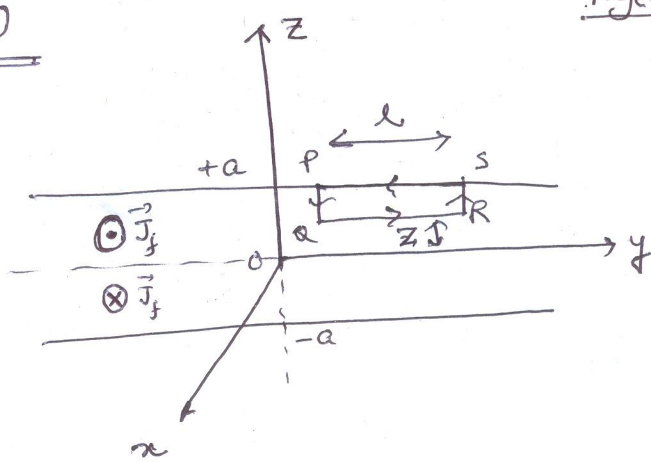
P.S #10



1. $\vec{J}_f = \frac{J_0 z}{a} \hat{x}$

We know that $\vec{B}$ due to a current sheet with current density $K\hat{x}$

$$\vec{B}(z) = \begin{cases} -\frac{\mu_0 K}{2}\hat{y} & \text{above the sheet} \\ \frac{\mu_0 K}{2}\hat{y} & \text{below the sheet} \end{cases}$$

Now if we have free current density $K\hat{x}$ on a sheet we'll have

$$\vec{H}(z) = \begin{cases} -\frac{K}{2}\hat{y} & \text{above the sheet} \\ \frac{K}{2}\hat{y} & \text{below the sheet} \end{cases}$$

- From symmetry $\vec{H}$ cannot depend on x, y, can depend only on z.
- If we think the slab as superposition of many current sheets and as each sheet produce $\vec{H}$ in $\pm\hat{y}$ dirn., the resultant $\vec{H}$ should also be only in $\pm\hat{y}$ direction.

$\Rightarrow \vec{H} = H(z)\hat{y} \rightarrow H$ due to the whole slab.

Total current in the slab $I = 0$

$\Rightarrow H = 0$ outside the slab (i.e for $z \geq a$, $z \leq -a$)

Now consider the Amperian loop PQRS as shown in fig.

$$\oint_{PQRS} \vec{H} \cdot \vec{dl} = \int_{z}^{a} \left(\frac{J_0 z'}{a}\right) l\, dz'$$

$$\Rightarrow H(z) l = \frac{J_0 l}{a}\left(\frac{a^2 - z^2}{2}\right) \Rightarrow H(z) = \frac{J_0 (a^2 - z_1^2)}{2a}$$

$$\Rightarrow \boxed{\vec{H}(z) = \frac{J_0 (a^2 - z^2)}{2a}\hat{y}} \quad \text{for } -a \leq z \leq a$$

This result holds throughout the slab, as it would be the same even if we extend the Amperian loop into the $z<0$ region.

$\vec{M} = \chi_m \vec{H}$

$\vec{B} = \mu_0 (1+\chi_m) \vec{H}$.

Outside the slab ~~[illegible]~~ $\vec{M} = 0$, $\vec{B} = 0$.

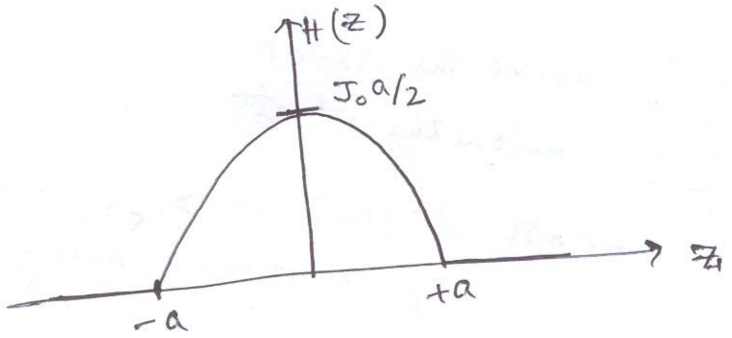
$\vec{K}_b = \vec{M} \times \hat{n} = 0$ as $\vec{H} = 0$ at $z = \pm a \Rightarrow \underline{\underline{M = 0}}$ at $z = \pm a$.

$\vec{J}_b = \vec{\nabla} \times \vec{M} = \chi_m \dfrac{J_0 z}{a} \hat{x}$

$= \chi_m \vec{J}_f$.

$\Rightarrow \int \vec{J}_b \cdot d\vec{a} = 0 \Rightarrow$ No net bound current.

