Table 8.3 Uncertainties in β

	Uncertainty		
	Random	Systematic	Total
	$s_{\overline{\beta}}$	b_β	$\pm u_\beta$ 95%
T [°C]	[K]	[K]	[K]
100	13.3	37.4	79.4
125	10.6	53.9	109.9
150	8.8	78.4	157.8

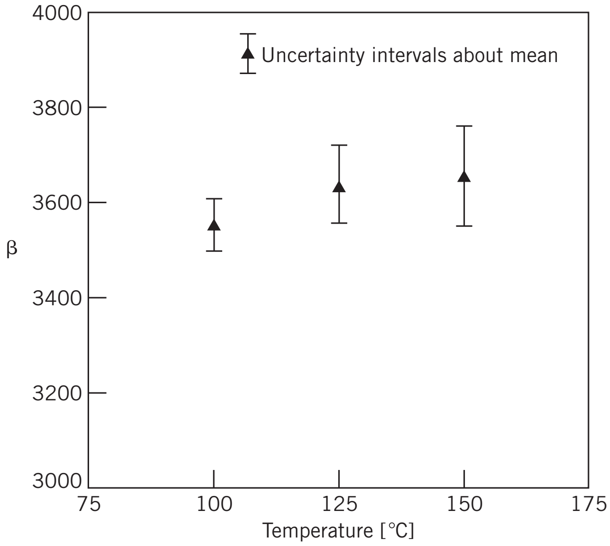

Figure 8.12 Measured values of β and associated uncertainties for three temperatures. Each data point represents $\overline{\beta} \pm u_\beta$.

The original results of the measured values of β must now be reexamined. The results from Table 8.3 are plotted as a function of temperature in Figure 8.12, with 95% uncertainty limits on each data point. Clearly, there is no justification for assuming that the measured values indicate a trend of changes with temperature, and it would be appropriate to use either the average value of β or the value determined from the linear least-squares curve fit.

8.5 THERMOELECTRIC TEMPERATURE MEASUREMENT

The most common method of measuring and controlling temperature uses an electrical circuit called a thermocouple. A *thermocouple* consists of two electrical conductors that are made of dissimilar metals and have at least one electrical connection. This electrical connection is referred to as a *junction*. A thermocouple junction may be created by welding, soldering, or by any method that provides good electrical contact between the two conductors, such as twisting the wires around one another. The output of a thermocouple circuit is a voltage, and there is a definite relationship between this voltage and the temperatures of the junctions that make up the thermocouple circuit. We will examine the causes of this voltage, and develop the basis for using thermocouples to make engineering measurements of temperature.

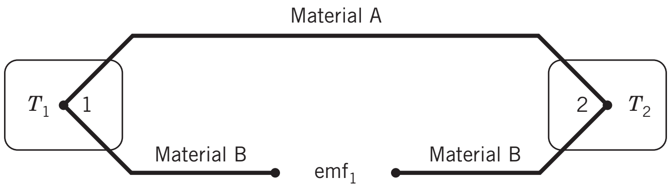

Figure 8.13 Basic thermocouple circuit.

Consider the thermocouple circuit shown in Figure 8.13. The junction labeled 1 is at a temperature T_1 and the junction labeled 2 is at a temperature T_2. This thermocouple circuit measures the difference between T_1 and T_2. If T_1 and T_2 are not equal, a finite open-circuit electric potential, emf$_1$, is measured. The magnitude of the potential depends on the difference in the temperatures and the particular metals used in the thermocouple circuit.

A thermocouple junction is the source of an *electromotive force* (emf), which gives rise to the potential difference in a thermocouple circuit. It is the basis for temperature measurement using thermocouples. The circuit shown in Figure 8.13 is the most common form of a thermocouple circuit used for measuring temperature.

It is our goal to understand the origin of thermoelectric phenomena and the requirements for providing accurate temperature measurements using thermocouples. In an electrical conductor that is subject to a temperature gradient, there will be both a flow of thermal energy and a flow of electricity. These phenomena are closely tied to the behavior of the free electrons in a metal; it is no coincidence that good electrical conductors are, in general, good thermal conductors. The characteristic behavior of these free electrons in an electrical circuit composed of dissimilar metals results in a useful relationship between temperature and emf. There are three basic phenomena that can occur in a thermocouple circuit: (1) the *Seebeck effect*, (2) the *Peltier effect*, and (3) the *Thomson effect*.

Under measurement conditions with no loading errors, the emf generated by a thermocouple circuit would be the result of the Seebeck effect only.

Seebeck Effect

The Seebeck effect, named for Thomas Johann Seebeck (1770–1831), refers to the generation of a voltage potential, or emf, in an open thermocouple circuit due to a difference in temperature between junctions in the circuit. The Seebeck effect refers to the case when there is no current flow in the circuit, as for an open circuit. There is a fixed, reproducible relationship between the emf and the junction temperatures T_1 and T_2 (Fig. 8.13). This relationship is expressed by the Seebeck coefficient, α_{AB}, defined as

$$\alpha_{AB} = \left[\frac{\partial(\text{emf})}{\partial T}\right]_{\text{open circuit}} \tag{8.15}$$

where A and B refer to the two materials that comprise the thermocouple. Since the Seebeck coefficient specifies the rate of change of voltage with temperature for the materials A and B, it is equal to the static sensitivity of the open-circuit thermocouple.

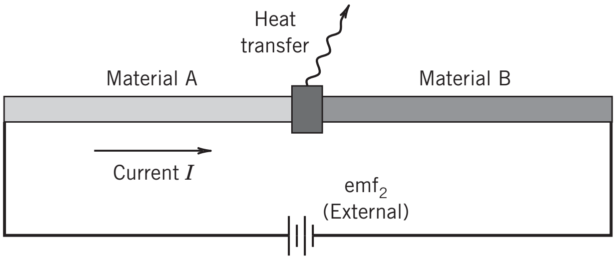

Figure 8.14 Peltier effect due to current flow across a junction of dissimilar metals.

