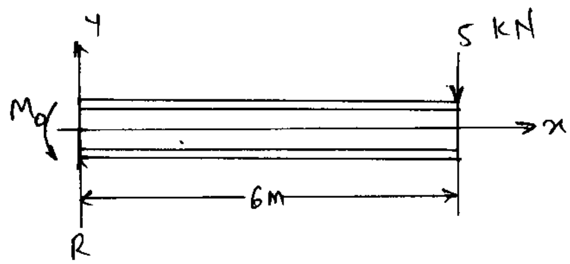
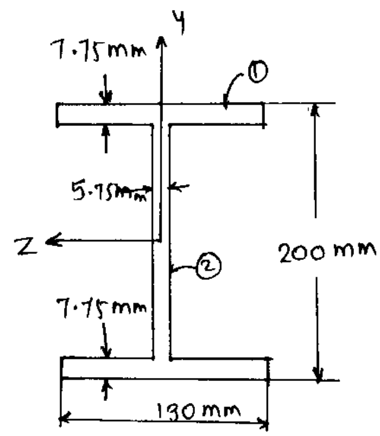
Solution of H/W and Practice problems from Chapter 7.

Solution to problem 7.14:

FBD





- Moment of Inertia:

$$I_{zz} = 2(I_{zz})_1 + (I_{zz})_2 .$$

$$(I_{zz})_1 = \frac{1}{12} \times 130 \times 7.75^3 + 130 \times 7.75 \times \left(100 - \frac{7.75}{2}\right)^2$$

$$= 9.31 \times 10^6 \ mm^4.$$

$$(I_{zz})_2 = \frac{1}{12} \times 5.75 \times (200 - 2 \times 7.75)^3 = 3 \times 10^6 \ mm^4.$$

$$\therefore \ I_{zz} = 21.62 \times 10^6 \ mm^4$$

- Equilibrium:

$$\Sigma F_y = 0 \Rightarrow R - 5 \times 10^3 N = 0 \Rightarrow R = 5 \times 10^3 N.$$

$$\Sigma M = 0 \Rightarrow M_o - 5 \times 10^3 \times 6 \times 10^3 = 0 \Rightarrow M_o = 30 \times 10^6 \ Nmm$$

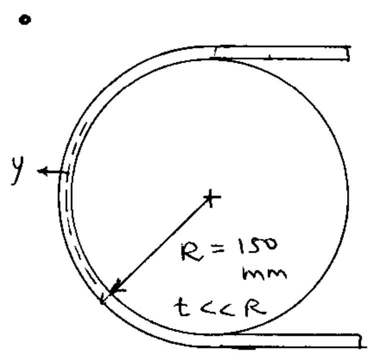
$$(M_b)_{max} = M_o = -30 \times 10^6 \ Nmm.$$

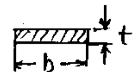
- Stress

$$(\sigma_x)_{max} = -\frac{M_{max} \, y_{max}}{I_{zz}} = -\frac{(-30 \times 10^6)(\pm 100)}{21.62 \times 10^6}$$

$$= \pm 138.76 \ N/mm^2.$$

Solution to problem 7.15





cross section.

Maximum strain $= \frac{y_{max}}{R}$

$= \frac{t/2}{R}$

$\therefore$ Maximum stress $= E\left(\frac{t/2}{R}\right)$

$= \frac{Et}{2R}$.

Given : maximum stress = 280 MN/m2.

$\therefore \quad \frac{Et}{2R} = 280 \Rightarrow t = \frac{280 \times 2 \times R}{E}$

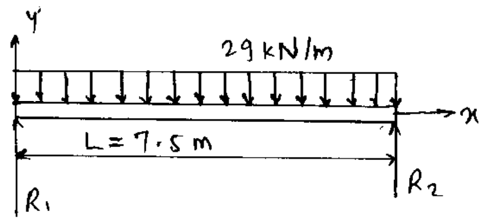
$= \frac{280 \times 2 \times 150}{210 \times 10^3} = 0.4$ mm.

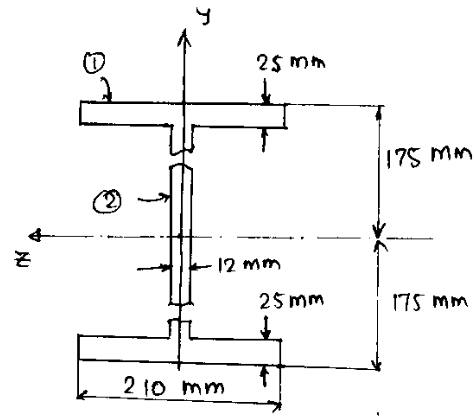
[Note: This is a plane stress solution (valid for b/t small). For real cases, b/t will usually be large and then a plane strain solution is more appropriate.]

- The maximum stress varies directly with t. Hence, if t is halved, the stress is halved to 140 MN/m^2.

