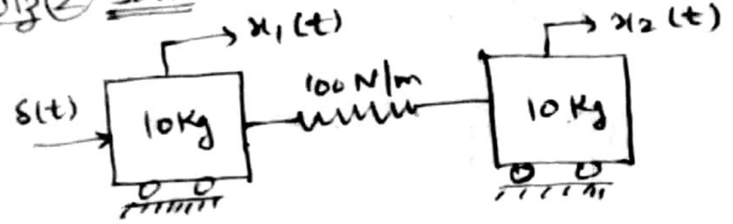
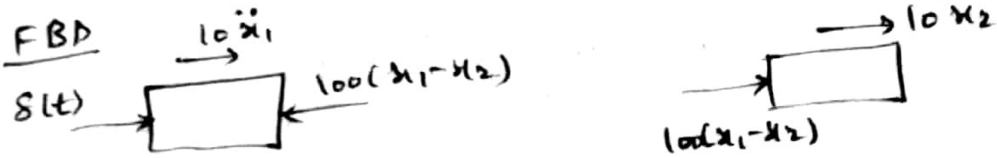
Quiz ② Soln.



$x_1(0) = x_2(0) = 0$

$\dot{x}_1(0) = \dot{x}_2(0) = 0$

FBD



$$100(x_1-x_2) = 10\ddot{x}_2$$

$$\delta(t) - 100(x_1-x_2) = 10\ddot{x}_1$$

$$10\ddot{x}_1 + 100(x_1-x_2) = \delta(t) \quad — ①$$

$$10\ddot{x}_2 - 100(x_1-x_2) = 0 \quad — ②$$

$$\begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 100 & -100 \\ -100 & 100 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{Bmatrix} \delta(t) \\ 0 \end{Bmatrix}$$

put $x = Xe^{i\omega t}$

we get the matrix $Z(\omega) = \begin{bmatrix} 100-10\omega^2 & -100 \\ -100 & 100-10\omega^2 \end{bmatrix}$

Finding eigen values.

$$\begin{vmatrix} 100-10\omega^2 & -100 \\ -100 & 100-10\omega^2 \end{vmatrix} = 0$$

$$(100-10\omega^2)^2 = (100)^2$$

$$100 - 10\omega^2 = \pm 100$$

$10\omega^2 = 200$ and $10\omega^2 = 0$

$\omega^2 = 20$ and $\omega^2 = 0$

$\omega = \sqrt{20}$ and $\omega = 0$

Say $\omega_1 = 0$, $\omega_2 = \sqrt{20}$ rad/sec

Finding eigen values, corresponding to $\omega_1^2 = 0$, $\omega_2^2 = 20$

$$\begin{bmatrix} 100 - 10\times 0 & -100 \\ -100 & 100-10\times 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow 100x_1 - 100x_2 = 0$$

$$\Rightarrow X_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\begin{bmatrix} 100 - 10\times 20 & -100 \\ -100 & 100-10\times 20 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0 \Rightarrow -100x_1 - 100x_2 = 0$$

$$\Rightarrow X_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

Normalising wrt mass mass matrix

$$\underline{X_i}^T \underline{M} \underline{X_i} = e_i^2$$

$$\Rightarrow X_1^T \underline{M} X_1 = e_1^2 \Rightarrow \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = e_1^2 = 20$$

$$\Rightarrow e_1 = \sqrt{20}$$

$$\Rightarrow \underline{X_2}^T \underline{M} \underline{X_2} = e_2^2 \Rightarrow \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = e_2^2 = 20$$

$$e_2 = \sqrt{20}$$

$$\therefore \phi_1 = \frac{X_1}{e_1} = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} ; \quad \phi_2 = \frac{X_2}{e_2} = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\therefore \underline{U} = [\phi_1 \,\vdots\, \phi_2] = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$Eq^n \rightarrow \quad M\ddot{x} + Kx = f(t)$

put $\underline{x} = \underline{U}\,\eta(t)$ and multiply by $\underline{U}^T$

we get $\quad \ddot{\eta}_i + \omega_i^2 \eta_i = \underline{U}^T f(t)$

$$\therefore \quad \ddot{\eta}_1 + \omega_1^2 \eta_1 = U^T f_0(t) \quad ; \quad \ddot{\eta}_2 + \omega_2^2 \eta_2 = U^T f_0(t)$$

$$\Rightarrow \quad \ddot{\eta}_1 = U^T \delta(t) \quad ; \quad \ddot{\eta}_2 + 20\eta_2 = U^T f_0(t)$$

$$\Rightarrow \quad \ddot{\eta}_1 = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} \delta(t) \\ 0 \end{Bmatrix} \quad ; \quad \ddot{\eta}_2 + 20\eta_2 = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} \delta(t) \\ 0 \end{Bmatrix}$$

$$\Rightarrow \boxed{\ddot{\eta}_1 = \frac{1}{\sqrt{20}} \delta(t)} \quad (3) \quad ; \quad \boxed{\ddot{\eta}_2 + 20\eta_2 = \frac{1}{\sqrt{20}} \delta(t)} \quad (4)$$

Converting the B.C. from physical co-ordinates to modal co-ordinates, we get

$$\eta_1(0) = \eta_2(0) = 0 \quad ; \quad \dot{\eta}_1(0) = \dot{\eta}_2(0) = 0$$

Soln. of eq (3)

$$\boxed{\eta_1 = \frac{1}{\sqrt{20}} t}$$

Soln. of eq 4

$$\boxed{\eta_2 = \frac{1}{\sqrt{20}} \left\{ \frac{1}{\sqrt{20}} \sin\sqrt{20}\, t \right\}}$$

Since, $\quad \underline{x} = \underline{U}\,\underline{\eta} \rightarrow \underline{x} = \frac{1}{\sqrt{20}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} t/\sqrt{20} \\ \sin\sqrt{20}\, t/20 \end{bmatrix}$

$$\Rightarrow x_1 = \frac{1}{\sqrt{20}} \left[\frac{t}{\sqrt{20}} + \frac{\sin\sqrt{20}\, t}{20} \right] ; \quad x_2(t) = \frac{1}{\sqrt{20}} \left[\frac{t}{\sqrt{20}} - \frac{\sin\sqrt{20}\, t}{20} \right]$$