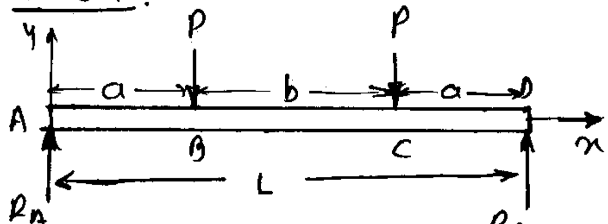
Solutions to Homework & Practice Problems of CHAPTER 3

Solution to problem 3.5

Sign convention :

F.B.D. :



$\Sigma F_y = 0 \Rightarrow$

$R_A - P - P + R_D = 0$

$\Sigma M_A = 0 \Rightarrow$

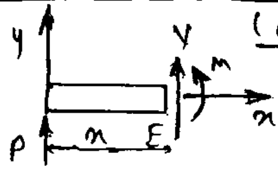
$-P \times a - P(a+b) + R_D \times L = 0.$

Above equations give

$R_A = P$, $R_D = P.$

Expressions For V & M :

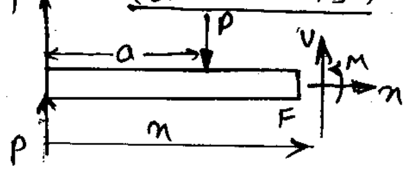
$(0 < x < a)$



$\Sigma F_y = 0 \Rightarrow V = -P$

$\Sigma M_E = 0 \Rightarrow M = P.x.$

$(a < x < a+b)$



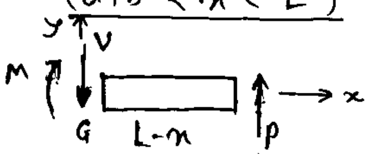
$\Sigma F_y = 0 \Rightarrow V + P - P = 0$

$\Rightarrow V = 0$

$\Sigma M_F = 0 \Rightarrow M = -P(-a+x) + Px$

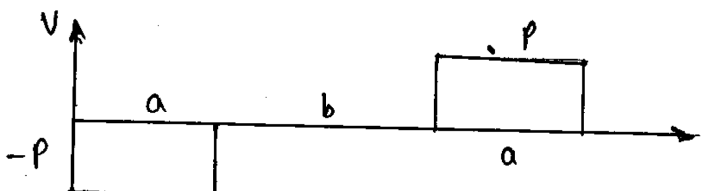
$M = P.a.$

$(a+b < x < L)$

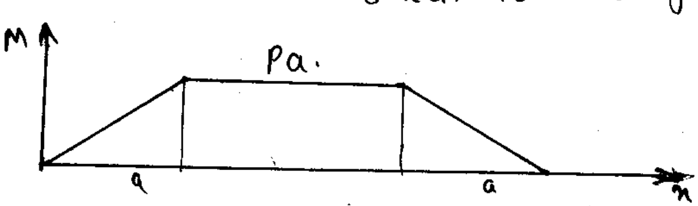


$\Sigma F_y = 0 \Rightarrow V = +P$

$\Sigma M_G = 0 \Rightarrow M = P(L-x).$



shear force diagram

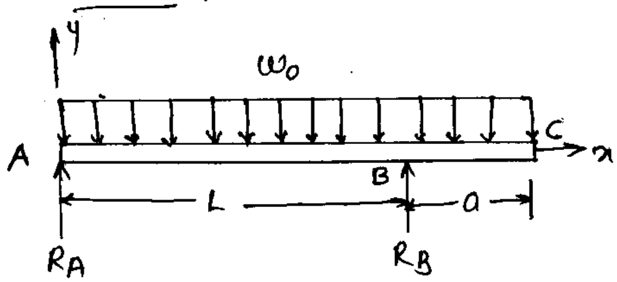


Bending Moment diagram.

Solution to problem 3.7

Sign convention:

F.B.D :



$\Sigma F_y = 0 \Rightarrow$

$R_A + R_B - w_o (L+a) = 0$

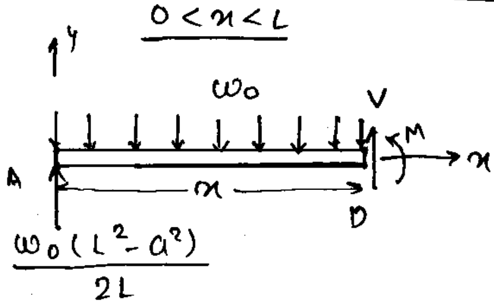
$\Sigma M_A = 0 \Rightarrow$

$-\frac{1}{2} W_o (L+a)^2 + R_B L$

$\therefore R_A = \dfrac{w_o (L^2 - a^2)}{2L}$, $R_B = \dfrac{w_o (L+a)^2}{2L}$.

Expressions for V and M :

$0 < x < L$



$\Sigma F_y = 0 \Rightarrow$

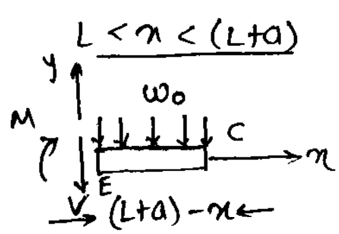
$V + \dfrac{w_o (L^2 - a^2)}{2L} - w_o x = 0.$

$\Rightarrow V = -\dfrac{w_o (L^2 - a^2)}{2L} + w_o x.$

$\Sigma M_D = 0. \Rightarrow M - \dfrac{w_o (L^2 - a^2)}{2L} . x + \frac{1}{2} w_o x^2 = 0$

$\Rightarrow M = \dfrac{w_o (L^2 - a^2)}{2L} . x - \frac{1}{2} w_o x^2.$

$L < x < (L+a)$

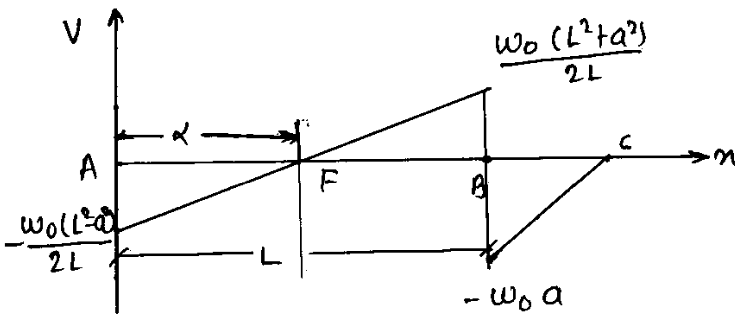


$\Sigma F_y = 0 \Rightarrow V + w_o [(L+a) - x] = 0$

$\Rightarrow V = -W_o [(L+a) - x]$

$\Sigma M_E = 0 \Rightarrow M = -\frac{1}{2} w_o [(L+a) - x]^2.$

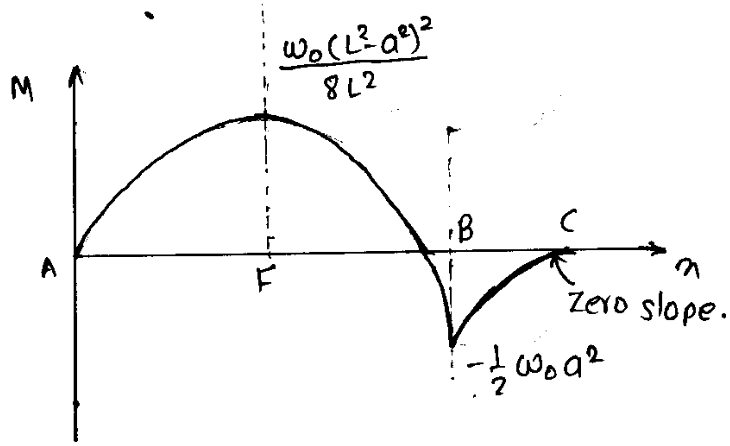
(Problem 3.7 contd).



