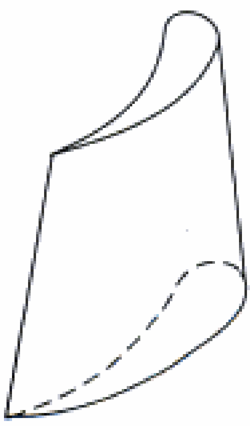
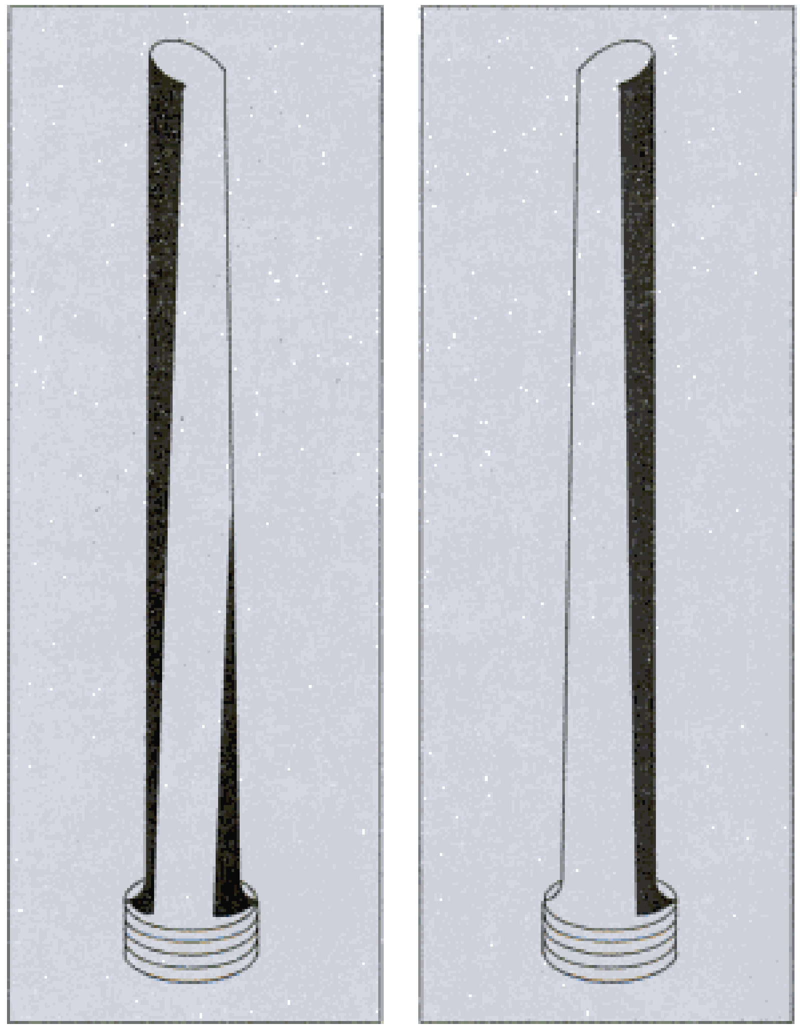


(a) Tapered blade



(b) Twisted blades

Fig. 7.40 *Tapered and twisted blades*

$$= \pi(D_m h_b)_{max} \sin \beta_1 \, k_{tb} \, V_{r1}$$

$$= \pi(D_m h_b)_{max} \, k_{tb} \, V_1 \sin \alpha \text{ [since } V_{r1} \sin \beta_1 = V_1 \sin \alpha] \qquad (7.91)$$

If impulse blading is used,

$$\frac{V_b}{V_1} = \frac{\cos \alpha}{2} \qquad \therefore V_1 = 700/\cos \alpha$$

From Eq. (7.91), for $\alpha = 20°$ and $k_{tb} = 0.9$ (assumed),

$$\text{Maximum volume flow} = \pi \times 2.23 \times 0.67 \times \frac{700}{\cos 20°} \sin 20° \times 0.9 = 1075 \text{ m}^3/\text{s}$$

The maximum mass flow of steam that the last stage can accommodate (Fig. 7.42).

$$(\omega_s)_{max} = \frac{1075}{v_2} \text{ kg/s}$$

For the turbine exhaust condition at 0.075 bar, 0.88 quality,

$$v_2 = 0.001 + 0.88 \times 19.24 = 16.93 \text{ m}^3/\text{kg}$$

$$\therefore \qquad (\omega_s)_{max} = \frac{1075}{16.93} = 63.5 \text{ kg/s}$$

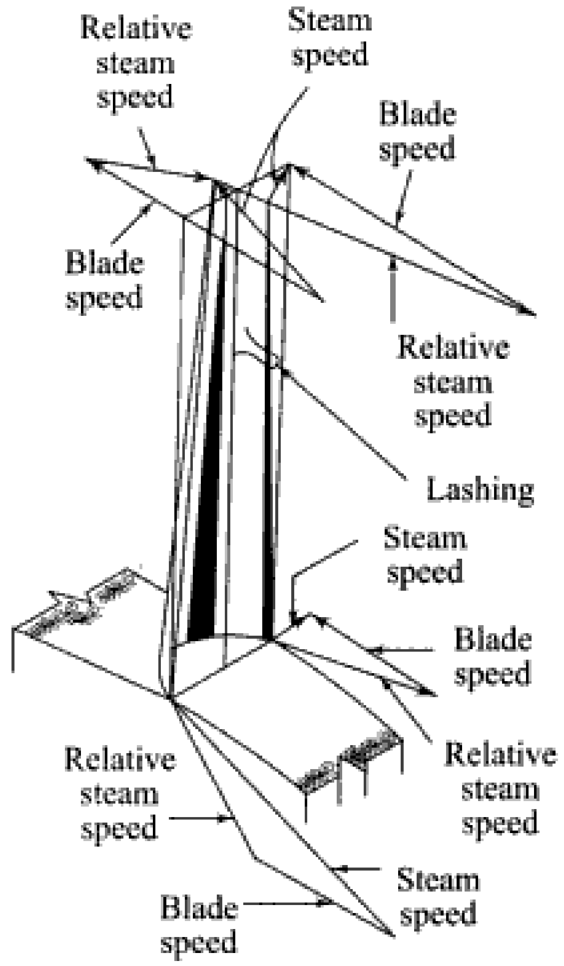


Fig. 7.41 *A twisted blade with impulse flow at root and reaction at tip*

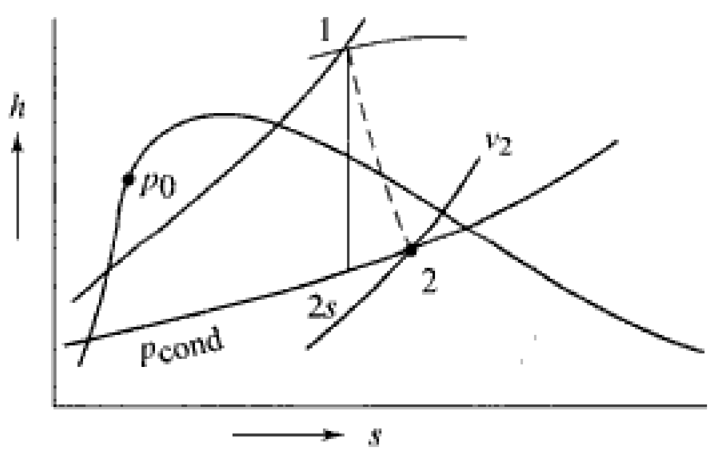


Fig. 7.42 *Specific volume at turbine exhaust*

The number of parallel exhausts or last stages required for a give steam flow rate of ω_s is

$$n = \frac{\omega_s}{(\omega_s)_{max}} = \frac{\omega_s}{63.5} \tag{7.92}$$

4. Casing arrangement If the number of parallel exhausts is estimated to be 4, i.e. $n = 4$, the casing arrangement of the turbine may be as shown in Fig. 7.43.

Steam first expands in the HP (high pressure) turbine (1–2), the exhaust from which is taken back to the steam generator for reheating (2–3). The reheated steam (3) then expands in the IP (intermediate pressure) turbine (3–4).

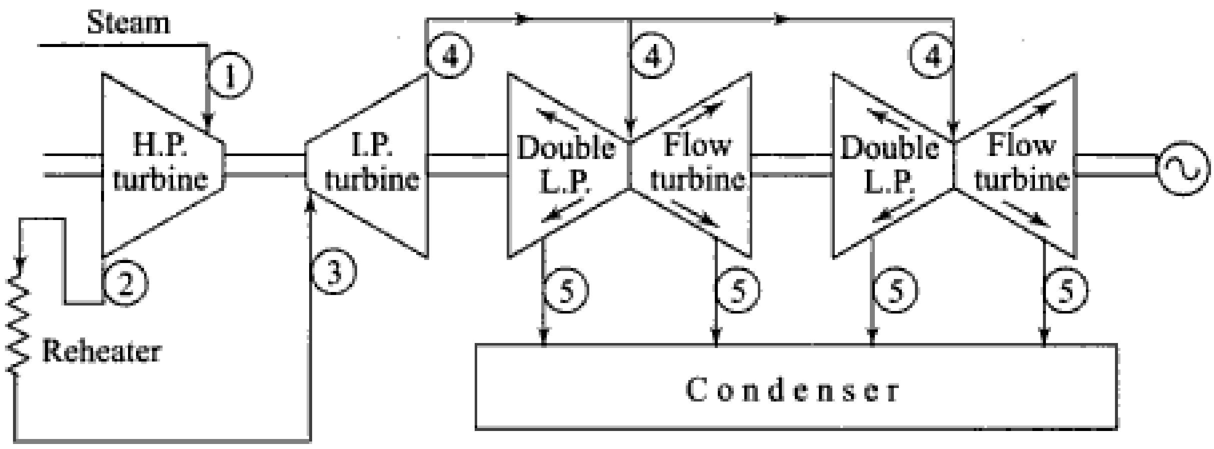


Fig. 7.43 *Casing arrangement of a steam turbine having four parallel exhausts*

The steam exiting the IP turbine (4) is split into four equal streams expanding in two DFLP (double flow low pressure) turbine (4–5). The four parallel exhaust streams from the LP turbines (5) then enter the condenser.

A double flow low pressure turbine (DFLPT) not only provides two parallel exhausts, but also helps the turbine contain the axial thrust. Equal and opposite axial thrusts operating in the two similar turbines of the DFLPT neutralize each other.

The HP and IP turbines are installed in the way as shown in order to reduce the axial thrust. Steam at states 1 and 3 is more or less at the same temperature. This will cause no thermal stress at H.P. and I.P turbine inlets. If the IP turbine is oriented in the opposite way, there will be a thermal gradient along the shaft, which would cause considerable thermal stresses.

The HP turbine outer casing is often made of double shell construction, with intermediate pressure steam filling the annular space so that the pressure difference across the casing wall is reduced.

To reduce the temperature gradient around and periphery and hence, the thermal stress at turbine entrance, steam is admitted at two or three feed points in both HP and IP turbines, instead of having single entry.

If $n = 3$, one DFLP turbine and one single flow LP turbine will be used to accommodate the required flow rate of steam (Fig. 7.44).

Figures 7.43 and 7.44 are called *tandem compounded* steam turbines with all turbine cylinders mounted on the same shaft. If the number of cylinders are large, or the cylinders are of heavy weights, then the cylinders may be mounted on two shafts, each coupled to a separate electric generator (Fig. 7.45). Such an arrangement of turbine cylinders on two shafts is called a *cross-compounded* steam turbine.

1. Internal losses, which are connected with the flow of steam.
2. External losses, which occur outside the turbine casing.

The internal losses may be enumerated as follows:

(a) Losses in regulating valves
(b) Nozzle friction losses
(c) Blade friction losses
(d) Disc friction losses
(e) Partial admission losses
(f) Gland leakage losses
(g) Residual velocity losses
(h) Carry-over losses

Table 7.1 *Turbine-Generator Configurations*

Fossil	*Fossil*	*Nuclear*
TC-2F LSB 26, 30 and 33.5 in Two casings 3600 r/min HI-IP LP G 125-400 MW	TC-6F LSB 26, 30 and 33.5 in Five casings 3600 r/min HP IP LP LP LP G 550-1000 MW	TC-4F LSB 38 and 43 in Three casings 1800 r/min HP LP LP G 450-1000 MW
TC-4F LSB 26, 30 and 33.5 in Three casings 3600 r/min HI-IP LP LP G 250-650 MW	TC-6F LSB 30 and 33.5 in Five casings 3600 r/min (double reheat) HP-IP IP LP LP LP G 450-725 MW	TC-6F LSB 38 and 43 in Four casings 1800 r/min HP LP LP LP G 600-1100 MW
TC-4F LSB 26, 30 and 33.5 in Four casings 3600 r/min HP IP LP LP G 550-850 MW LP LP G	CC-4F LSB 38 and 43 in Four casings 3600/1800 r/min HP IP G 3600 r/min LP LP G 1800 r/min 600 - 1250 MW	

Data provided by General Electric Company, TC = tandem compound, CC = Cross Compound, F = number of flow ducts to condenser, LSB = last-stage blade (1 in = 25.4 mm).

1. Losses in regulating valves Steam, before entering the turbine, passes through the main valve and the regulating valves, the flow through these being accompanied by pressure losses. Steam gets throttled adiabatically with constant enthalpy. However, the enthalpy drop in the turbine decreases yielding less specific output. Thus, some available energy of steam is lost due to the irreversible process of throttling. The pressure drop varies from 3 to 5% of the inlet steam pressure p_o.

2. Nozzle friction losses The friction losses in nozzles were mentioned earlier. The effect of friction is taken care of by the nozzle efficiency. Losses are due to the growth of boundary layer and the formation of eddies in the wake, apart from the frictional resistance of walls, which varies with the height and length of passage. Losses are higher in a turbulent boundary layer than in a laminar one. In reaction turbine where pressure or enthalpy drop per stage is less due to lower velocity, the laminar condition persists over a greater length of passage. So, the friction loss is less than the impulse stage. However, due to the large number of stages, the total surface area exposed to flow is more, which increases the friction loss. Thus, the nozzle loss depends on its size, surface roughness, its length, roundness of entrance, divergence angle, space between nozzles, moisture and trailing edge.

3. Blade friction losses Losses in moving blades are caused by various factors as enumerated below:

(a) *Impingement losses:* Steam issuing out from the nozzles meets the leading edges of the blades and energy may be lost if the entry is not smooth enough and eddies are formed.

(b) *Frictional losses:* Steam encounters these losses in the blade passages, which depends on the roughness of the blade surface.

(c) *Turning losses:* These occur as the steam turns in the blade passage.

(d) *Wake losses:* These occur at blade exit, depending on its shape and tip thickness.

The moving blade losses are taken care of by the blade friction coefficient ($k_b = V_{r2}/V_{r1}$) representing the reduction of relative velocity of steam from V_{r1} to V_{r2} due to friction.

4. Disc friction losses When the turbine disc rotates in the viscous steam, there is surface friction loss due to relative motion between the disc and steam particles. Due to centrifugal force, steam is thrown radially outward. The moving disc surface exerts a drag on the steam, sets it in motion from root to tip, and produces a definite circulation. Some part of the kinetic energy of steam is lost due to this friction.

5. Partial admission loss An impulse stage operating with partial admission, or an early stage in such a turbine with nozzles provided only over a part of the blade periphery, will have blades idle during part of the revolution. Some portion of kinetic energy of the incoming steam is spent in clearing away the steam existing within the blade passage. These are called "scavenging losses" which together with disc friction losses are often referred to as "windage losses" in which some kinetic energy is imparted to the fluid at the expense of the kinetic energy of the blades. Since reaction turbines are designed for full peripheral admission, the windage loss, as well as disc friction can be neglected.

6. Gland leakage losses Leakage of steam can occur between stages and along the shaft at inlet and exit ends of the casing. Diaphragm leakage takes place in both impulse and reaction stages through the radial clearance between the stationary nozzle diaphragm and the shaft or drum. Tip leakage occurs in reaction stages through the clearance between the outer periphery of the moving blades and the casing because of the pressure difference existing across the blades. Shaft leakage occurs through the radial clearance between the shaft and casing at bot high and low pressure ends of turbines. At the HP end, steam leaks out to the atmosphere, while at the LP end, the pressure being less than atmospheric, air leaks into the shell.

Since the leaked steam does not work on the blades, it represents energy loss. Both diaphragm and tip leakages can be minimized by reducing the radial clearances, but it must avoid rubbing or metal-to-metal contact. The clearance may be as low as 0.5 mm. However, proper balancing of the rotor, both static and dynamic, is a must to avoid any such rubbing. It is necessary to use seals or packing to further reduce the leakage flow. These seals may be labyrinths, carbon rings, water or steam seals, or gland leak-off. To prevent shaft leakage, labyrinths may be used with carbon rings and gland leak-off. Labyrinth seals consist of a series of thin strips fixed with the casing which maintain the smallest possible clearance with the shaft (Fig. 7.46). The small constrictions make the steam throttle to lower pressures many times, till only a very little quantity leaks out. Carbon ring seals, which consist of a ring of carbon divided into segments, have the rings fit snugly to the shaft by springs so as to prevent leakage, and may be used along with labyrinth glands in series in large turbines.

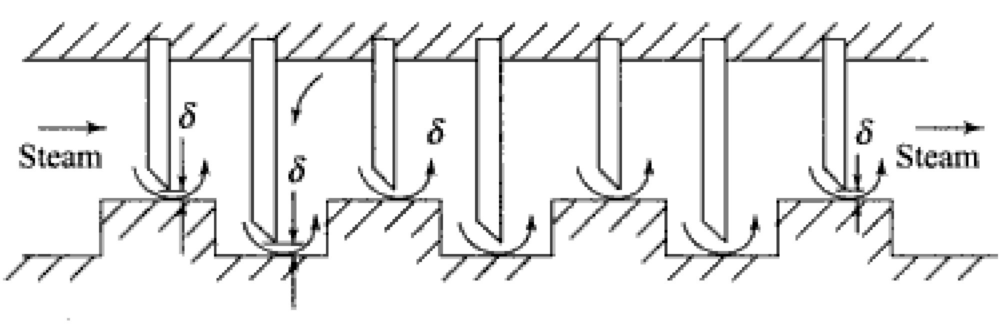


Fig 7.46 *Stepped labyrinth seals*

7. Residual velocity loss Steam leaving the last stage of the turbine has a certain velocity which represents an amount of kinetic energy that cannot be imparted to the turbine shaft and is thus wasted.

8. Carry-over losses Some energy loss takes place as steam flows from one stage to the next. The kinetic energy leaving one stage and available to the next is given by $\eta_{CO}(V_2^2/2)$, where η_{CO} is the carry-over efficiency.

In addition, there are some losses of energy due to wetness of steam (where the water particles are dragged along with steam at the expense of some K.E. of steam). If the quality of steam is less than 0.88, erosion and also corrosion can take place. Since the velocity of steam leaving the last stage of turbine is quite large (100–120 m/s), there will be energy losses due to friction in the exhaust hood of the turbine. Exhaust hoods to the condenser gradually increase in area like a diffusor and thus, there is a further decrease in velocity of steam and an increase in pressure as steam enters the condenser. Such hoods allow the turbine to operate down to a slightly lower pressure than that required by the condenser (depending on temperature and flow of cooling water, and air extraction from its shell), thus increasing the turbine work.

9. External losses There are some energy losses in the bearings and governing mechanisms which can be reduced by improving the lubrication systems. Some energy is consumed by oil pumps. Since the turbines are adequately insulated the surface heat loss by radiation and convection is small. Modern large electric generators are hydrogen cooled, well designed, and very efficient, where the energy losses are within 2 to 3 per cent.

7.3.6 Reheat Factor and Condition Line

Figure 7.47 shows the typical stage efficiencies of simple impulse, Curtis and reaction stages in contrast to Fig. 7.34 which demonstrates the variation of blading efficiency. As discussed in the earlier article, there are various losses in the stage and the portion of the available energy not converted to work and remaining in the fluid is termed as "reheat". A single-stage expansion with reheat is shown in Fig. 7.48.

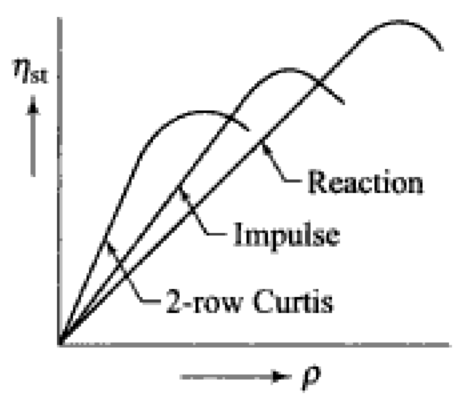


Fig. 7.47 *Typical stage efficiencies*

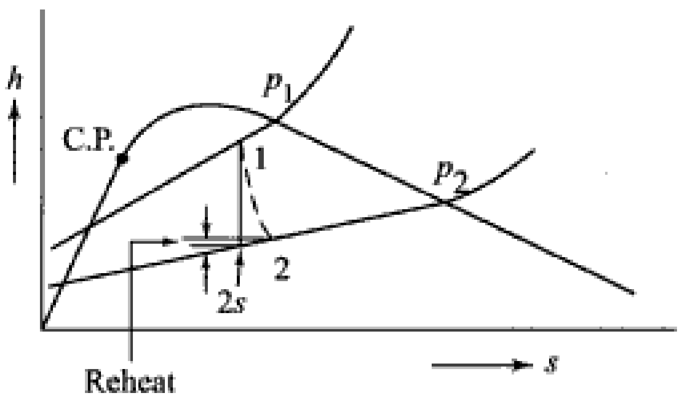


Fig. 7.48 *Expansion in a single stage with reheat*

$$\Delta h_s = h_1 - h_{2s} = \text{isentropic enthalpy drop}$$
$$\Delta h = h_1 - h_2 = \text{actual enthalpy drop}$$
$$\therefore \quad \text{Reheat} = h_2 - h_{2s}.$$

Figure 7.49 shows the expansion in a 4-stage turbine, taking into account the effects of reheat. The following conclusions are drawn:

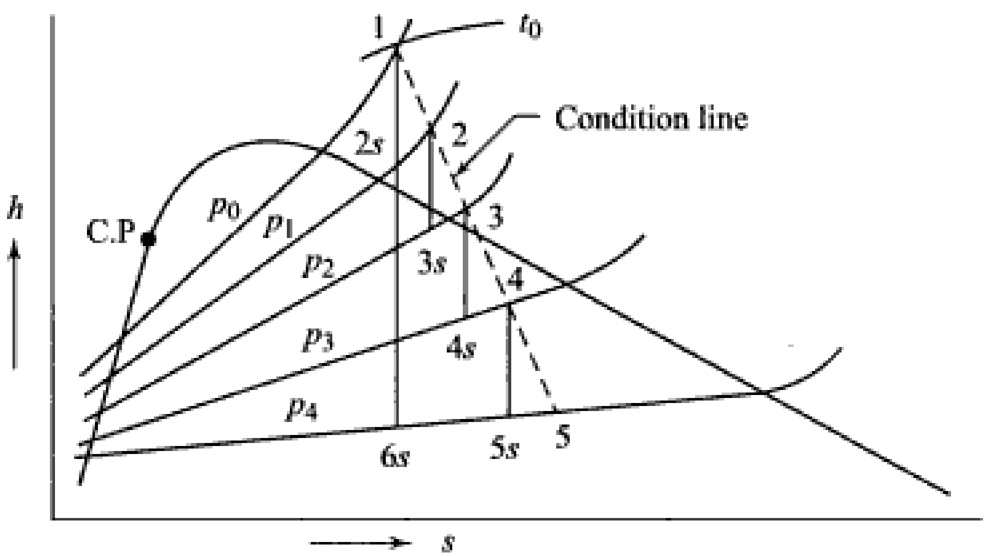


Fig. 7.49 *Condition line for a four-stage turbine*

1. Reheat takes place with increase in entropy.
2. The reheat in a given stage is available to do work in the succeeding stage except the last stage where the reheat is a loss.
3. The constant pressure lines diverge from one another, thereby increasing the enthalpy drop for the same pressure drop.
4. Because of (2) and (3) the sum of the available energies (isentropic enthalpy drops) for each stage is greater than the available energy (isentropic enthalpy drop) for the whole turbine.
5. The condition representing the actual expansion in the turbine is approximately the locus of points indicating the actual conditions of steam at the exit of each stage.

The effect of item 4 may be expressed, assuming equal available energy per stage, by a term called "*reheat factor*" (RF). The total isentropic enthalpy drop, $(h_1 - h_{6s})$, is divided equally into four parts in the four stages (Fig. 7.49) in the Mollier diagram, from which the interstage pressures p_1, p_2 and p_3 are noted, p_0 and p_4 being the boiler and condenser pressures, respectively. The isentropic enthalpy drop $(h_1 - h_{2s})$ in the first stage is multiplied by the stage efficiency to obtain the actual enthalpy drop $(h_1 - h_2)$, which is cut-off from 1 on the isobar at p_1 in the Mollier diagram. From 2, a vertical line is drawn which cuts the isobar at p_2 at state 3*s*. The isentropic enthalpy drop $(h_2 - h_{3s})$ is noted and the actual drop $(h_2 - h_3)$ is estimated by multiplying it with stage efficiency. In the same way, the enthalpy drops $(h_3 - h_{4s})$ and $(h_4 - h_{5s})$ are obtained from the Mollier diagram, and the state 5 is fixed. The states 2, 3, 4 and 5 record the actual conditions of steam at exit from each stage and the locus through these states is called the "*condition line*". The reheat factor is defined as

$$\text{RF} = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{h_1 - h_{6s}} \tag{7.93}$$

If the stage efficiency is assumed to be the same in all stages,

$$\eta_{st} = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{h_2 - h_3}{h_2 - h_{3s}} = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{h_4 - h_5}{h_4 - h_{5s}} \tag{7.94}$$

By substituting $h_1 - h_{2s} = (h_1 - h_2)/\eta_{st}$, $h_2 - h_{3s} = (h_2 - h_3)/\eta_{st}$ and so on in Eq. (7.93),

$$\text{RF} = \frac{\dfrac{h_1 - h_2}{\eta_{st}} + \dfrac{h_2 - h_3}{\eta_{st}} + \dfrac{h_3 - h_4}{\eta_{st}} + \dfrac{h_4 - h_5}{\eta_{st}}}{h_1 - h_{6s}}$$

or

$$\text{RF} = \frac{1}{\eta_{st}} \times \frac{h_1 - h_5}{h_1 - h_{6s}} = \frac{1}{\eta_{st}} \eta_{\text{internal}} \tag{7.95}$$

where the internal (or isentropic) efficiency of the turbine,

$$\eta_{\text{internal}} = \frac{h_1 - h_5}{h_1 - h_{6s}} \tag{7.96}$$

Thus, from Eq. (7.95)

$$\eta_{\text{internal}} = \text{R.F.} \times h_{\text{stage}} \tag{7.97}$$

Due to divergence of constant pressure lines, the reheat factor is greater than unity. The value of RF lies between 1.04 to 1.08. Therefore,

$$\eta_{\text{internal}} > \eta_{\text{stage}}$$

In other words, due to reheat, the expression $(h_2 - h_{2s}) + (h_3 - h_{3s}) + (h_4 - h_{4s}) + (h_5 - h_{5s})$ is greater than $(h_5 - h_{6s})$.

7.3.7 Design of Multi-Stage Turbines

The design of a steam turbine involves a judicious combination of theory with the results of experience, governed to a great extent by cost. The method of design outlined below is only illustrative of the theories discussed above. The following are specified to the designer: initial steam conditions, exhaust pressure, and the capacity in MW or kW. The turbine requires many stages which increase in diameter from inlet to the exit end. All wheels turn at the same speed (rpm), but V_b, V_1, k_{tb} k_{tn}, α, β, γ, leakage efficiency, disc friction and windage loss may all vary from stage to stage. The condition line, which is the logical starting point, can only be approximated until all the stage efficiencies are known.

The calculation for the casing arrangement of a multi-stage impulse turbine is made according to the Section 7.3.4.

The first stage is most often a two-row Curtis stage. In order to increase the height of the nozzles, the stage is usually given a partial admission. In large condensing steam turbines, where the specific volume at the end of expansion in the turbine becomes very large, long blades of special design are selected. The design of multi-stage turbines is usually started with initial design considerations of first, second and last stages, while the intermediate stages are designed later.

$$h_{b6} = h_n \times \frac{V_{1a}}{V_{4a}}$$

The ratio of axial components V_{1a}/V_{2a} and so on can be determined from the velocity diagrams. A constant blade diameter, D_m, has been assumed for the Curtis stage.

2. Second stage For impulse stages, the optimum velocity ratio

$$\rho_{opt} = \cos \alpha/2 = V_b/V_1$$

where α is the nozzle angle. For the actual stage, a value somewhat less than this optimum value may be assumed. If the average blade velocity is assumed, then the average nozzle exit velocity can be estimated.

Now $$V_1 = 44.72\,[\eta_n\,(\Delta h_s)_{stage}]^{1/2}$$

Assuming suitable value of η_n, the average drop of enthalpy per stage can be computed.

$$\text{Number of stages required} = \frac{(\Delta h_s)_{total}}{(\Delta h_s)_{stage}}$$

The absolute velocity of steam leaving the second row of moving blades (V_4) of the Curtis stage is known. From the energy balance across the nozzles of the second stage (impulse), the absolute velocity of steam (V_1) entering the blades can be estimated. By assuming suitable values of relevant parameters, the velocity diagrams can be plotted, from which η_{b1} can be estimated, and

$$\eta_{stage} = \eta_n \times \eta_{b1} \times (1 - h_{df})$$

can similarly be determined.

In the stage, $$\Delta h_{act} = \eta_{stage} \times (\Delta\, h_s)\text{stage}$$

Now, $$\omega_s = \frac{\pi D_m h_b \times k_{tb} V_1 \sin\alpha}{v_1}$$

Assuming a suitable value of (h_b/D_m), h_b and D_m can be estimated. The nozzle height will then be

$$h_n = h_b - 1.60 \text{ mm}$$

3. Last stage Assuming the maximum blade velocity consistent with the blade material (350–420 m/s), the last stage blade diameter is estimated. From stress consideration, the maximum height to diameter ratio for twisted or tapered blades can be assumed to be 0.3. If the blades are assumed to operate close to the maximum efficiency, the jet velocity of steam and hence, (Δh_s) can be determined. The velocity diagrams can be drawn from which η_{b_1} can be estimated.

Now, $$\omega_s = \frac{\pi D_m h_b \times k_{tb} V_1 \sin\alpha}{v_1}$$

from which both h_b and D_m are estimated.

The procedure has also been illustrated while discussing the casing arrangements earlier.

4. Intermediate stages The isentropic enthalpy drops that would take place in HP, IP and LP cylinders are known. Since the enthalpy drop per stage (Δh_s) is also known, the number of stages in each cylinder can be determined. The blade and nozzle dimensions can similarly be estimated.

By knowing the stage efficiencies of all the stages, the condition line can be drawn on the Mollier diagram from which the final condition (x, v) of steam can be noted. The turbine internal efficiency is then determined, and the steam flow rate (ω_s) is estimated. If this ω_s does not tally with the value used earlier, calculations have to be repeated.

7.3.8 Turbine Governing and Control

The function of a governor is to maintain the shaft speed constant as the load varies.

The simplest type of governor is the centrifugal flyball type (Fig. 7.50). The power available at the shaft is equal to $2\pi TN/60$, where T is torque and N is the rpm. As load (or torque) decreases, speed increases. Consequently, with the increase of centrifugal force, the flyballs fly apart and raise the sleeve which, operating through a lever and a fulcrum, actuates the main valve to close and reduce the mass flow of steam admitted to the turbine.

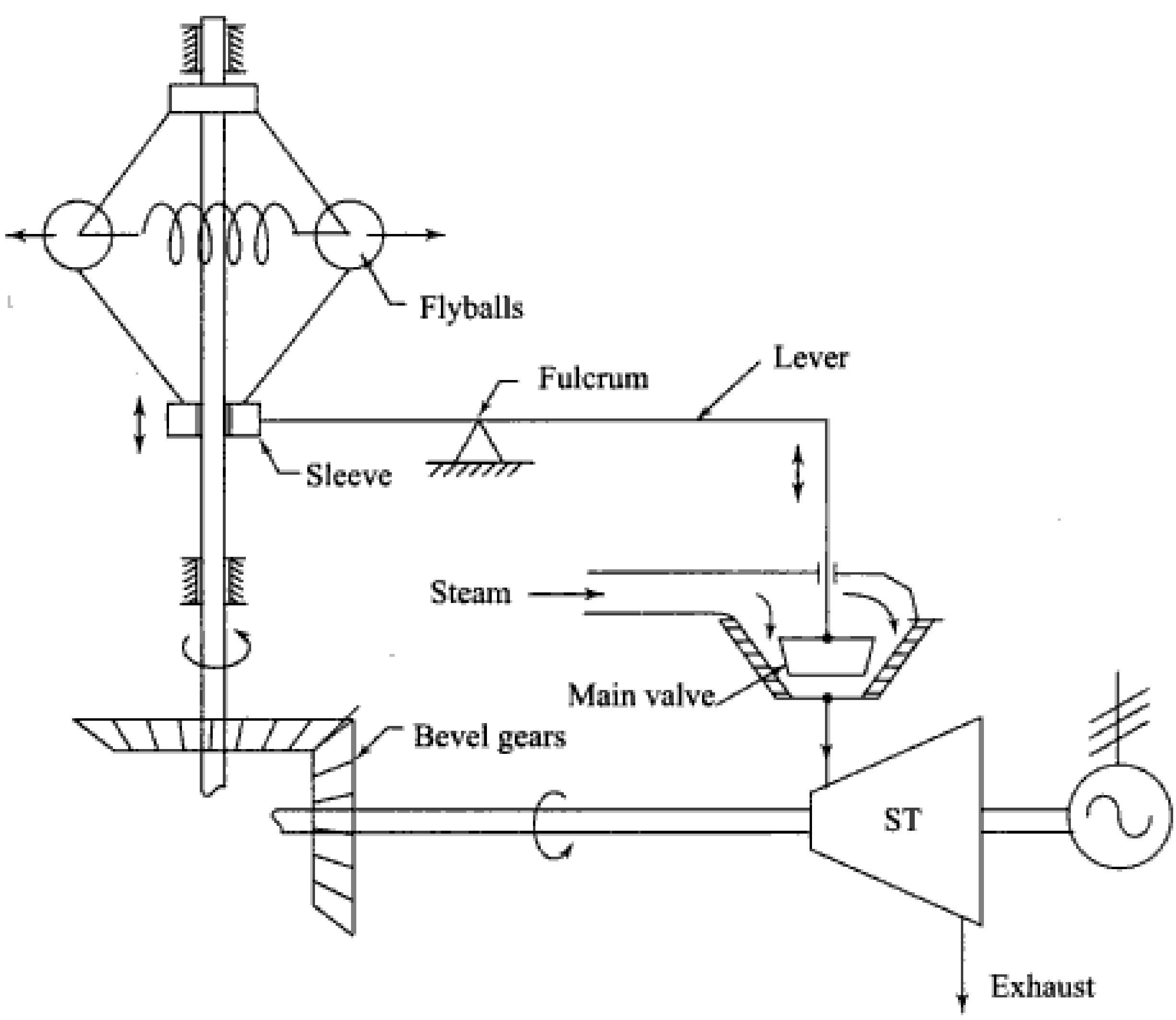


Fig. 7.50 *Centrifugal flyball governor*

An oil-operated servo system in addition may be used to enhance the sensitivity of the governor (Fig. 7.51). The governor force is amplified to move a light and almost frictionless pilot valve which controls the flow of high pressure oil to a piston. The piston powered by the oil can thus operate the governor valve as desired. The steady-state speed regulation R_s is given by

$$R_s = \frac{N_o - N}{N_r} \times 100 \tag{7.98}$$

where N_o = speed at no load,
N = speed at rated load, and
N_r = rated speed.

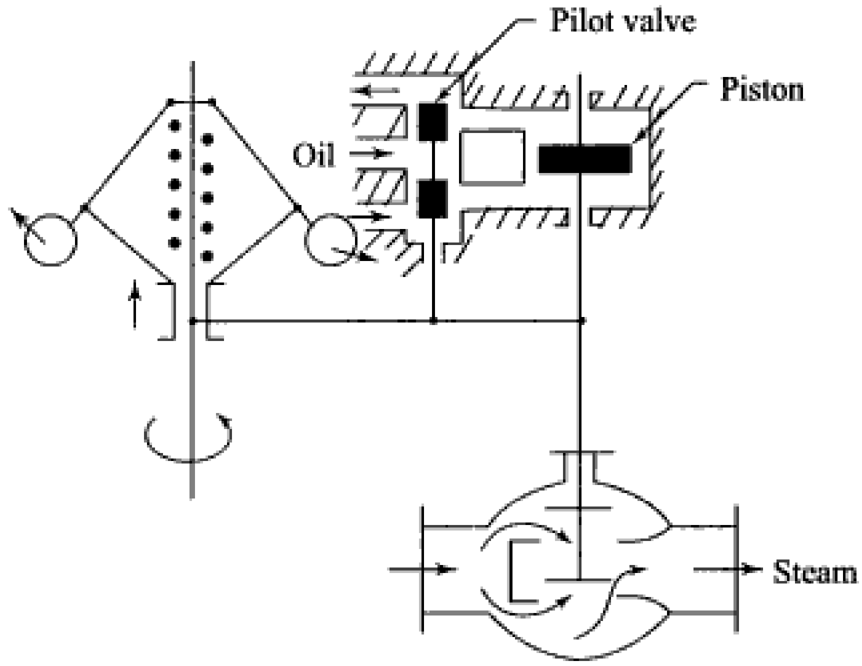


Fig. 7.51 *Centrifugal governor with hydraulic power amplifier*

Usually, the speed change from full load to no load is limited to 3–4% of the speed at rated load. If the rated speed is 3000 rpm, the speed at full load would be 3060 rpm and that at no load would be 2940 rpm. Therefore, in this case

$$R_s = \frac{3060 - 2940}{3000} \times 100 = 4\%$$

1. Throttle governing It was mentioned in the earlier Section that as the load decreases and shaft speed increases, the stop valve is partially closed to admit less steam to the turbine and to produce less power according to the demand. Due to restriction of passage in the valve, steam is throttled, say, from p_0 to p_{throttle} (Fig. 7.52). The specific ideal output of turbine thus reduces from $(h_1 - h_{2s})$ to $(h_3 - h_{4s})$. With further closure of the valve, p_{throttle} will still be less to produce a still lower output.

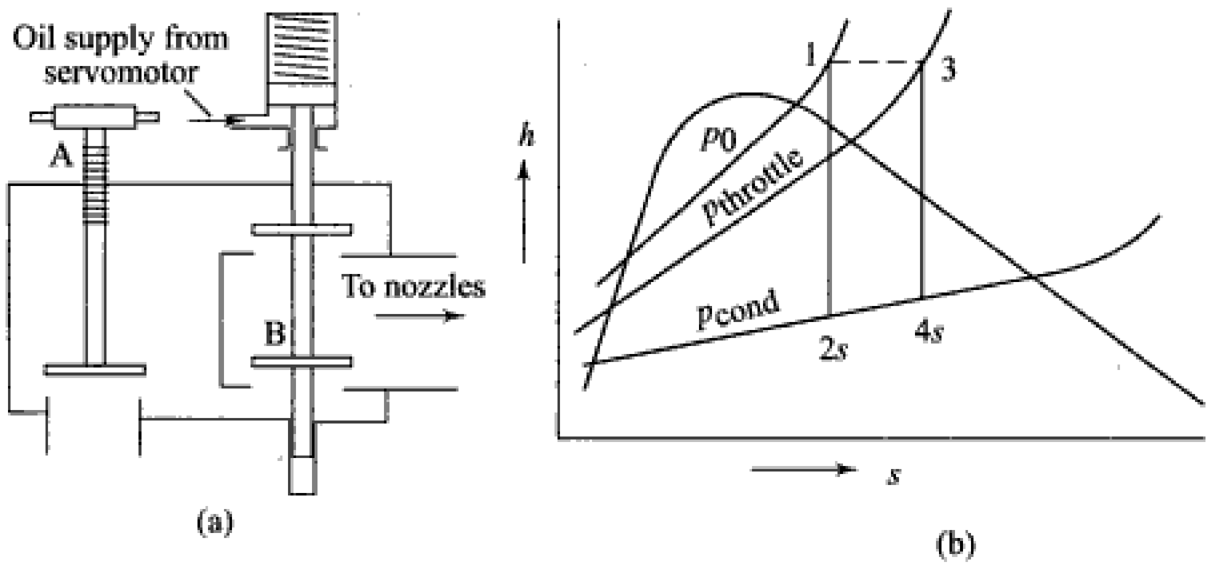


Fig. 7.52 *Throttling of steam*

The steam consumption plotted against the turbine load shows a linear relationship, which is called the *Willan's line* (Fig. 7.53), and given by

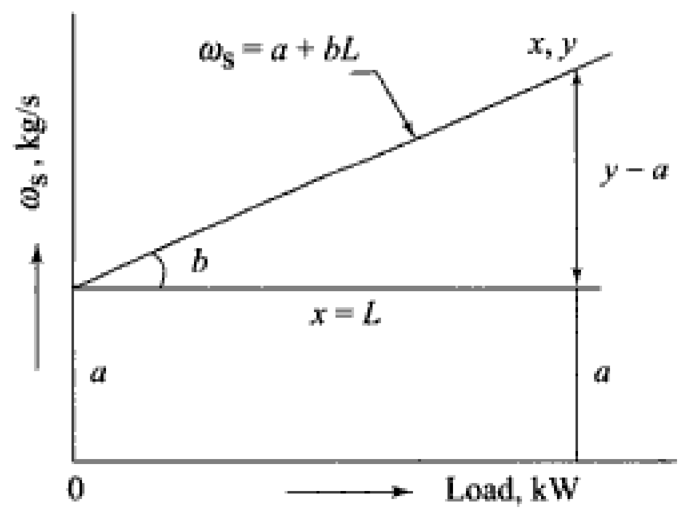


Fig. 7.53 *Willan's line*

$$\omega_s = a + bL \tag{7.99}$$

where a = no load steam consumption, kg/s;
b = steam rate (or specific steam consumption), kg/kWs; and
L = load, kW.

The throttle and stop valves are located in the steam supply line to the turbine. The stop valve is a hydraulically operated quick opening and shutting valve designed to be either fully open or shut. For small turbines, the stop valve may be manually operated. The throttle valve is used to regulate steam flow during starting or stopping.

2. *Nozzle governing* If throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control may then be a better method of governing. The nozzles are made up in sets, each set being controlled by a separate valve (Fig. 7.54). With the decrease of load, the required number of nozzles may be shut off.

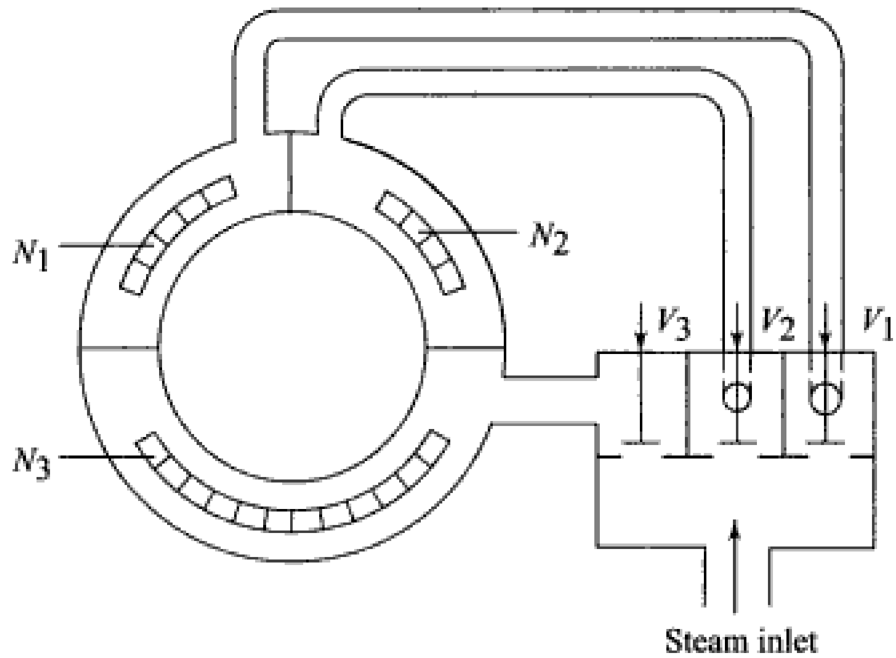


Fig. 7.54 *Nozzle governing*

3. By-pass governing To produce more power (when on overload), additional steam may be admitted through a by-pass valve to the later stages of the turbine (Fig. 7.55). By-pass regulation operates in a turbine which is throttle governed.

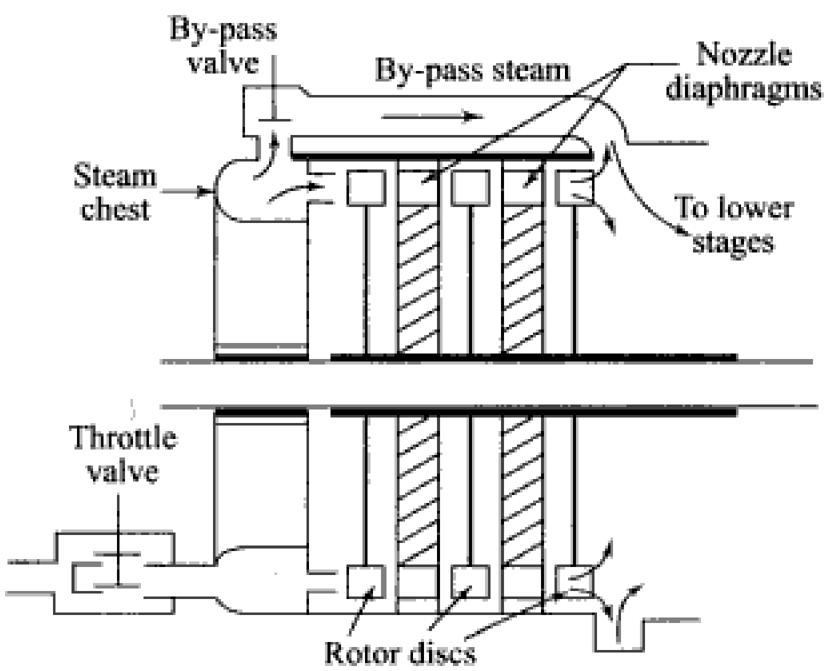


Fig. 7.55 *By-pass governing*

4. Emergency governor Every turbine is provided with some form of an emergency governor which trips the turbine (closes the stop valve and stops the steam supply) when

(a) shaft exceeds 110% of its rates value, i.e. 3300 rpm
(b) lubrication system fails
(c) balancing (static as well as dynamic) of turbine is not proper

(d) condenser becomes hot (due to inadequate cooling water circulation) or vacuum is less.

One common type of overspeed trip employs a pin or weight on the turbine shaft. Centrifugal force acting on the pin is opposed by a spring until about 10% overspeed is reached, whereupon, the centrifugal force overcomes the spring force and the pin flies out and strikes a trigger which, in turn, releases a spring to close the stop valve immediately.

7.3.9 Control and Supervisory Instruments

Certain control and supervisory instruments are provided for the safe and effective operation of a turbine given as follows.

1. *Pressure gauges* are provided to record the pressure of main steam at the stop valve, in the steam chest, at the first stage and the exhaust, the oil pressure to the bearings, the governor mechanism and the pressure of steam or water to the gland seals. For a condensing turbine, a vacuum gauge and a barometer are installed.
2. *Thermometers* are provided to record steam temperatures at the stop valve, in the steam chest, at the first stage, and at the gland. The oil temperatures entering and leaving the bearings are noted.
3. *A speed and cam-shaft position recorder* is required to record the turbine speed in rpm. During operation, the turbine speed is obtained from the generator frequency recorder. Thus, the speed recorder is used to record the cam-shaft position, which determines the opening of the valve and the load on the turbine.
4. *An eccentricity recorder* is provided to indicate and record the eccentricity of the shaft at the high pressure end of the turbine.
5. *A vibration amplitude recorder* is provided to record vibration of the rotor.
6. *An expansion indicator* is provided on the turbine control board to show the axial expansion of the turbine casing.
7. *A noise meter* on the control board is used to pick up and amplify the noise made by the moving parts of the turbine.
8. *Flow meters* are mounted on the turbine control board to indicate, record and integrate the mass rate of flow to the turbine, the steam bled at various points, and the flow to the condenser.
9. *Wattmeters, voltmeters* and *ammeters* are also provided on the turbine control board, which along with the flowmeters are used to determine the steam and heat rates of the unit.
10. Handwheels to operate the various drain valves are located at the turbine or on the turbine board.
11. Governor controls are located at the turbine or on the turbine control board for proper regulation of valves.
12. A trip lock lever for testing the overspeed trip is usually mounted on the turbine control board.

7.3.10 Dummy or Balance piston

In a pure impulse turbine the axial thrust on the blades is entirely due to the change in momentum of the steam in the axial direction and is usually very small. In the reaction turbine, however, there is a pressure drop across the blades of each ring, and so there can be a large force exerted on the blades in the axial direction. If it is not contained, the entire rotor may come out of the shaft and cause a serious accident.

As shown in Fig. 7.56 , the axial thrust produced is

$$T_1 = (p_1 - p_2)A - \omega_s(V_{a1} - V_{a2}) \tag{7.100}$$

where A is the annular area of the blade ring. Similarly, for all the rings of the rotor, the thrusts T_2, T_3, ... can be estimated, the total thrust will be the sum of all these thrusts.

Normally, the change in axial velocity components $(V_{a_1} - V_{a_2})$ in Eq. (7.100) is usually small and it is zero for 50% reaction blading. Therefore, the total thrust will be

$$T = \Sigma\,[(p_1 - p_2)\,A_1 + (p_2 - p_3)\,A_2 + (p_3 - p_4)\,A_3 + \dots] \tag{7.101}$$

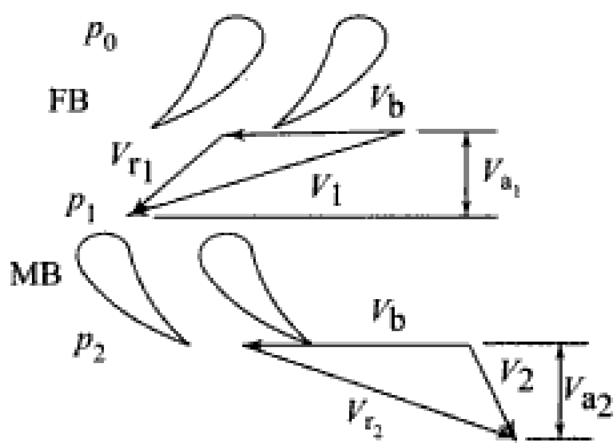


Fig. 7.56 *Axial thrust on blades of reaction turbines*

To balance this thrust, a few stages are provided on the other side of the cylinder (like a double-flow LP cylinder), which are designed in such a way that an equal and opposite thrust is produced. These stages which are used to balance the axial thrust constitute what is called a dummy or balance piston (Fig. 7.57).

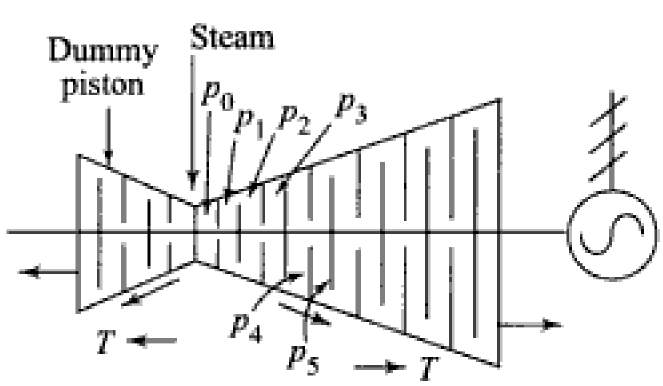


Fig. 7.57 *Dummy or balance piston for reaction turbine*

7.3.11 Blade Stresses

The determination of blade stresses is a critical factor in the design of blades. The severest stresses are imposed by the centrifugal forces due to high rotative speeds. Bending stresses are also imposed by centrifugal forces, fluid-pressure differences, and vibration.

Centrifugal stresses are a function of the mass of material in the blade, blade length and speed. The component of centrifugal force acting radially outward exerts a tensile stress at the root. Sufficient cross-sectional area must be provided in the blade at the root and a material capable of withstanding the stress without fatigue must also be provided.

The centrifugal force on an element dr at radius r (Fig. 7.58) is given by

$$dF = (\gamma a \, dr) \, w^2 r \tag{7.102}$$

where γ = specific weight of blade material, kg/m^3;

a = blade cross-sectional area, m^2; and

w = angular velocity, rad/s

Total centrifugal force exerted at the blade root is

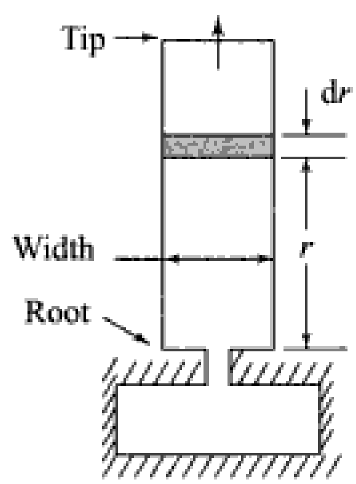


Fig. 7.58 *Centrifugal and bending forces on the turbine blade*

$$F_c = \int_{r_r}^{r_t} \gamma \, a \, dr \, \omega^2 r = \frac{\gamma a \omega^2}{2} \left(r_t^2 - r_r^2\right) \tag{7.103}$$

where r_t = tip radius, and r_r = root radius.

or $$F_c = \frac{\gamma a \omega^2}{2\pi} A = \frac{\gamma a A}{2\pi} \left(\frac{2\pi N}{60}\right)^2 \tag{7.104}$$

where A = annular area = $\pi \left(r_t^2 - r_r^2\right)$

The centrifugal or the tensile stress at the blade root is thus

$$S_c = F_c/a = \gamma A \left(\frac{N}{23.94}\right)^2 \tag{7.105}$$

If the blade is tapered, the mass of material is reduced, thereby reducing the centrifugal stress. Since the stress exerted at any section of the blading decreases radially, reaching a minimum near the tip, a constant cross-sectional area is not required for strength. Hence, where the centrifugal stresses are severe the blade is tapered by decreasing both its thickness and width.

Impulse blades are subject to bending from centrifugal stress and the tangential force exerted by the fluid. Reaction blades have an additional bending stress due to large axial thrust because of the pressure drop which occurs in the blades. All turbine blades may be subjected to bending because of vibration. The total stress at a given point on a turbine blade may be found by adding the centrifugal stress at that point to the bending stress.

7.3.12 Blade Fastenings

There are a number of methods for fixing turbine blades to the disc or drum. The selected type of fastening must be able to resist the centrifugal and bending forces to which the blades may be subjected. Blades which are loosely fastened to the disc or drum will amplify any vibrations induced in the blades, causing fatigue failure. The failure of one blade may lead to the destruction of the entire turbine.

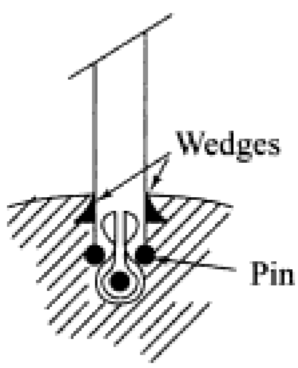


Fig. 7.59 *Bulb and shank fastening*

Figure 7.59 shows the bulb and shank type of fastening suitable for low pressure impulse steam turbines. Figure 7.60 illustrates a type of fastening known as straddle-T which is widely used on low pressure impulse turbines. Its modification, simply a straddle fastening (Fig. 7.61) is used for the long low-pressure blades of large steam turbines. The straddle fastening is slipped radially into the slot provided on the disc or drum at a point widened for this purpose. The blades are then pushed along in the slot until the last blade is inserted, whereupon a special stop piece is put in place.

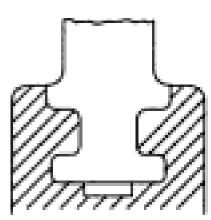


Fig. 7.60 *T-shaped attachment*

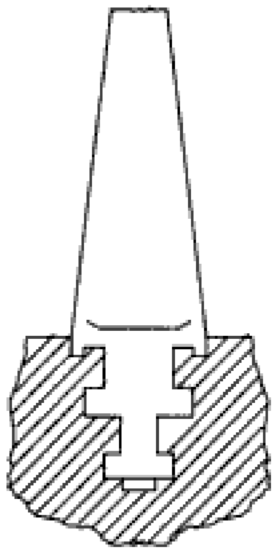


Fig. 7.61 *Straddle fastening*

A T-shaped attachment is used in the high pressure impulse stages of large turbines (Fig. 7.62). Figure 7.63 shows a fir-tree or Christmas tree attachment which is inserted on the disc axially.

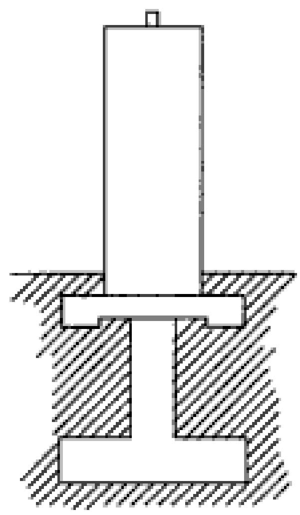


Fig. 7.62 *Stradle-T fastening*

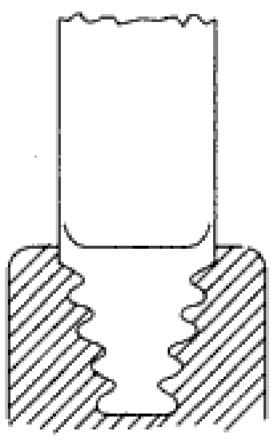


Fig. 7.63 *Fir-tree attachment*

7.3.15 Critical Speeds

Inspite of all the care taken in the construction and balancing of the turbine shaft and discs, due to some reason or other, the mass centre of the rotor does not coincide with the geometrical axis of the shaft and the distance between the two is known as eccentricity. During the shaft rotation even a small eccentricity gives rise to a transverse force that increases with shaft rpm and tends to deflect the shaft.

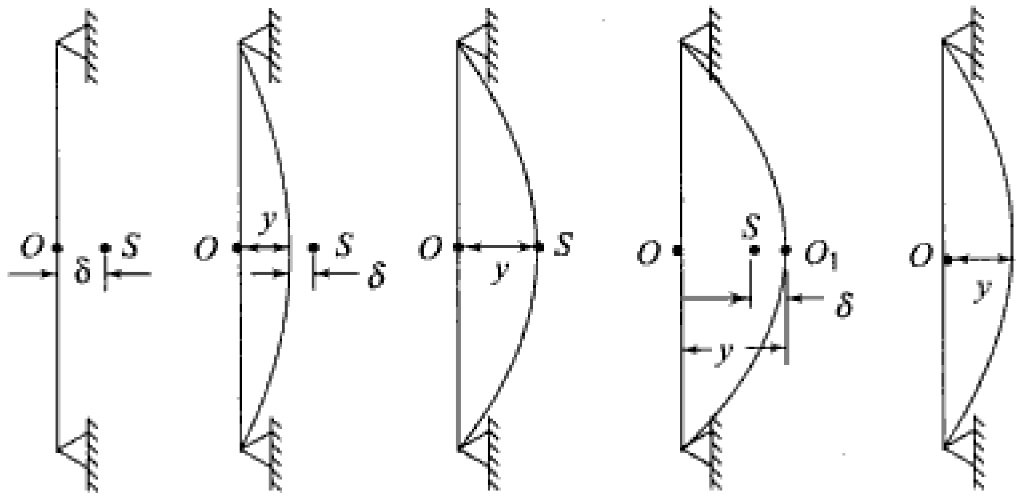


Fig. 7.66 *Position of centre of gravity of shaft depending on its rpm*

Let us consider a slightly out of balance shaft with an eccentricity δ (Fig. 7.66). The mass centre of the shaft is at a distance δ from the geometrical axis of the shaft. We shall consider the shaft to be supported in the vertical direction to avoid the shaft deflection due to its own weight.

The centrifugal force due to rotation is given by

$$F_c = m\omega^2(y + \delta) \qquad (7.106)$$

where y = deflection of rotor, mm;
m = mass of rotor, kg; and
ω = angular velocity, rad/s.

If F is the stiffness of shaft, i.e. the force that causes the shaft to deflect by 1 mm, then by force balance.

$$F_c = Fy \qquad (7.107)$$

$$\therefore \quad Fy = m\omega^2(y + \delta) \quad \text{or} \quad y = \frac{\delta}{F/(m\omega^2) - 1} \qquad (7.108)$$

From Eq. (7.108) it follows that each value of ω conforms to a definite deflection y, e.g., at $F/m\omega^2 = 1$, $y = \infty$. The angular velocity of the shaft at $m\omega^2 = F$, when $y = \infty$, is known as the *critical velocity*.

$$\therefore \qquad \omega_{cr} = \left[\frac{F}{m}\right]^{1/2} \tag{7.109}$$

Again,
$$\omega_{cr} = \frac{2\pi N_{cr}}{60} = \left[\frac{F}{m}\right]^{1/2}$$

or
$$N_{cr} = 9.55\left[\frac{F}{m}\right]^{1/2} \tag{7.110}$$

where m is the mass of the rotor in kg.

The rpm which numerically coincides with the natural frequency of transverse vibrations of the shaft is known as *critical speed*. Theoretically, at the critical speed the deflection of the shaft tends to infinity. Thus, operation of turbines at the critical speeds is to be avoided. The normal speed of rotation must not coincide with the critical speed of the shaft, i.e. with its natural frequency. For safe working of the turbine the critical speed should not differ from the normal speed by more than 20%.

Turbine shafts having critical speeds less than normal operating speed are known as *flexible shafts*, and those with critical speeds higher than the normal operating speed are known as *rigid shafts*. Rotors of impulse turbines are of both these types, flexible and rigid. If the critical speed is less than the operating speed, then while starting the turbine this speed must be passed over quickly so that there will be hardly any time for the deflection to grow, which, if allowed, could result in a bent shaft and damaged bearings.

At critical speed, the centre of gravity S coincides with the geometrical axis of the bent shaft (Fig. 7.66c). It is found from theoretical as well as practical considerations that when $N > N_{cr}$, the c.g. S would be between the vertical axis and the bent shaft (Fig. 7.66d). In this case, the equilibrium conditions for the forces will be expressed as

$$m\omega^2 (y - \delta) - Fy = 0$$

$$y = \frac{\delta}{1 - \dfrac{F}{m\omega^2}} \tag{7.111}$$

From Eq. (7.109), $\omega_{cr}^2 = \dfrac{F}{m}$

$$\therefore \qquad y = \frac{\delta}{1 - \left(\omega_{cr}^2/\omega^2\right)} \tag{7.112}$$

From this equation it is seen that with increase in ω, y has a value less than what it has at the critical speed, i.e., as ω increases, y decreases. When $\omega = \omega_{cr}$, $y = \infty$ and when $\omega = \infty$, $y = \delta$. Therefore, at infinite speed of rotation, the c.g. of the shaft S (Fig. 7.66e) coincides with the axis of rotation.

It is also seen from Eqs (7.108) and (7.112) that the deflection of the shaft y is a function of eccentricity δ and hence, while balancing the rotor it is advisable to reduce the eccentricity δ to the minimum possible.

If the shaft is in horizontal supports (Fig. 7.67), even under static conditions there will be some amount of deflection Δ caused by the weight of the shaft and the discs mounted on it. Thus, the shaft will always be slightly bent. Consequently, while in rotation there will be an additional amount of deflection y and the shaft begins to vibrate relative to static geometric axis (Fig. 7.67). The static deflection is given from

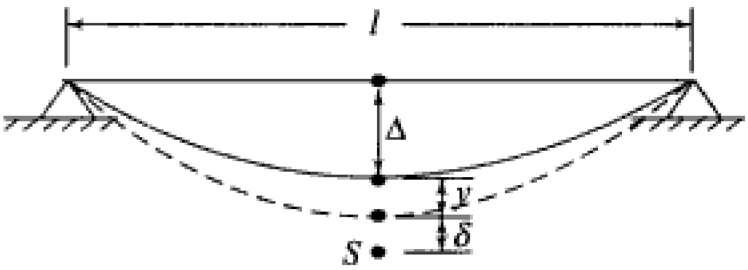


Fig. 7.67 *Position of c.g. of shaft in horizontal supports*

$$F \cdot \Delta = W = \text{weight of the rotor or shaft}$$

where F is the stiffness of the shaft (N/mm deflection)

$$\therefore \qquad \Delta = \frac{W}{F} \tag{7.113}$$

The static deflection depends on the stiffness of the shaft, the distance between the two supports and the load distribution. For simply supported shafts loaded at the centre, the deflection will be

$$\Delta = \frac{Wl^3}{48\,EI}$$

and for shafts with fixed ends and loaded at the centre

$$\Delta = \frac{Wl^3}{192\,EI}$$

where l = distance between the two supports
E = modulus of elasticity of the material of the shaft
I = moment of inertia of the shaft section, $\pi d^4/64$
d = shaft diameter.

We may rewrite the expression for shaft deflection as

$$\Delta = c\frac{Wl^3}{EI} \tag{7.114}$$

where c is a coefficient depending upon the type of support and the point at which the load is applied (in the cases referred above, c = 1/48 and 1/192).

Having determined the deflection Δ the critical speed can be easily determined from Eqs (7.110) and (7.113).

$$N_{cr} = \frac{9.55}{(\Delta)^{1/2}} \tag{7.115}$$

The diameter of the shaft is determined from the critical speed considerations and then checked for mechanical strength. From Eqs (7.114) and (7.115).

$$\Delta = \left(\frac{9.55}{N_{cr}}\right)^2 = c\frac{Wl^3}{EI}$$

$$I = \frac{cWl^3}{E}\left(\frac{N_{cr}}{9.55}\right)^2 = \frac{\pi}{64}d^4$$

$$\therefore \qquad d = \left[\frac{64\,cWl^3}{\pi E}\left(\frac{N_{cr}}{9.55}\right)^2\right]^{1/4} \tag{7.116}$$

The critical speed may be assumed for the calculation of shaft diameter. The coefficient c is obtained from the equations of strength of materials for various types of loading.

1. Other critical speeds At speeds higher than the first critical speed as shown in Fig. 7.68(a), Eq. (7.115), the shaft settles down until the second critical speed is approached when it commences to bend in the curve shown in Fig. 7.68(b) with a nodal point at the centre of its length. Half the length of the shaft is now under the same conditions as the whole shaft when passing through its first critical speed (Fig. 7.68a).

Since deflection $y \propto l^4$ and $N_{cr} \propto 1/\sqrt{y}$

$$\therefore \qquad N_{cr} \propto 1/l^2$$

For second critical speed, the length of shaft is $l/2$.

$$\therefore \qquad N_{cr2} : N_{cr1} = 4 : 1$$

Similarly, for the third critical speed, the shaft length is $l/3$.

$$\therefore \qquad N_{cr3} : N_{cr1} = 9 : 1$$

Hence,

$$N_{cr1} : N_{cr2} : N_{cr3} : N_{cr4} = 1 : 2^2 : 3^2 : 4^2 = 1 : 4 : 9 : 16$$

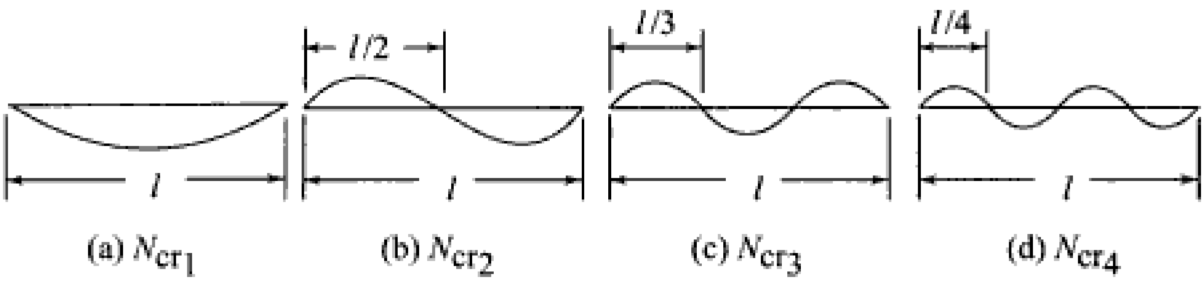


Fig. 7.68 *Form of whirling shaft at second, third and fourth critical speeds*

For further details of critical speed, the book by Kearton (1958) may be consulted.

7.3.16 Turning Gear

During shut-down or when the turbine is tripped, there is no steam supply to the turbine. If the turbine which was rotating at 3000 rpm is suddenly stopped, due to the inertia of motion, the rotor may get bent and distorted. There will also be thermal stresses developed due to non-uniform cooling. The reverse happens when the turbine is started. The turbine requires to be heated slowly and its rated speed is reached gradually in several steps.

The turning gear or the barring gear is a mechanism which keeps the turbine shaft rotating at about 1 to 20 rpm to avoid springing the shaft because of unequal expansions and contractions when warming or cooling the turbine. It consists of a gear integral with the turbine shaft which is driven by an electric motor through the necessary speed reduction equipment (Fig. 7.69).

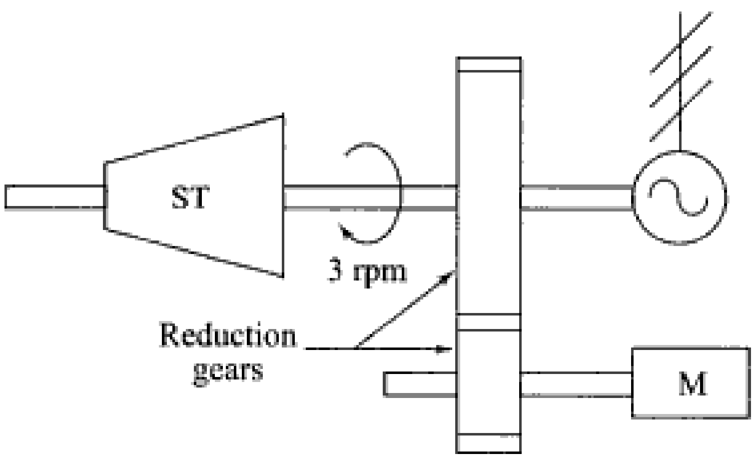


Fig. 7.69 *Turning gear operation during start-up and shut-down*

During shut-down or when tripped, the turbine is put on turning gear automatically and it keeps on rotating for about two days to cool down gradually and absorb the inertia of motion. Similarly, while starting the turbine, it is put on turning gear first to keep it in rotation at low rpm and then steam is admitted slowly by opening the stop valve till the rated speed is reached. When the first critical speed is approached, steam is admitted quickly to avoid this speed.

7.4 ELECTRICAL ENERGY GENERATION

The rotational mechanical energy of the turbine is converted to electrical energy in the generator by the rotation of the rotor's magnetic field.

The rotation of the turbine turns the rotor of the generator, producing electrical energy in the stator of the generator. The generator rotor consists of a steel forging with slots for conductors that are called the field windings. An electrical direct current is passed through the windings, causing a magnetic field to be formed in the roror, as shown in Fig. 7.70. This magnetic field is rotated by the turbine. The rotor is surrounded by the stator that includes copper

The power factor of a generator is defined as the ratio of kW to kVA and is the cosine of the angle describing the angular difference between phase current and phase voltage.

Heat is produced in a generator as a result of resistive losses caused by current flow in the stator and field windings, stator core magnetic losses and windage losses. This heating effect which depends on the load and power factor, is the limiting factor in generator rating. By providing forced cooling of the rotating and stationary components, the generator rating may be increased and the physical size of the components may be made smaller.

Large generators are cooled with hydrogen. The thermal properties of hydrogen (like specific heat and thermal conductivity) are superior to those of air and allow for reduced windage and better cooling. Windage and ventilating losses are lower because of the low density of hydrogen.

The generator losses may be reduced by using hydrogen at higher pressures, say 2 bar. The specific heat of hydrogen is the highest, since its molecular weight is the least,

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Since hydrogen is a diatomic gas, $\gamma = 1.4$. The characteristic gas constant

$$R = \frac{\text{universal gas constant}}{\text{molecular weight}} = \frac{8.3143}{2} = 4.157 \text{ kJ/kg K}$$

$$\therefore \qquad (c_p)_{H_2} = \frac{1.4 \times 4.157}{0.4} = 14.55 \text{ kJ/kg K}$$

It is 3.47 times the c_p of water.
The rate of heat removal from the armature = $\omega_g \, c_p \, (t_e - t_i)$ kW
where ω_g is the mass flow rate of hydrogen, and t_e and t_i are the exit and inlet temperature of hydrogen, respectively. The warm exiting hydrogen is cooled by water in a heat exchanger.

Since hydrogen gas explodes if it comes in contact with air, CO_2 gas, being heavier, is first used to purge the armature, i.e., drive away the air. When all the air has been removed, then only hydrogen gas is fed into the armature to take away the heat generated (I^2R).

Example 7.1 Steam is expanded in a set of nozzles from 10 bar. 300 °C to 2 bar. Are the nozzles convergent or convergent–divergent? Neglecting the initial velocity, find the minimum area of the nozzles to flow 1 kg/s of steam. Assume isentropic expansion.

Solution

$$p_0 = 10 \text{ bar}, \; t_0 = 300°\text{C}$$

Since the state of steam at nozzle inlet is in the superheated state, so the critical pressure $p^* = 0.546 \times 10 = 5.46$ bar.

$$h_{3s} = 2605 \text{ kJ/kg}$$
$$V_1 = 44.72\,[(h_1 - h_{2s}) + 0.85(h_{2s} - h_{3s})]^{1/2}$$

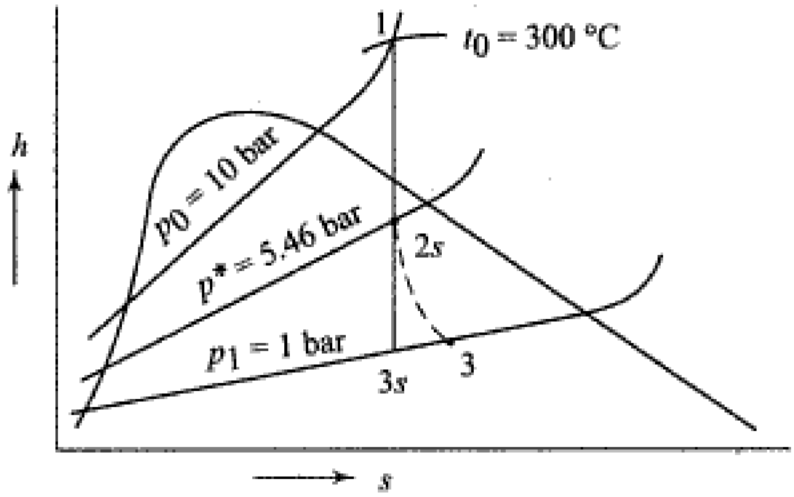


Fig. E7.2

$$= 44.72\,[136 + 0.85(311)]^{1/2} = 894.79 \text{ m/s}$$
$$s_1 = s_{3s} = 7.1276 \text{ kJ/kg K} = 1.3025 + x_{3s} \times 6.0579$$
$$x_{3s} = 0.962$$
$$h_{3s} = 417.46 + 0.962 \times 2258.01 = 2589.67 \text{ kJ/kg}$$
$$h_{2s} - h_3 = 0.85(2916.2 - 2589.67) = 277.53 \text{ kJ/kg}$$
$$h_3 = 2638.67 \text{ kJ/kg} = 417.46 + x_3\, 2258.01$$
$$x_3 = 0.984$$
$$v_3 = 0.001043 + 0.984 \times 1.694 = 1.688 \text{ m}^3\text{/kg}$$

By interpolation (at 5.46 bar, 226 °C),

$$v^* = v_{2s} = 0.416 \text{ m}^3\text{/kg}$$
$$w_s = 1 \text{ kg/s} = (A^* V^*)/v^* = (A^* \times 521.52)/0.416$$

$\therefore$ $A^* = 7.98 \text{ cm}^2$ (minimum area) *Ans.*

Also, $$1 \text{ kg/s} = A_1 V_1 / v_3 = \frac{A_1 \times 894.79}{1.688}$$

$\therefore$ $$A_1 = 18.64 \text{ cm}^2 = n \times \frac{\pi}{4}(2.5)^2$$

$n = 3.797 = 4$ *Ans.*

Example 7.3 Air at 7.8 bar and 180 °C expands through a convergent– divergent nozzle into a space at 1.03 bar. The flow rate of air is 3.6 kg/s. Assuming isentropic flow throughout and neglecting the inlet velocity, calculate the throat and exit areas of the nozzles.

Solution The critical pressure ratio (Fig. E7.3) for air ($\gamma = 1.4$) is

$$\frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)} = \left(\frac{2}{2.4}\right)^{1.4/0.4} = 0.528$$

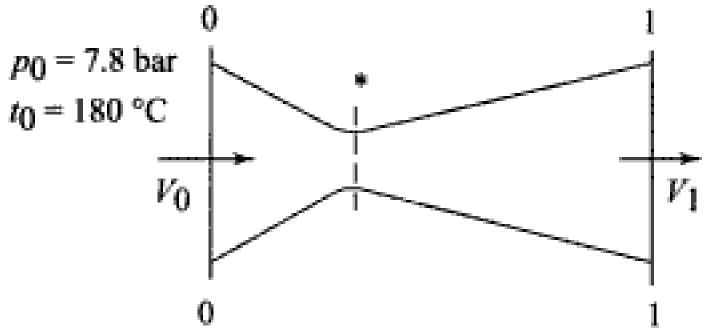


Fig. E7.3

$\therefore$ $$p^* = 0.528 \times 7.8 = 4.12 \text{ bar}$$

Now, $$\frac{T^*}{T_0} = \frac{2}{\gamma+1} = \frac{1}{1.2}$$

$\therefore$ $$T^* = \frac{180+273}{1.2} = 377.5 \text{ K}$$

$\therefore$ $$v^* = \frac{RT^*}{p^*} = \frac{287 \times 377.5}{4.12 \times 10^5} = 0.263 \text{ m}^3/\text{kg}$$

$\therefore$ $$V^* = (\gamma RT^*)^{1/2} = (1.4 \times 287 \times 377.5)^{1/2} = 389.46 \text{ m/s}$$

[Also, $$V^* = 44.72\,(h_0 - h^*)^{1/2} = 44.72 \times [c_p\,(T_0 - T^*)]^{1/2}$$
$$= 44.72 \times [1.005\,(75.5)]^{1/2} = 390.5 \text{ m/s}]$$

$$A^* = \frac{wv^*}{V^*} = \frac{3.6 \times 0.263}{389.46} = 0.002431 \text{ m}^2$$

$\therefore$ Area of throat = 2431 mm^2 *Ans.*

Now,

$$\frac{T_0}{T_1} = (p_0/p_1)^{(\gamma-1)/\gamma} = \left(\frac{7.8}{1.03}\right)^{0.4/1.4} = 1.784$$

$$T_1 = \frac{453}{1.784} = 253.9 \text{ K}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{287 \times 253.9}{1.03 \times 10^5} = 0.7075 \text{ m}^3/\text{kg}$$

$$V_1 = 44.72 \times [1.005\,(453 - 253.9)]^{1/2} = 634.17 \text{ m/s}$$

$\therefore$ $$A_1 = \frac{wv_1}{V_1} = \frac{3.6 \times 0.7075}{634.17} = 0.004016 \text{ m}^3$$

$\therefore$ Exit area = 4016 mm^2 *Ans.*

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$$T_{1s} = \frac{723}{1.3962} = 517.84 \text{ K}$$

$$h_n = \frac{(T_0 - T_1)}{(T_0 - T_{1s})} = \frac{723 - T_1}{723 - 517.84} = 0.93$$

$$T_1 = 532.2 \text{ K}$$

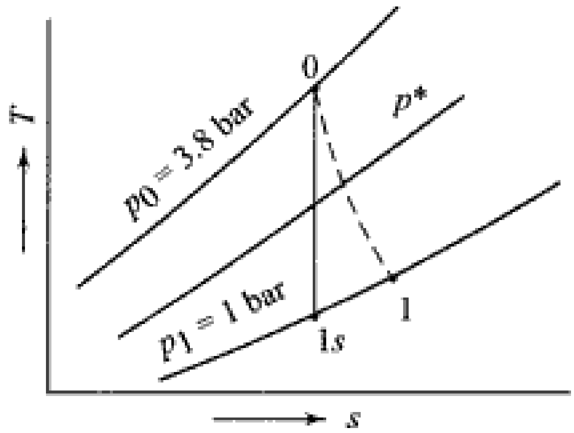


Fig. E7.4

$$v_1 = \frac{RT_1}{p_1} = \frac{277.5 \times 532.2}{10^5} = 1.48 \text{ m}^3/\text{kg}$$

$$V_1 = 44.72\ [1.11(723 - 532.2)^{1/2} = 650.8 \text{ m/s}$$

$$\therefore \qquad A_1 = \frac{\omega v_1}{V_1} = \frac{16.327 \times 1.48}{650.8} = 0.0371 \text{ m}^2$$

$\therefore$ Exit area = 0.0371 m^2 *Ans.*

Example 7.5 Steam at 20 bar and 300 °C enters a convergent–divergent nozzle at the rate of 0.3 kg/s with negligible inlet velocity and expands into a space at 3 bar.

(a) Assuming that the steam expands isentropicallly according to a law $pv^{1.3}$ = constant, estimate the throat and exit areas of the nozzles without using h–s chart.

(b) Re-calculate the throat and exit areas of the nozzle using the h–s chart and taking a coefficient of discharge of 0.98 and a coefficient of velocity as 0.92.

Solution At 20 bar, 300 °C, $v_0 = 0.1255$ m^3/kg

Since the steam is superheated, $p^*/p_0 = 0.546$.

$$\therefore \qquad p^* = 0.546 \times 20 = 10.92 \text{ bar}$$

(a) $$p^* v^{*k} = p_0 v_0^{k}$$

$$v^* = \left(\frac{p_0}{p^*}\right)^{1/k} v_0 = \left(\frac{1}{0.546}\right)^{1/1.3} \times 0.1255 = 0.2 \text{ m}^3/\text{kg}$$

$$V^* = (k\,p^*v^*)^{1/2} = (1.3 \times 10.92 \times 10^5 \times 0.2)^{1/2} = 532.8 \text{ m/s}$$

$$\therefore \quad A^* = \frac{\omega v^*}{V^*} = \frac{0.3 \times 0.2}{532.8} = 0.0001126 \text{ m}^2 = 112.6 \text{ mm}^2$$

$$v_1/v_0 = (p_0/p_1)^{1/k} = \left(\frac{20}{3}\right)^{1/1.3} = 4.309$$

$$v_1 = 0.54 \text{ m}^3/\text{kg}$$

$$\frac{V_1^2}{2} = \frac{k}{k-1}(p_0v_0 - p_1v_1)$$

$$= \frac{1.3 \times 10^5}{0.3}(20 \times 0.1255 - 3 \times 0.54) = 3.857 \times 10^5$$

$$\therefore \quad V_1 = 878.25 \text{ m/s}$$

$$\therefore \quad \text{Exit area, } A_1 = \frac{\omega v_1}{V_1} = \frac{0.3 \times 0.54}{878.25} = 0.0001844 \text{ m}^2$$

$$= 184.4 \text{ mm}^2 \qquad \textit{Ans.}$$

From Mollier chart, (Fig. E7.5),

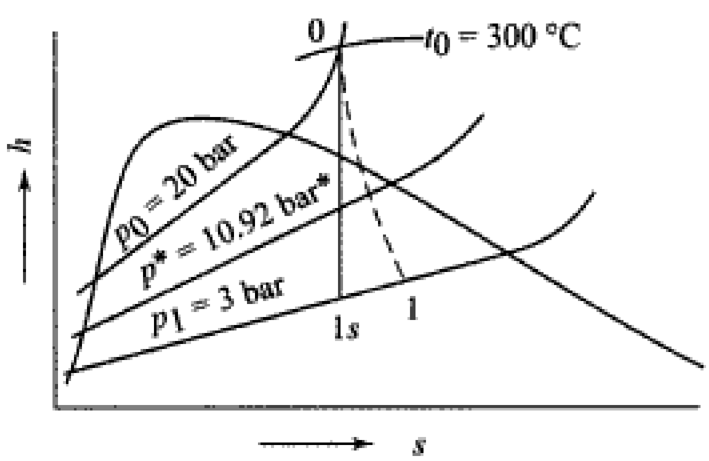


Fig. E7.5

$$h_0 = 3050 \text{ kJ/kg}, \; h^* = 2920 \text{ kJ/kg}, \; h_{1s} = 2650 \text{ kJ/kg}$$

$$c_d = \omega/\omega_s = 0.98 = 0.3/\omega_s$$

$$\therefore \quad \omega_s = 0.3061 \text{ kg/s}$$

v_s^*, as before, is 0.2 m^3/kg

$$V_s^* = 44.72\,(3050 - 2920)^{1/2} = 510 \text{ m/s}$$

$$V_{1s} = 44.72\,(3050 - 2650)^{1/2} = 894.4 \text{ m/s}$$

Coefficient of velocity, $\phi = V_1/V_{1s} = V_1/894.4 = 0.92$

$$V_1 = 822.8 \text{ m/s}$$

$$V_1 = 44.72\,(h_0 - h_1)^{1/2} = 822.8 \text{ m/s}$$

$$(h_0 - h_1) = \left(\frac{822.8}{44.72}\right)^2 = 338.52 \text{ kJ/kg}$$

$$h_1 = 3050 - 338.52 = 2711.48 \text{ kJ/kg}$$

Now,
$$h_1 = h_f + x_1 h_{fg}$$

$$2711.48 = 561.47 \times x_1 \times 2163.8$$

$$x_1 = 0.9936$$

$$v_1 = 0.001073 + 0.9936 \times 0.6047 = 0.6019 \text{ m}^3\text{/kg}$$

The exit area of nozzles is

$$\therefore \quad A_1 = \frac{\omega_s v_1}{V_1} = \frac{0.3061 \times 0.6019}{822.8} = 0.000224 \text{ m}^2$$

$$= 22.4 \text{ mm}^2 \qquad \textit{Ans.}$$

Taking the same velocity coefficient up to the throat

$$\phi = V^*/V_s^* = 0.92$$

$$V^* = 510 \times 0.92 = 469.2 \text{ m/s} = 44.72\,(h_0 - h^*)^{1/2}$$

$$h_0 - h^* = (469.2/44.72)^2 = 110.0$$

$$h^* = 3050 - 110 = 2940 \text{ kJ/kg}$$

From Mollier chart, $v^* = 0.22$ m^3/kg

The throat area is

$$A^* = \frac{\omega_s v^*}{V^*} = \frac{0.3061 \times 0.22}{469.2}$$

$$= 0.0001435 \text{ m}^2 = 143.5 \text{ mm}^2 \qquad \textit{Ans.}$$

Example 7.6 Dry saturated steam at 5 bar enters a convergent–divergent nozzle at a velocity of 100 m/s. The exit pressure is 1.5 bar. The throat and exit areas are 1280 mm^2 and 1600 mm^2, respectively. Assuming isentropic flow up to the throat and taking the critical pressure ratio as 0.58, estimate the mass flow rate and nozzle efficiency.

Solution In Fig. E7.6,

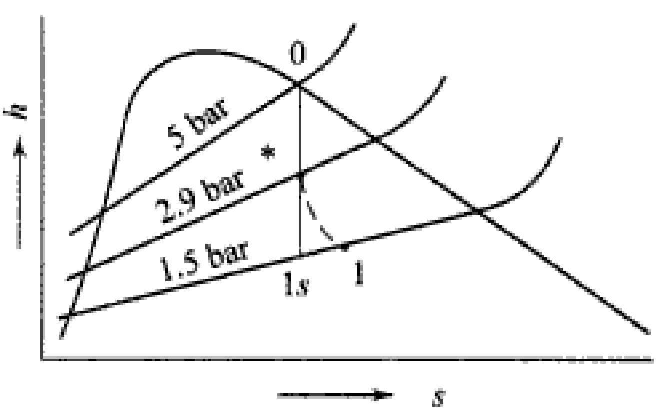


Fig. E7.6

$$h_0 = 2749 \text{ kJ/kg}, s_0 = 6.822 \text{ kJ/kg K}, p^* = 0.58 \times 5 = 2.9 \text{ bar}$$
$$s_0 = s^* = 6.822 = 1.660 + 5.344\, x^*$$
$$x^* = 0.966$$
$$\therefore \quad h^* = 556 + 0.966\,(2168) = 2650 \text{ kJ/kg}$$

The energy equation gives

$$h_0 + \frac{V_0^2}{2} = h^* + \frac{V^{*2}}{2}$$

$$2749 + \frac{100^2}{2} \times 10^{-3} = 2650 + \frac{V^{*2}}{2} \times 10^{-3}$$

$$\therefore \quad V^* = 456 \text{ m/s}$$
$$v^* = 0.966\,(0.6253) = 0.6040 \text{ m}^3\text{/kg}$$

$$\therefore \quad \omega = \frac{A^* V^*}{v^*} = \frac{1280 \times 10^{-6} \times 456}{0.6040} = 0.966 \text{ kg/s} \qquad \textit{Ans.}$$

Now, $s_{1s} = s^* = s_0 = 6.822 = 1.434 + x_{1s}\,(5.789)$

$$\therefore \quad x_{1s} = 0.931$$
$$h_{1s} = 467 + 0.931(2226) = 2539 \text{ kJ/kg}$$

Again, $\dfrac{V_1^2}{2} + h_1 = \dfrac{V^{*2}}{2} + h^*$

$$10^{-3} \times \frac{V_1^2}{2} + h_1 = \frac{456^2}{2} \times 10^{-3} + 2650 = 2754 \text{ kJ/kg} \qquad (1)$$

$$\omega = \frac{A_1 V_1}{v_1} = \frac{1600 \times 10^{-6} \times V_1}{v_1} = 0.966 \text{ kg/s}$$

$$V_1/v_1 = 603.8 \qquad (2)$$

The exit pressure is 1.5 bar. The solution of Eqs (1) and (2) has to be obtained by an iteration scheme, by assuming x_1. Let $x_1 = 0.94$, $v_1 = v_f + x_1\, v_{fg}$ (at 1.5 bar) $= 1.0895$ m^3/kg, $h_1 = h_f + x_1\, h_{fg} = 2559$ kJ/kg, $V_1 = 657.8$ m/s.

$$h_1 + (V_1^2/2) \times 10^{-3} = 2559 + \frac{(657.8)^2}{2} \times 10^{-3}$$

$$= 2775.35 \text{ kJ/kg } (2754 \text{ kJ/kg})$$

Assuming $x_1 = 0.92$, $v_1 = 0.92\,(1.1583) + 0.001 = 1.067$ m^3/kg

$$h_1 = 467.11 + 0.92 \times 2226.5 = 2515.5 \text{ kJ/kg}$$
$$V_1 = 644.3 \text{ m/s}$$

$$\therefore \quad h_1 + (V_1^2/2) \times 10^{-3} = 2723.1 \text{ kJ/kg } (2754 \text{ kJ/kg})$$

Finally, when $x_1 = 0.932$, $v_1 = 1.080$ m^3/kg, $h_1 = 2542$ kJ/kg

$$V_1 = 652.2 \text{ m/s}, h_1 + (V_1^2/2) \times 10^{-3}$$

$$= 2754.7 \text{ kJ/kg } (2754 \text{ kJ/kg})$$

$$\therefore \qquad h_{\text{nozzle}} = \frac{h^* - h_1}{h^* - h_{1s}} = \frac{2650 - 2542}{2650 - 2539} = \frac{108}{111} = 0.973 \qquad \textit{Ans.}$$

Example 7.7 A convergent–divergent nozzle receives steam at 5 bar, 200 °C and expands isentropically into a space at 2 bar. Neglecting the inlet velocity, calculate the exit area required for a mass flow of 0.3 kg/s in the following cases.

(a) when the flow is in equilibrium throughout,

(b) when the flow is supersaturated with $pv^{1.3}$ = constant. Calculate also for part (b):

(i) the degree of supercooling

(ii) the degree of supersaturation.

Solution (a) From steam tables, with reference to Fig. E7.7.

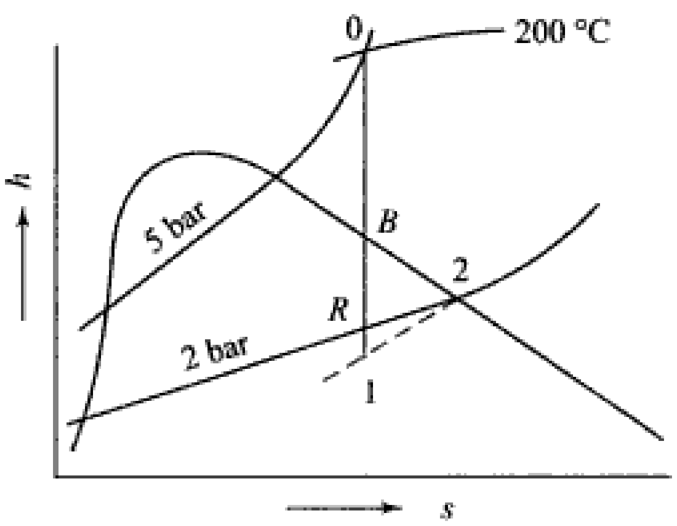


Fig. E7.7

$$v_0 = 0.4249 \text{ m}^3\text{/kg}, \; h_0 = 2855.4 \text{ kJ/kg}$$

$$s_0 = 7.0592 \text{ kJ/kg K} = s_1 = 1.4336 + x_1 (5.79)$$

$$x_1 = 0.972, \; h_1 = 504.7 + 0.972(2201.9) = 2645 \text{ kJ/kg}$$

$$v_1 = 0.972 \times 0.8857 = 0.86 \text{ m}^3\text{/kg}$$

$$V_1 = 44.72\,(h_0 - h_1)^{1/2} = 44.72\,(2855.4 - 2645)^{1/2}$$

$$= 648.67 \text{ m/s}$$

$$\therefore \qquad \omega_s = \frac{A_1 V_1}{v_1} = \frac{A_1 \times 648.67}{0.86} = 0.3 \text{ kg/s}$$

The exit area is

$$A_1 = 0.000398 \text{ m}^2 = 398 \text{ mm}^2 \qquad \textit{Ans. (a)}$$

(b)

$$dh = T ds + v\,dp = v\,dp \tag{1}$$

$$h_0 = h + \frac{V^2}{2} = \text{constant}$$

$$dh = -V\,d\,V \tag{2}$$

Equating (1) and (2),

$$V\,dV = -v\,dp$$

From $pv^{1.3} = p_1 v_1^{1.3} = \text{constant} = C = pv^n$, $n = 1.3$

$$\int_{V_0}^{V_R} V\,dV = \int_{p_0}^{p_1} -\left(\frac{C}{p}\right)^{1/n} dp = \int_{p_0}^{p_1} -(C^{1/n})\,p^{-1/n}\,dp$$

$$\frac{V_R^2 - V_0^2}{2} = -\,C^{1/n}\left(\frac{n}{n-1}\right)\left(p_1^{1-1/n} - p_0^{1-1/n}\right)$$

Since $V_0 = 0$,

$$\frac{V_R^2}{2} = \frac{n}{n-1}(p_0 v_0 - p_1 v_R)$$

Now, $$\frac{v_R}{v_0} = \left(\frac{p_0}{p_1}\right)^{1/n} = \left(\frac{5}{2}\right)^{1/1.3} = \left(\frac{5}{2}\right)^{0.769} = 2.023$$

$\therefore$ $$v_R = 2.023 \times 0.4249 = 0.86 \text{ m}^3/\text{kg}$$

$$\frac{v_R^2}{2} = \frac{1.3}{0.3}[500 \times 0.4249 - 200 \times 0.86]$$

$$= 175.28 \text{ kJ/kg} \times 1000 \text{ J/kJ}$$

$$V_R = 592 \text{ m/s}$$

$$A_1 = \text{exit area} = \frac{\omega v_R}{V_R} = \frac{0.3 \times 0.86}{592}$$

$$= 0.0004358 \text{ m}^2 = 435.8 \text{ mm}^2$$ *Ans.*

$$\frac{T_0}{T_R} = \left(\frac{p_0}{p_1}\right)^{(n-1)/n} = \left(\frac{5}{2}\right)^{0.3/1.3} = (2.5)^{0.23} = 1.2346$$

$$T_R = 473/1.2346 = 383.12 \text{ K}$$

$$t_R = 110.12\ ^\circ\text{C}$$

$$(t_{sat})_{p1} = 120.23\ ^\circ\text{C}$$

$\therefore$ Degree of subcooling $= 120.23 - 110.12 = 10.11^\circ$C *Ans.*

$$(p_{sat})_{tR = 110^\circ C} = 1.4327 \text{ bar}$$

$\therefore$ Degree of supersaturation $= \dfrac{2 \text{ bar}}{1.4327 \text{ bar}} = 1.396$ *Ans.*

Example 7.8 Recalculate the exit area, assuming a nozzle efficiency of 0.92 and a mean $c_p = 1.925$ kJ/kg K for supersaturated steam. Check the answer by assuming $pv \times 10^3 = 2.308\,(h - 1943)$, where p is in bar, v in m^{3}kg and h in kJ/kg.

Solution As shown in Fig. 7.8,

$$h_n = \frac{h_0 - h_Q}{h_0 - h_1}$$

Now, $h_0 - h_1 = 2855.4 - 2645 = 210.4$

$\therefore$ $h_0 - h_Q = 0.92 \times 210.4 = 193.57$ kJ/kg

$$V_Q = 44.72\,(193.57)^{1/2} = 622.2 \text{ m/s}$$

$$h_0 - h_Q = c_p(t_0 - t_Q) = 193.57$$

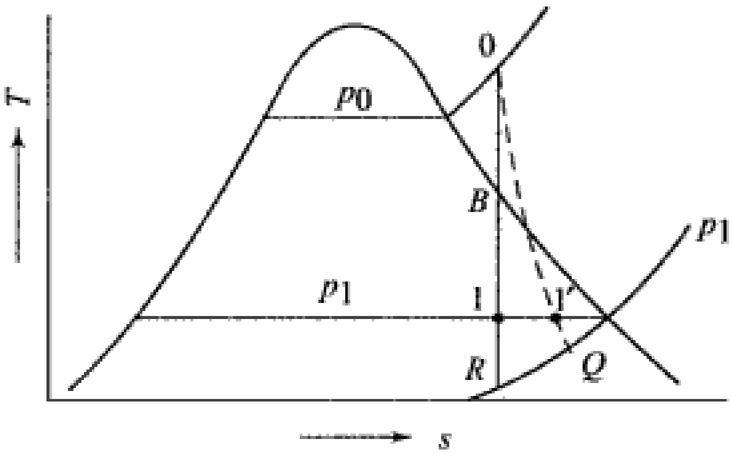


Fig. E7.8

$\therefore$ $$t_0 - t_Q = \frac{193.57}{1.925} = 100.56\ °\text{C}$$

$$t_Q = 200 - 100.56 = 99.44\ °\text{C} = 372.44 \text{ K}$$

$$\frac{p_Q v_Q}{T_Q} = \frac{p_0 v_0}{T_0} = \frac{500 \times 0.4249}{473}$$

$$v_Q = \frac{500 \times 0.4249}{473} \times \frac{372.44}{200} = 0.8364 \text{ m}^3/\text{kg}$$

$$A_1 = \frac{\omega v_Q}{V_Q} = \frac{0.3 \times 0.8364}{622.2}$$

$$= 0.0004033 \text{ m}^2 = 403.3 \text{ mm}^2$$ *Ans.*

Using approximate formula (Eastop and McConkey, 1986)

$$v_Q = \frac{2.308(h_Q - 1943)}{10^3 \times p_Q} \text{ m}^3/\text{kg}$$

where p_Q is in bar and h_Q is in kJ kg

$$v_Q = \frac{2.308(2855.4 - 193.6 - 1943)}{10^3 \times 2} \text{ m}^3/\text{kg} = 0.8295 \text{ m}^3/\text{kg}$$

$$A_1 = \frac{0.3 \times 0.8295}{622.2} = 0.00039995 \text{ m}^2 = 400 \text{ mm}^2$$ *Ans.*

Example 7.9 The velocity of steam entering a simple impulse turbine is 1000 m/s, and the nozzle angle is 20°. The mean peripheral velocity of blades is 400 m/s and the blades are symmetrical. If the steam is to enter the blades without shock, what will be the blade angles?

(a) Neglecting the friction effects on the blades, calculate the tangential force on the blades and the diagram power for a mass flow of 0.75 kg/s. Estimate also the axial thrust and diagram efficiency.

(b) If the relative velocity at exit is reduced by friction to 80% of that at inlet, estimate the axial thrust, diagram power and diagram efficiency.

Solution Given: $V_1 = 1000$ m/s, $V_b = 400$ m/s, $\alpha = 20°$, $\beta_1 = \beta_2$, $\omega_s = 0.75$ kg/s (Fig. E7.9)

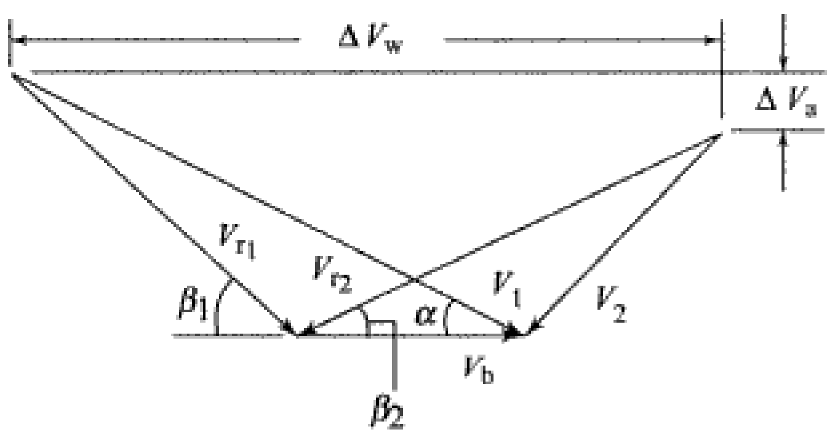


Fig. E7.9

(a)
$$k_b = 0$$
$$V_{r1} \sin \beta_1 = V_1 \sin \alpha$$
$$V_{r2} \cos \beta_1 = V_1 \cos \alpha - V_b$$
$$\beta_1 = \tan^{-1}\left[\frac{V_1 \sin\alpha}{V_1 \cos\alpha - V_b}\right] = \tan^{-1}\frac{1000 \sin 20°}{1000 \cos 20° - 400}$$
$$= \tan^{-1}\frac{342}{940 - 400} = 32.35° = \beta_2 \qquad \textit{Ans.}$$
$$V_{r1} \sin 32.35° = 342$$
$$\therefore \quad V_{r1} = 639.25 \text{ m/s} = V_{r2}$$
$$\Delta V_w = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2 = 2V_{r1} \cos \beta_1$$
$$= 2 \times 639.25 \times \cos 20° = 1080.07 \text{ m/s}$$
$$\Delta V_a = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = 0$$

Tangential thrust is $P_t = \omega_s \Delta V_\omega = 0.75 \times 1080.07 = 810.05$ N

Diagram power, $\dot{W}_D = P_t \times V_b = 810.05 \times 400 = 324.02$ kW *Ans.*

Diagram efficiency, $\eta_D = \dfrac{324.02 \text{ kW}}{\frac{1}{2} \times 0.75 \times 1000^2 \times 10^{-3} \text{ kW}}$

$= 0.864$ or 86.4% *Ans.*

Axial thrust, $P_a = \omega_s \Delta V_a = 0$ *Ans.*

(b) $k_b = 0.8$

$V_{r2} = 0.8\ V_{r1} = 0.8 \times 639.25 = 511.4$ m/s

$\Delta V_w = 639.25 \cos 32.35° + 511.4 \cos 32.35° = 972.06$ m/s

Axial thrust, $P_a = \omega_s (V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2)$

$= 0.75 \times 127.85 \sin 32.35° = 51.3$ N *Ans.*

Diagram power, $\dot{W}_D = 0.75 \times 972.06 \times 400 = 291.62$ kW *Ans.*

Diagram efficiency, $\eta_D = \dfrac{291620}{\frac{1}{2} \times 0.75 \times 1000^2}$

$= 0.7776$ or 77.76% *Ans.*

Example 7.10 An impulse steam turbine has a number of pressure stages, each having a row of nozzles and a single ring of blades. The nozzle angle in the first stage is 20° and the blade exit angle is 30° with reference to the plane of rotation. The mean blade speed is 130 m/s and the velocity of steam leaving the nozzles is 330 m/s.

(a) Taking the blade friction factor as 0.8 and a nozzle efficiency of 0.85, determine the work done in the stage per kg of steam and the stage efficiency.

(b) If the steam supply to the first stage is at 20 bar, 250 °C and the condenser pressure is 0.07 bar, estimate the number of stages required, assuming that the stage efficiency and the work done are the same for all stages and that the reheat factor is 1.06.

Solution

(a) Given: $\alpha = 20°$, $\beta_2 = 30°$, $V_b = 130$ m/s, $V_1 = 330$ m/s (Fig. E7.10a)

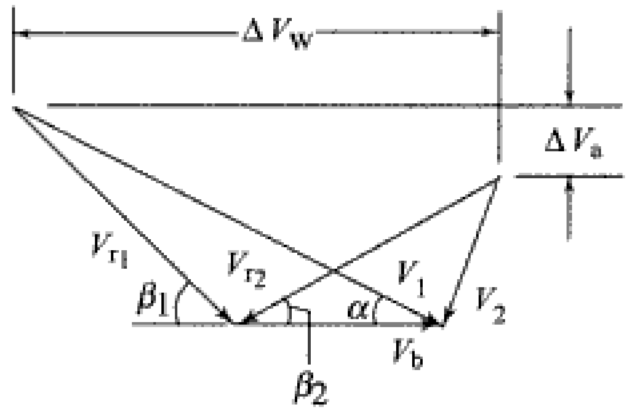


Fig. E7.10(a)

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{330 \sin 20°}{330 \cos 20° - 130} = 0.627$$

$$\beta_1 = 32.075°$$

$$V_{r1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{330 \times 0.342}{0.531} = 212.53 \text{ m/s}$$

$$V_{r2} = 0.8 \times 212.53 = 170.025 \text{ m/s}$$

$$\Delta V_w = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2$$

$$= 212.53 \cos 32.075° + 170.025 \cos 30° = 372.334 \text{ m/s}$$

$$W_D = \omega_s \, \Delta V_\omega \, V_b = 1 \times 327.334 \times 130 = 42.55 \text{ kJ/kg} \qquad \textit{Ans.}$$

$$\eta_{b1} = \eta_D = \frac{2\Delta V_w V_b}{V_1^2} = \frac{2 \times 327.334 \times 130}{330 \times 330}$$

$$= 0.7815 \quad \text{or} \quad 78.15\%$$

$$\eta_{stage} = \eta_n \times \eta_{b1} = 0.85 \times 0.7815 = 0.664 \quad \text{or} \quad 66.4\% \qquad \textit{Ans.}$$

(b) $$\eta_{internal} = \eta_{stage} \times \text{reheat factor} = 0.664 \times 1.06$$

$$= 0.7041 \quad \text{or} \quad 70.41\%$$

$$h_1 = 2902.3 \text{ kJ/kg (Fig. E7.10b)}$$

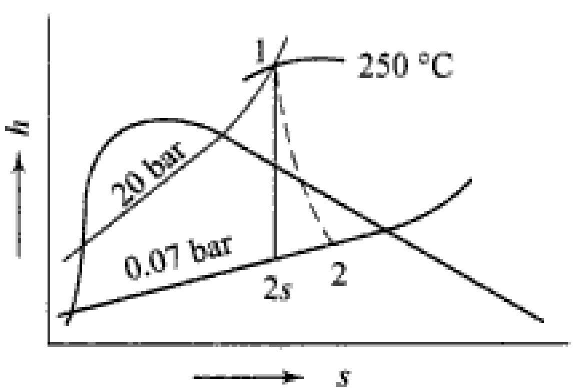


Fig. E7.10(b)

$$s_1 = 6.5466 = s_{2s} = 0.5582 + x_{2s}\, 7.7198$$

$$x_{2s} = 0.7757$$

$$h_{2s} = 163.16 + 0.7757 \,(2409.54) = 2032.29 \text{ kJ/kg}$$

$$h_1 - h_2 = 0.7041\,(h_1 - h_{2s}) = 0.7041\,(2902.3 - 2032.3)$$

$$= 612.57 \text{ kJ/kg}$$

$$\therefore \text{ Number of stages, } n = \frac{(\Delta h)_{total}}{(\Delta h)_{stage}} = \frac{612.57}{42.55}$$

$$= 14.39 \quad \text{or} \quad 15 \text{ stages} \qquad \textit{Ans. (b)}$$

Example 7.11 In a stage of an impulse turbine provided with a single row wheel, the mean diameter of the blade ring is 800 mm and the speed of rotation is 3000 rpm. The steam issues from the nozzles with a velocity of 300 m/s and the nozzle angle is 20°. The rotor blades are equiangular and the blade friction factor is 0.86. What is the power developed in the blading when the axial thrust on the blades is 140 newtons?

Solution

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 0.80 \times 3000}{60} = 125.6 \text{ m/s}$$

$$V_1 = 300 \text{ m/s}, \ \alpha = 20$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{300 \sin 20°}{300 \cos 20° - 125.6} = \frac{102.61}{281.91 - 125.6}$$

$$= 0.6565$$

$$\beta_1 = 33.3° = \beta_2$$

$$V_1 \sin \alpha = V_{r1} \sin \beta_1$$

$$102.61 = V_{r1} \sin 33.3°$$

$$\therefore \quad V_{r1} = 187 \text{ m/s}$$

$$V_{r2} = 0.86 \times 187 = 161 \text{ m/s}$$

Axial thrust, $$P_a = \omega_s (V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2)$$

$$= \omega_s V_{r1} \sin \beta_1 (1 - k_b)$$

$$= \omega_s \times 187 \sin 33.3° (1 - 0.86)$$

$$= \omega_s \times 14.3654 = 140 \text{ N}$$

$$\therefore \quad \omega_s = 9.7456 \text{ kg/s}$$

$$\Delta V_w = V_{r2} \cos \beta_2 + V_{r1} \cos \beta_1 = V_{r1} \cos \beta_1 (1 + k_b)$$

$$= 187 \cos 33.3° \times 1.86 = 290.71 \text{ m/s}$$

$$\therefore \text{ Power developed} = 9.7456 \times 290.71 \times 125.6 \times 10^{-3}$$

$$= 355.84 \text{ kW} \qquad \textit{Ans.}$$

Example 7.12 The nozzles of the impulse stage of a turbine receive steam at 15 bar and 300 °C and discharge it at 10 bar. The nozzle efficiency is 95% and the nozzle angle is 20°. The blade speed is that required for maximum work, and the inlet angle of the blades is that required for entry of the steam without shock. The blade exit angle is 5° less than the inlet angle. The blade friction factor is 0.9. Calculate for a steam flow of 1350 kg/h, (a) the axial thrust. (b) the diagram power, and (c) the diagram efficiency.

Solution

$$h_1 = 3038.9 \text{ kJ/kg (Fig. E7.12(a) \& (b))}$$

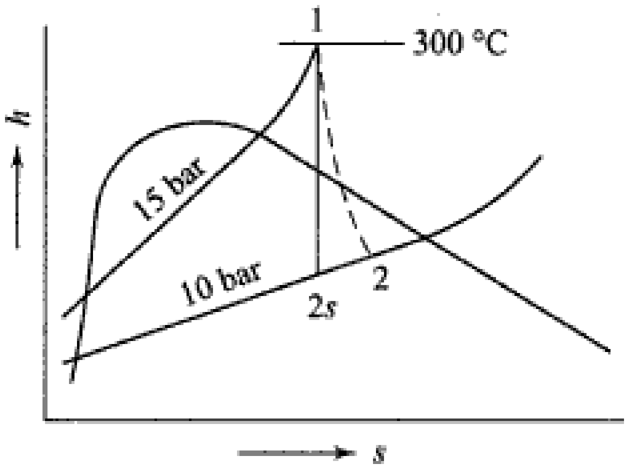


Fig. E7.12(a)

$$s_1 = 6.9224 \text{ kJ/kg K} = s_{2s}$$
$$(s_g)10 \text{ bar} = 6.5828 \text{ kJ/kg K}$$
$$s_{2s} > (s_g)10 \text{ bar}$$

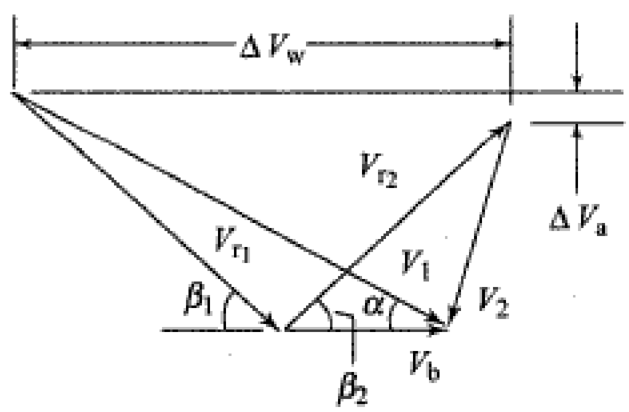


Fig. E7.12(b)

The state "2*s*" is in the superheated region

$$t_{2s} = 250 \text{ °C}, h_{2s} = 2943.1 \text{ kJ/kg}$$
$$V_1 = 44.72\,[(3038.9 - 2943.1)0.95]^{1/2} = 426.63 \text{ m/s}$$
$$\frac{V_b}{V_1} = \frac{\cos\alpha}{2} = \frac{\cos 20°}{2} = 0.4699$$
$$\therefore \quad V_b = 426.63 \times 0.4699 = 200.45 \text{ m/s}$$
$$\tan\beta_1 = \frac{V_1 \sin\alpha}{V_1\cos\alpha - V_b} = \frac{145.916}{200.45} = 0.7279$$
$$\beta_1 = 36°$$
$$\therefore \quad \beta_2 = 36 - 5 = 31°$$
$$V_{r1} = \frac{V_1 \sin\alpha}{\sin\beta_1} = 248.25 \text{ m/s}$$

$$V_{r2} = k_b V_{r1} = 223.42 \text{ m/s}$$
$$\Delta V_\omega = V_{r1} \cos \beta_1 + V_{r2} \cos \beta_2 = 392.35 \text{ m/s}$$
$$\Delta V_a = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2 = 30.85 \text{ m/s}$$

(a) Axial thrust, $P_a = \frac{1350}{3600} \times 30.85 = 11.57 \text{ N}$ *Ans.*

Tangential thrust, $P_t = \frac{1350}{3600} \times 392.35 = 147.13 \text{ N}$

(b) Diagram power, $\dot{W}_D = 147.13 \times 200.45 \times 10^{-3}$
$= 29.492 \text{ kW}$ *Ans.*

(c) Diagram efficiency $\eta_D = \dfrac{29492}{\frac{1}{2} \times \frac{1350}{3600} \times (426.63)^2}$
$= 0.864$ or 86.4% *Ans.*

Example 7.13 The following particulars refer to a two-row velocity-compounded impulse wheel:

Steam velocity at nozzle exit = 600 m/s
Nozzle angle = 16°
Mean blade velocity = 120 m/s
Exit angles: first row moving blades = 18°, fixed guide blades = 22°, second row moving blades = 36°
Steam flow = 5 kg/s
Blade friction coefficient = 0.85

Determine (a) the tangential thrust, (b) the axial thrust, (c) the power developed, and (d) the diagram efficiency.

Solution

V_b = 120 m/s, V_1 = 600 m/s, α = 16°, β_2 = 18°, k_b = 0.85, α_1 = 22°, β_4 = 36° (Fig. E7.13)

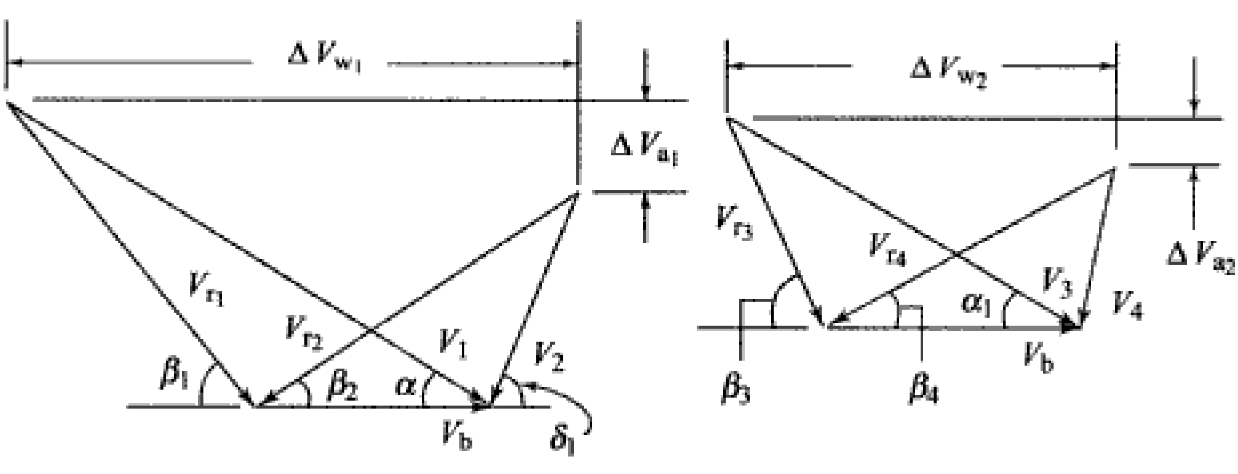


Fig. E7.13

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{600 \sin 16°}{600 \cos 16° - 120}$$

$$= \frac{165.382}{576.757 - 120} = 0.362$$

$$\beta_1 = 19.9°$$

$$V_{r1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{165.382}{0.34} = 485.78 \text{ m/s}$$

$$V_{r2} = 0.85 \times 485.78 = 412.91 \text{ m/s}$$

$$\tan \delta_1 = \frac{V_{r_2} \sin \beta_2}{V_{r_2} \cos \beta_2 - V_b} = \frac{412.9 \sin 18°}{412.9 \cos 18° - 120} = \frac{127.593}{272.69} = 0.468$$

$$\therefore \quad \delta_1 = 25.1°$$

$$V_2 = \frac{127.593}{\sin 25.1°} = 301.06 \text{ m/s}$$

$$V_3 = 0.85 \times 301.06 = 255.9 \text{ m/s}$$

$$\Delta V_{w_1} = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = V_1 \cos \alpha + V_2 \cos \delta_1$$

$$= 576.757 + 301.06 \cos 25.1° = 849.44 \text{ m/s}$$

$$\Delta V_{a_1} = V_1 \sin \alpha - V_2 \sin \delta_1 = 165.382 - 301.06 \sin 25.1°$$

$$= 37.79 \text{ m/s}$$

$$\tan \beta_3 = \frac{V_3 \sin \alpha_1}{V_3 \cos \alpha - V_b} = \frac{255.9 \sin 22°}{255.9 \cos 22° - 120}$$

$$= \frac{95.86}{117.266} = 0.8175$$

$$\beta_3 = 39.26°$$

$$V_{r_3} = \frac{V_3 \sin \alpha_1}{\sin \beta_3} = \frac{95.86}{\sin 39.26°} = 151.46 \text{ m/s}$$

$$V_{r_4} = 0.85 \times 151.46 = 128.74 \text{ m/s}$$

$$\Delta V_{w_2} = V_{r_3} \cos \beta_3 + V_{r_4} \cos \beta_4$$

$$= 151.46 \cos 39.26° + 128.74 \cos 36° = 221.42 \text{ m/s}$$

$$\Delta V_{a_2} = V_3 \sin \alpha_1 - V_{r4} \sin \beta_4 = 93.86 - 75.67 = 20.19 \text{ m/s}$$

$$\Sigma \Delta V_w = \Delta V_{w_1} + \Delta V_{w_2} = 849.44 + 221.42 = 1070.86 \text{ m/s}$$

$$\Sigma \Delta V_a = \Delta V_{a_1} + \Delta V_{a_2} = 37.79 + 20.19 = 57.98 \text{ m/s}$$

(a) Tangential thrust,

$$P_t = \omega_s \Sigma \Delta V_w = 5 \times 1070.86 \times 10^{-3} \text{ kN}$$

$$= 5.354 \text{ kN} \qquad \textit{Ans.}$$

(b) Axial thrust,

$$P_a = \omega_s \Sigma \Delta V_a = 5 \times 57.98 \times 10^{-3} \text{ kN}$$
$$= 0.29 \text{ kN} \qquad Ans.$$

(c) Power developed

$$\dot{W}_D = P_t \times V_b = 5.354 \times 120$$
$$= 642.48 \text{ kW} \qquad Ans.$$

(d) Diagram efficiency,

$$\eta_D = \frac{2\Delta V_\omega V_b}{V_1^2} = \frac{2 \times 1070.86 \times 120}{600 \times 600}$$
$$= 0.7139 \quad \text{or} \quad 71.39\% \qquad Ans.$$

Example 7.14 The following particulars apply to a two-row velocity compounded impulse stage of a turbine: nozzle angle 17°, mean blade speed 125 m/s; exit angles of the first row moving blades, the fixed blades, and the second row moving blades 22°, 26° and 30°, respectively; blade friction factor for each row 0.9. Assume that the absolute velocity of steam leaving the stage is in the axial direction. Draw the velocity diagrams for the stage and obtain (a) the absolute velocity of steam leaving the stage, (b) the diagram work, and (c) the diagram efficiency.

Solution

$\alpha = 17°$, $V_b = 125$ m/s, $\beta_2 = 22°$, $\alpha_1 = 26°$, $\beta_4 = 30°$, $k_b = 0.9$, $\delta_2 = 90°$

At first, the velocity diagrams for the second row of moving blades are drawn to scale (say, 1 cm = 50 m/s). Then those for the first row are drawn with the given particulars. (Fig. E7.14).

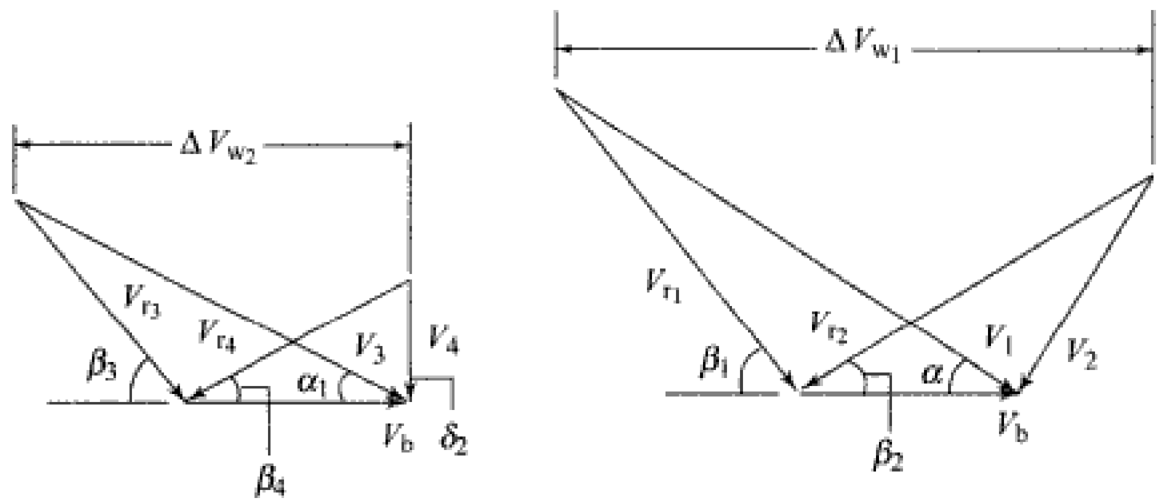


Fig. E7.14

ΔV_{w_1} and ΔV_{w_2} are measured from the diagrams.

$$\Delta V_{w_1} + \Delta V_{w_2} = (16 \text{ cm} + 4.8 \text{ cm}) \times 50 \frac{\text{m/s}}{\text{cm}} = 1040 \text{ m/s}$$

$$V_1 = 11.5 \text{ cm} \times 50 \frac{\text{m/s}}{\text{cm}} = 575 \text{ m/s}$$

(a) Velocity of steam exiting the stage, V_4 as read from the diagram

$$= 1.5 \text{ cm} \times 50 \frac{\text{m/s}}{\text{cm}} = 75 \text{ m/s} \qquad \textit{Ans.}$$

(b) Diagram work,

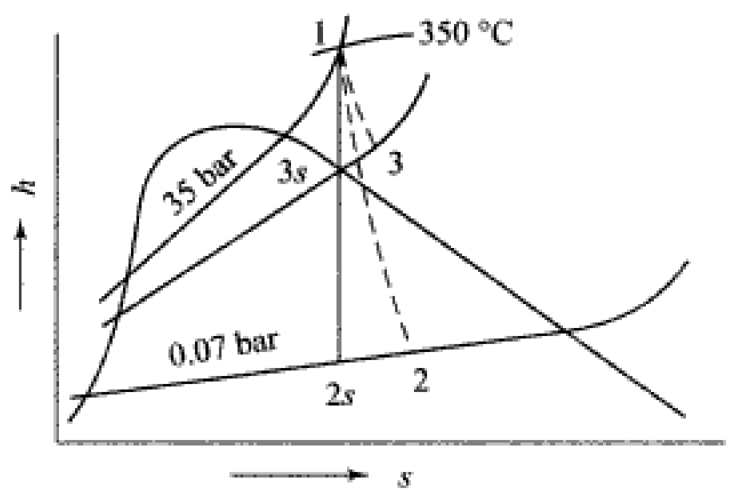
$$W_D = 1040 \times 1255 = 130{,}000 \text{ J/kg}$$
$$= 130 \text{ kJ/kg} \qquad \textit{Ans.}$$

(c) Diagram efficiency,

$$\eta_D = \frac{130{,}000}{\frac{1}{2} \times 575 \times 575}$$
$$= 0.7863 \quad \text{or} \quad 78.63\% \qquad \textit{Ans.}$$

Example 7.15 An impulse steam turbine is supplied with steam at 35 bar, 350 °C, the condenser pressure being 0.07 bar. The first stage of the turbine is velocity compounded with two rings of moving blades separated by a ring of fixed guide blades. The isentropic enthalpy drop for this stage is 1/4 of that for the whole turbine. The nozzle angle is 20° and the nozzle efficiency is 88%. The mean blade velocity of both the moving rings of blades is 0.2 of the velocity of steam leaving the nozzle. The exit blade angles for both fixed and moving blades are 30° and the blade friction coefficient for all blades is 0.9. If the internal efficiency of the turbine is 75%, calculate the efficiency of the first stage and the percentage of the total power developed by the turbine in this stage.

Solution From steam tables, (Fig. E7.15), h_1 = 3106.4, s_1 = 6.6643 kJ/kg K = $0.5582 + x_{2s} \times 7.7198$



(a)

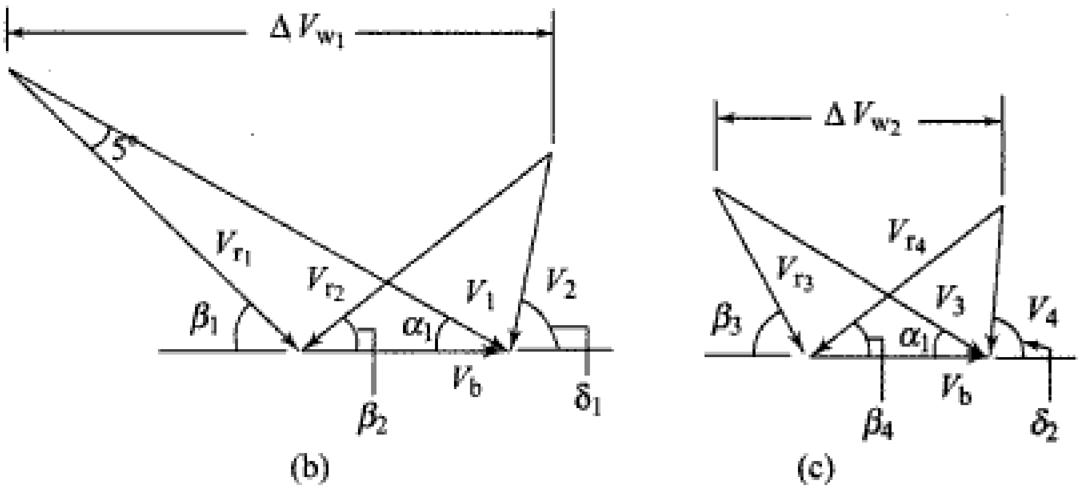


Fig. E7.15

$$\therefore \quad x_{2s} = 0.791$$

$$h_{2s} = 163.16 + 0.791 \times 2409.54 = 2069.11 \text{ kJ/kg}$$

$$(\Delta h_s)_{\text{total}} = h_1 - h_{2s} = 1037.29 \text{ kJ/kg}$$

Enthalpy drop in the 2-row velocity or Curtis stage is

$$\therefore \quad h_1 - h_{3s} = \frac{1}{4}\,(\Delta h_s)_{\text{total}} = \frac{1}{4} \times 1037.29 = 259.32 \text{ kJ/kg}$$

$$\therefore \quad h_1 - h_3 = 0.88 \times 259.32 = 228.2 \text{ kJ/kg}$$

$\therefore$ Velocity of steam leaving the nozzles,

$$V_1 = 44.72\,(228.2)^{1/2} = 675.56 \text{ m/s}$$

$$\therefore \quad V_b = 0.2 \times 675.56 = 135.112 \text{ m/s}$$

$$\alpha = 20°,\ \beta_2 = \beta_4 = 30°,\ k_b = 0.9$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{231.055}{499.709}$$

$$\therefore \quad \beta_1 = 24.815°$$

$$V_{r_1} = \frac{231.055}{\sin 24.815°} = 550.54 \text{ m/s}$$

$$V_{r_2} = 0.9 \times 550.54 = 495.486 \text{ m/s}$$

$$\Delta V_{w_1} = V_{r_1} \cos \beta_1 + V_{r_2} \cos \beta_2 = 497.709 + 429.10$$

$$= 928.81 \text{ m/s}$$

$$V_2 \cos \delta_1 = V_{r_2} \cos \beta_2 - V_b = 429.10 - 135.11 = 294 \text{ m/s}$$

$$V_2^2 = (V_{r_2} \sin \beta_2)^2 + (V_{r_2} \cos \beta_2 - V_b)^2$$

$$= 61376.59 + (294)^2$$

$$\therefore \quad V_2 = 384.46 \text{ m/s}$$

$$V_3 = 0.9 \times 384.46 = 346 \text{ m/s}$$

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 0.67 \times 1500}{60} = 52.62 \text{ m/s}$$

By sine law,

$$\frac{V_1}{\sin 145°} = \frac{V_b}{\sin 15°} = \frac{V_{r_1}}{\sin 20°}$$

$$V_1 = 52.62 \times \frac{0.5736}{0.2588} = 116.63 \text{ m/s} = V_{r_2}$$

$$V_{r_1} = 52.62 \times \frac{0.342}{0.2588} = 69.54 \text{ m/s} = V_2$$

$$\Delta V_w = V_1 \cos\alpha + V_2 \cos\delta = 116.63 \cos 20° + 69.54 \cos 35°$$

$$= 109.6 + 56.964 = 166.564 \text{ m/s}$$

$$v_1 = 0.001052 + 0.96 \times 1.15937 = 1.114 \text{ m}^3/\text{kg}$$

Now, $\omega_s = 3.6$ kg/s

$$= \frac{\pi D_m h_b V_1 \sin\alpha}{v_1} = \frac{\pi \times 0.67 \times h_b \times 116{\cdot}63 \sin 20°}{1.114}$$

$\therefore$ $h_b = 0.0478 \text{ m} = 47.8 \text{ mm}$ *Ans.*

Power developed by the ring

$$= \omega_s \Delta V_w V_b = 3.6 \times 166.564 \times 52.62 \times 10^{-3}$$

$$= 31.552 \text{ kW}$$ *Ans.*

Example 7.17 A Parsons reaction (50%) turbine running at 400 rpm develops 5 MW using 6 kg/kWh of steam flow. The exit angle of the blades is 20 ° and the velocity of steam relative to the blades at exit is 1.35 times the mean blade speed. At a particular stage in the expansion the pressure is 1.2 bar and the steam quality is 0.95. Calculate for this stage (a) a suitable blade height, assuming the ratio of D_m/h_b as 12, and (b) the diagram power.

Solution

$V_{r_1} = V_2$, $V_1 = V_{r_2} = 1.35\ V_b$, $\beta_1 = \delta$, $\beta_2 = \alpha = 20$ ° (Fig. E7.17)

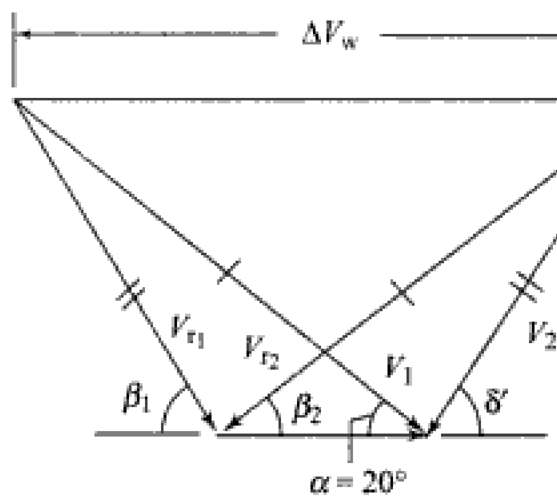


Fig. E7.17

$$\omega_s = 6 \text{ kg/kWh} = \frac{6 \times 5000}{3600} = \frac{25}{3} \text{ kg/s}$$

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi \times 12 h_b \times 400}{60} = 80\,\pi\, h_b$$

$$V_1 = 1.35 \times 80\, \pi h_b = 108\, \pi\, h_b$$

v_1 at 1.2 bar, 0.95 dry = 0.0010468 + 0.95 (1.454) = 1.381 m^3/kg Volume flow of steam,

$$\omega_s v_1 = \pi D_m\, h_b\, V_1 \sin \alpha\, k_{tb}$$

$$\frac{25}{3} \times 1.381 = \pi (12\, h_b)\, h_b\, (108\, \pi\, h_b) \sin 20°$$

$\therefore$ $h_b = 0.138$ m = 138 mm *Ans.* (a)

$$V_b = 80\, \pi \times 0.138 = 34.67 \text{ m/s}$$

$$V_1 = 1.35 \times 34.67 = 46.8 \text{ m/s}$$

$$\Delta V_w = 2V_1 \cos \alpha - V_b = 2 \times 46.8 \cos 20° - 34.67 = 53.28 \text{ m/s}$$

Diagram power,

$$\dot{W}_D = \omega_s\, \Delta V_W V_b$$

$$= \frac{25}{3} \times 53.28 \times 34.67 \times 10^{-3} \text{ kW}$$

$$= 15.39 \text{ kW}$$ *Ans.* (b)

Example 7.18 The speed of rotation of a blade group of a 50% reaction turbine is 3000 rpm. The mean blade speed is 100 m/s. The velocity ratio is 0.56 and the exit angle of the blades is 20°. If the mean specific volume of the steam is 0.65 m^3/kg and the mean height of the blade is 25 mm, calculate the mass flow of steam through the turbine in kg/h. Neglect the effect of blade thickness on the annulus area.

If there are five pairs of blades in the group, calculate the useful enthalpy drop required and the diagram power.

Solution

Given: $V_{r_2} = V_1$, $V_{r_1} = V_2$, $\alpha = \beta_2 = 20°$ (Fig. E7.18)

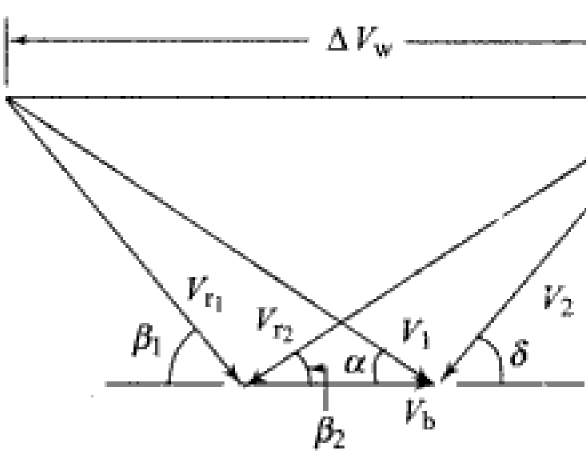


Fig. E7.18

$$V_b = 100 \text{ m/s}, V_b/V_1 = 0.56$$

$$V_1 = \frac{100}{0.56} = 178.57 \text{ m/s} = V_{r_2}$$

$$h_b = 25 \text{ mm}, v = 0.65 \text{ m}^3/\text{kg}, N = 3000 \text{ rpm}$$

$$V_b = \frac{\pi D_m N}{60} = \frac{\pi D_m \times 3000}{60} = 100 \text{ m/s}$$

$$D_m = \frac{2}{\pi} \text{ m}$$

$$\omega_s v = \pi D_m h_b k_{tb} V_1 \sin \alpha$$

$$\therefore \quad \omega_s = \frac{2 \times 0.025 \times 1 \times 178.57 \sin 20°}{0.65} = 4.698 \text{ kg/s}$$

$$= 16{,}912.8 \text{ kg/h} \qquad \textit{Ans.}$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{178.57 \sin 20°}{178.57 \cos 20° - 100} = \frac{61.075}{67.801}$$

$$\beta_1 = 42°$$

$$V_{r_1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{61.075}{\sin 42°} = 91.25 \text{ m/s}$$

$$\Delta h_{mb} = \frac{1}{2}\left(V_{r_2}^2 - V_{r_1}^2\right)\frac{\left(V_{r_2} + V_{r_1}\right)\left(V_{r_2} - V_{r_1}\right)}{2}$$

$$= \frac{(178.57 + 91.25)(178.57 - 91.25)}{2}$$

$$\therefore \quad \Delta h_{stage} = \Delta h_{fb} + \Delta h_{mb} = 2\Delta h_{mb} = (269.82 \times 87.32) \times 10^{-3}$$

$$= 23.56 \text{ kJ/kg}$$

For 5 pairs of blades,

$$(\Delta_h)_{total} = 5 \times 23.56 = 117.80 \text{ kJ/kg}$$

$\therefore$ Diagram power $= 4.698 \times 117.8 = 553.4$ kW *Ans.*

Example 7.19 A steam turbine is to develop 8 MW at 5000 rpm for driving a compressor. The steam enters at 40 bar, 500 °C and exhausts at 0.1 bar. The internal efficiency of the turbine is 0.85 and its mechanical efficiency is 0.96. Estimate (a) the number of impulse stages required, if similar impulse stages are used throughout, (b) the nozzle height for the first stage with full admission. Assume nozzle efficiency as 0.92, nozzle angle 15°, limiting blade velocity 300 m/s, and the blades operating at maximum efficiency.

Solution For maximum blading efficiency,

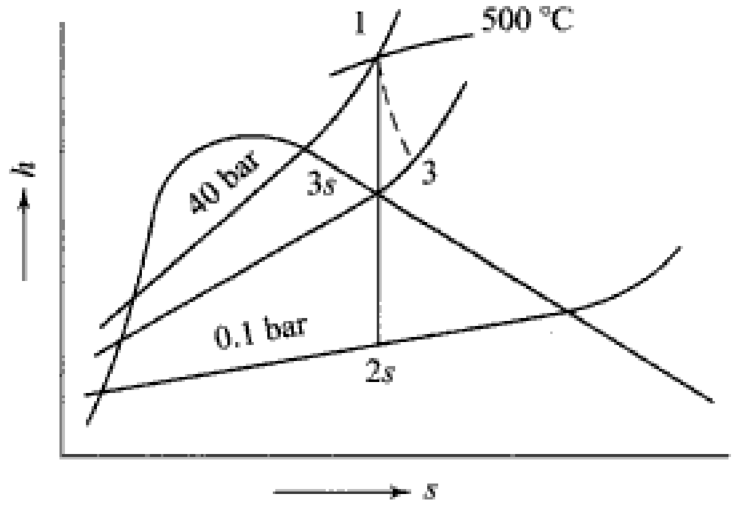


Fig. E7.19

$$\frac{V_b}{V_1} = \cos a/2$$

$$V_1 = \frac{2 \times 300}{\cos 15°} = 621.2 \text{ m/s}$$

Now, $$V_1 = 44.72\,[(\Delta h)_s h_n]^{1/2} = 621.2 \text{ m/s}$$

$\therefore$ $$(\Delta h)_s = \left(\frac{621.2}{44.72}\right)^2 \frac{1}{0.92} = 209.7 \text{ kJ/kg}$$

$$h_1 = 3445.3 \text{ kJ/kg (Fig. E7.19)}$$

$$s_1 = 7.0901 \text{ kJ/kg K} = s_2 = 0.6493 + x_{2s} \times 7.5009$$

$$x_2 = 0.86$$

$$h_{2s} = 191.83 + 0.86 \times 2392.8 = 2246.5 \text{ kJ/kg}$$

$$h_1 - h_{2s} = 1198.8 \text{ kJ/kg}$$

$\therefore$ Number of stages required

$$= \frac{h_1 - h_{2s}}{(\Delta h)_{1s}} = \frac{1198.8}{209.7} = 5.72 \quad \text{or} \quad 6 \text{ stages}$$ *Ans.* (a)

(b) Isentropic output

$$= \frac{8000}{0.85 \times 0.96} = \omega_s \times 1198.8$$

$\therefore$ $$\omega_s = \frac{8000}{0.85 \times 0.96 \times 1198.8} = 8.178 \text{ kg/s} = 29.44 \text{ t/h}$$

$$h_1 - h_3 = 0.92 \times 209.7 = 193 \text{ kJ/kg}$$

$$h_3 = 3252.3 \text{ kJ/kg}$$

From Mollier chart,

$$v_3 = 0.17 \text{ m}^3\text{/kg}$$

$$\omega_s = \frac{(A_1V_1)}{v_1} = \frac{A_1 \times 621.2}{0.17} = 8.178 \text{ kg/s}$$

$$A_1 = 0.002238 \text{ m}^2 = \pi D_m h_n \sin \alpha$$

$$V_b = 300 \text{ m/s} = \frac{\pi D_m \times 5000}{60}$$

$$\therefore \quad \pi D_m = 3.6 \text{ m}$$

$$\therefore \quad 3.6 \times h_n \times \sin 15° = 0.002238$$

$$h_n = 0.0024 \text{ m} = 2.4 \text{ mm} \qquad \textit{Ans.}$$

Example 7.20 At a certain point in a 50% reaction turbine, the steam leaving a moving blade row is at 1.5 bar, 0.90 dry. The steam flow rate is 7 kg/s and the turbine speed is 3000 rpm. At entry to the moving blade row, the axial velocity of flow is 0.7 times and at exit from the row 0.75 times the mean blade velocity. The exit angles of both fixed and moving blades are 20°, measured from the plane of rotation, and the height of moving blades at exit is 1/10 of the mean diameter. Determine the height of the moving blades at exit and the power developed in the blade row.

Solution

$\alpha = \beta_2 = 20°$, $V_1 \sin \alpha = 0.7\ V_b$, $V_2 \sin \delta = 0.75\ V_b$ (Fig. E7.20)

$$\pi D_m h_b V_2 \sin \delta = \omega_s v_{\text{exit}}$$

$$v_{\text{exit}} = 0.001052 + 0.9 \times 1.15937 = 1.045 \text{ m}^3\text{/kg}$$

$$\pi D_m (0.1\ D_m)\ 0.75 \frac{\pi D_m \times 3000}{60} = 7 \times 1.045$$

$$D_m^{\ 3} = \frac{7 \times 1.045 \times 60}{\pi \times 0.1 \times 0.75 \times \pi \times 3000} = 0.19764$$

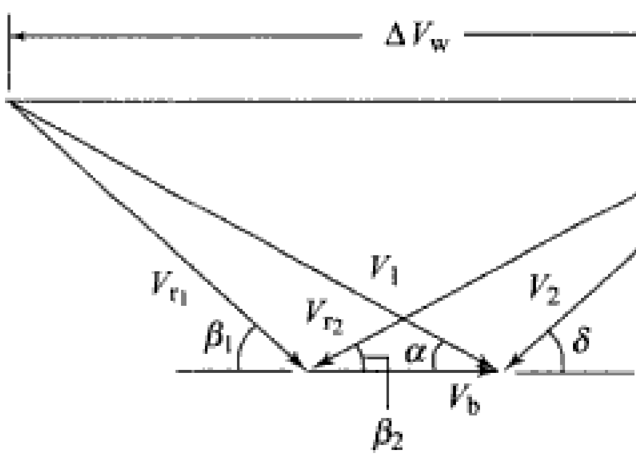


Fig. E7.20

$$D_m = 0.582 \text{ m}$$

$$\therefore \quad h_b = 0.0582 \text{ m} = 58.2 \text{ mm} \qquad \textit{Ans.}$$

$$V_b = \frac{\pi \times 0.582 \times 3000}{60} = 91.42 \text{ m/s}$$

$$\Delta V_w = 2V_1 \cos\alpha - V_b = 2\frac{0.7\,V_b}{\sin\alpha}\cos\alpha - V_b$$

$$= \left(\frac{1.4}{\tan 20^\circ} - 1\right) V_b = 260.22 \text{ m/s}$$

Power developed $= \omega_s\,\Delta V_w\,V_b = 7 \times 260.22 \times 91.42 \times 10^{-3}$

$= 166.53$ kW *Ans.*

Example 7.21 Steam expands in a turbine from 40 bar, 500 °C to 0.10 bar isentropically. Assuming ideal conditions, determine the mean diameter of the wheel if the turbine were of (a) single impulse stage, (b) single 50% reaction stage, (c) four pressure (or Rateau) stages, (d) one two-row Curtis stage, and (e) four 50% reaction stages. Take the nozzle angle as 16 ° and N as 300 rpm.

