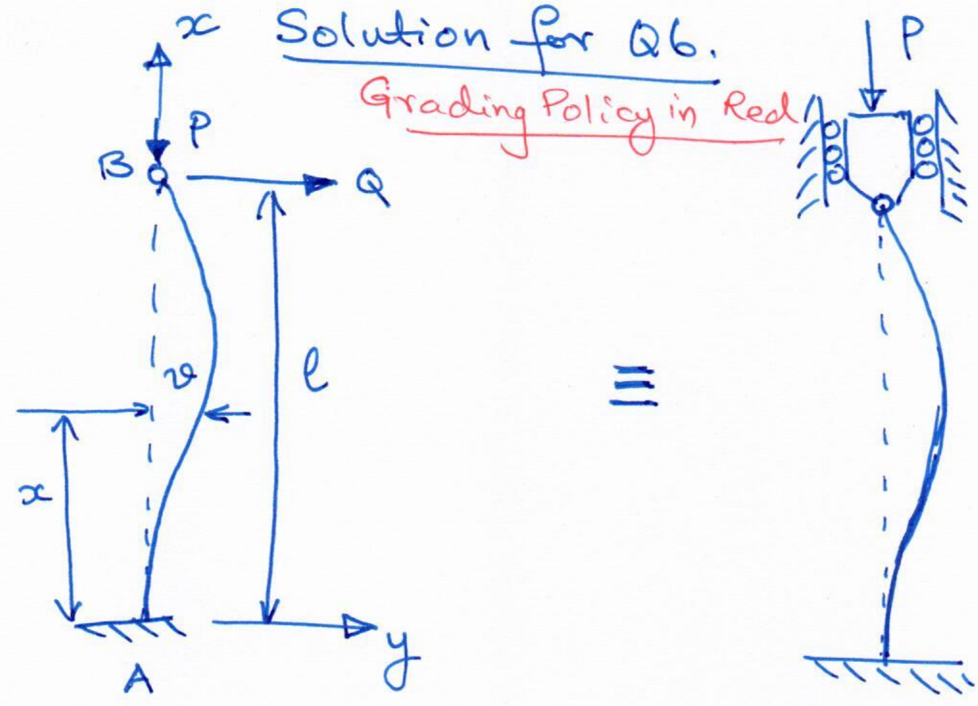
Solution for Q6.

Grading Policy in Red



Lower end A is built-in (clamped).

Upper end B is hinged & laterally supported such that a reaction Q is there when column buckles.

To find the first critical (lowest) value of P.

Two approaches can be used to solve this problem

Approach 1

The bending moment at section x is

$$M_z = Q(l-x) - Pv$$

The differential equation of the deflection curve is

$$EI\frac{\partial^2 v}{\partial x^2} = -Pv + Q(l-x) \qquad \boxed{3}$$

The general solution of above diff. eqn is

$$\boxed{v = C_1 \sin kx + C_2 \cos kx + \frac{Q}{P}(l-x)} \quad -(1)$$

where $k = \sqrt{P/EI}$ $\qquad \boxed{2}$

To determine the constants, C_1 & C_2 and the unknown reaction Q, we use the following boundary conditions.

BC1) $v = 0$ at $x = 0$

BC2) $\frac{\partial v}{\partial x} = 0$ at $x = 0$

BC3) $v = 0$ at $x = l$

Thus applying the BCs in equation 1 gives

BC1 $\Rightarrow$ $C_2 + \frac{Ql}{P} = 0$

Correct BCs + Correct eqn.

$$\Rightarrow \boxed{C_2 = -\frac{Ql}{P}} \quad — (2)$$

3

BC2 $\Rightarrow$ $\frac{\partial v}{\partial x} \equiv C_1 K \cos Kx - C_2 K \sin kx - \frac{Q}{P}$