Similarity of triangles

$$\frac{\alpha}{L-\alpha} = \frac{\frac{w_0(L^2-a^2)}{2L}}{\frac{w_0(L^2+a^2)}{2L}} \Rightarrow$$

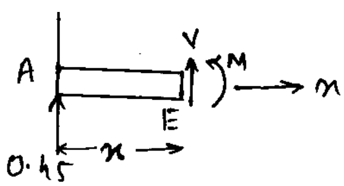
$$\alpha = \frac{L^2-a^2}{2L}.$$



M_F = area of V-diagram upto F

$$= \frac{1}{2}\alpha\left[w_0\frac{(L^2-a^2)}{2L}\right]$$

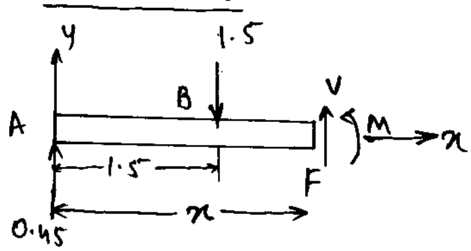
$$= \frac{w_0(L^2-a^2)}{8L^2}.$$



$\Sigma F_y = 0 \Rightarrow V = -0.45$

$\Sigma M_E = 0 \Rightarrow M = 0.45x.$

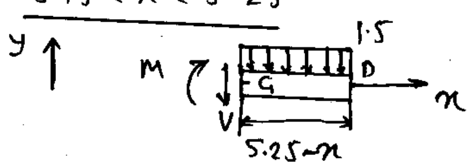
$1.5 < x < 3.75$



$\Sigma F_y = 0 \Rightarrow V = 1.05 \text{ kN}.$

$\Sigma M_F = 0 \Rightarrow M = -1.05x + 2.25.$

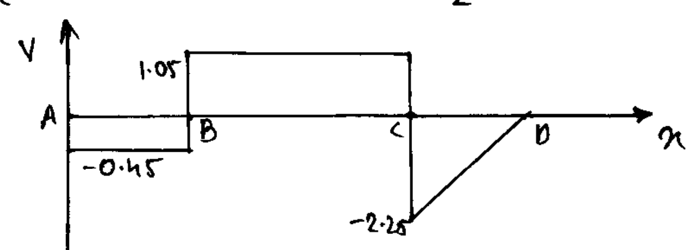
$3.75 < x < 5.25$

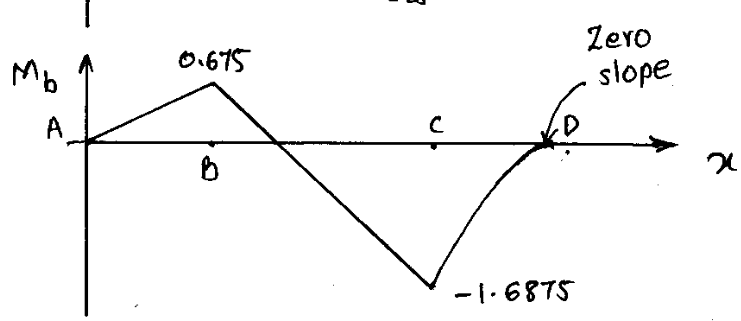


$\Sigma F_y = 0 \Rightarrow V = 1.5x - 7.875.$

$\Sigma M_G = 0 \Rightarrow M = -\frac{1}{2}(5.25 - x)^2 \times 1.5$

Shear Force and Bending Moment diagrams:

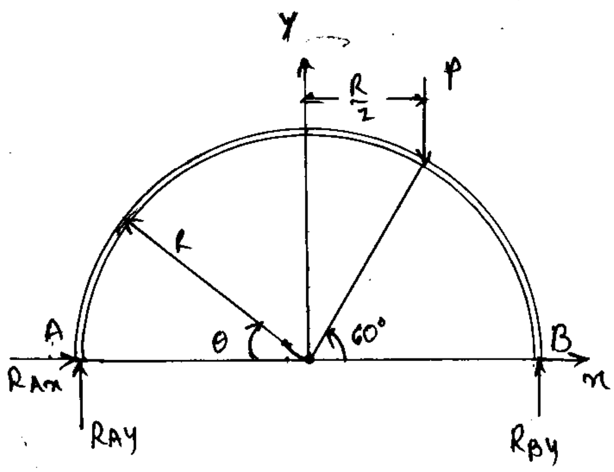




Solution to problem 3.11

Sign convention :

F.B.D



No moments at A, B ;
No horizontal force at B

$\Sigma (M_z)_A = 0 \Rightarrow$

$R_{By} \cdot (2R) - P\left(\frac{3R}{2}\right) = 0$

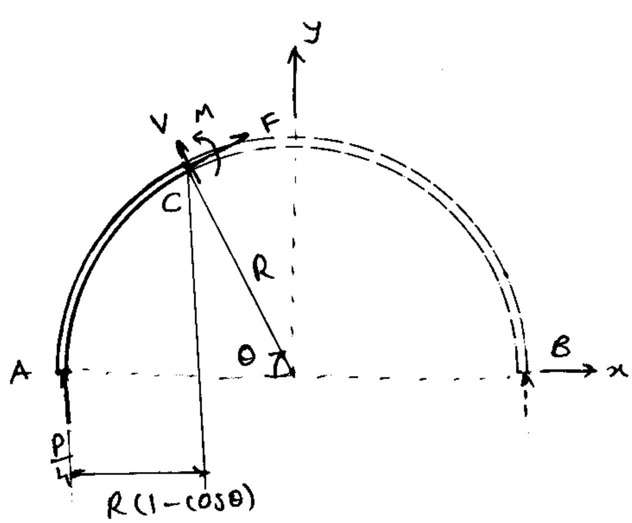
$\Rightarrow \quad R_{By} = \frac{3P}{4}.$

$\Sigma F_x = 0 \Rightarrow R_{Ax} = 0.$

$\Sigma F_y = 0 \Rightarrow$

$R_{Ay} + R_{By} - P = 0 \Rightarrow R_{Ay} = \frac{P}{4}.$

Free body for $\theta < 120°$.



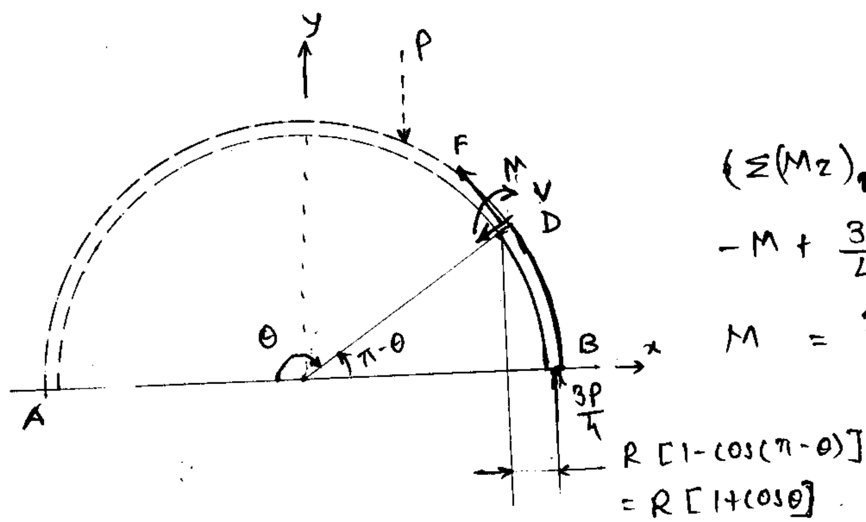
$\Sigma (M_z)_C = 0 \Rightarrow$

$M - \frac{P}{4}\left[R(1-\cos\theta)\right] = 0.$

$\Rightarrow \quad M = \frac{PR}{4}(1-\cos\theta), \quad \theta < 120°.$

(Problem 3.11 contd.)

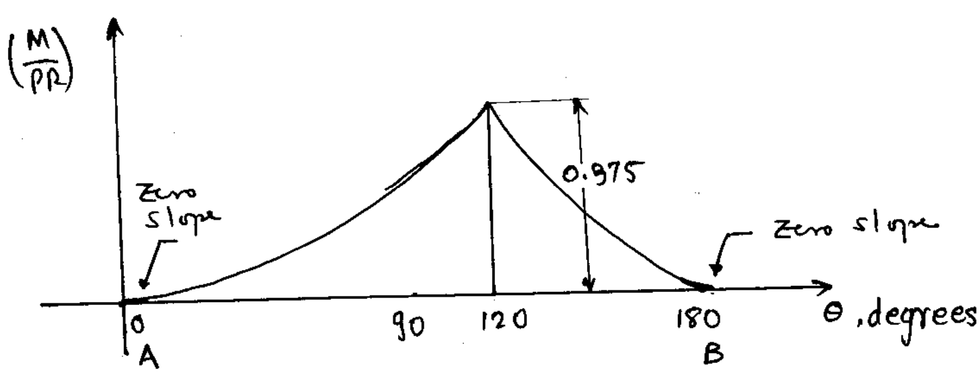
Free body: $\theta > 120°$.



