ME772A Mechatronics
Introduction to Signals and Systems

10-11-2018

What is a Signal?

Signal: Mathematically, it is a function of one or more independent variables

Signal=f(x1,x2,x3……)

Xi= Time, Distance, Temperature etc…(the number of independent variables are one or more)

Waveform: f(x) vs x

It can be 1D, 2D or Multidimensional…. Depends on number of independent variables.

Types of Signals:

Basically,

Natural and Synthetic.

Signals can be Continuous (Analog?) or discrete….

All continuous time signals are analog signals but not analog signals are continuous time signals.

Examples of Signal:

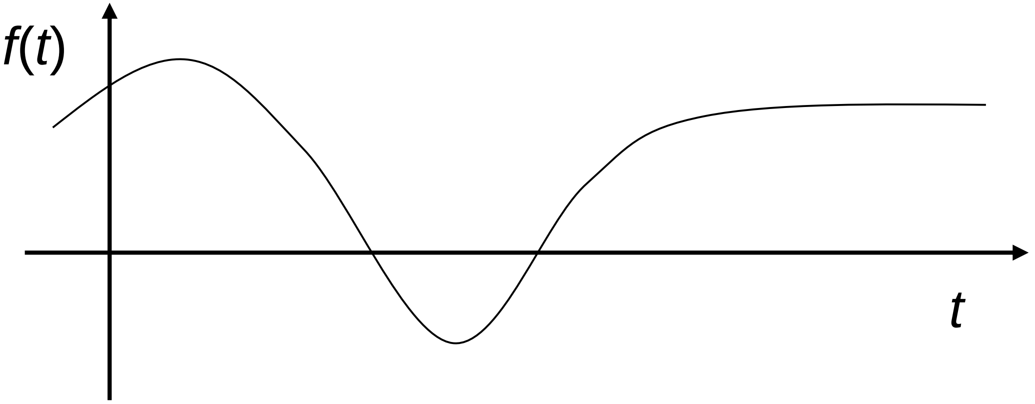
Electric signal in television: voltage and current in the circuit

Acoustic signal in speech: pressure over time

Mechanical signal in vibration: acceleration over time

How is a Signal Represented?

- In Communication systems, signal processing, and electrical engineering, a signal is a function that "conveys information about the behavior or attributes of some phenomenon".
- A **signal** is an electrical or electromagnetic current that is used for carrying data from one device or network to another.
- Signals can be one dimensional or multidimensional/ Single channel or multichannel/ Single variable or multivariable signal.
- For instance a black & white video signal intensity is dependent on x, y coordinates and time t, $f(x,y,t)$
- we shall be exclusively concerned with signals that are a function of a single variable: time
- According to Fourier analysis, any composite signal can be represented as a combination of simple sine waves with different frequencies, amplitude and phase.



What is System?

- A System process the input signals to produce output signals.
- A system is a combination of elements that manipulates one or more signals to accomplish a function and produces some output.

- System is defined by the type of input and output it deals with.
- Here, it would be mathematical model or a physical device or a black box.
- The input signal is known as excitation and the output is known as response.
- There are two kind of problems associated with System such as Analysis and Synthesis.

Examples:

–A circuit involving a capacitor can be viewed as a system that transforms the source voltage to the voltage (signal) across the capacitor

–A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker

–A communication system is composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver.

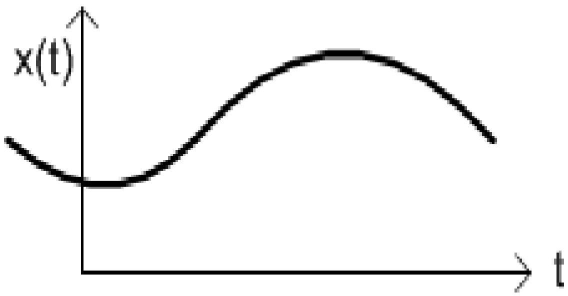
–Wheel suspension system comprises components such as wheel, tyre, Spring, shock absorber etc.

Classification of Signals

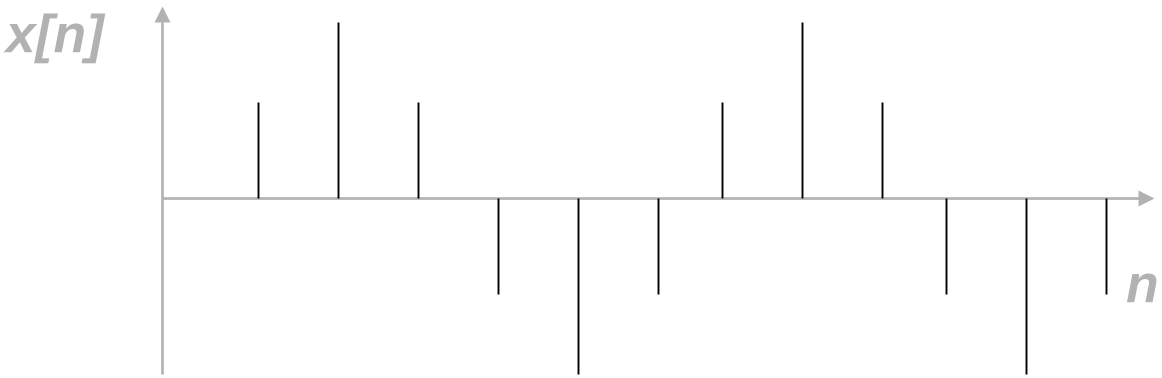
TIME SIGNALS DESCRIPTION

Mathematical expression $x(t)=A\sin(\omega t+\phi)$

Continuous (Analogue)



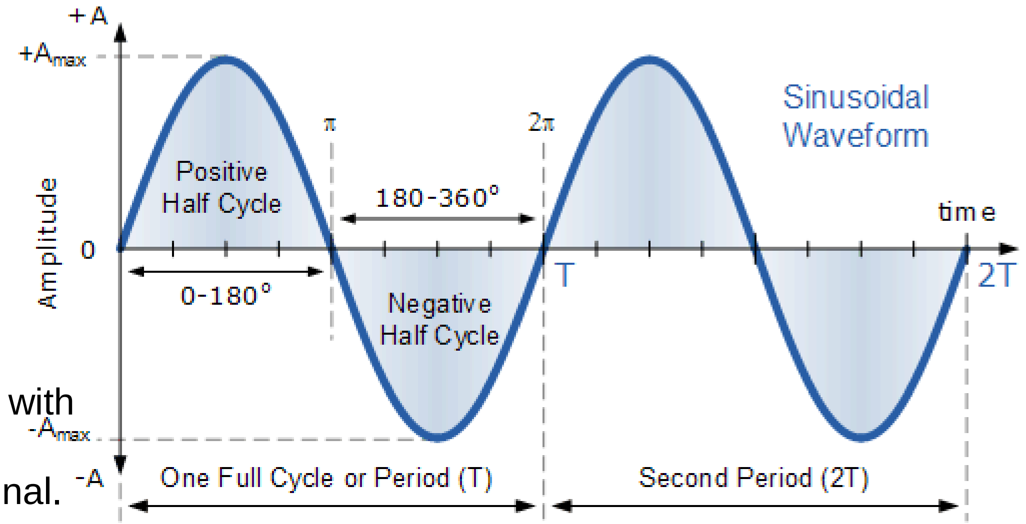
Discrete (Digital)



TIME SIGNALS DESCRIPTION

Further classification is based on various characteristics the signal possess.

Periodic



$x(t)= x(t+T_o)$, for all value of t

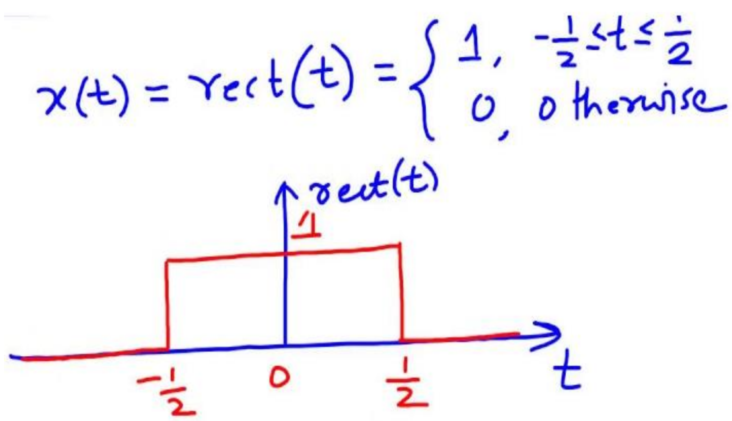
Fundamental Period = T_o

The Sine signal is periodic with periodicity value of 2π.
Periodic signals are deterministic signal.
Egs. Sine, Cosine and Sawtooth wave

Aperiodic

$x(t) \neq x(t+T_o)$, for all value of t

Periodic signals are power signals.
Non periodic signals are energy signals.
Aperiodic signals are Random signals.
Eg. All noise signal



Periodic and aperiodic signal (continue)

Any continuous time signal x(t) is classified as periodic if the signal satisfies the condition :

$$x(t) = x(t + nT) \qquad \text{where } n = 1, 2, 3 \ldots$$

The sum of two or more signals is periodic if the ratio (evaluation of two values) of their periods can be expressed as rational number. The new fundamental period and frequency can be obtained from a periodic signal.

A **rational number** is a number that can be written as a simple fraction (i.e. as a **ratio**).

The sum of two or more signals is aperiodic if the ratio (evaluation of two values) of their periods is expressed as irrational number and no new fundamental period can be obtained.

$$x(t) = \cos\frac{\pi}{3}t + \sin\frac{\pi}{4}t = x_1(t) + x_2(t)$$

$$\text{where} \quad x_1(t) = \cos(\pi/3)t = \cos\omega_1 t \to T_1 = \frac{2\pi}{\pi/3} = 6.$$

$$x_2(t) = \sin(\pi/4)t = \sin\omega_2 t \to T_2 = \frac{2\pi}{\pi/4} = 8.$$

$$\frac{T_1}{T_2} = \frac{6}{8} = \frac{3}{4} \text{ is a rational number}$$

$x(t)$ is periodic with fundamental period $T_0 = 4T_1 = 3T_2 = 24$.

Sum of discrete-time periodic signals is always periodic because the period ratios are always rational. But the sum of N-periodic continuous time signals is not necessarily periodic.

