ESO 201A: Thermodynamics
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Entropy: part 1

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Learning objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

The inequality of Clausius

Important inequality, introduced by R. J. E. Clausius $\oint \frac{\delta Q}{T} \leq 0$

Cyclic integral of δQ/T (i.e. over the entire cycle) is always less than or equal to zero. Valid for all cycles-reversible or irreversible

Consider reversible HE (Carnot) $\oint \delta Q = Q_H - Q_L > 0$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L} = 0$$

Since for reversible, $(Q_H/Q_L)=T_H/T_L$

Thus For reversible HE $\oint \delta Q \geq 0$ $\oint \frac{\delta Q}{T} = 0$

The inequality of Clausius

Now, consider an irreversible cyclic HE between the same T_H and T_L and receiving the same quantity of heat, Q_H

Comparing the irreversible cycle with the reversible one, we conclude from the second law that

$$W_{\text{irr}} < W_{\text{rev}}$$

Since $Q_H - Q_L = W$ for both the reversible and irreversible cycles, we conclude that

$$Q_H - Q_{L\ \text{irr}} < Q_H - Q_{L\ \text{rev}}$$

Therefore,

$$Q_{L\ \text{irr}} > Q_{L\ \text{rev}}$$

Consequently, for the irreversible cyclic engine,

$$\oint \delta Q = Q_H - Q_{L\ \text{irr}} > 0$$

$$\oint \frac{\delta Q}{T} = \frac{Q_H}{T_H} - \frac{Q_{L\ \text{irr}}}{T_L} < 0$$

Higher the irreversible

Q_H-$Q_{L,\text{irr}} \rightarrow 0$

Limiting case

$$\oint \delta Q = 0$$

$$\oint \frac{\delta Q}{T} < 0$$

The inequality of Clausius

Thus, we conclude that for all irreversible heat engine cycles

$$\oint \delta Q \geq 0$$

$$\oint \frac{\delta Q}{T} < 0$$

To complete the demonstration of the inequality of Clausius, consider both irreversible and irreversible refrigeration cycles.

For reversible refrigeration cycle

$$\oint \delta Q = -Q_H + Q_L < 0$$

and

$$\oint \frac{\delta Q}{T} = -\frac{Q_H}{T_H} + \frac{Q_L}{T_L} = 0$$

The inequality of Clausius

Reversible refrigeration cycles

$$\oint \delta Q \leq 0$$

$$\oint \frac{\delta Q}{T} = 0$$

For irreversible refrigerator operating between the same T_H and T_L and receive the same amount of heat Q_L

$$W_{\text{irr}} > W_{\text{rev}}$$

Since $Q_H - Q_L = W$

$$Q_{H\,\text{irr}} - Q_L > Q_{H\,\text{rev}} - Q_L$$

Therefore

$$Q_{H\,\text{irr}} > Q_{H\,\text{rev}}$$

$$\oint \delta Q = -Q_{H\,\text{irr}} + Q_L < 0$$

$$\oint \frac{\delta Q}{T} = -\frac{Q_{H\,\text{irr}}}{T_H} + \frac{Q_L}{T_L} < 0$$

Thus, for all irreversible refrigeration cycles,

$$\oint \delta Q < 0$$

$$\oint \frac{\delta Q}{T} < 0$$

Summarizing, we note that, in regard to the sign of $\oint \delta Q$, we have considered all possible reversible cycles (i.e., $\oint \delta Q \gtrless 0$), and for each of these reversible cycles

We have also considered all possible irreversible cycles for the sign of $\oint \delta Q$ (i.e, $\oint \delta Q \gtrless 0$), and for all these irreversible cycles