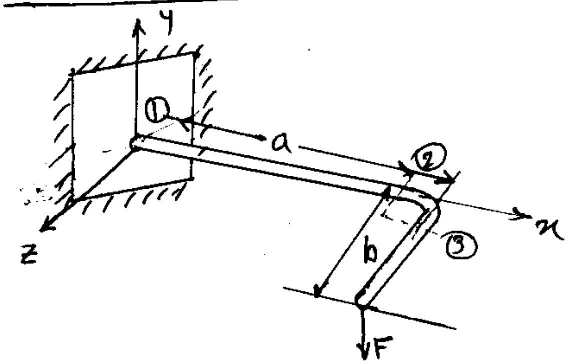
$(\Sigma M_z)_D = 0 \Rightarrow$

$$-M + \frac{3P}{4} R (1+\cos\theta) = 0.$$

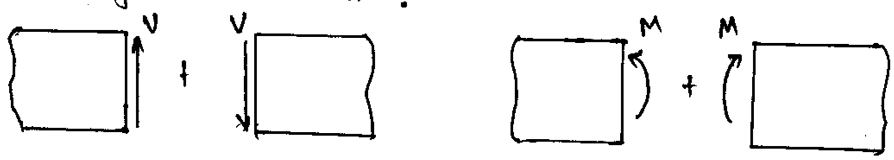
$$M = \frac{3PR}{4}(1+\cos\theta), \quad \theta > 120°.$$



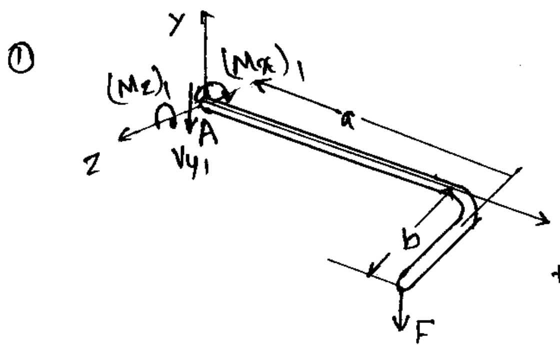
Solution to problem 3.13



Sign Convention :



FBD :(Section 1):



$\Sigma F_x = 0$, $\Sigma F_z = 0$, $\Sigma M_y = 0$ are satisfied identically.

$+\uparrow \Sigma F_y = 0 \Rightarrow$

$-V_{y_1} - F = 0 \Rightarrow V_{y_1} = -F.$ --1)

$+\circlearrowleft \Sigma M_{xA} = 0 \Rightarrow$

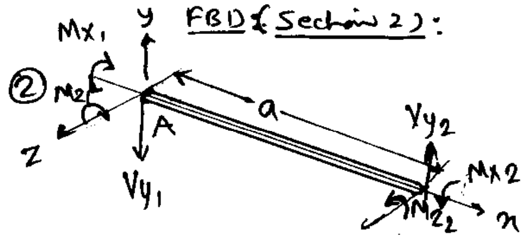
$-(M_x)_1 + Fb = 0 \Rightarrow (M_x)_1 = Fb$ --2)

$+\circlearrowleft \Sigma M_{zA} = 0 \Rightarrow$

$-(M_z)_1 - Fa = 0 \Rightarrow (M_z)_1 = -Fa$ --3)

note that ~~equations~~ expressions ①,②, ③ are for ~~finding~~ shearforce (V_{f_1}), twisting moment (M_{x_1}) and Bending moment (M_{z_1}) at section 1.

FBD (Section 2):



$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M_y = 0$ are identically satisfied.

Shear $\equiv +\uparrow \Sigma F_y = 0 \Rightarrow -V_{y_1} + V_{y_2} = 0 \Rightarrow V_{y_2} = V_{y_1} = -F$

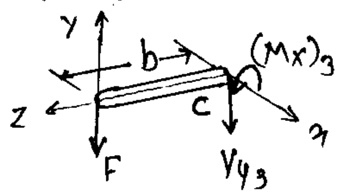
Twisting $\equiv +\circlearrowleft \Sigma M_{xA} = 0 \Rightarrow -(M_x)_1 + (M_x)_2 = 0 \Rightarrow (M_x)_2 = (M_x)_1 = Fb$

Bending $\equiv +\circlearrowleft M_{zA} = 0 \Rightarrow -(M_z)_1 + (V_y)_2 \times a + (M_z)_2 = 0 \Rightarrow$

$(M_z)_2 = (M_z)_1 - (V_y)_2 a = -Fa - (-F)a = 0.$

(Problem 3.13 contd.).

FBD : (Section 3) :



$\Sigma F_x = 0$, $\Sigma F_z = 0$, $\Sigma M_y = 0$, $\Sigma M_z = 0$ are satisfied identically.

Shear : $+\uparrow \Sigma F_y = 0 \Rightarrow -F - V_{y_3} = 0$

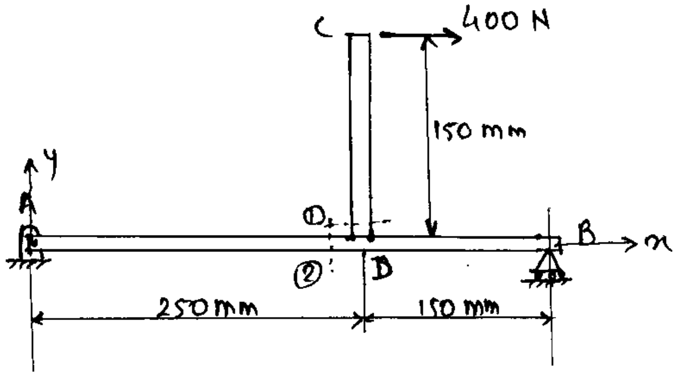
$$\Rightarrow V_{y_3} = -F$$

Bending : $+\circlearrowleft \Sigma M_{x_C} = 0 \Rightarrow$

$$F \times b - (M_x)_3 = 0$$

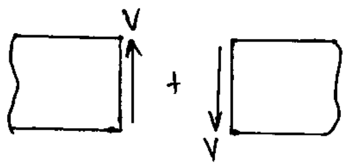
$$\Rightarrow (M_x)_3 = Fb.$$

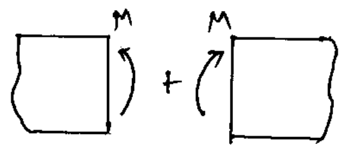
Solution to problem 3.14.



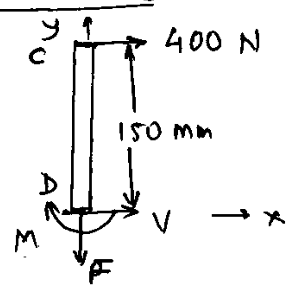
To find:
internal forces and moments at sections ①②

Sign convention:





Section 1:



$\Sigma F_y = 0 \Rightarrow F = 0$

$\Sigma F_x = 0 \Rightarrow V + 400 = 0$

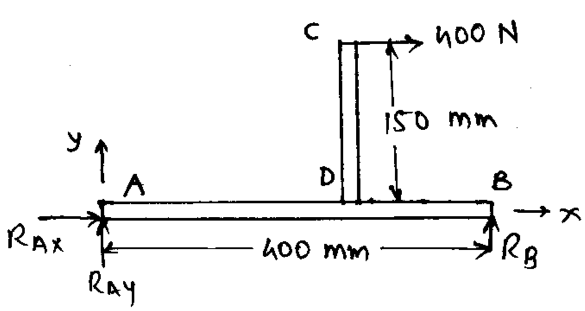
$\Rightarrow V = -400$ N.

$\Sigma M_D = 0 \Rightarrow M + 400 \times 0.15 = 0$

$\Rightarrow M = -60$ Nm.

Section 2:

Reactions:



$\Sigma F_x = 0 \Rightarrow$

$R_{Ax} + 400 = 0$

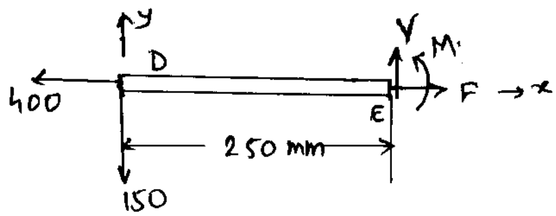
$\Rightarrow R_{Ax} = -400$ N

$\Sigma M_B = 0 \Rightarrow$

$400 \times 0.15 + R_{Ay} \times 0.4 = 0$

$\Rightarrow R_{Ay} = -150$ N.

(Problem 3.14 contd.)



$\Sigma F_x = 0 \Rightarrow$

$F - 400 = 0$

$F = 400$ N.

$\Sigma F_y = 0 \Rightarrow$

$V - 150 = 0$

$\Rightarrow V = 150$ N.