TIME SIGNALS DESCRIPTION

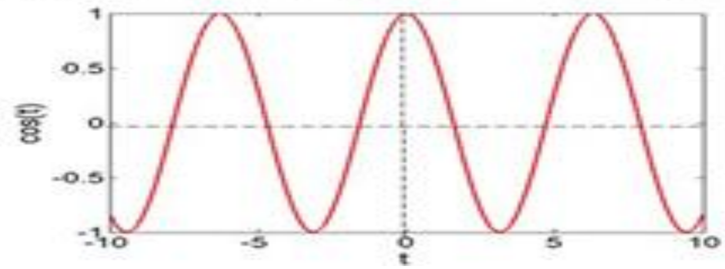
Even signal

An **even** signal is identical to its time reversed signal, i.e. it can be reflected in the origin and is equal to the original:

$$x(-t) = x(t)$$

Examples:

x(*t*) = cos(*t*)



Odd signal

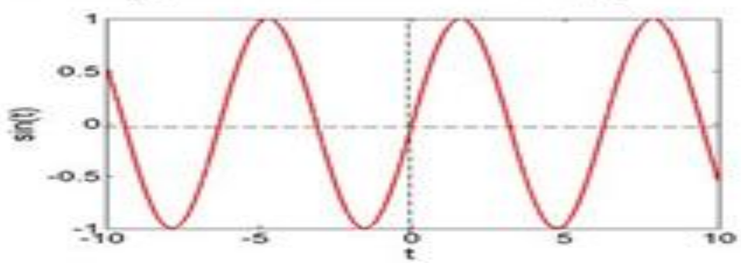
An **odd** signal is identical to its negated, time reversed signal, i.e. it is equal to the negative reflected signal

$$x(-t) = -x(t)$$

Examples:

x(*t*) = sin(*t*)

x(*t*) = *t*



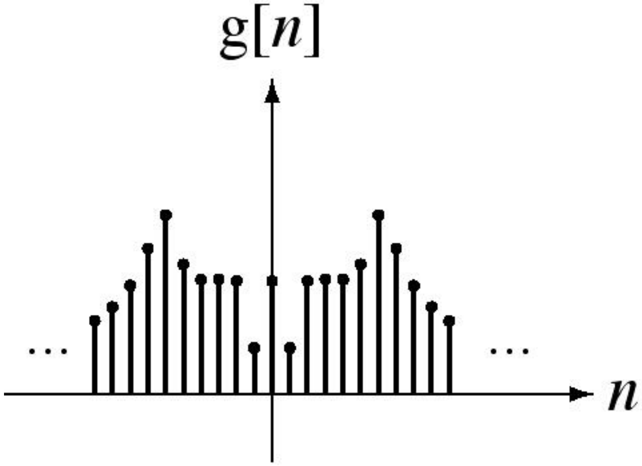
This is important because any signal can be expressed as the sum of an odd signal and an even signal.

Discrete Time Even and Odd Signals

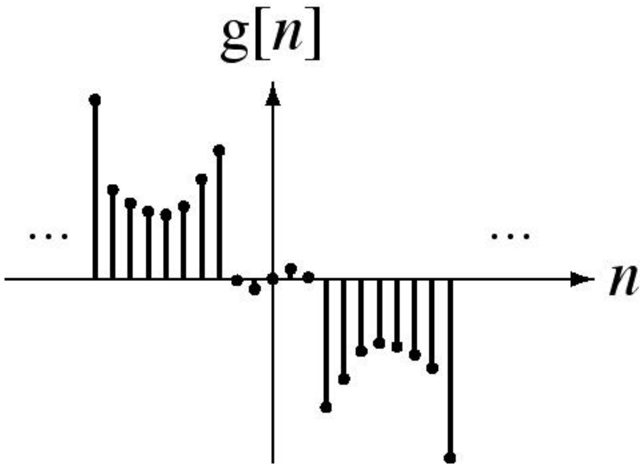
$$\mathrm{g}[n] = \mathrm{g}[-n]$$

$$\mathrm{g}[n] = -\mathrm{g}[-n]$$

Even Function



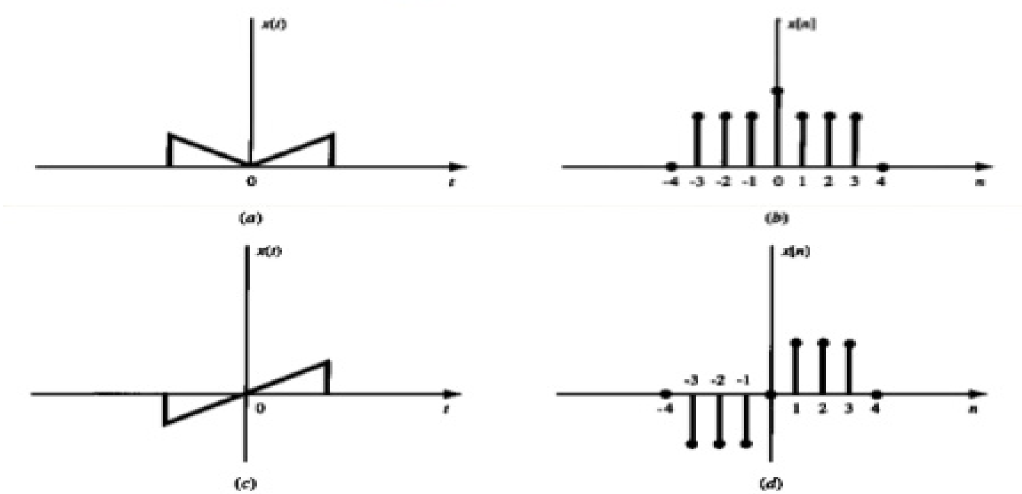
Odd Function

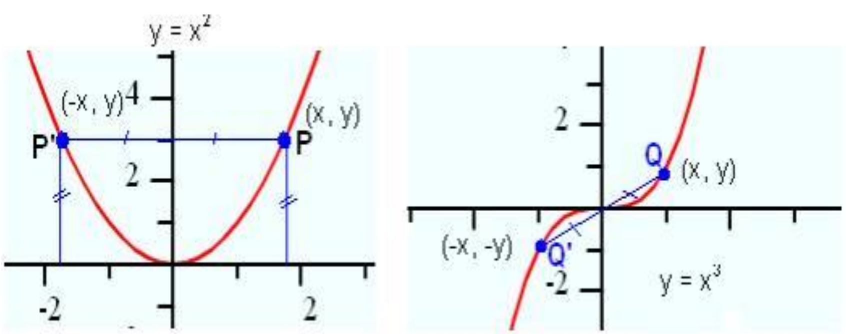


$$\mathrm{g}_e[n] = \frac{\mathrm{g}[n] + \mathrm{g}[-n]}{2}$$

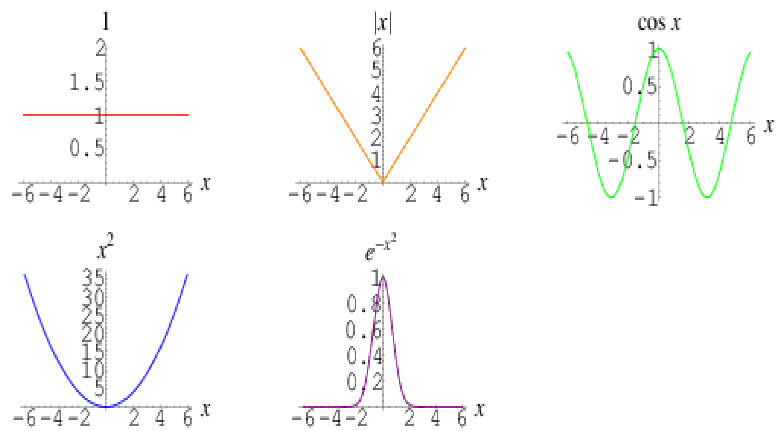
$$\mathrm{g}_o[n] = \frac{\mathrm{g}[n] - \mathrm{g}[-n]}{2}$$

Identify Even and Odd Signal



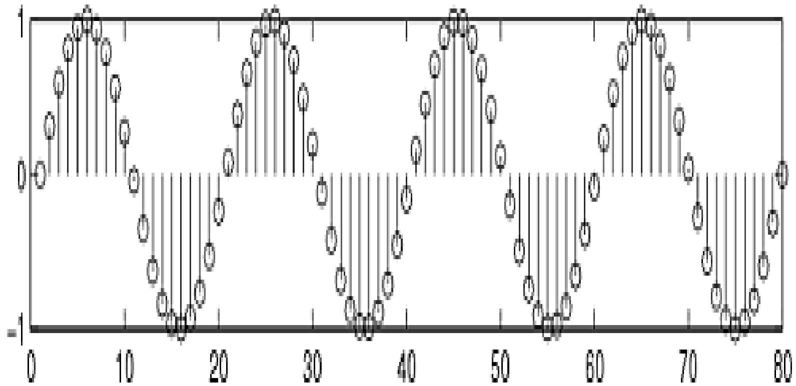


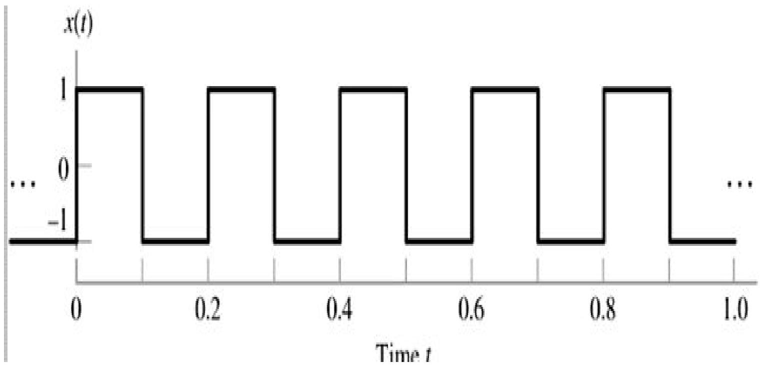
(i) (ii)

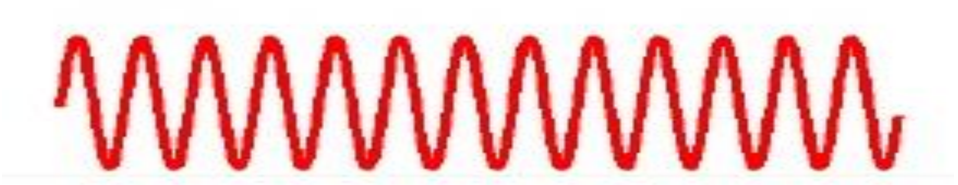


Deterministic & Non Deterministic Signals (Random Signal)