Solution to problem 7.16 :

FBD :





• **Moment of inertia:**

$$I_{zz} = 2(I_{zz})_1 + (I_{zz})_2$$

$$(I_{zz})_1 = \frac{1}{12} \times 210 \times 25^3 + 210 \times 25 \times \left(175 - \frac{25}{2}\right)^2$$

$$= 138.9 \times 10^6 \ mm^4.$$

$$(I_{zz})_2 = \frac{1}{12} \times 12 \times (2 \times 175 - 2 \times 25)^3$$

$$= 27 \times 10^6 \ mm^4.$$

$$\therefore I_{zz} = 2 \times 138.9 \times 10^6 + 27 \times 10^6$$

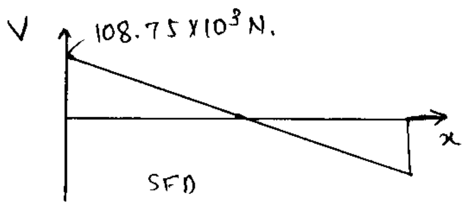
$$= 304.8 \times 10^6 \ mm^4.$$

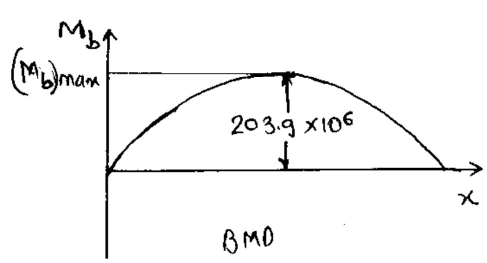
• **Equilibrium:**

From Symmetry : $R_1 = R_2$.

$$\Sigma F_y = 0 \Rightarrow R_1 = R_2 = \frac{29 \times 7.5 \times 10^3}{2} = 108.75 \times 10^3 \ N.$$

• BM :





Maximum Bending Moment:

$$(M_b)_{max} = R_1 \times \frac{L}{2} - 29 \times \frac{L}{2} \times \frac{L}{4}$$

$$= 108.75 \times 10^3 \times \frac{7.5 \times 10^3}{2} - 29 \times \frac{7.5}{2} \times 10^3 \times \frac{7.5 \times 10^3}{4}$$

$$= 407.8 \times 10^6 - 203.9 \times 10^6.$$

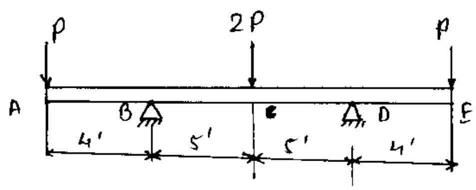
$$= 203.9 \times 10^6 \ Nmm.$$

• Stress :

$$\therefore (\sigma_x)_{max} = -\frac{(M_b)_{max} \, y_{max}}{I_{zz}} = -\frac{203.9 \times 10^6 \times (\pm 175)}{304.8 \times 10^6}$$

$$= \mp 117.07 \ N/mm^2.$$

Solution to problem 7.17

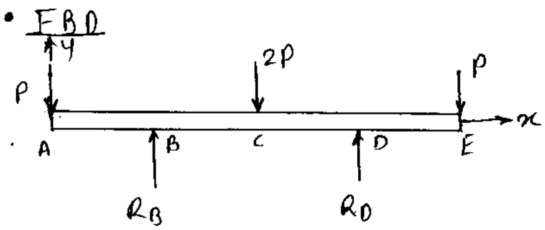


$\sigma_{all} = 5000$ psi in tension

$= 20000$ psi in compression.

To find: P_{max}.

• FBD



• Equilibrium:

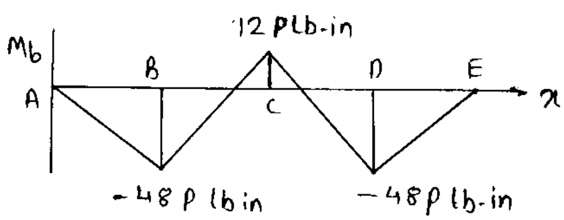
$\Sigma M_B = 0 \Rightarrow$

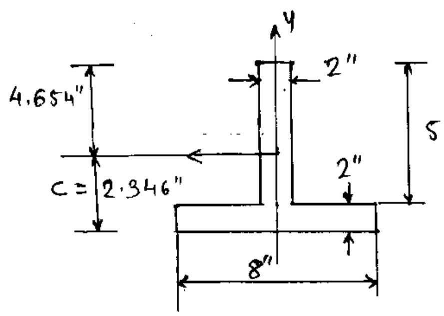
$10 R_D = 14P + 5(2P) - 4P = 20P$

$\Rightarrow R_D = 2P.$

$\Sigma F_y = 0 \Rightarrow R_B = 2P.$

• B.M.





• Centroid and Moment of Inertia:

$$c = \frac{5\times2\times4.5 + 8\times2\times1}{5\times2 + 8\times2} = 2.346''$$

$$I_{zz} = \left[\frac{2(5)^3}{12} + 5\times2\times(4.5-2.346)^2\right] + \frac{8(2)^3}{12} + \left[8\times2\times(2.346-1)^2\right]$$

$$= 101.55 \text{ in}^4$$

• Stress:

At B or D: Top fibre: $\sigma_x = -\dfrac{(-48P)\times(4.654)}{101.55} = +2.2P$ psi $\equiv \sigma_T$

bottom fibre: $\sigma_x = -\dfrac{(-48P)\times(-2.346)}{101.55} = -1.109$ psi $\equiv \sigma_C$

(problem 7.17 contd)

At C : Top fibre : $\sigma_x = -\dfrac{(12P)(4.654)}{101.55} = -0.55P$ psi.

bottom fibre : $\sigma_x = -\dfrac{(12P)(-2.346)}{101.55} = 0.277P$ psi.

- Determination of P

$\sigma_T = \sigma_{all}$ (in tension)

$\therefore 2.2P = 5{,}000 \Rightarrow P = 2{,}273$ lb.