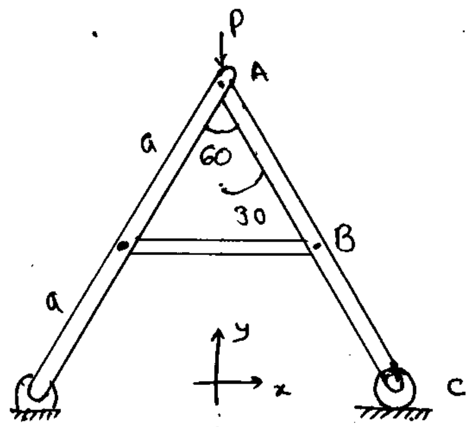
$\Sigma M_E = 0 \Rightarrow$

$M + 150 \times 0.25 = 0$

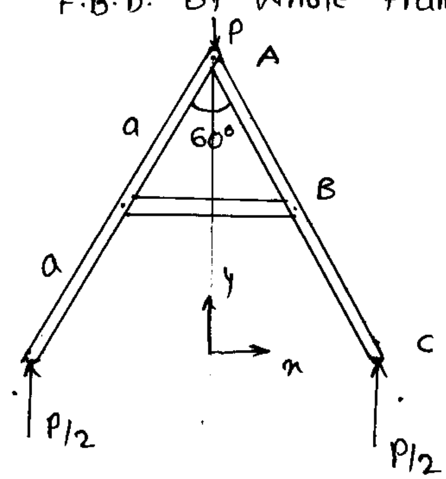
$\Rightarrow M = -37.5$ N.m.

Solution to problem 3.15.

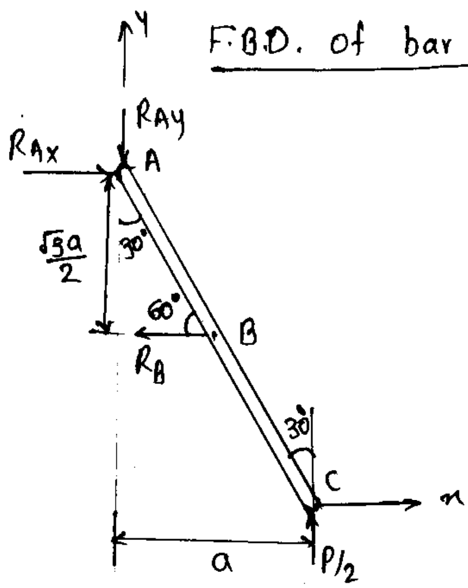
Sign convention:



F.B.D. of whole frame.



F.B.D. of bar AC :



$\Sigma M_A = 0 \Rightarrow$

$$\frac{P}{2}\cdot a - R_B \frac{\sqrt{3}}{2} a = 0$$

$$\Rightarrow R_B = \frac{P}{\sqrt{3}} = \frac{\sqrt{3}P}{3}.$$

$\Sigma F_x = 0 \Rightarrow$

$$R_{Ax} - R_B = 0 \Rightarrow$$

$$R_{Ax} = R_B = \frac{\sqrt{3}P}{3}.$$

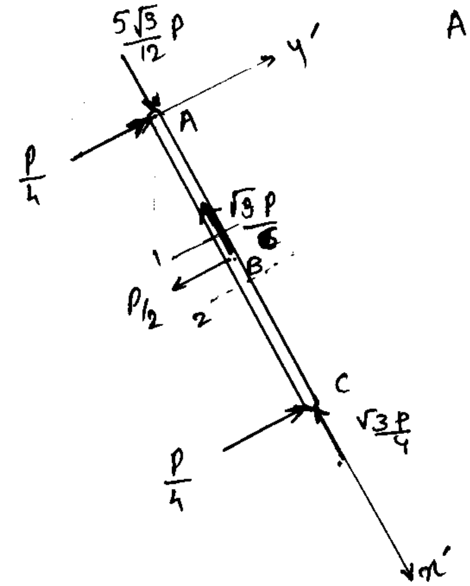
$\Sigma F_y = 0 \Rightarrow$

$$R_{Ay} - \frac{P}{2} = 0.$$

$$R_{Ay} = \frac{P}{2}.$$

(Problem 3.15 contd.)

Loading diagram:



A: R_{Ax}' (along the rod)

$= R_{Ay}\cos 30 + R_{Ax}\cos 60$

$= \frac{P}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3}P\frac{1}{2} = \frac{5\sqrt{3}P}{12}.$

R_{Ay}' (perpendicular to the rod)

$= -R_{Ay}\sin 30 + R_{Ax}\sin 60$

$= -\frac{P}{2}\cdot\frac{1}{2} + \frac{\sqrt{3}P}{3}\cdot\frac{\sqrt{3}}{2}. = \frac{P}{4}.$

B: R_{Bx}' (along the rod):

$= -R_B\cos 60 = -\frac{\sqrt{3}P}{3}\cdot\frac{1}{2} = -\frac{\sqrt{3}P}{6}.$

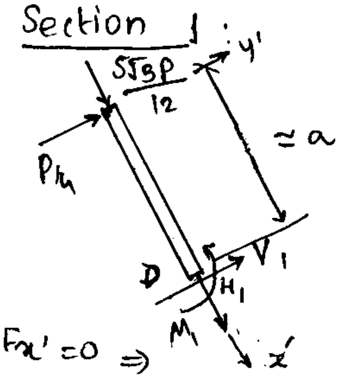
R_{By}' (perpendicular to the rod)

$= -R_B\sin 60 = -\frac{\sqrt{3}P}{3}\cdot\frac{\sqrt{3}}{2} = -\frac{P}{2}.$

C: R_{Cx}' (along the rod) $= -\frac{P}{2}\cos 30 = -\frac{\sqrt{3}}{4}P.$

R_{Cy}' (perpendicular to the rod) $= \frac{P}{2}\sin 30 = \frac{P}{4}.$

Section 1:



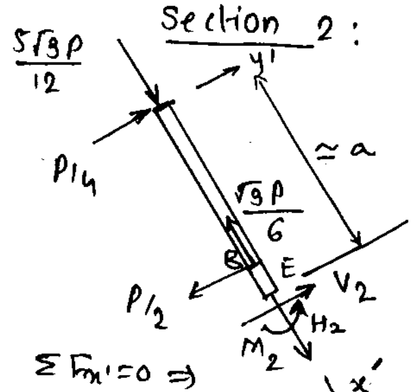
$\Sigma F_{x'} = 0 \Rightarrow$

$H_1 = -\frac{5\sqrt{3}}{12}P$

$\Sigma F_{y'} = 0 \Rightarrow V_1 = -\frac{P}{4}.$

$\Sigma M_D = 0 \Rightarrow M_1 = \frac{P\cdot a}{4}$

Section 2:



The distance BE is negligible

$\Sigma F_{x'} = 0 \Rightarrow$

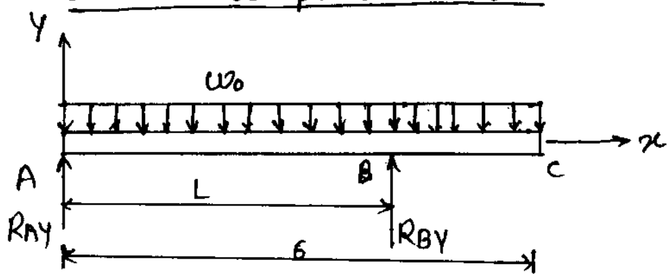
$H_2 = \frac{\sqrt{3}P}{6} - \frac{5\sqrt{3}P}{12} = -\frac{\sqrt{3}P}{4}$

$\Sigma F_{y'} = 0 \Rightarrow$

$V_2 = P/_2 - P/_4 = P/_4.$

$\Sigma M_E = 0 \Rightarrow M_2 = \frac{Pa}{4}.$

Solution to problem 3.17



$+\uparrow \Sigma F_y = 0 \Rightarrow$

$R_{AY} + R_{BY} - w_o \times 6 = 0.$

$\circlearrowleft \Sigma M_A = 0 \Rightarrow$

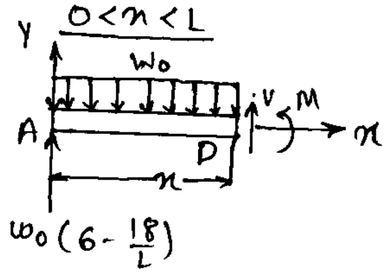
$R_{BY} \times L - \frac{1}{2} w_o \times 6^2 = 0.$

From above equations :