- *Deterministic signals*
- *Behavior of these signals is predictable w.r.t time*
- *There is no uncertainty with respect to its value at any time.*
- *These signals can be expressed mathematically.*
- *It behaves in a fixed known way with respect to time.*
- *If a signal is deterministic, whose future values can be predicted accurately.*
- *It can be modeled for both continuous as well as discrete time signals.*



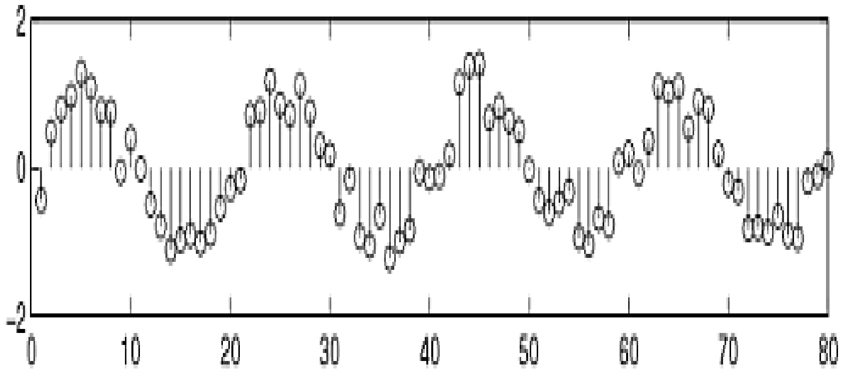


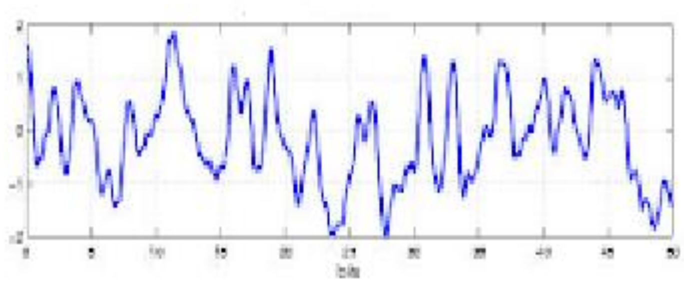


Deterministic & Non Deterministic Signals Continued…

Non Deterministic or Random signals

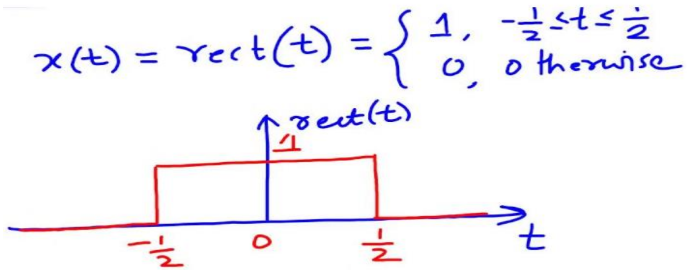
- **Behavior of these signals is random i.e. not predictable w.r.t time.**
- **There is an uncertainty with respect to its value at any time.**
- **These signals can't be expressed mathematically.**
- **If a signal is non- deterministic, whose future values can not be predicted accurately.**
- **Random signal analysis needs the knowledge of two areas such as Probability and Statistics.**
- **For example Thermal Noise generated is non deterministic signal.**



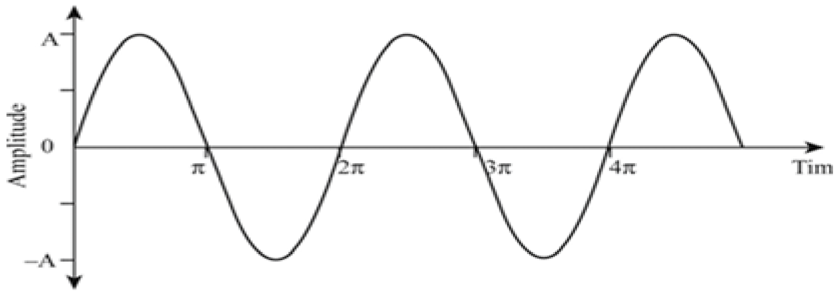


Energy and power signal

- The signals usually we considered are directly related to physical quantities captures power and energy.
- A signal is called energy signal only if its total energy is finite. For example rectangular pulse is an energy signal.



A signal is called power signal only if its average power is finite. For example Sinusoidal signals are power signals.



The energy signals have average power and power signals have infinite energy. It means both are mutually exclusive.

Energy and Power signal

Usually, the limits are taken over an infinite time interval

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, \mathrm{d}t \qquad E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

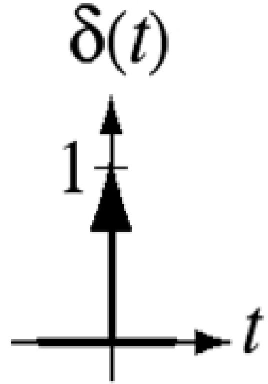
$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, \mathrm{d}t \qquad P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

- We will encounter many types of signals
- Some have infinite average power, energy, or both
- A signal is called an **energy signal** if $E_\infty < \infty$
- A signal is called a **power signal** if $0 < P_\infty < \infty$
- A signal can be an energy signal, a power signal, or neither type
- A signal can not be both an energy signal and a power signal

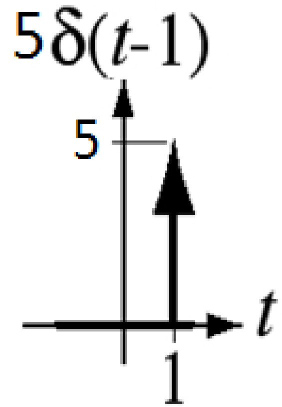
Some interesting Signals

Impulse Function

- *The area under which an impulse is called its strength or weight.*
- *It is represented graphically by a vertical arrow.*
- *An impulse with a strength of one is called a unit impulse.*

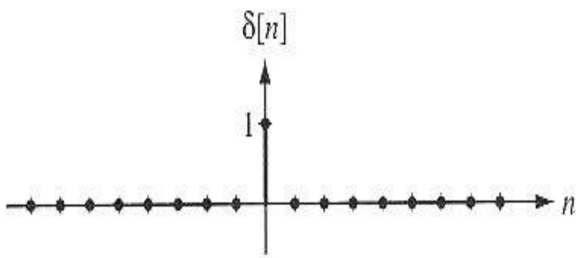


Representation of Unit Impulse



Shifted Impulse of Amplitude5

$$\delta[n] = \begin{cases} 1\ , n = 0 \\ 0\ , n \neq 0 \end{cases}$$



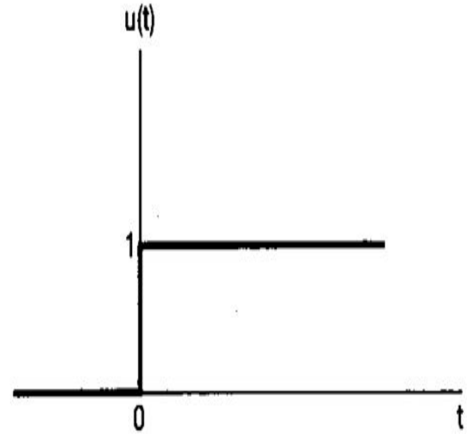
Unit Step Function

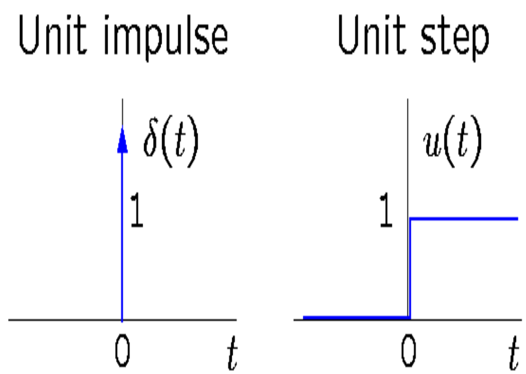
Integration of the unit impulse yields the unit step function

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau,$$

which is defined as

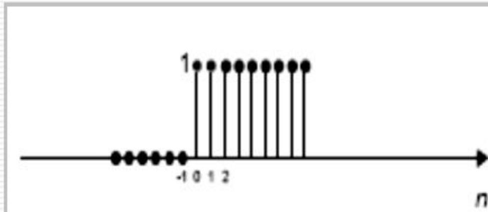
$$u(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$$





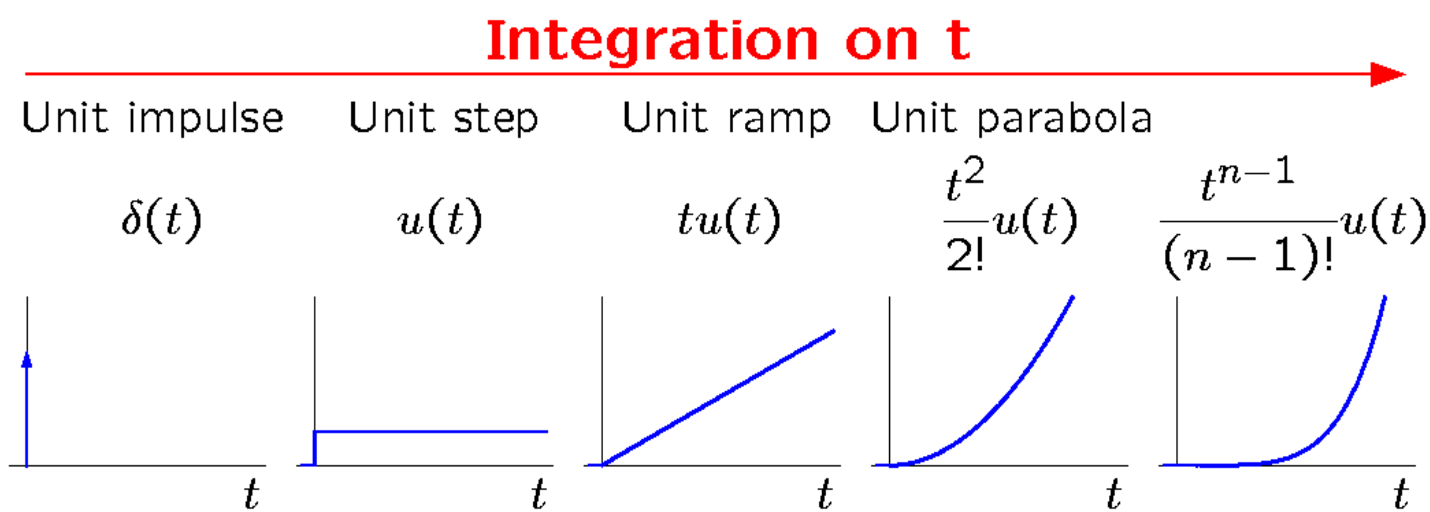
The discrete-time version of the unit-step function is defined by,

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



Successive Integrations of the Unit Impulse Function

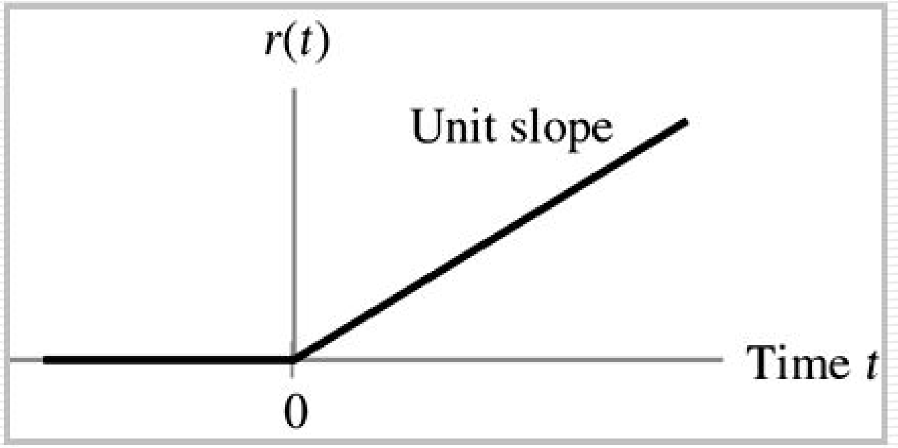
Successive integration of the unit impulse yields a family of functions.



