material			
	Steel	Cast Iron	Al Bronze	Tin Bronze
Steel, E = 207 GPa	191	$\boxed{166}$	162	158
Cast Iron, E = 131 GPa	166	149	149	145
Al Bronze, E=121 GPa	162	149	145	141
Tin Bronze, E=110 GPa	158	145	141	137

Contact ratio is given by:

$$CR_t = \left(\frac{\sqrt{(r_1+a)^2 - r_{b1}^2} + \sqrt{(r_2+a)^2 - r_{b2}^2} - (r_1+r_2)\sin\varphi}{\pi m \cos\varphi} \right)$$

Using standard tooth system with a= 1m$_n$, CR:

$$CR_t = \left(\frac{\sqrt{(93.51^2 - 80.88^2}}{\pi\, x 6.104 \cos 23.96^o} \right) + \left(\frac{\sqrt{(325.46^2 - 292.85^2}}{\pi\, x 6.104 \cos 23.96^o} \right)$$

$$- \left(\frac{408.97 \sin 23.96^o}{\pi\, x 6.104 \cos 23.96^o} \right) \quad = 1.3044$$

$$K_v = \left[\frac{78 + (200V)^{0.5}}{78} \right]^{0.5} = \left[\frac{78 + (200x\, 27.33)^{0.5}}{78} \right]^{0.5} = 1.396$$

K_v =1.396, K_o = 1.25, K_m =1.

$$\sigma_H = C_p \sqrt{\frac{F_t}{bdI} \left(\frac{\cos\psi}{0.95CR} \right) K_v K_o (0.93K_m)} \qquad (25)$$

$$= 166 \sqrt{\frac{5489}{35\, x\, 177.01\, x\, 0.1454} \left(\frac{\cos 35^o}{0.95\, x 1.3044} \right) 1.396\, x\, 1.25\, (0.93\, x\, 1.5)}$$

$$= 519.8\, MPa$$

Contact fatigue strength of pinion is:

$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T$

σ_{sf}' = surface fatigue strength of the material = 2.8 (Bhn) – 69 From Table 12.6

= 2.8x 380 -69

= 995 MPa

HELICAL GEAR – SURFACE FATIGUE STRENGTH

Table 12.6 Surface fatigue strength σ_{sf}' (MPa), for metallic spur gears, (10^7 cycle life 99% reliability and temperature < 120° C)

Material	σ_{sf}' (MPa)
Steel	$\boxed{2.8\,(\text{Bhn}) - 69\ \text{MPa}}$
Nodular Iron	$\boxed{0.95\,[\,2.8\,(\text{Bhn}) - 69\ \text{MPa}\,]}$
Cast Iron, grade 20	379
Cast Iron, grade 30	482
Cast Iron, grade 40	551
Tin Bronze, AGMA 2C (11% Sn)	207
Aluminium Bronze (ASTM B 148 – 52) (Alloy 9C – H.T.)	448

$K_L = 0.9$ for 10^8 cycles from Fig.12.10

$K_H = 1.005$ for K = 380/331 = 1.14 & i=4 from Fig.12.11

$K_R = 1.0$ for 99% reliability from Table 12.7

$K_T = 1.0$ assuming temp. < 120^0C

$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T$ = 995 x 0.9 x 1.005 x1 x 1

