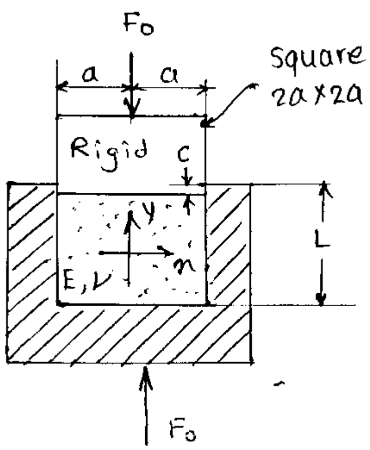
Solutions to H/W Problems of Chapter 5

Solution to problem 5.1:



Note:

$$\epsilon_{xx} = \epsilon_{zz} = 0 \quad — (1)$$

$$\epsilon_{yy} = -\frac{c}{L} \quad — (2)$$

$$\sigma_{yy} = -\frac{F_o}{(2a)^2} \quad — (3).$$

Stress - strain relationship:

$$\epsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right] \quad — (4)$$

$$\epsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right] \quad — (5)$$

$$\epsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right] \quad — (6).$$

eqns (1), (4), (6) $\Rightarrow$

$$\sigma_{xx} - \nu\sigma_{yy} - \nu\sigma_{zz} = 0 \quad — (7)$$

$$-\nu\sigma_{xx} - \nu\sigma_{yy} + \sigma_{zz} = 0 \quad — (8).$$

equations (7) & (8) $\Rightarrow \sigma_{zz} = \sigma_{xx} = \frac{\nu}{1-\nu}\sigma_{yy}$. — (9).

equations (5) and (4) $\Rightarrow$

$$\epsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\frac{2\nu}{1-\nu}\right)\sigma_{yy}\right].$$

$$= \frac{\sigma_{yy}}{E}\left(\frac{1-\nu-2\nu^2}{1-\nu}\right). \quad — (10).$$

Substituting (2), (3) into (10) we get

$$-\frac{c}{L} = -\frac{1-\nu-2\nu^2}{(1-\nu)E}\,\frac{F_o}{(2a)^2}.$$

$$\therefore F_o = \frac{(1-\nu)E}{1-\nu-2\nu^2}\cdot\frac{(2a)^2 c}{L} = \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\,\frac{4ca^2}{L}$$

Solution to problem 5.9:

Given: Flat steel plate loaded in xy plane

$\sigma_x = 145\ MN/m^2$, $E = 210\ GPa$

$\tau_{xy} = 42\ MN/m^2$, $\nu = 0.27$

$\epsilon_z = -3.6 \times 10^{-4}$;

Note: $\sigma_z = 0$ (Because of the state of plane stress in xy plane)

Stress - strain relationship:

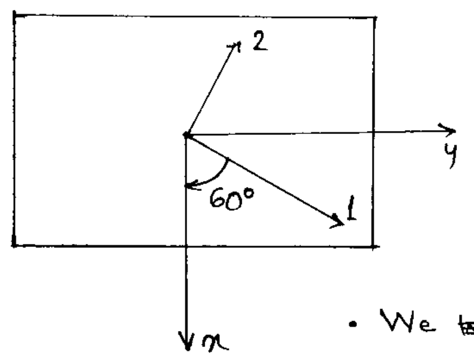
$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \nu(\sigma_x + \sigma_y)\right]$$

$$-3.6 \times 10^{-4} = \frac{1}{210 \times 10^9}\left[0 - 0.27\,(145 \times 10^6 + \sigma_y)\right]$$

$$\therefore \sigma_y = 135 \times 10^6\ N/m^2$$

$$= 135\ MN/m^2.$$

Solution to problem 5.10 :



Given:

- Flat 'Aluminium Plate' loaded in its plane.
- principal strains:

  $\epsilon_1 = 3.2 \times 10^{-4}$, $\epsilon_2 = -5.4 \times 10^{-4}$
- $E = 75$ GPa, $\nu = 0.33$

- We will first find σ_1 and σ_2 and use Mohr's circle to find σ_x, σ_y, τ_{xy}.