Unit Ramp Function

The integral of the step function u(t) is a ramp function of unit slope.

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



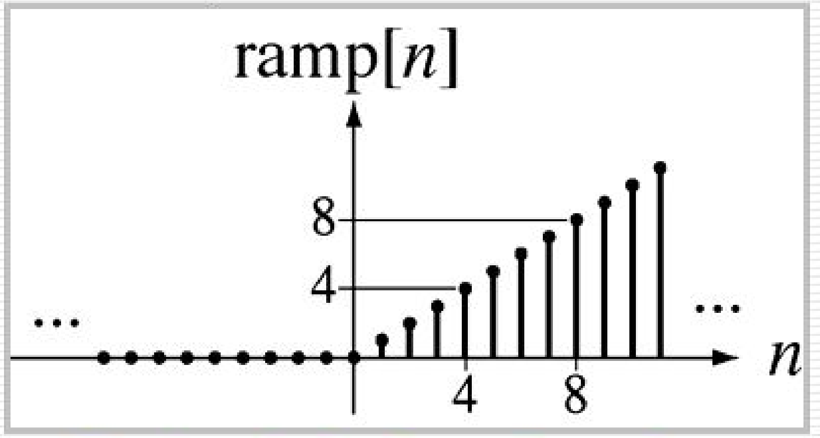
or

$$r(t) = tu(t)$$

The **discrete-time** version of the ramp function,

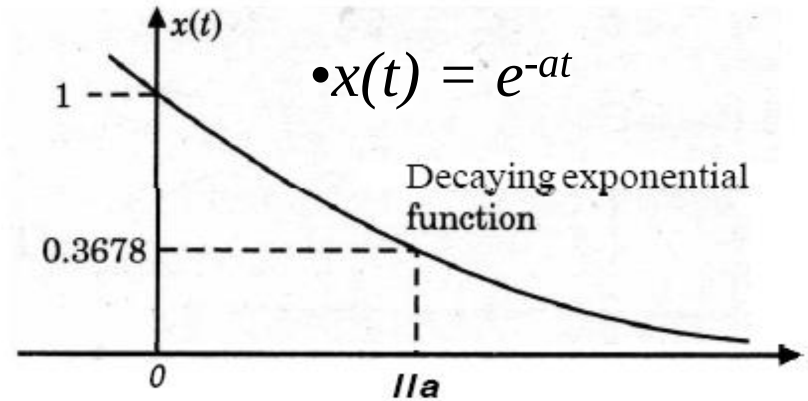
$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

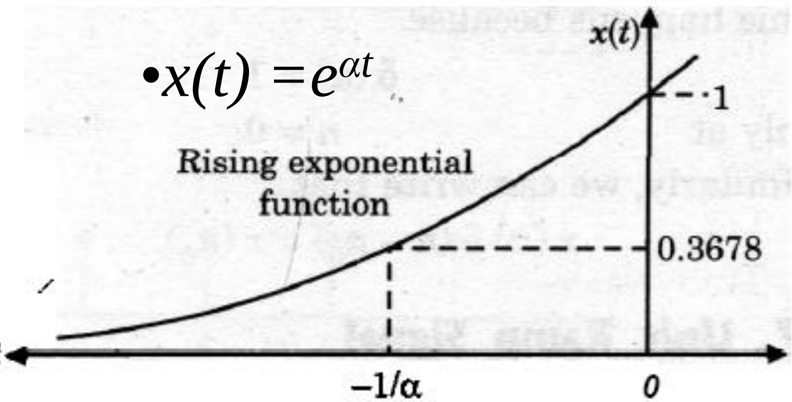
$$r[n] = nu[n]$$



Real Exponential Signals and damped Sinusoidal

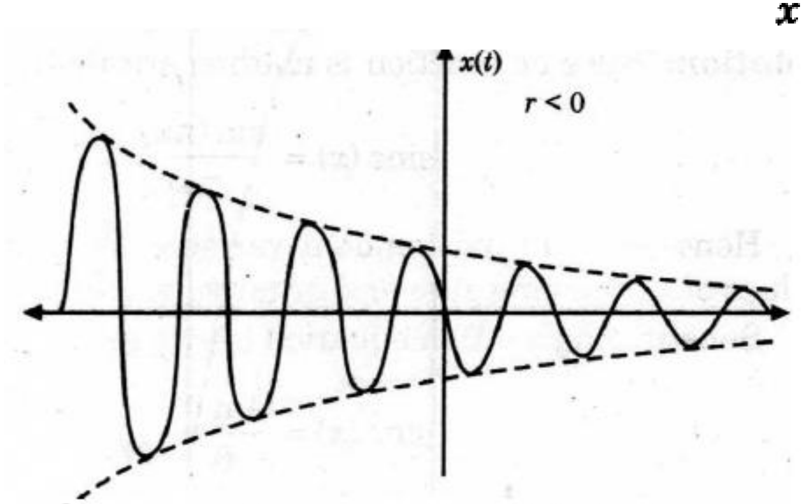
- $x(t) = e^{-at}$
- $x(t) = e^{\alpha t}$



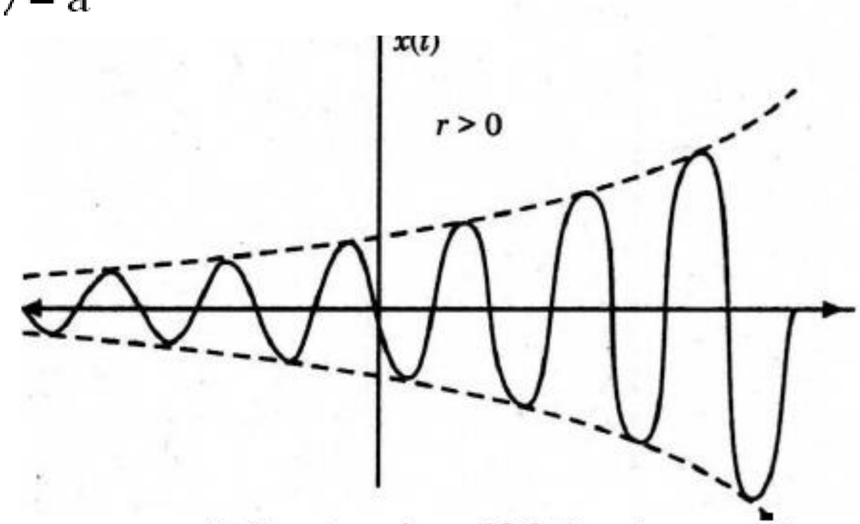


A discrete time exponential signal is expressed as

$$x(n) = a^n$$



(b) Decaying sinusoidal signal

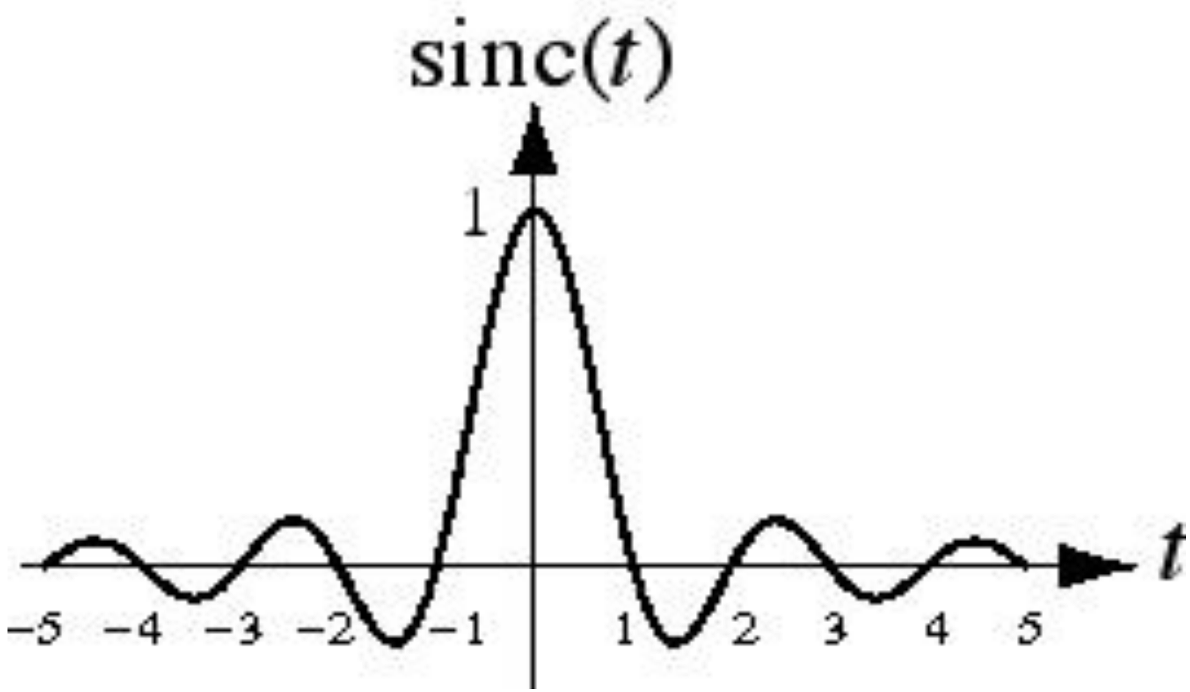


(a) Growing sinusoidal signal

$$e^{rt}\cos(\omega_0 t + \theta)$$

Sinc Function

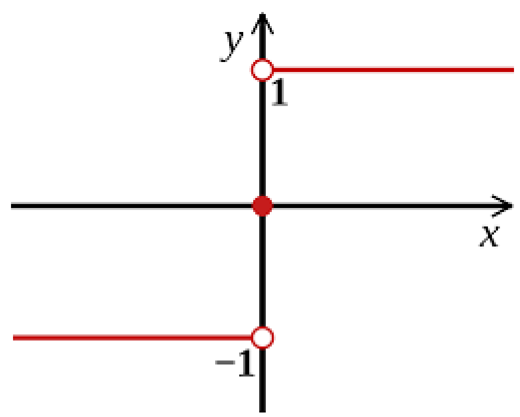
$$\operatorname{sinc}(t) = \frac{\sin(\pi t)}{\pi t}$$