Peltier Effect

A familiar concept is that of I^2R or joule heating in a conductor through which an electrical current flows. Consider the two conductors having a common junction, shown in Figure 8.14, through which an electrical current I flows due to an externally applied emf. For any portion of either of the conductors, the energy removal rate required to maintain a constant temperature is I^2R, where R is the resistance to a current flow and is determined by the resistivity and size of the conductor. However, at the junction of the two dissimilar metals the removal of a quantity of energy different than I^2R is required to maintain a constant temperature. The difference in I^2R and the amount of energy generated by the current flowing through the junction is due to the Peltier effect. The Peltier effect is due to the thermodynamically reversible conversion of energy as a current flows across the junction, in contrast to the irreversible dissipation of energy associated with I^2R losses. The Peltier heat is the quantity of heat in addition to the quantity I^2R that must be removed from the junction to maintain the junction at a constant temperature. This amount of energy is proportional to the current flowing through the junction; the proportionality constant is the Peltier coefficient π_{AB}, and the heat transfer required to maintain a constant temperature is

$$Q_\pi = \pi_{AB} I \tag{8.16}$$

caused by the Peltier effect alone. This behavior was discovered by Jean Charles Athanase Peltier (1785–1845) during experiments with Seebeck's thermocouple. He observed that passing a current through a thermocouple circuit having two junctions, as in Figure 8.13, raised the temperature at one junction, while lowering the temperature at the other junction. This effect forms the basis of a device known as a Peltier refrigerator, which provides cooling without moving parts.

Thomson Effect

In addition to the Seebeck effect and the Peltier effect, there is a third phenomenon that occurs in thermoelectric circuits. Consider the conductor shown in Figure 8.15, which is subject to a

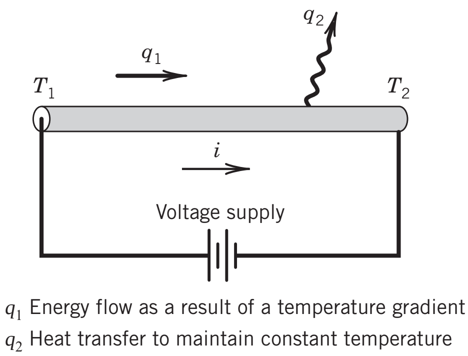

Figure 8.15 Thomson effect due to simultaneous flows of current and heat.

longitudinal temperature gradient and also to a potential difference, such that there is a flow of current and heat in the conductor. Again, to maintain a constant temperature in the conductor it is found that a quantity of energy different than the joule heat, I^2R, must be removed from the conductor. First noted by William Thomson (1824–1907, Lord Kelvin from 1892) in 1851, this energy is expressed in terms of the Thomson coefficient, σ, as

$$Q_\sigma = \sigma I(T_1 - T_2) \tag{8.17}$$

For a thermocouple circuit, all three of these effects may be present and may contribute to the overall emf of the circuit.

Fundamental Thermocouple Laws

The basic thermocouple circuit shown in Figure 8.16 can be used to measure the difference between the two temperatures T_1 and T_2. For practical temperature measurements, one of these junctions becomes a reference junction, and is maintained at some known, constant reference temperature, say T_2. The other junction then becomes the measuring junction, and the emf existing in the circuit provides a direct indication of the temperature of the measuring junction T_1.

The use of thermocouple circuits to measure temperature is based on observed behaviors of carefully controlled thermocouple materials and circuits. The following laws provide the basis necessary for temperature measurement with thermocouples:

1. ***Law of homogeneous materials:*** *A thermoelectric current cannot be sustained in a circuit of a single homogeneous material by the application of heat alone, regardless of how it might vary in cross section*. Simply stated, this law requires that at least two materials be used to construct a thermocouple circuit for the purpose of measuring temperature. It is interesting to note that a current may occur in an inhomogeneous wire that is nonuniformly heated; however, this is neither useful nor desirable in a thermocouple.
2. ***Law of intermediate materials:*** *The algebraic sum of the thermoelectric forces in a circuit composed of any number of dissimilar materials is zero if all of the circuit is at a uniform temperature*. This law allows a material other than the thermocouple materials to be inserted into a thermocouple circuit without changing the output emf of the circuit. As an example, consider the thermocouple circuit shown in Figure 8.16, where the junctions of the measuring device are made of copper and material B is an alloy (not pure copper). The electrical connection between the measuring device and the thermocouple circuit forms yet another thermocouple junction. The law of intermediate materials, in this case, provides that

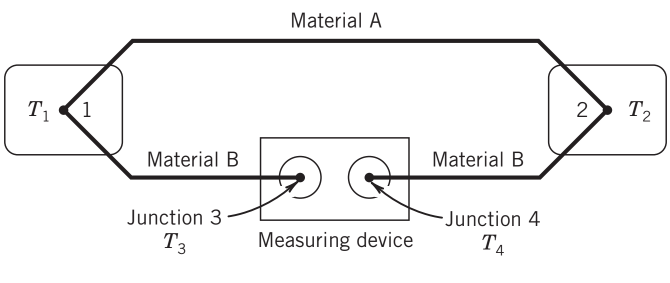

Figure 8.16 Typical thermocouple measuring circuit.

the measured emf will be unchanged from the open-circuit emf, which corresponds to the temperature difference between T_1 and T_2, if $T_3 = T_4$. Another practical consequence of this law is that copper extension wires may be used to transmit thermocouple emfs to a measuring device.

3. ***Law of successive or intermediate temperatures:*** *If two dissimilar homogeneous materials that form a thermocouple circuit produce emf$_1$ when the junctions are at T_1 and T_2 and produce emf$_2$ when the junctions are at T_2 and T_3, the emf generated when the junctions are at T_1 and T_3 will be emf$_1$ + emf$_2$.* This law allows a thermocouple calibrated for one reference temperature, say T_2, to be used at another reference temperature, such as T_3, to determine temperature T_1.

Basic Temperature Measurement with Thermocouples

Let's first examine a historically significant method of using a thermocouple circuit to measure temperature. Figure 8.17 shows two basic thermocouple circuits, using a chromel–constantan thermocouple and an ice bath to create a reference temperature. In Figure 8.17a, the thermocouple wires are connected directly to a potentiometer to measure the emf. In Figure 8.17b, copper extension wires are employed, creating two reference junctions. The law of intermediate materials ensures that neither the potentiometer nor the extension wires will change the emf of the circuit, as long as the two connecting junctions at the potentiometer and the two in the ice bath experience no temperature difference. All that is required to be able to measure temperature with this circuit is to know the relationship between the output emf and the temperature of the measuring junction, for the particular reference temperature. One method of determining this relationship is to calibrate the thermocouple. However, we shall see that for reasonable levels of uncertainty for temperature measurement, standard materials and procedures allow thermocouples to be accurate temperature measuring devices without the necessity of calibration.