$\sigma_c = \sigma_{all}$ (in compression)

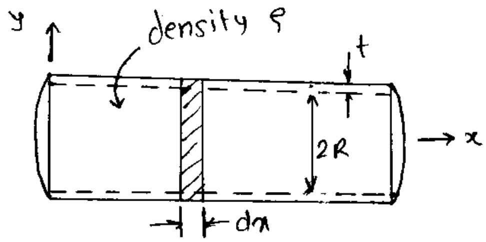
$1.109P = 20{,}000$

$P = 18{,}034$ lb $\rightarrow$ This will cause failure in tension.

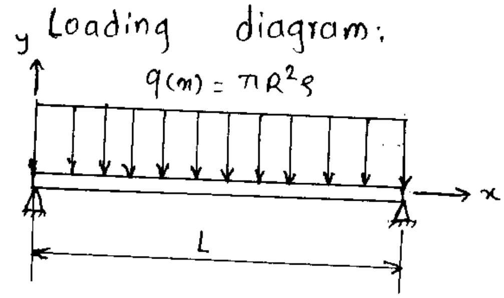
$\therefore P = 2273$ lb.

Solution to problem 7.23

We will consider the tank as a ~~freely~~ Simply supported beam loaded uniformly along its length. Since the tank weight is negligible, the intensity of loading (q) is the weight of liquid per unit length.



Loading diagram:



weight of liquid element

$= (\pi R^2 dx)\,\rho$

$\therefore\ q = \pi R^2 \rho$.

(intensity of loading)

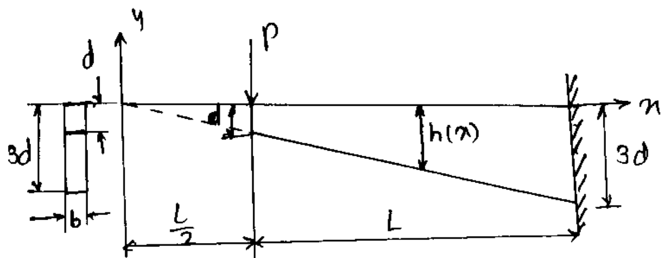
- $(M_b)_{max} = \frac{1}{8} q L^2 = \frac{\pi R^2 \rho L^2}{8}$.
- $I_{zz} = \pi R^3 t$

Maximum Bending Stress:

$$(\sigma_x)_{max} = -\frac{(M_b)_{max}\, y_{max}}{\pi R^3 t} = -\frac{\pi R^2 \rho L^2 (-R)}{8 \pi R^3 t} = \frac{\rho L^2}{8t}.$$

Thus, this stress is independent of R.

Solution to problem 7.24



$$h(x) = 2\frac{x}{L}d$$

• Assume that the expression for the bending stress σ_x is valid for the tapered beam also with σ_x being zero at $\frac{h(x)}{2}$ at any x. Then

$$(\sigma_x)_{max}(x) = \frac{M_b(x)\,\frac{h(x)}{2}}{I_{zz}(x)}$$

Now,

$$M_b(x) = P\left(x - \frac{L}{2}\right)$$

$$I_{zz}(x) = \frac{1}{12}bh^3(x) = \frac{1}{12}b\left[2\frac{x}{L}d\right]^3 = \frac{2}{3}\frac{bx^3d^3}{L^3}.$$

$$\therefore (\sigma_x)_{max}(x) = \frac{M_b(x)(h(x)/2)}{I_{zz}(x)} = \frac{P\left(x-\frac{L}{2}\right).\ \frac{xd}{L}}{\frac{2}{3}b\frac{x^3d^3}{L^3}}$$

$$= \frac{3}{2}\frac{PL^2}{bd^2}\frac{1}{x^2}\left(x - \frac{L}{2}\right).$$

• $(\sigma_x)_{max}(x)$ will be maximum at ~~a point~~ that x where:

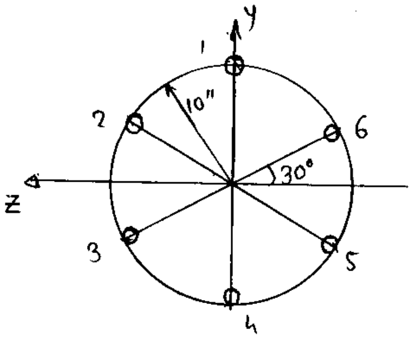
$$\frac{d}{dx}\left[\frac{1}{x^2}\left(x-\frac{L}{2}\right)\right] = 0 \quad \text{and} \quad \frac{d^2}{dx^2}\left[\frac{1}{x^2}\left(x-\frac{L}{2}\right)\right] < 0.$$

Above conditions are satisfied at $x = L$.

$$\therefore \text{maximum stress} = \frac{3}{2}\frac{PL^2}{bd^2}\frac{1}{L^2}\left(\frac{L}{2}\right).$$

$$= \frac{3}{4}\frac{PL}{bd^2}.$$

Solution to problem 7.25.



A = area of the bars.

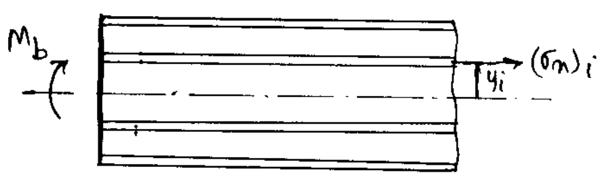
Neutral axis coincides with the centroidal axis because of the symmetry. Radius of curvature of the neutral axis $= \rho_0$

Assumptions: i) Only the bars carry bending stress.
ii) bending stress in each bar is uniform.

Strain: $(\epsilon_x)_i = -\,y_i/\rho_0$.