Signum function

In mathematics, the sign function or signum function is an odd mathematical function that extracts the sign of a real number

Actually signum function has three options as that of the boolean function.
Boolean functions has two possible outcomes such as yes/no but signum function has three outcomes.



$$\mathrm{sign}(x) = \begin{cases} 1 \ \ if \ \ x > 0 \\ 0 \ \ if \ \ x = 0 \\ -1 \ \ if \ \ x < 0 \\ \frac{x}{|x|} \ \ if \ \ x \ \ is \text{ complex} \end{cases}$$

Operation on Sequences

Addition, subtraction, multiplication, division, and scaling of sequences can be performed on a sample-by-sample basis:

$$y[n] = x_1[n] + x_2[n], \qquad \text{(signal addition)}$$

$$y[n] = x_1[n] - x_2[n], \qquad \text{(signal subtraction)}$$

$$y[n] = x_1[n] \cdot x_2[n], \qquad \text{(signal multiplication)}$$

$$y[n] = x_1[n]/x_2[n], \qquad \text{(signal division)}$$

$$y[n] = a \cdot x_2[n]. \qquad \text{(signal scaling)}$$

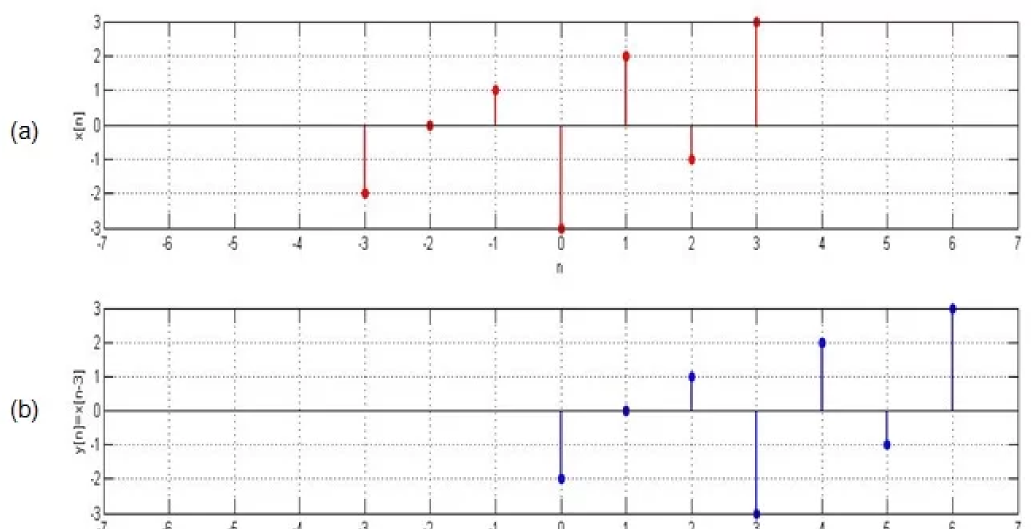
Basic Operations of Signals

- Sometimes a given mathematical function may be completely used to describe a signal .
- Different operations are required for different purposes of arbitrary signals.

The basic operations on signals can be,

- Time Shifting
- Time Scaling
- Time Inversion or Time Folding
- Convolution

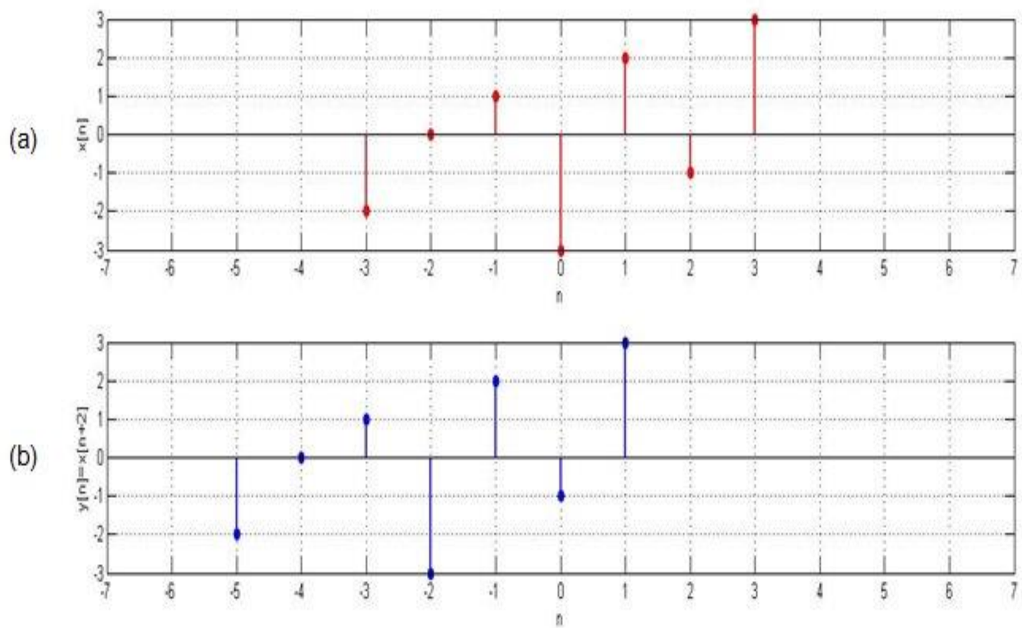
Time Shifting



Time delayed signals:

The time-shifting operation results in the change of just the positioning of the signal without affecting its amplitude.

Y[n]= X[n-3]



Time advanced signals

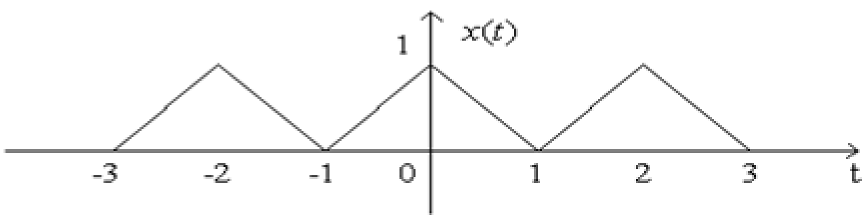
Y[n]=X[n+2]

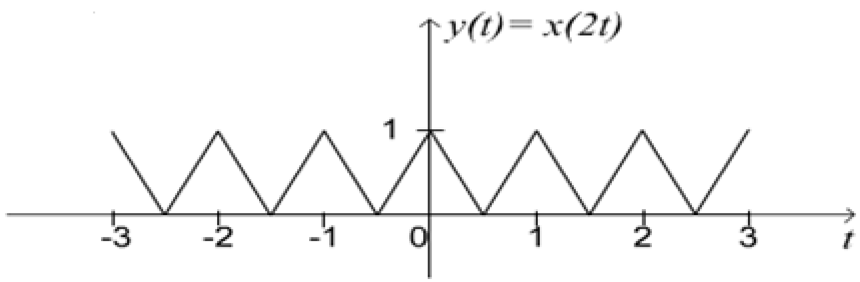
Time Scaling

For the given function x(t), x(at) is the time scaled version of x(t).

For a > 1,period of function x(t) reduces and function speeds up. Graph of the function shrinks.

For *Example: Given x(t) and we are to find y(t) = x(2t).* e function expands.