Solution

$$h_1 = 3445.3 \text{ kJ/kg},$$

$$s_1 = 7.0901 \text{ kJ/kg K} = s_2 = 0.6493 + x_{2s} \times 7.5009$$

$$x_{2s} = 0.86$$

$$h_{2s} = 191.83 + 0.86 \times 2392.8 = 2246.5 \text{ kJ/kg}$$

$$h_1 - h_{2s} = 1198.8 \text{ kJ/kg}$$

(a) $$V_1 = 44.72\,(1198.8)^{1/2} = 1548.37 \text{ m/s}$$

$$\frac{V_b}{V_1} = \frac{\cos\alpha}{2} = \frac{\cos 16^\circ}{2} = 0.4806$$

$$V_b = 744.2 \text{ m/s} = \frac{\pi D_m N}{60}$$

$D_m = 4.73$ m *Ans.*

(b) $$V_1 = 44.72\left[\frac{\Delta h_{stage}}{2}\right]^{1/2} = 44.72\left[\frac{1198.8}{2}\right]^{1/2}$$

$$= 1094.86 \text{ m/s}$$

$$\frac{V_b}{V_1} = \cos\alpha = \cos 16^\circ$$

$$V_b = 1052.4 \text{ m/s} = \frac{\pi D_m N}{60}$$

$\therefore$ $D_m = 6.7$ m *Ans.*

(c) $$V_1 = 44.72\left[\frac{1198.8}{4}\right]^{1/2} = 774.19 \text{ m/s}$$

$$\frac{V_b}{V_1} = \frac{\cos\alpha}{2},\ V_b = 372.07 \text{ m/s} = \frac{\pi D_m N}{60}$$

$D_m = 2.37$ m *Ans.*

(d) $$\frac{V_b}{V_1} = \frac{\cos\alpha}{4} = 0.2403,\ V_1 = 1548.37 \text{ m/s}$$

$$V_b = 372.1 \text{ m/s} = \frac{\pi D_m N}{60}$$

$D_m = 2.368$ m *Ans.*

(e) Four 50% reaction stages

$$\therefore \quad \Delta h_{stage} = \frac{1198.8}{4} = 299.7 \text{ kJ/kg}$$

$$V_1 = 44.72\left[\frac{\Delta h_{stage}}{2}\right]^{1/2} = 547.43 \text{ m/s}$$

$$\frac{V_b}{V_1} = \cos a$$

$$\therefore \quad V_b = 526.22 = \frac{\pi D_m N}{60}$$

$\therefore$ $D_m = 3.35$ m *Ans.*

Example 7.22 A steam turbine is to operate betwen 150 bar, 600 °C and 0.1 bar. The bucket velocity is limited to 300 m/s and the average nozzle efficiency is expected to be 95%, except for a 2-row Curtis stage for which it will be 90%. Nozzle angles will be assumed as 15° for impulse stages and 25° for reaction stages. All stages operate close to the speed of maximum efficiency. Estimate the number of stages required for each of the following arrangements:

(a) all simple impulse stages
(c) all 50% reaction stages
(c) a 2-row Curtis stage followed by simple impulse stages
(d) a 2-row Curtis stage followed by 50% reaction stages.

Solution With reference to Fig. E7.22.

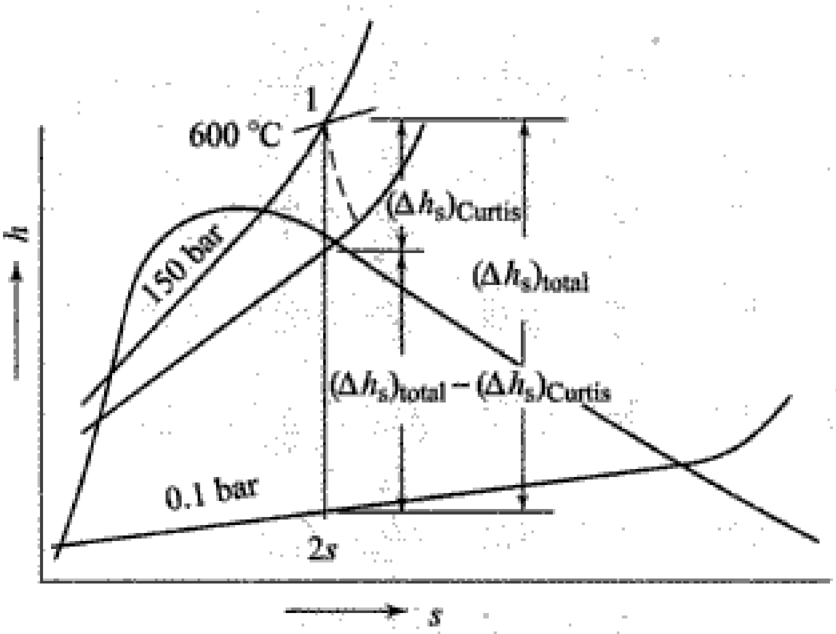


Fig. E7.22

$$h_1 = 3582.3 \text{ kJ/kg}$$
$$s_1 = 6.6776 \text{ kJ/kg K} = s_{2s} = 0.6493 + x_{2s}\, 7.5009$$
$$x_{2s} = 0.804,\ h_{2s} = 191.83 + 0.804 \times 2392.8 = 2114.9 \text{ kJ/kg}$$
$$h_1 - h_{2s} = (\Delta h_s)_{\text{total}} = 1467.4 \text{ kJ/kg}$$

(a) All simple impulse stages

$$\frac{V_b}{V_1} = \frac{\cos\alpha}{2} = \frac{\cos 15°}{2}$$

$$V_1 = \frac{300 \times 2}{\cos 15°} = 621.2 \text{ m/s}$$

$$V_1 = 44.72\,[(\Delta h_s)_{\text{stage}} \times h_n]^{1/2} = 621.2 \text{ m/s}$$

$$(\Delta h_s)_{\text{stage}} = \left(\frac{621.2}{44.72}\right)^2 \times \frac{1}{0.95} = 203.1 \text{ kJ/kg}$$

∴ Number of simple impulse stages required

$$= \frac{1467.4}{203.1} = 7.22 \quad \text{or} \quad 8 \text{ stages} \qquad \textit{Ans.}$$

(b) All 50% reaction stages

$$\frac{V_b}{V_1} = \cos a = \cos 25°,\ \eta_n = 0.90$$

$$V_1 = \frac{300}{\cos 25°} = 331 \text{ m/s}$$

$$V_1 = 44.72 \left[\frac{(\Delta h_s)_{\text{stage}}}{2} \times \eta_n\right]^{1/2} = 331 \text{ m/s}$$

$$(\Delta h_s)_{\text{stage}} = 115.34 \text{ kJ/kg}$$

∴ Number of 50% reaction stages required

$$= \frac{1467.4}{115.34} = 12.72 \quad \text{or} \quad 13 \text{ stages} \quad \textit{Ans.}$$

(c) A 2-row Curtis stage followed by simple impulse stages

$$\frac{V_b}{V_1} = \frac{\cos\alpha}{4}$$

$$V_1 = \frac{300 \times 4}{\cos 15°} = 1242.4 \text{ m/s}$$

$$V_1 = 44.72\,[(\Delta h_s)_{\text{stage}} \times \eta_n]^{1/2}$$

$$(\Delta h_s)_{\text{stage}} = \left(\frac{1242.4}{44.72}\right)^2 \times \frac{1}{0.90} = 857.58 \text{ kJ/kg}$$

$$\therefore (\Delta h_s)_{\text{impulse}} = 1467.4 - 857.6 = 609.8 \text{ kJ/kg}$$

$$\text{Number of impulse stages required} = \frac{609.8}{203.1} = 3 \quad \text{or} \quad 3 \text{ stages}$$

1, 2-row Curtis + 3 simple impulse stages *Ans.*

(d) A 2-row Curtis followed by 50% reaction stages

$$(\Delta h_s)_{\text{reaction}} = 609.8 \text{ kJ/kg}$$

∴ Number of 50% reaction stages required

$$= \frac{609.8}{115.34} = 5.28 \quad \text{or} \quad 6 \text{ stages}$$

∴ 1, 2 - row Curtis + 6 50% reaction stages *Ans.*

Example 7.23 Steam at 20 bar, 400 °C expands in a steam turbine to 0.1 bar. There are four stages in the turbine and the total enthalpy drop is divided equally among the stages. The stage efficiency is 75% and it is the same in all the stages. Determine the interstage pressures, the reheat factor and the turbine internal efficiency.

Solution From Mollier chart (Fig. E7.23),

$$h_1 - h_{6s} = 3250 - 2282 = 968 \text{ kJ/kg}$$

$$h_1 - h_{2s} = 968/4 = 242 \text{ kJ/kg}$$

The interestage pressures, p_2, p_3 and p_4 as read from the Mollier chart are 8 bar, 2.6 bar and 0.60 bar, respectively.

$$h_1 - h_2 = 0.75 \times 242 = 181.5 \text{ kJ/kg}$$

$$h_2 - h_{3s} = 3060 - 2800 = 260 \text{ kJ/kg}$$
$$h_2 - h_3 = 0.75 \times 260 = 195 \text{ kJ/kg}$$
$$h_3 - h_{4s} = 2870 - 2605 = 265 \text{ kJ/kg}$$

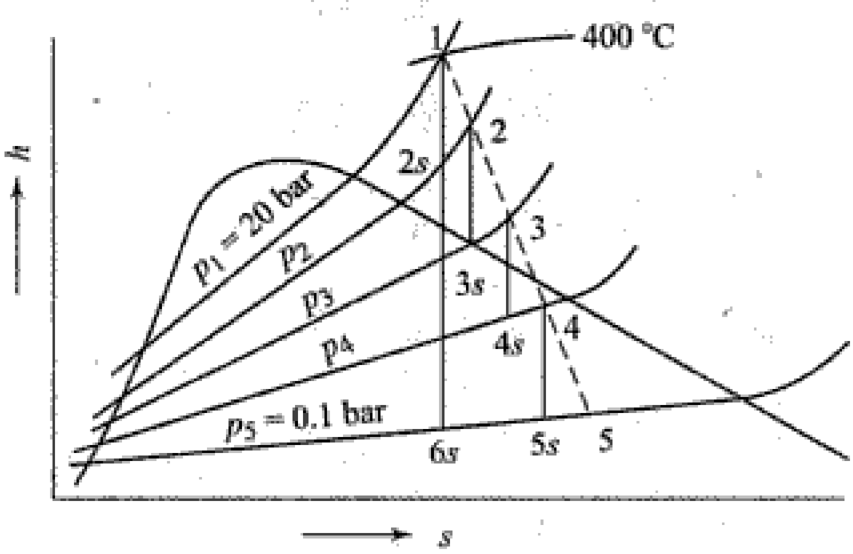


Fig. E7.23

$$h_3 - h_4 = 0.75 \times 270 = 202.5 \text{ kJ/kg}$$
$$h_4 - h_{5s} = 2680 - 2410 = 270 \text{ kJ/kg}$$
$$h_4 - h_5 = 0.75 \times 270 = 202.5 \text{ kJ/kg}$$
$$h_5 = 2470 \text{ kJ/kg}, x_5 = 0.958$$

$$\text{Reheat factor} = \frac{(h_1 - h_{2s}) + (h_2 - h_{3s}) + (h_3 - h_{4s}) + (h_4 - h_{5s})}{h_1 - h_{6s}}$$

$$= \frac{242 + 260 + 265 + 270}{984} = \frac{1037}{968} = 1.071$$

$$\eta_{\text{internal}} = \frac{h_1 - h_5}{h_1 - h_{6s}} = \frac{780}{968} = 0.805 \quad \text{or} \quad 80.5\%$$

Also, $\eta_{\text{internal}} = h_{\text{stage}} \times \text{RF} = 0.75 \times 1.071 = 0.803$ or, 80.3% *Ans.*

Example 7.24 Steam which is initially dry and saturated at an absolute temperature T_1, expands in a turbine to an absolute temperature T_2, the stage efficiency being η_s. Assuming a very large number of stages and that the condition curve on the T–s diagram is a straight line, show that the reheat factor,

$$R = \frac{T_1 + T_2}{2T_2 + \eta_s(T_1 - T_2)}$$

Is the actual reheat factor greater or less than this approximate value?

Solution By definition (Fig. E7.24)

(a) the flow rate of steam required, assuming that all stages develop equal work,
(b) the mean blade diameter,
(c) the speed of the rotor.

Solution

$$\eta_{\text{internal}} = \eta_{\text{st}} \times \text{RF} = 0.75 \times 1.04 = 0.78$$

From Mollier chart, $\Delta h_s = 855$ kJ/kg (Fig. E7.25)

$$\therefore \quad \Delta h_{\text{act}} = 0.78 \times 855 = 667 \text{ kJ/kg}$$

$$\omega_s (\Delta h)_{\text{act}} = 12 \text{ MW}$$

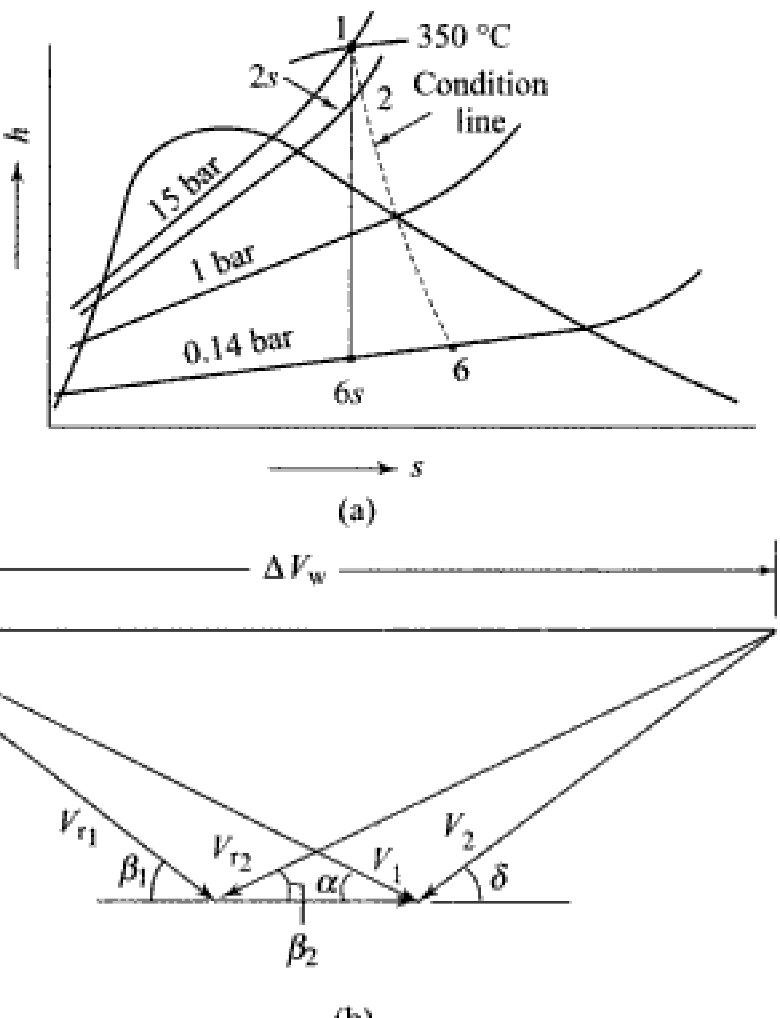


Fig. E7.25

$$\therefore \quad \omega_s = \frac{12000}{667} = 17.99 \text{ kg/s} \qquad \textit{Ans.} \text{ (a)}$$

Work done per kg of steam flow,

$$W_D = V_b (2V_1 \cos \alpha - V_b)$$

Now, $$\frac{V_b}{V_1} = 0.7, \alpha = 20°$$

$$W_D = V_b\left(\frac{2V_b}{0.7}\cos 20° - V_b\right) = 1.6848\ V_b^2 \text{ J/kg}$$

Enthalpy drop per stage = work done per stage = $\frac{667}{20}$ = 33.35 kJ/kg

$\therefore \quad 1.6848\ V_b^2 \times 10^{-3} = 33.35$

$\therefore \quad V_b = 141.4$ m/s

$$V_1 \sin\alpha = \frac{V_b}{0.7}\sin\alpha = \frac{141.4}{0.7}\sin 20° = 69.1 \text{ m/s}$$

Volume flow rate per sec at 1 bar

$$= \pi D_m h_b \times V_1 \sin\alpha = \pi D_m \frac{D_m}{12} \times 69.1 = 18.09\ D_m^2 \text{ m}^3\text{/s}$$

At 1 bar, $\quad v_g = 1.694$ m³/kg

$$\omega_s = \frac{18.09\ D_m^2}{1.694} = 17.99 \text{ kg/s}$$

$\therefore \quad D_m = 1.298$ m $\qquad$ *Ans.* (b)

$$V_b = 141.4 = \frac{\pi \times 1.298 \times N}{60}$$

$\therefore \quad N = 2081$ rpm $\qquad$ *Ans.* (c)

Example 7.26 The first stage of a steam turbine is a two-row velocity compounded impulse wheel. The steam velocity at inlet is 600 m/s and the mean blade velocity is 120 m/s. The nozzle angle is 16° and the exit angles of the first row of moving blades, fixed blades, and second row of moving blades are 18°, 21° and 35°, respectively. The steam flow rate is 5 kg/s and the nozzle height is 25 mm. Neglecting the nozzle wall thickness, estimate the length of the nozzle arc. The specific volume of steam leaving the nozzles is 0.375 m³/kg. Assuming that all the blades have a pitch of 25 mm and an exit tip thickness of 0.5 mm, calculate the blade height at exit from each row. Take k_b as 0.9 for all blades.

Solution By continuity equation,

$$\omega_s = \frac{x\pi D_m h_n V_1 \sin\alpha\ k_{in}}{v_1}$$

where $(x\,\pi D_m)$ is the length of the nozzle arc.

$$x\,\pi D_m = \frac{5 \times 0.375}{\sin 16° \times 600 \times 0.02} = 0.454 \text{ m} \qquad \textit{Ans.}$$

$$0 + t = p \sin\beta_2 \text{ (Fig. E7.26)}$$

$$0 = p\sin\beta_2 - t = \text{opening for steam flow}$$

$$A_b = 0 \times h_b \times z$$

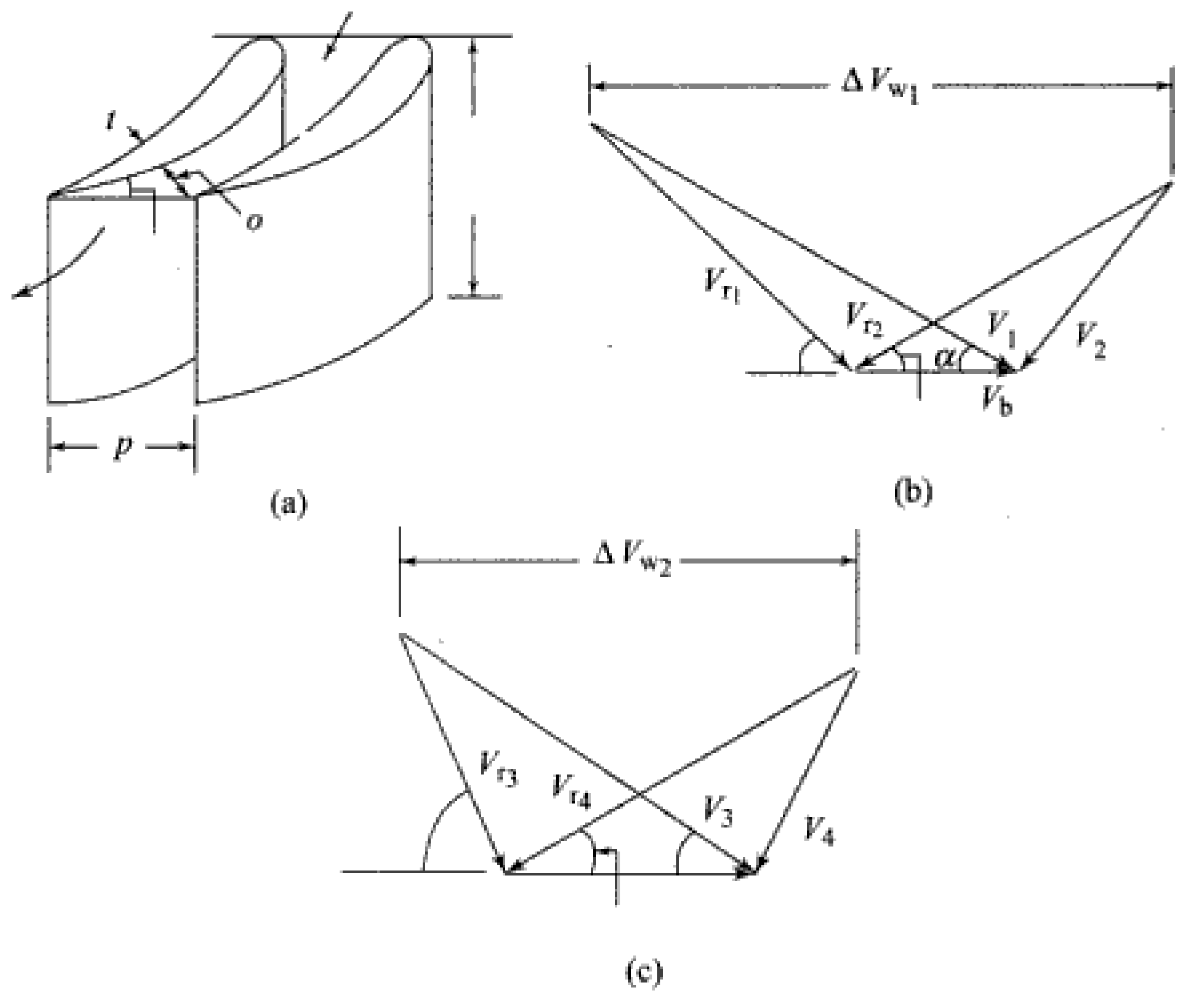


Fig. E7.26

where z = number of such openings = $(\pi D_m)/p$

$$A_b = \frac{\pi D_m}{.p} h_b (p \sin \beta_2 - t)$$

By continuity equation,

$$\omega v_2 = \frac{\pi D_m}{p} h_b (p \sin \beta_2 - t) V_{r_2} \quad (1)$$

for each row of blades.

Given: $\alpha = 16°$, $\beta_2 = 18°$, $\alpha_1 = 21°$, $\beta_4 = 35°$, $V_b = 120$ m/s,

$$V_1 = 600 \text{ m/s}$$

$$\tan \beta_1 = \frac{V_1 \sin \alpha}{V_1 \cos \alpha - V_b} = \frac{600 \sin 16°}{600 \cos 16° - 120} = \frac{165.38}{456.76}$$

$$\beta_1 = 19.9 = 20°$$

$$V_{r_1} = \frac{V_1 \sin \alpha}{\sin \beta_1} = \frac{600 \sin 16°}{\sin 20°} = 483.54 \text{ m/s}$$

$$V_{r_2} = 435.2 \text{ m/s}$$

$$V_2^2 = V_{r2}^2 + V_b^2 - 2V_{r2}V_b \cos\beta_2$$
$$= (435.2)^2 + (120)^2 - 2(435.2)(120)\cos 18°$$
$$= 189399 + 14400 - 99336 = 104463$$
$$V_2 = 323.2 \text{ m/s}, V_3 = 291 \text{ m/s}$$
$$\tan\beta_3 = \frac{V_3 \sin\alpha_1}{V_3\cos\alpha_1 - V_b} = \frac{291\sin 21°}{291\cos 21° - 120} = \frac{104.29}{151.67}$$
$$\beta_3 = 34.5°$$
$$V_{r_3} = \frac{V_3\sin\alpha_1}{\sin\beta_3} = \frac{291\sin 21°}{\sin 34.5°} = 184.13 \text{ m/s}$$
$$V_{r_4} = 166 \text{ m/s}$$

First row of moving blades: From Eq. (1),

$$5 \times 0.375 = \frac{0.454}{0.025}(0.025\sin 18° - 0.0005) \times h_{b_1} \times 435.2$$
$$h_{b_1} = \frac{5 \times 0.375 \times 0.025}{0.454 \times 0.00723 \times 435.2}$$
$$= 0.0331 \text{ m} = 33.1 \text{ mm} \qquad \textit{Ans.}$$

Fixed row of guide blades: From Eq. (1),

$$5 \times 0.375 = \frac{0.454}{0.025}(0.025\sin 21° - 0.0005) \times h_n \times 291$$
$$h_n = \frac{5 \times 0.375 \times 0.025}{0.454 \times 0.00846 \times 291} = 0.0420 \text{ m} \qquad \textit{Ans.}$$

Second row of moving blades: From Eq. (1),

$$5 \times 0.375 = \frac{0.454}{0.025}(0.025\sin 35° - 0.0005) \times h_{b_2} \times 166$$
$$\therefore \quad h_{b_2} = 0.0446 \text{ m} = 44.6 \text{ mm} \qquad \textit{Ans.}$$

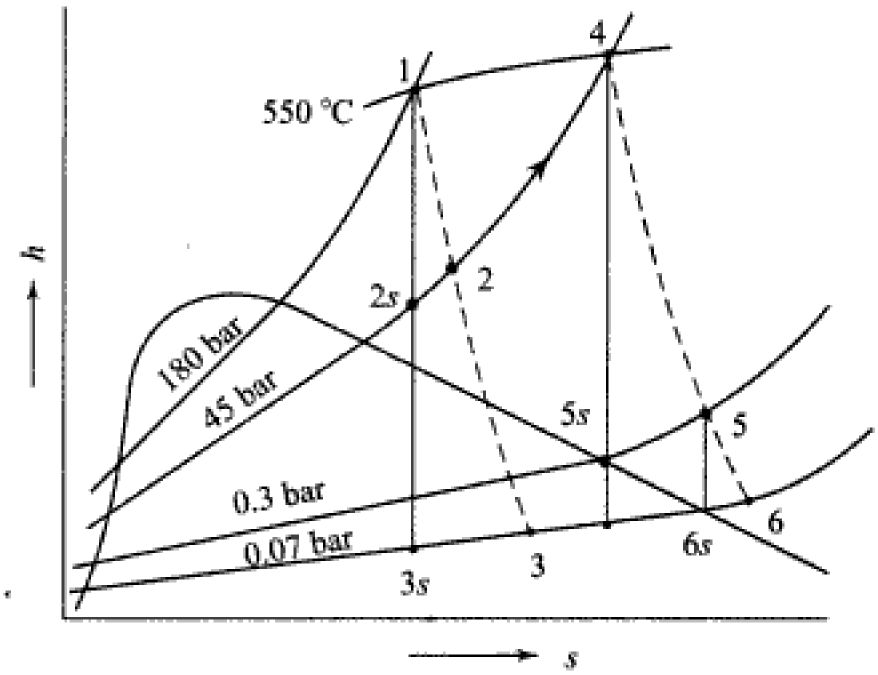
Example 7.27 Give the casing arrangements for turbines having the following ratings:

(a) 200 MW, steam condition at inlet – 180 bar, 550 °C condenser pressure–0.07 bar.

(b) 600 MW. steam condition at inlet – 300 bar, 580 °C condenser pressure–0.10 bar.

Assume 90% turbine efficiency in each case. Find also the intercasing steam condition.

Solution Steam expands in the HP turbine from state 1 to state 2, when it is reheated in the boiler till the original temperature of 550 °C is reached. The reheat pressure is assumed to be 1/4 th the boiler pressure (180 bar), i.e. 45 bar (Fig. E7.27a)



(a)

Fig. E7.27(a)

From Mollier chart,

$$h_1 = 3430 \text{ kJ/kg}, h_{2s} = 3040 \text{ kJ/kg}$$

$$h_1 - h_{2s} = 390 \text{ kJ/kg}$$

$$h_1 - h_2 = 0.9 \times 390 = 351 \text{ kJ/kg}$$

$$h_2 = 3070 \text{ kJ/kg}, v_2 = 0.06 \text{ m}^3\text{/kg}$$

$$h_4 = 3560 \text{ kJ/kg}, h_{3s} = 2000 \text{ kJ/kg}$$

$$\therefore \quad h_1 - h_{3s} = 3430 - 2000 = 1430 \text{ kJ/kg}$$

$$\therefore \quad h_1 - h_3 = 0.9 \times 1430 = 1287 \text{ kJ/kg}, v_3 = 21 \text{ m}^3\text{/kg}$$

$$\omega_s (h_1 - h_3) = 200 \times 10^3 \text{ kW}$$

$$\therefore \quad \omega_s = 155.4 \text{ kg/s}$$

Assuming $(V_b)_{max} = 350$ m/s and $\alpha = 25°$,

$$V_b = \frac{\pi D_m N}{60}, D_m = \frac{350 \times 60}{\pi \times 3000} = 2.215 \text{ m}$$

Assuming for the last stage,

$$(h_b/D_m)_{max} = 0.3 \text{ (with tapered/twisted blades)}$$

$$h_b = 0.3 \times 2.215 = 0.6645 \text{ m}$$

Flow area,

$$A_b = \pi D_m h_b k_{tb} \sin \alpha$$
$$= \pi \times 2.215 \times 0.6645 \times 0.9 \times 0.4226 = 1.76 \text{ m}^2$$

Assuming reaction blading,

$$\frac{V_b}{V_1} = \cos \alpha,$$

$$\therefore \qquad V_1 = 386.2 \text{ m/s}$$

Volume flow of steam that one last stage can accommodate

$$= A_b\, V_1 = 1.76 \text{ m}^2 \times 386.2 \text{ m/s} = 679.7 \text{ m}^3/\text{s}$$

Steam expands in the IP turbine from state 4 to a state 5, such that the last stage can accommodate the required volume flow.

Specific volume at the exhaust of a single casing should not exceed

$$= \frac{697.7 \text{ m}^3/\text{s}}{155.4 \text{ kg/s}} = 4.37 \text{ m}^3/\text{kg}$$

$$h_{5S} = 2456 \text{ kJ/kg},\ p_5 = 0.36 \text{ bar}$$

Steam expands in the IP turbine to a pressure of 0.36 bar.

$$h_4 - h_{5S} = 3560 - 2456 = 1104 \text{ kJ/kg}$$

$$h_4 - h_5 = 993.6 \text{ kJ/kg}$$

$$h_5 = 2566.4 \text{ kJ/kg},\ x_5 = 0.952$$

$$h_5 - h_{6s} = 2566.4 - 2340 = 226.4 \text{ kJ/kg}$$

$$h_5 - h_6 = 203.76 \text{ kJ/kg}$$

$$h_6 = 2362.64 \text{ kJ/kg},\ v_6 = 18 \text{ m}^3/\text{kg}$$

Maximum mass flow that one last stage can accommodate

$$= \frac{679.7}{18} = 37.76 \text{ kg/s}$$

$$\therefore \text{ Number of parallel exhausts} = \frac{155.4}{37.76} = 4.11$$

By suitable adjustment of exit blade angles, the number of parallel exhausts can be made to be 4.

The conditions of steam in between the casings are shown in the diagram (Fig. E7.27(b)).

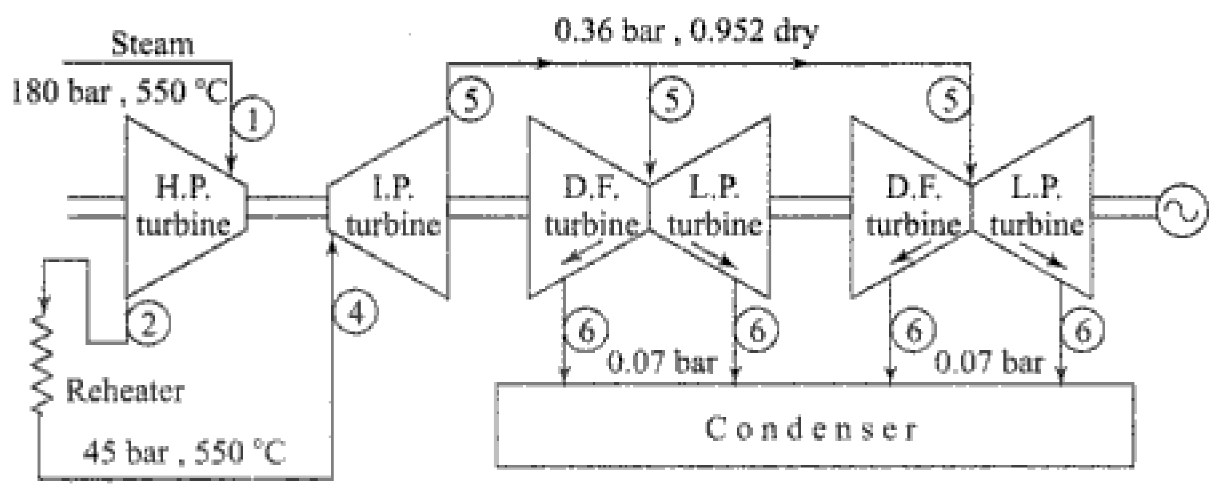


Fig. E7.27(b)

(b) The state of steam at turbine inlet: 300 bar, 580 °C.

Reheat pressure (assumed) $= \frac{1}{4} \times 300 = 75$ bar

$$h_1 = 3410,\ h_{2s} = 3015 \text{ kJ/kg}$$
$$h_1 - h_{2s} = 395 \text{ kJ/kg},$$
$$h_1 - h_2 = 0.9 \times 395 = 355.5 \text{ kJ/kg},\ v_2 = 0.035 \text{ m}^3\text{/kg}$$
$$h_4 = 3060 \text{ kJ/kg},\ h_{3s} = 1960 \text{ kJ/kg},\ h_1 - h_{3s} = 1450 \text{ kJ/kg}$$
$$h_1 - h_3 = 1305 \text{ kJ/kg},\ h_3 = 2105 \text{ kJ/kg}$$
$$\omega_s = \frac{600 \times 1000}{1305} = 459.8 \text{ kg/s}$$

Steam expands in the HP turbine up to the reheat pressure.

The maximum specific volume to which steam can expand in the IP turbine, the last stage of which has the maximum blade dimensions,

$$v_{max} = \frac{679.7}{459.8} = 1.478 \text{ m}^3\text{/kg}$$

Volume flow rate at the HP turbine exhaust

$$= \omega_s v_2 = 459.8 \times 0.035 = 16.09 \text{ m}^3\text{/s}$$

The pressure up to which steam can expand in the IP turbine cylinder as read from the Mollier Chart is 1.2 bar.

$$h_{5s} = 2300 \text{ kJ/kg},\ h_4 - h_{5s} = 3060 - 2300 = 760 \text{ kJ/kg}$$
$$h_4 - h_5 = 684 \text{ kJ/kg},\ h_5 = 2376 \text{ kJ/kg},\ v_5 = 1.25 \text{ m}^3\text{/kg},$$
$$x_5 = 0.86,\ h_{6s} = 2050,\ h_5 - h_{6s} = 2376 - 2050 = 326 \text{ kJ/kg}$$
$$h_5 - h_6 = 0.9 \times 326 = 293.4,\ h_6 = 2082.6 \text{ kJ/kg},$$
$$v_6 = 12 \text{ m}^3\text{/kg},\ x_6 = 0.792$$

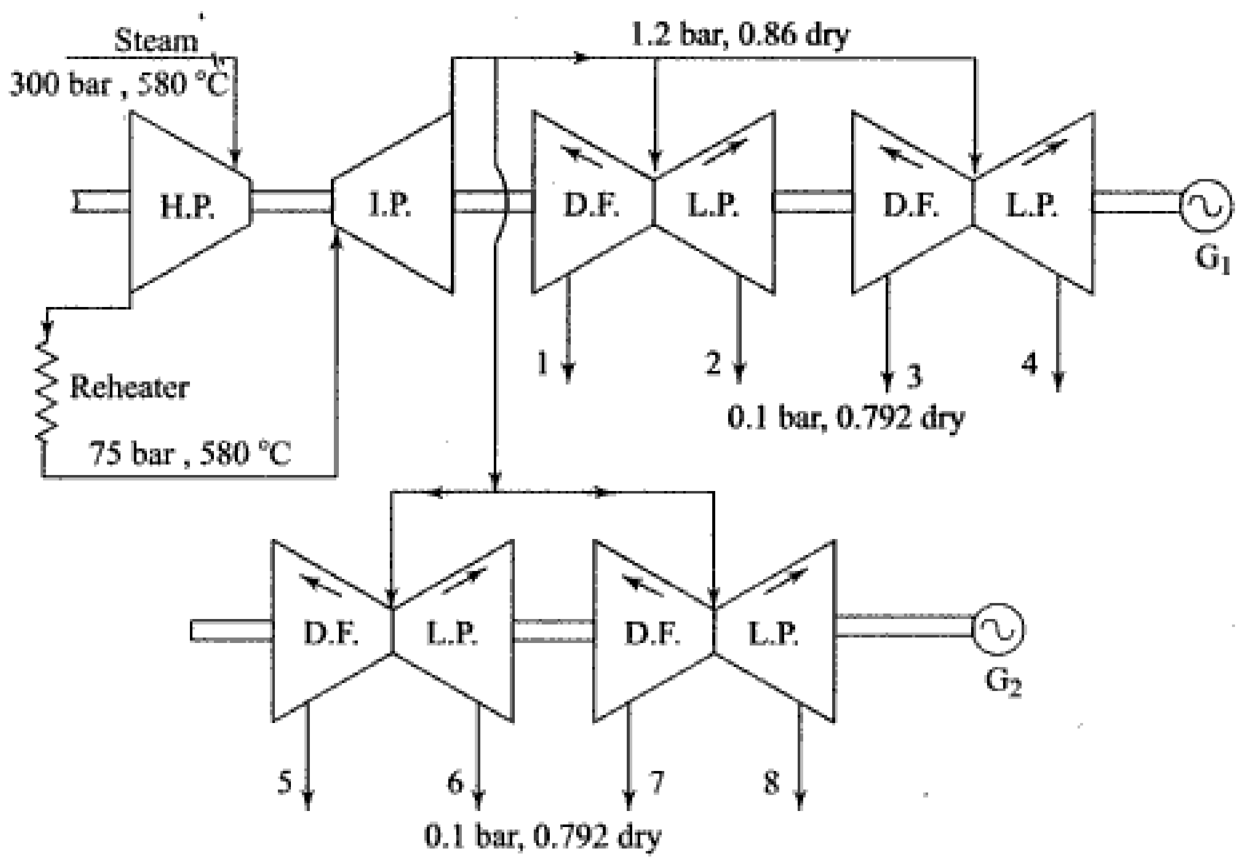
Maximum mass flow that one last stage can accommodate

$$= \frac{679.7}{12} = 56.64 \text{ kJ/kg}$$

$\therefore$ Number of parallel exhausts $= \frac{459.8}{56.64} = 8.11$

By adjusting blade exit angles and since the required mass flow will be somewhat less than estimated, the number of streams to which steam will be split in the LP turbines can be made eight. The maximum number of turbine cylinders that can be mounted on a single shaft in tandem arrangement is six. Therefore, the present turbine is cross-compounded with two shafts and two electric generators as shown (Fig. E7.27(c)).

Since the quality at turbine exhaust is only 0.792, there will be considerable erosion of turbine blades in later stages due to impact of entrained water particles. The cycle parameters need to be adjusted (two reheats may require to be adopted) so that the quality at turbine exhaust does not fall below 0.88.



(c)

Fig. E7.27(c) *Cross compounded octuple flow (8-exhausts) 6-casing, 2-shaft, 3000 rpm, 600 MW steam turbine with reheat*

Example 7.28 A 100 MW turbine generator unit is supplied with steam at 90 bar, 550 °C and the condenser pressure is 0.1 bar. At rated load the steam supplied is 500,000 kg/h and at zero load it is 25,000 kg/h. Determine (a) the steam rate in kg/kWh at 1/4, 1/2, 3/4 and full load, (b) the Rankine cycle efficiency, (c) the actual efficiency at full load of the plant assuming 100% boiler efficiency, (d) the turbogenerator efficiency at full load based on generator output.

Solution From Willan's line law, $\omega_s = a + bL$

where ω_s is the steam consumption rate in kg/h and L is the load in kW.

At no load, $25{,}000 = a + b \times 0$

$\therefore \quad a = 25{,}000$ kg/h

At full load,

$$500{,}000 = 25{,}000 + b \times 100{,}000$$

$$\therefore \quad b = 4.76 \text{ kg/kWh}$$

$$\omega_s = 25{,}000 + 4.75\, L$$

$$\therefore \quad \text{Steam rate} = \frac{\omega_s}{L} = \frac{25000}{L} + 4.75 \text{ kg/kWh}$$

(a) At one-fourth load,

$$L = 25{,}000 \text{ kW}$$

$$\therefore \qquad \text{Steam rate} = \frac{25{,}000}{25{,}000} + 4.75 = 5.75 \text{ kg/kWh}$$

At half the load, $L = 50{,}000$ kW

$$\therefore \qquad \text{Steam rate} = \frac{25{,}000}{50{,}000} + 4.75 = 5.25 \text{ kg/kWh}$$

At three-fourth load,

$$L = 75{,}000 \text{ kW}$$

$$\therefore \qquad \text{Steam rate} = \frac{25{,}000}{75{,}000} + 4.75 = 5.08 \text{ kg/kWh}$$

At full load, $L = 100{,}000$ kW

$$\therefore \qquad \text{Steam rate} = \frac{25{,}000}{100{,}000} + 4.75 = 5.0 \text{ kg/kWh}$$

(b)
$$h_1 = 3511 \text{ kJ/kg}, s_1 = 6.8142 \text{ kJ/kg K} = s_{2s}$$
$$= 0.6493 + x_{2s}\, 7.5009$$
$$x_{2s} = 0.822$$
$$h_{2s} = 191.83 + 0.822 \times 2392.8 = 2158.44 \text{ kJ/kg}$$

Neglecting pump work,

$$\eta_{\text{Rankine}} = \frac{3511 - 2158.44}{3511 - 191.83} = \frac{1352.56}{3319.17} = 0.407 \quad \text{or} \quad 40.7\%$$

(c)
$$\eta_{\text{actual}} = \eta_{\text{cycle}} \times \eta_{\text{boiler}} \times \eta_{\text{turbogenerator}}$$
$$= \frac{\omega_s(h_1 - h_{2s})}{\omega_s(h_1 - h_4)} \times 1 \times \frac{100 \times 1000 \text{ kW}}{\omega_s(h_1 - h_{2s})} = \frac{100 \times 1000}{\omega_s(h_1 - h_3)}$$
$$= \frac{100 \times 1000 \times 3600}{500{,}000 \times 3319.17} = 0.217 \quad \text{or} \quad 21.7\% \qquad \textit{Ans.}$$

(d)
$$\eta_{\text{TG}} = \frac{100{,}000}{\omega_s(h_1 - h_{2s})} = \frac{100{,}000 \times 3600}{500{,}000 \times 1352.56}$$
$$= 0.532 \quad \text{or} \quad 53.2\% \qquad \textit{Ans.}$$

SHORT-ANSWER QUESTIONS

7.1 How does a steam turbine convert energy in steam to shaft work?
7.2 What do you understand by (a) a nozzle, (b) a diffusor?
7.3 What is a compressible fluid?
7.4 What is the velocity of a pressure pulse in an isentropic flow?

7.5 What is the velocity of sound in an ideal gas? On what factors does it depend?
7.6 What is a stagnation state?
7.7 What are stagnation pressure and stagnation temperature?
7.8 Explain (a) supersonic nozzle, (b) subsonic nozzle, (c) subsonic diffusor, (d) supersonic diffusor.
7.9 What do you understand by (a) critical pressure ratio, (b) choked flow?
7.10 What is critical discharge?
7.11 What is the critical pressure ratio for (a) air, (b) dry saturated steam, and (c) superheated steam?
7.12 What is Zeuner's relation?
7.13 Explain (a) nozzle efficiency, (b) velocity coefficient, (c) coefficient of discharge.
7.14 What is shock? Where does it occur?
7.15 What do you understand by (a) Supersaturated flow, (b) metastable state?
7.16 What is Wilson line?
7.17 What is (a) degree of supersaturation, (b) degree of supercooling?
7.18 How does energy conversion occur in (a) impulse blades (b) reaction blades?
7.19 Define (a) diagram power, (b) diagram efficiency.
7.20 What is the optimum velocity ratio for (a) an impulse stage, (b) a two-row Curtis stage, (c) a 50% reaction stage?
7.21 Why are steam turbines compounded? What are the different methods of compounding?
7.22 How is the number of stages in a turbine estimated?
7.23 Why does the effectiveness of a Curtis stage decrease as the number of rows of moving blades increases?
7.24 How is degree of reaction defined? What is a 50% reaction turbine?
7.25 Why are reaction blades unsymmetrical?
7.26 Compare the diagram efficiencies of impulse, two-row Curtis and 50% reaction stages.
7.27 Compare the stage enthalpy drops of impulse, two-row Curtis and 50% reaction stages.
7.28 Why is a two-row Curtis stage most often used as the first stage in large steam turbines?
7.29 Show that the diagram work per unit mass of steam for maximum blading efficiency of a 50% reaction stage is V_b^2, where V_b is the mean blade velocity.
7.30 What do you understand by carryover efficiency?
7.31 How would you estimate nozzle and blade heights?
7.32 What do you understnad by partial admission of steam?
7.33 How are the last stage blade dimensions fixed up?
7.34 What are parallel exhausts? Why are these needed?
7.35 Enlist the various losses taking place in a steam turbine.
7.36 What do you understand by throttle governing and nozzle governing?
7.37 What is bypass governing?
7.38 What do you understand by Willan's line?
7.39 What is the function of a governor?
7.40 Why is an oil operated servo system added to a governor?

7.41 What is an emergency governor? When does it operate?
7.42 What is disc friction loss?
7.43 What is windage loss?
7.44 Why is there some energy loss due to wetness of steam?
7.45 What is leakage loss? How is it restricted?
7.46 What are labyrinth glands? Where are they used?
7.47 Name some supervisory and control instruments used in steam turbines.
7.48 What do you undersand by reheat factor and condition line? Why is RF greater than unity?
7.49 What is a dummy piston? Where is it used?
7.50 What are the stresses to which turbine blades are subjected?
7.51 Why are the turbine blades tapered towards the tip?
7.52 How are blades fastened to the disc or drum?
7.53 What are shrouds? Where are they used?
7.54 What are lacing wires? Why are they used?
7.55 What is critical speed?
7.56 What is (a) flexible shaft, (b) rigid shaft?
7.57 What is a turning gear? When is it used?
7.58 Why is hydrogen gas used for cooling of the generator? How is hydrogen filled into it?

PROBLEMS

7.1 Steam at 30 bar, 350 °C expands through a convergent–divergent nozzle. The exit plane pressure is 3 bar. The flow rate is 0.5 kg/s and the nozzle efficiency is 0.8. Assuming that the velocity at inlet is negligible, determine the throat and exit areas, steam velocity at the exit, and the quality of steam at the exit plane. The critical pressure ratio can be taken as 0.546.

[*Ans.* 127.6 mm^2, 323.5 mm^2, 922 m/s, $x_2 = 0.985$]

7.2 Dry saturated steam at 10 bar is expanded in a convergent-divergent nozzle. The velocity of steam at exit is 685 m/s, the flow rate is 7 kg/s and the nozzle efficiency is 85%. Assume the flow to be isentropic up to the throat. The critical pressure ratio can be taken as 0.54. Determine the throat and exit areas of the nozzle and the pressure at exit. Neglect the velocity of steam at inlet to the nozzle.

[*Ans.* 4870 mm^2, 7000 mm^2, 2.4 bar]

7.3 Dry saturated steam at 26 bar expands isentropically in a convergent-divergent nozzle to 12 bar. Determine the mass flow rate per cm^2 of throat area and the steam quality at the nozzle exit if the expansion is assumed to be
(a) in equilibrium, $n = 1.135$
(b) supersaturated, $n = 1.3$

[*Ans.* (a) 0.37 kg/s, 0.935, (b) 0.368 kg/s, superheated]

7.4 A convergent–divergent nozzle receives dry saturated steam and discharges it at a velocity of 800 m/s into a chamber at a pressure of 1.4 bar. The nozzle efficiency is 85% and $n = 1.135$. Estimate the pressure of steam supply. Neglect inlet velocity.
If the mass flow rate of steam is 10 kg/s, determine the throat and exit areas of the nozzle.

[*Ans.* 13 bar, 0.0054 m^2, 0.133 m^2]

7.5 Steam passes through a convergent–divergent nozzle from a pressure of 8 bar. The steam is initially dry saturated ($n = 1.135$). The nozzle efficiency is 90%. Given that the exit area = 2 × throat area, determine the pressure at exit. The inlet velocity is negligible.

[*Ans.* 1.2 bar]

7.6 Show that the maximum mass flow rate through a convergent nozzle passing air is given by

$$m = 4.04 \times 10^{-3} \frac{Ap_0}{\sqrt{T_0}} \text{ kg/s}$$

where, A = nozzle area (mm^2), p_0 = reservoir pressure (bar), and T_0 = reservoir temperature.
Air is discharged from a large container through a convergent nozzle of 10 mm diameter. The conditions in the container are 10 bar, 20 °C. The pressure at the nozzle is 6 bar. Calculate the mass flow rate and the flow Mach number at the nozzle.

[*Ans.* 0.184 kg/s, 0.89]

7.7 Steam at 7 bar, 290 °C expands in a convergent-divergent nozzle to a pressure of 0.55 bar. Assuming a negligible inlet velocity, calculate the mass flow and the nozzle exit area. The area of the nozzle throat is 970 mm^2 and the coefficient of discharge is 0.95. Take a velocity coefficient of 0.92 and the critical pressure ratio of 0.546.

[*Ans.* 0.875 kg/s, 2790 mm^2]

7.8 A convergent nozzle receives steam at 4 bar, 150°C and negligible inlet velocity, and expands it into a space at atmospheric pressure. Assuming supersaturated expansion and a nozzle efficiency of 0.9, calculate the nozzle throat area required for a mass flow of 1.2 kg/s.

[*Ans.* 2010 mm^2]

7.9 Calculate the throat and exit diameters of a convergent–divergent nozzle whcih will discharge 0.25 kg/s of steam from a pressure of 8 bar superheated to 250 °C into a chamber having a pressure of 1.5 bar. Friction loss in the divergent part of the nozzle may be taken as 0.15 of the enthalpy drop. The convergent part is sharp and frictionless. Neglect the inlet velocity of steam.

[*Ans.* 16.5 mm, 21.8 mm]

7.10 Show that the critical velocity at the throat is given by

$$V^* = (n\, p^*\, v^*)^{1/2}$$

where n is the polytropic index of expansion.

7.11 Steam at 7 bar, 200 °C expands isentropically in a convergent-divergent nozzle into a space at 3 bar. Neglecting the inlet velocity, estimate the exit area required for a mass flow of 0.1 kg/s when
(a) the flow is in equilibrium throughout,
(b) the flow is supersaturated with $pv^{1.2} = C$.
Find (b) the degree of supercooling and the degree of supersaturation.

[*Ans.* (a) 103.7 mm^2, (b) 101.5 mm^2, 17.5 °C, 1.713]

7.12 A stage of an impulse steam turbine operates close to the maximum blading efficiency. The blades are equiangular, and the friction effects in blades may be neglected. The mean blade velocity is 200 m/s and the steam flow rate is 0.75 kg/s. Find (a) the discharge angle at which the steam leaves the blades, (b) the diagram power.

[*Ans.* (a) 90°, (b) 60 kW]

7.13 The velocity of steam leaving the nozzle of an impulse turbine is 900 m/s and the nozzle angle is 20°. The blade velocity is 300 m/s and the blade friction factor is 0.7. Calculate for a mass flow rate of 1 kg/s and symmetric blading (a) the blade inlet angle, (b) the driving force on the wheel, (c) the axial thrust, (d) the diagram power, (e) the diagram efficiency.

[*Ans.* (a) 29°24′, (b) 927.7 N (c) 92.3 N, (d) 278.3 kW, (e) 68.7%]

7.14 In a stage of an impulse steam turbine the mean diameter of the blade ring is 800 mm and the speed of rotation is 3000 rpm. The direction of final absolute velocity of steam is axial. The inlet and exit angles of the blades are 30°. Assuming a blade friction factor of 0.85 and a steam flow rate of 1 kg/s, determine (a) the nozzle angle, (b) the absolute velocity of steam leaving the nozzle, (c) the enthalpy drop in the stage, (d) the tangential thrust, (e) the axial thrust, (f) the blading work, and (g) the blading efficiency.

Ans. (a) 17.33°, (b) 72.56 m/s, (c) 41.06 kJ/kg, (d) 273.5 N, (e) 12.8 N, (f) 34.37 kW, (g) 83.7%.

7.15 A single stage impulse turbine rotor has a mean blade ring diameter of 500 mm and rotates at a speed of 10,000 rpm. The nozzle angle is 20° and the steam leaves the nozzles with a velocity of 900 m/s. The blades are equiangular and the blade friction factor is 0.85. Construct the velocity diagrams for the blades and determine the inlet angle of the blades for shockless entry of steam. Determine (a) the diagram power for a steam flow of 750 kg/h, (b) the diagram efficiency, (c) the axial thrust, and (d) the loss of kinetic energy due to friction.

[*Ans.* 28°, (a) 56.8 kW, (b) 0.70, (c) 11.98 kW]

7.16 In an impulse turbine, the nozzle angle is a; the blade inlet and outlet angles are equal; the blade friction factor is k; and the steam velocity at nozzle outlet is V_1. Show that the optimum blade speed is given by $(V_1 \cos \alpha)/2$ and that the optimum blade efficency is $\frac{1+k}{2} \cos^2 \alpha$.

A turbine rotor has the mean diameter of 250 mm and the blade angles are equal. The nozzle angle is 20°, the steam speed at nozzle outlet is 930 m/s and the blade friction factor is 0.85. Find the best angle of the blades, the turbine speed in rpm, the steam consumption to generate 10 kW, and the blade efficiency.

[*Ans.* 36.1°, 33,300 rpm, 102.1 kg/h, 0.816]

7.17 A single stage simple impulse turbine generator set is to operate under the following conditions:

Steam at inlet	: 8 bar, dry saturated
Exhaust	: 0.2 bar
Nozzle efficiency	: 0.90
Nozzle angle	: 15°
Internal efficiency	: 0.75
Mechanical efficiency	: 0.92
Generator efficiency	: 0.90

Find (a) the mean blade speed for maximum blading efficiency, (b) the steam flow rate for developing 200 kW.

[*Ans.* (a) 490 m/s, (b) 0.564 kg/s]

7.18 Deduce a general expression for the blade efficiency of a stage of an impulse turbine with single row wheels, assuming equiangular blades, a nozzle angle α, and a blade friction factor k_b. What is the maximum efficiency if $\alpha = 20$ ° and $k_b = 0.83$? What is the velocity ratio? If the blade efficiency is 90% of the maximum value, what are the values of the velocity ratios? Draw the velocity diagrams for each case and state the blade angles.

[*Ans.* 80.7%, 0.47, 0.32 and 0.62, 36° and 48°6]

7.19 A stage of an impulse steam turbine is velocity compounded with two rows of moving blades. The isentropic enthalpy drop from the stage is 320 kJ/kg, the nozzle angle is 16°, blade speeed is 150 m/s, velocity coefficient is 0.95 and the blade friction factor is 0.9 (all blades). All blades are symmetrical and the steam flow rate is 20 kg/s. Determine the blade angles, power output, stage efficiency and kinetic energy of the steam leaving the stage.

[*Ans.* 19.8°, 26.8°, 42.4°, 4264 kW, 0.667, 234 kW]

7.20 An impulse steam turbine has nozzles inclined at 20° to the plane of rotation. The inlet and exit angles of the moving blades are equal, the blade friction factor is 0.8 and the mean diameter of the blades is 0.5 m. The steam leaves the nozzle with a velocity of 750 m/s. Determine the optimum value of the blade angles, the steam flow rate required to produce 20 kW and the blading efficiency.

[*Ans.* 36°4′, 323 kg/h, 0.795]

7.21 A stage of an axial-flow impulse steam turbine has two rows of moving blades separated by a row of fixed guide blades. The inlet and exit angles of all moving blades are 30°, measured from the plane of rotation. The blade speed of each moving blade row is 130 m/s. The direction of the final absolute velocity of steam leaving the second row of moving blades is axial. Assuming a blade friction coefficient of 0.85 for both fixed and moving blades, determine the velocity of steam leving the nozzles, the work done per kg of steam and the blade efficiency.

[*Ans.* 650 m/s, 149.5 kJ/kg, 0.707]

7.22 The following particulars refer to a two-row velocity-compounded impulse wheel which forms the first stage of a combination turbine:

Steam velocity at nozzle outlet	: 630 m/s
Mean blade velocity	: 125 m/s
Nozzle angle	: 16°
Outlet angle, first row of moving blades	: 18°
Outlet angle, fixed guide blades	: 22°
Outlet angle, second row of moving blades	: 36°
Steam flow rate	: 2.6 kg/s

The ratio of the relative velocity at outlet to that at inlet is 0.84 for all the blades. Determine (a) the velocity of whirl, (b) the tangential thrust on the blades, (c) the axial thrust on the blades, (d) the power developed, and (e) the blading efficiency.

[*Ans.* (a) 1127.5 m/s, (b) 2.93 kN, (c) 188.5 N, (d) 366.4 kW, (e) 0.71]

7.23 Show that in a 50% reaction steam turbine stage, the maximum stage efficiency is

$$\frac{2\cos^2\alpha}{1+\cos^2\alpha}$$

where α is the nozzle angle.

In a particular stage the mean diameter is 500 mm and the blade height is 30 mm. The blade angles are 60° at inlet and 160° at outlet. The density of steam is 2.7 kg/m^3, the speed is 3000 rpm. Calculate the mass flow rate of steam, power developed and stage efficiency.

[*Ans.* 4.61 kg/s, 43.57 kW, 0.915]

7.24 In a reaction stage of a steam turbine the nozzle angle is 20° and the absolute velocity of steam at inlet to the moving blades is 240 m/s. The blade velocity is 210 m/s. If the blading is designed for 50% reaction, determine (a) the blade height at inlet and exit, (b) the enthalpy drop per kg of steam in the moving blades and in the complete stage, (c) the diagram power for a steam flow of 1 kg/s, and (d) the diagram efficiency.

7.25 A 50% reaction turbine is supplied with steam at 60 bar, 600 °C. The condenser pressure is 0.07 bar. If the reheat factor is assumed to be 1.04 and the stage efficiency is constant throughout at 80%. Calculate the steam flow required for a diagram power of 25 MW.

[*Ans.* 21 kg/s]

7.26 Steam at 20 bar, 400 °C expands in a 50% reaction turbine to a pressure 0.2 bar. The steam leaving the turbine is dry saturated. The reheat factor is 1.05 and the isentropic efficiency of each stage is the same throughout. There are 14 stages and the enthalpy drop is the same in each. All the blades have an exit angle of 22° and the mean value of blade velocity ratio is 0.82. The mass flow rate of steam is 34,000 kg/h. The turbine speed is 2400 rpm. Calculate

(a) the stage efficiency, (b) the diagram power, (c) the drum diameter, (d) the blade height for the last row of moving blades, and (e) the pressure at the entry to the last stage.

[*Ans.* (a) 67.7% (b) 6025 kW, (c) 1.34 m, (d) 175 mm, (e) 0.32 bar]

7.27 A stage of a 50% reaction turbine delivers dry saturated steam at 2.7 bar from the fixed blades at 90 m/s. The mean blade height is 40 mm, and the moving blade exit angle is 20°. The axial velocity of steam is 3/4 of the mean blade velocity. Steam flow rate is 9000 kg/h. The effect of blade tip thickness on the annulus area can be neglected. Calculate (a) the wheel speed in rpm, (b) the diagram power, (c) the diagram efficiency, and (d) the enthalpy drop of steam in this stage.

[*Ans.* (a) 1824 rpm, (b) 13.14 kW, (c) 0.787 and (d) 5.26 kJ/kg]

7.28 The nozzles of a single stage impulse turbine discharge the working fluid at an angle of 25° to the plane of rotation of the blades. The fluid leaves the blades with an absolute velocity of 290 m/s and a trailing angle of 60° to the plane of rotation. The blades have equal inlet and outlet angles and there is 5% reduction in the axial velocity component over the stage.

(a) Determine: (i) the blade angles, (ii) the blade work per kg fluid, (iii) the blade velocity coefficient, (iv) the blading efficiency.

(b) If the fluid is steam and the nozzle inlet conditions are 1.5 bar. 150 °C, determine the pressure at the moving blades and express as a percentage of the isentropic enthalpy drop (i) the nozzle loss, (ii) the loss in blade channels, (iii) the disc friction loss, (iv) the residual velocity loss, (v) the net work available.

The nozzle efficiency is 87% and the disc friction loss is 3% of the work done on the blades. Show the losses on an *h–s* diagram for the stage.

7.29 A steam turbine is to operate between 140 bar, 560 °C and 0.075 bar. The maximum blade velocity is 320 m/s and the nozzle efficiency in all stages is 0.90. Nozzle angles will be 15° for impulse stages and 25° for reaction stages. All stages operate close to the maximum efficiency. Estimate the number of stages required for each of the following arrangements:

(a) all simple impulse stages,
(b) all 50% reaction stages,
(c) a two-row Curtis stage followed by simple impulse stages
(d) a two-row Curtis stage followed by 50% reaction stages.

7.30 A five-stage steam turbine working between 20 bar, 350 °C and 2 bar has equal enthalpy drops in all stages, and the stage efficiency is 75% in all stages. Plot on the Mollier diagram the condition line. Find the interstage pressures, reheat factor, and turbine internal efficiency using properties from the diagram.

7.31 In a four-stage pressure-compounded impulse turbine steam at 23 bar, 345 °C expands to 0.07 bar. The internal efficiency of the turbine is 0.72. Assuming that the work is shared equally among the stages and the condition line is straight, estimate the stage pressures, efficiency of each stage and reheat factor.

[*Ans.* 7.25 bar, 1.9 bar, 0.39 bar; 61.68%, 68.14%, 70.41%, 72.84%; 1.0574]

7.32 Give the casing arrangements for the turbines having the following ratings

(a) 75 MW, steam condition at inlet– 100 bar, 550 °C, condenser pressure– 0.1 bar

(b) 300 MW, steam condition at inlet – 180 bar, 550 °C, condenser pressure– 0.07 bar

Assume 90% turbine efficiency in each case. Find also the intercasing condition of steam.

7.33 A 3 MW steam turbine consumes 4.611 kg/s at full load and 2.53 kg/s at half load. Using Willan's line estimate the steam consumption at no load and the consumption at a load of 2.5 MW.

[*Ans.* 0.444 kg/s, 3.917 kg/s]

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8

Condenser, Feedwater and Circulating Water Systems

8.1 NEED OF A CONDENSER

A condenser where the exhaust steam from the turbine is condensed operates at a pressure lower than atmosphere. There are two objects of using a condenser in a steam plant:

1. To reduce the turbine exhaust pressure so as to increase the specific output of the turbine (Fig. 8.1). If the circulating cooling water temperature in a condenser is low enough (say 30 °C), it creates a low back pressure (vacuum) for the turbine. This pressure is equal to the saturation pressure corresponding to the condensing steam temperature (say 0.074 bar at 40 °C, Fig. 8.1), which, in turn, is a function of the cooling water temperature. It is known that the enthalpy drop or turbine work per unit pressure drop is much greater at the low pressure end than

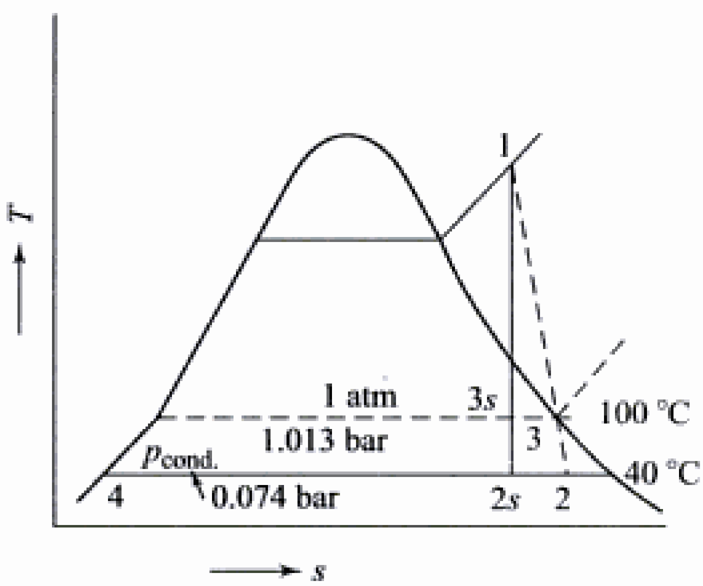

Fig. 8.1 *The use of condenser increases the specific work output of turbine from* $(h_1 - h_3)$ *to* $(h_1 - h_2)$

at the high pressure end of a turbine. A condenser by lowering the back pressure, say, from 1.013 to 0.074 bar, thus increases the plant efficiency and reduces the steam flow for a given output. The lower the pressure, the greater the output and efficiency. Hence, it is important to use the lowest possible cooling water temperature. This restricts the temperature rise of cooling water in the condenser tubes to 5–8 °C so that the tube outer surface temperature remains low and consequently, the condensing steam temperature is low and vacuum is high.

2. To recover high quality feedwater in the form of condensate and feed it back to the steam generator without any further treatment.

As a result, only the makeup water to replenish the water losses in the cyclic plant needs be treated.

8.1.1 Types

There are two broad classes of condensers:

(a) *Direct contact type condensers*, where the condensate and cooling water directly mix and come out as a single stream.
(b) *Surface condensers*, which are shell-and-tube heat exchangers where the two fluids do not come in direct contact and the heat released by the condensation of steam is transferred through the walls of the tubes into the cooling water continuously circulating inside them.

8.2 DIRECT CONTACT CONDENSERS

These can be of three types:

(a) Spray condenser
(b) Barometric condenser
(c) Jet condenser

In a spray condenser, the cooling water is sprayed into the steam. Steam by mixing directly with cold water gets condensed. The exhaust steam from the turbine at state 2 mixes with cooling water at state 5 to produce saturated water at state 3, which is pumped to state 4 (Fig. 8.2).

Part of the condensate (ω_4), equal to the turbine exhaust flow (ω_2) is sent back to the plant as feedwater. The remainder is cooled in a dry cooling tower to state 5, and is then sprayed on to the turbine exhaust. Since the cooling water mixes with the steam and part of the condensate is used as feedwater, the water must be of high purity.

In a geothermal or OTEC (ocean thermal energy conversion) plant, only vacuum is required to be maintained in the condenser and no feedwater is needed. Hence, the mixture at state 4 is discarded.

For the spray condenser (Fig. 8.2), a mass balance and an energy balance give the following equations

$$\omega_2 = \omega_4, \qquad \omega_3 = \omega_2 + \omega_5$$

$$\omega_2 h_2 + \omega_5 h_5 = \omega_3 h_3$$

from which
$$\frac{\omega_5}{\omega_2} = \frac{h_2 - h_3}{h_3 - h_5} \tag{8.1}$$

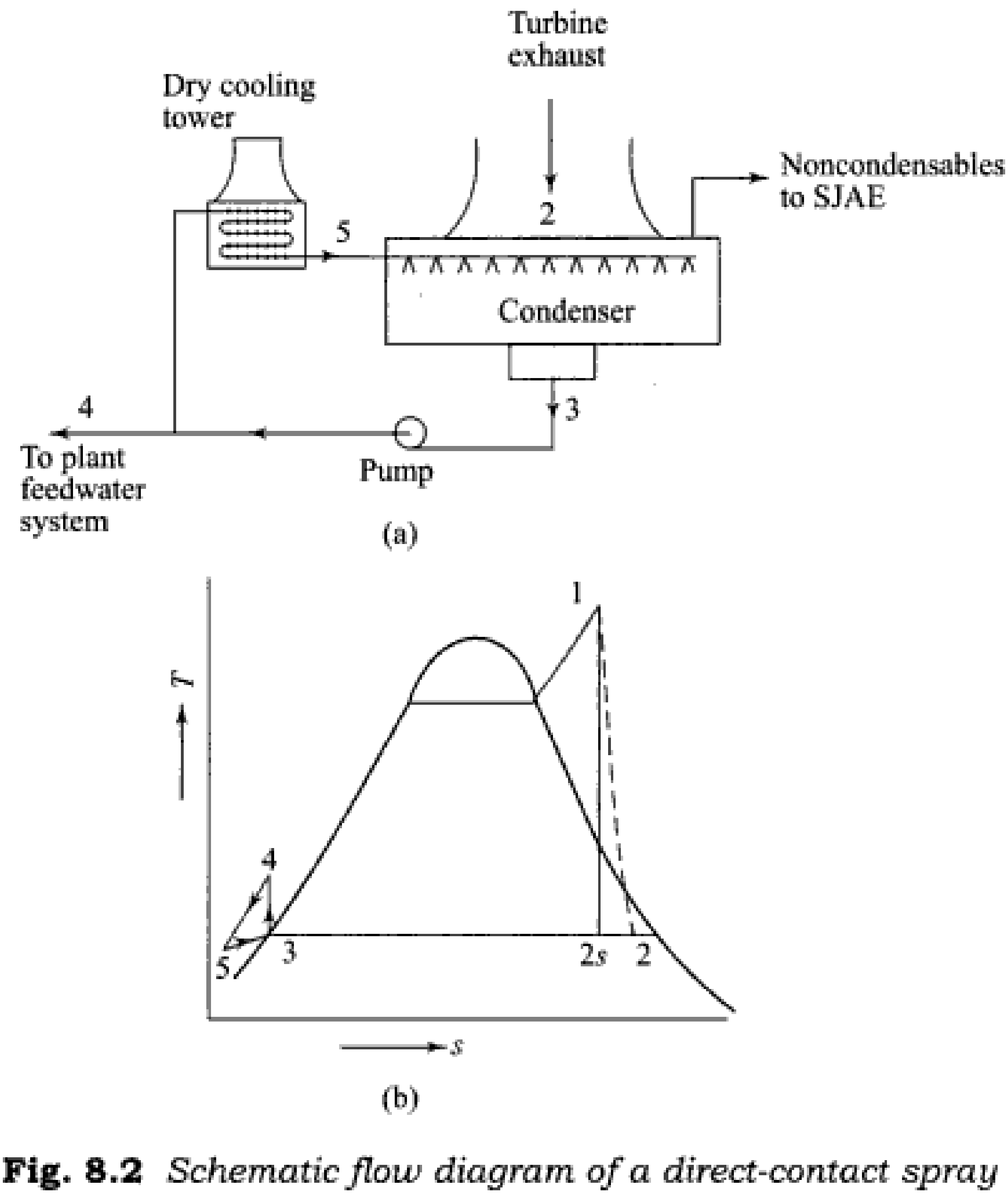

Fig. 8.2 *Schematic flow diagram of a direct-contact spray condenser (a) and the corresponding* T–S *diagram (b)*

Since $h_2 - h_3$ is much greater than $h_3 - h_5$, the circulating water flow (ω_5) is much larger than the steam flow (ω_2).

In a barometric condenser (Fig. 8.3a), the cooling water is made to fall in a series of baffles to expose large surface area for the steam fed from below to come in direct contact. The steam condenses and the mixture falls in a tail pipe to the hot well below. By virtue of its static head, the tail pipe compresses the mixture to atmospheric pressure. Thus,

$$p_{atm} - p_{cond} + \Delta p_f = \rho g H \tag{8.2}$$

where ρ = density of mixture, H = height of tail pipe, and Δp_f is the pressure drop due to friction.

For low values of Δp_f, H is around 9.5 m. Higher is the value of H, higher is the friction. Friction is lowered by increasing the tail pipe diameter, which results in a tall and heavy system.

In the jet condenser (Fig. 8.3b), the height of the tail pipe is reduced by replacing it with a diffusor. The diffusor helps raising the pressure in a short distance than a tail pipe.

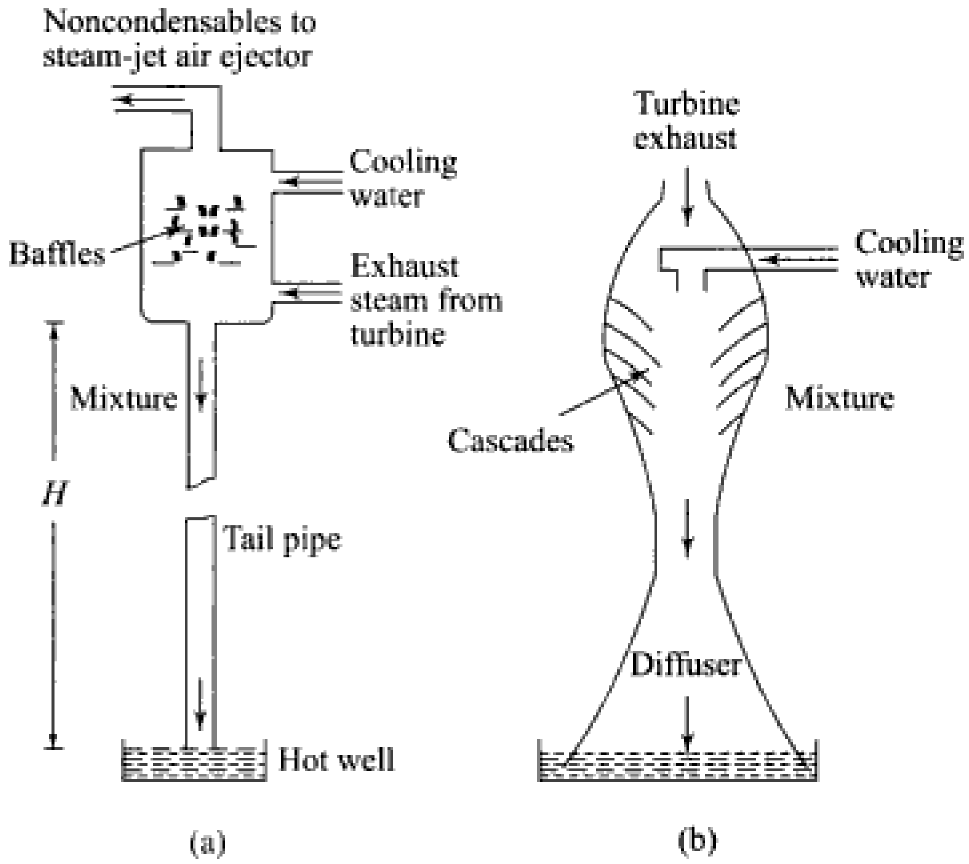

Fig. 8.3 *Schematics of direct-contact condensers: (a) barometric, (b) diffuser or jet*

In spray type condensers, the non-condensable gases are usually removed with a steam jet air ejector (SJAE), as discussed later.

8.3 SURFACE CONDENSERS

Surface condensers are mostly used in power plants. They are essentially shell-and-tube heat exchangers. For the convenience of cleaning and maintenance, cooling water flows through the tubes and steam condenses outside the tubes. Figure 8.4 shows a surface condenser with two passes on the water side. It consists of a steel shell with water boxes on each side. The right water box is divided to allow for two water passes. At each end there are tube sheets into which the water tubes are rolled. This prevents leakage of circulating water into the steam. An expansion joint allows for the different rates of expansion between the tubes and shell. There are vertical plates at intermediate points between the two tube sheets to provide support to the long tubes and to prevent tube vibration. The hot well acts as a reservoir of the condensate with a capacity equal to the total condensate flow for a certain period of time, say 5 min.

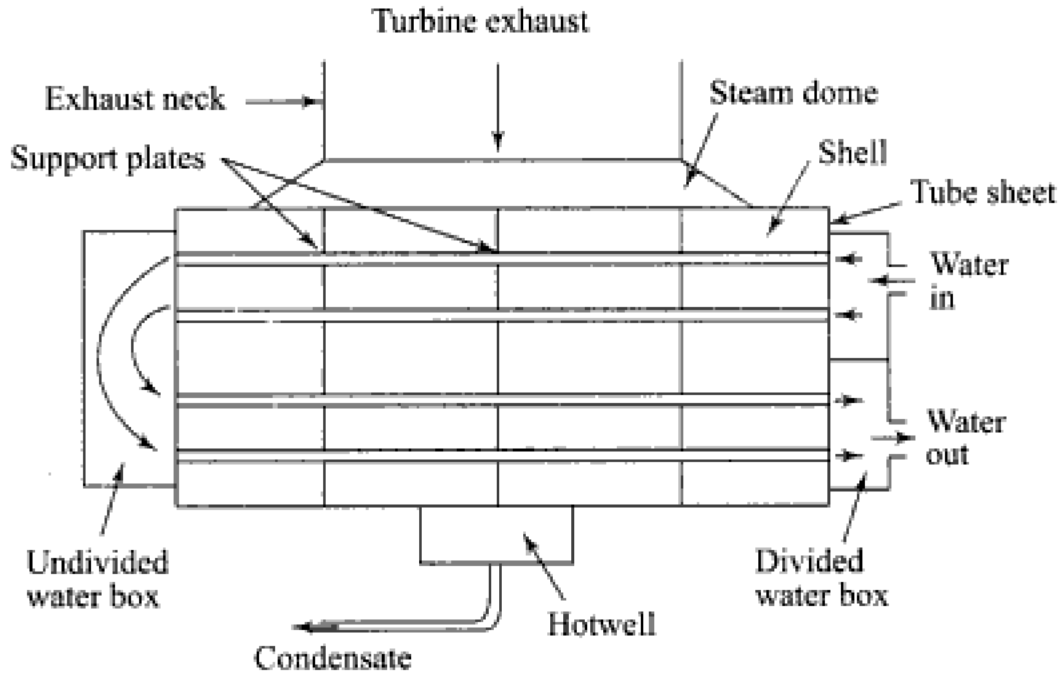

Fig. 8.4 *Schematics of a two-pass surface condenser*

In a single pass condenser (Fig. 8.5), cooling water flows through the tubes once, from one end to the other. Compared to a two-pass condenser, a single-pass condenser with the same number and size of the tubes and with the same water velocity requires twice as much water flow but results in half the water temperature rise and therefore, lower condenser pressure. Thus, a single-pass condenser is good for overall plant efficiency and reduces thermal pollution, but requires more than twice the water flow and hence, four times the pumping power.

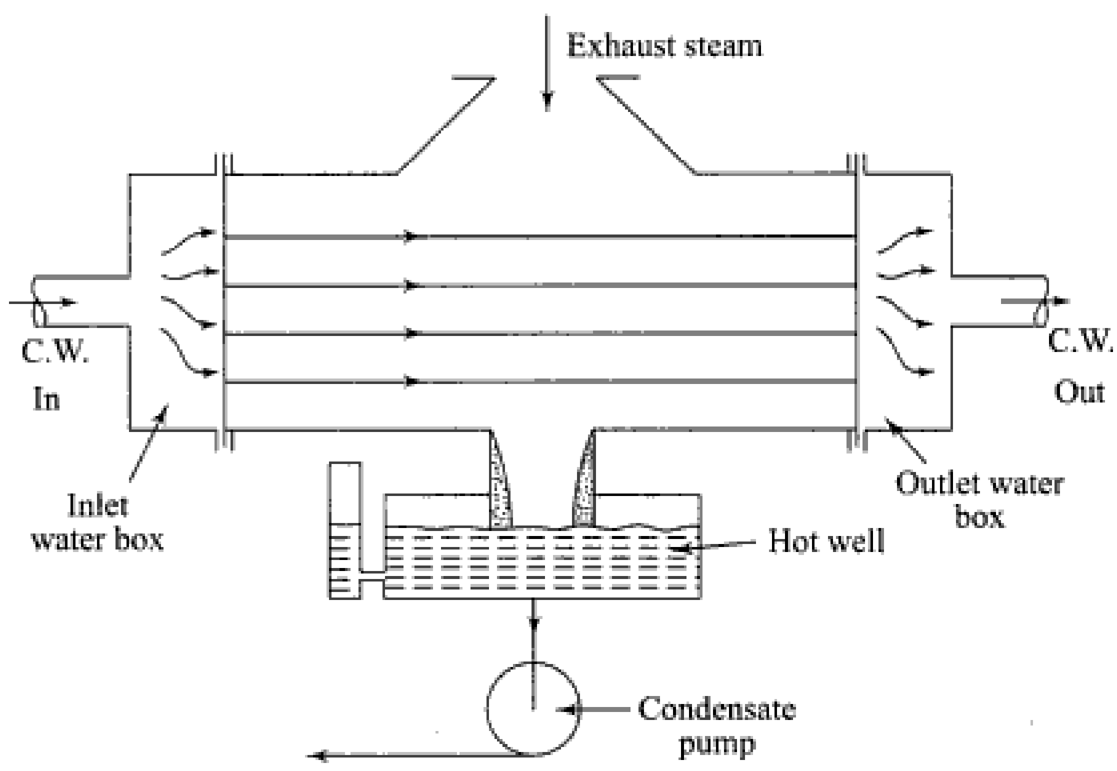

Fig. 8.5 *Single-pass condenser with divided water boxes*

Apart from the division required to allow for a number of water passes, the water boxes are often further divided. For example, in a single-pass condenser, both the inlet and outlet water boxes are divided by a partition. This permits to take off half the condensing surface out of service for cleaning while water flows through the other half to keep the unit running at half load (Fig. 8.5).

The tube arrangement in the condenser shell controls its effectiveness. Older designs tended to crowd a maximum number of tubes inside the shell to make available as much surface area as possible. Modern designs have steam lanes between tube banks to get maximum steam flow with least pressure drop and uniform distribution of steam in the shell. Tube lengths of 9–15 m are now quite common. Such long tubes result in a large rise in temperature of cooling water and the condensing ability decreases at the exit ends of the tubes. Thus, the tubes are closer at the cold end than at the hot end. To minimise the unequal distribution of steam flow from the turbine exhaust duct to condenser tubes, a well-tapered steam dome is added above the tube bundles. An expansion joint provided between the turbine exhaust and the condenser inlet permits the condenser to be rigidly mounted on the floor.

Figure 8.6 shows a typical modern two-pass surface condenser for a large steam power plant. Steam enters the tube bundles in two separate sections from the top, sides and bottom, and flows toward the centre of the tube nest in each section. At that point most of steam has condensed leaving only air and other non-condensable gases which are cooled and removed by SJAE, as explained later.

The tube material can be (a) cupronickel (70% copper, 30% nickel) (b) aluminium brass (76% copper, 22% zinc and 2% aluminium), (c) aluminium bronze (95% copper and 5% aluminium), (d) muntz metal (60% copper, 40% zinc), (e) admiralty alloy (71% copper, 28% zinc and 1% tin), or (f) stainless steel. The outside diameter of tubes is either 22 mm, 23 mm or 25.4 mm. The length varies from 9 to 15 m.

8.3.1 Design Calculations

When wet steam comes in contact with a cold surface, the temperature of which is below the saturation temperature at the exhaust pressure of steam, it cannot but condense rejecting the latent heat of condensation (Fig. 8.7). For filmwise condensation, the average heat transfer coefficient for a horizontal tube as given by Nusselt is

$$h_{av} = 0.725 \left[\frac{k_f^3 \rho_f^2 g h_{fg}}{N \mu_f d_0 \theta} \right]^{1/4} \tag{8.3}$$

where N = number of horizontal tubes in a vertical tier and $\theta = t_{sat} - t_w$. So, $h_{av} \propto 1/N^{1/4}$, $h_{av} \propto 1/\theta^{1/4}$ and $h_{av} \propto h_{fg}^{1/4}$.

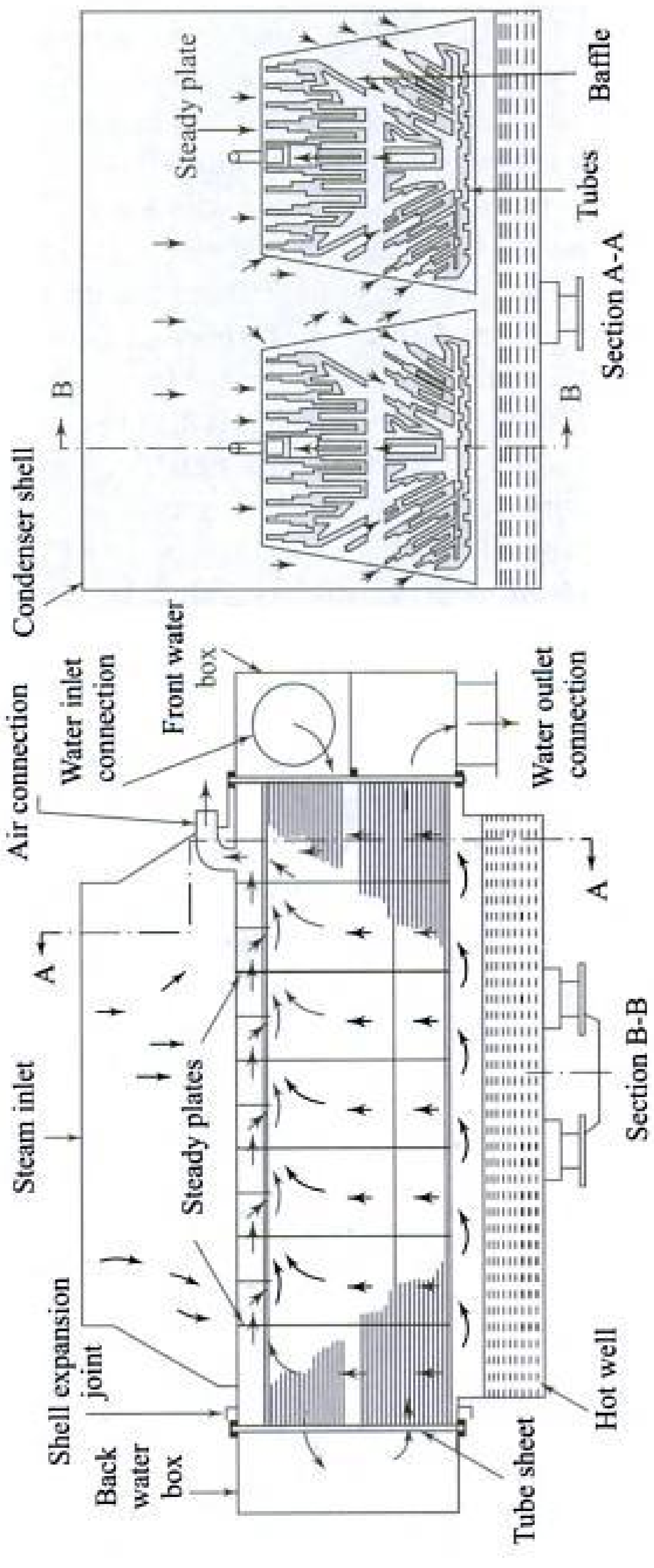

Fig. 8.6 *Sections through a typical two-pass surface condenser for a large steam power plant*

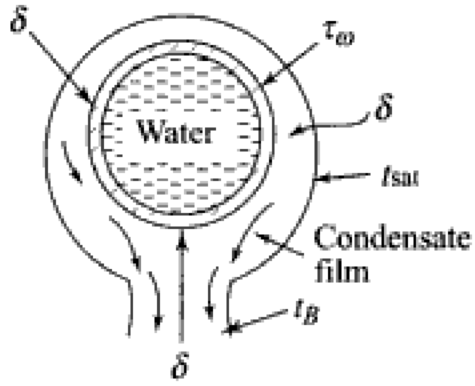

Fig. 8.7 *Film condensation of steam on a horizontal tube*

The bulk temperature of the condensate lies between t_{sat} and t_{ω}. So, it is less than t_{sat} and subcooled. This subcooling is not desired as it increases the heat supply in the boiler. By making some exhaust steam flow upward through the condensate its temperature is increased. Nusselt's equation gives only a conservative value for the condensing film coefficient of heat transfer, which is also influenced by vapour superheat, vapour velocity, turbulence, and the inside air.

The inside heat transfer coefficient on the water side may be obtained with the help or Dittus–Boelter equation,

$$Nu_d = 0.023\ \mathrm{Re}_d^{0.8}\ \mathrm{Pr}^{0.4} \tag{8.4}$$

Where Re_d is the Reynolds number $\left(=\frac{Vd}{\nu}\right)$ and Pr is the Prandtl number $\left(=\frac{c_p\mu}{k}\right)$

For water, $h_i \propto V^{0.8}$, V being the water velocity. Higher water velocity will improve heat transfer but increases pumping power also. The optimum water velocity varies between 2.0 to 2.5 m/s.

The overall heat transfer coefficient for a condenser tube is

$$\frac{1}{U_0 A_0} = \frac{1}{h_i A_i} + \frac{1}{h_{s_i} A_i} + \frac{x_w}{k_w A_{lm}} + \frac{1}{h_0 A_0} \tag{8.5}$$

Since condenser tubes are thin and made of good thermal conductivity, the tube wall resistance can be ignored, then

$$\frac{1}{U_0} = \frac{1}{h_i} + \frac{1}{h_{s_i}} + \frac{1}{h_0} \tag{8.6}$$

Here h_0 is much larger than h_i, and U_0 mainly depends on water velocity as given by

$$\frac{1}{U_0} = A + B\frac{1}{V^{0.8}} \tag{8.7}$$

where $$A = \frac{1}{h_{s_i}} + \frac{1}{h_0} \quad \text{and} \quad B = \frac{1}{0.023\dfrac{k_f\ \mathrm{Pr}^{0.4}}{d_i^{0.2} v_f^{0.8}}}$$

By estimating the overall heat transfer coefficient at different water velocities, a plot (Wilson plot) can be made on log–log coordinates from which the intercept A and slope B can be obtained (Fig. 8.8). The rate of heat transfer from the condensing vapour to the cooling water is

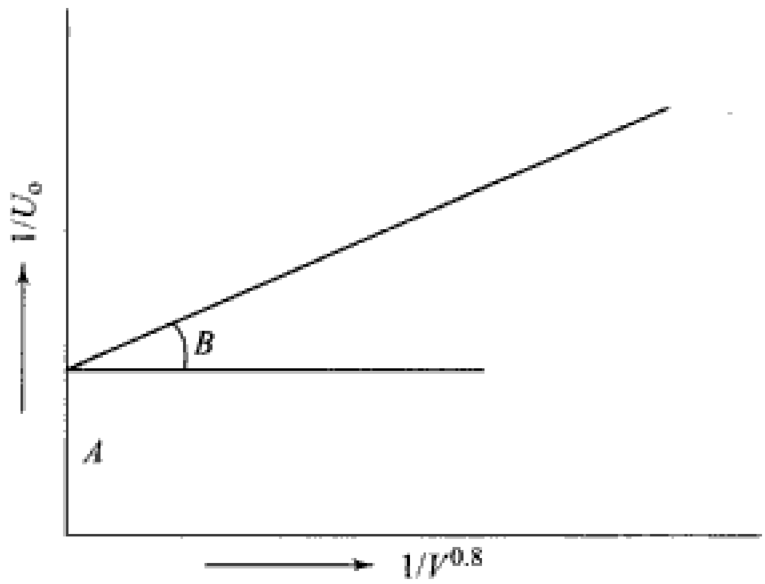

Fig. 8.8 *Plot of* $\frac{1}{U_0}$ vs $\frac{1}{V^{0.8}}$

$$Q = \omega_s (h_2 - h_3) = \omega_c c_{pc} (t_{C_2} - t_{C_1}) = U_0 A_0 \Delta t_{1.m} \tag{8.8}$$

where $\Delta t_{1.m} = (\Delta t_i - \Delta t_e)/\left(\ln \frac{\Delta t_i}{\Delta t_e}\right)$ is the logarithmic mean temperature difference (Fig. 8.9). The temperature difference at exit, Δt_e, is the terminal temperature difference (TTD) of the condenser. A small TTD results in a large condenser but reduced water flow and higher exit water temperature. It increases the capital cost but reduces the operating cost. The cooling water inlet temperature should be sufficiently low to have a good vacuum in the condenser shell. It is usually recommended that Δt_i should lie between 11 to 17 °C and that Δt_e or TTD should not be less than 3 °C.

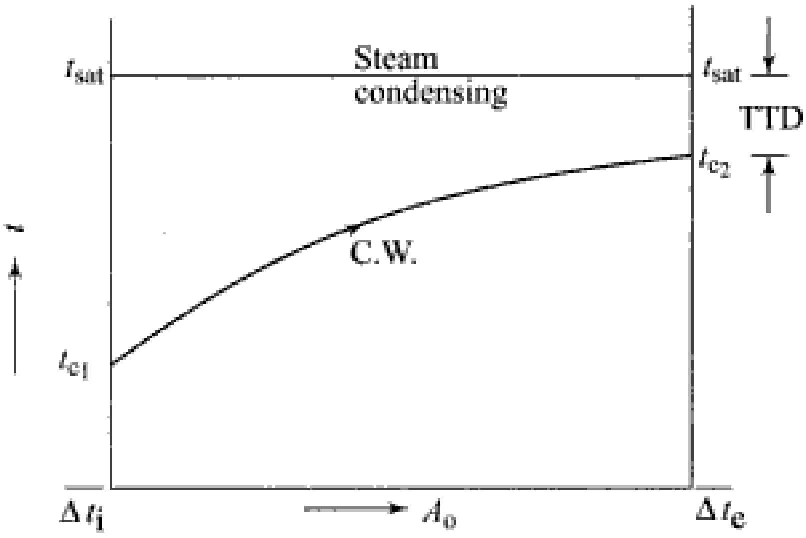

Fig. 8.9 *Temperature profiles in a condenser*

From Eq. (8.8), the water mass flow rate is

$$\omega_c = \frac{\omega_s(h_2 - h_3)}{c_{p_c}(t_{C_2} - t_{C_1})} \tag{8.9}$$

The rise in water temperature is limited to about 8–10 °C. So, for every kg of steam condensed, 75 to 100 kg of cooling water is required. Thus, to meet the huge water demand, the power plant is located where water is available in plenty.

The surface area needed by the condenser is obtained from Eq. (8.8).

$$A_0 = \frac{\omega_s(h_2 - h_3)}{U_0 \Delta t_{l.m}} = n\pi d_0 l \tag{8.10}$$

where n = number of tubes, and l = length of one tube (for a single-pass condenser). The water flows through the tubes. Therefore,

$$\omega_s = \left(n\frac{\pi}{4}d_i^2\right)\rho V \tag{8.11}$$

where ρ is the density of water (1000 kg/m^3) and V is the water velocity (1.8– 2.5 m/s). Therefore, the length and number of tubes can be estimated from the above two equations. For a modern condenser of a utility plant the number of tubes may be as high as 50,000 (22–23 mm o.d.).

The pressure drop in the condenser consists of (1) the pressure drop in the water boxes and (2) the pressure drop due to friction in the tubes $\left(\frac{fL}{d} \times \frac{\rho V^2}{2}\right)$.

The pumping power required is then given by

$$P = \omega_c v \Delta p = \frac{\omega_C \Delta p}{\rho} \tag{8.12}$$

where Δp = total pressure head to be developed by the pump in overcoming the losses.

8.3.2 Air Removal

Air leaks into the condenser shell through flanges. Some air also comes along with steam, which has leaked into the exhaust end of the turbine along the shaft. This air affects the condenser performance badly because of the following reasons.

1. It reduces the heat transfer considerably.
2. It reduces the condenser vacuum and increases the turbine exhaust pressure thus reducing the turbine output.

As air–water vapour mixture approaches the cold tube surface, water vapour condenses. Air, being non-condensable, forms an air film around the condensate film. Since air has a low thermal conductivity, the heat transfer is greatly reduced.

The amount of air infiltrating into the shell may be estimated by Dalton's law of partial pressures.

$$p_{sh} = p_{air} + p_{st} \tag{8.13}$$

where p_{st} is the saturation pressure at the measured shell temperature, t_{sh}, p_{air} is the partial pressure of air inside the shell, and p_{sh} is the measured total pressure of the shell (Fig. 8.10). Assuming that air behaves as an ideal gas at such a low pressure, we have

$$p_{air}\omega_s v_2 = \omega_a R_a (t_{sh} + 273) \tag{8.14}$$

where v_2 = specific volume of exhaust steam, ω_a = rate of air leakage, R_a = characteristic gas constant of air = 0.287 kJ/kg K. With p_{air} being known from Eq. (8.13), the rate of air leakage can be estimated from Eq. (8.14). This air has to be continuously removed from the condenser shell.

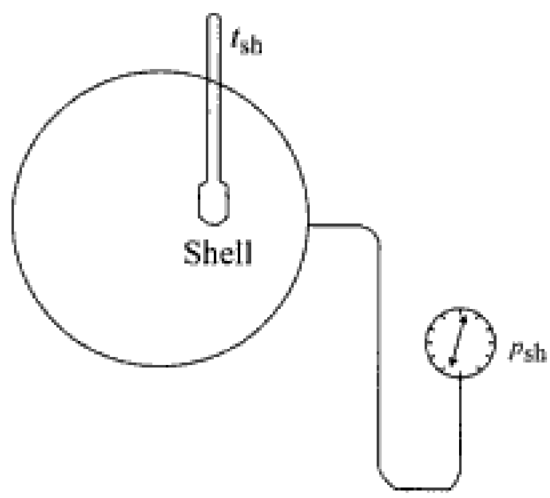

Fig. 8.10 *Condenser shell pressure and temperature*

The pressure in the condenser (p_{sh}) is approximately constant, and steam and air enter the condenser in fixed proportions when steady conditions prevail. As some of the steam is condensed, the partial pressure of the remaining steam (p_{st}) decreases and hence, the partial pressure of the air increases to maintain the same total pressure (Eq. 8.13). At reduced partial pressure, the steam has a saturation temperature which is below that of the incoming steam. Hence, condensation proceeds at progressively lower temperatures.

For the convenience of air removal an air cooler section is provided in the condenser shell. In Fig. 8.11(a), most of the condensation is carried out on the main bank of tubes and the air is drawn over another smaller bank which is shielded from the main bank by a baffle and is called the *aircooler*. Here, further condensation takes place at a lower temperature and thus, there is saving in feedwater as well as in air ejection load. In Fig. 8.11(b), the air cooling tubes are in the centre of the condenser and air is removed from this section. The incoming steam passes all around the bank of tubes and some is drawn upward to the centre. In doing so, it meets the subcooled condensate falling below and thus reduces the amount of subcooling by heating it.

A steam jet air ejector, SJAE, is mostly used to remove air from the condenser shell (Fig. 8.12). As explained above, air is extracted from the coldest part of the condenser to reduce the loss of vapour accompanying it and hence, to reduce the ejector load. Baffles keep off the air-cooler section from the main

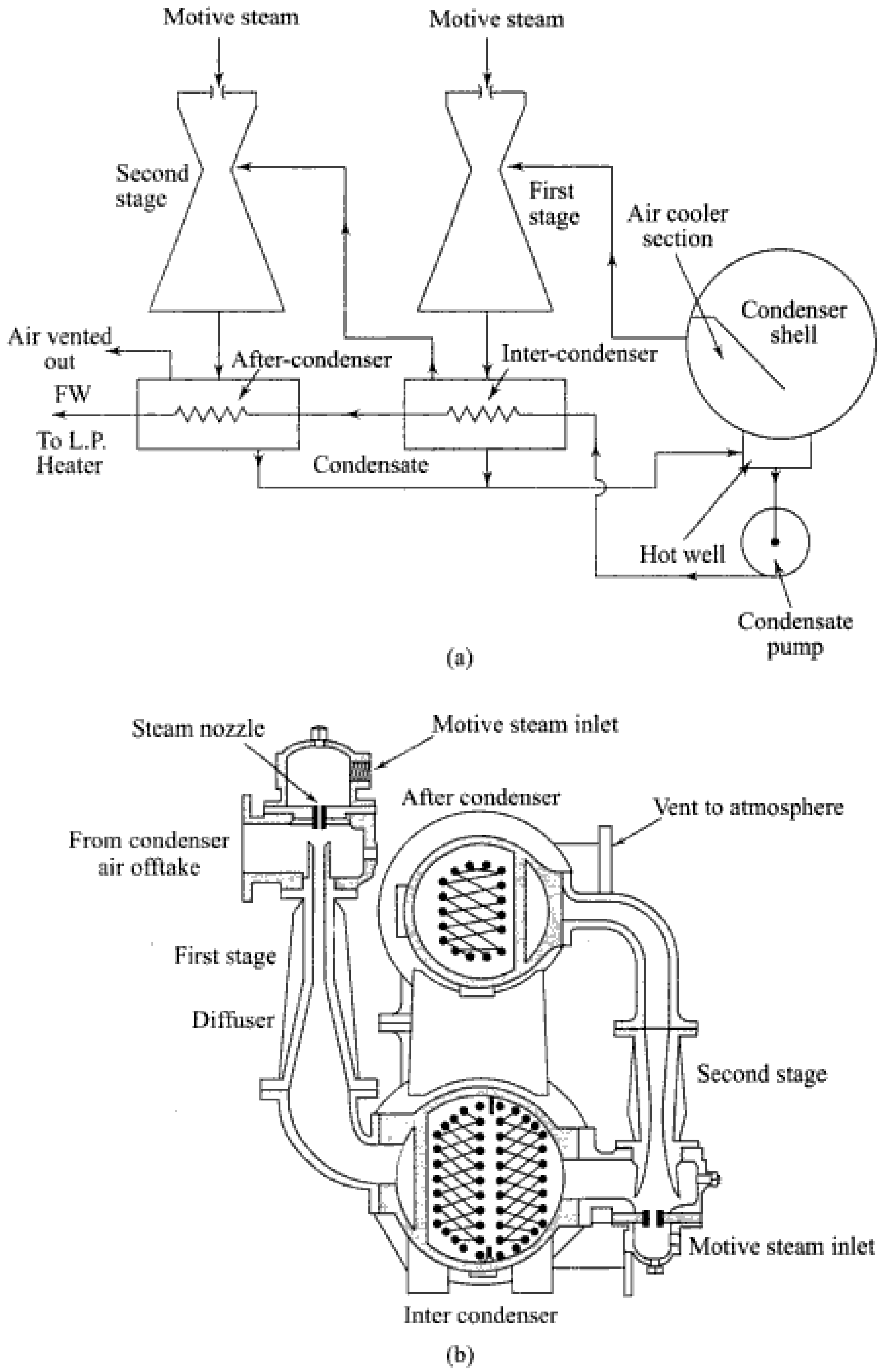

Fig. 8.12 *Two-stage steam jet air ejector with inter-condenser and after-condenser*

8.4 FEEDWATER HEATERS

As discussed in Chapter 2 regenerative feedwater heaters are always used in steam power plants to improve the cycle efficiency. They raise the temperature of the feedwater before it enters the economiser. Both open and closed type heaters are used. In small industrial plants, only one open feedwater heater may be used. But in large industrial and utility plants, five to seven closed heaters and one open heater are used. The open heater acts as a deaerator.

8.4.1 Closed Feedwater Heater

Closed feedwater heaters are shell-and-tube heat exchangers. They are basically small condensers which operate at higher pressures than the main condenser because bled steam is condensed on the shell side, whereas the feedwater, acting like circulating cooling water in the condenser, is heated on the tube side.

It was shown in Chapter 2 that the temperature rise in each heater and economiser is equal for maximum cycle efficiency. Thus the heaters receive bled steam from the turbine at pressures determined roughly by equal temperature rise from the condenser to the boiler saturation temperature. They are classified as low pressure (LP) and high pressure (HP) heaters depending upon their locations in the cycle. The LP heaters are usually located between the condensate pump and the deaerator, which is followed by the boiler feed pump (BFP). The HP heaters are located between the BFP and the economiser.

When bled steam entering a feedwater heater is superheated, as in a HP heater, the heater includes a desuperheating zone where steam is cooled to its saturation temperature. It is followed by a condensing zone where the steam is condensed to a saturated liquid rejecting the latent heat of condensation. This liquid, called heater drain, is then cooled below its saturation temperature in a subcooling zone or a drain cooling zone before the drain is cascaded backward or pumped forward.

Figure 8.13 shows the schematic diagram and the temperature profiles of a three-zone closed feedwater heater. There are, however, two-zone heaters that include a desuperheating and a condensing zone or a condensing and a subcooling zone. There are also single-zone heaters that include only a condensing zone. A drain-cooling zone, instead of being a part of the shell, may be located outside it. It is then called a drain cooler.

Closed feedwater heaters may be either horizontal or vertical, depending upon space availability. Vertical heaters occupy less space. Fig. 8.14 shows a typical horizontal three-zone closed feedwater heater. The feedwater tubes are usually in the form of U-tube bundles. The feedwater enters a divided water box and flows through the subcooling zone and then through the condensing zone, and leaves to the water box through the desuperheating zone. The bled steam first flows through the desuperheating zone separated by a shroud. The vertical baffles provide good heat transfer and tube support. The condensing zone is the major portion of the heater. The subcooling zone is separated from the rest of the heater by an end plate. Pressure drops of the feedwater in heaters are usually large due to friction in long small diameter tubes of the heaters.

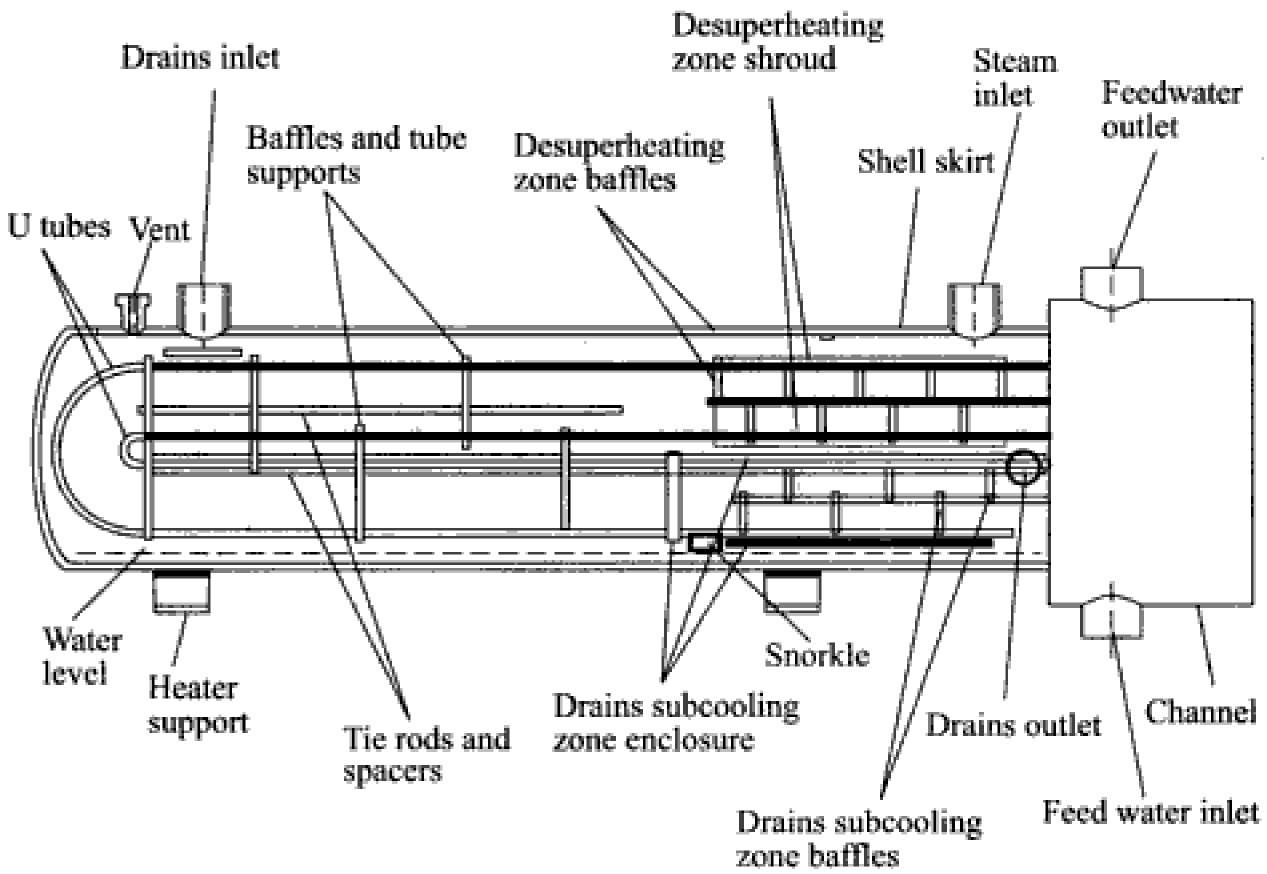

Fig. 8.13(a) *A three-zone horizontal closed-type feedwater heater*

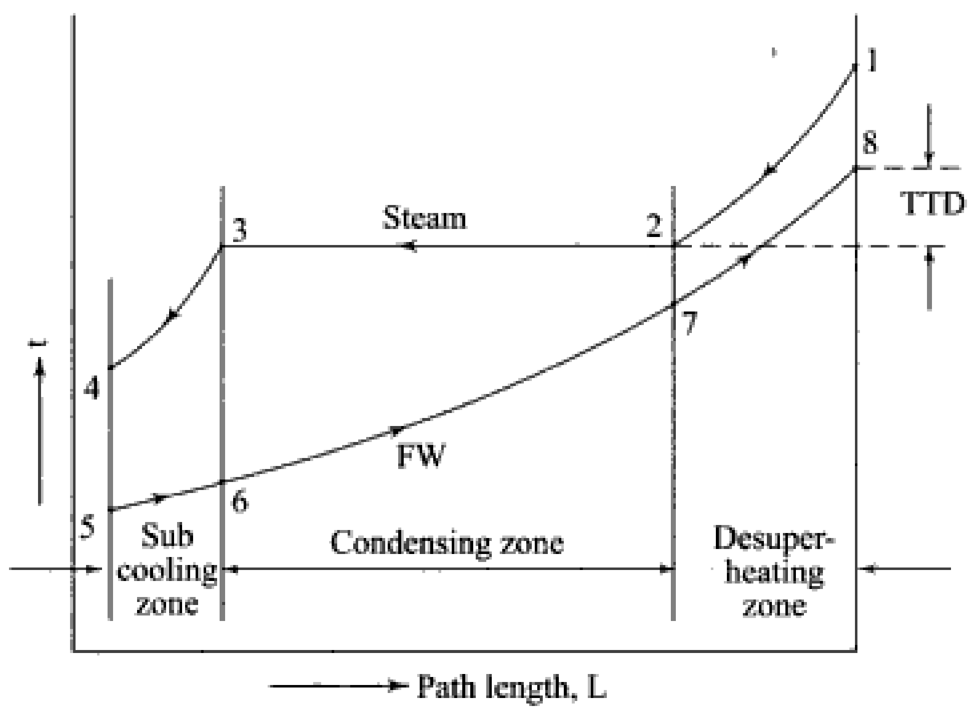

Fig. 8.13(b) *Temperature profiles along the path length in a three-zone feedwater heater*

8.4.2 Open Feedwater Heater

An open feedwater heater or deaerator is one in which the feedwater is heated by direct mixing with the steam bled from the turbine. It is used to remove dissolved gases in feedwater (Chapter 2). It is located at a sufficient height (20–25 m) above the boiler feed pump so that the suction pressure does not fall below saturation pressure to prevent cavitation.

There are three types of deaerating heaters.

(a) *Spray-type deaerators*: Here, feedwater is sprayed through nozzles into the heater from the top and bled steam is fed from the bottom. Water is heated and scrubbed to release the dissolved gases.

(b) *Tray-type deaerators*: Feedwater here falls through a series of cascading horizontal trays. As water falls from tray to tray, it comes in direct contact with the upflowing bled steam, and gets heated and scrubbed to release the dissolved gases.

(c) *Combined spray-tray deaerators*: Feedwater is first sprayed and then made to cascade down a series of trays and bled steam flows upward. These types are now preferred in power plants. A typical heater of this type is shown in Fig. 8.15.

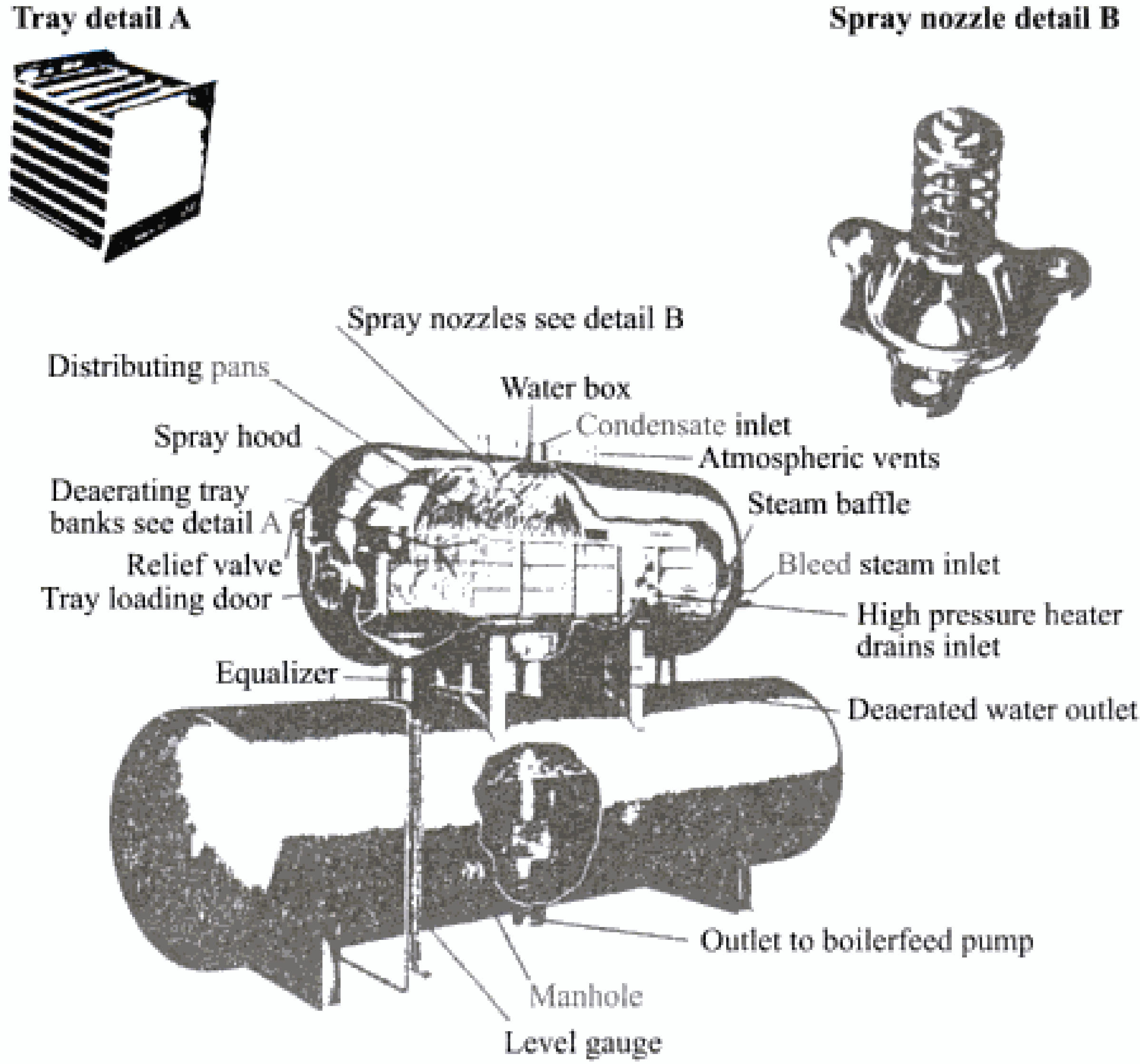

Fig. 8.15 *A typical open-type deaerating feedwater heater*

8.5 CIRCULATING WATER SYSTEM

The circulating water system supplies cooling water to the turbine condensers and thus acts as a medium through which heat is rejected from the steam cycle to the environment. Cooling water can flow through the condenser in two methods: (1) Once-through system, and (2) Closed loop system.

Once-through system (Fig. 8.16) is used when there is a large source of water available. Water is taken from a natural body of water like a lake, river, or ocean and pumped through the condenser, where it is heated, and then discharged back to the source.

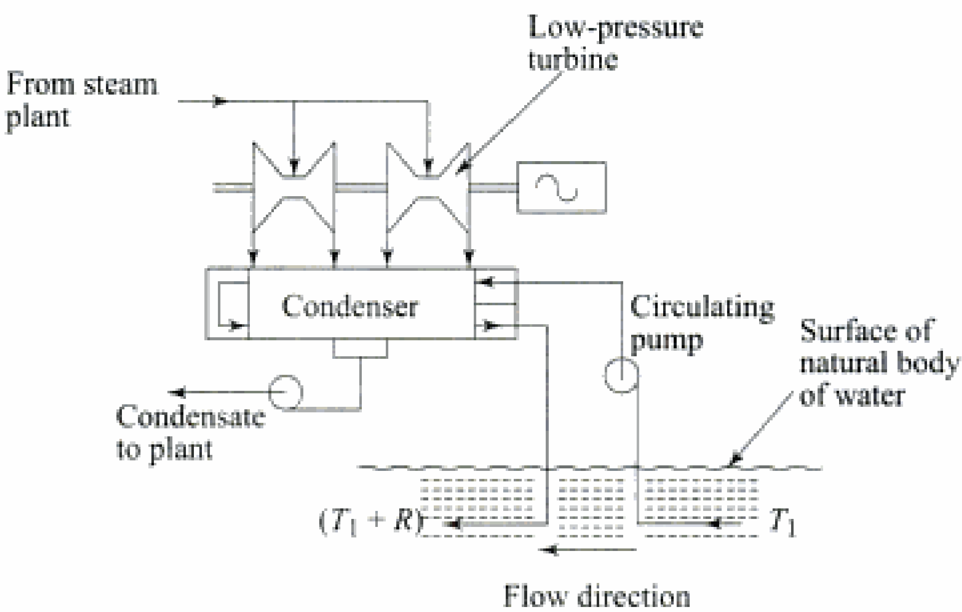

Fig. 8.16 *Schematic of a once-through circulating water system*

In closed loop systems, warm water from the condenser is passed through a cooling device like a cooling tower or a spray pond and the cooled water is then pumped back for condenser circulation (Fig. 8.17). However, a natural body of water is still necessary nearby to supply the makeup water to replace the loss due to evaporation, blowdown and so on.

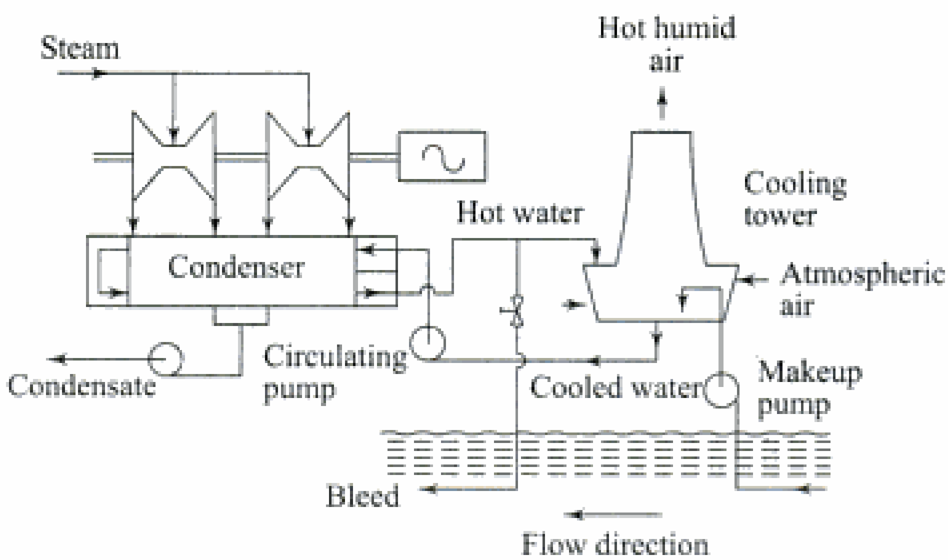

Fig. 8.17 *Schematic usual view of a wet cooling tower operating in the closed mode*

The once-through system, though more efficient, causes thermal pollution. In addition, availability of huge quantity of water is shrinking. Closed loop systems are now almost universally preferred.

8.6 COOLING TOWERS

Cooling towers cool the warm water discharged from the condenser and feed the cooled water back to the condenser. They, thus, reduce the cooling water demand in the power plant. They can be either wet type or dry type.

8.6.1 Wet Cooling Towers

Wet cooling towers have a hot water distribution system that showers or sprays water evenly over a lattice of horizontal slats or bars called fill or packing (Fig. 8.18). The fill thoroughly mixes the falling water with air moving through the fill as the water splashes down from one fill level to another by gravity. Outside air enters the tower through louvres on the side of the tower. Intimate mixing of water and air enhances heat and mass transfer (evaporation), which cools the water. More the water evaporates, more will be the cooling since the latent heat of evaporation is taken from water itself (evaporative cooling). Cold water is collected in a concrete basin at the bottom of the tower, from where it is pumped back to the condenser. Hot and moist air leaves the tower from the top.

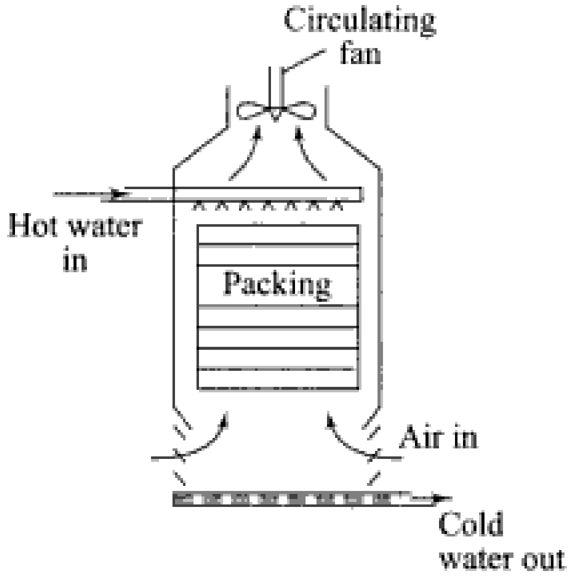

Fig. 8.18 *Packing or fill in a wet cooling tower*

Air entering the tower is unsaturated and as it comes in contact with the water spray, water continues to evaporate till the air becomes saturated. So, *the minimum temperature to which water can be cooled is the adiabatic saturation or wet bulb temperature of the ambient air.* At this temperature (WBT), air is 100% saturated and cannot absorb any more water vapour. Hence, there will be no further evaporation and cooling. The humid air while moving up comes in contact with warm water spray and so the air temperature rises.

A cooling tower is specified by (a) approach, (b) range, and (c) cooling efficiency. The approach (A) is defined as the difference between the exit temperature of cooling water and the wet bulb temperature of the ambient air, or

$$A = t_{C_2} - t_{wb} \tag{8.15}$$

Warm water from the condenser enters the cooling tower at temperature t_{C_1} and is cooled to temperature t_{C_2}, higher than the minimum value, the wet bulb temperature, t_{wb}, and this unattainable temperature difference is the approach. The approach varies from 6 °C to 8 °C.

The cooling range or simply range (R) is defined as the difference in temperatures of the incoming warm water (t_{C_1}) and the exiting cooled water (t_{C_2}), or

$$R = t_{C_1} - t_{C_2} \tag{8.16}$$

It is the range by which warm water from the condenser is cooled. The range varies from 6 °C to 10 °C.

The cooling efficiency is defined as the ratio of the actual cooling of water to the maximum cooling possible, or

$$\eta_{\text{cooling}} = \frac{\text{actual cooling}}{\text{maximum cooling possible}} = \frac{t_{C_1} - t_{C_2}}{t_{C_1} - t_{wb}} \tag{8.17}$$

The approach, range and cooling efficiency are the performance parameters of cooling towers.

Wet cooling towers can be either mechanical draught or natural draught cooling towers. In mechanical draught cooling towers, air is moved through the fill by one or more fans driven by motors. As in steam generators, the fans could be of the forced draught (FD) type or induced draught (ID) type. The FD fan is mounted on the lower side of the tower (Fig. 8.19). Since it operates on cooler air, it consumes less power. However, it has the disadvantages of (a) air distribution problems in the fill, often causing channeling of air flowing through paths of less flow resistance, (b) leakage and (c) recirculation of the hot and moist air back to the tower.

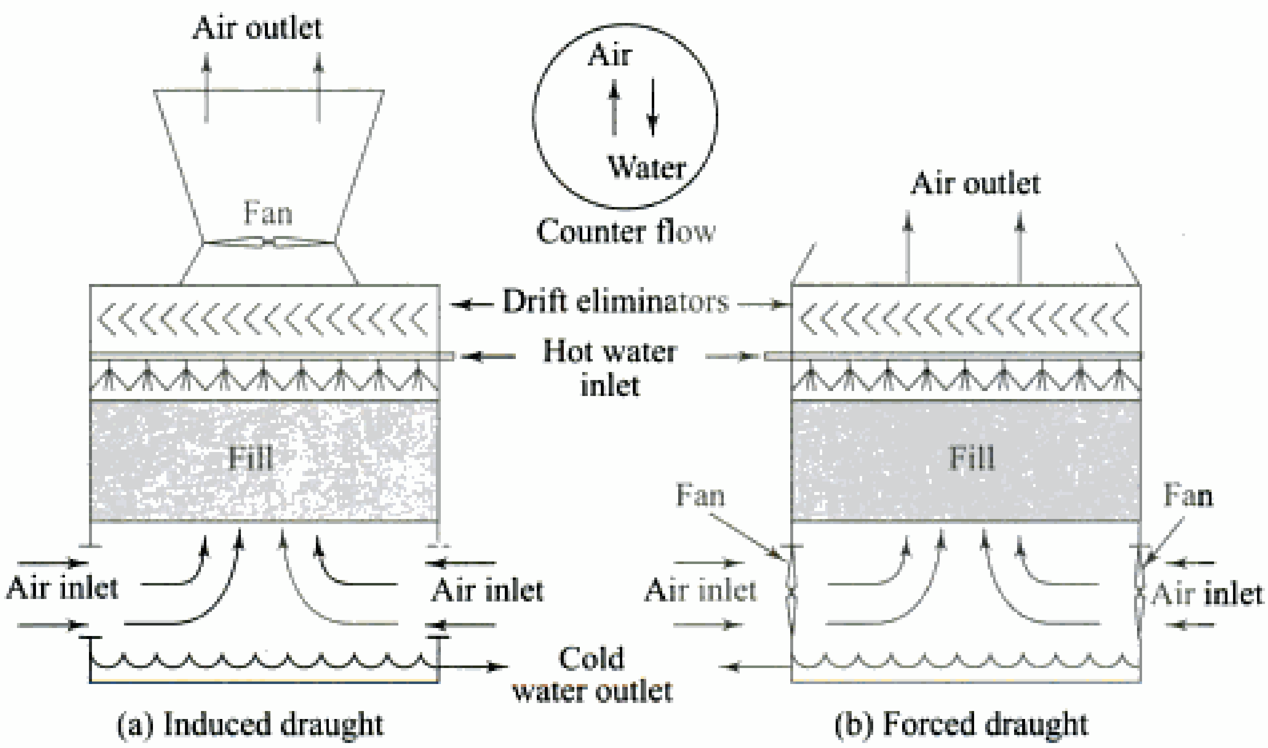

Fig. 8.19 *Induced draught counterflow cooling towers*

Most of the mechanical draught cooling towers for utility applications are of the induced draught type. The ID fan is located at the top of the tower Fig. 8.19(a). Air enters the sides of the tower through large openings at low velocity and passes through the fill. Hot humid air is exhausted by the fan at the top to the atmosphere. It maintains the tower at a negative pressure thereby reducing leakage. The ID fans are large, having 0.6 to 10 m in diameter. They are driven by electric motors at low speeds through reduction gearing. They are of the propeller type which deliver large volume flow of air at lower static pressures. The blades are usually made of cast aluminium, stainless steel, or fibre glass so as to protect them from corrosion.

The air flow into the tower is more or less horizontal. However, in the fill the flow can be horizontal or vertical, in which case it is either a cross-flow or a counter-flow cooling tower. The main advantages of mechanical draught cooling towers are:

1. Low capital and construction costs.
2. Assured supply of the required quantity of air at all loads and climatic conditions,
3. Small physical structure.

In natural draught cooling towers, the flow of air occurs due to the natural pressure head caused by the difference in density between the cold outside air and the hot humid air inside (Fig. 8.20(a) and (b)). Thus, the pressure head developed is

$$\Delta p_d = (\rho_0 - \rho_i)\, gH \qquad (8.18)$$

where H = height of the tower above the fill, ρ_0 = density of outside air, and ρ_i = density of inside air.

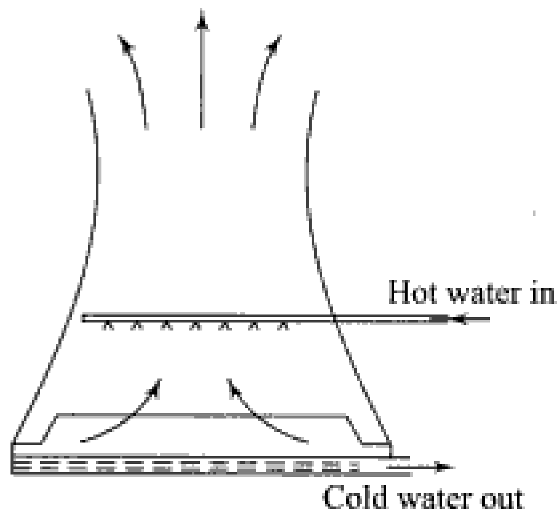

Fig. 8.20(a) *Natural draught cooling tower*

Because of relatively small density difference, $\rho_o - \rho_i$, H must be large so as to result in the desired Δp_d, which must

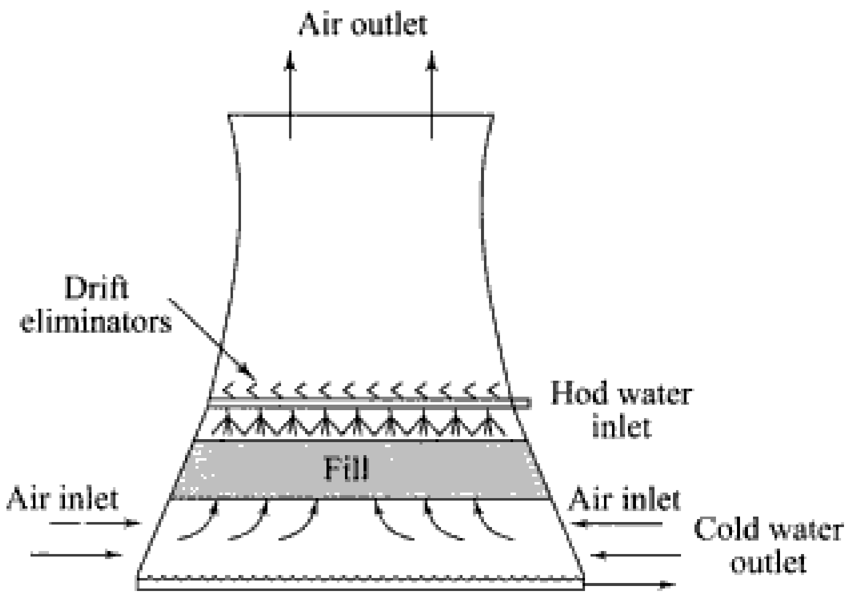

Fig. 8.20(b) *Counterflow hyperbolic natural draught cooling tower*

balance the air pressure losses in the tower. Natural draught cooling towers are, therefore, very tall. The tower body, above the water distribution system and the fill, is an empty shell of circular cross-section, but with a hyperbolic vertical profile. The hyperbolic profile offers superior strength and the greatest resistance to outside wind loading compared to other forms. Natural draught cooling towers are, therefore, often termed as hyperbolic towers. Made of reinforced concrete, they are an imposing sight and are conspicuous from a distance.

Mechanical draught cooling towers are preferred when the approach is low and a broad range of water flow is expected. The broad range is possible since they are made of multicell units with a variable air-flow fan. These towers are, therefore, more versatile and respond readily to changes in cooling parameters and demands.

Natural draught cooling towers are chosen (1) in cool, humid climates (low wet bulb temperature and high relative humidity), (2) when the wet bulb temperature is low and high condenser water inlet and outlet temperature, i.e. a broad range and a long approach, or (3) in heavy winter loads. However, their initial capital cost is high and occupy more space.

There is always some water loss in the cooling tower due to

(a) evaporation
(b) drift
(c) blowdown

Water that evaporates leaves the tower along with air in the form of water vapour. The evaporation loss rate is 1–1.5 per cent of the total circulating water flow rate. Drift is fine water droplets entrained and carried by the air. This water is thus lost to the circulating water system. Drift eliminators are provided at exit to minimise the drift loss (Fig. 8.21). The baffles force the air to make a sudden change in direction. Heavier water particles separate out by gravity. Thus the drift loss is much less, about 0.03 per cent.

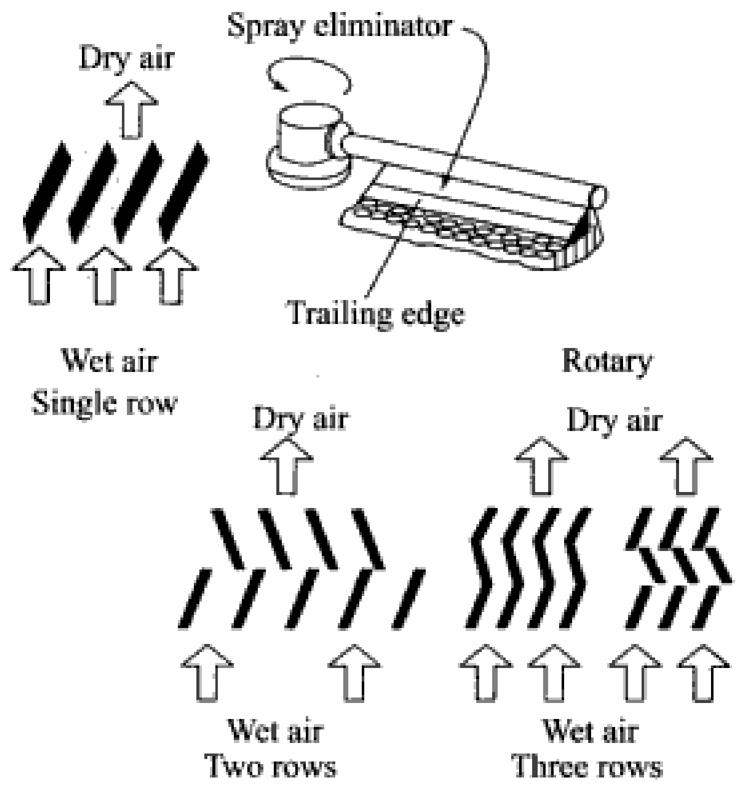

Fig. 8.21 *Types of drift eliminators*

To maintain a certain solid concentration, blowdown is necessary from the cold water basin at the bottom of the tower. The blowdown loss of water is also 1–1.5 per cent of the total water flow. To replenish these losses, makeup water (2–2.2% of water flow) is added.

8.6.2 Dry Cooling Towers

Dry cooling towers are employed where cooling water is not available in plenty, even for the use of makeup. In a dry tower, warm water from the condenser flows through finned tubes over which the cooling air is passed. Heat is rejected to the air as water is cooled.

There are two basic types of dry cooling towers: direct and indirect. In a direct dry cooling tower (Fig. 8.22), turbine exhaust steam flows into a large

flows through finned tubes and is cooled by atmospheric air blown over the tubes. There are two heat exchangers in series, one between steam and water in the condenser and the other between water and air in the tower.

The second design uses a direct contact spray condenser (Fig. 8.24). The turbine exhaust steam enters the open condenser and the cold circulating water is sprayed into the steam for intimate mixing. The condensate falls into a bottom receiver from which a part is fed to the plant as feedwater, and the remainder is pumped into finned tubes cooled by air flow to return to the condenser sprays. Condensate polishers may be used to maintain the desired quality of feedwater.

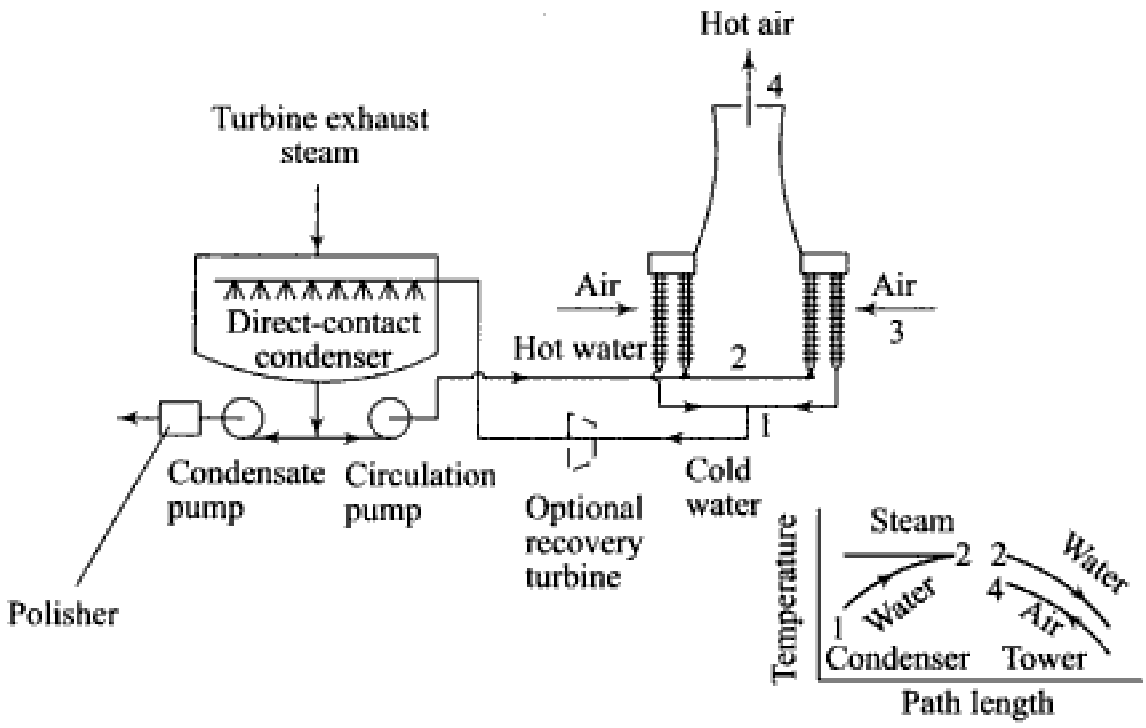

Fig. 8.24 *Schematic of an indirect dry-cooling tower with an open-type condenser*

The third design of an indirect dry cooling tower employs ammonia as the coolant for the condenser (Fig. 8.25). In the steam condenser ammonia evaporates. In the dry tower ammonia condenses, rejecting the latent heat of condensation to the air which is heated. Saturated liquid ammonia is then pumped back to the condenser.

Dry cooling towers have attracted much attention now-a-days. Plants can be erected without regard for large supplies of cooling water. Typical sites are near the sources of abundant fuel where there is no sufficient water. Their disadvantages are that they are not so efficient as evaporative cooling, and that there is an increase in turbine exhaust pressure and a decrease in cycle efficiency. However, as power plants grow bigger and water sources are dwindling, they are likely to receive greater attention in future.

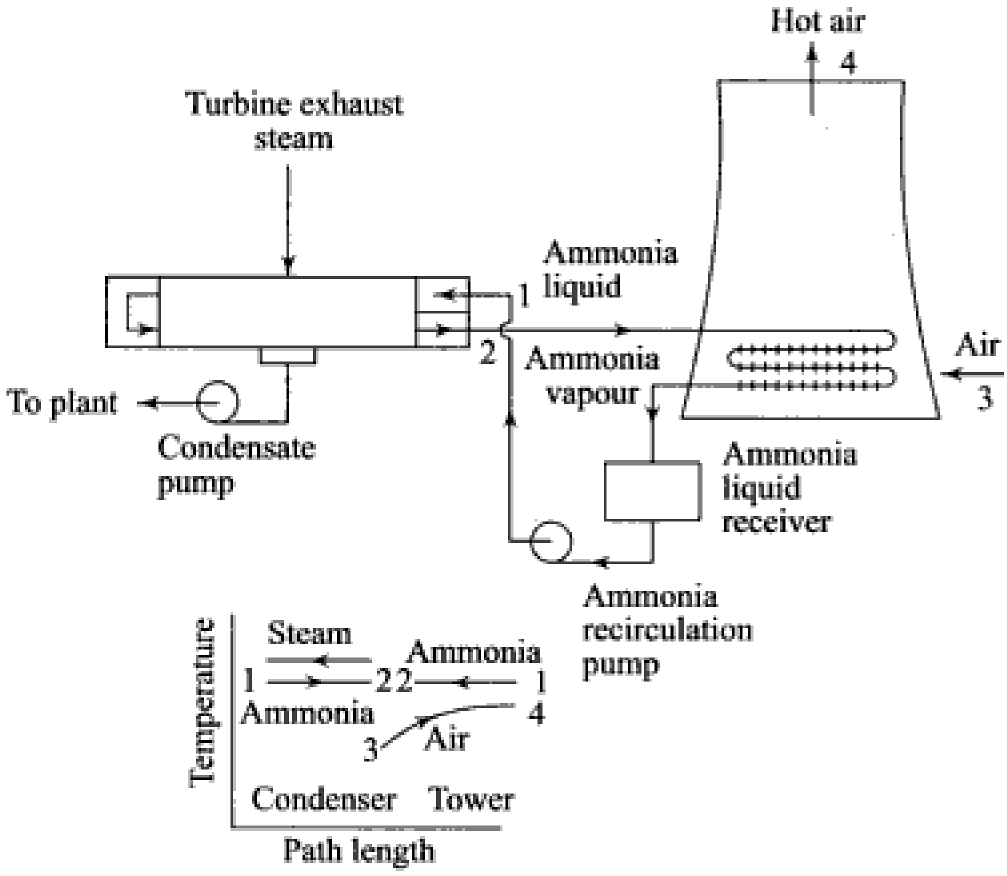

Fig. 8.25 *Schematic of an indirect dry-cooling tower with a surface condenser having ammonia as the coolant*

8.7 COOLING TOWER CALCULATIONS

In a wet cooling tower, ambient air is used to cool the warm water exiting the condenser. Properties associated with air–water vapour mixture may now be discussed.

Atmospheric air is considered to be a mixture of dry air and water vapour. If p_a and p_w are the partial pressures of dry air and water vapour, respectively, then by Dalton's law of partial pressures,

$$p_a + p_w = p$$

where p is the atmospheric pressure. Since p_w is very small, the saturation temperature of water vapour at p_w is less than atmospheric temperature, t_{atm} (Fig. 8.26). So, the water vapour in air exists in the superheated state and air is said to be unsaturated. Saturated air holds the maximum water vapour at the given temperature. If the temperature is increased, then only it can accept more water vapour (till it is again saturated). If the temperature of saturated air is decreased, some water vapour will condense and the new cooler air would also be saturated. At 15 °C, the partial pressure of water vapour in saturated air (p_s) is equal to 1.705 kPa from steam tables. So, the partial pressure of dry air (p_a) is (101.325 – 1.705) or 99.62 kPa.

Dew point temperature (dpt, t_{dp}) is the saturation temperature at the partial pressure of water vapour. When air is cooled at constant pressure, the temperature at which water vapour starts condensing is called the dew point temperature (Fig. 8.26).

Dry bulb temperature (dbt, t_{db}) is the temperature recorded by a thermometer with a dry bulb.

Wet bulb temperature (wbt, t_{wb}) is the temperature recorded by a thermometer when the bulb is enveloped by a cotton wick saturated with water. A psychrometer measures both dbt and wbt. If air flowing over the bulb is relatively dry, water in the wick would evaporate at a rapid rate, cooling the bulb and resulting in a much lower reading than if the bulb were dry. If the air is humid, the evaporation rate is slow, the lowest temperature recorded by the moistened bulb is the wbt. If air is saturated, i.e. $\phi = 100\%$, wbt = dbt. The wbt is also called adiabatic saturation temperature.

Psychrometric chart (Fig. 8.27) is a graphical plot with specific humidity and partial pressure of water vapour as ordinates and dbt as abscissa. The volume of the mixture, wbt, relative humidity and enthalpy of the mixture appear as parameters. Any two of these parameters fix the condition of the mixture. The chart is plotted for one barometeric pressure, say 760 mm Hg. The constant wbt line represents the adiabatic saturation process and coincides with the constant enthalpy line.