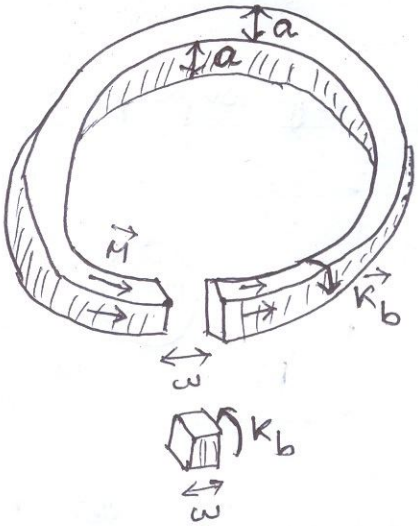
$\left(\text{as } \int \vec{J}_f \cdot d\vec{a} = 0 \right)$



2.

$\vec{K}_b = \vec{M} \times \hat{n}$

$$\begin{cases} = M \text{ on the surface of the rod} \\ = 0 \text{ at the ends.} \end{cases}$$



$\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$

$\Rightarrow$ Field inside a complete ring

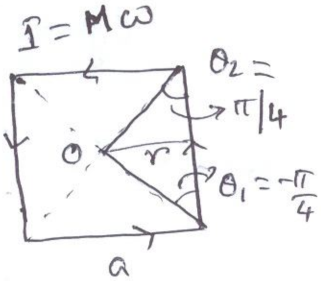
$\equiv$ field inside a solenoid with surface current M ($K_b = M$)

$= \mu_0 M$ along the axis of the rod.

Ring with a gap $\equiv$ a complete ring with $K_b = M$ and a small square loop of width ω superposed on the ring with the same K_b but in opposite direction.

Magnetic field due to current carrying wire

$$\vec{B} = \frac{\mu_0 I}{4\pi r}(\sin\theta_2 - \sin\theta_1)$$



At the centre of the square loop ($I = M\omega$)

$\theta_2 = \pi/4$, $\theta_1 = -\pi/4$, $r = a/2$

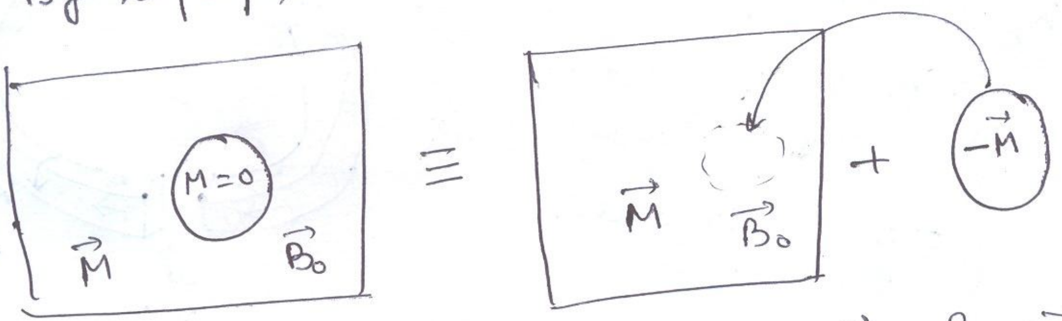
Considering four sides, total $\vec{B}$ at the centre

$$|\vec{B}| = 4 \times \frac{\mu_0 (M\omega)}{4\pi (a/2)}(2 \sin \pi/4) = \frac{2\sqrt{2}\,\mu_0 M\omega}{\pi a}$$

Net field inside the gap

$$\vec{B}_{Net} = \mu_0 \vec{M} - \frac{2\sqrt{2}\,\mu_0 \vec{M}\omega}{\pi a} = \mu_0 \vec{M}\left(1 - \frac{2\sqrt{2}\,\omega}{\pi a}\right).$$

3. (a) By superposition



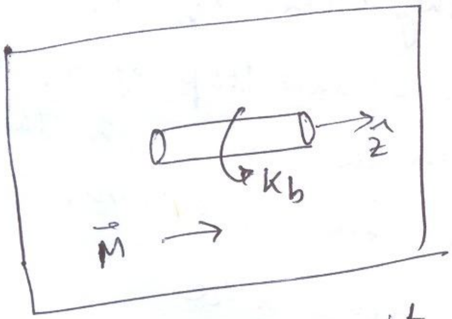
Field due to a magnetized sphere $\vec{B} = \frac{2}{3}\mu_0\vec{M}$

$\Rightarrow$ $\vec{B}$ inside the cavity $\vec{B} = \vec{B}_0 - \frac{2}{3}\mu_0\vec{M}$

Inside the cavity $\vec{M} = 0 \Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0} - \cancel{\vec{M}}^{\,0} = \frac{\vec{B}_0}{\mu_0} - \frac{2}{3}\vec{M}$

$= \vec{H}_0 + \vec{M} - \frac{2}{3}\vec{M} = \vec{H}_0 + \frac{1}{3}\vec{M}$.