$R_{BY} = \frac{18 w_o}{L}.$

$R_{AY} = w_o \left(6 - \frac{18}{L} \right).$

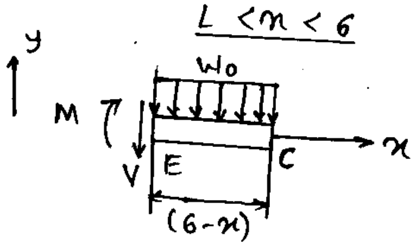
$0 < x < L$



$\Sigma M_D = 0 \Rightarrow$

$M = w_o \left(6 - \frac{18}{L} \right) . x - \frac{1}{2} w_o x^2.$

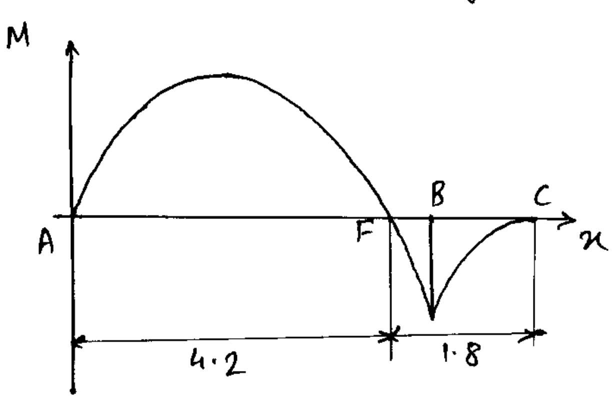
$L < x < 6$



$\Sigma M_E = 0 \Rightarrow$

$M = -\frac{1}{2} w_o \, (6-x)^2.$

Bending moment diagram.



The bending moment is zero at point F. (betweem points A & B)

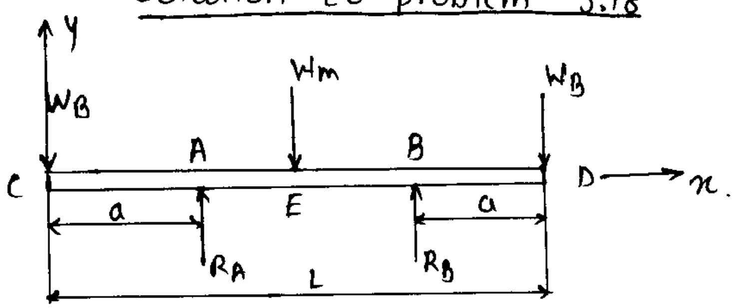
Bending moment equation for AB gives

$$w_o\left(6 - \frac{18}{L}\right)x - \frac{1}{2} w_o x^2 \Big|_{x=4.2} = 0$$

$$w_o \left(6 - \frac{18}{L}\right) . 4.2 - \frac{1}{2} w_o . 4.2^2 = 0.$$

$\Rightarrow \underline{L = 4.615 \, m.}$

Solution to problem 3.18



a) Apparently he is not minimizing the bending moment by putting the bricks at the end. The bending moment would be less if he puts the bricks right above the support.

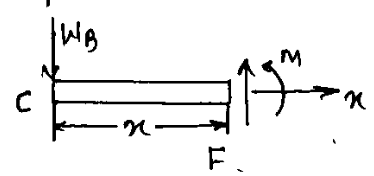
b) $+\uparrow \Sigma F_y = 0 \Rightarrow R_A + R_B - W_m - 2W_B = 0.$

$+\circlearrowleft \Sigma M_A = 0 \Rightarrow W_B \cdot a - W_m \cdot (\frac{L}{2} - a) + R_B \cdot (L - 2a) - W_B \cdot (L - a) = 0.$

from above equations: $R_B = W_B + \frac{W_M}{2}$

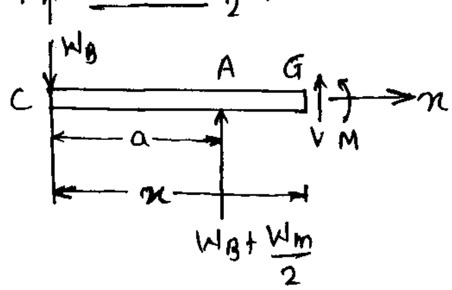
$R_A = W_B + \frac{W_M}{2}.$

$0 < x < a$



$\Sigma M_F = 0 \Rightarrow M = -W_B \cdot x.$

$a < x < \frac{L}{2}.$



$\Sigma M_G = 0 \Rightarrow$

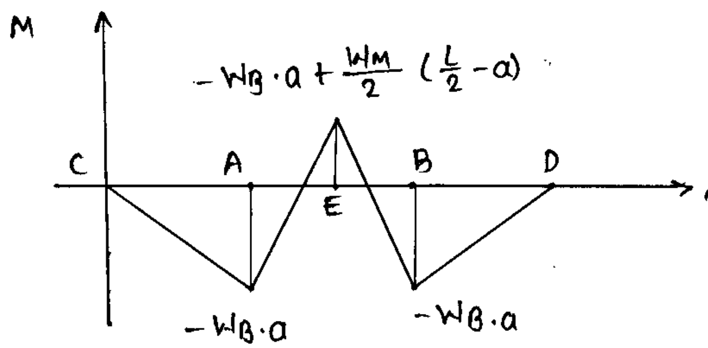
$$M = -W_B x + (W_B + \frac{W_m}{2}) \cdot (x - a)$$

$$= \frac{W_m}{2} \cdot x - (\frac{W_m}{2} + W_B) \cdot a$$

Symmetry will give the same expressions for M in EB and BD.

(problem 9.18 contd.)

Bending moment diagram:



$$M\big|_{x=\frac{L}{2}} = \frac{W_m}{2}\cdot\frac{L}{2} - \left(\frac{W_m}{2}+W_B\right)\cdot a$$

$$= -W_B \cdot a + \frac{W_m}{2}\left(\frac{L}{2}-a\right).$$

(Assumed positive)

To minimize the maximum Bending Moment:

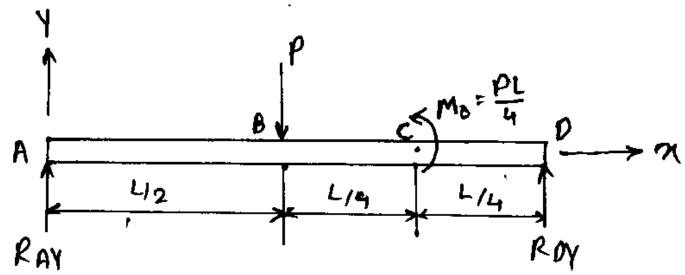
magnitude of M at A = Magnitude of M at E.

$$\therefore\ W_B \cdot a = -W_B \cdot a + \frac{W_m}{2}\left(\frac{L}{2}-a\right).$$

$$\therefore\ 2W_B \cdot a = \frac{W_m}{2}\left(\frac{L}{2}-a\right)$$

$$\therefore\ W_B = \frac{W_m}{4a}\left(\frac{L}{2}-a\right).$$

Solution to problem 3.22



$+\uparrow \Sigma F_y = 0 \Rightarrow R_{AY} - P + R_{DY} = 0.$

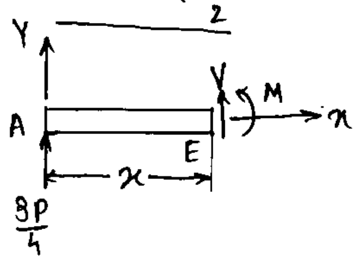
$+\circlearrowleft \Sigma M_A = 0 \Rightarrow -P\left(\frac{L}{2}\right) + M_0 + R_{DY}(L) = 0.$ $\quad M_0 \to \frac{PL}{4}$

