

## ME361 – Manufacturing Science Technology

Practice problems on bulk deformation processes and machining (conventional)

This handout is for your self-study. Solutions will not be posted, and answers will not be graded.

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### Section: Machining

#### Problem 1

A set of orthogonal tests were conducted to identify shear angle, friction coefficient, and shear stress. Steel was cut and the cutting conditions, measured forces, and chip thicknesses are tabulated below. Rake angle,  $\alpha_r$ , was zero. Width of cut was  $b = 5$  mm, and the cutting speed was  $V = 240$  m/min. Density of the steel that was cut =  $\rho = 7800$  kg/m<sup>3</sup>

Feed rate, $h$ [mm/rev]	Tangential force, $F_t$ [N]	Feed force, $F_f$ [N]	Measured chip thickness $h_c$ [mm]
0.02	350	290	0.050
0.03	480	350	0.058
0.04	590	400	0.074
0.05	690	440	0.083
0.06	790	480	0.102
0.07	890	505	0.116
0.08	980	540	0.131

Evaluate:

- The cutting coefficients  $K_{tc}$  and  $K_{fc}$  and the edge coefficients  $K_{te}$  and  $K_{fe}$  by a linear regression of the measured forces.
- The shear angle,  $\phi_c$ , shear stress,  $\tau_s$ , and average friction coefficient,  $\mu_a$  for each test, and express them as an empirical function of uncut chip thickness,  $h$ , to form an orthogonal cutting database.
- The shear strain and strain rate for each test at the primary shear zone

Predict:

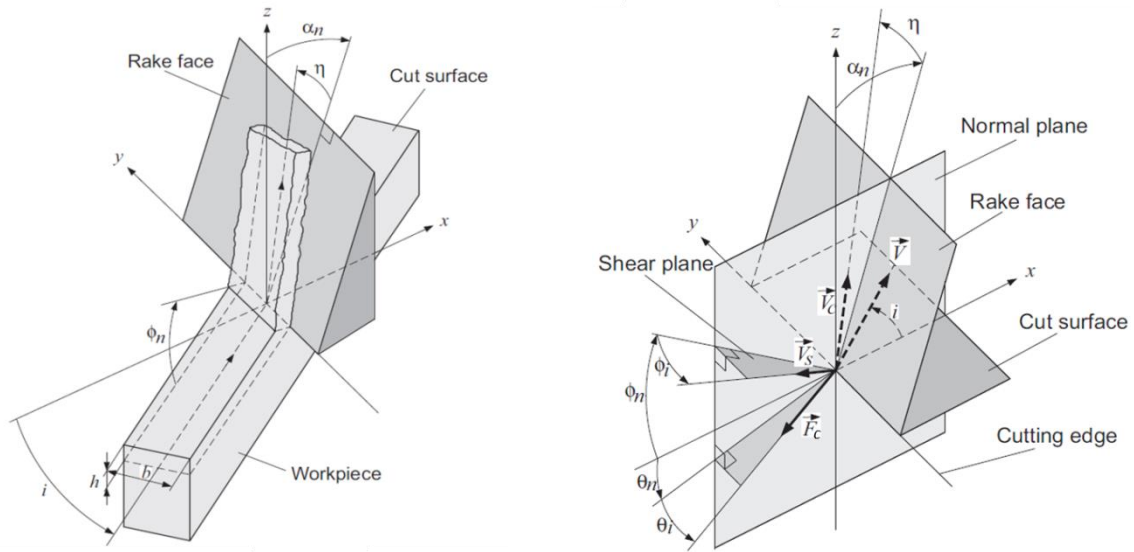
- The cutting force coefficients  $K_{tc}$  and  $K_{fc}$  using empirically expressed  $\tau_s$ ,  $\phi_c$ , and  $\mu_a$  and compare them against the values identified from the mechanistic linear regression

Construct:

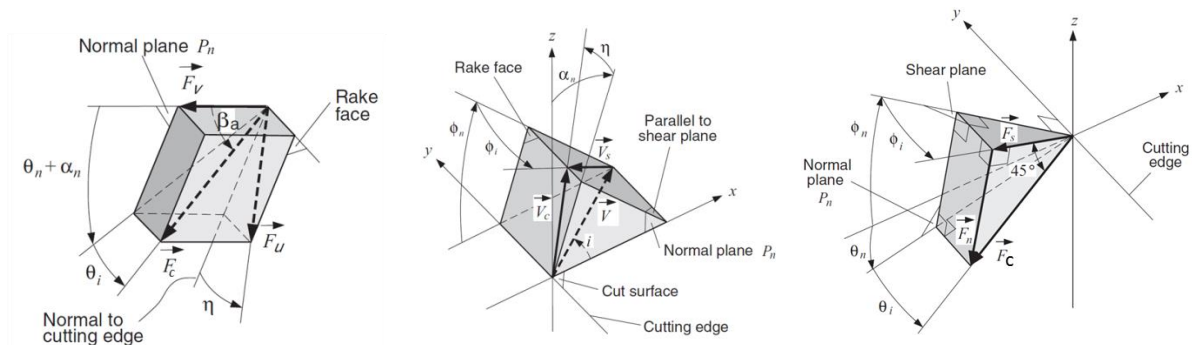
- The force circle diagram for any one test, and compare graphically obtained shear angle and friction coefficient with those evaluated in (ii) for the same test.