Stress: $(\sigma_x)_i = -\dfrac{E y_i}{\rho_0}$, — ①

where, y_i = co-ordinate of centroid of the bar. i.



Equilibrium:

$$M_b = \sum_{i=1}^{6} -(\sigma_x)_i\, A y_i .$$

$$= \sum_{i=1}^{6} \frac{E y_i}{\rho_0} A y_i .$$

$$= \frac{EA}{\rho_0} \sum_{i=1}^{6} y_i^2 .$$

$$\therefore \frac{E}{\rho_0} = \frac{M_b}{(A \sum_{i=1}^{6} y_i^2)} \quad \text{— ②}.$$

$$\therefore \text{① and ②} \Rightarrow (\sigma_x)_i = -\frac{M_b y_i}{A \sum_{i=1}^{6} y_i^2} = -\frac{M_b y_i}{I_{zz}} .$$

Where $I_{zz} = A \sum_{i=1}^{6} y_i^2$.

(problem 7.25 contd.)

$A = 1'' \times \frac{1}{2}'' = \frac{1}{2}$ sq. in

$y_i = +10'' \sin 30 = 5''$ for bar 2 and 6.

$= -10'' \sin 30 = -5''$ for bar 3 and 5.

$y_i = 10''$ for bar 1

$= -10''$ for bar 4.

$\therefore \; I_{zz} = \frac{1}{2}\left[2(10)^2 + 4(5)^2\right] = \frac{1}{2}\left[200 + 100\right]$

$= 150 \text{ in}^4.$

$\therefore (\sigma_x)_{max} = -\dfrac{M_b (y_i)_{max}}{I_{zz}} = -\dfrac{100\,000(-10)}{150}$

$= 6667$ psi.

Solution to problem 7.29:

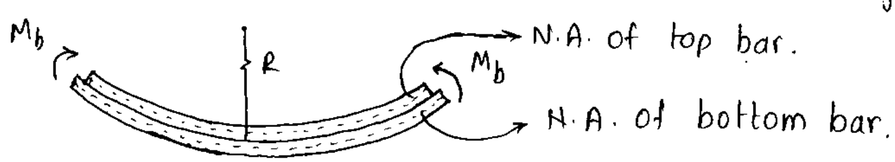
a) Beam of 2 identical bars soldered togather.

$$\frac{1}{\rho} = \frac{M_b}{EI_{zz}}$$

It bends as one unit.

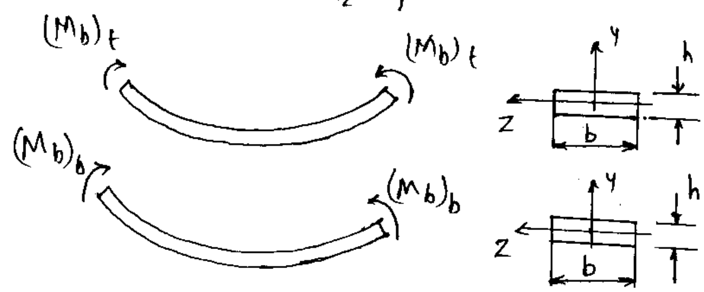
$$\left(\frac{M_b}{d\phi/ds}\right)_1 = \left(M_b\rho\right)_1 = E\,I_{zz} = \frac{Eb(2h)^3}{12} = \frac{2Ebh^3}{3}. \quad \text{--- (1)}$$

Beam of 2 indentical bars not soldered togather.



Each bar bends as a beam such that curvatures are ~~equal~~ compatible.

$$\left.\begin{array}{l} \rho_{\text{top bar}} = R - \frac{h}{2} \\ \rho_{\text{bottom bar}} = R + \frac{h}{2} \end{array}\right\} \Rightarrow \rho_{\text{top bar}} \cong \rho_{\text{bottom bar}} = \rho, \text{ say.}$$



$$(M_b)_t\,\rho = E\,I_{zz} = E\frac{bh^3}{12}$$

$$(M_b)_b\,\rho = E\,I_{zz} = \frac{Ebh^3}{12}.$$

$$M_b = (M_b)_t + (M_b)_b.$$

$$\therefore (M_b\rho)_2 = (M_b)_t\,\rho + (M_b)_b\,\rho$$

$$\therefore (M_b\rho)_2 = E\frac{bh^3}{12} + \frac{Ebh^3}{12} = \frac{Ebh^3}{6}. \quad \text{--- (2)}$$

(Problem 7.29 contd.)

(1) and (2) $\Rightarrow$

$$\frac{(M_b \rho)_1}{(M_b \rho)_2} = \frac{2Ebh^3}{3} \cdot \frac{6}{Ebh^3} = 4.$$

b] For a beam of two identical bars soldered togather:

$$\sigma_x = -\frac{M_b y}{I_{zz}} = -\frac{M_b y}{\frac{2bh^3}{3}} = -\frac{3}{2}\frac{M_b y}{bh^3}.$$

$$(\sigma_x)_{max1} = -\frac{3M_b y_{max}}{2bh^3} = \frac{-3M_b(\pm h)}{2bh^3} = \mp\frac{3}{2}\frac{M_b}{bh^2} \quad \text{—— (3)}$$

For a beam in which bars are not soldered.