$\Rightarrow$

$\therefore \quad R_{AY} = \frac{3P}{4}$

$R_{DY} = \frac{P}{4}.$

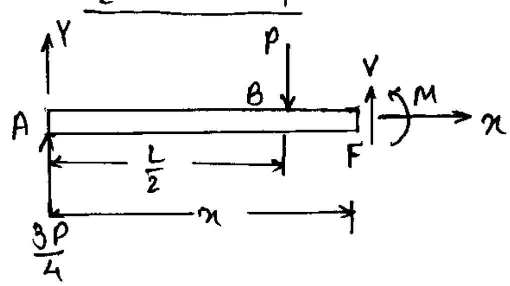
$0 < x < \frac{L}{2}$



$\Sigma F_y = 0 \Rightarrow V = -\frac{3P}{4}.$

$\Sigma M_E = 0 \Rightarrow M = \frac{3P}{4}x.$

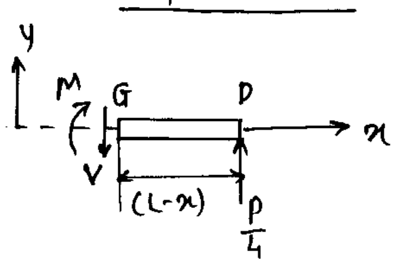
$\frac{L}{2} < x < \frac{3L}{4}$



$\Sigma F_y = 0 \Rightarrow V = \frac{P}{4}$

$\Sigma M_F = 0 \Rightarrow M = \frac{PL}{2} - \frac{Px}{4}.$

$\frac{3L}{4} < x < L$

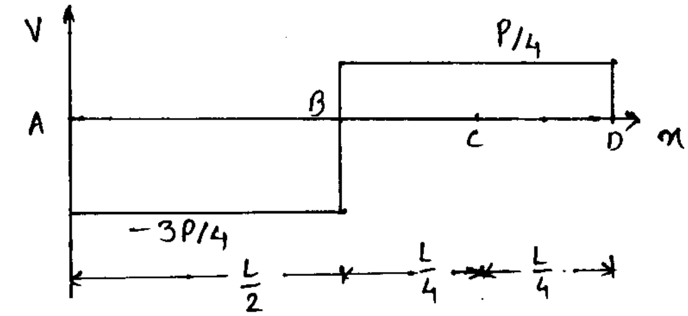


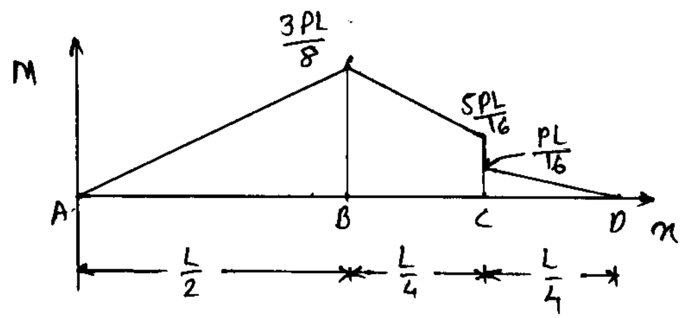
$\Sigma F_y = 0 \Rightarrow V = \frac{P}{4}$

$\Sigma M_G = 0 \Rightarrow M = \frac{P}{4}(L-x).$

(Problem 3.22 contd.)

Shear force and Bending moment diagrams:



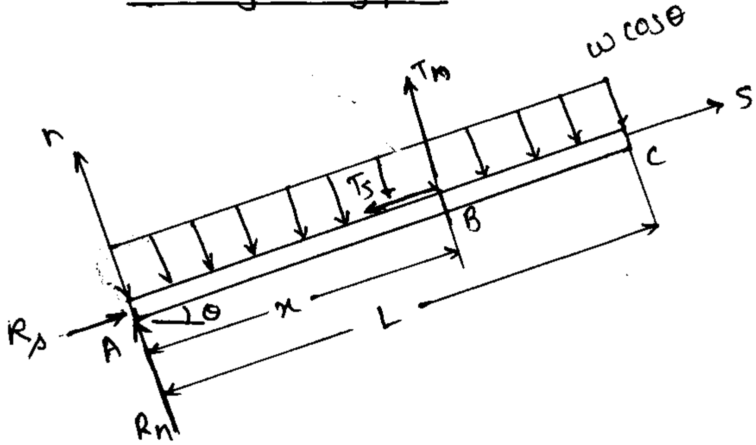


Note: At point C, there is a concentrated moment of magnitude $\frac{PL}{4}$ in ccw direction. ~~And~~ Therefore, there is a jump in the bending moment diagram of the same magnitude in the downward direction

Solution to problem 3.26

FBD of Flag pole:

$L = 75'$



(The component w sinθ is not shown)

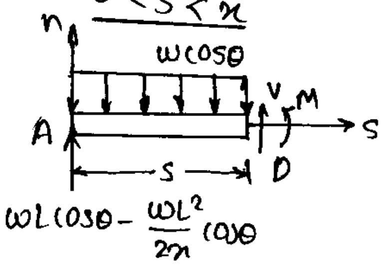
If the flagpole doesn't bend to an appreciable extent, then bending moment can only be caused by forces normal to flagpole. These are: i) R_n, ii) T_n, 3) Normal component of $w = w\cos\theta$.

$$\Sigma(M_z)_A = 0 \Rightarrow T_n = \frac{wL^2}{2x}\cos\theta.$$

$$\Sigma F_n = 0 \Rightarrow R_n - w\cos\theta \cdot L + T_n = 0.$$

$$\Rightarrow R_n = wL\cos\theta - \frac{wL^2}{2x}\cos\theta.$$

$0 < s < x$

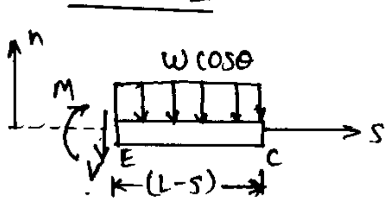


$$\Sigma F_n = 0 \Rightarrow V = -wL\cos\theta + \frac{wL^2}{2x}\cos\theta + w\cos\theta . s \quad \text{—— (1)}$$

$$\Sigma M_D = 0 \Rightarrow$$

$$M = (wL\cos\theta - \frac{wL^2}{2x}\cos\theta). s - w\cos\theta . \frac{s^2}{2}. \quad \text{—— (2)}$$

$x < s < L$



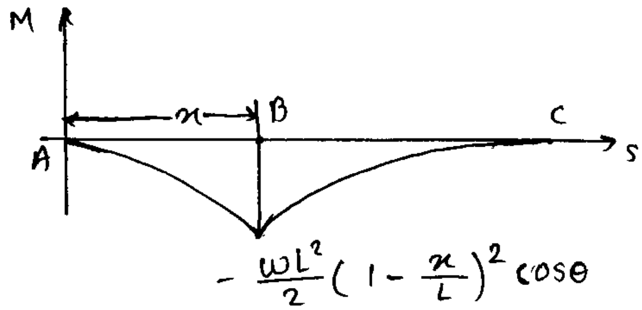
$$\Sigma F_n = 0 \Rightarrow V = -w\cos\theta(L-s) \quad \text{—— (3)}$$

$$\Sigma M_E = 0 \Rightarrow$$

$$M = -\frac{1}{2} w\cos\theta(L-s^2). \quad \text{—— (4)}$$

(problem 3.26 contd.)

Bending moment diagram for $x < \frac{L}{2}$ (case 1):

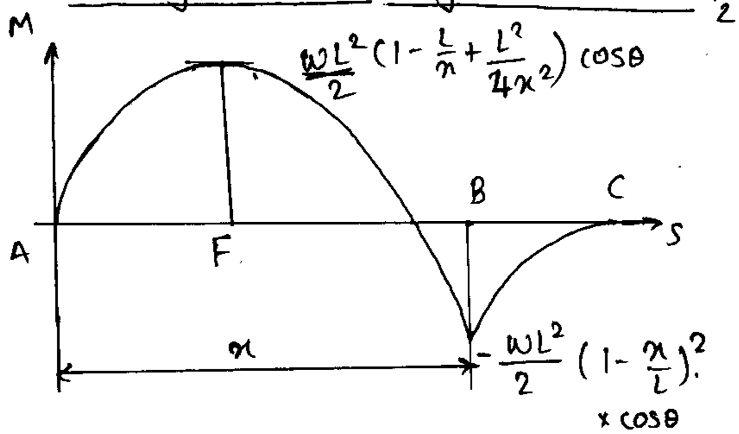


$$M|_x = wLx\cos\theta - w\cos\theta\,\frac{x^2}{2} - \frac{wL^2}{2x}\cdot x\cos\theta \quad \text{(eq. 2)}$$

$$= -\frac{wL^2}{2}\cos\theta\left(1-\frac{x}{L}\right)^2.$$

Maximum bending moment becomes minimum when $x = L$. But it is outside the range. ∴ This is not an acceptable solution.