## Problem 2

Shown below is the geometry of the oblique cutting process:



Shown below are the force and velocity diagrams in oblique cutting.



Show the steps involved in obtaining the following five equations:

$$\sin \theta_i = \sin \beta_a \sin \eta \quad (1)$$

$$\tan(\theta_n + \alpha_n) = \tan \beta_a \cos \eta \quad (2)$$

$$\tan \eta = \frac{\tan i \cos(\phi_n - \alpha_n) - \cos \alpha_n \tan \phi_i}{\sin \phi_n} \quad (3)$$

$$F_s = F_c [\cos(\theta_n + \phi_n) \cos \theta_i \cos \phi_i + \sin \theta_i \sin \phi_i] \quad (4)$$

$$F_{tc} = F_c (\cos \theta_i \cos \theta_n \cos i + \sin \theta_i \sin i) \quad (5)$$

### Problem 3

Consider oblique cutting of a material that has a yield strength of 750 MPA with a tool that has a normal rake angle of  $\alpha_n = 10^\circ$  and an angle of obliquity of  $i = 20^\circ$ . Assume a coefficient of friction,  $\mu = 0.5$ . Using the empirical approach of Armarego and Whitfield (discussed in class and given in the notes), Eq. (1 - 3) from Problem 2 transform to:

$$\tan(\phi_n + \beta_n) = \frac{\cos \alpha_n \tan i}{\tan \eta - \sin \alpha_n \tan i} \quad (1)$$

$$\tan \beta_n = \tan \beta_a \cos \eta \quad (2)$$

$$\phi_n = \tan^{-1} \left( \frac{r_c (\cos \eta / \cos i) \cos \alpha_n}{1 - r_c (\cos \eta / \cos i) \sin \alpha_n} \right) \quad (3)$$

Wherein  $\beta_n = \theta_n + \alpha_n$ , and  $r_c$  is the chip thickness ratio that can be evaluated from any one of the tests listed in Problem 1. Please state the  $r_c$  value assumed by you.

- Solve the above three equations numerically to find the three unknown angles of:  $\beta_n$ ,  $\phi_n$ , and  $\eta$ .
- Adopt Stabler's empirical chip rule for simplification, i.e.  $\eta = i$ , and calculate  $\beta_n$ ,  $\phi_n$ , and  $\eta$  again.
- Compare calculations from (i) and (ii). Comment on the reasons for discrepancies, if any.
- Using the parameters obtained in (ii), estimate the force components in directions of cutting ( $F_{tc}$ ), the thrust force ( $F_{fc}$ ), and the normal force ( $F_{rc}$ ). Assume that the material fails as per the von Mises criterion.
- Calculate the cutting force coefficients  $K_{tc}$ ,  $K_{fc}$ , and  $K_{rc}$  for oblique cutting
- If this happened to be orthogonal cutting, how different would the cutting force coefficients be? Calculate them. Make assumptions as necessary, and state them.

### Problem 4

A steel shaft of diameter 57 mm is turned with a carbide tool that has a side rake angle of  $\alpha_f = 5^\circ$ , a back rake angle of  $\alpha_p = -5^\circ$ , and zero approach angle. The nose radius of the insert is  $r = 0.8$  mm. The radial depth of cut, the feed rate, and the cutting speed are:  $a = 1$  mm,  $c = 0.06$  mm/rev, and  $V = 240$  m/min, respectively.

The orthogonal parameters for the same steel and carbide tool with a rake angle  $\alpha_r = -5^\circ$  are:

$$\text{Shear stress: } \tau_s = 1400h + 0.327V + 507 \text{ [N/mm}^2\text{]}$$

$$\text{Friction angle: } \beta_a = 33.69 - 12.16h - 0.0022V \text{ [degree]}$$

$$\text{Chip ratio: } r_c = 2.71h + 0.00045V + 0.227$$

wherein the units for  $h$  are mm and for  $V$  are in m/min.

- i. Evaluate the distribution of chip thickness along the curved chip length.
- ii. Evaluate the cutting coefficients along the curved chip length using the oblique cutting transformation.
- iii. Evaluate the distribution of tangential, radial, and feed cutting forces ( $F_t$ ;  $F_r$ ;  $F_f$ ) along the curved chip.
- iv. Find the total cutting forces ( $F_x$ ;  $F_y$ ;  $F_z$ ), torque, and power required to turn the shaft.