$$(M_b)_t = (M_b)_b = \frac{M_b}{2}.$$

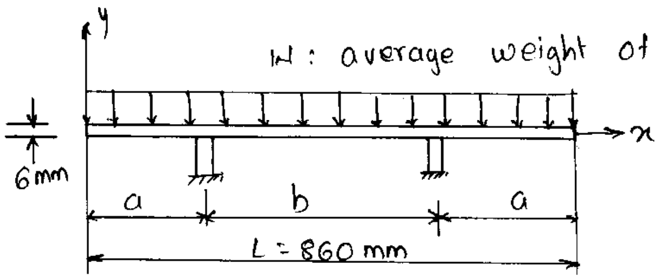
$$\sigma_x = -\frac{M_b/2\ y}{I_{zz}} = -\frac{M_b/2\ y}{\frac{bh^3}{12}} = -\frac{6M_b y}{bh^3}.$$

$$(\sigma_x)_{max2} = -\frac{6M_b y_{max}}{bh^3} = -\frac{6M_b\left(\pm\frac{h}{2}\right)}{bh^3} = \mp\frac{3M_b}{bh^2} \quad \text{—— (4)}$$

(3) and (4) $\Rightarrow$

$$\therefore \frac{(\sigma_x)_{max1}}{(\sigma_x)_{max2}} = \frac{3M_b}{2bh^2} \cdot \frac{bh^2}{3M_b} = \frac{1}{2}.$$

Solution to problem 7.35

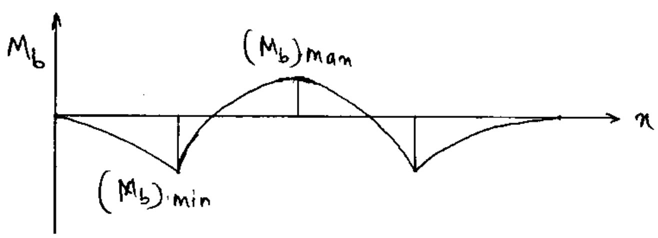


W : average weight of books per unit length.

- Support in optimal position.
- $\sigma_{all} = 7$ MPa in tension.
- To determine: W.

B.M.

· For optimal position of supports, $b = 0.586\,L$ $(L-b = 0.414\,L)$.
(See Prob 3.25)

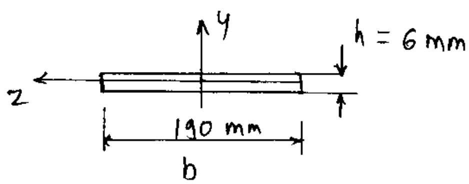


$$(M_b)_{max} = |(M_b)_{min}| = \frac{W(L-b)^2}{8}$$

$$= \frac{W(0.414\times 860)^2}{8}$$

$$= 15845.56\ W \text{ N mm}$$

MOI :



$$I_{zz} = \frac{bh^3}{12} = \frac{190\,(6)^3}{12}$$

$$= 3420 \text{ mm}^4.$$

Stress :

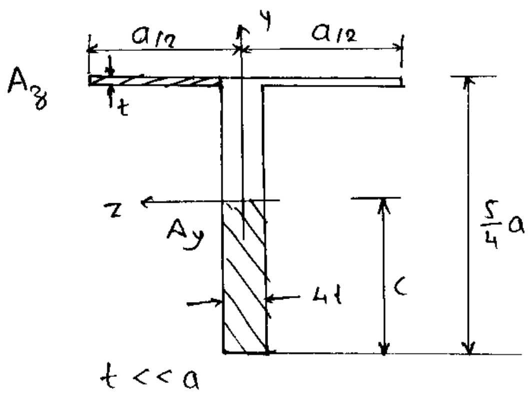
$$(\sigma_x)_{max} = -\frac{M_b\,y_{max}}{I_{zz}}$$

$$\therefore\ 7 = -\frac{15845.56\,W\,(-3)}{3420}$$

$$\therefore W = 0.504 \text{ N/mm}$$

$$= 504 \text{ N/m}.$$

Solution to problem 7.40.



Centroid:

$$c = \frac{a.t\ \frac{5a}{4} + \frac{5a}{4}\,4t\ \frac{5a}{8}}{at + \frac{5a}{4}\,4t.}$$

$$= \frac{10\,ta^2 + 25\,ta^2}{8(6at)}$$

$$= \frac{35\,a^2t}{48\,at} = \frac{35}{48}a.$$

Maximum bending stress:

a) flange: $$|\sigma_x|_{max} = \frac{M_b\left(\frac{5a}{4} - \frac{35}{48}a\right)}{I_{zz}} = \frac{M_b\,\frac{25a}{48}}{I_{zz}}.$$

b) Stem: $$|\sigma_x|_{max} = \frac{M_b\,\frac{35a}{48}}{I_{zz}}$$

$$\frac{|\sigma_x|_{max\ stem}}{|\sigma_x|_{max\ flange}} = \frac{35}{25} = 1.4$$

Maximum shear stress:

a) Flange: $$|\tau_{xz}|_{max} = \frac{V_y Q_z}{t\,I_{zz}} = \frac{V_y\,\frac{a}{2}t\left(\frac{5a}{4} - \frac{35a}{48}\right)}{t\,I_{zz}}$$ (Q_z = first moment of A_z about z-axis)

$$= \frac{25}{96}a^2\,\frac{V_y}{I_{zz}}.$$

b) Stem: $$|\tau_{xy}|_{max} = \frac{V_y|Q_y|}{4t\,I_{zz}} = \frac{V_y\,4t\left(\frac{35}{48}a\right)\left(\frac{35}{2\times 48}a\right)}{4t\,I_{zz}} = \frac{35^2}{(48)(96)}a^2\,\frac{V_y}{I_{zz}}.$$