Bending Moment diagram for $x > \frac{L}{2}$ (case 2):



At F, $S = L - \frac{L^2}{2x}$ from the condition $V|_F = 0$ (eq. 1)

$$M|_{S = L - \frac{L^2}{2x}} = \left(wL - \frac{wL^2}{2x}\right)\left(L - \frac{L^2}{2x}\right)\cos\theta - \frac{w}{2}\left(L - \frac{L^2}{2x}\right)^2\cos\theta$$

$$\therefore\ M|_{S = L-\frac{L^2}{2x}} = \frac{wL^2}{2}\left(1 - \frac{L}{x} + \frac{L^2}{4x^2}\right)\cos\theta. \quad \text{(eq. 2)}$$

To minimize the maximum bending moment:

magnitude of M at F = magnitude of M at B.

$$\therefore\ \frac{wL^2}{2}\left(1-\frac{L}{x}+\frac{L^2}{4x^2}\right)\cancel{\cos\theta} = \frac{wL^2}{2}\left(1-\frac{x}{L}\right)^2 .\cos\theta.$$

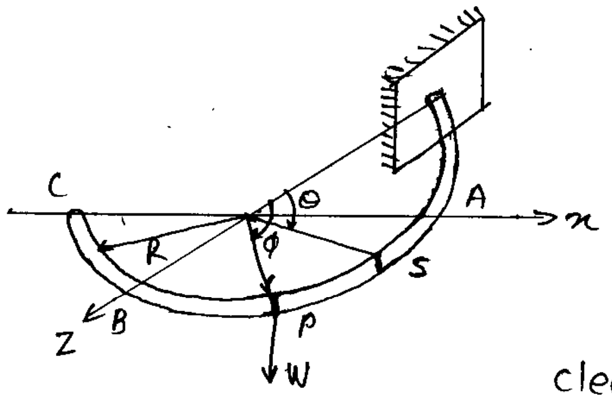
$$\Rightarrow \frac{x^4}{L^4} - \frac{2x^3}{L^3} + \frac{x}{L} - \frac{1}{4} = 0 \qquad \Rightarrow \frac{x}{L} = 1.7071, -0.707, \underline{0.707}, 0.293.$$

As x is between $L/2$ to L

$x = L \times 0.707 = 53.03'$

Solution to problem 3.27

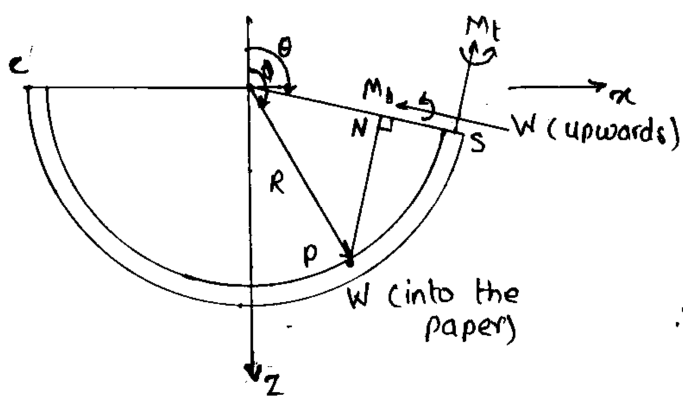
Sign conventions:



Consider the general case where a load W is applied at a point P (angle ϕ) and we wish to find the bending and twisting moments for any point S at angle θ.

Clearly, for $\theta > \phi$, $M_b = M_t = 0$.

Projection of free body:



For $\theta \leq \phi$

(from adjecent fig.)

$M_b = W(PN)$

$M_t = W(NS)$

$PN = R\sin(\phi - \theta)$

$NS = R[1 - \cos(\phi - \theta)]$

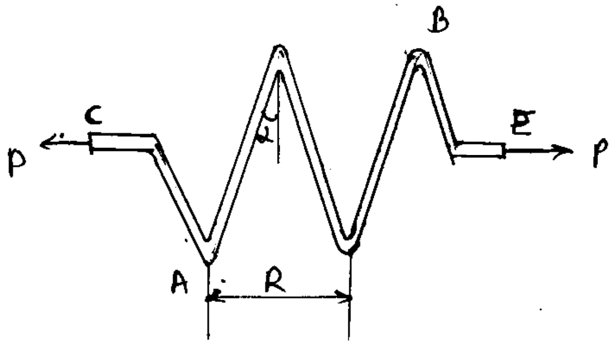
$$\therefore \left.\begin{aligned} M_b &= WR\sin(\phi-\theta) \\ M_t &= WR[1-\cos(\phi-\theta)] \end{aligned}\right\} \theta \leq \phi$$

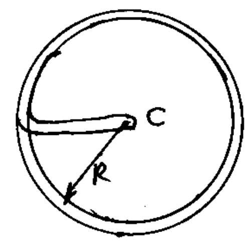
1. load at A, $\phi = 90°$: $M_b = WR\cos\theta$; $(M_b)_{max} = WR$ at $\theta = 0°$
   $M_t = WR(1 - \sin\theta)$; $(M_t)_{max} = WR$ at $\theta = 0°$.

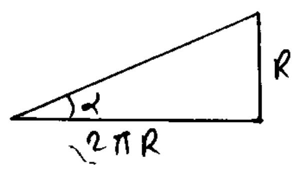
2. Load at B, $\phi = 180°$: $M_b = WR\sin\theta$; $(M_b)_{max} = WR$ at $\theta = 90°$
   $M_t = WR[1 + \cos\theta]$; $(M_t)_{max} = 2WR$ at $\theta = 0°$.

3. Load at C, $\phi = 270°$: $M_b = -WR\cos\theta$; $(M_b)_{max} = WR$ at $\theta = 0°, 180°$
   $M_t = WR[1 + \sin\theta]$; $(M_t)_{max} = 2WR$ at $\theta = 90°$.

Solution to problem 3.29







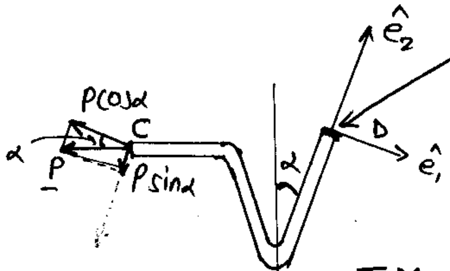
$$\tan\alpha = \frac{1}{2\pi}.$$

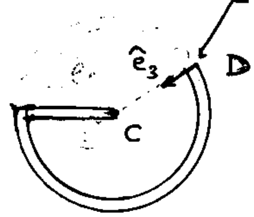
$$\tan^2\alpha + 1 = \frac{1}{\cos^2\alpha}$$

$$\therefore \frac{1}{4\pi^2} + 1 = \frac{1}{\cos^2\alpha}.$$

$$\cos\alpha = \frac{2\pi}{\sqrt{1+4\pi^2}}, \quad \sin\alpha = \frac{1}{\sqrt{1+4\pi^2}}.$$

Moment $\underset{\sim}{M}$ on this section





$\hat{e}_1, \hat{e}_2, \hat{e}_3$ is right handed triad.

$$\Sigma \underset{\sim}{M}_D = \underset{\sim}{0} \Rightarrow \underset{\sim}{M} - \underset{\sim}{DC} \times \underset{\sim}{P} \Rightarrow \underset{\sim}{M} = \underset{\sim}{DC} \times \underset{\sim}{P}$$

$$\therefore \underset{\sim}{M} = R\,\hat{e}_3 \times (-P\cos\alpha\,\hat{e}_1 - P\sin\alpha\,\hat{e}_2)$$

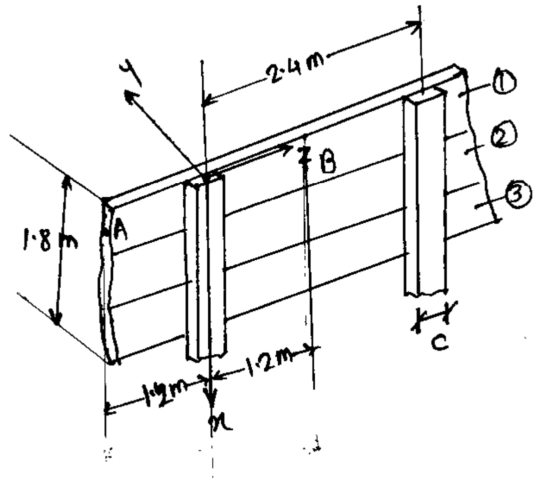
$$= -PR\cos\alpha\,\hat{e}_2 + PR\sin\alpha\,\hat{e}_1.$$

This expression is valid at every point from A to B.

M_t (twisting Moment) := coefficient of $\hat{e}_2 = -PR\cos\alpha$

M_b (Bending Moment) = coefficient of $\hat{e}_1 = PR\sin\alpha$.

Solution to problem 3.34



The slices in the planks are symmetrical about the x-axis. Thus there can be no shear ~~stresses~~ force V_{zy} at A & B in plank ① and similar points in the other planks.

Now consider the y-direction equilibrium of the plank. The y-direction forces are:

i) Linearly distributed water pressure.

ii) Force exerted by the upright (unknown distribution)

By writing $\Sigma F_y = 0$ and $\Sigma M_z = 0$ we can find magnitude and point of application of the resultant force between upright and plank. The distribution cannot be determined by simple statics. However, since we need to know the distribution to calculate M and V in uprights, we will assume that it is the same as waterpressure distribution.

