Practice Problems Set -2

ME-231A

P1. A velocity field is by $\vec{V} = axi - btyj$ where a = 1 s^{-1} and b = 1 s^{-2}. Find the equation of the streamlines at anytime t.

P2. The velocity for a steady, incompressible flow in the *x*-*y* plane is given by $\vec{V} = i\frac{A}{x} + j\frac{Ay}{x^2}$ where A = 2 m^2/s, and the coordinates are measured in meters. Obtain an equation for the streamline that passes through the point (*x*, *y*) = (1, 3). Calculate the time required for a fluid particle to move from *x* = 1 m to *x* = 2 m in this flow field.

P3.A flow field is given by $\vec{V} = Axi + 2Ayj$ where A = 2 s^{-1}.Verify that the parametric equations for particle motion are given by $x_p = c_1e^{At}$ and $y_p = c_1e^{2At}$. Obtain the equation for the path line of the particle located at the point (*x*, *y*) = (2, 2) at the instant t = 0. Compare this path line with the streamline through the same point.

P4. Consider the flow described by the velocity field, $\vec{V} = A(1 + Bt)i + Ctyj$ with A = 1 m/s, B = 1 s^{-1}, and C = 1 s^{-2}. Coordinates are measured in meters. Plot the path line traced out by the particle that passes through the point (1, 1) at time t = 0. Compare with the streamlines plotted through the same point at the instants *t* = 0, 1, and 2 s.

P5. Streak lines are traced out by neutrally buoyant marker fluid injected into a flow field from a fixed point in space. A particle of the marker fluid that is at point (*x*, *y*) at time t must have passed through the injection point (x_0, y_0) at some earlier instant t = τ. The time history of a marker particle may be found by solving the path line equations for the initial conditions that $x=x_0$, $y=y_0$ when t=τ. The present locations of particles on the streak line are obtained by setting τ equal to values in the range $0 \leq \tau \leq t$. Consider the flow field $\vec{V} = ax(1 + bt)i + cyj$ where a = c = 1 s and b = 0.2 s^{-1}. Coordinates are measured in meters. Plot the streakline that passes through the initial point (x_0, y_0) = (1, 1), during the interval from t = 0 to t = 3 s. Compare with the streamline plotted through the same point at the instants t = 0, 1, and 2 s.

P6. Consider the flow field $\vec{V} = i\ axt + j\ b$, where a = 1/4 s^2 and b = 1/3 m/s. Coordinates are measured in meters. For the particle that passes through the point (x, y) = (1, 2) at the instant t = 0, plot the path line during the time interval from t = 0 to 3 s. Compare this pathline with the streakline through the same point at the instant t = 3 s.

P7.Consider a flow with velocity components u = z $(3x^2 - z^2)$, v = 0, and w = $x(x^2 - 3z^2)$.

(a) Is this a one-, two-, or three-dimensional flow?

P8. Consider the velocity field $\vec{V} = A\left(x^4 - 6x^2y^2 + y^4\right)i + A\left(4xy^3 - 4x^3y\right)j$ in the xy plane, where $A = 0.25\ m^{-3}.s^{-1}$, and the coordinates are measured in meters. Calculate the acceleration of a fluid particle at point (x, y) = (2, 1).

P9. Consider the velocity field $\vec{V} = \frac{Ax}{\left(x^2 + y^2\right)}i + \frac{Ay}{\left(x^2 + y^2\right)}j$ in the *x-y* plane, where A = 10 m^2/s, and x and y are measured in meters. Is this an incompressible flow field? Derive an expression for the fluid acceleration. Evaluate the acceleration along the x axis, the y axis, and along a line defined by y = x. What can you conclude about this flow field?

P10. Consider the two-dimensional flow field in which u = Axy and v = By^2, where A = $1m^{-1}.s^{-1}$; B = $-1/2m^{-1}.s^{-1}$; and the coordinates are measured in meters. Show that the velocity field represents a possible incompressible flow. Determine the rotation at point (*x*, *y*) = (1, 1).

P11. Consider a velocity field for motion parallel to the x axis with constant shear. The shear rate is du/dy = A, where A = 0.1 s^{-1}. Obtain an expression for the velocity field, $\vec{V}$. Calculate the rate of rotation.

P12. Consider a flow field represented by the stream function $\vec{V} = -i\frac{Ay}{\left(x^2 + y^2\right)^2} + j\frac{Ax}{\left(x^2 + y^2\right)^2}$ where A = constant. Is the flow irrotational?

P13. Consider the pressure-driven flow between stationary parallel plates separated by distance 2b. Coordinate y is measured from the channel centerline. The velocity field is given by $u = u_{max}[1 - (y/b)^2]$. Evaluate the rates of linear and angular deformation. Obtain an expression for the vorticity vector, $\vec{\xi}$: Find the location where the vorticity is a maximum.

P14. The velocity profile for fully developed flow in a circular tube is $V_z = V_{max}[1 - (r/R)^2]$. Evaluate the rates of linear and angular deformation for this flow. Obtain an expression for the vorticity vector.

P15. Consider a plane Couette flow of a viscous fluid confined between two flat plates at a distance *b* apart (see Figure P15). At steady state the velocity distribution is

$$u = Uy/b \quad \text{and} \quad v = 0$$

where the upper plate at y = *b* is moving parallel to itself at speed *U*, and the lower plate is held stationary. Find the rate of linear strain, the rate of shear strain, and vorticity.

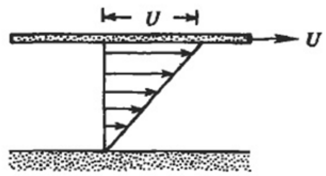

Fig.P15

P16. Consider the two-dimensional flow from a line source given in cylindrical coordinates by $V_r = Q/2\pi r$, $V_z = V_\theta = 0$, where Q is constant. Compute the components of the strain rate tensor for this flow.

P17. A three-dimensional velocity field is given by

$$u(x, y, z) = cx + 2w_o y + u_o$$
$$\mathrm{v}(x, y, z) = cy + v_o$$
$$\mathrm{w}(x, y, z) = -2cz + w_o$$

Where $c, w_{o,} u_o$ and v_o are constants. Find the components of (i) rotational velocity, (ii) vorticity and (iii) the strain rates for the above flow field.

P18. The position of a fluid particle in a Lagrangian system is described as:

$x = x_0 e^{kt}, y = y_0 e^{kt}$; where x_0, y_0 and k are constants. State with proof about the temporal state (steady or unsteady) of the flow field.