ESO202A Quiz 1 Solution

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Given three identical rods at an angle of 120 degrees with respect to each other pinned to each other and to 3 supports.

Equilibrium

Assumption 1: *The deflection of point A is negligible in the equilibrium configuration.*
Hence, the given configuration is assumed to stay approximately the same after loading.

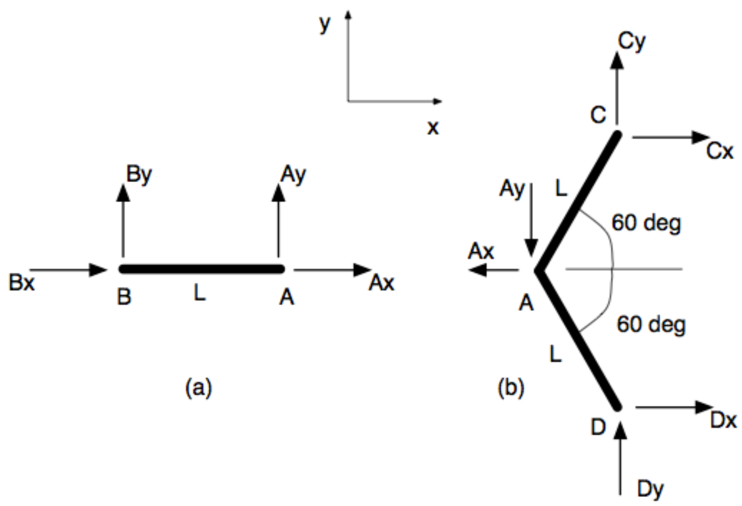

Figure 1: FBD of AB (a) and AC & AD (b).

The FBD of rod AB is Fig. 1(a) using the assumption 1.
$\sum F_x = 0 \implies A_x + B_x = 0$,
$\sum F_y = 0 \implies A_y + B_y = 0$,
$\sum M_{z/B} = 0 \implies A_y L = 0$.
Hence, it is concluded that $A_y = B_y = 0$, i.e. the rod AB is a 2-force member.

Let the longitudinal force $A_x = -B_x = F_{AB}$.
Similarly, it is found that rods AC and AD are also 2-force members, let the respective longitudinal forces be F_{AC} and F_{AD}.
Using the results so far, the FBD of the three rods pinned at A must be Fig. 2 using the assumption 1.
$\sum F_x = 0 \implies F_{AC} \cos 60^\circ + F_{AD} \cos 60^\circ - F_{AB} = 0$,
$\sum F_y = 0 \implies F_{AC} \sin 60^\circ - F_{AD} \sin 60^\circ = 0$,
$\sum M_{z/A} = 0 \implies 0 = 0$.
Hence, it is concluded that $F_{AB} = F_{AC} = F_{AD}$, i.e. the three rods experience a tensile or compressive force of

the same magnitude. Assume each of these forces to be tensile in nature with magnitude equal to F (negative value implies compression).
Given that the diameter d of the cross-sectional area (circular) of the rods is same, the area of cross-section is $A_{AB} = A_{AC} = A_{AD} = A = \frac{\pi}{4}d^2$. By the definition of the stress, it is found that it is longitudinal and its longitudinal (axial) component for each rod is given by

$$\sigma_{AB} = \sigma_{AC} = \sigma_{AD} = \sigma = \frac{F}{A}, \tag{1}$$

where

$$A = \frac{\pi}{4}d^2 \tag{2}$$

All other components of the stress are zero.

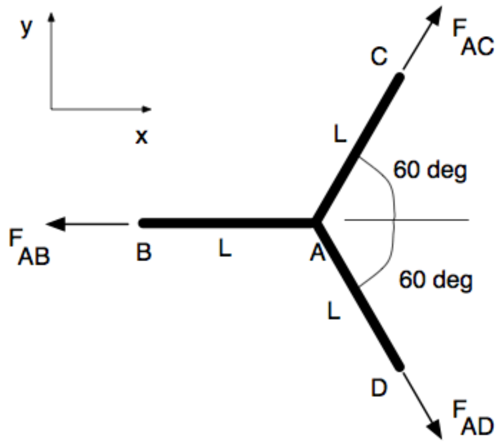

Figure 2: FBD of three rods.