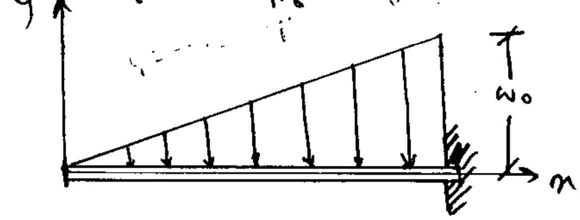
Thus if q = load per meter of upright then

$q = C \times p$ where p is water pressure $= \gamma x$, c = width of upright

$\therefore q = w_0 \frac{x}{1.8}$ where $w_0 \equiv$ constant $= c\gamma(1.8)$

Negative sign because q is along $-$ve y-axis

$q = -w_0 \frac{x}{1.8}$



mean pressure over 1.8 m deapth

= pressure at 0.9 m

$= 1000 \times 9.81 \times 0.9$

$= 8.83 \times 10^3$ N/m2.

Total force on 1.8×2.4 section of planks

$= 1.8 \times 2.4 \times 8.83 \times 10^3 = 38.14$ KN.

(Problem 3.34 contd.)

$\therefore$ Total load on upright $= \int_0^{1.8} w_0 \left(\frac{x}{1.8}\right) dx = 38.14$ kN.

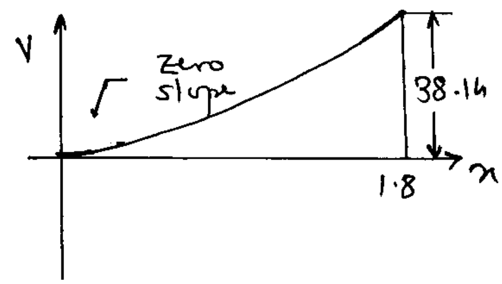
$\Rightarrow \quad 0.9\, w_0 = 38.14 \quad \Rightarrow \quad w_0 = 42.38$ kN/m.

Sign convention:

$\Rightarrow \quad q(x) = -\frac{42.38}{1.8} x = -23.54 \times 10^3 x$

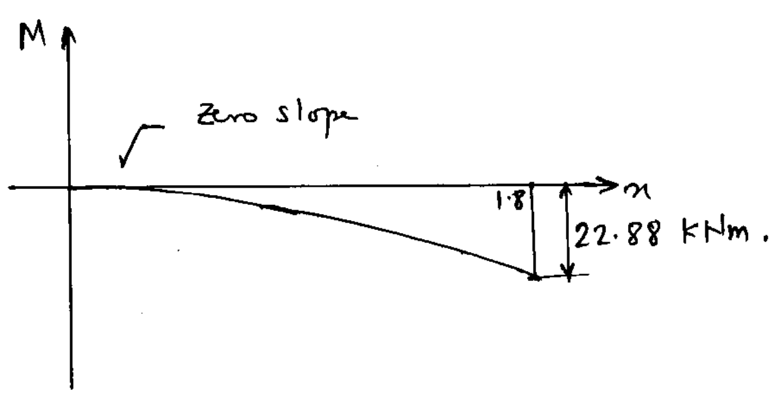
Shear force distribution:

$$V(x) = -\int_0^x q(x)\, dx = \frac{23.54 \times 10^3}{2} x^2$$



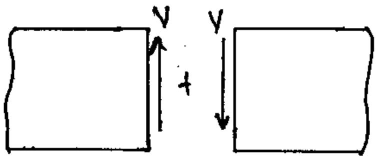
Bending moment distribution

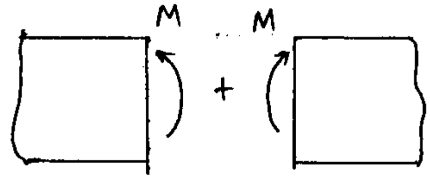
$$M(x) = -\int_0^x V(x)\, dx = -\frac{23.54 \times 10^3}{6} x^3.$$

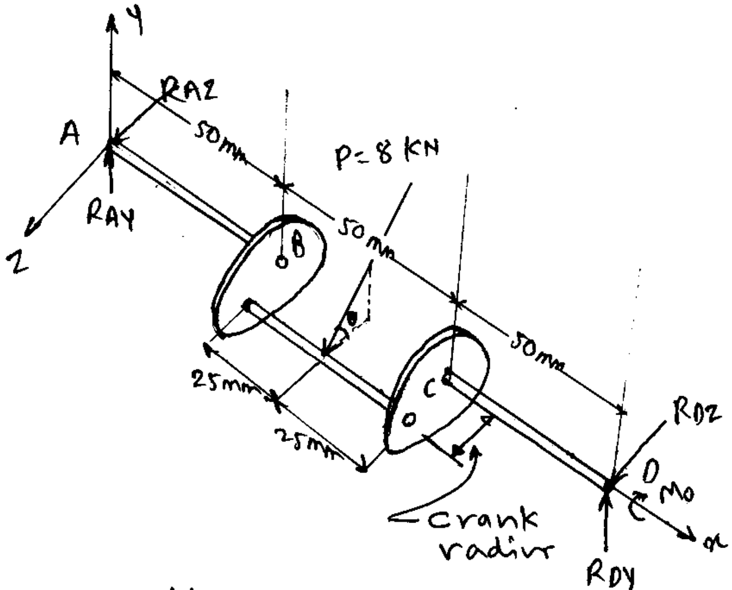


Solution to problem 3.40:

Sign convention:







Crank radius

$= \frac{1}{2} \times$ stroke

$= \frac{75}{2}$ mm.

Assume that bearings do not transmit moments to shaft.

connecting rod length = 125 mm.

$\cos\theta = \dfrac{75/2}{125} \therefore \theta = 72.54°$ (In one particular position)

Overall equilibrium:

$\Sigma M_x = 0 \Rightarrow M_o - P\cos\theta \times 0 - P\sin\theta \times \frac{75}{2} = 0$. (mm)

$\therefore M_o = P\sin\theta \times \frac{75}{2} = 8 \times \sin 72.54° \times \frac{75}{2}$

$= 286.2$ ~~kN.mm~~ N.m

$\Sigma (M_y)_A = 0 \Rightarrow -R_{DZ} \times 150 - P\cos\theta \times 75 = 0$

$\Rightarrow R_{DZ} = -1.2$ KN

$\Sigma (M_z)_A = 0 \Rightarrow R_{DY} \times 150 - P\sin\theta \times 75 = 0$.

$\Rightarrow R_{DY} = 3.8$ KN

By symmetry: $R_{AZ} = R_{DZ} = -1.2$ kN

$R_{AY} = R_{DY} = 3.8$ kN.

(Problem 3.40 contd.)

It is only necessary to sketch shear force & Bending moment diagram for AB. For CD, they are identical except that the sign of V is reversed.

Net V = $\sqrt{V_y^2 + V_z^2}$

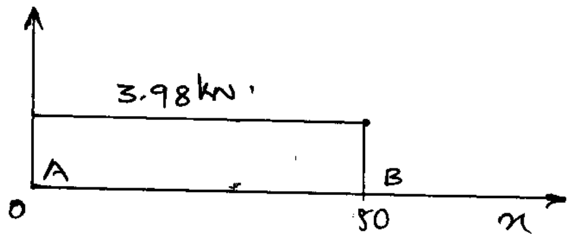
Net reaction at A:

$R_A = \sqrt{R_{Ay}^2 + R_{Az}^2}$

$= \sqrt{(3.8)^2 + (-1.2)^2}$

$= 3.98$ kN

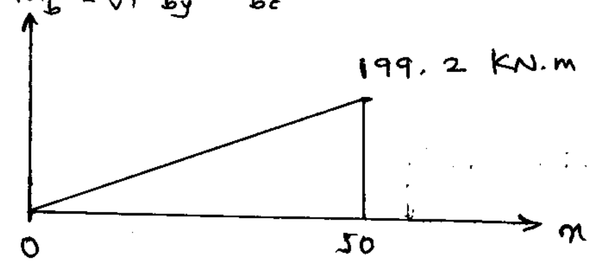
i)



Shear force for AB, CD.

Net $M_b = \sqrt{M_{by}^2 + M_{bz}^2}$

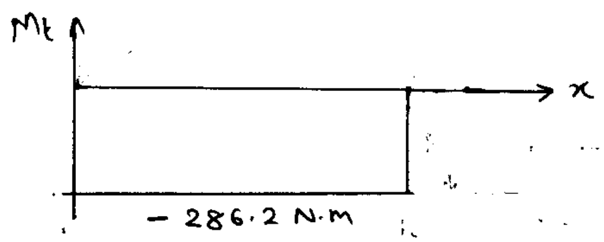
ii)



Bending moment for AB, CD

iii) Twisting moment for AB = 0.

iv)



Twisting momement for CD.

Sign convention for twisting moment