= 900 MPa

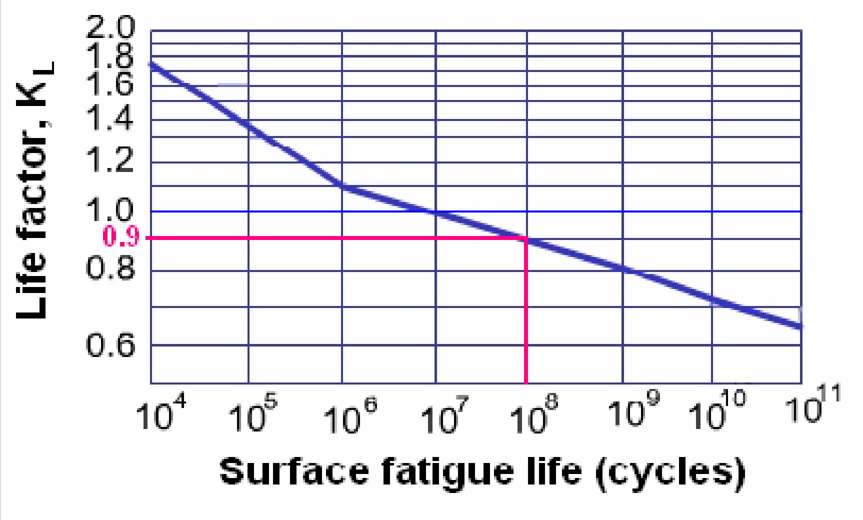


Fig.12.10 Life Factor K_L

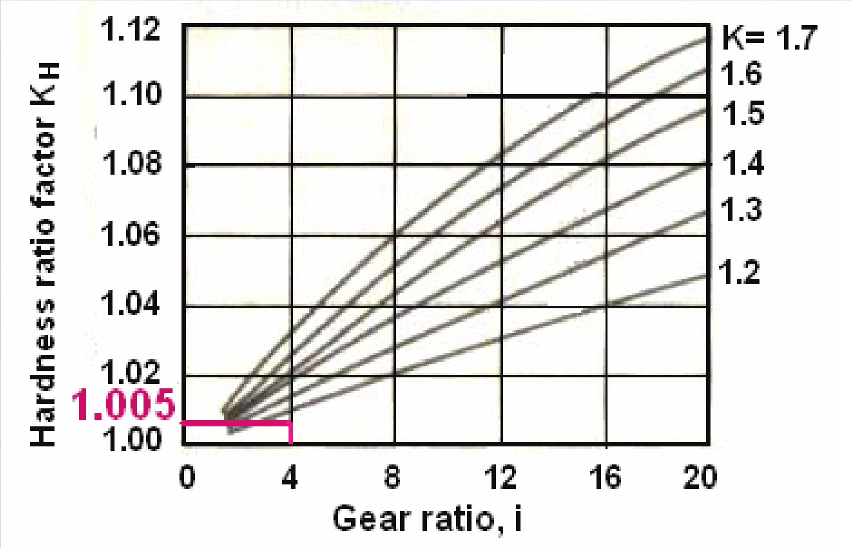


Fig.12.11 Hardness ratio factor, K_H K = Brinell hardness ratio of pinion and gear, K_H = 1.0 for values of K below 1.2

Table 12.7 Reliability factor K_R

Reliability (%)	K_R
50	1.25
99	1.00
99.9	0.80

K_T = temperature factor,

= 1 for T≤ 120°C based on Lubricant temperature.

Above 120°C, it is less than 1 to be taken from AGMA standards.

HELICAL GEAR – ALLOWABLE SURFACE FATIGUE STRESS (AGMA)

Allowable surface fatigue stress for design is given by

$[\sigma_H] = \sigma_{Sf} / s_H$

Design equation is: $\boxed{\sigma_H \leq [\sigma_H]}$

For gear: $\sigma_{sf}' = 0.95[2.8(Bhn)-69] = 0.95[2.8x331-69] = 815$ MPa

$K_L = 0.97$ for $2.5x10^7$ cycles from Fig.12.10

$K_H = 1.005$ for K = 380/331 = 1.14 & i=4 from Fig.12.11

$K_R = 1.0$ for 99% reliability from Table 12.10

$K_T = 1.0$ assuming temp. < 120^0C

$\sigma_{sf} = \sigma_{sf}' K_L K_H K_R K_T = 815 \times 0.97 \times 1.005 \times 1 \times 1 = 795$ MPa

Factor of safety for the pinion against pitting:

$s_{H1} = \sigma_{Sf} / \sigma_H = 900 / 519.8 = 1.73$

Factor of safety for gear against pitting:

$s_{H2} = \sigma_{Sf} / \sigma_H = 795 / 519.8 = 1.53$

Table 12.4 Guidance on the necessary factor of safety

Factor of safety against	Long life gearing	Finite life gearing
Tooth breakage $S_B \geq$.	1,8 ··· 4	1,5 ··· 2
Pitting S_G	1,3 ··· 2,5	0,4 ··· 1
Scoring S_F	3 ··· 5	3 ··· 5

As per the Niemen guidance for factor of safety given in Table 12.4, for long life gearing the factor of safety has to be more than 1.3 in pitting. Since for both gear and pinion the factor of safeties is more than 1.3, the design is adequate.

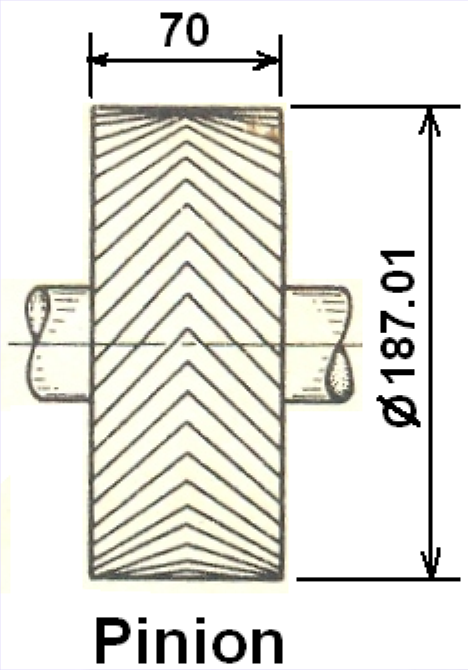
The final specifications of the pinion and gear are:

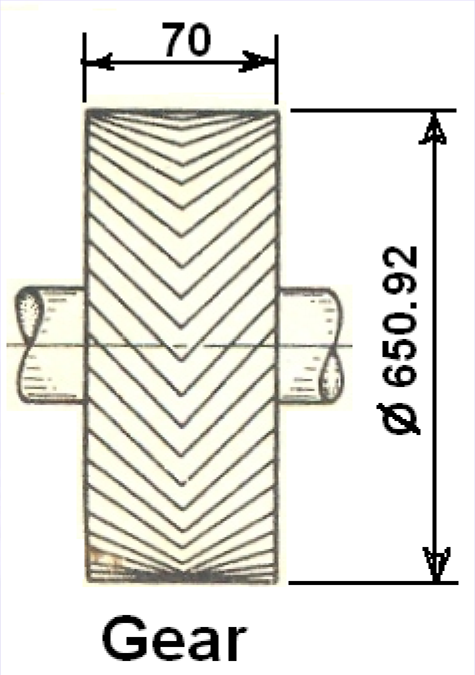
20^o pressure angle involute teeth with helix angle of 35^o, $h_a = 1m_n$, $h_f = 1.25m_n$

	Z	m_n mm	d mm	d_a mm	d_b mm	d_r mm	m_t mm
Pinion	29	5	177.01	187.01	161.76	164.51	6.104
Gear	105	5	640.92	650.92	585.69	628.42	6.104

	Φ_n	φ_t	b mm	p_t mm	p_a mm
Pinion	20^o	23.96^o	**35**	**19.165**	**27.37**
Gear	20^o	23.96^o	**35**	**19.165**	**27.37**

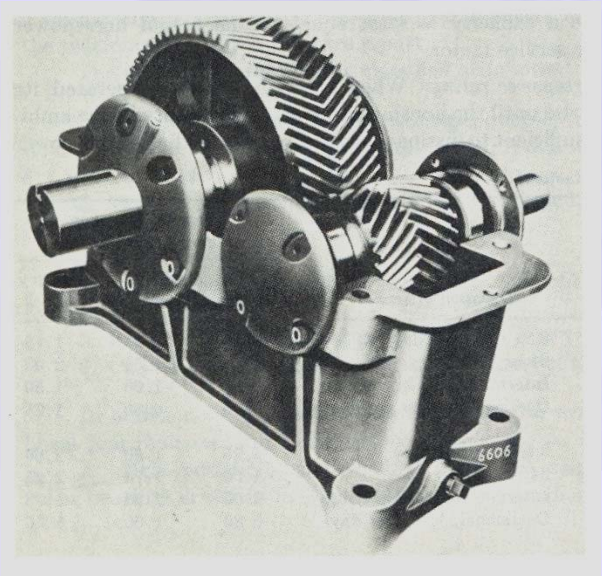
	CR_t	CR_a	CR	FS s_b	FS s_H
Pinion	1.3044	1.2787	2.583	1.99	1.73
Gear	1.3044	1.2787	2.583	1.89	1.53





(a) (b)

Fig. 12.12 Dimensional sketch of the pinion and the gear.
(All dimensions are in mm and not to scale.)



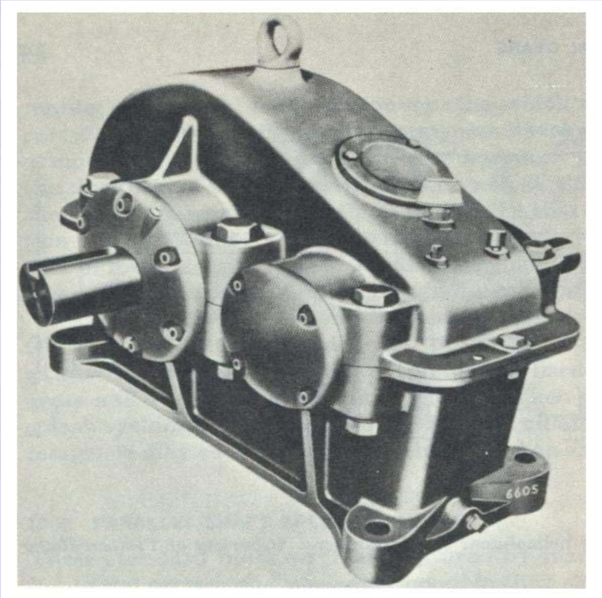


Fig. 12.13 Assembly drawing of the double helical gearbox