- Since $\sigma_3 = 0$ (Plane stress in x-y plane),

$$\epsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2) \quad — ①$$

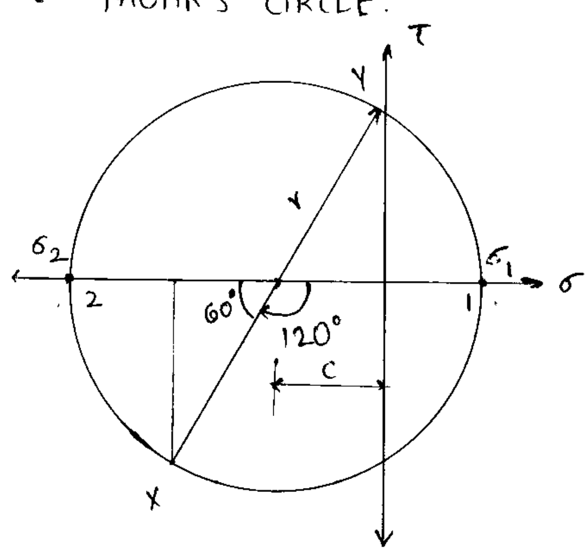
$$\epsilon_2 = \frac{1}{E}(\sigma_2 - \nu\sigma_1) \quad — ②.$$

equations ① and ② $\Rightarrow$

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2) = \frac{75 \times 10^9}{1-0.33^2}\left[3.2 \times 10^{-4} - 0.33(5.4 \times 10^{-4})\right] = 11.9 \text{ MN/m}^2$$

$$\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2 + \nu\epsilon_1) = \frac{75 \times 10^9}{1-0.33^2}\left[-5.4 \times 10^{-4} + 0.33(3.2 \times 10^{-4})\right] = -36.5 \text{ MN/m}^2$$

- MOHR'S CIRCLE:



$$c = \frac{\sigma_1 + \sigma_2}{2} = \frac{-36.5 + 11.9}{2} = -12.3$$

$$r = \frac{\sigma_1 - \sigma_2}{2} = \frac{11.9 + 36.5}{2} = 24.2$$

$\therefore$ State of stress at XY.

$$\sigma_x = -r\cos 60 + c = -24.4 \text{ MN/m}^2.$$

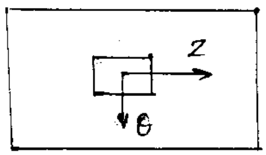
$$\sigma_y = c + r\cos 60 = -12.3 + 24.2\cos 6\text{[illegible]}$$
$$= -0.2 \text{ MN/m}^2.$$

$$\tau_{xy} = r\sin 60 = 20.96 \text{ MN/m}^2.$$

(Positive as per the sign convention)

Solution to problem 5.12 :

Given: Long, thin-walled closed-ended cyllinder.



$p, r, t.$

$\epsilon_z = \epsilon_0.$

To find : p.

- Stresses: $\sigma_\theta = \frac{pr}{t}, \quad \sigma_z = \frac{pr}{2t}, \quad \sigma_r \simeq 0$

ϵ_z :
$$\epsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_\theta)$$
$$= \frac{1}{E}\left(\frac{pr}{2t} - \nu\frac{pr}{t}\right).$$
$$= \frac{pr}{tE}(0.5 - \nu)$$

$$\therefore \epsilon_0 = \frac{pr}{tE}(0.5 - \nu)$$

$$\therefore p = \frac{E}{(0.5-\nu)}\frac{t}{r}\epsilon_0$$

Solution to problem 5.17

- Heating the pulley or cooling the shaft will be equally effective, although it may be easier to heat the pulley. Also, when the pulley is to be fixed on the shaft at some distance from the shaft ends, it will be much more convenient to heat the pulley.

- Pulley hole dia. = 24.950 mm.

  Shaft dia. = 25.000 mm.

Clearance after heating = 0.025 mm.

$\therefore$ Total thermal expansion of hole diameter = (25.000 − 24.950) + 0.025

= 0.075 mm.

= Δd

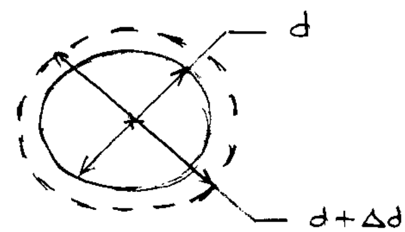
$$\therefore \epsilon_\theta = \frac{\Delta d}{d} = \frac{0.075}{24.95} = 3\times10^{-3}$$

- Strain – temperature relation:

$$\epsilon_\theta = \alpha \Delta T$$

$\alpha = 12\times10^{-6}/^\circ C$ for steel

$$\therefore \Delta T = \frac{\epsilon_\theta}{\alpha} = \frac{3\times10^{-3}}{12\times10^{-6}} = 250^\circ C$$