Reference Junction

The provisions for a reference junction should provide a temperature that is accurately known, stable, and reproducible. A very common reference junction temperature is provided by the ice point, 0°C, because of the ease with which it can be obtained.

The creation of a reference junction temperature of 0°C is accomplished in either of two basic ways. Prior to the development of an electronic means of creating a reference point in the electric circuit, an ice bath served to provide the reference junction temperature. An ice bath is typically made by filling a vacuum flask, or Dewar, with finely crushed ice, and adding just enough water to create a transparent slush. When done correctly, the method is surprising accurate and reproducible. A few ice cubes floating in water do not create a 0°C environment! Ice baths can be constructed to provide a reference junction temperature to an uncertainty within ±0.01°C.

Electronic reference junctions provide a convenient means of the measurement of temperature without the necessity to construct an ice bath. Numerous manufacturers produce commercial

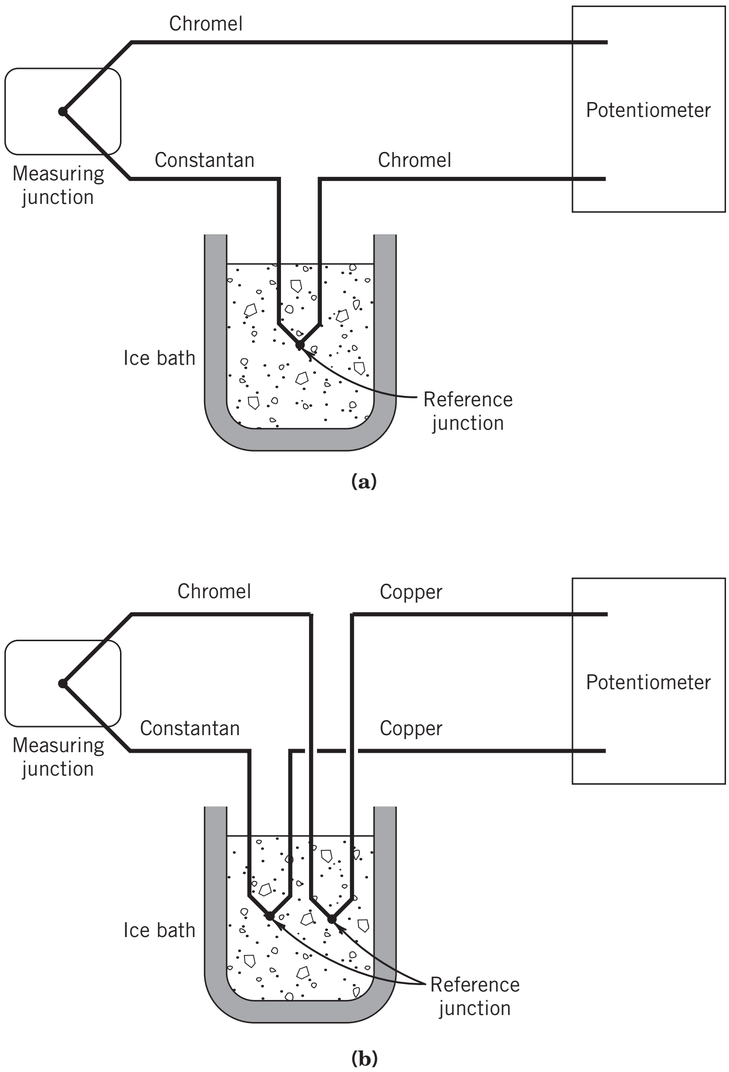

Figure 8.17 Thermocouple temperature measurement circuits.

temperature measuring devices with built-in reference junction compensation, and many digital data acquisition cards for personal computers include built-in reference junction compensation. The electronics generally rely on a thermistor, a temperature-sensitive integrated circuit, or an RTD to determine the reference junction temperature, as shown for a thermistor in Figure 8.18.

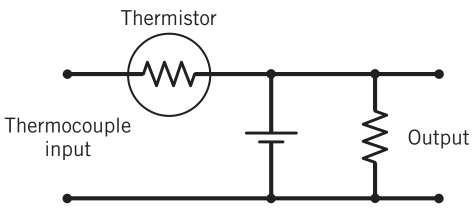

Figure 8.18 Basic thermistor circuit for thermocouple reference junction compensation.

Table 8.4 Thermocouple Designations

	Material Combination		
Type	Positive	Negative	Applications
E	Chromel(+)	Constantan(−)	Highest sensitivity (<1000°C)
J	Iron(+)	Constantan(−)	Nonoxidizing environment (<760°C)
K	Chromel(+)	Alumel(−)	High temperature (<1372°C)
S	Platinum/ 10% rhodium	Platinum(−)	Long-term stability high temperature (<1768°C)
T	Copper(+)	Constantan(−)	Reducing or vacuum environments (<400°C)

Uncertainties for the reference junction temperature in this case are on the order of ±0.1 °C, with ±0.5°C as typical.

Thermocouple Standards

The National Institute of Standards and Technology (NIST) provides specifications for the materials and construction of standard thermocouple circuits for temperature measurement (10). Many material combinations exist for thermocouples; these material combinations are identified by a thermocouple type and denoted by a letter. Table 8.4 shows the letter designations and the polarity of common thermocouples, along with some basic application information for each type. The choice of a type of thermocouple depends on the temperature range to be measured, the particular application, and the desired uncertainty level.

To determine the emf output of a particular material combination, a thermocouple is formed from a candidate material and a standard platinum alloy to form a thermocouple circuit having a 0°C reference temperature. Figure 8.19 shows the output of various materials in combination with platinum-67. The notation indicates the thermocouple type. The law of intermediate temperatures then allows the emf of any two materials, whose emf relative to platinum is known, to be determined. Figure 8.20 shows a plot of the emf as a function of temperature for some common thermocouple material combinations. The slope of the curves in this figure corresponds to the static sensitivity of the thermocouple measuring circuit.

