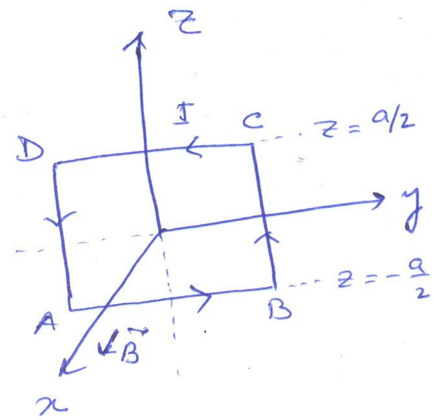
PHY 103

P.S # 8



Q.1.

$\vec{B} = k z \hat{x}$

$\vec{F} = \int I (\vec{dl} \times \vec{B})$

Force on side AB: $(\vec{dl} = dy\,\hat{y})$

$$\vec{F}_{AB} = I \int_{-a/2}^{+a/2} dy\,\hat{y} \times kz\hat{x} \Big|_{z=-a/2}$$

$$= \frac{1}{2} I k a^2 \hat{z}$$

Force on side CD: $\vec{dl} = -dy\,\hat{y}$

$$\vec{F}_{CD} = -I \int_{-a/2}^{+a/2} dy\,\hat{y} \times kz\hat{x} \Big|_{z=+a/2} = \frac{1}{2} I k a^2 \hat{z}$$

Force on BC: $\vec{dl} = dz\,\hat{z}$

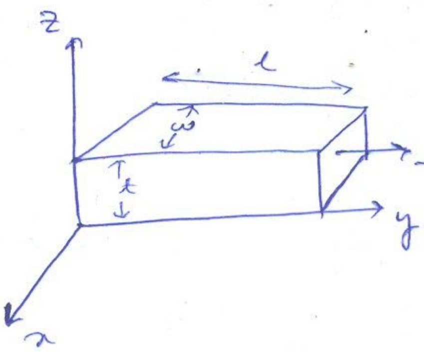
$$\vec{F}_{BC} = I \int_{-a/2}^{a/2} dz\,\hat{z} \times kz\hat{x} = Ik\hat{y}\int_{-a/2}^{+a/2} z\,dz = 0$$

Force on AD: $\vec{dl} = -dz\,\hat{z} \Rightarrow \vec{F}_{AD} = 0$

Total force $\vec{F} = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{AD} = \underline{\underline{I k a^2 \hat{z}}}$

Area of the loop $\vec{a} = a^2 \hat{x}$

$$\vec{\nabla}(I\vec{a}\cdot\vec{B}) = \vec{\nabla}(I k a^2 z) = \hat{z}\, I k a^2 = \vec{F}.$$



Q. 2 $\quad \vec{B} = B\hat{x}$

a) moving charges are positive, say $+q$.

charges are moving in the direction of I

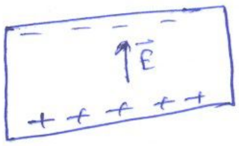
i.e in y-direction.

Magnetic force $\vec{F}_{mag} = q\vec{v} \times \vec{B}$

$$= qv\hat{y} \times B\hat{x}$$

$$= -qvB\hat{z}$$

$\Rightarrow$ charges will be deflected downward.



b) In eqlbrm $\quad qE = qvB \quad E = |\vec{E}|$

$$|\vec{E}| = vB \quad — (1)$$

V = Hall voltage.

$|\vec{E}| = E = \left|\frac{dV}{dz}\right|$

$I = nqvA \quad$ n = number of charge q flowing per second/area
$\quad A$ = area of cross-section $= tw$

$= nqvtw$

$\Rightarrow v = \frac{I}{nqtw} \quad \Rightarrow E = \left|\frac{dV}{dz}\right| = vB$ from (1)

$$= \frac{I}{nqtw}B$$

$\Rightarrow V = \frac{I}{nqtw}B\int dz = \frac{IB}{nq\cancel{t}w}\cdot\cancel{t} = \frac{IB}{nqw}$

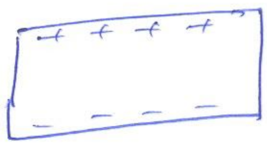
Lower edge accumulates +ve charges $\Rightarrow$ has higher potential than the upper surface.

c) for negative charge $-q \Rightarrow$ direction of flow of the charges
$= (-)$ve of direction of $I = -\hat{y}$

$\Rightarrow \vec{v} = -v\hat{y}$

$\vec{F}_{mag} = -q(-v\hat{y}) \times B\hat{x} = -qvB\hat{z} \longrightarrow$ same as for $+q$.

the charges will be deflected downward



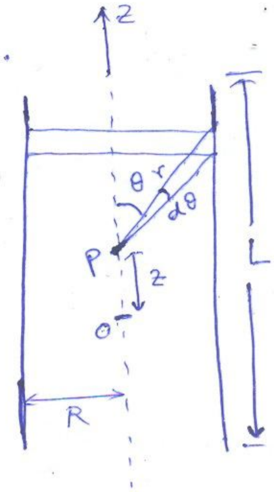
Again the charges will be deflected downward

Again the lower surface now accumulates (−)ve charges

$\Rightarrow$ lower surface will have lower potential,

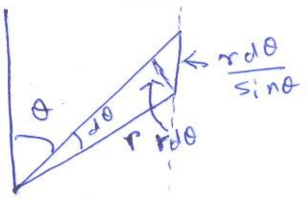
and thus upper surface will have higher potential.

Q.3 Solenoid $\equiv$ Superposition of current rings.