### Problem 5

Consider orthogonal cutting using a tool that has a rake angle of  $20^\circ$ . The cutting and thrust force components are measured to be  $F_{tc} = 1000$  N and  $F_{fc} = 100$  N.

- a. By constructing (to scale) Merchant's Force Circle Diagram, graphically estimate the normal and in-plane rake-face force components, as well as the resultant machining force.
- b. Graphically estimate the normal and in-plane shear plane force components. Assume the Merchant's minimum energy model to be valid.
- c. Using the relations of both the rake face and the shear plane force components to the cutting and thrust force components, evaluate:  $F_s$ ,  $F_n$ ,  $F_v$ , and  $F_u$  to check your graphical estimates of parts (a) and (c). Comment on differences if any.

### Problem 6

Consider oblique cutting with a tool that has a normal rake angle of  $\alpha_n = 10^\circ$  and an angle of obliquity of  $i = 20^\circ$ . Assume a coefficient of friction,  $\mu = 0.5$ . The following five equations are given to you. Three are obtained from force and velocity diagrams in oblique cutting and two are obtained by applying the minimum energy principle to orthogonal cutting.

$$\sin \theta_i = \sin \beta_a \sin \eta \quad (1)$$

$$\tan(\theta_n + \alpha_n) = \tan \beta_a \cos \eta \quad (2)$$

$$\tan \eta = \frac{\tan i \cos(\phi_n - \alpha_n) - \cos \alpha_n \tan \phi_i}{\sin \phi_n} \quad (3)$$

$$\frac{\partial P_t'}{\partial \phi_n} = 0; \quad (4)$$

$$\frac{\partial P_t'}{\partial \phi_i} = 0 \quad (5)$$

Wherein the normalized power is:  $P_t' = \frac{P_{tc}}{V \tau_s b h} = \frac{\cos \theta_n + \tan \theta_i \tan i}{[\cos(\theta_n + \phi_n) \cos \phi_i + \tan \theta_i \sin \phi_i] \sin \phi_n}$

- i. Find the five unknown oblique angles:  $\phi_n$ ,  $\phi_i$ ,  $\eta$ ,  $\theta_i$ ,  $\theta_n$  using the numerical iteration technique discussed in class and given in the notes.

Adopt this procedure:

Start with Stabler's rule that states:  $\eta = i$ . After calculating the initial values of  $\phi_n$ ,  $\phi_i$ ,  $\theta_i$  and  $\theta_n$ , using Eq. (1-2), and Eq. (4-5), calculate the cutting power  $P_t'$ . Having calculated the

power  $P_t'$ , then change the shear angles slightly, i.e.  $\phi_n + \Delta\phi_n$  and  $\phi_i + \Delta\phi_i$ . Evaluate the steepest descent direction, i.e.  $\Delta P_t' / \Delta\phi_n$ ,  $\Delta P_t' / \Delta\phi_i$ . Change the shear angles by a step  $\delta$  in the steepest descent direction such that the cutting energy approaches a minimum value as follows:

$$\begin{Bmatrix} \phi_n(k) \\ \phi_i(k) \end{Bmatrix} = \begin{Bmatrix} \phi_n(k-1) \\ \phi_i(k-1) \end{Bmatrix} - \delta \begin{Bmatrix} \Delta P_t' / \Delta\phi_n \\ \Delta P_t' / \Delta\phi_i \end{Bmatrix}.$$

$k$  is the iteration counter. Continue the numerical iteration until the normalized power converges to a minimum value. Plot the change in power as a function of number of steps. Report the shear angles for the minimum power.

- ii. Use the empirical approach of Armarego and Whitfield discussed in class and given in the notes to find the three unknown angles of:  $\beta_n$ ,  $\phi_n$ , and  $\eta$ . Adopt Stabler's empirical chip rule for simplification, i.e.  $\eta = i$ . Calculate the chip thickness ratio,  $r_c$  from any one of the tests listed in Problem 1.
- iii. Compare calculations for the normal shear angle using Armarego and Whitfield's approach with that obtained using the numerical iteration scheme in (i). Comment on the reasons for discrepancies, if any.

### Problem 7

Consider face milling of mild steel with a cutter that has eight teeth. The cutter diameter is 200 mm, and the workpiece is 100 mm square. The cutter is centered on the workpiece during cutting.

- a. What are the entry and exit angles?
- b. What are the maximum/minimum number of teeth that can be cutting at the same time?

For the remainder of the questions, let there be only four teeth on the cutter so that at most there is only one tooth cutting at any time.

- c. For this tool and workpiece situation, consider the magnitudes of the  $x, y, \text{ and } z$  force components acting on a tooth and choose (by circling your choice) which of the following is true.

