Department of Physics, IIT Kanpur

Tutorial Problem Set-7

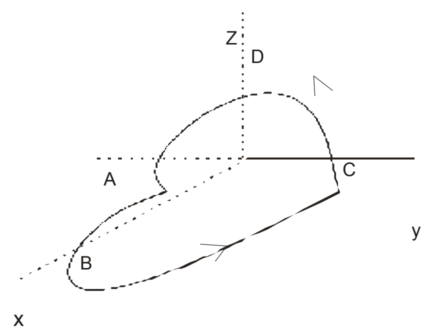
PHY103 HCV/SMT **8th Sept 2015**

It is proposed to set up a current density $\mathbf{J} = J_0 \dfrac{\mathbf{r}}{R}$ in a region where J_0 and R are constants.

(a) Does it represent a steady current? (b) Find the rate at which the charge density is changing at $\mathbf{r}$.

2. Show that magnetic moment of a plane current loop can be written as $\mathbf{m} = \dfrac{1}{2}\oint_{loop} \mathbf{r} \times I d\mathbf{l}$. This can be generalized for a current distribution in volume τ as $\mathbf{m} = \dfrac{1}{2}\int_{volume} \mathbf{r} \times \mathbf{J} d\tau$.

3. Wire loop ABCDA carries a current *I*. As suggested in the figure ABC is a semicircle of radius R in the x-y plane and CDA is a semicircle of radius a in the y-z plane . Calculate the magnetic dipole moment of the loop.



4. A particle having mass m and charge q is released from the origin in a region where fields $\mathbf{E} = E_0\hat{k}$ and $\mathbf{B} = B_0\hat{j}$ exist.

(a) Discuss qualitatively the nature of the path.
(b) Find the speed of the particle as a function of its z- coordinate.
(c) Find velocity of the particle as a function of time.
(d) Find the position of the particle as a function of time.
(e) Find the value of z for which the particle's velocity becomes perpendicular to the electric field.

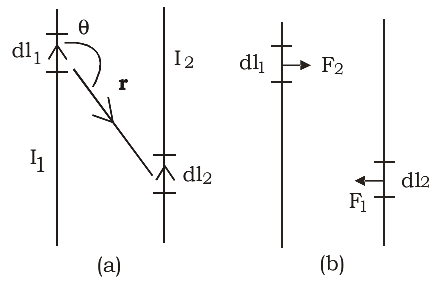
More Problems

1. A region has uniform electric and magnetic fields given by $\mathbf{E} = E_0\,\hat{i}$, $\mathbf{B} = B\hat{j}$. A particle having charge q and mass m is started from the origin with an initial velocity $\boldsymbol{\upsilon}_0 = \upsilon_0\hat{j}$. Find the position as function of time and sketch the path.

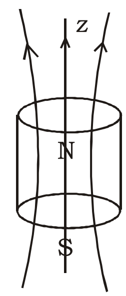
[Ans. (b) $x = \dfrac{\upsilon_0}{\omega}\sin\omega t - \upsilon_0 t,\ y = \upsilon_0 t,\ z = \dfrac{\upsilon_0}{\omega}(1-\cos\omega t)$]

2.A uniformly charged solid sphere with total charge q and mass m is spinning with an angular velocity ω about one of its diameter. Find (a) the magnetic moment of this sphere (b) the angular momentum of the sphere. [Ans.(a) $\dfrac{q\omega R^2}{5}\hat{k}$]

3.Consider two parallel wires carrying currents I_1 and I_2 placed parallel to each other at separation d. Consider two elements dl_1 and dl_2 in the two wires (Figure). What is the force on the element dl_2 due to the current in dl_1? What is the force on the element dl_1 due to the current in dl_2? Is this consistent with Newton's third law?



4.A cylindrical magnet is kept with its axis along the z-axis. Close to the axis, the z-component of the field outside of the magnet can be approximated as $B_z = B_0 e^{-\alpha z^2}$. Using the symmetry, the ϕ component of the magnetic field is zero. Find the s-component of the field as a function of z for points close to the axis.



[Ans. $B_0\alpha s z e^{-\alpha z^2}$]

5.Charge is distributed uniformly with density ρ in a long cylindrical region of radius R. The whole distribution rotates with a constant angular velocity ω about its axis. Find the magnetic field everywhere due to this rotating distribution.

[Ans. $\dfrac{1}{2}\mu_0\rho\omega\left(R^2 - s^2\right)$ for $s < R$, zero for $s > R$]

6.An infinite cylinder of radius a carries a current with current density $\mathbf{J} = \mathbf{J}_0\left(\dfrac{s}{a}\right)\hat{k}$. A thin wire placed along the axis of the cylinder, carries a current in the opposite direction equal to that carried by the cylinder.

a) Compute the magnetic field $\mathbf{B}$ everywhere.
b) Show explicitly that divergence of this magnetic field is zero.

[Ans. $\dfrac{2\pi J_0}{3a}\left(s^3 - a^3\right)$]