The cooling tower utilizes the phenomenon of evaporative cooling to cool the warm water below the dbt of air. However, water never reaches the minimum temperature, i.e., the wbt since an excessively large cooling tower would then be required. Also, since warm water is continuously introduced to the tower (Fig. 8.28), the equilibrium conditions are not achieved, and the dbt of air is increased. Hence, while the water is cooled, the air is heated and humidified.

If x is the make-up water supplied to replenish the evaporation loss (Fig. 8.28), then

$$x = G(W_2 - W_1) \tag{8.21}$$

where G = the mass flow rate of dry air, kg/s, W = specific humidity, kg water vapour per kg dry air.

By energy balance,

$$G_1 h_1 + w_{c_3} h_{w_3} + x h_w = G_2 h_2 + w_{c_4} h_{w_4}$$

$$w_C(h_{w_3} - h_{w_4}) = G(h_2 - h_1) - G(W_2 - W_1)h_w \tag{8.22}$$

where $\omega_{c_3} = \omega_{c_4} = \omega_c$ = circulating water flow rate, kg/s; h_w = enthalpy of circulating water, kJ/kg; and h = enthalpy of dry air, kJ/kg

$$\text{Range } (R) = t_{w_3} - t_{w_4} = \frac{G}{c_{p_w} \omega_c}[(h_2 - h_1) - (W_2 - W_1) h_w] \tag{8.23}$$

where c_{p_w} is the specific heat of water and h_w is the enthalpy of makeup water.

$$\text{Approach } (A) = t_{w_3} - t_{wb_1} \tag{8.24}$$

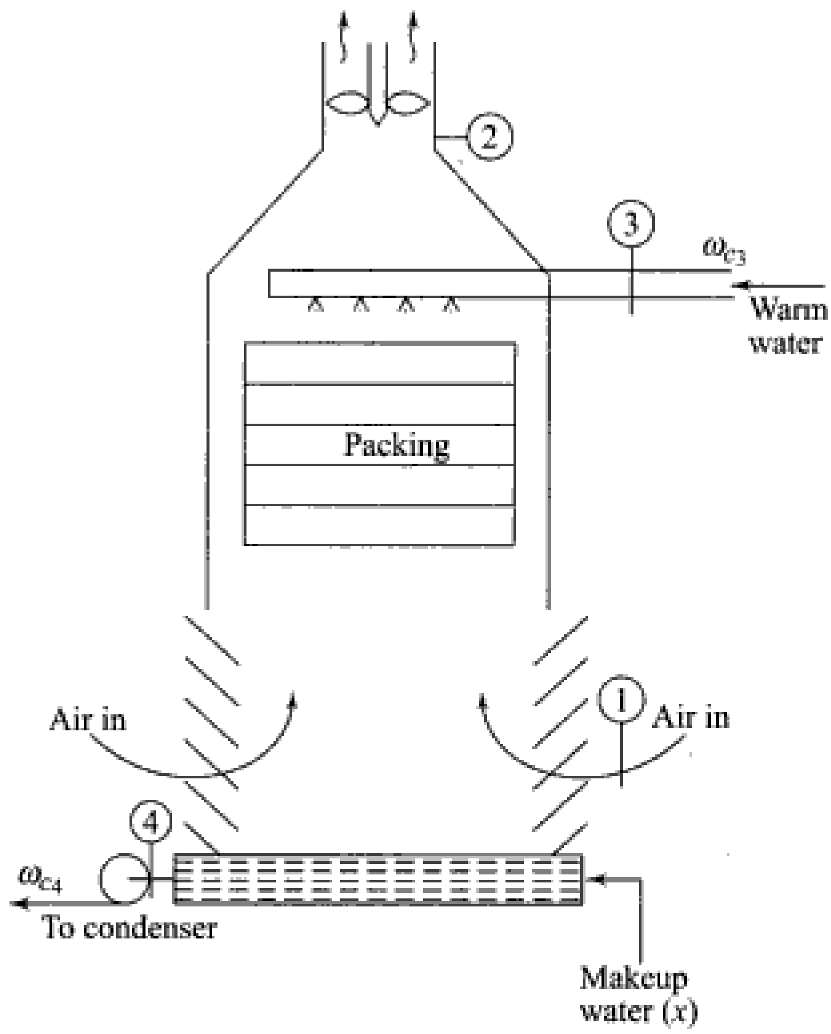

Fig. 8.28 *Cooling tower calculations*

Most towers are rated in practice for wet-bulb approach between 6 °C and 8 °C. Values of 6 to 10 °C are common for the cooling range. The relationships among t_{c_1}, t_{c_2}, R and A are shown in Fig. 8.29. Vaporization, hence cooling, takes place as long as the partial pressure of water vapour at the cooling water surface exceeds the partial pressure of water vapour in the bulk air stream.

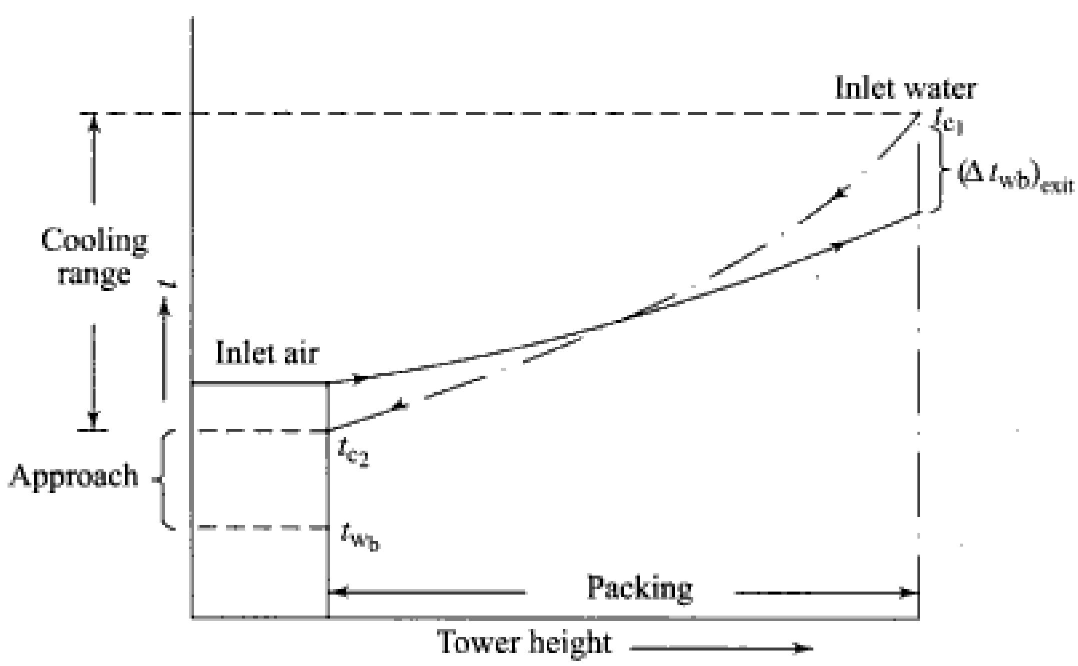

Fig. 8.29 *Temperature relationship in a counterflow cooling tower*

$$\Delta t_{1.m} = \frac{\Delta t_i - \Delta t_e}{\ln(\Delta t_i / \Delta t_e)} = \frac{8-2}{\ln \frac{8}{2}} = 4.33\,°\text{C}$$

$$Q = U_0 A_0 \Delta t_{1.m} = \omega_s (h_2 - h_3)$$

$$2.6 \times A_0 \times 4.33 = \frac{250 \times 1000}{3600} \times 2118.16$$

$$A_0 = 13066 \text{ m}^2$$

$$\omega_c = \left(n \frac{\pi}{4} d_i^2 \right) \rho V = 5855.2 \text{ kg/s}$$

$$n \frac{\pi}{4} (25.4 - 2.5)^2 \times 10^{-6} \times 1000 \times 1.8 = 5855.2$$

$$n = \frac{5.8552 \times 4 \times 10^6}{524.41 \times \pi \times 1.8} = 7898$$ *Ans.* (d)

Again, $$A_0 = n \pi d_0 l = 13066 \text{ m}^2$$

$$l = \frac{13066}{7898 \times \pi \times 25.4 \times 10^{-3}} = 20.73 \text{ m}$$ *Ans.* (c)

Example 8.2 Steam enters the condenser at 35°C. The condenser vacuum is 70 cm of mercury when the barometer reads 75.5 cm of Hg. Determine the vacuum efficiency. Estimate the mass of air present in the condenser per kg of steam.

Solution Saturation pressure of steam at 35°C = 0.05622 bar and $v_g = 25.24 \text{ m}^3/\text{kg}$
1std. atm. pr = 76 cm Hg = 1.013 bar.

$\therefore$ $$p_{sat} = 0.05622 \text{ bar} = 4.2717 \text{ cm Hg}$$

$$P_{abs} = 75.5 - 70 = 5.5 \text{ cm Hg}$$

Vacuum gauge corrected to standard atmosphere

$$= 76 - 5.5 = 70.5 \text{ cm Hg}$$

$\therefore$ $$\text{Vacuum efficiency} = \frac{70.5}{76 - 4.2717} = 0.9829 \text{ or } 98.29\%$$ *Ans.*

Absolute pressure inside the condenser = 5.5 cm Hg

Partial pressure of steam at 35°C = 4.2717 cm Hg

$\therefore$ Partial pressure of air = 5.5 − 4.2717

= 1.2283 cm Hg

$\therefore$ Mass of air associated with 1 kg steam

$$= \frac{pv}{Rt} = \frac{1.2283}{76} \times \frac{1.013 \times 10^5 \times 25.24}{287 \times (273 + 35)}$$

$$= 0.467 \text{ kg air/kg steam}$$ *Ans.*

Example 8.3 Exhaust steam having a quality of 0.9 enters a surface condenser at an absolute pressure of 0.13 bar and comes out as water at 45°C. The circulating water enters at 30°C and leaves at 40°C. Estimate the quantity of circulating water and the condenser efficiency.

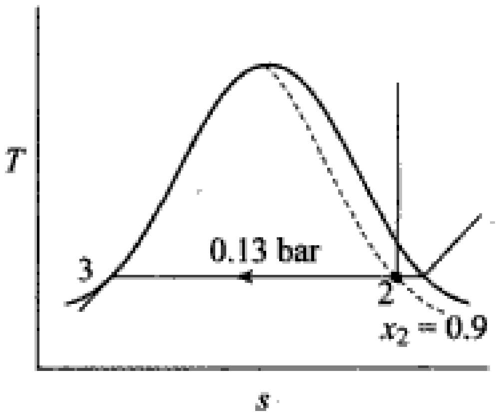

Fig. E8.3

Solution At 0.13 bar, $t_{sat} = 51.06°C$

$$h_2 = h_f + x_2\, h_{fg}$$
$$= 213 + 0.9 \times 2380.3$$
$$= 2355.97 \text{ kJ/kg}$$
$$h_f \text{ at } 45°C = 188.35 \text{ kJ/kg}$$

By energy balance,

$$\dot{m}_w\ c_{p_w}(t_{w_2} - t_{w_1}) = \dot{m}_s\ (h_2 - h_3)$$

$$\therefore \qquad \dot{m}_w / \dot{m}_s = \frac{2355.97 - 188.35}{4.182 \times (40 - 30)} = 51.77\ \frac{\text{kg water}}{\text{kg steam}} \qquad \textit{Ans.}$$

$$\text{Condenser efficiency} = \frac{40 - 30}{51.06 - 30} = 0.475 \text{ or } 47.5\% \qquad \textit{Ans.}$$

Example 8.4 The following readings were taken during a test on a surface condenser:
Mean condenser temperature = 35°C, Hot well temperature = 30°C, condenser vacuum = 69 cm Hg, barometer reading 76 cm Hg. Condensate collected 16 kg/min. Cooling water enters at 20°C and leaves at 32.5°C, flow rate being 37,500 kg/h. Calculate (a) mass of air present per cubic metre of condenser, (b) quality of steam at condenser inlet, (c) vacuum efficiency, and (d) condenser efficiency.

Solution Absolute pressure inside the condenser = 76 – 69 = 7 cm Hg = 0.0933 bar

$$p_{sat} \text{ at } 35°C = 0.05622 \text{ bar}$$

$$\therefore \quad \text{Partial pressure of air, } p_a = 0.0933 - 0.05622$$
$$= 0.03708 \text{ bar}$$

$$\therefore \quad \text{Mass of air present, } m_a = \frac{pv}{RT}$$

$$\text{or, } m_a = \frac{0.03708 \times 10^5 \times 1}{287 \times (273 + 35)} = 0.042 \text{ kg/m}^3 \qquad \textit{Ans. } (a)$$

Let x be the quality of steam at condenser inlet

$$\dot{m}_s\ [x h_{fg} + c_p\,(t_{sat} - 30)] = \dot{m}_w\ c_{p_w}(t_{w_2} - t_{w_1})$$
$$16 \times 60\ [x + 2418.8 + 4.182\ (35 - 30)] = 37{,}500 \times 4.182 \times (32.5 - 20)$$
$$= 1.96 \times 10^6$$

$\therefore$ $x = 0.836$ *Ans.* (b)

$$\text{Vacuum efficiency} = \frac{\text{actual vacuum}}{\text{ideal vacuum}}$$

$$= \frac{69}{79 - 4.22} = 096.13 \text{ or } 96.13\% \qquad \textit{Ans. } (c)$$

$$\text{Condenser efficiency} = \frac{\text{actual temperature rise}}{\text{maximum temperature rise}}$$

$$= \frac{12.5}{35 - 20} = 0.833 \text{ or } 83.33\% \qquad \textit{Ans. } (d)$$

Example 8.5 A surface condenser receives 20 t/h of dry saturated steam at 40 °C. The air leakage to the condenser is estimated to be 0.35 kg per 1000 kg of steam. The condensate leaves at a temperature of 38 °C Makeup water is supplied at 10 °C. The cooling water enters at 32 °C and leaves at 38 °C. A separate air extraction pump (SJAE) is added and from the air cooler section air along with some steam leaves at 27 °C. The pressure in the condenser is assumed to remain constant. Calculate (a) the rate of saving of condensate and the rate of saving in the heat supply in the boiler due to separate air extraction pump, (b) the percentage reduction in air ejector load due to this separate air extraction method, (c) the rate of cooling water flow.

Solution At 40 °C, $p_{sat} = 0.07384$ bar and $v_g = 19.52$ m^3/kg (Fig. E8.5a)
From Eq. (8.14),

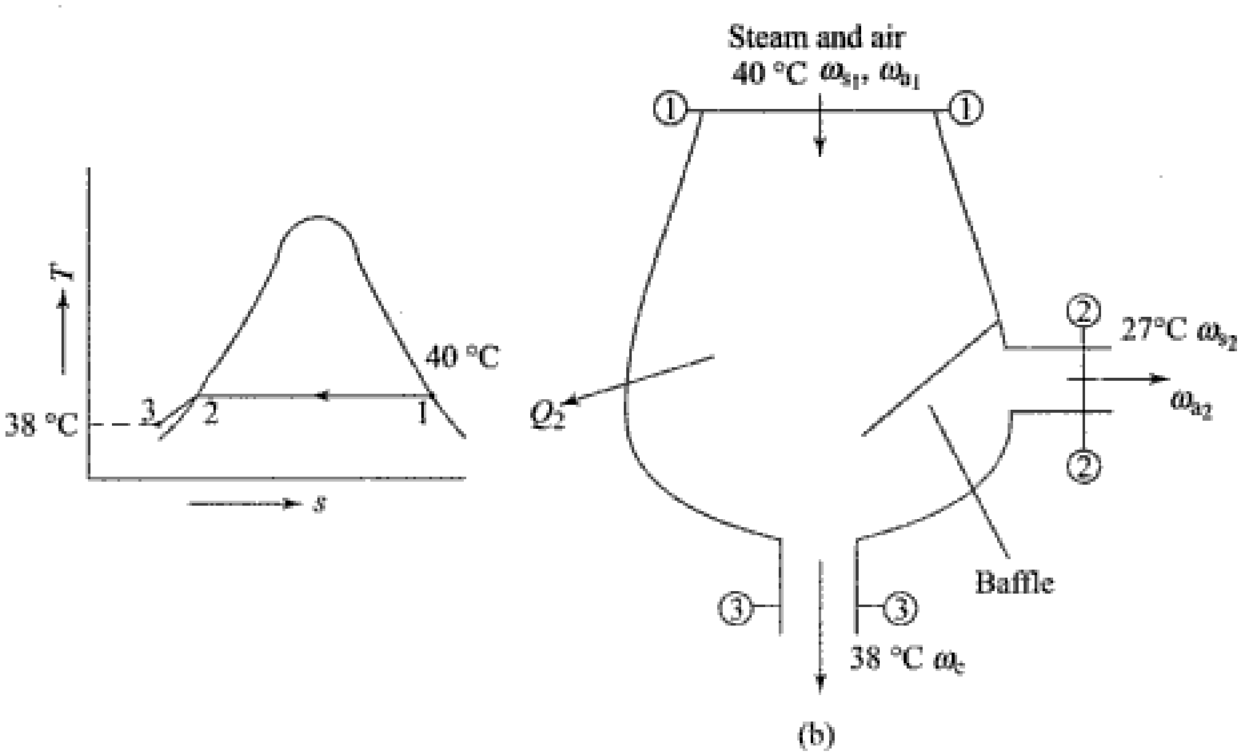

Fig. E8.5

$$p_{air}\, \omega_s\, v_1 = \omega_a\, R_a\, (t_{sh} + 273)$$

$$p_{air} = \frac{0.35 \times 0.287 \times 313 \times 10^3}{1000 \times 19.52}\ \text{N/m}^2$$

$$= 1.61\ \text{N/m}^2 = 1.61 \times 10^{-5}\ \text{bar}$$

This is very small and can be neglected.
Condensate leaves at 38 °C. At 38 °C, $p_{sat} = 0.06624$ bar and $v_g = 21.63$ m^3/kg.
The total pressure in the condenser is 0.07384 bar

$$p = p_{st} + p_{air}$$

$$0.07384 = 0.06624 + p_{air} \quad \text{or} \quad p_{air} = 0.00760\ \text{bar}$$

Mass of air removed per hour,

$$\omega_a = 20{,}000 \times \frac{0.35}{1000} = 7\ \text{kg/h}$$

Volume of air removed per hour,

$$V = \frac{\omega_a R_a (273 + 38)}{p_{air}} = \frac{7 \times 287 \times 311}{760} = 822.1\ \text{m}^3\text{/h}$$

The mass of steam accompanying this air is

$$\omega_s = \frac{822.1}{21.63} = 38\ \text{kg/h}$$

Separate air extraction pump: Air along with some steam leaves the air cooler section at 27 °C.

At 27°C, $p_{sat} = 0.03564$ bar and $v_g = 38.81$ m^3/kg

$$p_{air} = 0.07384 - 0.03564 = 0.03820\ \text{bar}$$

Therefore, the volume of air removed,

$$V = \frac{\omega_a R_a T}{p_{air}} = \frac{7 \times 287 \times 300}{3820} = 157.77\ \text{m}^3\text{/h}$$

Mass of steam accompanying this air,

$$\omega_s = \frac{157.77}{38.81} = 4.07\ \text{kg/h}$$

Hence, the saving in condensate by using separate extraction is

$$38 - 4.07 = 33.93\ \text{kg/h}$$ *Ans.* (a)

Saving in heat supply in the boiler

$$= 33.98 \times 4.187\ (38 - 10)$$

$$= 3977.8\ \text{kJ/h} = 1.105\ \text{kW}$$ *Ans.* (a)

Air ejector capacity without air cooler = 822.1 m^3/h
Air ejector capacity with the air cooler = 157.77 m^3/h
Percentage reduction in air ejector load

$$= \frac{822.1 - 157.77}{822.1} \times 100 = 80.8\% \qquad \textit{Ans. (b)}$$

By making an energy balance for the condenser (Fig. E8.2b), with subscripts s, a and c being used for steam, air and condensate respectively,

$$Q_2 = \omega_{s_1} h_{s_1} + \omega_{a_1} h_{a_1} - (\omega_{s_2} h_{s_2} + \omega_{a_2} h_{a_2}) - \omega_c h_c$$

Here,

$$\omega_{a_1} = \omega_{a_2} = 7 \text{ kg/h}, \omega_{s_2} = 4.07 \text{ kg/h},$$
$$\omega_c = 20{,}000 - 4.07 = 20{,}000 \text{ kg/h} = \omega_{s_1}$$
$$h_c = h_f \text{ at } 38\text{ °C} = 159.3 \text{ kJ/kg},$$
$$h_{s_1} = h_g \text{ at } 40\text{ °C} = 2574.3 \text{ kJ/kg},$$
$$h_{s_2} = h_g \text{ at } 27\text{ °C} = 2550.3 \text{ kJ/kg}$$
$$Q_2 = \omega_{s_1}(h_{s_1} - h_c) - \omega_{a_1}(h_{a_1} - h_{a_2}) - \omega_{s_2} h_{s_2}$$
$$= 20{,}000\,(2574.3 - 159.3)$$
$$- 7 \times 1.005(40 - 27) - 4.07 \times 2550.3$$
$$= 48{,}300{,}000 - 91.5 - 10379.7$$
$$= 48389529 \text{ kJ/h} = 13441.5 \text{ kW}$$

Now,

$$Q_2 = \omega_c c_p (t_{c_2} - t_{c_1})$$
$$13441.5 = \omega_c \times 4.187 \times 6$$
$$\omega_c = 535 \text{ kg/s} \qquad \textit{Ans. (c)}$$

If we neglect the energy leaving with the flow to the ejector,

$$Q_2 = \omega_{s_1}(h_{s_1} - h_c) = \omega_c c_p (t_{c_2} - t_{c_1})$$
$$48{,}300{,}000 = w_c \times 4.187 \times 6$$
$$\omega_c = 534 \text{ kg/s} \qquad \textit{Ans. (c)}$$

Example 8.6 Water at 30 °C flows into a cooling tower at the rate of 1.15 kg per kg air. Air enters the tower at the dbt of 20 °C and a relative humidity of 60 % and leaves it at a dbt of 28 °C and 90% relative humidity. Makeup water is supplied at 20 °C. Determine (a) the temperature of water leaving the tower, (b) the fraction of water evaporated, and (c) the approach and range of the cooling tower.

Solution Properties of air entering and leaving the tower (Fig. 8.28) are obtained from psychrometric chart.

$$t_{wb_1} = 15.2\text{ °C}, t_{wb_2} = 26.7\text{ °C},$$
$$h_1 = 43 \text{ kJ/kg dry air}, h_2 = 83.5 \text{ kJ/kg dry air},$$
$$W_1 = 0.0088 \text{ kg water vapour/kg dry air}$$
$$W_2 = 0.0213 \text{ kg water vapour/kg dry air}$$

Enthalpies of water entering the tower and the makeup water are

$$h_{w_3} = 125.8 \text{ kJ/kg}, h_w = 84 \text{ kJ/kg}$$

From the energy balance Eq. (8.22),

$$h_{w_3} - h_{w_4} = \frac{G}{\omega_C}[(h_2 - h_1) - (W_2 - W_1)\,h_w]$$

$$= \frac{1}{1.15}\,[(83.5 - 43) - (0.0213 - 0.0088)84]$$

$$= 34.2 \text{ kJ/kg}$$

$$t_{w_3} - t_{w_4} = \frac{34.2}{4.19} = 30 - t_{w_4}$$

$$t_{w4} = 21.8\ °\text{C} \qquad \textit{Ans.}\ \text{(a)}$$

$$\text{Approach} = t_{w_4} - t_{wb_1} = 21.8 - 15.2 = 6.6\ °\text{C} \qquad \textit{Ans.}\ \text{(c)}$$

$$\text{Range} = t_{w_3} - t_{w_4} = 30 - 21.8 = 8.2\ °\text{C} \qquad \textit{Ans.}\ \text{(c)}$$

Fraction of water evaporated.

$$x = G(W_2 - W_1)$$

$$= 1(0.0231 - 0.0088)$$

$$= 0.0125 \text{ kg/kg dry air} \qquad \textit{Ans.}\ \text{(b)}$$

Example 8.7 Warm water at 45 °C enters a cooling tower at the rate of 6 kg/s. An ID fan draws 10 m^3/s of air through the tower and absorbs 4.90 kW. The air entering the tower is at 20 °C dbt and 60% relative humidity. The air leaving the tower is assumed to be saturated and its temperature is 26 °C. Calculate the final temperature of the water and the amount of makeup water required per second. Assume that the pressure remains constant throughout the tower at 1.013 bar.

Solution At inlet, RH $\phi = p_W/p_s = 0.6$, At 20 °C, $p_s = 0.0234$ bar

$$p_{s_1} = 0.6 \times 0.0234 = 0.01404 \text{ bar}$$

$$p_{a_1} = 1.013 - 0.01404 = 0.99896 \text{ bar}$$

Dry air flow,

$$G_1 = \frac{10^5 \times 0.99896 \times 10}{0.287 \times 10^{-3} \times 293} = 11.8795 \text{ kg/s} = 11.88 \text{ kg/s}$$

Moisture flow,

$$\omega_1 = \frac{0.01404 \times 10^5 \times 10}{0.4619 \times 10^3 \times 293} = 0.1037 \text{ kg/s}$$

$$W_1 = \frac{0.1037}{11.8795} = 0.00874 \text{ kg vap/kg dry air}$$

At exit, at 26 °C,

$$p_s = 0.0336 \text{ bar}, \phi = 100\ \%$$

$$p_{w_2} = 0.0336 \text{ bar}$$

$$W_2 = W_s = 0.622\,\frac{p_{w_2}}{p - p_{w_2}} = 0.622\,\frac{0.0336}{1.013 - 0.0336}$$

$$= 0.02133 \text{ kg vap./kg dry air}$$

Now, $W_2 = \dfrac{G_2}{\omega_2} = 0.02133$ kg vap/kg dry air

$$G_1 = G_2$$

Moisture flow at exit,

$$\omega_2 = 0.02133 \times 11.88 = 0.2534 \text{ kg/s}$$

Makeup water required

$$= \omega_2 - \omega_1 = 0.2534 - 0.1037$$
$$= 0.1497 \text{ kg/s} \quad \textit{Ans.}$$
$$\omega_{c_1} = 6 \text{ kg/s} = \text{cooling water inflow}$$
$$\omega_{c_2} = 6 - 0.1497 = 5.8503 \text{ kg/s} = \text{cooling water outflow}$$

Applying steady flow energy equation (Fig. 8.22),

$$\omega_{c_1} h_{w_3} + G_1(h_1 + W_1 h_{w_1}) + W_s = \omega_{c_2} h_{w_4} + G_2(h_2 + W_2 h_{w_2})$$

where W_s = shaft work input to the fan = 4.90 kW.

$$h_{w_3} = 4.187 \times 45 = 188.4 \text{ kJ/kg}$$
$$h_{w_1} = h_g + c_p(t - t_{sat})$$

At 20 °C, $h_g = 2538.1$ kJ/kg

At $p_{w_1} = 0.01404$ bar, $t_{sat} = 12$ °C

$$h_{w_1} = 2538.1 + 1.88(20 - 12) = 2553.14 \text{ kJ/kg}$$
$$h_{w_2} = h_g \text{ at } 26\text{ °C} = 2548.4 \text{ kJ/kg}$$
$$\omega_{c_1} h_{w_3} - \omega_{c_2} h_{w_4} = G[h_2 - h_1) + W_2 h_{w_2} - W_1 h_{w_1}] - W_s$$
$$6 \times 188.4 - 5.85\, h_{w_4} = 11.88\,[1.005\,(26 - 20) + 0.02133 \times 2548.4$$
$$- 0.00874 \times 2553.14] - 4.90$$
$$h_{w_4} = 116.752 \text{ kJ/kg}$$

Exit water temperature $= \dfrac{116.752}{4.187} = 27.88$ °C *Ans.*

SHORT-ANSWER QUESTIONS

8.1 What are the functions of a condenser in a steam power plant?
8.2 Why is the temperature rise of cooling water restricted?
8.3 What are the different classes of condensers?
8.4 What is a spray condenser? Where is it used?
8.5 Explain a barometric condenser. What is the function of the tail pipe?
8.6 How is a jet condenser different from a barometric condenser?
8.7 What is a surface condenser? Why does cooling water flow inside the tubes and steam condense outside the tubes?
8.8 What are single-pass and two-pass condensers?
8.9 What is a hot well?
8.10 Explain the importance of tube arrangement in the condenser shell. What are steam lanes?

8.11 Why is an air-cooling section provided in the condenser?
8.12 Why is the bulk temperature of the condensate less than the saturation temperature?
8.13 Why is the condensate subcooling not desired? How is it overcome?
8.14 Which parameter affects the overall heat transfer coefficient most?
8.15 What is TTD? How does it influence the condenser design?
8.16 How is the water velocity in a condenser tube optimized?
8.17 How is the pumping power required by a condenser estimated?
8.18 How does air leakage affect the condenser performance?
8.19 Explain how the rate of air leakage into the condenser can be estimated.
8.20 Explain the objective of the steam jet air ejector.
8.21 How does an SJAE operate?
8.22 What are intercondenser and aftercondenser?
8.23 Why are feedwater heaters used?
8.24 Why is one of the heaters always an open heater?
8.25 Where are LP and HP heaters located?
8.26 What is a 3-zone heater? What are the three zones?
8.27 What is a drain cooler?
8.28 Why is the deaerator located at a sufficient height above the BFP?
8.29 What are the different types of deaerators?
8.30 What do you understand by once-through and closed loop circulating water systems?
8.31 Why is open loop system not used?
8.32 What is the function of a cooling tower?
8.33 How does a cooling tower operate?
8.34 What is the need of fill in a tower?
8.35 What do you mean by evaporative cooling?
9.36 What is the minimum temperature to which water can be cooled?
9.37 Define (a) approach, (b) range, and (c) cooling efficiency of a cooling tower.
8.38 What are the different types of cooling towers?
8.39 What is an FD cooling tower? Mention its merits and demerits.
8.40 Why are ID cooling towers preferred in utility plants?
8.41 What are the main advantages of mechanical draught cooling towers?
8.42 What are cross-flow and counter-flow cooling towers?
8.43 What is a natural draught cooling tower? What is the reason of its hyperbolic shape?
8.44 When is a natural draught cooling tower a good choice?
8.45 How does water loss occur in a cooling tower?
8.46 What is drift? How is the drift eliminated?
8.47 Define a dry cooling tower. When is it recommended?
8.48 What are the different types of dry cooling towers?
8.49 Explain an indirect dry cooling tower where a direct contact spray type (open) condenser is used.
8.50 Explain an indirect dry cooling tower where ammonia is used as the coolant in the condenser.

8.6 Water at 60 °C leaving the condenser at the rate of 22.5 kg/s is sprayed into a natural draught cooling tower and leaves it at 27 °C. Air enters the tower at 1.013 bar, 13 °C and 50% relative humidity and leaves it at 38 °C, 1.013 bar and saturated, Calculate (a) the air flow rate required in m^3/s, and (b) the makeup water required in kg/s.

[*Ans.* (a) 21 m^2/s, (b) 1 kg/s]

8.7 Water from a cooling system is itself to be cooled in a cooling tower at a rate of 2.78 kg/s. The water enters the tower at 65 °C and leaves a collecting tank at the base at 30 °C. Air flows through the tower, entering the base at 15 °C, 0.1 MPa, 55% RH and leaving the top at 35 °C, 0.1 MPa, saturated. Makeup water enters the collecting tank at 14 °C. Determine the air flow rate into the tower in m^3/s and the makeup water flow rate in kg/s.

[*Ans.* 3.438 m^3/s, 0.129 kg/s]

8.8 Cooling water enters a cooling tower at a rate of 1000 kg/h and 70 °C. Water is pumped from the base of the tower at 24 °C and some makeup water is added afterwards. Air enters the tower at 15 °C, 50% RH, 1.013 bar, and is drawn from the tower saturated at 34 °C, 1 bar. Calculate the flow rate of the dry air in kg/h and the makeup water required per hour.

[*Ans.* 2088 kg/h, 62.9 kg/h]

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9 Nuclear Power Plants

The unit cost per kWh of electricity generation in nuclear power plants is now comparable to or even lower than the unit cost in coal-fired power plants in most parts of the world. In addition, the problems associated with environmental pollution, mine safety, fuel transportation and so on are much less severe in nuclear power stations. Nuclear power utilization can help save a considerable amount of fossil fuels which can be used in other areas of utility.

In recent years, a strong public opinion has grown against the use of nuclear energy for power generation due to the problems related to nuclear safety, radioactive waste disposal and nuclear weapons proliferation. Despite these difficulties, the future of large capacity electricity generation includes nuclear energy as one of the main sources. In many countries like France, Japan, the U.K. and Russia, the bulk of the electricity is produced in nuclear power plants. In India also, the Nuclear Power Corporation has been forging ahead with installation of new plants all around the country.

9.1 STRUCTURE OF THE ATOM

All matter is composed of unit particles called *atoms*. An atom consists of a relatively heavy, positively charged *nucleus* and a number of much lighter negatively charged, *electrons* orbiting around the nucleus. The nucleus consists of *protons* and *neutrons*, which together are called *nucleons*. Protons are positively charged, while the neutrons are electrically neutral. The electric charge on the proton is equal in magnitude but opposite in sign to that on an electron. The atom as a whole is electrically neutral, since the number of protons is equal to the number of electrons in orbit.

The number of protons in the nucleus is called the *atomic number*, Z. The total number of nucleons in the nucleus is called the *mass number*, A. Nuclear symbols are written conventionally as

$$_{Z}X^{A} \quad \text{or} \quad ^{A}_{Z}X$$

where X is the usual chemical symbol. Most of the weight of an atom is concentrated in the nucleus. The radius of a nucleus is of the order of 10^{-16} m and that of a atom is 10^{-11} m. Figure 9.1 illustrates the atomic structure of some simple atoms. The masses of the three primary atomic subparticles are:

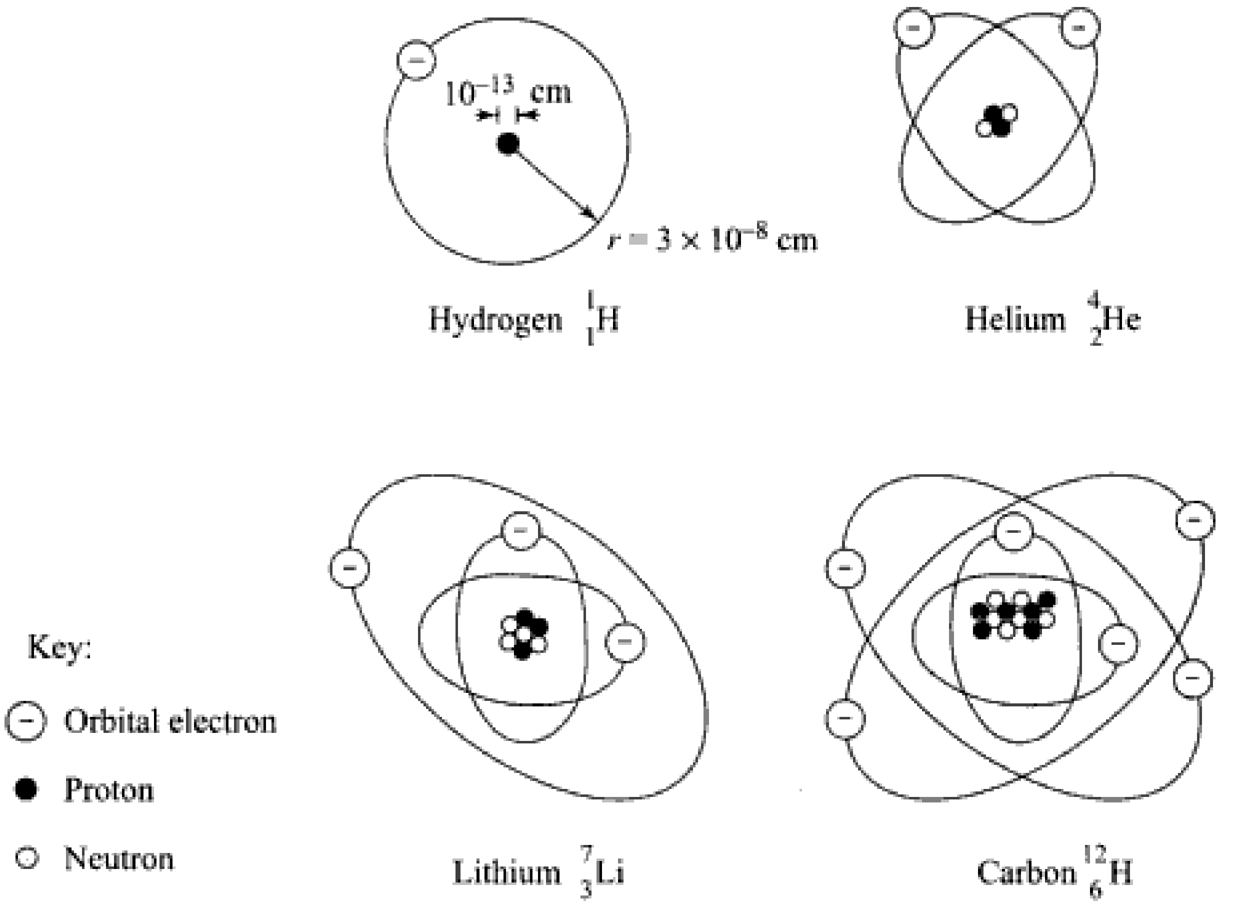

Fig. 9.1 *Structure of the atom*

$$\text{Neutron mass, } m_n = 1.008665 \text{ amu} = 1.674 \times 10^{-27} \text{ kg}$$
$$\text{Proton mass, } m_p = 1.007277 \text{ amu} = 1.673 \times 10^{-27} \text{ kg}$$
$$\text{Electron mass, } m_e = 0.0005486 \text{ amu} = 9.109 \times 10^{-31} \text{ kg}$$

The *atomic mass unit*, amu, is a unit of mass approximately equal to 1.66×10^{-27} kg.

Hydrogen (${}_1H^1$) has a nucleus composed of one proton, no neutron, and one orbital electron ($Z = 1, A = 1$). It is the only atom that has no neutron. Deutereium (${}_1H^2$) has one proton and one neutron in its nucleus and one orbital electron ($Z = 1$, $A = 2$). Helium (${}_2He^4$) has two protons, two neutrons and two electrons ($Z = 2$, $A = 4$).

Atoms with nuclei having the same number of protons have similar chemical and physical properties and differ mainly in their masses. They are called isotopes. For example, deuterium, often called heavy hydrogen, is an isotope of hydrogen. When combined with oxygen, ordinary hydrogen and deuterium form ordinary water (H_2O) and heavy water (D_2O) respectively.

Natural uranium is composed of 99.282 % U^{238}, 0.712 % U^{235} and 0.006 % U^{234}, the atomic number being 92 in all cases. Many isotopes do not appear in nature and are synthesized in the laboratory or in nuclear reactors.

Two other particles are of importance, *viz*., the *positron* and *neutrino*. The positron is a positively charged electron having the symbols ${}_{+1}e^0$, e^+ or β^+, the symbol for electron being ${}_{-1}e^0$, e^- or β^-. The neutrino is a tiny, electrically neutral particle, ejected along with β particle during nuclear fission. The ejected neutrinos (ν) carry some 5% of the total energy produced in fission.

Electrons that orbit in the outermost shell of an atom are called valence electrons which decide the chemical properties of an element.

9.2 CHEMICAL AND NUCLEAR REACTIONS

Atoms are combined or separated in a chemical reaction. In the reaction,

$$C + O_2 = CO_2$$

an energy of 4 electron volt (eV) is released. In nuclear engineering, the unit of energy is electron volt, 1 eV = 1.6021×10^{-19} J = 4.44×10^{-26} kWh. It is the energy acquired by an electron when it is accelerated across a potential difference of 1 volt.

In chemical reactions, although the molecules change, each atom participates as a whole and retains its identity. Only the valence electrons are shared or exchanged. The nuclei do not change. In a chemical reaction, the number of atoms of each element in the products is equal to the number in the reactants.

In nuclear reactions, the products do not have the reactant nuclei but some other nuclei. The number of nucleons in the products are the same as those in the reactants. If A, B, C and D represent the chemical symbols, the corresponding nuclear equation may be written as

$$_{Z1}A^{A1} + {}_{Z2}B^{A2} \longrightarrow {}_{Z3}C^{A3} + {}_{Z4}D^{A4}$$

where $Z_1 + Z_2 = Z_3 + Z_4$ and $A_1 + A_2 = A_3 + A_4$

Sometimes, electromagnetic radiation (γ-rays) and neutrino (ν) are often emitted but they do not affect the above balance, since they carry only energy and have zero Z and A.

9.3 NUCLEAR STABILITY AND BINDING ENERGY

The sum of the masses of the protons and neutrons that comprise the nucleus exceeds the mass of the atomic nucleus. This difference in mass is called the *mass defect*. The mass defect (Δm) is found by adding up all the individual particle weights and subtracting the actual mass (m) of the atom:

$$\Delta m = n_n m_n + (m_p + m_e)Z - {}_Z m^A \qquad (9.1)$$

where n refers to the number and m the mass of particles. The mass defect is converted to energy in a nuclear reaction as given by Einstein's law:

$$\Delta E = \Delta m \cdot C^2 \qquad (9.2)$$

where E = energy, J; C = velocity of light = 3×10^8 m/s; and Δm = mass defect, kg.

The energy associated with the mass defect is known as the *binding energy* (BE) of the nucleus. It acts as a "glue" which binds the protons and neutrons together in the nucleus. The energy equivalent of 1 g of mass is

$$\Delta E = 1 \times 10^{-3}\ \text{kg} \times (3 \times 10^8\ \text{m/s})^2 = 9 \times 10^{13}\ \text{J}$$

Similarly, the energy equivalent of 1 amu of mass is

$$\begin{aligned}\Delta E &= 1.66 \times 10^{-27}\ \text{kg} \times (3 \times 10^8\ \text{m/s})^2 \\ &= 14.94 \times 10^{-11}\ \text{J} = 9.31 \times 10^8\ \text{eV} \\ &= 931\ \text{MeV} \qquad (9.3)\end{aligned}$$

Therefore, if 1 amu of mass could be completely converted to energy, 931 MeV would be yielded.

The binding energy per nucleon (i.e., proton and neutron) determines the stability of the nucleus. Let us consider a helium nucleus as a simple example.

$$\text{He nucleus: } 2\,{}_0n^1 + 2\,{}_1p^1 \longrightarrow {}_2\text{He}^4$$

Experimental mass (by mass spectrography) of helium atom – mass of two orbital electrons = 4.00387 – 2 × 0.00055 = 4.00277 amu

$$\text{Calculated mass} = 2\,m_p + 2m_n = 2 \times 1.00759 + 2 \times 1.00898 = 4.03314\ \text{amu}$$

$$\Delta m = \text{mass defect} = 0.03037\ \text{amu}$$

and $\quad \Delta E = 0.03037 \times 931 = 28.2$ MeV

This is the energy released when two protons and two neutrons are bound together. If we were to change the helium nucleus back into its constituents, we would have to give back this 28.2 MeV to the nucleus. The binding energy per nucleon is then

$$\text{BE/nucleon} = 28.2/4 = 7.05\ \text{MeV}$$

For deuterium it is 1.115 MeV/nucleon. In this way, the binding energy per nucleon can be calculated for all the isotopes. Higher the binding energy per nucleon, higher is the stability of the nucleus (Fig. 9.2). The binding energy curve shows that the most stable elements (like iron, cobalt, nickel etc.) are in the intermediate mass number range. If elements of low mass number are fused together, it would lead to more stable elements. The elements of higher mass number are less stable and if they are fissioned, they would form elements of less mass number, which are more stable. Thus light isotopes like hydrogen, deuterium and so on are good for fusion reactions, while the heavier isotopes like uranium are suitable for fission reaction.

For most medium and heavy nuclei, the binding energy per nucleon falls roughly between 7.5 and 8.7 MeV. Thus, if a nucleus is to expel one nucleon, say a neutron, it should first have a minimum excitation energy of between 7.5 and 8.7 MeV. Only in such an excited state a nucleus can emit a neutron.

It was found that the nuclei of the even-even type, i.e., having an even number of protons and even number of neutrons, are very stable, Therefore, a ${}_{92}\text{U}^{238}$ atom having 92 protons and 146 neutrons is quite stable and requires very high energy neutrons for fission, whereas a ${}_{92}\text{U}^{235}$ atom having 92 protons and 143 neutrons can be fissioned even by low energy neutrons.

Except for light nuclei, where $n_p = n_n$, the number of neutrons (n_n) of known isotopes exceeds the number of protons (n_p). Thus, for heavier atoms, more neutrons are necessary to shield the protons and overcome the electrical repulsive forces between them in the nucleus.

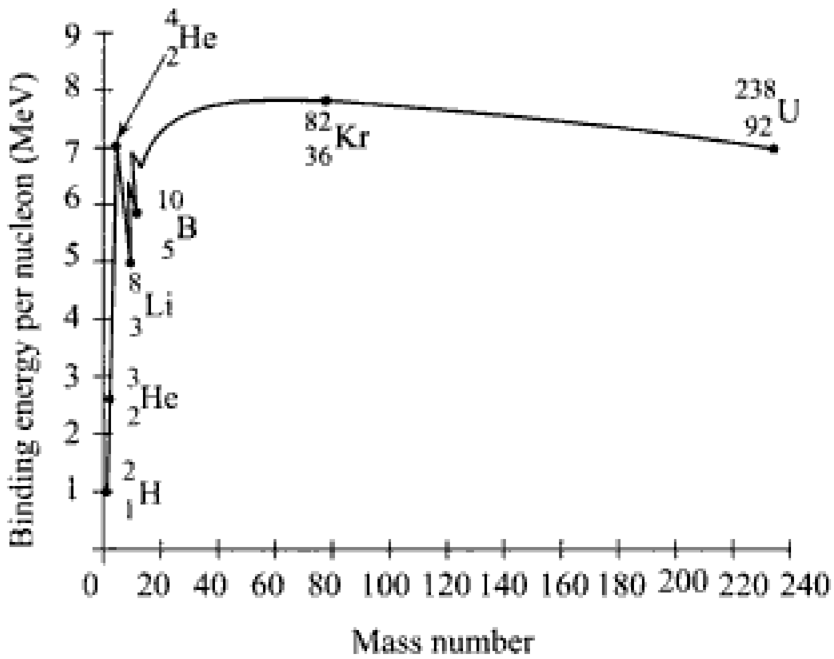

Fig. 9.2 *Binding energy per nucleon varying with mass number*

9.4 RADIOACTIVE DECAY AND HALF LIFE

Most isotopes that occur in nature are stable. Some isotopes of heavy elements like thallium ($Z = 81$), lead ($Z = 82$) and bismuth ($Z = 83$), and all isotopes of heavier elements starting with polonium ($Z = 84$) are not stable (the binding energy per nucleon being small) and emit radiation till a more stable nucleus is reached. Thus, a spontaneous disintegration process, called *radioactive decay*, occurs. The resulting nucleus is called the *daughter* and the original nucleus is called the *parent*. The daughter product may be stable or radioactive. A few lower mass isotopes are also naturally radioactive, such as K^{40}, Rb^{87} and In^{115}.

Radioactive isotopes, both natural and man-made, are commonly called *radioisotopes*. An example of radioactivity is

$$_{49}In^{115} \longrightarrow {}_{50}Sn^{115} + {}_{-1}e^0 \tag{9.4}$$

where In^{115} is a naturally occurring radioisotope and its daughter, Sn^{115}, is stable. Radioactivity is always accompanied by a decrease of mass or liberation of energy. The energy thus liberated shows up in the form of KE of the emitted particles and as electromagnetic radiation (γ-rays).

Naturally occurring radioisotopes emit (1) α particles, (2) β particles, (3) γ radiation; undergo, (4) Positron decay, (5) orbital electron absorption, called K capture, and also emit (6) neutrons and neutrinos.

1. *Alpha decay* Alpha particles are helium nuclei, commonly emitted by heavier radioactive nuclei and are accompanied by γ-radiation. For example,

$$_{94}Pu^{239} \xrightarrow{\alpha} {}_{92}U^{235} + {}_2He^4 \tag{9.5}$$

2. *Beta decay* It is equivalent to the emission of an electron and raises the atomic number by one, while the mass number remains the same. It is usually accompanied by the emission of neutrino (ν) and γ radiation.

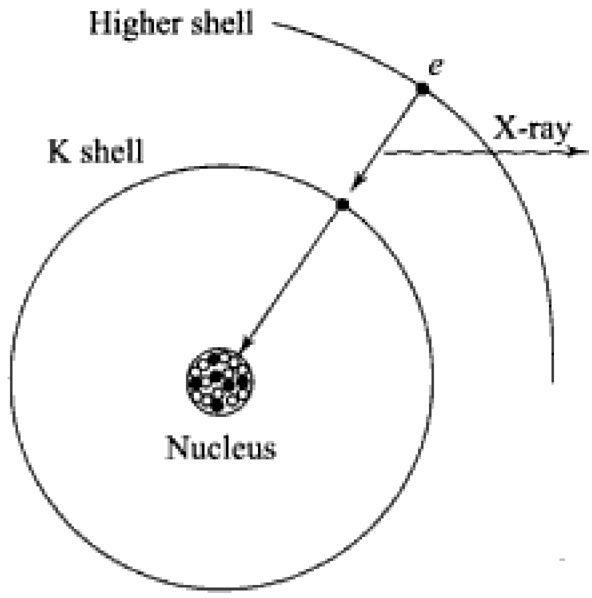

Fig. 9.3 *K capture*

In neutron decay the daughter is an isotope of the parent. Though it occurs rarely, it, however, comes about in nuclear reactors yielding delayed fission neutrons which greatly influence the reactor control.

The rate of decay is a function only of the number of radioactive nuclei present at a time, provided that the number is large. It does not depend on temperature, pressure or the physical and chemical states of matter, i.e. whether it is in solid, liquid or gaseous phase, or in chemical combination with other atoms.

If N be number of radioactive nuclei of one species at any time θ, the rate of decay

$$-\frac{dN}{d\theta} = \lambda N \tag{9.12}$$

where λ is a constant of proportionality, called the *decay constant*, having different values for different isotopes, with the dimension s^{-1}. By integrating the above, we obtain a simple exponential relation,

$$N = N_0 e^{-\lambda\theta} \tag{9.13}$$

where N_0 = radioactive atoms present at time $\theta = 0$, and N = radioactive atoms present at time θ

The rate of decay ($-dN/d\theta$) is also called *activity*, A, and has the dimension of disintegration per second or dis/s or s^{-1}. Thus, from Eqs (9.12) and (9.13),

$$A = -\frac{dN}{d\theta} = \lambda N = \lambda N_0 e^{-\lambda\theta} = A_0 e^{-\lambda\theta} \tag{9.14}$$

The decay rate is often expressed in the form of *half-life*, $\theta_{1/2}$, i.e. the time during which one-half of the number of radioactive species decays. Thus,

$$\frac{N}{N_0} = \frac{A}{A_0} = \frac{1}{2} = e^{-\lambda\theta_{1/2}}$$

or

$$\theta_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda} \tag{9.15}$$

Thus, the half-life is inversely proportional to the decay constant. Starting with N_0, half of N_0 decay after one half-life; one-half of the remaining atoms or 1/4 of N_0 decay during the second half-life and so on (Fig. 9.4).

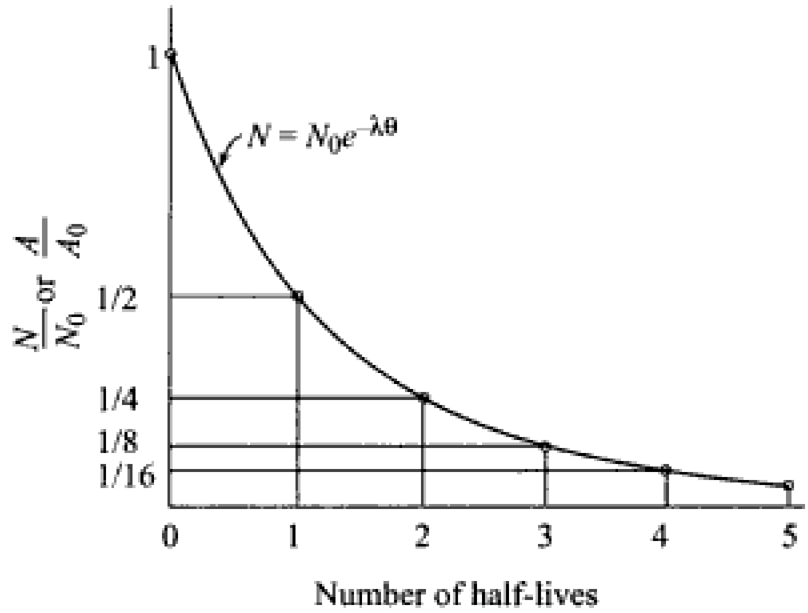

Fig. 9.4 *Radioactive decay rate and half-life*

Half-lives of radioactive isotopes differ by a wide range, varying from fractions of a microsecond to billions of years (Table 9.1). No. two radioisotopes

Table 9.1 *Half-lives of some radioisotopes*

Isotope	$\theta_{1/2}$	*Activity*
Tritium (H^3)	12.26 yr	β
Carbon 14	5730 yr	β
Krypton 87	76 min	β
Strontium 90	28.1 yr	β
Xenon 135	9.2 h	β and γ
Barium 139	82.9 min	β and γ
Radium 223	11.43 days	α and γ
Radium 226	1600 yr	α and γ
Thorium 232	1.41×10^{10} yr	α and γ
Thorium 233	22.1 min	β
Protactinum 233	27.0 days	β and γ
Uranium 233	1.65×10^5 yr	α and γ
Uranium 235	7.1×10^8 yr	α and γ
Uranium 238	4.51×10^9 yr	α and γ
Neptunium 239	2.35 days	β and γ
Plutonium 239	2.44×10^4 yr	α and γ

have exactly the same half-lives. Thus, half-lives are considered "finger-prints" to identify a radioisotope.

Readily fissionable isotopes U-233, U-235 and Pu-239 have extremely long half-lives, showing that they can be stored practically indefinitely.

The unit of radioactivity is curie (Ci),

$$1 \text{ curie} = 3.615 \times 10^{10} \text{ dis/s}$$

It was based on measurement of the activity of 1 g of radium 226. Curie has now been superseded by the SI unit, becquerel (Bq), which is defined as one disintegration per second. Since this is very small, the levels of radioactivity are expressed in kBq or MBq. Another unit, called roentgen (r), is used to provide some measure of the extent of biological injury, say due to X-rays and γ-rays.

9.5 NUCLEAR FISSION

Fission can be caused by bombarding with high energy α-particles, protons, deuterons, X-rays as well as neutrons. However, neutrons are most suitable for fission, They are electrically neutral and thus require no high KE to overcome electrical repulsion from positively charged nuclei. Two or three neutrons are usually released for each one absorbed in fission, and can thus keep the reaction going (Fig. 9.5). Isotopes like U-233, U-235 and Pu-239 can be fissioned by neutrons of all energies, whereas isotopes U-238, Th-232 and Pu-240 are fissionable by high energy (14 MeV) only. As shown in Fig. 9.5,

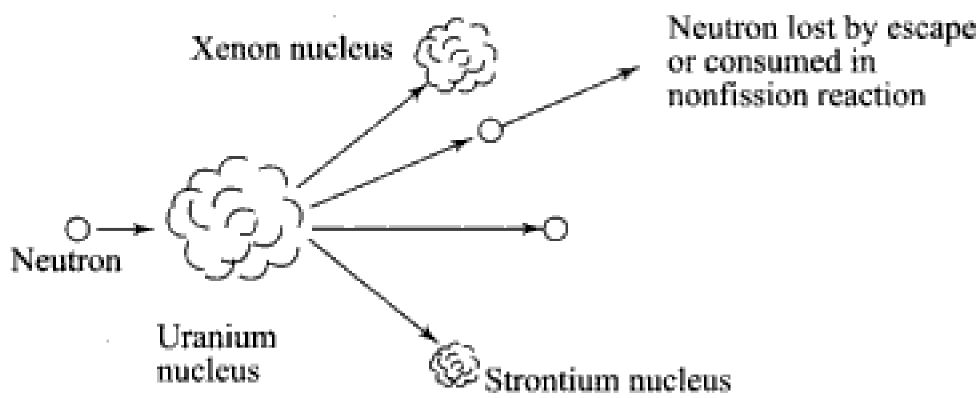

Fig. 9.5 *A fission reaction*

$$_{92}U^{235} + {_0n^1} \longrightarrow {_{54}Xe^{140}} + {_{38}Sr^{94}} + 2{_0n^1} \tag{9.16}$$

The immediate (prompt) products of a fission reaction, such as Xe^{140} and Sr^{94} are called *fission fragments*, which along with other decay products (α, β, γ etc.) are called *fission products*.

When a neutron collides with and is absorbed by a fissionable nucleus, the latter is transformed into a compound nucleus in an excited state, e.g.,

$$_{92}U^{235} + {_0n^1} \longrightarrow [{_{92}U^{236}}]^* \longrightarrow \text{fission}$$

which then undergoes fission. If the excitation energy is not sufficiently large, the nucleus may not udergo fission and may emit only γ-radiation or eject a particle. Such absorption of a neutron in a non-fission reaction occurs about 16% of the time in all neutron absorptions by U-235.

The two fission fragments in Eq. (9.16) are not equal in size and are radioactive. The original nuclei of U-235 have neutron–proton ratio of 1.55. Their fission fragments have also similar n–p ratios, which for stable nuclei are, however, 1.2 to 1.4. The fission fragments, therefore, undergo several stages of β decay (converting neutrons into protons) until a stable product is formed in each case.

$$_{54}Xe^{140} \xrightarrow[16s]{\beta} {}_{55}Cs^{140} \xrightarrow[66s]{\beta} {}_{56}Ba^{140} \xrightarrow[12.8\text{ days}]{\beta} {}_{57}La^{140} \xrightarrow[40\text{ hrs}]{\beta} {}_{58}Ce^{140}$$
(stable)

and

$$_{38}Sr^{94} \xrightarrow[2\text{ min}]{\beta} {}_{39}Y^{94} \xrightarrow[17\text{ min}]{\beta} {}_{40}Zr^{94}\text{ (stable)} \tag{9.17}$$

This series is called a *fission chain*. Since β-decay is usually accompanied by γ-radiation, suitable shielding against γ-rays as well as neutrons must be provided in a reactor. Figure 9.6 shows fission product data for U-235 by thermal and fast (14 MeV) neutrons and for U-233 and Pu-239 by thermal neutrons. The most probable fission products have mass numbers in the ranges 85 to 105 and 130 to 150, meaning that the products are not equal in size.

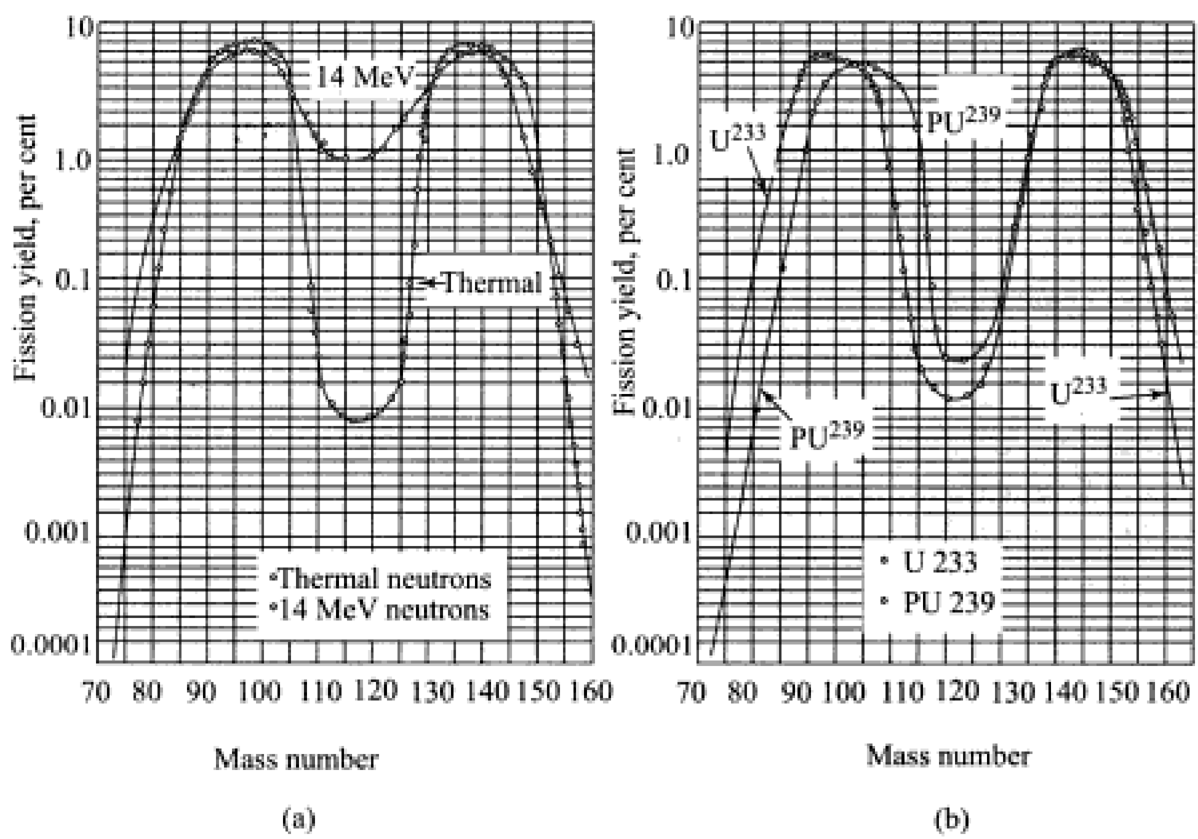

Fig. 9.6 *Fission product data for (a) U-235 by thermal and 14 MeV neutrons, and (b) U-233 and Pu-239 by thermal neutrons*

9.5.1 Energy from Fission and Fuel Burnup

There are many fission reactions which release different amounts of energy, For the reaction,

$$_{92}U^{235} + {}_0n^1 \longrightarrow {}_{56}Ba^{137} + {}_{36}Kr^{97} + 2{}_0n^1 \tag{9.18}$$

has the mass balance

$$235.0439 + 1.00867 \longrightarrow 138.9061 + 96.9212 + 2 \times 1.00867$$

or, $$236.0526 \longrightarrow 235.8446 \text{ amu}$$

There is thus a reduction in mass, which appears in the form of energy (exothermic). The mass defect is

$$\Delta m = 235.8446 - 236.0526 = -0.2080 \text{ amu}$$

Therefore,

$$\Delta E = -0.2080 \times 931 = -193.6 \text{ MeV}$$

The fission of U-235 yields on an average about 193 MeV. which is the same for the fission of U-233 and Pu-239. This amount of energy is prompt, i.e. released during the fission process. More energy is, however, produced due to (i) the slow decay of the fission fragments, and (ii) the non-fission capture of excess neutrons. The total energy released per fission reaction is about 200 MeV. The complete fission of 1 g of U-235 nuclei thus produces

$$\frac{\text{Avogadro constant}}{\text{Mass of U-235 isotope}} \times 200 \text{ MeV}$$

$$= \frac{6.023 \times 10^{23}}{235.0439} \times 200 \text{ MeV} = 5.126 \times 10^{23} \text{ MeV}$$

$$= 8.19 \times 10^{10} \text{ J} = 2.276 \times 10^{24} \text{ kWh} = 0.984 \text{ MW-day}$$

Thus, a reactor burning 1 g of U-235 generates nearly 1 MW-day of energy. This is referred to by the term "*fuel burnup*", which is the amount of energy in MW-days produced of each metric ton of fuel.

The complete fission of all U-235 nuclei in a fuel mass is impossible, since many of the fission products capture neutrons in a non-fission reaction. In time, the number of neutrons so captured becomes great enough because of the accumulation of products, and the fission chain can no longer be sustained. Depending upon fuel enrichment, this happens when only a small percentage, often less than 1% of the fissionable nuclei in the fuel has been consumed. Further use of this poisoned fuel can only be made by removing the fission products and reprocessing.

9.6 CHAIN REACTION

The number of newly born fission neutrons in a single fission for U^{235} nuclei is either 2 or 3 and on an average 2.47. In a reactor where controlled and sustained energy production is desired, conserving neutrons is a vital matter.

There are two reasons why not all the fission neutrons cause further fission.

1. Non-fission capture or absorption of some neutrons by the fission products, non-fissionable nuclei in the fuel, structural material, coolant, moderator and so on.

2. Leakage of neutrons escaping from the core.

The smaller the surface–volume ratio of the core, i.e. the larger its size, the lower is the percentage of leakage of neutrons. The core size is increased to the point where a chain reaction is possible. This size is called the *critical size*. The mass of fuel in such a core is called the *critical mass*.

In a reactor using U^{235} as fuel, 100/2.47 or about 40.5 of each 100 fission neutrons must ultimately engage in fission to keep the reactor critical. However, only about 84% of the neutrons that get absorbed in U^{235}cause fission. The remaining 16% neutrons reacting with it produce U^{236} (non-fission capture), an isotope of no particular importance. Therefore, a total of about 40.5/0.84 or 48 neutrons must be absorbed in U^{235} to cause fission. Thus, a maximum of about 52 neutrons may be allowed to leak out of the core and be absorbed in other core materials.

9.7 NEUTRON ENERGIES

The kinetic energy of a neutron, KE_n or simply E_n, is given by

$$E_n = \frac{1}{2} m_n V^2$$

where m_n is the mass of neutron (1.008665 amu), and V the speed of neutron. Thus,

$$E_n = \frac{1}{2} \times 1.008665\ V^2 \times \frac{1}{0.965 \times 10^{18}}$$

$$= 5.227 \times 10^{-19}\ V^2\ \text{MeV} = 5.227 \times 10^{-13} V^2\ \text{eV} \qquad (9.19)$$

where V is in cm/s.

The newly born fission neutrons have energies varying between 0.075 to 17 MeV. As these neutrons travel through matter, they collide with other nuclei and get slowed down. This process is called *scattering*. The neutron gives up some of its energy with each successive collision.

Neutrons are classified into three general categories according to their energy as fast, intermediate and slow.

Classification	*Neutron energy (eV)*	*Corresponding velocity (m/s)*
Fast	$> 10^5$	$> 4.4 \times 10^6$
Intermediate	$1–10^5$	$(1.38 \text{ to } 4.4) \times 10^6$
Slow	< 1	$< 1.38 \times 10^4$

Newly born fission neutrons carry, on an average, about 2% of a reactor fission energy in the form of KE. As stated earlier, they can be (1) Prompt neutrons, emitted within 10^{-14} s after fission occurs from the fission fragments, and (2) Delayed neutrons, produced in radioactive decay reactions of the fission fragments and their products. Though the energies of delayed neutrons are relatively small, they play a vital role in nuclear reactor control.

9.7.1 Neutron Scattering

In the scattering process, the energy balance of colliding particles before and after collision gives

$$(E_n + KE_c)_1 = (E_n + KE_c + E_c^*)_2 \tag{9.20}$$

where the subscripts *n* and *c* denote neutron and nucleus, and 1 and 2 denote before and after collision. E_c^* is the excitation energy of the struck nucleus. Scattering can be of two types:

1. *Inelastic scattering*, in which momentum and total energy of the particles before and after collision are conserved. However, *KE* is not conserved. Part of the *KE* before collision is absorbed by the nucleus to have the excitation energy E^*_c. Thus, for a neutron to engage in inelastic scattering, it has to possess an initially high KE (> E^*_c).
2. *Elastic scattering*, in which a neutron does not possess the necessary minimum *KE* and E^*_c is zero. Both momentum and *KE* of the colliding particles are conserved.

In each scattering process, a part of the *KE* of the neutron is transferred to the initial relatively stationary and heavier nucleus, thereby slowing down the neutron. The amount of energy lost by a neutron in each collision depends upon the mass of the nucleus and the angle of scatter. The maximum energy is lost in a head-on collision. If a neutron possesses an initial *KE* $E_{n,i}$, its *KE* after a head-on collision E_n, min is given by

$$E_{n,min} = E_{n,i}\left(\frac{M - m_n}{M + m_n}\right)^2 \tag{9.21}$$

where M and m_n are the nucleus and neutron masses, respectively. It can be approximately expressed as

$$\frac{E_{n,\,min}}{E_{n,\,i}} = \left(\frac{A-1}{A+1}\right)^2 \tag{9.22}$$

where A is the mass number of the nucleus. A neutron may lose a maximum of less than 2% in a collision with U^{238} nucleus, but about 28% with a carbon nucleus and all its energy in a single collision with a hydrogen nucleus ($A = 1$).

The average neutron energy lost per elastic collision is expressed in terms of a quantity called the *logarithmic energy decrement*, ξ, defined by

$$\xi = \ln E_{n,i} - \ln E_{n,av} = \ln\frac{E_{n,\,i}}{E_{n,\,av}} \tag{9.23}$$

where $E_{n,av}$ is the average energy of the neutron after a single collision. ξ is given by

$$\xi = 1 - \left[\frac{(A-1)^2}{2A}\ln\frac{A+1}{A-1}\right] \tag{9.24}$$

where A is the mass number of the struck nucleus (moderator). From the above equation, it is seen that as $A \to 1$, $\xi = 1$. Thus, if a neutron collides with a hydrogen nucleus ($A = 1$), the average neutron energy after one collision is $1/e$ of its initial energy. This is the maximum possible decrease of neutron energy.

Thus, the number of collisions, n, required to slow down a neutron from initial energy $E_{n,i}$ to a final energy $E_{n,f}$ in elastic scatter is given by

$$n = \frac{\ln \frac{E_{n,i}}{E_{n,f}}}{\xi} \tag{9.25}$$

A *moderator* is used to slow down the neutron in a reactor. Thus, smaller the nucleus, better the moderator. Table 9.2 gives the values of n to bring down the neutron energies from 2 MeV to 0.025 eV in elastic collisions. However, n is not the sole criterion of moderator effectiveness. Other aspects, such as the probability of collision, the probability of absorption and scattering, as well as the number of moderator nuclei in a given volume also influence the moderator efflectiveness.

Table 9.2 *Number of elastic collisions (n) between 2 MeV and 0.025 eV*

Nucleus	A	ξ	n
H	1	1.000	18
D	2	0.725	25
Be	9	0.208	86
C	12	0.158	114
Al	27	0.074	246
Fe	56	0.038	472
Zr	91	0.021	866
U	238	0.004	4480

9.7.2 Thermal Neutrons

When a large number of neutrons are slowed down in a medium, such as a moderator, the lowest energies that they can attain are those that put them in thermal equilibrium with the molecules of that medium. In this state (ground state) they become thermalized and are called *thermal* (or slow) *neutrons*. A reactor primarily utilizing thermal neutrons for fission is called a *thermal reactor*.

From kinetic theory of gases, it is known that at a certain temperature the molecules of a gas have a wide range of velocity varying from zero to infinity (or velocity of light?) demonstrating Maxwell–Boltzmann velocity distribution (Nag, 1981). The thermalized neutrons will have similar distributions of velocity and also of energy. The most probable velocity of a neutron is thus,

$$V_m = \left[\frac{2\,KT}{m_n}\right]^{1/2} \tag{9.26}$$

where K = Boltzmann constant = 1.38×10^{-23} J/ molecule-K; m_n = mass of neutron = 1.674×10^{-27} kg, and T = absolute temperature. K.

On substitution,

$$V_m = 128.4\ T^{1/2}\ \text{m/s} \tag{9.27}$$

The most probable kinetic energy of a neutron will be

$$KE_m = \frac{1}{2} m_n V_m^2$$

$$= \frac{1}{2} \times \frac{1.674 \times 10^{-27} \times (128.4)^2 T}{1.6021 \times 10^{-19}}\ \text{eV}$$

$$= 8.613 \times 10^{-5}\ T\ \text{eV} \tag{9.28}$$

At 20 °C, $KE_m = 0.02524$ eV and $V_m = 2198$ m/s $\cong$ 2200 m/s

Table 9.3 gives thermal neutron energies and speeds at different temperatures.

Table 9.3 *Thermal neutron energies and speeds*

Temperature (°C)	KE_m(eV)	V_m (m/s)
20	0.0252	2200
260	0.0459	2964
537.8	0.0699	3656
1000	0.0097	4580

Neutrons having energies greater than thermal such as those slowing down in a thermal reactor are called epithermal neutrons.

9.8 NUCLEAR CROSS-SECTIONS

If a group of neutrons travel with the same *KE* and the corresponding speed is v cm/s and if their volume density in the beam at a particular point is n neutrons/cm^3, then the product nv is equal to the number of neutrons crossing a unit target area of 1 cm^2 per second and is called the *neutron flux*. ϕ. Therefore,

$$\phi = nv \tag{9.29}$$

Its dimension is neutrons/cm$^2\cdot$ s. However, the speeds of neutrons vary widely (Maxwell-Boltzmann velocity distribution).
Then,

$$\phi = \int_{v}^{\infty} n(v)\ dv \tag{9.30}$$

where $n(v)$ dv is the number of neutrons per unit volume whose speeds vary between v and v + dv.

The interaction rate between a beam of neutrons and the nuclei in a target material has been experimentally observed to be proportional to (i) the neutron flux, and (ii) the number of atoms (or nuclei) in the target.

Let us consider a beam of neutrons, all of speed V cm/s and density n neutrons/cm^3, incident on a target area of A cm^2 and thickness dx and containing N nuclei/cm^3 (Fig. 9.7). The interaction rate, being proportional to the neutron flux, ϕ, and the number of nuclei on the target material, NA dx, is thus

$$\text{Interaction rate} = \sigma\,\phi\,NA\,dx$$
$$= \sigma\,\phi\,NV_0 \qquad (9.31)$$

where σ is the constant of propor-tionality, called the microscopic cross-section of the isotope concerned, and V_0 the volume of the target (A dx). σ has the unit of cm^2/nucleus, and it can be regarded as the area presented by each nucleus to neutrons to cause a reaction. The radius of a nucleus is given by

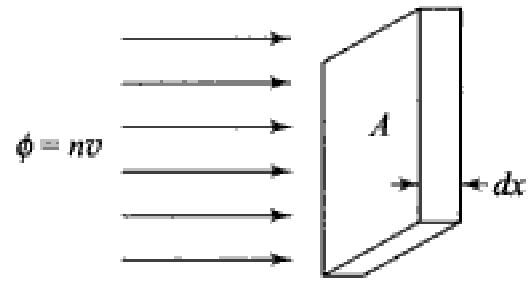

Fig. 9.7 *Interaction rate of neutrons*

$$r_c = 1.4 \times 10^{-13}\,A^{1/3} \qquad (9.32)$$

where A is the mass number. The radii of nuclei vary roughly between 1.4×10^{-13} and 10^{-12} cm. Therefore, the cross-sectional area of an average nucleus (πr_c^2) is equal to 10^{-24} cm^2. This value is taken as the unit of microscopic cross-section and is called one *barn*.

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 \qquad (9.33)$$

The total cross-section of all nuclei in unit volume of a material is called the macroscopic cross-section, Σ, and has units cm^2/cm^3 or cm^{-1}.

$$\Sigma = N\sigma\ \frac{\text{nuclei}}{\text{cm}^3}\ \text{cm}^2 \quad \text{or} \quad \text{cm}^{-1} \qquad (9.34)$$

and the interaction rate per unit volume,

$$F = \Sigma\,\phi\ \frac{\text{neutrons}}{\text{cm}^3\text{s}} \qquad (9.35)$$

The probability that a neutron entering the target will collide or interact within a distance dx,

$$= \frac{\sigma\phi NA dx}{\phi A} = \sigma N dx = \Sigma dx \qquad (9.36)$$

Thus, the macroscopic cross-section can be explained as the probability per unit length that a neutron will collide, i.e. the collision cross-section.

As the neutrons collide with target nuclei, they get removed from the group. For neutrons which survive collision in an element of thickness dx,

$$\text{Rate of collision} = \text{neutron flux (in–out)}\ A$$
$$\sigma\,\phi\,N\,A\,dx = -A\,d\phi$$

The negative sign indicates that the flux is decreasing. Rearranging,

$$\frac{d\phi}{\phi} = -\sigma\,N\,dx = -\Sigma\,dx$$
$$\phi(x) = \phi_0 e^{-\Sigma x} \qquad (9.37)$$

where ϕ_0 is the incident neutron flux (at $x = 0$). This is referred to as the "survival equation", i.e., the neutron flux which has survived collision after traveling a distance x in the target.

The average distance that a neutron travels without making a collision or interaction with a target nucleus is called the mean free path, λ,

$$\lambda = \frac{\int_0^\infty x \mathrm{d}\phi(x)}{\phi_0}$$

Using Eq. (9.37)

$$\lambda = \frac{\int_0^\infty x\left(-\phi_0 e^{-\Sigma x}\Sigma\right)\mathrm{d}x}{\phi_0} = \frac{1}{\Sigma} \tag{9.38}$$

Thus the mean free path is the reciprocal of the macroscopic cross-section.

Neutrons have as many cross-sections as there are reactions. The reactions can be scattering, capture and fission. Thus, the rates at which elastic scattering, inelastic scattering, capture and fission take place are characterized by the elastic scattering cross-section, σ_s, the inelastic scattering cross-section, σ_i, the capture cross-section σ_C, and the fission cross-section σ_f. For non-fissionable isotopes σ_f is zero. The total cross-section, σ_t, is the sum of these cross-sections.

$$\sigma_t = \sigma_s + \sigma_i + \sigma_c + \sigma_f \tag{9.39}$$

The absorption cross-section, σ_a, is the sum of the capture and fission cross-sections, or

$$\sigma_a = \sigma_c + \sigma_f$$

If an isotope has capture and elastic scattering cross-sections of 0.1 and 10 barns, respectively, it is obvious that elastic scattering is the most probable reaction and capture is almost negligible, occurring in less than 1% of all reactions.

Macroscopic cross-sections for mixtures and compounds can be calculated from a knowledge of the cross-sections and numbers of atoms per unit volume of the constituents. If a mixture or compound contains N_1, N_2, N_3, etc. atoms per cm^3 of elements whose microscopic cross-sections are σ_1, σ_2, σ_3 and so on, the macroscopic cross-section of the mixture or compound, Σ, is given by

$$\Sigma = N_1\sigma_1 + N_2\sigma_2 + N_3\sigma_3 + \ldots \tag{9.40}$$

9.9 NEUTRON FLUX AND REACTION RATES

The neutron flux ϕ is given by the equation

$$\phi = nV \tag{9.41}$$

where n is the volume density of neutrons in the beam (neutrons/m^3) and V the average velocity (m/s) of the neutrons.

The neutron flux ϕ varies within the reactor core depending on the energy of neutrons. It is maximum at the core geometric centre and minimum near the core edges. In a thermal reactor, where high-energy fission neutrons are born in the fuel and thermalized in the moderator, fast-neutron fluxes peak in the fuel and thermal-neutron fluxes peak in the moderator. At any position in the fuel, the neutron flux is proportional to the power. It is maximum during full power operation and zero during shutdown.

If a target medium containing nuclei of density N is subjected to a neutron flux ϕ, the reaction rate between the nuclei and neutrons is given by $\sigma N(nV)$, where σ is the microscopic cross-section of the particular reaction (i.e., scatter, absorption etc.). It is equal to $\Sigma\phi$ and hence, is inversely proportional to the mean free path of the neutrons.

9.10 MODERATING POWER AND MODERATING RATIO

It was shown earlier that the logarithmic energy decrement factor ξ represents the effect of nucleus size on the average number of collisions required to slow down a neutron over a prescribed energy range. The hydrogen moderator would slow a neutron from 2 MeV to 0.025 eV in 18 collisions, deuterium in 25 collisions and so on. However, aspects like the probability of scattering and absorption, and the number of moderator nuclei per unit volume, N, are also important. These along with ξ are grouped together in two parameters, called

$$\text{Moderating power} = \xi N\sigma_s = \xi\, \Sigma_s \tag{9.42}$$

$$\text{Moderating ratio} = \xi\,\frac{\sigma_s}{\sigma_a} = \xi\frac{\Sigma_s}{\Sigma_a}$$

Since ξ is the average loss in the ln E_n per collision and Σ_s is the probability of scatter collision/m, the slowing-down power is equal to the average loss in ln E_n per m. It has the unit (m^{-1}) and should be as large as possible for good moderation. It may be noticed that a high nuclear density N is essential since there will be more reactions if there are more nuclei to react. Thus, hydrogen and deuterium are not suitable as moderators, in gaseous forms, instead they are used in light and heavy water. Similarly, when CO_2 is a coolant, graphite is the moderator.

The moderating ratio is a relative measure of the ability of a moderator to scatter neutrons without appreciably absorbing them. It should also be as high as possible for good moderation.

The selection of a moderator also depends on cost, chemical and structural considerations. Heavy water is an excellent moderator, but is extremely costly. Light water is cheap, but has a small neutron absorption cross-section. It is used as a coolant and a moderator when enriched uranium is used as fuel. Graphite is low in cost but is structurally weak. Liquid metal-cooled fast reactors need no moderator.

9.11 VARIATION OF NEUTRON CROSS-SECTIONS WITH NEUTRON ENERGY

The neutron cross-section for any nucleus depends upon the energy of the neutron reacting with it. Plots of σ vs. E_n are usually made on log-log coordinates. In most cases, scattering cross-sections, σ_s, are so small compared with absorption cross-sections, σ_a, that the total cross-section α_t is

$$\sigma_t \cong \sigma_a$$

Also, for most nuclei, σ_s does not change much with neutron energy E_n.

$$\sigma_a = \sigma_f + \sigma_c \quad \text{and} \quad \sigma_s = \sigma_t - \sigma_a$$

Variations of absorption cross-sections, σ_a, with neutron energy, E_n, can be divided into three regions: (1) $1/V$ region, (2) resonance region, and (3) fast neutron region (Figs 9.8 and 9.9).

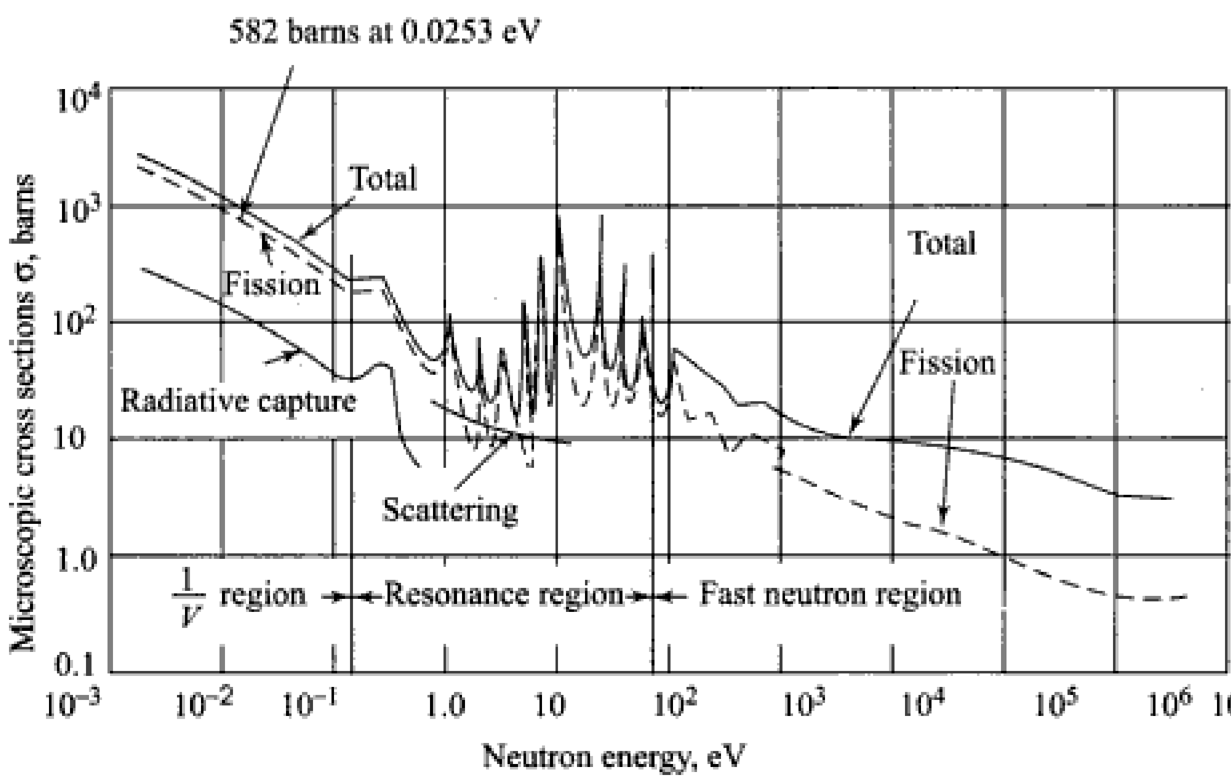

Fig. 9.8 *Neutron cross-section for U-235*

1. *1/V region* In the low energy region, σ_a is inversely proportional to the square root of the neutron energy E_n.

$$\sigma_a = C_1 \left(\frac{1}{E_n}\right)^{0.5} \tag{9.43}$$

Thus,

$$\sigma_a = C_1 \left(\frac{1}{m_n \frac{V^2}{2}}\right)^{0.5} = C_2 \frac{1}{V} \tag{(9.44)}$$

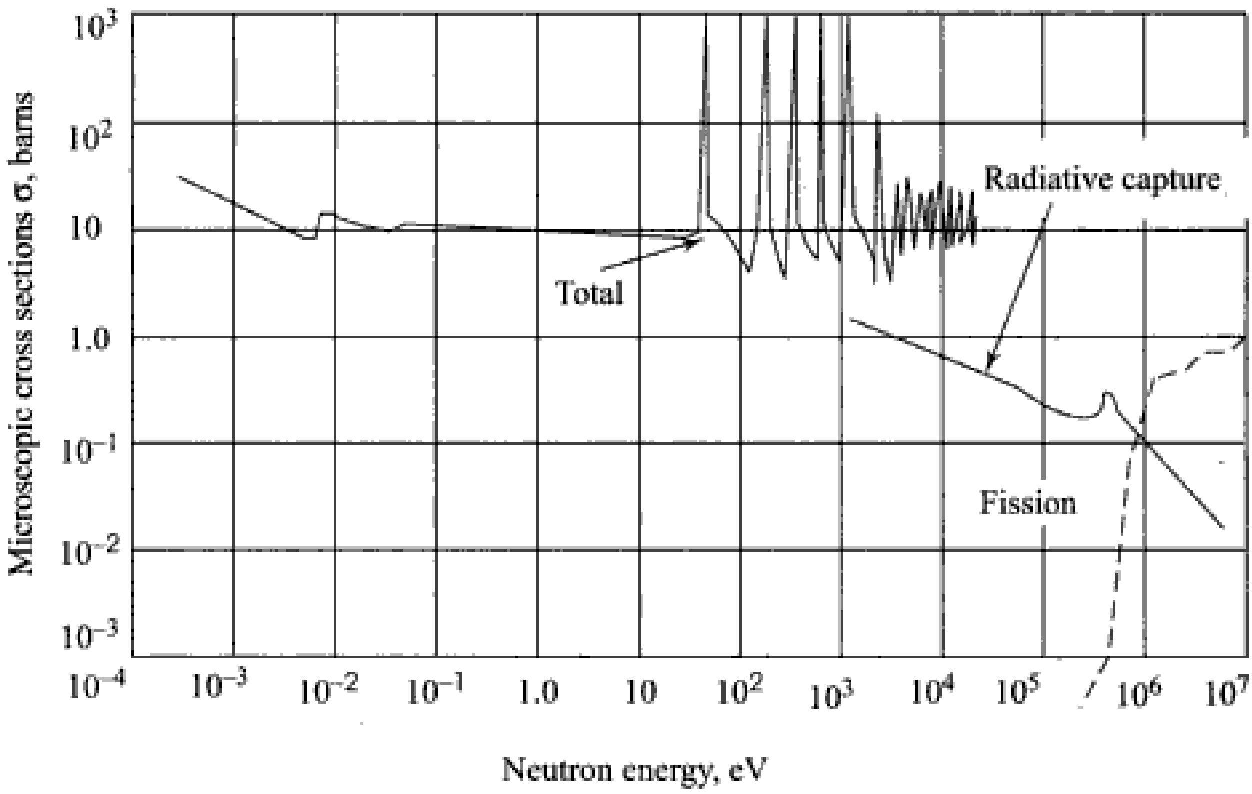

Fig. 9.9 *Neutron cross-section for U-238*

where C_1 and C_2 are constants, m_n is the neutron mass and V is the neutron velocity.

This relationship is known as the $1/V$ law. It indicates that lower is the velocity of the neutron, longer will be the time a neutron spends in the vicinity of the nucleus, and higher will be the probability of its absorption by the nucleus. It is a straight line with a slope of about – 0.5 (Fig. 9.8 and 9.9).

2. *Resonance region* Most neutron absorbers, after the $1/V$ region, show one or more σ_a peaks occurring at definite neutron energies. These are called resonance peaks. Indium has only one peak, whereas U-235 and U-238 have many. U-238 has very high resonance σ_a, with the highest peak, about 4000 barns, occurring at about 7 eV. The design of thermal reactors is affected by this fact, since U-238 absorbs many of the neutrons passing through the region and affects the reactor neutron balance. Many elements, particularly those of low mass numbers and low σ_a, do not exhibit resonance absorption and hence, can be used as reactor construction materials.

3. *Fast neutron region* As the neutron energies increase beyond the resonance region, the absorption cross-sections gradually decrease. At very high values of E_n.

$$\sigma_t = \sigma_a + \sigma_s \longrightarrow 2 \times \text{cross-sectional area of the target nucleus}$$

$$\sigma_t = 2\,\pi\, r_c^2$$

Combining with Eq. (9.32),

$$\sigma_t = 2\pi (1.4 \times 10^{-13})^2 A^{2/3} \text{ cm}^2$$
$$= 0.125 A^{2/3} \text{ barns} \quad (9.45)$$

where 1 barn = 10^{-24} cm^2 and A is the mass number of the target nucleus.

Thus, in the very high neutron energy range, σ_t is very low, usually less than 5 barns for heavier nuclei.

9.12 NEUTRON LIFE CYCLE

In a reactor core, neutrons are born at all times and in all places having fissionable material and diffuse in all directions. We will examine the life cycle of a group of neutrons, all assumed to be born at the same time, which undergo scatter, leakage, absorption and other reactions, and finally cause fission and attain the same energy levels simultaneously. This group of neutrons is called a *generation*. The series of events or processes that such a group of neutrons undergoes from birth until a new generation is born by fission is called a *life cycle of neutrons*.

Figure 9.10 shows the events that the generation of N fission neutrons produced in U-235 undergoes in a thermal reactor.

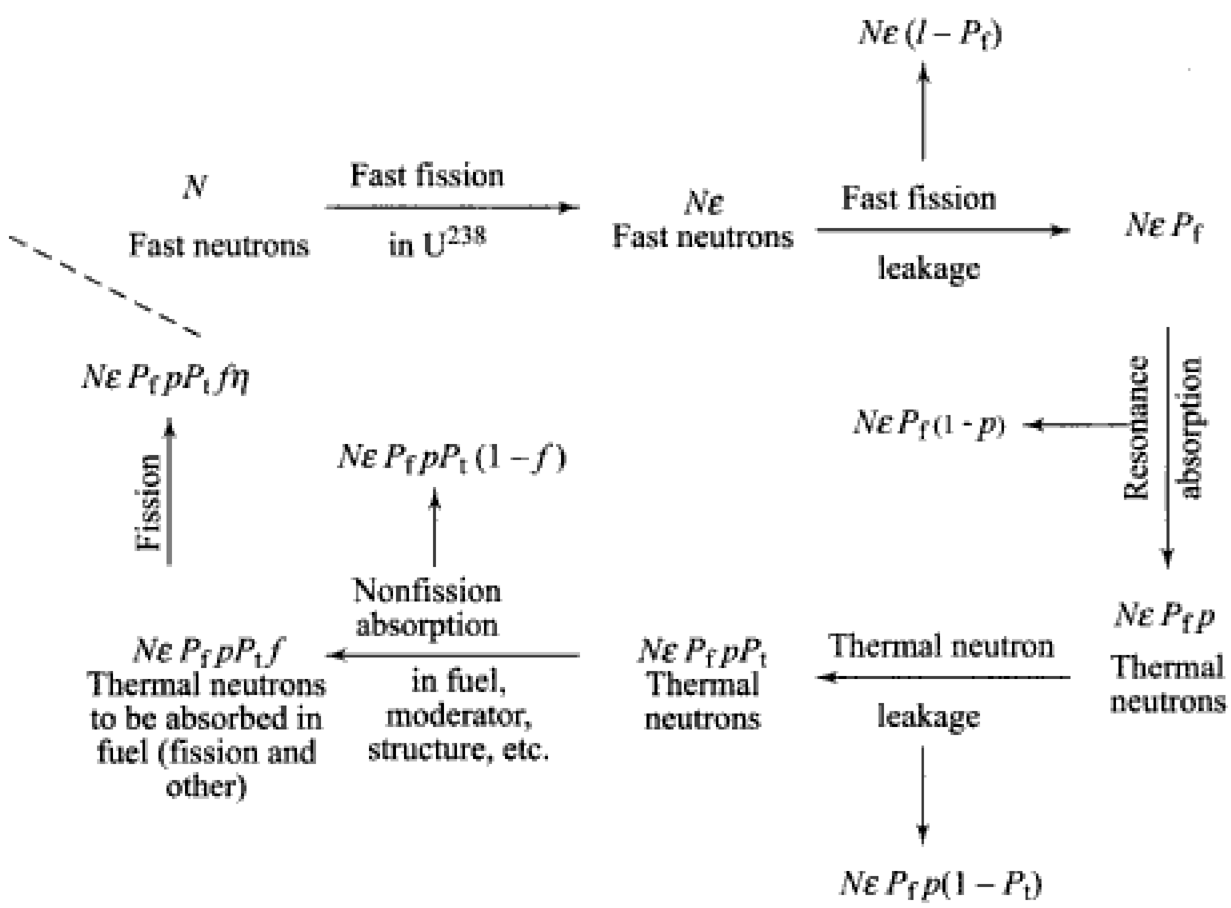

Fig. 9.10 *Neutron life cycle in a thermal reactor*

explosive rate as in an atom bomb. If $k_{eff} < 1$, the reactor is subcritical and the chain reaction decreases and eventually dies out.

Let us consider a chain reaction in an infinitely large system by adding more and more fuel such that the neutron leakage becomes negligibly small and $P = 1$, Therefore,

$$k_{eff} = k_{\infty} = \varepsilon\, p\, f\, \eta \tag{9.47}$$

where k_{α} is called the infinite multiplication factor. This is a *four-factor formula*, where ε, p and f are functions of the fuel and the internal configuration of the core, called the *lattice constants*, and η depends on the kind of fuel and is a *fuel constant*.

In a critical reactor, the rate at which neutrons are used up or lost must be exactly equal to the rate at which they are produced by fission. The theory of nuclear reactors is concerned with the analysis of all the processes which take place in the core of a reactor, and in particular with the slowing down, diffusion and absorption of neutrons. Only by analyzing these processes in detail can accurate calculations be made to determine the multiplication factor, critical mass of fuel or size of a reactor (permitting a certain amount of neutron leakage).
The reactivity ρ is defined as

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \tag{9.48}$$

The neutron life time, τ^*, is defined as the average time between successive neutron generations (prompt neutrons). The number of neutrons existing in a unit volume is n. Then, the time rate of change of the neutron density is

$$\frac{dn}{d\tau} = \frac{n\rho}{\tau^*}$$

or

$$\frac{dn}{n} = \frac{\rho}{\tau^*} d\tau$$

or

$$\ln \frac{n}{n_0} = \frac{\rho}{\tau^*} \tau$$

where n and n_0 represent the neutron density at time τ and $\tau = 0$. It can also be expressed as

$$n = n_0 e^z \tag{9.49}$$

where,

$$z = \frac{\rho}{\tau^*} \tau$$

9.13 REFLECTORS

Surrounding the reactor core exists a reflector, which is a medium of low neutron absorption and high neutron scattering cross-sections. Some of the neutrons leaking out of the core are scattered back into it by the reflector nuclei. This

conductivity, good corrosion resistance, good mechanical strength at high temperatures and a high limiting temperature for operation. The *cladding* serves three functions.

1. To provide structural support and strength for the fuel and prevent distortion.
2. To prevent the release of radioactive fission products into the coolant stream.
3. In certain types of reactors (mainly gas-cooled) to provide extended surfaces in the form of fins to promote more heat transfer to the coolant.

Materials suitable for cladding should have low neutron capture cross-section, high thermal conductivity, good mechanical strength at high temperatures and chemical compatibility with the fuel and coolant. The most common cladding materials are aluminium, magnesium alloys (Magnox), stainless steel and alloys of zirconium (zircaloy).

The rate of heat release by fission per unit volume of fuel is called the volumetric thermal source strength, q_G, given by

$$q_G = G\,N\,\sigma_f\,\phi \text{MeV/m}^3\text{s} \tag{9.50}$$

where G = energy per fission (~180–190 MeV), N = number of fissionable nuclei/m^3, σ_f = microscopic fission cross-section of that fuel, m^2, and ϕ = neutron flux per m^2s.

9.14.1 Heat Conduction in Fuel Elements

Fourier's heat conduction equation in three dimensions is given by [7],

$$\nabla^2 T + \frac{q_G}{k} = \frac{1}{\alpha} \times \frac{\partial T}{\partial t} \tag{9.51}$$

where T is the temperature (°C), q_G the volumetric source strength (W/m^3), k the thermal conductivity of the solid (W/mK), α the thermal diffusivity of the solid (m^2/s) and t the time (s).

For a plate type fuel element (Fig. 9.12), in which the fuel of thickness $2a$ is enclosed on both sides by a cladding of thickness b, the dimensions of the plate in the y and z directions are large compared with the values of a and b. Consequently, heat conduction may be assumed to be in the direction of the x-axis only. In the fuel, Eq. (9.51) reduces to

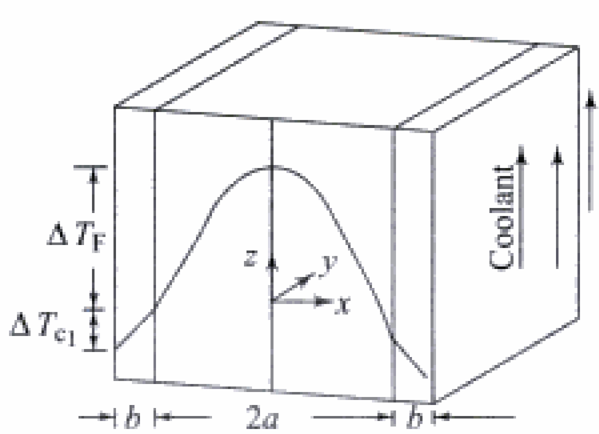

Fig. 9.12 *Temperature distribution in a plate type fuel element*

$$\frac{d^2T}{dx^2} = -\frac{q_G}{k_F}$$

or

$$\frac{dT}{dx} = -\frac{q_G}{k_F}\,, x + C,$$

k_F being he thermal conductivity of fuel.
When $x = 0$, $dT/dx = 0$, therefore $C = 0$, Integrating again between $x = 0$ and $x = a$, we get

$$\Delta T_F = \frac{q_G a^2}{2k_F} \tag{9.52}$$

where ΔT_F is the temperature drop from the centre to the surface of the fuel. The temperature distribution within the fuel is parabolic.

In the cladding there is no energy release, and the heat conducted per unit area through the cladding on each side of the fuel is $q_G \cdot a$ W/m^2. Therefore,

$$q = -k_{cl}\frac{dT}{dx} = q_G \cdot a$$

or
$$\frac{dT}{dx} = -\frac{q_G \cdot a}{k_{cl}}$$

Integrating from $x = a$ to $x = a + b$, we get

$$\Delta T_{cl} = \frac{q_G ab}{k_{cl}} \tag{9.53}$$

where ΔT_{cl} is the temperature drop through the cladding, The total temperature drop from the centre of the fuel to the surface of the cladding is

$$\Delta T_F + \Delta T_{cl} = q_G \cdot a\left(\frac{a}{2k_F} + \frac{b}{k_{cl}}\right) \tag{9.54}$$

If a coolant is now considered to be flowing along the cladding, heat will be transferred from the cladding surface to the coolant. Let the cladding surface temperature be T_s and the bulk fluid temperature be T_c. Then the heat flow per unit area,

$$q = q_G \cdot a = h(T_s - T_c)$$

or
$$T_s - T_c = \frac{q_G a}{h} = \Delta T_c \tag{9.55}$$

= temperature drop in the coolant.

Therefore, the total temperature drop from the centre of the fuel to the bulk fluid is

$$\Delta T_F + \Delta T_{cl} + \Delta T_c = q_G\, a\left[\frac{a}{2k_F} + \frac{b}{k_{cl}} + \frac{1}{h}\right] \tag{9.56}$$

For a cylindrical fuel rod of radius a surrounded by a cladding of thickness b, the heat conduction along the rod is negligible (Fig. 9.13) and the heat conduction equation in cylindrical coordinates [7] is

$$\frac{d^2T}{dr^2} + \frac{1}{r}\times\frac{dT}{dr} = -\frac{q_G}{k_F}$$

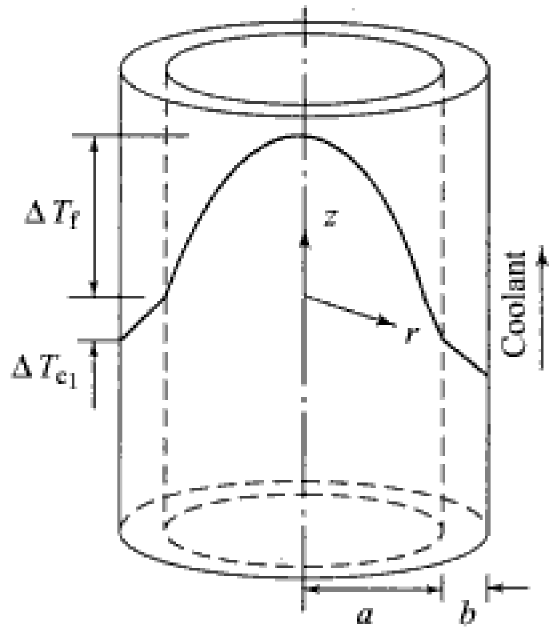

Fig. 9.13 *Temperature distribution in cylindrical fuel element*

or $$\frac{d}{dr}\left[r\frac{dT}{dr}\right] = -\frac{q_G r}{k_F}$$

or $$\frac{dT}{dr} = -\frac{q_G r}{2k_F} + \frac{C_1}{r} \tag{9.57}$$

When $r = 0$, $\frac{dT}{dr} = 0$, therefore, $C_1 = 0$.

Integrating between $r = 0$ and $r = a$, the temperature drop from the centre to the surface of the fuel is given by

$$\Delta T_F = \frac{q_G a^2}{4k_F} \tag{9.58}$$

The heat conducted through the cladding per unit length of fuel element $= \pi a^2 q_G$. The radial heat conduction at any radius is

$$Q = -k\, 2\pi rL\, \frac{dT}{dr}$$

or $$dT = -\frac{Q}{2\pi kL} \times \frac{dr}{r}$$

Integrating between the limits of $r = a$ to $r = a + b$ in the cladding and substituting Q/L as $\pi a^2 q_G$, we get the temperature drop through cylindrical wall of cladding as

$$\Delta T_{cl} = \frac{q_G a^2 \ln\left(\frac{a+b}{a}\right)}{2k_{cl}} \tag{9.59}$$

The total temperature drop from the centre of the fuel to the surface of the cladding is:

$$\Delta T_F + \Delta T_{c1} = \frac{q_G a^2}{2}\left[\frac{1}{2k_F} + \frac{\ln\left(\frac{a+b}{b}\right)}{k_{cl}}\right] \tag{9.60}$$

If we consider the coolant flowing with bulk temperature T_C, the heat transfer to the fluid per unit length is given by

$$Q = \pi a^2 q_G = h 2\pi (a+b)(T_s - T_c) \tag{9.61}$$

where h is the heat transfer coefficient (W/m^{2}K) and T_s the cladding surface temperature.

The temperature drop from the cladding surface to the bulk fluid is

$$\Delta T_c = T_s - T_c = \frac{q_G a^2}{2h(a+b)} \tag{9.62}$$

The total temperature drop from the centre of the fuel to the bulk fluid is then

$$\Delta T_F + \Delta T_{c1} + \Delta T_c = \frac{q_G a^2}{2}\left[\frac{1}{2k_F} + \frac{\ln\left(\frac{a+b}{b}\right)}{k_{cl}} + \frac{1}{h(a+b)}\right] \tag{9.63}$$

9.14.2 Axial Temperature Distribution of Coolant and Fuel Element

The temperatures expressed by the relationship of the previous section are valid if q_G is uniform. Let us consider a single fuel element whose core height is equal to the length L (Fig. 9.14). The variation of neutron flux ϕ in the axial direction is assumed to be a cosine function of z such that $\phi = 0$ at $z = \pm L/2$, L being the extrapolated height and $z = 0$ being the core centre plane and

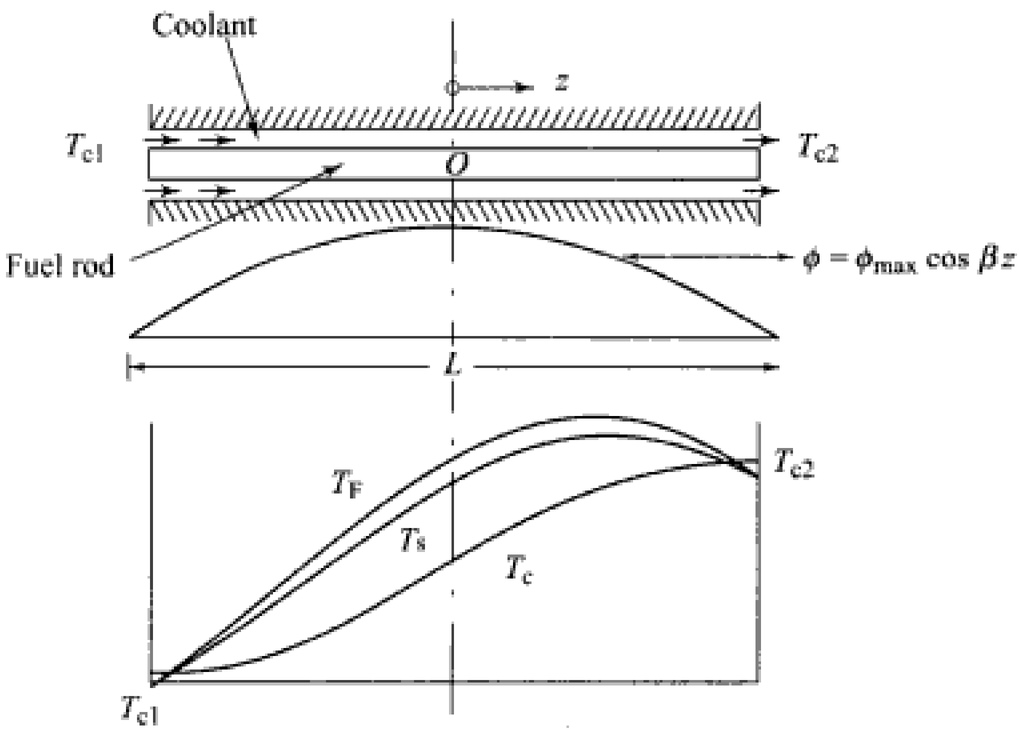

Fig. 9.14 *Coolant flow and temperatures in the central fuel channel*

$$\phi = \phi_c \cos \frac{\pi z}{L}$$

where ϕ_c is the maximum neutron flux at the core centre. Therefore, the volumetric thermal source strength q_G may be assumed to vary in a similar way,

$$q_G = (q_G)_{max} \cos \frac{\pi z}{L} \tag{9.64}$$

where $(q_G)_{max}$ is the source strength at the core centre. An energy balance for a differential section of the fuel element of length dz at height z is given by

$$m_c\, c_c\, dT_c = q_G \cdot A_c dz$$

where m_c = mass flow rate of coolant, kg/s; c_c = specific heat of the coolant, kJ/kg K; dT_c = temperature rise of coolant in the length dz, °C; and A_c = cross-sectional area of the fuel element, m^2.

The temperature of the coolant at any point in the channel can now be found by integration

$$\int_{T_{C_1}}^{T_C(z)} dT_c = \frac{(q_G)_{max} \pi a^2}{m_c c_c} \cdot \int_{-L/2}^{z} \cos \frac{\pi z}{L} dz$$

Therefore,

$$T_c(z) - T_{c_1} = \frac{(q_G)_{max} \pi a^2 L}{\pi m_c c_c} \left(\sin \frac{\pi z}{L} + \sin \frac{\pi L}{2L} \right) \tag{9.65}$$

The temperature rise of the coolant in the whole channel is

$$T_{c_2} - T_{c_1} = \frac{2(q_G)_{max} \pi a^2 L}{\pi\, m_c c_c} \sin \frac{\pi L}{2L} \tag{9.66}$$

$$= \Delta T_c$$

using this result, $T_c(z)$ can be expressed in the following form

$$T_c(z) = T_{c_1} + \frac{\Delta T_c}{2} \left(1 + \frac{\sin \beta z}{\sin \beta L/2} \right) \tag{9.67}$$

where $\beta = \pi/L$.

The temperatures at the surface of the cladding, T_s, and at the centre of the fuel, T_F, can be found at any position z as follows:

$$T_s(z) = T_c(z) + \theta_c(z)$$

$$= T_{c_1} + \frac{\Delta T_c}{2} \left(1 + \frac{\sin \beta z}{\sin \beta L/2} \right) + \theta_{CO} \cos \beta z \tag{9.68}$$

$$T_F(z) = T_{c_1} + \frac{\Delta T_c}{2} \left(1 + \frac{\sin \beta z}{\sin \beta L/2} \right)$$

$$+ \theta_{CO} \cos \beta z \left[1 + \frac{bh \ln\left(\dfrac{a+b}{a}\right)}{K_{cl}} + \frac{bh}{2k_E} \right] \tag{9.69}$$

where $\theta_{CO} = \dfrac{(q_G)_{max} a^2}{2h(a+b)}$, the temperature drop from the surface of the cladding to the coolant at the midpoint of the channel.

Figure 9.14 shows the variation of T_c, T_s and T_F along the coolant channel and shows the existence of maximum values in T_s and T_F. Differentiating Eq. (9.66) and equating to zero, the location of the maximum cladding surface temperature is given by the equation:

$$z = \frac{1}{\beta} \tan^{-1} \frac{\Delta T_c/2}{\theta_{CO} \sin(\beta L/2)} \tag{9.70}$$

Substituting this expression for z into Eq. (9.68), the maximum cladding surface temperature is given by:

$$T_s\,(\max) = T_{c_1} + \frac{\Delta T_c}{2}\left[1 + \left\{\operatorname{cosec}^2 \frac{\beta L}{2} + \left(\frac{\theta_{CO}}{\Delta T_c/2}\right)\right\}^2\right]^{1/2} \tag{9.71}$$

By the same procedure, the maximum fuel temperature may be found as

$$T_F(\max) = T_{c_1} + \frac{\Delta T_c}{2}\left[1 + \left\{\operatorname{cosec}^2 \frac{\beta L}{2} + \left(\frac{C}{\Delta T_c/2}\right)\right\}^2\right]^{1/2} \tag{9.72}$$

where,
$$C = \theta_{CO} \left[1 + \frac{bh \ln\left(\dfrac{a+b}{a}\right)}{k_{cl}} + \frac{bh}{2k_F} \right]$$

These temperatures fix the maximum coolant temperature that can be allowed.

The heat transfer coefficient for turbulent flow can be estimated from the Dittus–Boelter equation,

$$\text{Nu}_d = 0.023\,(\text{Re}_d)^{0.8}\,\text{Pr}^{0.4} \tag{9.73}$$

where the fluid properties are evaluated at the bulk temperature T_b given by

$$T_b(z) = T_{c_1} + \frac{1}{mc_p} \int_{\text{inlet}}^{z} q_L \,dz \tag{9.74}$$

where q_L is the heat transferred to the fluid per unit length of the duct. For non-circular ducts, the equivalent diameter should be used for Reynolds and Nusselt numbers. For liquid metals, the following correlation is often used

$$\text{Nu} = 7 + 0.025\,(\text{Pe})^{0.8} \tag{9.75}$$

where Pe = Peclet number = (Re)(Pr)

Table 9.4 gives typical values of heat transfer coefficients for gases, water and liquid metals under reactor operating conditions.

Table 9.4 *Typical values of Prandtl number and heat transfer coefficient of reactor coolants*

Coolants	*Prandtl Number*	h(W/m^2K)
Gases	0.8	50 to 500
Water	1 to 7	2000 to 20,000
Liquid metals	0.01	5000 to 50,000

Since h for gases is low, large surface area is required for a certain heat transfer duty in gas cooled reactors and cladding is most often finned. The criteria for the choice of coolants for reactors are: (1) Low neutron capture cross-section, (2) High specific heat, density, thermal conductivity and heat transfer coefficient, (3) Good chemical stability, and (4) Low neutron induced radioactivity.

9.14.3 Pumping Power

The pumping work, W, required by the circulating coolant to overcome pressure losses through the loop is given by

$$W = \Delta p \cdot A_c V \tag{9.76}$$

where, Δp = pressure drop through the loop, N/m^2; A_c = cross-sectional area of coolant passage, m^2; and V = coolant speed, m/s. In turbulent flow,

$$\Delta p = \frac{fL}{D_e} \times \frac{\rho V^2}{2} \tag{9.77}$$

where f = friction factor, D_e = equivalent diameter, $4A_c/P$, P being the wetted perimeter, L = channel length , and ρ = density of coolant. The well-known Moody chart can be used to find f.

9.15 TYPES OF REACTORS

Reactors can be heterogeneous or homogeneous. A heterogeneous reactor has a large number of fuel rods with the coolant circulating around them and carrying away the heat released by nuclear fission. In a homogeneous reactor, the fuel and moderator are mixed, e.g. a fissionable salt of uranium like uranium sulphate (or nitrate) dissolved in the moderator like H_2O or D_2O. The solution is critical in the core. Due to difficulties in component maintenance, induced radioactivity, erosion and corrosion, homogeneous reactors are not common. Present day nuclear reactors are of the heterogeneous class. These reactors are again classified according to the type of fuel used, the neutron flux spectrum, the coolant, and the moderator, if used (Table 9.5).

Table 9.5 *Reactor classification*

Neutron flux spectrum	*Moderator*	*Coolant*	*Fuel material*
Thermal	Light water	Light water	Enriched uranium
	Heavy water	Heavy water	Natural uranium
	Graphite	Gas (CO_2)	Natural or enriched uranium
Fast	Nil	Liquid metal (Na, K)	Plutonium, thorium

Light water-cooled and moderated reactors (LWR) using slightly enriched uranium fuel are the type most commonly used for power production. These reactors are further divided into:

1. Pressurized water reactor (PWR)
2. Boiling water reactor (BWR)

High temperature gas-cooled reactors (HTGCR) have been used in countries like the UK, France and Germany. Fast reactors which use high energy neutrons for fission and require no moderator utilize a liquid metal as a coolant with either plutonium or a plutonium–uranium mixture for fuel. Liquid metal fast breeder reactors (LMFBR) are likely to be the source of electrical power for the future. A breeder reactor produces more fissionable isotope then what it consumes. The characteristics of different reactor systems are provided in Table 9.6. The schematics for nuclear steam supply systems (NSSS) of five reactor types are shown in Fig. 9.15.

Table 9.6 *Characteristics of typical power reactors*

	PWR	*BWR*	*LMFBR*	*HTGR*
Electric power (MWe)	1300	1050	1000	330
Thermal power (MWth)	3800	3000	2750	842
Specific power (kWth/kg)	33	26	575	50
Power density (kWth/m^3)	100	60	300	10
Core height (m)	4.25	3.75	1.50	5.0
Core diameter (m)	3.50	4.90	3.25	5.9
Coolant	H_2O	H_2O	liq.Na	He
Pressure (MPa)	15.5	7.2	0.8	4.8
Inlet temperature (°C)	280	275	330	400
Outlet temperature (°C)	310	285	500	770
Coolant flow rate (Mg/s)	20	12	11	0.45
Average linear heat rate (kW/m)	22.5	20	30	—

9.16 PRESSURIZED WATER REACTOR (PWR)

The excellent properties of water as a moderator and coolant make it a natural choice for power reactors, and the PWR has been extensively developed in the

USA. The most important limitation on a PWR is the critical temperature of water, 374°C. This is the maximum possible temperature of the coolant in the reactor, and in practice it is considerably less, possibly about 300 °C, to allow a margin of safety. In a PWR, the coolant pressure must be greater than the saturation pressure at, say, 300 °C (85.93 bar) to suppress boiling. The pressure is maintained at about 155 bar so as to prevent bulk boiling.

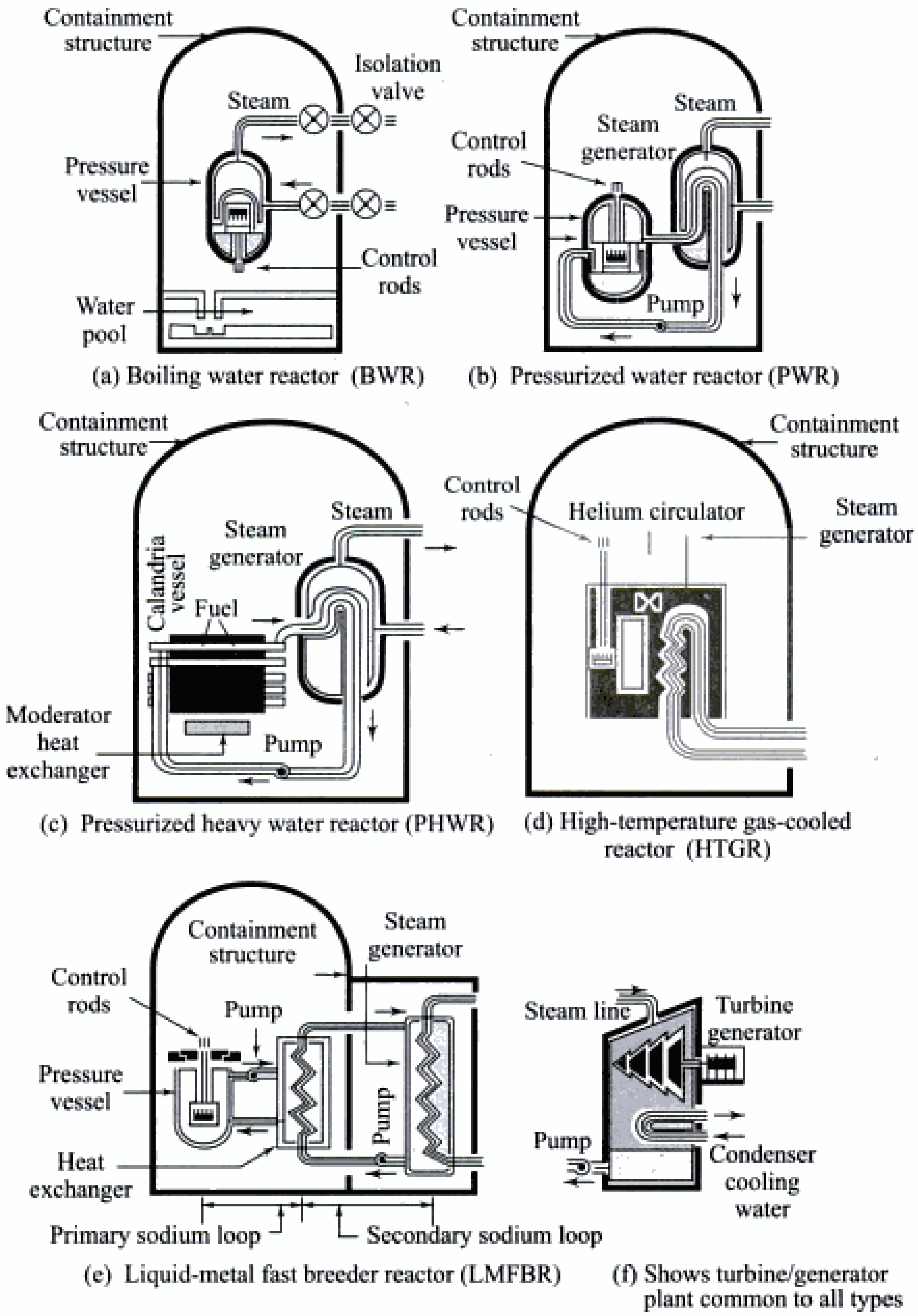

Fig. 9.15 *Schematics for nuclear steam supply systems*

A PWR power plant is composed of two loops in series, the coolant loop, called the primary loop, and the water–steam or working fluid loop (Fig. 9.16). The coolant picks up heat in the reactor and transfers it to the working fluid in the steam generator. The steam is then used in a Rankine type cycle to produce electricity.

The fuel in PWRs is slightly enriched uranium in the form of thin rods or plates. The cladding is either of stainless steel or zircaloy. Because of very high coolant pressure, the steel pressure vessel containing the core must be about 20 to 25 cm thick. A typical PWR contains about 200 fuel assemblies, each assembly being an array of rods. In a typical fuel assembly, there are 264 fuel rods and 24 guide tubes for control rods. Grid spacers maintain a separation between the fuel rods to prevent excessive vibration and allow some axial thermal expansion.

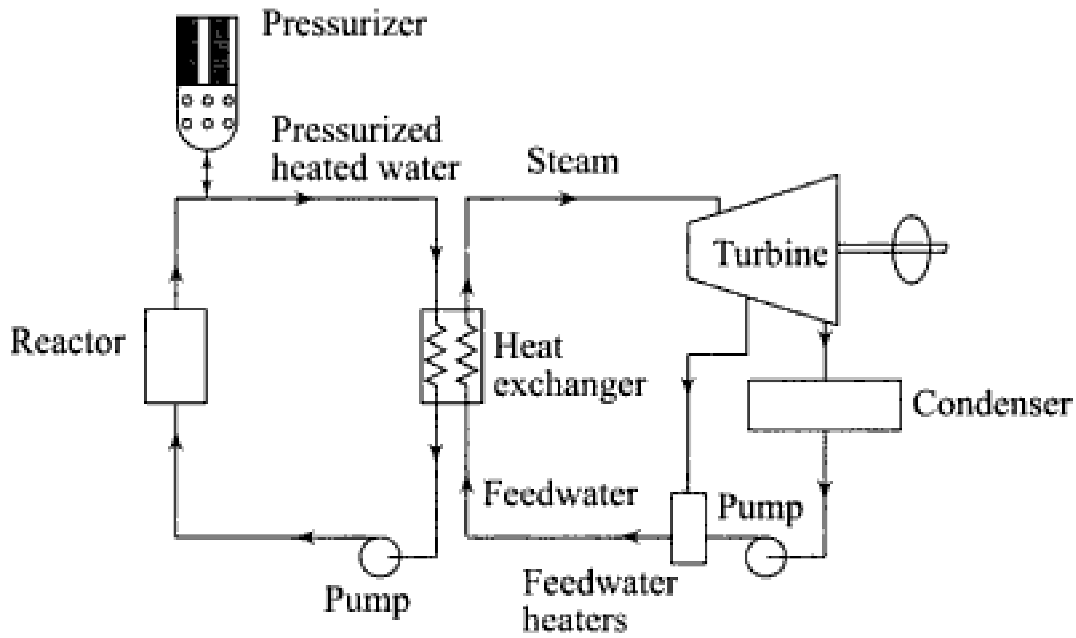

Fig. 9.16 *Schematic of a PWR power plant*

The coolant leaving the reactor enters the steam generator which can be either shell and tube type with U-tube bundles or once-through type, the former being more common. In the U-tube steam generator, the hot coolant enters an inlet channel head at the bottom, flows through the U-tubes, and reverses direction to an outlet at the bottom. It can produce only saturated steam. In the once-through design, the primary coolant enters at top, flows downward through tubes and exits at the bottom to the main pumps. Feedwater is on the shell side. A dry or low degree of superheat steam is possible.

The first land-based PWR for power generation was built at Shippingport, USA in 1957. Its thermal output is 231 MW, the pressure in the primary circuit is 141 bar, and the water temperature at outlet from the reactor is 282 °C. Dry saturated steam is generated in the heat exchangers at 41 bar, 252 °C. For a gross electrical output of 68 MW, the thermal efficiency is 29.4%.

The Shippingport cycle has been modified in the Indian Point (USA) PWR by the inclusion of an oil-fired superheater between the main heat exchangers and the turbines (Fig. 9.17). There is also an economiser along with some feedwater heaters. The steam condition improves to 25.5 bar and 538 °C at turbine inlet, and so the cycle efficiency increases.

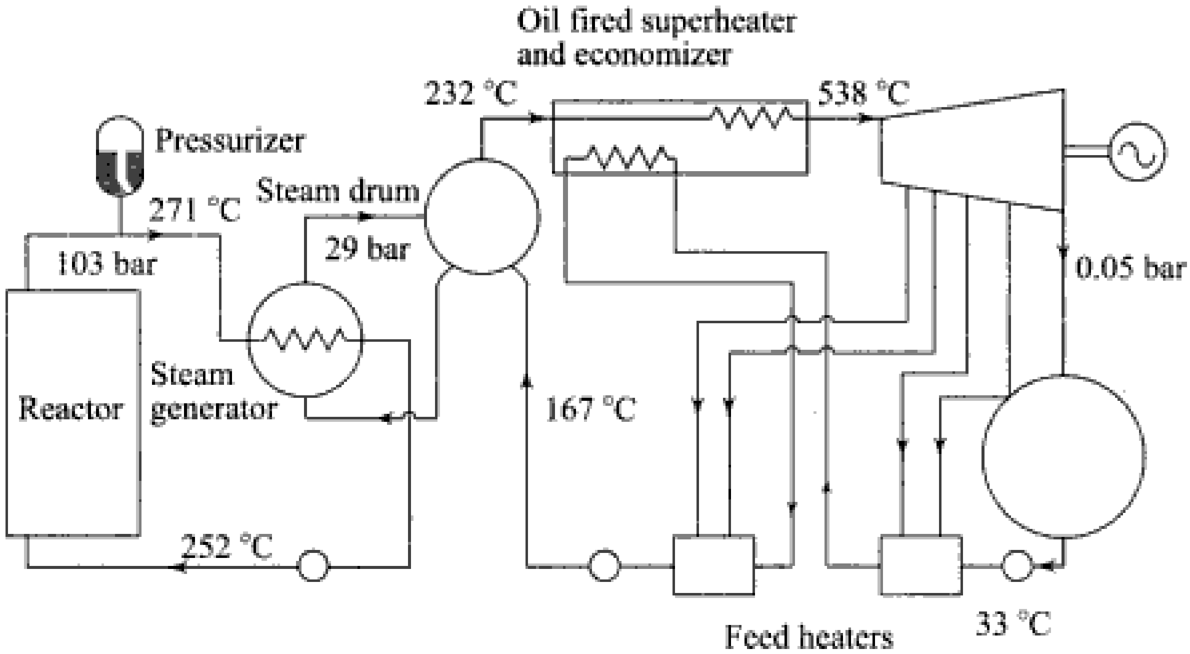

Fig. 9.17 *The Indian Point (USA) PWR with oil-fired superheater*

9.16.1 Pressurizer

The coolant in the PWR primary loop is maintained at a pressure (about 155 bar) greater than the saturation pressure corresponding to the maximum coolant temperature in the reactor to prevent bulk boiling. Because liquids are incompressible, small changes in volume due to changes in coolant temperatures because of either load variation or sudden nuclear reactivity insertions cause severe or oscillatory pressure changes, due to which pressure may increase or decrease. If the pressure increases, some water will flash into steam and it will affect the reactor performance, often leading to its burnout. If the pressure decreases, there may be cavitation. It is thus necessary to provide a surge chamber that will accommodate the coolant volume changes while maintaining pressure within permissible limits. Such a chamber is called a *pressurizer*.

Figure 9.18 shows a vapour pressurizer which is essentially a small boiler where the liquid, the same as the coolant, is kept at a constant temperature by controlled electric heating. There is, thus, a constant vapour pressure above its surface. This pressure is the same as that of the primary coolant at the junction between the pressurizer and hot leg of the primary loop. Thus, the pressurizer temperature is higher than the primary coolant temperature because the latter is subcooled. A spray nozzle located at the top of the pressurizer is connected to the cold leg of the primary coolant system after the pump. The pressurizer is half full with water and half full with vapour.

When there is a positive surge, the volume of coolant increases and the vapour in the top is compressed. Some coolant is then sprayed to condense some of the vapour and thus check the pressure rise. If there is a negative surge, coolant volume decreases, there is a momentary decrease of pressure as a result of which some liquid flashes into vapour. Also, the electric heaters operate. A relief valve is installed at the top to protect against pressure surges beyond the capacity of the pressurizer. It discharges steam into a pressurizer relief tank containing water in which it condenses.

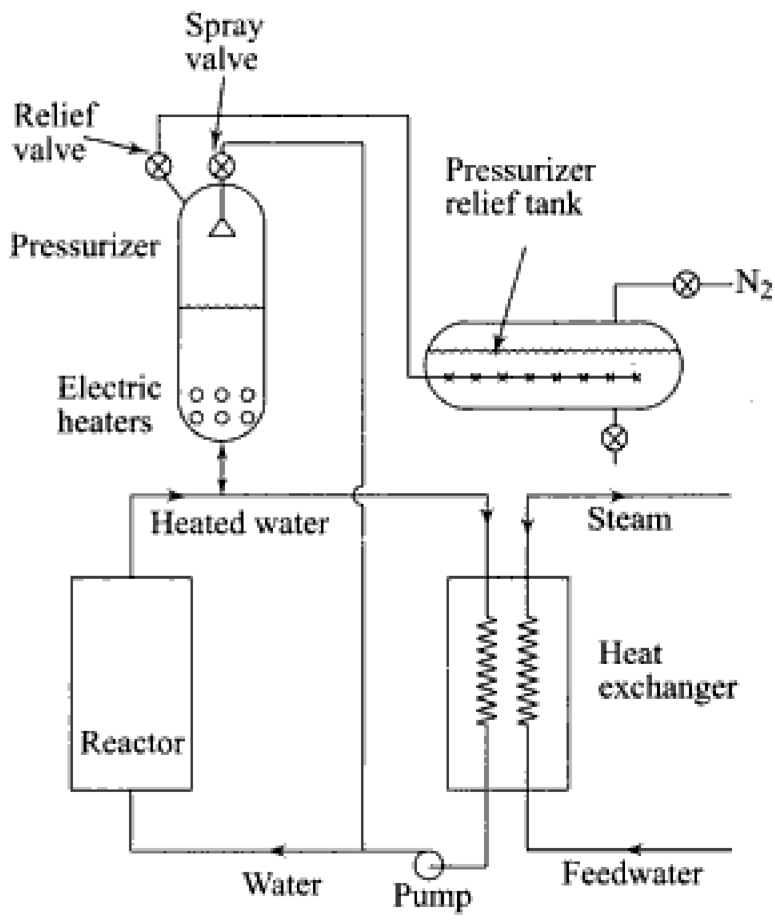

Fig. 9.18 *A PWR primary loop with a vapour-type pressurizer system*

9.17 BOILING WATER REACTOR (BWR)

A BWR differs from the PWR in that the steam flowing to the turbine is produced directly in the reactor core. Steam is separated and dried by mechanical devices located in the upper part of the pressure vessel assembly. The dried steam is sent directly to the high pressure turbine thus eliminating the need for steam generators (Fig. 9.19). The coolant thus serves the triple function of coolant, moderator and working fluid. Since the coolant boils in the reactor itself, its pressure is much less than that in a **PWR** and it is maintained at about 70 bar with steam temperature around 285 °C. However, an increase in the boiling rate displaces water (moderator) in the core and reduces the ability of the moderator to thermalize neutrons and hence, reduces the reactor power level. At power levels above 60% of the nominal, the fraction of steam in the core can be kept nearly constant by varying the coolant circulation rate.

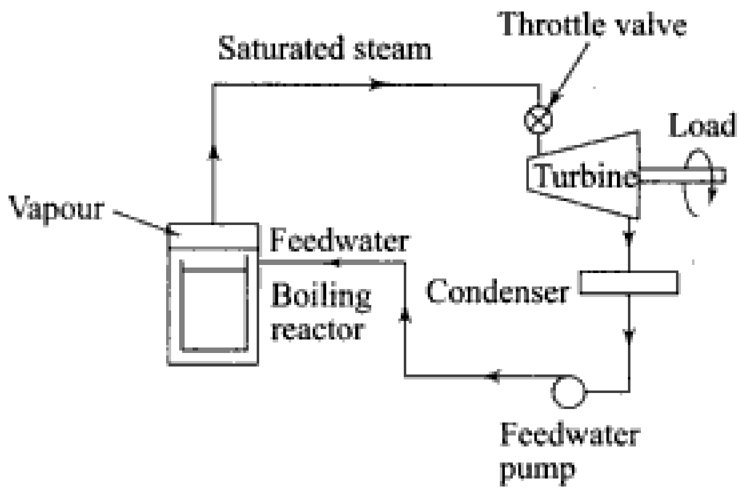

Fig. 9.19 *Schematic of a direct-cycle BWR plant*

The saturated liquid that separates from the vapour at the top of the reactor in a steam separator flows downward either internally within the reactor or externally outside the reactor and mixes with the return condensate (Fig. 9.20). This recirculating coolant again either flows naturally due to density difference or by a forced circulation pump. The ratio of the recirculated coolant to the saturated vapour produced is called the circulation (or recirculation) ratio(as defined earlier in Chapter 6). It is a function of the core average exit quality. The BWR core exit quality varies from 10 to 14%, so that circulation ratio is of the range 6–10. This is necessary to avoid large void fractions in the core, which would reduce the moderating power of the coolant resulting in low heat transfer coefficient or vapour blanketing and burnout.

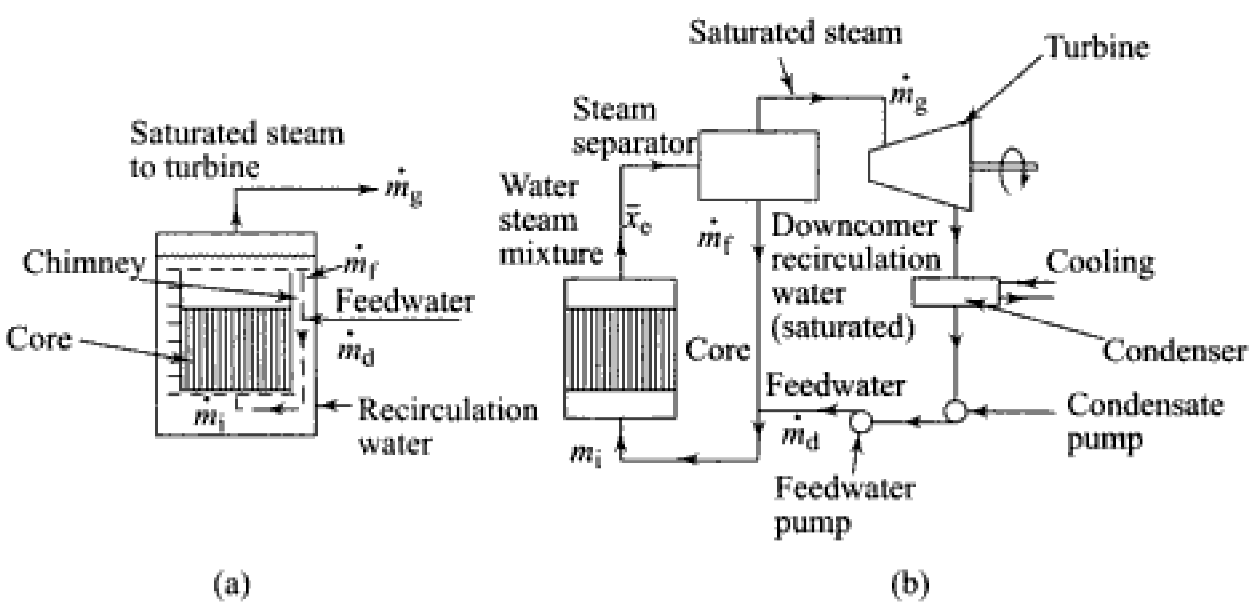

Fig. 9.20 *A BWR system with (a) internal and (b) external circulation*

A slightly subcooled liquid enters the core bottom at a rate of m_i and rises through the core and chimney, if any. The chimney is an unheated section above the core that helps increase the pressure for natural circulation. The resulting vapour separates and goes to the turbine at a rate of m_g, while saturated liquid recirculates via the downcomer at the rate of m_f. There it mixes with the cold return feedwater m_d to form the subcooled inlet liquid m_i.

An overall mass balance in the reactor core gives

$$m_d = m_g$$

$$m_g + m_f = m_i$$

The quality of the liquid–vapour mixture at the core exit x_e is given by

$$x_e = \frac{m_g}{m_f + m_g} = \frac{m_d}{m_d + m_f} = \frac{m_d}{m_i}$$

The circulation ratio R is then

$$R = \frac{m_f}{m_g} = \frac{1 - x_e}{x_e}$$

Neglecting any heat loss and KE and PE changes, an energy balance of the core gives

$$m_i h_i = m_f h_f + m_d h_d$$

where h_i, h_f and h_d are the enthalpies of the inlet, recirculated and incoming feedwater, respectively. On rearrangement,

$$h_i = (1 - x_e)h_f + x_e h_d$$

or $$x_e = \frac{h_f - h_i}{h_f - h_d}$$

The enthalpy of subcooling of liquid entering the bottom of the core is

$$\Delta h_{sub} = h_f - h_i = x_e (h_f - h_d)$$

or the degree of subcooling is given by

$$\Delta t_{sub} = t_f - t_i$$

The total heat generation Q_t then becomes

$$Q_t = m\,[h_f + x_e h_{fg}) - h_i]$$
$$= m_g (h_g - h_d)$$

A number of BWR power plants have used a dual pressure, direct cycle arrangement, e.g. Dresden 1 (USA) as shown in Fig. 9.21. There is a decrease in the temperature of the water entering the reactor and thus, the power output increases with the unaltered exit condition. However, there is the disadvantage of increased complexity of the plant.

The active or fueled core region of a BWR consists of about 800 fuel assemblies. Each typically contains an 8 by 8 array of fuel rods. The zircaloy channel around the fuel rods prevents cross flow in the core. BWR fuel rods are slightly larger than PWR fuel rods. A typical pellet diameter is 10.6 mm with an outside cladding diameter of 12.5 mm. The average fuel enrichment varies from 1.9 to 2.6 %.

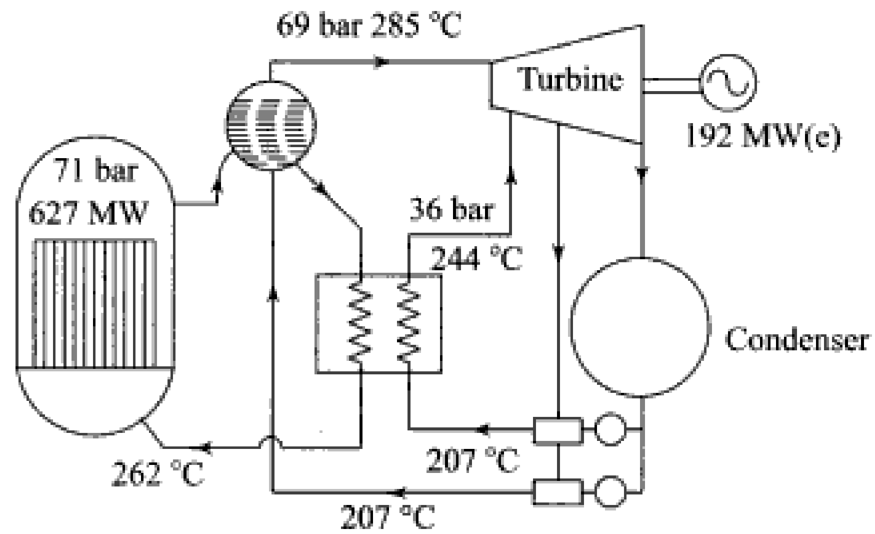

Fig. 9.21 *Dresden (USA) dual-pressure direct-cycle BWR system*

9.18 GAS-COOLED REACTORS

The first gas-cooled reactors with CO_2 gas (at a pressure of 16 bar) as coolant and graphite as moderator were developed in Britain during 1956–69. The fuel was a natural uranium, clad with an alloy of magnesium called Magnox (Fig. 9.22).

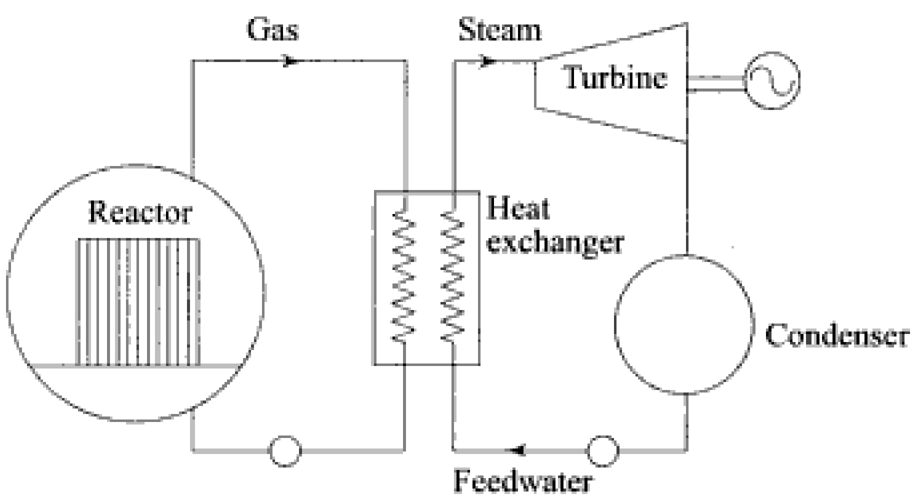

Fig. 9.22 *Schematic of a gas-cooled reactor plant*

Several types of gas-cooled reactors have been designed and built, with England developing an advanced gas-cooled reactor (AGR) system, and Germany and the USA developing helium-cooled. graphite-moderated systems (HTGR). The AGR uses UO_2 as the fuel clad in stainless steel tubes with CO_2 gas as coolant and graphite as moderator.

The graphite moderated helium-cooled HTGR is designed to use U-233 as the fissile material and thorium as fertile material. Initially, the system would have to be fuelled with U-235, until sufficient U-233 is available for makeup fuel. Because of the very high melting point of graphite, these fuel elements can operate at very high temperatures, and it is possible to generate steam at conditions equivalent to those in modern coal-fired power plant. The basic fuel forms are small spheres of fissile and fertile material as carbides, UC_2 or ThC_2. The fissile spheres are 0.35 to 0.50 mm in diameter and the fertile spheres are 0.6 to 0.7 mm in diameter. Each sphere is coated with two to three layers of carbon and silicon carbide to prevent fission products from escaping from the particles. Helium is a suitable coolant in the sense that it is chemically inert, has good heat transfer characteristics and low neutron absorption. Being a monatomic gas, it can produce more power for given temperatures in the Brayton cycle and higher efficiency.

A direct cycle HTGR gas turbine plant is shown in Fig. 9.23. It incorporates a regenerator and multi-stage compression with intercooling. Typical figures for such a cycle are: pressure ratio 4, turbine inlet pressure 50 bar, turbine inlet (reactor outlet) temperature 900 °C, compressor inlet temperature 50 °C. The temperatures at other points are shown in the figure, With U-233/Th-232 fuel, the HTGR functions as a thermal breeder reactor.