For  $F_x$ : (i)  $F_x$  at entry  $<$   $F_x$  at exit (ii)  $F_x$  at entry  $>$   $F_x$  at exit (iii)  $F_x$  at entry  $=$   $F_x$  at exit

For  $F_y$ : (i)  $F_y$  at entry  $<$   $F_y$  at exit (ii)  $F_y$  at entry  $>$   $F_y$  at exit (iii)  $F_y$  at entry  $=$   $F_y$  at exit

For  $F_z$ : (i)  $F_z$  at entry  $<$   $F_z$  at exit (ii)  $F_z$  at entry  $>$   $F_z$  at exit (iii)  $F_z$  at entry  $=$   $F_z$  at exit

- d. When one tooth is at an immersion angle of  $\phi = 55^\circ$ , the (tooth-local) forces acting on the tooth are:  $F_t = 640$  N,  $F_r = 375$  N, and  $F_a = 200$  N. What are the force components in the  $x, y, \text{ and } z$  acting on the workpiece?

### Problem 8

It is desired to determine the effects of spindle speed (RPM), feed rate (mm/rev) and corner radius (mm) on the surface finish of a turned surface. Design an experiment to determine a model as a function of un-coded variables. Include in the model all effects that appear to be significant. Ignore any effect that is computed to be  $\leq 0.2$ .

Take the low/high levels for three factors as:

Spindle speed,  $n_s$ : 500/1000 RPM; Feed rate,  $f_r$ : 0.05/0.3 mm/rev; Corner radius,  $r_e$ : 0.5/1 mm

Take the un-coded and coded variables to be:  $n_s: X_1 \rightarrow x_1$ ;  $f_r: X_2 \rightarrow x_2$ ;  $r_e: X_3 \rightarrow x_3$ ;

The experiment design matrix in the coded space is given below. Also given is the response, i.e. surface roughness that was measured for each case.

	Main effects			Interaction effects				Response
Test	$x_1$	$x_2$	$x_3$	$x_1x_2$	$x_1x_3$	$x_2x_3$	$x_1x_2x_3$	$y$ ( $\mu\text{m}$ )
1	-1	-1	-1	+1	+1	+1	-1	$y_1 = 1.3$
2	+1	-1	-1	-1	-1	+1	+1	$y_2 = 1.6$
3	-1	+1	-1	-1	+1	-1	+1	$y_3 = 5.4$
4	+1	+1	-1	+1	-1	-1	-1	$y_4 = 5.2$
5	-1	-1	+1	+1	-1	-1	+1	$y_5 = 0.8$
6	+1	-1	+1	-1	+1	-1	-1	$y_6 = 0.7$
7	-1	+1	+1	-1	-1	+1	-1	$y_7 = 2.8$
8	+1	+1	+1	+1	+1	+1	+1	$y_8 = 2.7$

Hint: Solve in the coded space, and transform to un-coded space later. The transformation to go from coded to un-coded space is:  $x = \frac{2X - X^+ - X^-}{X^+ - X^-}$ ; wherein  $X^+ \rightarrow$  high level of variable, and  $X^- \rightarrow$  low level of variable. For the final form of the response model, keep in mind the geometry and process mechanics during turning with a nose radius.

### Problem 9

Assume steel was cut in an orthogonal setup with an uncut chip thickness,  $h = 0.2$  mm. The workpiece diameter was 100 mm, and the spindle speed was 100 m/min. Tool rake angle,  $\alpha_r$  was  $10^\circ$ . Width of cut was  $b = 6$  mm. Shear stress,  $\tau_s$  of the material is given to be 760 MPa. Measured chip thickness was found to be 0.45 mm. Assume a coefficient of friction,  $\mu = 0.5$ . Assume no edge rubbing takes place during cutting.

Determine:

- The tangential and feed cutting force coefficients. (Hint: think about the chip ratio and what it gives you that will serve as an input for these calculations).
- Using the coefficients calculated in (a), predict tangential and feed forces.
- Using results of (a) and (b), find all the forces in the primary and secondary shear zones.
- Find the shearing power, power consumed to overcome friction and the total power

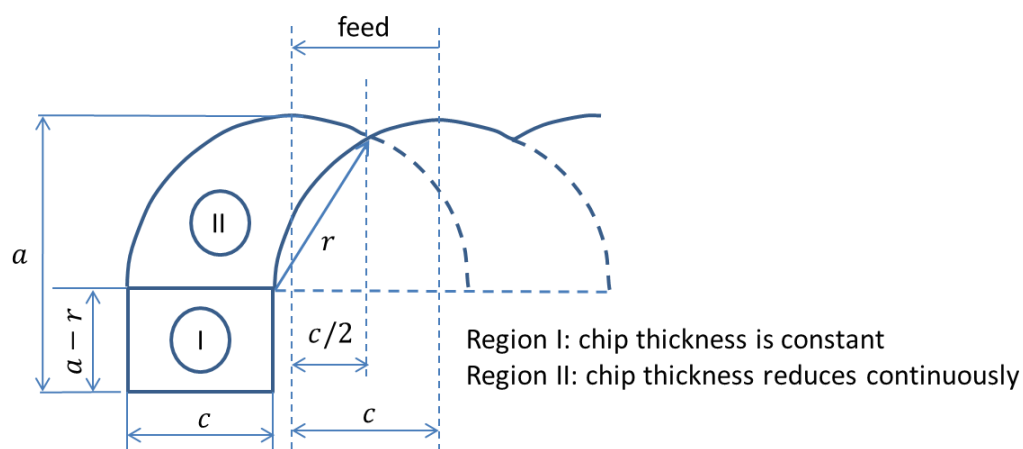
- In (a), instead of using the chip ratio and what it gives you, if you had instead used the minimum energy principle to determine the shear angle, how different would the coefficients turn out to be? Please quantify the difference, if any. If any different, how much would the power consumed change by? Change in power, if any, you may address qualitatively.
- If you assume the shear plane thickness to be 0.02 mm, what is the strain and strain rate for results obtained in (a) and (e). Do you observed any dependence of strain rate on shear angle? Briefly comment on why/why not.
- Briefly describe two changes (specify increase or decrease) to the tool geometry that would reduce the cutting power.