Standard Thermocouple Voltage

Table 8.5 provides the standard composition of thermocouple materials, along with standard limits of error for the various material combinations. These limits specify the expected maximum errors resulting from the thermocouple materials. The NIST uses high-purity materials to establish the standard value of voltage output for a thermocouple composed of two specific materials. This results in standard tables or equations used to determine a measured temperature from a measured value of

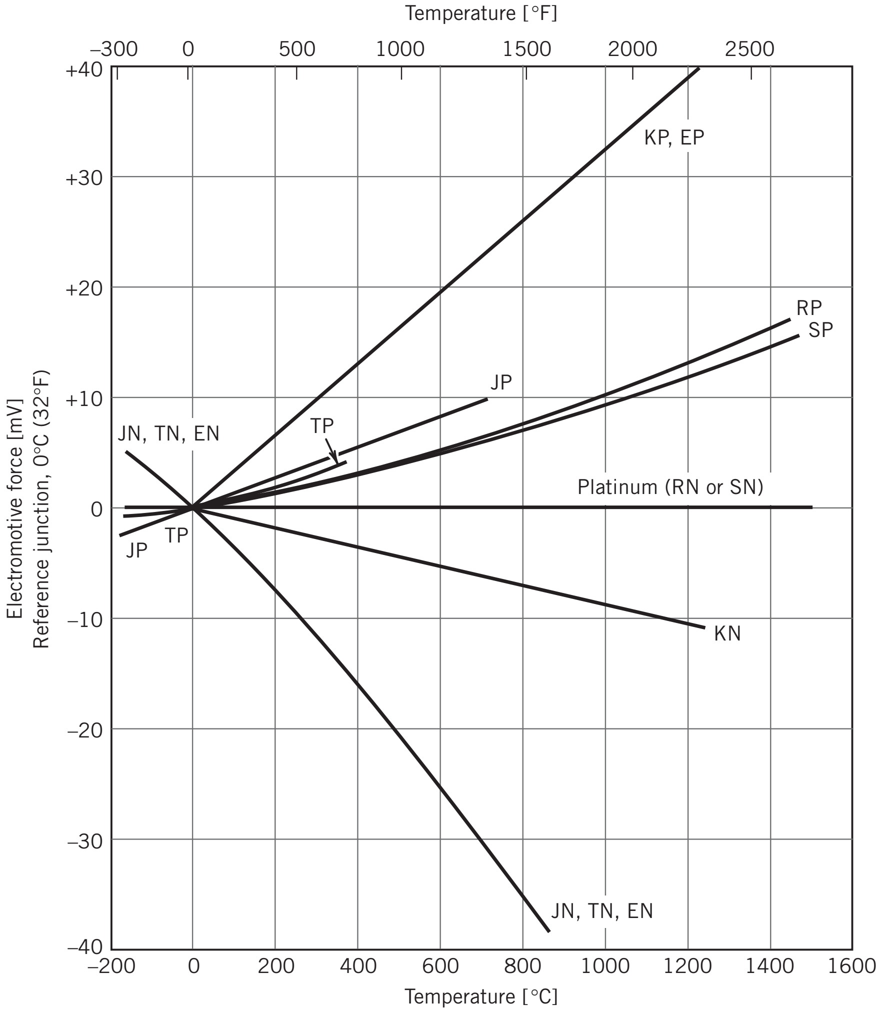

Figure 8.19 Thermal emf of thermocouple materials relative to platinum-67. Note: For example, JP indicates the positive leg of a J thermocouple, or iron. JN indicates the negative leg of a J thermocouple, or constantan. All other notations are similar for each type of thermocouple. (From Benedict, R. P., *Fundamentals of Temperature, Pressure, and Flow Measurements,* 3rd ed. Copyright© 1984 by John Wiley and Sons, New York. Reprinted by permission.)

emf (10). An example of such a table is provided in Table 8.6 for an iron/constantan thermocouple, usually referred to as a J-type thermocouple. Table 8.7 provides the corresponding polynomial equations that relate emf and temperature for standard thermocouples. Because of the widespread need to measure temperature, an industry has grown up to supply high-grade thermocouple wire. Manufacturers can also provide thermocouples having special tolerance limits relative to the NIST standard voltages with uncertainties in temperature ranging from $\pm 1.0°C$ to perhaps $\pm 0.1°C$. Thermocouples constructed of standard thermocouple wire do not require calibration to provide measurement of temperature within the tolerance limits given in Table 8.5.

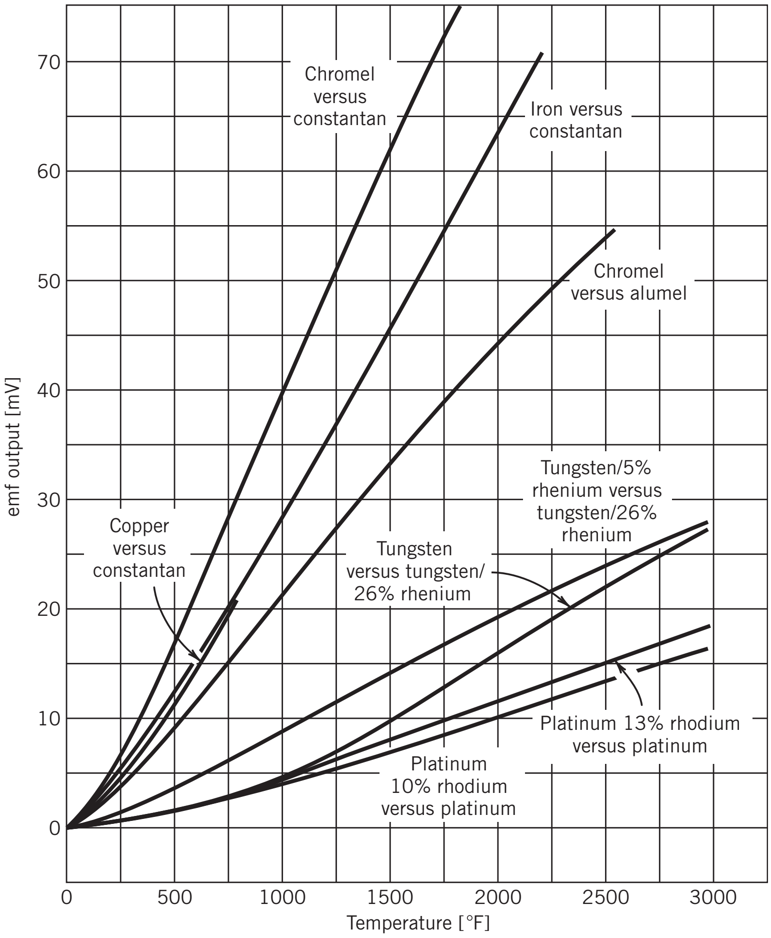

Figure 8.20 Thermocouple voltage output as a function of temperature for some common thermocouple materials. Reference junction is at 32°F (0°C). (From Benedict, R. P., *Fundamentals of Temperature, Pressure and Flow Measurements,* 3rd ed. Copyright © 1984 by John Wiley and Sons, New York. Reprinted by permission.)