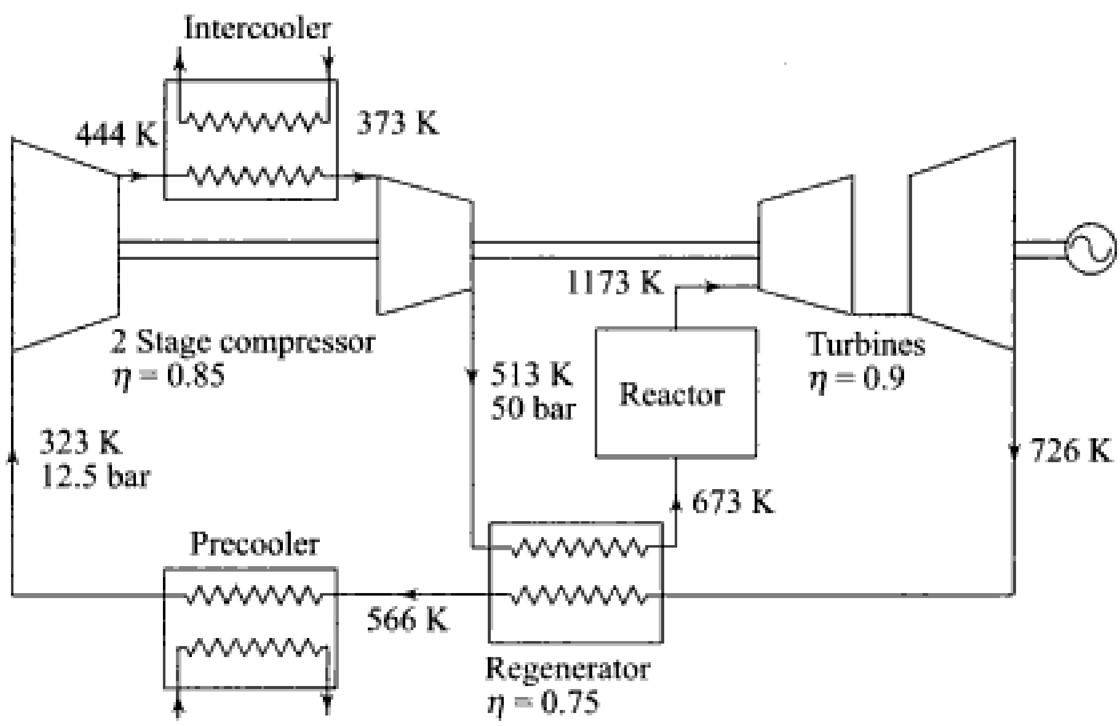

Fig. 9.23 *A–HTGR direct cycle gas turbine plant using helium*

9.19 LIQUID METAL FAST BREEDER REACTOR

Fast breeder reactors are designed to create or breed new fissile material, while producing useful electric power. Most produce fissile plutonium from fertile uranium 238. The fuel rods in the core region thus contain a mixture of fissile Pu-239 and U-238. The active core region is surrounded by a blanket of fertile U-238. This blanket region captures neutrons that would otherwise be lost through leakage, thus producing additional fissile material. A fast neutron reaction with U-238 producing Pu-239 is shown as follows.

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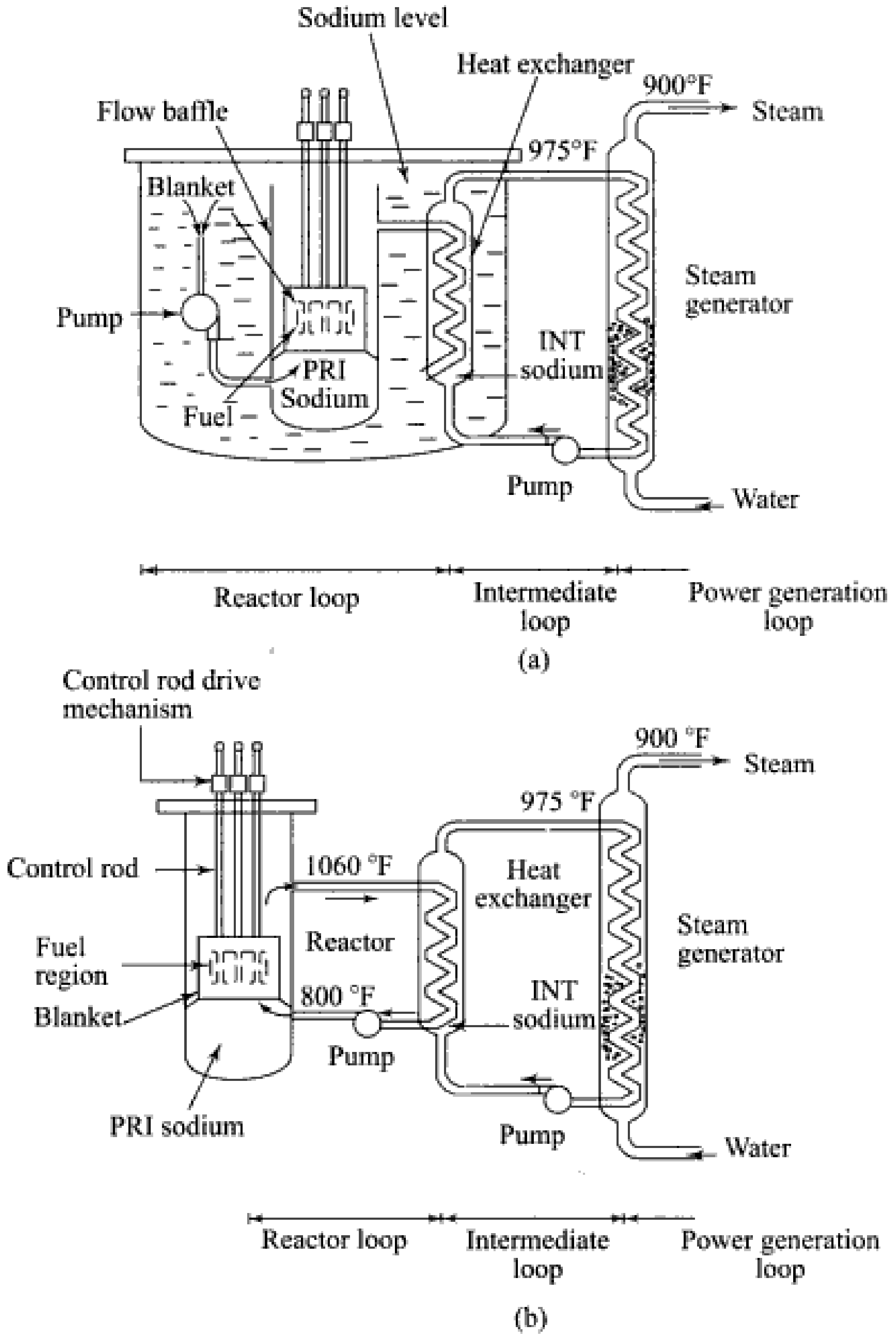

Fig. 9.25 *Liquid metal fast breeder reactor (a) Pool type, (b) Loop type*

Heavy water-moderated and cooled reactors have been extensively developed in Canada, and form the basis of the nuclear power programme in that country. They are called CANDU-PHW (Canadian Deuterium Uranium Pressurized Heavy Water). The CANDU reactors have several features that distinguish them from other types. The moderator is contained in a cylindrical steel vessel, called the calandria, with a large number of zircaloy tubes through it parallel to its axis, which is horizontal (Fig. 9.26). The active core region is approximately 6 m high with a diameter of 7 to 8 m. The D_2O coolant enters the regular array of pressure tubes at 260 °C and 110 bar, flows through the fuel elements, and leaves the pressure tubes at 320 °C, and the net efficiency is about 29%. Like PWR, there is no bulk boiling of coolant.

The heavy water coolant pressure in the reactor is 88.3 bar, and the inlet and outlet temperatures are 250 °C and 290 °C, respectively. In heat exchangers, steam is generated at 41 bar pressure, 251 °C. The thermal power of each reactor (there are 8 at Pickering at Canada) is 1744 MW, and the net electrical output is 515 MW, giving a thermal efficiency of 29.5% (Fig. 9.26).

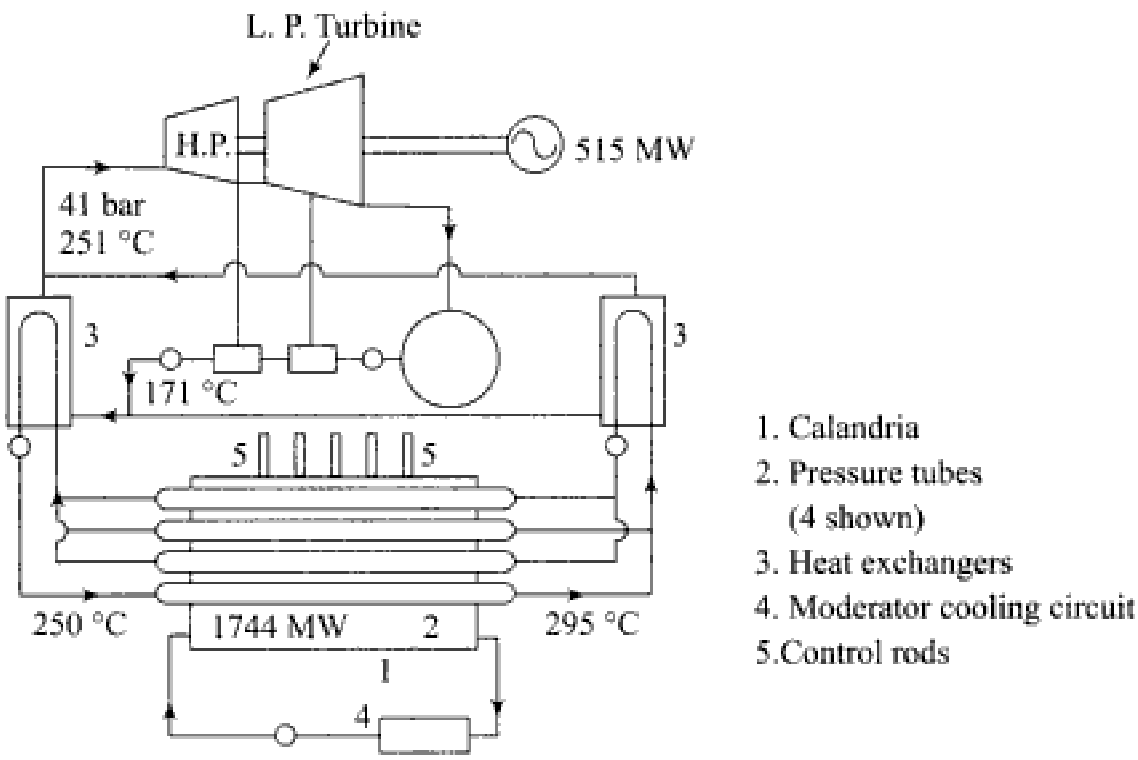

Fig. 9.26 *Layout of the calandria, heat exchangers and simplified steam cycle of the CANDU reactor*

The calandria contains up to 380 horizontal pressure tubes, called calandria tubes, which are welded to the tube sheets at each end of the vessel. The moderator temperature is maintained at about 70 °C and low pressure to reduce heavy water losses. The fuel assembly contains 37 fuel rods. as shown in Fig. 9.27. Each rod contains natural uranium dioxide (UO_2) fuel pellets with 0.38 zircaloy cladding. Each rod bundle is about 0.1 m in diameter and 0.5 m long.

9.21 India's Nuclear Power Programme

From the several reactor types available, India has selected Pressurized Heavy Water Reactors (PHWR) because of several inherent advantages. A PHWR uses natural uranium as fuel with heavy water as moderator and coolant. Natural uranium being easily available in India, helps cut heavy investments for enriched uranium, the import of which is very difficult due to restrictive international trade practices. Besides, the PHWR core containing natural uranium is safer with its lesser reactivity and has on-power refuelling facility. Due to its excellent neutron economy, a PHWR has a greater yield of plutonium.

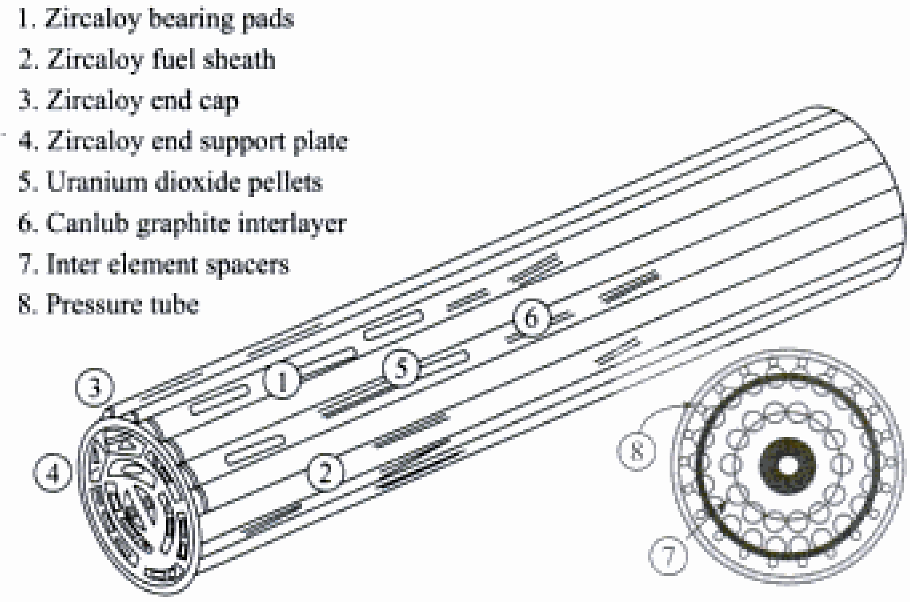

Fig. 9.27 *CANDU fuel element assembly*

India has planned a three stage programme to benefit from the atom:

1st stage: PHWRs use natural uranium fuel to produce electricity and plutonium fuel with 96% unused uranium (U-238).

2nd stage: The use of fast breeder reactors with plutonium as fuel will generate electricity and more plutonium from U-238, kept as a blanket. It will also produce U-233 fuel from thorium, used as a blanket material.

3rd stage: U-233 will be used as fuel and thorium as blanket, producing more U-233 fuel than the fuel consumed in fast and thermal reactors.

India's nuclear power programme up to 2000 A.D. as envisaged by the Nuclear Power Corporation (Government of India) is given in Table 9.7.

Table 9.7

Operational units	*Station capacity MWe*	*Cumulative capacity MWe*	*Year of commissioning*
Tarapur (BWR) 1 and 2	2 × 210	420	1969
Rajasthan 1 and 2	2 × 220	860	1973, 1981
Madras 1 and 2	2 × 235	1330	1983, 1985
Narora 1 and 2	2 × 235	1800	1990, 1991
Kakrapar 1 and 2	2 × 235	2270	1991, 1992
Under construction			
Kaiga 1 and 2	2 × 235	2740	1995, 1996
Rajasthan 3 and 4	2 × 235	3210	1995, 1996
Under sanction			
Kaiga 3, 4, 5, and 6	4 × 235	4150	1996, 1997
Tarapur 3 and 4	2 × 500	5150	1997, 1998
Rajasthan 5, 6, 7 and 8	4 × 500	7150	1998, 2000
Planned			
Kudankulam (PWR) 1 and 2	2 × 1000	9150	1998, 1999
New Projects	6 × 500	12150	1998, 2000

Since the resources of fossil fuels are fast depleting, India has to depend more on nuclear power. Besides, it is economical in regard to the cost of generation of electricity. An optimal mix of thermal, hydel and nuclear sources is required to help India achieve self-sufficiency in power generation for years to come.

9.22 FUSION POWER REACTORS

Fusion of light nuclei to form a heavy nucleus also releases energy. To cause fusion, it is necessary to accelerate the positively charged nuclei to high kinetic energies, in order to overcome electrical repulsive forces, by raising their temperature to hundreds of millions of degrees resulting in a plasma. The plasma must be prevented from contacting the walls of the container, and must be confined for about tenth of a second having a density of around 10^{15} ions/cm^3.

There are several possible reactions between the nuclei of light elements that can be the basis for controlled fusion. Deuterium, a stable heavy isotope of hydrogen, present in natural water, is the main fuel for a fusion reactor. Four reactions involving deuterium are given below:

	Fusion reaction	Energy per reaction
1.	${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1$ $(D + D \longrightarrow He^3 + n)$	3.2 MeV
2.	${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1p^1$ $(D + D \longrightarrow T + p)$	4.0 MeV
3.	${}_1H^2 + {}_1H^3 \longrightarrow {}_2He^4 + {}_0n^1$ $(D + T \longrightarrow {}_2He^4 + n)$	17.6 MeV
4.	${}_1H^2 + {}_2He^3 \longrightarrow {}_2He^4 + {}_1p^1$ $(D + He^3 \longrightarrow He^4 + p)$	18.3 MeV

where n, p, D and T are the symbols for neutron, proton, deuterium (${}_1H^2$) and tritium (${}_1H^3$), respectively.

Fusion reaction occurs most easily between deuterium and tritium (D + T), which is self-sustaining at a temperature of 50×10^6 K releasing 17.6 MeV per reactions. The first two D–D reactions occur at 500×10^6 K and release less energy (3.2 and 4.0 MeV). The fourth reaction releases very high energy (18.3 MeV), but it requires a very high temperature (1000×10^6 K) also for fusion.

Tritium does not occur abundantly in nature. It can, however, be produced in a lithium "breeding blanket" that surrounds the plasma core of the fusion reactor.

Figure 9.28 shows the schematic diagram of a futuristic deuterium–tritium fusion reactor (Sorensen, 1983). The plasma is contained inside an evacuated tube of about 4 m. The surrounding vacuum wall through which 14 MeV neutrons from the plasma pass, is maintained at about 750 °C. Outside this wall are two concentric regions, *viz*., the lithium breeding moderator and the magnetic shield. Tritium is manufactured in the lithium blanket. Large cryogenic superconducting magnets of 7 to 8 m diameter maintain the magnetic shield. The binary vapour power cycle consists of a potassium topping cycle and a conventional steam cycle. It includes a tritium recovery system.

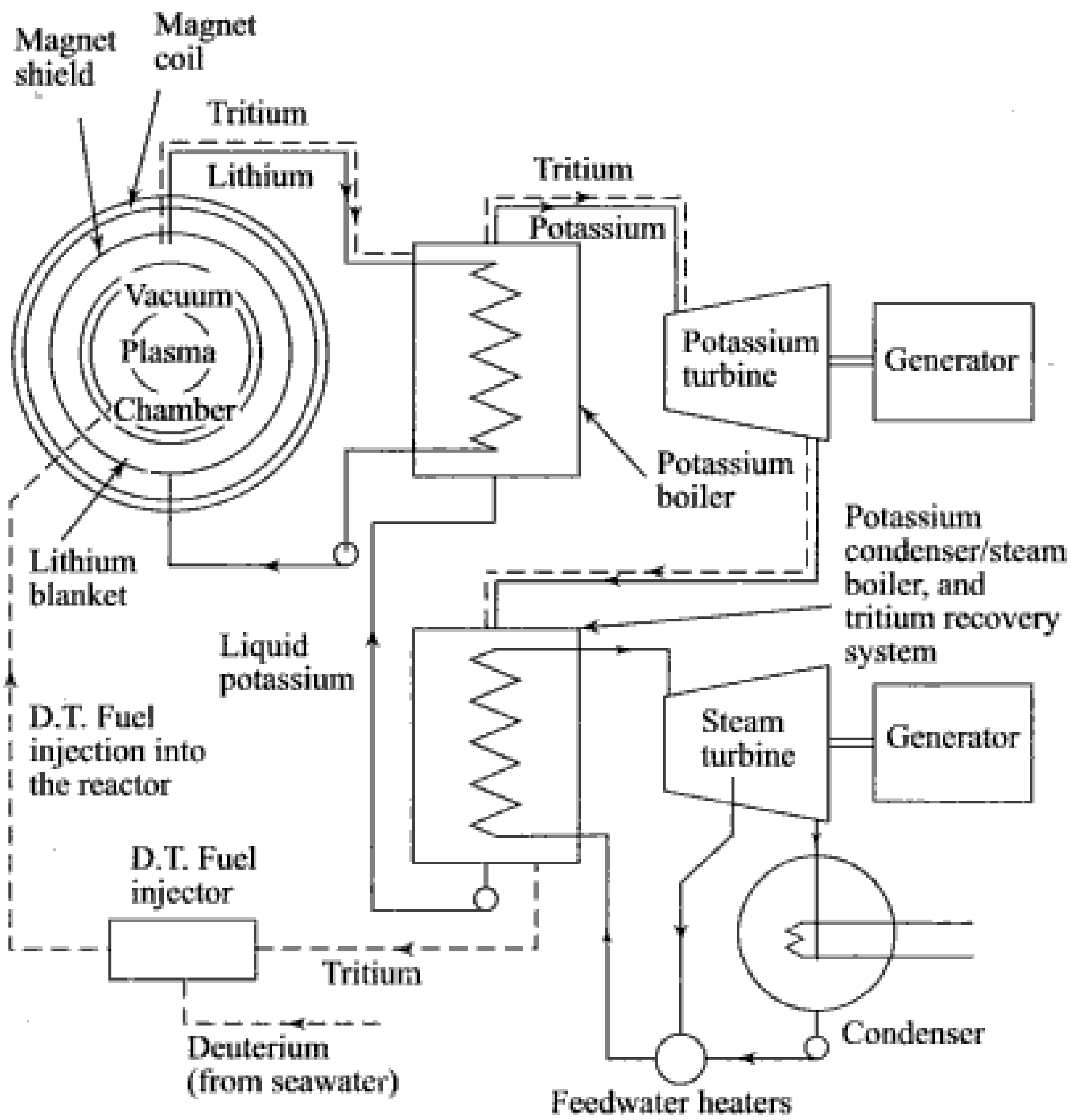

Fig. 9.28 *Deuterium–tritium (D–N) fusion power plant conceptual design*

Significant advantages of the fusion power plant are:

1. The supply of deuterium is almost inexhaustible.
2. Radioactive wastes are not produced.
3. It is very safe to operate.
4. High energy conversion efficiency (~60%) can be achieved.
5. Low heat rejection to the environment takes place per kW of electricity generated.

It is almost certain that large, practical fusion power plants will be built in the twenty-first century. Once this technology is developed, an almost unlimited supply of energy will be available for the world's needs ushering in a better living standard for the human kind all over the world.

Example 9.1 Calculate the mass defect and binding energy per nucleon of oxygen. Given: $m_p = 1.007277$ amu, $m_n = 1.008665$ amu, $m_e = 0.00055$ amu, atomic mass of oxygen ~ 16 = 15.99491 amu.

Solution A molecule of oxygen has 8 protons, 8 electrons, and 8 neutrons. Therefore, the mass defect is

$$\Delta m = 8 \times 1.007277 + 8 \times 0.00055 + 8 \times 1.008665 - 15.99491$$
$$= 0.13701 \text{ amu}$$

Binding energy $= 0.13701 \times 931 = 127.6$ MeV

Binding energy per nucleon $= 127.6/16 = 7.97$ MeV *Ans.*

Example 9.2 The half-life of radium 226 (atomic mass = 226.095) is 1620 yrs. Compute (a) the decay constant, and (b) the initial activity of 1 g of radium 226.

Solution From Eq. (9.15),

$$\theta_{1/2} = \frac{0.6931}{\lambda}$$

(a) The decay constant

$$\lambda = \frac{0.6931}{1620 \times 365 \times 24 \times 3600} = 1.3566 \times 10^{-11} \text{ s}^{-1}$$

(b) Number of atoms per g of radium 226

$$= \frac{\text{Avogadro's constant}}{\text{Atomic mass}} = \frac{6.023 \times 10^{23}}{226.095} = 2.6645 \times 10^{21}$$

Initial activity $A_0 = \lambda N_0 = 1.3566 \times 10^{-11} \times 2.6645 \times 10^{21}$

$$= 3.615 \times 10^{10} \text{ dis/s}$$

Example 9.3 Each fission of U-235 yields 190 MeV of useful energy. Assuming that 85% of neutrons absorbed by U-235 cause fission, the rest being absorbed by non-fission capture to produce an isotope U-236, estimate the fuel consumption of U-235 per day to produce 3000 MW of thermal power.

Solution Each fission yields 190 MeV $\times 1.60 \times 10^{-13}$ J/MeV or 3.04×10^{-11} J of useful energy.

Number of fissions required to obtain W-s of energy

$$= \frac{1}{3.04 \times 10^{-11} \text{ J}} = 3.3 \times 10^{10}$$

In one day's operation (i.e., 86,400 s) of a reactor per MW of thermal power, the number of U-235 nuclei burned is

$$\frac{(10^6 \text{ W})(3.3 \times 10^{10} \text{ fission/W-s})(86{,}400 \text{ s/day})}{0.85 \text{ fission/absorption}}$$
$$= 3.35 \times 10^{21} \text{ absorptions/day}$$

Mass of U-235 consumed to produce 1 MW power is

$$\frac{(3.35 \times 10^{21} \text{ day}^{-1})(235 \text{ g/g mol})}{6.023 \times 10^{23} (\text{nuclei/g mol})} = 1.3 \text{ g/day}$$

Therefore, the fuel consumption of U-235 to produce 3000 MW is 3.9 kg/day. *Ans.*

To produce the same energy by use of fossil fuels, millions of times as much weight would be required.

Example 9.4 A certain nucleus has a cross-section of 10 barns for 2200 m/s neutrons. Find the cross-section if the KE of the neutrons increases to 0.1 eV. The two neutron energies are within 1/V range of the nucleus.

Solution At 2200 m/s, $E_n = 0.02525$ eV

In the low energy region,

$$\sigma_a = C\left(\frac{1}{E_n}\right)^{0.5} = C_1\frac{1}{V}$$

or
$$\frac{\sigma_{a_1}}{\sigma_{a_2}} = \frac{V_2}{V_1} = \left(\frac{E_{n_1}}{E_{n_2}}\right)^{0.5} = \left(\frac{0.1}{0.02525}\right)^{0.5} = 1.98$$

$$\sigma_{a2} = \frac{10}{1.98} = 5.01 \text{ barns} \qquad \textit{Ans.}$$

Example 9.5 Calculate the microscopic absorption cross-section of natural uranium, which consists of 99.285% U-238 and 0.715% U-235. The microscopic cross-sections for 0.025 eV neutrons are:

U-238: $\sigma_c = 2.72$, barns $\sigma_f = 0$ U-235: $\sigma_c = 101$ barns $\sigma_f = 579$ barns

Solution For natural uranium, $\sigma_a = \sigma_c + \sigma_f$

$$\sigma_a = 0.99285\,(2.72 + 0) + 0.00715\,(101 + 579)$$
$$= 7.6 \text{ barns} \qquad \textit{Ans.}$$

Example 9.6 Calculate the macroscopic capture cross-section of water of density 1 g/cm^3. The microscopic capture cross-sections of hydrogen and oxygen are 0.332 barn and 0.0002 barn, respectively.

Solution Number of molecules of water per cm^3 is

$$= \frac{6.023\times10^{23}\,\dfrac{\text{molecules}}{\text{g mol}}}{18\,\text{g/g mol}} \times 1\frac{\text{g}}{\text{cm}^3} = 3.35\times10^{22}$$

$$\Sigma_c \text{ for water } (H_2O)) = N_1\sigma_{c_1} + N_2\sigma_{c_2}$$
$$= 2\times3.35\times10^{22}\times0.332\times10^{-24} + 3.35\times10^{22}\times0.0002\times10^{-24}$$
$$= 0.0222 \text{ cm}^{-1} \qquad \textit{Ans.}$$

Example 9.7 A 230 g piece of boron (mol wt. 10) absorbs thermal neutrons at the rate of 9.57×10^{13} per (cm$^3\cdot$s). Boron density is 2.3 g/cm^3. Find (a) the thermal neutron flux, and (b) the average distance that a neutron travels before it is absorbed. For thermal neutrons, $\sigma_a = 755$ barns and $\sigma_s = 4.0$ barns.

Solution

$$\sigma_t = \sigma_a + \sigma_s = 755 + 4 = 759 \text{ barns}$$

The number density of neutrons is

$$N = \frac{\rho}{M} N_a = \frac{2.3\ \text{g/cm}^3}{10\ \text{g/g mol}} \times 6.023 \times 10^{23}\ \frac{\text{molecules}}{\text{g mol}}$$

$$= 1.3853 \times 10^{24}\ \text{cm}^{-3}$$

$$\Sigma_t = N\sigma_t = 1.3853 \times 10^{23} \times 759 \times 10^{-24}\ \text{cm}^{-1}$$

$$= 105.144\ \text{cm}^{-1}$$

Reaction rate, $R = \phi\Sigma = 9.57 \times 10^{13}\ \text{cm}^{-3}\ \text{s}^{-1}$

Neutron flux is $\phi = \dfrac{9.57 \times 10^{13}}{105.144}\ \text{cm}^{-2}\ \text{s}^{-1} = 0.091 \times 10^{13}\ \text{cm}^{-2}\ \text{s}^{-1}$ *Ans.* (a)

Average distance a neutron travels before it is absorbed,

$$\lambda = \frac{1}{\Sigma} = \frac{1}{105.144} = 9.51 \times 10^{-3}\ \text{cm} \qquad \textit{Ans.}\ \text{(b)}$$

Example 9.8 A newly born neutron of 4.8 MeV is to be slowed to 0.025 eV in a graphite moderator. Assuming all collisions to be elastic, calculate the logarithmic energy decrement representing the neutron energy loss per elastic collision and the number of collisions necessary.

Solution From Eq. (9.24), the logarithmic energy decrement ξ is given by

$$\xi = 1 - \left[\frac{(A-1)^2}{2A} \ln \frac{A+1}{A-1}\right]$$

where A is the mass number of the nucleus (graphite) with which the neutron collides. For graphite (carbon), $A = 12$.

$$\xi = 1 - \left[\frac{(12-1)^2}{2 \times 12} \ln \frac{12+1}{12-1}\right] = 0.158$$

The number of collisions required to slow down the neutron from 4.8 MeV to 0.025 eV is given by Eq. (9.25).

$$n = \frac{\ln \dfrac{E_{n,i}}{E_{n,f}}}{\xi} = \frac{\ln \dfrac{4.8 \times 10^6}{0.025}}{0.158} = 120.72 = 121 \qquad \textit{Ans.}$$

Example 9.9 A reactor is fuelled with 100 tonnes of natural uranium (atomic mass 238.05) in which the average thermal neutron (2200 m/s) flux is 10^{13} neutrons/cm^{2}s, The 2200 m/s cross-section of U-235 (atomic mass 235.04) are; $\sigma_f = 579$ barns and $\sigma_c = 101$ barns. The energy release per fission is 200 MeV and 0.715% of natural uranium is U-235. Calculate (a) the rating of the reactor in MW/tonne, (b) the rate of consumption of U-235 per day.

Solution The number of U-235 atoms in the reactor

$$= \frac{10^5 \text{ kg} \times 6.023 \times 10^{26} \dfrac{\text{atoms}}{\text{kg mol}} \times 0.00715}{238.05 \text{ kg/kg mol}}$$

$$= 1.81 \times 10^{27} \text{ atoms}$$

The rate of fission in the reactor is given by Eq. (9.31)

$$= \sigma \phi N V_0$$

$$= 579 \times 10^{-24} \text{ cm}^2 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2\text{s}} \times 1.81 \times 10^{27}$$

$$= 1.05 \times 10^{19} \text{ fissions/s}$$

The rate of energy release or thermal power of the reactor

$$= 1.05 \times 10^{19} \times 200 \text{ MeV/s}$$

$$= 1.05 \times 10^{19} \times 200 \times 1.602 \times 10^{-19} \text{ MW} = 336 \text{ MW}$$

Rating of the reactor = 336/100 = 3.36 MW/tonne *Ans.* (a)

Rate of consumption of U-235 by fission

$$= \frac{1.05 \times 10^{19} \times 235.04 \times 60 \times 60 \times 24}{6.023 \times 10^{26}}$$

$$= 0.353 \text{ kg/day or } 353 \text{ g/day} \qquad \textit{Ans. (b)}$$

Thus complete fissioning of 1 g of U-235 releases about 1 MWd of thermal energy. For the same amount of energy release in a combustion process, 3 tonnes of coal are required, If 15% of the neutrons absorbed result in non-fission capture to produce U-236, the total consumption rate of U-235 becomes

$$\frac{0.353}{0.85} = 0.415 \text{ kg/day}$$

The burnup of nuclear fuel is a measure of the total amount of energy released by fission per unit mass of fuel over a period of time. If the reactor operates at steady and continuous power for one year the burnup is 3.36 * 365 = 1226 MWd/tonne. The fraction of U-235 consumed, both by fission and neutron capture, in one year is

$$\frac{0.415 \times 365 \times 6.023 \times 10^{26}}{235.04 \times 1.81 \times 10^{27}} = 0.214$$

Example 9.10 The fuel density N for a uranium oxide fuel is given by

$$N = 2.373 f \times 10^{22} \text{ U-235 nuclei/cm}^3$$

where f is the mass fraction of U-235 in the fuel.

Determine for a light water moderated uranium reactor the specific energy release rate (in W/cm^3) for the following conditions:

$$\phi = \text{neutron flux} = 10^{13}/\text{cm}^2\text{s}$$

$$G = \text{energy per fission} = 180 \text{ MeV}$$

$$f = 3.5\% \text{ U-235 enrichment, i.e. } 0.035$$

$$\sigma_f = 577 \text{ barns}$$

Solution

$$N = 2.372 f \times 10^{22} = 2.372 \times 0.035 \times 10^{22}$$

$$= 8.302 \times 10^{20} \text{ nuclei/cm}^3$$

From Eq. 9.48, the volumetric source strength,

$$q_G = \text{rate of energy release} = GN\sigma_f\,\phi$$

$$= 180(8.302 \times 10^{20})(577 \times 10^{-24})\,10^{13}$$

$$= 8.622 \times 10^{14} \text{ MeV/cm}^3\text{s}$$

Since $1 \text{ MeV} = 1.602 \times 10^{-13}$ J,

$$q_G = 138.13 \text{ W/cm}^3 \qquad \textit{Ans.}$$

Example 9.11 A reactor is operating at a low power of 1W. It then becomes suppercritical with $k_{eff} = 1.0015$. The average neutron life is 0.0001 s for prompt neutrons. Determine the reactor power level at the end of 1 s.

Solution

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{1.0015 - 1}{1.0015} = 0.0014978$$

From Eq. (9.49)

$$z = \frac{\rho}{\tau^*}\tau = \frac{0.0014978}{1.0001} \times 1 = 14.978$$

$$n/n_0 = e^z = e^{14.978} = 3.198 \times 10^6$$

The neutron density increases by 3.198×10^6 times in 1 s . Since the reactor power is proportional to neutron density, it is increased from 1 W to 3.198 MW within a period of 1 s.

SHORT-ANSWER QUESTIONS

9.1 Briefly describe the structure of an atom.
9.2 What is the difference between atomic number and mass number?
9.3 What is amu?
9.4 What do you understand by an "isotope"? What are the isotopes of hydrogen?
9.5 Explain the difference between chemical and nuclear reactions.
9.6 What do you mean by mass defect and binding energy?
9.7 What is nuclear stability? Why are elements of higher mass number not stable?
9.8 What do you understand by radioactive decay? What are radioisotopes?
9.9 What do naturally occurring radioisotopes emit? What is K capture?

9.10 What do you mean by half-life? Why are half-lives regarded as "fingerprints" of radioisotopes?
9.11 What is a curie? What is a roentgen?
9.12 What are fission fragments and fission products?
9.13 Explain a fission chain with an example.
9.14 What is the average energy released per fission for U-233 and Pu-239?
9.15 Explain the term "fuel burnup". What is fuel poisoning?
9.16 What do you mean by critical size and critical mass?
9.17 How are neutrons classified according to neutron energy?
9.18 What are prompt and delayed neutrons?
9.19 Explain inelastic and elastic scattering. What is logarithmic energy decrement?
9.20 Explain the function of a moderator. What is the criterion of its effectiveness?
9.21 What is a thermal reactor? What is a fast reactor?
9.22 What is neutron flux? How would you define nuclear cross-section? What is a barn?
9.23 On what factors does the nuclear reaction rate depend?
9.24 What do you mean by moderating power and moderating ratio?
9.25 Briefly explain how the neutron cross-section varies with the neutron energy.
9.26 Define the life cycle of neutrons. What is the four factor formula?
9.27 What is multiplication factor? Explain the subcritical and supercritical chain reactions
9.28 What are the functions of a reflector?
9.29 Explain the function of a cladding. What are the criteria of selecting a suitable cladding?
9.30 Define the volumetric thermal source strength.?
9.31 What are homogeneous and heterogeneous reactors?
9.32 Explain the characteristic features of a PWR.
9.33 What is the function of pressurizer in a PWR?
9.34 Explain the characteristic features of a BWR. What do you mean by external and internal circulation?
9.35 What is an HTGR? Why is it called Magnox? Explain its main features.
9.36 What is an LMFBR? Why is a liquid metal the preferred coolant in a fast reactor? What is its drawback?
9.37 Explain the terms: (a) breeding ratio, (b) burner, (c) converter, (d) breeder, (d) doubling.
9.38 What is a CANDU-type reactor? Explain with a sketch its main features. What is a calandria?
9.39 What are the three stages in India's nuclear power programme?
9.40 What is the basis of energy release by fusion power?
9.41 What are the four reactions involving deuterium in a fusion reactor? Which one is achieved quite easily?
9.42 What are the sources of deuterium and tritium?

PROBLEMS

9.1 The number density of fuel atoms (N) of U-235 is 0.048×10^{24} cm^{-3} and the microscopic cross-section for absorption (σ_a) is 681×10^{-24} cm^2. With a neutron flux of 2×10^{13} cm^{-2}s^{-1}, estimate the reaction rate for absorption, What is the average distance travelled before striking a U-235 nucleus?

[*Ans.* 6.5×10^{14} cm^{-3} s^{-1}, 0.03058 cm]

9.2 Calculate the energy yield from the reaction

$$_{91}U^{235} + {}_0n^1 \longrightarrow {}_{55}Cs^{140} + {}_{37}Rb^{92} + 4{}_0n^1 + E$$

using atomic masses 139.91711 for cesium and 91.91914 for rubidium.

9.3 Calculate the time required for the reactor power to double for (a) assumed fission with prompt neutrons ($\tau^* = 0.0001$ s) and (b) actual fission with prompt and delayed neutrons ($\tau^* = 0.1$ s). Take $k_{eff} = 1.002$.

[*Ans.* (a) 0.0347s, (b) 34.73s]

9.4 A nuclear power plant is operated continually for one year producing 500 MW. The reactor contained 75 tonnes of 3% enriched uranium dioxide fuel. Assuming the power plant efficiency to be 33%, calculate (a) the mass of U-235 consumed in kg, and (b) the fuel burnup in MWd/tonne.

9.5 Boron 10 is used in reactor cores as a control rod material. Natural boron has an atomic mass of 10.8110 amu, a density of 2.3 g/cm^3 and contains 19.78 atomic per cent of B-10 which has an atomic mass of 10.0194 amu and a microscopic absorption cross-section for 2200 m/s thermal neutrons of 3837 barns. Calculate the number of such neutrons absorbed per sec by 1 kg of natural boron.

9.6 Calculate the power generated in MeV/cm^3 and kW/m^2 for a 3.5% enriched uranium dioxide fuel element in a thermal reactor if the effective fission cross-section is 350 barns and the neutron flux per (cm^{2}s) is 10^{14}. The density of UO_2 is 10.5 g/cm^3.

9.7 A PWR has inlet and exit water at 290 and 320 °C respectively. It has a 30 m^3 vapour pressurizer which is normally 60% full of water at a pressure of 140 bar. A case of an insurge occurred during which 0.25 m^3 of water entered the pressurizer from the primary circuit hot leg, 0.05 m^3 entered through the spray, and 50 kWh was added by electric heaters. Determine the internal energy of the contents of the pressurizer before and after the event. Neglect heat losses to the surroundings.

9.8 A BWR operating at a pressure of 70 bar produces 1200 kg/s of saturated steam from feedwater at 200 °C. The average core exit quality is 10%. Calculate (a) the recirculation ratio, (b) the core inlet enthalpy and temperature. (c) the degree of subcooling, and (d) the heat generated in the reactor.

9.9 A fast breeder reactor generates 3000 MW of heat. The fuel is composed of 20% $Pu^{239}O_2$, 80% $U^{238}O_2$ by mass. The average neutron flux is 10^{16}. Estimate the total mass of the fuel material in the core. Ignore fast fission in U-238 and take neutron losses by leakage and parasitic absorption as 0.25 per neutron absorbed.

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10 Hydroelectric Power Plant

10.1 INTRODUCTION

In hydroelectric power plants the energy of water is utilized to drive the turbine which, in turn, runs the generator to produce electricity. Rain falling upon the earth's surface has potential energy relative to the oceans towards which it flows. This energy is converted to shaft work where the water falls through an appreciable vertical distance. The hydraulic power is thus a naturally available renewable energy source given by Eq. (10.1).

$$P = g\rho QH \tag{10.1}$$

Here P is the hydraulic power in Watts, g is 9.81 m/s^2 (the acceleration due to gravity), ρ is the water density, 1000 kg/m^3, Q is the flow or discharge, m^3/s and H is the height of fall of water or head, m. The electrical energy produced in kWh can then be written in the form of Eq. (10.2).

$$\begin{aligned} W &= 9.81 \times 1000 \times Q \times H \times \eta \times t \\ &= 9.81\ QH\ \eta t \text{ kWh} \end{aligned} \tag{10.2}$$

where t is the operating time in hours (8760 h/year) and η is the efficiency of the turbine-generator assembly, which varies between 0.5 and 0.9. The power developed thus depends on quantity (Q) and head (H) of water.

Hydro or water power is important only next to thermal power. Nearly 20 per cent of the total power of the world is met by hydropower stations. There are some countries like Norway and Switzerland where the hydropower forms almost the total installed capacity.

Hydroelectric power was initiated in India in 1897 with a run-of-river unit near Darjeeling. However, the first major plant was the Sivasamudram Scheme in Mysore of 4.5 MW capacity commissioned in 1902. Khopoli project of 50 MW in Maharashtra was put into operation in 1914 to supply power to Mumbai city. Since independence a substantial growth in hydropower has occurred with the commissioning of large multipurpose projects like

Damodar Valley Corporation (DVC), Bhakra Nangal, Hirakud, Nagarjunsagar, Mettur, Koyna, Rihand and so on.

10.2 ADVANTAGES AND DISADVANTAGES OF WATER POWER

These have been stated point by point as below.

10.2.1 Advantages of Water Power

Hydropower have some inherent advantages which make it very attractive.

1. Water source is perennially available. No fuel is required to be burnt to generate electricity. It is aptly termed as 'the white coal'. Water passes through turbines to produce work and downstream its utility remains undiminished for irrigation of farms and quenching the thirst of people in the vicinity.
2. The running costs of hydropower installations are very low as compared to thermal or nuclear power stations. In thermal stations, besides the cost of fuel, one has to take into account the transportation cost of the fuel also.
3. There is no problem with regards to the disposal of ash as in a thermal station. The problem of emission of polluting gases and particulates to the atmosphere also does not exist. Hydropower does not produce any greenhouse effect, cause the pernicious acid rain and emit obnoxious NO.
4. The hydraulic turbine can be switched on and off in a very short time. In a thermal or nuclear power plant the steam turbine is put on turning gear for about two days during start-up and shut-down.
5. The hydraulic power plant is relatively simple in concept and self-contained in operation. Its system reliability is much greater than that of other power plants.
6. Modern hydropower equipment has a greater life expectancy and can easily last 50 years or more. This can be compared with the effective life of about 30 years of a thermal or nuclear station.
7. Due to its great ease of taking up and throwing off the load, the hydro-power can be used as the ideal spinning reserve in a system mix of thermal, hydro and nuclear power stations.
8. Modern hydro-generators give high efficiency over a considerable range of load. This helps in improving the system efficiency.
9. Hydro-plants provide ancillary benefits like irrigation, flood control, afforestation, navigation and aqua-culture.
10. Being simple in design and operation, the hydro-plants do not require highly skilled workers. Manpower requirement is also low.

10.2.2 Disadvantages of Water Power

Major disadvantages of water power are the following:

1. Hydro-power projects are capital-intensive with a low rate of return. The annual interest of this capital cost is a large part of the annual cost of hydro-power installations.
2. The gestation period of hydro projects is quite large. The gap between the foundation and completion of a project may extend from ten to fifteen years.
3. Power generation is dependent on the quantity of water available, which may vary from season to season and year to year. If the rainfall is in time and adequate, then only the satisfactory operation of the plant can be expected.
4. Such plants are often far way from the load centre and require long transmission lines to deliver power. Thus the cost of transmission lines and losses in them are more.
5. Large hydro-plants disturb the ecology of the area, by way of deforestation, destroying vegetation and uprooting people. Strong public opinion against erection of such plants is a deterrent factor. The emphasis is now more on small, mini and micro hydel stations.

10.3 OPTIMIZATION OF HYDRO-THERMAL MIX

A hydroelectric power plant was earlier used as an exclusive source of power. However, it suffers seasonal variation of output proportional to the variation of water flow. To meet the variable load demand, large amount of water requires to be stored. At the times of low water flow rates the hydro plants cannot meet the maximum load. Again, if the maximum capacity of the station is based on the minimum water flow, this will prove uneconomical. There will be a great wastage of water over the dam for greater part of the year. Hence, the present trend is to use hydroelectric power in conjunction with thermal power in an interconnected system. This hydro-thermal mix is optimized to achieve minimum cost of power generation, which may be 30 per cent hydro-70 per cent thermal or 35 per cent hydro-65 per cent thermal. Load sharing by hydro is maximum when the available flow of water is maximum, say during the monsoon months. As long as there is planety of water stored in the reservoir the hydro part of the system carries the base load, with thermal plants taking the peaks. When water availability is low, say during the dry months of winter and spring, the steam plants take the base load and hydro plants meet the peak load (Fig. 10.1). By interconnecting hydropower with steam, a great deal of saving in cost can be effected by way of the following.

(i) Reduction in necessary reserve capacity.
(ii) Diversity of construction programmes.
(iii) Higher utilization factors of hydro-plants.
(iv) Higher capacity factors of thermal plants.

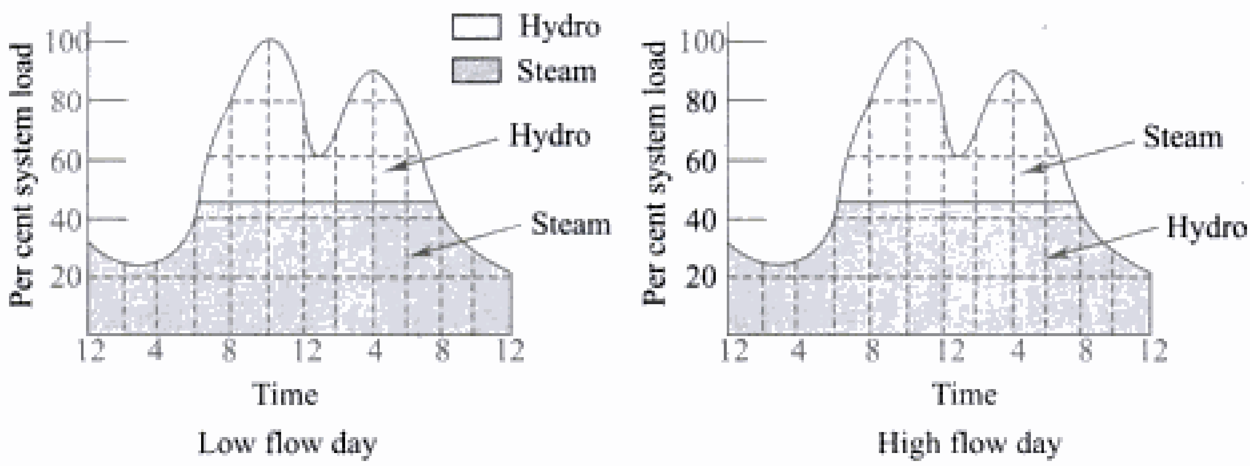


Fig. 10.1 *Typical division of load on a hydro-steam system*

10.4 SELECTION OF SITE FOR A HYDROELECTRIC PLANT

The following factors should be considered while selecting the site for hydroelectric power plant.

1. Availability of water
2. Water storage capacity
3. Available water head
4. Accessibility of the site
5. Distance from the load centre
6. Type of land of site

1. *Availability of water* The design and capacity of the hydro-plant greatly depends on the amount of water available at the site. The run-off data along with precipitation at the proposed site with maximum and minimum quantity of water available in a year should be made available to

(a) decide the capacity of the plant,
(b) set up the peak load plant such as steam, diesel or gas turbine plant,
(c) provide adequate spillways or gate relief during flood period.

2. *Water storage capacity* Since there is a wide variation in rainfall all round the year, it is always necessary to store the water for continuous generation of power. The storage capacity can be estimated with the help of mass curve.

3. *Available water head* In order to generate the desired quantity of power it is necessary that a large quantity of water at a sufficient head should be available. An increase in effective head, for a given output, reduces the quantity of water required to be supplied to the turbines.

4. *Accessibility of the site* The site should be easily accessible by rail and road. An inaccessible terrain will jeopardize the movement of men and material.

5. *Distance from the load centre* If the site is close to the load centre, the cost of transmission lines and the transmission losses will be reduced.

6. *Type of the land of the site* The land of the site should be cheap and rocky. The dam constructed at the site should have large catchment area to store water at high head. The foundation rocks of the masonry dam should be strong enough to withstand the stresses in the structure and the thrust of water when the reservoir is full.

10.5 HYDROLOGICAL CYCLE

Hydrology is the science that deals with the processes governing depletion and replenishment of water resources over and within the earth's surface. With the knowledge of hydrology at a certain site it is possible to design the irrigation and flood control works, power projects, water supply schemes, navigation works, etc.

As water vapour in atmospheric air goes up it cools, condenses and falls as rain, hail, snow or sleet. When this precipitation falls on hills and mountains and converges to form streams and rivers, it can be used for power generation. Intensity of rainfall, season and topography largely determine the usefulness of rainfall for power purposes. Light falls aid the growth of vegetation but do not contribute to stream flow. When total monthly precipitation concentrates in one or more storms, the *runoff* will increase greatly though vegetation may suffer. Distribution of precipitation may be classified as (i) direct evaporation (ii) absorption and transpiration by vegetation. (iii) seepage and storage; and (iv) direct surface runoff, eventually forming rivers (Fig. 10.2).

(i) A major part of precipitation on land areas that reaches the soil re-evaporates to the atmosphere, the rate being large from surfaces of lakes, ponds and swamps. A rise in temperature and drop in humidity increase the evaporation rate with the wind aiding it.
(ii) Plants absorb water through their roots and *transpire* it as vapour through their leaves to the atmosphere.
(iii) Precipitation absorbed by the soil *seeps* or percolates into the ground, forming bodies of water called the *water table* or *ground storage*. It is also called "infiltration" which is a process by which water enters the surface strata of the soil and makes its way downwards to the water table. The amount of seepage or infiltration depends on the geological character of the surface and subsoil.
(iv) The remaining water flows over the ground surface as direct runoff to form brooks and rivers (Fig. 10.2). The amount of runoff from a given rainfall depends on the nature of precipitation. Short, hard showers may produce relatively little runoff, whereas long rainfall saturates the soil lowering seepage rate and slows down evaporation by increased humidity and thus produces more runoff.

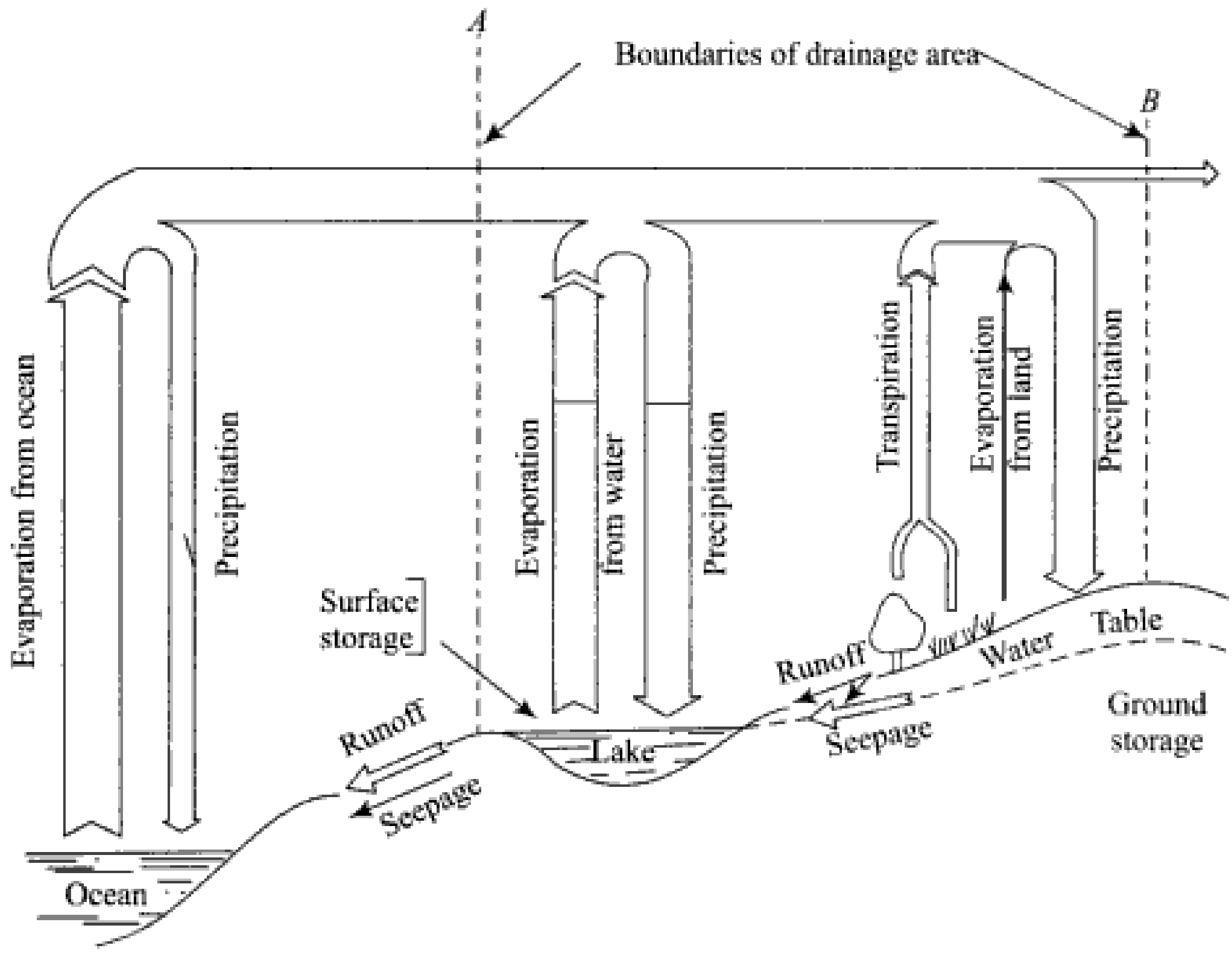


Fig. 10.2 *Graphic portrayal of the water cycle*

The water equation summarizing the disposal of the rainfall over a certain area during a given period is given by

Runoff + Seepage + Evaporation + Transpiration = Precipitation ± Change in storage

The best way to study the rainfall pattern is with the help of graphical plots. The *hyetographs* are the rainfall intensity-time curves which indicate the variation of the rate of rainfall with respect to time. The cumulative value of rainfall plotted against time represents the mass curve of rainfall.

10.6 HYDROGRAPHS

The variation of stream flow at a given site depends on the geographical, geological and topographical features of the drainage area feeding the river as well as the magnitude of the area rainfall. *Hydrographs* show the variation of river flow (discharge) with time. Runoff may be plotted as *flow duration curves* (Fig. 10.3 a), which show the time when a stream flow rate is equalled or exceeded in any period (daily, weekly or monthly basis). The area under the flow duration curve represents the average yield from the stream. By changing the ordinate to power (kW) instead of discharge (m^3/s) in Fig. 10.3 (a), the *power duration curve* is obtained and the area under the curve would then represent the average yield of power from the hydro-power project. It can be noted in Fig. 10.3. (b) that Q_m is the minimum flow rate that would be available

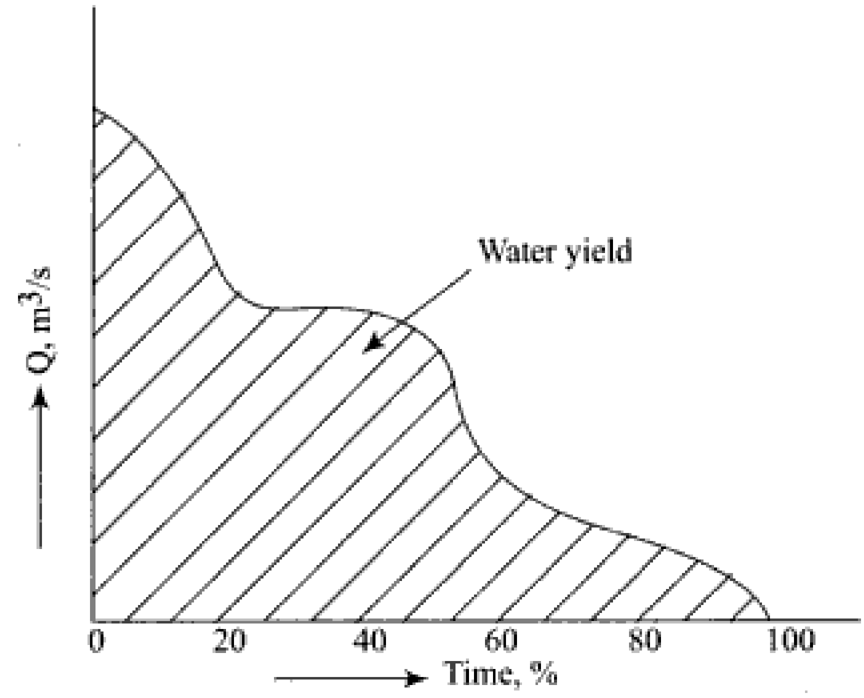


Fig. 10.3 (a) *Flow duration curve with % time on x-axis and Run-off on y-axis*

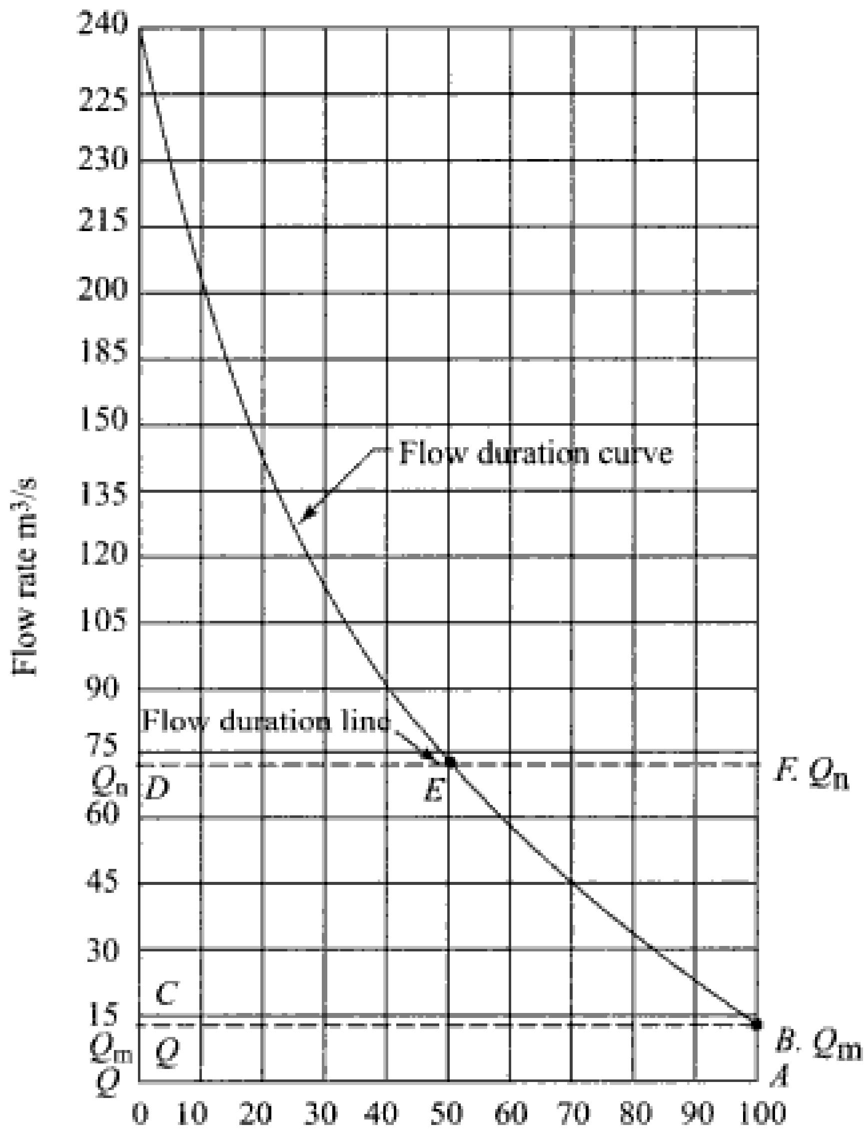


Fig. 10.3 (b) *Flow duration curve of a typical river having a low flow*

for all the times (i.e. for 100 per cent of time) and the area *OABC* would represent the firm yield of water or power, often termed as *primary power*. The additional output available at higher water flows is called *secondary power*. If a flow rate of Q_n is required for all the times as indicated by the area under the flow demand line *DEF*, then it would be possible to meet this uniform demand of flow rate (or power) for all the times only if storage equal to area *BEF* is provided. An alternative to this is to install a thermal power unit of *BF* capacity to work as a supplement to the hydro-power unit. The curve also shows that natural flow sufficient to meet the flow demand Q_n is available for 53.5 per cent of time or 195 days in the year of the lowest flow of the record. In the absence of any storage, area *BCDE* represents the secondary power that would be available from the river.

In order to facilitate the storage computation, mass curves are commonly used. A *mass curve* is a plot of accumulated flow (in hectare-metre) against time, made from the records of mean monthly flows of a stream (Fig. 10.4). The slope of the curve at any point indicates the rate of flow at that particular time. If the curve is horizontal, the flow is zero and if there is a high rate of flow the curve rises steeply. Relatively dry periods are indicated as concave depressions on the mass curve.

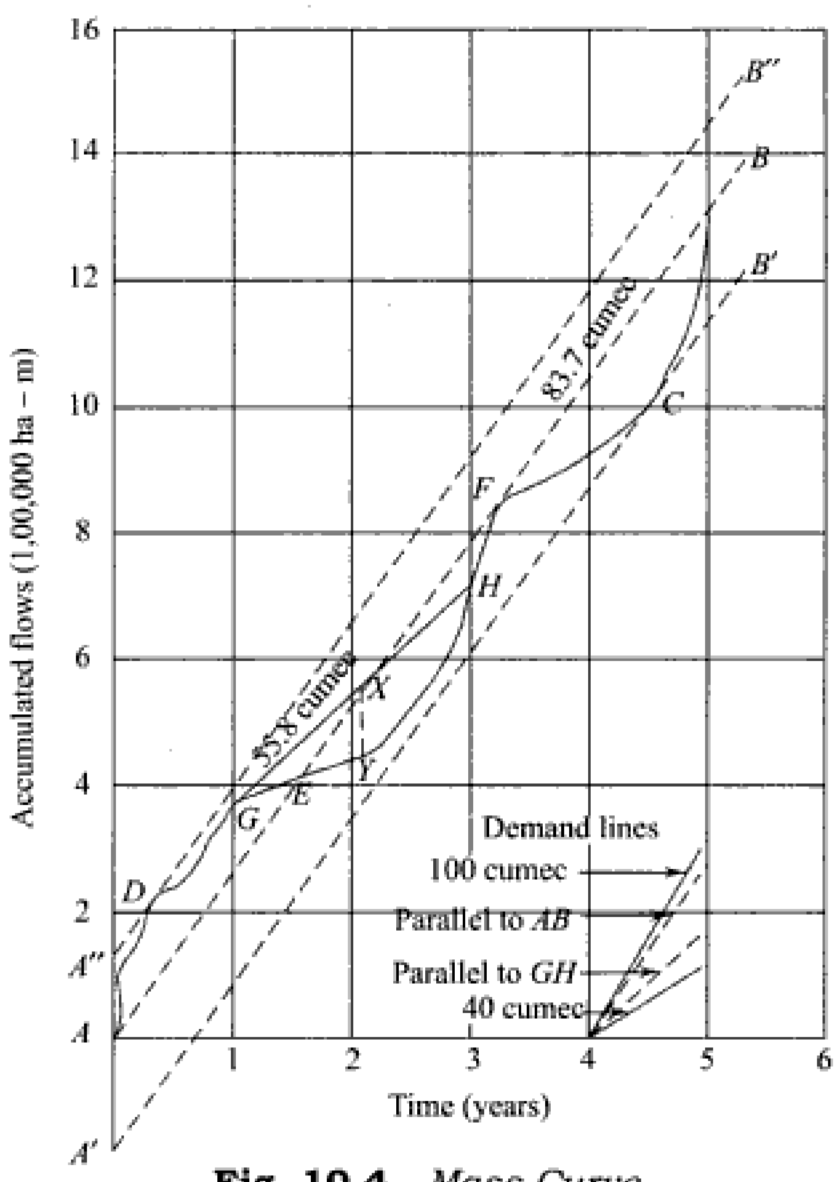


Fig. 10.4 *Mass Curve*

10.7 STORAGE AND PONDAGE

As stated earlier the flow rate of a stream varies considerably with time. For example, during rainy season when the stream is in floods it carries a huge quantity of water as compared to other times of the year when the quantity of water carried by it is considerably less. However, the demands for power ordinarily do not correspond to such variations of the natural flow of the stream. As such some arrangement in the form of storage and pondage of water is required for the regulation of the flow of water so as it make it available in requisite quantity to meet the power demand at a given time.

Storage may be defined as impounding of a considerable amount of excess run off during seasons of surplus flow for use in dry seasons. This is accomplished by constructing a dam across the stream at a suitable site and building a storage reservoir on the upstream side of the dam.

Pondage may be defined as a regulating body of water in the form of a relatively small pond or reservoir provided at the plant. The pondage is used to

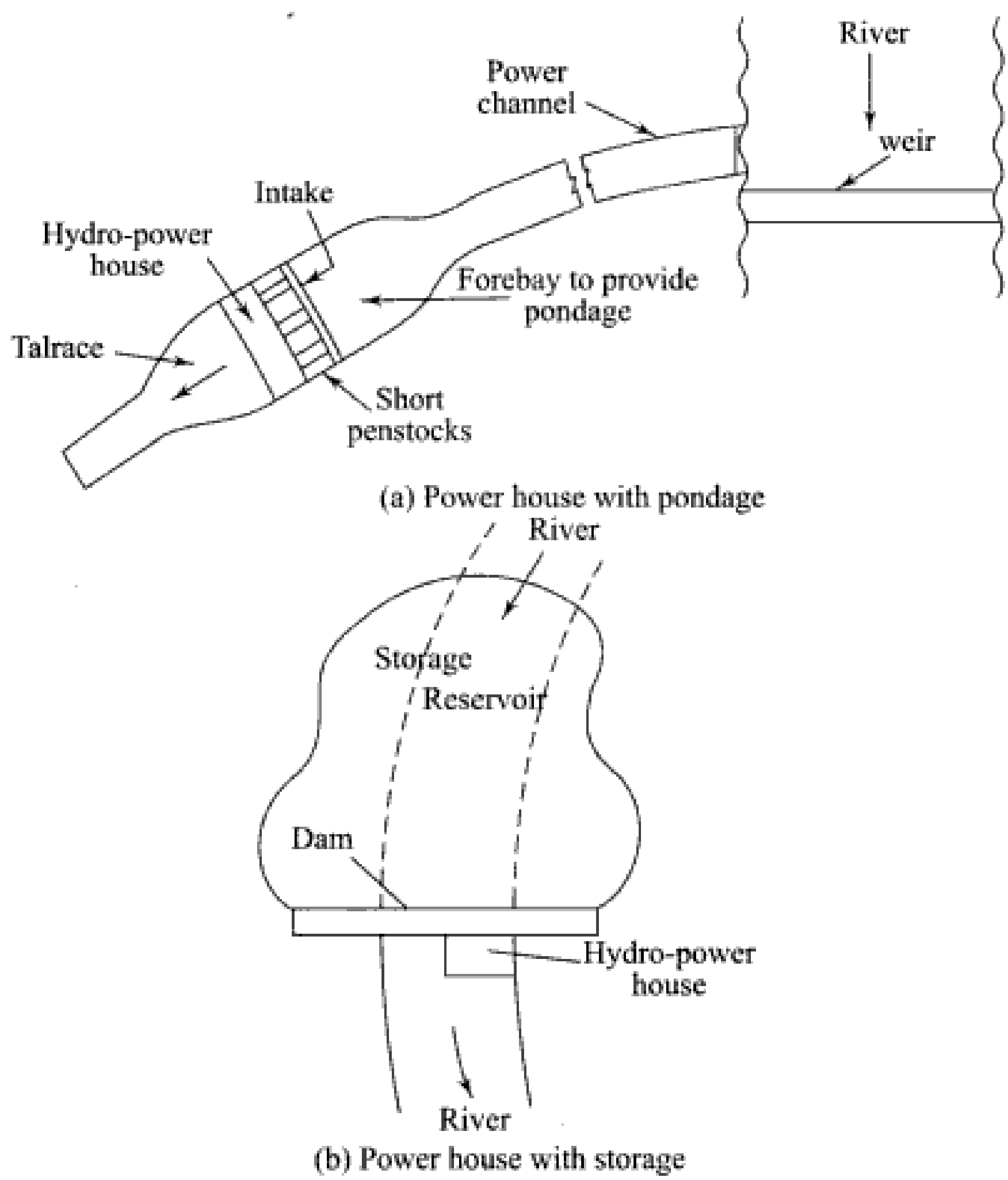


Fig. 10.5 *Hydro-power units with pondage and storage*

regulate the variable water flow to meet power demand. It caters for short-term fluctuations which may occur due to (a) sudden increase or decrease of load on the turbine (b) sudden changes in the inflow of water, say by breaches in the conveyance channel (c) change of water demand by turbines and the natural flow (supply) of water from time to time. The turbines are often required to meet the power demand higher than the average load when the pondage supplies the excess quantity of water required during that period. Figure 10.5 shows the locations of power houses with storage and pondage. Pondage increases the capacity of a river over a short-time, such as a week. Storage, however, increases the capacity of a river over an extended period of 6 months to as much as 2 years.

10.8 ESSENTIAL ELEMENTS OF A HYDROELECTRIC POWER PLANT

Figure 10.6 gives the flow diagram of a typical hydroelectric power plant. The essential elements of such a plant are the following.

1. Catchment area
2. Reservoir
3. Dam
4. Spillways
5. Conduits
6. Surge tanks
7. Draft tubes
8. Powerhouse
9. Switch yard for transmission of power.

10.8.1 Catchment Area

The whole area behind the dam draining into a stream or river across which the dam has been constructed is called the catchment area. The characteristics of the catchment include its size, shape, surface, orientation, altitude, topography and geology. The bigger the catchment, steeper is the slope, higher is the altitude, and greater is the total runoff of water.

10.8.2 Reservoir

Storage during times of plenty for subsequent use in times of scarcity is fundamental to the efficient use of water resources. The management of

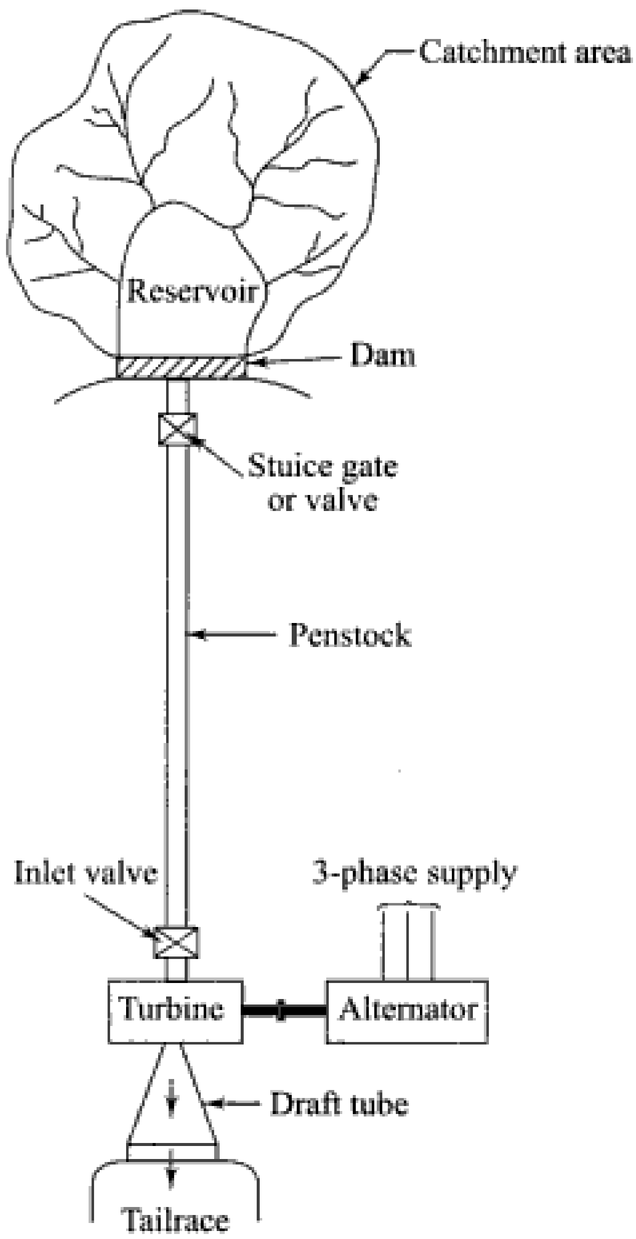


Fig. 10.6 *Flow sheet of a hydroelectric power plant*

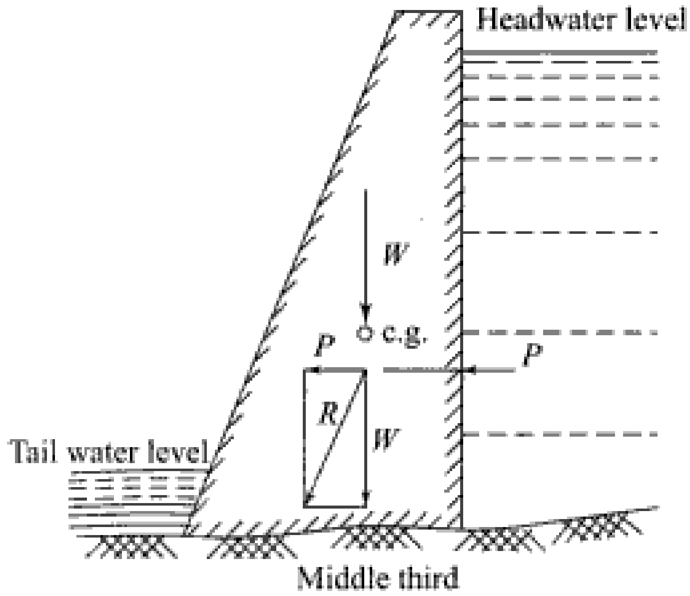


Fig. 10.8 *Cross-section of solid gravity type of masonry dam*

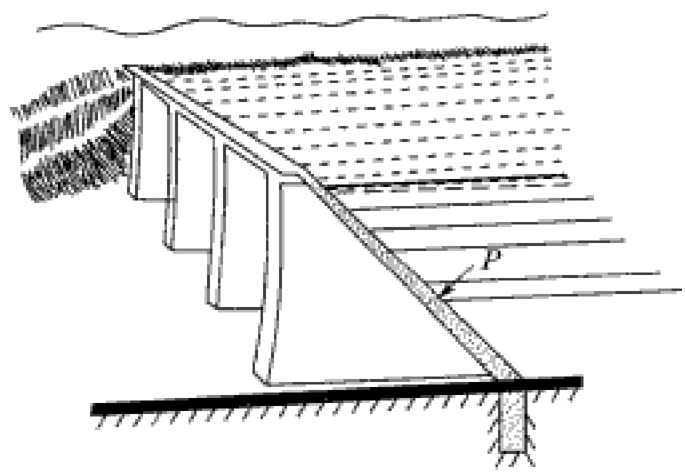


Fig. 10.9 *Buttress or hollow gravity type of masonry dam with flat deck*

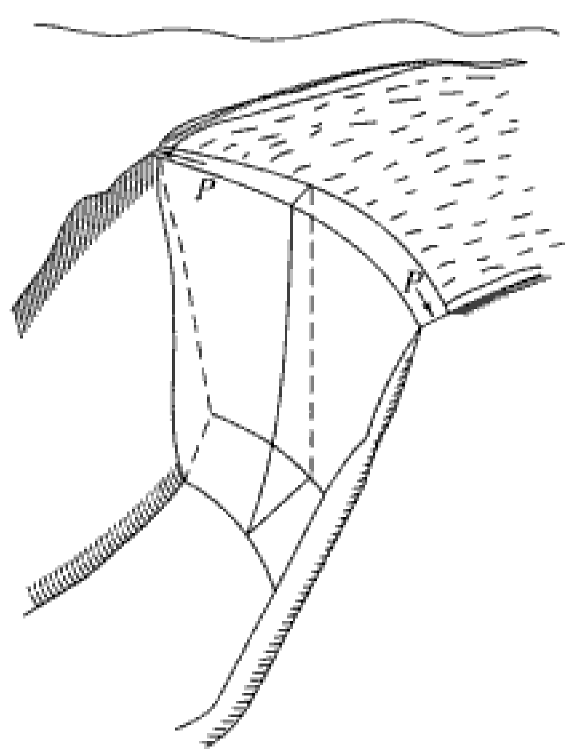


Fig. 10.10 *Arch type of masonry dam*

Earth dams For a small project of up to 70 m in height, dams constructed of earth fill or embankment are used. A large volume of material is required and it should be available in the vicinity. The dam construction varies with the height and the side slopes are flatter (Fig. 10.11). It is cheaper than masonry dam, but has more seepage losses. There may be serious damage from erosion by water overtopping the dam or seeping through it.

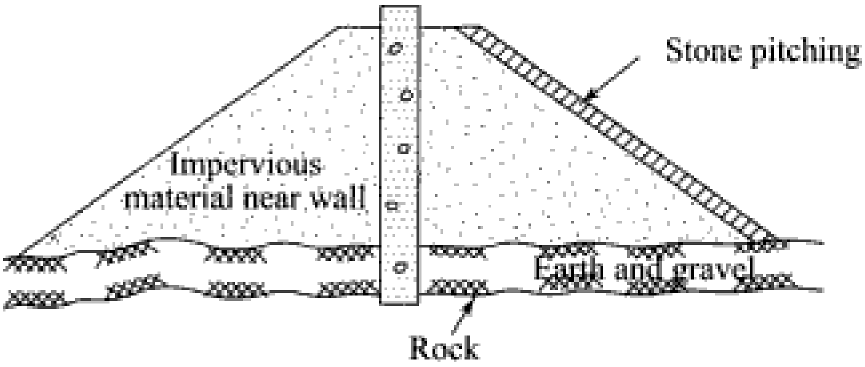


Fig. 10.11 *Earth dam*

Rock-fill dams It is made of loose rock of all sizes and has a trapezoidal shape with a wide base, having a watertight section to reduce seepage. It is used in mountainous region where rock is available.

10.8.4 Spillways

When the water level in the reservoir basin rises, the stability of the dam structure is endangered. To relieve the reservoir of this excess water, a structure is provided in the body of a dam or close to it. This safeguarding structure is called a *spillway*. It provides structural stability to the dam under conditions of floods without raising reservoir level above H.F.L. (high flood level). Following are the various types of spillways.

1. ***Overall spillway*** It is also called solid gravity spillway. It is provided in concrete and masonry dams (Fig. 10.12). Water spills and flows over the crest in the form of a rolling sheet of water. The bucket at the lower end changes the direction of the fast moving water, destroying its excess energy.

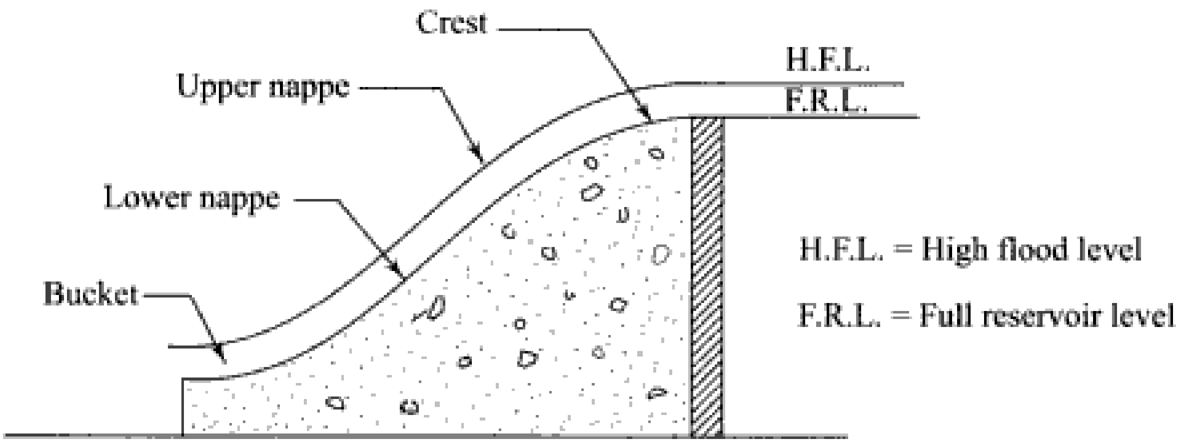


Fig. 10.12 *Overall spillway*

2. ***Chute or trough spillway*** It is suitable when the valley is too narrow to accommodate the solid gravity spillway in the body of the dam. After crossing over the crest the water shoots down a channel or trough to meet the river downstream of the dam.

3. ***Side channel spillway*** This is used when the valley is too narrow and in non-rigid dams where the flood water is not desired to flow over the dam. When there is no room to provide chute spillway, the side channel spillway (Fig. 10.13) is used. Here, after crossing the crest water flows parallel to it.

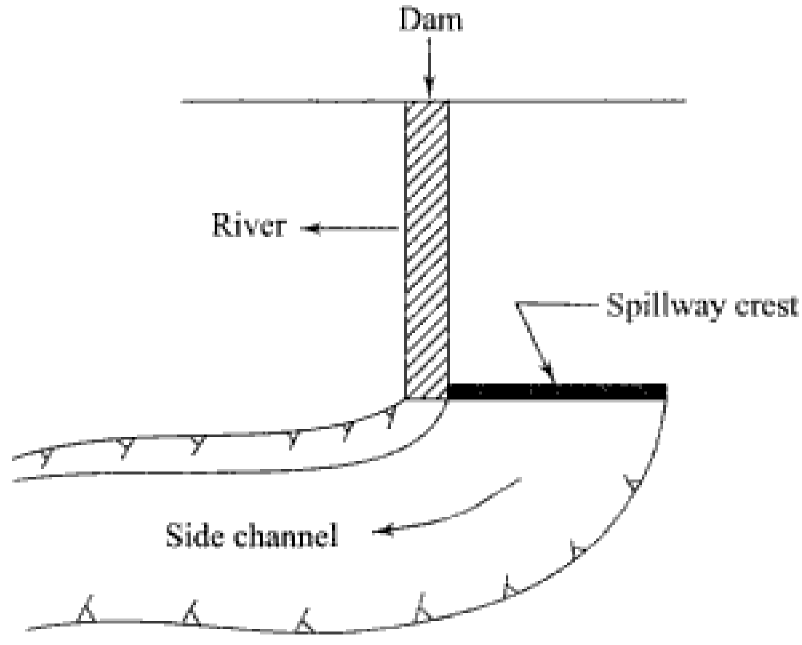


Fig. 10.13 *Side channel spillway*

4. ***Saddle spillway*** When conditions are not favourable for any of the above types of spillway, a saddle spillway is used. Some natural depression or saddle on the periphery of the reservoir basin away from the dam is used as the spillway, with the bottom of the depression being at the full reservoir level (Fig. 10.14).

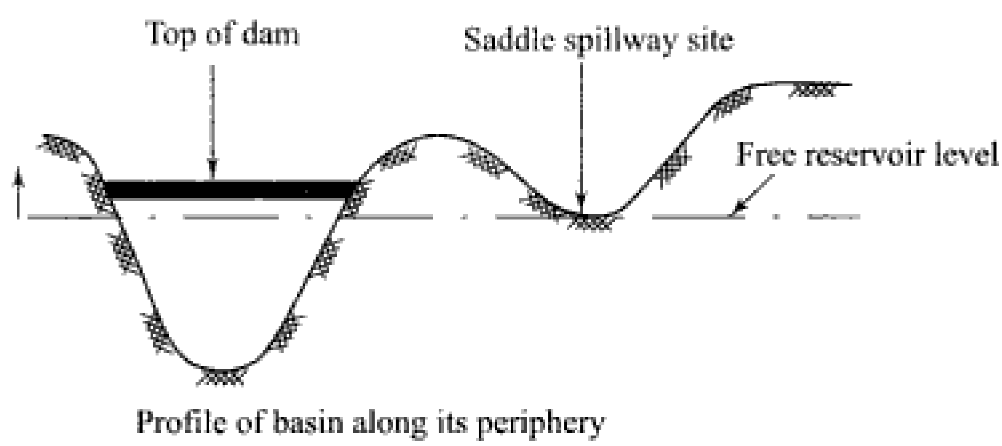


Fig. 10.14 *Saddle spillway*

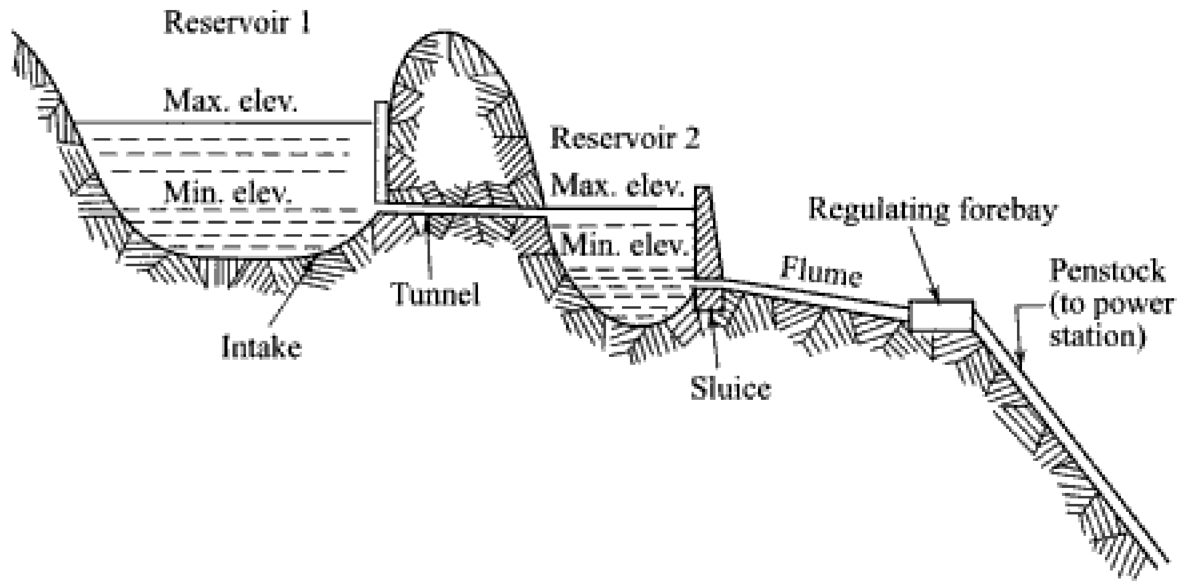


Fig. 10.16 *Combination of tunnel, flume and penstocks at a high head power plant*

10.8.6 Surge Tanks

A surge tank is a small reservoir in which the water level rises or falls to reduce the pressure swings so that they are not transmitted to the closed conduit. If the power house is located within a short distance of the headworks, surge tanks are not necessary. Thus for run off plants and medium head schemes no surge tank is needed. Surge tanks are required for high head plants where water is taken to the power house through tunnels and penstocks. A typical arrangement is shown in Fig. 10.17, where the surge tank is a vertical standpipe connected to the penstock

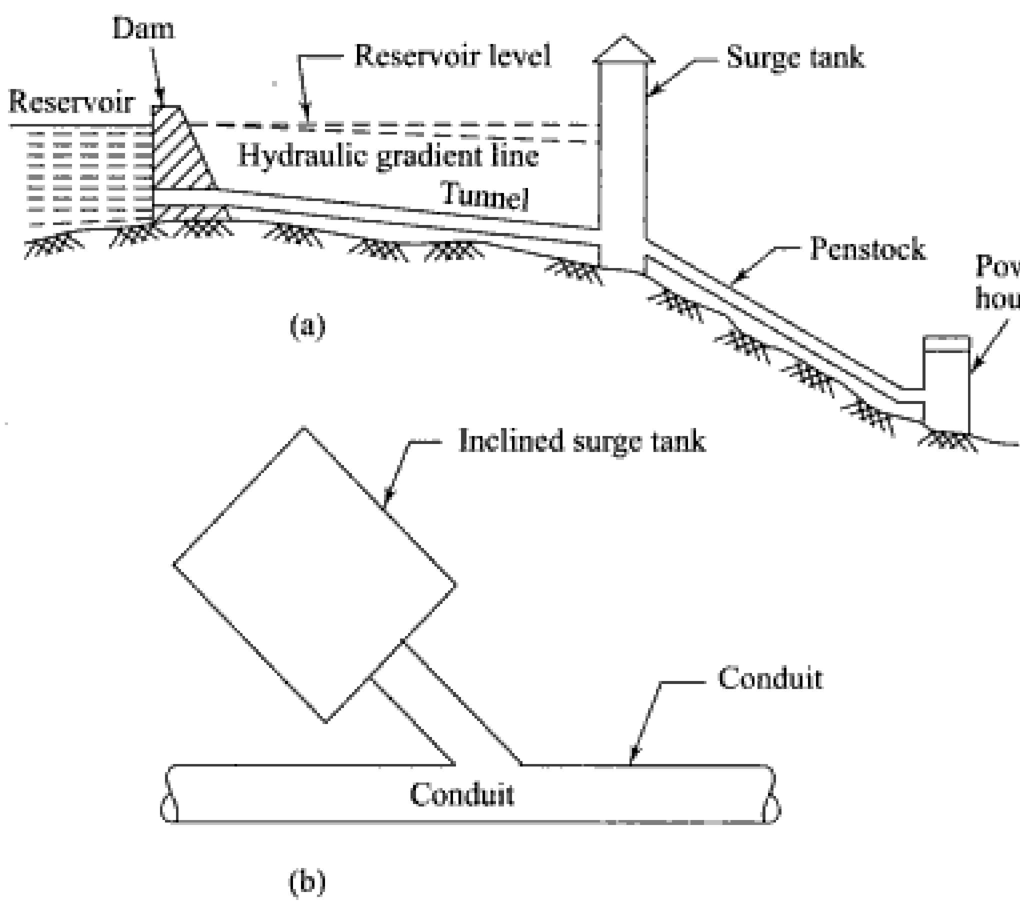


Fig. 10.17 *(a) Surge tank on ground level, (b) Inclined surge tank*

with no overflow of water. A further discussion on surge tanks has been made in Section 10.23.

10.8.7 Draft Tubes

The draft tube allows the turbine to be set above the tailrace to facilitate inspection and maintenance and by diffuser action regains the major portion of the kinetic energy or velocity head at runner outlet, which would otherwise go waste as an exit loss. The draft tube can be a straight conical tube (Fig. 10.18 a) or an elbow tube (Fig. 10.18 b). The conical type is used for low power units, while the elbow type is more common. In the elbow type energy is regained in the vertical portion which flattens in the elbow section to discharge water horizontally to the tailrace.

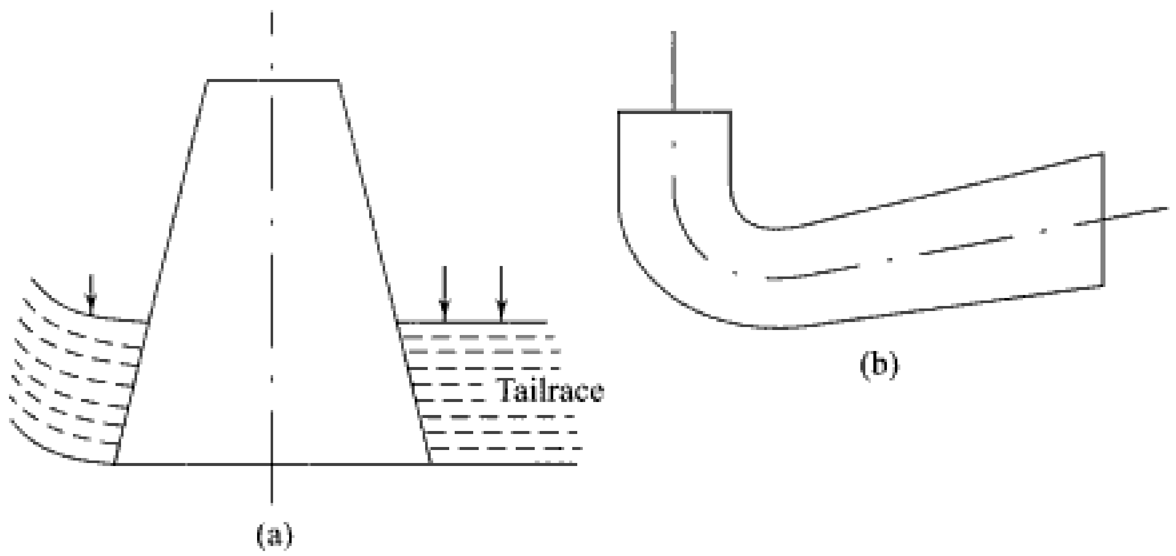


Fig. 10.18 *(a) Straight conical draft tube (b) Elbow type draft tube*

10.8.8 Powerhouse

A powerhouse should have a stable structure and its layout should be such that adequate space is provided around the equipment for convenient dismantling and repair. The equipment provided in the powerhouse includes the following.

(i) Hydraulic turbines
(ii) Electric generators
(iii) Governors
(iv) Gate valves
(v) Relief valves
(vi) Water circulation pumps
(vii) Air duct
(viii) Switch board and instruments
(ix) Storage batteries
(x) Cranes

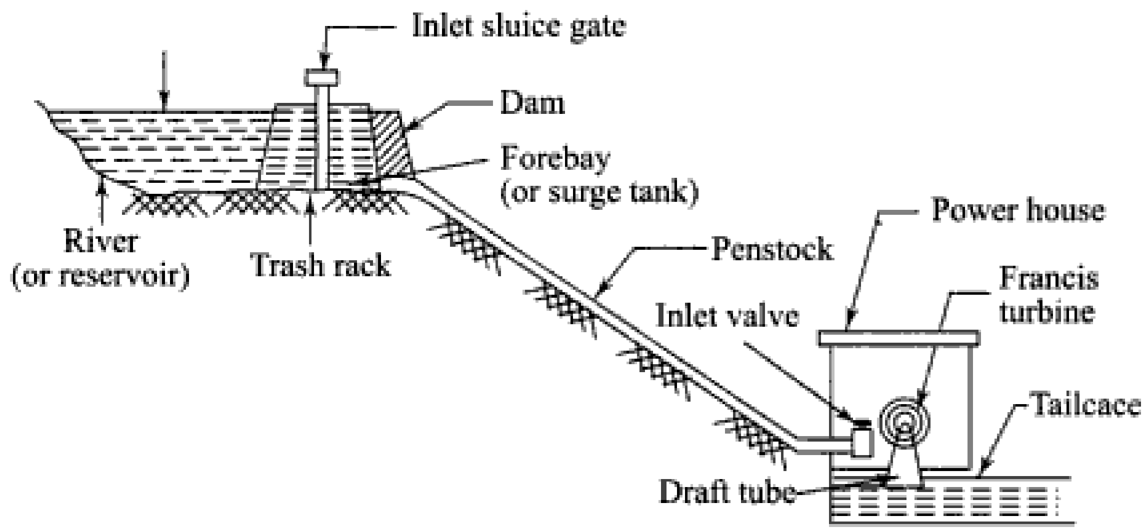


Fig. 10.20 *Medium head power plant*

10.9.3 Low Head Power Plants

A dam is constructed across a river and a sideway stream diverges from the river at the dam. Later this channel joins the river further downstream (Fig. 10.21). Francis turbine or Kaplan turbine is used for power generation.

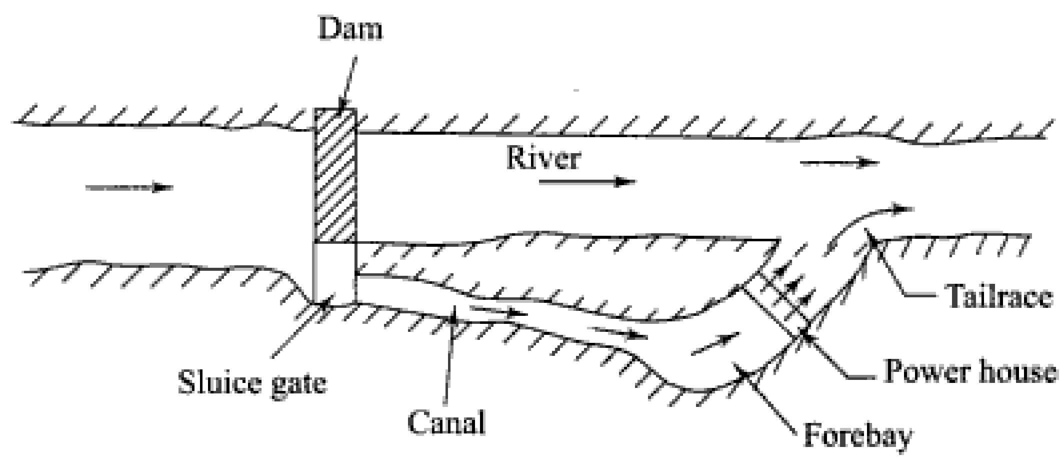


Fig. 10.21 *Low head power plant*

10.9.4 Base Load Plants

These plants are required to supply constant power to the grid. They run continuously without any interruption and are mostly remote controlled.

10.9.5 Peak Load Plants

They only work during certain hours of a day when the load is more than the average. Thermal stations work with hydel plants in tandem to meet the base load and peak load during various seasons.

10.9.6 Run-Of-River Plants with or Without Pondage

Such a plant works daily according to the nature and limit to the flow in the river. Power generated depends on the quantity of flow. Sometimes, a small storage reservoir or pond is built, which can store a few hours' supply of water to the plant, when the river flow exceeds the amount required by the plant. Such a scheme is called a run-of-river plant with pondage. The pondage or stored water is used in generating power during the hours when the demand is in excess of the flow of the river at the moment.

10.9.7 Hydroelectric Plants with Storage Reservoir

These plants are most common in India. During the rainy season water is stored in reservoirs so that it can be utilized during other seasons to supplement the flow of the river whenever the flow in the river falls below a specified minimum. Power can be generated directly from the reservoir. Sometimes canals are constructed to convey water from the reservoir for irrigation purposes.

10.9.8 Pumped Storage Plants

Water after working in turbines is stored in the tailrace reservoir. During *low load*, say night time, the water is pumped back from the tail to the head reservoir drawing excess electricity from the grid or from the nearby steam plant. During *peak load*, this water is used to work on turbines to produce electricity (Fig. 10.22). It is always economical to run the steam power plants all the time at full plant capacity factor. Whenever the load demand is less than the full plant capacity, the surplus energy instead of being wasted is transmitted to a pump is installed at the tailrace of the hydroelectric plant. The advantages of such a plant can be summarized as follows:

(a) Substantial increase in peak load capacity at low cost (b) High operating efficiency (c) Better load factor (d) Independence of stream flow conditions.

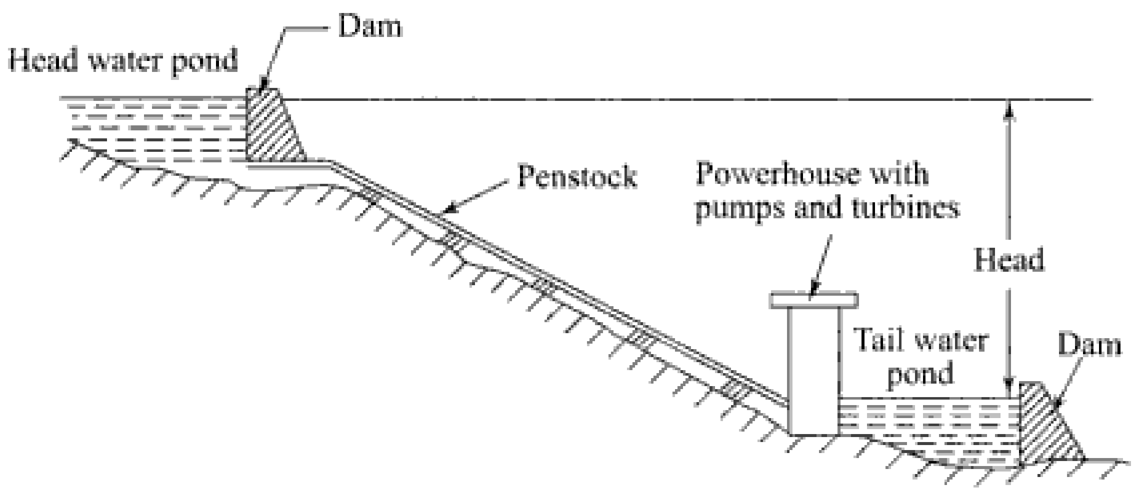


Fig. 10.22 *Pumped storage plant*

About 70 per cent of power used in pumping is recovered. A motor-generator set and a reversible turbine-pump unit can be profitably used as the same machine can be operated as a motor or generator and similarly as a pump or a turbine (Deriaz turbine).

10.9.9 Mini and Micro-Hydel Plants

More emphasis is now being given on such plants. The natural water source in hilly terrain can be utilized for power generation with low-head standardized turbo-generator units. Its adverse effect on ecology is negligible. The mini-plants operate with 5 m–20 m head producing about 1 MW to 5 MW of power, while micro-plants are still smaller and work under a head of less than 5 m and generate electricity between 0.1 MW to 1 MW. The potential energy source in India in this category is around 20,000 MW.

10.10 HYDRAULIC TURBINES

Hydraulic turbines convert the potential energy of water into shaft work, which, in turn, rotates the electric generator coupled to it in producing electric power. Historically, hydraulic turbines of today are derived from the waterwheels of the middle ages used for flour mills (to grind wheat) and ore-crushing. One such waterwheel (pan-chakki) can still be seen at Aurangabad, which is, at least, four hundred years old. Modern turbines have undergone many technological advances in diverse areas like fluid mechanics, metallurgy and mechanical engineering.

10.10.1 Classification of Hydraulic Turbines

The hydraulic turbines can be classified according to the (a) head and quantity of water available (b) name of the originator (c) nature of working on the blades (d) direction of flow of water (e) axis of the turbine shaft (f) specific speed.

1. ***According to the head and quantity of water available*** The difference in elevation of water surface between upstream and downstream of the turbine is the head under which the turbine acts (Fig. 10.23). The turbines work under a wide range of heads varying from 2 to 2000 m. A classification of turbine based on head as follows.

Low head	2–15 m
Medium head	16–70 m
High head	71–500 m
Very high head	Above 500 m

For low heads, only Kaplan or propeller turbines are used. For medium heads either Kaplan or Francis turbines are used. For high heads either Francis or Pelton turbines are used. For very high heads, invariably Pelton turbines are used. Deriaz turbines are used up to a head of 300 m. Their use is, however, restricted under reversible flow conditions (i.e. pumped-storage plants where the turbine also works as a pump).

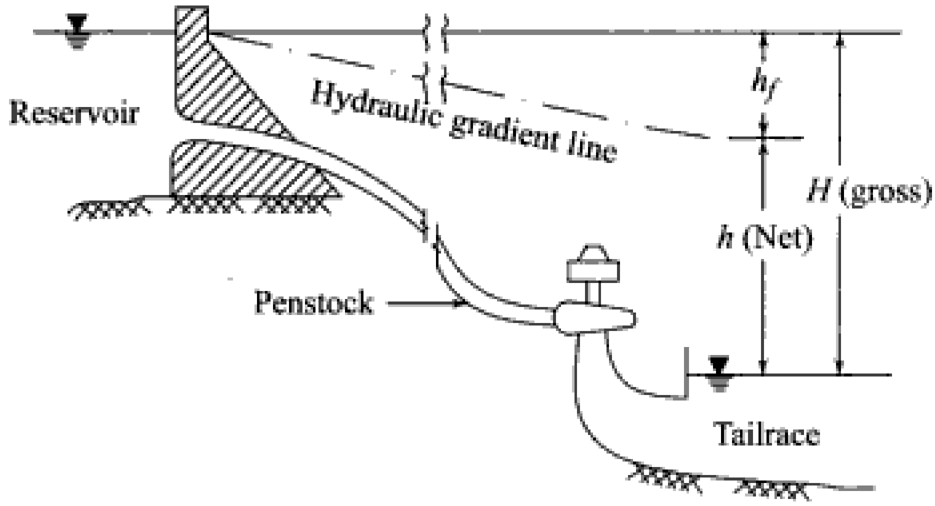


Fig. 10.23 *Head on the turbine*

Turbines can also be classified as low discharge, medium discharge and high discharge turbines, depending on the flow available. Pelton turbines are relatively low discharge turbines. Kaplan turbines are high discharge turbines, while Francis turbines occupy an intermediate position in this regard.

2. ***According to the name of the originator*** (i) Pelton turbine-named after Lester Allen Pelton of the USA, an impulse turbine used for high head and low discharge.

(ii) Francis turbine—named after James B. Francis, a reaction turbine used for medium head and medium discharge.

(iii) Kaplan turbine—named after Dr. Victor Kaplan, a reaction turbine used for low head and large discharge.

(iv) Deriaz turbine—named after the Swiss engineer Deriaz, a reversible turbine-pump used up to a head of 300 m.

3. ***According to the nature of working on the blades*** Turbines are classified as impulse and reaction turbines depending on the mode of energy conversion of potential energy of water into shaft work. In an impulse turbine all the available head of water is converted into kinetic energy in a nozzle. The water shoots out of the nozzle in a free jet into a bucket which revolves round a shaft. During this action, the water is in contact with air all the time and the water discharged from bucket falls freely through the discharge passage into the tailwater. The free jet is at atmospheric pressure before and after striking the vanes. These are pressureless or impulse turbines. Pelton wheel belongs to this category.

In reaction turbines, the entire flow from the headwater to the tailwater takes place in a closed conduit system which is not open to the atmosphere at any point in its passage. At the entrance to the runner, only a part of P.E. is converted into K.E. and the remaining into pressure energy. The runner converts both K.E. and pressure energy into mechanical energy. Such turbines are called reaction or pressure

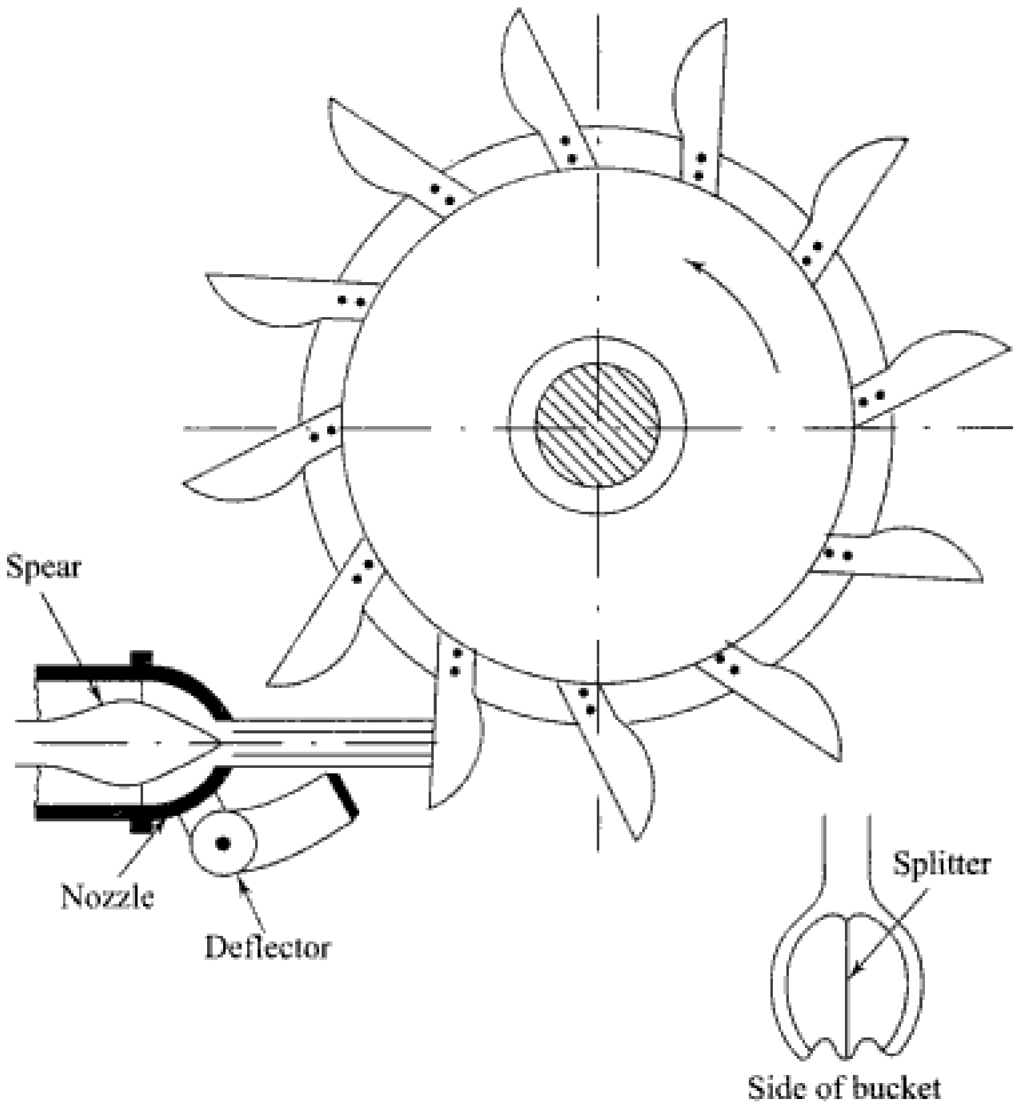


Fig. 10.24 *(a) Pelton wheel*

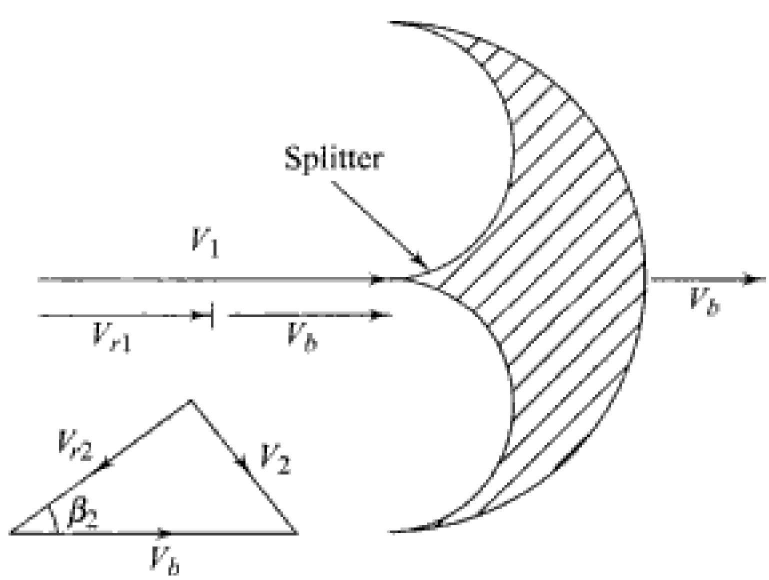


Fig. 10.24 *(b) Velocity diagrams and section through a bucket*

The nozzle directs the flow on the wheel. It also governs the quantity of flow with the help of a *spear valve* controlled by the governor action. In the simple arrangement there is a single nozzle feeding water to the turbine. However, for larger discharge there are turbines having up to six jets, all symmetrically arranged and causing rotation in the same direction. Figure 10.25 shows a multi-jet arrangement with four jets. Multi-jet machines usually have vertical shafts.

The specific speed of a multi-jet machine, i.e., N_{smj} is given by the following reaction.

$$N_{S_{MJ}} = \sqrt{n}\, N_{S_{SJ}} \tag{10.5}$$

where $N_{S_{SJ}}$ is the specific speed for a single-jet machine and n is the number of jets. Thus the specific speed of a given wheel can be increased by using multi-jet arrangement. The maximum number of jets used so far is six and the maximum speed for a single jet is of the order of 30. Thus the maximum specific speed for multi-jet machines is about 70.

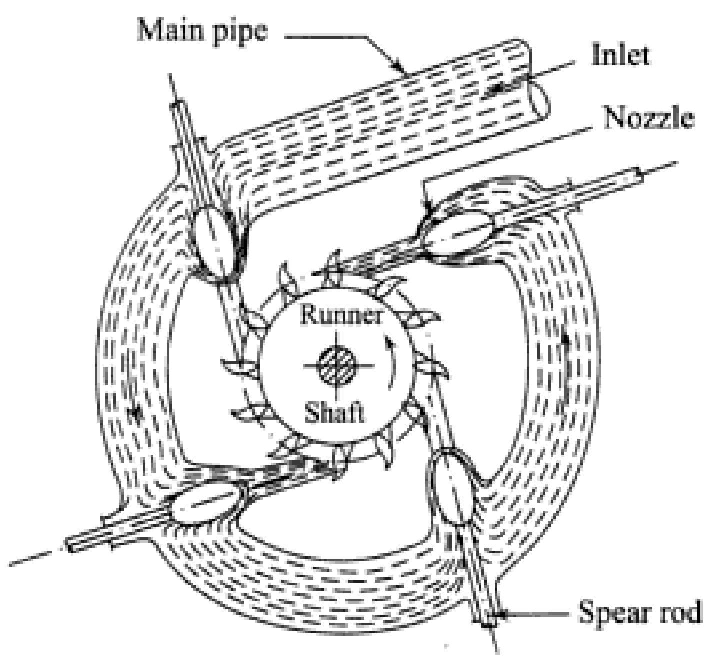


Fig. 10.25 *Multi-jet Pelton wheel arrangement*

It may be noted that the bucket deflection angle is of the order of 165° (Fig. 10.26). The slightly oblique direction of existing water allows it to escape freely without hitting the back of the next bucket. The water after leaving the bucket drops freely into the tailrace.

The jet moves in a tangential plane before and after striking the wheel and the bucket moves at a speed given by Eq. (10.6)

$$V = \omega r = \frac{\pi D N}{60} \tag{10.6}$$

where r and D are the bucket circle radius and diameter respectively and ω is the angular velocity given by $\frac{2\pi N}{60}$, N being the rpm.

With the nozzle diameter d, D/d is a size parameter for the turbine. This is known as *jet ratio*, *m*, having a value in the range of 10 to 24. The net head available at the nozzle is equal to the gross head less losses in the pipeline. If it is equal to H, the velocity of jet issuing from the nozzle is as follows.

$$V_1 = C_v\,[2gH]^{1/2} \tag{10.7}$$

where C_v is the coefficient of velocity (0.97 – 0.99).

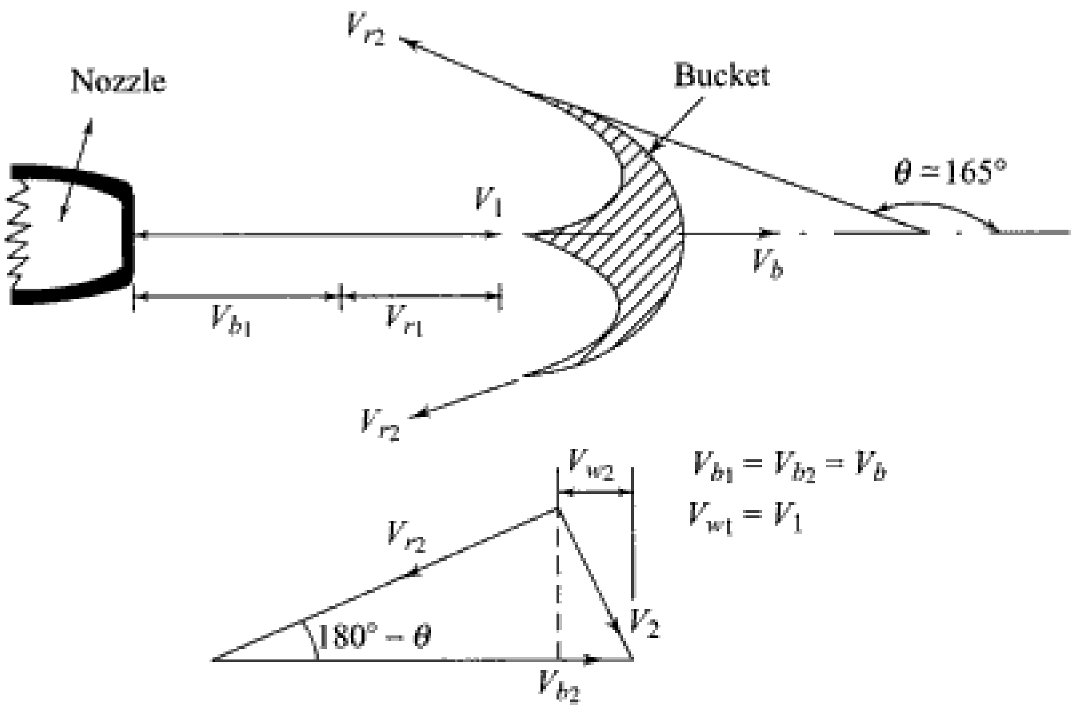


Fig. 10.26 *Bucket deflection angle and velocity diagrams*

Total energy transferred to the wheel (Fig. 10.26) is given by Euler equation (Eq. 10.8).

$$E = (V_{w_1} V_{b_1} - V_{w_2} V_{b_2})/g = \frac{V_b}{g}\,(V_{w_1} - V_{w_2}) \tag{10.8}$$

where subscript 1 represents the condition at inlet and subscript 2 the condition of water at outlet of the bucket, V_w is the velocity of whirl (tangential component) and V_b is the bucket velocity given by $V_{b1} = V_{b2} = \dfrac{\pi DN}{60}$

Now, from the exit velocity diagram (Fig. 10.26), we get

$$V_{w2} = V_b - V_{r2}\cos(180 - \theta) = V_b + V_{r2}\cos\theta \tag{10.9}$$

where θ is the bucket deflection angle (~ 165°).

Now, V_{r2} = relative velocity of water at exit

$$= kV_{r1} = k(V_1 - V_b) \tag{10.10}$$

where k is the blade friction coefficient, V_1 is the absolute velocity of water from the jet, and V_{r1} is the relative velocity of water at inlet.

$$V_{w2} = V_b + k(V_1 - V_b)\cos\theta \tag{10.11}$$

Substituting V_{w2} from Eq. (10.11) in Eq. (10.8) and since $V_{w1} = V_1$, we get

$$E = \frac{V_b}{g}\,[V_1 - V_b - k(V_1 - V_b)\cos\theta]$$

$$= \frac{V_b}{g}(V_1 - V_b)(1 - k\cos\theta)$$

$$= \frac{1 - k\cos\theta}{g}\left(V_1 V_b - V_b^2\right) \tag{10.12}$$

For given values of V_1, k and θ, there is a certain value of V_b for which E is maximum. Differentiating E with respect to V_b and putting it equal to zero, we get

$$\frac{dE}{dV_b} = \frac{1 - k\cos\theta}{g}(V_1 - 2V_b) = 0$$

$$V_b = \frac{V_1}{2} \tag{10.13}$$

Therefore, the optimum bucket velocity for maximum work output is half the jet velocity. On its substitution, we get

$$E_{max} = \frac{1 - k\cos\theta}{g}\frac{V_1^2}{4} \tag{10.14}$$

The kinetic energy of the input jet $= \dfrac{V_1^2}{2g}$.

Therefore, the blading or diagram or hydraulic efficiency of the wheel is given by

$$\eta_D = \frac{E}{V_1^2/2g} = \frac{1 - k\cos\theta}{g}\left(V_1 V_b - V_b^2\right)\frac{2g}{V_1^2}$$

$$= 2(1 - k\cos q)(\rho - \rho^2) \tag{10.15}$$

where ρ is the velocity ratio, $\dfrac{V_b}{V_1}$ (Fig. 10.27)

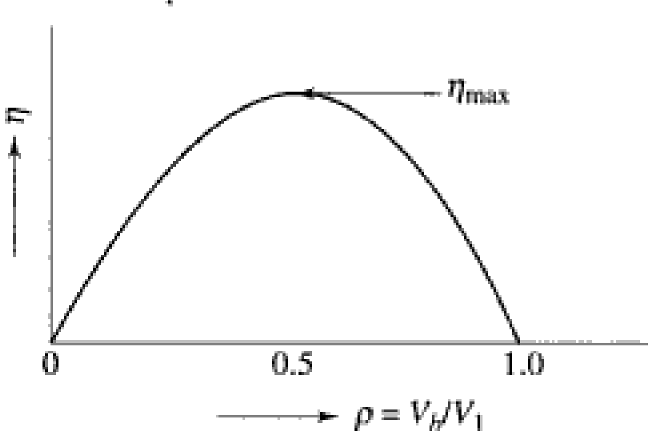


Fig. 10.27 *Variation of x with velocity ratio*

Making $\frac{d\eta_D}{d\rho} = 0_1$, we get

$$2(1 - k \cos q)(1 - 2\rho) = 0$$

or $$(\rho_{opt})_{max\ effy} = 1/2 \tag{10.16}$$

Thus, for maximum hydraulic efficiency also, the wheel velocity is half the jet velocity.

Substituting results of Eq. (10.16) in Eq. (10.15), we get

$$(\eta_D)_{max} = 2(1 - k \cos \theta)(1/2 - 1/4)$$

$$= \frac{1 - k \cos \theta}{2} \tag{10.17}$$

Again, the diagram efficiency for maximum work is given by

$$(\eta_D)_{max\ work} = \frac{E_{max}}{V_1^2/2g} = \frac{1 - k \cos \theta}{g} \frac{V_1^2}{4} \times \frac{2g}{V_1^2}$$

$$= \frac{1 - k \cos \theta}{2} = (\eta_D)_{max} = \eta_{max} \tag{10.18}$$

If $k = 1$, i.e. there is no energy loss due to friction, then

$$\eta_{max} = \frac{1 - \cos \theta}{2}$$

If $\theta = 180°$, $\eta_{max} = 1$ or 100%.

However, k lies between 0.8 and 0.85 and $\theta \cong 165°$, so that the exiting water does not hit the following bucket.

$$\eta_{max} = \frac{1 - 0.8 \cos 165°}{2} \cong 0.886$$

In practice, $\rho_{opt} \cong 0.46$, instead of 0.5.

If we plot η *vs* ρ, we get Fig. 10.27. Now,

V_b is constant and $V_1 = c_v\,[2gH]^{1/2}$, which depends on net head H. Then the discharge is given by the following equation.

$$Q = A\, c_v\, [2gH]^{1/2} \text{ m}^3/\text{s} \tag{10.19}$$

where the flow area A is controlled by the spear to regulate Q.

The velocity of wheel V_b is given by $V_b = \phi\, \sqrt{2gH}$

where ϕ = speed ratio, which varies from 0.43 to 0.48.

The minimum number of buckets in the wheel is approximately given by

$$Z = \frac{m}{2} + 15 \tag{10.19 a}$$

where m is equal to the jet ratio, D/d.

The erosion of the Pelton wheel occurs (i) on the buckets due to erosive effect of flow and (ii) at the nozzle due to cavitation effect (discussed later). To protect the buckets from wear and tear, chrome alloy steel or stainless steel is used. In India, many Pelton turbines are in operation such as at Koyna (475 m head, 4 jets), Sharavathi (570 m head, 4 jets), Kundah I (360 m head, 5 jets) and Kundah II (690 m, 3 jets). The world's largest Pelton turbine is a 6-jet, 840 m head turbine at Aurland-2 in Brazil producing 243 MWe.

10.13 DEGREE OF REACTION

By applying Bernoulli's equation to the inlet and outlet of a turbine, we get

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g} = E+\frac{p_2}{\rho g}+\frac{V_2^2}{2g} \tag{10.20}$$

where E is the energy transferred from fluid to the rotor. Therefore,

$$E = \frac{p_1-p_2}{\rho g}+\frac{V_1^2-V_2^2}{2g} \tag{10.21}$$

The first term on the R.H.S. is the energy transfer due to drop in static pressure and the second term represents the energy transfer due to drop in velocity head.

If $p_1 = p_2$, i.e. if pressure is constant,

$$E = \frac{V_1^2-V_2^2}{2g} \tag{10.22}$$

This happens in the case of impulse turbine, i.e. Pelton wheel where the pressure is atmospheric.

If $V_1 = V_2$,

$$E = \frac{p_1-p_2}{\rho g} \tag{10.23}$$

This holds good for a pure reaction turbine, where the wheel rotates only due to pressure drop across it exerting a reaction by Newton's third law of motion.

Degree of reaction, R, is defined in the following manner.

$$R = \frac{\text{Energy transfer due to pressure drop}}{\text{Total energy transfer}}$$

$$= \frac{(p_1-p_2)/\rho g}{E}$$

$$= \frac{E-\dfrac{V_1^2-V_2^2}{2g}}{E}$$

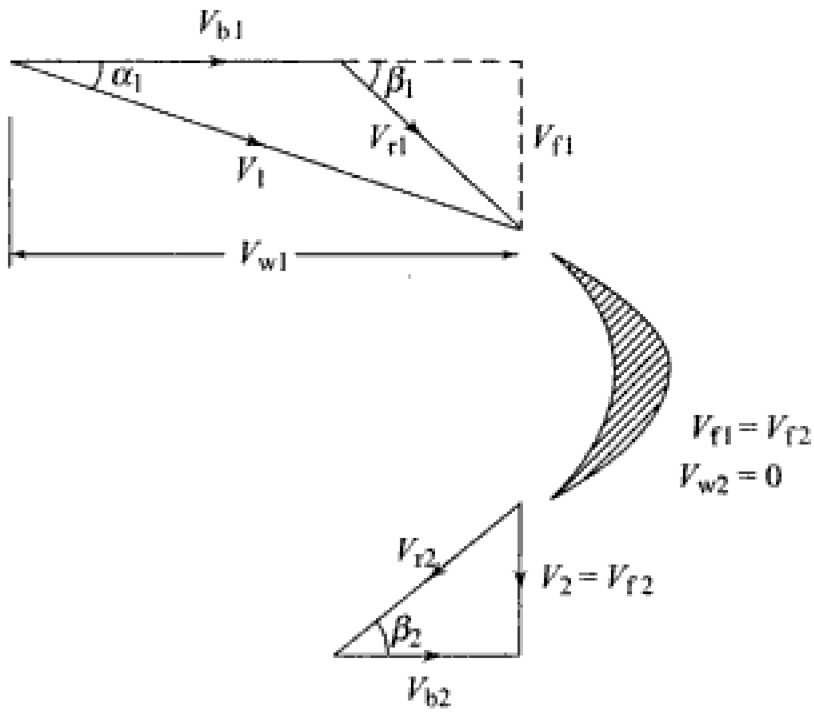


Fig. 10.29 *Velocity triangles of a Francis turbine blade*

Blading or diagram efficiency, $\eta_D = \dfrac{E}{E + E_l} = 1 - \dfrac{E_l}{E + E_l}$

$$= 1 - \frac{1}{1 + 2\cot\alpha_1(\cot\alpha_1 - \cot\beta_1)} \qquad (10.27)$$

Degree of reaction, $\quad R = 1 - \dfrac{V_{f_1}^2 \cot^2\alpha_1}{E}$

Hydraulic efficiency, $\quad \eta_h = \dfrac{E}{H} = \dfrac{V_{w1}V_{b1}}{gH} \qquad (10.28)$

Overall efficiency, $\quad \eta_0 = \dfrac{P}{\rho Q g H} \qquad (10.29)$

where P is the total power output.

10.15 PROPELLER AND KAPLAN TURBINES

The propeller turbine is a reaction turbine used for low heads (4 m – 80 m) and high specific speeds (300 – 1000). It is an axial flow device providing large flow area utilizing a large volume flow of water with low flow velocity. It consists of an axial-flow runner usually with four to six blades of airfoil shape (Fig. 10.30). The spiral casing and guide blades are similar to those in Francis turbines. In propeller turbines as in Francis turbines the runner blades are fixed and non-adjustable.

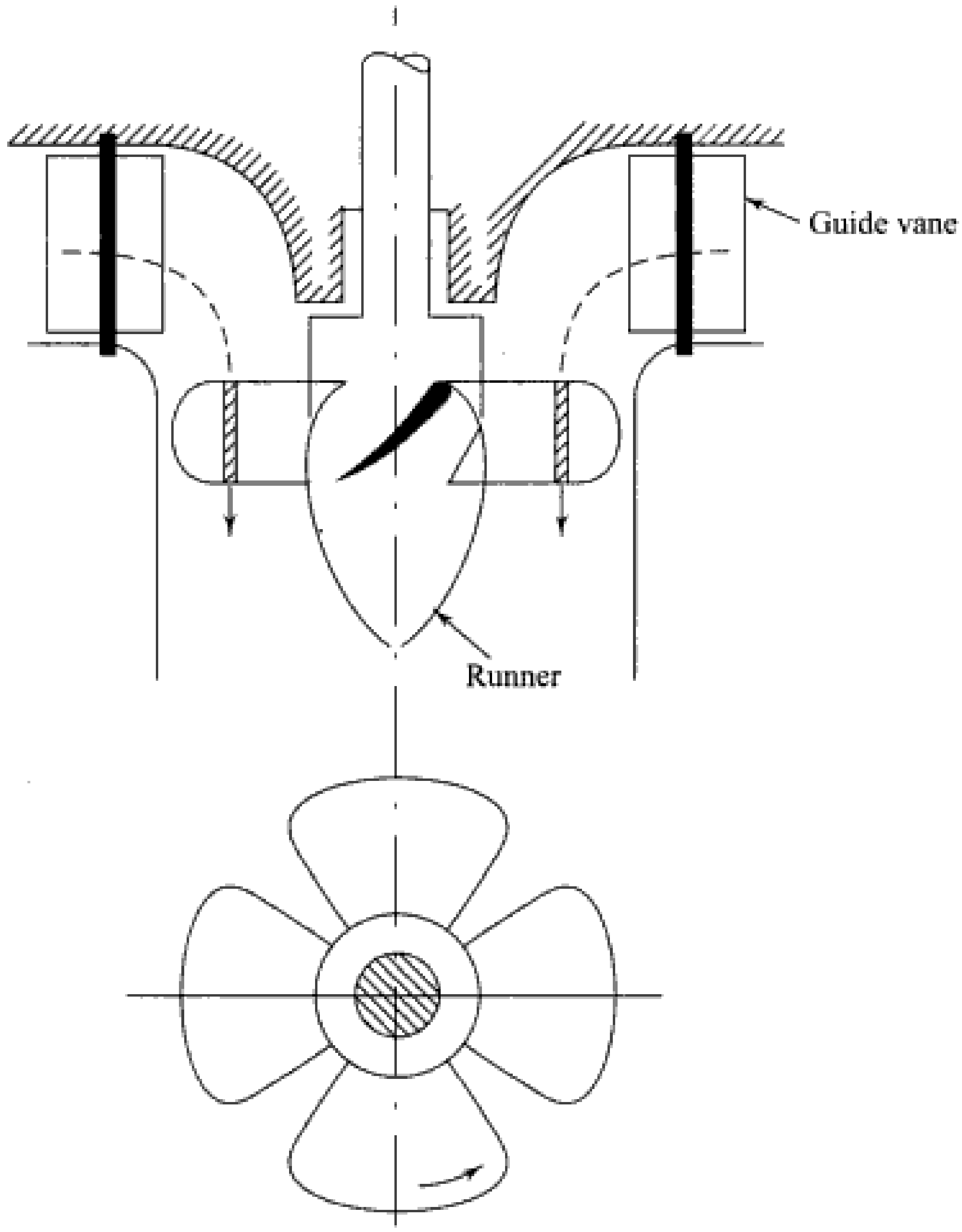


Fig. 10.30 *Schematic view of propeller turbine*

A special type of a propeller turbine is the Kaplan turbine in which the individual runner blades are pivoted on the hub (Fig. 10.31) so that their inclination may be adjusted during operation responding to changes in load. The blades are adjusted automatically rotating about pivots with the help of a governor servo-mechanism. The efficiency of a reaction turbine depends on the inlet blade angle. In fixed blade runners, it is not possible to vary the inlet blade angle for varying demands of power (load). So such turbines are designed for maximum efficiency only for a particular load. At all other loads their efficiency is less than this. In the Kaplan turbine, because of the arrangement for automatic variation of inlet blade angle with variation in load, the turbine can be run at maximum efficiency at all loads.

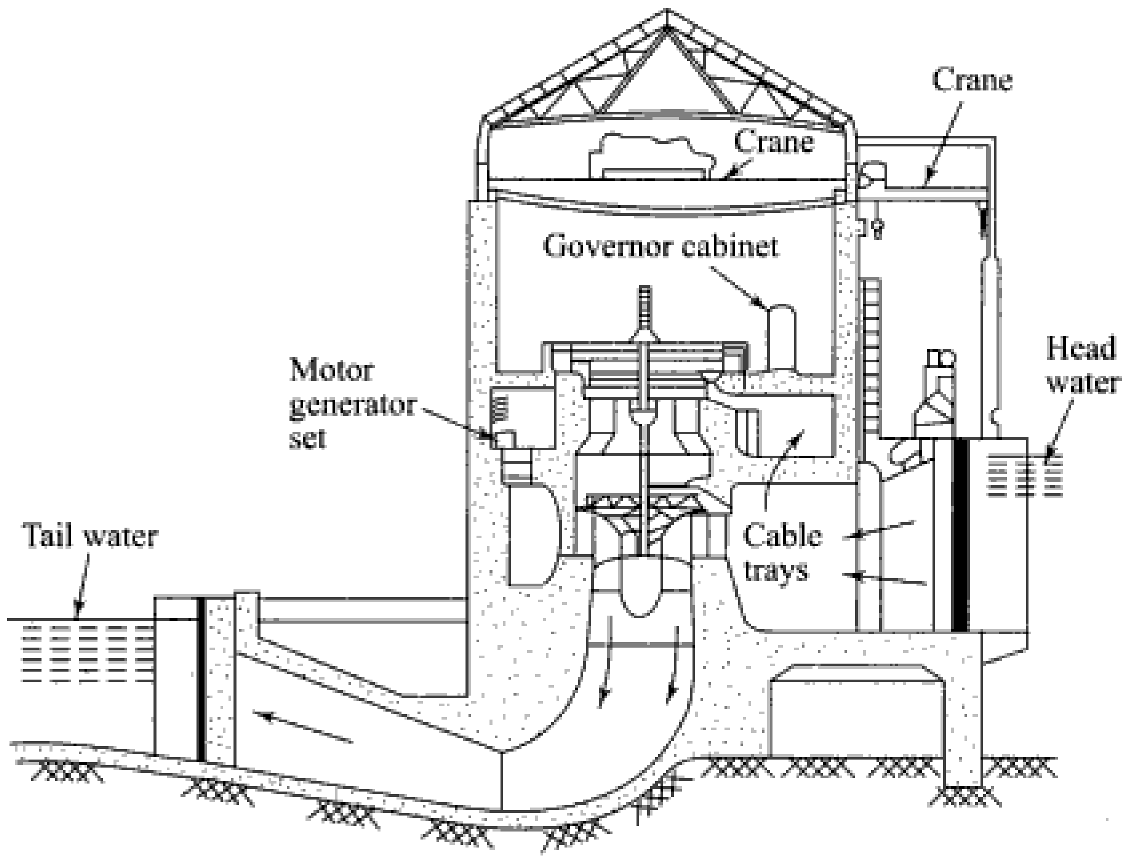


Fig. 10.31 *(c) Cross-section of typical low head concrete spiral-case setting with Kaplan turbine*

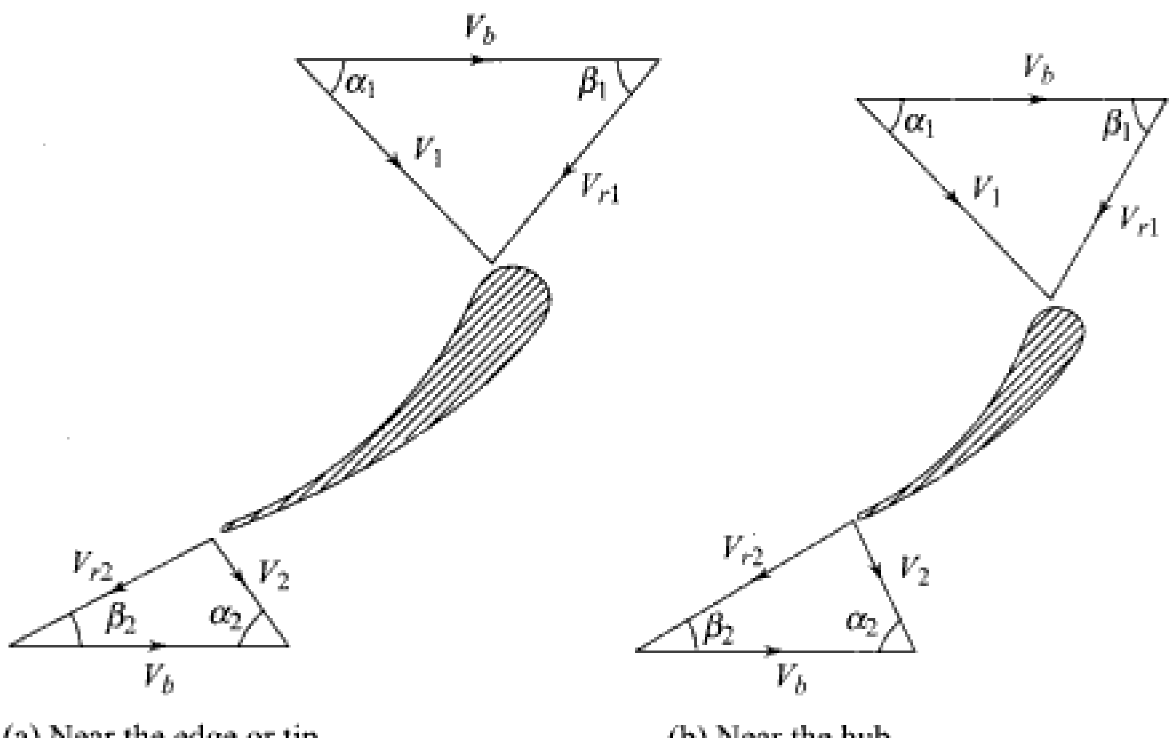


Fig. 10.32 *Velocity triangles for propeller or Kaplan runner blade*

10.16 DERIAZ TURBINE

The Deriaz turbine is also known as the 'diagonal turbine'. The flow over the runner is at an angle of 45° to the axis (Fig. 10.33). It has adjustable blades like

Kaplan turbines. At the same time the flow is diagonal or mixed as in Francis turbines. It can be described as a cross between the two turbines (Kaplan and Francis) and can be used for heads up to 200 m. The number of blades varies from 10 to 12. The guide blades and the stay vanes are also inclined.

The Deriaz runner is particularly suited for reversible flow conditions when the turbine also has to work as a pump as in pumped-storage power plants.

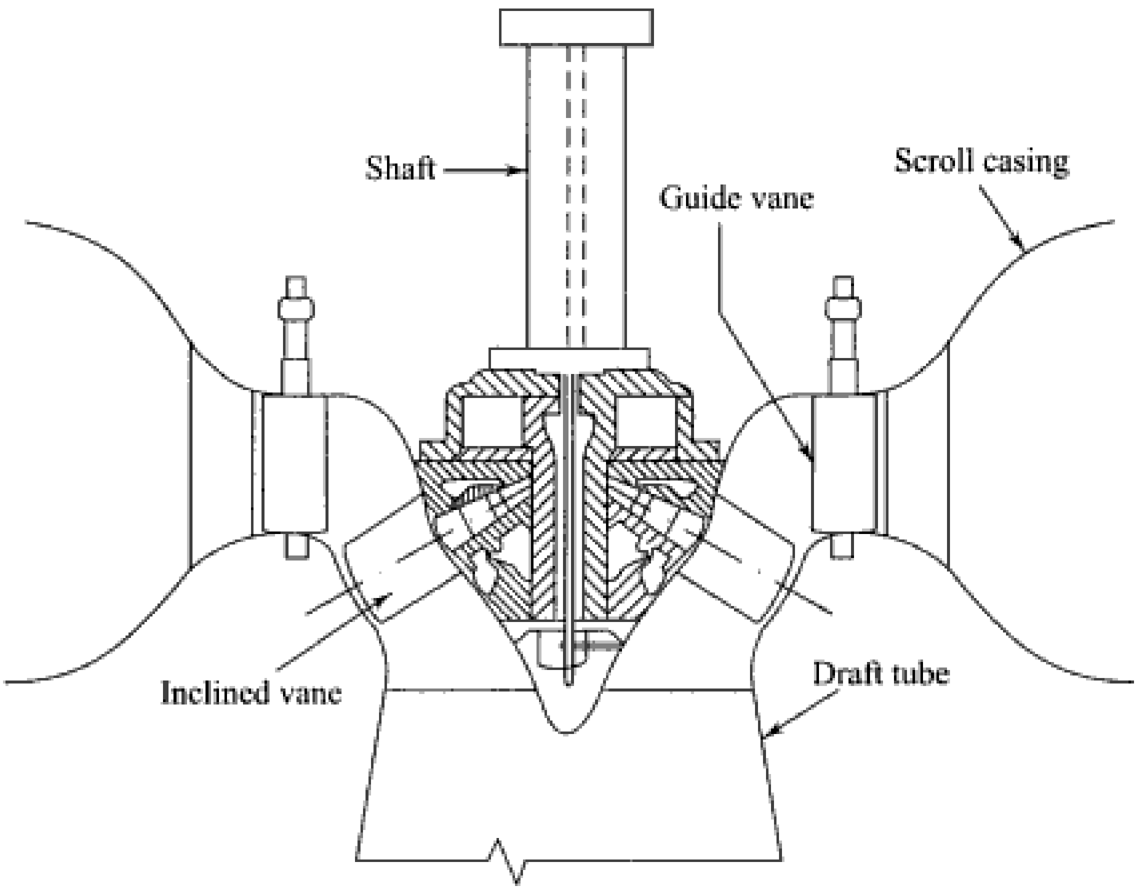


Fig. 10.33 *Deriaz turbine*

10.17 BULB TURBINE

Tubular or bulb turbines are small fixed axial flow propeller turbines operating under low heads. The turbo-generator is housed in an enclosed bulb-shaped casing, which is installed right in the middle of the flow passage. The bulb and the propeller form an integral unit followed by a straight conical flaring draft tube (Fig. 10.34). Bulb turbines are suitable for tidal power plants.

10.18 SPECIFIC SPEED

To analyse hydroelectric schemes it is economical to make a scale model and perform necessary hydraulic tests on it in order to predict what will happen in the prototype or full-sized system under similar operating conditions. The suitability of a turbine for a particular application depends on (a) head of water (b) rotational speed (c) power developed, which together fix a parameter called 'specific speed'.

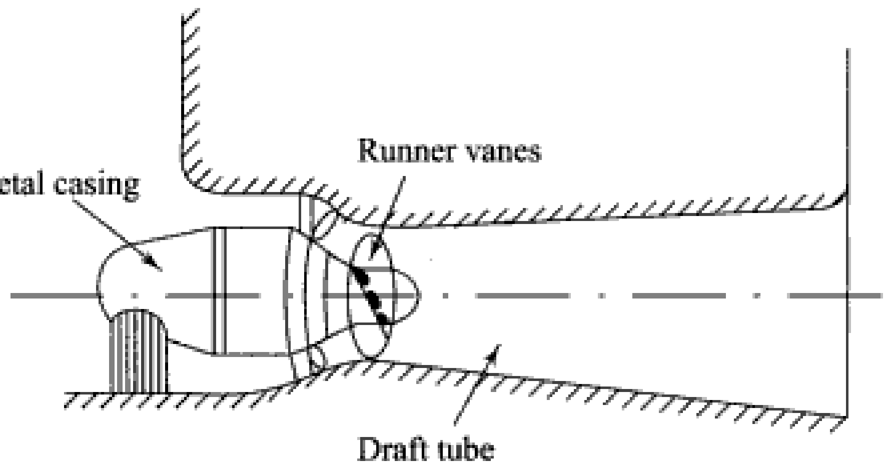


Fig. 10.34 *Bulb turbine*

The specific speed of a turbine is defined as the speed of operation of a *geometrically similar model* of the turbine which is so proportioned that it produces 1 kW *power* when operating under 1m *head*.

We know,

$$\text{Power, } P = \rho Q g H$$

Therefore, $P \propto QH$, since the density of water and acceleration due to gravity g are constant,

or $$P \propto (AV)\, H \tag{10.30}$$

where A is the cross-sectional flow area and V is the water velocity.

or $$P \propto D^2\, [2gH]^{1/2}\, H$$

or $$P \propto D^2 \times H^{3/2} \tag{10.31}$$

where D is the wheel diameter through which the water flows axially and H is the net head of water. Again, the blade velocity is

$$V_b \propto V \tag{10.32}$$

or $$DN \propto [2gH]^{1/2}$$

or $$D \propto \frac{H^{1/2}}{N} \tag{10.33}$$

Substituting Eq. (10.33) in Eq. (10.31), we get

$$P \propto \frac{H}{N^2} \cdot H^{3/2}$$

$\therefore$ $$N^2 \propto \frac{H^{5/2}}{P}$$

$\therefore$ $$N \propto \frac{H^{5/4}}{P^{1/2}} \tag{10.34}$$

or $$N = \frac{KH^{5/4}}{P^{1/2}}$$

or $$K = \frac{NP^{1/2}}{H^{5/4}}$$

If $P = 1$ kW, $H = 1$ m, then K is equal to N, called the specific speed, N_s.

$$\therefore \quad N_s = \frac{NP^{1/2}}{H^{5/4}} \tag{10.35}$$

The classification of turbines on the basis of specific speeds has been discussed in Section 10.10. The ranges of specific speeds of different turbines are given in Table 10.2. The non-dimensional form of specific speed is given by the following equation which is Eq. (10.4) derived earlier.

$$N_s' = \frac{NP^{1/2}}{\rho^{1/2}(gH)^{5/4}}$$

It is also known as the shape number of the turbine.