### Problem 10

A steel shaft of diameter 75 mm is turned with a carbide tool that has a side rake angle of  $\alpha_f = 7^\circ$ , a back rake angle of  $\alpha_p = -8^\circ$ . The nose radius of the insert is  $r = 0.8$  mm. The radial depth of cut, the feed rate, and the cutting speed are:  $a = 2$  mm,  $c = 0.1$  mm/rev, and  $V = 500$  m/min, respectively.

Orthogonal cutting was carried out previously with a carbide tool on the same material, and the friction angle  $\beta_a$  was found to be  $25^\circ$ , and the shear stress was found to be  $\tau_s = 450$  MPa.

Divide the tool-workpiece contact into two regions as shown below. Ignore any edge rubbing effects.



- In the first region, the approach angle,  $\psi_r = 0^\circ$ . Evaluate the cutting coefficients using the orthogonal to oblique cutting transformation. Assume that Stabler's rule holds, and that angle of obliquity,  $i = \eta$ , the chip flow angle. Calculate the cutting forces in this region
- In the second region, evaluate chip thickness distribution along the curved chip length.
- Assume that the second region can be discretized into three elements. Evaluate the cutting coefficients for each element, and then calculate the tangential, radial, and feed cutting forces ( $F_t, F_r, F_f$ ) in each element. Again, use the orthogonal to oblique transformation and assume that Stabler's rule holds.

Find the total cutting forces ( $F_x, F_y, F_z$ ), torque, and power required to turn the shaft

**Problem 11**

Consider an orthogonal cutting process. Feed is 0.2 mm/rev. Width of cut is 2 mm. Cutting speed is 3 m/sec. Rake angle is  $20^\circ$ . The coefficient of friction on the rake face is measured to be 0.3. The cutting force is measured to be 600 N, and the thrust force is measured to be 120 N. The cut chip thickness is measured to be 0.48 mm. Merchant's or any other such shear angle prediction models are thought to be not valid for this material.

- Find the shear plane angle
- Other than directly measuring the cut chip thickness or shear angle, briefly describe one way the cut chip thickness can be experimentally measured.
- Find the cutting force coefficient (specific cutting energy).
- Briefly describe two changes to the tool geometry that would reduce the cutting power
- Briefly describe two reasons why empirically obtained material properties from tensile tests conducted at room temperatures are not very useful in cutting process analyses.

**Problem 12**

A set of orthogonal cutting tests are performed on steel. Cutting conditions for the first set are as follows: depth of cut = 1.7 mm, feed rate = 0.2 mm/rev, spindle speed = 300 rev/min, workpiece diameter = 100 mm. Tool's rake angle =  $+5^\circ$ . From experiments, the deformed chip thickness is observed to be = 0.44 mm, and the feed and tangential forces are measured to be 600 N and 1200 N respectively. For the above conditions:

- First sketch the idealized 2D view of the orthogonal cutting process. Then,
- Derive an expression for the chip thickness ratio i.e. the ratio between the uncut and the cut chip thickness expressed as a function of the shear angle and the rake angle. Show all the steps involved. Then,
- Determine the shear angle, and the friction coefficient

Now, in the next set of tests, the feed rate is increased from 0.2 mm/rev to 0.5 mm/rev in steps of 0.1 mm/rev. For these cutting conditions:

- Sketch the tangential force behaviour as a function of increasing uncut chip thickness (sketch the trend only).
- When the uncut chip thickness = 0, what will be the tangential force? Provide only a qualitative analysis.
- For the feed rate of 0.2 mm/rev, describe qualitatively in %, how much energy is consumed in shearing and how much in friction.



## Machining: Short questions

### Problem 13

Consider a face milling process with a face mill that has five teeth with a uniform pitch and straight teeth. Down milling is carried out with this cutter with the radial immersion of the cut being  $1/4^{\text{th}}$  that of the diameter of the cutter. Cutter diameter is 50 mm. Assume the chip load  $c$  to be 0.2 mm.

