Assignment 1

ME-354 Vibration and control (Due: January 19, 2018)

January 12, 2018

1. A buoy of uniform cross-sectional area A and mass m is depressed a distance x from the equilibrium position as shown in Fig.1, and then released. Derive the differential equation of motion and obtain the natural frequency of oscillation. The mass density of the liquid in which the buoy floats is ρ.

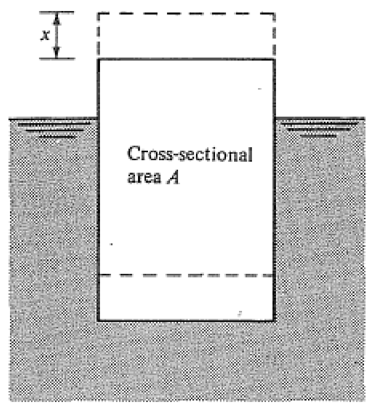

Fig.1

2. Derive the differential equation of motion for the system shown in Fig.2 and obtain the period of oscillation. Denote the mass density of the liquid by ρ and the total length of the column of liquid by L.

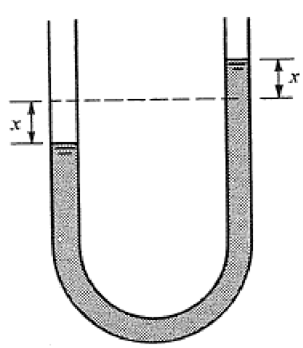

Fig.2

3. The hinges of the rectangular door shown in Fig.3 are mounted on a line making an angle α with respect to the vertical. Assume that the door has uniform mass distribution and determine the natural frequency of oscillation.

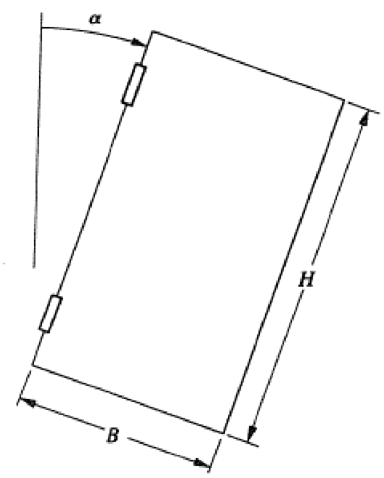

Fig.3

4. A bead of mass m is suspended on a massless string as shown in Fig.4. Assume that the string is subjected to the tension T, and this tension does not change throughout the motion, and derive the differential equation for small motion y from equilibrium, as well as the natural frequency of oscillation.

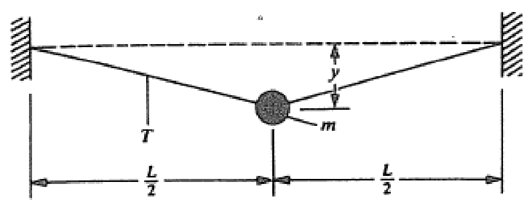

Fig.4

5. A mass m is suspended on a massless beam of bending stiffness EI through a spring of stiffness k as shown in Fig.5. Derive the differential equation of motion and determine the natural frequency of oscillation.

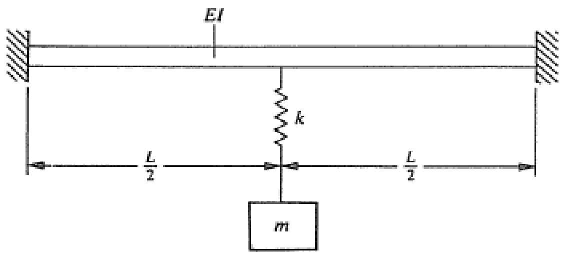

Fig.5

6. A uniform rigid bar of mass m is suspended by two inextensible massless strings of length L (see Fig.6). Such a system is referred to as bifilar pendulum. Derive the differential equation for the oscillation θ about the vertical axis through the bar center. Note that the mass moment of inertia of the bar about its center is $\frac{1}{3}ma^2$.

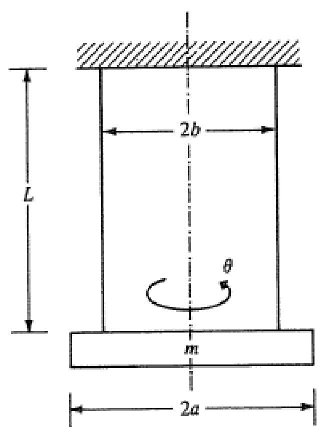

Fig.6

7. Obtain the natural frequency of the system as shown in Fig.7. The spring is linear and the pulley has a mass moment of inertia I about the center O. Take $k = 4.3782 \times 10^5$ N/m, $I = 67.79$ Nms^2, $m = 437.82$ Kg and $R = 0.51$ m.

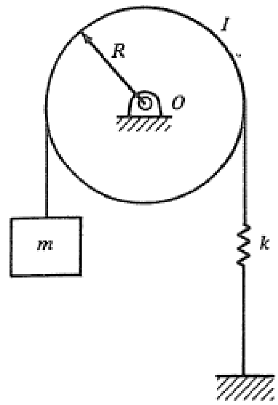

Fig.7

8. A uniform disk of radius r rolls without slipping inside a circular of radius R as shown in Fig.8. Derive the equation of motion for arbitrarily large angles θ. Then, show that in the neighbourhood of the trivial equilibrium $\theta = 0$ the system behaves like a harmonic oscillator, and determine the natural frequency of oscillation.