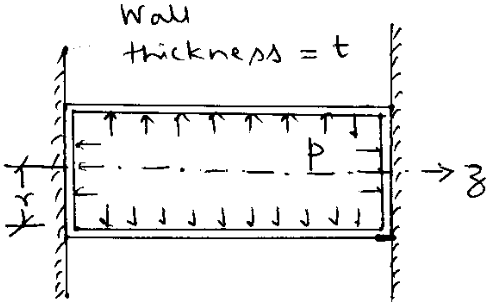


d = hole diameter of the pulley before heating,

$d+\Delta d$ = hole diameter of the pulley after heating,

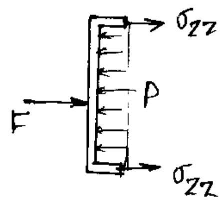
$$\therefore \epsilon_\theta = \frac{\pi(d+\Delta d)-\pi d}{\pi d} = \frac{\Delta d}{d}$$

Solution to problem 5.20 :



• Equilibrium :

Let F be the force exerted by the wall on the tank, then



$$\sigma_{zz}(2\pi r t) = p\pi r^2 - F.$$

$$\therefore \sigma_{zz} = \frac{pr}{2t} - \frac{F}{2\pi r t}.$$

Also, $\sigma_{\theta\theta} = \frac{pr}{t}$, $\sigma_{rr} \simeq 0$.

Stress - strain Relations :

$$\epsilon_{zz} = \frac{\sigma_{zz} - \nu\sigma_{\theta\theta}}{E}$$

$$= \frac{1}{E}\left[\frac{pr}{2t} - \frac{F}{2\pi r t} - \nu\frac{pr}{t}\right]$$

$$= \frac{1}{E}\left[\frac{pr}{t}(0.5-\nu) - \frac{F}{2\pi r t}\right].$$

Compatibility condition:

$$\epsilon_z = 0$$

$$\therefore F = 2\pi r t \cdot \frac{pr}{t}(0.5-\nu)$$

$$= \pi r^2 p(1-2\nu).$$

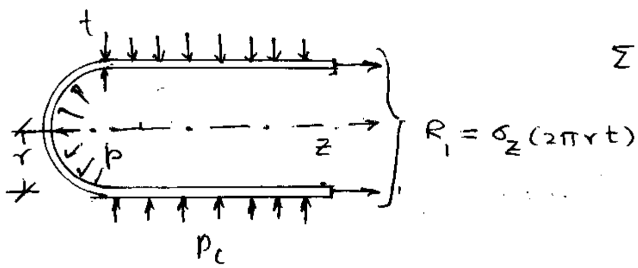
Solution to problem (5.21):

Idealisation: zero friction between tank and wall of cavity.

Let p_c = contact pressure between tank and cavity.

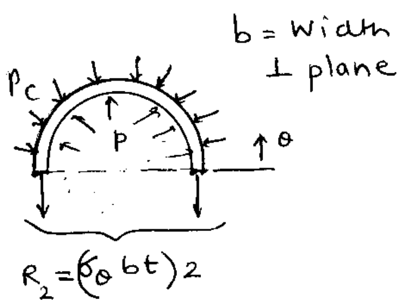
$\epsilon_\theta = 0$ in tank wall.

Equilibrium:



$\Sigma F_z = 0 \Rightarrow p\pi r^2 = R_1 = \sigma_z(2\pi r t)$.

$\therefore \sigma_z = \dfrac{pr}{2t}$.



$\Sigma F_{vertical} = 0 \Rightarrow$

$R_2 = 2\sigma_\theta tb = (p - p_c)\cdot 2rb$

$\therefore \sigma_\theta = (p - p_c)\dfrac{r}{t}$.

σ_r is between $-p$ and $-p_c$. Assuming that p_c is of the order of p we will neglect σ_r $(t/r \ll 1)$, i.e. $\sigma_r \simeq 0$

Stress - strain:

$$\epsilon_\theta = \frac{1}{E}\left[\sigma_\theta - \nu(\cancel{\sigma_r}^{\,0} + \sigma_z)\right] = 0.$$