- a. Sketch the top view of this process showing the entry and exit conditions. Calculate the entry and exit angles
- b. Express the chip thickness variation as a function on engagement conditions (entry/exit) with an appropriate expression and estimate the chip thickness at entry and exit
- c. Derive an expression for estimating the mean value of the chip thickness during cut. Then calculate the mean chip thickness.

### Problem 14

- a. Sketch the idealized 2D orthogonal cutting process, and overlay the idealized distribution of friction power over the rake face of the tool. Comment on why the distribution takes the shape that it does.
- b. Sketch the relationship between temperatures (y axis) along the chip-tool contact (x axis) for general free-machining steels. On the same plot overlay the behaviour for Aluminium and Titanium. Comment on how/if the behaviour changes with different materials.

### Problem 15

State if following are true or false and justify your selection in a statement (or more). (Correct selection of true/false without proper reasoning will not be considered for a full grade)

In modelling and estimating the temperature field in the chip and tool during orthogonal machining,

- a. We assume that no heat escapes from the shear plane into the workpiece (T/F)
- b. Heat conduction cannot be neglected as compared to heat mass transfer in the direction of motion of the chip motion (T/F)
- c. The cutting force coefficient (specific cutting force/energy) is proportional to shear-plane temperature and friction heat input into the chip and the tool (T/F)
- d. Shear-plane temperature is not inversely proportional to the specific heat per volume (T/F)

### Problem 16

Consider a boring operation with a bar that has two cutting teeth (inserts) spaced  $180^\circ$  apart. The first tooth makes a  $30^\circ$  angle with respect to the positive X axis.

- a. Sketch the boring bar with its teeth, looking at it in the X-Y plane. Remember that the Z axis is along the axis of the spindle/boring bar.
- b. Show the radial and tangential cutting forces acting on each tooth.
- c. Resolve the forces in the X and Y directions.
- d. Comment on the nature of the vector sum of forces.

### Problem 17

Consider orthogonal cutting. Sketch it and derive an expression for the chip thickness ratio  $r_c$  (i.e. the ratio between the uncut and the cut chip thickness) expressed as a function of the shear angle,  $\phi_c$  and the rake angle,  $\alpha_r$ . Show all the steps involved.

### Problem 18

Sketch a 2D view of the idealized orthogonal cutting process. Show the following on the sketch: (a) uncut chip thickness, (b) rake angle, (c) clearance angle, (d) nose radius, (e) shear plane, (f) shear zone, (g) ploughing zone, (h) frictional zone

### Problem 19

List three assumptions in orthogonal cutting process modelling, and briefly comment on their validity/goodness.

### Problem 20

In orthogonal cutting, what variables are needed to relate the chip velocity and shear velocity to the cutting velocity, and where from (what model) can the values of these variables be obtained?

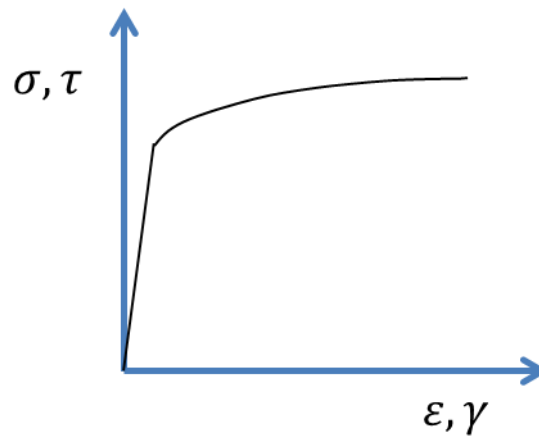
### Problem 21

Of the many assumptions in orthogonal cutting, one of the most important ones has something to do with the stress-strain relationship in the shear zone. What is this assumption? Sketch the stress-strain diagram for this assumption.

### Problem 22

On the following stress-strain diagram, sketch the influence on increasing temperature during machining, and the influence of increasing strain rates seen in machining. Comment briefly, on why

empirical values for stress-strain relationships obtained from room-temperature quasi-static tensile tests are not useful in cutting-process analysis.



### Problem 23

In turning with a non-zero finite nose radius, explain with a sketch, what parameters does the machined surface quality depend on?

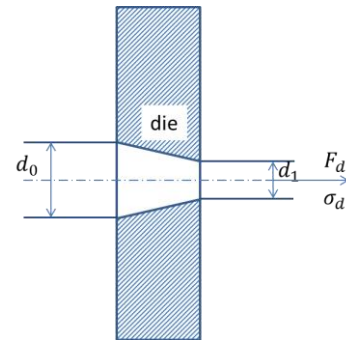
## Section: Bulk deformation processes

### Problem 1

The figure shows a wire being drawn through a frictionless conical die. It is reduced from an initial diameter  $d_0$  to the final diameter  $d_1$ . The drawing force is  $F_d$ , and the stress in the drawn wire is  $\sigma_d$ .