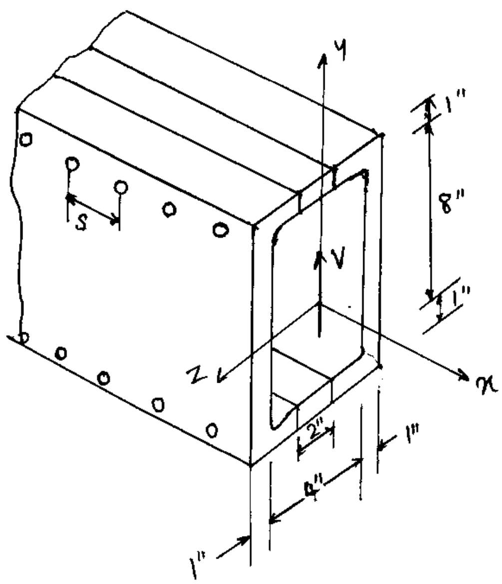
(Q_y = first moment of A_y about z-axis)

(problem 7.40 contd.)

$$\therefore \quad \frac{(\tau_{xy})_{max\ stem}}{(\tau_{xz})_{max\ flange}} = \frac{35^2}{48(96)} a^2 \, \frac{96}{25 a^2} = \frac{35^2}{48(25)}$$

$$= 1.02.$$

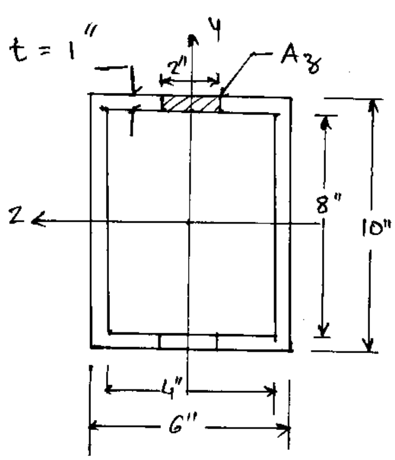
Solution to problem 7.41



Given:

- build up beam
- $\frac{1}{4}$" bolts with spacing s
- Each bolt can resist a shear force of 400 lb
- V = 10,000 lb

To find: spacing s.



* t = 1"

* $I_{zz} = \frac{6.(10)^3}{12} - \frac{4(8)^3}{12}$

$= 329.33 \text{ in}^4$

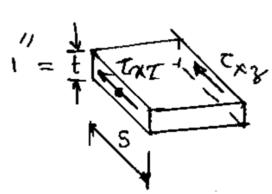
* Q_z = First moment of A_z about z-axis (shaded area)

$= 2 \times 1 \times 4.5$

$= 9 \text{ in}^3$.

$\tau_{xz} = \frac{VQ_z}{2t\, I_{zz}}$ (shear acts on two faces)

$= \frac{10{,}000 \times 9}{2 \times 1 \times 329.33} = 136.64 \text{ lb/in}^2$



· Force on 1 bolt $= \tau_{xz}\,(s)(t)$

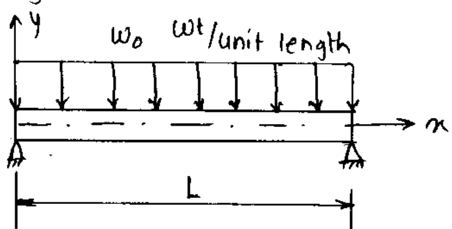
$= 136.64\, s$ lb

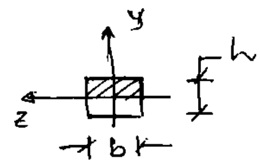
· For safe design $136.64 s = 400$

$\therefore s = 2.93"$

Solution to problem 7.44

- loading

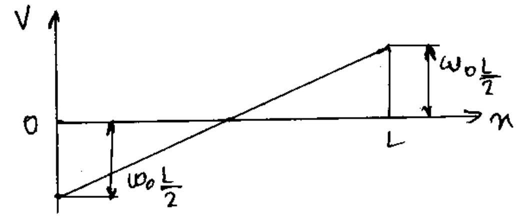




$(Q_y)_{max}$

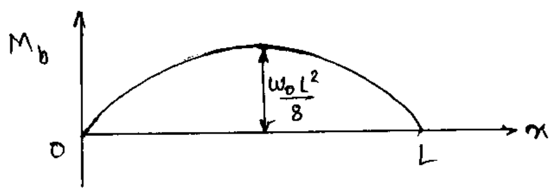
= First moment of shaded area about z-axis

- SF and BM Diagrams



Maximum shear force

$$V_{max} = w_0 \frac{L}{2}$$



maximum bending moment

$$(M_b)_{max} = w_0 \frac{L^2}{8}.$$

Maximum bending stress =

$$(\sigma_x)_{max} = \frac{-(M_b)_{max}\left(-\frac{h}{2}\right)}{I_{zz}} = \frac{1}{I_{zz}} \frac{w_0 L^2 h}{16}.$$