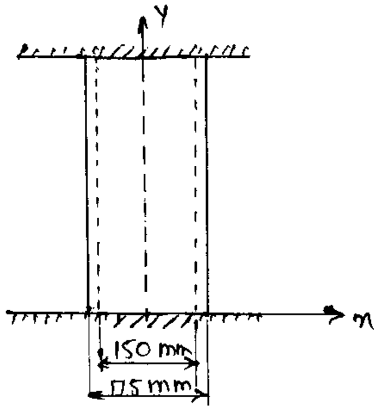
Compatibility:

$$\epsilon_\theta = 0$$

$$\therefore \sigma_\theta = \nu\sigma_z = \frac{\nu pr}{2t}$$

[Note: Then $p_c = (1 - \frac{\nu}{2})p$. Thus p_c is of the order of p]

Solution to problem 5.40 :



Away from ends, assume only normal stress in y-direction

i.e. $\sigma_x = \sigma_z = 0$, $\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$.

$$\therefore\ \epsilon_x = -\frac{\nu \sigma_y}{E} + \alpha\,\Delta T$$

$$\epsilon_y = \frac{\sigma_y}{E} + \alpha\,\Delta T$$

$$\epsilon_z = -\frac{\nu \sigma_y}{E} + \alpha\,\Delta T$$

$$\gamma_{xy} = \gamma_{yz} = \gamma_{xz} = 0.$$

Because of constraint by wall,

$$\epsilon_y = 0 \Rightarrow \sigma_y = -\alpha E \Delta T.$$

$E = 210 \times 10^9\ N/m^2$, $\alpha = 12 \times 10^{-6}\ /^\circ C$ $\quad \nu = 0.27$ (steel)

$$\Delta T = (-15) - 20 = -35\ ^\circ C.$$

$$\sigma_y = -\left[12 \times 10^{-6} \times 210 \times 10^9 \times (-35)\right]$$

$$= 88.2 \times 10^6\ MN/m^2.$$

$$\epsilon_x = -\frac{\nu \sigma_y}{E} + \alpha \Delta T = \frac{\nu \alpha E \Delta T}{E} + \alpha \Delta T$$

$$= \alpha \Delta T (1+\nu)$$

$$= 12 \times 10^{-6} \times (-35)(1 + 0.27)$$

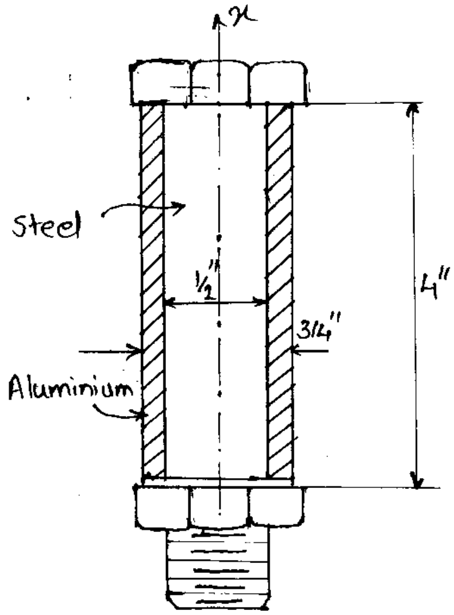
$$= -5.3 \times 10^{-4},$$

Similarly,

$$\epsilon_y = \alpha \Delta T (1+\nu) = 12 \times 10^{-6} \times (-35)(1+0.27)$$

$$= -5.3 \times 10^{-4}.$$

Solution to problem (5.41):



The problem is solved by superposition of following two problems –

i) Determination of stresses due to turning of the nut.

ii) Determination of stresses due to temperature rise from 60° to 100° F.

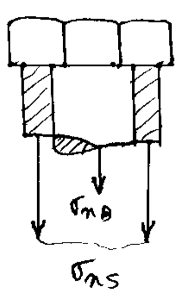
In both the cases, the state of stress is 1-D i.e. there is only σ_x.

For Sleeve : $E_S = 10 \times 10^6$ lb/in², $\alpha_S = 12 \times 10^{-6}$ /°F

For Bolt : $E_B = 30 \times 10^6$ lb/in², $\alpha_B = 6.5 \times 10^{-6}$ /°F.

(I) Stresses due to turning of nut:

F.B.D.