Thermocouple Voltage Measurement

The Seebeck voltage for a thermocouple circuit is measured with no current flow in the circuit. From our discussion of the Thomson and Peltier effects, it is clear that the emf is different from the open-circuit value when there is a current flow in the thermocouple circuit. As such, the best method for the measurement of thermocouple voltages is a device that minimizes current flow. For many years, the potentiometer was the laboratory standard for voltage measurement in thermocouple circuits. A potentiometer, as described in Chapter 6, has nearly zero loading error at a balanced condition. However, modern voltage-measuring devices, such as digital voltmeters or data-acquisition cards, have sufficiently large values of input impedance that they can be used with minimal loading error. These devices can also be used in either static or dynamic measuring situations where the loading error created by the measurement device is acceptable for the particular application. For such needs,

Table 8.5 Standard Thermocouple Compositions[a]

	Wire		
Type	Positive	Negative	Expected Systematic Uncertainty[b]
S	Platinum	Platinum/10% rhodium	±1.5°C or 0.25%
R	Platinum	Platinum/13% rhodium	±1.5°C
B	Platinum/30% rhodium	Platinum/6% rhodium	±0.5%
T	Copper	Constantan	±1.0°C or 0.75%
J	Iron	Constantan	±2.2°C or 0.75%
K	Chromel	Alumel	±2.2°C or 0.75%
E	Chromel	Constantan	±1.7°C or 0.5%

Alloy Designations

Constantan: 55% copper with 45% nickel

Chromel: 90% nickel with 10% chromium

Alumel: 94% nickel with 3% manganese, 2% aluminum, and 1% silicon

[a]From Temperature Measurements ANSI PTC 19.3-1974.

[b]Use greater value; these limits of error do not include installation errors.

Table 8.6 Thermocouple Reference Table for Type-J Thermocouple[a]

Temperature (°C)								Thermocouple emf (mV)		
	0	−1	−2	−3	−4	−5	−6	−7	−8	−9
−210	−8.095									
−200	−7.890	−7.912	−7.934	−7.955	−7.976	−7.996	−8.017	−8.037	−8.057	−8.076
−190	−7.659	−7.683	−7.707	−7.731	−7.755	−7.778	−7.801	−7.824	−7.846	−7.868
−180	−7.403	−7.429	−7.456	−7.482	−7.508	−7.534	−7.559	−7.585	−7.610	−7.634
−170	−7.123	−7.152	−7.181	−7.209	−7.237	−7.265	−7.293	−7.321	−7.348	−7.376
−160	−6.821	−6.853	−6.883	−6.914	−6.944	−6.975	−7.005	−7.035	−7.064	−7.094
−150	−6.500	−6.533	−6.566	−6.598	−6.631	−6.663	−6.695	−6.727	−6.759	−6.790
−140	−6.159	−6.194	−6.229	−6.263	−6.298	−6.332	−6.366	−6.400	−6.433	−6.467
−130	−5.801	−5.838	−5.874	−5.910	−5.946	−5.982	−6.018	−6.054	−6.089	−6.124
−120	−5.426	−5.465	−5.503	−5.541	−5.578	−5.616	−5.653	−5.690	−5.727	−5.764
−110	−5.037	−5.076	−5.116	−5.155	−5.194	−5.233	−5.272	−5.311	−5.350	−5.388
−100	−4.633	−4.674	−4.714	−4.755	−4.796	−4.836	−4.877	−4.917	−4.957	−4.997
−90	−4.215	−4.257	−4.300	−4.342	−4.384	−4.425	−4.467	−4.509	−4.550	−4.591
−80	−3.786	−3.829	−3.872	−3.916	−3.959	−4.002	−4.045	−4.088	−4.130	−4.173
−70	−3.344	−3.389	−3.434	−3.478	−3.522	−3.566	−3.610	−3.654	−3.698	−3.742
−60	−2.893	−2.938	−2.984	−3.029	−3.075	−3.120	−3.165	−3.210	−3.255	−3.300
−50	−2.431	−2.478	−2.524	−2.571	−2.617	−2.663	−2.709	−2.755	−2.801	−2.847
−40	−1.961	−2.008	−2.055	−2.103	−2.150	−2.197	−2.244	−2.291	−2.338	−2.385
−30	−1.482	−1.530	−1.578	−1.626	−1.674	−1.722	−1.770	−1.818	−1.865	−1.913
−20	−0.995	−1.044	−1.093	−1.142	−1.190	−1.239	−1.288	−1.336	−1.385	−1.433
−10	−0.501	−0.550	−0.600	−0.650	−0.699	−0.749	−0.798	−0.847	−0.896	−0.946
0	0.000	−0.050	−0.101	−0.151	−0.201	−0.251	−0.301	−0.351	−0.401	−0.451

(*continued*)

Table 8.6 (*Continued*)

