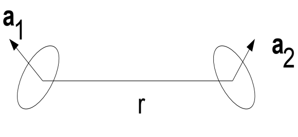
IITK/PHY103, **Problem set-12** **Date: March 27, 2015** [DpC/TKG]

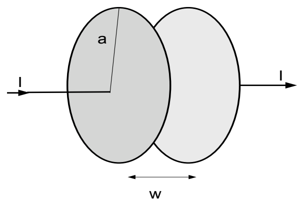
1. Two tiny wire loops, with area $\vec{a_1}$ and $\vec{a_2}$ as shown in the figure, are situated a displacement r apart (as shown in Fig. (1)).
   (a) Find their mutual inductance.
   (b) Suppose a current I_1 is flowing in loop 1 and we propose to turn on a current I_2 in loop 2. How much work must be done against the mutually induced emf, to keep the current I_1 flowing in loop 1?

2. Consider an iron ring wound with N turns of wire carrying current I. Assume the radius r of the ring is much larger that the dimensions of its cross-sectional area A. How much energy must be supplied to carry the iron through one complete hysteresis cycle?

3. Consider a parallel plate capacitor with each plate of radius a and a constant current I flows as shown in Fig.(2). Assume that the current flows out over the plates in such a way that the surface charge is uniform, at any given time, and is zero at $t = 0$. Gap between the plates $w << a$.
   (a) Find the electric field between the plates, as a function of time.
   (b) Find the displacement current through a circle of radius s in the plane midway between the plates. Using the circle as your "Amperian loop" and the flat surface that span it, find the magnetic field at a distance s from the axis.
   (c) Repeat (b), but this time use the cylindrical surface of radius s which extends to the left through the plate and terminates outside the capacitor.



(1)



(2)

4. Since $\vec{E}$ inside a conductor is zero, by Farady's law $\vec{B}$ has to be constant (time independent) inside the conductor. A *superconductor* is a perfect conductor with additional property that $\vec{B} = 0$ inside(*known as Meissner effect*). You have a sphere of radius

a which becomes superconductor below a certain critical temperature T_c. Suppose, you held it in uniform magnetic field $B_0\hat{k}$ while cooling it below T_c. Find the surface current density $\vec{K}$ induced on the sphere, as a function of polar angle θ.

Practice Problems

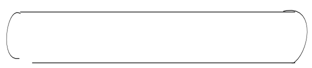
1. Compute the self inductance of a long hairpin of length l(Fig. (3)). Ignore the fields from the bends and any flux through the wire itself.

2. A circular loop of wire with radius$a$ and electrical resistance R lies in the xy plane . A uniform magnetic field is turned on at $t = 0$, for $t > 0$ the field is

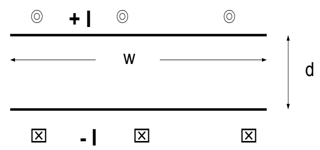
$$\vec{B}(t) = \frac{B_0}{\sqrt{2}}(\hat{y} + \hat{k})(1 - e^{-\lambda t})$$

Determine the current $I(t)$ induced in the loop. Sketch $I(t)$ as a function of t.

3. Equal and opposite currents $+I$ and $-I$ flow in two long parallel plates(Fig.(4)). The plates have width w and separation d where d is small.
(a) Neglecting the edge effects, find the magnetic field between the plates.
(b) Calculate the magnetic field energy per unit length.
(c) Show that the self inductance per unit length is $\mu_0 d/w$.



(3)



(4)

4. Show that the Biot-Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{J}(\mathbf{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d\tau'$$

obeys Ampere's law with Maxwell's displacement current term.