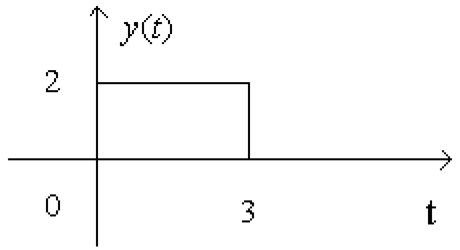


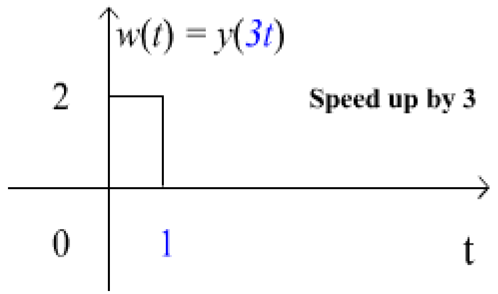
Time scaling Continued…

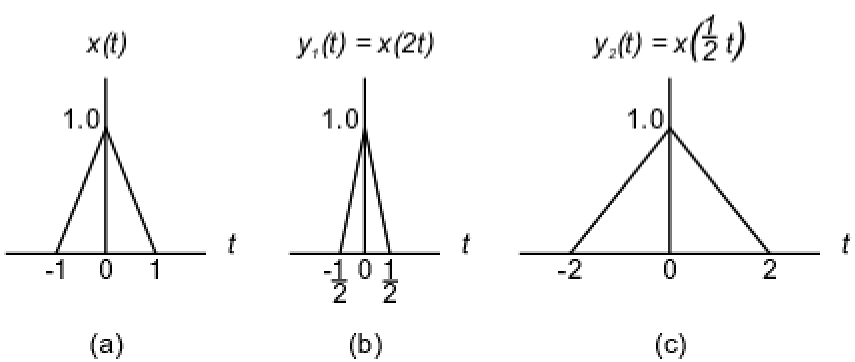
Given $y(t)$,

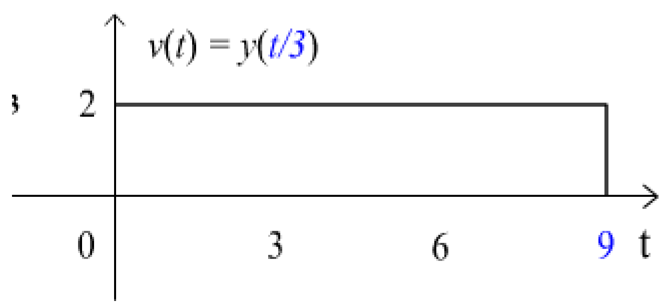
find $w(t) = y(3t)$

and $v(t) = y(t/3)$







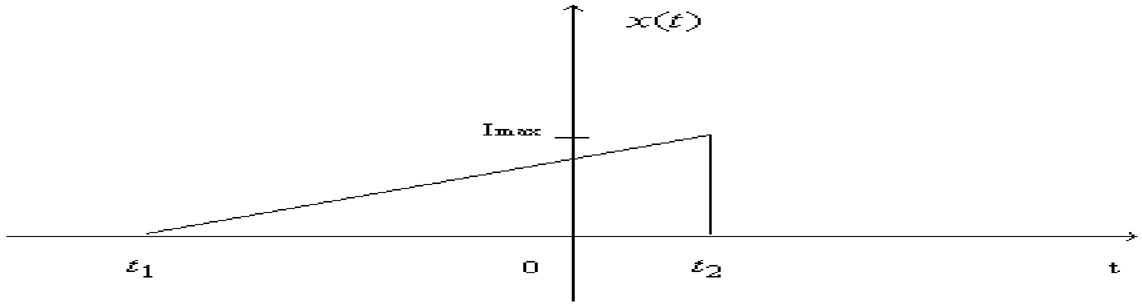


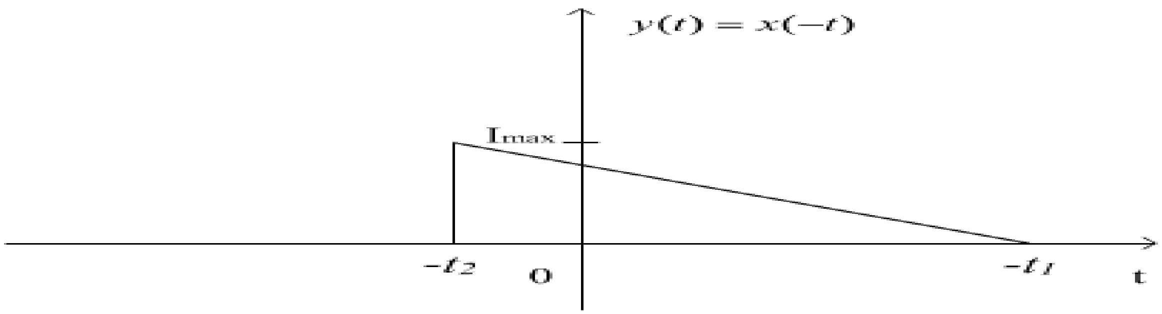
Time Reversal

Time reversal is also called time folding.

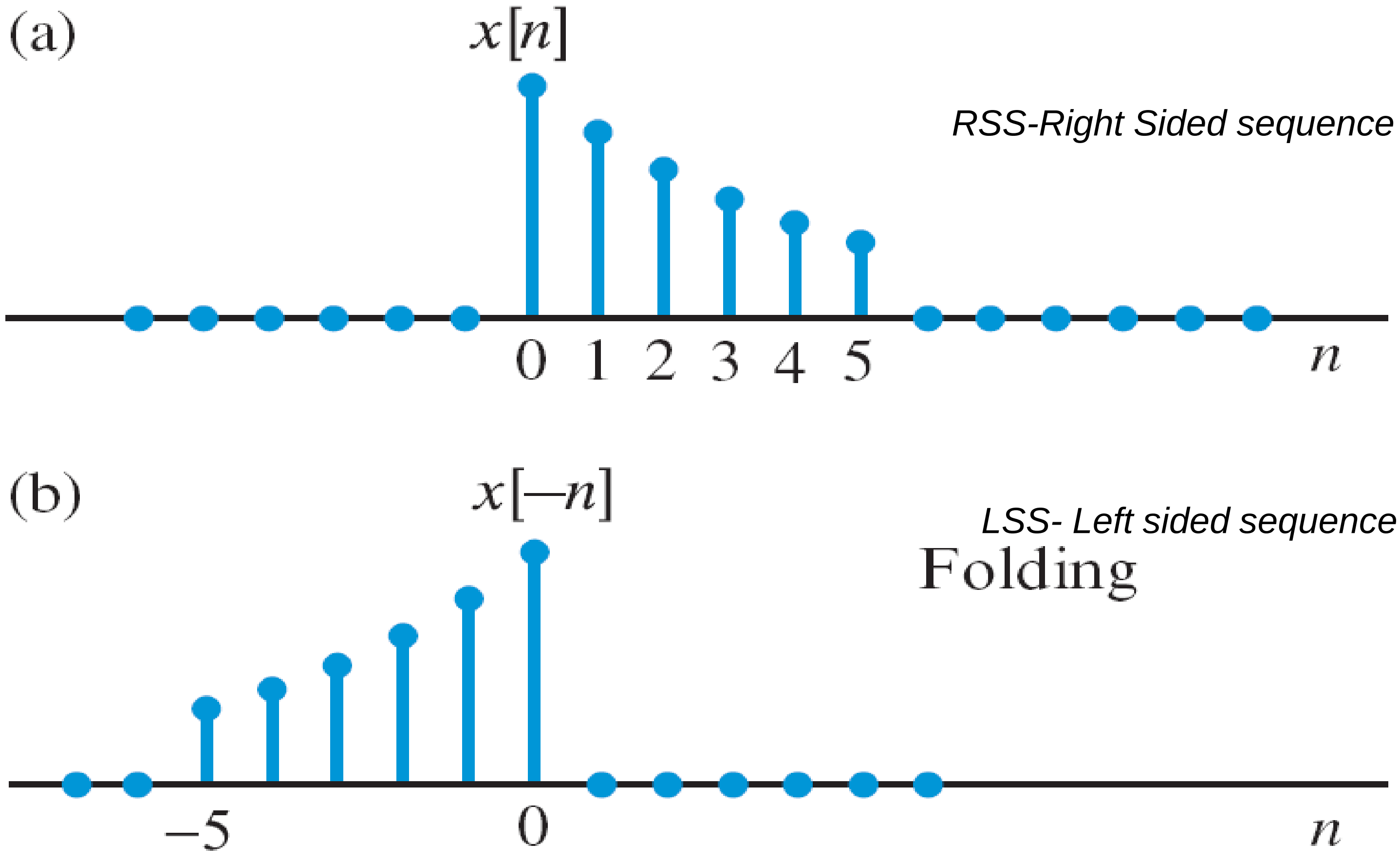
In Time reversal signal is reversed with respect to time.

i.e. y(t) = x(-t) is obtained for the given function.





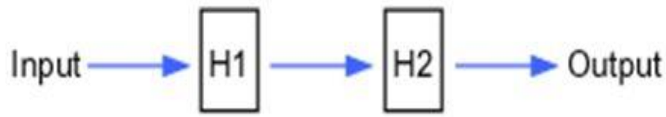
For a given signal x[n],



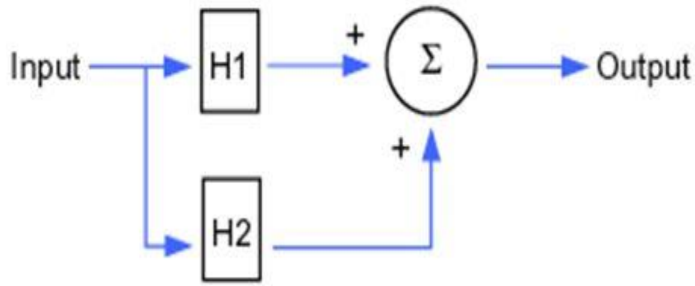
Classification of Systems

Interconnection of Systems

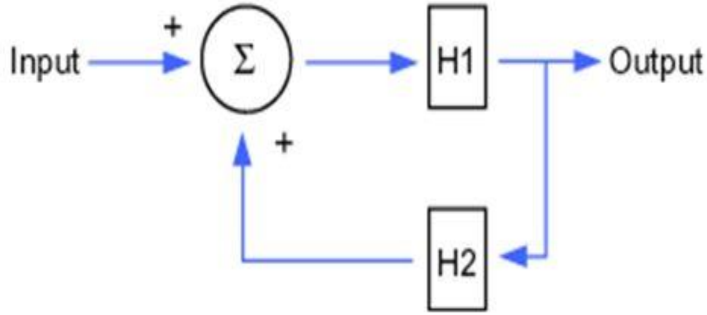
1. Cascaded (Series) Configuration:



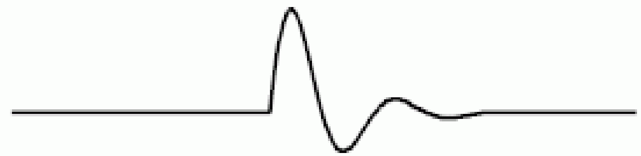
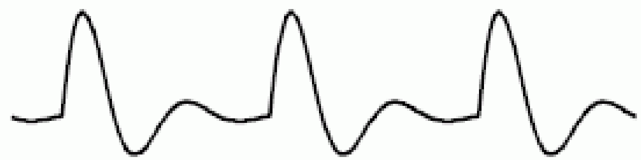
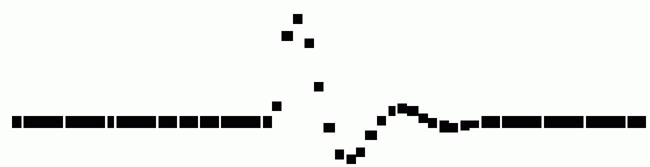
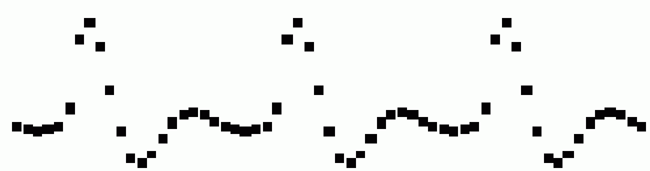
2. Parallel Configuration:



3. Feedback (Series-Parallel) Configuration:



Types of Transforms

Type of Transform	Example Signal
Fourier Transform *signals that are continious and aperiodic*	
Fourier Series *signals that are continious and periodic*	
Discrete Time Fourier Transform *signals that are discrete and aperiodic*	
Discrete Fourier Transform *signals that are discrete and periodic*	

