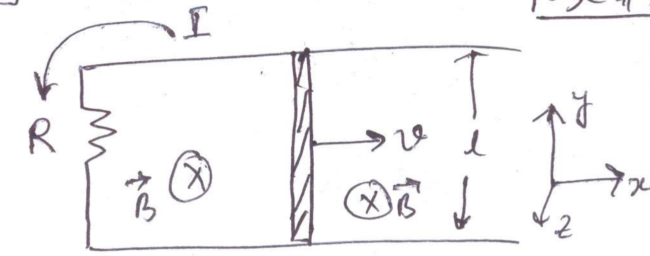


1. a) emf

$$\mathcal{E} = -\frac{d\phi}{dt}$$

$$= -B\frac{da}{dt} = -B\frac{d}{dt}(lx)$$

$$= -Blv$$

$$I = |\mathcal{E}/R| = \frac{Blv}{R}$$

$\vec{v}\times\vec{B}$ = upward in the bar

⇒ current will flow in the upward ($\hat{y}$) direction in the bar

& ⇒ downward direction in the resistance R.

b) Magnetic force $\vec{F}_{mag} = q\,\vec{v}'\times\vec{B}$ ($\vec{v}'$ = velocity of the charge q)

$$= \int(\vec{I}\times\vec{B})\,d\vec{l}$$

$$= Il\,\hat{y}\times B(-\hat{z})$$

$$= -IBl\,\hat{x} = -\left(\frac{Blv}{R}\right)Bl\,\hat{x}$$

$$= -\frac{B^2l^2v}{R}\hat{x}$$

c) $\vec{F} = m\dfrac{d\vec{v}}{dt}$

$$\Rightarrow -\frac{B^2l^2v}{R}\hat{x} = m\frac{dv}{dt}\hat{x}$$

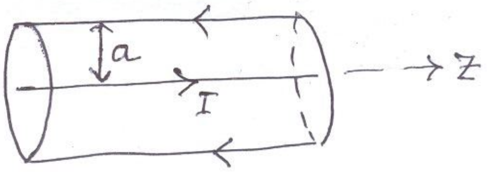
$$\Rightarrow \int_{v_0}^{v_t}\frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt$$

$$\Rightarrow \boxed{v_t = v_0\,e^{-\frac{B^2l^2}{mR}t}} = v_0\,e^{-t/\tau} \quad \text{where } \left(\tau = \frac{mR}{B^2l^2}\right)$$

d) $$\frac{dw}{dt} = I^2R = \left(\frac{Blv_t}{R}\right)^2 R = \frac{B^2l^2v_0^2}{R}e^{-2t/\tau}$$

Total energy delivered $$W = \frac{B^2l^2v_0^2}{R}\int_{t=0}^{\infty} e^{-2t/\tau} = \frac{1}{2}mv_0^2$$

2. (a) $I = I_0 \cos(\omega t)$



In quasistatic approximation

$$B_\phi(s) = \begin{cases} \dfrac{\mu_0 I}{2\pi s} & s<a \\ 0 & s>a \end{cases}$$

B is circumferential.

$$\left.\begin{aligned} \nabla\times\bar{B} &= \mu_0 \vec{J} \\ \bar{\nabla}\cdot\bar{B} &= 0 \end{aligned}\right\} \text{analogous to } \begin{cases} \nabla\times\bar{E} = -\dfrac{\partial \bar{B}}{\partial t} \\ \bar{\nabla}\cdot\bar{E} = 0 \end{cases}$$

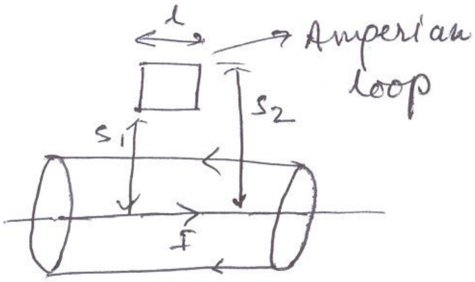
here $\bar{B} \equiv$ current in a solenoid

$\Rightarrow$ induced $\vec{E}$ will be in axial direction.

b) By symmetry E depends only on "s"

$\Rightarrow \vec{E} = E(s)\,\hat{z}$ for $s<a$

$= 0$ for $s>a$ $\Rightarrow$



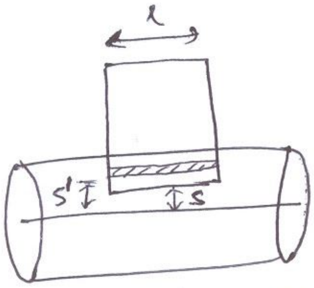
$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt} = 0$$

$E(s_1) = E(s_2)$

as $E(\infty) = 0$ $\quad E(s_1) = E(s_2) = E(\infty) = 0$

$\Rightarrow E(s) = 0$ for $s>a$

for $s<a$



Consider the Amperian loop as shown.

$$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{a} = -\frac{d}{dt}\int_{s'=s}^{a} \frac{\mu_0 I}{2\pi s'}\,(l\,ds')$$

$$(=El) \quad = -\frac{d}{dt}\left[\frac{\mu_0 I l}{2\pi}\ln(a/s)\right] = -\frac{\mu_0 l}{2\pi}\ln(a/s)\,\frac{dI}{dt}$$

$$\Rightarrow E = \frac{\mu_0 I_0 \omega}{2\pi}\ln(a/s)\,\sin(\omega t)$$

$$\Rightarrow \vec{E} = \frac{\mu_0 I_0 \omega}{2\pi}\ln(a/s)\,\sin(\omega t)\,\hat{z} \text{ for } s<a.$$