Temperature (°C)										Thermocouple emf (mV)
	0	+1	+2	+3	+4	+5	+6	+7	+8	+9
0	0.000	0.050	0.101	0.151	0.202	0.253	0.303	0.354	0.405	0.451
10	0.507	0.558	0.609	0.660	0.711	0.762	0.814	0.865	0.916	0.968
20	1.019	1.071	1.122	1.174	1.226	1.277	1.329	1.381	1.433	1.485
30	1.537	1.589	1.641	1.693	1.745	1.797	1.849	1.902	1.954	2.006
40	2.059	2.111	2.164	2.216	2.269	2.322	2.374	2.427	2.480	2.532
50	2.585	2.638	2.691	2.744	2.797	2.850	2.903	2.956	3.009	3.062
60	3.116	3.169	3.222	3.275	3.329	3.382	3.436	3.489	3.543	3.596
70	3.650	3.703	3.757	3.810	3.864	3.918	3.971	4.025	4.079	4.133
80	4.187	4.240	4.294	4.348	4.402	4.456	4.510	4.564	4.618	4.672
90	4.726	4.781	4.835	4.889	4.943	4.997	5.052	5.106	5.160	5.215
100	5.269	5.323	5.378	5.432	5.487	5.541	5.595	5.650	5.705	5.759
110	5.814	5.868	5.923	5.977	6.032	6.087	6.141	6.196	6.251	6.306
120	6.360	6.415	6.470	6.525	6.579	6.634	6.689	6.744	6.799	6.854
130	6.909	6.964	7.019	7.074	7.129	7.184	7.239	7.294	7.349	7.404
140	7.459	7.514	7.569	7.624	7.679	7.734	7.789	7.844	7.900	7.955
150	8.010	8.065	8.120	8.175	8.231	8.286	8.341	8.396	8.452	8.507
160	8.562	8.618	8.673	8.728	8.783	8.839	8.894	8.949	9.005	9.060
170	9.115	9.171	9.226	9.282	9.337	9.392	9.448	9.503	9.559	9.614
180	9.669	9.725	9.780	9.836	9.891	9.947	10.002	10.057	10.113	10.168
190	10.224	10.279	10.335	10.390	10.446	10.501	10.557	10.612	10.668	10.723
200	10.779	10.834	10.890	10.945	11.001	11.056	11.112	11.167	11.223	11.278
210	11.334	11.389	11.445	11.501	11.556	11.612	11.667	11.723	11.778	11.8.34
220	11.889	11.945	12.000	12.056	12.111	12.167	12.222	12.278	12.334	12.389
230	12.445	12.500	12.556	12.611	12.667	12.722	12.778	12.833	12.889	12.944
240	13.000	13.056	13.111	13.167	13.222	13.278	13.333	13.389	13.444	13.500
250	13.555	13.611	13.666	13.722	13.777	13.833	13.888	13.944	13.999	14.055
260	14.110	14.166	14.221	14.277	14.332	14.388	14.443	14.499	14.554	14.609
270	14.665	14.720	14.776	14.831	14.887	14.942	14.998	15.053	15.109	15.164
280	15.219	15.275	15.330	15.386	15.441	15.496	15.552	15.607	15.663	15.718
290	15.773	15.829	15.884	15.940	15.995	16.050	16.106	16.161	16.216	16.272
300	16.327	16.383	16.438	16.493	16.549	16.604	16.659	16.715	16.770	16.825
310	16.881	16.936	16.991	17.046	17.102	17.157	17.212	17.268	17.323	17.378
320	17.434	17.489	17.544	17.599	17.655	17.710	17.765	17.820	17.876	17.931
330	17.986	18.041	18.097	18.152	18.207	18.262	18.318	18.373	18.428	18.483
340	18.538	18.594	18.649	18.704	18.759	18.814	18.870	18.925	18.980	19.035
350	19.090	19.146	19.201	19.256	19.311	19.366	19.422	19.477	19.532	19.587
360	19.642	19.697	19.753	19.808	19.863	19.918	19.973	20.028	20.083	20.139
370	20.194	20.249	20.304	20.359	20.414	20.469	20.525	20.580	20.635	20.690
380	20.745	20.800	20.855	20.911	20.966	21.021	21.076	21.131	21.186	21.241
390	21.297	21.352	21.407	21.462	21.517	21.572	21.627	21.683	21.738	21.793

Table 8.6 (*Continued*)

Temperature (°C)										Thermocouple emf (mV)
400	21.848	21.903	21.958	22.014	22.069	22.124	22.179	22.234	22.289	22.345
410	22.400	22.455	22.510	22.565	22.620	22.676	22.731	22.786	22.841	22.896
420	22.952	23.007	23.062	23.117	23.172	23.228	23.283	23.338	23.393	23.449
430	23.504	23.559	23.614	23.670	23.725	23.780	23.835	23.891	23.946	24.001
440	24.057	24.112	24.167	24.223	24.278	24.333	24.389	24.444	24.499	24.555
450	24.610	24.665	24.721	24.776	24.832	24.887	24.943	24.998	25.053	25.109
460	25.164	25.220	25.275	25.331	25.386	25.442	25.497	25.553	25.608	25.664
470	25.720	25.775	25.831	25.886	25.942	25.998	26.053	26.109	26.165	26.220
480	26.276	26.332	26.387	26.443	26.499	26.555	26.610	26.666	26.722	26.778
490	26.834	26.889	26.945	27.001	27.057	27.113	27.169	27.225	27.281	27.337
500	27.393	27.449	27.505	27.561	27.617	27.673	27.729	27.785	27.841	27.897
510	27.953	28.010	28.066	28.122	28.178	28.234	28.291	28.347	28.403	28.460
520	28.516	28.572	28.629	28.685	28.741	28.798	28.854	28.911	28.967	29.024
530	29.080	29.137	29.194	29.250	29.307	29.363	29.420	29.477	29.534	29.590
540	29.647	29.704	29.761	29.818	29.874	29.931	29.988	30.045	30.102	30.159
550	30.216	30.273	30.330	30.387	30.444	30.502	30.559	30.616	30.673	30.730
560	30.788	30.845	30.902	30.960	31.017	31.074	31.132	31.189	31.247	31.304
570	31.362	31.419	31.477	31.535	31.592	31.650	31.708	31.766	31.823	31.881
580	31.939	31.997	32.055	32.113	32.171	32.229	32.287	32.345	32.403	32.461
590	32.519	32.577	32.636	32.694	32.752	32.810	32.869	32.927	32.985	33.044
600	33.102	33.161	33.219	33.278	33.337	33.395	33.454	33.513	33.571	33.630
610	33.689	33.748	33.807	33.866	33.925	33.984	34.043	34.102	34.161	34.220
620	34.279	34.338	34.397	34.457	34.516	34.575	34.635	34.694	34.754	34.813
630	34.873	34.932	34.992	35.051	35.111	35.171	35.2.30	35.290	35.350	35.410
640	35.470	35.530	35.590	35.650	35.710	35.770	35.830	35.890	35.950	36.010
650	36.071	36.131	36.191	36.191	36.252	36.373	36.433	36.494	36.554	36.615
660	36.675	36.736	36.797	36.858	36.918	36.979	37.040	37.101	37.162	37.22.3
670	37.284	37.345	37.406	37.467	37.528	37.590	37.651	37.712	37.773	37.835
680	37.896	37.958	38.019	38.081	38.142	38.204	38.265	38.327	38.389	38.450
690	38.512	38.574	38.636	38.698	38.760	38.822	38.884	38.946	39.008	39.070
700	39.132	39.194	39.256	39.318	39.381	39.443	39.505	39.568	39.630	39.693
710	39.755	39.818	39.880	39.943	40.005	40.068	40.131	40.193	40.256	40.319
720	40.382	40.445	40.508	40.570	40.633	40.696	40.759	40.822	40.886	40.949
730	41.012	41.075	41.138	41.201	41.265	41.328	41.391	41.455	41.518	41.581
740	41.645	41.708	41.772	41.835	41.899	41.962	42.026	42.090	42.153	42.217
750	42.281	42.344	42.408	42.472	42.536	42.599	42.663	42.727	42.791	42.855
760	42.919	42.983	43.047	43.110	43.174	43.238	43.303	43.367	43.431	43.495