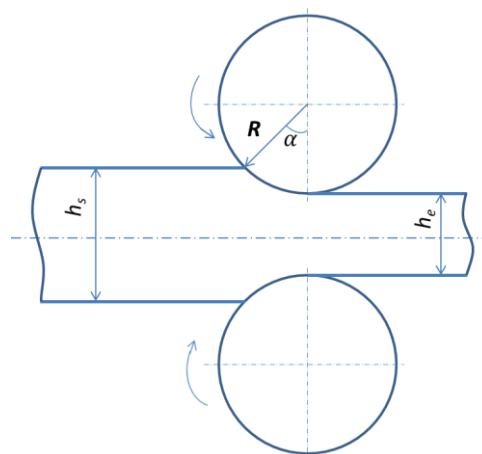
- Find the total true strain produced in this operation (in terms of  $d_0$  and  $d_1$ ).
- Assume the material deforms with an idealized rigid, linearly strain hardening behaviour. Take the initial yield strength to be  $Y_0$ . Write the expression relating stress to strain for this material behaviour. Assume this to be a cold working process.
- Find the specific plastic work done in this operation

For a material deforming with an idealized rigid-plastic, non-strain hardening behaviour, what is the limiting drawing ratio,  $d_0/d_1$ ?



### Problem 2

Shown below is a schematic of a cold rolling process with friction. The roll radius is  $R$ . The starting thickness at the entry is  $h_s$ , and the exit thickness is  $h_e$ . The width  $w$  of the strip is considerably greater than its thickness. The roll velocity is  $v_r$ . This is a case of plane-strain yielding with rigid, linearly strain hardening type behaviour.



- Write a simplified expression for velocity  $v$  of the strip at any location between entry and exit (The velocity of the strip varies from entry to exit).
- What is the expression for velocity at the neutral point?
- Sketch the velocity profile as it changes from entry to exit.

- d. Show the forces and stresses acting on an element in rolling at the entry zone and at the exit zone, i.e. show one element each at the entry and exit.
- e. Sketch the typical pressure distribution in rolling as a function of direction of rolling, i.e. from entry to exit. Discuss how this curve may change for an increase/or decrease in the coefficient of friction.
- f. For the case of an element at the entry side, find, on both rolls, expressions for the frictional force, and for the contact pressure force

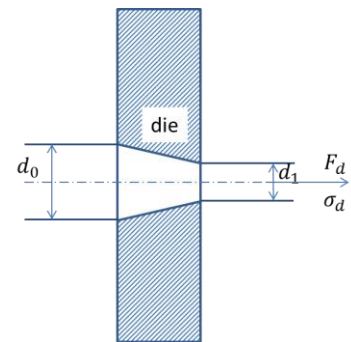
### Problem 3

In a single pass rolling operation, a 20 mm thick plate of width 100 mm, is reduced to 18 mm. the roller radius is 250 mm and rotational speed is 10 RPM. The average flow stress for the plate material is 300 MPa. Estimate the total power required for the rolling operation.

### Problem 4

The figure shows a wire being drawn through a conical die. It is reduced from an initial diameter  $d_0$  to the final diameter  $d_1$ . The drawing force is  $F_d$ , and the stress in the drawn wire is  $\sigma_d$ .

- d. Using the slab method, show the stresses acting on an element in drawing
- e. Sketch on separate figures the trend of variation in (i) the drawing stress and (ii) the die contact pressure along the deformation zone. Comment on what happens to the die pressure as the drawing pressure increases, and relate it your sketch(es)



### Problem 5

(i) Consider the case of rolling a billet of rectangular cross section between two cylindrical rollers. Assume that  $l$  is the length of the billet,  $h_0$  is the thickness at entry,  $h_1$  is the thickness at exit,  $b_0$  is the width at entry, and  $b_1$  is the width at exit. At the start of the rolling process, a billet of size  $l_0, b_0, h_0$  is held against the rolls touching them at points A and B respectively. The rolls exert a force of  $N$  normal to their surface and another force  $T$  due to friction acting at right angles.

- (a) Sketch the rolling process as described above, and show the forces acting at points A and B.
- (b) Also show the contact angle, i.e. the angle which the normal to the roll surface at point A makes with the line of centres.
- (c) Resolve the resulting forces into their horizontal component
- (d) Describe the condition (in terms of  $T$ ,  $N$ , and the contact angle) for rolling to proceed. Assume that the vertical components of forces are responsible for deformations and the horizontal components for drawing the billet through the rolls