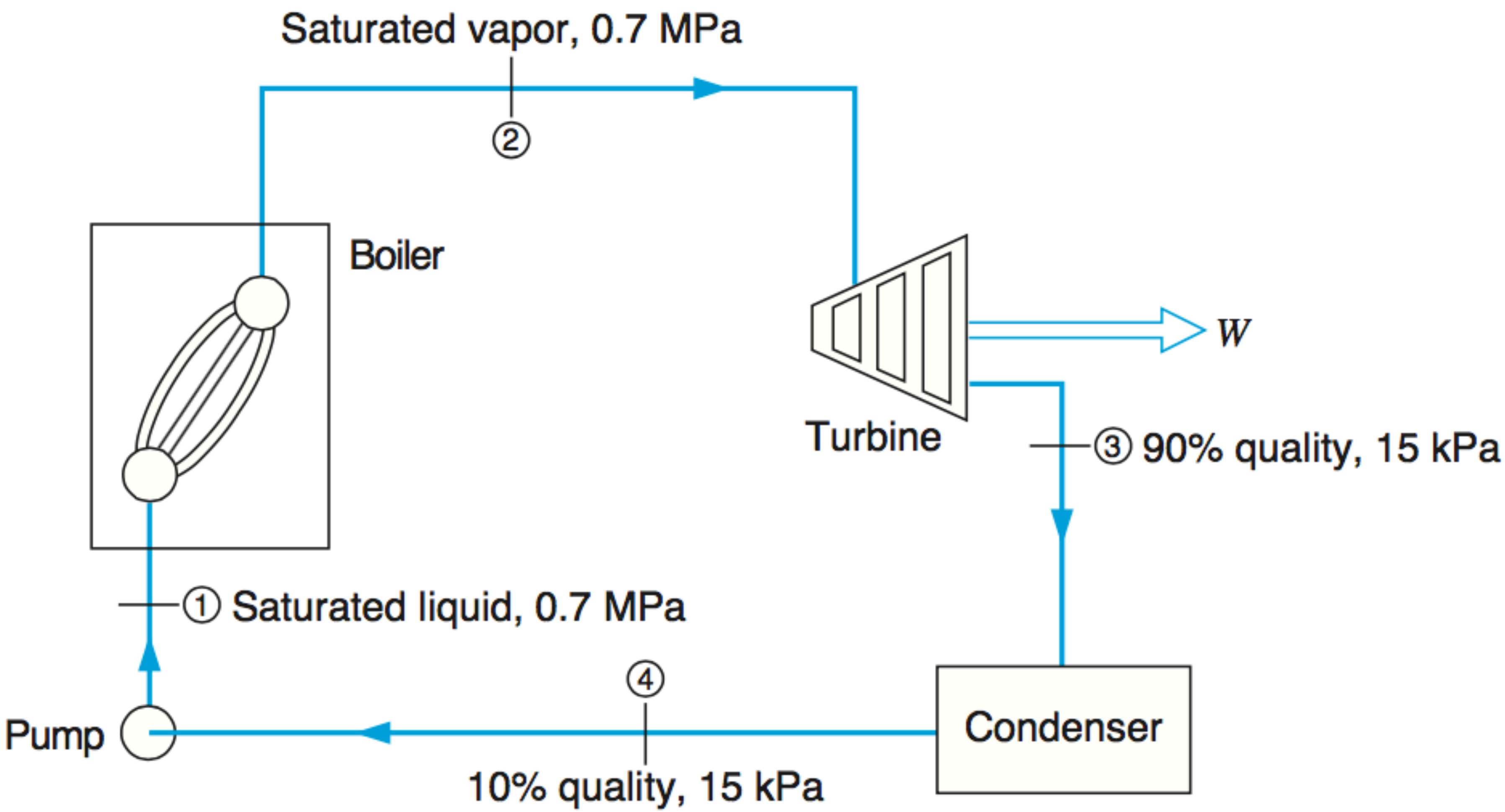
$$\oint \frac{\delta Q}{T} < 0$$

Thus, for all cycles we can write

$$\oint \frac{\delta Q}{T} \leq 0$$

Inequality of clausius

Does this process violate the second law of thermodynamics?



Does this cycle satisfy the inequality of Clausius?

Does this process violate the second law of thermodynamics?

$$\oint \frac{\delta Q}{T} = \int \left(\frac{\delta Q}{T}\right)_{\text{boiler}} + \int \left(\frac{\delta Q}{T}\right)_{\text{condenser}}$$

$$\oint \frac{\delta Q}{T} = \frac{1}{T_1}\int_1^2 \delta Q + \frac{1}{T_3}\int_3^4 \delta Q = \frac{{}_1Q_2}{T_1} + \frac{{}_3Q_4}{T_3}$$

Let us consider a 1 kg mass as the working fluid. We have then

$${}_1q_2 = h_2 - h_1 = 2066.3\ \text{kJ/kg}, \qquad T_1 = 164.97^{\circ}\text{C}$$

$${}_3q_4 = h_4 - h_3 = 463.4 - 2361.8 = -1898.4\ \text{kJ/kg}, \qquad T_3 = 53.97^{\circ}\text{C}$$

Therefore,

$$\oint \frac{\delta Q}{T} = \frac{2066.3}{164.97 + 273.15} - \frac{1898.4}{53.97 + 273.15} = -1.087\ \text{kJ/kg K}$$

Thus, this cycle satisfies the inequality of Clausius, which is equivalent to saying that it does not violate the second law of thermodynamics.

Next lecture

- Apply the second law of thermodynamics to processes.
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