Equilibrium:

$$\sigma_{xs} A_s + \sigma_{xB} A_B = 0$$

$$\therefore \sigma_{xs} \frac{\pi}{4}\left[\left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2\right] + \sigma_{xB} \frac{\pi}{4}\left(\frac{1}{2}\right)^2 = 0.$$

$$\therefore 5\,\sigma_{xs} + 4\,\sigma_{xB} = 0 \qquad \text{— 1)}$$

Stress strain Relations:

$$\epsilon_{xs} = \frac{\sigma_{xs}}{E_s}, \quad \epsilon_{xB} = \frac{\sigma_{xB}}{E_B}.$$

(Problem 5.41 contd.)

$$\therefore \epsilon_{xS} = \frac{\sigma_{xS}}{10\times10^6} \quad \text{—— 2)}$$

$$\epsilon_{xB} = \frac{\sigma_{xB}}{30\times10^6} \quad \text{—— 3).}$$

Compatibility :

$\delta_{xB} + |\delta_{xS}| = \text{Nut travel}$ (Similar to problem 2.24)

$\delta_{xB} - \delta_{xS} = \frac{1}{4}\times\frac{1}{16}$ ($\delta_{xS} < 0$, $\frac{1}{4}$ turn, and 16 threads per inch).

$$\therefore \frac{\delta_{xB}}{L} - \frac{\delta_{xS}}{L} = \frac{\frac{1}{4}\times\frac{1}{16}}{L}$$

$$\therefore \epsilon_{xB} - \epsilon_{xS} = \frac{1}{256} \quad \text{— 4)} \qquad (L = 4'').$$

Solution:
equations 2), 3), 4) $\Rightarrow$

$$\frac{\sigma_{xB}}{30\times10^6} - \frac{\sigma_{xS}}{10\times10^6} = \frac{1}{256}.$$

$$\therefore \sigma_{xS} - \frac{\sigma_{xB}}{3} = -3.9\times10^4 \quad \text{— 5)}$$

equations 1) and 5) $\Rightarrow$

$$\sigma_{xS} = -2.75\times10^4 \ \text{lb/in}^2.$$

$$\sigma_{xB} = 3.44\times10^4 \ \text{lb/in}^2$$

(II) Stresses due to temperature rise:

FBD and equilibrium ~~conditions~~ equations are same as for 1st part.

$$\therefore 5\sigma_{xST} + 4\sigma_{xBT} = 0 \quad \text{— a)}$$

(problem 2.41 contd.)

Stress strain temperature relations: $\Delta T = 40^\circ F$

$$\epsilon_{xs} = \frac{\sigma_{xST}}{E_S} + \alpha_S \Delta T = \frac{\sigma_{xST}}{10 \times 10^6} + 4.8 \times 10^{-4} \quad - b)$$

$$\epsilon_{xB} = \frac{\sigma_{xBT}}{E_B} + \alpha_B \Delta T = \frac{\sigma_{xBT}}{30 \times 10^6} + 2.6 \times 10^{-4} \quad - c)$$

compatibility:

$$\epsilon_{xST} = \epsilon_{xBT} \quad - d)$$

Solution:

b), c), d) $\Rightarrow$

$$\frac{\sigma_{xST}}{10 \times 10^6} + 4.8 \times 10^{-4} = \frac{\sigma_{xBT}}{30 \times 10^6} + 2.6 \times 10^{-4}$$

$$\Rightarrow 3\sigma_{xST} - \sigma_{xBT} = -6.6 \times 10^3 \quad - e)$$

a) and e) $\Rightarrow$

$$\sigma_{xST} = -1.55 \times 10^3 \text{ lb/in}^2$$
$$\sigma_{xBT} = 1.94 \times 10^3 \text{ lb/in}^2$$

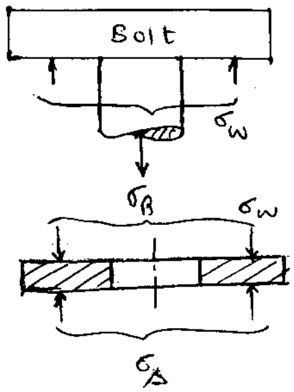
(III) Superposition:

Stress in sleeve $= \sigma_{xs} + \sigma_{xST} = (-2.75 - 0.155) \times 10^4$
$= -2.905 \times 10^4$ psi

Stress in bolt $= \sigma_{xB} + \sigma_{xBT} = 3.44 \times 10^4 + 0.194 \times 10^4$
$= 3.634 \times 10^4$ psi.

Solution to problem 5.44 :

FBD's



(A) $\boxed{\text{Initial tightening}}$ produces a strain ϵ_B in bolt.