10.18.1 Scale Ratio

The model of a turbine and its prototype are in definite geometric ratio depending on their respective heads and the rotative speeds. The ratio of blade velocity V_b and the water velocity V is called the speed ratio, which has a definite value for a particular turbine (0.42 to 0.47 for a Pelton turbine, from 0.55 to 1.00 or more for a Francis turbine and 1.5 to 3.00 or more for a propeller turbine).

$$\therefore \quad V_b \propto V$$

$$\therefore \quad DN \propto \sqrt{H}$$

Using subscript m for the model turbine and p for the prototype, we get

$$\frac{D_m N_m}{D_p N_p} = \sqrt{\frac{H_m}{H_p}}$$

$$\frac{D_m}{D_p} = \sqrt{\frac{H_m}{H_p}}\,\frac{N_p}{N_m} \tag{10.36}$$

This is called the *scale ratio* which represents the ratio of the diameters of the model turbine and the prototype turbine.

10.18.2 Unit Speed, Unit Power and Unit Discharge

The terms 'unit speed', 'unit power' and 'unit discharge' are frequently used to express the operational characteristics of hydraulic turbines.

The unit speed N_u is defined as the speed of a geometrically similar turbine working under a head of 1 m,

$$V_b \propto V$$

$$DN \propto [2gH]^{1/2}$$

$$N \propto \sqrt{H}$$

$$\therefore \quad N = K\sqrt{H}$$

where K is a constant.

When $\quad H = 1$ m, $N = N_u = K$

$$\therefore \quad N_u = \frac{N}{\sqrt{H}} \tag{10.37}$$

The *unit power* P_u is the kW of power generated by a geometrically similar turbine working under a head of 1 m.

$$P = \rho Q g H = \rho (AV) g H$$

$$= \rho A [2gH]^{1/2} \cdot gH$$

or $\quad P \propto H^{3/2}$

When $\quad H = 1$ m, $P = P_u = K$

$$\therefore \quad P_u = \frac{P}{H^{3/2}} \tag{10.38}$$

The unit discharge, Q_u is the flow rate the turbine would have under a head of 1 m.

$$Q = AV = A\,[2gH]^{1/2}$$

$$Q = K\sqrt{H}$$

where K is a constant.

When $H = 1$ m, $Q = Q_u = K$

$$\therefore \quad Q_u = \frac{Q}{H^{1/2}} \tag{10.39}$$

10.19 COMPARISON OF TURBINES

The characteristic features of common types of turbine are summarized in Table 10.4.

10.20 CAVITATION

When the velocity of a fluid increases its pressure falls. In any turbine part if the pressure drops below the vapour pressure at that temperature some of the liquid flashes into vapour. The bubbles formed during vaporization are carried by the water stream to higher pressure zones, where the bubbles condense into liquid forming a cavity or vacuum. The surrounding liquid rushes towards the cavity giving rise to a very high local pressure which may be as high as 7000 atm. The formation of such a cavity and high pressure occurs repeatedly hundreds of times in a second. This phenomenon is known as cavitation, which causes pitting on the metallic surface of runner blades and draft tube. It is accompanied by considerable vibration and noise.

Table 10.4 *Comparison of common turbines*

		Pelton wheel	*Francis turbine*	*Kaplan/ Propeller turbine*
1.	Flow	Tangential, single stage, impulse	Inward radial flow, single stage, reaction	Axial flow, single stage, reaction
2.	Maximum capacity	250 MW	720 MW	225 MW
3.	Number of jets/kind of blades	1 to 6 Maximum 2 for horizontal and 6 for vertical shaft	Fixed blades	Propeller turbines have fixed blades, while Kaplan turbines have adjustable blades
4.	Head	100–1750 m	30–550 m	1.3–77.5 m
5.	RPM	75–1000	93.8–1000	72–600
6.	Hydraulic efficiency	Single jet 85–90%	90–94%	85–93%
7.	Specific speed	6–60	50–400	280–1100
8.	Regulation mechanism	Spear nozzle and deflector plate	Guide vanes	Blade stagger

This table can be used in selecting a turbine for a specific application.

Cavitation should be minimised or avoided by selecting proper material like stainless steel or alloy steel, by adequate polishing of the surface, by selecting a runner of low specific speed or by keeping the runner under water.

10.21 GOVERNING OF HYDRAULIC TURBINES

Hydraulic turbines are directly coupled to the electric generators. The generators are always required to run at a constant speed irrespective of the variations in the load. This constant speed (rpm) of the generator is given by

$$N = \frac{120 f}{p} \tag{10.40}$$

where f is the frequency for power generated in cycles per second and p is the number of poles for the generator. The speed of the generator can be maintained at a constant level only if the speed of the turbine runner is constant as given by Eq. (10.40). It is known as the synchronous speed of the turbine runner for which it is designed.

If the load on the generator goes on varying and if the input for the turbine remains the same, then the speed of the runner tends to increase if the load goes down or it tends to decrease if the load on the generator goes up. Therefore, the speed of the generator and hence, the frequency will vary accordingly, which is not desired. Therefore, the speed of the runner is always required to be maintained at a constant level at all loads.

It is done automatically by a governor which regulates the quantity of water flowing through the runner in proportion to the load.

10.21.1 Governing of Impulse Turbine

In a Pelton turbine, water flow to the runner is regulated by the combined action of the spear and the deflector plate. There is a centrifugal governor, as in the case of a steam turbine, where its sensitivity to load variation is augmented by an oil-operated servo-mechanism (Fig. 10.35). When the load on the generator drops, the speed of turbine runner increases. The flyballs of the centrifugal governor fly outward due to more centrifugal force (due to higher rpm). The sleeve moves up, the portion of the lever to the right of the fulcrum moves down pushing the piston rod of the control valve downwards.

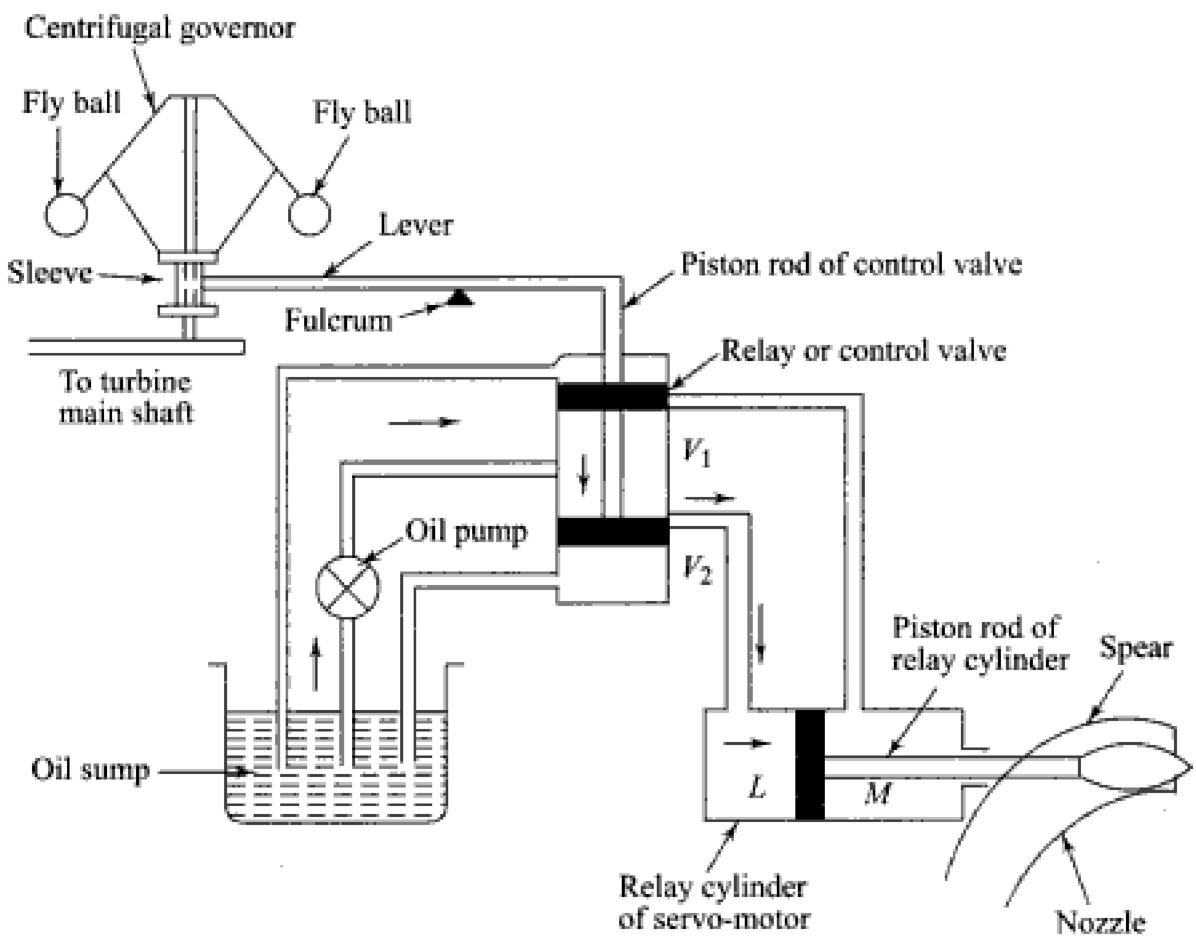


Fig. 10.35 *Governing of Pelton turbine*

With the downward motion of the piston rod, valve V_1 closes and valve V_2 opens as shown in Fig. 10.35. A gear pump pumps oil from the oil sump to the control or relay valve. Oil flows through valve V_2 and exerts force on the face L of the piston of the relay cylinder. The piston (or spear) rod along with the spear moves to the right, thus decreasing the flow area and hence, the rate of water flow to the turbine. The speed of the turbine falls till it becomes normal when the flyballs, sleeve, lever, etc. also come to normal position. The reverse happens when the load on the generator increases, speed decreases, flyballs fly inward with less centrifugal force (due to less rpm), the sleeve moves down, the piston rod of control valve goes up, valve V_1 opens and valve V_2 closes, the oil under pressure flows through valve V_1 and exerts a force on the face M of the piston. The piston rod and the spear move to the left as a result of which more water flows to the turbine to take up more load and the speed becomes normal, i.e. attains its rated value.

The spear or needle valve is used normally for small load fluctuations. When there is a sudden fall of load, the spear has to move rapidly to close the nozzle. This rapid closing may cause water hammer. It is quite serious in large capacity plants with long penstocks. To avoid the water hammer effects during a sudden fall of load, a deflector is introduced in the system, which is not shown in Fig. 10.35. The function of the deflector is to deflect some water from the jet advancing to the turbine runner when the load on the turbine suddenly decreases. The quantity of water flowing through the nozzle remains the same, but a certain part of water coming out from the nozzle is deflected and is not allowed to strike the buckets. The deflected water goes waste into the tailrace level.

10.22 GOVERNING OF REACTION TURBINES

The governing of Francis turbine is similar to the governing of Pelton wheel except that the motion of the piston in servo-motor is used to partially close or open the guide vanes gate through which the water is supplied to the turbine (instead of the spear in the nozzle of the Pelton turbine). The working diagram of the governor is shown in Fig. 10.36. The position of the control valve and the servo-motor correspond to the design load on the turbine and operate in the same way as in the case of Pelton wheel. A compensating device is, however, added to prevent the governor from overshooting. When the servo-motor piston moves to the right, the bell-crank lever EFG is rotated downward about F and the arm G is lowered. This pulls down the pivot A, which, in turn, lowers the fulcrum B. Thus the relay port 'b' is partially or fully closed, restricting the piston motion to the right.

The governor is always operated with a pressure relief valve (not shown). A sudden closure of wicket gates will open the relief valve due to the sudden increase in pressure and protect the conduit from inertia effects of speeding water. The relief valve consists of a spear and is held by fluid pressure to close the bypass of water from the spiral casing to the tailrace at design load. When the load decreases suddenly, a bell-crank lever opens the pilot valve of the pressure chamber so that the pressure on the spear is reduced, thereby permitting the spear to be lifted up and allowing a portion of water to flow directly from the spiral casing to the tailrace though the bypass without striking the turbine runner. Thus, both the deflector of Pelton wheel and the relief valve of Francis or Kaplan turbine perform the same function of protecting the system from water hammer effects when the load suddenly decreases.

In the case of Kaplan turbine, in addition to guide vanes the runner vanes are also adjustable and hence the governor is required to operate both sets of vanes simultaneously. The runner vanes are also operated by a separate servo-motor and a control valve which are interconnected with those of the guide vanes to ensure that for a given guide vane opening there is a definite runner vane inclination.

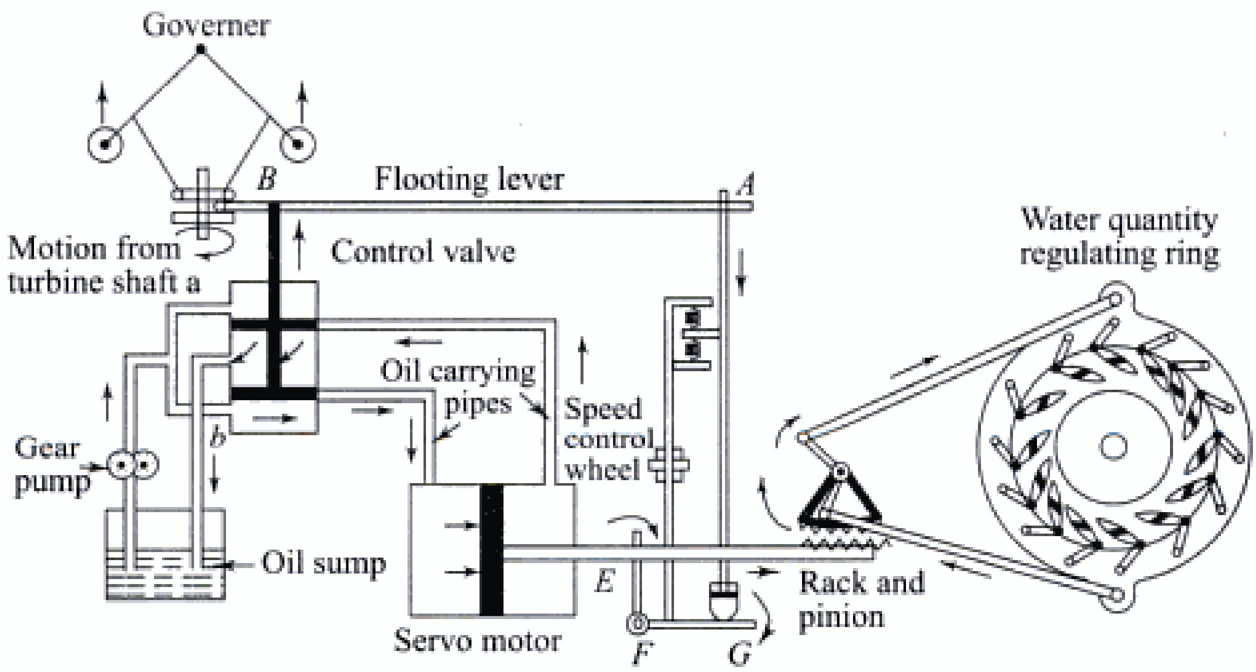


Fig. 10.36 *Governing of Francis turbine*

10.23 SURGE TANKS

A preliminary discourse on surge tanks was made in Section 10.8.6. When the load on the generator decreases the governor reduces the rate of flow of water striking the runner in order to maintain the constant speed of the runner. But the sudden reduction of the rate of flow in the penstock may build a water hammer in the pipe, which may cause excessive inertia pressure in the pipeline due to which the pipe may burst. Two devices, viz. the deflector and the relief valve, as described earlier, are provided to avoid the sudden reduction of the rate of flow in the penstock. But neither of these devices is of any help when the load on the generator increases and the turbine is in need of more water. Thus, in order to fulfil both the above objectives, in addition to the deflector or the relief valve, certain other devices such as *surge tank* and *forebay* are provided. Surge tanks are employed in the case of high head and medium head power plants where the penstock is very long and forebays are suitable for medium head and low head power plants where the length of the penstock is short.

An ordinary surge tank is a cylindrical open-topped storage reservoir, as shown in Fig. 10.37, which is connected to the penstock at a point as close as possible to the turbine. The upper lip of the tank is kept well above the maximum water level in the supply reservoir.

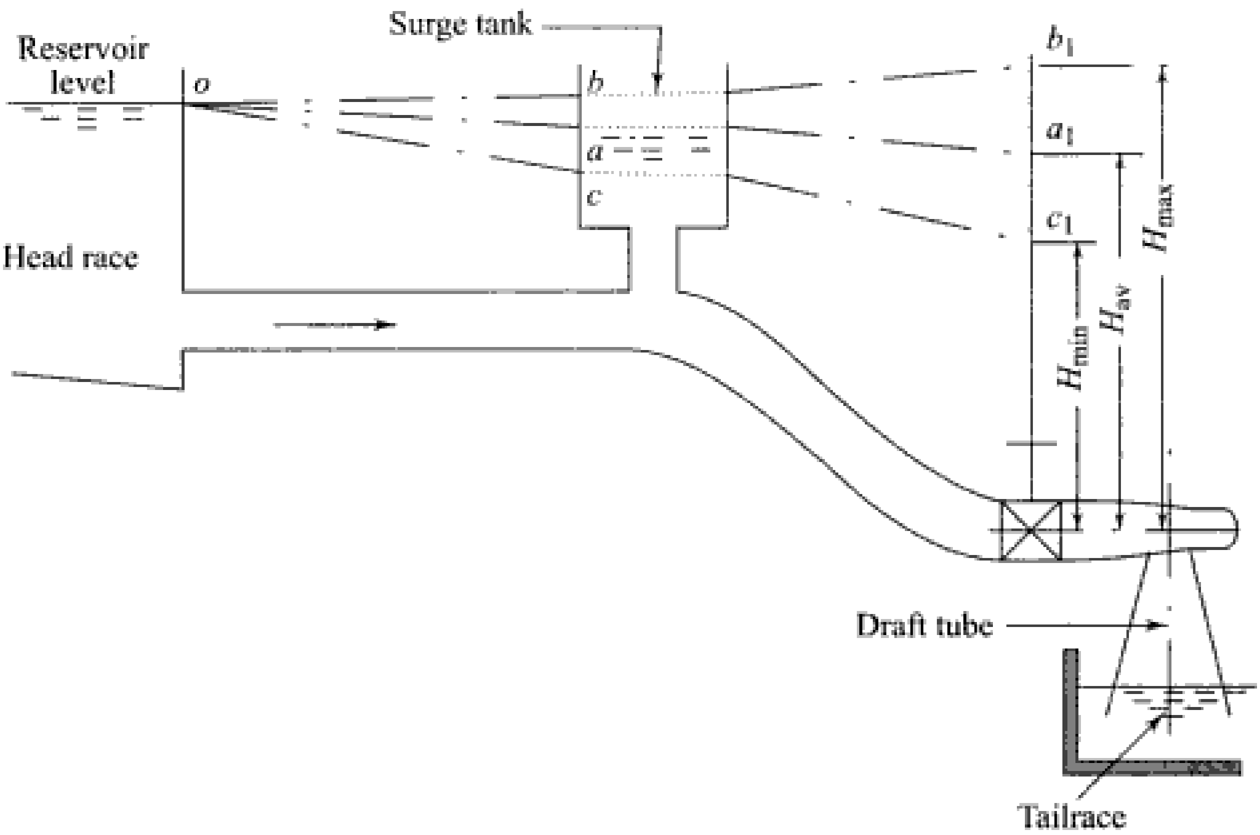


Fig. 10.37 *Ordinary surge tank*

When the load on the turbine is normal and steady, there are no velocity variations in the pipeline and the normal pressure gradient is oaa$_1$ (Fig. 10.37). The water surface in the surge tank is lower than the reservoir surface by an amount equal to the friction head loss in the pipe connecting the reservoir and the surge tank. When the load on the generator decreases, the turbine gates are partially closed and the excess water moving towards the turbine is stored in the surge tank in the space between the levels *a* and *b* and a rising pressure gradient abb$_1$ develops. The resulting retarding head reduces the velocity of flow in the pipeline corresponding to the reduced discharge required by the turbine.

When the load on the generator increases, the governor opens the turbine gates to increase the rate of flow entering the runner. The increased demand of water by the turbine is partly met by the water stored between levels *a* and *c* in the surge tank (Fig. 10.37). As such the water level in the surge tank falls and a falling pressure gradient Occ$_1$ is developed. The surge tank thus provides an accelerating head which increases the velocity of flow in the pipeline corresponding to the increased demand by the turbine.

Various other types of surge tanks are also shown in Fig. 10.38. Type (a) is a conical type surge tank. Type (b) has an internal bell-mouth spillway which permits the overflow to be easily disposed of Type (c) is a differential surge tank, which has a central riser pipe having small ports at its lower end. It

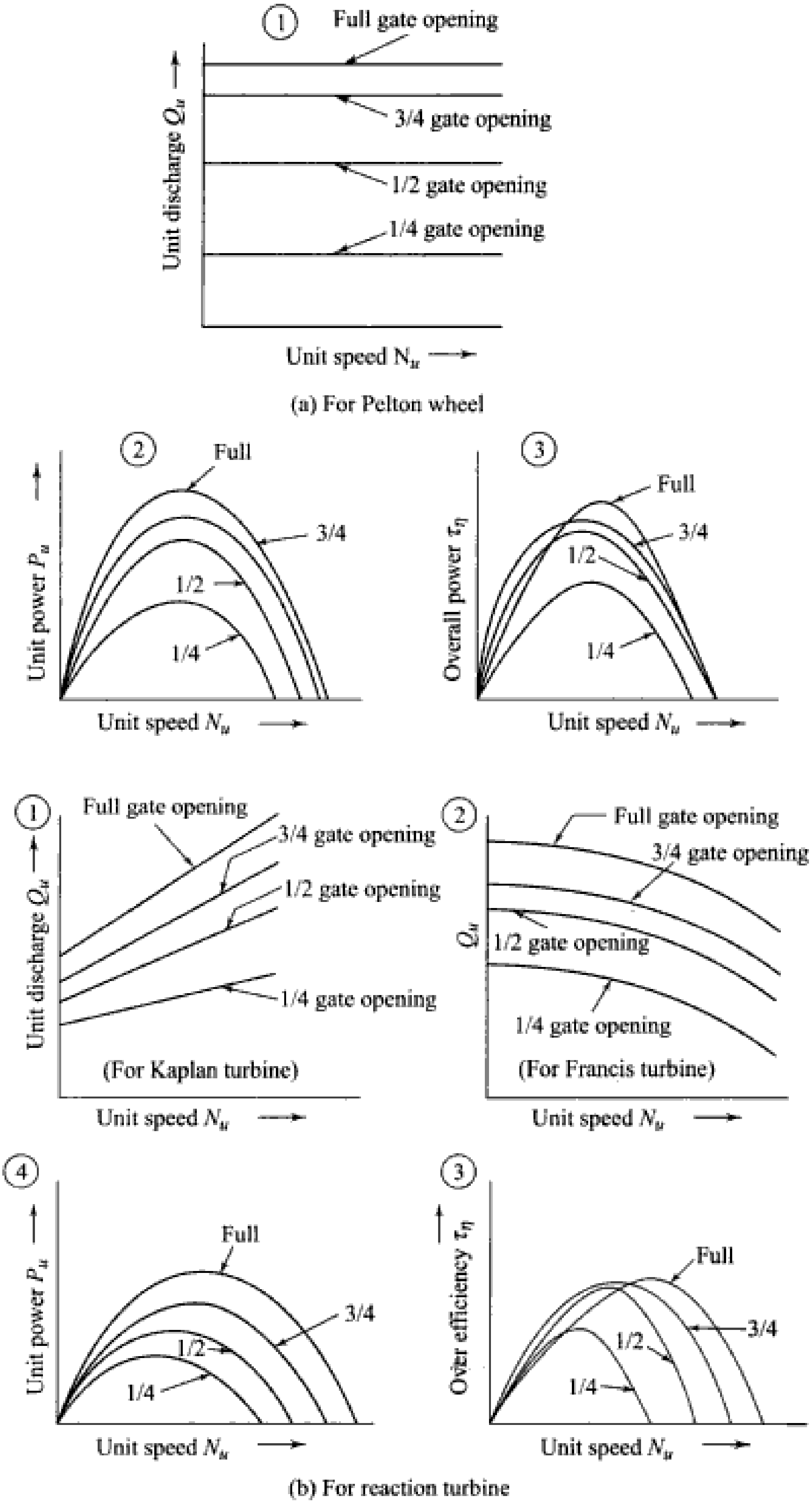


Fig. 10.39 *Constant head characteristics of Pelton wheel and reaction turbines*

(b) *Constant speed characteristic curves* In these tests the constant speed is attained by regulating the gate opening (i.e. discharge) as the load varies. The head may or may not remain constant. The characteristic curves of efficiency against load for different turbines are shown in Figs 10.40 (a) and (b). The efficiency increases with load and reaches the maximum at the full or rated load. It is observed that the Kaplan turbine and the Pelton wheel maintain a high efficiency over a longer range of part load as compared with either the Francis or the fixed blade propeller turbine.

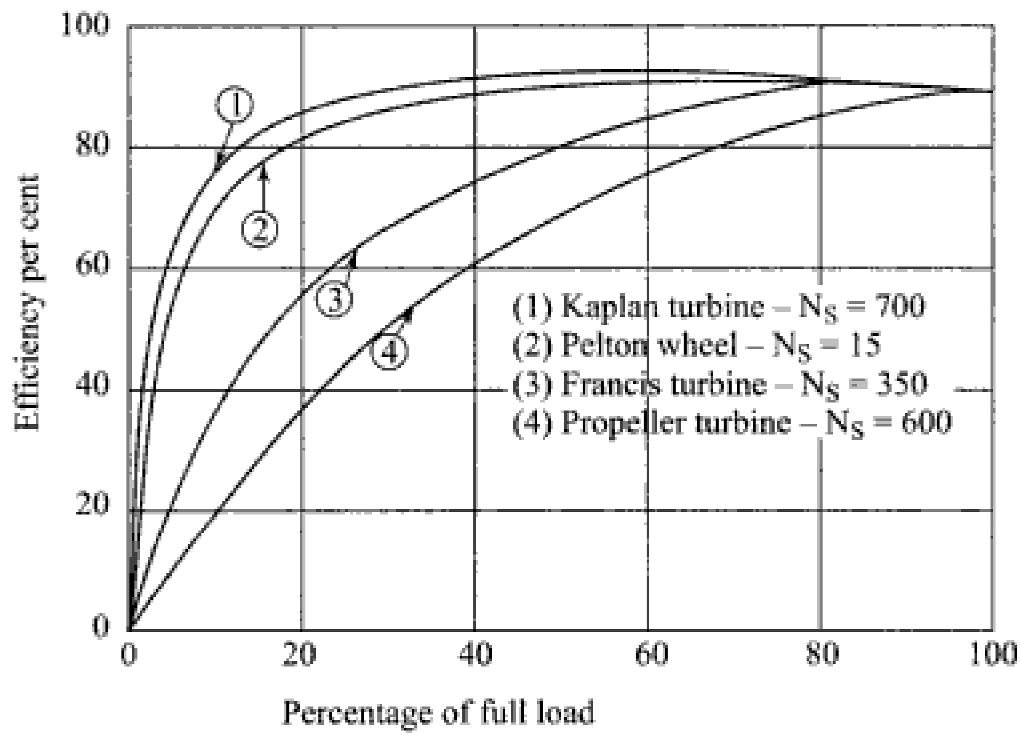


Fig. 10.40 *(a) Overall efficiency variation with load for various turbines*

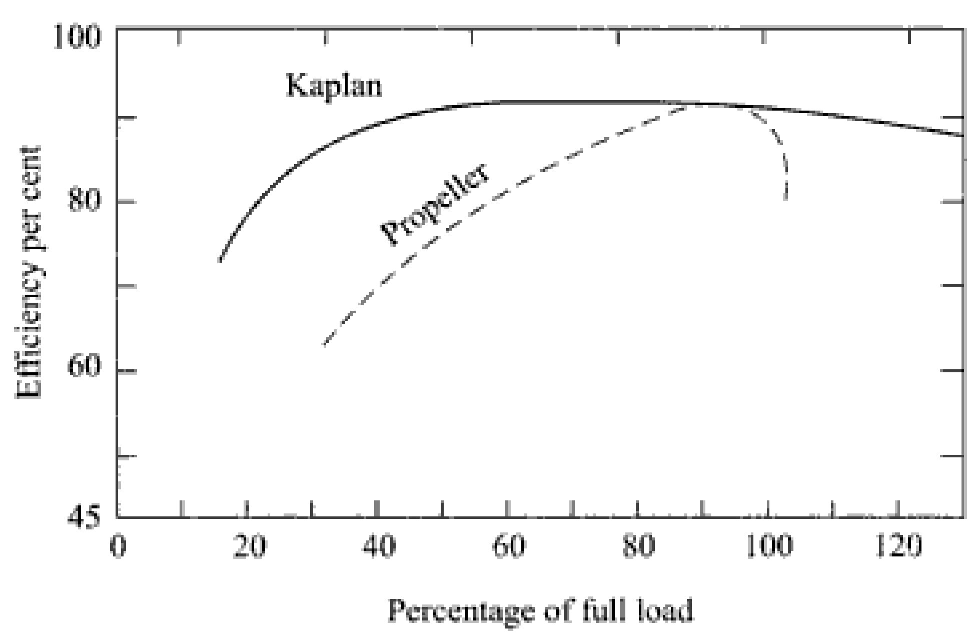


Fig. 10.40 *(b) Efficiency variation with load for propeller and Kaplan turbines*

Figure 10.41 shows the plots of efficiency and power varying with discharge, where Q_0 is the maximum discharge required to initiate the motion of the turbine runner from the state of rest. Since the power ($P = \rho QgH$) is directly proportional to discharge if the head is constant, the P vs Q plot is a straight line. However, the overall efficiency increases with discharge and becomes more or less constant beyond a certain value of discharge.

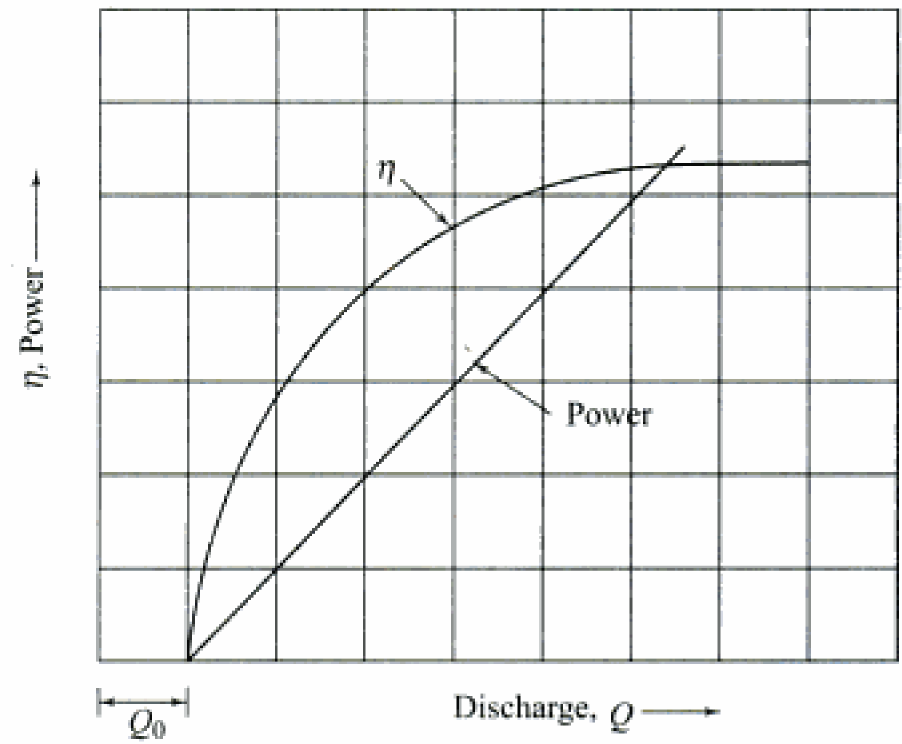


Fig. 10.41 *Overall efficiency and brake power varying with discharge*

(c) *Constant efficiency curves* Figure 10.42 shows the constant efficiency curves for all conditions of running, which are also called the universal characteristic curves of the turbine. The inner most curve represents the highest efficiency of the turbine and the outer curves represent lower efficiencies. If a vertical line is drawn at a certain Q_u, it will intersect an efficiency curve at two points and it will also touch some other efficiency curve of higher η at one point. Thus, for a unit discharge (or power) the vertical line touches the curve of maximum efficiency at only one point. Now, if these points are joined together by a smooth curve, we obtain the best performance curve for the turbine. By drawing a horizontal line for a given N_u (at certain H and N) which cuts this best performance curve, the point of maximum efficiency is known, corresponding to which Q_u or P_u can be obtained and hence Q and P can be estimated at which the turbine efficiency is maximum for the given H and N.

There is a term called '*runway speed*' which is the maximum speed of the turbine under no load and no governing action. The hydraulic design is for optimum speed, but it must also satisfy structurally the safety conditions at runaway speed, it is about 1.8 to 2.3 times the optimum speed.

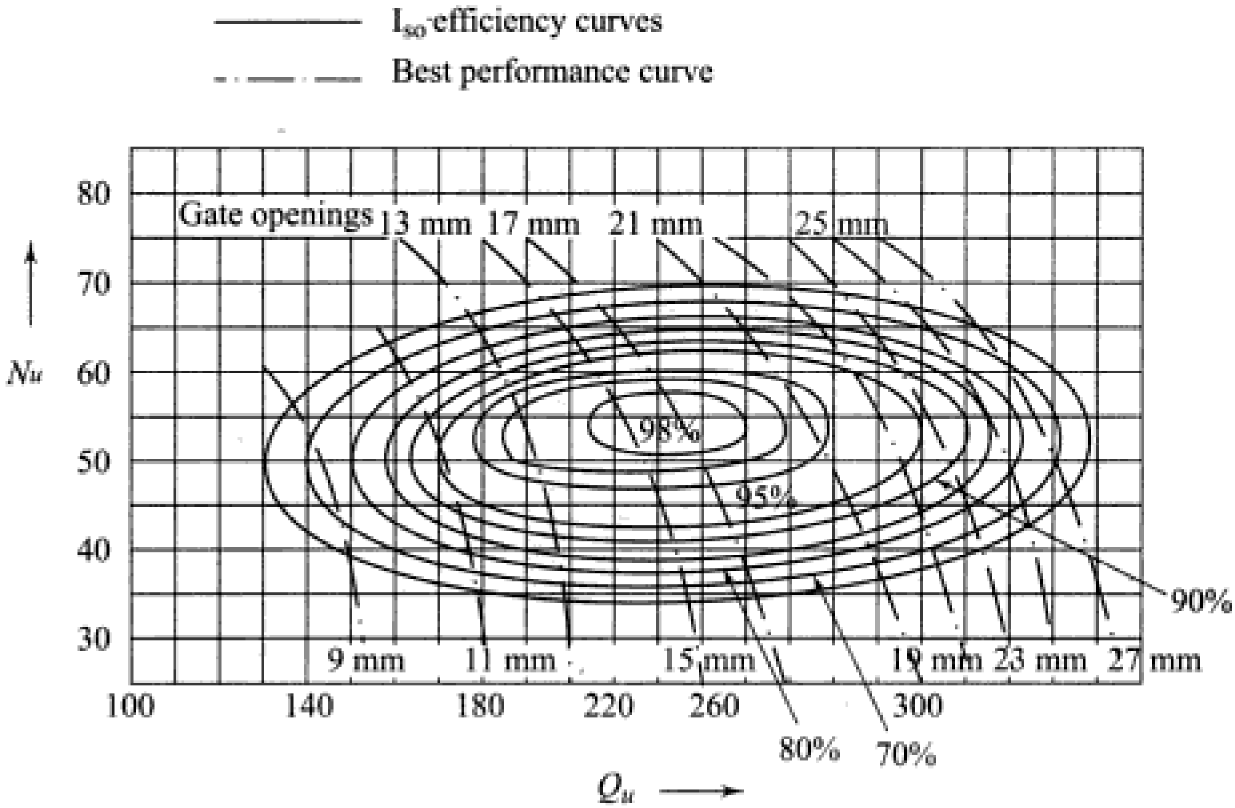


Fig. 10.42 *Constant efficiency and best performance curves (universal characteristics) of a Francis turbine*

10.25 SELECTION OF TURBINES

The hydraulic turbine is selected according to the specific conditions under which it has to operate and attain the maximum possible efficiency. The choice depends on the head available, power to be developed and the speed at which it has to run. The following factors basically govern the selection of a suitable type of turbine.

(a) *Operating head* The present practice is to use Kaplan and Propeller type of turbines for heads up to 50 m. For head from 50 to 400 m, Francis turbines are used. For heads greater than 400 m, impulse or Pelton turbines are used. The range of heads as mentioned is not rigid and may change if other conditions dominate to achieve economy.

(b) *Specific speed* It is better to choose turbines of high specific speeds. High speed turbines mean small sizes of turbines, generators, power house, etc. and are therefore, more economical. The range of specific speeds of the turbines should correspond to the synchronous speed of the generator, $N = \dfrac{120f}{p}$, where f is the frequency and p the number of poles.

(c) *Height of installation* It is better to install the turbines as high above the tail water level (TWL) as possible. This saves the cost of excavation for the draft tube. Care should be taken to ensure that cavitation does not occur.

(d) *Performance characteristics of turbine* The performance characteristics of turbines as discussed in Section 10.24 should be studied carefully before recommending the type of turbine to be used. A turbine has the maximum efficiency at a certain load. When a turbine has to operate mostly at part loads, only those turbines whose efficiencies do not fall appreciably with part loads should be selected. Kaplan and Pelton turbines are better than Francis and propeller turbines in this respect.

(e) *Size of turbine* It is better to go in for as large a size of turbine as possible since this results in economy of size of the power house, the number of penstocks, the generator, etc. Bigger size means less number of runners. However, the number of runners should not be less than two so that at least one unit is always available for service in the case of a plant breakdown.

Example 10.1 A Pelton wheel driven by two similar jets transmits 4000 kW to the shaft when running at 400 rpm. The head from the reservoir level to the nozzle is 200 m and the efficiency of power transmission through the pipelines and nozzles is 90 per cent. The jets are tangential to a 1.50 m diameter circle. The relative velocity decreases by 10 per cent as the water traverses the buckets, which are so shaped that they would, if stationary, deflect the jet by 165°. Neglecting windage losses, estimate (a) the efficiency of the runner and (b) the diameter of each jet.

Solution

Velocity of fluid at inlet to the bucket is,

$$V_1 = \sqrt{2gH.k_v} = \sqrt{2\times 9.81\times 200\times 0.9} = 59.43 \text{ m/s}$$

$$V_b = \frac{\pi DN}{60} = \frac{\pi \times 1.5\times 400}{60} = 31.42 \text{ m/s}$$

$$\eta = \frac{(1-k\cos\theta)(V_1 - V_b)V_b}{g\times V_1^2/2g} = \frac{2(1-k\cos\theta)(V_1-V_b)V_b}{V_1^2}$$

$$= \frac{2(1-0.9\cos 165°)(59.43-31.42)\,31.42}{(59.43)^2}$$

$$= 0.9312 \text{ or } 93.12\%$$ *Ans.* (a)

Power developed, is

$$P = \frac{4000}{0.9312} = 4295.53 \text{ kW}$$

Power developed per jet is,

$$= \frac{4295.53}{2} = 2147.77 \text{ kW}$$

$$\therefore \quad 2147.77 = (\rho A_1 V_1)\frac{V_1^2}{2} = \rho\frac{\pi d^2}{4}\frac{V_1^3}{2}$$

$$= 1000 \times \frac{\pi}{8} d^2 \times (59.43)^3 \times 10^{-3}$$

$$\therefore \quad d = \sqrt{\frac{2147.77 \times 8}{\pi \times (59.43)^3}} = 0.1614 \text{ m}$$

= diameter of each jet *Ans.* (b)

Example 10.2 A Pelton wheel has to be designed for the following specifications. Power to be developed = 6000 kW. Net head available = 300 m. Speed = 550 rpm. Ratio of jet diameter to wheel diameter = 1/10. Hydraulic efficiency = 0.85. Assuming the velocity coefficient $C_v = 0.98$ and speed ratio $f = 0.46$, find (a) the number of jets (b) diameter of each jet (c) diameter of the wheel and (d) the quantity of water required.

Solution

$$V_1 = C_v \sqrt{2gH} = 0.98 \sqrt{2 \times 9.81 \times 300}$$

$$= 75.19 \text{ m/s}$$

$$V_b = 0.46\sqrt{2 \times 9.81 \times 300} = 35.29 \text{ m/s}$$

$$\eta_0 = \frac{P}{\rho Q g H} = \frac{6000 \times 10^3}{1000 \times Q \times 9.81 \times 300} = 0.85$$

$$\therefore \quad Q = \frac{20}{9.81 \times 0.85} = 2.4 \text{ m}^3/\text{s}$$ *Ans.* (d)

$$V_b = 35.29 \text{ m/s} = \frac{\pi D \times 550}{60}$$

D = diameter of the wheel

$$= \frac{35.29 \times 6}{\pi \times 55} = 1.23 \text{ m}$$ *Ans.* (c)

$$\frac{d}{D} = \frac{1}{10}$$

$\therefore$ d = diameter of each jet = 0.123 m *Ans.* (b)

$$\text{Number of jet's} = \frac{Q}{V_1 \times \frac{\pi}{4} d^2}$$

$$= \frac{2.4 \times 4}{75.19 \times \pi \times (0.12)^2}$$

= 2.822, i.e. 3 jets *Ans.* (a)

Example 10.3 A single jet impulse turbine of 10 MW capacity is to work under a head of 500 m. If the specific speed of the turbine is 10, the overall efficiency is 80 per cent and the coefficient of velocity is 0.98, find the diameters of the jet and the bucket wheel. Assume the speed of the bucket wheel as 0.46 of the velocity of jet.

Solution

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$\therefore \quad N = \frac{N_s H^{5/4}}{\sqrt{P}} = \frac{10 \times (500)^{5/4}}{\sqrt{10.000}}$$

$$= 236.4 \text{ rpm}$$

Velocity of jet, $V = C_v\sqrt{2gH}$

$$= 0.98\sqrt{2 \times 9.81 \times 500}$$

$$= 97.06 \text{ m/s}$$

Speed of bucket wheel, $V_b = 0.46 \times 97.06$

$$= 44.65 \text{ m/s}$$

$$V_b = \frac{\pi DN}{60} = 44.65$$

$$\therefore \quad D = \frac{60 \times 44.65}{\pi \times 236.4} = 3.61 \text{ m} \qquad \textit{Ans.}$$

$$\eta_0 = \frac{P}{\rho Q g H} = \frac{P}{\rho \times \frac{\pi}{4} d^2 \times V \times gH}$$

$$0.80 = \frac{10{,}000 \times 10^3}{1000 \times \frac{\pi}{4} d^2 \times 97.06 \times 9.81 \times 500}$$

$$d^2 = \frac{80}{0.8 \times \pi \times 97.06 \times 9.81} = 0.0334 \text{ m}^2$$

$\therefore$ Diameter of jet, $d = 0.183$ m *Ans.*

Example 10.4 Show that the specific speed of a single jet Pelton wheel is about 202 (d/D) where d and D represent the jet and bucket wheel diameters respectively. Take $C_v = 0.97$, $f = 0.45$ and $h = 0.85$.

Solution

$$Q = aV = n\frac{\pi}{4}d^2 C_v\sqrt{2gH}\text{ , where } n = \text{number of jets.}$$

$$P = \rho Q g H \eta = \rho \eta \frac{\pi}{4} d^2 C_v \sqrt{2gH} . gH\eta \times 10^{-3} \quad \text{where } P \text{ is in kW.}$$

The peripheral velocity V_b of the Pelton wheel of diameter D, is

$$V_b = \frac{\pi D N}{60} = \phi\sqrt{2gH} \text{ , where } f \text{ is the speed ratio.}$$

$$N = \frac{60\phi\sqrt{2gH}}{\pi D}$$

$$N_s = N \frac{\sqrt{P}}{H^{5/4}}$$

$$= \frac{60\phi\sqrt{2gH}}{\pi D} \frac{\sqrt{\rho n \frac{\pi}{4} d^2 C_v \sqrt{2gH} \cdot gH\eta \times 10^{-3}}}{H^{5/4}}$$

or,

$$N_s = \frac{60}{\pi} \cdot \phi\sqrt{2g} \cdot \frac{d}{D} \sqrt{\rho n \frac{\pi}{4} C_v \sqrt{2g} \cdot g\eta \times 10^{-3}}$$

Substituting $f = 0.45$, $C_v = 0.97$, $h = 0.85$, $\rho = 1000$ kg/m^3, we get

$$N_s = \frac{60}{\pi} \times 0.45\sqrt{2g} \frac{d}{D} \sqrt{n \frac{\pi}{4} \times 0.97\sqrt{2g} \times g \times 0.85}$$

$$= 38.07 \frac{d}{D} \cdot 5 \cdot 30 \cdot \sqrt{n}$$

$$= 202 \sqrt{n} \frac{d}{D}$$

For a single jet turbine, $n = 1$. Thus the specific speed is given by

$$N_s = 202 \frac{d}{D} \text{. Hence proved.}$$

Example 10.5 Four jets each of 60 mm diameter strike the buckets of an impulse wheel and each gets deflected by an angle of 165°. The speed of the bucket wheel is 45 m/s. Find the velocity of the jet for maximum efficiency, power developed and the hydraulic efficiency. Assume that the bucket moves linearly.

Solution

For maximum efficiency, the jet velocity is,

$$V_1 = 2\, V_b = 2 \times 45 = 90 \text{ m/s} \qquad \textit{Ans.}$$

Flow through the jet,

$$Q = \frac{\pi}{4} d^2 \times V_1$$

$$= \frac{\pi}{4} \times (0.06)^2 \times 90$$
$$= 0.2545 \text{ m}^3/\text{s}$$

Power developed, P, is given by Eq. (10.14), i.e. $= \frac{1-k\cos\theta}{g} \cdot \frac{V_1^2}{4}$ w.g.

Taking friction coefficient k to be unity,

$$P = (1 - \cos 165°) \cdot \frac{90^2}{4} \times 0.2545 \times 1000 \times 10^{-3} \text{ kW} = 1013 \text{ kW}$$

For four jets

$$P = 4 \times 1013 = 4052 \text{ kW} \quad \textit{Ans.}$$

The maximum efficiency is given by Eq. (10.17), i.e.

$$(\eta_D)_{max} = \frac{1-k\cos\theta}{2}$$
$$= \frac{1-\cos 165°}{2} = 0.983 \text{ or } 98.3\% \quad \textit{Ans.}$$

Example 10.6 The peripheral velocity of the wheel of an inward flow reaction turbine is 20 m/s. The velocity of whirl of the inflowing water is 17 m/s and the radial velocity of flow is 2 m/s. If the flow is 0.7 m^3/s and the hydraulic efficiency is 80 per cent, find the head on the wheel, the power generated by the turbine and the angles of the vanes. Assume radial discharge.

Solution

The velocity triangles for the moving vanes are shown in Fig. E10.6. Since the discharge is radial, Vw_2 is zero.

$\therefore$ Hydraulic efficiency, $\eta_h = \frac{Vw_1 \cdot V_b}{gH}$

$$\therefore \quad 0.8 = \frac{17 \times 20}{9.81 \times H}$$

$\therefore$ Head on the wheel, $H = 43.3$ m *Ans.*

Power generated is,

$$P = \rho Q g H\, \eta_h = 10^3 \times 0.7 \times 9.81 \times 43.3 \times 0.8 \times 10^{-3}$$
$$= 238 \text{ kW}$$

Exit angle of guide vanes $= \alpha_1$.

Now,

$$\alpha_1 = \tan^{-1} \frac{Vf_1}{Vw_1} = \tan^{-1} \frac{2}{17}$$
$$= 180 - 6.71 = 173.29° \quad \textit{Ans.}$$

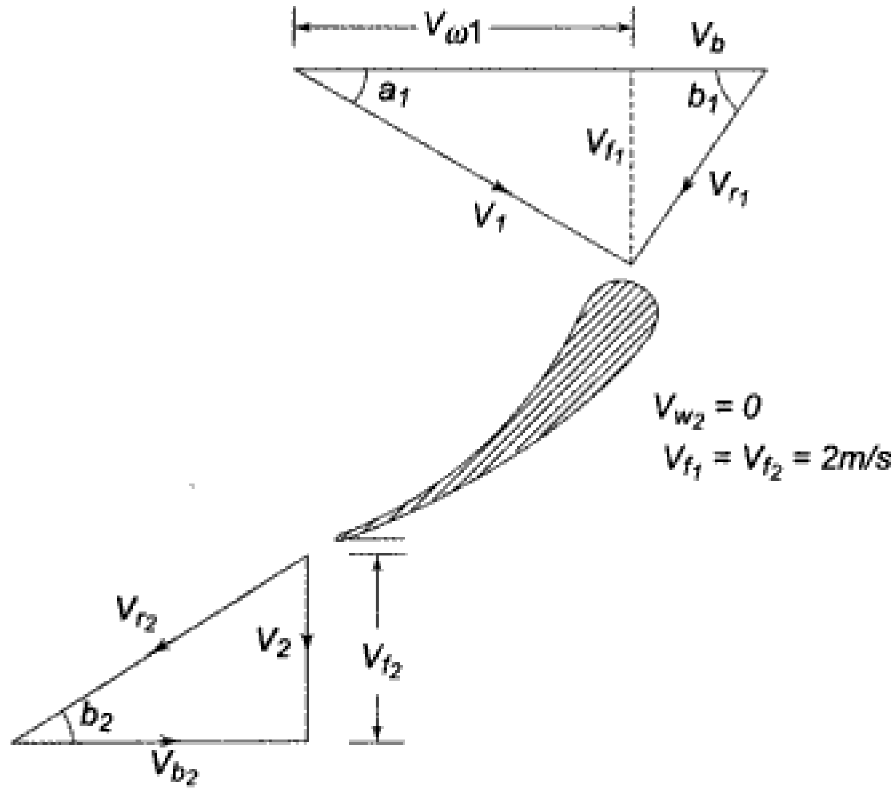


Fig. E10.6

Inlet blade angle = β_1

$$\beta_1 = \tan^{-1}\frac{V_{f1}}{V_{b_1} - V_{\omega_1}} = \tan^{-1}\frac{2}{20-17}$$

$$= \tan^{-1} 0.6667 = 33.7° \qquad \textit{Ans.}$$

Example 10.7 A runner of a Francis turbine having 1.50 m outer diameter and 0.75 m inner diameter operates under a head of 150 m with a specific speed of 120 and generates 14 MW. If the water enters the wheel at angle of 11°20′ and leaves the blade radially with no velocity of whirl, what will be the inlet and outlet blade angles? Assume the hydraulic efficiency to be 92 per cent.

Solution

$$N_s = \frac{N\sqrt{P}}{H^{5/4}}$$

$$120 = \frac{N\sqrt{14000}}{(150)^{5/4}}$$

$$\therefore \qquad N = 532 \text{ rpm}$$

$$V_{b1} = \frac{\pi DN}{60} = \frac{\pi \times 1.5 \times 532}{60} = 41.76 \text{ m/s}$$

$$\eta_h = 0.92 = \frac{V_{w1}V_{b1} - V_{w2}V_{b2}}{gH}$$

$$= \frac{V_{w1}V_{b1}}{gH}, \text{ since } V_{w2} = 0$$

$$V_{w1} = \frac{0.92 \times 9.81 \times 150}{41.76} = 32.42 \text{ m/s}$$

$$\tan \alpha_1 = \tan 11°20' = 0.2 = \frac{V_{f_1}}{V_{w_1}}$$

$$\therefore \quad V_{f_1} = 0.2 \times 32.42$$
$$= 6.49 \text{ m/s} = V_{f_2}$$

$$\tan \beta_1 = \frac{V_{f_1}}{V_{b1} - V_{w1}} = \frac{6.49}{41.76 - 32.42} = 0.6955$$

$$\therefore \quad \beta_1 = 34°49' \qquad \textit{Ans.}$$

Since the inner diameter is half the outer diameter,

$$V_{b2} = \frac{V_{b1}}{2} = \frac{41.76}{2} = 20.88 \text{ m/s}$$

$$\tan \beta_2 = \frac{V_{f_2}}{V_{b2} - V_{w2}} = \frac{6.49}{20.88 - 0} = 0.3108$$

$$\therefore \quad \beta_2 = 17.27 = 17°16'$$

Example 10.8 The following data relate to a Francis turbine: Net head = 70 m; Speed = 700 rpm; Overall efficiency = 85 per cent; Shaft power = 350 kW; Hydraulic efficiency = 92 per cent; Flow ratio, $V_{f1}\sqrt{2gH} = 0.22$; Breadth ratio, $B/D = 0.1$; Outer diameter of the runner = 2 × inner diameter of runner; Velocity of flow, V_f = constant; Outlet discharge = radial. The thickness of vanes occupies 6 per cent of circumferential area of the runner. Determine (a) the guide vane angle (b) the runner vane angles at inlet and outlet (c) the diameters of the runner at inlet and outlet; (d) the width of the wheel at inlet.

Solution

The velocity triangles are shown in Fig. E10.8.

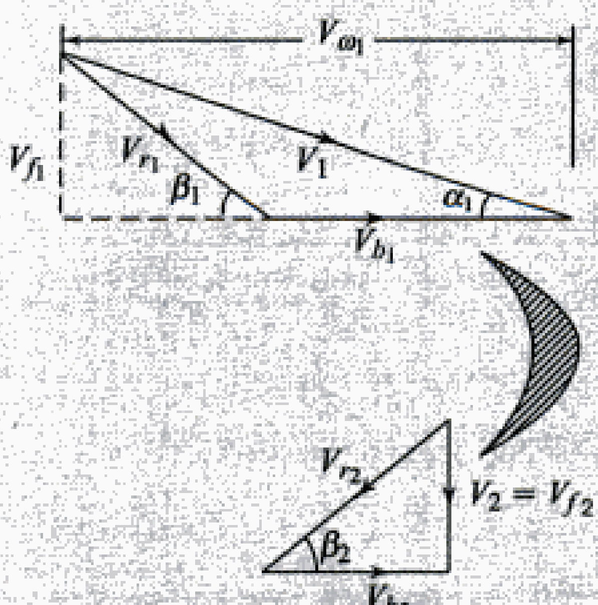


Fig. E10.8

Flow ratio, $\dfrac{V_{f1}}{\sqrt{2gH}} = 0.22$

$$\therefore \quad V_{f1} = 0.22\sqrt{2 \times 9.81 \times 76} = 8.153 \text{ m/s}$$
$$= V_{f2}$$
$$A_b = 0.94\, \pi D_1 B_1$$

Since discharge at outlet is radial,

$$V_{w2} = 0,\ V_{f2} = V_2 = 8.153 \text{ m/s}$$

Now $$\eta_0 = \frac{\text{shaft power}}{\text{water power}} = \frac{350 \text{ kW}}{1000 \times Q \times 9.81 \times 70 \times 10^{-3}}$$

$$\therefore \quad Q = \frac{350}{9.81 \times 70 \times 0.85} = 0.5996 \text{ m}^3\text{/s} \cong 0.6 \text{ m}^3\text{/s}$$
$$Q = 0.94\, \pi D_1 B_1 \times V_{f1}$$
$$0.6 = 0.94\, \pi D_1 \times 0.1\, D_1 \times 8.153$$
$$\therefore \quad D_1 = 0.4992 \text{ m} \cong 0.5 \text{ m} \qquad \textit{Ans.} \text{ (c)}$$
$$\therefore \quad B_1 = 0.1 \times 0.5 = 0.05 \text{ m} = 5 \text{ m} \qquad \textit{Ans.} \text{ (d)}$$
$$D_2 = \frac{0.5}{2} = 0.25 \text{ m} \qquad \textit{Ans.} \text{ (c)}$$
$$V_{b1} = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.5 \times 700}{60} = 18.33 \text{ m/s}$$
$$\eta_h = \frac{V_{w1} V_{b1}}{gH} = \frac{V_{w1} \times 18.33}{9.81 \times 70} = 0.92$$
$$\therefore \quad V_{w1} = 34.47 \text{ m/s}$$
$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{8.153}{34.47} = 0.2365$$
$$\therefore \quad \alpha = 13.3° = 13°\ 18' \qquad \textit{Ans.} \text{ (a)}$$
$$\tan\beta_1 = \frac{V_{f1}}{V_{w1} - V_{b1}} = \frac{8.153}{34.47 - 18.33} = 0.5051$$
$$\therefore \quad \beta_1 = 26.8° = 26°\ 148' \qquad \textit{Ans.} \text{ (b)}$$
$$\tan\beta_2 = \frac{V_f}{V_{b2}}$$
$$\frac{V_{b1}}{r_1} = \frac{V_{b2}}{r_2} = \omega$$
$$\therefore \quad V_{b2} = \frac{D_2}{D_1} V_{b1} = \frac{18.33}{2}$$
$$= 9.17 \text{ m/s}$$

$$V_b = \frac{\pi DN}{60} = 45.41$$

$\therefore$ Pitch circle diameter of the wheel, D is,

$$D = \frac{45.41 \times 60}{\pi \times 300} = 2.89 \text{ m} \qquad \textit{Ans.} \text{ (d)}$$

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{260.000}}{(475)^{5/4}} = 68.98 \qquad \textit{Ans.} \text{ (e)}$$

Jet ratio,

$$\frac{D}{d} = \frac{2.89}{0.227} = 12.73$$

$\therefore$ Number of buckets $= \dfrac{D}{2d} + 15 = \dfrac{12.73}{2} + 15 = 21.37$ or 22 Ans. (f)

Work done per kg, is

$$E = \frac{(V_1 - V_{b1})(1 - k\cos\theta)V_{b1}}{g}$$

$$= \frac{(94.6 - 45.41)(1 - 0.98\cos 165°) \times 45.41}{9.81}$$

$$= 443.24 \text{ kg–m/kg} \qquad \textit{Ans.} \text{ (g)}$$

$$\eta_h = \eta_{\text{head}} \times \eta_{\text{dis}} = \frac{443.24}{475} \times 0.9975$$

$$= 0.93 \text{ or } 93\% \qquad \textit{Ans.} \text{ (h)}$$

Example 10.10 Water is supplied to an axial flow turbine under a gross head of 35 m. The mean diameter of the runner is 2 m and it rotates at 145 rpm. Water leaves the guide vanes at 30° to the direction of the runner rotation and at mean radius the angle of the runner blade at outlet is 28°. If 7 per cent of the gross head is lost in the casing and guide vanes, and the relative velocity is reduced by 8 per cent due to friction in the runner, determine the blade angle at inlet and the hydraulic efficiency of the turbine.

Solution

Net heat $H = 0.93 \times 35$

$= 32.6$ m

$\therefore$ $V_1 = \sqrt{2gH} = \sqrt{2 \times 9.81 \times 32.6}$

$= 25.3$ m/s

$$V_b = \frac{\pi DN}{60} = \frac{\pi \times 2 \times 145}{60} = 15.2 \text{ m/s}$$

With Reference to Fig. E10.10,

$$V_{r1} \sin \beta_1 = V_1 \sin \alpha$$

$$V_{r1} \cos \beta_1 = V_1 \cos \alpha - V_b$$

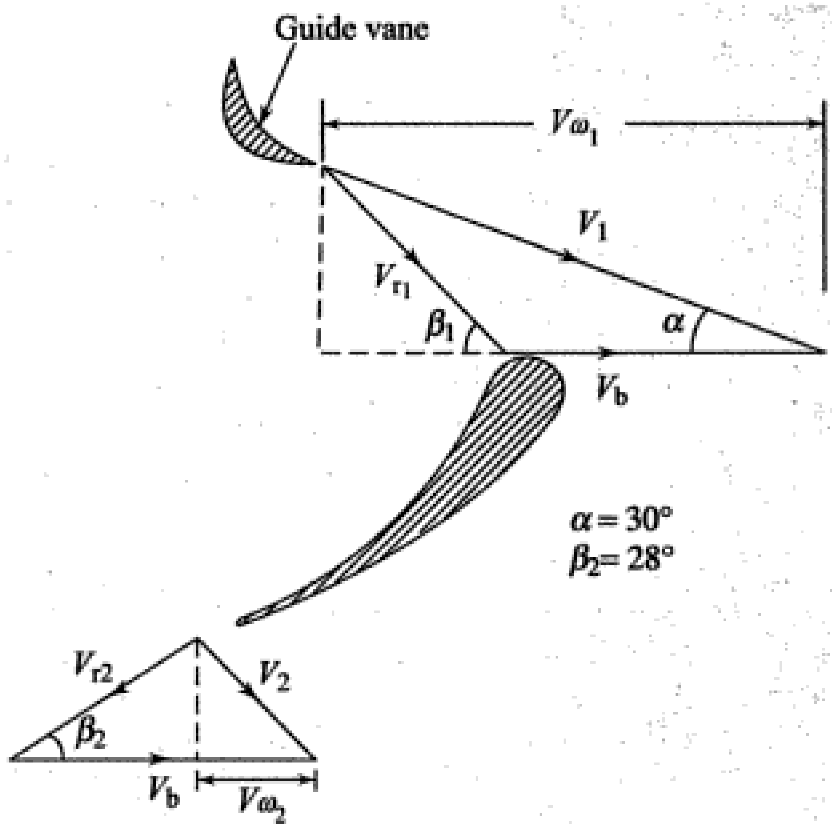


Fig. E10.10

$$\therefore \quad \tan\beta_1 = \frac{V_1 \sin\alpha}{V_1\cos\alpha - V_b} = \frac{25.3\ \sin\ 30°}{25.3\cos 30° - 15.2} = 1.8852$$

$$\therefore \quad \beta_1 = 62.1°$$

$$V_{r1} = \frac{25.3 \sin 30°}{\sin 62.1°} = 14.31 \text{ m/s}$$

$$V_{r2} = 0.92 \times 14.31 = 13.17 \text{ m/s}$$

$$V_{w1} = V_1 \cos 30° = 21.91 \text{ m/s}$$

$$V_{w2} = V_b - V_{r2}\cos 28°$$

$$= 15.2 - 13.17 \times 0.883 = 3.57 \text{ m/s}$$

$$E = \frac{V_b(V_{w1} - V_{w2})}{g} = \frac{15.2(21.91 - 3.57)}{9.81} = 28.42 \text{ m}$$

$$\therefore \quad \eta_h = \frac{28.42}{35} = 0.812 \quad \text{or} \quad 81.2\% \quad \textit{Ans.}$$

Example 10.11 A Kaplan turbine develops 10000 kW under a head of 12 m when the following conditions prevail. Speed ratio = 2, flow ratio = 0.65, diameter of hub = 0.3 times the external diameter of the vane and the overall efficiency = 94 per cent. Estimate (a) the speed (b) the diameter of the runner and (c) the specific speed.

Solution

$$P = \rho QgH\eta \times 10^{-3}$$

$$10000 = Q \times 9.81 \times 12 \times 0.94$$

$$Q = 90.37 \text{ m}^3/\text{s}$$

Flow ratio,

$$\phi' = \frac{V_{f1}}{\sqrt{2gH}} = 0.65$$

$$V_{f1} = 0.65\sqrt{2 \times 9.81 \times 12} = 9.97 \text{ m/s}$$

Area of flow,

$$A_b = \frac{90.37}{9.97} = 9.064 \text{ m}^2$$

$$A_b = \frac{\pi}{4}\left(D^2 - d_h^2\right) = 9.064$$

$$D^2 - (0.3D)^2 = \frac{9.064 \times 4}{\pi} = 11.54$$

$$D^2 = \frac{11.54}{0.91} = 12.682$$

Runner diameter, $D = 3.56$ m *Ans.* (b)

Speed ratio $\quad \phi = 2 = \dfrac{V_b}{\sqrt{2gH}} = \dfrac{V_b}{\sqrt{19.62 \times 12}}$

$$V_b = 30.69 \text{ m/s} = \frac{\pi DN}{60}$$

$$N = \frac{30.69 \times 60}{\pi \times 3.56} = 164.6 \quad \text{or} \quad 165 \text{ rpm}$$

Synchronous speed,

$$N = \frac{120 f}{p}$$

If we take $\dfrac{120 f}{p} = 165$, then $p = \dfrac{120 \times 50}{165} = 36.36$

Let 36 poles or 18 pairs of poles are taken.

Then $\quad N = \dfrac{120 \times 50}{36} = 166.7$ rpm *Ans.* (a)

Specific speed $\quad N_s = \dfrac{N\sqrt{P}}{H^{5/4}}$

$$= \frac{166.7\sqrt{10000}}{25^{5/4}} = 746 \quad \textit{Ans. (i)}$$

$$\frac{180\sqrt{30000}}{H_p^{5/4}} = 210$$

$$\therefore \quad H_p^{5/4} = \frac{6\sqrt{30000}}{7} \quad \therefore H_p = 54.61 \text{ m}$$

Substituting, $$\frac{D_p}{D_m} = \frac{285}{180}\sqrt{\frac{54.61}{4.5}} = 5.516 \qquad \textit{Ans.}$$

Flow through the turbine

$$Q_p = \frac{P_p}{\rho g H_p \eta} = \frac{30000 \times 1000}{1000 \times 9.81 \times 54.61 \times 0.88}$$

$$= 63.6 \text{ m}^3/\text{s} \qquad \textit{Ans.}$$

Example 10.15 Tests conducted on a one-fifth scale model of a Francis turbine under a head of 1.5 m indicated that it could develop 5 kW power at 450 rpm. Determine the speed and power of a full sized turbine while working under a head of 30 m.

Solution

$$\frac{D_m}{D_p} = \frac{N_p}{N_m}\sqrt{\frac{H_m}{H_p}}$$

$$\frac{1}{5} = \frac{N_p}{450}\sqrt{\frac{1.5}{30}}$$

$$\therefore \quad N_p = 90\sqrt{20} = 402 \text{ rpm} \qquad \textit{Ans.}$$

$$N_s = \frac{N_m\sqrt{P_m}}{H_m^{5/4}} = \frac{450\sqrt{5}}{(1.5)^{5/4}} = 606.16$$

Again, $$N_s = \frac{N_p\sqrt{P_p}}{H_p^{5/4}} = 606.16 = \frac{402\sqrt{P_p}}{30^{5/4}} = 606.16$$

$$\therefore \quad P_p = \text{Power of the full sized turbine} = 11208 \text{ kW} \qquad \textit{Ans.}$$

Example 10.16 A turbine works under a head of 19 m and has a maximum flow rate of 3 m^3/s and a speed of 600 rpm. If it has to work in another plant under a head of 5 m, at what speed must the turbine run in order to attain approximately the same efficiency and what will be the maximum flow rate?

Solution

$$\frac{D_m}{D_p} = \sqrt{\frac{H_m}{H_p}} \cdot \frac{N_p}{N_m}$$

Since the same turbine is used in both places, the diameter is the same.

Kaplan turbines $820 = \dfrac{250\sqrt{P}}{30^{5/4}}$

$\therefore$ $P = 53033.57$ kW

$\therefore$ Number of turbines $= \dfrac{89614.35}{53033.57} = 1.69$

i.e. 2 turbines *Ans.* (b)

Example 10.18 The following data refers to a proposed hydroelectric power plant:

Available head = 27 m, Catchment area 430 sq. km, Rainfall = 150 cm/year, Percentage of total rainfall utilized = 65%, Penstock efficiency = 95%, Turbine efficiency = 80%, Generator efficiency = 86% and Load factor = 0.45.

(a) Calculate the power developed.

(b) Suggest suitable turbines for the plant.

Solution

Quantity of water available per year

$$= (430 \times 10^6)\ \text{m}^2 \times 1.50\ \text{m} \times 0.65$$
$$= 419.25 \times 10^6\ \text{m}^3$$

Quantity of water available per second

$$Q = \frac{419.25 \times 10^6}{365 \times 24 \times 3600} = 13.29\ \text{m}^3$$

Power developed

$$P = h_p \times h_1 \times \eta_G\, \rho Q g H$$
$$= 0.95 \times 0.8 \times 0.86 \times 1000 \times 13.29 \times 9.81 \times 27 \times 10^{-3}$$
$$= 2300\ \text{kW}$$ *Ans.* (a)

$$\text{Load factor} = \frac{\text{Average load}}{\text{Peak load}} = 0.45$$

$$\therefore \quad \text{Peak load capacity} = \frac{2300}{0.45} = 5111\ \text{kW}$$

If two machines of equal capacity are provided,

$$\text{Capacity of each unit} = \frac{5111}{2 \times 0.86} = 2971.5\ \text{kW}$$

As the available head is low, Kaplan turbines are suggested. Two such turbines, each of 3000 kW capacity, may be installed.

Example 10.19 The run off data of a river at a particular site is tabulated in Table E10.19(a).

Table E10.19 (a)

Month	*Mean discharge (millions of cu.m.)*	*Month*	*Mean discharge (millions of cu.m.)*
January	30	July	80
February	25	August	100
March	20	September	110
April	0	October	65
May	10	November	45
June	50	December	30

(a) Draw the hydrograph and find the mean flow.
(b) Draw the flow duration curve.
(c) Find the power developed if the head available is 90 m and the overall efficiency of generation is 86 per cent. Assume each month of 30 days.

Solution

The hydrograph of the given data is shown in Fig. E10.19 (a).

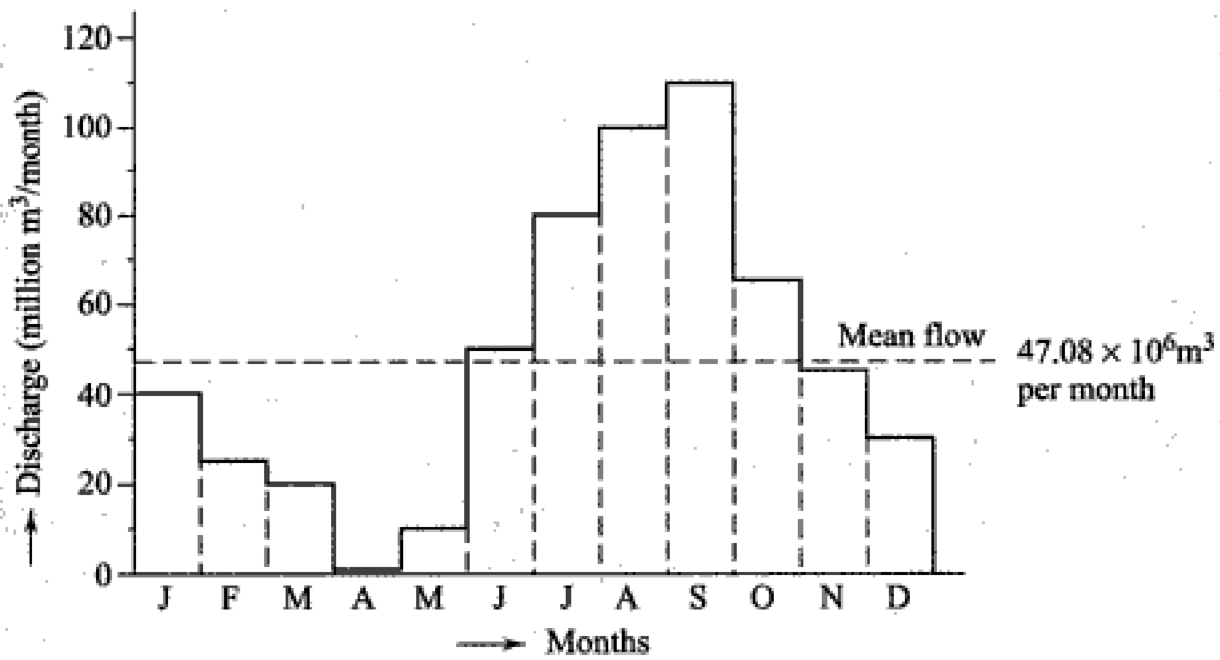


Fig. E10.19 (a) *Hydrograph*

The mean discharge

$$= \frac{30 + 25 + 20 + 0 + 10 + 50 + 80 + 100 + 110 + 65 + 45 + 30}{12}$$

$$= \frac{565}{12} = 47.08 \text{ million m}^3\text{/s}$$

To obtain the flow duration curve, it is necessary to find the lengths of time during which certain flows are available as given in Table E10.19 (b).

Table E10.19 (b)

Discharge per month (million m^3)	*Total number of months during which flow is available*	*Percentage time*
0	12	100
10	11	91.7
20	10	83.3
25	9	75.0
30	8	66.7
40	6	50.0
50	5	41.7
60	4	33.3
70	3	25.0
80	3	25.0
90	3	25.0
100	2	16.7
110	1	8.3

The flow duration curve from the above data shown in Fig. E10.19 (b).

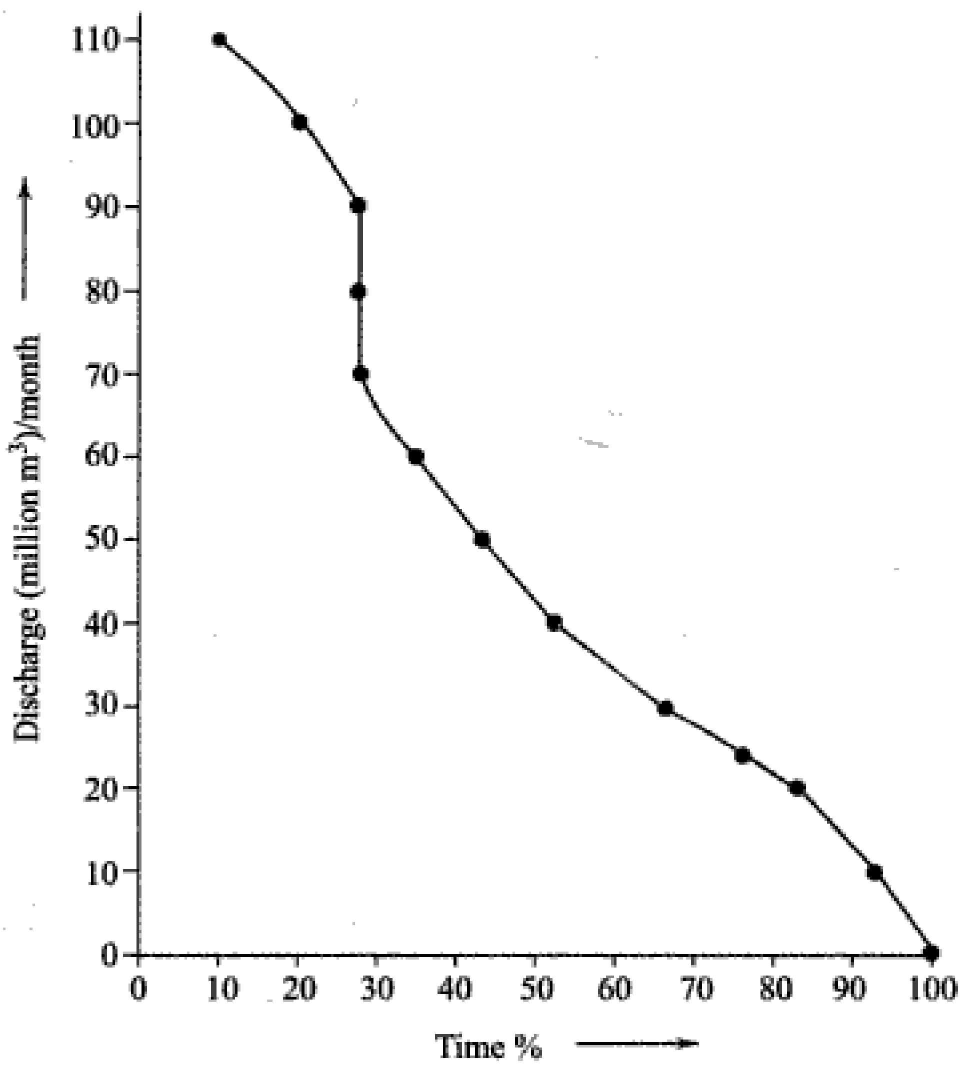


Fig. E10.19 (b) *Flow Duration Curve*

Power developed,

$$P = rQgH\eta_0 \times 10^{-3} \text{ kW}$$

$$= \frac{47.08 \times 10^6 \times 9.81 \times 90 \times 0.86}{30 \times 24 \times 3600}$$

$$= 13.79 \text{ MW}$$ *Ans.*

SHORT-ANSWER QUESTIONS

10.1 What are the two key parameters of water on which the magnitude of hydro-power depends?
10.2 Enlist the advantages and disadvantages of water power.
10.3 Explain the optimization of hydro-thermal mix in meeting the power demand of a certain region.
10.4 Discuss the factors which should be considered while selecting a site for a hydroelectric plant.
10.5 What do you understand by hydrology? Explain the hydrological cycle.
10.6 What do you mean by (i) hydrograph (ii) Flow duration curve and (iii) power duration curve? What is primary power and secondary power?
10.7 What is a mass curve? What does the slope of the curve at a point indicate?
10.8 Explain what you mean by storage and pondage. Why are they required?
10.9 State the essential elements of a hydroelectric power plant.
10.10 What is a catchment area? Why is a reservoir required?
10.11 State the functions of a dam. How are dams classified? Briefly describe a few important types of dams. How would you select the site and the type of the dam?
10.12 What is a spillway? Why are spillways required? What are the different types of spillways?
10.13 Explain the following terms
(i) Headrace
(ii) Tailrace
(iii) Canal
(iv) Flume
(v) Tunnel
(vi) Pipeline
(vii) Penstock
10.14 What is a surge tank? Why is it important in a hydro-plant?
10.15 What is the function of a draft tube? Briefly explain the different types of draft tubes?
10.16 Enlist the various equipment provided in a powerhouse.
10.17 Explain the different methods of classifying a hydroelectric power plant. What is a run off river plant?
10.18 Explain with a neat sketch a pumped storage plant. What are its advantages?
10.19 What are mini and micro-hydel plants? Why are they important these days?

10.20 How does a hydraulic turbine convert energy? What is a waterwheel?
10.21 Describe the classification of hydraulic turbines in different categories.
10.22 How is the size of turbine ascertained?
10.23 Classify the hydro-turbines according to head, power, size and specific speed.
10.24 What type of turbine would you recommend for the following heads and why?
(a) 1000 m
(b) 150 m
(c) 20 m
10.25 Explain with a neat sketch the principle of operation of a Pelton turbine.
10.26 What are the functions of (a) spear and (b) deflector plate in a Pelton wheel?
10.27 Deduce the ratio between the peripheral velocity of the runner and the velocity of the jet for attaining (a) maximum efficiency and (b) maximum power.
10.28 Explain the following terms.
(i) Jet ratio
(ii) Speed ratio
10.29 How are the number of jets in a Pelton wheel ascertained?
10.30 "The number of buckets in a Pelton wheel is a function of the jet ratio". Explain.
10.31 How is the degree of reaction, R, of a hydraulic turbine defined? Explain the cases for $R = 0$, $R = 0.50$ and $R = 1$.
10.32 Explain with a neat schematic diagram the operation of a Francis turbine. What are its advantages?
10.33 Draw the velocity diagrams of an inward-flow Francis turbine and derive the expression of blading efficiency in terms of vane angles.
10.34 What are Kaplan turbines? How is a Kaplan turbine different from a propeller turbine? Explain the characteristic features of a Kaplan turbine.
10.35 What is a Deriaz turbine? What is its importance?
10.36 What is a bulb turbine? Where is it used?
10.37 Define specific speed of a turbine. Derive its expression in terms of speed, power and head.
10.38 What is scale ratio? What is its importance?
10.39 Define unit speed, unit power and unit discharge and derive their relevant relations.
10.40 What do you understand by cavitation? What are its effects? How can it be minimized?
10.41 Write short notes on the following.
(i) pitting of turbine blades, and its prevention (ii) servo-motors.
10.42 What is the synchronous speed of the turbine runner? How is it estimated?
10.43 Explain with a neat sketch the governing principle of an impulse turbine. What are the functions of needle valve and the deflector?
10.44 How is the governing of a reaction turbine carried out? Explain with a neat sketch. What is the function of relief valve?

10.45 When and why are surge tanks and forebays provided? Explain a few types of surge tanks.

10.46 Discuss with neat sketches the characteristic curves related to the performance of hydraulic turbine.

10.47 What is "runaway speed"? How does it affect the turbine design?

10.48 How is the type of turbine selected in a certain hydro-plant? Discuss the effects of head, specific speed, height of installation, the operating characteristics and the capacity on the selection process.

PROBLEMS

10.1 A Pelton wheel is required to develop 4500 kW at 400 rpm operating under an available head of 360 m. There are two equal jets and the bucket angle is 170°. The bucket pitch circle diameter is 1.82 m. Taking k for the buckets as 0.85, determine (a) the efficiency of the runner and (b) the diameter of each jet.

Ans. (a) 0.9106 (b) 0.103 m

10.2 In a Pelton wheel the diameter of the bucket circle is 2 m and the deflecting angle of the bucket is 162°. The jet has 165 mm diameter, the pressure behind the nozzle is 700 kPa and the wheel rotates at 320 rpm. Neglecting friction, find the power developed by the wheel and the hydraulic efficiency.

Ans. 351.9 kW, 0.616

10.3 A Pelton wheel develops 8 MW under a net head of 130 m at a speed of 200 rpm. Assuming $c_v = 0.98$, hydraulic efficiency = 87 per cent, speed ratio = 0.46 and the ratio of jet-to-wheel diameter = 1/9, determine (a) the flow required (b) the diameter of the wheel (c) the diameter and number of jets needed.

Ans. (a) 7.51 m^3/s,(b) 2.17 m, (c) 0.242 m, 3.

10.4 A Pelton wheel has to develop 12 MW under a head of 300 m at a speed of 500 rpm. If the diameter of the jet is not to exceed 1/9 of the wheel diameter, estimate the number and diameter of the jets, diameter of the bucket wheel and the quantity of flow. Assume overall efficiency = 88 per cent, $C_v = 0.97$ and $\phi = 0.45$.

Ans. $D = 1.32$ m, $d = 0.147$ m, $n = 4$ and $Q = 4.63\ m^3/s$

10.5 A Pelton wheel to be designed is to run at 300 rpm under an effective head of 150 m. The ratio of the nozzle diameter to the pitch circle diameter is 1/12. Assuming efficiency = 84%, $C_v = 0.98$ and speed ratio = 0.45, determine (a) the diameter of the wheel, (b) diameter of the jet (c) the quantity of water flow (d) the minimum number of buckets required; and (e) the power developed.

Ans. (a) 1.55 m (b) 0.129 m (c) 0.694 m^3/s (d) 21 (e) 858 kW.

10.6 A jet of 75 mm diameter strikes the bucket of an impulse wheel and gets deflected by an angle of 165°. The speed of the bucket is 45.5 m/s. Find the velocity of the jet for maximum efficiency and the power developed.

Ans. 91 m/s, 1545 kW.

10.7 An inward flow reaction turbine having an overall efficiency of 75 per cent delivers 132 kW. The head H is 9 m, velocity of the periphery of the wheel is 54 m/s and the radial velocity is 18 m/s. The wheel makes 120 rpm. The hydraulic losses in the turbine are 20 per cent of the available energy. Determine the discharge at the inlet, the guide blade angle, the wheel blade angle and the diameter and width of the wheel. Assume radial discharge.

Ans. 2 m^3/s, $a_1 = 56° \, 49'$ $\beta_1 = 23°$, $D = 2.86$ m $B = 0.037$ m.

10.8 In a Francis turbine of low specific speed, the velocity of flow from inlet to exit of the runner remains constant. If the turbine discharges radially, show that the degree of reaction R can be expressed as

$$R = \frac{1}{2} - \frac{1}{2}\left[\frac{\cot \beta_1}{\cot \alpha - \cos \beta_1}\right]$$

where α and β_1 are the guide and runner vane angle respectively and the degree of reaction is equal to the ratio of pressure drop to the hydraulic work done in the runner, assuming that the losses in the runner and negligible.

10.9 A Francis turbine with an overall efficiency of 76 per cent is required to produce 180 kW. It is working under a head of 8 m. The peripheral velocity is $0.25\,(2\,gH)^{1/2}$ and the radial velocity of flow is $0.95\,(2\,gH)^{1/2}$. The wheel runs at 150 rpm and the hydraulic losses in the turbine are 20 per cent of the available energy. Assuming radial discharge, determine (a) the guide vane angle (b) the wheel vane angle at inlet (c) the diameter of the wheel at inlet; and (d) the width of the wheel at inlet.

Ans. (a) 30° 45′, (b) 35° 12′, (c) 0.398 m, (d) 0.203 m

10.10 An inward flow reaction turbine works under a head of 22.5 m. The external and internal diameters of the runner are 1.35 m and 1 m respectively. The angle of guide vanes is 15° and the moving vanes are radial at inlet. Radial velocity of flow through the runner is constant and there is no velocity of whirl at outlet. Determine the speed of the runner in rpm and the angle of vane at outlet. If the turbine develops 375 kW, find the specific speed. Neglect friction losses.

Ans. 206.5 rpm, 19°53′, 81.6

10.11 Two inward flow reaction turbines have the same runner diameter of 0.60 m and the same efficiency. They work under the same head and they have the same velocity of flow of 6 m/s. One of the runners A revolves at 520 rpm and has an inlet vane angle of 65°. If the other runner B has an inlet vane angle of 110°, at what speed should it run?

Ans. 600 rpm.

10.12 Water enters an inward flow turbine at an angle of 22° to the tangent to the outer rim and leaves the turbine radially. If the speed of the wheel is 300 rpm and the velocity of flow is constant at 3 m/s, find the necessary angles of blades when the inner and outer diameters of the turbine are 0.3 m and 0.6 m respectively. If the width of the wheel at inlet is 0.15 m, calculate the power developed. Neglect the thickness of blades.

Ans. $\beta_1 = 59°54'$, $\beta_2 = 32°32'$, $P = 61.15$ kW.

size if (a) Francis turbines having specific speed not greater than 200 or (b) Kaplan turbines of specific speed not greater than 600, are used.

Ans. (a) 16, (b) 3.

10.21 A run off of 30 m^3/s is available at 7.5 m head for generating the desired power. The turbine efficiency is 85 per cent. (a) Is it feasible to develop the desired power by two turbines with 50 rpm and the specific speed of turbine not greater than 450? (b) What type of runner is required to be used? (c) What is the diameter of the runner if the speed ratio is 0.85?

Ans. (a) $As/N_s = 203$, two turbine units can be used. (b) Francis turbine, (c) 3.93 m.

10.22 From the following table of mean monthly discharge, draw the following curves.

Month	*Discharge (m^3/s)*	*Month*	*Discharge (m^3/s)*
January	100	July	1100
February	325	August	1300
March	400	September	1000
April	700	October	800
May	850	November	600
June	900	December	300

(a) the hydrograph (b) the flow duration curve.

10.23 From the following table of the mean monthly discharge for 12 months of a river at a site, draw (a) the hydrograph and find the average monthly flow, and the power available at mean flow of water for head 90 m and overall efficiency of generation 90 per cent. Assume 30 days in each month.

Month	April	May	June	July	August	September
$Q - m^3 \times 10^6$	500	200	1500	2500	3000	2400
Month	October	Nov.	Dec.	January	Feb.	March
$Q - m^3 \times 10^6$	2000	1500	1500	1000	800	600

(b) Draw the flow duration curve from the data in the hydrograph.

10.24 The following data pertain to a hydroelectric plant. Available head = 140 m, catchment area = 2000 sq. km; annual average rainfall = 145 cm, turbine efficiency = 85%, generator efficiency = 90%, percolation and evaporation losses = 16%. Determine the power developed and suggest the type of turbine to be used if the runner speed is to be kept below 240 rpm.

Ans. 8.11 MW, Pelton turbine with 4 jets may be used.

10.25 (a) Discuss the differences between Kaplan, Francis and Pelton turbines and state the types of power plants they are suitable for.

(b) At a particular hydroelectric power plant site the discharge of water is 400 m^3/s and the head is 25 m. The turbine efficiency is 88 per cent. The generator is directly coupled to the turbine having frequency of generation as 50 cycles/s and number of poles as 24. Calculate the least number of turbines required if (a) a Francis turbine is used with a specific speed of 300 (b) a Kaplan turbine with a specific speed of 750 is used.

11

Diesel Engine and Gas Turbine Power Plants

Diesel electric plants in the range of 2 to 50 MW capacity are used as central stations for small supply authorities and works and they are universally adapted to supplement hydroelectric or thermal power stations where standby generating plants are essential for starting from cold or under emergency conditions.

11.1 APPLICATIONS OF DIESEL ENGINES IN POWER FIELD

The diesel electric power plants are chiefly used in the following field.

(a) *Peak load plant* Diesel plants can be used in combination with thermal or hydro-plants as peak load units. They can be easily started or stopped at a short notice to meet the peak demand.

(b) *Mobile plant* Diesel plants mounted on trailers can be used for temporary or emergency purposes such as for supplying power to large civil engineering works.

(c) *Standby unit* If the main unit fails or cannot cope up with the demand, a diesel plant can supply the necessary power. For example, if water available in a hydro-plant is not adequately available due to less rainfall, the diesel station can operate in parallel to generate the short fall in power.

(d) *Emergency plant* During power interruption in a vital unit like a key industrial plant or a hospital, a diesel electric plant can be used to generate the needed power.

(e) *Nursery station* In the absence of main grid, a diesel plant can be installed to supply power in a small town. In course of time, when electricity from the main grid becomes available in the town, the diesel unit can be shifted to some other area which needs power on a small scale. Such a diesel plant is called a "nursery station".

(f) *Starting stations* Diesel units can be used to run the auxiliaries (like *FD* and *ID* fans, *BFP*, etc.) for starting a large steam power plant.

(g) *Central stations* Diesel electric plants can be used as central station where the capacity required is small.

11.2 ADVANTAGES AND DISADVANTAGES OF DIESEL ENGINE POWER PLANT

Following are the advantages of diesel electric stations.

1. It is easy to design and install these electric stations.
2. They are easily available in standard capacities.
3. They can respond to load changes without much difficulty.
4. There are less standby losses.
5. They occupy less space.
6. They can be started and stopped quickly.
7. They require less cooling water.
8. Capital cost is less.
9. Less operating and supervising staff required.
10. High efficiency of energy conversion from fuel to electricity.
11. Efficiency at part loads is also higher.
12. Less of civil engineering work is required.
13. They can be located near the load centre.
14. There is no ash handling problem.
15. Easier lubrication system.

Following are some of the disadvantages in installing diesel units for power generation.

1. High operating cost.
2. High maintenance and lubrication cost.
3. Capacity is restricted. Cannot be of very big size.
4. Noise problem.
5. Cannot supply overload.
6. Unhygienic emissions.

11.3 TYPES OF DIESEL PLANTS

In a diesel engine, air is first compressed to a high pressure and a small volume (volumetric compression ratio varying between 13 and 22) at which the hot air temperature is more than the self ignition temperature of the fuel oil, which is sprayed into the compressed air in fine atomized form. The combustion products expand doing work on the piston till the exhaust valve opens. Exhaust of the products then takes place, at the end of which fresh air is again taken into the cylinder and the cycle repeats itself. There is no spark plug. Fuel oil spray burns in the hot compressed air. Hence, a diesel engine is also called a compression ignition or a C.I. engine.

Diesel engines can be four-stroke and two-stroke, horizontal and vertical, single-cylinder and multi-cylinder, naturally aspirated and supercharged. A textbook on internal combustion engines (like Obert, Lichty, Taylor, Rogowski, Mathur and Sharma, Ganesan, etc. as given in the bibliography at the end of the chapter can be consulted for a good understanding of analysis and operation of diesel engines. Here only some of the salient features of diesel engines are being discussed.

11.4 GENERAL LAYOUT

The cross-section of an air cooled IC engine with principal parts is shown in Fig. 11.1. The essential components of a diesel electric plant are shown in Fig. 11.2. It consists of the following elements.

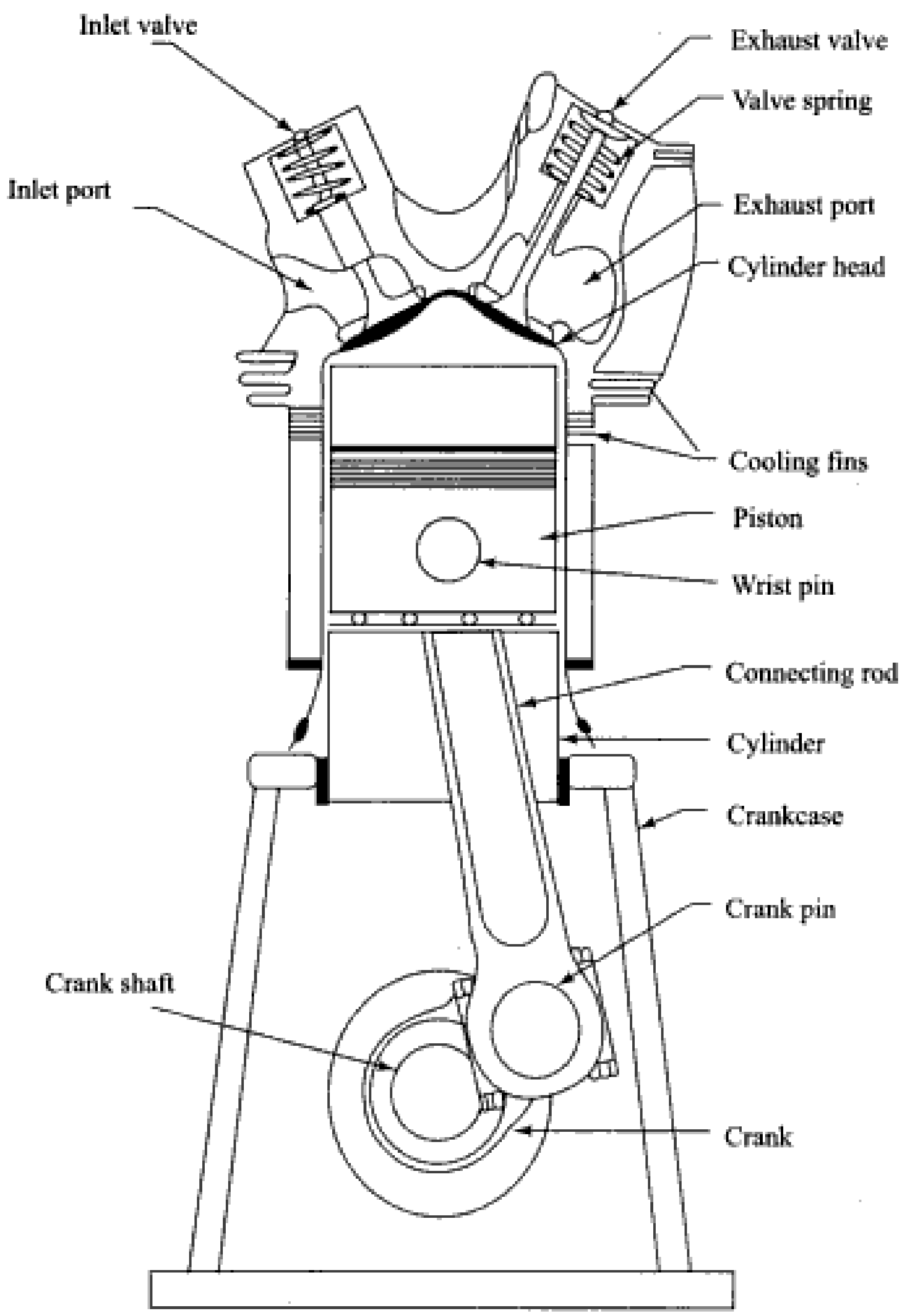


Fig. 11.1 *General View of an air-cooled IC engine*

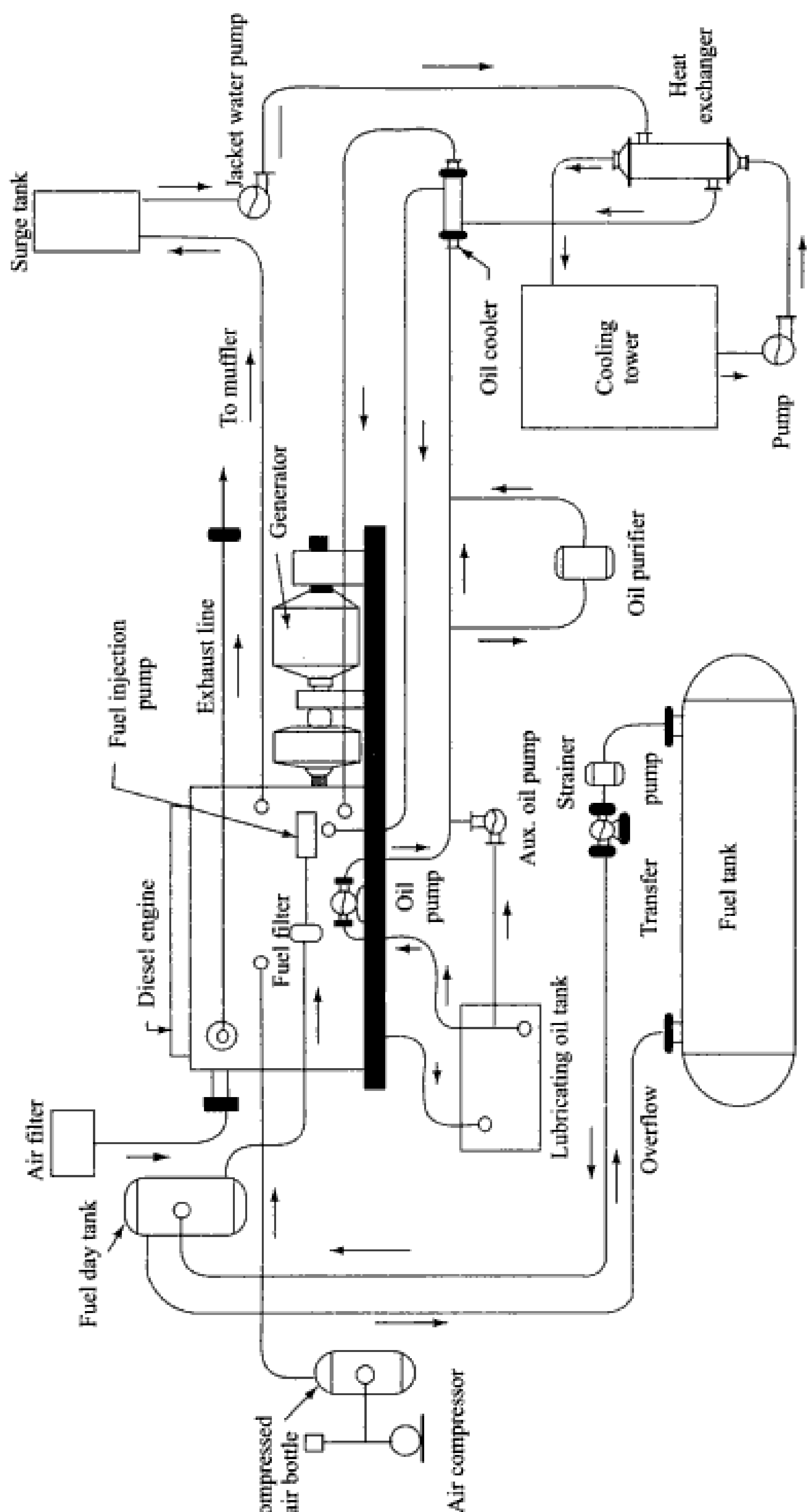


Fig. 11.2 *Schematic arrangement of a diesel engine power plant*

1. *Engine* It is the main component of the plant and is directly coupled to the generator.

2. *Air intake system* It conveys fresh air through louvres and air filter that removes dirt, etc. causing wear of the engine. Supercharger, if fitted, is generally driven by the engine itself and it augments the power output of the engine.

3. *Exhaust system* It discharges the engine exhaust to the atmosphere. The exhaust manifold connects the engine cylinder exhaust outlets to the exhaust pipe which is provided with a muffler or silencer to reduce pressure on the exhaust line and eliminate most of the noise which may result if gases are discharged directly to the atmosphere. The exhaust pipe should have flexible tubing system to take up the effects of expansion due to high temperature and also isolate the exhaust system from the engine vibration (Fig. 11.3).

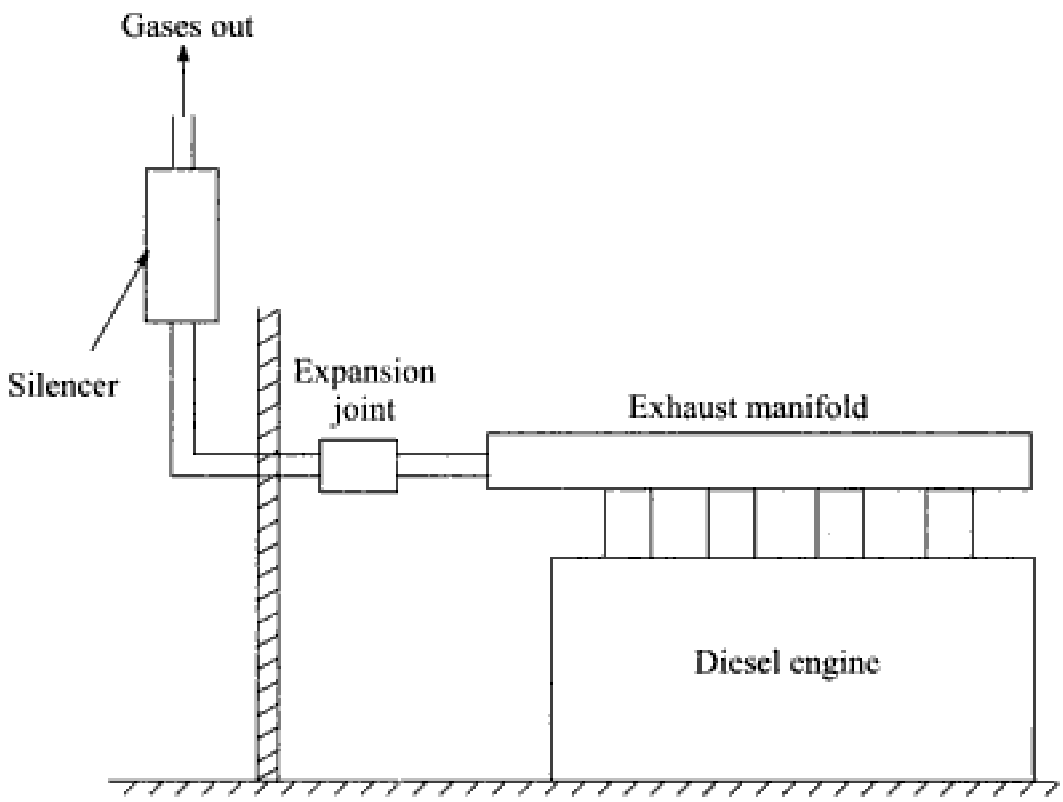


Fig. 11.3 *Diesel engine exhaust system*

There is scope of waste heat utilization from the diesel engine exhaust by installing a waste heat boiler to raise low pressure steam which can be used for any process, purpose or for generating electricity. The hot exhaust may also be utilized to heat water in a gas-to-water heat exchanger which can be in the form of a water coil installed in the exhaust muffler. It can also be used for air heating where the exhaust pipe is surrounded by the cold air jacket.

4. *Fuel system* Fuel oil may be delivered at the plant site by trucks, railway wagons or barges and oil tankers. An unloading facility delivers oil to the main storage tanks from where oil is pumped to small service storage tanks known as engine day tanks, which store oil for approximately eight hours of operation (Fig. 11.4). Coils heated by hot water or steam reduce oil viscosity to reduce pumping power.

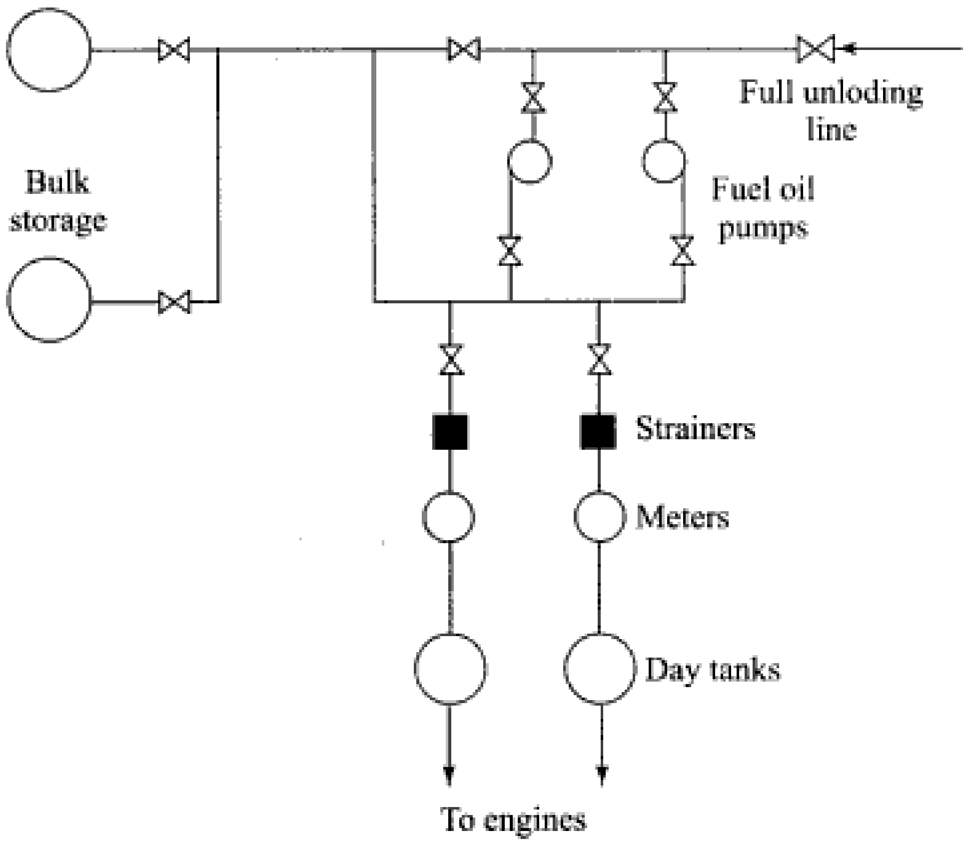


Fig. 11.4 *Fuel storage in a diesel engine power plant*

The fuel injection system is the heart of a diesel engine. Engines driving electric generators have lower speeds and simple combustion chambers that promote good mixing of fuel and air. The fuel injection system performs the following functions.

(a) Filter the fuel
(b) Meter the correct quantity of the fuel to be injected
(c) Time the injection process
(d) Regulate the fuel supply
(e) Secure fine atomization of fuel oil
(f) Distribute the atomized fuel properly in the combustion chamber.

Oil is atomized either by air blast or pressure jet. Early diesel engines used air blast fuel atomization where compressed air at about 70 bar was used to atomize as well as to inject the fuel oil. For this an air compressor and a storage tank are needed, which becomes expensive. In pressure jet atomization the fuel oil is forced to flow through spray nozzles at a pressure above 100 bar. It is known as solid injection, which is more common. Solid injection systems may be classified as follows.

(a) Common rail injection system
(b) Individual pump injection system
(c) Distributor system

(a) *Common rail injection system* A single pump supplies fuel under high pressure to a fuel header or common rail (Fig. 11.5). The high pressure in the header forces the fuel to each of the nozzles located in the cylinders. At the proper time a mechanically operated valve (by means of a push rod and a rocker arm) allows the fuel to enter the cylinder through the nozzle. The amount of fuel entering the cylinder is regulated by varying the length of the push rod stroke.

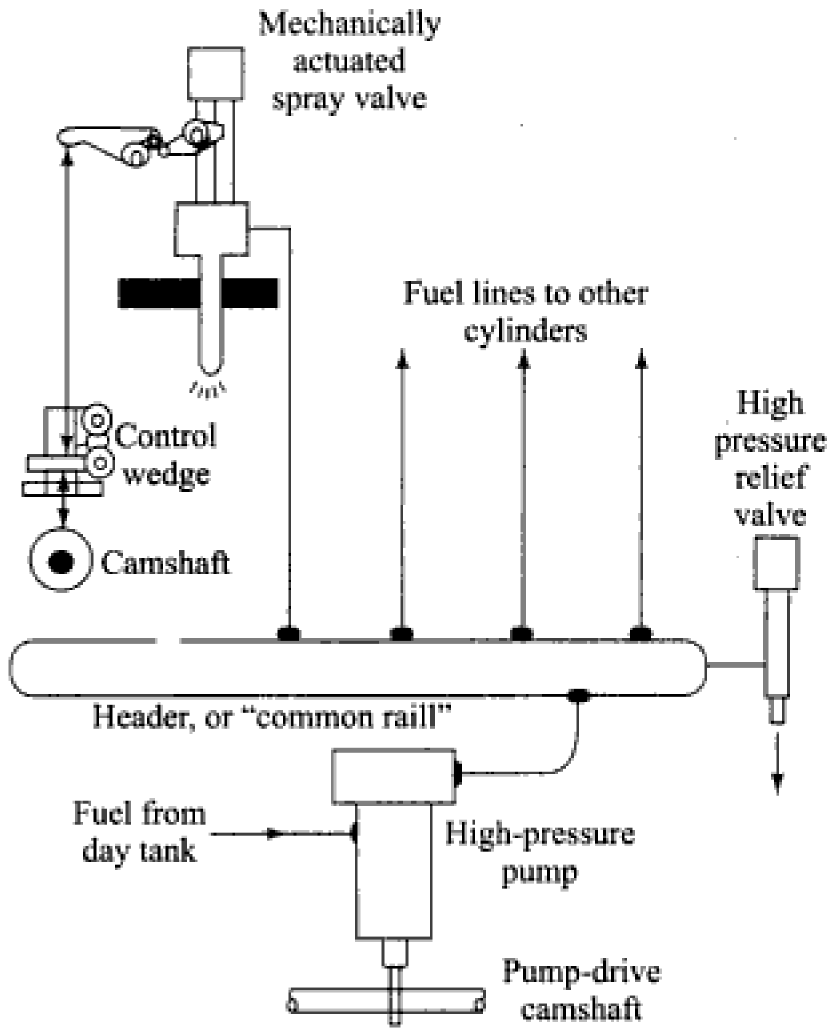


Fig. 11.5 *Common-rail injection system*

(b) *Individual pump injection system* Each cylinder is provided with one pump and one injector (Fig. 11.6). The pump directly feeds oil to the cylinder, meters the oil and controls the injection timing. The nozzle contains a delivery valve actuated by fuel oil pressure.

(c) *Distributor system* In this system the fuel is metered at a central point. A pump pressurises, meters the fuel and times the injection (Fig. 11.7). The fuel is then distributed to cylinders in correct firing order by cam-operated poppet valves which open to admit oil to the fuel nozzle.

Fuel pump It consists of a plunger (L) driven by a cam and tappet mechanism, which reciprocates inside a barrel B (Fig. 11.8). A vertical groove in the plunger leads to a helical groove. The delivery valve (V) lifts off its seat under oil pressure against the spring force (S). When the plunger is at the bottom, the supply port Y and the spill port (SP) are uncovered and low pressure filtered oil is forced into the barrel. As the plunger moves up, the ports Y and SP are closed and oil gets compressed lifting the delivery valve

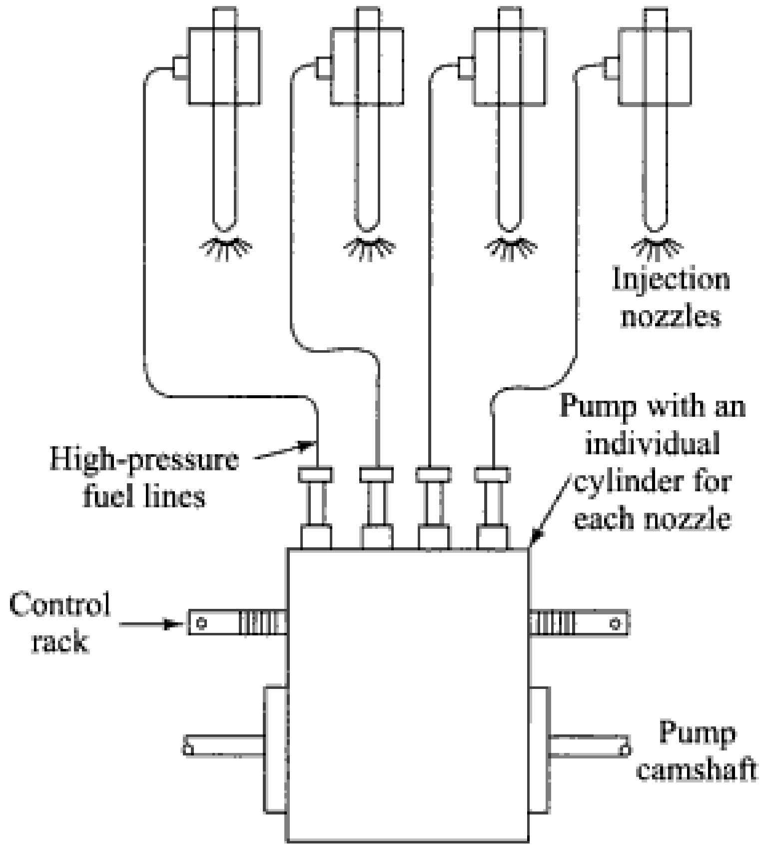


Fig. 11.6 *Individual pump injection system*

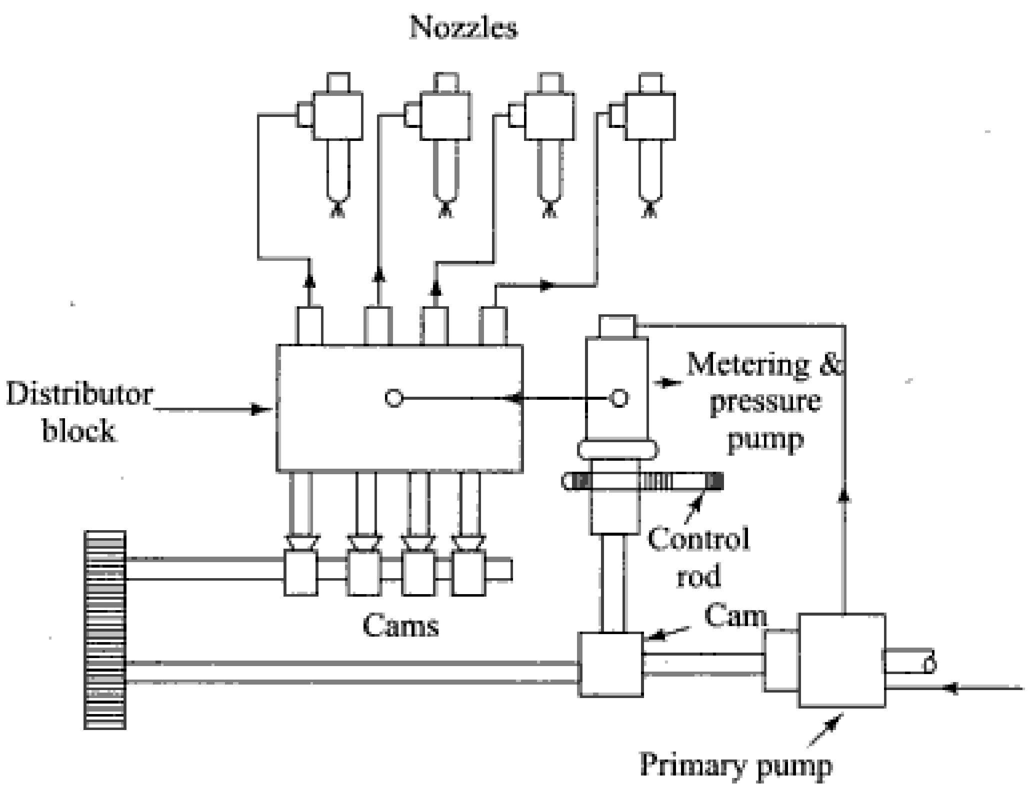


Fig. 11.7 *Distributor fuel injection system*

V to enter the injector nozzle through the passage *P*. When the plunger moves up further, the port *SP* gets connected to the fuel at its top through the vertical groove resulting in a sudden drop in pressure and the delivery valve falls back to its seat against the spring force. The plunger is rotated by the rack *R* operated by a governor. By rotating the plunger the position of the helical groove relative to the supply port *Y* can be varied. The length of stroke during which the oil is delivered is varied and so the quantity of fuel delivered to the engine also varies accordingly.

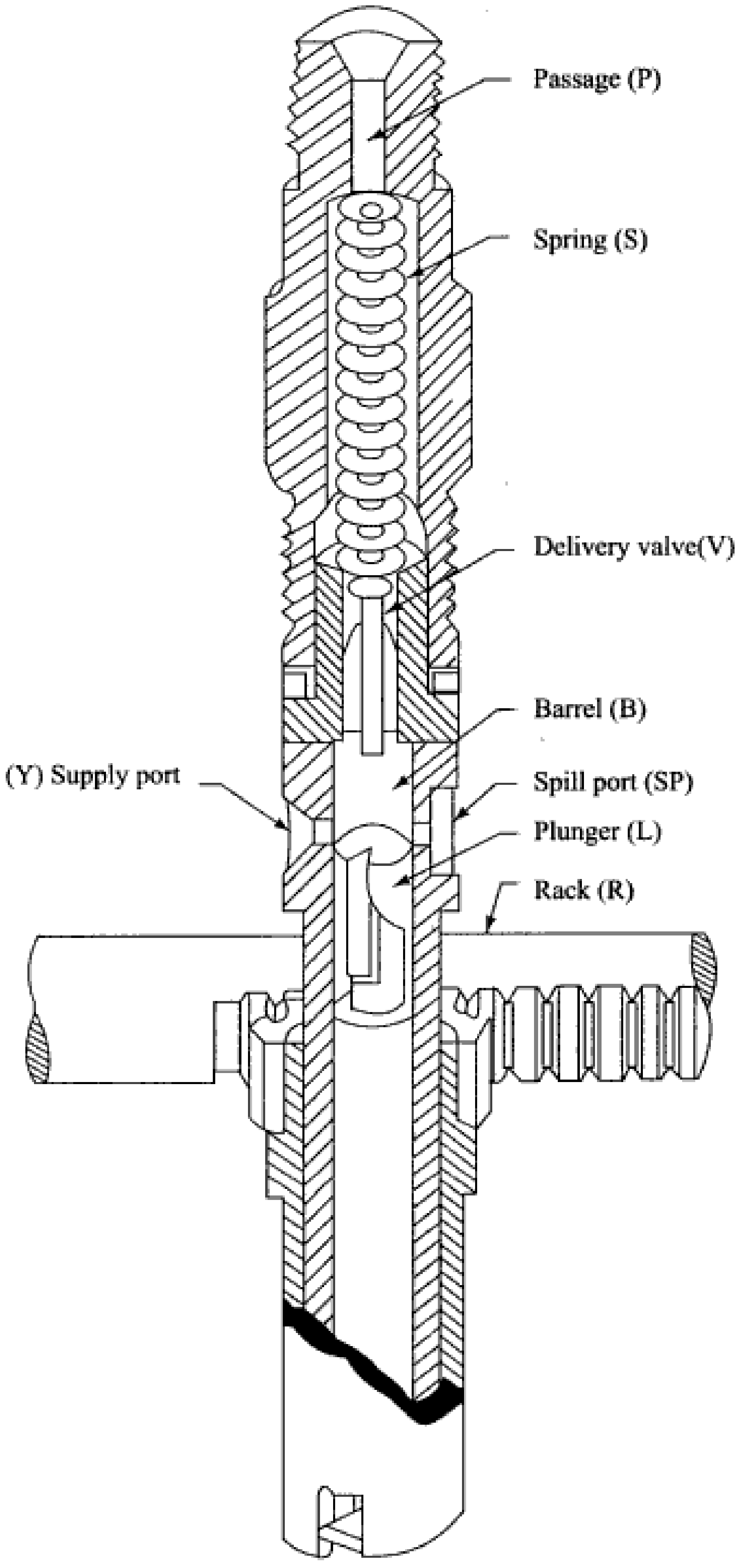


Fig. 11.8 *Fuel pump*

Fuel injector It consists of a nozzle valve (*NV*) fitted in the nozzle body (*NB*) (Fig. 11.9). The nozzle valve is held on its seat by a spring force (*S*) acting through

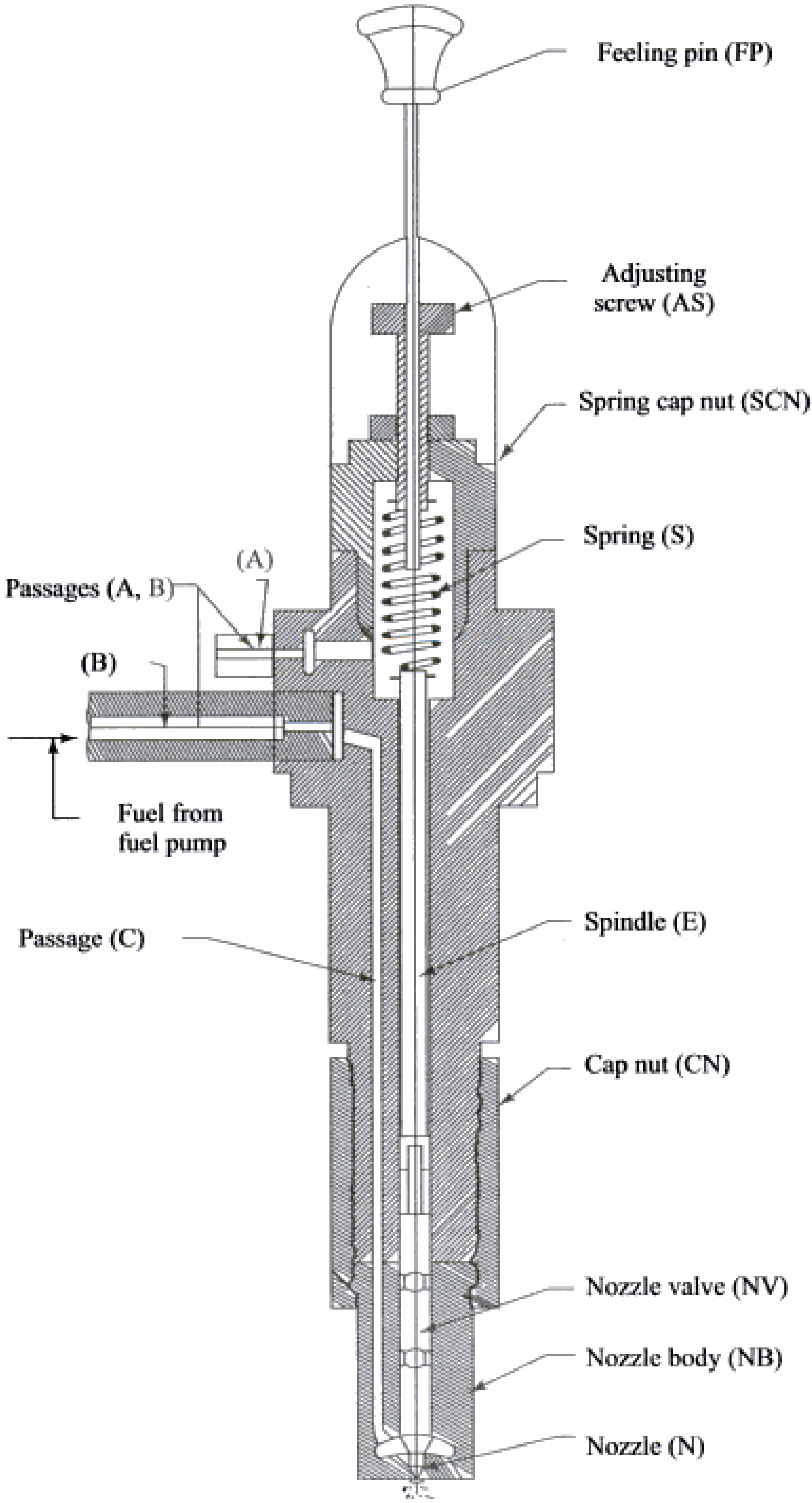


Fig. 11.9 *Fuel injector*

the spindle (*E*). A feeling pin (*FP*) at the top indicates whether the valve is working properly or not. The high pressure oil from the fuel pump enters the injector through the passages *B* and *C* and lifts the nozzle valve to admit oil into the fuel nozzle that injects oil to the cylinder in fine atomized spray. As the oil pressure falls, the nozzle valve comes back to its seat under spring force and the fuel supply is cut off. Any leakage of fuel (due to wearing out of valve) accumulated above the valve returns to the fuel tank through the passage *A*.

Various types of nozzles are used in CI engines, but the most common types are the single-orifice, multi-orifice and pintle nozzles as shown in Fig. 11.10. A pintle nozzle is clogged less by carbon particles and is thus less expensive to maintain.

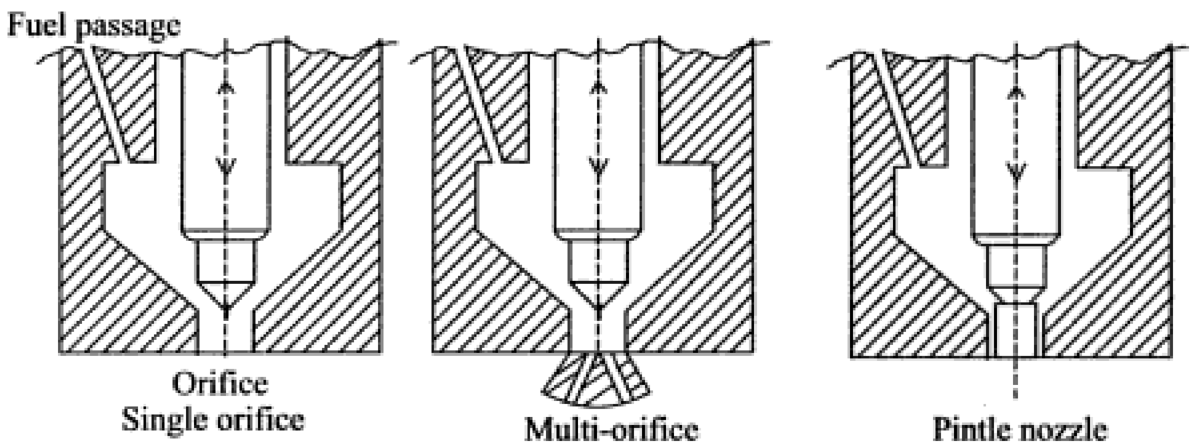


Fig. 11.10 *Vairous types of nozzles*

5. *Cooling system* The temperature of the gases inside the cylinder may be as high as 2750°C. If there is no external cooling, the cylinder walls and piston will tend to assume the average temperature of the gases which may be of the order of 1000° to 1500°C. The cooling of the engine is necessary for the following reasons.

(a) The lubricating oil used determines the maximum engine temperature that can be used. This temperature varies from 160°C to 200°C. Above these temperatures the lubricating oil deteriorates very rapidly and may evaporate and burn damaging the piston and cylinder surfaces. Piston seizure due to overheating may also occur.

(b) The strength of the materials used for various engine parts decreases with increase in temperature. Local thermal stresses can develop due to uneven expansion of various parts, often resulting in cracking.

(c) High engine temperatures may result in very hot exhaust valve, giving rise to pre-ignition and detonation or knocking.

(d) Due to high cylinder head temperature, the volumetric efficiency and hence power output of the engine are reduced.

Following are the two methods of cooling the engine.

(i) Air cooling
(ii) Water cooling

Air cooling is used in small engines, where fins are provided to increase heat transfer surface area.

Big diesel engines are always water cooled. The cylinder and its head are enclosed in a water jacket which is connected to a radiator. Water flowing in the jacket carries away the heat from the engine and becomes heated. The hot water then flows into the radiator and gets cooled by rejecting heat to air from the radiator walls. Cooled water is again circulated in the water jacket.

Various methods used for circulating the water around the cylinder are the following.

(a) *Thermosiphon cooling* In this method water flow is caused by density difference (Fig. 11.11). The rate of circulation is however slow and insufficient.

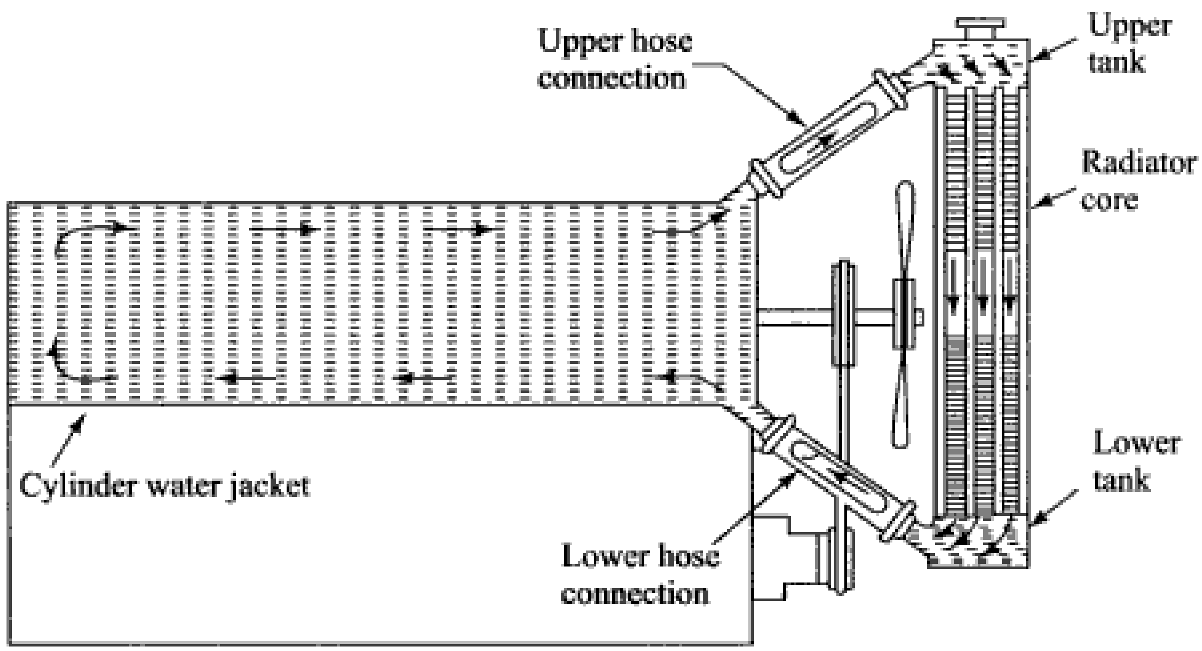


Fig. 11.11 *Thermosiphon cooling*

(b) *Forced cooling by pump* In this method a pump, taking power from the engine, forces water to circulate, ensuring engine cooling under all operating conditions. There may be overcooling which may cause low temperature corrosion of metal parts due to the presence of acids.

(c) *Thermostat cooling* This is a method in which a thermostat maintains the desired temperature and protects the engine from getting overcooled (Fig. 11.12).

(d) *Pressurized water cooling* In this method a higher water pressure, 1.5 to 2 bar, is maintained to increase heat transfer in the radiator. A pressure relief valve is provided against any pressure drop or vacuum.

(e) *Evaporative cooling* In this method water is allowed to evaporate absorbing the latent heat of evaporation from the cylinder walls. The cooling circuit is such that the coolant is always liquid and the steam flashes in a separate vessel (Fig. 11.13).

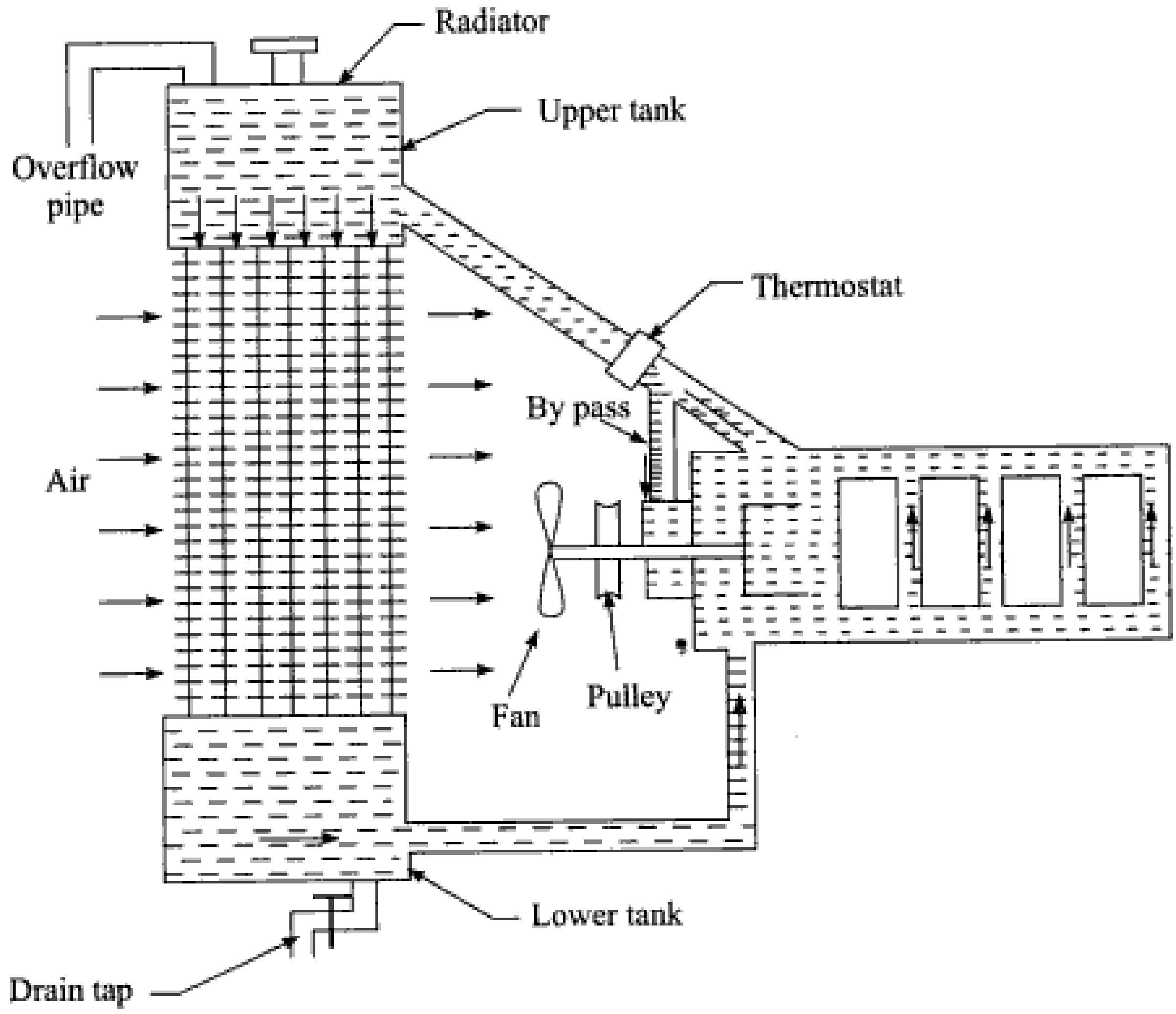


Fig. 11.12 *Thermostat cooling*

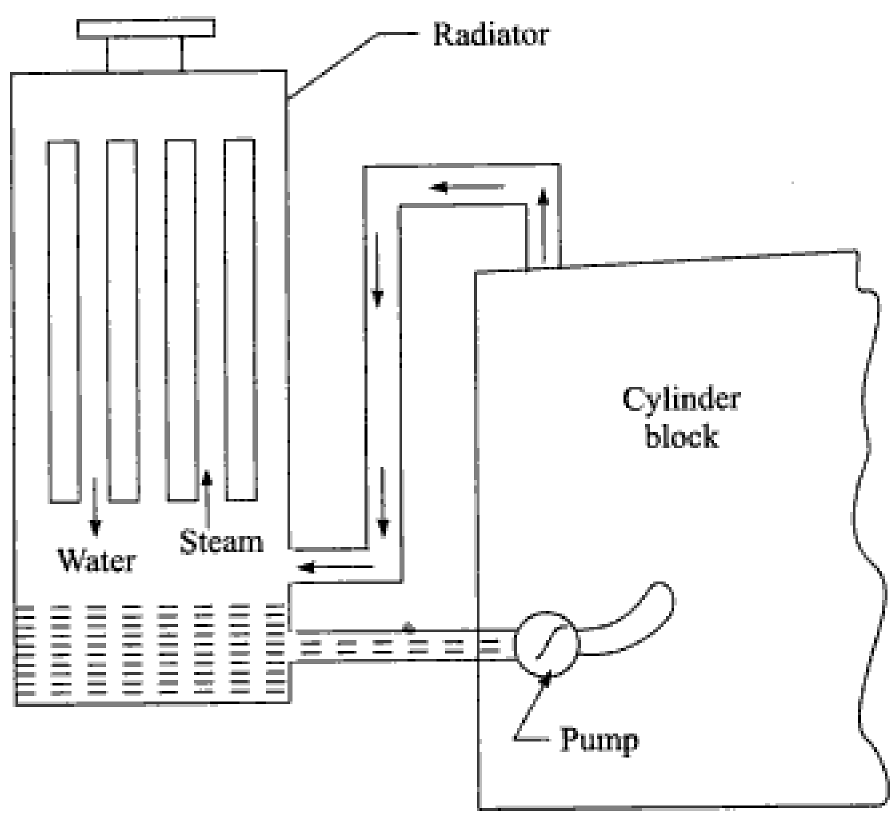


Fig. 11.13 *Evaporative cooling*

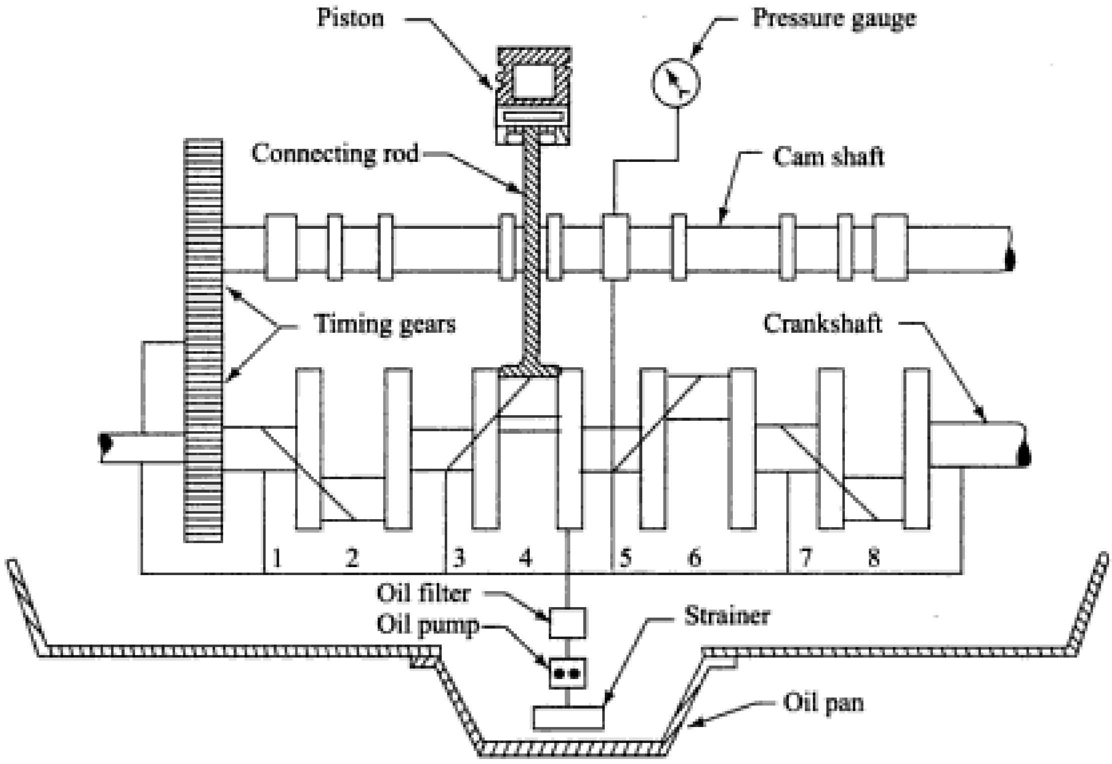


Fig. 11.15 *Full pressure lubrication system*

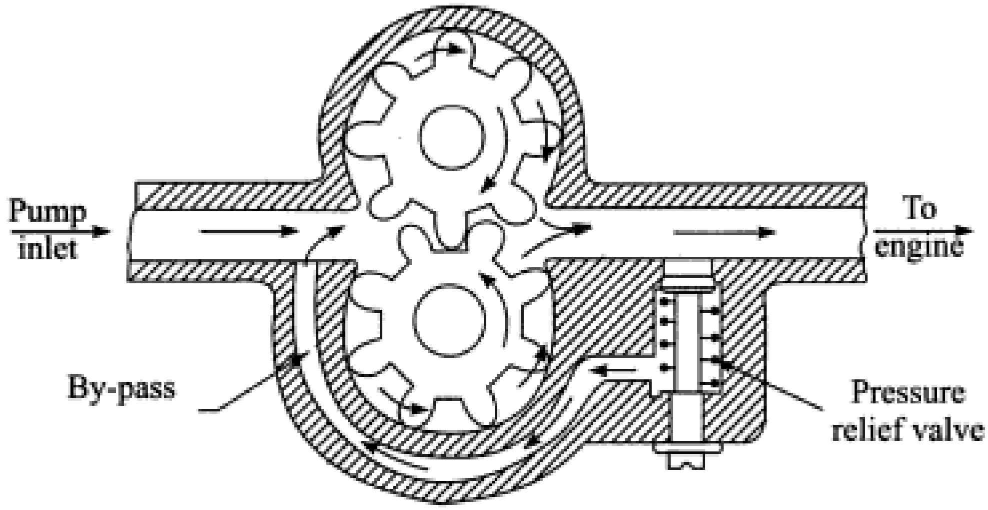


Fig. 11.16 *Gear oil pump with relief valve*

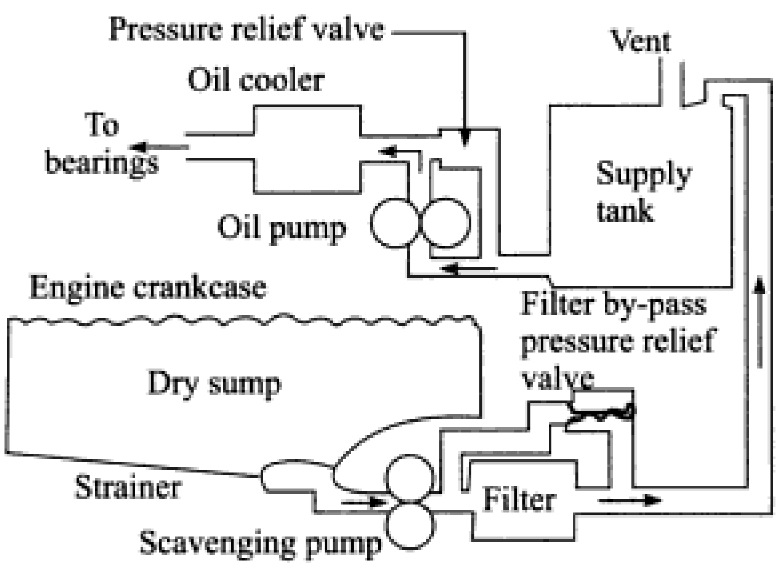


Fig. 11.17 *Full flow dry sump lubrication*

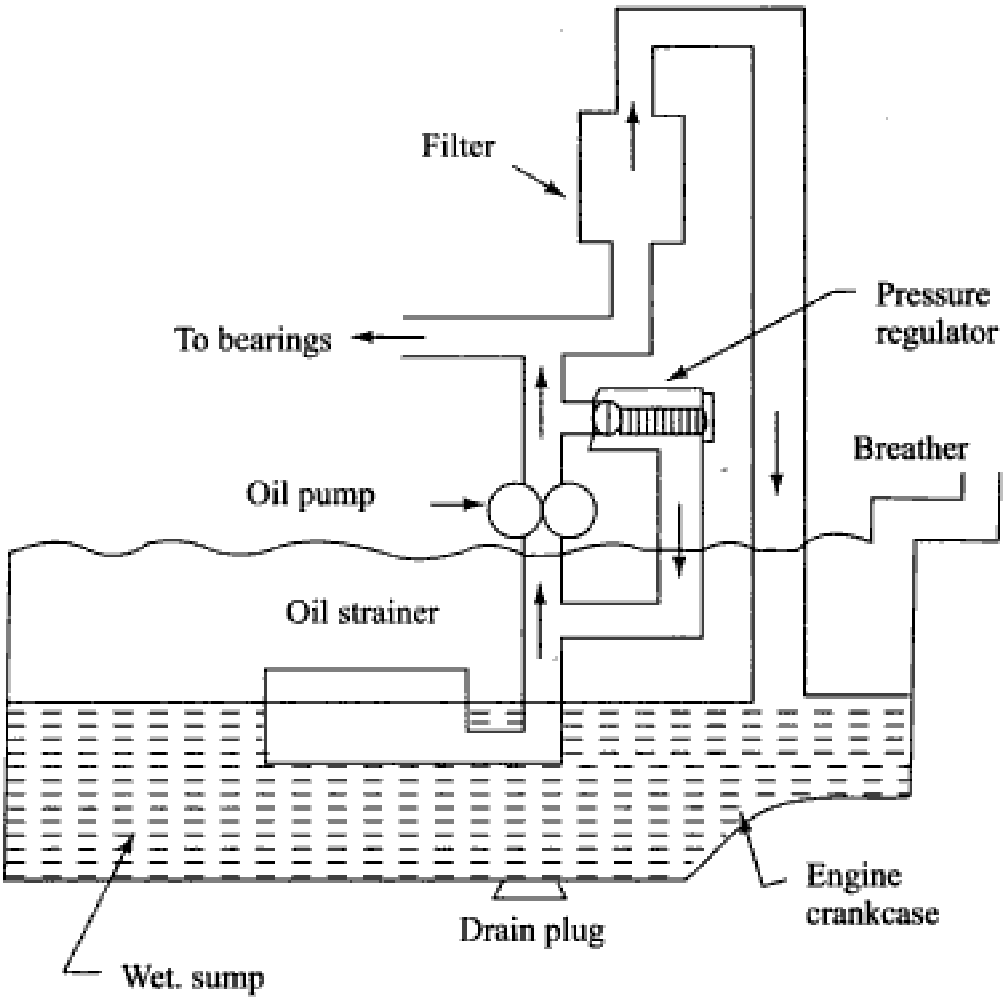


Fig. 11.18 *By-pass type wet sump lubrication*

7. Starting of engine Following are the three common methods of starting an engine.

(i) By an auxiliary engine, which is mounted close to the main engine and drives the latter through a clutch and gears.