Order of a System

- *The order of a continuous-time system corresponds to the highest derivative of the output signal that appears in input-output differential equation.*
- *The order of a discrete-time system corresponds to largest number of units of delay of the output discrete–time signal appearing in the input-output difference equations.*
- *The input-output equation of a continuous –time system is given by as follows. Here, y(t) is the output of the system and x(t) is the input of the system.*

$$\frac{d^3}{dt^3}y(t) + \frac{d^2}{dt^2}y(t) = 6\frac{d^2}{dt^2}x(t) + 5\frac{d}{dt}x(t) + x(t)$$

$$\frac{d^2}{dt^2}y(t) + \frac{d}{dt}y(t) - 6y(t) = 2.4\frac{d}{dt}x(t) + 8.1x(t)$$

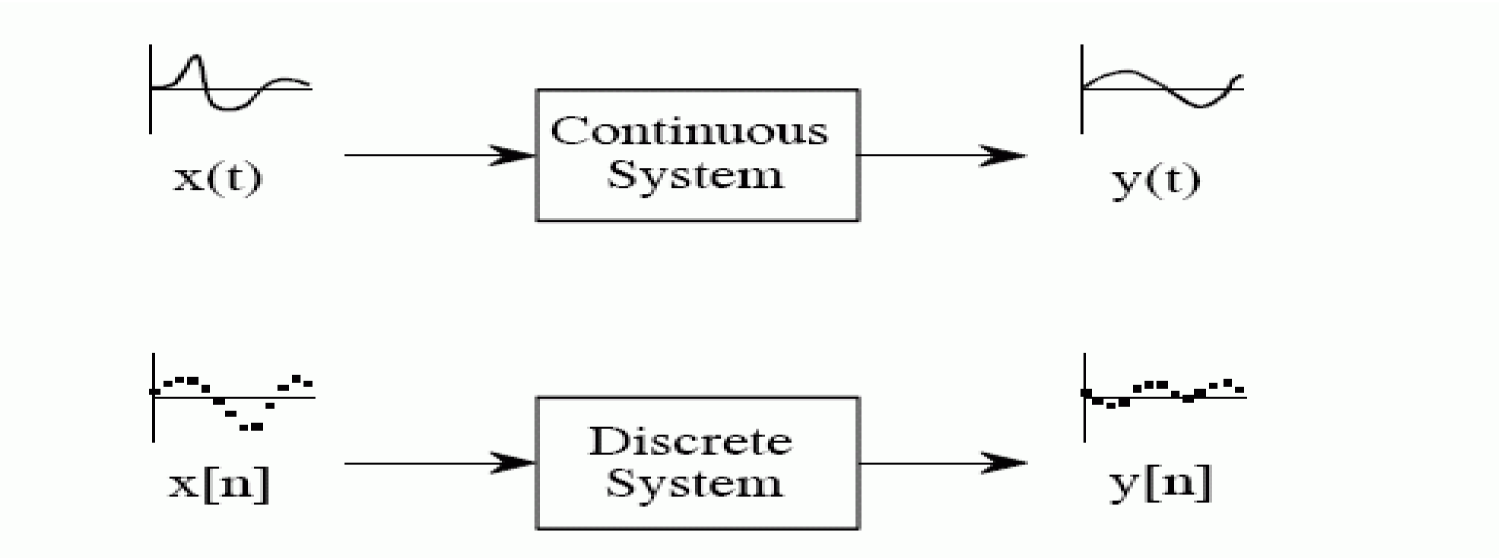
The input-output equation of a discrete time system is given as follows. Here, y[n] is the output of the system and x[n] is the input of the system.

$$y[n] - 3y[n-1] - 4y[n-2] = x[n] + 2x[n-1].$$

Continuous and Discrete time Systems

If the input and output signals x and y are continuous-time signals, then the system is called a ***continuous-time system***

If the input and output signals are discrete-time signals or sequences, then the system is called a ***discrete-time system***



Static & Dynamic Systems

- A static system is memoryless system
- It has no storage devices
- Its output signal depends on present values of the input signal

For example

$$i(t) = \frac{1}{R} v(t)$$

- A dynamic system possess memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal.
- An accumulator and delay elements are the examples for dynamic system

- Examples of memoryless systems:

$$y(t) = Rx(t) \qquad \text{or} \qquad y[n] = (2x[n] - x^2[n])^2.$$

- Examples of systems with memory:

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau \qquad \text{or} \qquad y[n] = x[n-1].$$

Causal & Noncausal Systems

Causal system : A system is said to be *causal* if the present value of the output signal depends only on the present and/or past values of the input signal.

Example: $y[n]=x[n]+1/2x[n-1]$

Non causal system : A system is said to be *anticausal or Noncausal system* if the present value of the output signal depends only on the future values of the input signal.

Some examples of causal systems are given below:

1) $y(n) = x(n) + x(n-2)$
2) $y(n) = x(n-1) – x(n-3)$
3) $y(n) = 7x(n-5)$

some examples of non-causal systems are given below:

1) $Y(n) = x(n) + x(n+1)$
2) $Y(n) = 7x(n+2)$
3) $Y(n) = x(n) + 9x(n+5)$

Significance: Generally all real time systems are causal systems. s**ince future samples are not present, causal system is memoryless system.**
Since non-causal system contains future samples, it is practically not realizable.

Linear & Non Linear Systems

A system which obeys the superposition principle is called Linear system, otherwise is known as non-Linear system

- If the operator T in $y = \mathrm{T}x$ satisfies the following two conditions, then T is called a linear operator and the system represented by a linear operator T is called a linear system:

  1. Additivity:

  Given that $\mathrm{T}x_1 = y_1$, and $\mathrm{T}x_2 = y_2$, then

  $$\mathbf{T}\{x_1 + x_2\} = y_1 + y_2$$

  2. Homogeneity (or Scaling):

  $$\mathbf{T}\{\alpha x\} = \alpha y$$

  for any signals x and any scalar α .

- Can be combined into a single condition as

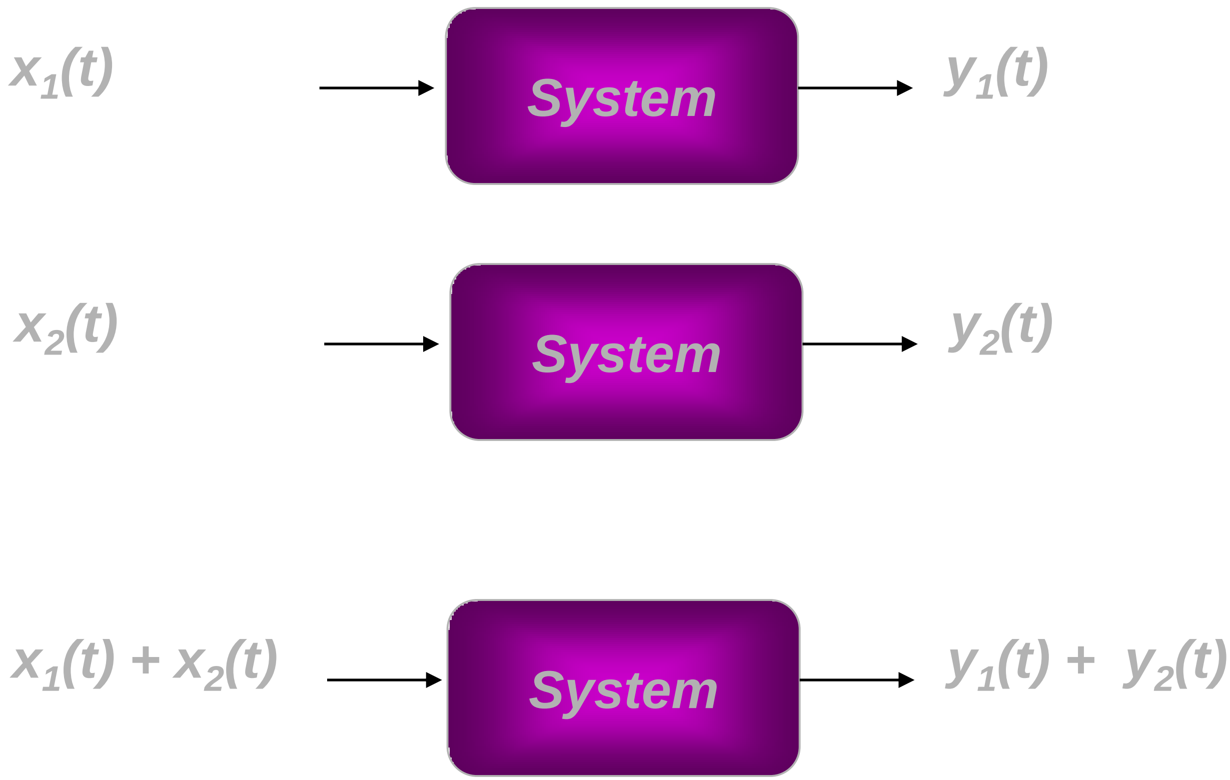
  $$\mathbf{T}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$$

SYSTEM DESCRIPTION

Linearity

IF

THEN



SYSTEM DESCRIPTION

Homogeneity

IF $x_1(t) \rightarrow$ ***System*** $\rightarrow y_1(t)$

THEN $ax_1(t) \rightarrow$ ***System*** $\rightarrow ay_1(t)$

Where a is a constant

Linear and Non-linear system continued…

$y(t) = x^2(t)$

The output of the system to two inputs $x_1(t)$ and $x_2(t)$ becomes,

$$y_1(t) = f[x_1(t)] = x_1^2(t)$$

$$y_2(t) = f[x_2(t)] = x_2^2(t)$$

Hence linear combination of these outputs become,

$$\begin{aligned} y_3(t) &= a_1 y_1(t) + a_2 y_2(t) \\ &= a_1 x_1^2(t) + a_2 x_2^2(t) \end{aligned}$$

Now let us find the response of the system to linear combination of inputs. i.e.,

$$\begin{aligned} y'_3(t) &= f[a_1 x_1(t) + a_2 x_2(t)] \\ &= [a_1 x_1(t) + a_2 x_2(t)]^2 \\ &= a_1^2 x_1^2(t) + a_2^2 x_2^2(t) + 2 a_1 a_2 x_1(t) x_2(t) \end{aligned}$$

Here note that $y'_3(t) \neq y_3(t)$. Hence this is not linear system.

Time Invariant and Time Variant Systems

- Conceptually, a system is called time invariant if the behavior and input-output characteristics of the system does not change with time.

- A system is said to be *time invariant* if a time delay or time advance of the input signal leads to a identical time shift in the output signal.

- Algebraic, differential and difference which are the mathematical models for time invariant systems.

