Name: Roll no:

ME623: Finite Element Methods in Engineering

Mid-Semester Examination, 24th February, 2018

Duration: 2 hours

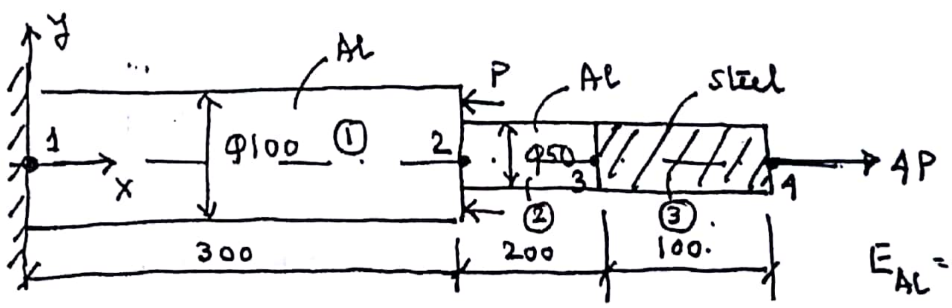
Full Marks: 40

Important note:

- *Write the answer sought inside the box provided. Answers written elsewhere will not be counted.*
- *Nothing other than the specific answer must be written inside the boxes. All other calculations must be done outside the box.*
- *All calculations have to be shown. If workings are not shown in the space provided below each question, you will not be given any credit even if your answer is correct.*
- *You may use your laptop, coursenotes and programmes like Matlab.* **But no books are allowed..**

"Tricks and treachery are the practise of fools, that don't have brains enough to be honest"
— Benjamin Franklin.

Pr 1: Consider the structure shown in the figure below. The structure has 3 elements and 4 nodes as shown. You have to solve the problem with as few total degrees of freedom as possible.



All dimensions in mm

$E_{Al} = 70$ GPa

$E_{St} = 210$ GPa

$P = 450$ kN.

Scanned by CamScanner

a. Write the local stiffness matrices in the global (x, y, z) coordinate system for each element. Note the units specified.

$$K_1 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{1.83259} \text{MN/mm}, K_2 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{.687223} \text{MN/mm}, K_3 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}^{4.12334} \text{MN/mm}$$

[Marks: 2 × 3 = 6]

b. Assemble the global stiffness matrix in the box below.

$$K = \begin{pmatrix} 1.83259 & -1.83259 & 0 & 0 \\ -1.83259 & 2.5198 & -.6872 & 0 \\ 0 & -.6872 & 4.806 & -4.12334 \\ 0 & 0 & -4.12334 & 4.12334 \end{pmatrix} \text{MN/mm}$$

[Marks: 4]

c. Write the global force vector in units of MN.

Scanned by CamScanner

$$F = \begin{Bmatrix} 0 \\ -.45 \\ 0 \\ 1.8 \end{Bmatrix} \text{ MN}$$

[Marks: 2]

d. Solve for the displacements in the x direction for nodes 1-4.

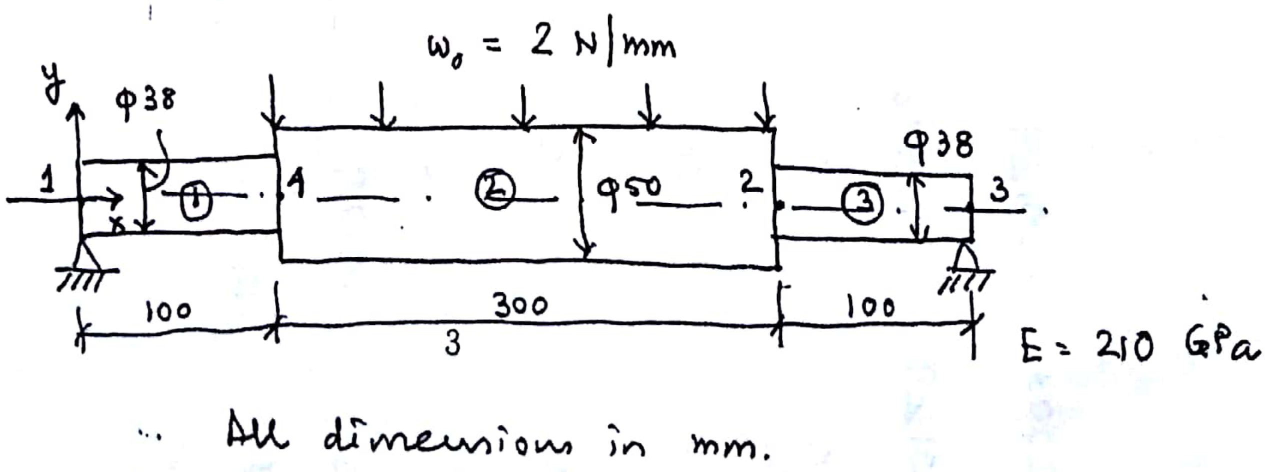
$u_1 =$ 0 mm, $u_2 =$ .736 mm, $u_3 =$ 3.355 mm, $u_4 =$ 3.792 mm

0.736 3.355 3.792

[Marks: 3]

2: Consider the structure shown below. The elements and nodes are numbered as shown. We need to find the vertical displacements at each node. Choose an appropriate type of element.

a. Write the consistent equivalent global load vector.