(ii) By using an electric motor, in which a storage battery of 12 to 36 volts is used to supply power to an electric motor that derives the engine.

(iii) By compressed air system, in which compressed air at about 17 bar supplied from an air tank is admitted to a few engine cylinders making them work like reciprocating air motors to run the engine shaft. Fuel is admitted to the remaining cylinders and ignited in the normal way causing the engine to start. The compressed air system is commonly used for starting large diesel engines employed for stationary power plant service.

11.5 COMBUSTION IN A CI ENGINE

In a CI engine combustion of fuel occurs due to the high temperature produced by the compression of air and hence it is an auto-ignition engine. For this, a minimum compression ratio of 12 is required. The efficiency of the cycle increases with higher values of compression ratio, but the maximum pressure reached in the cylinder also increases. This requires heavier construction. The upper limit of compression ratio is a compromise between high efficiency and low weight and cost. The normal compression ratios are in the range of 14 to 17, but they may be up to 23. The air fuel ratios used in CI engines lie between 18 and 25 as against about 15 in the SI engine. So, for same power CI engines are bigger and heavier than SI engines.

In a CI engine the intake is air alone and the fuel is injected at high pressure in the form of fine droplets near the end of compression. This leads to the delay period, as explained below.

Due to the practical limitations caused by smoke at engine exhaust (smoke limit), CI engines are operated at air fuel ratios higher than the stoichiometric requirement. Due to shortcomings of distribution and limited intermixing of fuel with air within the combustion chamber, CI engines always operate with excess air (unlike SI engines).

In the SI engine there is an ignition delay which occurs between the time the spark is produced and the time when the "actual burning" phase of combustion commences. In a CI engine, the fuel does not ignite immediately upon injection into the combustion chamber (CC). There is a certain period of apparent inactivity between the time when the first droplet of fuel hits the hot air in the CC and the time when it starts through the "actual burning" phase. This period is known as "ignition delay" or "ignition lag". During this period there is no pressure rise within the cylinder. This delay is indicated on the pressure-time diagram (Fig. 11.19), as the distance between points "*a*" and "*b*". Point "*a*"

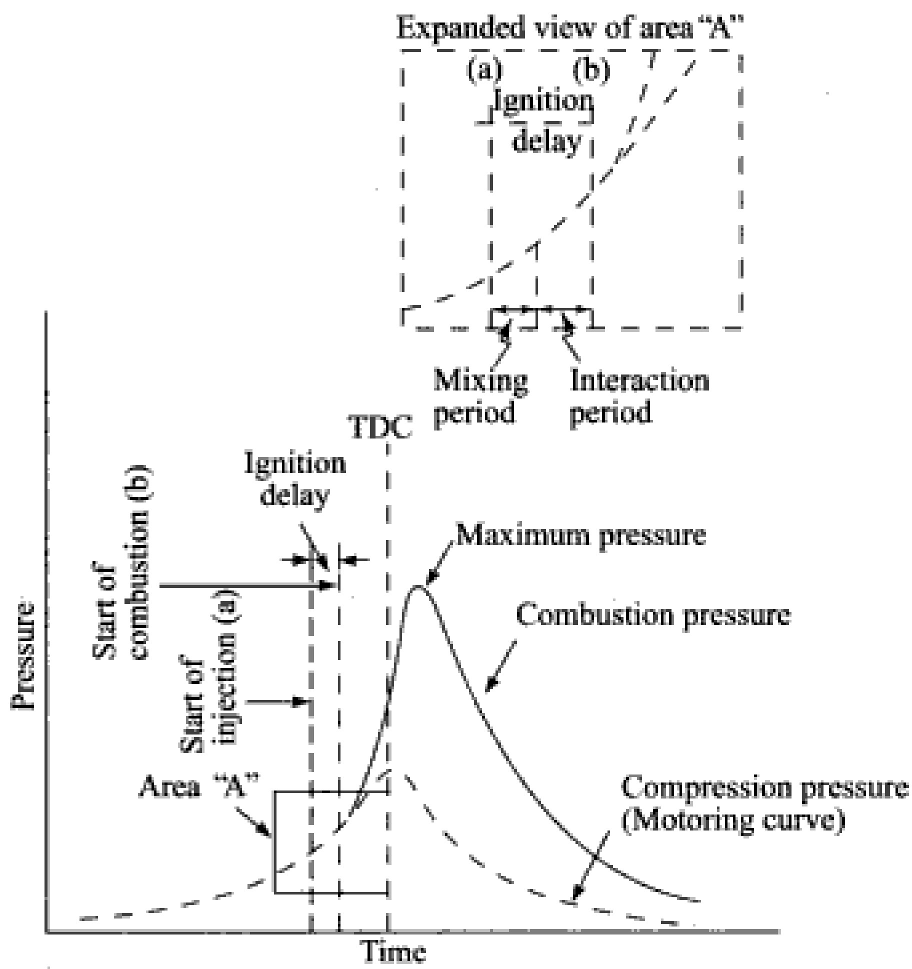


Fig. 11.19 *Pressure-time diagram illustrating ignition delay in a CI engine*

represents the time of injection and point "*b*" represents the time at which the pressure curve, caused by combustion, first separates from the compression pressure (non-firing or motoring) curve. The ignition delay period is divided into two parts—(*a*) the mixing period, which is the time required for atomization and evaporation of the fuel, and physical mixing with the air and (*b*) the interaction period, in which molecular interaction prepares the mixture for and initiates, the "actual burning" phase of combustion, which is longer (of the two periods).

Ignition delay has a great influence on combustion rate and on detonation. If the ignition delay is short, there will be smooth operation as shown in Fig. 11.19. However, if the ignition delay is long, fuel droplets accumulate in the CC. When the actual burning commences, there is a very rapid rate of pressure rise, resulting in "jamming" of forces against the piston and rough engine operation. Such a situation produces the extreme pressure differentials and violent gas vibrations known as detonation and evidenced by audible knock. The phenomenon is similar to that in the SI engine. However, in the SI engine detonation occurs near the end of combustion, whereas in the CI engine, detonation occurs near the beginning of combustion (Fig. 11.20).

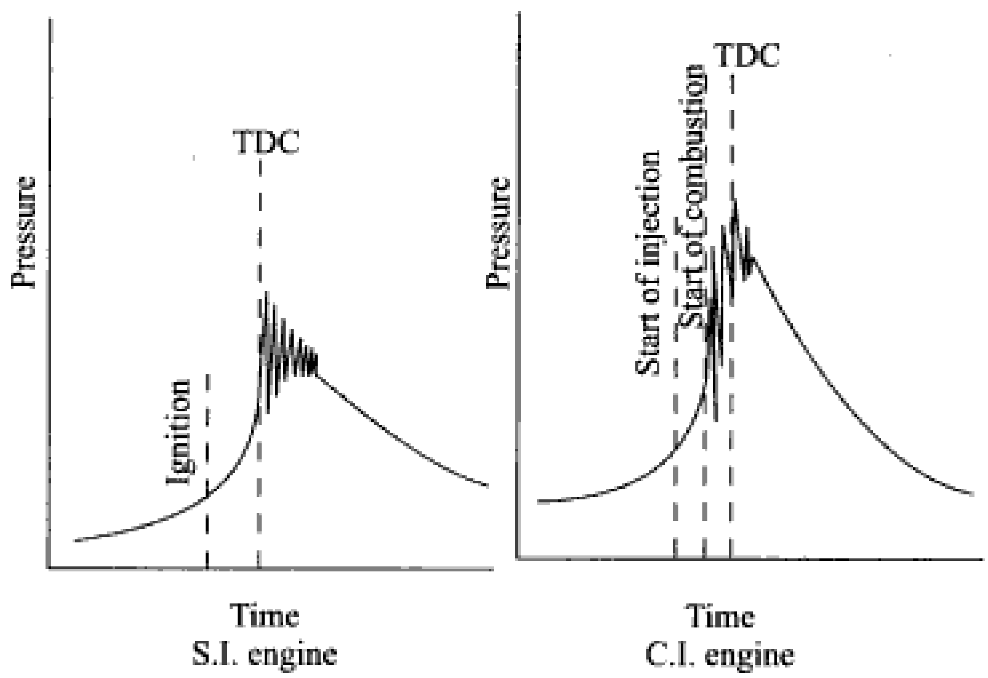


Fig. 11.20 *Diagrams illustrating knocking combustion in SI and CI engines*

It is necessary to decrease the ignition delay, which varies with the fuel and is measured in terms of cetane number. Ignition delay can be decreased by adding small amounts of certain compounds such as ethyl nitrate, amyl thionitrite and others. The engine variables which affect ignition delay are (a) compression ratio (b) inlet air temperature (c) coolant temperature and (d) engine speed. An increase in compression ratio or inlet air temperature or coolant temperature or engine speed, decreases the ignition delay or knock.

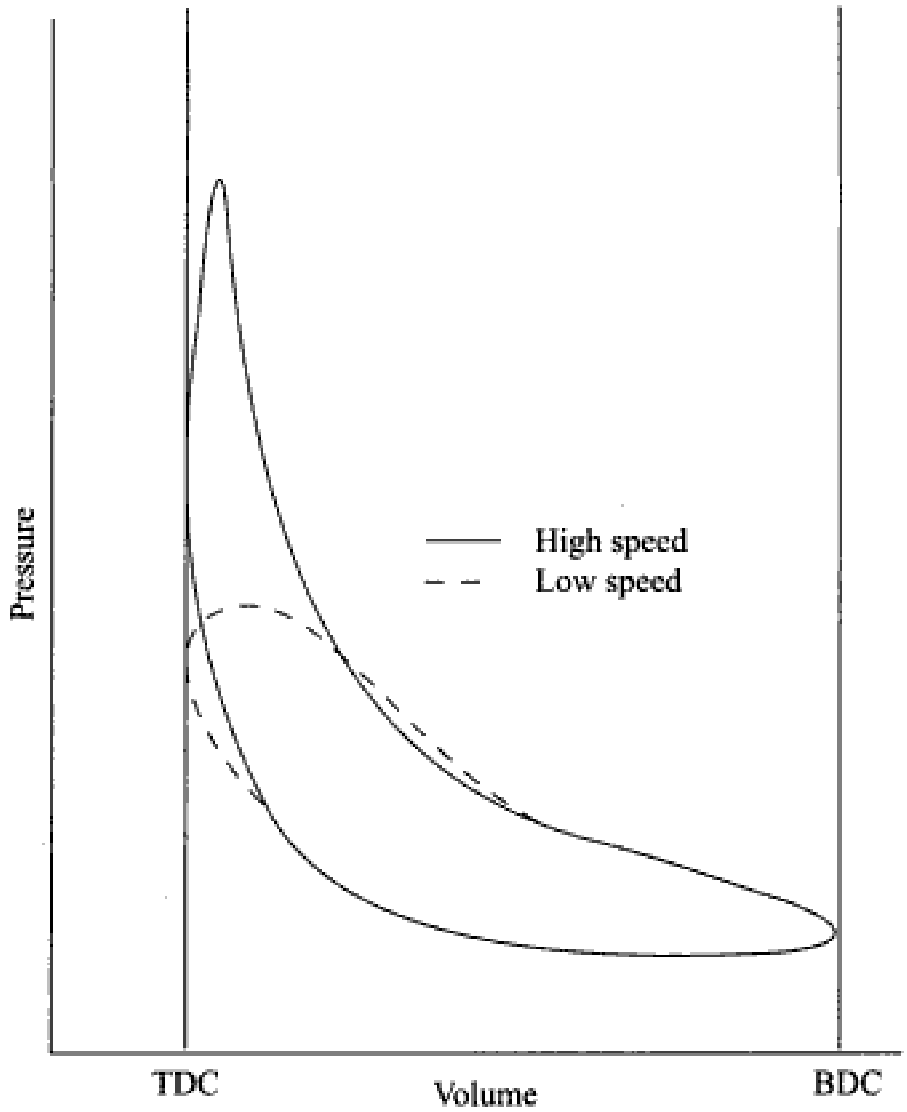


Fig. 11.21 *Diagram illustrating the difference between a high speed and a low speed four-stroke cycle unsupercharged CI engine*

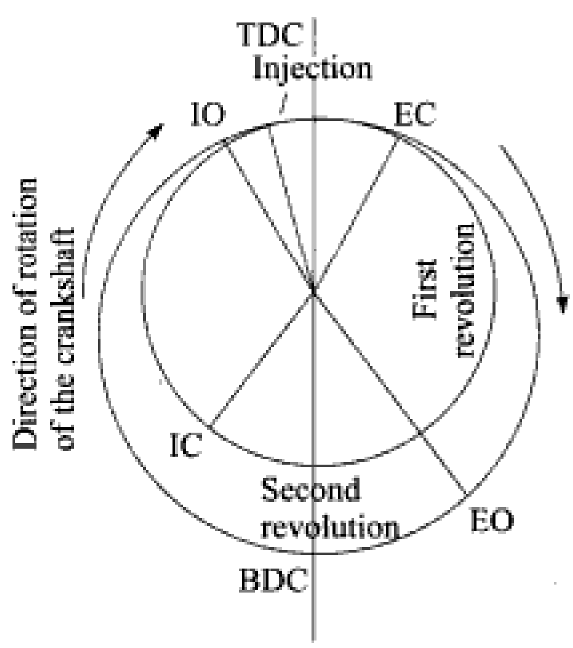


Fig. 11.22 *Valve timing diagram for a four-stroke CI engine*

Brake power (bp) is the output of the engine at the shaft measured by a dynamometer. Absorption dynamometers which are more common can be (a) friction type like prony brake (b) hydraulic and (c) electrical. The brake power is

$$bp = \frac{2\pi TN}{60} \tag{11.3}$$

where T is the torque measured.

Power required to overcome the frictional resistance is the friction power (fp) given by

$$fp = ip - bp \tag{11.4}$$

The mechanical efficiency (η_M) of the engine is defined as

$$\eta_M = bp/ip \tag{11.5}$$

which lies between 80 and 90 per cent.

The fp is very nearly constant at a given engine speed. If the load is decreased giving lower values of bp, then the variation in η_M with bp is as shown in Fig. 11.25.

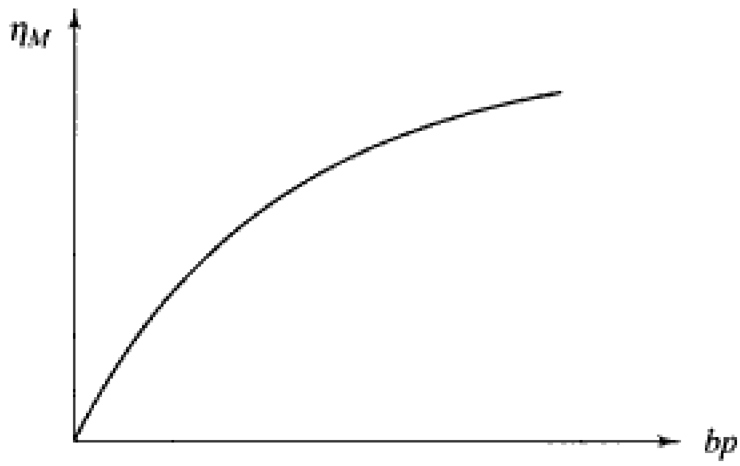


Fig. 11.25 *Variation of mechanical efficiency*

The Morse test can be used to measure the ip of multi-cylinder engines. The engine, say having four cylinders, is run at the required speed and the torque is measured. One cylinder is cut out by disconnecting the injector of a CI engine (or by shorting the spark plug of an SI engine). The speed falls because of the loss of power with one cylinder cut out, but is restored by reducing the load. When the speed has reached the original value, the torque is again measured. It is repeated by cutting out other cylinders one by one. If the values of ip of the cylinders are denoted by I_1, I_2, I_3 and I_4 and the power losses in each cylinder are denoted by L_1, L_2, L_3 and L_4, then the value of bp, B, at the test speed with all cylinders firing is given by

$$B = (I_1 - L_1) + (I_2 - L_2) + (I_3 - L_3) + (I_4 - L_4) \tag{i}$$

If number 1 cylinder is cut out, then the contribution I_1 is lost. If the losses due to that cylinder remain the same as when it was firing, then the bp, B_1, obtained at the same speed is

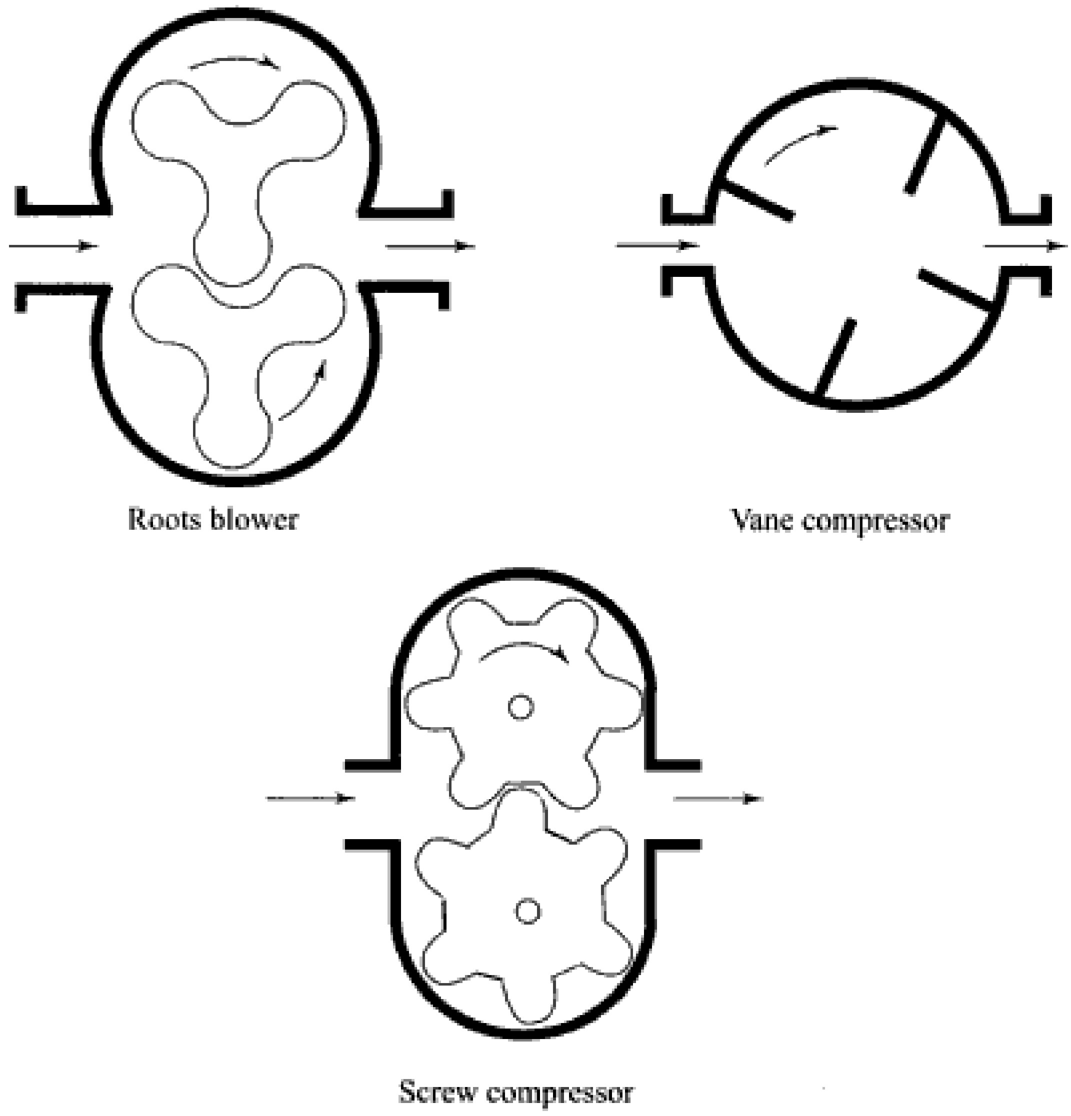


Fig. 11.33 *Types of positive displacement compressor*

(ii) *Non-positive displacement* type like axial and radial compressors (Fig. 11.34). Turbochargers are superior to superchargers, since the former use the exhaust gas energy during blowdown and the latter consume a part of the engine output.

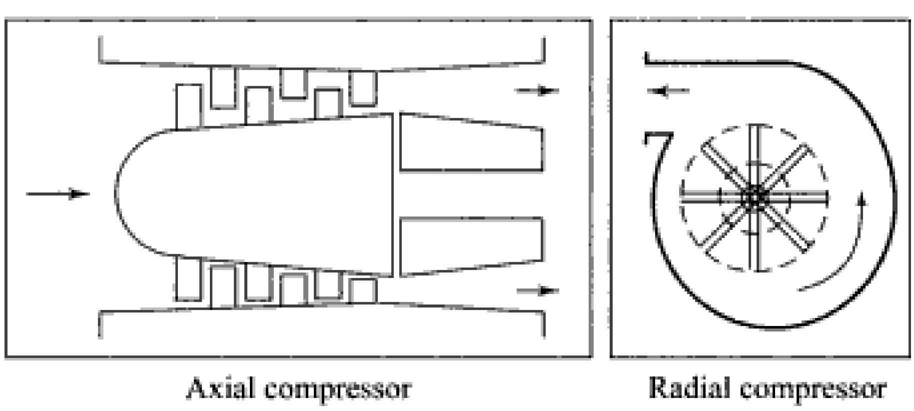


Fig. 11.34 *Types of dynamic or non-positive displacement compressor*

Figure 11.35 shows the continuous running performance characteristics of a diesel engine in three modes, i.e. (a) normally aspirated (b) turbocharged and (c) turbocharged with intercooling for a specific engine as given by Eastop and

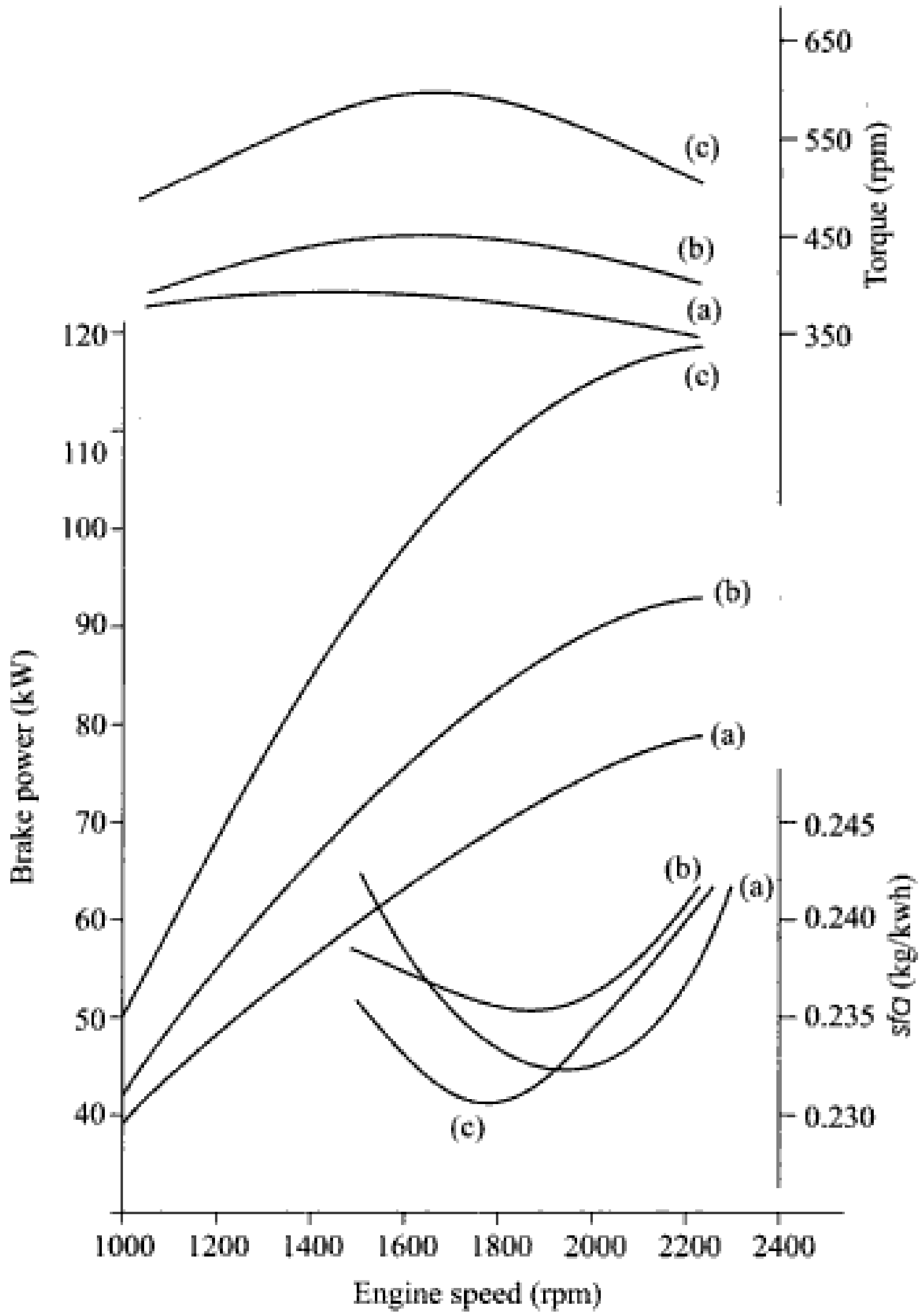


Fig. 11.35 *Continuous running performance characteristics of a CI engine in three modes: (a) Normally aspirated, (b) Turbocharged and (c) Turbocharged with intercooling*

McConkey. It is seen that intercooling of air has a very good influence on the engine performance. Typical performances for a CI engine at full load and part load are shown in Fig. 11.36.

11.8 LAYOUT OF A DIESEL ENGINE POWER PLANT

The layout of a diesel engine power plant is shown in Fig. 11.37. Diesel engine units are installed side by side with some room left for extension in the future. The repairs and usual maintenance works require some space around the units. The air intakes and filters and exhaust mufflers are located outside. Adequate space for oil storage, repair shop and office are provided as shown. Bulk storage of oil may be outdoor.

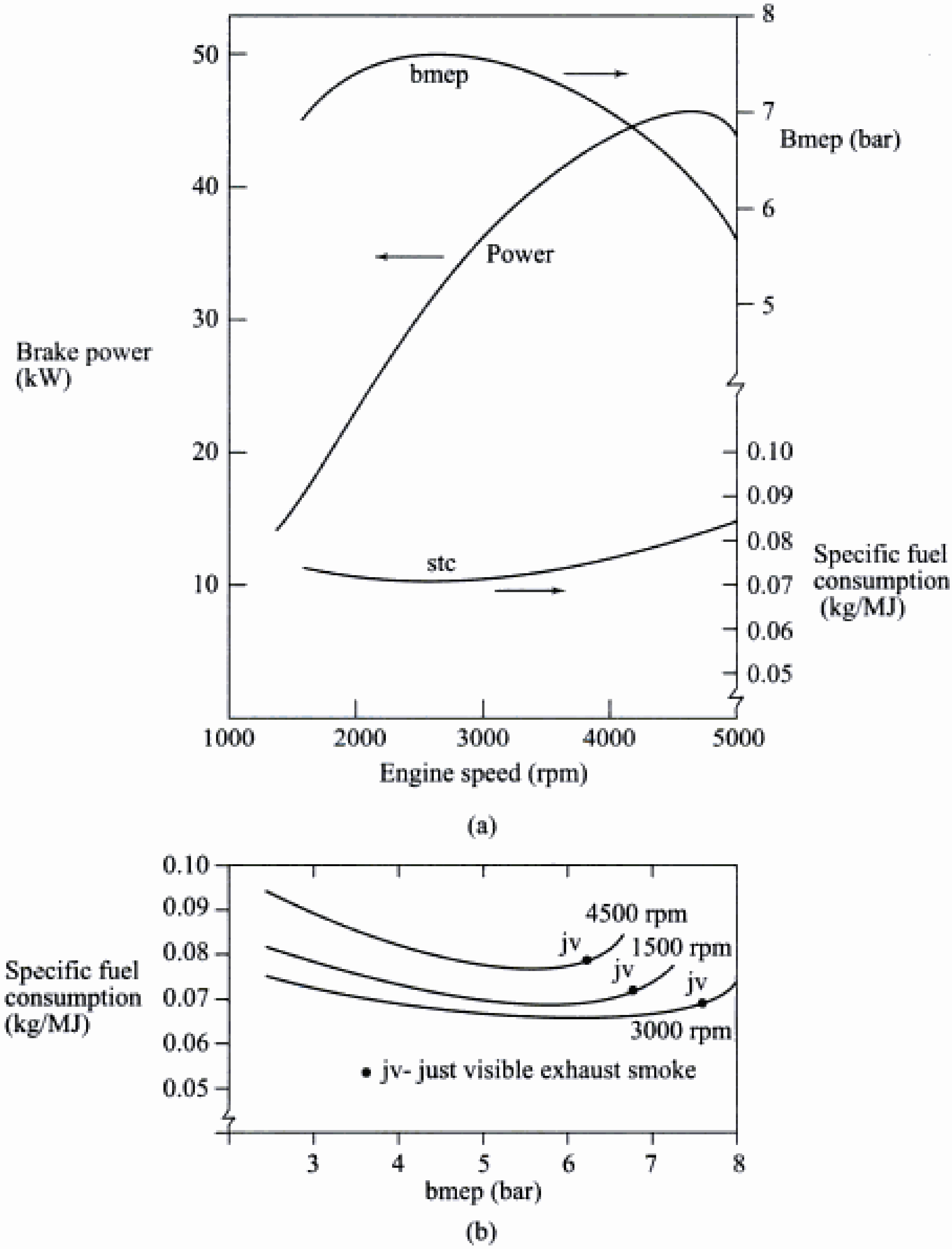


Fig. 11.36 *Typical performance for a CI engine at (a) Full load and (b) Part load at different speeds*

Example 11.1 A four-stroke CI engine of 3.5 litre capacity develops indicated power on average of 13.1 kW/m^3 of free air induced per minute, while running at 3600 rpm and having a volumetric efficiency of 82 per cent, referred to free air conditions of 1.013 bar and 25°C. A blower driven mechanically from the engine is proposed to be installed for supercharging. It works through a pressure ratio of 1.75 and has an isentropic efficiency of 70 per cent. Assume that at the end of the intake

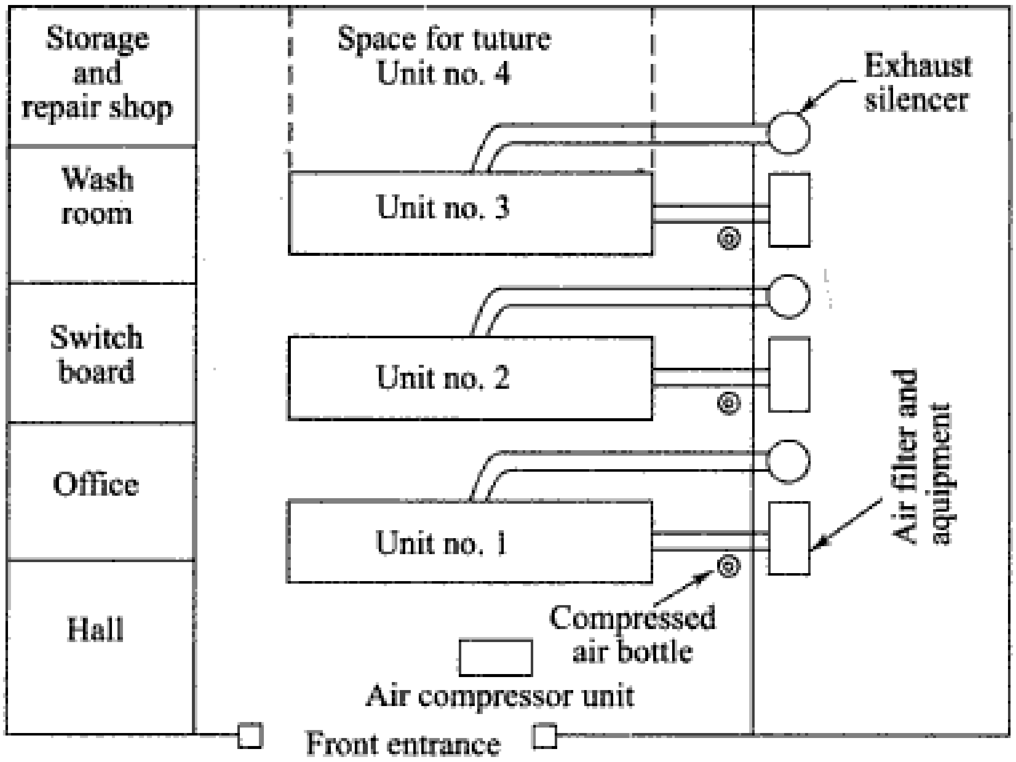


Fig. 11.37 *Typical layout of a diesel engine power plant*

stroke the cylinders contain a volume of charge equal to the swept volume, at the pressure and temperature of the delivered air from the blower. Taking all mechanical efficiencies to be 80 per cent, estimate the net increase in brake power of the engine due to supercharging.

Solution

Engine capacity = 3.5 litres = 0.0035 m^3
Swept volume = 3600/2 × 0.0035 = 6.3 m^3/min
Unsupercharged induced volume = 0.82 × 6.3 = 5.166 m^3/min
Blower delivery pressure = 1.75 × 1.013 = 1.7728 bar

$$T_{2s}/T_1 = [p_2/p_1]^{(\gamma-1)/\gamma}$$
$$= (1.75)^{(1.4-1)/(1.4)} = 1.173$$
$$T_{2s} = 1.173 \times 298 = 349.67 \text{ K}$$
$$[T_{2s} - T_1]/[T_2 - T_1] = \eta_c = 0.7$$
$$T_2 - T_1 = \frac{349.67 - 298}{0.7} = 73.8$$
$$T_2 = 371.8 \text{ K} = \text{Blower delivery temperature}$$

The blower delivers 6.3 m^3/min at 1.7728 bar and 371.8 K.
Equivalent volume at 1.013 bar and 25°C

$$= \frac{6.3 \times 1.7728 \times 298}{1.013 \times 371.8} = 8.837 \text{ m}^3/\text{min}$$

Increase in induced volume = 8.837 – 5.166 = 3.671 m^3/min
Increase in indicated power due to extra air induced

$$= 13.1 \times 3.671 = 48.1 \text{ kW}$$

$p_1 = 0.97$ bar, $T_1 = 303$K, $p_2 = 2.1$ bar, $A/F = 18$, $T_3 = 580°$C, $p_3 = 1.9$ bar, $p_4 = 1.06$ bar.

If the compression were isentropic (Fig. E11.2).

$$T_{2s}/T_1 = [p_2/p_1]^{(\gamma-1)/\gamma}$$

$$T_{2s} = 303 \left[\frac{2.1}{0.97}\right]^{(1.4-1)/0.4} = 377.8 \text{ K}$$

$$[T_{2s} - T_1]/[T_2 - T_1] = 0.75$$

$$T_2 - T_1 = [377.8 - 303]/0.75$$

$$T_2 = 402.73 \text{ K}$$

(a) Exit temperature of air from the compressor

$$T_2 = 129.73°\text{C}$$ *Ans.*

If the turbine were isentropic,

$$T_3/T_{4s} = [p_3/p_4]^{(\gamma ex - 1)/\gamma ex}$$

$$= \frac{580+273}{T_{4S}} = [1.9/1.06]^{(1.33-1)/1.33} = 1.1558$$

$$T_{4s} = 853/1.1558 = 738 \text{ K}$$

$$T_3 - T_4 = (T_3 - T_{4s})\eta_T$$

$$853 - T_4 = (853 - 738) \times 0.85 = 97.75$$

$$T_4 = 755.25 \text{ K}$$

(b) Turbine exhaust temperature

$$T_4 = 482.25°\text{C}$$ *Ans.*

Compressor power

$$\dot{W}_C = \dot{m}_a\, c_{pa}\, (T_2 - T_1)$$

$$= \dot{m}_a \times 1.01\, (129.73 - 30)$$

$$= 100.73\, \dot{m}_a \text{ kW}$$

Rate of flow of exhaust gases,

$$\dot{m}_{ex} = \dot{m}_a + \dot{m}_f = \dot{m}_a\, [1 + F/A]$$

$$= \dot{m}_a\, [1 + 1/18] = 1.056\, \dot{m}_a$$

Turbine power,

$$\dot{W}_T = \dot{m}_{ex} \times c_{pex} \times (T_3 - T_4)$$

$$\dot{W}_T = \dot{m}_a \times 1.056 \times 1.15\, (853 - 755.25)$$

$$= 118.7\, \dot{m}_a \text{ kW}$$

Thus, the mechanical power loss as a percentage of the power generated in the turbine is

$$\frac{118.7 - 100.73}{118.7} \times 100 = 15.14\%$$ *Ans.*

Example 11.3 During a test on a diesel engine used for driving a dc generator, the following observations were made.

The output of the generator was 215 A at 210 V, the efficiency of the generator being 85%. The quantity of fuel supplied to the engine was 11.8 kg/h, the calorific value of fuel being 43 MJ/kg. The air-fuel ratio was 18:1.

The exhaust gases were passed through an exhaust gas calorimeter for which the observations were as follows:

Water circulated through the calorimeter = 560 litres/h, Temperature rise of water = 38°C, Temperature of exhaust gases at exit from calorimeter = 97°C, Specific heat of exhaust gases = 1.04 kJ/kgK, Ambient temperature = 30°C.

If the heat lost to the jacket cooling water was 32% per cent of the total energy released by combustion, draw up an energy balance sheet of the engine.

Solution

Total power generated = 215 × 210 = 45150 W = 45.15 kW

Brake power of the engine = 45.15/0.85 = 53.12 kW

Energy supplied to the engine = (11.8/3600) × 43000

= 140.94 kW

Rate of flow of exhaust gases

$$\dot{m}_g = \dot{m}_f + \dot{m}_a$$

$$= \dot{m}_f (1 + A/F) = (11.8/3600) \times 19$$

$$= 0.0623 \text{ kg/s}$$

Heat carried away by exhaust gases

$$= 0.0623 \times 1.04 \times (97 - 30) + (560/3600) \times 4.187 \times 38$$

$$= 4.341 + 24.75 = 29.091 \text{ kW}$$

Heat lost to jacket cooling water = 0.32 × 140.94 = 45.1 kW.

Table E11.3 *Energy Balance Sheet*

Heat supplied to the engine 140.94 kW			
1.	Brake power	53.12 kW	37.69%
2.	Heat carried away by exhaust gases	29.09 kW	20.64%
3.	Heat lost to jacket cooling water	45.10 kW	32.00%
4.	Heat loss unaccounted (by difference)	13.63 kW	9.67%
		140.94 kW	100.00%

Example 11.4 Following are the observations made for a 20 minute trial of a two-stroke diesel engine.

Net brake load = 680 N, mep = 3.0 bar, N = 360 rpm, Fuel consumption = 1.56 kg, Cooling water = 160 kg, Water inlet temperature = 32°C, Water outlet temperature = 57°C, Air used/kg fuel = 30 kg, Room temperature = 27°C, Exhaust gas temperature = 310°C, Cylinder dimensions = 210 mm bore × 290 mm stroke, Brake diameter = 1m, Calorific value of fuel = 44 MJ/kg, Steam formed per kg fuel in the exhaust = 1.3 kg, specific heat of steam in exhaust = 2.093 kJ/kgK, Specific heat of dry exhaust gases = 1.01 kJ/kgK.

Calculate the indicated power and the brake power and make an energy balance of the engine.

Solution

For a two-stroke engine,

$$ip = \frac{p_i LAN}{60} = \frac{3.0 \times 100 \times 0.29 \times \frac{\pi}{4} \times (0.21)^2 \times 360 \times 1}{60}$$

$$= 18.08 \text{ kW} \qquad \textit{Ans.}$$

$$bp = \frac{2\pi TN}{60} = \frac{2\pi \times (680 \times 1/2) \times 360}{60} \times 10^{-3} = 12.818 \text{ kW}$$

$$\eta_M = \frac{bp}{ip} = 0.7089 \text{ or } 70.89\%$$

Heat supplied during the trial

$$= 1.56 \times 44{,}000 = 68{,}640 \text{ kJ}$$

Energy equivalent of *ip* in trial period

$$= 18.08 \frac{\text{kJ}}{s} \times 20 \times 60 = 21{,}696 \text{ kJ}$$

Energy carried away by cooling water

$$= 160 \times 4.187 \times (57 - 32) = 16748 \text{ kJ}$$

Total mass of exhaust gas

$$= 1.56 \times 30 = 46.8 \text{ kg}$$

Mass of steam formed = $1.3 \times 1.56 = 2.028$ kg

Mass of dry exhaust gas = $46.8 - 2.028 = 44.772$ kg

Energy carried away by dry exhaust gases

$$= 44.772 \times 1.01 \times (310 - 27) = 12{,}797 \text{ kJ}$$

Energy carried away by steam

$$= 2.028\,[4.187\,(100 - 27) + 2257.9 + 2.093\,(310 - 100)]$$

$$= 2.028\,(305.65 + 2257.9 + 439.53)$$

$$= 6090 \text{ kJ}$$

Total energy carried away by exhaust gases = 12,797 + 6090 = 18887 kJ

Table E11.4 *Energy Balance Sheet (for trial period)*

Energy released by combustion of fuel	*68,640 kJ*	
Energy equivalent of ip	21,696 kJ	31.61%
Energy carried away by cooling water	16,748 kJ	24.40%
Energy carried away by exhaust gases	18,887 kJ	27.51%
Unaccounted for energy loss	11,309 kJ	16.47%
(by difference)	68,640 kJ	100.00%

11.9 GAS TURBINE POWER PLANT

The economics of power generation by gas turbines is now quite attractive due to its low capital cost and its high reliability and flexibility in operation. Another outstanding feature is its capability of quick starting and using a wide variety of fuels from natural gas to residual oil or powdered coal. Due to better materials being made available and with the use of adequate blade cooling, the inlet gas temperature to gas turbine (GT) blades can now exceed 1200 °C, as a result of which the overall efficiency of a GT plant can be about 35 per cent, almost the same as that of a conventional steam power plant.

Because of its low weight per unit power, gas turbine is exclusively used to drive aviation systems of all kinds of aircraft. It is also being increasingly used in land vehicles like buses and trucks and also to drive locomotives and marine ships. In oil and gas industries, the gas turbine is widely employed to drive auxiliaries like compressors, blowers and pumps.

11.9.1 Closed Cycle and Open Cycle Plants

The essential components of a gas turbine (GT) power plant are the compressor, combustion chamber (CC) and the turbine. The air standard cycle of a GT plant is the Brayton cycle.

A GT plant can either be open or closed. Figure 11.38 shows the arrangement of an open cycle plant which is more common, where the combution products after doing work in the turbine are exhausted to atmosphere. In a closed cycle plant (Fig. 11.39), the working fluid (air, helium, argon, carbon dioxide, etc.) is externally heated (by burning fuel or by nuclear reactor) and cooled and it operates in a closed cycle.

The specific advantages and disadvantages of a GT plant for a utility system have been discussed in Section 3.6. Following is given a more detailed account in this regard.

$$\eta_{\text{cycle}} = 1 - \frac{1}{r_p^{(\gamma-1)/\gamma}} = 1 - \frac{T_1}{T_{4s}} r_p^{(\gamma-1)/\gamma} \tag{11.13}$$

As r_p increases, η_{cycle} increases till Carnot cycle is reached (Fig. 11.41).

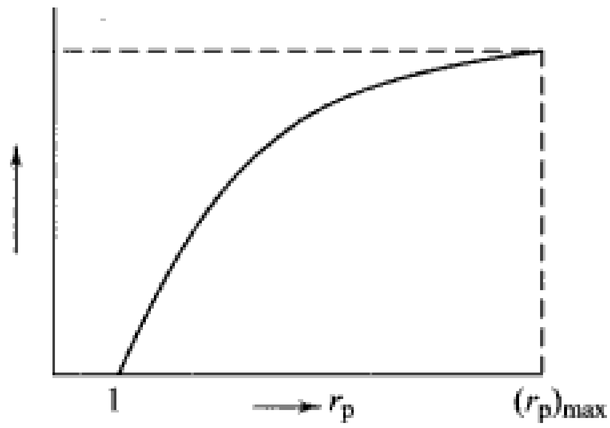


Fig. 11.41 *Variation of cycle efficiency with r_p*

$$(r_p)_{max} = [T_3/T_1]^{1/2} = [T_{max}/T_{min}]^{1/2} \tag{11.14}$$

There is a particular value of r_p when W_{net}, i.e. $W_T - W_C$ becomes maximum (Fig. 11.42).

$$W_{net} = Q_1 - Q_2 = W_T - W_C$$

$$= m_a c_p [T_3 - T_{2s} - T_{4s} + T_1] W_{net} \tag{11.15}$$

Substituting T_{2s} and T_{4s} in terms of r_p (Eq. 11.12) and since T_3 (= T_{max}) and T_1 (= T_{min}) are fixed, on differentiation of W_{net} with respect to r_p and making dW_{net}/dr_p equal to zero, we get

$$(r_p)_{opt} = [T_{max}/T_{min}]^{\gamma/2(\gamma-1)} \tag{11.16}$$

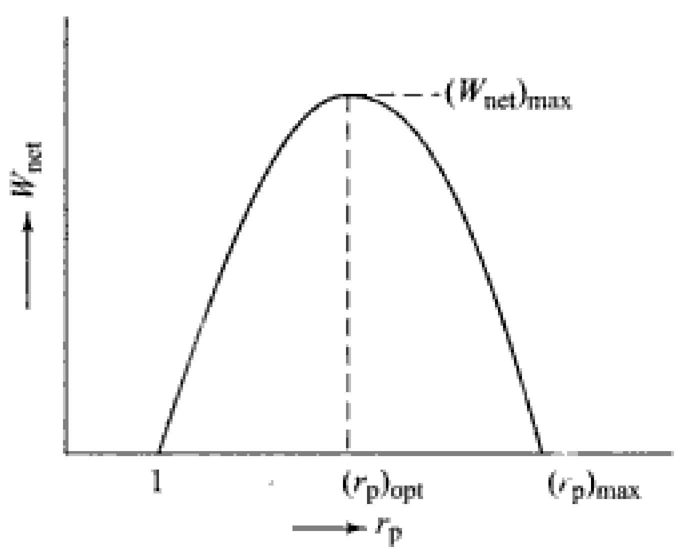


Fig. 11.42 *Variation of net cycle work with r_p*

$$(r_p)_{opt} = [(r_p)_{max}]^{1/2} \tag{11.17}$$

On substitution in Eq. (11.14),

$$W_{net} = m_a c_p [(T_{max})^{1/2} - (T_{min})^{1/2}]^2 \tag{11.18}$$

Effect of intercooling By staging the compression process (1-2 and 3-4) with perfect intercooling (2-3), the cycle efficiency decreases, as shown in Fig. 11.44, where the small cycle 1-2-3-4-4′-1 is added to the basic cycle 1-4′-5-6–1, without intercooling. However, it permits more heat recovery from hot gases exiting the turbine at state 6 by heating air leaving the compressor at state 4. For minimum work of compression, the intercooler pressure $p_i = [p_1 p_2]^{1/2}$, where p_1 and p_2 are suction and discharge pressures respectively.

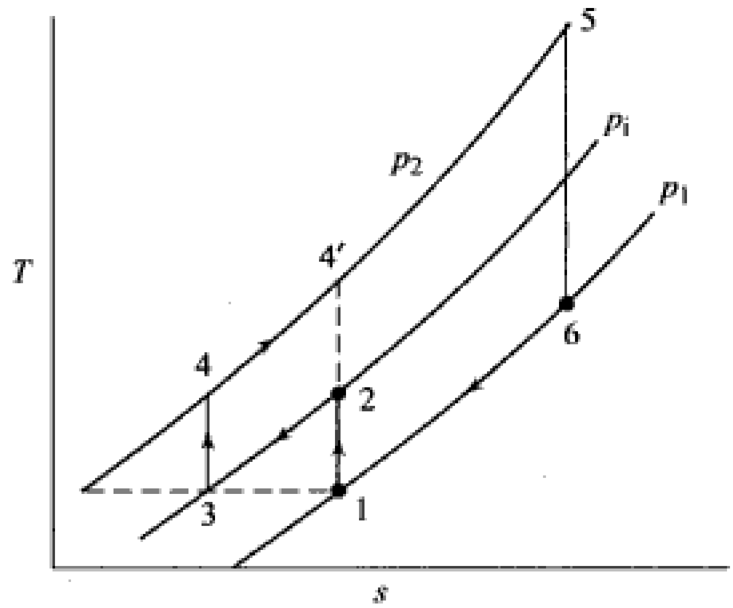


Fig. 11.44 *Effect of intercooling on brayton cycle*

Effect of reheating Similarly, by staging the heat supply process with a combustor and a reheater, the cycle efficiency decreases, but it permits more heat recovery from the turbine exhaust gases (Fig. 11.45) (since $T_6 > T_4$), with the result that reheating along with regeneration may bring about an improvement in cycle efficiency. It can be shown that the optimum reheate pressure for maximum work is

$$p_r = [p_1 p_2]^{1/2} \tag{11.24}$$

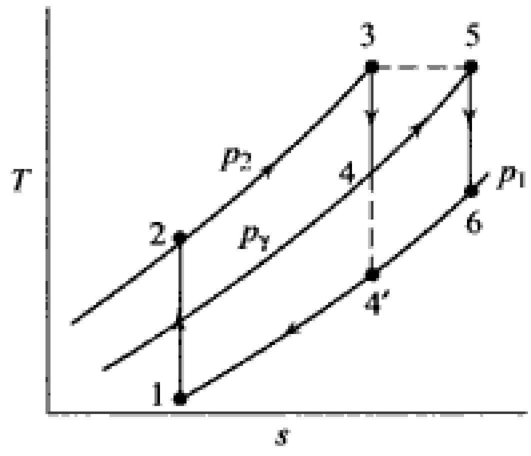


Fig. 11.45 *Effect of reheat on brayton cycle*

Figure 11.46 (a) and (b) shows the flow and T–s diagrams of a GT plant with intercooling, reheating and regeneration.

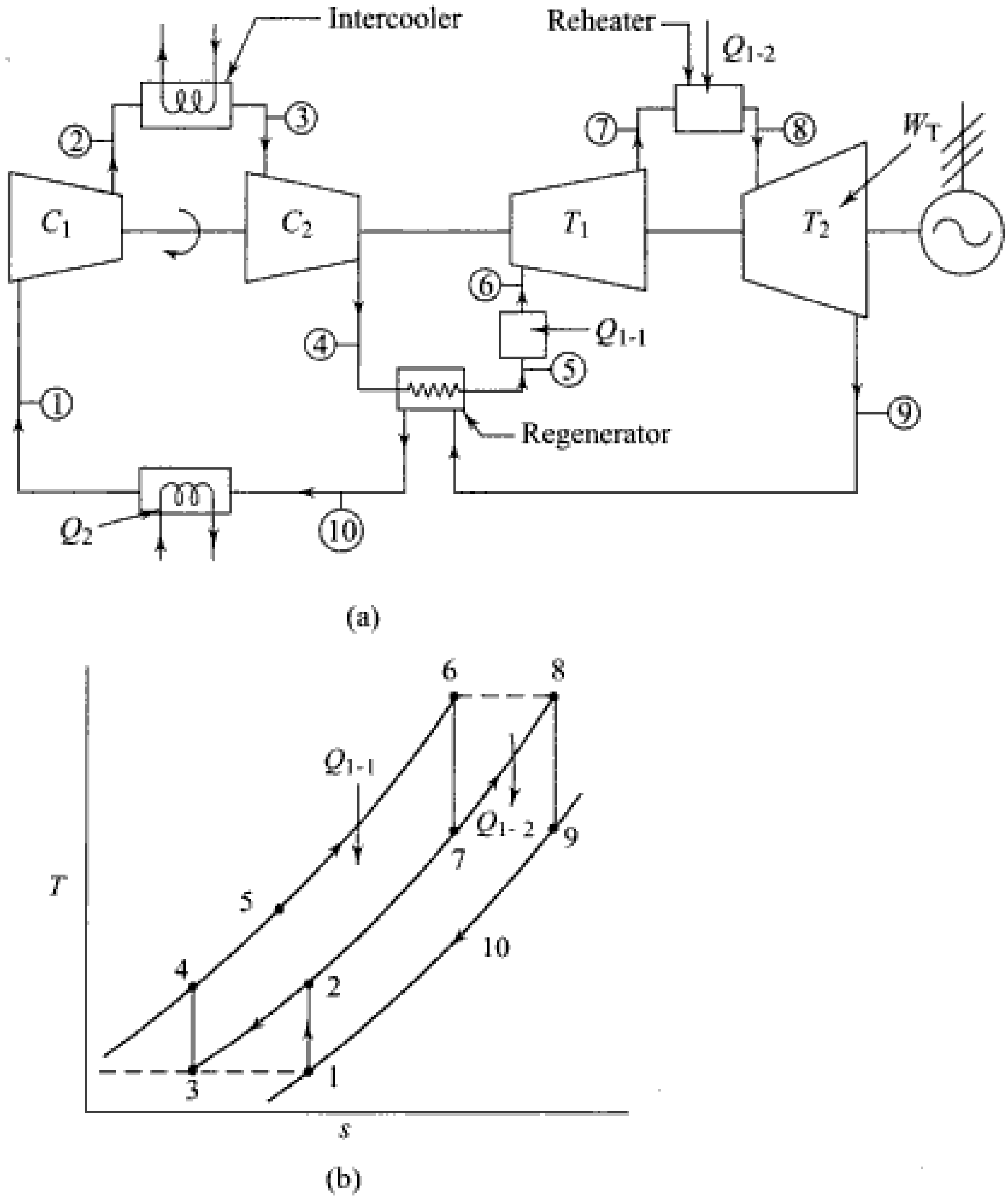


Fig. 11.46 *Brayton cycle with intercooling, reheat and regeneration*

The network of a GT plant is given by

$$W_{net} = W_T - W_C$$

$$= (\dot{m}_a + \dot{m}_f)c_{p_g}\ (T_3 - T_4) - \dot{m}_a\, c_{p_a}\ (T_2 - T_1) \quad (11.25)$$

and the heat supply is

$$Q_1 = \dot{m}_f \times \text{CV}$$

Therefore, the overall plant efficiency,

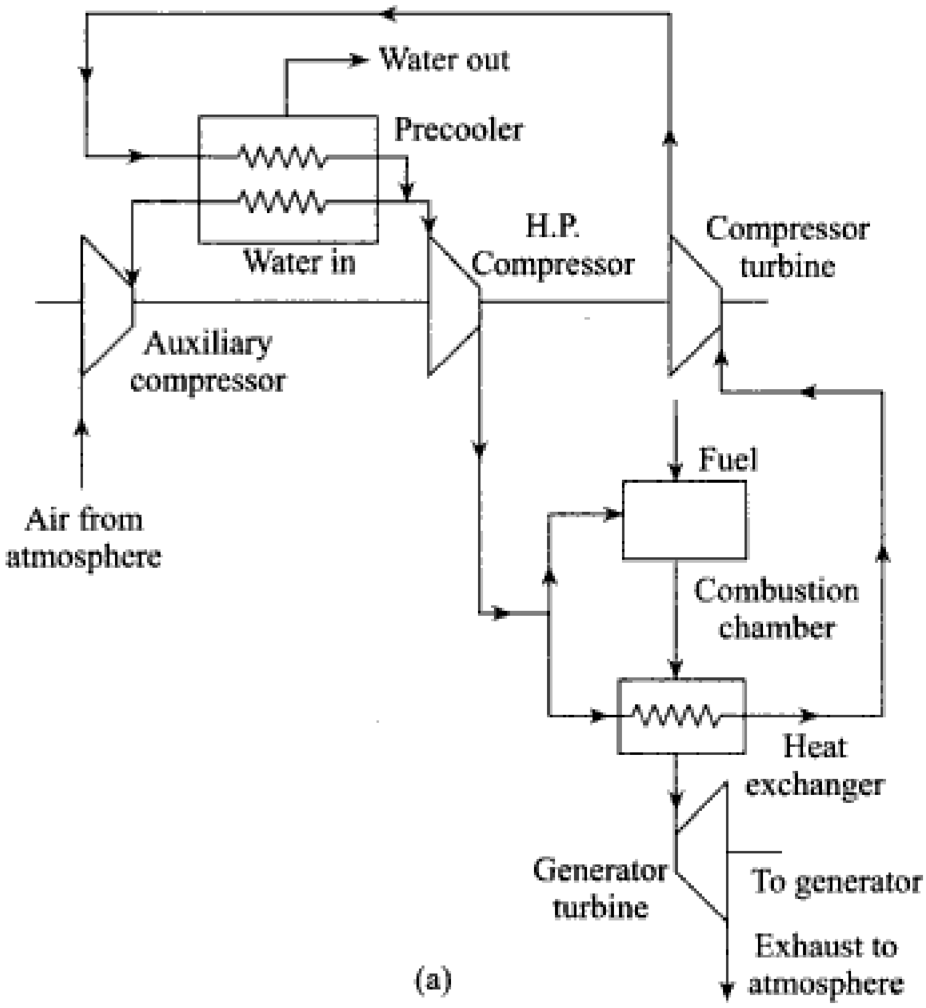
$$\eta_0 = \frac{W_{net}}{\dot{m}_f \times \text{CV}} \quad (11.26)$$

11.9.5 Semi-closed Cycle Gas Turbine Plant

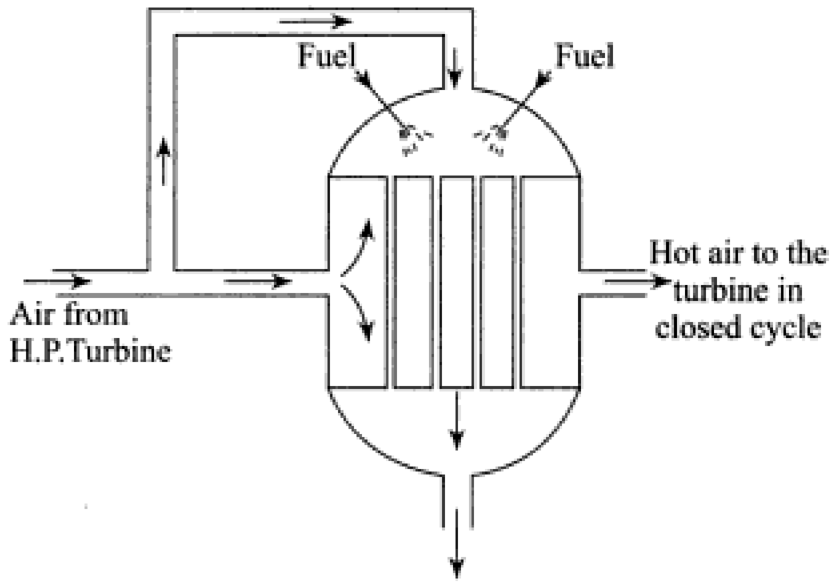
The advantages of the open cycle plant, viz. quick and easy starting and the closed cycle plant, viz. constant efficiency at all loads and higher unit rating permitting the use of higher back pressure, are combined in a semi-closed cycle gas turbine power plant. Here, part of the compressed air is heated by the gases exiting the combustion chamber (CC) and then expanded in an air turbine which drives the compressor, thus operating in a closed cycle. The remaining air is used in the CC to burn fuel, and the combustion products after heating the air expand in a gas turbine to drive the generator before exhausting to the atmosphere (Fig. 11.47a). Figure 11.47 (b) shows a combined combustion chamber and a heat exchanger, where hot gases of combustion leave to expand in the gas turbine in the open cycle and the heated air flows to the air turbine in the closed cycle.

11.9.6 Performance of Gas Turbine Power Plants

The gas turbine plant works under variable load conditions. It is thus necessary to study the effect of load on the cycle efficiency which is directly concerned with the running cost of the plant.



(a)



(b)

Fig. 11.47 *(a) Semi-closed cycle gas turbine plant*
(b) Combined combustion chamber and air heater

It is necessary to study the effect of pressure ratio on the thermal efficiency, air mass flow and specific fuel consumption with regenerative reheat and intercooled cycle, because smaller mass flow rate for the given output reduces the component sizes and the plant capital costs. Lower fuel consumption reduces the running cost of the plant. Some of these characteristics are represented graphically and also discussed.

(a) *Part load efficiency* The part load efficiencies for open cycle, closed cycle and semi-closed cycle are shown in Fig. 11.48. The part load performance of the semi-closed cycle is seen to be the best.

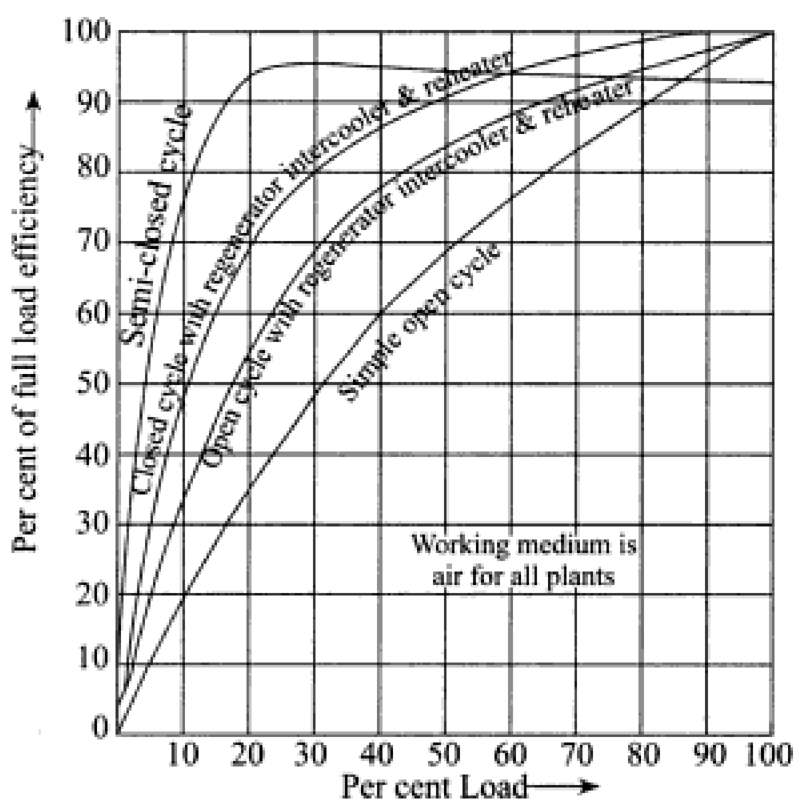


Fig. 11.48 *Part load efficiencies of different plants*

(b) *Fuel consumption* The effect of pressure ratio on the specific fuel consumption (sfc) of an open cycle plant with the degree of regeneration as a parameter is shown in Fig. 11.49. It shows that for each degree of regeneration there is an optimum pressure ratio for minimum sfc.

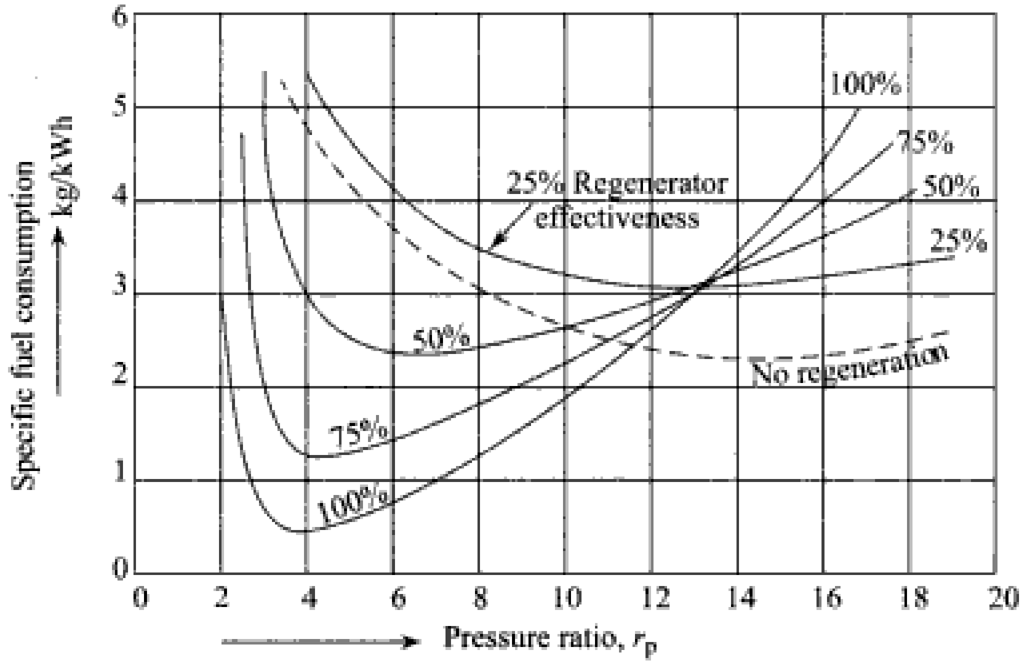


Fig. 11.49 *Effect of regenerator effectiveness on specific fuel consumption*

(c) *Air flow rate* The effect of pressure ratio on the air mass flow rate for an open cycle plant with the turbine inlet temperature as a parameter is shown in Fig. 11.50. It indicates optimum pressure ratio for different turbine inlet temperatures requiring minimum air flow rates.

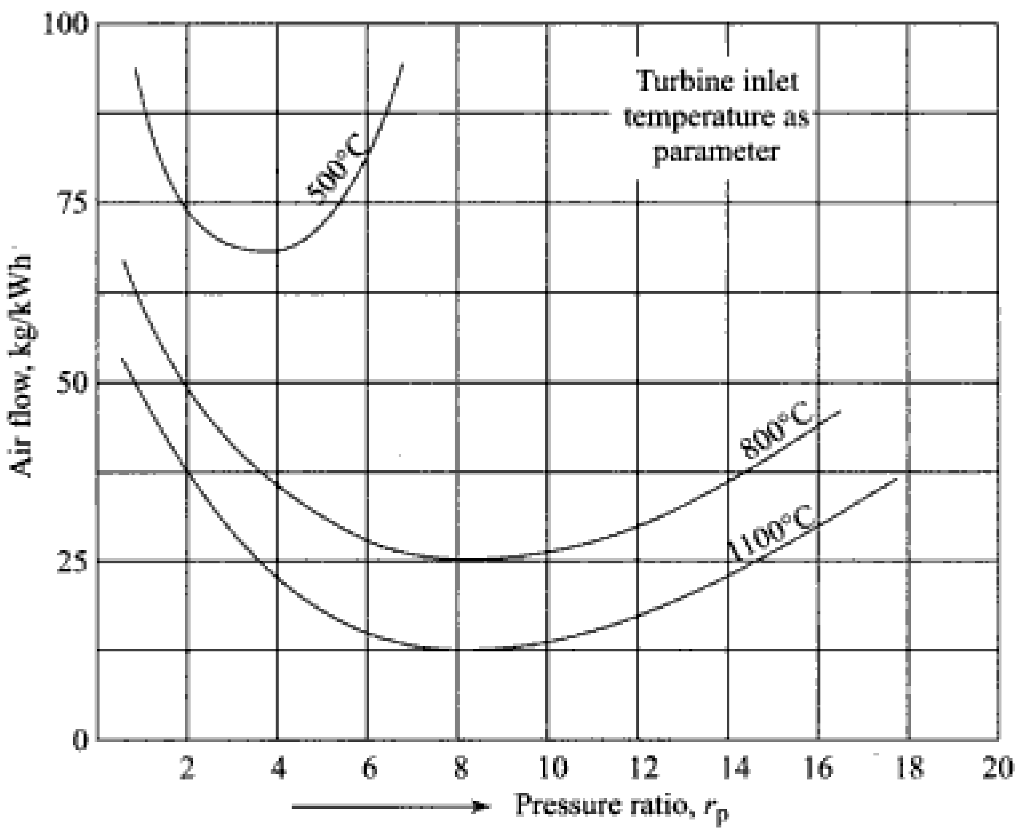


Fig. 11.50 *Effect of pressure ratio on air mass flow per unit output*

(d) *Thermal efficiency* The effect of pressure ratio of a simple open cycle plant with turbine inlet temperature as a parameter is shown in Fig. 11.51(a) and with compressor inlet temperature as a parameter in Fig. 11.51 (b).

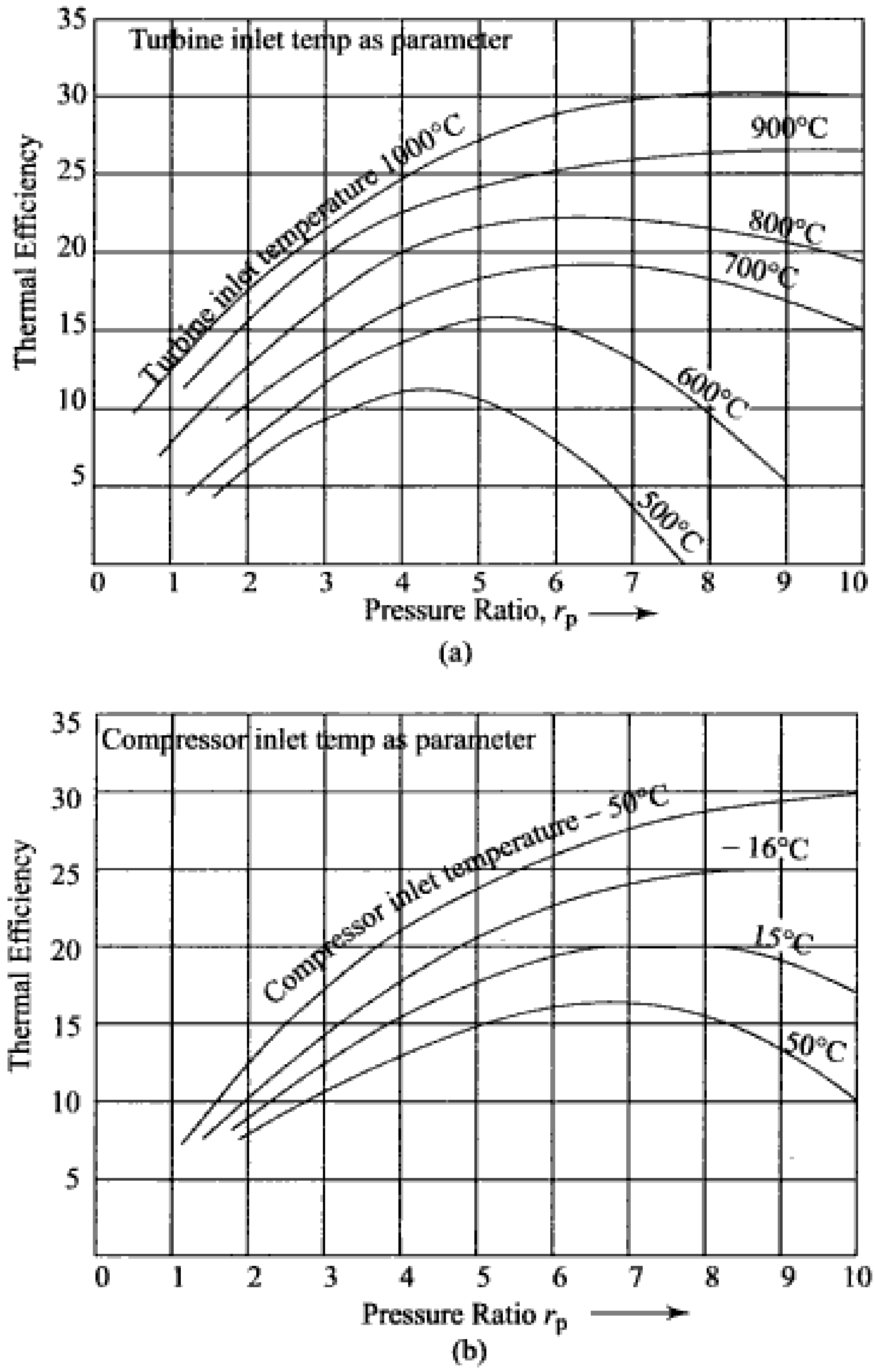
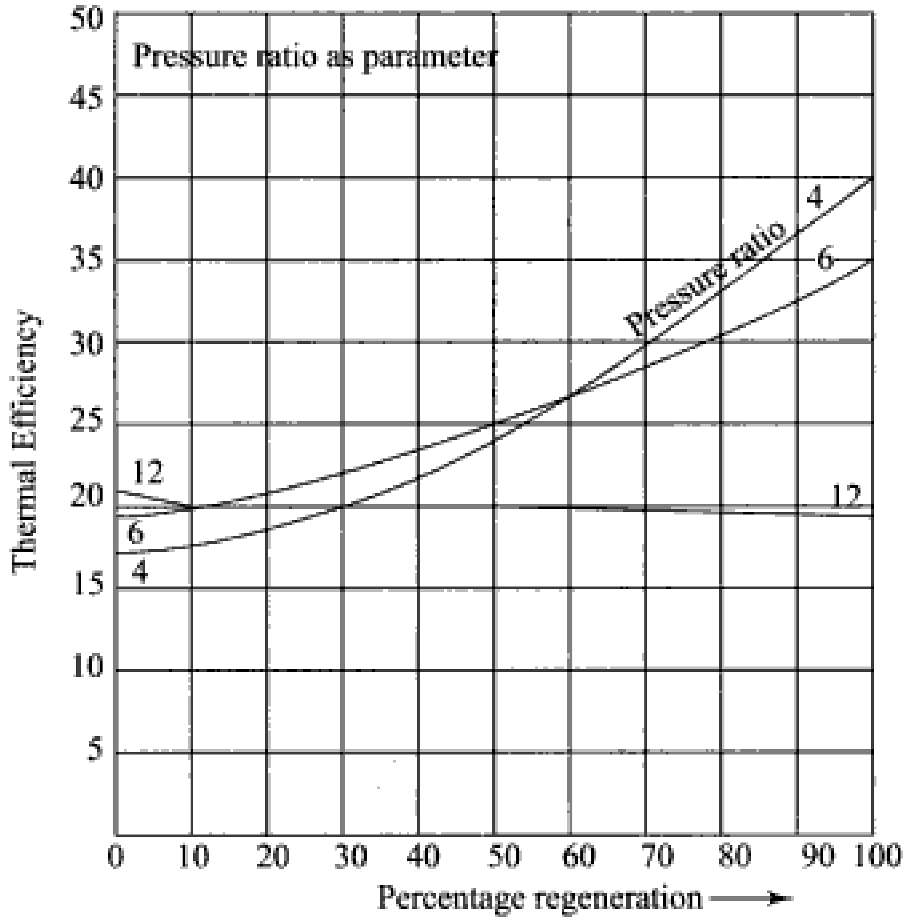
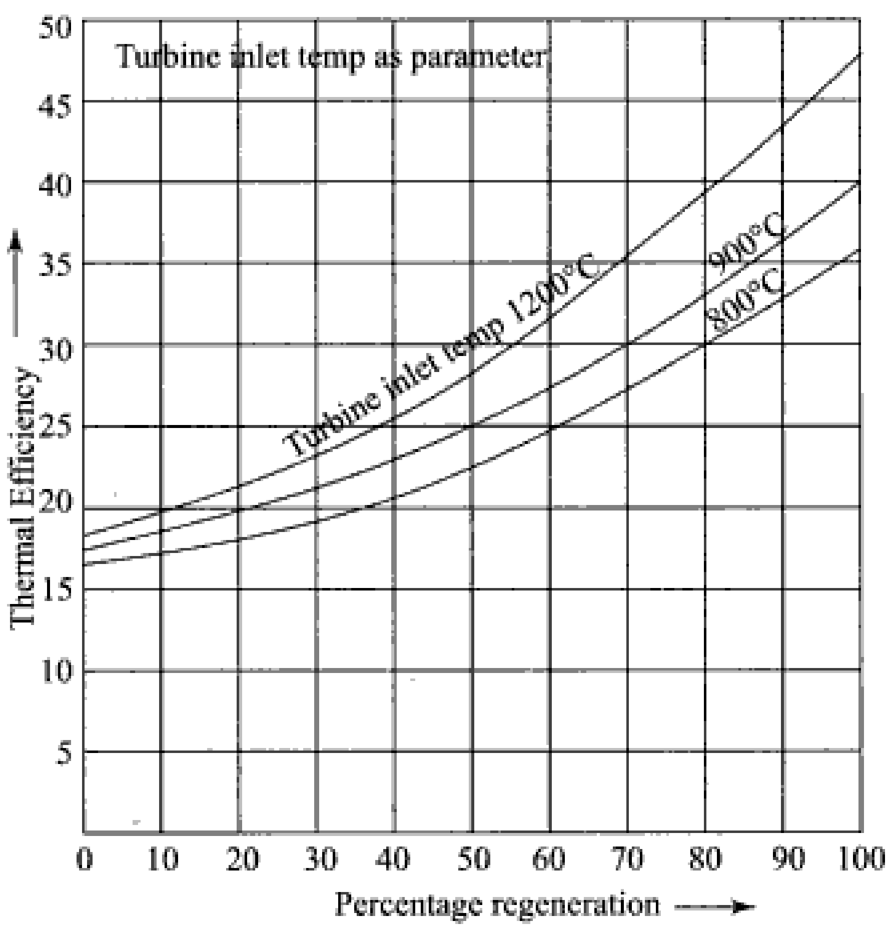


Fig. 11.51 *Effect of pressure ratio on thermal efficiency of a simple open cycle plant with (a) Turbine inlet temperature and (b) Compressor inlet temperature as parameters*

(e) *Regeneration* The effect of regeneration on thermal efficiency of a simple cycle taking pressure ratio and the turbine inlet temperature as parameters is shown in Fig. 11.52 (a) and Fig. 11.52 (b) respectively.



(a)



(b)

Fig. 11. 52 *Effect of regeneration on thermal efficiency of simple cycle with (a) Presssure ratio and (b) Turbine inlet temperature as parameters*

11.10 COMPONENTS OF GAS TURBINE PLANT

The construction and operation of the components of a gas turbine plant are necessary for proper understanding and design.

(a) Compressor The high flow rates of air through the turbine and the relatively moderate pressure ratios necessitate the use of rotary compressors. The types of compressor commonly used are the following.

1. Centrifugal compressors
2. Axial flow compressors

A centrifugal compressor consists of an impeller with a series of curved radial vanes as shown in Fig. 11.53. Air is sucked in near the hub, called the impeller eye and is whirled round at high speed by the vanes on the impeller rotating at high rpm. The static pressure of air increases from the eye to the tip of the impeller. Air leaving the impeller tip flows through diffuser passages (scroll) which convert the kinetic energy to pressure energy (Fig. 11.54). The compressors may have single inlet or double inlet. In a double inlet impeller having an eye on either side, air is drawn in on both sides (Fig. 11.55). The impeller is subjected to approximately equal forces in the axial direction. About half the pressure rise occurs in the impeller vanes and half in the diffuser passages.

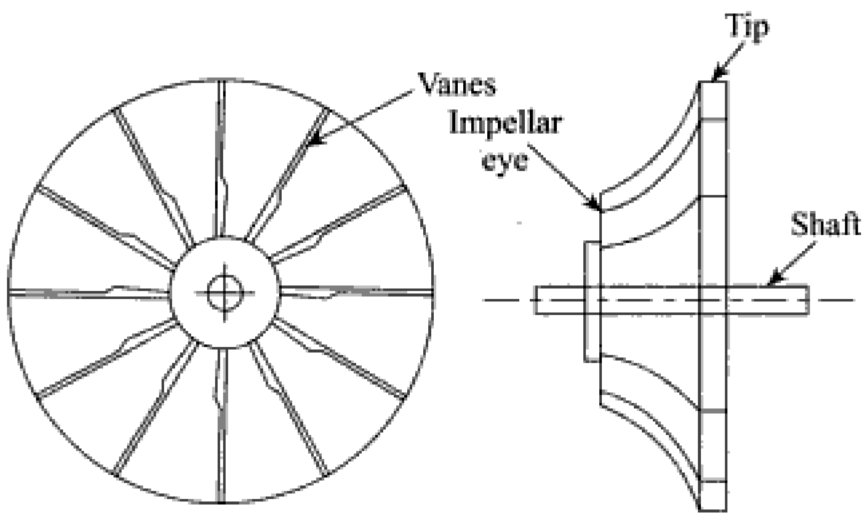


Fig. 11.53 *Centrifugal compressor impeller*

If the air flow into the impeller eye is in the axial direction (Fig. 11.53), the blade velocity diagram at inlet is shown in Fig. 11.56 (a). By using fixed guide blades, the inlet velocity to the impeller eye is inclined at an angle, known as pre-whirl (Fig. 11.56 (b)).

At exit from the impeller the flow is in the radial direction and the blade velocity V_{b_2} is larger, since the radius of the impeller is larger at outlet. The blade velocity diagram is shown in Fig. 11.57 (a) being the case of radially inclined blades and (b) being that of blades inclined backwards at an angle β_2.

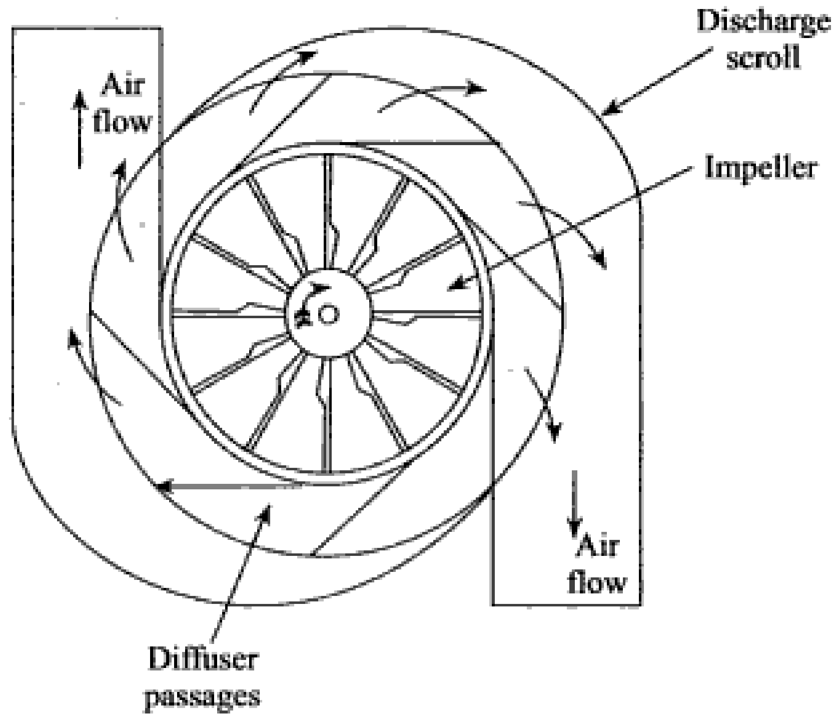


Fig. 11.54 *Centrifugal compressor showing discharge scroll*

The inertia of the air trapped between the impeller blades, however, causes the actual whirl velocity V'_{w_2} to be less than V_{w_2}. It is known as *slip*.

$$\text{Slip factor} = V'_{w2}/V_{w2} = V'_{w2}/(V_{b2} - V_{f2} \cot b_2) \tag{11.27}$$

$$\text{Power input} = \dot{m}(V_{b2}\, V'_{w2} - V_{b1}V_{w1}) \tag{11.28}$$

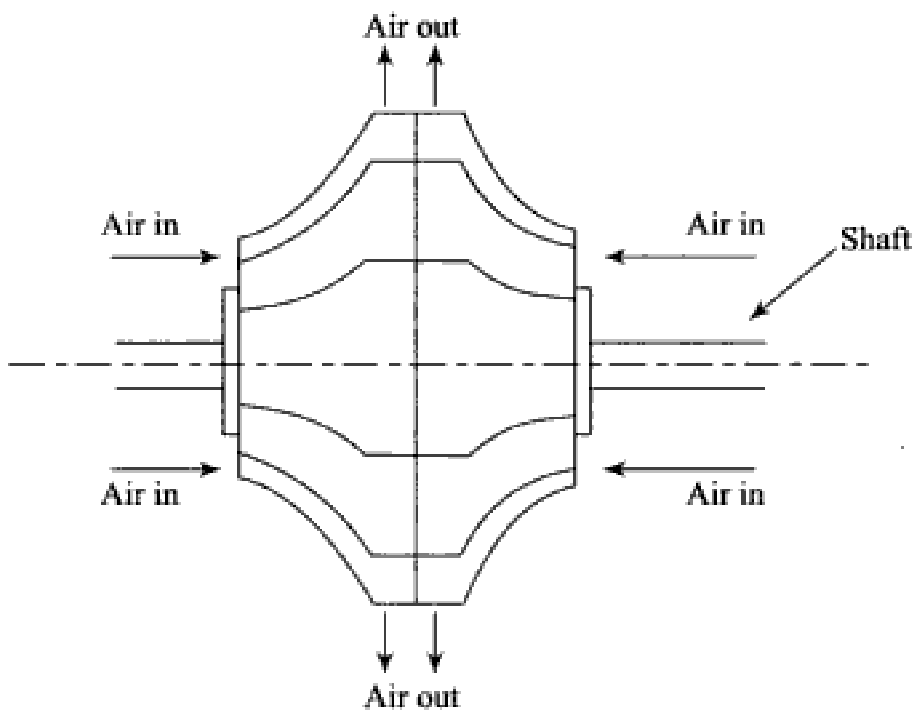


Fig. 11.55 *Double-sided impeller of a centrifugal compressor*

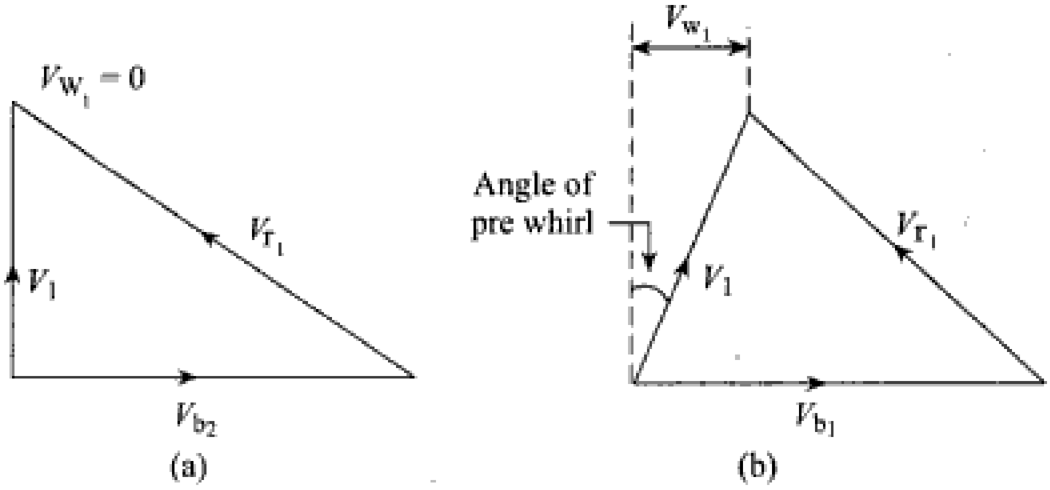


Fig. 11.56 *Blade velocity diagrams at blade inlet of a centrifugal compressor (a) Without and (b) With pre-whirl*

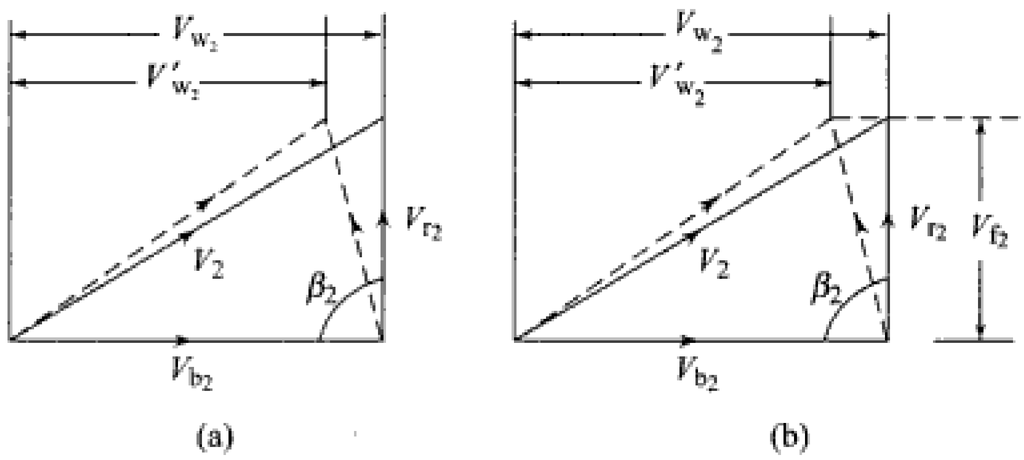


Fig. 11.57 *Blade velocity diagrams at blade outlet of a centrifugal compressor for (a) Radially inclined and (b) Backward inclined blading*

For low pressure ratios (less than 4/1) the centrifugal compressor is lighter and is able to operate effectively over a wider range of mass flows at any speed. Using titanium alloys pressure ratios above eight have now been achieved.

For larger units with higher pressure ratios the axial-flow compressor is more efficient and is usually preferred. For industrial and large marine gas turbine plants axial compressors are normally used, although some units may employ two or more centrifugal compressors with intercooling between stages. Centrifugal compressors are cheaper to produce, more robust and have a wider operating range than the axial-flow type.

An axial-flow compressor is similar to an axial-flow turbine with a succession of moving blades on the rotor shaft and fixed blades arranged around the stator (casing). Air flows axially through the moving and fixed blades, with diffuser passages throughout which continuously increase the pressure and decrease the velocity. Stationary guide vanes are provided at entry to the first row of moving blades (Fig. 11.58). The work input to the rotor shaft is transferred by the moving blades to the air, thus accelerating it. The spaces between the blades as well as the stator blades from diffusing passages decreasing velocity and increasing pressure. There can be a large number of stages (5 to 14) with a constant work input per stage.

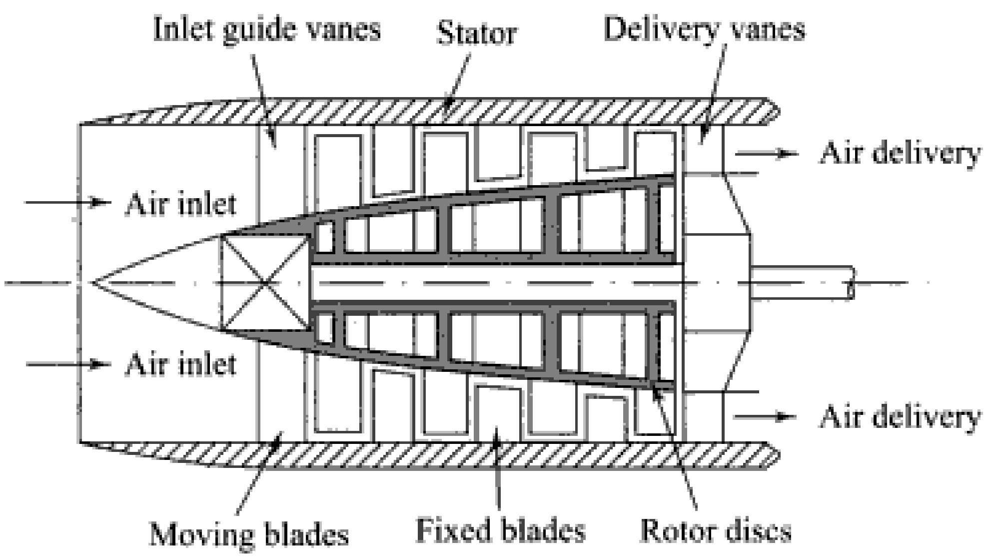


Fig. 11.58 *Axial flow compressor*

An equal temperature rise in the moving and fixed blades is usually maintained. The axial velocity of air is also kept constant throughout the compressor. A diffusing flow is less stable than a converging flow as in a turbine and for this reason the blade shape and profile are more important for a compressor than for a reaction turbine.

Typical blade sections of an axial-flow compressor are shown in Fig. 11.59 (a) and the corresponding velocity diagrams in Fig. 11.59 (b).

$$\text{Power input} = \dot{m} V_b \Delta V_w \tag{11.29}$$

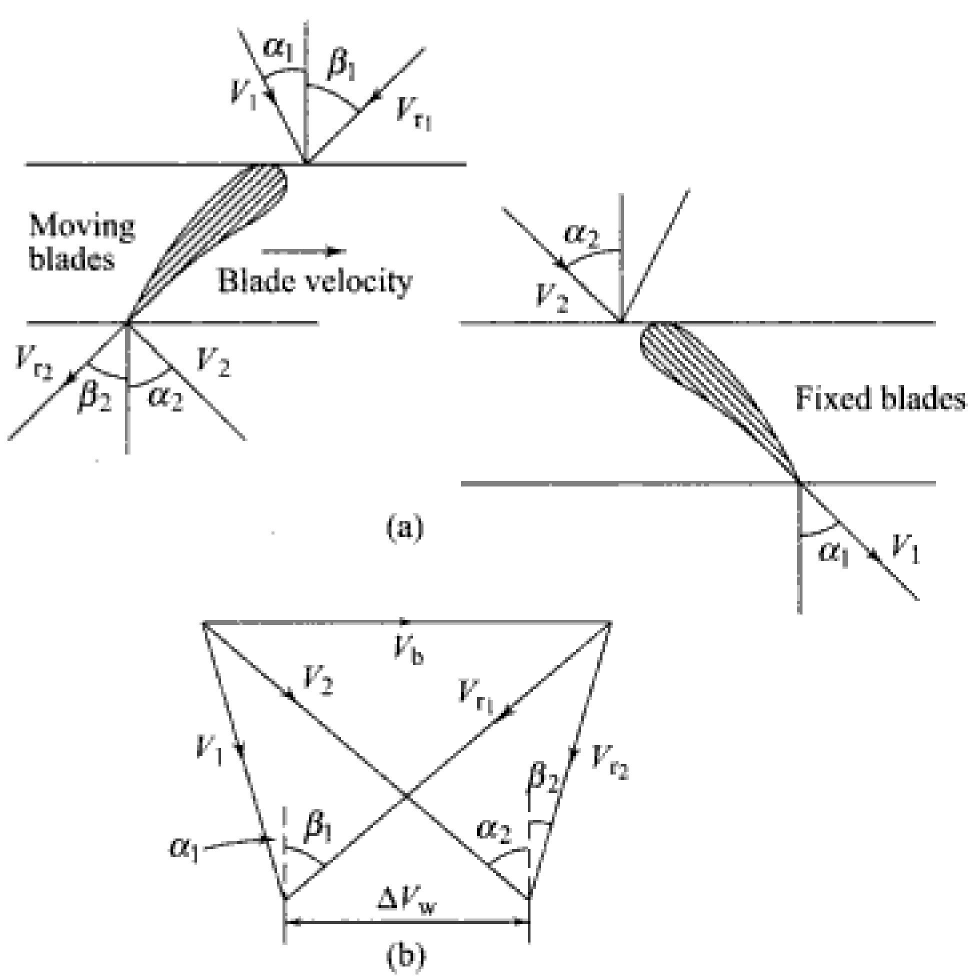


Fig. 11.59 *(a) Typical blade sections (b) Blade velocity diagrams for an axial-flow compressor*

From the geometry of the diagram,

$$\Delta V_w = V_{r1} \sin \beta_1 - V_{r2} \sin \beta_2$$

$$= V_f (\tan \beta_1 - \tan \beta_2) \tag{11.30}$$

Degree of reaction,

$$R = \frac{\text{Enthalpy rise in rotor}}{\text{Enthalpy rise in the stage}}$$

$$= \frac{h_1 - h_0}{h_2 - h_0} = \frac{V_{r_1}^2 - V_{r_2}^2}{2V_b \Delta V_w}$$

By re-arrangement,

$$R = \frac{V_f^2 (\sec^2 \beta_1 - \sec^2 \beta_2)}{2V_b V_f (\tan \beta_1 - \tan \beta_2)}$$

$$= \frac{V_f}{2V_b} - (\tan \beta_1 + \tan \beta_2) \tag{11.31}$$

Blades are usually of twisted section designed according to free vortex theory (see Cohen et al.).

Due to nonuniformity of the velocity profile in the blade passages the work that can be put into a given blade passage is less than that given by the ideal diagram. It is taken care of by introducing a work done factor, y, defined as

Work done factor, $$y = \frac{\text{Actual power input}}{\dot{m} V_b \Delta V_w}$$

which is about 0.85 for a compressor stage.

(b) *Combustion chamber* In an open cycle GT plant combustion may be arranged to take place in one or two large cylindrical can-type combustion chambers (CC) with ducting to convey the hot gases to the turbine. Combustion is initiated by an electric spark and once the fuel starts burning, the flame is required to be stabilized. A pilot or recirculated zone is created in the main flow to establish a stable flame which helps to sustain combustion continuously. The common methods of flame stabilization are by swirl flow and by bluff body.

Figure 11.60 shows a can-type combustor with swirl flow flame stabilization. About 20 per cent of the total air from the compressor is directly fed through a swirler to the burner as primary air, to provide a rich fuel-air mixture in the primary zone, which continuously burns, producing high temperature gases. Air flowing through the swirler produces a vortex motion creating a low pressure zone along the axis of the CC to cause reversal of flow. About 30 per cent of total air is supplied through dilution holes in the secondary zone through the annulus round the flame tube to complete the combustion. The secondary air must be admitted at right points in the CC, otherwise the cold injected air may chill the flame locally thereby reducing the rate of reaction. The secondary air not only helps to complete the combustion process but also helps to cool the flame tube. The remaining 50 per cent of air is mixed with burnt gases in the tertiary zone to cool the gases down to the temperature suited to the turbine blade materials.

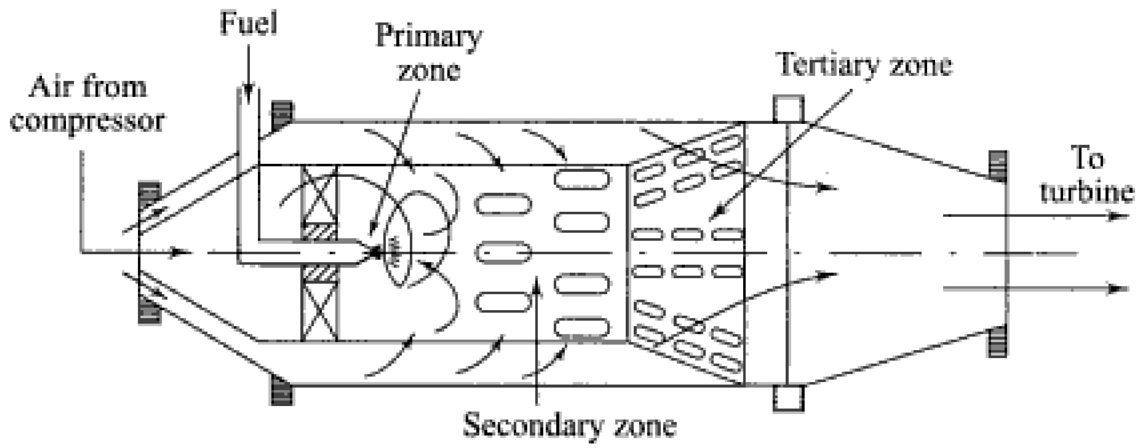


Fig. 11.60 *Can-type combustor with swirl flow flame stabilizer*

Figure 11.61 shows a can-type combustor with a bluff body stabilizing the flame. The fuel is injected upstream into the air flow and a sheet metal cone and perforated baffle plate ensure the necessary mixing of fuel and air. The low pressure zone created downstream side causes the reversal of flow along the axis of the CC to stabilize the flame. Sufficient turbulence is produced in all three zones of the CC for uniform mixing and good combustion.

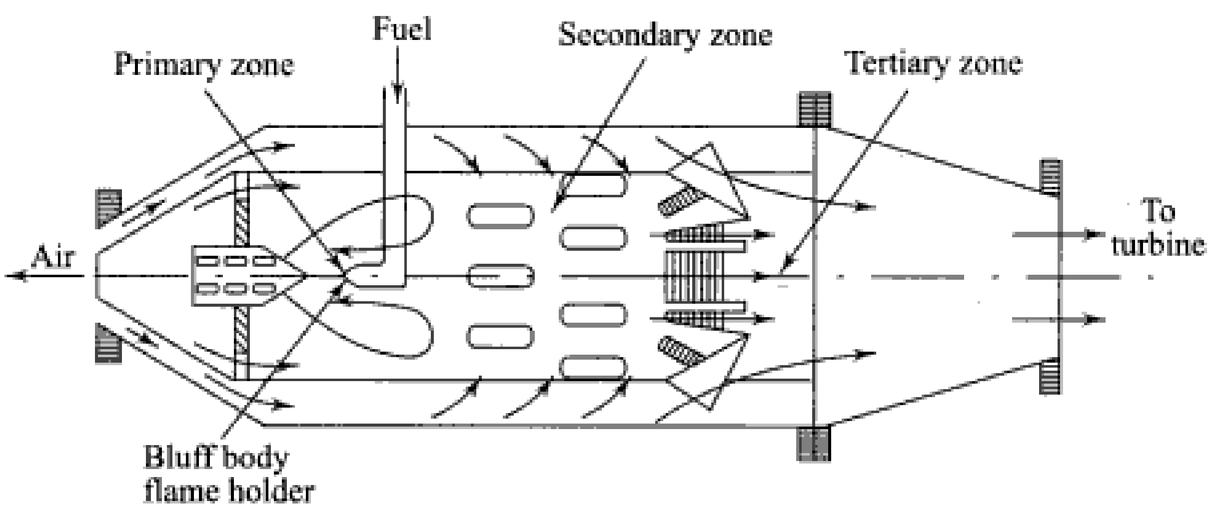


Fig. 11.61 *Can-type combustor with bluff-body flame stabilizer*

The air-fuel ratio in a GT plant varies from 60/1 to 120/1 and the air velocity at entry to the CC is usually not more than 75 m/s. There is a rich and a weak limit of flame stability and the limit is usually taken at flame blowout. Instability of the flame results in rough running with consequent effect on the life of the CC.

Because of the high air-fuel ratio used, the gases entering the HP turbine contain a high percentage of oxygen and therefore if reheating is performed, the additional fuel can be burned satisfactorily in HP turbine exhaust, without needing further air for oxygen.

A term "combustion efficiency" is often used in this regard, which is defined as follows.

$$\text{Combustion efficiency} = \frac{\text{Theoretical fuel–air ratio for actual temperature rise}}{\text{Actual fuel air ratio for actual temperature rise}}$$

Theoretical temperature rise depends on the calorific value of the fuel used, the fuel-air ratio and the initial temperature of air. To evaluate the combustion efficiency, the inlet and outlet temperatures and the fuel and air mass flow rates are measured. The fuel used in aircraft gas turbine is a light petroleum distillate or kerosene of gross calorific value of 46.4 MJ/kg. For gas turbines used in power production or in cogeneration plants, the fuel used can be natural gas.

In order to give a comparison of combustion chambers operating under different ambient conditions, a combustion intensity is defined as the following.

$$\text{Combustion intensity} = \frac{\text{Heat release rate}}{\text{Volume of CC} \times \text{inlet pressure}}$$

The lower the combustion intensity, the better the design. In aircraft a figure of about 2 kW/(m^3 atm) is normal, whereas in large industrial plants it is about 0.2 kW/(m^3 atm).

(c) Gas turbines Like steam turbines, gas turbines are also of the axial-flow type (Fig. 11.62). The basic requirements of the turbines are light weight, high efficiency, reliability in operation and long working life. Large work output can be obtained per stage with high blade speeds when the blades are designed to sustain higher stresses. More stages are always preferred in gas turbine power plants, because it helps to reduce the stresses in the blades and increases the overall life of the turbine. The cooling of gas turbine blades is essential for long life as it is continuously subjected to high temperature gases.

Blade angles of gas turbines follow the axial-flow compressor blading (Fig. 11.59 (a)), where the degree of reaction is not 50 per cent. It is usually assumed for any stage that the absolute velocity at inlet to each stage (V_2) is equal to the absolute velocity at exit from the moving blades (i.e. V_2) and that the same flow velocity V_f is constant throughout the turbine.

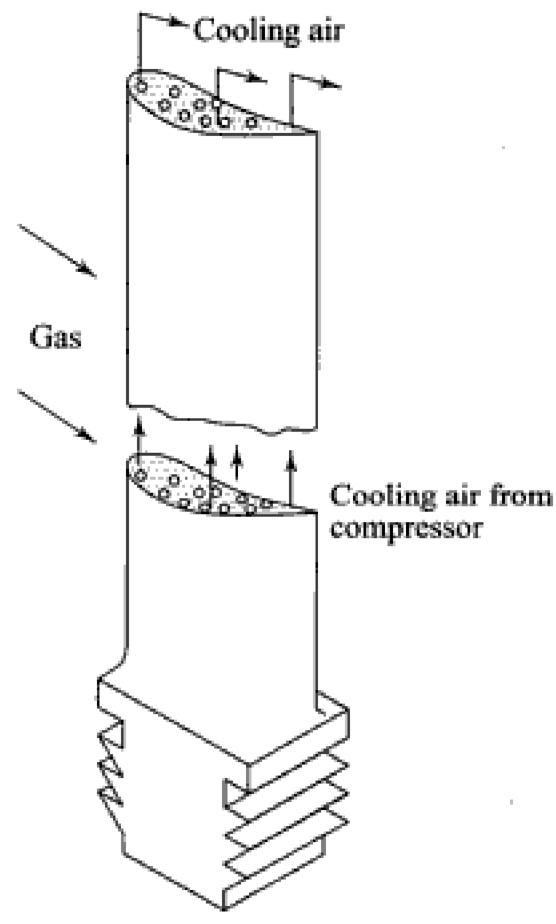


Fig. 11.62 *Typical air-cooled gas turbine blade*

The degree of reaction, R, as defined for a steam turbine, is valid for gas turbines also. It is the ratio of the enthalpy drop in the moving blades to the enthalpy drop in the stage. As shown in Fig. 11.59 (a), we have

$$R = \frac{V_{r_2}^2 - V_{r_1}^2}{2V_b \Delta V_w} = \frac{V_f^2(\sec^2\beta_2 - \sec^2\beta_1)}{2V_b(V_f\tan\beta_2 + V_f\tan\beta_1)}$$

$$= \frac{V_f(\tan^2\beta_2 - \tan^2\beta_1)}{2V_b(\tan\beta_2 + \tan\beta_1)} = \frac{V_f}{2V_b}(\tan\beta_2 - \tan\beta_1) \quad (11.32)$$

Putting $R = 0.5$ in Eq. (11.32), we get

$$V_f(\tan\beta_2 - \tan\beta_1) = V_b$$

or

$$V_b + V_f\tan\alpha_2 - V_f\tan\beta_1 = V_b$$

$$\alpha_2 = \beta_1$$

It also follows that $\alpha_1 = \beta_2$. The fixed and moving blades have the same cross-section and the diagram is symmetrical.

(d) *Vortex blading* is the name given to the twisted blades which are designed by using three dimensional flow equations with a view to decrease fluid flow losses. A radial equilibrium equation can be derived (*see the book of Cohen et al.*) and it can be shown that one set of conditions which satisfies this equation is as follows.

(a) Constant axial velocity along the blades, i.e.

$$V_f = \text{constant.}$$

(b) Constant specific work over the annulus, i.e.

$$V_b\,\Delta V_w = \text{constant.}$$

(c) Free vortex at entry to the moving blades, i.e. $V_{w1}r$ = constant, where r is the blade radius at any point.
Since the specific work output is constant over the annulus, it can be calculated at the mean radius, and multiplied by the mass flow rate it becomes the power for the stage. Since the fluid density varies along the blade height, the density at the mean radius can be used, so that $\dot{m} = \rho_m V_f A$, where A is the blade annular area.

(d) *Duct work* The duct work consists of ducts between the compressor and the combustion chamber, combustion chamber to the turbine, and the exhaust duct. The ducts must be sized to minimize the pressure losses, as the loss in pressure directly reduces the capacity of the plant.

Ducts should be supported from the floor to reduce vibration. Expansion joints must be provided to allow for dimensional changes due to temperature variation.

11.11 GAS TURBINE FUELS

Gas turbines are basically designed to operate on petroleum-based fuels like natural gas, kerosene, aviation fuel and residual fuel oil. Other fuels like powdered coal, sewage gas, etc. are also being actively considered.

All these required properties cannot be obtained in one material. Therefore, the selection of material for each component is a difficult job.

1. *Metals for turbine rotor discs* The turbine rotor disc is subjected to centrifugal and thermal stresses. The thermal stresses (due to temperature gradient) can be reduced by using an alloy of high conductivity.

The disc hub stresses tend to cause tensile deformation. This can be minimised by using a material of low expansion coefficient. Austenitic steels with 12 to 18 per cent chromium, 8 to 12 per cent nickel and small percentages of tungsten, molybdenum and titanium are used for turbine rotor discs.

These days the turbine discs are cooled by tapping compressed air from the compressor. Therefore, less expensive materials can be used. Ferritic steels having higher creep strength at low temperature (up to 600°C) can be used for the central portion, whereas austenitic steel is used on the outer surface of the ferritic rotor disc.

2. *Material for turbine rotor blade* Blades are subjected to the highest stresses and temperatures. Most satisfactory materials for blades are the stainless steel alloys and 8-20 nickel chromium alloys, known as Nimonic alloys. These alloys have high resistance to oxidation, scaling and deformation and have good creep and fatigue properties. A uniform coating with ceramics (silicon carbide, silicon nitride, aluminium nitride, etc.) on the blades of nimonic alloys provides better mechanical properties. Blades are cooled by compressed air taken by a bleed from the compressor.

3. *Material for combustion chamber* The gas turbine combustion chamber is generally made of Nimonic 75 alloy. This alloy has an excellent creep resistance, capacity to withstand heavy thermal shocks, and high resistance to oxidation.

4. *Material for compressor* The impeller of centrifugal compressor is subjected to high centrifugal and thermal stresses, the latter being due to the temperature difference between the air inlet and air discharge temperatures. To minimise centrifugal stresses lighter materials like aluminium alloys are used. These alloys suffer from high thermal expansion, for which allowance is provided.

The axial flow compressor blades are now made of titanium alloys, which are of low density, possess good strength at high temperatures (400-500°C) and are strongly resistant to corrosion. Light weight, good creep strength and fatigue resistance are attractive features of titanium alloys.

11.13 FREE PISTON ENGINE PLANT

The free piston engine is usually constructed as an opposed piston, two-stroke diesel cycle with a conventional fuel injection system. It could be used in a gas turbine plant to replace the air compressor and combustion chamber. The operation of such a plant is shown in Fig. 11.63. It comprises five cylinders with two assemblies of pistons that move opposite to each other. A diesel cylinder located at the centre powers the pistons in opposite direction. These pistons are

11.13.1 Advantages of a Free Piston Engine

1. It is very compact in space and thus has an economical building cost.
2. It has high availability and can run up from cold to full load hardly within 15 minutes.
3. The exhaust gases coming out from the free piston engine are rich in oxygen due to the scavenging air and are at a lower temperature. The plant output can be increased by injecting additional fuel, the combustion of which raises the temperature to a level acceptable to the turbine.
4. The plant can be run at 20–25 per cent overload.
5. It can be built in high capacities.
6. It is heavier than the gas turbine, but lighter than diesel engine.
7. The full load efficiency of the plant is as good as a normal diesel engine.
8. The compressor and the combustor in a gas turbine plant are replaced by the free piston engine which acts as a gasifier converting the fuel and air into a steady supply of hot gas at moderate pressure and temperature (4.5 bar and 500°C). The gas turbine being unburdened with a compressor can develop about three times more of useful power than a conventional gas turbine plant of the same size.

11.13.2 Disadvantages of a Free Piston Engine

1. Lack of design and manufacturing techniques is a deterrent factor for its wide use.
2. There are some problems of starting and controlling the output.
3. There is a problem of synchronization of the free piston engine and the gas turbine.

Example 11.5 The blade velocity at the mean diameter of a gas turbine stage is 360 m/s. The blade angles at inlet and exit are 20° and 52° respectively and the blades at this section are designed to have a degree of reaction of 50 per cent. The mean diameter of the blades is 0.450 m and the mean blade height is 0.08 m. Assuming that the blades are designed according to vortex theory, calculate (a) the flow velocity (b) the blade angles at the tip and the root (c) the degree of reaction at the tip and at the root of the blades.

Solution

Given: $\beta_1 = 20° = \alpha_2$, $\beta_2 = 52° = \alpha_1$, $V_{bm} = 360$ m/s, $D_m = 0.450$ m
$h_b = 0.08$ m (Fig. E11.5 (a)).

$$V_f \tan\beta_2 - V_f \tan\beta_1 = V_{bm} = 360$$

$$V_f(\tan 52° - \tan 20°) = 360$$

$$V_f = \frac{360}{1.2799 - 0.364} = 393 \text{ m/s} \qquad \textit{Ans. } \text{(a)}$$

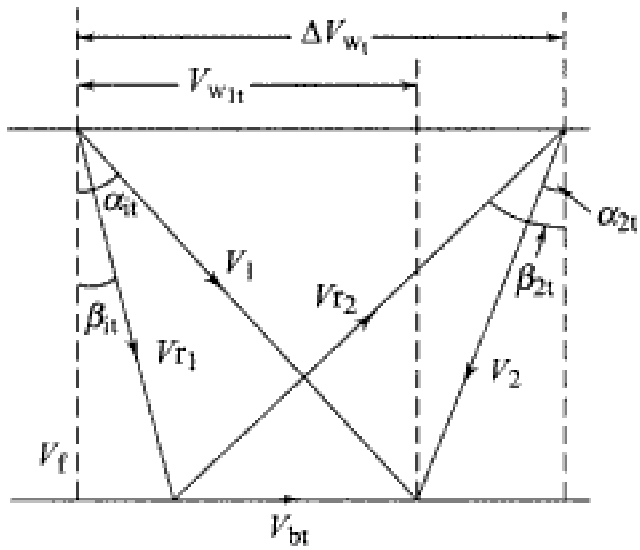


Fig. 11.5 (b)

$$V_f \tan \alpha_{1r} = V_{w_{1r}}$$
$$393 \tan \alpha_{1r} = 611.76$$
$$\alpha_{1r} = 57.28°$$
$$V_f \tan \alpha_{2r} = \Delta V_{w_r} - V_{w1r} = 785.79 - 611.76$$
$$= 174.03$$
$$\alpha_{2r} = 23.88°$$

Fixed blades (root): $\alpha_{1r} = 57.28°$, $\alpha_{2r} = 23.88°$ *Ans.* (b)

For moving blades

$$V_f \tan \beta_{1r} = V_{w_{1r}} - V_{b_r} = 611.76 - 296 = 315.76$$
$$\beta_{1r} = 38.78°$$
$$V_f \tan \beta_{2r} = V_{b_r} + V_f \tan \alpha_{2r} = 296 + 174.03$$
$$= 470.03$$
$$\beta_{2r} = 50.1°$$

Moving blades (root): $\beta_{1r} = 38.78°$, $\beta_{2r} = 50.1°$ *Ans.* (b)

$$V_f \tan \alpha_{1t} = V_{w1t}$$
$$393 \tan \alpha_{1t} = 427$$
$$\alpha_{1t} = 47.37°$$
$$V_f \tan \alpha_{2t} = DV_{w_t} - V_{w_{1t}} = 548.57 - 427$$
$$= 121.57$$
$$\tan \alpha_{2t} = .\frac{121.57}{393}$$
$$\alpha_{2t} = 17.19°$$

Fixed blades (tip) $\alpha_{1t} = 47.37°$, $\alpha_{2t} = 17.19°$ *Ans.*

Similarly for moving blades,

$$V_f \tan \beta_{1t} = V_{w_{1t}} - V_{b_t}$$

$$393 \tan \beta_{1t} = 427 - 424$$
$$\beta_{1t} = 0.44°$$
$$V_f \tan \beta_{2t} = 424 + 121.57 = 545.57$$
$$\beta_{2t} = 54.23°$$

Moving blades (tip) $\beta_{1t} = 0.44°$, $\beta_{2t} = 54.23°$ *Ans.*

Blade root

$$r_r = 0.225 - 0.04 = 0.185 \text{ m}$$
$$V_{b_r} = 360 \times \frac{0.185}{0.225} = 296 \text{ m/s}$$
$$V_{w_{1m}} r_m = V_{w_{1r}} r_r$$
$$V_{w_{1r}} = \frac{503 \times 0.225}{0.185} = 611.76 \text{ m/s}$$
$$\Delta V_{wr} = \frac{\Delta V_{w_m} V_{b_m}}{V_{b_r}} = \frac{232593.68}{296} = 7285.79 \text{ m/s}$$

Blade velocity diagram at the blade root is shown in Fig. E11.5 (c).

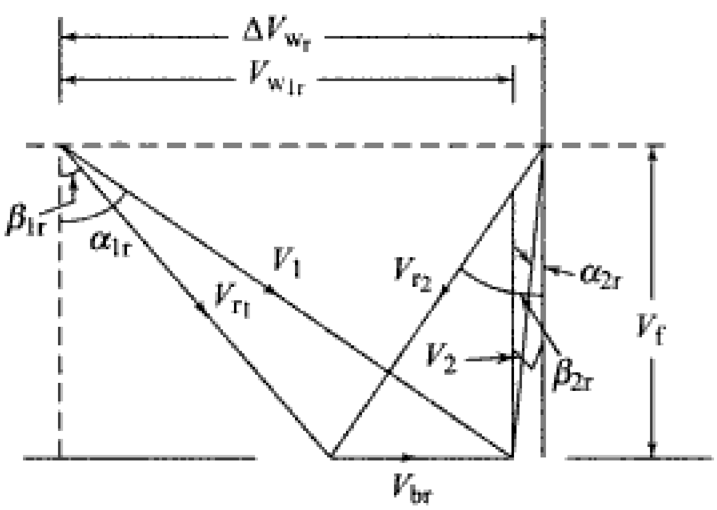


Fig. E11.5 (c)

The degree of reaction is given by

$$R = \frac{V_f (\tan \beta_2 - \tan \beta_1)}{2V_b}$$

At the tip, $$R = \frac{393(\tan 54.23° - \tan 0.44°)}{2 \times 424} = 0.64 \text{ or } 64\%$$ *Ans.*

At the root, $$R = \frac{393(\tan 50.1° - \tan 38.78°)}{2 \times 296} = 0.26 \text{ or } 26\%$$ *Ans.*

Example 11.6 A centrifugal compressor running at 16 000 rpm takes in air at 17°C and compresses it through a pressure ratio of 4 with an isentropic efficiency of 82 per cent. The blades are radially inclined and the slip factor is 0.85. Guide vanes at inlet give the air an angle of pre-whirl of 20° to the axial direction. The mean diameter of the impeller eye is 200 mm and the absolute air velocity at inlet is 120 m/s. Calculate the impeller tip diameter. Take $c_p = 1.005$ kJ/kgK and $\gamma = 1.4$.

Solution With reference to Fig. E11.6 (a).

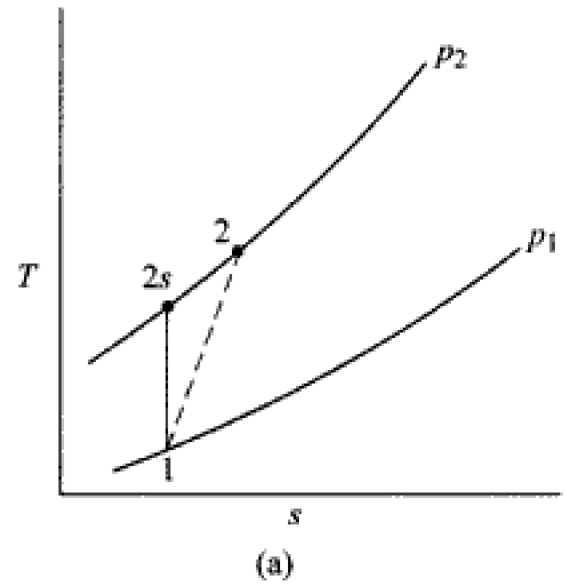


Fig. E11.6 (a)

$$T_{2s}/T_1 = (4)^{(1.4-1)/1.4} = 4^{0.286} = 1.487$$
$$T_{2s} = 290 \times 1.487 = 431 \text{ K}$$
$$\Delta T_s = T_{2s} - T_1 = 431 - 290 = 141 \text{ K}$$
$$\Delta_T = 141/0.82 = 171.95 \text{ K}$$
$$W_c = mc_p\Delta T = 1 \times 1.005 \times 171.95$$
$$= 172.81 \text{ kJ/kg} = \text{power input per kg}$$

Absolute air velocity at inlet (Fig. E11.6 (b))

$$V_1 = 120 \text{ m/s}$$

$$V_{b1} = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.2 \times 16000}{60} = 167.55 \text{ m/s}$$

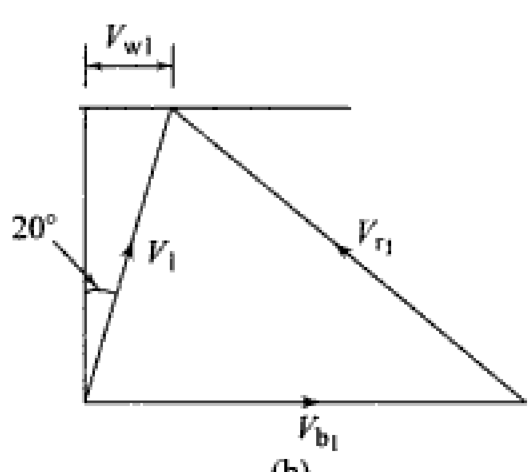
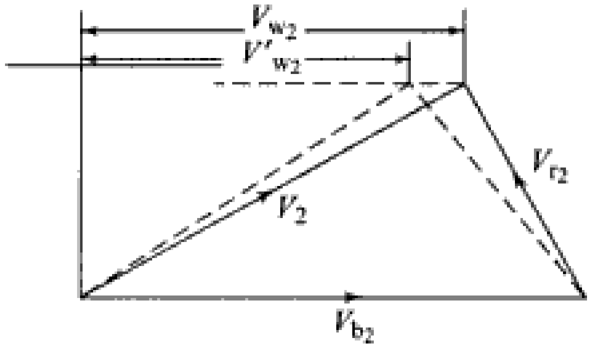


Fig. E11.6 (b)

Pre-whirl angle = 20°

$$V_{w_1} = V_1 \sin 20° = 120 \sin 20° = 41.04 \text{ m/s}$$

At exit of the vanes (Fig. E11.6 (c)),



(c)

Fig. E11.6 (c)

$$V_{w_2} = V_{b_2}$$

$$\text{Slip factor} = V'_{w_2}/V_{w_2} = 0.85$$

$$V'_{w_2} = 0.85\, V_{w_2} = 0.85\, V_{b_2} \quad (1)$$

$$\text{Power input per kg} = V_{b_2} V'_{w2} - V_{b_1} V_{w_1}$$

$$172.81 \times 10^3 = V_{b_2} \times 0.85\, V_{b2} - 167.55 \times 41.05$$

$$= 0.85\, V^2_{b_2} - 6877.93$$

$$V_{b_2} = 459.78 \text{ m/s} = \frac{\pi d_2 \times 16000}{60}$$

Tip diameter, $d_2 = 0.5488$ m = 549 mm *Ans.*

Example 11.7 Air enters an axial flow compressor at 25°C and undergoes a pressure increase 6 times that at inlet. The mean velocity of rotor blades is 220 m/s. The inlet and exit angles of both the moving and fixed blades are 45° and 15° respectively. The degree of reaction at the mean diameter is 50 per cent and there are 10 stages in the compressor. If the isentropic efficiency of the compressor is 83 per cent and the axial velocity is taken constant throughout, find the work done factor of the compressor.

Solution

Given: $V_b = 220$ m/s, $\beta_1 = 45° = \alpha_2$, $\beta_2 = 15° = \alpha_1$

$V_{r_2} = V_1$, $V_{r_1} = V_2$.

From Fig. E11.7, $V_1 \cos \alpha_1 = V_2 \cos \alpha_2$

$$V_b = V_{r_2} \sin \beta_2 + V_2 \sin \alpha_2$$

$$= V_1 \sin \beta_2 + V_1 \cos \alpha_1 \tan \alpha_2$$

$$220 = V_1 [\sin 15° + \cos 15° \tan 45°]$$

$$= V_1 (0.2588 + 0.9659)$$

$$V_1 = 179.64 \text{ m/s}$$

$$V_2 = \frac{179.64 \times \cos 15°}{\cos 45°} = \frac{173.52}{0.7071} = 245.4 \text{ m/s}$$

$$\Delta V_w = V_2 \sin \alpha_2 - V_1 \sin \alpha_1$$

Solution The arrangement of the power plant and the corresponding T–s diagram are shown in Fig. E11.8 (a) and (b).

For perfect intercooling, the intercooler pressure p_i is given by

$$p_i = [p_1 p_2]^{1/2} = [1 \times 6]^{1/2} = 2.45 \text{ bar}$$

$$T_{2s}/T_1 = [p_i/p_1]^{(\gamma-1)/\gamma} = (2.45)^{0.4/1.4} = 1.292$$

$$T_{2s} = 1.292 \times 293 = 378.59 \text{ K}$$

$$\frac{T_{2s} - T_1}{T_2 - T_1} = \eta_c = 0.82$$

$$T_2 - T_1 = \frac{378.59 - 293}{0.82} = 104.38 \text{ K}$$

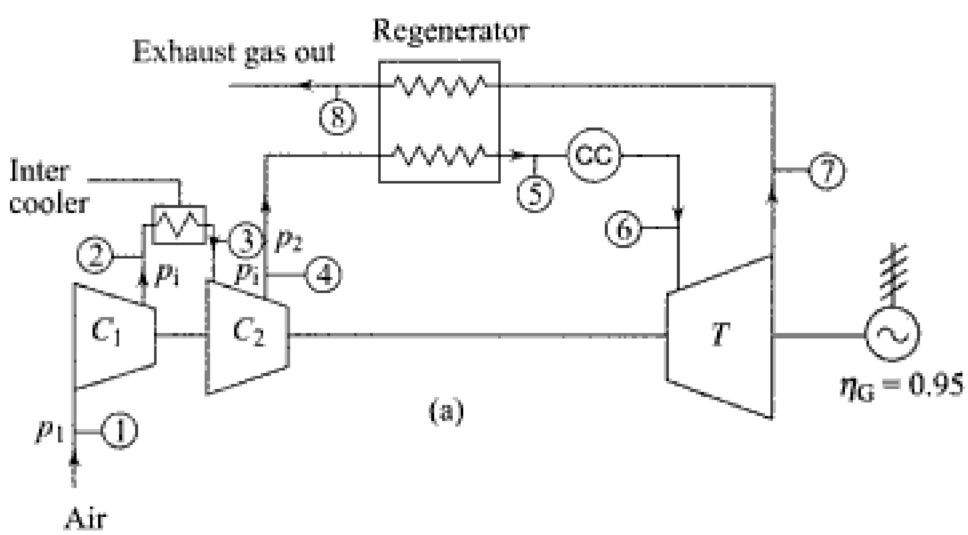
$$T_2 = 397.38 \text{ K}$$

$$T_{4s}/T_3 = [p_2/p_i]^{(\gamma-1)/\gamma}$$

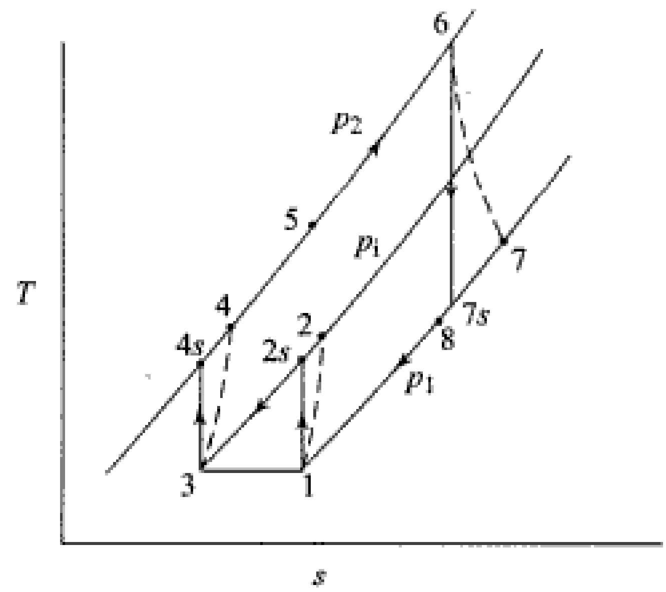
$$T_{4s} = 293 \times (6/2.45)^{0.286} = 378.54 \text{ K}$$

$$T_4 - T_3 = \frac{378.54 - 293}{0.82} = 104.32$$

$$T_4 = 397.32 \text{ K}$$



(a)



(b)

Fig. E11.8

$$T_6/T_{7s} = [p_2/p_1]^{(\gamma-1)/\gamma}$$
$$1173/T_{7s} = 6^{0.33/1.33} = 6^{0.248} = 1.5595$$
$$T_{7s} = 752.17 \text{ K}$$
$$T_6 - T_7 = \eta_T(T_6 - T_{7s}) = 0.92\,(1173 - 752.17) = 1173 - T_7$$
$$T_7 = 1173 - 387.16 = 785.84 \text{ K}$$

Regenerator effectiveness

$$\varepsilon = \frac{T_5 - T_4}{T_7 - T_4}$$

$$0.7 = \frac{T_5 - 397.32}{785.84 - 397.32}$$

$$T_5 = 669.28 \text{ K}$$

Making an energy balance for the combustion chamber,

$$m_f \times CV \times \eta_{comb} = c_p\,(1 + m_f)\,(T_6 - T_5)$$

where m_f is the mass of fuel consumed per kg of air.

$$m_f \times 40{,}800 \times 0.95 = 1.005\,(1 + m_f)\,(1173 - 669.28)$$
$$m_f = (1 + m_f)\,0.0131$$

$$\frac{1}{m_f} + 1 = 76.56$$

$$1/m_f = 75.56$$

$$m_a/m_f = \text{air-fuel ratio} = 75.56$$ *Ans.* (a)

$$W_{GT} = (m_a + m_f)c_{pg}\,(T_6 - T_7)$$
$$= \left(1 + \frac{1}{75.56}\right) \times 1.08\,(1173 - 785.84)$$
$$= 423.67 \text{ kJ/kg air}$$
$$W_c = m_a\,c_p\,[(T_2 - T_1) + (T_4 - T_3)]$$
$$= 1 \times 1.005\,[(397.38 - 293) + (397.32 - 293)]$$
$$= 209.74 \text{ kJ/kg air}$$
$$W_{net} = 423.67 - 209.74 = 213.93 \text{ kJ/kg air}$$

$$\text{Heat supplied} = \frac{40{,}800 \times 0.95}{75.56} = 512.97 \text{ kJ/kg air}$$

$$\eta_{cycle} = \frac{213.93}{512.97} = 0.417 \text{ or } 41.7\%$$ *Ans.* (b)

Since air flow rate is 210 kg/s, the power output of the plant at generator terminals

$$= 213.93 \times 210 \times 0.96 \times 0.95$$
$$= 40971.82 \text{ kW or } 40.972 \text{ MW}$$ *Ans.* (c)

$$\text{Fuel consumption per hour} = 210 \times 3600 \times \frac{1}{75.56} = 10005.29 \text{ kg}$$ *Ans.* (d)

Specific fuel consumption

$$= \frac{10005.29}{40971.82} = 0.244 \text{ kg/kWh} \qquad \textit{Ans.} \text{ (d)}$$

SHORT-ANSWER QUESTIONS

11.1 What are the applications of diesel electric power plants?
11.2 Enlist the advantages and disadvantages of diesel engine power plants.
11.3 Which are the different types of diesel engine?
11.4 Name the essential components of a diesel electric plant.
11.5 How is noise of the engine reduced?
11.6 Explain the scope of utilizing the waste heat in the engine exhaust.
11.7 What is an engine day tank? State the functions of a fuel injection system.
11.8 What is solid injection? Briefly explain the different systems of solid injection.
11.9 Explain the operation of a fuel pump. How is the fuel supply to the engine regulated?
11.10 What is a fuel injector? Explain a pintle nozzle.
11.11 Explain the necessity of the cooling system in a diesel engine. What are the methods of cooling the engine?
11.12 Explain the important functions of the lubrication system.
11.13 Discuss the wet sump lubrication system pertaining to a diesel engine.
11.14 How is a diesel engine started?
11.15 What do you mean by auto-ignition? Why is excess air always used in a CI engine?
11.16 Explain the ignition delay in a CI engine. What do you mean by the mixing period and the interaction period?
11.17 Explain the phenomenon of detonation in a CI engine. On what factors does it depend?
11.18 Define cetane number of a fuel. How can fuel knock be controlled?
11.19 Give the typical valve timing diagram of a four-stroke oil engine.
11.20 What is mep? How does an electronic engine indicator help in measuring *ip*?
11.21 What is a dynamometer? What does it measure? What are the different types of dynamometers?
11.22 Explain the Morse test. What is Willan's line?
11.23 Define (a) bmep (b) brake thermal efficiency and (c) bsfc.
11.24 What is volumetric efficiency? What are the variables which influence the volumetric efficiency of an engine?
11.25 Give an energy balance of a CI engine.
11.26 Explain the performance characteristics of a CI engine. What do you understand by "Smoke limit"?
11.27 What is the objective of supercharging? Why is it more beneficial in a CI engine compared to an SI engine?

11.28 Explain the main features of supercharging with the help of p-V diagram. What do you mean by mechanical supercharging and turbocharging? What is the effect of intercooling in turbocharging?
11.29 Which are the different types of compressors used for supercharging? Why are turbochargers superior to superchargers?
11.30 Give the layout of a diesel engine power plant.
11.31 Why is power generation by gas turbines attractive these days?
11.32 Give the specific advantages and disadvantages of a gas turbine plant for a utility system.
11.33 Bring out the difference between the closed cycle and open cycle gas turbine power plants.
11.34 Explain the effect of regeneration in a gas turbine plant.
11.35 Discuss the effect of pressure ratio on Brayton cycle output and efficiency.
11.36 Derive the optimum pressure ratio in an ideal gas turbine plant for maximum network. How is the expression modified when compressor and turbine efficiencies are taken into consideration? What is the corresponding maximum net work and the cycle efficiency?
11.37 Discuss the effect of intercooling and reheating in a gas turbine plant.
11.38 What is a semi-closed cycle gas turbine plant? Explain it with the help of a sketch of the plant.
11.39 Discuss the performance characteristics of a gas turbine power plant.
11.40 Explain with a neat sketch the operation of a centrifugal compressor. What is pre-whirl? Why is it provided?
11.41 What is slip? Define slip factor. State the applications of centrifugal compressor.
11.42 What is an axial-flow compressor? What are its applications? What is the basis of its design? How is the degree of reaction defined? What is work done factor? What is its approximate value?
11.43 Explain with a neat sketch the combustion chamber of a gas turbine plant. What are dilution holes? How is flame stabilization secured by (a) a swirler (b) a bluff body?
11.44 Explain the staging of air supply in the forms of primary, secondary and tertiary air in the CC of a gas turbine plant. Why is a large air-fuel ratio used?
11.45 Explain the terms (a) combustion efficiency (b) combustion intensity.
11.46 What are the essential features of gas turbine blades? How are blades cooled?
11.47 Briefly describe the different fuels which can be burnt in gas turbine plants.
11.48 Discuss the materials which are used for gas turbines and compressors. What properties should the blade material possess?
11.49 With the help of neat sketches explain the operation of a free piston engine plant. Explain its applications. What are its advantages and disadvantages?

PROBLEMS

11.1 During a 60–minute trial of a single cylinder four-stroke oil engine, the following observations were made. Engine dimension = 0.3 m bore × 0.45 m stroke, Fuel consumption 11.4 kg, Calorific value of fuel = 42 MJ/kg, imep =

6 bar, Net load on brakes = 1500 N, N = 300 rpm, Brake drum diameter = 1.8 m, Brake rope diameter = 20 mm, Quantity of jacket cooling water = 600 kg, Temperature rise of cooling water = 55°C, Quantity of air as measured = 250 kg, Exhaust gas temperature = 420°C, Specific heat of exhaust gas = 1.0 kJ/kgK, Ambient temperature = 20°C. Estimate (a) the indicated power (b) the brake power and (c) the indicated thermal efficiency. Draw up an energy balance sheet of the engine.

[*Ans.* (a) 47.71 kW (b) 42.88 kW (c) 35.86%]

11.2 The following test data refer to a four-stroke four-cylinder diesel engine. Cylinder diameter = 0.35 m, Stroke = 0.40 m, N = 315 rpm, imep = 7 bar; *bp* of the engine = 250 kW, Fuel consumption = 80 kg/h, Calorific value of fuel = 44 MJ/kg, Hydrogen content in fuel = 13%, Air consumption = 30 kg/min, Cooling water circulated = 90 kg/min, Rise in cooling water temperature = 38°C, Exhaust gas temperature = 324°C, Specific heat of air = 1.005 kJ/kgK, Specific heat of exhaust gas = 1.05 kJ/kgK, Ambient air temperature = 24°C, Specific heat of superheated steam = 2.093 kJ/kgK, Partial pressure of steam in exhaust gases = 0.03 bar. Find (a) mechanical efficiency (b) indicated thermal efficiency, and (c) bsfc. Draw up a heat balance of the engine.

11.3 A four-cylinder, four-stroke diesel engine develops 83.5 kW at 1800 rpm with a bsfc of 0.231 kg/kWh and air-fuel ratio of 23/1. The analysis of the fuel is 87% carbon and 13% hydrogen. The calorific value of fuel is 43.5 MJ/kg. The jacket cooling water flows at 0.246 kg/s and its temperature rise is 50°C. The exhaust temperature is 316°C. Draw up an energy balance for the engine. Take R = 0.302 kJ/kgK and c_p = 1.09 kJ/kgK for the dry exhaust gas and c_p = 1.86 kJ/kgK for superheated steam. The room temperature is 17.8°C and the exhaust gas pressure is 1.013 bar.

[*Ans. bp* 35.8%, cooling water 22.1%, exhaust 25.3%,
radiation and unaccounted 16.9%]

11.4 A six-cylinder, four-stroke CI engine of 75 mm bore and 100 mm stroke has a brake power output of 110 kW at 3750 rpm. The volumetric efficiency at this operating condition referred to ambient conditions of 1.013 bar and 20°C is 80 per cent.

The engine is now fitted with a mechanically driven supercharger which has an isentropic efficiency of 70 per cent and a pressure ratio of 1.6. The supercharged version has a volumetric efficiency of 100 per cent referred to the supercharger delivery pressure and temperature. If it is assumed that the *ip* developed per unit volume flow rate of induced air at ambient conditions is the same for normal aspiration and supercharging, calculate the net increase in *bp* to be expected from the supercharged engine. Take the mechanical efficiency of the engine as 80 per cent in both cases and the mechanical efficiency of the drive from the engine to the supercharger is 95 per cent.

[*Ans.* 64.1 kW]

11.5 A turbocharged six-cylinder four-stroke diesel engine has a swept volume of 39 litres. The inlet manifold conditions are 2.0 bar and 53°C. The volumetric efficiency of the engine is 95 per cent, and it is operating at a load of 16.1 bar

Determine the power input to the air and the direction of the air at entry to and exit from the rotor and stator blades. Assume air as an ideal gas with c_p = 1.005 kJ/kgK and γ = 1.4.

[*Ans.* 773 kW]

11.16 An axial flow gas turbine stage is to be designed in accordance with the following data: Blade height = 0.10 m, Rotor speed = 15,000 rpm, Mean blade ring diameter = 0.45 m, Inlet gas pressure = 3.5 bar, Absolute gas velocity at inlet and exit = 280 m/s, Stage isentropic efficiency = 0.90, Inlet gas temperature = 870°C, Exit gas pressure = 1 bar, Degree of reaction at mean blade height = 0.50.

The axial velocity can be assumed constant throughout the rotor and free vortex conditions exist in the space between the stator and rotor. Find (a) the blade angles at mean blade height at stator exit, at rotor inlet and at rotor exit and (b) the degree of reaction at the blade root.

11.17 The polytropic efficiency η_p of an expansion or compression process is the isentropic efficiency of an infinitely small stage, so that for an ideal gas $\eta_p = dh/dh_s = dh/v\,dp = (c_p dT.p)/(RT\,dp)$. Show that in an expansion process $T_1/T_2 = (p_1/p_2)^{(\gamma-1)\eta_p/\gamma}$ and for a compression process $T_2/T_1 = (p_2/p_1)^{(\gamma-1)/(\gamma\eta_p)}$.

11.18 The gas turbine in a cogeneration plant has an output of 150 MW with a thermal efficiency of 35 per cent. The fuel oil used has a calorific value of 43 MJ/kg. The exhaust gas flow rate from the gas turbine is 400 kg/s and its temperature is 550°C. The exhaust gas from the gas turbine passes through a boiler plant and leaves at 90°C. The steam generated is at a pressure of 100 bar and a temperature of 450°C. The feedwater temperature to the boiler is 140°C. The generated steam passes through a steam turbine of 86 per cent isentropic efficiency to exhaust at 5 bar. The boiler has an efficiency of 92 per cent. Determine (a) the mass of fuel used by the gas turbine in t/h (b) the mass flow of steam from the boiler in t/h (c) the output of the steam turbine in MW and (d) the overall efficiency of the total plant. Take c_p of gas turbine exhaust gases as 1.1 kJ/kgK.

[*Ans.* (a) 35.9 t/h, (b) 252.4 t/h, (c) 40.58 MW (d) 44.5%]

11.19 The air in a gas turbine plant is taken in the LP compressor at 20°C and 1.05 bar. After compressor it is passed through an intercooler where its temperature is reduced to 27°C. The cooled air is further compressed in the HP compressor and then passed to the combustion temperature where its temperature is increased to 750°C by burning the fuel. The combustion products expand in the HP turbine which runs the compressor. Further expansion of the gas continues in the LP turbine which drives the alternator. The gases coming out from the LP turbine at 1.05 bar are used for heating the incoming air from the HP compressor. The pressure ratio of each compressor is 2, isentropic efficiency of each compressor and each turbine is 0.82, the air flow rate is 16 kg/s and the calorific value of fuel is 42 MJ/kg. Neglecting the mechanical, pressure and heat losses in the plant, determine (a) the power output (b) the overall thermal efficiency and (c) the specific fuel consumption. Take $c_p = 1.0$ kJ/kgK and $g = 1.4$ for air and $c_p = 1.15$ kJ/kgK and $\gamma = 1.33$ for gases.

[*Ans.* (a) 2.05 MW, (b) 30.9%, (c) 0.277 kg/kWh]

11.20 A gas turbine power plant of 10 MW capacity works in a closed cycle using air as the working medium. The plant having a regenerator is designed for maximum specific work output. The inlet air temperature is 300 K and the maximum temperature in the cycle is 960 K. Taking the isentropic efficiency of compressor as 0.8, that of turbine as 0.9, the mechanical efficiency and generator efficiency each as 0.95, the regenerator effectiveness as 0.7, the combustion efficiency as 0.96 and assuming that 90 per cent of the heat released by combustion is transferred to air, determine the fuel burning rate and the air-fuel ratio. Take the calorific value of fuel used as 37 MJ/kg.