(b)

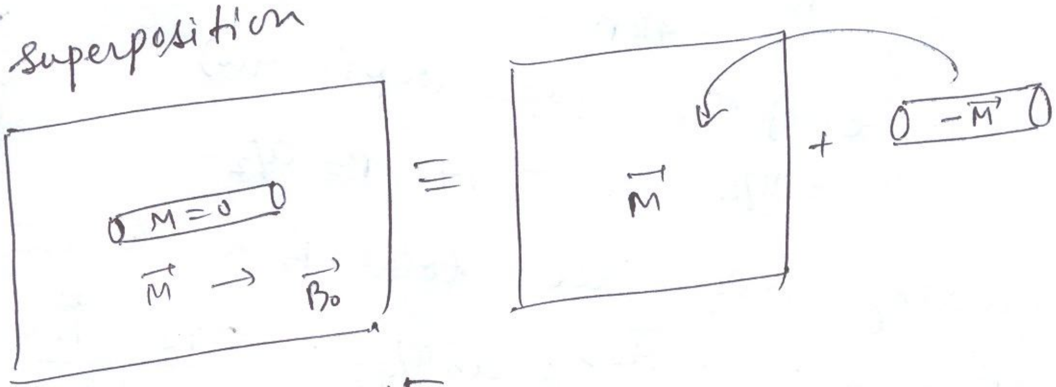


$\vec{J}_b = \nabla \times \vec{M} = 0$

$\vec{K}_b = \vec{M} \times \hat{n} = M\hat{\varphi}$

cylindrical cavity ≡ solenoid $\Rightarrow \vec{B} = \mu_0 M\hat{z}$ inside the solenoid.

Again by superposition

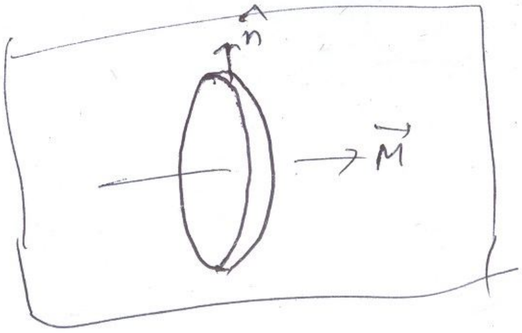


$\Rightarrow$ Field inside the cavity

$\vec{B} = \vec{B}_0 - \mu_0\vec{M}$

$\vec{H} = \frac{B}{\mu_0} - \cancel{\vec{M}}^{\,0} = \frac{\vec{B}_0}{\mu_0} - \vec{M} = \vec{H}_0 + \vec{M} - \vec{M} = \vec{H}_0$

3.(e)



$\vec{J}_b = \vec{\nabla} \times \vec{M} = 0$

$K_b = \vec{M} \times \hat{n}$

$\neq 0$ only on the curved surface.

thin wafer $\Rightarrow$ surface area of the curved surface is very small.

$\Rightarrow$ Total surface current = very small & negligible.

$\vec{B} = \vec{B}_0$ inside the cavity [boundary condition of $\vec{B}$]

$$\vec{H} = \vec{B}/\mu_0 - \cancel{\vec{M}}^{0} = \frac{\vec{B}_0}{\mu_0} = \vec{H}_0 + \vec{M}.$$

4. $\vec{M}$ = magnetization of the sphere.

$\vec{M} = \chi_m \vec{H}$

$\vec{B} = \vec{B}_0 + \frac{2}{3}\mu_0 \vec{M}$ = field inside the sphere.

$\Rightarrow \vec{B} = \mu_0(1+\chi_m)\vec{H} \quad \Rightarrow \vec{H} = \frac{\vec{B}}{\mu_0(1+\chi_m)} = \frac{(\vec{B}_0 + \frac{2}{3}\mu_0\vec{M})}{\mu_0(1+\chi_m)}$

$$\Rightarrow \vec{M} = \chi_m \cdot \frac{(\vec{B}_0 + \frac{2}{3}\mu_0\vec{M})}{\mu_0(1+\chi_m)}$$

$$\Rightarrow \vec{M} = \frac{3\chi_m}{\mu_0(3+\chi_m)}\vec{B}_0$$

Hence $\vec{B} = \vec{B}_0 + \frac{2}{3}\mu_0\vec{M} = \vec{B}_0 + \frac{2}{\cancel{3}} \frac{\cancel{3}\chi_m \cancel{\mu_0} \vec{B}_0}{\cancel{\mu_0}(3+\chi_m)}$

$$= \frac{3(1+\chi_m)}{3+\chi_m}\vec{B}_0$$