$\therefore$ BC2 $\Rightarrow$ $C_1 K - \frac{Q}{P} = 0$

$$\Rightarrow \boxed{C_1 = \frac{Q}{KP}} \quad — (3)$$

3

BC3 $\Rightarrow$ $C_1 \sin Kl + C_2 \cos Kl = 0$

3

Substituting for C_1 & C_2 from 2 & 3

$$\frac{Q}{KP} \sin Kl - \frac{Ql}{P} \cos Kl = 0$$

$$\frac{Q}{P}\left[\frac{1}{K} \cdot \sin Kl - l \cos Kl\right] = 0$$

For non-trivial Q,

$$\frac{1}{K} \sin Kl - l \cos Kl = 0$$

$$\Rightarrow \quad \boxed{\tan Kl = Kl} \quad — (4) \quad \boxed{2}$$

The lowest non-zero solution of the above eqn. is

$$\boxed{Kl = 4.49} \quad \boxed{2}$$

$$\Rightarrow \quad K = \frac{4.49}{l}$$

Since $K = \sqrt{P/EI}$, we can write

$$\frac{P}{EI} = \left(\frac{4.49}{l}\right)^2 =$$

Thus the critical compressive load,

$$\boxed{P_{cr} = \frac{20.2\, EI}{l^2}} \quad \boxed{2}$$

Approach 2

Using the generalised governing equation for no transverse load,

$$EI\frac{\partial^4 v}{\partial x^4} + P\frac{\partial^2 v}{\partial x^2} = 0$$

for which the general solution is

$$\boxed{v = c_1 + c_2 x + c_3 \sin kx + c_4 \cos kx} \quad — (1)$$

where $k = \sqrt{\frac{P}{EI}}$

Correct governing equation + correct general solution $\boxed{2}$

To determine the 4 constants c_1, c_2, c_3, c_4, we apply the following Boundary Conditions.

BC1) $v = 0$ at $x = 0$

BC2) $\frac{\partial v}{\partial x} = 0$ at $x = 0$

BC3) $v = 0$ at $x = l$

BC4) $\frac{\partial^2 v}{\partial x^2} = 0$ at $x = l$ (Moment = 0) ∴ it is hinged.

Thus applying the BCs in equation (1)

Correct Boundary Condn & correct eqn.

BC1 $\Rightarrow$ $c_1 + c_4 = 0$

$\Rightarrow \boxed{c_1 = -c_4} \quad — (2)$

$\boxed{3}$

$$\frac{\partial v}{\partial x} = c_2 + c_3 k \cos kx - c_4 k \sin kx$$

BC2 $\Rightarrow$ $c_2 + c_3 k = 0$

3

$\Rightarrow$ $\boxed{c_2 = -c_3 k}$ — (3)

BC3 $\Rightarrow$

$\boxed{c_1 + c_2 l + c_3 \sin kl + c_4 \cos kl = 0}$ — (4)

3

$$\frac{\partial^2 v}{\partial x^2} = -k^2 (c_3 \sin kx + c_4 \cos kx)$$

BC4 $\Rightarrow$

$\boxed{c_3 \sin kl + c_4 \cos kl = 0}$ — (5)

3

Substituting for c_1 & c_2 from (2) & (3) in (4)

we get

$\boxed{c_3 (\sin kl - kl) + c_4 (\cos kl - 1) = 0}$ — (6)

writing (5) & (6) in a matrix form

$$\begin{bmatrix} \sin kl & \cos kl \\ \sin kl - kl & \cos kl - 1 \end{bmatrix} \begin{Bmatrix} c_3 \\ c_4 \end{Bmatrix} = 0$$

This is the classic eigenvalue problem where

$$[A]\{x\} = 0$$

& for non-trivial solution of $\{x\}$

$$|A| = 0$$

Thus,

$$\begin{vmatrix} \sin kl & \cos kl \\ \sin kl - kl & \cos kl - 1 \end{vmatrix} = 0$$

$\Rightarrow \quad \sin kl(\cos kl - 1) - \cos kl(\sin kl - kl) = 0$

$\Rightarrow \quad \boxed{kl = \tan kl} \quad — \quad (7) \qquad \boxed{2}$

The lowest non-zero solution

is $\quad kl = 4.49 \qquad \boxed{2}$

and as before in Approach 1.

$$\boxed{P_{cr} = \frac{20.2\,EI}{l^2}} \qquad \boxed{2}$$