[*Ans.* 4400 kg/h, 92.5]

11.21 A simple open cycle gas turbine plant works between the pressures of 1 bar and 6 bar and temperatures of 300 K and 1023 K. The calorific value of fuel used is 44 MJ/kg. If the mechanical efficiency and the generator efficiency are 95 per cent and 96 per cent respectively and for an air flow rate of 20 kg/s, calculate (a) the air-fuel ratio (b) the thermal efficiency and (c) the power output.

11.22 An open cycle constant pressure gas turbine plant consists of two compressors with perfect intercooling and a two stage turbine with a reheater. Air enters at 1 bar, 15°C. The maximum pressure ratio and the maximum temperature of the cycle are limited to 5 and 800°C respectively. The reheating takes place at 2.3 bar to 800°C. The isentropic efficiencies of each compressor and each turbine are 0.8 and 0.9 respectively. The calorific value of fuel is 42 MJ/kg. Taking $c_p = 1.005$ kJ/kgK and $\gamma = 1.4$ for air, and $c_p = 1.15$ kJ/kgK and $\gamma = 1.33$ for gases and neglecting pressure and heat losses and if the air flow rate is 25 kg/s, estimate (a) the overall thermal efficiency of the plant (b) the air-fuel ratio (c) the specific fuel consumption, and (d) power output of the plant.

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12 Energy Storage

The demand for electricity in a utility system varies hourly, daily and even from season to season, whereas the supply is fixed according to the installed capacity of the system, which corresponds to the maximum demand plus a reasonable excess to take care of scheduled and unscheduled shutdowns. The result is a large, expensive power plant that operates below its capacity most of the time, thus causing high operating and capital costs.

The objective of energy storage is to offset this adverse effect of fluctuating demand of electricity and to assure a steady output from existing power plants. When the demand is lower than the capacity, energy is stored. When the demand is higher than the capacity, the stored energy is released. It is thus possible to supply electricity reliably, efficiently and economically, meeting the peak electrical demand on short notice.

The need for energy storage was not actually felt till the generating plants were relatively cheap and there was abundant fuel supply. Nature herself stored energy in the form of fuels. The energy density of fossile fuels is about 37×10^6 kJ/m^3 (at 70 bar), while for natural uranium (0.071%, U–235) it is about 10^{14} kJ/m^3. With shrinking fuel availability, the need to conserve natural resources is paramount, for which the following suitable methods of energy management must be adopted.

(a) Meet the peak load demand by interconnecting power networks that might have different power demands on them.
(b) Use modern, more efficient power plants for base-load power generation and old, less efficient power plants for peak-power generation.
(c) Install smaller low capital cost power plants, which may not be highly efficient, as peaking units, e.g. gas turbine unit and small hydroelectric plant.
(d) Incorporate energy storage systems.

Reliability and economy of electricity supply are achieved by having a mix of three types of power plants: a base-load plant, an intermediate plant and a peaking plant. Base-load plants providing the base electrical load to the grid are usually large, i.e. efficient steam power stations powered by fossil or nuclear fuels and they operate continuously except for scheduled maintenance or forced outages. Intermediate plants are older, less efficient steam plants or repowered combined cycle plants, which operate primarily during hours of high load demand. Peaking plants are specifically designed to provide power during peak demand periods.

The variation of load throughout the day, week and year stimulates a demand for storage especially when the increase in installed capacity of large coal or nuclear plants, (designed to operate at maximum efficiency on their rated power output), exceeds the base load demand and when a future increase in utilization of intermittent and variable energy sources (such as solar, wind or ocean energy) exceeds the utilities' reserve capacities.

Figure 12.1 shows a typical weekly load curve of a utility with and without energy storage. As illustrated by Fig. 12.1 (a) curve, intermediate and peaking power involves extensive generating capacity. The load variation shown here is typical for any European or US utility, but it applies to most other countries, where cheap off-peak electricity rates exist. In countries where this is not the case the daily variation tends to be larger. In any case it appears to be the fact all over the world that installed capacity is about double the yearly average load (Jensen, 1980).

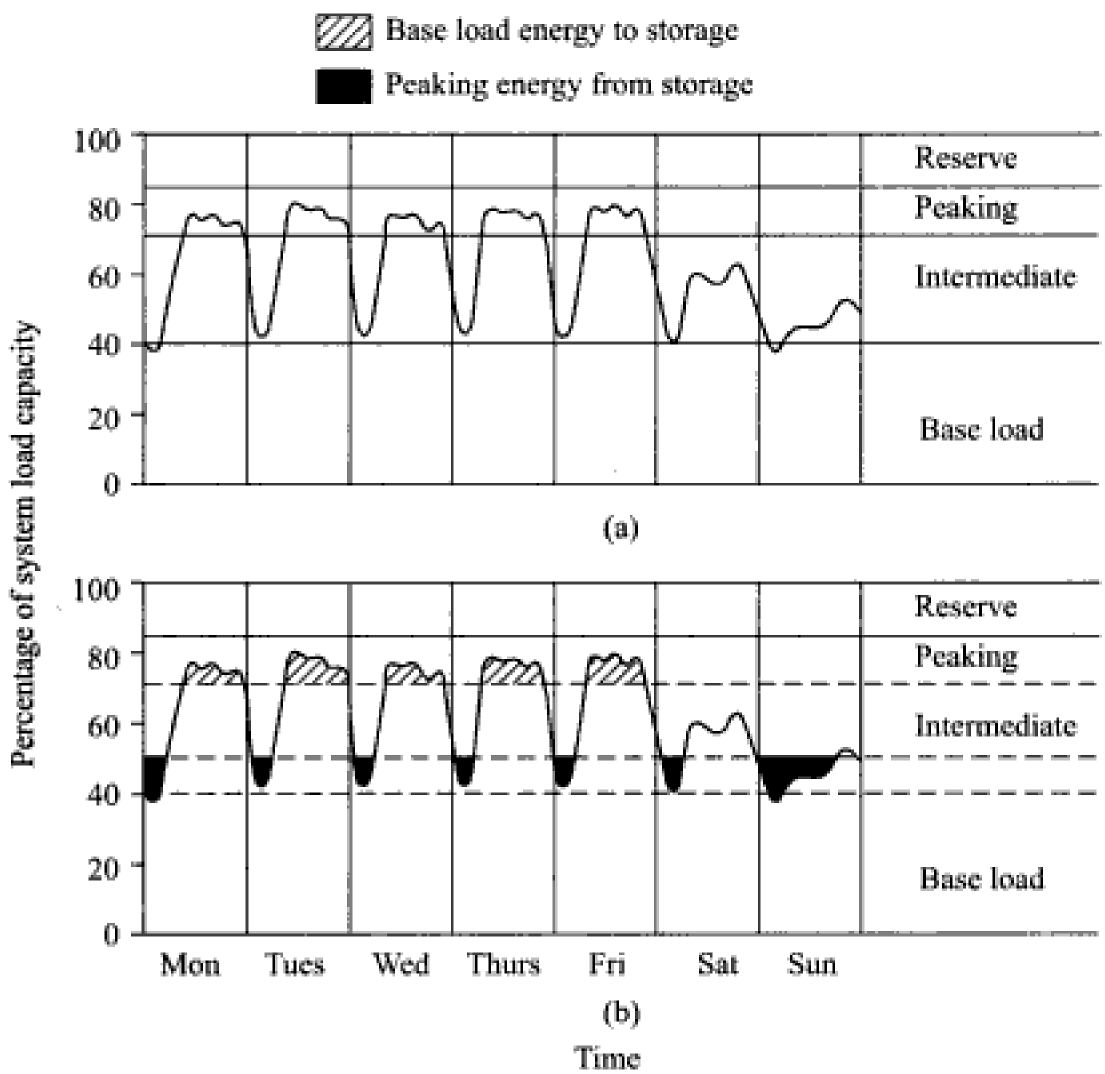


Fig. 12.1 *Weekly load curve for a typical power utility*
(a) Current generation mix
(b) Generation mix with energy storage

If large-scale energy storage were available as illustrated by the curve in Fig. 12.1 (b), then the relatively efficient and economical base-load generation could be increased and the excess beyond off-peak demand (lower shaded areas) could be used to charge the storage system.

Discharge of the stored energy (upper shaded areas) during periods of peak load demand would then reduce or replace fuel-burning peaking plant capacity, thus conserving fuel (mostly oil-based) resources. Use of energy storage to generate peaking power in this manner is termed "*peak shaving*". The higher base-load level may replace part of the intermediate generation thus performing load-levelling and enabling the more extensive use of storage to eliminate most or all conventional intermediate cycling equipment, thus reducing installed capacity and saving cost and fuel. Smoothing of the daily load curve leads to reduced stresses of load following operation by the steam plant and consequently reduces maintenance costs.

As shown in Fig. 12.2 the power plant is continuously operated during a day in a base-load mode, as a result of which excess electricity ($ab + cd$) is produced during the off-peak hours.

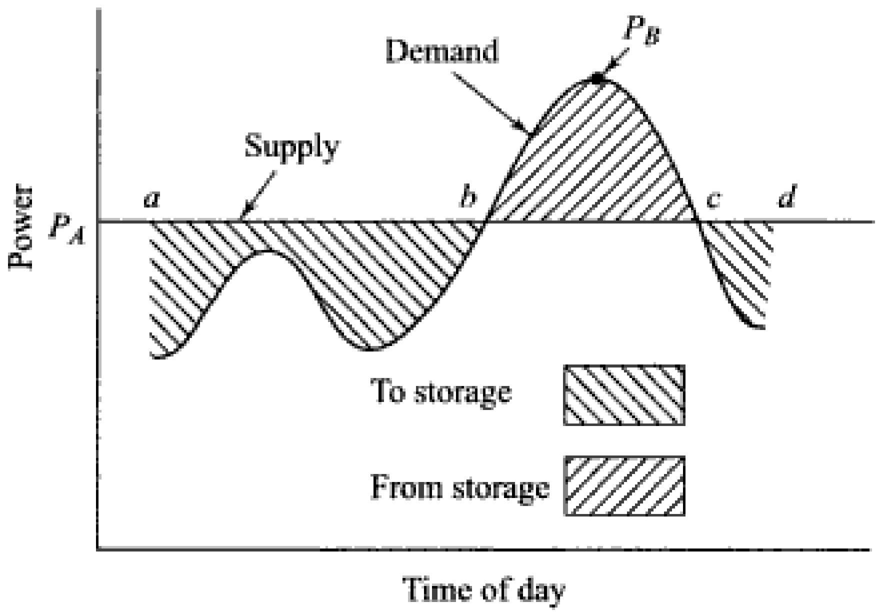


Fig. 12.2 *Energy storage with constant thermal input as from fossil or nuclear fuel*

This excess electricity is stored and then released during the peak demand period bc. Stored energy (area below $ab + cd$) is greater than that supplied (area above bc) since there are conversion losses to and from storage. Various energy storage schemes are the following.

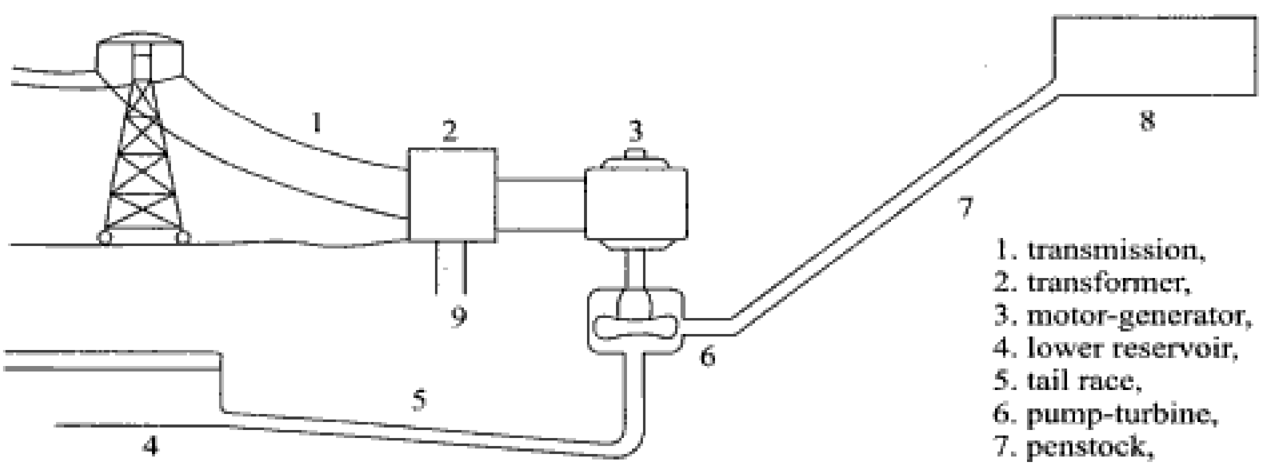
(a) Pumped hydro
(b) Compressed air energy storage
(c) Energy storage by flywheels
(d) Electrochemical energy storage
(e) Magnetic energy storage
(f) Thermal energy storage

(g) Chemical energy storage
(h) Hydrogen energy

12.1 PUMPED HYDRO

Pumped hydro storage is the only large scale energy storage method which is highly developed and used in power systems. For decades, utilities have used pumped hydro storage as an economical way to utilize off-peak energy, by pumping water to a reservoir at a higher level. During peak load periods the stored energy is discharged through the pumps, then acting as turbines, to generate electricity to meet the peak demand. Energy is thus stored as hydraulic potential energy by pumping water from a low-level into a higher level reservoir. When the discharge of the energy is required, the water is returned to the lower reservoir through turbines which drive electricity generators.

Pumped hydro storage usually comprises the following. An upper reservoir, waterways, a pump, a turbine, a motor, a generator and a lower reservoir as shown schematically in Fig. 12.3.



1. transmission,
2. transformer,
3. motor-generator,
4. lower reservoir,
5. tail race,
6. pump-turbine,
7. penstock,
8. upper reservoir,
9. to loads

Fig. 12.3 *Pumped hydroelectric energy storage*

Potential energy stored by raising mass of water m to an elevation H is

$$PE = mgH \tag{12.1}$$

The operating heads on the pump-turbine in the pumping mode Hp and in the turbine mode H_T are

$$H_P = H + H_l \tag{12.2}$$

and

$$H_T = H - H_l \tag{12.3}$$

where H is the static head and H_l represents the losses during flow.

The pumping power P_P and the generating power P_T are

$$P_P = \frac{Q_P \, \rho \, g \, H_p}{\eta_p} \tag{12.4}$$

and $$P_T = [\rho\, g\, H_T\, Q_T]\eta_T \tag{12.5}$$

where Q represents the volumetric flow rate (m^3/s), ρ is the density of water ($1000\ kg/m^3$) and η is the efficiency.

1000 kg of water raised by 100 m will store (1000) kg × (9.81) m/s^2 × (100) m or 9.81×10^5 J or 0.2725 kWh of energy. Thus large masses of water must be elevated to sufficiently large heights to store large quantities of energy.

One or both of these reservoirs may be artificially excavated or may be a natural river or lake. Pumped hydro systems may be above ground which can be of high head and medium head, or underground. When topography does not permit the former, underground reservoir may be used. A typical above ground pumped hydro system of a conventional design is shown in Fig. 12.4. A surge tank is built near the mouth of the pressure tunnel to relieve the pipes of undue inertia pressure set up in the tunnel when the flow is checked following a reduction of load. Should the pressure exceed a certain predetermined value, water merely spills over the lip of the surge tank. The surge tank also provides a reservoir of water that can be drawn upon when the load on the turbine suddenly increases.

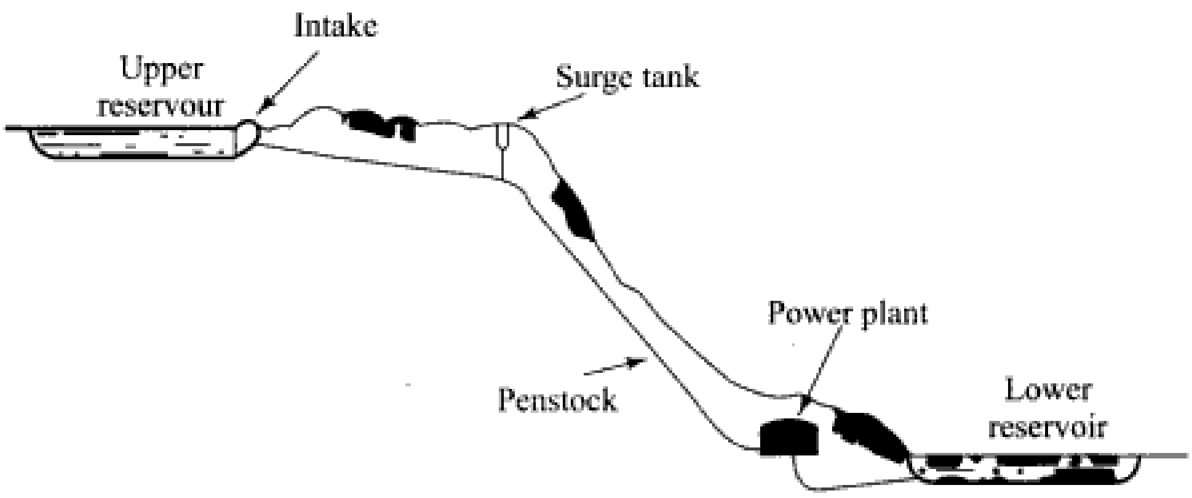


Fig. 12.4 *Schematic of a conventional above ground pumped hydro storage system*

The losses in pumped hydro systems include the following:

(a) motor and pump losses
(b) flow losses during upflow
(c) seepage into ground
(d) leakage of water from pipes and equipment
(e) evaporation during storage
(f) turbine and generator losses
(g) flow losses during downflow

The combined efficiency of a pumped hydro system, called the *turnaround efficiency*, η_{TA}, is defined as

$$\eta_{TA} = \frac{\text{Total energy output}}{\text{Total energy input during a charge} - \text{discharge cycle}} \tag{12.6}$$

In most plants, it is around 65 per cent.

Figure 12.5 gives the outline of the 400 MWe. Crauchan Pumped Hydrostation in Scotland in conjunction with a thermal power plant. Pumping is carried out mostly at night and at weekends to meet day time peak load on next day.

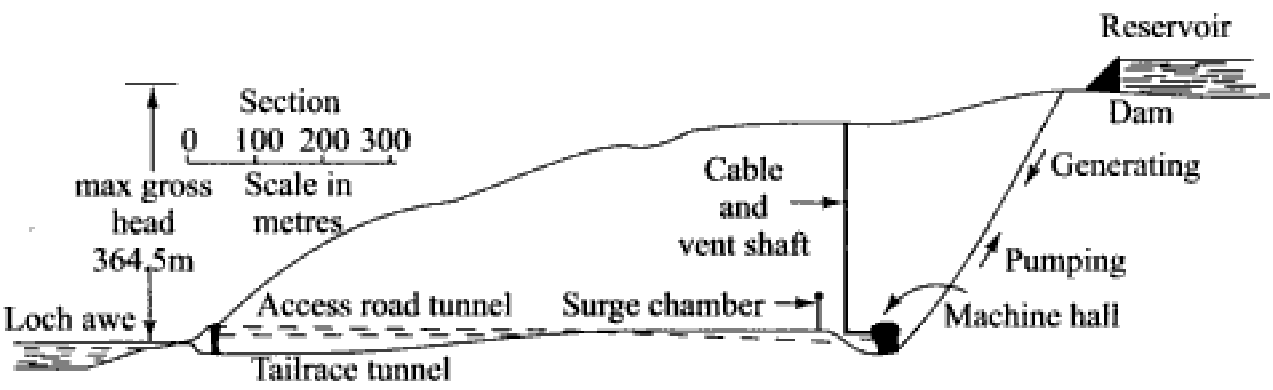


Fig. 12.5 *Crauchan station of the north of Scotland Hydroelectric Board*

12.2 COMPRESSED AIR ENERGY STORAGE (CAES)

During off-peak hours, excess energy is used to compress air and store it in reservoirs, aquifers or caverns. The stored energy is then released during periods of peak demand by expansion of the air through an air turbine. The turnaround efficiency is the same as that of a pumped hydro system, i.e. 65 per cent.

Three types of reservoirs can be used to store compressed air. They are salt caverns, aquifers and hard-rock caverns. Aquifers are naturally occurring porous rock formations. These underground reservoirs are, however, subjected to fluctuations in pressure, temperature and humidity.

When air is compressed for storage, its temperature will rise according to Eq. (12.7).

$$T_2 = T_1\, [p_2/p_1]^{(n-1)/n} \tag{12.7}$$

The heat of compression may be retained in the compressed air. This is called adiabatic storage and results in high storage efficiency, since more energy is recovered by expansion, with pressure lines diverging. If the heat of compression is allowed to dissipate, additional heat could be added by burning fuel to retain the high storage efficiency. This is called a hybrid system.

12.2.1 Adiabatic Storage System

A simple adiabatic compressed air energy storage (CAES) system is shown in Fig. 12.6. During off-peak hours, electrical energy from the main power plant is used by the motor-generator (MG) set operating in the motor mode to drive the compressor C. The compressed air passes through a packed bed P for sensible thermal energy storage and then to a constant pressure underground reservoir (R). The constant pressure is obtained by displacing water to a pressure-compensation pond that has a constant head above the reservoir. During peak hours, air from the reservoir flows through the packed bed (*P*) picking back sensible heat, then through the air turbine that now drives the *MG* set in the generator mode. Clutches separate the compressor during the peak periods and the turbine during off-peak periods.

Air reservoir volume of the cavern (*V*) greatly depends on the storage pressure. For a peak load capacity of 1500 MWh, the required volume is

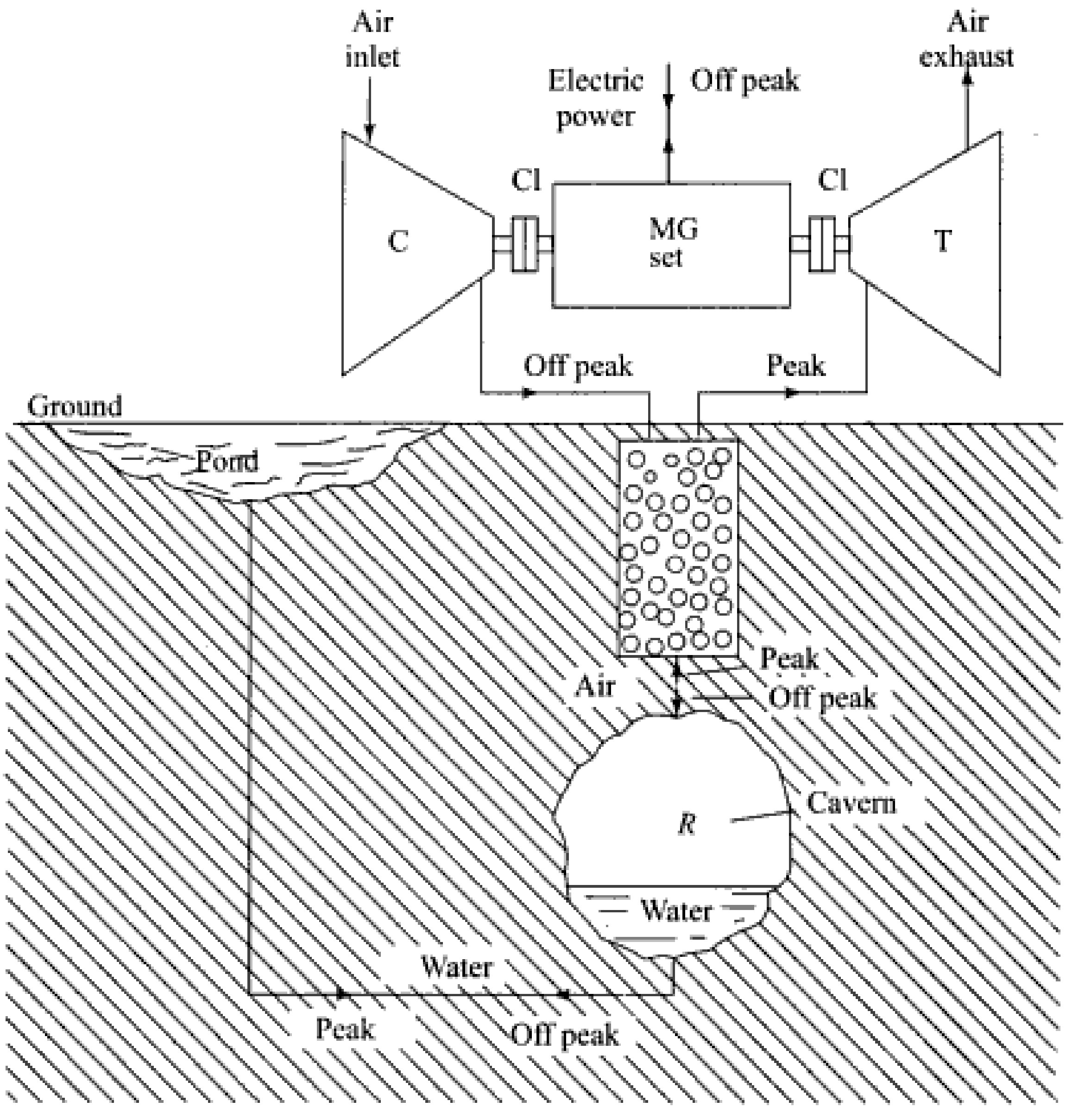


Fig. 12.6 *Simple single-stage adiabatic compressed air energy storage system with pressure compensation pond*

$$V = 2 \times 10^6 \text{ m}^3 \text{ for 10 bar storage pressure}$$

and $$V = 64,000 \text{ m}^3 \text{ for 100 bar storage pressure}$$

In most cases, the volume of packed bed (*P*) is about one-tenth of the reservoir volume (*R*).

12.2.2 Hybrid System

In a simple gas turbine power plant comprising a compressor, a combustion chamber and a turbine, about two-thirds of the power generated is consumed by the compressor and only one-thirds of the total power is available in the form of electricity ($\eta \sim 33\%$). In a hybrid system, fuel is burnt in the compressed air. As shown in Fig. 12.7, during off-peak hours at night, the MG set operating as a motor drives the compressor. The compressed air is stored in a pressure vessel or cavern. During peak hours on the next day, air is taken from the store, heated in a combustor by burning fuel and then expanded in a turbine to the ambient pressure. The MG set now operating in the generator mode supplies electricity to the power system.

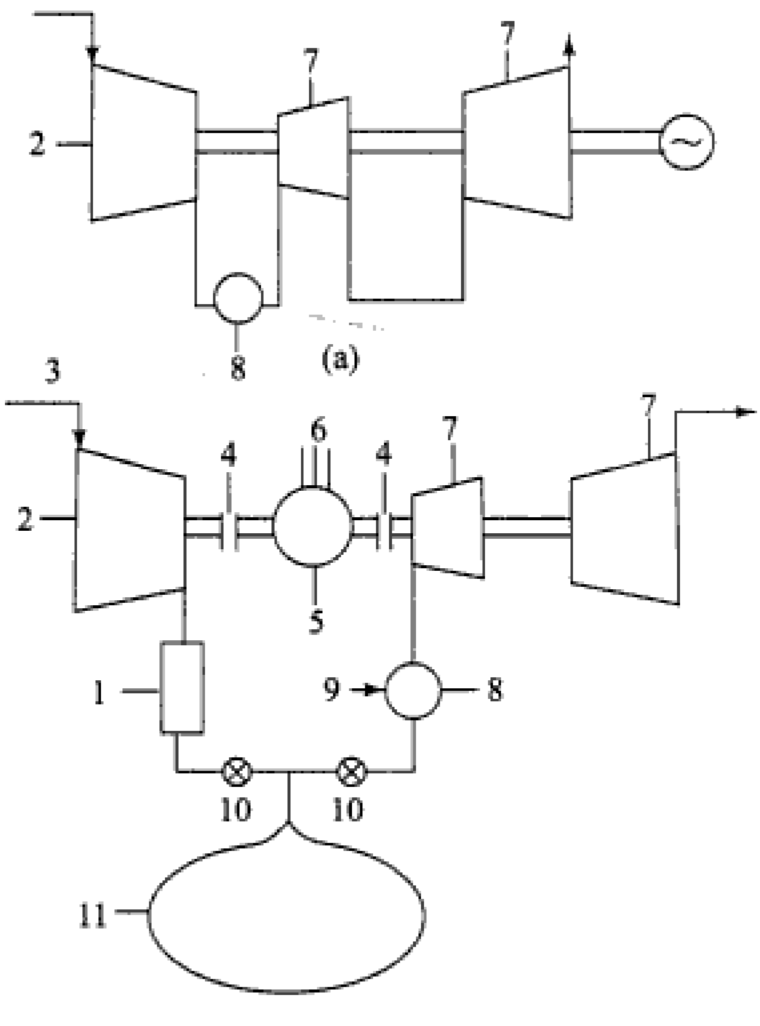


Fig. 12.7 *Development of the CAES concept (a) Simple gas turbine cycle (b) Gas turbine cycle modified to CAES configuration*

In a hybrid CAES scheme all the energy produced by the turbine is available for electricity generation during peak hours, since the compressor has already done its work during off-peak hours and thus does not consume any power from the turbine output now.

The first CAES system to be built was a 290 MW plant designed by Brown Boveri and built at Huntorf, Germany for a utility of Humburg which is in operation since 1978. It uses two salt caverns with a total volume of 300000 m^3

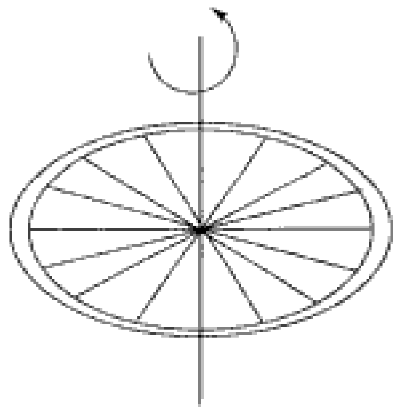


Fig. 12.9 *Flywheel with mass concentrated in the rim*

and $$W = \frac{1}{2} mr^2 \omega^2 \qquad (12.10)$$

Thus the energy content depends on the total mass to the first power and on the angular velocity (or rpm) to the second power. In order to obtain high energy content, high angular velocity is more important than the total mass of the rotary system.

The energy density W_m, i.e. the amount of energy per kg, is thus

$$W_m = \frac{1}{2} \omega^2 r^2 \qquad (12.11)$$

The volume energy density W_v is then

$$W_v = \frac{1}{2} \rho \omega^2 r^2 \qquad (12.12)$$

where ρ is the mass density

High angular velocity depends on the strength of the material. The tensile stress σ in the rim is given by

$$\sigma = \rho \omega^2 r^2 \qquad (12.13)$$

Now the maximum kinetic energy per unit volume will be

$$(W_v)_{max} = \frac{1}{2} \sigma_{max} \qquad (12.14)$$

If the dimensions of the flywheel are fixed, the main requirement is high tensile strength. Also,

$$(W_m)_{max} = \frac{1}{2} \frac{\sigma_{max}}{\rho} \qquad (12.15)$$

A light material (low density) with a high tensile stress is suitable. Fibre composites with strengths higher than steel and much lower mass densities are an obvious choice. The tensile strength and mass densities of some fibres are given in Table 12.1.

Table 12.1 *Strength and density of some fibres*

Fibre	*Strength (GN/m²)*	*Mass density (kg/m³)*
Glass	3.5	2500
Silica	6.0	2200
Carbon	2.6	1900
Chrysolite asbestos	4.5	2500

For comparison the value of strength for steel wire (0.9% carbon) is 4.2, but the mass density is rather high (7.9).

The factor 0.5 in Eq. (12.15) relates to a simple rim flywheel. But the expression is valid for any flywheel, made from material of uniform mass density ρ so that

$$(W_m)_{max} = k_m \frac{\sigma_{max}}{\rho} \tag{12.16}$$

The value of k_m depends on the geometry of the flywheel and k_m is called the shape factor or the mass efficiency factor. The value of k_m comes from the expression of moment of inertia I, the values of which for some shapes of rotating bodies are shown in Fig. 12.10. Values of k_m for a number of flywheel shapes are given in Table 12.2.

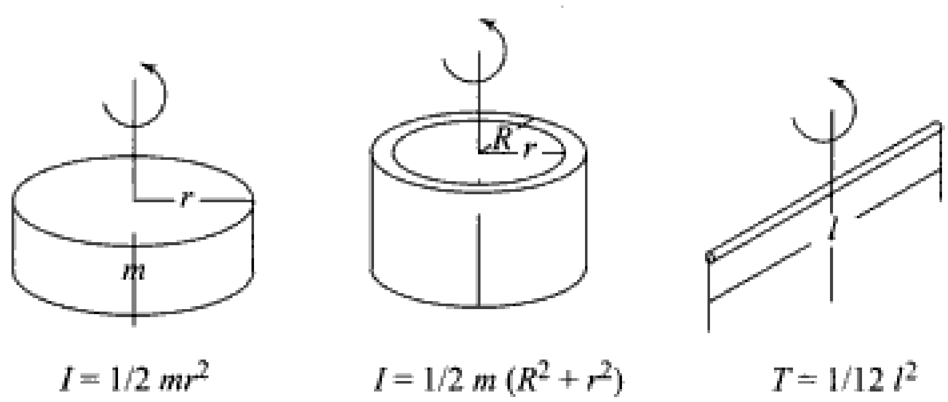


Fig. 12.10 *Moment of inertia for disc, cylinder and bar shaped flywheel*

Table 12.2 *Flywheel shape factors*

Flywheel shape	k_m
Constant stress disk	0.931
Flat unpierced disk	0.606
Thin rim	0.500
Rod or circular brush	0.333
Flat pierced disk (o.d./i.d. = 1.1)	0.305

The shape factor k_m is a measure of the efficiency with which the flywheel's geometry uses the material's strength. The ideal shape would be a constant stress disc where all the material is uniformly stressed biaxially and $k_m = 1$, which is not the case for a solid flywheel.

In order to obtain maximum energy storage density, a special design has been proposed, where maximum stress is obtained throughout the flywheel. Such flywheels are thickest near the axis and thinnest near the rim. The shape factor of these truncated conical discs is about 0.8.

Other designs have to be applied to flywheels made of fibre materials, where the tensile strength is high only in one direction. Composite materials have poor transverse strength and a radially thin rim has the material in hoop stress utilizing its tangential stress capability.

The energy stored in a flywheel is

$$KE = \frac{1}{2} I\omega^2 = \frac{1}{2} mR^2 (2\pi N)^2$$

$$= 2\pi^2 mR^2 N^2 \tag{12.17}$$

where m = mass of flywheel, R = radius of gyration and N = rps.

The energy absorbed (or released) by a flywheel between speeds N_1 and N_2,

$$\Delta E = 2\pi^2 mR^2 (N^2_2 - N^2_1)$$

$$= 2\pi^2 mR^2 (N_2 + N_1)(N_2 - N_1) \tag{12.18}$$

The ratio of the variation in rotational speed to the mean speed (N) is called the coefficient of speed fluctuation, k_s.

$$k_s = \frac{N_2 - N_1}{N} = \frac{(N_2 - N_1)^2}{N_1 + N_2}$$

where $N = (N_1 + N_2)/2$

$$\Delta E = 4\pi^2 mR^2 k_s N^2 \tag{12.19}$$

The value of k_s depends upon the desired closeness of speed regulation. For engines, k_s may vary between 0.005 and 0.2.

An important consideration in flywheel design is the stress level a flywheel rotating at a very high speed is subjected to. The theoretical *maximum specific energy* W_m (energy stored per unit mass, E/m) is given by Eq. (12.16). The maximum volumetric specific energy, $(W_v)_{max}$ or $(E/V)_{max}$ as given by Eq. (12.14) may be written for any flywheel as

$$(E/V)_{max} = (W_v)_{max} = k_v\, \sigma_{max} \tag{12.20}$$

where k_v is the volume efficiency ratio, the value of which indicates how well a particular flywheel design utilizes the material strength and fills the cylindrical volume around the flywheel.

The principal parameters that determine the suitability of flywheels for energy storage are the two efficiency ratios k_m and k_v as well as the stress and density. The values of k_m and k_v depend upon the type of material (isotropic, uniaxial composite, variable density) as well as flywheel shape (disc, drum, rod). The strength-density ratio, σ/ρ, is high for such materials as glass or silica fibres as given in Table 12.1.

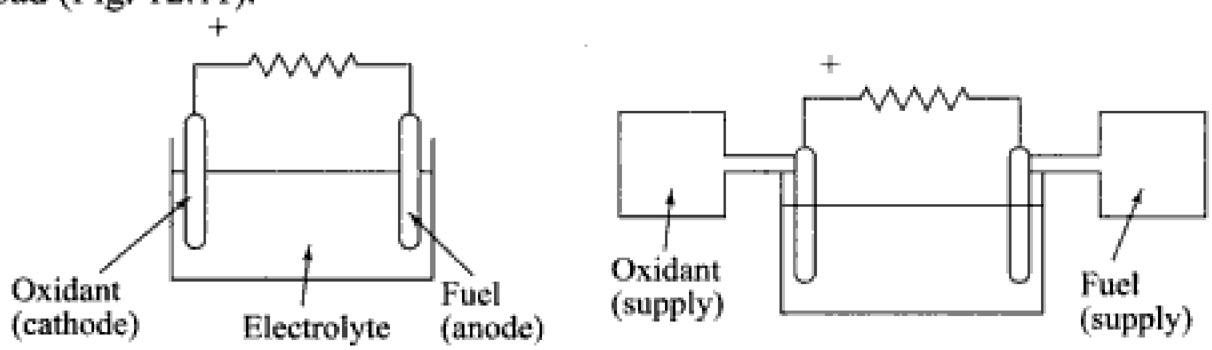
The high specific design stresses of carbon fibre/epoxy and Kevlar-fibre/epoxy allow storage of a large amount of energy in a relatively light flywheel. The minimum energy capacity of 3.6×10^9 J required by a power system could be achieved by multi-ring flywheel energy storage made of hoop-wound Kevlar-fibre/epoxy material. The angular speed of this flywheel would be about 3000 rpm. The diameter of the largest ring would be about 5 m, the length about 5 m and the total mass of the flywheel would be 130×10^3 kg.

The turnaround efficiency η_{TA} of a flywheel energy storage system during the charge-store-discharge period will depend on the duration of the store period. There are two main sources of losses in the flywheel, windage and bearing. Windage losses can be reduced to a low level by running the flywheel in a vacuum chamber. Bearing losses for a typical 200t rotor have been estimated at 2×10^5 J/s. Based on this figure, the η_{TA} is about 85 per cent for discharge immediately after charge, but falls to 78 per cent after five hours and 45 per cent after 24 hours of the keeping regime.

12.4 ELECTROCHEMICAL ENERGY STORAGE

The most traditional of all energy storage devices for power systems is electrochemical energy storage, which can be classified into three categories, primary batteries, secondary batteries and fuel cells. The common feature of these devices is that stored chemical energy is converted to electrical energy. The main attraction of the process is that its efficiency is not second law or Carnot cycle limited, unlike thermal processes. Primary and secondary batteries utilize the chemicals built into them, whereas fuel cells have chemically bound energy supplied from the outside in the form of synthetic fuel (hydrogen, methanol or hydrazine). Unlike secondary batteries, primary batteries cannot be reacharged when the built-in active chemicals have been used and therefore strictly they cannot be considered as genuine energy storage. The term "batteries" will refer to secondary batteries in the following text.

Batteries and fuel cells consist of two electrode systems and an electrolyte, placed together in a special container and connected to an external source or load (Fig. 12.11).



(a) Primary or secondary cell (b) Fuel cell

Fig. 12.11 *Comparison of a battery and a fuel cell*

These two electrodes, fitted on both sides of an electrolyte and exchanging ions with the electrolyte and electrons with the external circuit, are called the anode (–) and cathode (+) respectively (Fig. 12.12).

used for generation in small, local DC power systems were usually shut down at night and the demand was met by lead-acid batteries which were charged during the day. These batteries were also used in several US towns to feed DC electricity to electric street cars during rush hour. The growth of large centralized AC power systems and cheap coal and oil generated electricity relegated batteries to emergency standby duty for DC auxiliaries.

The battery market is still dominated by the lead-acid battery invented by Plante in 1859. It is the oldest chemical storage device. The battery (Fig. 12.13) consists of alternate pairs of plates, one pure lead in spongy form and the other lead coated with lead dioxide, immersed in a dilute solution of sulphuric acid which serves as an electrolyte. During discharge both electrodes are converted into lead sulphate ($PbSO_4$). Charging restores the positive electrode to lead dioxide and the negative electrode to metallic lead. The battery deteriorates gradually in performance due to irreversible physical changes in the electrodes and ultimately fails after 1000–2000 cycles.

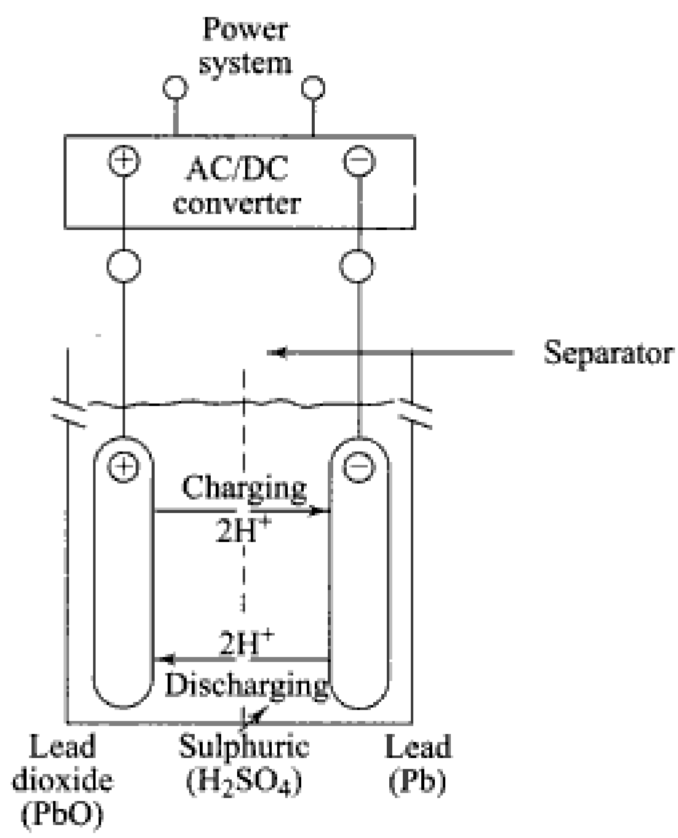


Fig. 12.13 *Lead-acid storage battery*

The overall cell reaction is as follows.

$$\begin{array}{lll} \text{Anode} & & \text{Cathode} \\ - & & + \\ Pb & + 2H_2SO_4 & + PbO_2 \end{array} \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \begin{array}{lll} PbSO_4 & + 2H_2O & + PbSO_4 \\ + & & - \\ \text{Anode} & & \text{Cathode} \end{array}$$

During discharge the cathode is positive and the anode negative. The reasons for its wide use are the following:

(a) Relatively high nominal voltage, 2.0 V/Cell

(b) Versatility in providing high or low current
(c) High degree of reversibility, capable of hundreds of charge/discharge cycles
(d) Relatively cheap and easy to fabricate.

The main drawbacks of these batteries are, however, low energy density, long charge time and the need for careful maintenance. About half the weight of a lead-acid battery is occupied by inert materials, e.g. grid metal, water, separators, connectors, terminals and cell containers. Attempts to reduce the weight and hence to increase the energy density have made the use of low-density grid materials. The use of carbon fibres in the positive electrode has resulted in reduction in weight and also in increased power capability.

Industrial development on alkaline electrolyte batteries such as Ni-Zn, Fe-Ni, etc. aims to produce improved energy storage systems for traction applications. The nickel-zinc battery is analogous to the much more expensive nickel-cadmium battery. The cell reaction is the following:

Cathode Anode

$$2NiOOH + 2H_2O + Zn \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} 2Ni(OH)_2 + Zn(OH)_2$$

The Ni-Zn cell's main drawbacks are its short cycle life, separator stability, temperature control, high cost and mass production problems. Poor cycle life is caused by the high solubility of reaction products at the zinc electrodes. Redeposition of zinc during charging results in the growth of dendrites which penetrate the separators and cause internal short circuit. Attempts have been made to suppress the growth of zinc dendrites during charging by vibrating the zinc electrode.

The *nickel-iron battery* is an alkaline storage battery using KOH as the electrolyte. The cell reaction is:

Cathode Anode

$$2NiOOH + 2H_2O + Fe \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} Ni(OH)_2 + Fe(OH)_2$$

The main drawback of Ni—Fe batteries for electric vehicle applications has been their low energy density. It also suffers from a poor peaking capability and low cell voltage. Recently, however, some improvements have been brought about.

The iron-air battery consists of an anode using iron as active material and a cathode taking oxygen from the air. The cell reaction is:

Anode Cathode

$$Fe + H_2O + \frac{1}{2}O_2 \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} Fe(OH)_2$$

The battery suffers from high self-discharge of the iron electrode at low temperatures, poor charge efficiency and limited power capacity (max 30–40 W/kg).

The zinc-air battery has a highly concentrated KOH electrolyte and the electrochemical reaction is between oxygen from the air and zinc metal. The cell reaction is:

Anode Cathode

$$Zn + \frac{1}{2}O_2 \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} ZnO$$

As with the iron-air system, the zinc-air couple has a poor overall charge-discharge efficiency due to the polarization losses associated with the air electrode.

The zinc-chlorine battery attempts to overcome the replating problem of zinc by using an acid electrolyte. The cell reaction is

Anode Cathode

$$Zn + Cl_2 \cdot 6H_2O \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} ZnCl_2 + 6H_2O$$

The main problem of this battery is its heavy weight, a 10^{12} J storage device requiring 60 tonnes of Cl_2, which itself needs utmost safety.

Advanced batteries like sodium-sulphur couple using a solid state electrolyte and lithium-sulphur couple using a fused salt electrolyte are still under development.

The *sodium-sulphur* battery has a ceramic electrolyte (β-alumina) which can conduct sodium ions. The cell reaction is

Anode Cathode

$$\times Na + y S \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} Na_xS_y$$

Advanced batteries have provided the stimulus for more interesting work in solid state chemistry and electrochemistry in recent years. The seminal discovery was the observation of very high sodium ion mobility above room temperature in sodium beta-alumina. The beta-alumina was thought to be an isomorph of aluminium oxide, and its crystal structure was determined before the Second World War. However, its unusual electrical properties were discovered only recently. When the electrical conductivity of beta alumina was measured at 300 °C, it became clear that the charge carriers were exclusively sodium ions and not electrons. The application of such materials to a modern generation of power batteries was quickly realised. In the sodium-sulphur battery, patented by Ford, instead of solid electrodes separated by a liquid electrolyte, as in the conventional lead-acid car battery, sodium beta-alumina is used as a solid electrolyte, specifically conducting sodium ions, between liquid electrodes of sodium metal and sulphur (Fig. 12.14). The cell voltage, 2.08V, is derived from the chemical reaction between sodium and sulphur to produce sodium polysulphide and the theoretical energy density, about 750 Wh/kg (or 2.7×10^6 J/kg), is much higher than 170 Wh/kg (or 0.61×10^6 J/kg) of the lead-acid battery.

The lithium-sulphur battery consists of liquid lithium and sulphur electrodes and an electrolyte of molten LiCl-KCl eutectic at an operating temperature in the 380–450°C range. The cell reaction is

Anode		Cathode		

$$2Li \quad + \quad S \quad \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \quad Li_2S$$

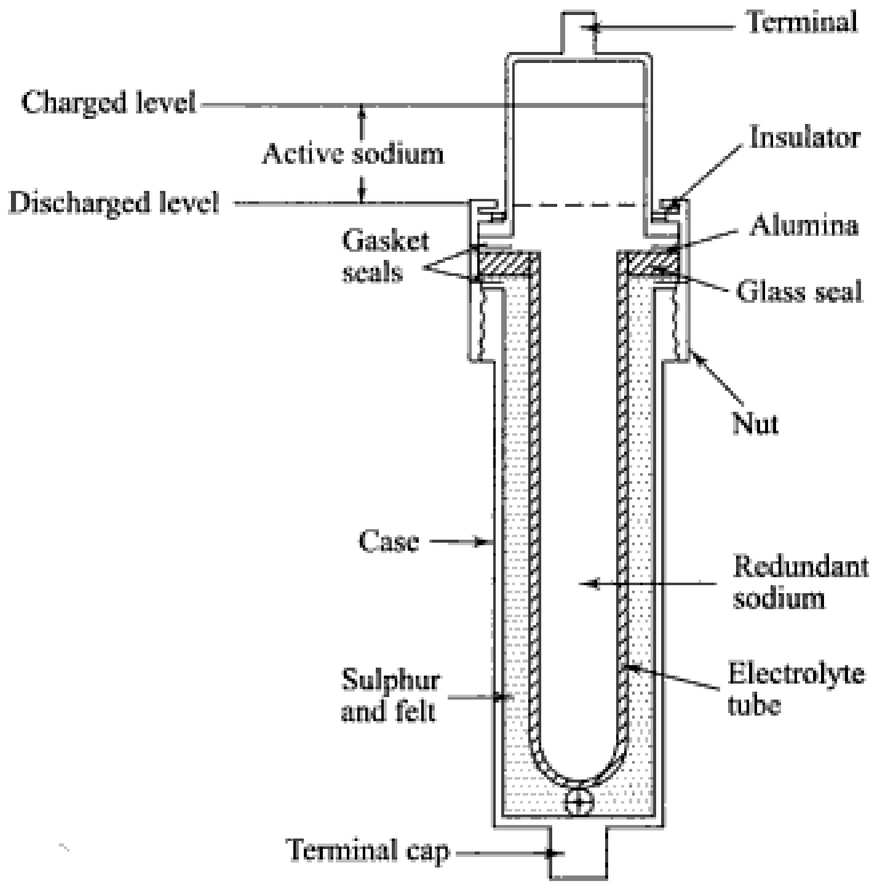


Fig. 12.14 *Schematic diagram of the sodium-sulphur battery which uses a sodium beta alumina solid electrolyte as the separator between liquid electrodes (sodium anode and sulphur cathode). The operation temperature is 300–400°C*

Highly corrosive liquid lithium attacks the ceramic insulators and separators and shortens the cell's life. Efficiency is not very high because of self-discharge caused by lithium dissolving in the molten LiCl-KCl electrolyte.

The use of lithium-aluminium alloys and iron sulphide as electrodes has led to the development of more efficiency Li-S cells with good energy densities. The reactions are the following.

Anode Cathode

$$4LiAl \quad + \quad FeS_2 \quad \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \quad Fe + 2Li_2S + 4Al$$

and

Anode Cathode

$$4LiAl \quad + \quad FeS \quad \underset{\text{Charge}}{\overset{\text{Discharge}}{\rightleftharpoons}} \quad Fe + 2Li_2S + 2Al$$

The lithium-titanium disulphide cell has a lithium metal anode and a cathode of TiS_2. The electrochemical discharge reaction proceeds via. the insertion of lithium ions between adjacent sulphur layers

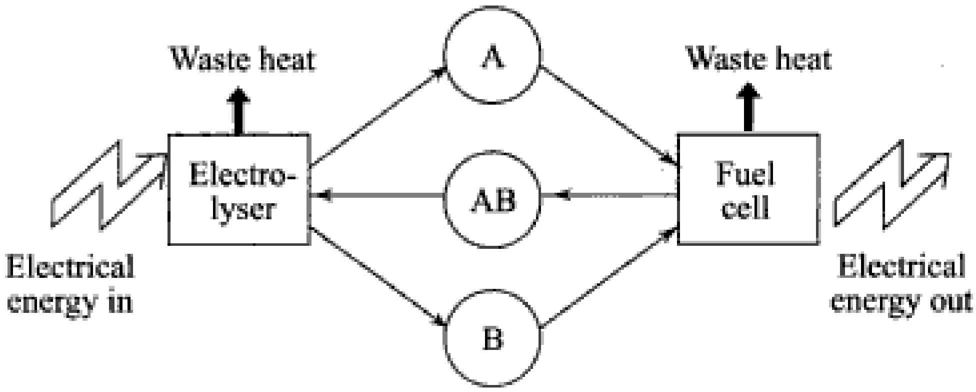


Fig. 12.15 *Schematic diagram of an electrochemical fuel cell*

where n is the number of electrons needed to get one atom or molecule of X into its ionic form in the electrolyte and a_1 and a_2 are the activities at electrodes 1 and 2. F is the Faraday constant (96 000 coulomb/mol).

We can make use of such a cell in the following ways.

(i) If $a_1 > a_2$ and X is continuously added on the left and removed on the right, we have a source of energy—a concentration fuel cell.

(ii) If T and a_1 are known, we can measure a_2—we have an electrochemical sensor.

(iii) If we apply a greater voltage than E in the opposite sense, we can drive X from one side to the other—hence we have an ion pump or an electrolyser.

As a fuel cell is an electrochemical cell which can continuously change the chemical energy of a fuel and oxidant to electrical energy with high efficiency, a variety of synthetic fuels have been tried, such as hydrogen, methanol, ammonia and methane. A diagram of hydrogen-oxygen fuel cell is shown in Fig. 12.16.

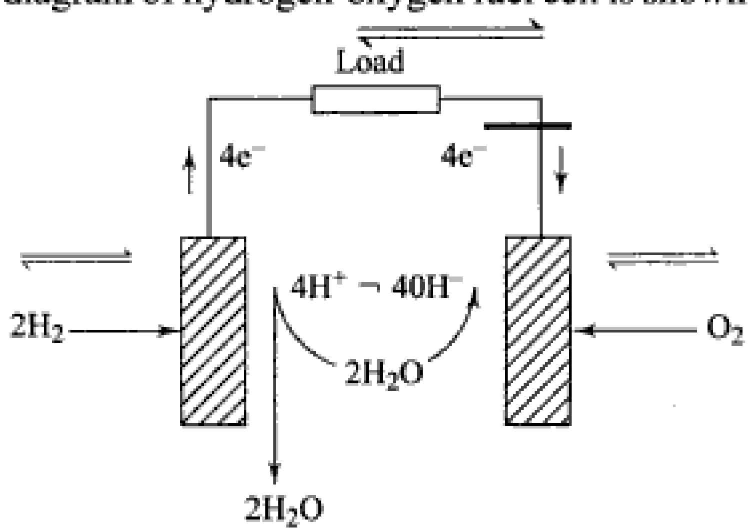


Fig. 12.16 *A hydrogen-oxygen fuel cell*

Hydrogen-oxygen fuel cells with the overall reaction

$$H_2 + \frac{1}{2} O_2 \rightarrow H_2O$$

are attractive because of their high energy density, no pollution and high cell efficiency. A good hydrogen or oxygen conductor should be used as the electrolyte.

The maximum energy available from a chemical reaction is $-\Delta G$, the change in the Gibbs free energy for the reaction. The corresponding ideal voltage for a reversible reaction is

$$V_0 = \frac{-\Delta G}{n\mathrm{F}} \tag{12.22}$$

where n is the number of electrons transferred in the reaction and F is the Faraday constant (96 000 coulomb/mol). Now,

$$G - G^{\circ} = n\,\bar{R}\,T \ln a \tag{12.23}$$

where a is the activity given by f/f°, f being the fugacity (see Nag, 1995).

$$\text{Per unit mol, } -\Delta G = \bar{R}\,T \ln \frac{a_2}{a_1} \tag{12.24}$$

On substituting result of Eq. (12.24) in Eq. (12.22), it becomes $V_0 = \bar{\mathrm{R}}\,T/n\mathrm{F}$ $\ln \frac{a_2}{a_1}$, which is the same as Eq. (12.21).

12.5 MAGNETIC ENERGY STORAGE

Electrical resistance of metals depends on temperature. In 1911, Kammerlingh Onnes found that electrical resistance of mercury drops to zero in the neighbourhood of absolute zero temperature. Onnes called this phenomenon "*superconductivity*". All metals exhibit this property. The temperature below which a metal becomes a superconductor is called the transition or critical temperature. All superconducting metals have transition temperatures in the cryogenic range (0-123 K).

Superconducting electromagnets were constructed in 1970 for magnetohydrodynamic power generation. These were also used in bubble chambers to cool electric generators, motors and transformers. Electric power transmission and distribution can best be accomplished by the use of high-purity aluminium cables operating at liquid hydrogen temperatures (~ 20 K). Commercial success depends upon whether the metal, the gas and the refrigeration systems can be acquired and operated economically.

The concept of superconducting magnetic energy storage is based on the principle that energy can be stored in the magnetic field associated with a coil. If a coil is made of a material in a superconducting state (i.e. maintained below the critical temperature), then once it is charged, the current will not decay and the magnetic energy can be stored indefinitely. The stored energy can be released back to the network by discharging the coil.

Energy stored in a coil in which the current I flows is

$$E = \frac{1}{2} LI^2 \tag{12.25}$$

where E is in J, L is the inductance in henry $(V - s/A)$ and I is the current in A. The inductance L of a coil depends on its dimensions, which for a coil with conductors of a rectangular cross-section (Fig. 12.17) may be characterized by

$$\xi = 2R/(ab)^{1/2} \tag{12.26}$$

$$\delta = a/b$$

and

$$V = 2\ \pi Rab = \frac{8\pi R^3}{\xi^2} \tag{12.27}$$

where R = mean radius of coil, m; a and b = width and depth of conductor, m; V = volume of conductor in one coil turn, m^3.
The inductance L is given by

$$L = f(\xi, \delta)\ RN^2 \tag{12.28}$$

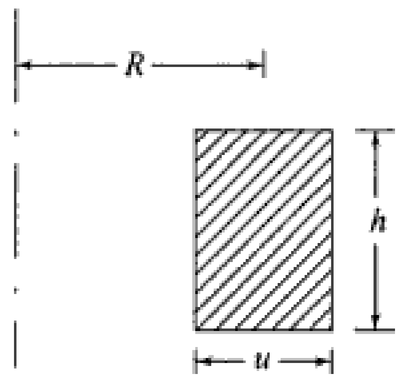


Fig. 12.17 *Dimensions of a cylindrical coil with rectangular conductors*

where $f(\xi, \delta)$ = form function, V.s/A.m; N = number of turns of coil.
Energy stored in a coil, from Eq. (12.25).

$$E = \frac{1}{2}\ f(\xi, \delta)\ RN^2I^2 \tag{12.29}$$

The current density j is

$$j = NI/ab \tag{12.30}$$

Substituting Eq. (12.30) in Eq. (12.29) and simplifying

$$E = \frac{1}{4}\ \pi^{-5/3} f(\xi, \delta)\ \xi^{-2/3}\ V^{5/3}\ j^2 \tag{12.31}$$

The coil that gives the maximum value of inductance per unit volume, i.e. maximum L/V ratio is called the Brooks coil. It has

$$a = b,\ R = \frac{3}{2}b,\ d = 1,\ x = 3$$

Energy stored in a Brooks coil is given by

$$E_B = 3.028 \times 10^{-8}\ V^{5/3}\ j^2 \tag{12.32}$$

For a cylindrical coil, other than Brooks, the energy stored is

$$E = F\ E_B \tag{12.33}$$

where F is less than 1.0 and is a function of ξ and δ. An important parameter is the volume of material needed per unit of energy stored, i.e.

$$\frac{V}{E} = \frac{V}{F E_B} = \frac{0.33 \times 10^8}{F V^{2/3} j^2} \tag{12.34}$$

For a Brooks coil, $F = 1$. The cost of the coil is proportional to its volume. Therefore, from Eq. (12.34), it is seen that the cost per unit energy stored is inversely proportional to $V^{2/3}$ and to j^2. For stability consideration, the current density j is limited to values between 50×10^6 and 100×10^6 A/m^2. Thus, huge structural mass is required to contain the magnetic field energy. For stainless steel, it would be about 160 kg/kWh, which is cost prohibitive. Much of research on magnetic energy storage for utility use is in progress in the USA and Japan.

12.6 THERMAL ENERGY STORAGE

Direct storage of heat in insulated solids or fluids is possible even at low temperatures, but energy can only be recovered effectively as heat. Thermal energy storage (TES) is ideally suited for applications such as space heating, where low quality (temperature) energy is required. It has found wide use in many industrial applications such as the manufacture of cement, iron and steel, glass, aluminium, paper, plastics and rubber and in food processing.

Following are the two distinct thermal energy storage mechanisms.

(a) Sensible heat storage, based on the heat capacity of the storage medium.
(b) Latent heat storage, based on the energy associated with a change of phase for the storage medium (melting evaporation or structural change).

12.6.1 Sensible Heat Energy Storage

Energy can be stored as sensible heat by virtue of a rise in temperature of the storage medium, such as water, liquid or a solid.

Storage density (kJ/m^3) is = Temperature difference (°C) × Specific heat (kJ/kg °C) × Density of material (kg/m^3)

$$= \rho \cdot c_p \cdot \theta$$

This system is simple in concept, but has the disadvantage of variable temperature operation and low storage density. Sensible energy storage could employ one of the following devices.

(a) Pressurized water storage
(b) Organic liquid storage
(c) Packed solid beds
(d) Fluidized solid beds

An example of pressurized water sensible heat storage system in a power plant is shown in Fig. 12.18, where the primary heat source is either a fossil fuelled furnace or a nuclear reactor. The base load plant is capable of supplying more steam than needed during periods of low demand. The excess steam is fed from the turbine at high pressure (as in feedwater heating) during these periods of

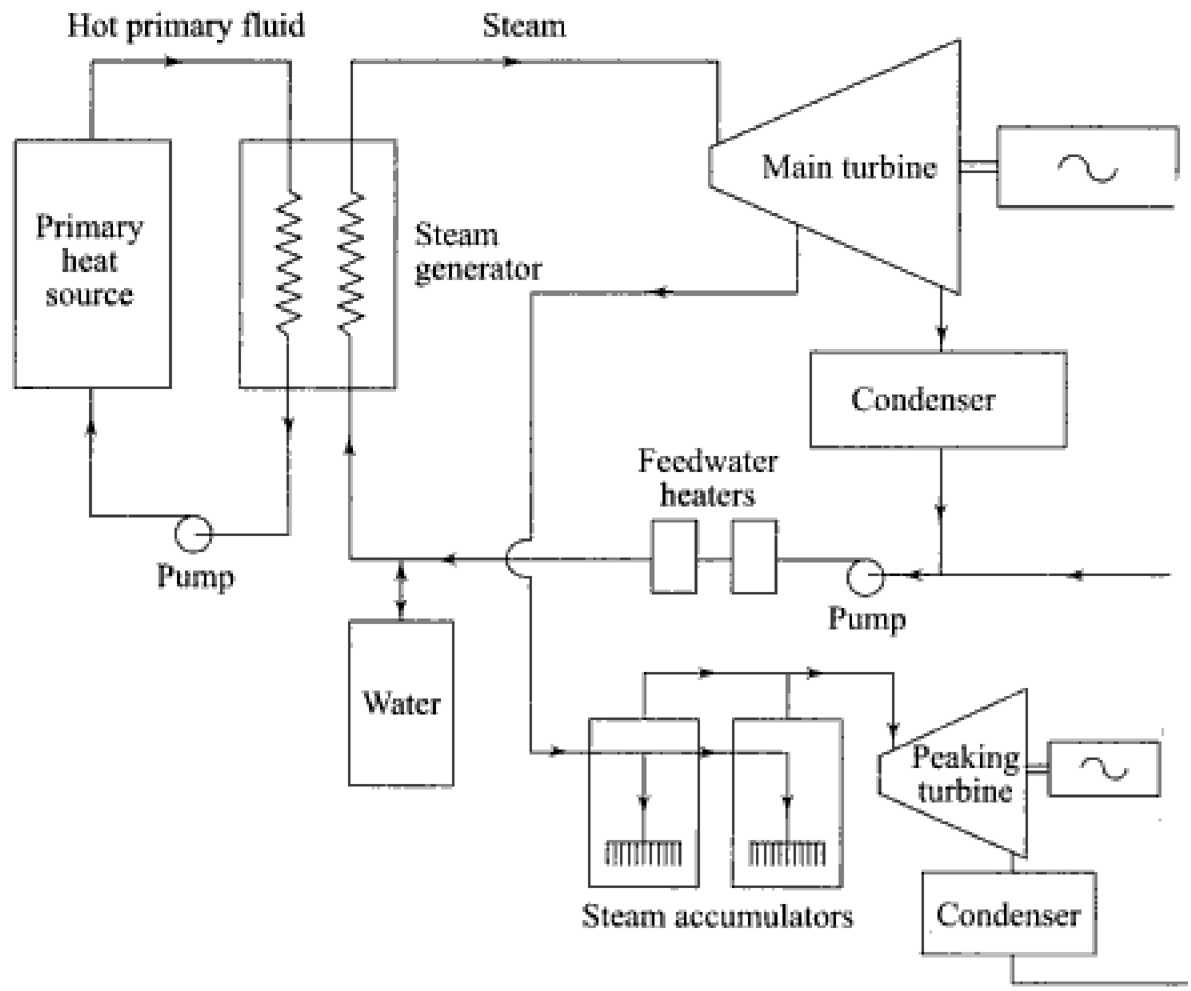


Fig. 12.18 *Schematic flow diagram of a power plant with a pressurized water sensible energy storage system*

low demand. This extracted steam is fed to steam accumulators and mixed v water, thus producing saturated pressurized water. The accumulators are l discharged during periods of high demand through a small peaking turbi Discharge continues until a low specified pressure is reached in the accumulat Steam at low and varying temperature enters the peaking turbine.

Typical values of accumulator high and low pressures are 20 bar (t_{sa} 212 °C) and 2 bar (t_{sat} = 120 °C).

Steam condenses in water during accumulator charge and re-evapora during discharge. The storage medium is the pressurized water in accumulators and operates over a relatively wide temperature range.

Storage density of thermal energy utilized in the peaking turbine per u volume of the high pressure saturated water is

$$= \frac{1}{v_{f,1}} (h_{f,1} - h_{f,2}) \quad (12.3$$

where v_f and h_f are the specific volume and enthalpy of saturated wa respectively, and the subscripts 1 and 2 referred to the high (stored) and l (emptied) pressures. Therefore, for 20 bar and 2 bar pressures,

$$\text{Storage density} = \frac{908.5 - 504.8}{0.0011766} = 343\ 107\ \text{kJ/m}^3 = 95.3\ \text{kWh/m}^3$$

$$\Delta T = 212 - 120 = 92°\text{C}$$

∴ Storage density is approximately 1 kWh/(m^3 °C) over the temperature range.

The electric-energy density obtained by the peaking turbine-generator depends upon

1. Thermal turnaround efficiency, η_{TA}
2. Peaking turbine-generator efficiency, η_{PT}

The value of η_{TA} is a function of the following:

(a) Losses associated with sensible heat transfer to and from the steel walls, structural members of the accumulators and the interconnecting pipework.
(b) The transient heat losses to the environment.

Now, η_{TA} is the ratio of

$$\frac{\text{Energy stored in the accumulator structure}}{\text{Energy stored in the contained water at a given pressure}}$$

which is

$$\frac{(\pi D L t)\,\rho_s c_s}{\left(\dfrac{\pi}{4} D^2 L\right)\rho_f c_f} = \frac{4t}{D}\frac{\rho_s c_s}{\rho_f c_f} = 2(P/\sigma)\,\frac{\rho_s c_s}{\rho_f c_f} \tag{12.36}$$

since $\sigma = PD/2t$, or $2t/D = P/\sigma$

where D is the diameter of cylindrical accumulator, L its length, t its thickness, P the pressure, σ the wall stress, ρ the density, c the specific heat and the subscripts s and f denote solid and liquid respectively. The volumetric heat capacities are roughly equal, i.e. $\rho_f c_f = \rho_s c_s$. The ratio of P/σ is of the order of 0.03 for steel. Thus the contribution of losses by sensible heat transfer to the walls of the vessel is very small and may be ignored.

The convective heat losses from the water to the environment are therefore the major contributor to the thermal turnaround efficiency. They vary with time and depend upon the water temperature and the overall heat transfer coefficient U between the water and the outside environment.

Let us consider saturated water of volume V, at instantaneous temperature T, while the surroundings are at temperature T_∞ (Fig. 12.19). Let $\theta = T - T_\infty$, so that $d\theta = dT$. Assuming a lumped capacity system with low Biot number,

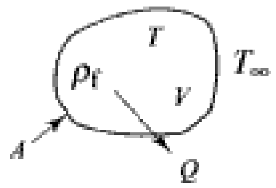


Fig. 12.19

Rate of decrease of internal energy of water

= Rate of heat loss to the surroundings

$$\rho_f V c_f \frac{d\theta}{dt} = -U A\,\theta$$

where A is the heat transfer surface area and t the time.

$$\frac{d\theta}{\theta} = \frac{UA}{\rho_f c_f V} dt$$

which on integration gives

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-(UAt)/(\rho_f c_f V)} \tag{12.37}$$

Let the time constant τ be defined by

$$\tau = \frac{\rho_f c_f V}{UA} = \rho_f c_f \left[\frac{\pi}{4} D^2 L\right] \frac{1}{\pi DLU}$$

$$= \frac{D \rho_f c_f}{4U} \tag{12.38}$$

Equation (12.37) can be re-arranged as

$$1 - \frac{T - T_\infty}{T_i - T_\infty} = 1 - e^{-t/\tau}$$

$$\frac{T - T_i}{T_\infty - T_i} = 1 - e^{-t/\tau} \tag{12.39}$$

If the liquid at fully charged condition is at temperature T_1 (= T_i), its temperature T decreases with time t due to heat losses (Fig. 12.20), so that

$$\frac{T(t) - T_1}{T_\infty - T_1} = 1 - e^{-t/\tau} \tag{12.40}$$

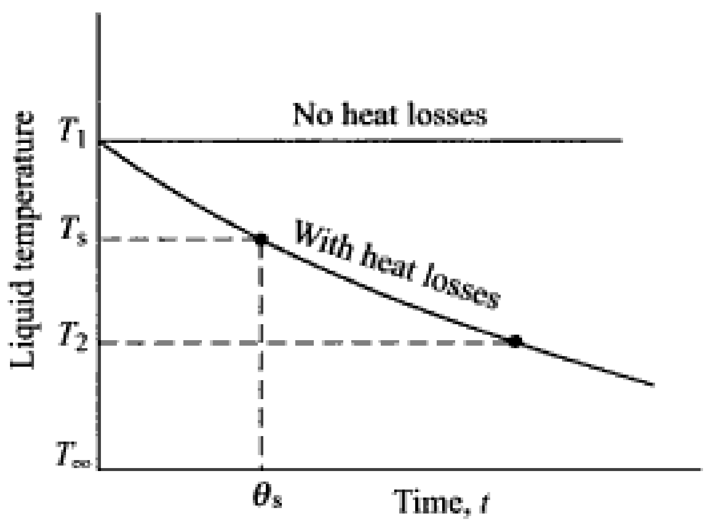


Fig. 12.20 *Stored liquid temperature-time change in sensible energy storage.* T_1 *= fully charged,* T_2 *= end of energy withdrawal,* T_∞ *= ambient*

Assuming a storage period of t_s at which the water temperature decreases to T_s.

$$\frac{T_s - T_1}{T_\infty - T_1} = 1 - e^{-ts/\tau} \tag{12.41}$$

The thermal turnaround efficiency is given as

$$\eta_{TA} = \frac{\text{energy left in storage at } T_s \text{ (after heat losses)}}{\text{original energy stored}}$$

$$= \frac{h_s - h_2}{h_1 - h_2} \cong \frac{T_s - T_2}{T_1 - T_2}$$

$$\therefore \qquad 1 - \eta_{TA} = 1 - \frac{T_s - T_2}{T_1 - T_2} = \frac{T_1 - T_s}{T_1 - T_2}$$

$$= \frac{T_1 - T_s}{T_1 - T_2}\left[1 - e^{-t_s/\tau}\right]\left[\frac{T_\infty - T_1}{T_s - T_1}\right]$$

$$= \frac{T_1 - T_\infty}{T_1 - T_2}\,[1 - e^{-t_s/\tau}]$$

$$\therefore \qquad \eta_{TA} = 1 - \frac{T_1 - T_\infty}{T_1 - T_2}\,[1 - e^{-t_s/\tau}] \qquad (12.42)$$

Thus η_{TA} strongly depends on the ratio t_s/τ, where t_s is several hours in a daily storage system. The only variable in τ is U, which heavily depends on accumulator insulations, design and location. High values of τ will result for low values of U.

Accumulators may be constructed above ground or underground. Underground accumulators are more costly, but have heavier insulation and higher T_∞ and hence higher η_{TA}. The choice of above ground on underground accumulators is decided by cost, efficiency, operational problems and safety.

The efficiency of the peaking turbogenerator η_{PT} is low because of variable inlet conditions, the use of low temperature saturated steam, small size and absence of feedwater heating. An efficiency of 20 to 25 per cent is reasonable, while the base load plant has an efficiency of 33 to 40 per cent.

There are three main ways of operating the accumulator, viz. variable pressure, expansion or displacement.

(a) ***Variable Pressure Accumulator*** It is also called Ruths accumulator (Fig. 12.21). When fully charged, almost all its volume is filled with saturated hot water, with a small cushion of saturated steam above it. In the discharge mode, steam is drawn off from the top and as the pressure in the steam cushion decreases, some water flashes into steam.

(b) ***Expansion accumulator*** It is shown in Fig. 12.22. When fully charged, the accumulator is almost full of hot water with a small steam cushion, as in the variable pressure mode. As hot water is drawn from the bottom during discharge, some water flashes to steam, which reduces the pressure and temperature of saturated water and steam. All the water can be removed to the flash evaporator with a reduction of pressure of about 30 per cent. The water from the last flash evaporator is collected and stored.

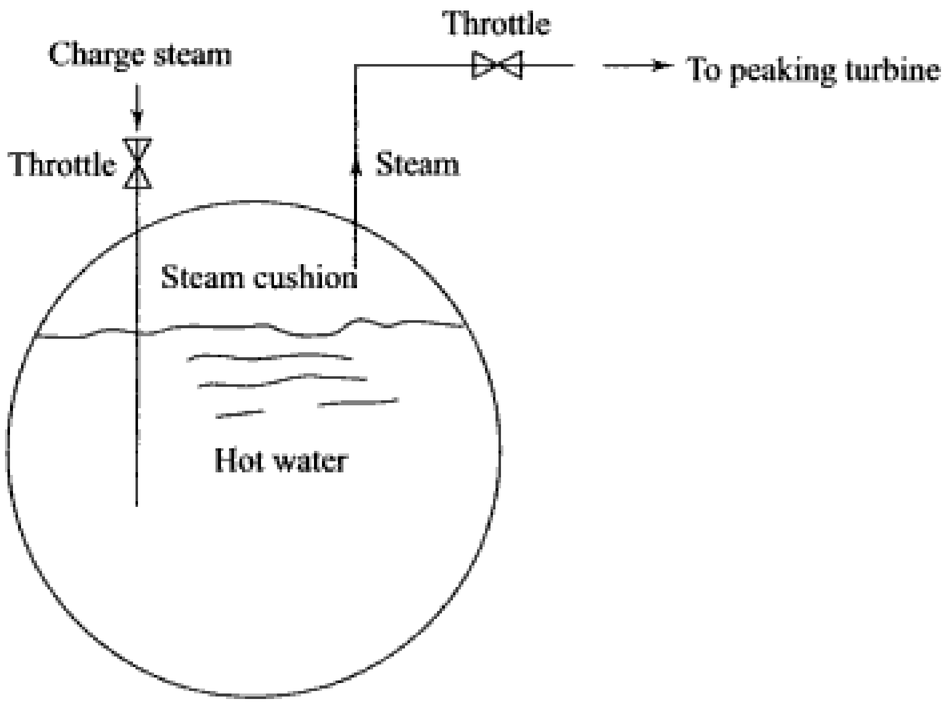


Fig. 12.21 *Variable pressure accumulator*

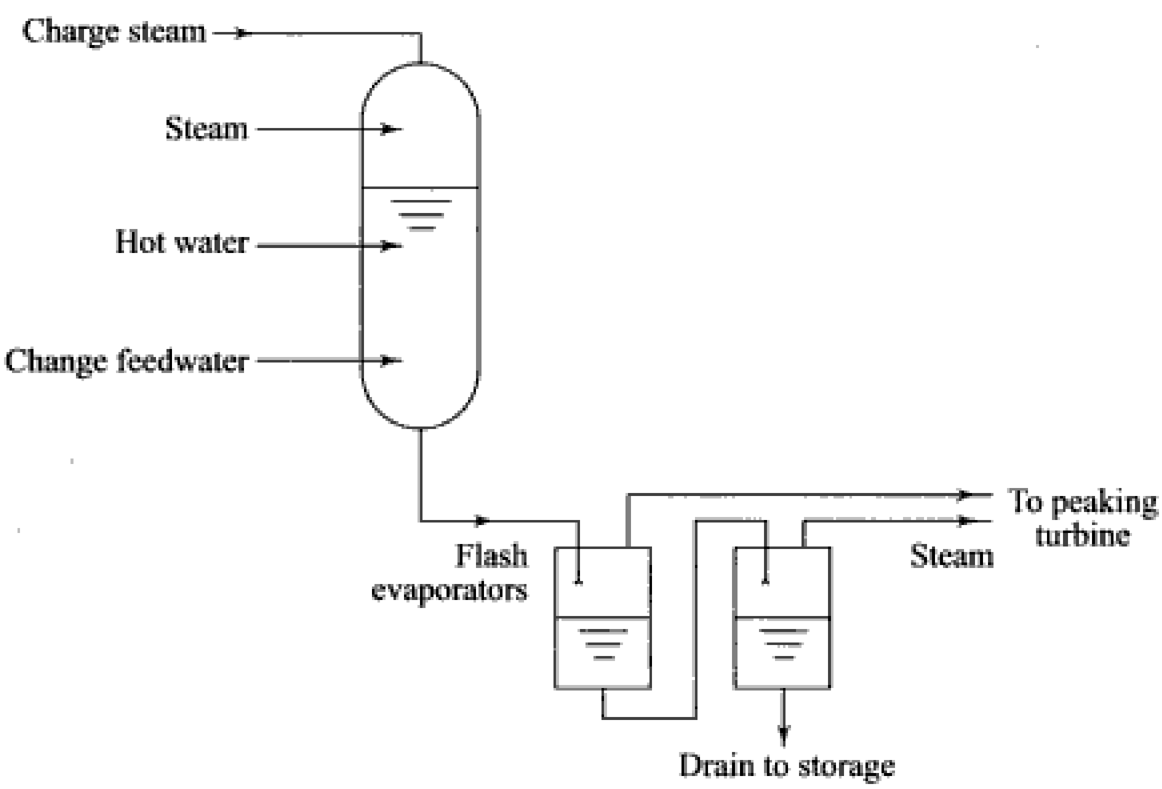


Fig. 12.22 *Expansion accumulator with flash evaporator*

(c) *Displacement accumulator* It is always completely full of water. When fully charged, it contains hot water at the desired temperature. When fully discharged, all the water is cold. As shown in Fig. 12.23, hot water is injected at the top during charge and removed from the top during discharge. Cold water leaves and enters at the bottom. Since hot water has lower density than cold water, it will float at the top. A sharp temperature gradient or thermocline separates the hot and cold water. During discharge, one or more flash evaporators are used to

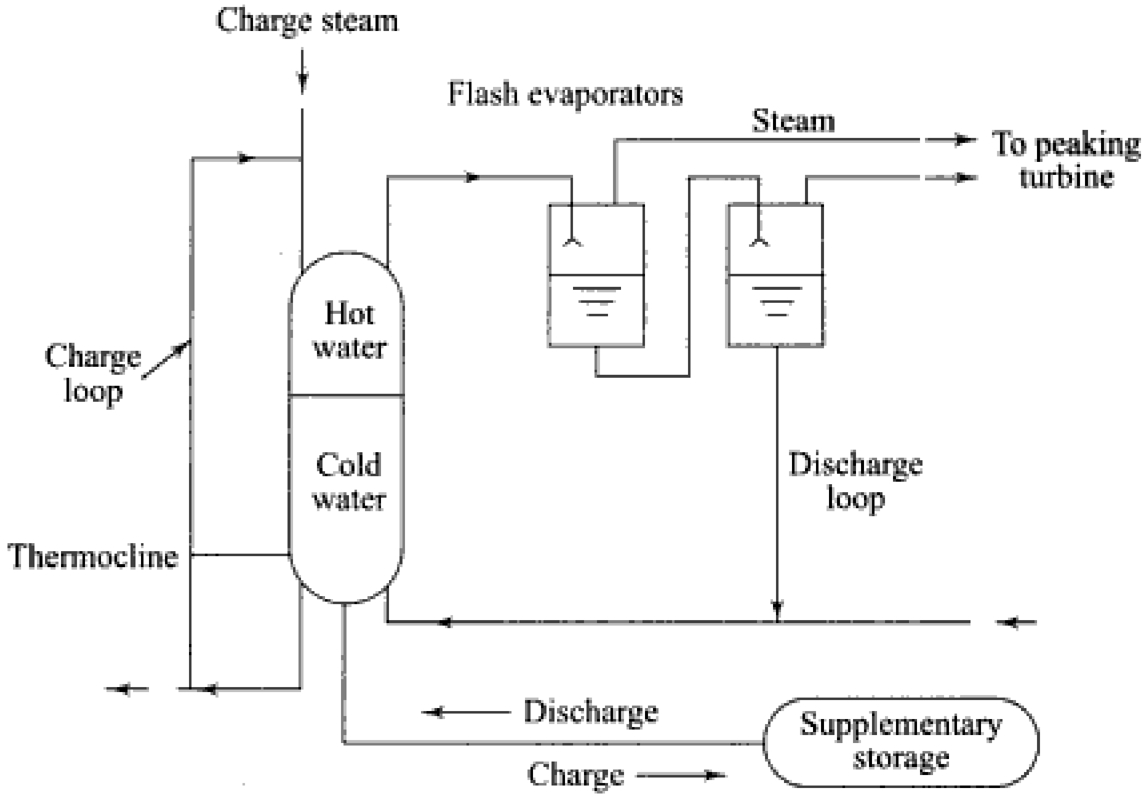


Fig. 12.23 *Displacement accumulator with flash evaporators*

generate steam for peaking turbines. The water drained from the evaporators and the condensate from the turbine are returned to the accumulator as cold water. During charge, steam is mixed with cold water taken from the bottom to raise the temperature to the desired level. Cold water equal in mass to the steam is returned to the boiler inlet feedwater to generate more steam.

12.6.2 Latent Heat Energy Storage

Energy is stored in the form of the latent heat caused by phase change either by melting a solid or vaporizing a liquid. Energy release is accomplished by reversing the process, i.e. by solidifying the liquid or condensing the vapour.

Storage density = latent heat × density

Since the latent heats are much larger than the specific heats, the energy storage density is larger than that in sensible heat storage. The system has the additional advantage of operating at constant temperature. It also has the advantage of a wide choice of materials with different fusion and evaporation temperatures to suit the peaking turbine.

Storage materials must possess, in addition to proper phase transition temperature and high latent heat, good thermal conductivity, containability, chemical stability, nontoxicity and low cost. No material meets all these requirements, but some fluoride salts meet some of them. One of the most suitable salts for latent heat storage is the 70% NaF-30% FeF_2 eutectic salt, which has a fusion temperature of 680 °C and the highest storage density (of all salts) of 1500 MJ/m^3. $ZnCl_2$ is another salt with a fusion temperature of about 370 °C and an energy density of about 400 MJ/m^3. The following fluoride mixtures in Table 12.4 are chemically stable and can be contained in chromium nickel steel, suitable as storage for heat engines.

Table 12.4 *Melting points of some storage media*

Storage media	*Melting point (°C)*
Sodium-magnesium fluoride, NaF/MgF_2	832
Lithium-magnesium fluoride, LiF/MgF_2	746
Sodium-calcium-magnesium fluoride, $NaF/CaF_2/MgF_2$	745
Lithium-sodium-magnesium fluoride, $LiF/NaF/MgF_2$	632

12.7 CHEMICAL ENERGY STORAGE

The heat of reaction of reversible chemical reactions is used to store thermal energy during endothermic reactions and to release it during exothermic reactions. Like latent heat energy storage, this form also offers large energy storage densities. The following reversible reaction along with some others as given in Table 12.5 have been suggested for energy storage.

$$CO + 3H_2 \underset{\text{Endo}}{\overset{\text{Exo}}{\rightleftharpoons}} CH_4 + H_2O$$

For reaction from right to left at 298K, 1 atm

Energy balance is

$$H_R + Q = H_P$$

$$\left(\bar{h}_f^o\right)_{CH_4} + \left(\bar{h}_f^o\right)_{H_2O} + Q = \left(\bar{h}_f^o\right)_{CO} + 3 \times 0$$

$$-74.9 + (-286) + Q = -110.6 + 0$$

$$\therefore \quad Q = 250.3 \text{ kJ/gmol}$$

The reaction is endothermic and energy is absorbed. The reverse reaction from left to right results in $Q = -250.3$ kJ/gmol, i.e. energy is released and the reaction is exothermic. The endothermic reaction is called reformation and the exothermic reaction is called methanation.

Table 12.5 *Reversible chemical reactions under consideration for energy storage*

Reaction		*Temperature range, K*	*Heat of reaction at 298K, kJ/gmol*
1. $CO + H_2$	$\rightleftharpoons CH_4 + H_2O$	700 –1200	250.3
2. $2CO + 2H_2$	$\rightleftharpoons CH_4 + CO_2$	700 –1200	247.4
3. $C_6H_6 + 3H_2$	$\rightleftharpoons C_6H_{12}$	500 –750	207.2
4. $C_7H_8 + 3H_2$	$\rightleftharpoons C_7H_{14}$	450 –700	213.5
5. $C_{10}H_8 + 5H_2$	$\rightleftharpoons C_{10}H_{18}$	450 –700	314.0
6. $C_2H_4 + HCl$	$\rightleftharpoons C_2H_5Cl$	420 –770	56.1
7. $CO + Cl_2$	$\rightleftharpoons COCl_2$	–	112.6

A schematic of a power plant with a chemical storage system is shown in Fig. 12.24. During periods of low demand, some heat from the primary heat source is diverted to the reformer (endothermic reactor) to convert the products $CH_4 + H_2O$ to the reactants $CO + 3H_2$, which are stored in a vessel at high pressure, about 70 bar, but at ambient temperature. During periods of high demands, these reactants are fed to the methanator (exothermic reactor) where heat is generated to run a peak turbine. In the methanator the reactants are converted to the products $CH_4 + H_2O$, which are stored in a separate vessel for later use in the reformer during periods of low demand. A thermal turnaround efficiency of this system is estimated as 85 to 90 per cent.

The two storage vessels and the two reactors operate at different pressures. Storage pressures need to be high to minimize vessel size and cost and the reformer has to operate at low pressure to maximize the rate of endothermic reaction.

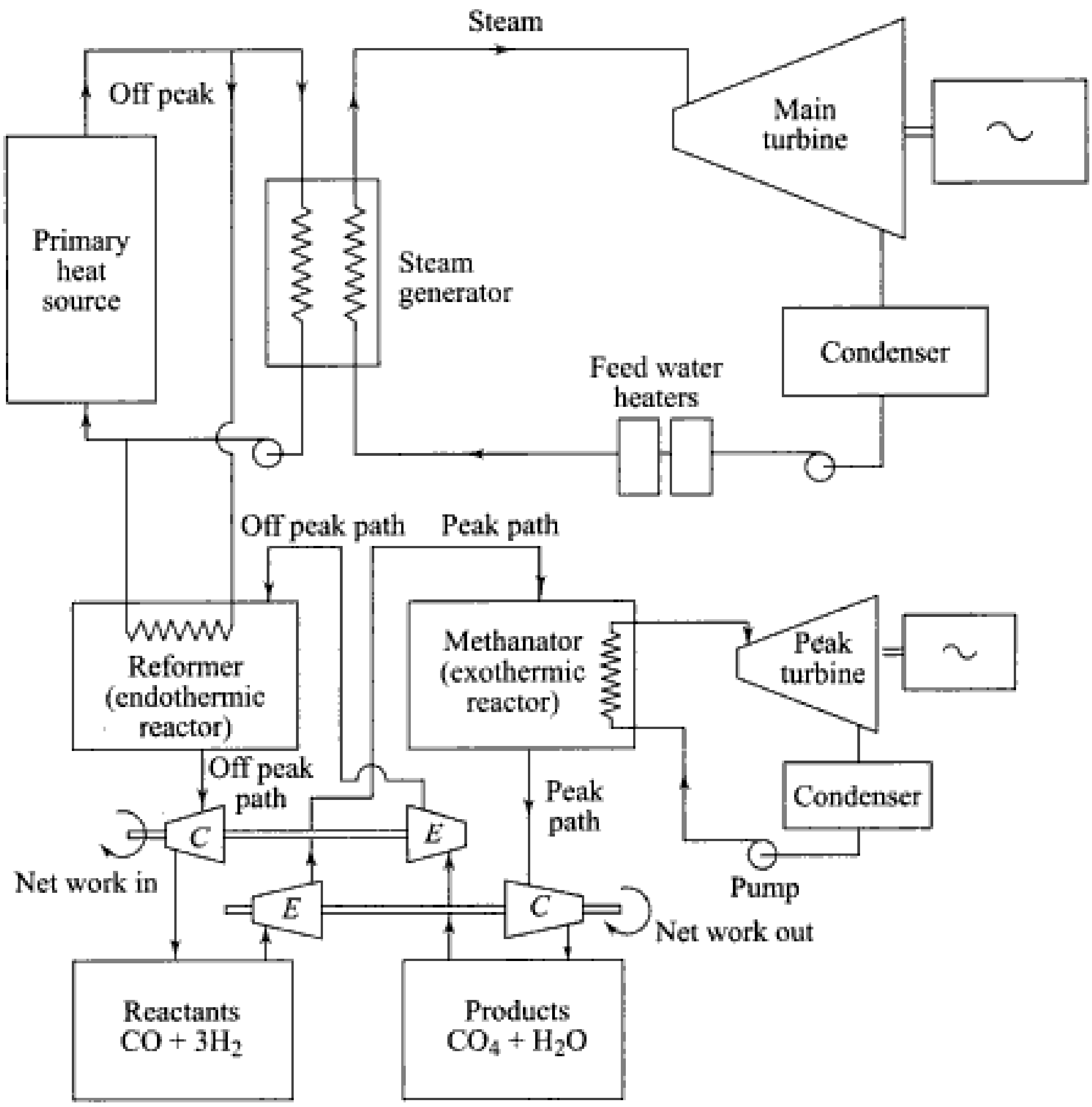


Fig. 12.24 *Schematic flow diagram of a power plant with a chemical storage system using the reaction $CO + 3H_2 \rightleftharpoons CH_4 + H_2O$*

12.8 HYDROGEN ENERGY

Hydrogen is widely regarded as the ultimate fuel and energy storage medium for future centuries. It can be derived from water (by electrolysis) using any source of high quality energy and it can be combusted back to water in a closed chemical cycle without any pollution. The potential of hydrogen for the storage and cheap transmission of energy over long distances has led to the concept of hydrogen economy (Fig. 12.25).

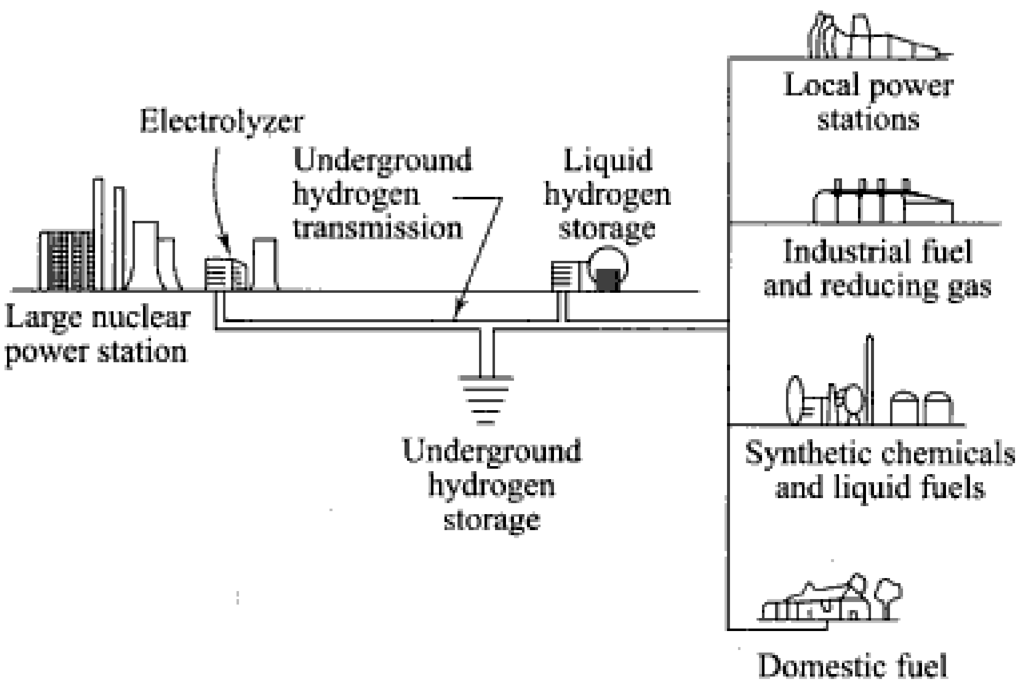


Fig. 12.25 *The hydrogen economy fuel system*

Hydrogen can be produced by the following methods.

1. Catalytic steam reforming of natural gas
2. Chemical reduction of coal (water gas reaction)

$$C + H_2O \rightarrow CO + H_2$$

or

$$CO + H_2O \rightarrow CO_2 + H_2$$

3. Industrial photosynthesis
4. Ultraviolet radiation
5. Partial oxidation of heavy oils
6. Electrolytic decomposition of water
7. Thermal decomposition of water, utilizing thermochemical cycles.

The electrolytic decomposition of water (electrolysis) comprises two processes.

$$\text{cathode} \quad : \quad 2H_2O + 2e^- \rightarrow H_2 + 2OH^-$$

$$\text{anode} \quad : \quad 2OH^- \rightarrow \frac{1}{2}O_2 + H_2O + 2e^-$$

According to Faraday's law of electrolysis the mass of hydrogen m discharged may be obtained from

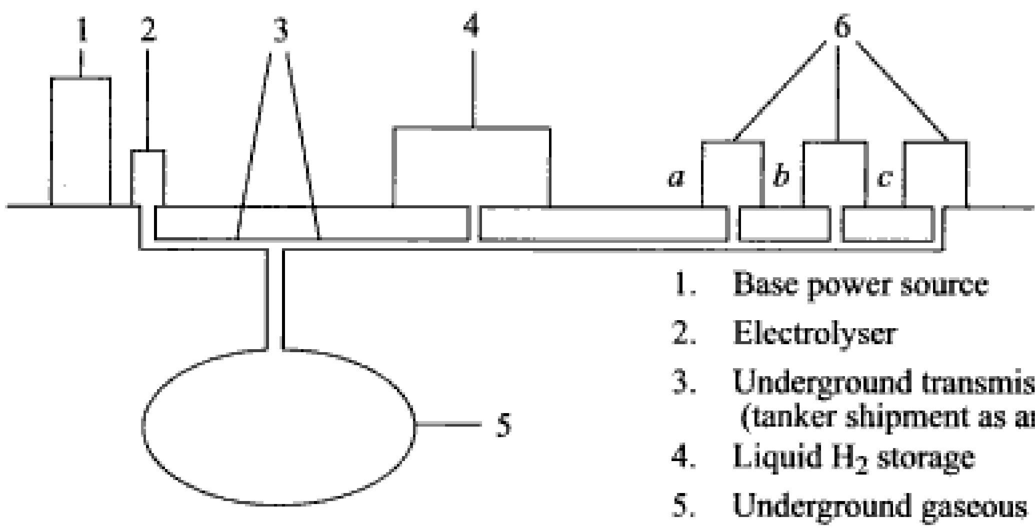
$$m = \frac{A\,I\,t}{F\,Z} \tag{12.43}$$

where A = atomic weight, I = electric current through the electrolyte, t = duration of electrolysis, Z = valency and F = Faraday constant = 96500 coulomb/kgm. The overall reaction is

$$H_2O \rightarrow H_2 + \frac{1}{2}\,O_2 - 237 \text{ kJ/kmol}$$

Hydrogen may be used as a primary fuel for peak power generation producing water in the reaction. During off-peak hours hydrogen may be produced by electrolysis of water with excess power available. The hydrogen based power utility concept is shown schematically in Fig. 12.26.

The main drawback of hydrogen is its extreme flammability and the problem of storing the gas under pressure. Liquefaction could simplify storage but it consumes a lot of energy.



1. Base power source
2. Electrolyser
3. Underground transmission (tanker shipment as an alternative)
4. Liquid H_2 storage
5. Underground gaseous H_2 storage
6. H_2 consumers:
   (a) Small conventional power plants
   (b) Synthetic fuel industry
   (c) Domestic fuel

Fig. 12.26 *Hydrogen-based power utility concept*

Methane may be produced by reactions with hydrogen.

$$CO + 3H_2 = CH_4 + H_2O \qquad Q = -205 \text{ kJ}$$

or

$$C + 2H_2 = CH_4 \qquad Q = -73 \text{ kJ}$$

Methane may be used by consumers as an ordinary fuel, without being highly inflammable as hydrogen.

Ammonia may be more attractive as a fuel, which can be produced from hydrogen by combining it with nitrogen, under 200–500 bar pressure and at 720 K in presence of a catalyst (Haber process)

$$N_2 + 3H_2 = 2NH_3 \qquad Q = -90 \text{ kJ}$$

Dissociating ammonia back into hydrogen and nitrogen can be done by passing the gas through a hot tube. The advantages of storing ammonia, relative to storing hydrogen, are the following.

(a) safer storage
(b) energy density is higher
(c) easier to liquefy

12.8.1 Storage of Hydrogen

Hydrogen may be stored as (i) compressed gas (ii) chemical compound (iii) liquid (iv) metallic hydrides.

(i) Since the cost of liquefaction is high, the bulk storage of hydrogen can be made in the form of compressed gas in underground caverns, where it can be stored like natural gas.
(ii) Hydrogen in chemical combination with other elements like methane and ammonia is more amenable to storage.
(iii) Liquid hydrogen has a mass energy density three times greater than oil. Its use is attractive for heavy surface transport and aircraft. For energy storage, liquid hydrogen is not so attractive because of its low density. Ammonia and methane, as liquids, are more efficient in this aspect.
(iv) The principal disadvantages of gaseous hydrogen as a storage medium is that it takes up large space, it is explosive and it is not leak-proof. Liquid hydrogen, highly cryogenic and inflammable, is costly too. These drawbacks can be fought back if hydrogen is stored in hydride form (Table 12.6). The aim is to select a hydride which can be thermally decomposed in a reversible manner so that hydrogen may be withdrawn or replenished from or to the vessel when necessary. Some of the desired features of a suitable hydride store are the following:

1. High hydrogen content per unit mass of metal.
2. Low dissociation pressure at moderate temperatures.
3. Constancy of dissociation pressure during the decomposition time.
4. Safe on exposure to air.
5. Low cost.

Table 12.6 *Mass and volume energy densities*

	W_m (kJ/kg)	W_v (kJ/dm^3)
Gas at 150 atm, 20°C	140000	1700
Liquid –252°C	140000	10500
Metal hydride (including metal)	1400–1100	17500–21000
Oil (for comparison)	44000	40000

The exothermic chemical reaction of hydride formation from metals (Me) and hydrogen (H_2) is as follows:

$$H_2 + Me \underset{\text{Discharging (heat added)}}{\overset{\text{Charging (heat released)}}{\rightleftharpoons}} \text{hydride} + \text{heat}$$

Low temperature FeTi hydride, with a low energy requirement for hydrogen release, has been developed by Brookhaven National Laboratory, USA. Magnesium-based high temperature hydride has attracted interest because magnesium is a readily available and cheap metal. Both materials release hydrogen endothermically, thus creating no safety problem. The mass energy densities (W_m) of hydrides based on Ti and Mg are the following:

$FeTiH_{1.7}$	$\rightarrow$	$FeTiH_{0.1}$	516 Wh/kg	(1856 kJ/kg)
Mg_2NiH_4	$\rightarrow$	$Mg_2NiH_{0.3}$	1121 Wh/kg	(4036 kJ/kg)
MgH_2	$\rightarrow$	$MgH_{0.005}$	2555 Wh/kg	(9198 kJ/kg)

When hydrides are used as hydrogen stores for heat engines or for domestic heaters, the waste heat from them can be returned to the hydride. If the amount of waste heat is less than the heat needed for hydrogen release, then the waste heat energy can be stored in the metal hydride, acting as a thermal storage device. A combination of hydrides with different release temperatures may be used for different applications such as heat pumps, central air conditioning and even for renewable energy storage systems.

Example 12.1 In a simple CAES system, the average air flow into a cavern of 64,000 m^3 is at the rate of 8300 m^3/h. Air enters the compressor at 1 bar, 20°C and leaves at 100 bar, the polytropic compressor efficiency being 70 per cent. If the peaking turbine efficiency is 60 per cent and air is stored in the cavern at 100 bar, 20°C, estimate (a) the compressed air temperature (b) the storage time (c) the total energy storage in MWh (d) the total energy delivered by the peaking turbine. Take $\gamma = 1.4$, $c_p = 1.005$ kJ/kg K and $R = 0.287$ kJ/kgK for air.

Solution With reference to Fig. E12.1

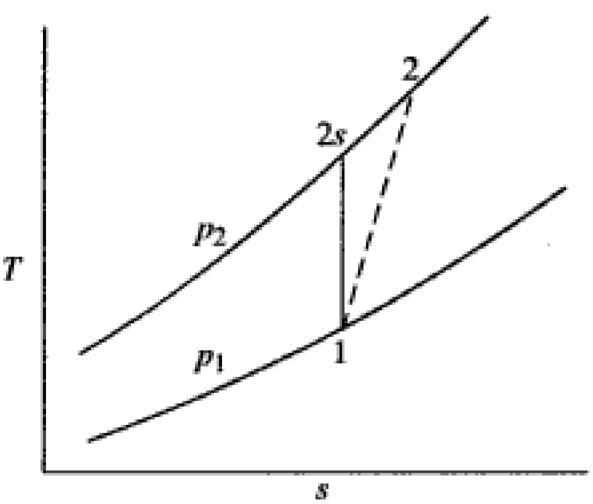


Fig. E12.1

$$\frac{T_{2s}}{T_1} = (p_2/p_1)^{(\gamma-1)/\gamma} = (100)^{0.4/1.4} = 3.7325$$

$$\therefore \quad T_{2s} = 1093.62\text{K}$$

$$\frac{T_{2s}-T_1}{T_2-T_1}=0.7$$

$$\therefore \qquad T_2-T_1=\frac{800.62}{0.7}=1143.74$$

$$\therefore \qquad T_2=1436.74\,\text{K}=1163.7°\text{C}$$ *Ans.* (a)

Specific volume of air at 100 bar, 20°C

$$=\frac{RT}{p}=\frac{0.287\times 293}{100\times 100}=84.09\times 10^{-4}\,\text{m}^3/\text{kg}$$

Mass flow rate of air into the cavern

$$=\frac{8300}{84.09\times 10^{-4}\times 3600}=274.18\,\text{kg/s}$$

Rate of energy storage

$$=\dot{m}_a\, c_p(T_2-T_1)=274.18\times 1.005\times 1143.74\times 10^{-3}$$

$$=315.16\,\text{MW}$$

Storage time

$$=\frac{64000}{8300}=7.71\text{h}$$ *Ans.* (b)

∴ Total energy storage

$$=315.16\times 7.71$$

$$=2429.88\cong 2430\,\text{MWh}$$ *Ans.* (c)

Total energy delivered by the peaking turbine

$$=2430\times 0.6=1458\,\text{MWh}$$ *Ans.* (d)

Example 12.2 In a pressurized water sensible heat storage system, steam is extracted from the turbine during off-peak hours and fed into steel accumulators of volume 175000 m^3. The accumulators are 4 m in diameter each and are well insulated so that U = 1.5 W/m^{2}K. The maximum and minimum storage pressures are 20 bar and 2 bar. The ambient temperature is 20°C. The specific heat of water may be taken as 4.35 kJ/kgK. Assuming the thermal turnaround efficiency of 96 per cent and a peaking plant efficiency of 25 per cent, calculate (a) the storage time (b) the total energy stored in the accumulators and (c) the total energy that can be delivered by the peaking turbine.

Solution At 20 bar

$$T_1=212.37°\text{C},\ h_{f,1}=908.5\ \text{kJ/kg},$$

$$v_{f,1}=0.0011766\ \text{m}^3/\text{kg}$$

At 2 bar,

$$T_2 = 120.23°\text{C}, h_{f,2} = 504.8 \text{ kJ/kg},$$
$$v_{f,2} = 0.0010605 \text{ m}^3/\text{kg}$$

At $\quad 20°\text{C}, h_f = 293 \text{ kJ/kg}$

Taking an average density of water as

$$\frac{1}{2}\left[\frac{1}{v_{f,1}} + \frac{1}{v_{f,2}}\right] = 896.43 \text{ m}^3/\text{kg}$$

The time constant $\tau = \dfrac{D\rho_f c_f}{4U}$

$$= \frac{4\text{m} \times 869.43 \dfrac{\text{kg}}{\text{m}^3} \times 4.35 \times 1000 \dfrac{\text{J}}{\text{kg k}}}{4 \times 1.5 \dfrac{\text{W}}{\text{m}^2\text{K}} \times \dfrac{3600\, s}{1\text{h}}}$$

$$= 700.374 \text{ h}$$

The thermal turnaround efficiency

$$\eta_{TA} = 1 - \frac{T_1 - T_\infty}{T_1 - T_2} = [1 - e^{-t_s/\tau}]$$

$$0.96 = 1 - \frac{212.37 - 20}{212.37 - 120.23}\,[1 - e^{-t_s/\tau}]$$

$$1 - 2.0878\,[1 - e^{-t_s/\tau}] = 0.96$$
$$2.0878\,[1 - e^{-t_s/\tau}] = 0.04$$
$$1 - e^{-t_s/\tau} = 0.019159$$
$$e^{-t_s/\tau} = 0.980841$$
$$e^{t_s/\tau} = 1.0193$$

$\therefore \quad t_s/\tau = 0.0193$

$\therefore \quad$ Storage time $= 0.0193 \times 700.374 = 13.517$ h *Ans.* (a)

Total volume of accumulators = 175000 m^3.

Mass of water needed to be flashed

$$m = \frac{175000}{v_{f,1}} = \frac{175000}{0.0011766} = 1.4873 \times 10^8 \text{ kg}$$

Total energy stored in accumulators

$$= m(h_{f,1} - h_{f,2})$$
$$= 1.4873 \times 10^8\,(908.5 - 504.8)$$
$$= 600.423 \times 10^8 \text{ kJ} = 0.166784 \times 10^8 \text{ kWh}$$
$$= 0.166784 \times 10^5 \text{ MWh} = 16678.4 \text{ MWh}$$ *Ans.* (b)

Total energy that can be delivered by the peaking turbine

$$= 16{,}678.4 \times 0.96 \times 0.25 = 4002.82 \text{ MWh}$$ *Ans.* (c)

12.25 What is a fuel cell? How is it different from a battery? Give the essential functions of a fuel cell. What is the emf of a fuel cell? On what factors does it depend?

12.26 Explain a hydrogen-oxygen fuel cell? Why is it attractive?

12.27 What do you mean by superconductivity? What is the concept of superconductive magnetic energy storage?

12.28 What is a Brooks coil? What is the amount of energy stored in a Brooks coil?

12.29 Why is huge structural mass required to contain magnetic energy?

12.30 What are the two distinct thermal energy storage systems? What is the typical value of storage density of sensible heat energy storage?

12.31 What is the thermal turnaround efficiency? On what factors does it depend?

12.32 How is convective heat loss from the stored saturated water estimated? Why is the energy stored in the accumulator structure very small compared to energy stored in water?

12.33 Show that the thermal turnaround efficiency of sensible heat energy storage system is given by

$$\eta_{TA} = 1 - \frac{T_1 - T_\infty}{T_1 - T_2} [1 - e^{-t_s/\tau}]$$

where t_s is the duration of energy storage and τ is the time constant.

12.34 Explain with a neat sketch a pressurized sensible heat storage system where the primary heat source is either a fossil-fueled furnace or a nuclear reactor.

12.35 What are the three main ways of operating an accumulator? Explain a Ruths accumulator. Why is steam cushion provided?

12.36 Explain the operation of an expansion accumulator.

12.37 Explain a latent heat energy storage system. What are its advantages and disadvantages compared to sensible heat energy storage?

12.38 What should be the properties of storage materials for storing latent heat? What are the suitable salts?

12.39 Explain the principle of chemical energy storage.

12.40 For the reversible chemical reaction

$$CO + 3H_2 \rightleftharpoons CH_4 + H_2O$$

what do you mean by reformation and methanation?

12.41 Explain with a neat diagram of a power plant having a chemical energy storage system utilizing the reaction given in the Question 12.40.

12.42 What is the advantage of hydrogen as an energy storage medium? What is hydrogen economy?

12.43 What are the different methods of producing hydrogen?

12.44 What are Faraday's laws of electrolysis?

12.45 How can hydrogen be used for peak power generation? What are the drawbacks of hydrogen as a fuel?

12.46 What are the advantages of storing ammonia. How can ammonia be produced from hydrogen?

12.47 Describe the different methods of storing hydrogen. Explain how hydrogen is stored in hybride form.
12.48 What are the desirable features of a hydride to store hydrogen?
12.49 How is hydrogen released from a hydride?
12.50 What are the advantages of magnesium-based hydrides? Give the names of some hydrides based on Ti and Mg.

PROBLEMS

12.1 A pumped-hydro energy storage system with an elevation of 40 m is considered for a power grid which has a load pattern of 600 MW during 18h and 1200 MW during 6h for one day. Calculate (a) the power output of a power plant in MW that would meet the load demand with and without energy storage and (b) the volume of water in m^3 that must be pumped to meet storage demand. The electric generator efficiency is 0.8 and density of water is 1000 kg/m^3.

12.2 Calculate the air flow, compressed air temperature and storage volume for a 1500 MWh peaking unit charging for 7.5 h. Assume compressor inlet at 1 bar, 20°C, compressor exit at 100 bar, a compressor polytropic efficiency of 70 per cent, a peaking turbine efficiency of 60 per cent and a constant specific heat of air c_p = 1.05 kJ/kgK. Take R = 284.75 J/kgK for air.

[*Ans.* 1162°C, 7.5 × 10^6 kg, 62,575 m^3]

12.3 An adiabatic CAES system is required to generate 250 MW during 6 peak hours from storage during 12 h of operation of the power plant. There is no intercooling or reheat used. The compressor and turbine have polytropic exponents of 1.5 and 1.3 and pressure ratios of 100 and 80, respectively and mechanical efficiencies of 0.92 each. The MG set has a combined mechanical electrical efficiency of 0.97. Atmospheric air is at 1 bar, 20°C. Air enters the heat storage packed bed at 90 bar, 100°C. The packed bed has heat losses of 10 per cent and air leakage losses of 1.5 per cent. Calculate (a) the turbine air mass flow rate during peak operation in kg/s (b) the minimum volume of air storage cavern in m^3 (c) the energy storage capacity in MWh (d) the compressor power input, in MW and (e) the turnaround efficiency of the system. Take c_p = 1.05 kJ/kgK for air.

12.4 A magnetic energy storage coil is constructed of a conductor of square cross-section 400 mm × 400 mm and a mean diameter of 32 m. Calculate the number of turns necessary for a stored energy of 5000 MWh if the current is 160000 A. Take the energy stored $E = FE_B$, where E_B is the energy stored in a Brooks coil and $F = 0.15$.

[*Ans.* 11 turns]

12.5 A flywheel in the form of a disc 8 m in diameter and 3 m thick runs at 3000 rpm. It is made of an anisotropic filament composite material of a uniform density 2160 kg/m^3. Calculate (a) the energy in the flywheel, in kWh (b) the change in rotational speed and corresponding energy that can be extracted from the flywheel if the coefficient of speed fluctuation may not exceed 0.01.

13

Non-conventional Power Generation: Direct Energy Conversion

Fossil fuels (coal, fuel oil and natural gas) still meet the major part of our energy demand and are getting fast depleted. Moreover, there are serious pollution hazards like greenhouse effect and global warming which occur due to fossil fuel burning. The use of nuclear power too has its own problems and nuclear fusion is yet to be realized in practice. We have thus been forced to look for nonconventional power generation systems so as to reduce fossil fuel consumption by increasing the conversion efficiency from fuel to electricity. A substantial fuel economy can be achieved by converting "heat" (internal energy) directly to electricity by eliminating the link process of producing mechanical energy via steam (Rankine cycle). Major *direct energy-conversion devices* are magnetohydrodynamic, thermionic and thermoelectric generators and fuel cells. In this chapter we will describe the following unconventional energy-conversion systems which have considerable influence on the energy scenario of the future:

1. Magnetohydrodynamic (MHD) power generation
2. Thermionic power generation
3. Thermoelectric power generation
4. Fuel cells
5. Geothermal energy
6. Hydrogen energy system

13.1 MAGNETOHYDRODYNAMIC (MHD) POWER GENERATION

Of all the direct energy conversion methods exploitable, the MHD power generation seems to be the most promising for a utility system. The maximum limiting temperature for turbine blades being 750–800°C, the MHD generator is capable of tapping the vast potential offered by modern furnaces, which can reach temperatures of more than 2500 K, and up to 3000 K with preheating of air.

Faraday's law of electromagnetic induction states that when a conductor and a magnetic field move relative to each other, an electric voltage is induced in the conductor. The conductor may be a solid, liquid or gas. In an MHD generator, the hot ionized gas replaces the copper windings of an alternator. When a gas is heated to high temperatures, the valence electrons of the excited atoms move on to higher quantized orbits and ultimately, at certain energy levels they fly off and become free electrons. For a gas to be conducting, a certain number of free electrons must be present along with an equal number of ions and the main body of neutral atoms. Since a very high temperature is required to ionize a gas (thermal ionization) which cannot be endured by the materials available, the hot gas is seeded with an alkali metal, such as cesium or potassium (K_2CO_3 or KOH) having a low ionization potential (energy needed to ionize one g mol of atoms) before the gas enters the MHD duct. An adequate electrical conductivity of the order of 10 mho/m can thus be realized at somewhat lower temperatures in the range 2200–2700 °C.

A simple view of the MHD generator is shown in Fig. 13.1. The duct through which the electrically conducting ionized gas flows has two sides supporting a

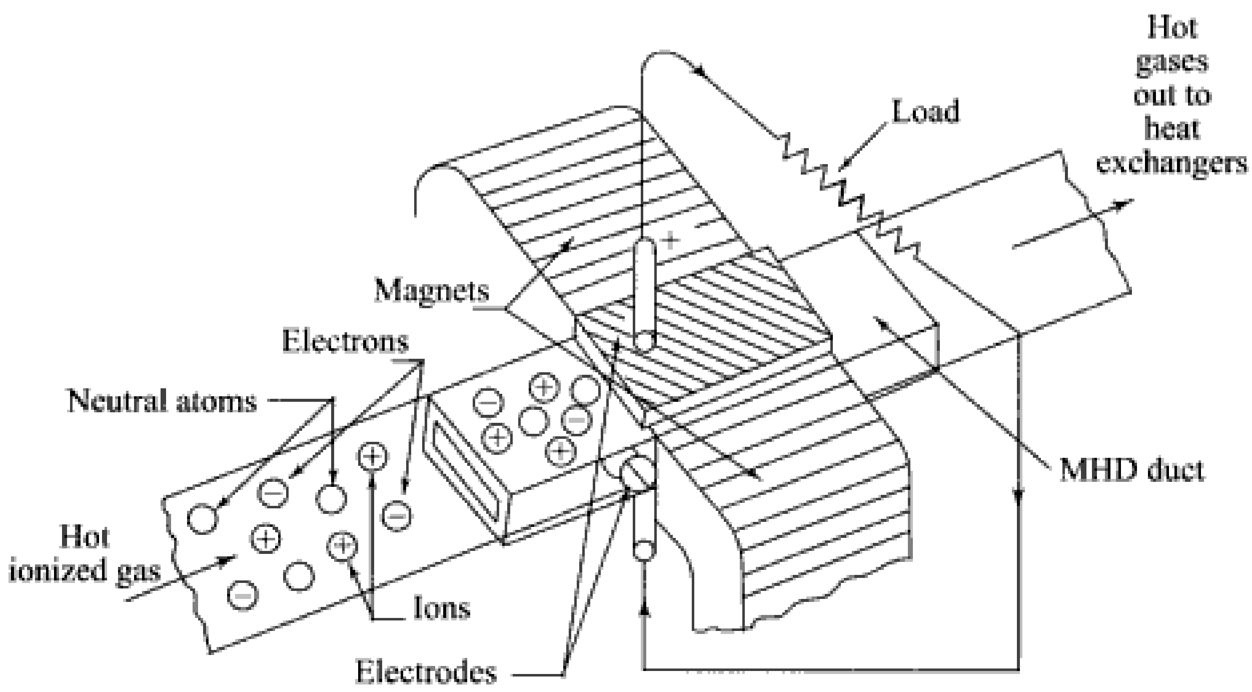


Fig. 13.1 *Simplifed view of an MHD generator*

strong transverse magnetic field of 4 – 5 tesla (1 tesla = 10^4 gauss) at right angles to the flow and the other sides forming the faces of electrodes which are joined through an electrical circuit. As the hot ionized gas or plasma enters the MHD duct, due to the effect of the strong magnetic field and the consequent Lorentz force, there is a decrease in the kinetic energy of the plasma, and the electrons and ions get deposited on the opposite electrodes. The power generated per unit length is approximately proportional to $\sigma u B^2/\rho$, where σ is the electrical conductivity, u is the velocity of the gas, B is the magnetic field strength and ρ is the density. The power produced being dc, the conversion to ac is done by an inverter. Figure 13.2 shows the principal components of a typical MHD plant and its cycle of operations on T–s diagram. It is a Brayton cycle with MHD generator replacing the combustor of the conventional GT plant.

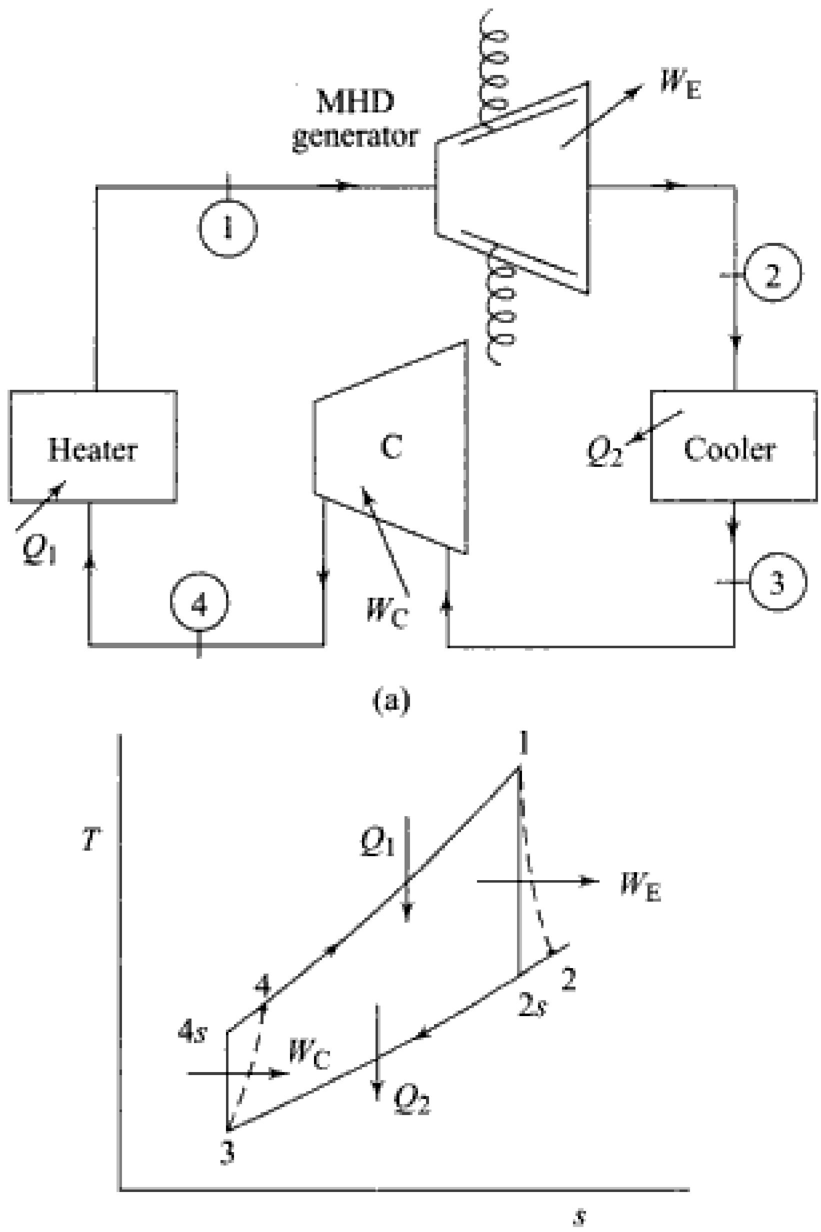


Fig. 13.2 *MHD plant as a cyclic heat engine*

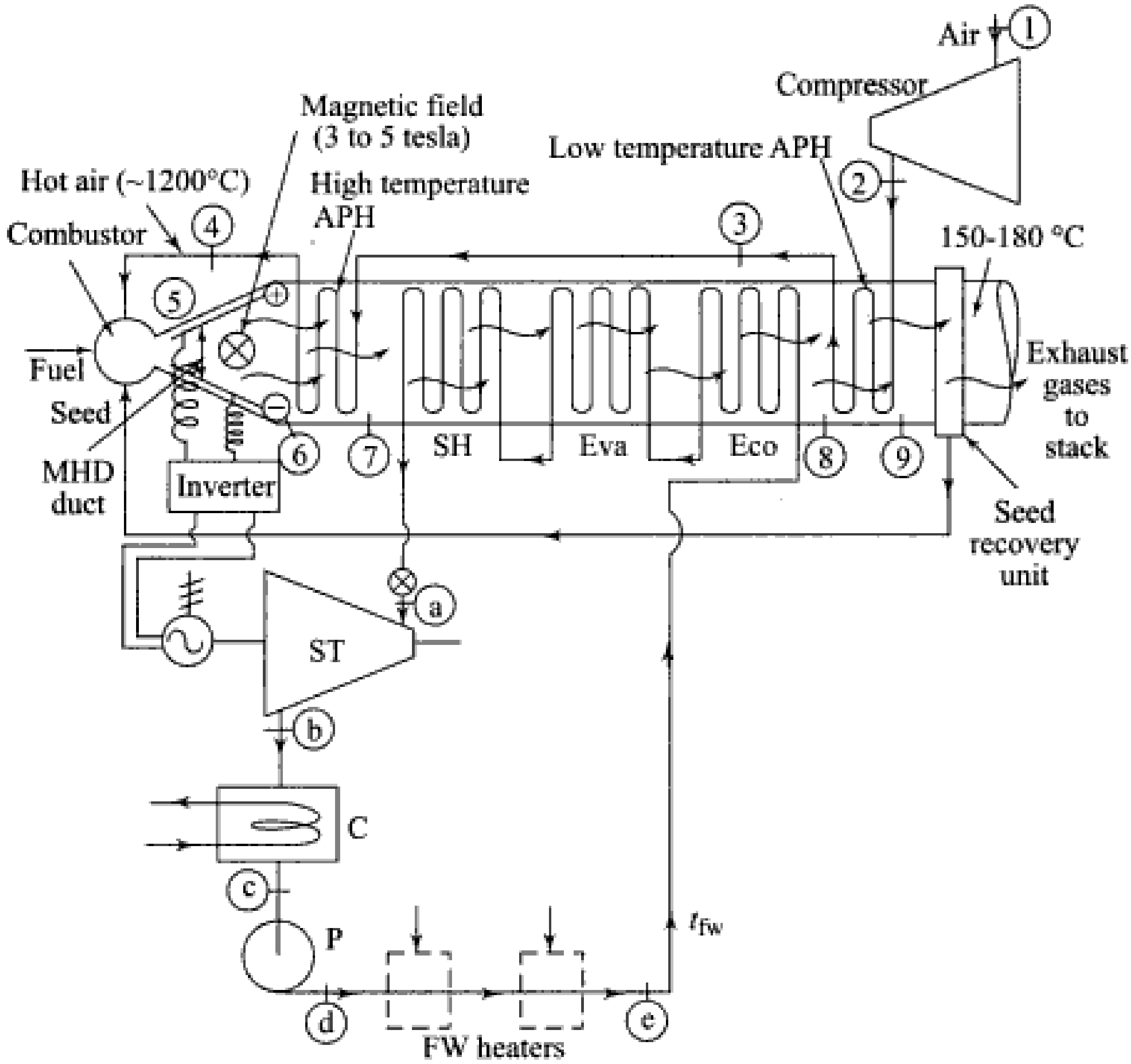
13.1.1 Combined MHD-Steam Power Plant

If the gas entering the MHD duct at about 3000 °C could be expanded to the ambient temperature of 30 °C, the Carnot efficiency would have reached 90%. Unfortunately, the MHD power output is restricted because by the time the gas temperature falls to 2000 °C the electrical conductivity becomes very low with the electrons combining with ions to form neutral atoms, and the generator then ceases to operate satisfactorily. Therefore, the MHD generator is used as a topping unit and the MHD exhaust at about 2000 °C is utilized in raising steam to drive turbine and generate electricity in a conventional steam power plant used as a bottoming unit (Fig. 13.3). If the fraction z of the fuel energy is directly converted to electricity in the MHD generator, the remainder $(1 - z)$ is converted with an efficiency η' in the bottoming steam plant so that the overall efficiency is

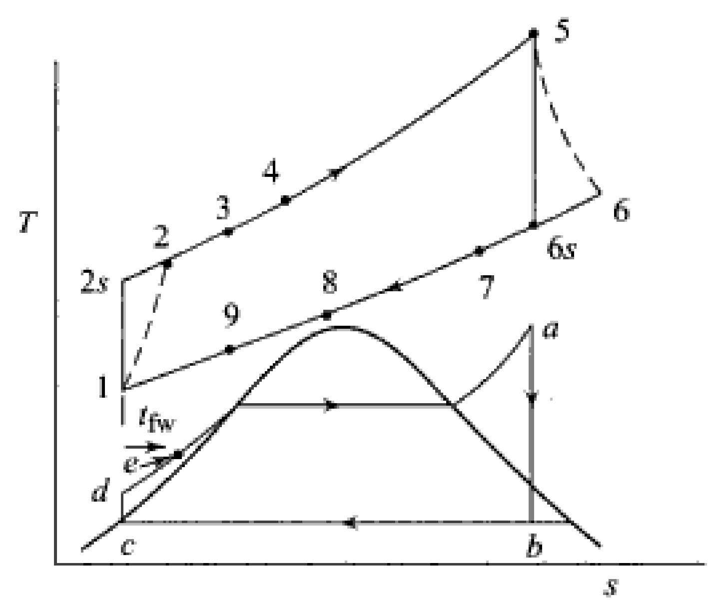
$$\eta = z + \eta' (1 - z) \qquad (13.1)$$

If $z = 0.3$ and $\eta' = 0.4$, then $\eta = 0.58$, which is a good power plant efficiency.

MHD-topped steam plants can operate either in an open cycle or in a closed cycle. An open cycle scheme is shown in Fig. 13.3. The products of combustion



(a)



(b)

Fig. 13.3 *An open cycle MHD-steam power plant* with highly preheated air are seeded with 1% potassium before they enter the MHD duct at about 2500 – 3000 K, where some part of the internal energy of the gas (plasma) is directly converted to dc electricity, and then by dc – ac inverter to ac power. The high temperature exhaust from the MHD duct is then used in the preheating of air and in raising steam. In the steam cycle, the feedwater heaters and reheaters have not been shown. The combustion air can also be preheated indirectly using an auxiliary combustion system. Oxygen-enriched air is used when the preheated air temperature is not high enough. Since the products of combustion are exhausted to atmosphere, the oxides and hydroxides of the seeding element cause severe air pollution. The use of an electrostatic precipitator helps in the recovery of the seed, which can be used again, and also in the abatement of atmospheric pollution. An MHD–steam power plant with coal as the fuel is shown in Fig. 13.4.

In the closed cycle scheme, helium (or argon) gas seeded with cesium is heated in a nuclear reactor, passed into the MHD duct and then into the steam generating system (Fig. 13.5). A gas turbine plant can also be used as a bottoming unit (Fig. 13.6). Since the combined plant operates over a larger temperature difference, the efficiency will obviously be higher.

For MHD duct walls, the material has to stand up to temperatures above 2200 °C and the corrosive atmospheres of alkali-seeded gases. The duct wall will also need to be an electrical insulator at these temperatures. Materials used are magnesium oxide, strontium zirconate and hafnia. Electrodes in the dc MHD generator perform the same function as brushes in a conventional dc generator. Tungsten or carbon electrodes have been used. Electrodes are often segmented to reduce energy losses due to Hall effect [20]. To produce a strong magnetic field, electromagnets used consume a lot of electricity. To reduce the power consumption of these electromagnets, cryogenic or superconducting coils at liquid helium temperatures have been suggested.

Although most of the developmental efforts on MHD were based on fuels like natural gas, kerosene, benzene, toluene, fuel oil, etc., coal is inherently a better fuel than others, because it contains less hydrogen, and thus the sink of electrons in the flow created by the OH-ions is reduced. The only fuel which has better characteristics than coal is char, which contains almost no hydrogen and, in general, results in a 25% increase in the performance of the generator (Womack, 1969). It was found that there was no deterioration of the channel through chemical or thermal action of coal-slag deposits, or of the electrical performance of the duct, and there was no loss of the insulator property through penetration of the potassium seed by the coal combustion products. In fact, the slag deposits protect the electrodes and insulators and improve the breakdown properties of the channel by reducing the electrical field gradient in the flow direction.

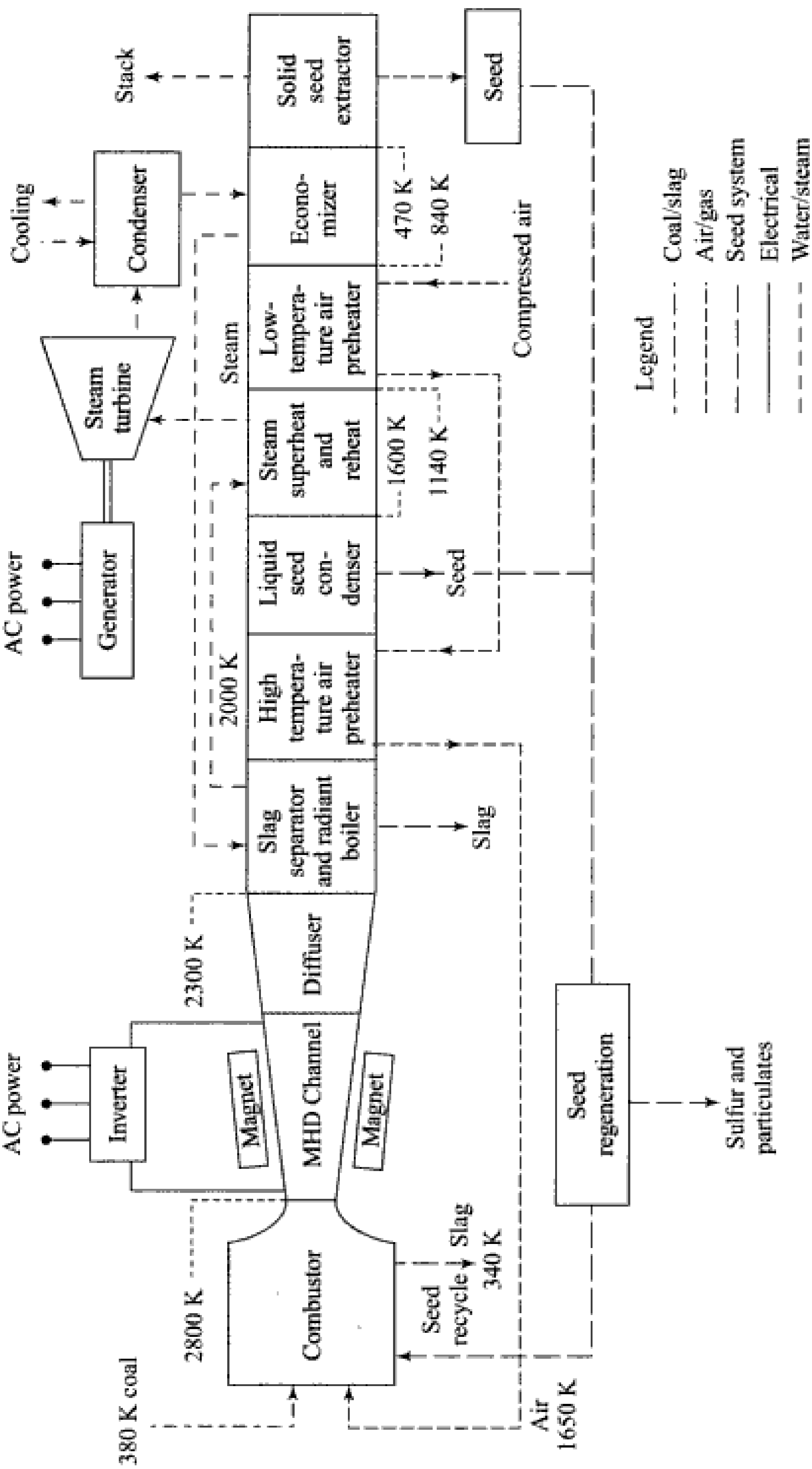


Fig. 13.4 *Component arrangement and system temperatures for a coal-fired MHD/steam power plant. (A. W. Postelethwaite and M. M. Sluyter. Mechanical Engineering, March 1978.[3])*

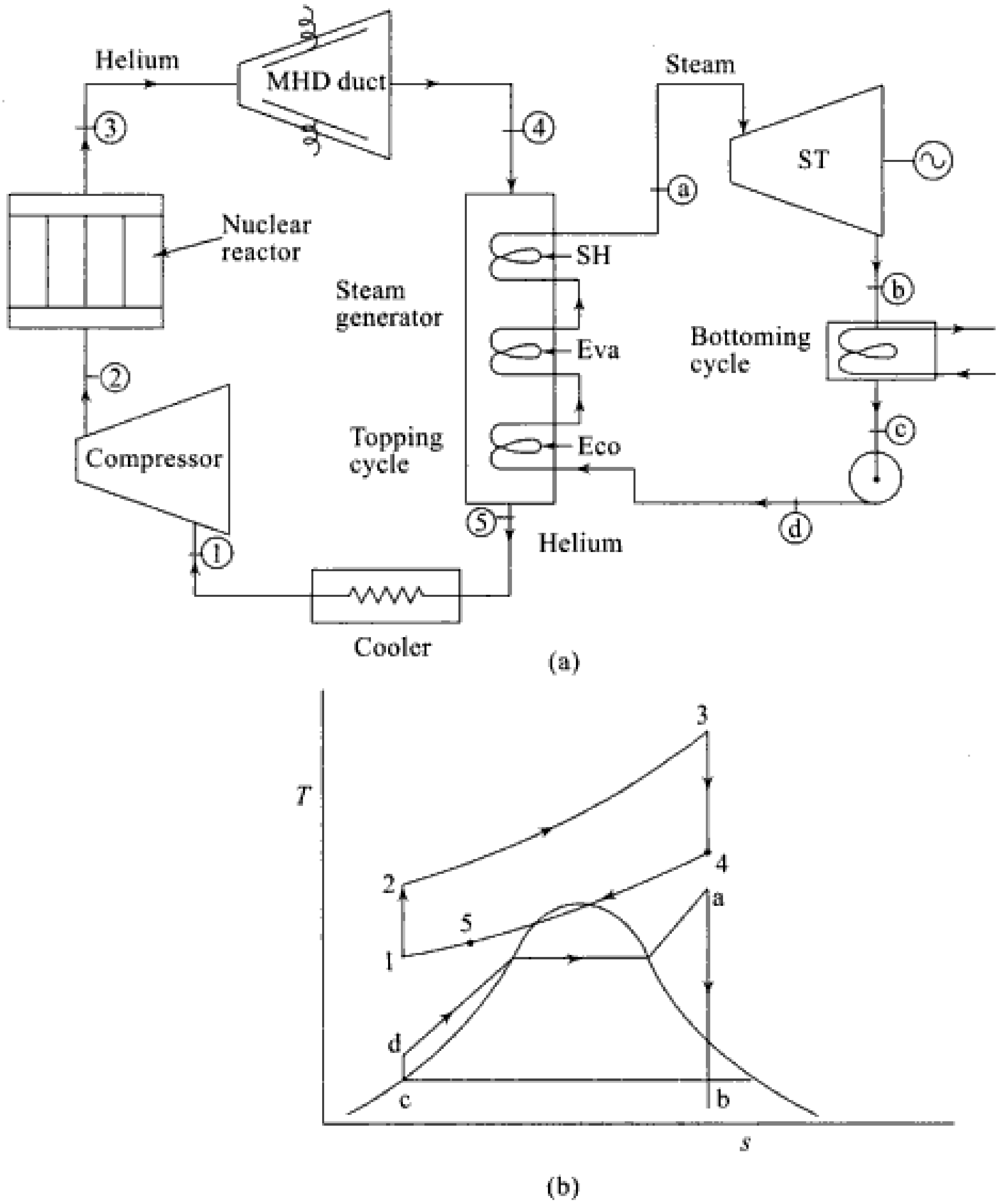


Fig. 13.5 *Closed cycle MHD-steam power plant*

The oldest MHD-steam power plant is the 75 MW unit (U–25 facility) in the erstwhile USSR, of which 25 MW is generated by MHD means. In 1981 another 1000 MW plant was commissioned near Moscow. Under the joint Indo-USSR program, a prototype of an MHD generator having 5 MW thermal input with coal gas as the fuel using 40% enriched air, 5 bar combustor pressure, a magnetic field of 5 tesla, and a seed of 50% K_2CO_3 in water has been built up at BHEL, Tiruchirapalli, in collaboration with BARC, Bombay [1]. The technology of MHD power generation is poised for a big leap, and is now a major contender for future power plant schemes.

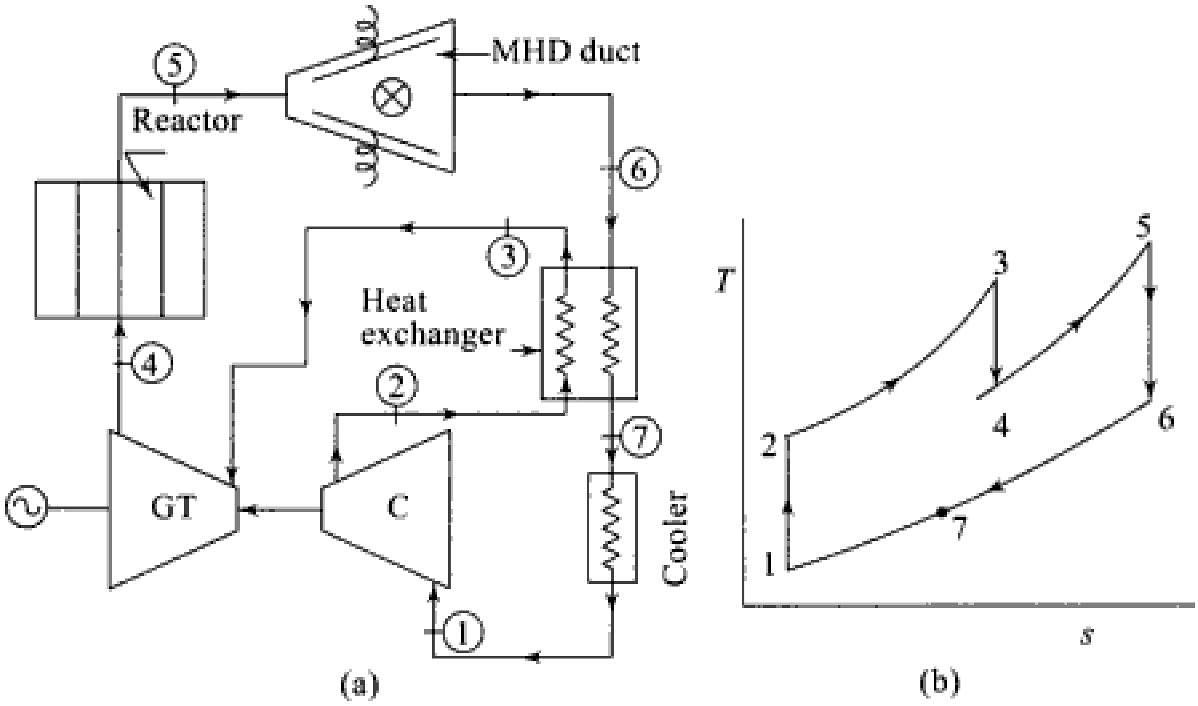


Fig. 13.6 *Closed cycle MHD–gas turbine power plant*

13.2 THERMICONIC POWER GENERATION

A thermionic generator transforms "heat" directly into electrical energy by utilizing thermionic emission. Any metal has free electrons. A metal electrode, which is called the *emitter*, is heated until it is hot enough to release electrons from its surface. The electrons cross a small gap and accumulate on a cooled metal electrode, called the *collector*. To minimize energy losses as electrons cross the gap, the space between the electrodes is either maintained at a high vacuum or filled with a highly conducting plasma like ionized cesium vapour. The electrons enter the collector and return through an external load to the emitter, thereby producing electrical power (Fig. 13.7). The emitter is positively charged, called the cathode, and the collector is negatively charged, called the

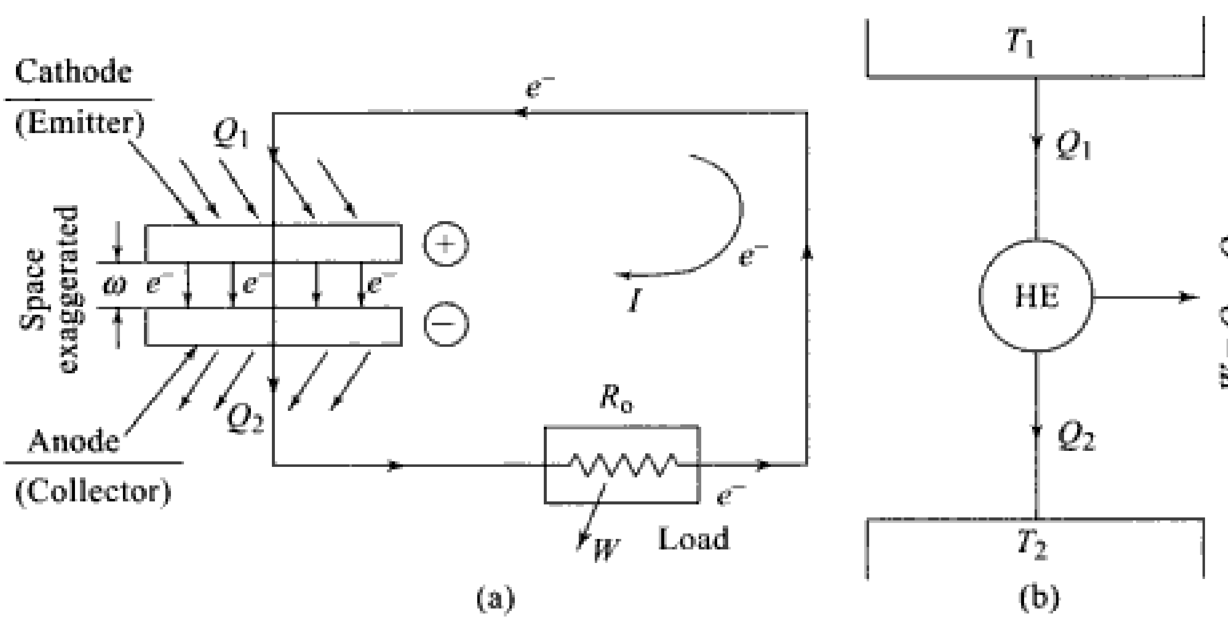


Fig. 13.7 *A basic thermionic generator*

anode. A thermionic generator is thus a cyclic heat engine and its maximum efficiency is limited by Carnot's law. It is essesntially a low-voltage high-current device, where current densities of 20 – 50 amp/cm^2 have been achieved at voltages from 1 to 2 V. Thermal efficiencies of 10–20% have been realized, but higher values are certainly possible in the near future.

The emission of an electron from a metal surface is opposed by a potential barrier equal to the difference between the energies of an electron outside and inside the metal. Thus, a certain amount of energy must be spent to release the electron from the surface, which is referred to as surface work function (ϕ). Based on statistical mechanics, the maximum electron current per unit area that an emitting surface can provide is given by the Richardson-Dushman equation:

$$J = A_1 T^2 \exp\left(-\frac{\phi}{KT}\right) \tag{13.2}$$

where J is the current density (amp/m^2), T is the absolute temperature (K), ϕ is the work function (eV), K is the Boltzmann constant (1.38×10^{-23} J/molecule-K) and A_1 is the emission constant which is equal to 120 amp/(cm^2–K^2). For different materials, the work function varies from 1–5 eV (1 eV = 1.602×10^{-19} J).