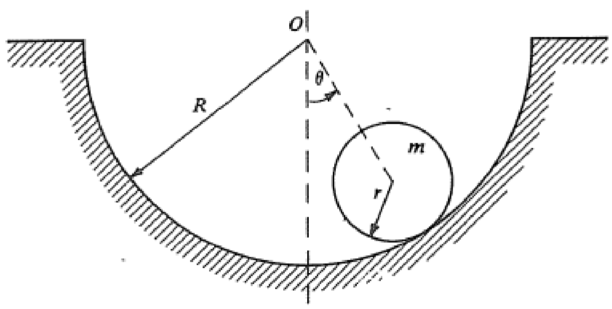

Fig.8

9. A uniform bar of total mass m and length l rotates with the constant angular velocity Ω about a vertical axis as shown in Fig.8. Denote by θ the angle between vertical axis and the bar, and :

(a) Determine the equilibrium position as expressed by the constant angle θ_0.

(b) Derive the differential equation for small motion θ_1 about θ_0.

(c) Determine a stability criterion for each equilibrium position based on the requirement that the motion θ_1 be harmonic.

(d) Calculate the natural frequency of oscillation θ_1 for the special cases.

(e) Determine the natural frequency for very large Ω and draw conclusions.

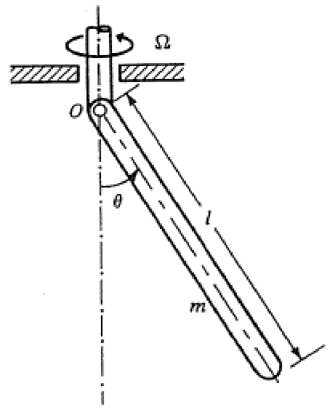

Fig.9

10. The simple pendulum of Fig.10 is immersed in viscous fluid so that there is a force $c\dot{\theta}$ resisting the motion. Derive the equation of motion for arbitrary amplitudes θ, then linearize the equation and obtain the frequency of the damped oscillation.

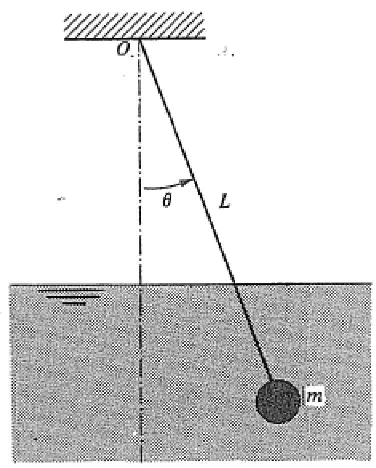

Fig.10

11. Calculate the frequency of damped oscillation of the system shown in Fig.11 for the values of $k = 7.0051 \times 10^5$ N/m, $c = 3502.54$ Ns/m, $m = 1751.27$ Kg, $a = 1.27$ m, and $L = 2.54$ m. Determine the value of the critical damping.

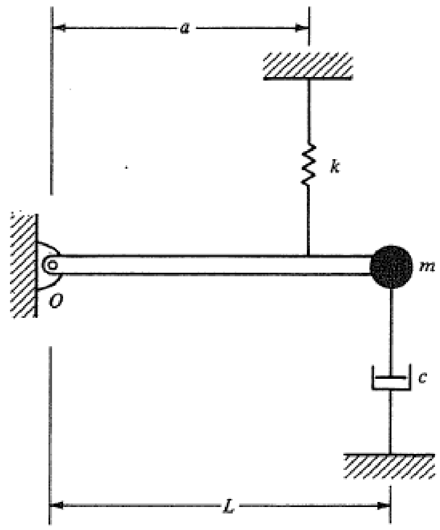

Fig.11

12. Find the fourier cosine and sine series expansion of the function shown in Fig.12 for $A = 2$ and $T = 1$.

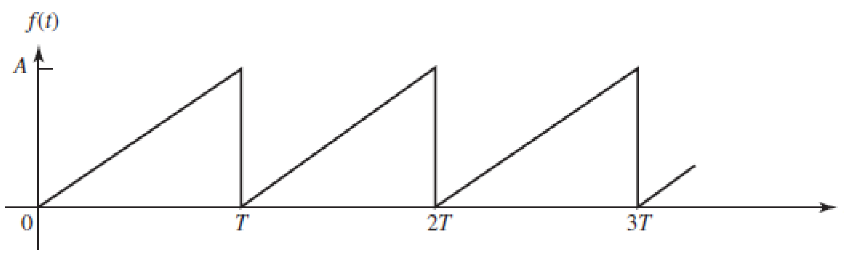

Fig.12

13. Express the following function as a sum of sine and cosine functions:

$$f(t) = 5\sin(10t - 2.5)$$

14. Using Laplace transform solve the following differential equation and discuss the behaviour of the solution, with graphs of y against t and with a phase space graph:

$$y" + 3y' + 2y = -5\sin(t) + 5\cos(t)$$

$$Given : y(0) = 5, y'(0) = -3$$

15. Find the Inverse Laplace transform of the following:

(a) F(s)= $\dfrac{5s^2 + 8s - 5}{s^2(s^2 + 2s + 5)}$

(b) F(s)= $\dfrac{s(1 + e^{-1.5s} + e^{-2.2s}) + e^{-1.5s}}{s(s + 2)}$