Scanned by CamScanner

$$F = \begin{Bmatrix} 0 \\ 0 \\ -.3 \\ +15 \\ 0 \\ 0 \\ -.3 \\ -15 \end{Bmatrix} \begin{matrix} \text{kN} \\ \text{kN-mm} \end{matrix}$$

[Marks: 4]

Write the global stiffness matrix.

$$K = \begin{pmatrix} \underline{.2579} & 12.8965 & 0 & 0 & 0 & 0 & -.2579 & 12.896 \\ 12.897 & \underline{859.7725} & 0 & 0 & 0 & 0 & -12.8965 & 429.886 \\ 0 & 0 & \underline{.2866} & 8.6014 & -.2579 & 12.8965 & \cancel{-12.8965}\ -.0286 & -4.29514 \\ 0 & 0 & 8.6014 & \underline{1718.802} & -12.8965 & 429.886 & \cancel{-.0286}\ 4.2951 & 429.514 \\ 0 & 0 & -.2579 & -12.8965 & \underline{.2579} & -12.8965 & \cancel{4.295}\ 0 & 0 \\ 0 & 0 & 12.8965 & 429.886 & -12.8965 & \underline{859.773} & 0 & 0 \\ -.2579 & -12.8965 & -.0286 & 4.29514 & 0 & 0 & \underline{.2865} & -8.6014 \\ 12.8965 & 429.886 & -4.2951 & 429.515 & 0 & 0 & -8.6014 & \underline{1718.801} \end{pmatrix}$$

~~MN/mm.~~

MN/mm.

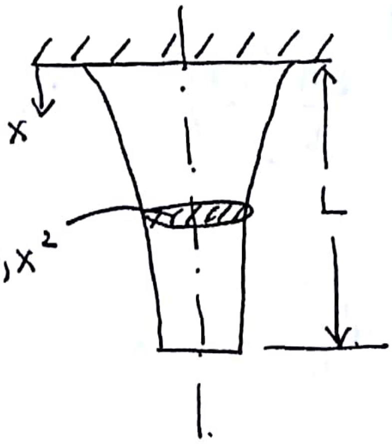
Scanned by CamScanner

c. Solve for the vertical displacements at each node. ~~$\theta_4 = .0035 \times 10^{-8}$~~

$v_1 =$ 0 mm, $v_2 =$ -1.513×10^{-2} mm mm, $v_3 =$ 0 mm, $v_4 =$ -1.513×10^{-2} mm

[Marks: 3]

The tapered bar is made up of an isotropic, linear elastic material with Young's modulus E and Poisson's ratio ν. Considering the deformation to be uniaxial,



Area varies as:

$A(x) = A_0 + A_1 x^2$

a. Derive an expression for the potential energy Π of the structure. It is loaded by self weight. Density of the material is ρ. Choose appropriate boundary conditions.

$$\Pi = \int_0^L \frac{EA}{2}\left(\frac{du}{dx}\right)^2 dx - \int_0^L \rho g A u \, dx$$

[Marks: 3]

Scanned by CamScanner

WORKING:

$$\Pi = \int_0^L \frac{EA}{2}\left(\frac{du}{dx}\right)^2 dx - \int_0^L \rho g A\, u\, dx$$

$$\Pi = \int_0^L \left[\frac{EA}{2}(u')^2 - \rho g A u \right] dx$$

$$F = \frac{EA}{2}(u')^2 - \rho g A u$$

Euler Lagrange Equation:

$$\frac{dF}{du} - \frac{d}{dx}\left(\frac{dF}{du'}\right) = 0$$

$$-\rho g A - \frac{d}{dx}\left(\frac{EA}{2}\, 2u'\right) = 0$$

$$\boxed{\rho g A + E\frac{d}{dx}(A u') = 0}$$

Scanned by CamScanner

b. Using the Euler Lagrange equation, derive the equation of motion for the structure.

$$(\rho g A \quad) + E\frac{d}{dx}(\quad A\frac{du}{dx} \quad) = 0.$$

[Marks: 3]

Pr.4: Consider the following weak form for a variable $\phi(x, y)$ in the 2-d domain V bounded by the surface ∂V.

$$\delta\Pi = \int_V \delta\phi \left[-\frac{\partial}{\partial x}F_1 - \frac{\partial}{\partial x}F_2 + a_{00}\phi - f \right] dxdy = 0,$$

with

$$F_1 = a_{11}\frac{\partial \phi}{\partial x} + a_{12}\frac{\partial \phi}{\partial y}, F_2 = a_{21}\frac{\partial \phi}{\partial x} + a_{22}\frac{\partial \phi}{\partial y}.$$

Here $\delta\phi$ is a variation in ϕ. Show that on the boundary ∂V, we may have (you need to complete the expressions within the round brackets):

$$n_x(\quad F_1 \quad) + n_y(\quad F_2 \quad) = 0$$

[Marks: 4]

WORKING:

$$\delta\Pi = \int_V \delta\phi \left[-\frac{\partial F_1}{\partial x} - \frac{\partial F_2}{\partial x} + a_{00}\phi - f \right] dx\, dy = 0.$$

$$= \int_V \left[-\frac{\partial}{\partial x}(F_1\delta\phi) + F_1\frac{\partial \delta\phi}{\partial x} - \frac{\partial}{\partial y}(F_2\delta\phi) + F_2\frac{\partial\delta\phi}{\partial y} + a_{00}\phi\,\delta\phi - f\,\delta\phi \right] dx\,dy = 0$$

$$= \underbrace{-\int_V \frac{\partial}{\partial x}(F_1\delta\phi) + \frac{\partial}{\partial y}(F_2\delta\phi)\, dx\,dy}_{} + \iint_V \left[F_1\frac{\partial\delta\phi}{\partial x} + F_2\frac{\partial\delta\phi}{\partial y} + a_{00}\phi\,\delta\phi - f\,\delta\phi \right] dx\,dy = 0$$

Apply Gauss Divergence Theorem:

$$-\int_S \left[F_1 n_x \delta\phi + F_2 n_y \delta\phi \right] ds = 0 \Rightarrow F_1 n_x + F_2 n_y = 0$$

Scanned by CamScanner