[a]Reference junction at 0°C.

Table 8.7 Reference Functions for Selected Letter Designated Thermocouples

The relationship between emf and temperature is provided in the form of a polynomial in temperature [10]

$$E = \sum_{i=0}^{n} c_i T^i$$

where E is in μV and T is in °C. Constants are provided below.

Thermocouple Type	Temperature Range	Constants
J-type	−210 to 760°C	$c_0 = 0.000\ 000\ 000\ 0$ $c_1 = 5.038\ 118\ 7815 \times 10^{1}$ $c_2 = 3.047\ 583\ 693\ 0 \times 10^{-2}$ $c_3 = -8.568\ 106\ 572\ 0 \times 10^{-5}$ $c_4 = 1.322\ 819\ 529\ 5 \times 10^{-7}$ $c_5 = -1.705\ 295\ 833\ 7 \times 10^{-10}$ $c_6 = 2.094\ 809\ 069\ 7 \times 10^{-13}$ $c_7 = -1.253\ 839\ 533\ 6 \times 10^{-16}$ $c_8 = 1.563\ 172\ 569\ 7 \times 10^{-20}$
T-type	−270 to 0°C	$c_0 = 0.000\ 000\ 000\ 0$ $c_1 = 3.874\ 810\ 6364 \times 10^{1}$ $c_2 = 4.419\ 443\ 434\ 7 \times 10^{-2}$ $c_3 = 1.184\ 432\ 310\ 5 \times 10^{-4}$ $c_4 = 2.003\ 297\ 355\ 4 \times 10^{-5}$ $c_5 = 9.013\ 801\ 955\ 9 \times 10^{-7}$ $c_6 = 2.265\ 115\ 659\ 3 \times 10^{-8}$ $c_7 = 3.607\ 115\ 420\ 5 \times 10^{-10}$ $c_8 = 3.849\ 393\ 988\ 3 \times 10^{-12}$ $c_9 = 2.821\ 352\ 192\ 5 \times 10^{-14}$ $c_{10} = 1.425\ 159\ 477\ 9 \times 10^{-16}$ $c_{11} = 4.876\ 866\ 228\ 6 \times 10^{-19}$ $c_{12} = 1.079\ 553\ 927\ 0 \times 10^{-21}$ $c_{13} = 1.394\ 502\ 706\ 2 \times 10^{-24}$ $c_{14} = 7.979\ 515\ 392\ 7 \times 10^{-28}$
T-type	0 to 400°C	$c_0 = 0.000\ 000\ 000\ 0$ $c_1 = 3.874\ 810\ 636\ 4 \times 10^{1}$ $c_2 = 3.329\ 222\ 788\ 0 \times 10^{-2}$ $c_3 = 2.061\ 824\ 340\ 4 \times 10^{-4}$ $c_4 = -2.188\ 225\ 684\ 6 \times 10^{-6}$ $c_5 = 1.099\ 688\ 092\ 8 \times 10^{-8}$ $c_6 = -3.081\ 575\ 877\ 2 \times 10^{-11}$ $c_7 = 4.547\ 913\ 529\ 0 \times 10^{-14}$ $c_8 = -2.751\ 290\ 167\ 3 \times 10^{-17}$

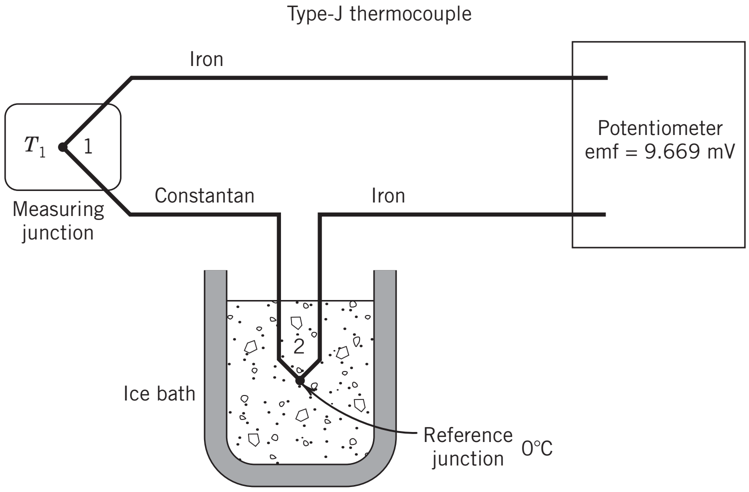

Figure 8.21 Thermocouple circuit for Example 8.7.

high-impedance voltmeters have been incorporated into commercially available temperature indicators, temperature controllers, and digital data-acquisition systems (DAS).

Example 8.7

The thermocouple circuit shown in Figure 8.21 is used to measure the temperature T_1. The thermocouple reference junction labeled 2 is at a temperature of 0°C, maintained by an ice-point bath. The voltage output is measured using a potentiometer and found to be 9.669 mV. What is T_1?

KNOWN A thermocouple circuit having one junction at 0°C and a second junction at an unknown temperature. The circuit produces an emf of 9.669 mV.