### Problem 6

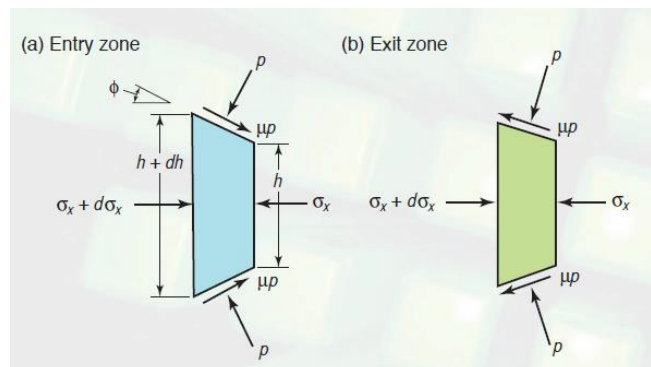
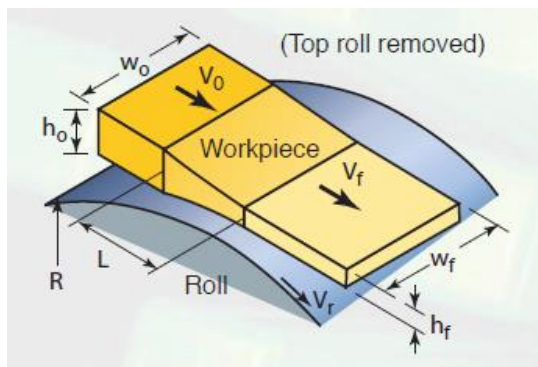
Using the slab method of analysis for forging of a cylindrical specimen, derive:  $p = Y e^{\frac{2\mu(r-x)}{h}}$ ; wherein  $Y$  is the yield pressure,  $\mu$  is the coefficient of friction,  $r$  is the radius,  $x$  any radius, and  $h$  is the workpiece thickness.

### Problem 7

Consider rolling. Take an element at the center of deformation zone in flat rolling. Assuming that all the stresses acting on this element are principal stresses, indicate the stress qualitatively, and state whether they are tension or compression stresses. Explain your reasoning. Is it possible for all three principal stresses to be equal to each other in magnitude? Explain.

### Problem 8

For rolling shown below, show how:  $h = h_f + 2R(1 - \cos \phi)$  approximates to  $h = h_f + R\phi^2$ ;



### Problem 9

A flat rolling operation is being carried out where  $h_o = 10 \text{ mm}$ ,  $h_f = 8 \text{ mm}$ ; width of strip is 250 mm, roll radius is also 250 mm,  $\mu = 0.2$ ; and the average flow pressure is 275 MPa. Estimate the roll force and torque.

### Problem 10

Using the slab method as for wire drawing, show that the drawing stress,  $\sigma_d$ , in plane-strain drawing of a flat sheet or plate is given by the following expression:

$$\sigma_d = Y \left( 1 + \frac{\tan \alpha}{\mu} \right) \left[ 1 - \left( \frac{h_f}{h_o} \right)^{\mu \cot \alpha} \right]$$

Wherein  $h_f$  and  $h_o$  are the final and original thickness respectively.

## Bulk deformation: Short questions

### Problem 11

- a. Sketch the idealized stress-strain behaviour for the following material behaviour:
  - i. Perfectly elastic
  - ii. Rigid, perfectly plastic
  - iii. Elastic, perfectly plastic
  - iv. Rigid, linearly strain hardening
  - v. Elastic, linearly strain hardening
- b. Of the above, which model is most often used to describe the stress-strain behaviour in hot working? Write the corresponding stress-strain expression for this model.

### Problem 12

In plastic deformation, assume that the strains are not determined so much so by the stresses, as they are by the entire history of loading. So, if a rod was first elongated by a tension to a length  $l_1 = 1.3l_0$ , and subsequently compressed to the original length  $l_0$ , what will be the strain?

### Problem 13

A strip of metal is originally 1.5 m long. It is stretched in three steps: first to a length of 1.75 m, then to 2.0 m, and finally to 3.0 m. Show that the total true strain is the sum of the true strains in each step, i.e, that the strains are additive. Show that, using engineering strains, the strain for each step cannot be added to obtain the total strain. Comment on your findings.

### Problem 14

Take a cubic piece of metal with a side length  $l_0$  and deform it plastically to the shape of a rectangular parallelepiped of dimensions  $l_1$ ,  $l_2$ , and  $l_3$ . Assuming that the material is perfectly plastic:

- a. Sketch the idealized stress strain behaviour, and
- b. Show that volume constancy requires the following expression be satisfied:  $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$ .

### Problem 15

Consider cold rolling of a very wide plate to reduce its thickness. Is this a plane strain deformation process? Please explain with a sketch. Show the directions of principal stresses.

**Problem 16**

Match the following:

Metal forming process    Associated force

- |   |              |   |                   |
|---|--------------|---|-------------------|
| A | Wire drawing | 1 | Shear force       |
| B | Extrusion    | 2 | Tensile force     |
| C | Blanking     | 3 | Compressive force |
| D | Bending      | 4 | Spring back force |

**Problem 17**

Assume that a rod-drawing operation can be carried out either in one pass or in two passes in tandem. If all the die angles are the same and the total reduction is the same, will the drawing forces be different? Explain.