- Thus, for a continuous-time system, the system is time-invariant if

$$\mathbf{T}\{x(t-\tau)\} = y(t-\tau)$$

- For a discrete-time system, the system is time-invariant (or shift-invariant) if

$$\mathbf{T}\{x[n-k]\} = y[n-k]$$

- Else the systems are known as the time varying systems

Continued…..

$$y(t) = \sin x(t)$$

Let us determine the output of the system for delayed input $x(t-t_1)$. i.e.,

$$y(t, t_1) = f[x(t-t_1)]$$

$$= \sin x(t-t_1) \qquad \text{... (1.7.6)}$$

Here $y(t, t_1)$ represents output due to delayed input.

Now delay the output $y(t)$ by t_1. Hence we have to replace t by $t-t_1$ in $y(t)=\sin x(t)$. i.e.,

$$y(t-t_1) = \sin x(t-t_1)$$

On comparing the above equation with equation 1.7.6 we find that,

$$y(t, t_1) = y(t-t_1)$$

This satisfies equation 1.7.5. Hence the system is time invariant.

SYSTEM DESCRIPTION

Time-invariance: System does not change with time

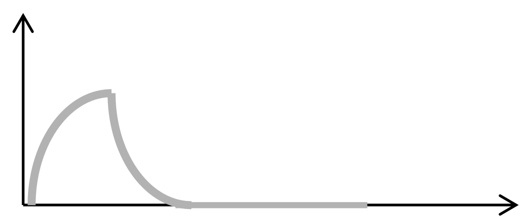
IF $x_1(t)$  $y_1(t)$

THEN $x_1(t-\tau)$  $y_1(t-\tau)$

$x_1(t)$

t

$y_1(t)$



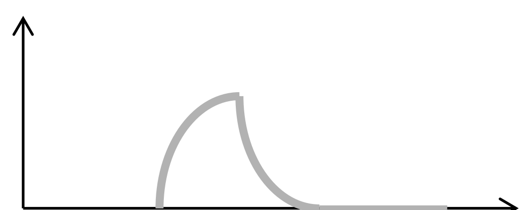
t

$x_1(t-\tau)$

τ

t

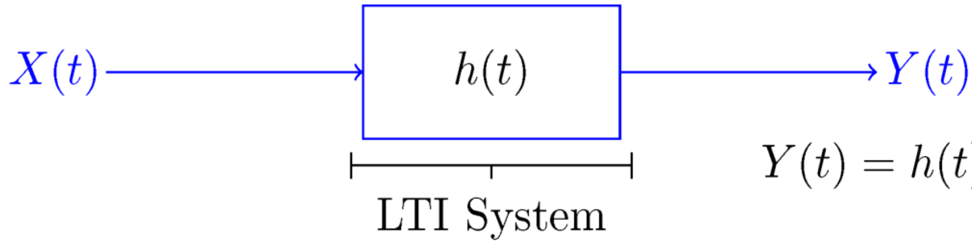
$y_1(t-\tau)$



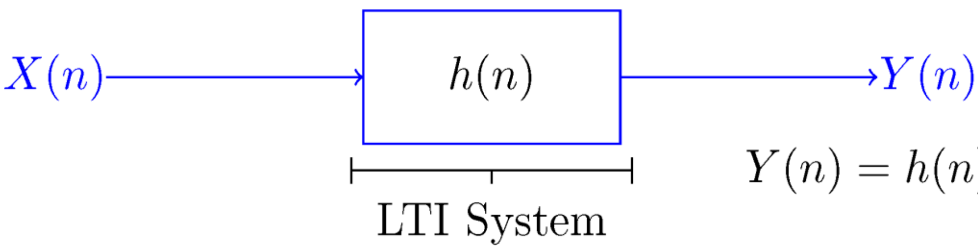
τ

t

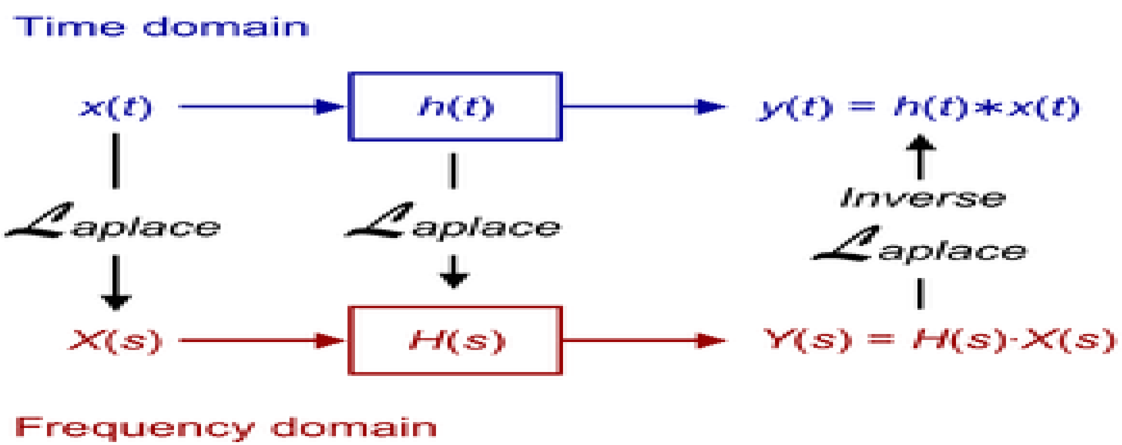
LTI (Linear Time Invariant) system



$$Y(t) = h(t) * X(t)$$



$$Y(n) = h(n) * X(n)$$



Stable & Unstable Systems

A system is said to be *bounded-input bounded-output stable* (BIBO stable) if every bounded input results in a bounded output.

The output of a stable system settles back to the quiescent state (e.g., zero) when the input is removed

The output of an unstable system continues, often with exponential growth, for an indefinite period when the input is removed

Example: The system represented by

$$y(t) = A\,x(t) \text{ is unstable ; } A>1$$

Reason: let us assume $x(t) = u(t)$, then at every instant $u(t)$ will keep on multiplying with A and hence it will not be bounded.

- For a stable system, the output to bounded inputs is also bounded. Example: pendulum at bottom equilibrium
- For an unstable system, the output diverges to infinity or to values causing permanent damage. Example: Inverted pendulum.

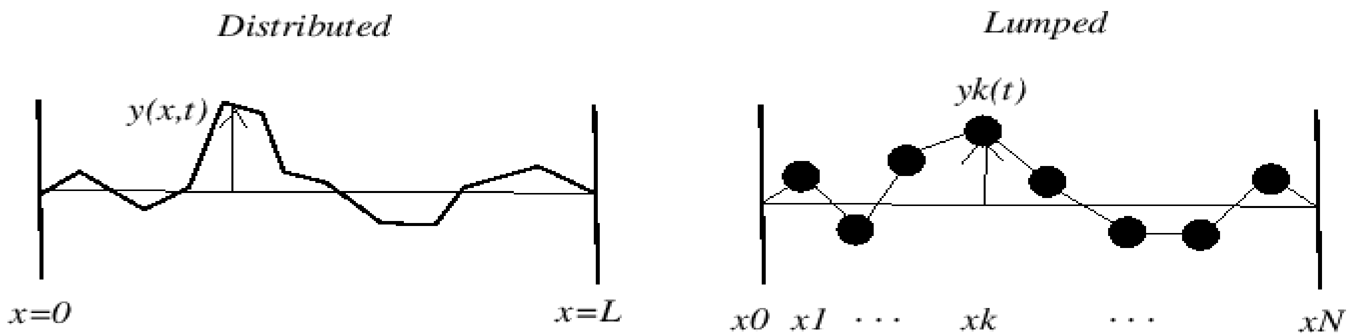
Distributed and Lumped parameter system

A lumped system, in which the dependent variables of interest are a function of time alone.

A *distributed* system is one in which all dependent variables are functions of time *and* one or more spatial variables.

Lumped systems are said to be described by ordinary differential equations while the latter is said to be described by partial differential equations.

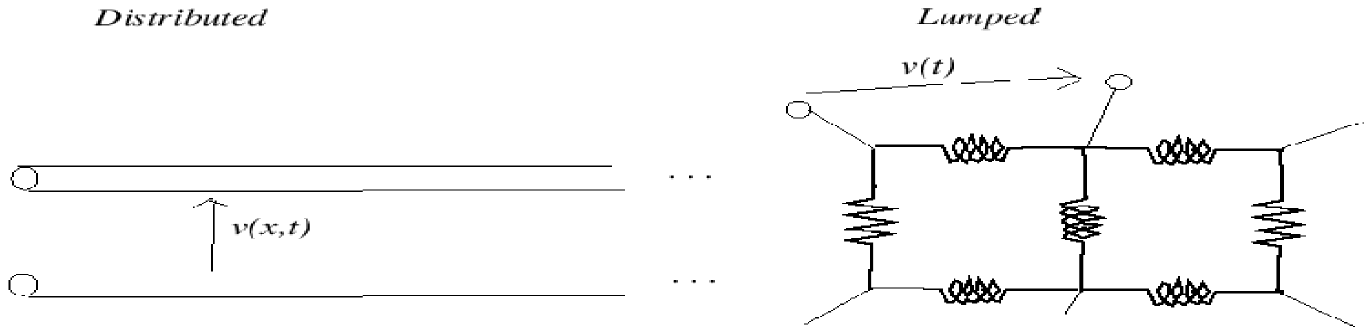
For example, consider the following two systems,



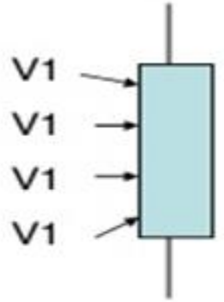
- In distributed system, it consists of infinitely thin string supported at both ends, the dependent variable, the vertical position of the string is indexed continuously in both space and time.
- In Lumped system, series of beads connected by string segments constrained to move vertically.

Distributed and Lumped parameter system Continued…

In electrical systems,

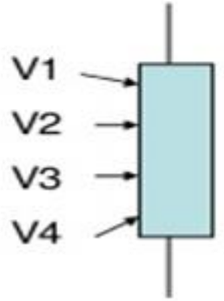


- If the component used in system has **identical** values of physical parameters(current, voltages etc) throughout its area and can be considered as a single point (node) in the system , it is called as lumped parameter system
- e.g normal components like resistor, capacitor in low frequency applications etc



- If the component used in system has **different** values of physical parameters (current, voltages etc) throughout its area and **cannot** be considered as a single point (node) in the system.

- E.g. transmission lines, microwave tubes which normally used in high frequency



Thank you