Stress-strain relation

Let the longitudinal strain in the rods AB, AC, and AD be denoted by $\epsilon_{AB}, \epsilon_{AC}$, and ϵ_{AD}, respectively.
Let T_0 be the initial temperature of the system and T_f be the final temperature.
Given that the rods are made of steel with the Young's modulus E_{steel}.
Using the Hooke's relation $\epsilon_{longitudinal} = \frac{1}{E_{steel}}\sigma_{longitudinal} + \alpha_{steel}\delta T$, i.e.

$$\begin{aligned} \epsilon_{AB} &= \frac{1}{E_{steel}}\sigma_{AB} + \alpha_{steel}\delta T, \\ \epsilon_{AC} &= \frac{1}{E_{steel}}\sigma_{AC} + \alpha_{steel}\delta T, \\ \epsilon_{AD} &= \frac{1}{E_{steel}}\sigma_{AD} + \alpha_{steel}\delta T, \end{aligned} \tag{3}$$

where α_{steel} is the coefficient of thermal expansion for steel and $\delta T = T_f - T_0$ is the change in temperature.
By equations (1) and (3),

$$\epsilon_{AB} = \epsilon_{AC} = \epsilon_{AD} = \epsilon = \frac{1}{E_{steel}}\sigma + \alpha_{steel}\delta T. \tag{4}$$

Geometric compatibility

Assumption 2: *None of the rods buckles if* $F < 0$. This is closely related to assumption 1.
Assuming that the point A moves to A' when T_0 is changed to T_f. By rotating the FBD shown in Fig. 2 by $\pm 120°$, it is found that the displacement vector $\vec{AA'}$ also changes but this is not possible since the FBD after rotation is indistinguishable from the one before rotation.

$$\text{Hence, the displacement of point } A \text{ is zero using the assumption 2.} \tag{5}$$

Since the given information in the problem does not specify whether the rods are free of stress before the temperature is applied, it is possible that the stress-free length is L_0 and it is not same as L.
Assumption 3: *All three rods have identical stress-free length* L_0.
Assumption 4: $|L - L_0| \ll 1$. This is in addition to assumption 1.
Therefore, the longitudinal strain in the rods with respect to the unpinned configuration at T_0, before the change in temperature, is $\delta L / L$ where $\delta L = L - L_0$. Hence, using equation (5),

$$\epsilon = \frac{\delta L}{L}, \tag{6}$$

(due to assumption 4, alternatively $\epsilon \approx \delta L / L_0$).

Final calculation

Substituting equation (6) in (4), it is found that $\frac{1}{E_{steel}}\sigma + \alpha_{steel}\delta T = \frac{\delta L}{L}$, so that

$$\sigma = E_{steel}\left(\frac{\delta L}{L} - \alpha_{steel}\delta T\right). \tag{7}$$

Hence, equations (1), (2), and (7) imply that

$$F = \frac{\pi}{4} d^2 E_{steel}\left(\frac{\delta L}{L} - \alpha_{steel}\delta T\right) = F_0 + \delta F,$$

where $F_0 = \frac{\pi}{4} d^2 E_{steel} \frac{\delta L}{L}$. The force δF develops in each rod in the pinned configuration after the change in temperature from T_0 to T_f. Using the values given for $d, E_{steel}, \alpha_{steel}$, and δT, it is found that

$$\delta F = -\frac{\pi}{4} d^2 E_{steel} \alpha_{steel} \delta T = -58.91 kN. \tag{8}$$

In particular, each of the three rods develops a compressive force of same magnitude $58.91kN$ after the change in temperature from T_0 to T_f assuming $L_0 = L$, i.e. $\delta L = 0$.

Remark: Note that the expression for δF (8) does not involve L. However, in practice, when assuming $L_0 = L$, it is likely that assumption 2 is violated hence the calculations are not meaningful. However, a pre-stretch can always be chosen for given δT such that $F = F_0 + \delta F$ remains positive (or small even if negative).

Marks distribution:

FBD: 3 marks,
Equilibrium: 4 marks, (the equations should be consistent with given FBD)
Stress-strain relation: 4 marks,
Geometric compatibility: 4 marks
Final calculation: 5 marks (including 2 marks for correct F).