Stress – strain Relation :

$$\epsilon_B = \frac{\sigma_B}{E_B} \Rightarrow \sigma_B = \epsilon_B E_B.$$

Equilibrium in Bolt :

$$\sigma_W A_W - \sigma_B A_B = 0.$$

$$\therefore \sigma_W = \frac{\sigma_B A_B}{A_W} = \frac{\epsilon_B E_B A_B}{A_W}$$

$$= \frac{0.0005 \times 210 \times 10^9 \times 310 \times 10^{-6}}{625 \times 10^{-6}}$$

$$= 52 \times 10^6 \text{ N/m}^2. \text{ (compressive)} \quad — (A)$$

(Note : No compatibility condition required)

(B) Stresses due to $\boxed{\text{temp. change}}$: Let temperature change produce tensile stresses.

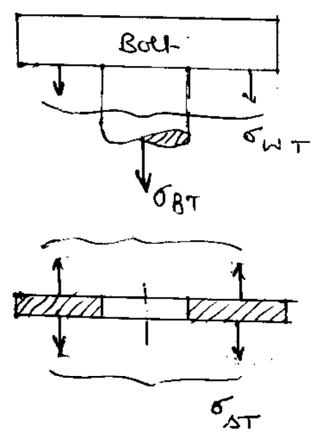
Stress – Strain – Temp Relations :

$$\epsilon_{BT} = \frac{\sigma_{BT}}{E_B} + \alpha_B \Delta T \quad — (1)$$

$$\epsilon_{WT} = \frac{\sigma_{WT}}{E_W} + \alpha_W \Delta T \quad — (2)$$

$$\epsilon_{ST} = \frac{\sigma_{ST}}{E_S} + \alpha_S \Delta T. \quad — (3)$$

FBD's



Equilibrium of Bolt :

$$\sigma_{BT} A_B + \sigma_{WT} A_W = 0$$

$$\Rightarrow \cancel{\sigma_{WT} = - \frac{\sigma_{BT} A_B}{A_W}}.$$

$$\Rightarrow \sigma_{BT} = - \frac{\sigma_{WT} A_W}{A_B} \quad — (4)$$

Equilibrium of washer :

$$\sigma_{ST} A_S - \sigma_{WT} A_W = 0. \Rightarrow \sigma_{ST} = \frac{\sigma_{WT} A_W}{A_S} \quad — (5).$$

$\therefore$

Compatibility condition: $\delta_{BT} = \delta_{\omega T} + \delta_{ST}$.

$$\therefore \; \epsilon_{BT} L_B = \epsilon_{\omega T} L_\omega + \epsilon_{ST} L_S \quad \text{— (6)}$$

From equations (1), (2), (3), (6)

$$L_B\left(\frac{\sigma_{BT}}{E_B} + \alpha_B \Delta T\right) = L_\omega\left(\frac{\sigma_{\omega T}}{E_\omega} + \alpha_\omega \Delta T\right) + L_S\left(\frac{\sigma_{ST}}{E_S} + \alpha_S \Delta T\right) \quad \text{— (7)}$$

From equation (4), (5), (7)

$$\sigma_{\omega T}\left(-\frac{L_B A_\omega}{E_B A_B} - \frac{L_\omega}{E_\omega} - \frac{L_S A_\omega}{E_S A_S}\right) = \left(L_\omega \alpha_\omega + L_S \alpha_S - L_B \alpha_B\right)\Delta T.$$

$L_\omega = 1.5\,mm$, $L_S = 150\,mm$. $L_B = 151.5\,mm$.

$\alpha_\omega = \alpha_B = 12 \times 10^{-6}\ /^\circ C$, $\alpha_S = 22 \times 10^{-6}\ /^\circ C$.

$A_\omega = A_S = 625\ mm^2$ $\quad A_B = 310\ mm^2$.

$E_\omega = E_B = 210 \times 10^{+9}\ N/m^2$ $\qquad E_S = 75 \times 10^9\ N/m^2$

$= 210 \times 10^3\ N/mm^2$ $\qquad = 75 \times 10^3\ N/mm^2$.

$\Delta T = 95^\circ C$.

Substituting the values we get

$$\sigma_{\omega T} = -41.1\ N/mm^2$$
$$= 41.1 \times 10^6\ N/m^2 \ \text{(compressive)}. \quad \text{— (B)}$$

(c) $\boxed{\text{Total stress}}$ in washer : $\sigma_\omega + \sigma_{\omega T}$ [Add (A) & (B)]

$$= \cancel{58}\ (52 + 41.17) \times 10^6$$

$$= 93.17 \times 10^6\ N/m^2 \ \text{(compressive)}.$$