The kinetic energy of the free electrons at absolute zero would occupy quantum states, or discrete energy levels from zero up to some maximum value defined by the Fermi energy level, ε_f [24]. Each energy level contains a limited number of free electrons just like an electron orbit containing a limited number of electrons. Above absolute zero temperature, some electrons may have energies higher than the Fermi level. The electrons may be assumed to be vibrating about ε_f with an amplitude of vibration depending on the temperature. The energy that must be supplied to overcome the weak attractive force on the outermost orbital electrons is the work function ϕ, so that the electron leaving the emitter has an energy level $\phi + \varepsilon_f$.

When heat is supplied to the emitter, some of the high-energy free electrons at the Fermi level get the necessary energy—energy equal to emitter work function ϕ_c to escape the emitter surface—move through the gap, strike the collector and give up their K.E. ($\varepsilon_f(a)$) plus the energy equal to collector (anode) work function ϕ_a. This energy is rejected as heat from the low temperature collector.

The electron energy is reduced to the Fermi energy level of the anode, $\varepsilon_f(a)$, but this energy state is higher than that of the electron at the Fermi energy level of the cathode, $\varepsilon_f(c)$, so that the electron can pass through the external load from the anode to the cathode. Cathode materials should thus have low Fermi levels while anode materials must have high Fermi levels. In an electron beam, the average kinetic energy of an electron is given by $2KT$, K being the Boltzmann constant.

The positively charged cathode tends to pull the electrons back and the electrons already in the gas exert a retarding force on the electrons trying to cross. This produces a space charge barrier.

Figure 13.8 shows a thermionic generator with an interspace retarding potential equivalent to δ volts above the anode work function ϕ_a. Due to this potential barrier, $V_c > \phi_c$ and $V_a > \phi_a$, and the current density from cathode to anode and that from anode to cathode (back emission) are given by

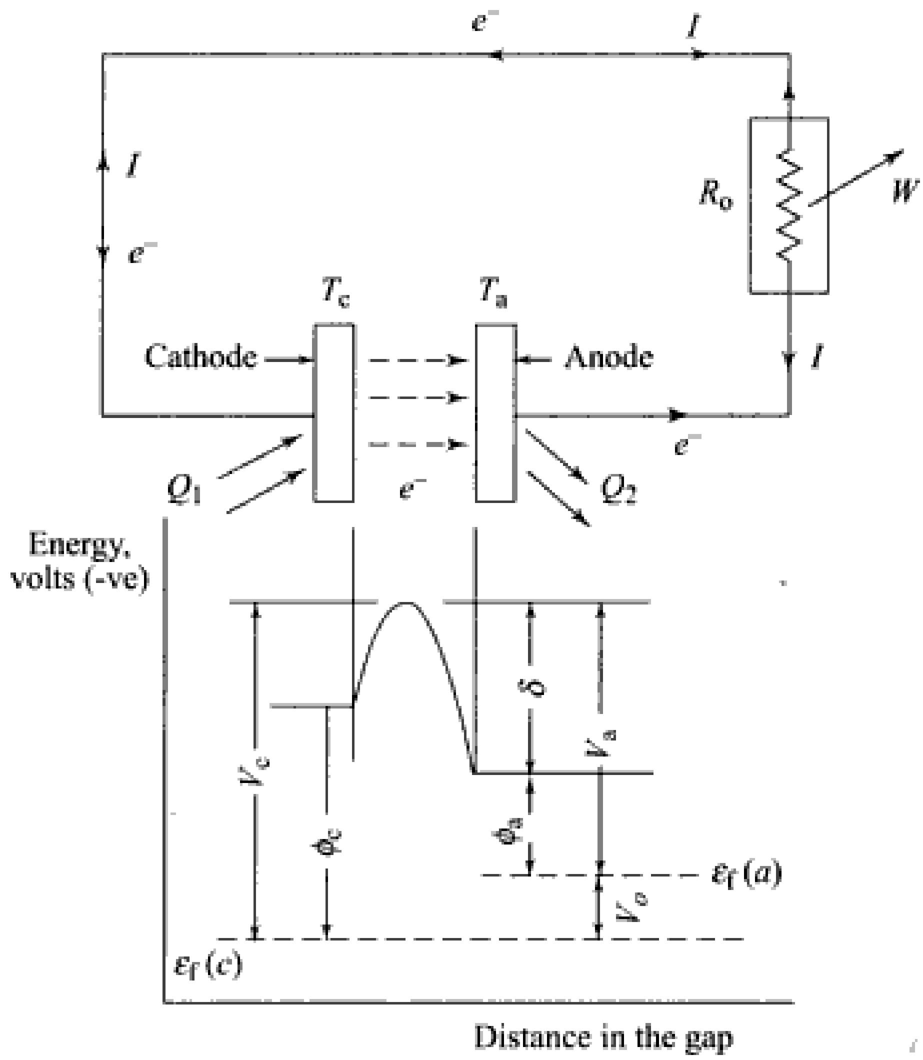


Fig. 13.8 *A thermionic generator with interspace retarding potential*

$$\left.\begin{aligned} J_c &= A_1\, T_C^2 \exp\left(-\frac{V_c}{KT_c}\right) \text{ amp/cm}^2 \\ J_a &= A_1\, T_a^2 \exp\left(-\frac{V_a}{KT_a}\right) \text{ amp/cm}^2 \end{aligned}\right\} \tag{13.3}$$

The output voltage across the electrical resistance (R_o) is

$$V_o = V_c - V_a = \phi_c - \phi_a = \frac{1}{e}\,[\varepsilon_f(a) - \varepsilon_f(c)] \tag{13.4}$$

Each electron must overcome the interspace potential ($V_c - \phi_c$) and the work function ϕ_c as it leaves the cathode. In doing this work, it carries away the net energy

$$Q_{1c} = J_c\,(V_c - \phi_c + \phi_c) = J_c V_c \text{ W/cm}^2$$

Each electron also carries away its own K.E. which is 2 KT_c. This component of energy transfer is

$$Q_{2c} = J_c \frac{2\,KT_c}{e} \text{ W/cm}^2$$

The back emission from the anode must similarly carry energy to the cathode. The net rate of energy supply to the cathode would thus be

$$Q_1 = J_c\left(V_c + \frac{2\,KT_c}{e}\right) - J_a\left(V_a + \frac{2\,KT_a}{e}\right) \tag{13.5}$$

where $e = 1.602 \times 10^{-19}$ coulomb.

The power output from the generator is

$$W = V_o\,(J_c - J_a) \tag{13.6}$$

The thermal efficiency of the thermionic generator would be given by

$$\eta = \frac{V_o(J_c - J_a)}{J_c\left(V_c + \dfrac{2KT_c}{e}\right) - J_a\left(V_a + \dfrac{2KT_a}{e}\right)} \tag{13.7}$$

where $V_o = V_c - V_a$. Substituting

$$\frac{V_c}{KT_c} = \beta_c, \quad \frac{V_a}{KT_a} = \beta_a \quad \text{and} \quad \frac{T_a}{T_c} = \theta,$$

The efficiency can be written as

$$\eta = \frac{(\beta_c KT_c - \beta_a KT_a)(J_c - J_a)}{J_c\left(\beta_c KT_c + \dfrac{2KT_c}{e}\right) - J_a\left(\beta_a\, KT_a + \dfrac{2KT_a}{e}\right)}$$

$$= \frac{(\beta_c - \theta\beta_a)[1 - \theta^2 \exp(\beta_c - \beta_a)]}{(\beta_c + 2) - \theta^2\,(\beta_c + 2\theta)\exp(\beta_c - \beta_a)} \tag{13.8}$$

It is found that for all values of θ, the efficiency curve peaks are very near to the value of β_a equal to β_c [5].

Putting $\beta_c = \beta_a$, Eq. (13.8) becomes

$$\eta_{max} = [1 - \theta]\,\frac{\beta}{\beta + 2}\left[\frac{1 - \theta^2}{1 - \theta^2\,(\beta + 2\theta)/(\beta + 2)}\right] \tag{13.9}$$

It is interesting to note that the final bracketed term is very nearly equal to unity so that

$$\eta_{max} = [1 - \theta)\,\frac{\beta}{\beta + 2} \tag{13.10}$$

Here, $(1 - \theta)$, i.e. $(1 - T_a/T_c)$ is the Carnot cycle efficiency. If $\beta = 18$,

$$\eta_{max} = 0.9(1 - \theta)$$

Again, η_{max} occurs when $\beta_a = \beta_c$, i.e.,

$$\frac{V_c}{T_c} = \frac{V_a}{T_c} \tag{13.11}$$

To reduce the space charge barrier and to promote electron emission from the cathode, ionized cesium vapour is made to fill the gap. To achieve a higher degree of ionization of the cesium, the emitter temperature must be of the range 1500 – 1600 °C.

13.2.1 Thermionic Generator as Topping Unit

Attempts are being made to use thermionic generators for utility applications. The fuel elements of nuclear reactors may be the most suitable high temperature heat sources for thermionic generators (Fig. 13.9). The fuel element 1 containing the fissile material carries the cathode 2 which is surrounded by the anode 3 separated from the cathode by a space 4 filled with ionized cesium vapour. The anode is cooled on the outside by the coolant flowing through the annulus 5. Some of the energy released by nuclear fission is thus directly converted to electricity by thermionic means, and the remainder is converted in a bottoming steam plant, yielding a higher overall plant efficiency.

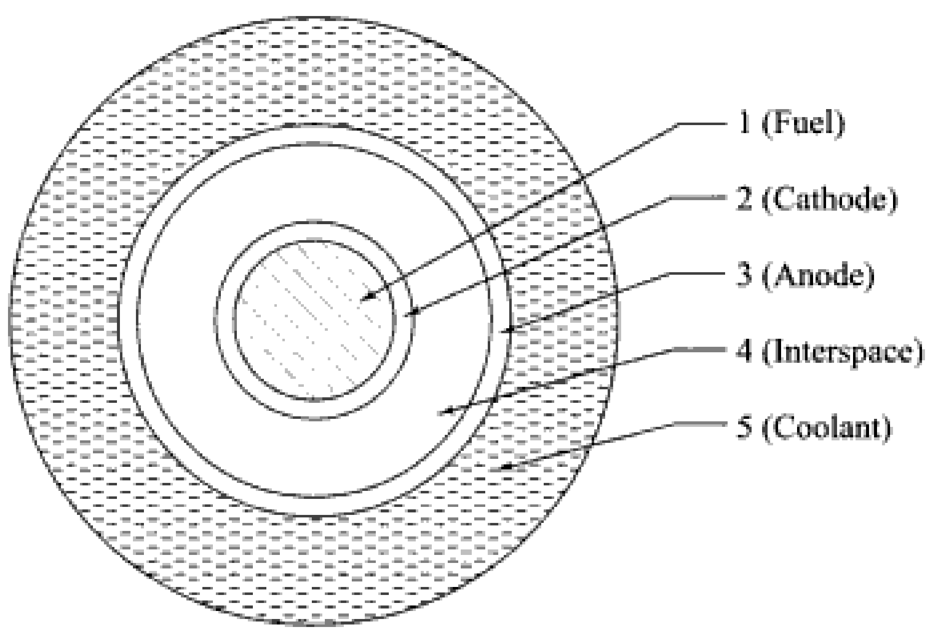


Fig. 13.9 *Schematic of a thermionic generator in a nuclear reactor*

In the steam generator of a fossil-fueled steam power plant, the riser tubes are located in the radiant zone of the furnace. The energy of high temperature combustion gases can be partly converted to electricity if the riser tubes are provided with cathode and anode of a thermionic generator with the interspace filled, as usual, with ionized cesium vapour (Fig. 13.10). The use of the hot combustion gases to produce extra energy before the steam cycle in a topping unit improves the overall plant efficiency.

Another interesting application is in a topping cycle combining an MHD generator with a thermionic generator (Fig. 13.11 and Fig. 13.11a). The waste

High temperature emitters may be made of materials like tungsten or rhenium. Ceramic shields must be provided to protect them from the corrosive combustion gases. Collectors may be made of molybdenum coated with cesium.

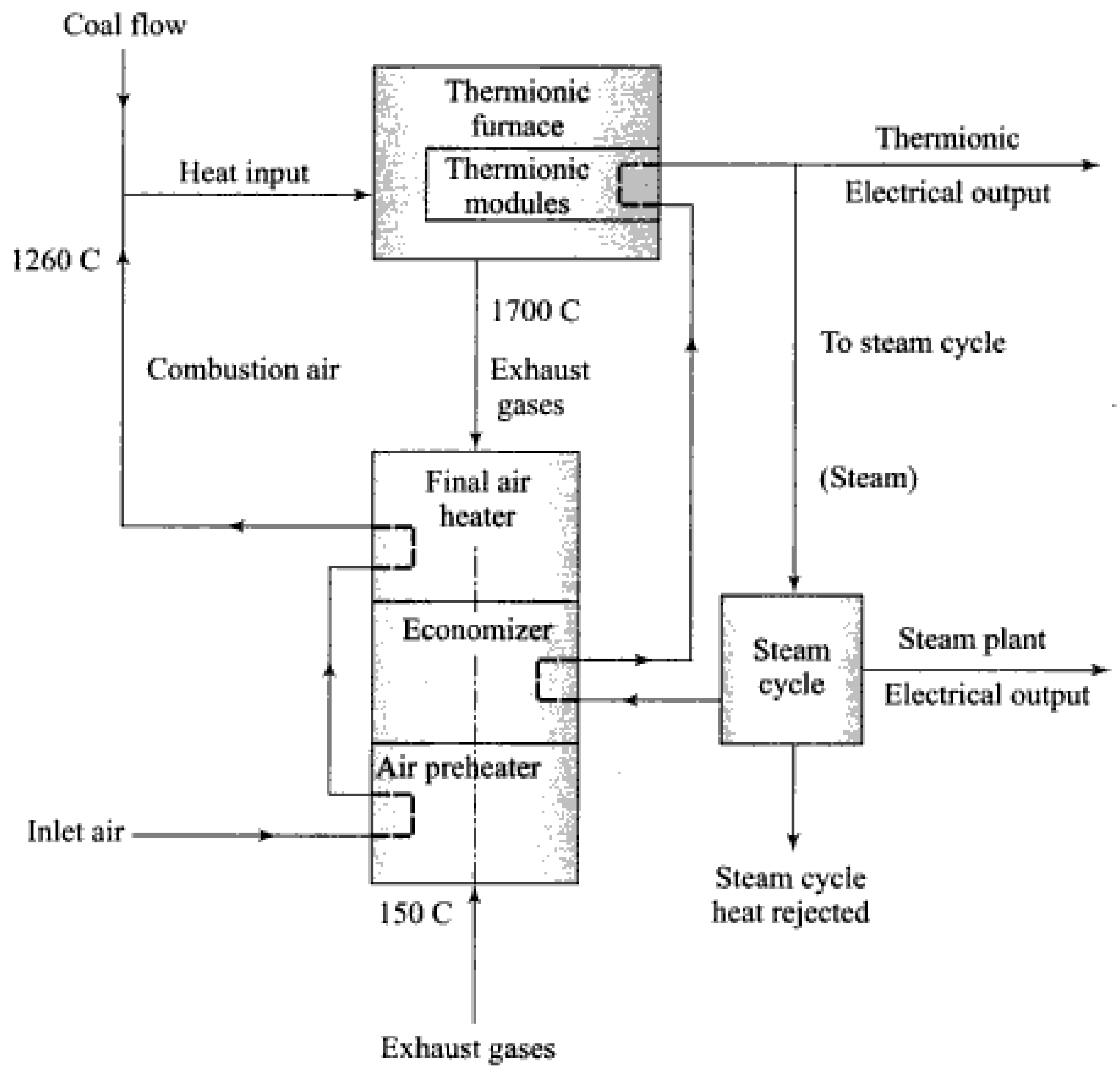


Fig. 13.11a *Block diagram for a thermionic topped steam power plant*

13.3 THERMOELECTRIC POWER GENERATION

When the junctions of two dissimilar wires A and B are maintained at two different temperatures a potential difference is developed (Fig. 13.12). It is called the *Seebeck effect* and it is the basis of temperature measurement by a thermocouple. The Seebeck coefficient or thermoelectric power is defined as

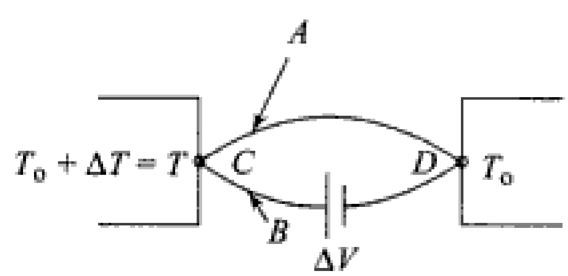


Fig. 13.12 *A thermocouple*

$$\alpha_{A,B} = \lim_{\Delta T \to 0} \frac{\Delta V}{\Delta T} \tag{13.12}$$

Let m be the resistance ratio defined by

$$m = \frac{R_L}{R_p + R_n} \tag{13.23}$$

$$\text{so that } 1 + m = \frac{R_p + R_n + R_L}{R_p + R_n}$$

and from Eq. (13.22),

$$I = \frac{\alpha_{p,n}\,(T_1 - T_o)}{\left(R_p + R_n\right)(1 + m)} \tag{13.24}$$

The useful power is then (from Eq. 13.20),

$$W_L = \frac{\alpha_{p,n}^2\,(T_1 - T_o)^2}{\left(R_p + R_n\right)^2\,(1 + m)^2}\,(R_p + R_n)m = \frac{m}{(1+m)^2}\,\frac{\alpha_{p,n}^2\,(T_1 - T_o)^2}{R_p + R_n} \tag{3.25}$$

and the heat input (from Eq. 13.19) is

$$\dot{Q}_1 = \alpha^2{}_{p,n}\,\frac{T_1\,(T_1 - T_o)}{\left(R_p + R_n\right)(1+m)} - \frac{1}{2}\frac{\alpha_{p,n}^2\,(T_1 - T_o)^2}{(1+m)^2(R_p + R_n)} + (K_p + K_n)(T_1 - T_o) \tag{13.26}$$

The efficiency of the thermoelectric generator is given as

$$\eta = \frac{\dot{W}_L}{\dot{Q}_1} = \frac{\dfrac{m}{(1+m)^2}\cdot\dfrac{\alpha_{p,n}^2}{R_p + R_n}\,(T_1 - T_o)^2}{\alpha_{p,n}^2\,\dfrac{T_1(T_1 - T_o)}{\left(R_p + R_n\right)(1+m)} - \dfrac{1}{2}\dfrac{(T_1 - T_o)^2\,\alpha_{p,n}^2}{(1+m)^2\left(R_p + R_n\right)} + (K_p + K_n)(T_1 - T_o)}$$

$$= \frac{T_1 - T_o}{T_1}\,\frac{m}{(1+m)\dfrac{T_1 - T_o}{T_1} - \dfrac{1}{2}\dfrac{T_1 - T_o}{T_1} + \dfrac{(K_n + K_p)(R_n + R_p)(1+m)^2}{\alpha_{p,n}^2\cdot T_1}} \tag{13.27}$$

Let z, called the figure of merit, be defined by

$$z = \frac{\alpha_{p,n}^2}{(K_n + K_p)(R_n + R_p)} \tag{13.28}$$

It consists of only material properties of the two semiconductors. It can be seen from Eq. (13.27) that as z increases, η will increase. If $R = R_n + R_p$ and $K = K_n + K_p$, for a pair of materials when the product $(R \cdot K)$ is minimum, z (and hence η) is maximum.

Now,

$$R \cdot K = \left[\frac{\rho_p L_p}{A_p} + \frac{\rho_n L_n}{A_n}\right]\left[\frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}\right] \tag{13.29}$$

Let $A_p/L_p = \gamma_p$ and $A_n/L_n = \gamma_n$, so that

$$\mathrm{R}\cdot\mathrm{K} = \left[\frac{\rho_p}{\gamma_p} + \frac{\rho_n}{\gamma_n}\right](k_p\,\gamma_p + k_n\,\gamma_n)$$

$$= k_p\,\rho_p + k_p\rho_n\frac{\gamma_p}{\gamma_n} + k_n\rho_p\frac{\gamma_n}{\gamma_p} + k_n\,\rho_n \tag{13.30}$$

Once the materials have been selected, the variables are γ_n and γ_p, the area-to-length ratios of the legs. To minimise R · K,

$$\frac{\mathrm{d}(\mathrm{R}\cdot\mathrm{K})}{\mathrm{d}(\gamma_n/\gamma_p)} = 0 = k_n\rho_p - k_p\rho_n\left(\frac{\gamma_n}{\gamma_p}\right)^{-2}$$

$$\therefore \qquad \frac{\gamma_n}{\gamma_p} = \left[\frac{k_n\rho_p}{k_p\rho_n}\right]^{1/2} \tag{13.31}$$

On substitution in Eq. (13.30),

$$\mathrm{R}\cdot\mathrm{K}_{\min} = k_p\,\rho_p + k_p\,\rho_n\left[\frac{k_n\rho_p}{k_p\rho_n}\right]^{1/2} + k_n\,\rho_p\left[\frac{k_n\rho_p}{k_p\rho_n}\right]^{1/2} + k_n\,\rho_n$$

$$= \left[\sqrt{(\rho_p\,k_p)} + \sqrt{(\rho_n\,k_n)}\right]^2 \tag{13.32}$$

and the maximum figure of merit is given by

$$z_{\max} = \left[\frac{\alpha_{p,n}}{\sqrt{(\rho_p\,k_p)} + \sqrt{(\rho_n\,k_n)}}\right]^2 \tag{13.33}$$

From Eq. (13.27).

$$\eta = \frac{T_1 - T_o}{T_1}\,\frac{m}{(1+m) - \dfrac{1}{2}\dfrac{T_1 - T_o}{T_1} + \dfrac{(1+m)^2}{z\,T_1}} \tag{13.34}$$

For given values of T_1 and T_o, and with the maximum z for a given pair of materials, η depends on m, and to find the optimum value of m,

$$\frac{\mathrm{d}\eta}{\mathrm{d}m} = 0, \text{ which gives}$$

$$m_{opt} = \left[1 + z\frac{T_1 + T_o}{2}\right]^{1/2} = m_o, \text{ say} \tag{13.35}$$

By rearrangement,

$$m_o + 1 = \frac{z}{m_o - 1}\,\frac{T_1 + T_o}{2}$$

From this, it would seem that the greater the number of stages, the greater the efficiency. However, the maximum stage efficiency is limited by the temperature range over which it works, as given by Eq. (13.36). If each stage is optimized for geometry and resistance ratio, each stage efficiency can be improved. The greater the number of optimized stages the great the efficiency, and hypothetical infinite staging should produce a theoretical maximum value of efficiency over a given temperature range.

Figure 13.15 presents a 3-stage thermoelectric generator with the stages operating at different temperatures and the sizes successively reduced.

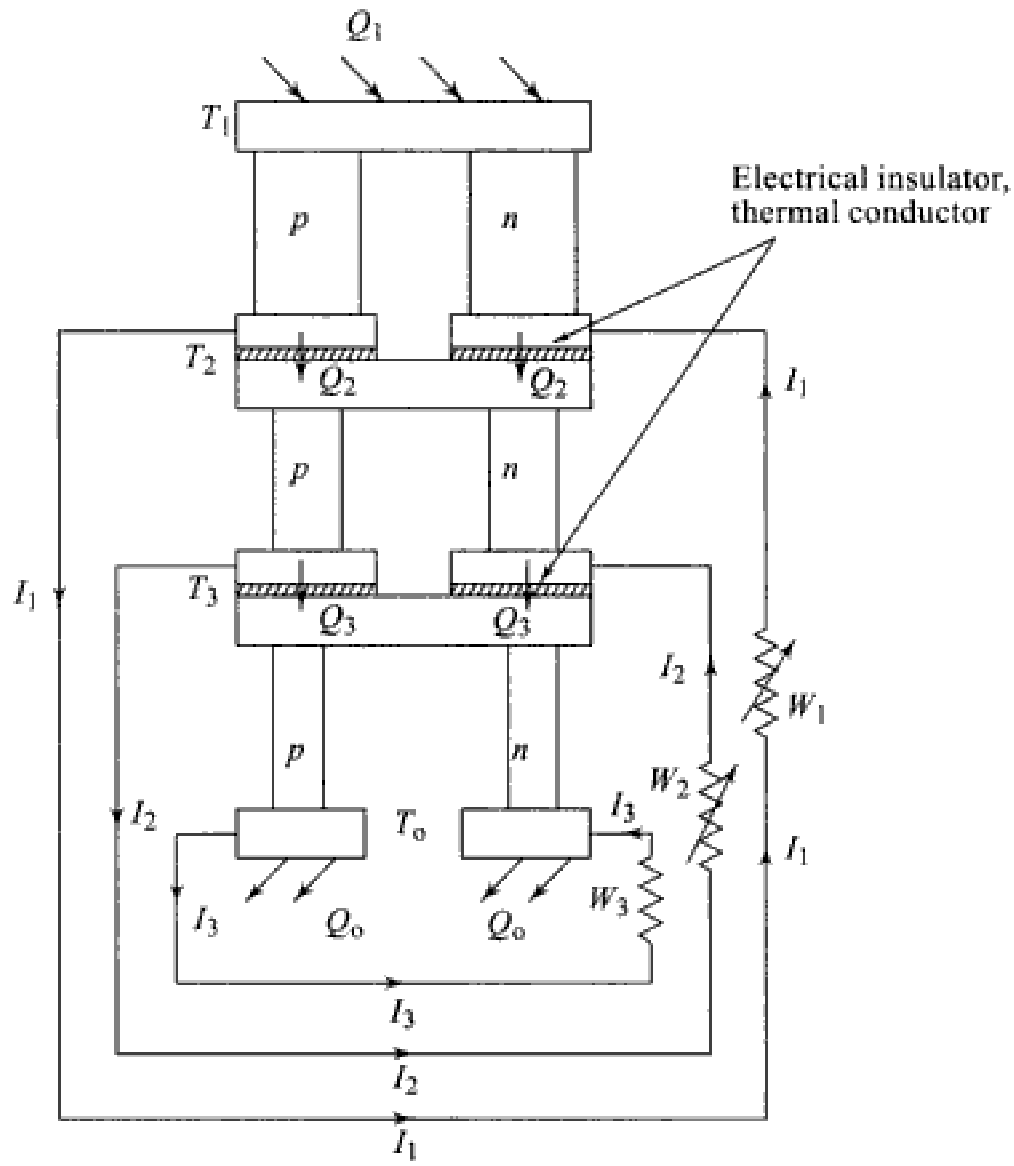


Fig. 13.15 *A simple 3-stage cascade thermoelectric generator*

The output voltage can be increased by putting a number of couples in series (Fig. 13.16). Such an arrangement is called a *thermopile* which can measure an emf signal even for a small temperature difference.

13.3.2 Materials of Thermoelectric Elements

Bismuth telluride, lead telluride, germanium and other semiconductor materials have properties suitable for thermoelectric generation. Alloying and "doping" make it possible to produce *p*-type and. *n*-type materials. In the temperature

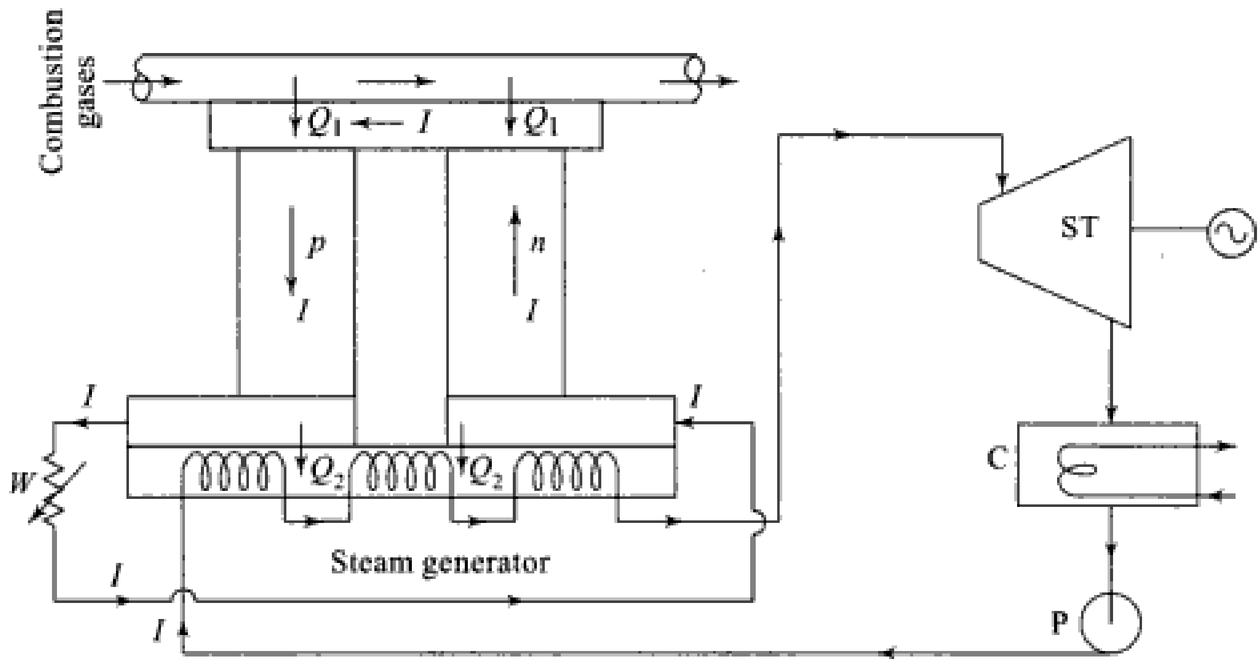


Fig. 13.18 *A schematic of combined thermoelectric-steam power plant*

The waste heat of gas turbines and diesel engines can be utilized for thermoelectric power generation. Even stack gases can be used to produce electricity by thermoelectric means (Fig. 13.19). The metal stack consists of a series of rings of two alternate metals connected alternately at the inner and outer annular edges, and are thermally and electrically insulated. Pairs of a series of thermoelectrodes produce comulative voltage.

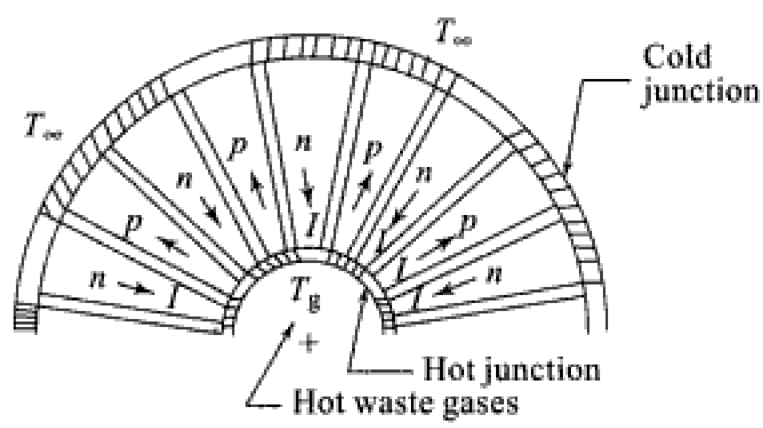


Fig. 13.19 *Thermoelectric waste heat stack*

The decay heat of radioisotopes has been used for the operation of small (0.1 kW) thermoelectric generators.

Notwithstanding their comparatively low thermal efficiency, thermoelectric generators are already recognised as very convenient direct energy conversion systems, due to their simple and compact construction and the absence of moving parts, and have a promising future in utility systems as well as for waste heat recovery.

13.4 FUEL CELLS

The fuel cell converts chemical energy directly into electrical energy in a reaction that eliminates combustion of the fuel. Unlike a heat engine that operates on a thermodynamic power cycle, the performance of the fuel cell is not restricted by the second law of thermodynamics.

Some discussion on fuel cells was undertaken in the previous chapter. The sign convention for the cathode (+) and the anode (–) is the same for batteries and fuel cells, and for thermoelectric, thermionic and MHD generators; negative ions or electrons flow from the cathode to the anode within the device, so that the conventional current flow is from the cathode to the anode in the external circuit. The elemental particles are referred to as charge carriers. The negative charge carriers may consist of electrons or of atoms or molecules with negative charges or electrons. The positive charge carries may consist of atoms or molecules that have lost some of their electrons, or may be an *electron hole* (space left by the departure of an electron).

The charge of an electron (e) is -1.60×10^{-19} coulomb.

To move one electronic charge over a distance with a potential difference of – 1 volt, an energy of 1 eV (electron volt) is required.

$$1 \text{ eV} = -1.602 \times 10^{-19} \times -1 \text{ coulomb-volt}$$
$$= 1.602 \times 10^{-19} \text{ joule}$$

A *dry cell battery* is shown in Fig. 13.20. The chemical equations show the separate reactions at the anode and at the cathode, and also the overall reaction of the whole cell.

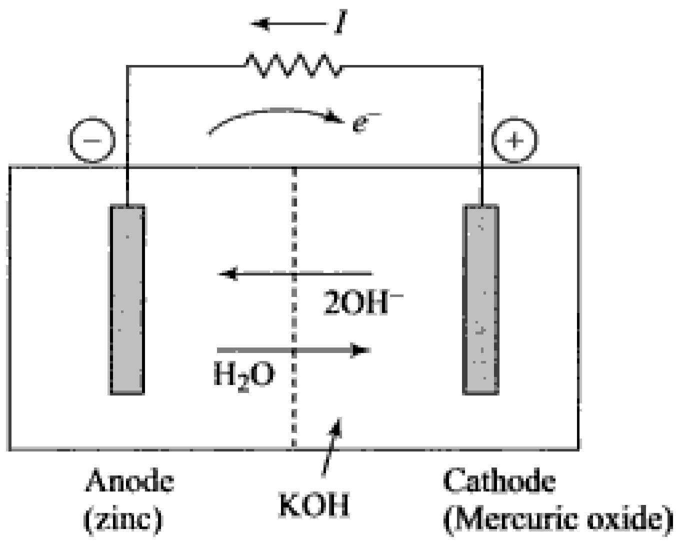


Fig. 13.20 *Typical electric battery*

Anode reaction: Electrons lost to external circuit (oxidation)

$$Zn + 2OH^- \rightarrow Zn(OH)_2 + 2e^-$$
$$\rightarrow ZnO + H_2O + 2e^-$$

Cathode reaction: Electrons gained from the external circuit (reduction)

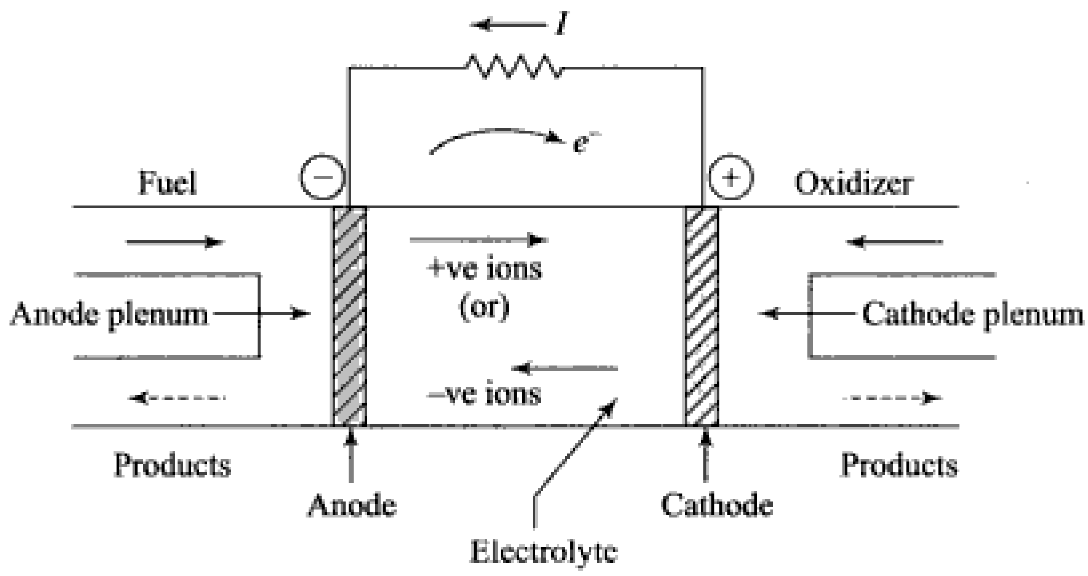
$$HgO + H_2O + 2e^- \rightarrow Hg + 2OH^-$$

Cell reaction: Anode and cathode materials eventually depleted.

$$Zn + HgO \rightarrow ZnO + Hg$$

The anode reaction is essentially oxidation of zinc. This can be imagined as taking place in two steps as shown. The cathode reaction is essentially reduction of the mercuric oxide to mercury. It is typical of batteries that the electrodes and sometimes the electrolyte are chemically changed and exhausted so that eventually the reaction must come to a stop.

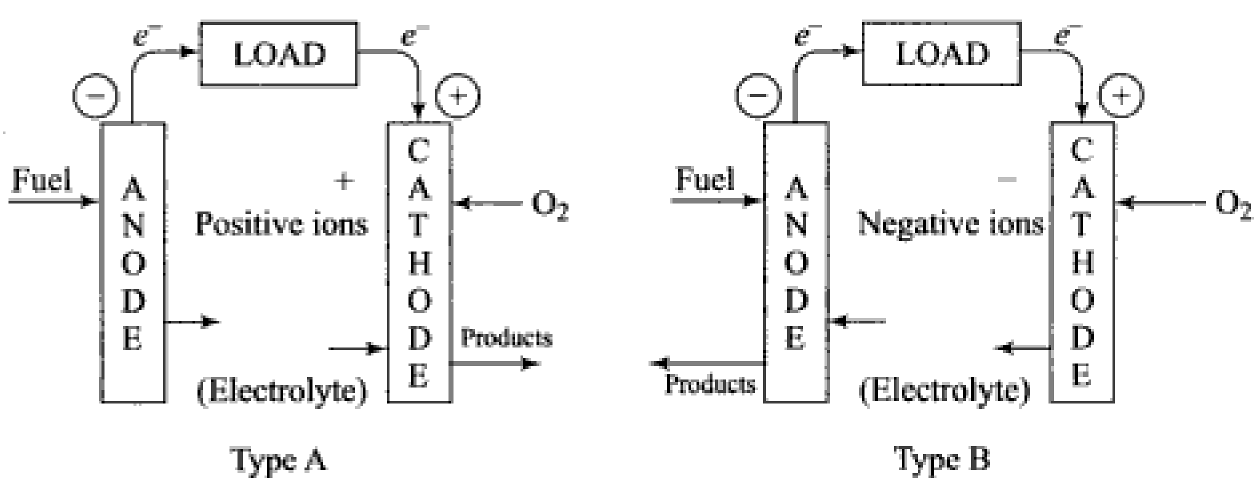
A fuel cell could be considered as an electric battery in which both the fuel and the oxidizer are continuously replaced (Fig. 13.21). The anode and the cathode material do not normally enter into the chemical reactions although they may act as catalysts. The two electrodes must also serve the function of preventing the non-ionized fuel and oxidizer into the electrolyte between the two.

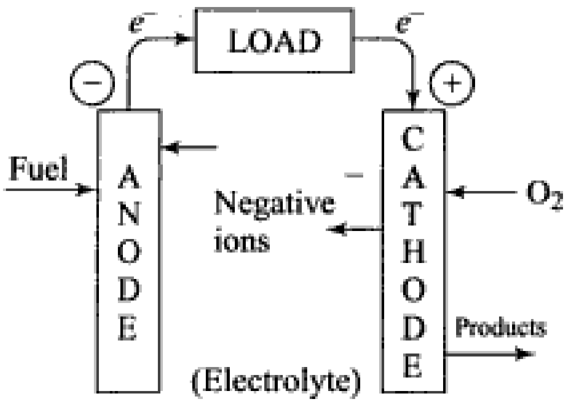


One product exhaust line of the other may not be required

Fig. 13.21 *Fuel cell operation*

Fuel cells might be divided into basic categories according to whether the product of the overall reaction must be disposed of in the cathode plenum space or in the anode plenum space, and whether the current flow through the electrolyte is a transfer of negative ions from the cathode to the anode or a transfer of positive ions in the opposite direction. Three types are illustrated in Fig. 13.22. More complicated fuel cells may actually operate as combinations of these basic types. Fuel cells are also classified according to the temperatures at which they operate.





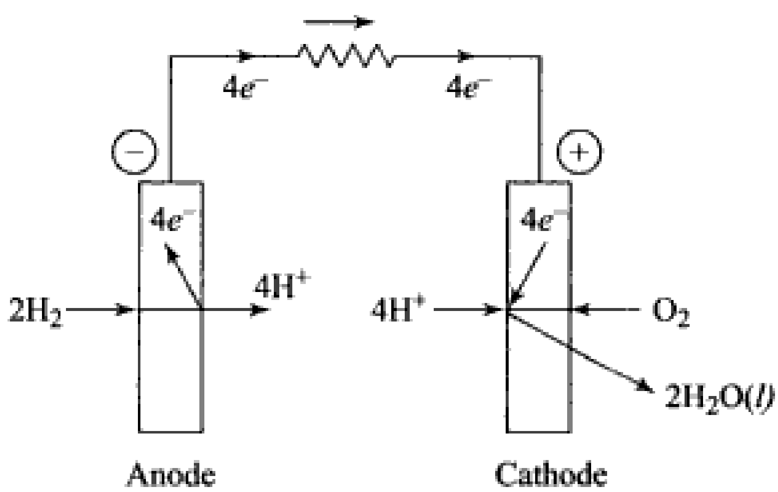
Type C

Fig. 13.22 *Three different types of fuel-cell reactions*

13.4.1 Typical Fuel-Cell Reactions

In a fuel cell, the type of reactions taking place is determined by the fuel and oxidizer combination, by the composition of the electrolyte, and by the materials and the catalytic effect of cathode and the anode surfaces.

In Figs. 13.23, 13.24, 13.25 and 13.26, four different fuel cell reactions are presented. In each figure, the anode, cathode and overall cell reactions are given and the sides from which the products are removed are noted. The first two figures of this group are both hydrogen-oxygen cells. Although the product is water in both examples, the reactions are quite different. In Fig. 13.23, the charge carrier through the electrolyte is a positive hydrogen ion, while in Fig. 13.24, it is a negative hydroxyl ion.



Anode: $2H_2 \rightarrow 4H^+ + 4e^-$

Cathode: $4e^- + O_2 + 4H^+ \rightarrow 2H_2O(l)$

Cell: $2H_2 + O_2 \rightarrow 2H_2O(l)$

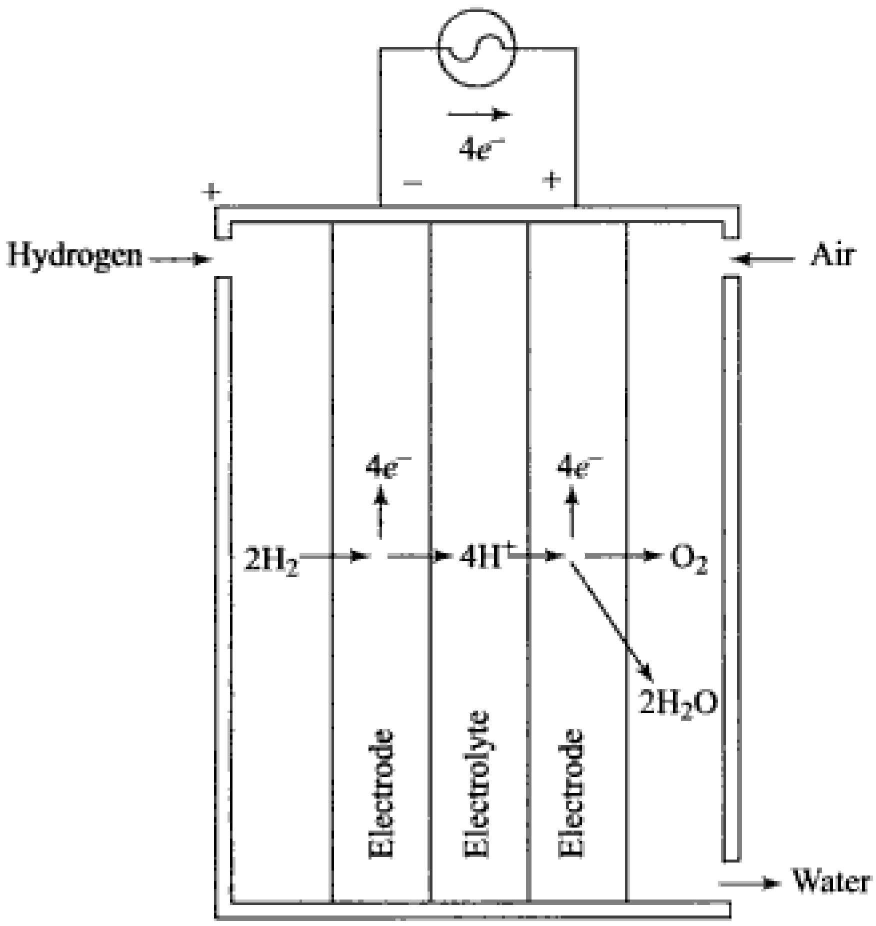


Fig. 13.23 *Hydrogen-oxygen fuel cell, Type A*

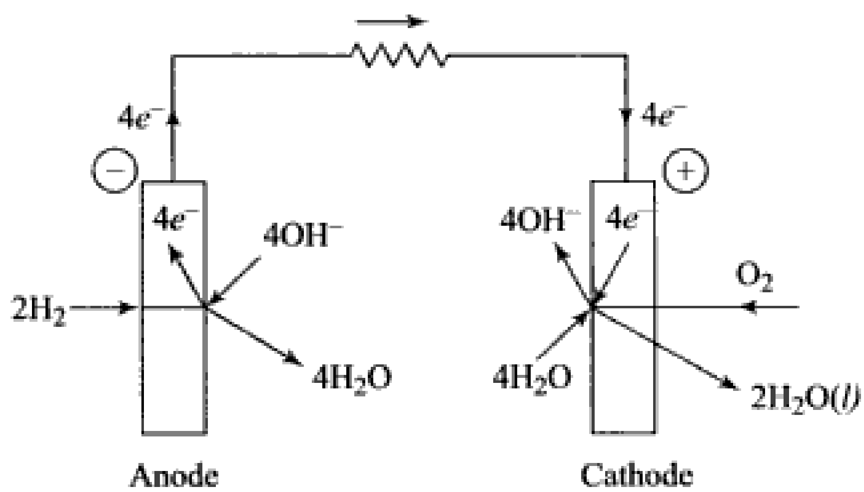


Fig. 13.24 *Hydrogen-oxygen fuel cell, Type C*

Anode: $2H_2 + 4OH^- \rightarrow 4H_2O + 4e^-$

Cathode: $2O_2 + 4H_2O + 4e^- \rightarrow 2HO^-_2 + 2OH^- + 2H_2O \rightarrow 4OH^- + O_2 + 2H_2O(l)$

$\therefore \quad O_2 + 4H_2O + 4e^- \rightarrow 4OH^- + 2H_2O(l)$

Cell: $2H_2 + O_2 \rightarrow 2H_2O(l)$

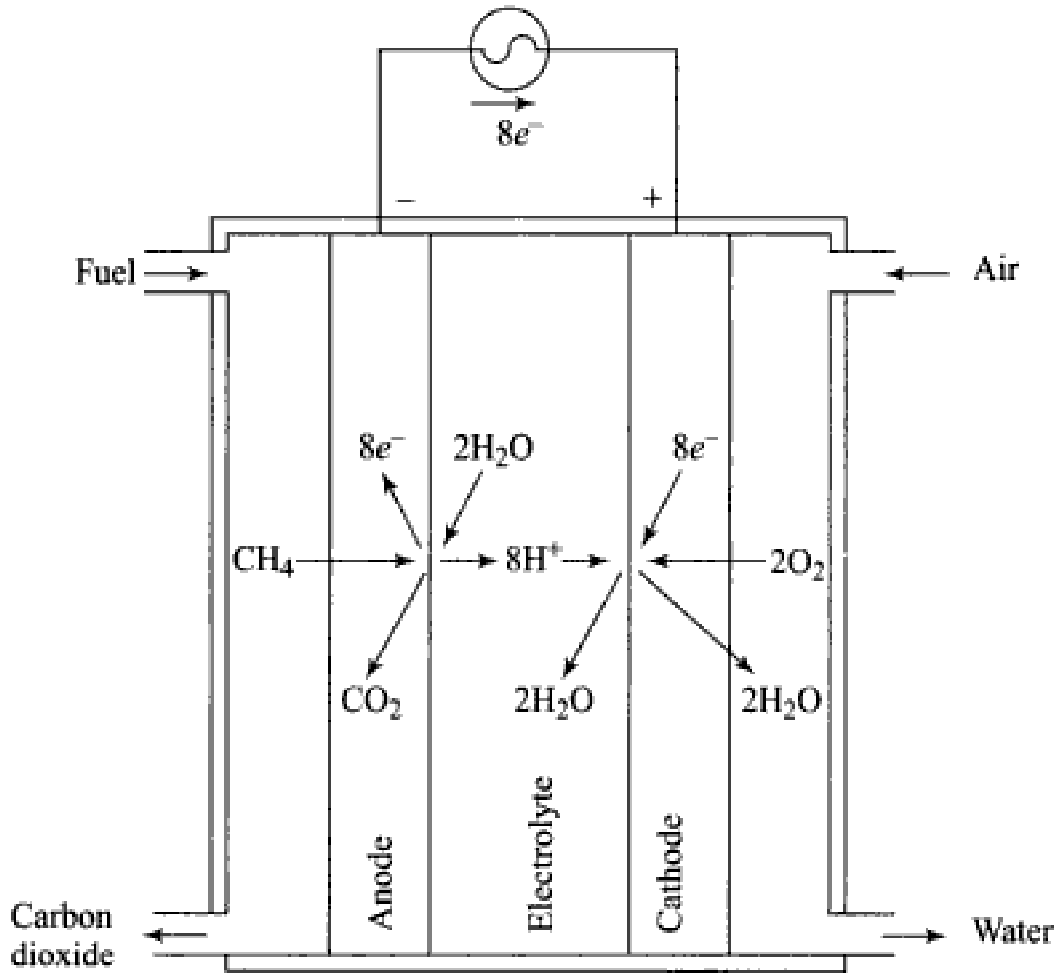


Fig. 13.26 *Schematic representation of the methane-oxygen fuel cell*

13.4.2 Thermodynamics of Fuel Cell Reactions

The fuel cell operates as a steady state, steady flow system (Fig. 13.27) in which fluid enters at state 1 and leaves at state 2. Ideally the system should operate isothermally.

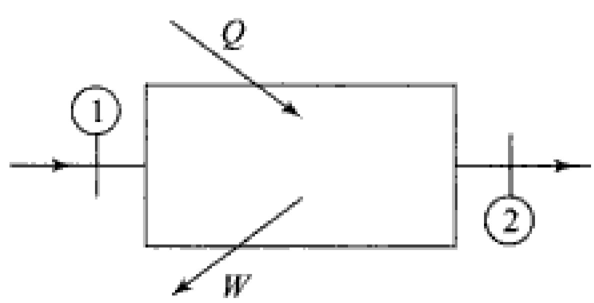


Fig. 13.27 *Ideal fuel cell*

Writing the S.F.E.E.,

$$H_1 + Q = H_2 + W$$

$$\therefore \quad W = H_1 - H_2 + Q \quad ...(13.39)$$

For ideal isothermal heat transfer,

$$Q = T(S_2 - S_1) \quad ...(13.40)$$

In terms of Gibbs function,

$$G_2 - G_1 = H_2 - H_1 - T(S_2 - S_1) \quad ...(13.41)$$

The work for an isothermal reversible system

$$W(\max) = G_1 - G_2 = -\Delta G \quad ...(13.42)$$

Thermal efficiency of an ideal fuel cell

Hydrogen-Oxygen Fuel Cell

$$H_2(g) + \frac{1}{2} O_2(g) \rightarrow H_2O(l)$$

$$\Delta G = \sum_P \Delta G - \sum_R \Delta G, \text{ where } P = \text{products}, \quad R = \text{reactants}$$

$$= -237.3 - 0 = -237.3 \text{ kJ/kg mol}$$

Similarly, $\Delta H = -285.99$ kJ/kg mol

$$\therefore \quad \eta_i = \frac{237.3}{285.99} = 0.83 \text{ or } 83\%$$

$$V_i = \frac{\Delta G}{nj\mathscr{F}} = \frac{-237.3}{1 \times 2 \times 96.529} = -1.23 \text{ V}$$

Carbon-Oxygen Cell

$$\Delta G = -394.57 \text{ kJ/kg mol}, \qquad \Delta H = -393.7 \text{ kJ/kg mol}$$

$$\eta_i \cong 100\%, \; V_i = \frac{394.57}{1 \times 4 \times 96.529} = -1.02 \text{ V}$$

Methane-Oxygen Cell

$$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O\,(l)$$

$$\Delta G = -818.48 \text{ kJ/kg mol}, \Delta H = -890.91 \text{ kJ/kg mol}$$

$$\eta_i = \frac{818.48}{890.91} = 0.92 \text{ or } 92\%$$

$$V_i = \frac{-818.48}{8 \times 96.529} = -1.06 \text{ V}$$

If 1 gmol of CH_4 is consumed in 1 min, estimate the ideal power output.

$$P_i = \dot{n}j\mathscr{F}\, V_i$$

$$= \frac{1 \times 8 \times 96.529 \times 1.06}{60} \frac{\text{gmol}}{s} \times \frac{\text{J}}{\text{V} \times \text{gmol}} \times \text{V}$$

$$= 13.643 \text{ W}$$

13.4.3 Different Fuel Cells

Fuel cells were originally used for manned space missions where the hydrogen and oxygen were stored in their pure form as liquids and the resulting combustion product, namely, water, was then used by astronauts for drinking. However, for terrestrial uses, in addition to utilizing the fuel and oxidant in liquified form, cheaper methods are also utilized. These involve using hydrogen-containing gases or liquids for the fuel at the anode and air-containing oxygen at the cathode. Such cells are not as efficient as using H_2 and O_2 in pure form, but they are much cheaper.

Examples of hydrogen-containing gases and liquids which have been used and can still be used are as follows:

Hydrocarbons: methane CH_4, ethane, C_2H_6, acetylene C_2H_2, ethylene C_2H_4, propylene C_3H_6, propane C_3H_8, methanol CH_3OH, hexane C_6H_{14}, butene C_4H_8, butane C_4H_{10}, pentane C_5H_{12}, benzene C_6H_6, toluene C_7H_8, heptane C_7H_{16}, octane C_8H_{18}, nonane C_9H_{20}, decane $C_{10}H_{22}$.

Nitrogenous : ammonia NH_3, hydrazine N_2H_4

The hydrocarbons are 'cracked' with steam giving rise to CO, CO_2 and H_2. When fuel H_2 is blown through a porous metallic electrode consisting of catalysts such as platinum and noble metals, the hydrogen molecule loses two electrons ($2e^-$) and becomes a doubly charged ion ($2H^+$). This is an oxidation process. Because of electrons accumulating on he surface of the metallic anode and the electrolyte acquiring positively charged ions adjacent to the electrode, a charge separation occurs resulting in a potential difference, positive on the electrolyte side and negative on the anode side, much like the plates of a capacitor. The H^+ ions pass through the electrolyte such as KOH, in which the bond is ionic with K^+ and OH^- ions being present.

At the cathode, electrons returning from the external circuit combine with oxygen and react with water in the KOH solution of the electrolyte to form hydroxyl ions:

$$O_2 + 4e^- + 2H_2O \rightarrow 4OH^-$$

The oxygen suffers a reduction process through combining with the electrons. These enter the electrolyte and maintain the strength of the KOH, transporting the electrons from the cathode to the anode. The H^+ and OH^- ions combine to form H_2O and go into solution.

The electrodes must be good electrical conductors and highly resistant to corrosive environment. They must also be catalytic to perform charge separation, but not take part in any chemical reaction themselves. Because fuel cells work best with platinum and other precious metals, nearly 25% of the cost of the cell is in these electrodes. The electrolyte is the carrier of charges and can be either acidic or alkaline, and be in liquid or solid state. Regeneration in which the product materials can be re-converted to fuel and oxidant reduces costs.

Five types of fuel cells developed so far are as follows:

(a) Phosphoric Acid Fuel Cell (PAFC)

The cell operates at about 200°C, H_2–O_2 cell, high pressure, efficient, 1 MW and above, 13.8 kV, platinum electro-catalyst.

(b) Alkaline Fuel Cell (AFC)

H_2 and O_2 in pure form, KOH electrolyte: electrodes porous Ni substrate with Pt support.

(c) Solid Polymer Electrolyte Fuel Cell (SPEFC)

Operates at temperatures below 100°C, high polymer electrolyte and Pt electro-catalyst.

(d) Molten Carbon Fuel Cell (MCFC)

Operates at high pressure and temperature. Electrolyte consists of molten carbonate of sodium or potassium ($NaCO_3$ or KCO_3). Electrodes are made of Ni for the anode and Ag for the cathode.

(e) High Temperatures Solid Oxide Fuel Cell (HTSOFC)

13.9 What do you understand by (a) surface work function, (b) Fermi energy?
13.10 What is interspace retarding potential?
13.11 What is Richardson-Dushman equation? What is its relevance in thermionic power generation?
13.12 What is the reason of filling the gap with ionized cesium vapour?
13.13 How does a thermonic generator fit as a topping unit in a (a) nuclear reactor, (b) riser tube of a boiler?
13.14 Explain a combined MHD-thermionic-steam power plant.
13.15 What are the suitable materials for the emitters and collectors of a thermionic generator?
13.16 What do you understand by (a) Seebeck effect, (b) thermoelectric power?
13.17 What do you mean by (a) Peltier heat, (b) Peltier coefficient?
13.18 Explain the principle of thermoelectric power generation.
13.19 What do you understand by "figure of merit"? When does its value become the maximum?
13.20 What is the optimum resistance ratio for (a) maximum power, (b) maximum efficiency?
13.21 Explain the cascade multi-stage operation of thermoelectric generators.
13.22 What are the suitable materials for thermoelectric elements?
13.23 How can a thermoelectric generator be incorporated in the fuel elements of nuclear reactors?
13.24 Explain how a thermoelectric generator can be used as the topping unit of a steam power plant.
13.25 Explain how the waste heat of gas turbines and diesel engines can be utilized for thermoelectric power generation.
13.26 Briefly discuss the merits of thermoelectric power generators.
13.27 How does a fuel cell operate? How is it different from a battery?
13.28 Explain the reactions in a dry cell battery.
13.29 What are the three types of full cell reactions? Give the hydrogen-oxygen, carbon-oxygen and methane-oxygen fuel cell reactions.
13.30 Explain the thermodynamics of fuel-cell reactions.
13.31 What is a Faraday? What do you mean by the ideal voltage of a fuel cell?
13.32 Explain the PAFC, AFC, MCFC and HTSOFC.
13.33 Give the applications of fuel cells.
13.34 Explain geothermal heat as an energy source. What are the sites where geothermal power plants have been established?
13.35 Give the five types of geothermal systems in commercial use.
13.36 Enumerate the advantages of hydrogen as a fuel.
13.37 Explain the different methods of producing hydrogen.

PROBLEMS

13.1 The duct of an MHD generator has a constant spacing between electrodes of 0.4 m. Each electrode has an area of 0.5 m^2. Ionized gas with an electrical conductivity of 30 $(\text{ohm-m})^{-1}$ flows through the duct at an average velocity of

800 m/s. A cross magnetic flux of 2.5 Wb/m^2 is applied, and an external resistance (load) of 0.04 ohm is connected across the electrodes. What is the theoretical power output?

13.2 A thermionic generator with a cathode work function of 2.2 V and an anode work function of 1.6 V operates at a cathode temperature of 1400 K. For ideal values, assume zero retarding voltage in the interspace. (a) What should be the approximate anode temperature? (b) What would be the ideal current density? (c) Calculate the ideal power output per cm^2 and the thermal efficiency.

13.3 Design a thermoelectric generator to operate from a heat source of 1000 K and to reject heat at 600 K. The required output is 50 W at 6 V. The properties of the materials to be used are: $\alpha_{p,n} = 0.001$ V/K, $k_p = 0.03$ W/cm-K, $k_n =$ 0.02 W/cm-K, $\rho_p = 0.005$ ohm-cm, $\rho_n = 0.006$ ohm-cm. Assume the thermoelectric elements to be 1 cm in length.

[*Ans.* 0.51 cm^2, 0.685 cm^2]

13.4 Certain elements *A* and *B* have the following properties in the temperature range of interest:

$$\alpha_{p,n} = 0.003 \text{ V/K},$$
$$K_p, K_n = 0.04 \text{ W/K}$$
$$R_p, R_n = 0.025 \text{ ohm}$$

The elements operate between junction temperatures of 1250 K and 750 K. Determine (a) the maximum output and the efficiency at maximum output, (b) input power and terminal voltage at no load, and (c) the input power and current under short-circuit condition.

[*Ans.* (a) 11.25, 12.4%, (b) 40 W, 1.5 V, (c) 130 W, 30 A]

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14

Environmental Degradation and Use of Renewable Energy

Fossil fuels are primarily used to generate electricity, to produce heat in industrial processes as well as to drive engines for transport purposes in land, water and air. The products of combustion of such fuels are CO_2, CO, H_2O, SO_2, NO_2, NO, etc. For many years, CO_2 was regarded as a benign gas as long as it was not inhaled in great amounts, for it is not poisonous. Since CO_2 is released into the atmosphere, photosynthesis in which green plants use CO_2 to produce sugars like glucose, kept levels of CO_2 in balance to sustain the biosphere. Indeed, plants and therefore life, could not survive without CO_2. But gradually, as the decades went by, scientists began to realise that the massive increase of CO_2 in the atmosphere due to industrial activities worldwide and deforestation has been causing a great harm to the world. This CO_2 gas acts like the glass in a greenhouse, trapping the sun's heat and causing the earth's atmosphere to warm up.

Unlike CO_2, carbon monoxide is poisonous. It asphyxiates, or suffocates, people who inhale it. It causes headache, dizziness and confusion. It tricks red blood cell which feed CO to the brain, instead of oxygen, leading to its malfunction.

Nitrogen and oxygen combine during combustion (at high temperature) to form NO, NO_2, N_2O, and so on, termed together as NOX. These pollute the air which when inhaled, turns into acid in the lungs which eats away at the lungs spongy structure causing it to lose the capacity to absorb oxygen from the air. Less oxygen is thus passed into the blood stream as a result of which a person suffers shortness of breath, and lung and bronchial illnesses, leading often to death.

If there is sulphur in fuel, it is emitted in the form of sulphur oxide like SO_2 or SO_3, termed together as SOX. It causes similar health problems as NOX especially those of the respiratory organs. In the atmosphere it may combine with rainwater to form sulphuric acid turning the rainwater acidic. When this rainwater falls on the earth, it scorches the earth making the soil absolutely infertile and destroys aquatic life in rivers and lakes.

Pollutants in the air (acids, smog, dust, etc.) clog the tiny pores in leaves, stopping trees from photosynthetic activity and respiration which helps plants breakdown food to give them energy.

A large source of pollution comes from burning wastes in incinerators. If burning occurs without any effort to collect the dust and debris, these rise high in the air and stay there, becoming part of the dirt contained in smog, eventually falling as part of the dust and grit, along with chemicals released from plastics, packing and other disposable articles. Thus, incinerators, manufacturing chemineys belching out smoke and debris, exhaust pipes of cars and trucks, and stacks of electric power plants are sources of air pollution.

14.1 GREENHOUSE EFFECT

A greenhouse is designed with a transparent roof and side panels, normally made of glass, that allow sunlight to reach the plants kept inside, keeping them warm, so that in the middle of a frigid winter outside, a temperatre or tropical climate keeps the plants lush and green inside. The greenhouse effect for the earth works in the same way. Certain gases in the atmosphere act as a transparent roof, the most abundandant of these 'greenhouse gases' being carbon dioxide. The others are methane, carbon monoxide, hydrocarbons and chlorofluorocarbons (CFCs). These gases are transparent, so that the high-temperature radiation from the sun passes through the atmosphere and reaches the earth, but the low-temperature radiation from the earth is prevented from escaping into the outer space by the greenhouse gases (Fig. 14.1). The energy absorbed and trapped by the CO_2 gas heats up the atmosphere causing *global warming*.

With the increasing energy needs of a growing population and economy, the increase of fossil fuel consumption has caused the atmospheric CO_2 level to rise, which in turn causes the earths' temperature to rise. This has started to melt the ice caps at the North and South Poles, retreating the glaciers and snowlines and causing the ocean levels to rise. It is projected that at this rate of CO_2 release, the oceans will rise 2.0 – 2.5 metres by the end of this century, as a result of which the coastal plains, which are the most fertile, will be severely flooded with the consequent shortage of habitable and agricultural land.

A great deal of CO_2 is dissolved in the sea. If the temperature rises due to increasing CO_2 levels, then the top 70 m of seas will also heat up, releasing some of the dissolved CO_2 which will further add to the greenhouse effect. This will increase the temperature of the seas even more, releasing even more CO_2, and so on. This is called the *catastrophic greenhouse effect*. Also, an increase in the sea's surface temperature would substantially increase the number and severity of hurricanes leading to hundreds of deaths, millions becoming homeless and also great property damage.

The greenhouse effect causes heavy rainfall and consequent flooding in one part of the world, while droughts occur in other parts of the world to balance the water cycle. Both cause hardships to society and enormous economic losses.

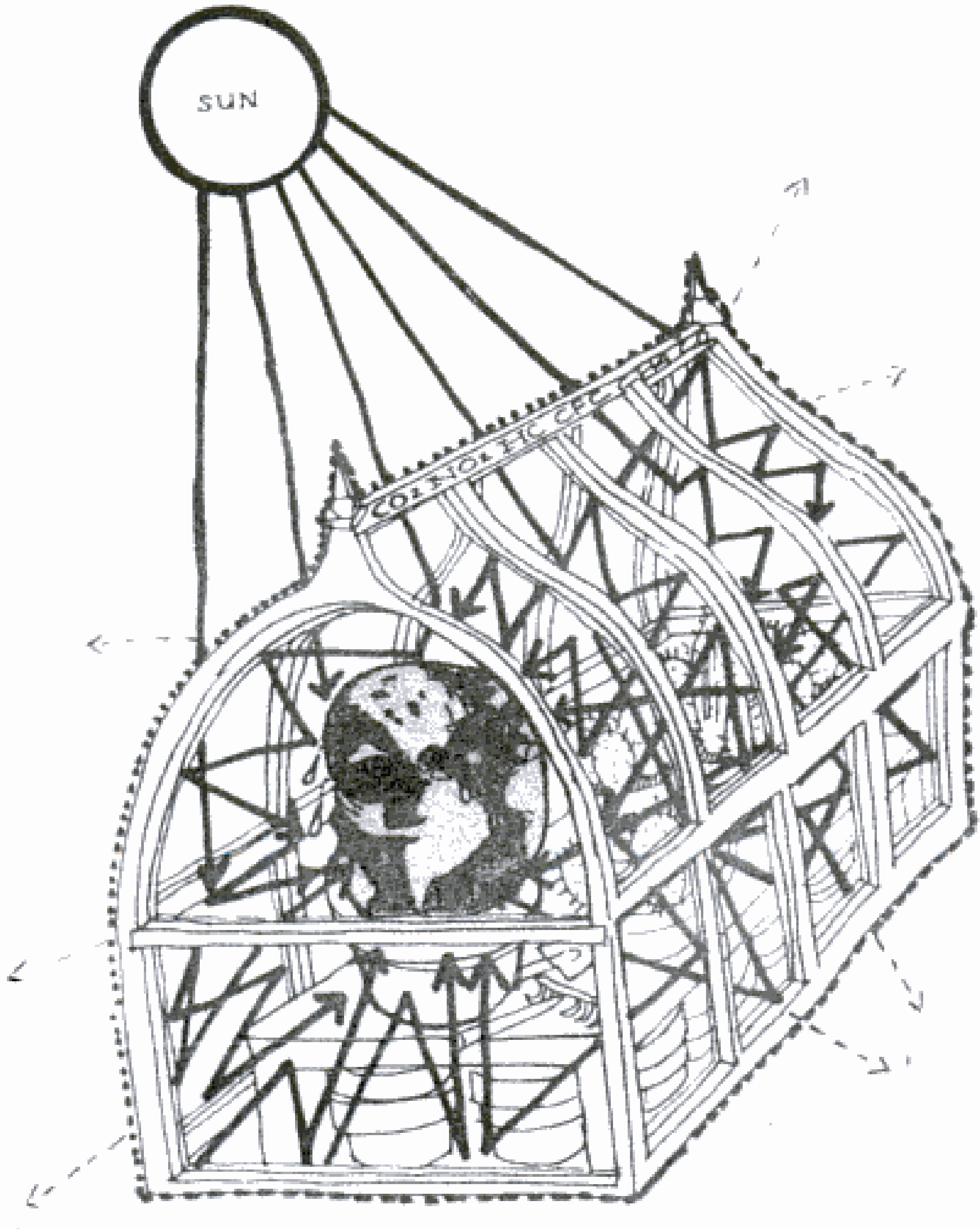


Fig. 14.1 *A greenhouse*

14.2 ACID RAIN

When water vapour in clouds condenses into water droplets, they fall as rain through the atmosphere mixing with polluting gases such as oxides of sulphur, nitrogen and carbon, forming dilute acids-sulphuric acid, nitric acid and carbonic acid. The first two are very strong corrosive acids and are cited as the main culprits causing acid rains. However, the third one, carbonic acid, although much weaker than the first two, could actually be more damaging because it is produced in much greater quantities.

The rain containing these acids falls everywhere—into lakes, rivers and oceans, over forests, fields and farms and on to homes, buildings and structures. Everything which comes into contact with rainwater is subjected to the corrosive effects of the acids, which are harmful to everything, both to living beings and to material objects, and as the acid concentrations increase, the rainwater becomes more destructive and the acid concentrations increase as fossil fuel consumption increases.

As a result of acid rains, the waters in lakes, ponds or estuaries become more and more acidic, affecting greatly the flora and fauna, decimating fish and all aquatic organisms. It causes irrepairable damage to forests and farms, affecting the quality and quantity of farm produce. Acid rain damages the protective coatings of paint and exterior metallic trims on cars, buses and other vehicles. Acidified drinking water can cause various ailments, especially in the kidneys and urinary tracts. Hundreds of species of plants and animals become extinct every year due to the effects of fossil fuel generated pollutants and acid rains.

14.3 SMOG

The basic difference between fog (water vapour condensing on solid particles) and smog in which a complex organic compound, peroxyacyl nitrate (PAN) condenses or gets adsorbed on pieces of particles like flyash or on dust floating in the air. It causes irritation to the eyes, attacks bronchial tubes and even causes death of a person having a respiratory disease, if the smog is thick enough.

14.4 NUCLEAR RADIATION

A properly constructed well-maintained nuclear reactor is harmless. However, nuclear plants are not always well managed as it happened in Three-Mile Island (USA) and Chernobyl (erstwhile USSR). Emitting radiations of high as well as low intensity may be extremely hazardous to human life and animals. So, nuclear power plants may also not be so benign as they are thought out to be. Any kind of failure, either human or otherwise, may cause a great catastrophe.

14.5 SOLAR ENERGY

The radiation continuously showered on earth by the sun represents the most basic and inexhaustible source of energy which is the mother of all forms of energy—conventional or non-conventional, renewable or nonrenewable, the only exception being nuclear energy. Plants use solar energy to effect photosynthesis, converting the carbon derived from atmospheric CO_2 to plant tissues. This gives rise to *plant biomass*. Animals including humans use plant biomass to get food energy. The *animal biomass* results from the animal tissues generated by food consumption. This plant or animal biomass becomes food energy for carnivores and the source of work. Apart from food, the plant biomass is also a major energy source—it is either directly converted into heat by burning or is converted to chemicals like methane, methanol, coke, etc. which in turn become sources of energy.

The heat from the sun causes continuous evaporation of water from the oceans, lakes, rivers, plants and soil. The sun also heats up the air. Due to the differences in the nature of terrains, altitudes and distances from the sun, this heating is not uniform. The air acquires different temperatures at different points horizontally as well as vertically. This leads to winds—slow and fast, providing *wind energy*. The evaporation of water coupled with action of winds drives what we call *hydrological cycle* leading to rains and changes in weather. Water stored at an elevation when allowed to flow by gravity, is a major source of energy—*hydropower* or *hydel energy*. The waves generated in oceans by winds, and the gravitational pull of the sun and moon contain in them *wave energy*. The sun heats up the top layer of oceans (up to the depths to which its light penetrates). The bottom darker layers in oceans remain cooler than the top layers. This difference in temperatures is used to generate *OTEC-Ocean Thermal Energy Conversion.*

Thus all renewable energy forms, with the exception of breeder reactors, are solar in origin.

The nonrenewable sources such as petroleum, coal and lignite, called fossil fuels, are also solar in origin—generated by the action of heat, pressure and time on forests and animals that existed some millions of years ago.

Solar energy is thus very large and inexhaustible: The power from the sun intercepted by the earth is about 1.8×10^{11} MW, which is many thousands of times larger than the present consumption rate of all commercial energy sources. Thus solar energy, in principle, could supply all the present and future energy needs of the world.

In addition to its size, solar energy has two other big advantages. Firstly, unlike fossil fuels and nuclear power, it is an environmentally clean source of energy. Secondly, it is free and available in adequate quantities in almost all parts of the world.

However, there are certain problems associated with its use, such as (1) it is a *dilute source* of energy, hardly exceeding 1 kW/m^2. Thus large collecting areas are required in many applications resulting in excessive costs. (2) Availability of solar energy varies widely with time in the day-night cycle as well as from season to season. Consequently, the energy collected when the sun is shining must be stored for use during periods when it is not available.

Various methods of solar energy utilization are given in Table 14.1. It is seen that the energy from the sun can be used directly and indirectly. The direct means include thermal and photovoltaic conversion, while the indirect means include the use of water power, the winds, biomass and OTEC.

Table 14.1 *Methods of Solar Energy Utilization*

- Solar Energy Utilization
  - Direct Methods
    - Thermal
    - Photovoltaic
  - Indirect Methods
    - Water power
    - Wind
    - Biomass
    - OTEC
    - Wave energy

energy can be stored as sensible heat or latent heat. Sensible heat storage is usually done in an insulated container containing a liquid like water or a porous solid in the form of pebbles or rocks (Fig. 14.6). The first type is preferred with liquid collectors, while the second type is compatible with air heaters. In the case of latent heat storage, heat is stored in a substance when it melts and extracted when the substance freezes.

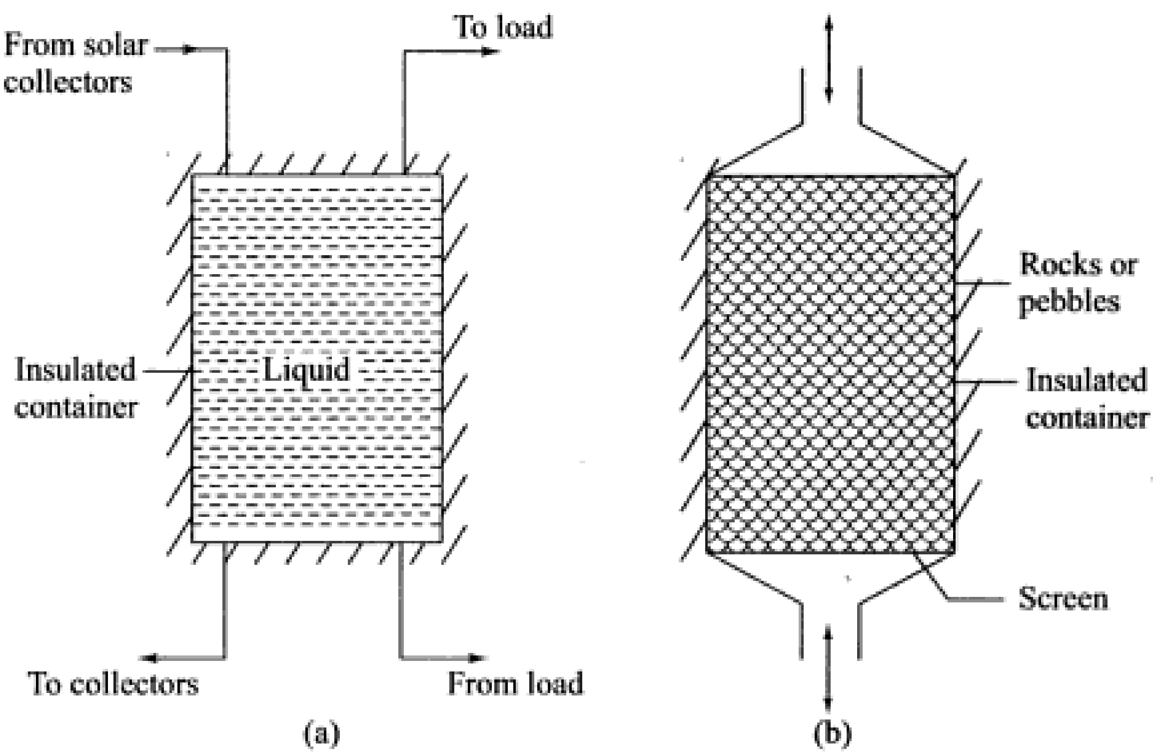


Fig. 14.6 *Schematic of two forms of sensible heat storage: (a) liquid, (b) porous solid*

A novel device which combines the functions of both collection and storage is the *solar pond*. It consists of an expanse of water about a metre or two in depth in which salts like sodium or magnesium chloride are dissolved. The concentration of the salt is more at the bottom and less at the top. Because of this, the bottom layers of water are denser than the surface layers even if they are hotter and thus natural convection does not occur. The heat from the sun's rays absorbed at the bottom of the pond is retained in the lower depths and the upper layers of water act like a thermal insulation.

14.5.2 Solar Thermal Power Generation

Solar thermal power cycles can be broadly classified as low, medium and high temperature cycles. Low temperature cycles generally use flat-plate collectors so that maximum temperatures are limited to about 100°C. Medium temperature cycles work in the range of 150°C to 300°C, while high temperature cycles work at maximum temperatures above 300°C.

(a) Low Temperature Rankine Cycle A typical system working on a low temperature Rankine cycle is shown in Fig. 14.7. The energy of the sun is collected by water flowing through an array of flat-plate collectors. To get still higher temperature, booster mirrors which reflect radiation on to the flat-plate collectors are sometimes used. The hot water at about 100°C is stored in a well-insulated thermal storage tank. From here, it flows through a vapour generator through which the working fluid of the Rankine cycle is also passed. The working fluid has a low boiling point. Consequently, vapour at about 90°C and a pressure of a few atmospheres leaves the vapour generator. This vapour then executes a regular Rankine cycle by flowing through a prime mover, a condenser and a pump. The working fluids normally used are organic fluids like methyl chloride and toluene and refrigerants like *R*-11, *R*-113 and *R*-114. Since the cycle operates through a small temperature difference ($t_{sat.vap} - t_{cond} \simeq$ 55°C), the Rankine efficiency hardly exceeds 7-8%. The efficiency of the collector system is of the order of 25%. Hence an overall efficiency of only about 2% is obtained.

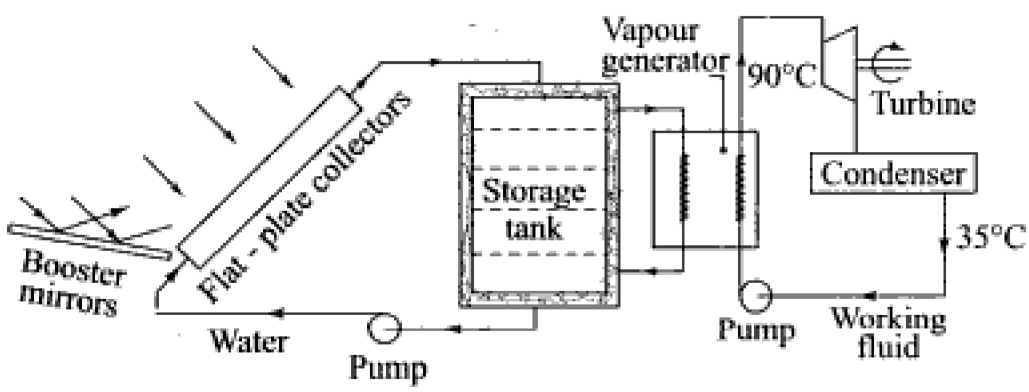


Fig. 14.7 *Low temperature Rankine cycle*

Plants of this type of French design having capacities up to about 50 kW have been installed in many parts of the world. However, such plants have been found to be costly, because of the large collector area required. If it is coupled to solar ponds, it may be cost effective, as done in Israel [1].

(b) Medium Temperature Rankine Cycle Temperatures around 150 to 300°C can be attained by using arrays of cylindrical parabolic focussing collectors. The axes of these collectors could be oriented north-south or east-west and tracking about these axes would be required. The fluid flowing through the absorber tubes is a high-boiling-point liquid. It is stored in tanks and drawn through a heat-exchanger in which it transfers heat to high pressure water which is converted to steam. The high pressure steam executes a Rankine cycle. Fluids other than steam have been considered. Fluorine-based fluids are quite suitable, yielding a cycle efficiency of about 20% and an overall efficiency of 10%.

(c) High Temperature Rankine Cycle For generating temperatures higher than 400°C and for operating Rankine cycles with efficiencies comparable to those obtained in conventional power plants, a concept which is being seriously considered both in the United States and Europe is the *Central Tower Concept* (Fig. 14.8). Solar radiation reflected from heliostats, i.e. arrays of large mirrors, is here concentrated on a boiler situated at the top of a supporting tower. This thermal input to the boiler is used to operate a high-temperature Rankine cycle. At the moment, a few prototypes are either in operation or under construction.

Using solar energy, hydrogen can be produced in four different ways—direct heat, thermochemical, electrolytic and photolytic.

In the direct-heat method, water is heated to form steam and then steam is superheated to about 1400°C or more, at which stage the molecules of steam start splitting apart to form hydrogen and oxygen. As the temperatures are further increased, the rate at which the steam molecules split apart increases. The same effect can also be achieved by reducing the steam pressure. Thus higher temperatures and lower pressures are the best ways to produce hydrogen using the direct-heat method. Large concentrating parabolic mirrors would then be used to focus the solar energy on to containers of water. Such a system is called a *solar furnace*, since it produces very high temperature—steam without pollution.

In the thermochemical method, temperatures as high as 2500-3000°C are not required to split steam. If much cooler steam at 300-1000°C is passed over powdered iron, the iron soaks up the oxygen, forming iron oxide—rust—and leaving hydrogen. The rust can be then heated to make it release the oxygen, leaving us with unrusted, powdered iron again. By doing this over and over again, with large amounts of powdered iron, we could obtain a supply of hydrogen gas.

In the electrolytic method, cells similar to the cells in a car battery are used to produce hydrogen and oxygen from water. Each cell consists of two electrodes immersed in an electrolyte of water plus some chemicals that conduct electricity well, and is connected to a direct current (d.c.) electricity supply. When enough electricity is supplied between the electrodes to cause a current to flow, oxygen is produced at one end (the anode) and hydrogen at the other (the cathode). Instead of splitting steam with heat, we are splitting water with electricity.

In the photolytic method, the sun's energy is directly used to split water into hydrogen and oxygen. Water 'soaks up' minute light particles in the sun's rays, called photons. When water has absorbed enough photons, it splits into hydrogen and oxygen. This phenomenon is called *photolysis*. The photons in the ultraviolet portion of sunlight have the higher energy needed for the direct photolysis of water. However, most of the ultraviolet radiation is absorbed in the upper atmosphere by the ozone layer. Consequently, not much UV radiation reaches the earth.

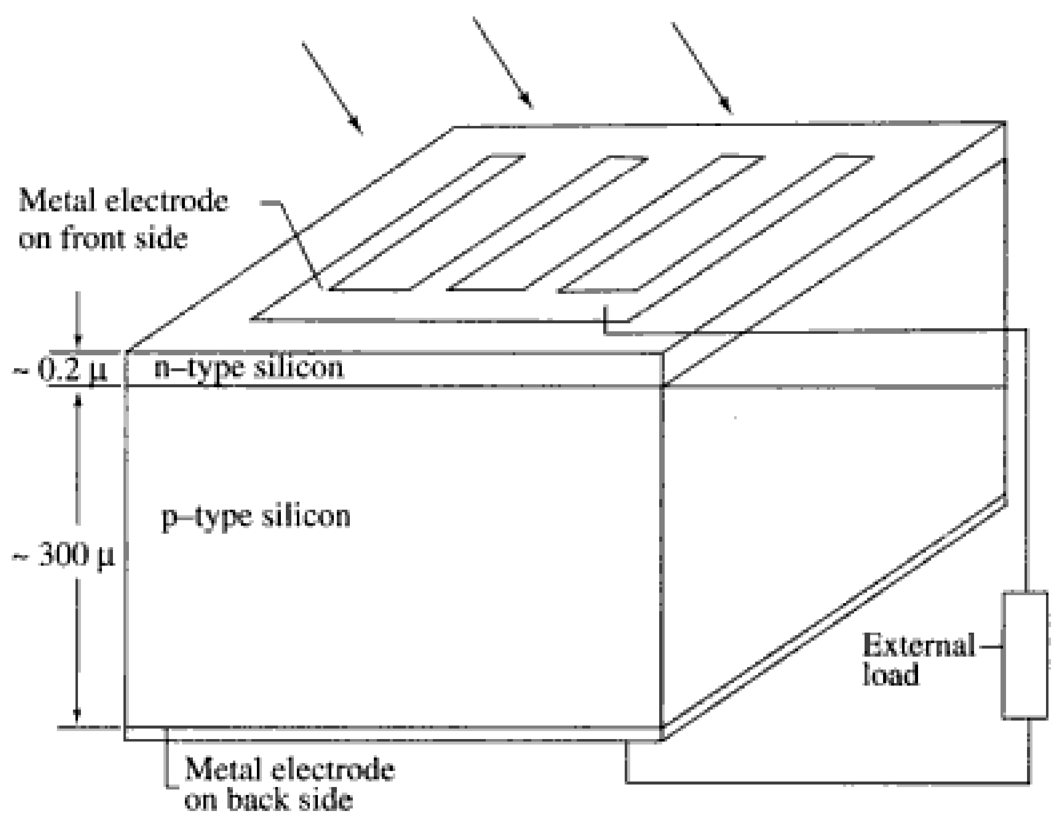
For large-scale storage of solar hydrogen, unused caverns resulting from mining, etc., can be used (see Chapter 12). On a small-scale, hydrogen can be transported and distributed as a gas in a tanker truck, but for industries requiring lots of energy, pipelines are the most economical way to transport and distribute large quantities of hydrogen.

Hydrogen can be converted to electricity using three different systems—gas turbines, steam turbines and fuel cells (Fig. 14.11). Gas turbines can be run on hydrogen gas instead of natural gas, generating mechanical or electrical energy clearly and efficiently. Steam could also be produced cleanly by burning hydrogen in pure oxygen, which can be operated in a conventional Rankine cycle. There is also a third method of converting hydrogen into electricity that is

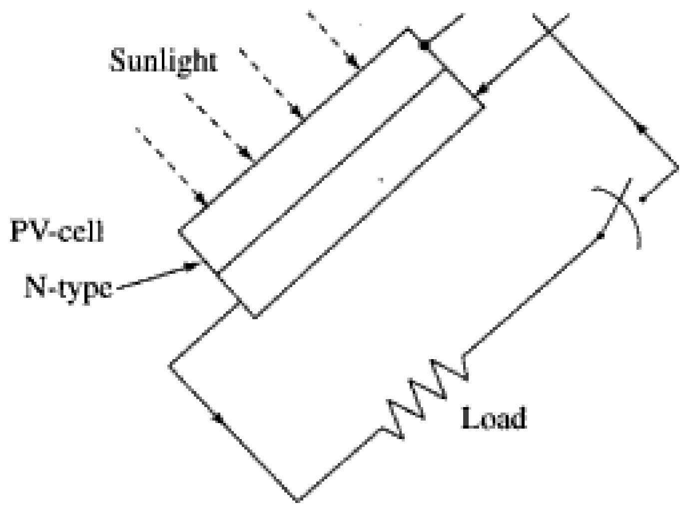
Hydrogen is the best fuel for fuel cells. If this hydrogen is obtained from a fossil fuel or an alcohol fuel like methanol, then we will still have the pollutants that cause the greenhouse effect, the acid rains and the smog. But when hydrogen is obtained from water, we don't get those pollutants. One day, every house, factory, shop, office block, etc., could be powered by its own fuel cell, but we must ensure that the hydrogen used to fuel the cell is obtained from the safest source—water.

14.5.4 Photovoltaic Conversion

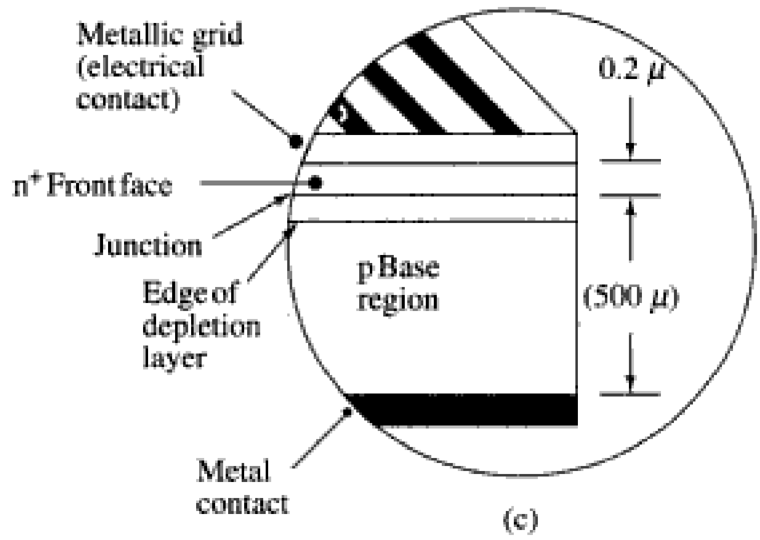
In photovoltaic conversion, solar radiation falls on semi-conductor devices called solar cells which convert the sunlight directly into electricity. A schematic diagram of a photovoltaic cell (PV-cell) or solar cell is given in Fig. 14.12. It relies on the effect that light has on the junction between two types of semiconductors called *p*-type and *n*-type. *N*-type has an excess of electrons and *p*-type has a shortage of electrons. When a bright light shines on a cell, energy from the light (photons) enables electrons to break free from the junction between them. This is called the *photoelectric effect*. For single-crystal silicon (4 valence electrons), '*p*' is obtained by doping silicon with boron (3 valence electrons) and is typically 1 μm thick; '*n*' is obtained by doping with arsenic or phosphorous (5 valence electrons) and is typically 800 μm thick. The sun's photons strike the cell on the microthin *p*-side and penetrate to the junction to generate electron-hole pairs. When the cell is connected to a load, as shown, the electrons will diffuse from *n* to *p*. The direction of the current *I* is in the opposite



(a)



(b)



(c)

Base region : Resistivity (p) = 1 Ω cm
Mean diffusion length (L) = 200 μ

Fig. 14.12 *Diagrams of (a) a silicon cell, (b) another view of a PV cell (c) a solar cell*

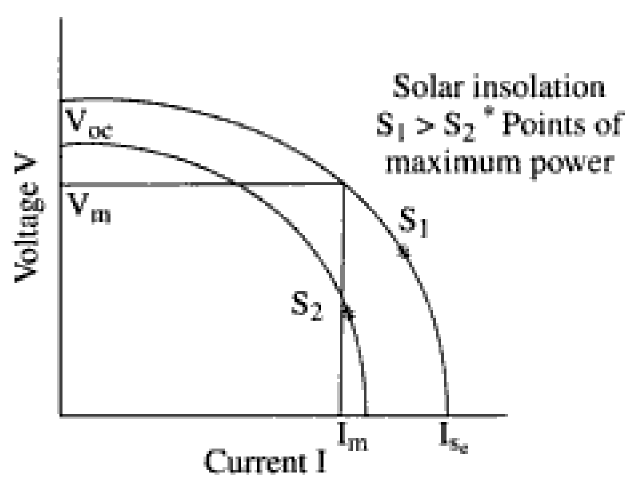


Fig. 14.13 *Typical performance characteristics of a silicon solar cell at two solar radiations*

direction of the electrons. Typical voltage current characteristics are shown in Fig. 14.13 at two different solar radiation levels, for each of which V_{OC} = open-circuit voltage, I_{SC} = short-circuit current. The ideal power of the cell is $V_{OC} \cdot I_{SC}$. The maximum useful power is the area of the largest rectangle that can be formed under the *I-V* curve. If the voltage and current corresponding to this situation are denoted by V_m and I_m then the maximum useful power is V_m I_m. The ratio of the maximum useful power to the ideal power is called the *fill factor* (k). Typical values of these factors for a silicon cell are:

$$V_{OC} = 450 \text{ to } 400 \text{ mV}, I_{OC} = 30 \text{ to } 50 \text{ mA/cm}^2, K = 0.65 \text{ to } 0.80.$$

Solar cells in the form of thin films or wafers convert from 3% to less than 30% of incident solar energy into d.c. electricity. Connection of such cells into series-parallel configurations permits the design of solar 'panels' with high voltages as high as several kilovolts. Combined with energy-storage and power-conditioning equipment, these cells can be used as an integral part of a complete solar-electric conversion system.

The remarkable simplicity of a solar-voltaic system would make it appear a highly desirable energy system for terrestrial purposes, apart from its use in space applications. The principal advantages are that they have no moving parts, require little maintenance, work quite satisfactorily with beam or diffuse radiation and can work as modular systems ranging from a few watts to megawatts. However, the extremely high costs of development and fabrication of solar arrays have discouraged widespread use of such cells.

Monocrystalline silicon is costly to produce since it cannot be made in a continuous automated fabrication process. Considerable work is in progress to develop continuous production processes for cell manufacture. One of the most interesting applications for the large scale use of photovoltaic cells has been suggested by Glaser [4]. He had proposed a concept of satellite power station in which a satellite would be placed in a geosynchronous orbit far above the earth (Fig. 14.14). The satellite would consists of a large array of solar cells, many square kilometres in area. The d.c. power generated by these cells, when solar radiation falls on them, would be fed to microwave oscillators and large antennas would beam the output of the oscillators towards the earth. Antennas on the earth would receive the microwave beam energy, which would then be converted back to d.c. power. The array is space would not be subject to any day-night cycles and it would receive a much higher intensity of solar radiation than an earth, which is claimed to be a big advantage.

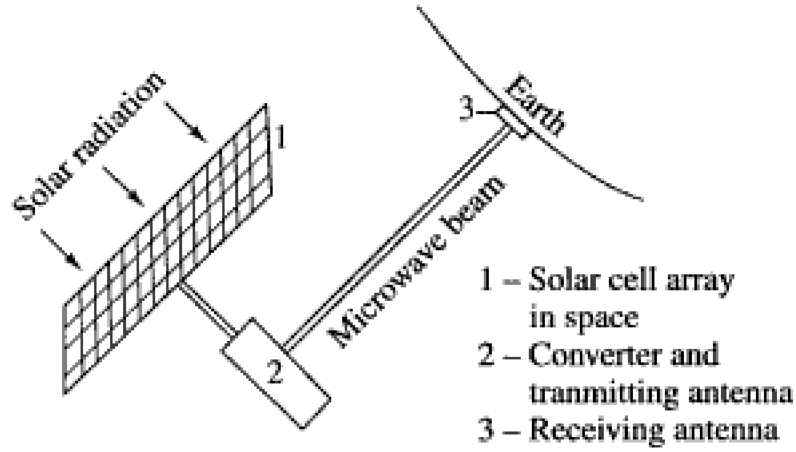


Fig. 14.14 *Satellite solar power station concept*

14.6 WIND ENERGY

Wind is induced chiefly by the uneven heating of the earth's crust by the sun. Thus wind energy is rightly an indirect form of solar energy. Winds can be classified as *planetary* and *local*. Planetary winds are caused by greater solar heating of the earth's surface near the equator than near the north or south poles. This causes warm tropical air to rise and flow through the upper atmosphere towards the poles and cold air from the poles to flow back to the equator nearer to the earth's surface. The direction of motion of the planetary winds is affected by the rotation of the earth. Local winds are caused by differential heating of land and water, and also by hills and mountain sides. Windmills played an important role in water pumping throughout the world. Recent development of wind energy has concentrated on the generation of electricity.

The function of a windmill is to extract energy from the wind and to produce mechanical energy which may then be converted to electricity. Many types of windmills have been designed and developed. However, only a few have been found to be practically suitable and useful. Some of these are: (i) Multiblade type, (ii) Sail type, (iii) Propeller type, (iv) Savonius type, and (v) Darrieus type. The first three have a horizontal axis, while the last two have a vertical axis (Fig. 14.15). A sketch of a historic four-blade Dutch windmill is also shown in the same figure. Both the multiblade and sail-type mills run at low speeds of 60 to 80 rpm. The propeller type has two or three aerofoil blades and run at speeds

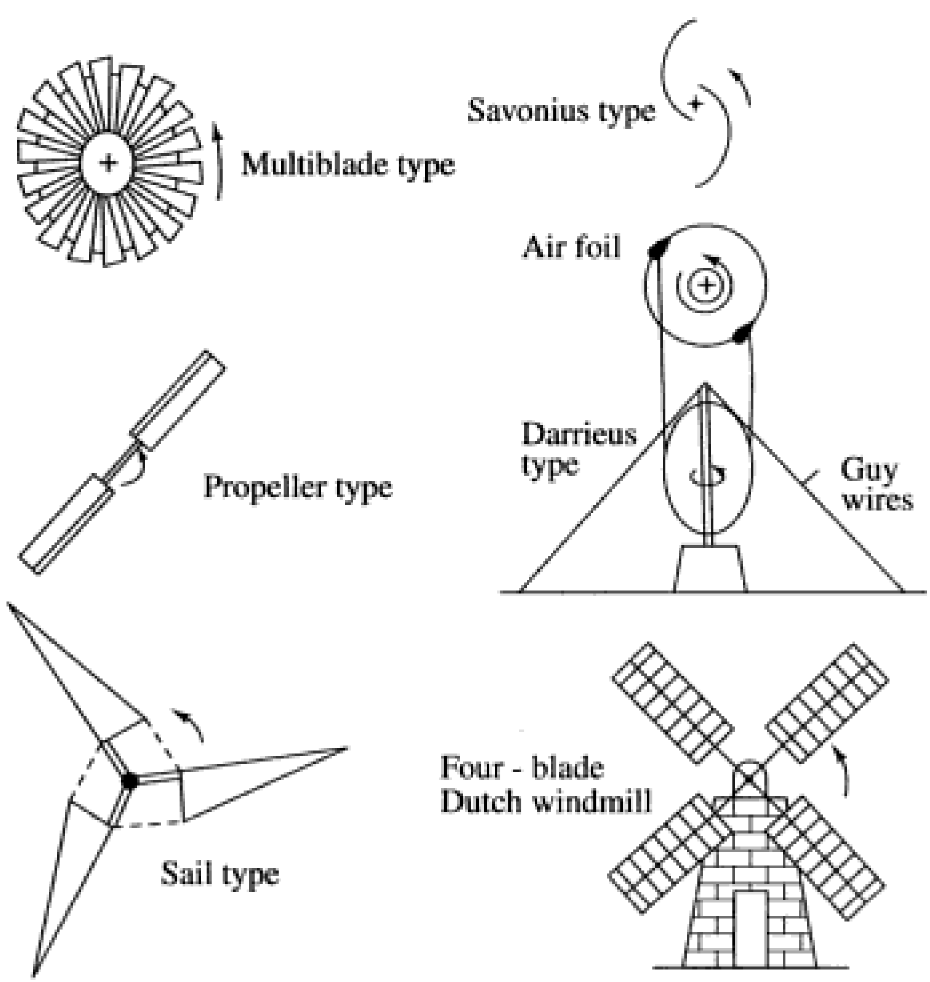


Fig. 14.15 *Types of windmill rotors*

and Savonius types are much lower than the values for the propeller and the Darrius types. It is also seen that the highest values of C_p are obtained with the propeller type. The variation of C_p for an ideal rotor without having any losses is also shown. It is assumed that only the wind blowing in the area A_i is blown over the turbine and it is streamlined (Fig. 14.16a). Some thermal augmenters are often used to increase the pressure difference between the inlet and outlet sides of turbine rotor, e.g., air on the downstream side is heated by burning waste.

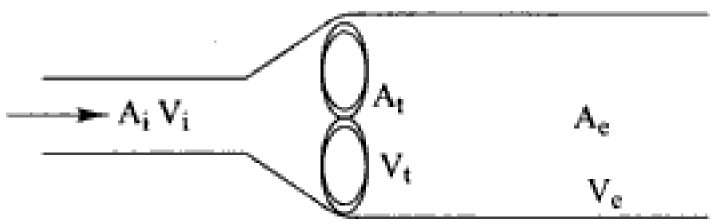


Fig. 14.16 (a) *Theory of wind energy conversion*

The following properties hold:

1. Mass of air flowing past an area A_i per second

$$\dot{m}_i = \rho A_i V_i = \rho A_t V_t \text{ kg/s}$$

2. Kinetic energy per second = power available in wind $P_i = \frac{1}{2} \dot{m}_i \ V_i^2 = \frac{1}{2} \rho A_i V_i^3$, W

3. Power available in the wind at exit

$$P_e = \frac{1}{2} \rho A_e V_e^3 = \frac{1}{2} \rho A_t V_t V_e^2$$

4. Power developed by an ideal rotor or turbine

$$P_t = P_i - P_e = \frac{1}{2} \rho A_t V_t (V_i^2 - V_e^2)$$

5. We can eliminate V_e by assuming

$$a = \frac{V_t}{V_i}$$

and $$V_t = \frac{1}{2} (V_e + V_i) \text{ giving } V_e = 2V_t - V_i$$

$$\therefore \quad V_i^2 - V_e^2 = (V_i + V_e)(V_i - V_e) = 2V_t (V_i - 2V_t + V_i)$$

$$= 2V_t \cdot 2 (V_i - V_t) = 4 V_t V_i \left(1 - \frac{V_t}{V_i}\right)$$

$$= 4 \frac{V_t V_i}{V_i^2} \left(1 - \frac{V_t}{V_i}\right) V_i^2 = 4a (1 - a) V_i^2$$

6. $\therefore P_t = \frac{1}{2} \rho A_t V_t \cdot 4a (1 - a) V_i^2$

A square float moves up and down with the water, guided by four vertical manifolds that are part of a platform (Fig. 14.18). The platform is stabilized within water by four large underwater floatation tanks so that it is supported by buoyancy forces and no significant vertical or horizontal displacement of the platform due to wave motion occurs. A piston attached to the float moves up and down inside a cylinder. The piston-cylinder arrangement is used as a reciprocating compressor. The downward motion of the piston draws air into the cylinder via an inlet check valve. This air is compressed by upward motion of the piston and is supplied to the four underwater floatation tanks through an outlet check valve via the four manifolds. The four floatation tanks thus serve the dual purpose of buoyancy and air storage, and the four vertical manifolds serve the dual purpose of manifolds and float guides. An air turbine is run by the compressed air which is stored in the buoyancy-storage tanks, which in turn drives an electrical generator, producing electricity which is then transmitted to the shore through an underwater cable.

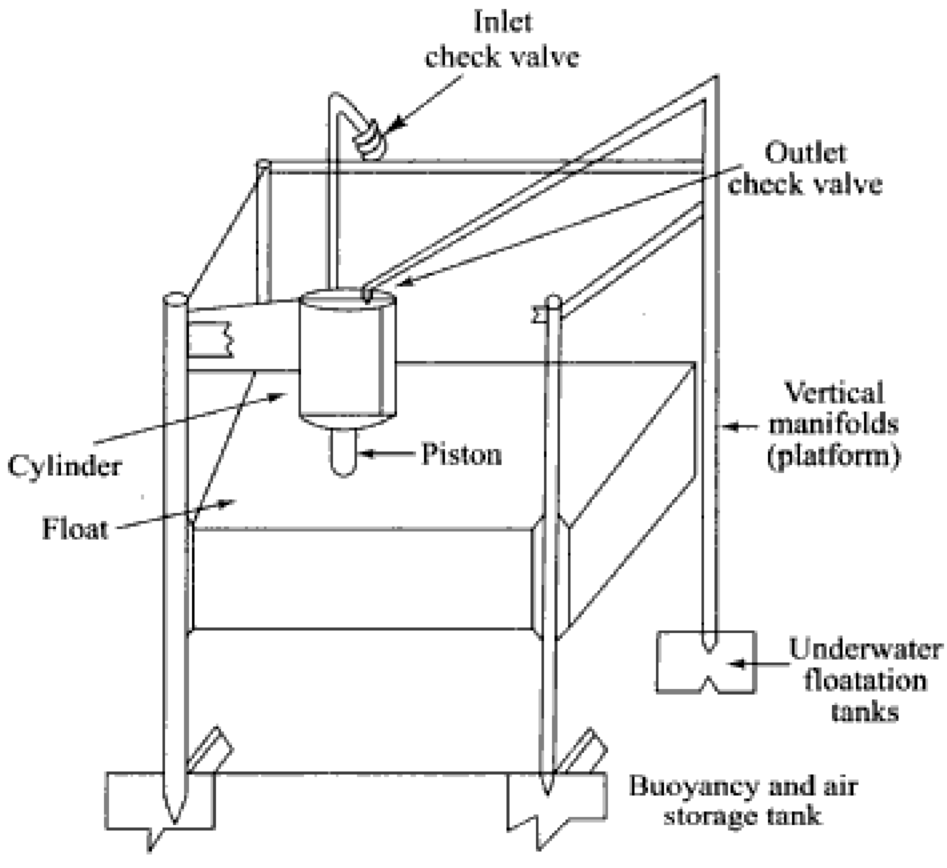


Fig. 14.18 *Schematic of a float wave-power conversion machine*

In a *high-level reservoir wave machine* (Fig. 14.19), a magnification piston is used and the pressurized water is elevated to a natural reservoir above the wave generator which has to be near a shoreline or to an artificial water reservoir. The water in the reservoir is made to flow through a turbine coupled to an electric generator and then back to the sea level. Calculations show that a 20 m diameter generator can produce 1 MW power.

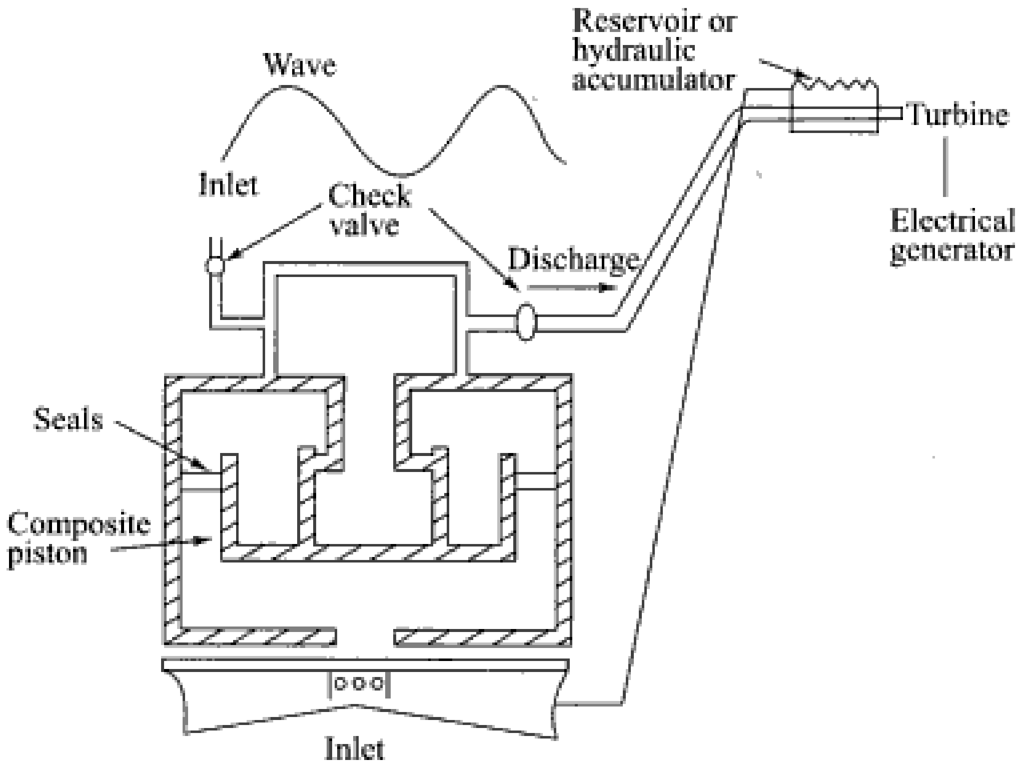


Fig. 14.19 *Schematic of a high-level reservoir wave machine (hydraulic accumulator wave machine)*

A *Dolphin-Type Wave-Power machine* (Fig. 14.20), designed by Tsu Research Laboratories in Japan, consists of a dolphin, a float, a connecting rod and two electric generators. The float has two motions. The first is a rolling motion about its own fulcrum with the connecting rod. The other is a nearly vertical or heaving motion about the connecting rod fulcrum. It causes relative revolving movements between the connecting rod and the stationary dolphin. In both cases, the movements are amplified and converted by gears into continuous rotary motions that drive the two electrical generators.

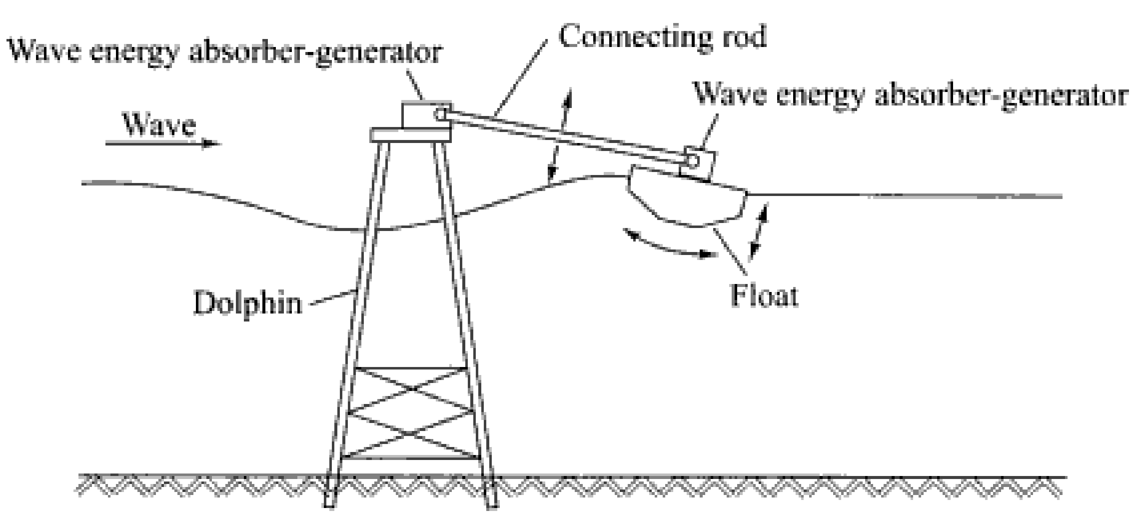


Fig. 14.20 *Schematic of a Dolphin type wave generator*

Hydraulic accumulator wave machines are also used, where instead of compressing air, the water itself is pressurized and stored in a high-pressure accumulator or pumped to a high-level reservoir, from which it flows through a water turbine electric generator. A different idea of extracting energy from ocean

earth every 24 h 50 min. During this time the tide rises and falls twice, resulting in a *tidal cycle* that lasts12 h 25 min. The *tidal tange R* is defined as

R = water elevation at high tide – water elevation at low tide.

The rise and fall of the water level follows a sinusoidal curve, shown with point A indicating the high tide point and point B indicating the low tide point (Fig. 14.24). The average period of time for the water level to fall from A to B and then rise from B to C is each approximately equal to 6 h 12.5 min.

At times during full or new moon, when the sun, moon and earth are approximately in one line, the gravitational forces of the sun and moon are enhanced. These high tides are called *spring tides*.

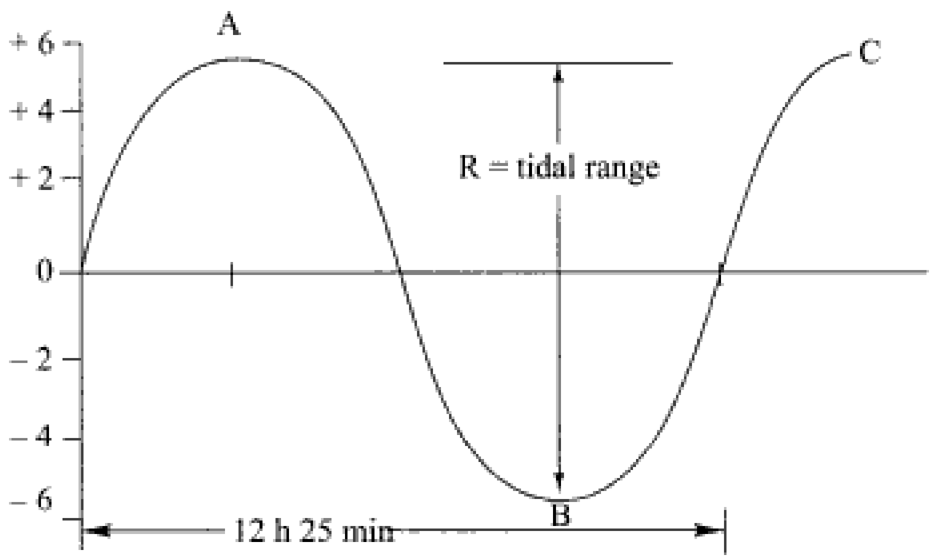


Fig. 14.24 *Tides of sea*

On the other hand, near the first and third quarters of the moon, when the sun and moon are at right angles with respect to the earth, *neap tides* occur. The tidal range is then very small, the high tides are lower and the low tides are higher than the average. Thus the range varies during the 29.5 day lunar month (Fig. 14.25). It is the maximum at the time of the new and full moons (spring tides) and the minimum at the time of the first and third quarter moons (neap tides). The spring-neap tidal cycle lasts one-half of a lunar month. A typical mean range is roughly one-third of the spring range.

The variations in the periodicity and monthly and seasonal ranges must be taken into account in the design and operation of tidal power plants. The tides are, however, predictable, and fairly accurate tide tables are usually available. Tidal ranges vary from one location to another. They have to be very large to justify the huge costs of building dams and associated hydro-electric power plants.

The tides along most coastlines are about a metre high, but in constructed areas they may rise by 10 metres or more. In these constricted areas the most effective tidal power plants are located. A dam or sluice gate is placed across an ocean bay or estuary. An incoming tide fills up the enclosed basin while passing through a row of hydroulic turbines.

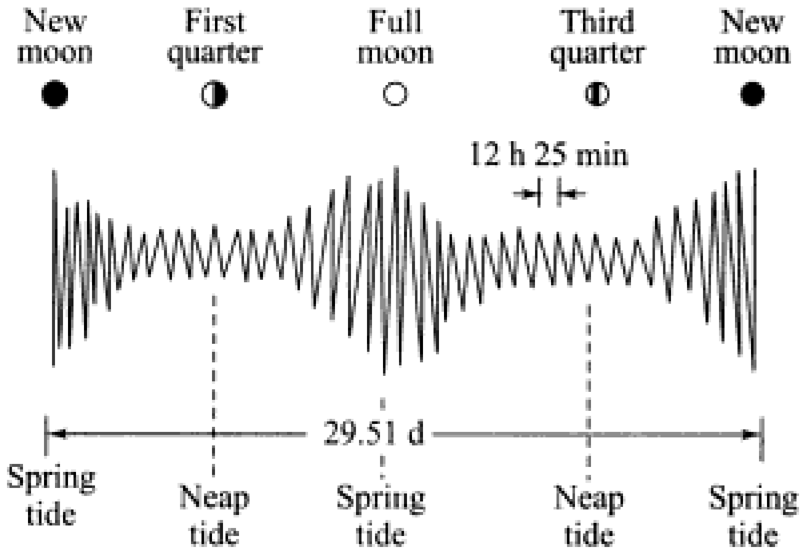


Fig. 14.25 *Relative high and low tides showing variation in range during lunar month*

After the basin is filled with water, the gates are closed and the turbines are shut down. Then the turbine blades are reversed and the gates are opened again to let the water surge out. Thus turbines would be rotated either way to generate electric power.

A tidal power development scheme essentially involves the construction of a long barrier across a bay or estuary to create a large basin on the landward side. The barrier includes dykes, gate-controlled sluices, and the power house. Tidal power schemes may have the following different configurations:

1. Single basin, single-effect tidal power scheme.
2. Single basin, double effect tidal power scheme.
3. Linked basin scheme

In the *Single Basin, Single-Effect Tidal Power Scheme*, the basin is filled by keeping the sluices open and letting the water flow from the sea to the basin during the high tide (Fig. 14.26). Power is generated by letting the water flow from the basin to the sea through the turbines during the low tide.

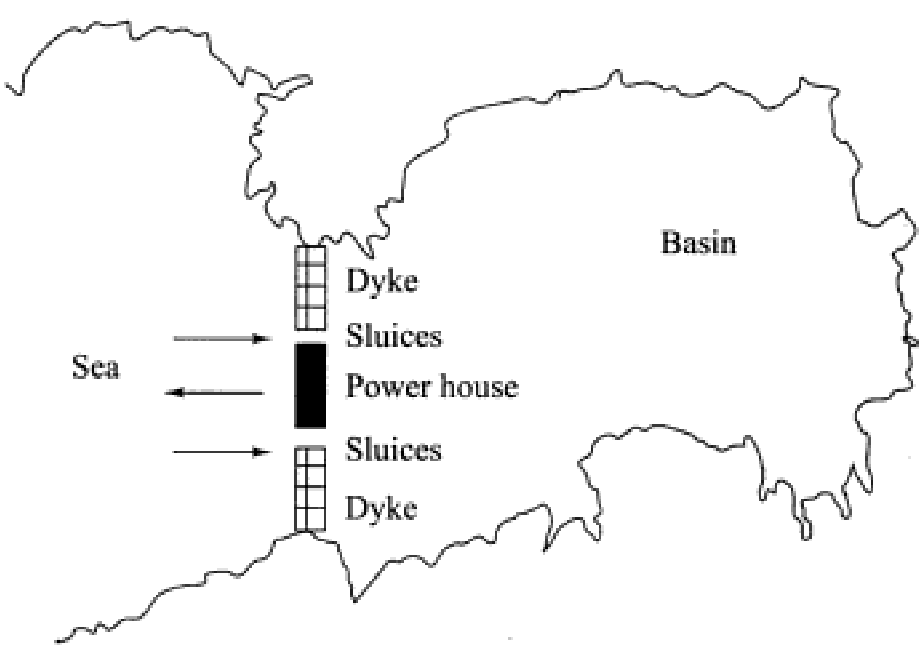


Fig. 14.26 (a) *Single basin, single effect tidal power scheme*

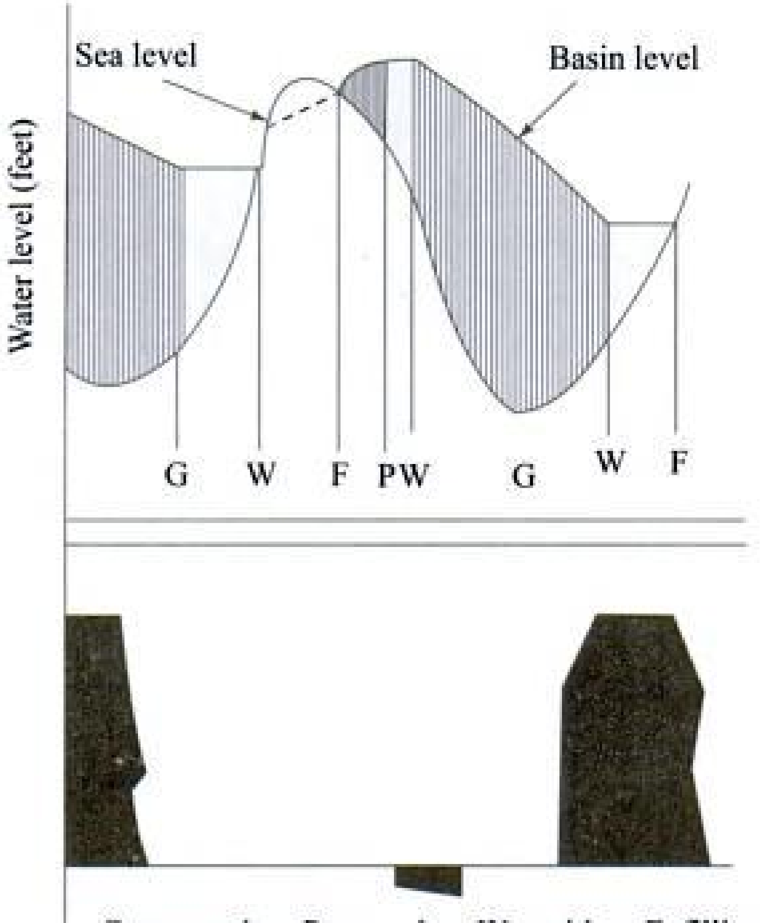


Fig. 14.26 (b) *Single basin, single effect tidal power scheme: schematic top view*

In a *Single Basin Double-Effect Tidal power Schemes*, power is generated during flood (high) tide, with water flowing from the sea to the basin through the turbines and also during ebb (low) tide, with water flowing from the basin to the sea through the turbines (Fig. 14.27). In this case, turbine blades should be reversible with proper blade angles depending upon the direction of flow.

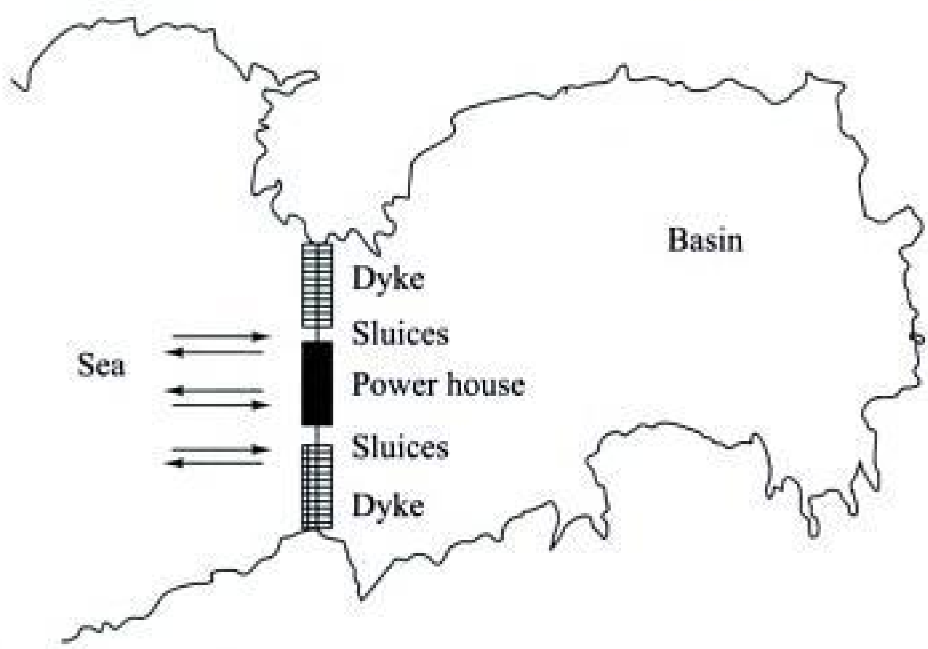


Fig. 14.27 (a) *Single basin, double-effect tidal power scheme*

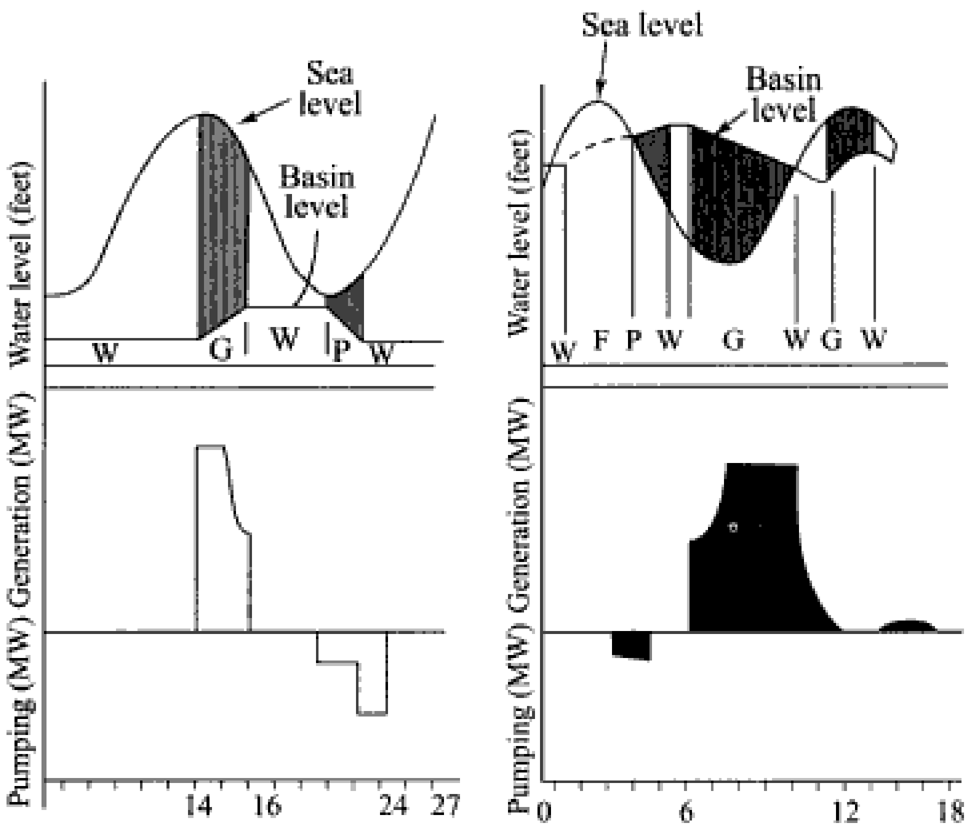


Fig. 14.27 (b) *Single basin, double-effect tidal power scheme: schematic top view*

In a *Linked Basin* (Double Basin Single Effect Tidal) Power Scheme (Fig. 14.28), there are two basins on the landward side with the powerhouse located in the barrier between the two basins. Power is generated by water flowing from the high basin to the low basin through the turbines and water flowing from the low basin to the sea during ebb tide. Turbo-generators should be capable of efficient generation at low heads, and consequently, of handling large discharges. The layout of a typical tidal power plant is shown in Fig. 14.29.

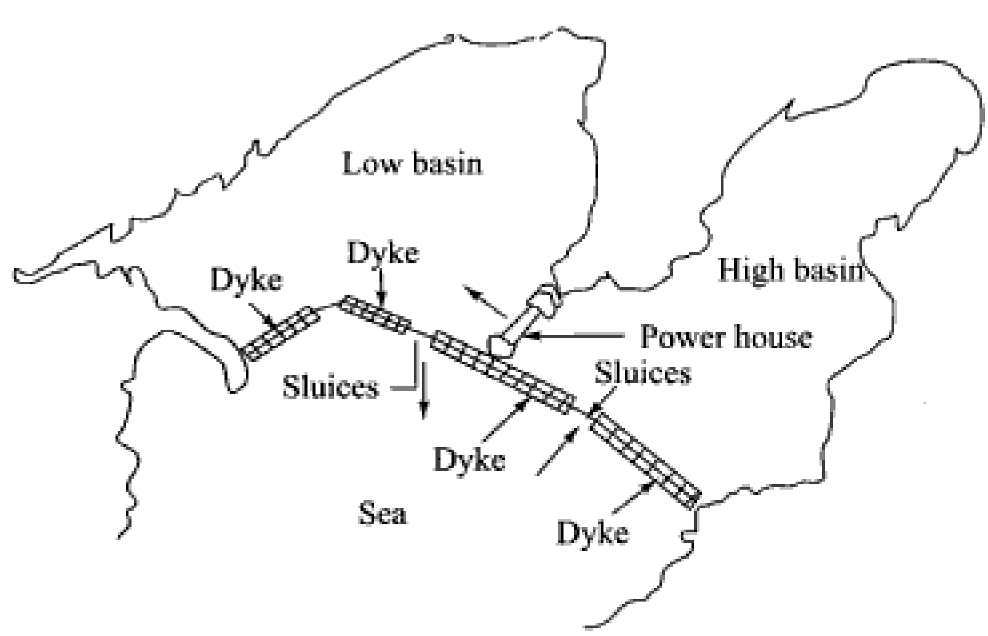


Fig. 14.28 (a) *Linked basin scheme*

having 24 bulb turbines of 10 MW each. In India, the prospective sites for tidal power exist in Gujarat and West Bengal (Sunderbans).

14.8 GEOTHERMAL ENERGY

Geothermal energy is primarily energy from the earth's own interior. The natural heat in the earth has manifested itself for thousands of years in the form of volcanoes, lava flows, hot springs and geysers. The interior of the earth is thought to consist of a central molten core surrounded by a region of semifluid material called the mantle (Fig. 14.30). This is covered by the *crust*, which has a depth of about 30 to 90 km. The temperature in the crust increases with the depth at the rate of about 30°C/km. Below the crust, the molten mass, called

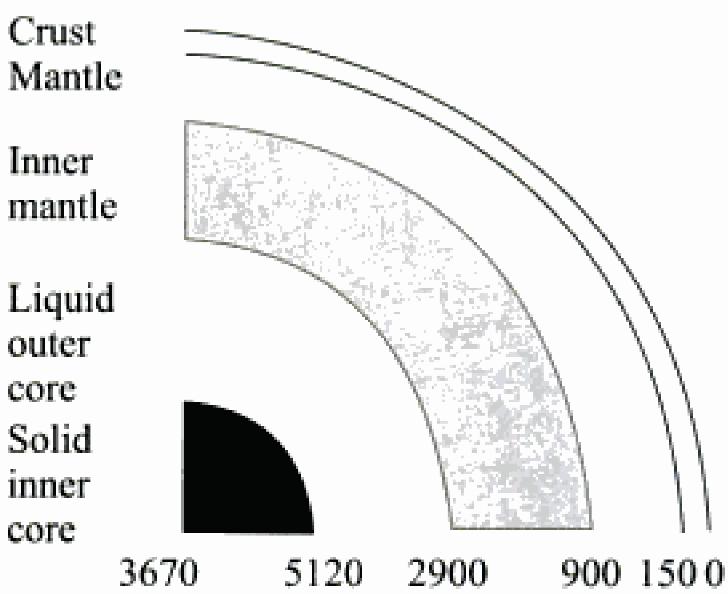


Fig. 14.30 *Different layers in the cross-section of the earth*

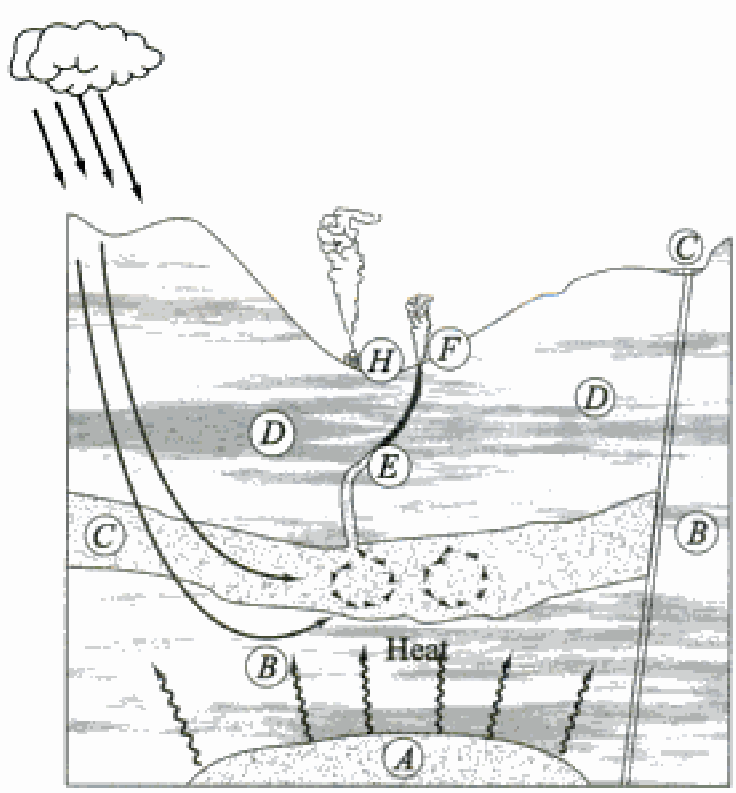


Fig. 14.31 *A typical geothermal field*

magma, is in the process of cooling at the rate of 0.063 W/m^2. Figure 14.31 shows a typical geothermal field. The hot magma near the surface (*A*) solidifies into igneous rock (*B*) or volcanic rock. Groundwater that finds its way down to this rock through fissures in it will be heated by the heat of the rock or by mixing with hot gases and steam emanating from the magma. The heated water will then rise connectively upward and into a porous and permeable reservoir (*C*) above the igneous rock. This reservoir is capped by a layer of impermeable solid rock (*D*) that traps the hot water in the reservoir. The solid rock has fissures (*E*) that act as vents of the giant underground boiler. The vents show up at the surface as geysers, fumaroles (*F*), or hot springs (*G*). A well (*H*) taps steam from the fissure for use in a geothermal power plant. Geothermal steam is of two kinds: *magmatic steam* that originates from the magma itself and *meteoritic steam* with groundwater heated by the magma. The latter is the largest source of geothermal steam.

Not all geothermal sources, however, produce steam. Some are lower in temperature so that there is only hot water. Some receive no groundwater at all and contain only hot rock. Geothermal sources are therefore of three kinds: (1) hydrothermal, (2) geopressurized, and (3) petrothermal.

Hydrothermal systems are those in which water is heated by contact with the hot rock which can be either vapour-dominated or liquid-dominated.

In *vapour-dominated systems* the water is vaporized into steam that reaches the surface in a relatively dry condition at about 250°C and rarely above 8 bar. This steam is suitable for use in power plants with the least cost. However, corrosive gases and erosive material are discouraging.

In *liquid-dominated systems* the hot water trapped underground is at a temperature range of 174°C to 315°C. When tapped by wells drilled, the water flows either naturally to the surface or pumped up to it. The drop in pressure to about 8 bar or less causes it to flash to a two-phase mixture of low quality i.e. liquid-dominated. It contains large concentrations of dissolved solids ranging from 3000 to 25,000 ppm. Power production is adversely affected because these solids precipitate and cause scaling in pipes and heat transfer surfaces. Liquid-dominated systems, are however, much more plentiful, and the US Geological Survey estimated 900 to 1400 quads Q ($1\ Q = 10^{15}$ Btu, about 10^{18} J) of energy available in these systems.

Geopressurized systems are sources of water or brine that has been heated in a manner similar to hydrothermal water, except that this water is trapped in much deeper underground aquifers (2400 to 9100 m deep) at relatively low temperature (~ 160°C) and very high pressure (> 1000 bar) with high salinity (*H*–10%) and is often referred to as *brine*. Also, it is saturated with natural gas, mostly methane, thought to be the result of decomposition of organic metter. There is economic feasibility of generating electricity by a combined cycle, one that involves the combustion of methane as well as heat from the thermal energy of hot water.

In *petrothermal systems*, magma lying close to the earth's surface heats overlying rock and when no underground water exists, there is simply hot dry rock (HDR). The temperatures of HDR vary between 150°C to 290°C. This energy, called petrothermal energy, represents by far the largest resource of geothermal energy. Since the HDR is largely impermeable, to make, it permeable, fracturing methods are considered which involve drilling wells into the rock and then fracturing by high-pressure water or nuclear explosives.

Figures 14.32 and 14.33 show a schematic and *T-s* diagrams of vapour-dominated power system. The steam at the well (1) at about 200°C is nearly saturated and may have a shut-off pressure up to 35 bar. Pressure drops through the well cause it to slightly superheat at the well head (2) where the pressure rarely exceeds 7 bar. It then goes through a centrifugal separation to remove particulate matter and then enters the turbine after an additional pressure drop (3). The steam expands through the turbine and enters the condenser at 4. Direct-contact condensers are more effective and less expensive than surface-type condensers. The turbine exhaust steam at 4 mixes with cooling water (7) that comes from a cooling tower. The mixture of 7 and 4 is saturated water (5) that is pumped to the cooling tower (6). A steam-jet ejector (SJE) is used to rid the condenser of the noncondensable gases and to minimize their corrosive effect. Examples of vapour-dominated system are the plants at the Geysers, USA, Larderello, Italy and Matsukawa, Japan.

Liquid-dominated systems can be of two types:

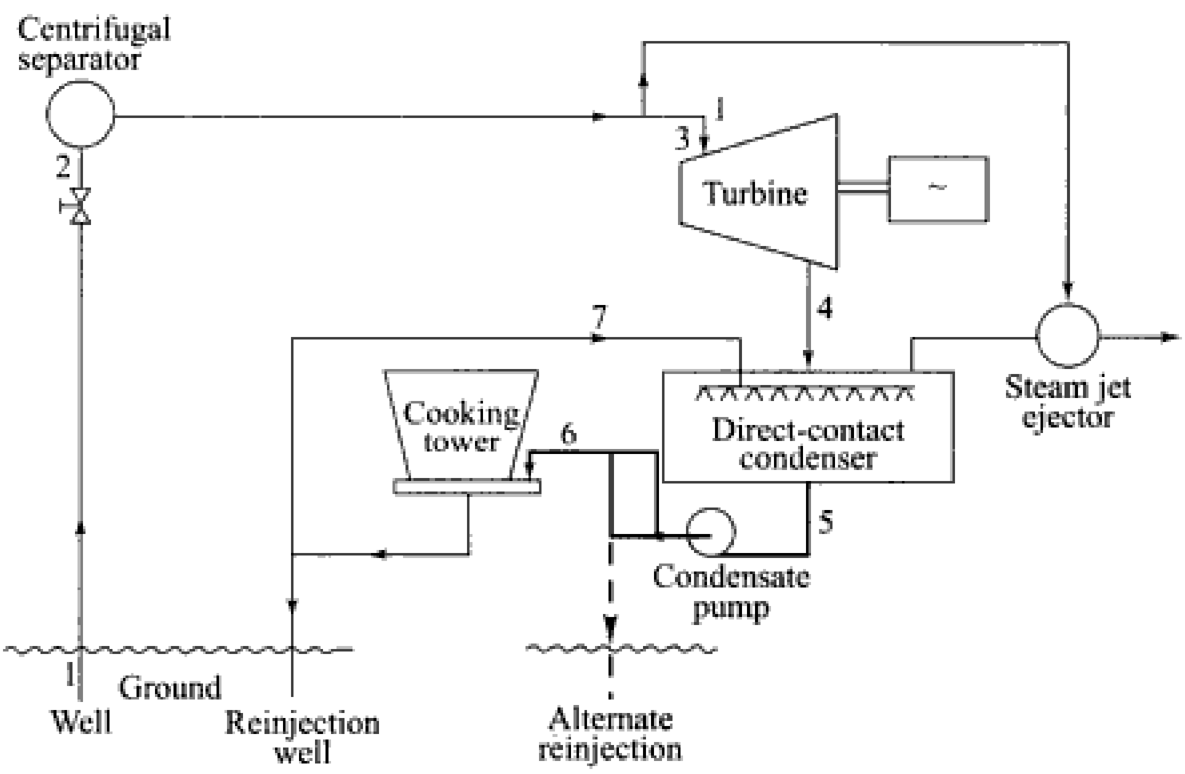


Fig. 14.32 *Schematic of a vapour-dominated powerplant*

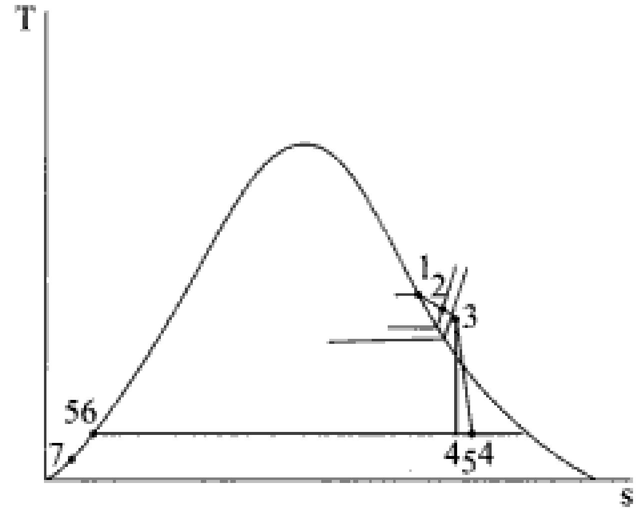


Fig. 14.33 *T-s diagram of the cycle shown in Fig. 14.32*

1. Flashed steam system, 2. Binary cycle system. Flashed steam system is illustrated by the flow and *T-s* diagrams in Figs. 14.34 and 14.35. Water from the underground reservoir at 1 reaches the well head at 2 at a low pressure. Process 1-2 is a throttling process that results in a two-phase mixture of low quality at 2. This is throttled further in a flash separator to state 3. The mixture is now separated into dry saturated steam at 4 and saturated brine at 5. The latter is reinjected into the ground. The dry steam at about 8 bar is expanded in a turbine to 6 and mixed with cooling water in a direct-contact condenser with the mixture at 7 going to a cooling tower. Flashed-steam systems have been used widely in Japan, New Zealand, Italy, Mexico and elsewhere.

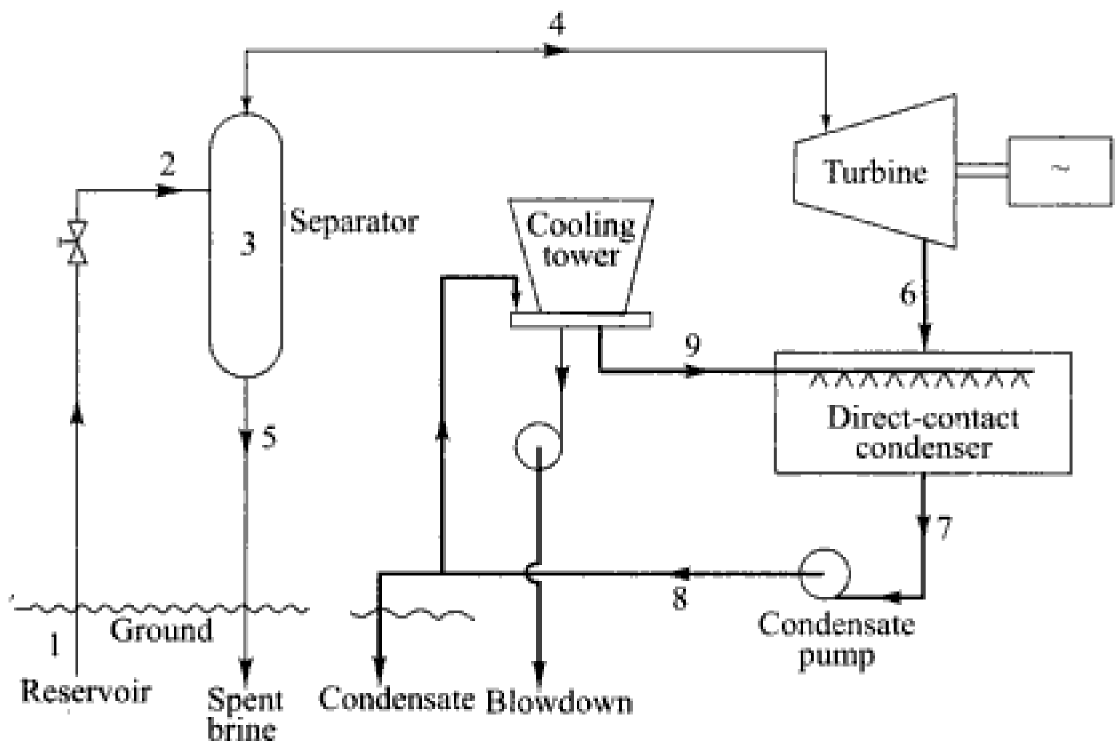


Fig. 14.34 *Schematic of a liquid-dominated single-flash steam system*

14.29 Explain the scope of generating electricity from biomass. With the help of a typical biogas plant, explain the functions of the digestor and the dome and how are animal wastes utilized to produce biogas. How can India benefit from installation of biogas plants?

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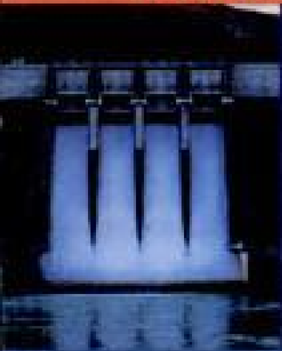

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