Maximum shear stress =

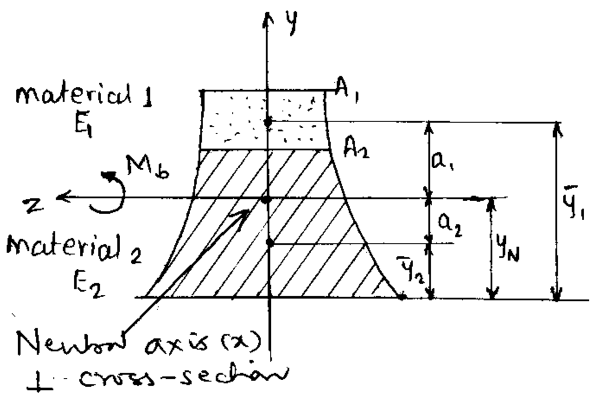
$$(\tau_{xy})_{max} = \frac{V_{max}(Q_y)_{max}}{b\, I_{zz}} = \frac{1}{I_{zz}} \frac{V_{max}}{b}\, b\left(\frac{h}{2}\right).\frac{h}{4}$$

$$= \frac{1}{I_{zz}} \frac{w_0 L h^2}{16}.$$

$$\therefore \frac{(\sigma_x)_{max}}{(\tau_{xy})_{max}} = \frac{1}{I_{zz}} \frac{w_0 L^2 h}{16} * \frac{I_{zz} \cdot 16}{w_0 L^2 h}$$

$$= \frac{L}{h}.$$

Solution to problem 7.46



ρ = radius of curvature of the deformed neutral axis

a_1 = distance of centroid of A_1 from NA
$= \bar{y}_1 - y_N$

a_2 = distance of centroid of A_2 from NA
$= y_N - \bar{y}_2$.

Strain: $\epsilon_x = -y/\rho$ (y measured from neutral axis) —— (1)

Stress: $$\left.\begin{aligned}\sigma_x &= -E_1 \frac{y}{\rho} \quad \text{in material 1} \\ &= -E_2 \frac{y}{\rho} \quad \text{in material 2.}\end{aligned}\right\} \quad \text{—— (2)}$$

(a) Location of Neutral axis.

$$\int_A \sigma_x \, dA = 0 \Rightarrow \int_{A_1} -E_1 \frac{y}{\rho} dA + \int_{A_2} -E_2 \frac{y}{\rho} dA = 0$$

$$\Rightarrow -\frac{1}{\rho}\left[E_1(\bar{y}_1 - y_N) A_1 + E_2 \underbrace{(\bar{y}_2 - y_N)}_{\text{Negative}} A_2\right] = 0$$

$$\Rightarrow y_N = \frac{E_1 A_1 \bar{y}_1 + E_2 A_2 \bar{y}_2}{E_1 A_1 + E_2 A_2} . \quad \text{—— (3)}$$

(b) Expression for curvature:

$$\int_A \sigma_x \, y \, dA = -M_b \Rightarrow \int_{A_1} -E_1 \frac{y^2}{\rho} dA + \int_{A_2} -E_2 \frac{y^2}{\rho} dA = -M_b$$

$$\Rightarrow \frac{1}{\rho}\left[E_1 (I_{zz})_1 + E_2 (I_{zz})_2\right] = M_b$$

(Here, $(I_{zz})_1$ & $(I_{zz})_2$: MOI of A_1 and A_2 about z-axis of Neutral surface)

$$\Rightarrow \frac{1}{\rho} = \frac{M_b}{E_1 (I_{zz})_1 + E_2 (I_{zz})_2} . \quad \text{—— (4)}$$

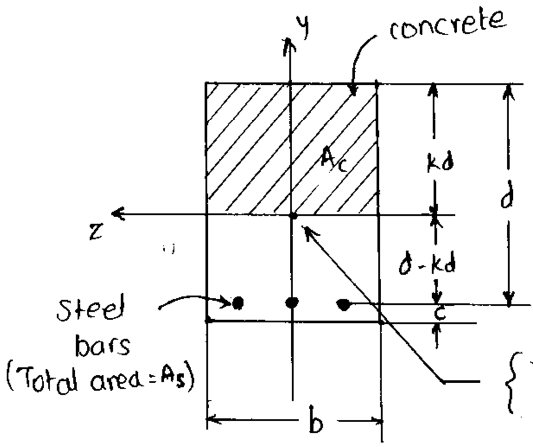
(problem 7.46 contd.)

(c) Stress: Equations (2) and (4) $\Rightarrow$

$$(\sigma_{x})_i = -E_i \frac{M_b y}{E_1 (I_{zz})_1 + E_2 (I_{zz})_2} \quad \text{——— (5)}$$

where i takes on the value 1 or 2, depending on which material we are interested in.

Solution to problem 7.51



- Assume:
  i) No tensile stress is carried by concrete.
  ii) Tensile stress in bars is uniform.
- ρ = radius of curvature of the deformed neutral axis

Strain: $\epsilon_x = -y/\rho$ (y measured from the neutral axis) —— (1)

Stress: $$\left.\begin{aligned}\sigma_x &= -E_c \frac{y}{\rho} && \text{for concrete} \\ &= -E_s \frac{y}{\rho} && \text{for steel}\end{aligned}\right\} \text{—— (2)}$$

- Location of neutral axis:

$$\left[\int_{A_c} y\,dA = [b(kd)]\left(\frac{kd}{2}\right)\right]$$

$$\int_A \sigma_x \, dA = 0 \Rightarrow \int_{A_c} -E_c \frac{y}{\rho}\, dA - \frac{E_s}{\rho}[-(d-kd)]A_s = 0.$$

$$\Rightarrow -\frac{E_c}{\rho}\left\{[b\,(kd)]\left(\frac{kd}{2}\right)\right\} + \frac{E_s A_s}{\rho}(d-kd) = 0.$$

$$\Rightarrow E_s A_s (d-kd) - \frac{1}{2} E_c b (kd)^2 = 0. \text{ —— (3)}$$

- Expression for curvature:

$$\left[\int_{A_c} y^2 dA = \frac{1}{12} b(kd)^3 + [b(kd)]\left(\frac{kd}{2}\right)^2\right]$$

$$\int_A \sigma_x \, y \, dA = -M_b$$

$$\Rightarrow \int_{A_c} -E_c \frac{y^2}{\rho}\, dA - \frac{E_s}{\rho}(d-kd)^2 A_s = -M_b$$

$$\Rightarrow -\frac{E_c}{\rho}\left[b\frac{(kd)^3}{12} + b(kd)\left(\frac{kd}{2}\right)^2\right] - \frac{E_s}{\rho}(d-kd)^2 A_s = -M_b.$$

$$\Rightarrow \frac{1}{\rho}\left[E_c b \frac{(kd)^3}{3} + E_s A_s (d-kd)^2\right] = M_b.$$