FIND The temperature T_1.

ASSUMPTION Thermocouple follows NIST standard.

SOLUTION Standard thermocouple tables such as Table 8.6 are referenced to 0°C. The temperature of the reference junction for this case is 0°C. Therefore, the temperature corresponding to the output voltage may simply be determined from Table 8.6, in this case as 180.0°C.

COMMENT Because of the law of intermediate metals, the junctions formed at the potentiometer do not affect the voltage measured for the thermocouple circuit, and the voltage output reflects accurately the temperature difference between junctions 1 and 2.

Example 8.8

Suppose the thermocouple circuit in the previous example (Ex. 8.7) now has junction 2 maintained at a temperature of 30°C, and produces an output voltage of 8.132 mV. What temperature is sensed by the measuring junction?

KNOWN The value of T_2 is 30°C, and the output emf is 8.132 mV.

ASSUMPTION Thermocouple follows NIST standard emf behavior.

FIND The temperature of the measuring junction.

SOLUTION By the law of intermediate temperatures the output emf for a thermocouple circuit having two junctions, one at 0°C and the other at T_1, would be the sum of the emfs for a thermocouple circuit between 0° and 30°C and between 30°C and T_1. Thus,

$$\text{emf}_{0-30} + \text{emf}_{30-T_1} = \text{emf}_{0-T_1}$$

This relationship allows the voltage reading from the nonstandard reference temperature to be converted to a 0°C reference temperature by adding $\text{emf}_{0-30} = 1.537$ to the existing reading. This results in an equivalent output voltage, referenced to 0°C as

$$1.537 + 8.132 = 9.669 \text{ mV}$$

Clearly, this thermocouple is sensing the same temperature as in the previous example, 180.0°C. This value is determined from Table 8.6.

COMMENT Note that the effect of raising the reference junction temperature is to lower the output voltage of the thermocouple circuit. Negative values of voltage, as compared with the polarity listed in Table 8.4, indicate that the measured temperature is less than the reference junction temperature.

Example 8.9

A J-type thermocouple measures a temperature of 100°C and is referenced to 0°C. The thermocouple is AWG 30 (American wire gauge [AWG]; AWG 30 is 0.010-in. wire diameter) and is arranged in a circuit as shown in Figure 8.17a. The length of the thermocouple wire is 10 ft in order to run from the measurement point to the ice bath and to a potentiometer. The resolution of the potentiometer is 0.005 mV. If the thermocouple wire has a resistance per unit length, as specified by the manufacturer, of 5.6 Ω/ft, estimate the residual current in the thermocouple when the circuit is balanced within the resolution of the potentiometer.

KNOWN A potentiometer having a resolution of 0.005 mV is used to measure the emf of a J-type thermocouple that is 10 ft long.

FIND The residual current in the thermocouple circuit.

SOLUTION The total resistance of the thermocouple circuit is 56 Ω for 10 ft of thermocouple wire. The residual current is then found from Ohm's law as

$$I = \frac{E}{R} = \frac{0.005 \text{ mV}}{56\,\Omega} = 8.9 \times 10^{-8} \text{ A}$$

COMMENT The loading error due to this current flow is $\sim 0.005 \text{ mV}/54.3\ \mu\text{V}/^\circ\text{C} \approx 0.09^\circ\text{C}$.

Example 8.10

Suppose a high-impedance voltmeter is used in place of the potentiometer in Example 8.9. Determine the minimum input impedance required for the voltmeter that will limit the loading error to the same level as the potentiometer.

KNOWN Loading error should be less than 8.9×10^{-8} A.

FIND Input impedance for a voltmeter that would produce the same current flow or loading error.

SOLUTION At 100°C a J-type thermocouple referenced to 0°C has a Seebeck voltage of $E_s =$ 5.269 mV. At this temperature, the required voltmeter impedance to limit the current flow to 8.9×10^{-8} A is found from Ohm's law:

$$\frac{E_s}{I} = 5.269 \times 10^{-3}\ \mathrm{V}/8.9 \times 10^{-8}\ \mathrm{A} = 59.2\ \mathrm{k}\Omega$$

COMMENT This input impedance is not at all high for a modern microvoltmeter and such a voltmeter would be a reasonable choice in this situation. As always, the allowable loading error should be determined based on the required uncertainty in the measured temperature.

Multiple-Junction Thermocouple Circuits

A thermocouple circuit composed of two junctions of dissimilar metals produces an open-circuit emf that is related to the temperature difference between the two junctions. More than two junctions can be employed in a thermocouple circuit, and thermocouple circuits can be devised to measure temperature differences, or average temperature, or to amplify the output voltage of a thermocouple circuit.

Thermopiles

Thermopile is the term used to describe a multiple-junction thermocouple circuit that is designed to amplify the output of the circuit. Because thermocouple voltage outputs are typically in the millivolt range, increasing the voltage output may be a key element in reducing the uncertainty in the temperature measurement, or may be necessary to allow transmission of the thermocouple signal to the recording device. Figure 8.22 shows a thermopile for providing an amplified output signal; in this case the output voltage would be N times the single thermocouple output, where N is the number of measuring junctions in the circuit. The average output voltage corresponds to the average

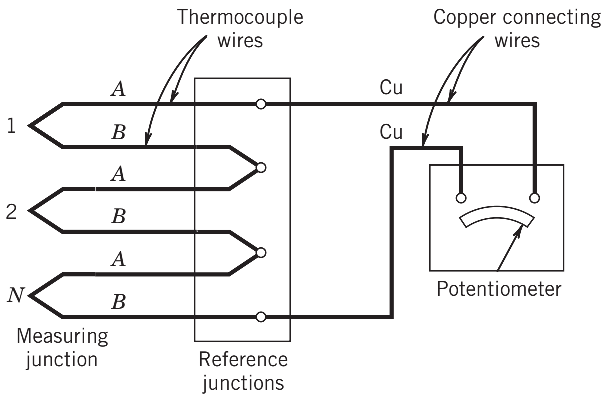

Figure 8.22 Thermopile arrangement. (From Benedict, R. P. *Fundamentals of Temperature, Pressure and Flow Measurements,* 3rd ed. Copyright © 1984 by John Wiley and Sons, New York. Reprinted by permission.)