Consider the contribution from a circular ring making an angle θ and $\theta + d\theta$ to the point ~~Point at a distance z~~ P on the axis at a height z from the centre.

$\Rightarrow$ Thickness of the ring



$$dr_\perp = \frac{r\,d\theta}{\sin\theta}$$

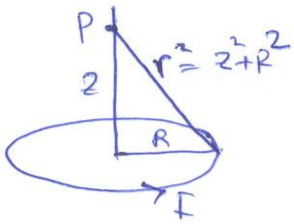
Current in the ring $dI = (nI)\,dr_\perp = \dfrac{nI\,r\,d\theta}{\sin\theta}$

& $\sin\theta = \dfrac{R}{r} \Rightarrow r = \dfrac{R}{\sin\theta}$

We know ~~for a ring~~ the magnetic field on the axis of a ring

$$B(z) = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\hat{z}$$

$\rightarrow$ only z-component is nonzero.

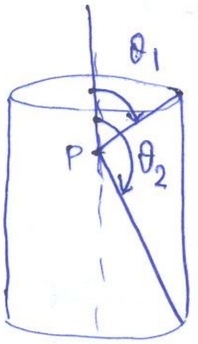


$$\Rightarrow dB_z = \frac{\mu_0}{2}(dI)\frac{R^2}{r^3}$$

$$= \frac{\mu_0}{2}\left(nI\frac{r\,d\theta}{\sin\theta}\right)\frac{R^2}{r^3} = \frac{\mu_0}{2}nI\left(\frac{\cancel{R}}{\sin\theta}\right)\frac{d\theta}{\sin\theta}\frac{\cancel{R^2}\sin^3\theta}{\cancel{R^3}}$$

$$[\because r = R/\sin\theta]$$

$$= \frac{\mu_0}{2}nI\sin\theta\,d\theta$$

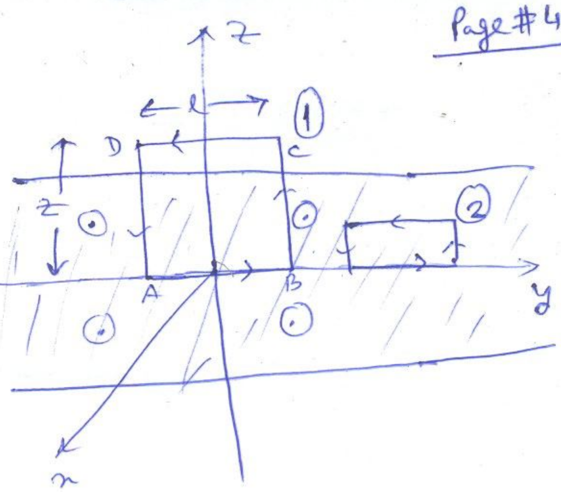


$$\Rightarrow B_z = \int_{\theta_1}^{\theta_2}\frac{\mu_0 nI}{2}\sin\theta\,d\theta = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$$

For an infinitely long solenoid $\underline{\theta_1 = 0,\ \theta_2 = \pi}$

$$\Rightarrow B_z = \frac{\mu_0 nI}{2}(1+1) = \mu_0 nI.$$

Q.4 $\vec{J} = J_0 \hat{i}$



from symmetry:
B can not depend on x, y
$\Rightarrow B = B(z)$.

Magnetic field due to a thin ~~sheet~~ sheet with current $\vec{J} = J_0 \hat{i}$ $\Rightarrow$ constant in magnitude and in $-y$ direction above the sheet
in $+y$ " below " ".
$\Rightarrow$ The field due to a stack of such sheets must also be $\parallel^l$ to y-axis.
$\Rightarrow \vec{B} = B(z)\hat{j}$

$\Rightarrow$ Note $B = 0$ on the plane $z = 0$ [field due to current sheet at $+z_i$ cancels the field due to symmetrically opposite one at $-z_i$]

consider, Amperian loop 1 with $z > a$.

$$\oint \vec{B}\cdot\vec{dl} = -B(z)\cdot l = \mu_0 (l\cdot a) J_0$$

[$-ve$ sign due to $\vec{dl}$ is in $-y$ dirn for CD.]

$$\Rightarrow B(z) = -\mu_0 J_0 a.$$

$$\Rightarrow \boxed{B(z) = -\mu_0 J_0 a\,\hat{j}} \quad \text{for } z \geq a$$

Similarly for $z < a$, consider the Amperian loop 2

$$\oint \vec{B}\cdot\vec{dl} = -B(z)\cdot l = \mu_0 (l z) J_0$$

$$\Rightarrow B(z) = -\mu_0 J_0 z \quad \Rightarrow \boxed{B(z) = -\mu_0 J_0 z\,\hat{j}} \quad \text{for } 0 \leq z \leq a$$

Similarly considering the regions for $z < 0$

we can write

$$B(z) = \begin{cases} -\mu_0 J_0 a\,\hat{j} & \text{for } z \geq a \\ \mu_0 J_0 a\,\hat{j} & \text{for } z \leq a \end{cases} = -\mu_0 J_0 \frac{z}{|z|}\hat{j} \quad \text{for } |z| \geq a$$

and

$$B(z) = \begin{cases} -\mu_0 J|z|\,\hat{j} & \text{for } 0 \leq z \leq a \\ \mu_0 J|z|\,\hat{j} & \text{for } -a \leq z \leq 0 \end{cases} = -\mu_0 J_0 z\,\hat{j} \quad \text{for } |z| \leq a$$