(problem 7.51 contd.)

$$\Rightarrow \frac{1}{\rho}\Big[\underbrace{2E_sA_s(d-kd)\frac{kd}{3}}_{\text{using eq.(3)}} + E_sA_s(d-kd)^2\Big] = M_b.$$

$$\Rightarrow \frac{1}{\rho} = \frac{3M_b}{E_sA_s(d-kd)(3d-kd)} \qquad \text{———— (4)}$$

- **Stress:** Equations (2) and (4) $\Rightarrow$

* **steel:**

$$\sigma_x = -E_s\frac{3M_b y}{E_sA_s(d-kd)(3d-kd)}$$

$$= -\frac{3M_b[-(d-kd)]}{A_s(d-kd)(3d-kd)} \qquad \left(\begin{matrix}\text{Since,}\\ y=-(d-kd)\end{matrix}\right)$$

$$= \frac{3M_b}{A_s.3d(1-k/3)}$$

$$= \frac{M_b}{A_s d(1-k/3)}.$$

* **concrete:**

$$\sigma_x = -E_c\frac{3M_b y}{E_sA_s(d-kd)(3d-kd)}$$

$$= -E_c\frac{3M_b y}{\frac{1}{2}E_c b(kd)^2(3d-kd)} \qquad \left(\begin{matrix}\text{using eq.(3):}\\ E_sA_s(d-kd)=\frac{1}{2}E_c b(kd)^2\end{matrix}\right)$$

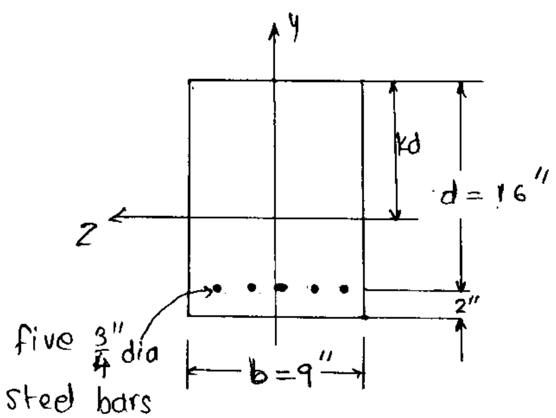
$$= -\frac{6M_b y}{bk^2d^2.3d(1-k/3)}$$

$$= -\frac{2M_b y}{bk^2d^3(1-k/3)}.$$

$$(\sigma_x)_{max} = -\frac{2M_b kd}{bk^2d^3(1-k/3)} \qquad \left(\begin{matrix}(\sigma_x)_{max}\text{ at}\\ y=kd\end{matrix}\right)$$

$$= -\frac{2M_b}{bkd^2(1-k/3)}$$

Solution to problem 7.52



Given:

- $E_s = 30 \times 10^6$ psi, $E_c = 1.5 \times 10^6$ psi
- Steel: $\sigma_{all} = 20{,}000$ psi in tension
- concrete: $\sigma_{all} = 1{,}350$ psi in compression

To find: $(M_b)_{max}$.

- Note:

$$A_s = 5\,[\pi(3/8)^2] = \frac{45\pi}{64}\ \text{in}^2$$

- location of NA:

$$E_s A_s d(1-k) - \frac{1}{2} E_c b k^2 d^2 = 0.$$

$$\Rightarrow 30\times10^6 \times \frac{45\pi}{64} \times 16(1-k) - \frac{1}{2}\, 1.5\times10^6 \times 9\, k^2\, 16^2 = 0.$$

$$\Rightarrow 1.629\, k^2 + k - 1 = 0$$

$$\Rightarrow k = -1.1485,\ 0.5345.$$

only possible value is +ve value $\therefore k = 0.5345$.

- Max. Stress:

* Steel:
$$\sigma_x = \frac{M_b}{A_s d(1-k/3)} = \frac{M_b}{\frac{45\pi}{64}\, 16\left(1 - \frac{0.5345}{3}\right)} = 0.0344\, M_b.$$

$$\therefore 20000 = 0.0344\, M_b \Rightarrow M_b = 5.814\times10^5\ \text{lb.in} \quad \text{—(1)}$$

* concrete:
$$(\sigma_x)_{max} = \frac{2M_b}{bd^2k(1-k/3)} = \frac{2M_b}{9\times16^2\times0.5345\left(1-\frac{0.5345}{3}\right)}$$
$$= 1.976\times10^{-3}\, M_b.$$

$$\therefore 1350 = 1.976\times10^{-3} M_b \Rightarrow M_b = 6.832\times10^5\ \text{lb.in} \quad \text{—(2)}$$

* Maximum M_b is the smallest of (1) and (2).

$$\therefore (M_b)_{max} = 5.814\times10^5\ \text{lb.in.}$$