ME351 - Design Exercise #5

Given data: Maximum pressure - $p = 5\,MPa$
Design factor $n_d = 2$
Sealing diameter $D_s = 144\,mm$

Stud details: ISO 4.8 $S_p = 310\,MPa$
$S_y = 340\,MPa$
$S_{ut} = 420\,MPa$

Size - $M10 \times 1.5$ — $A_t = 58\,mm^2$

Bolt circle dia $D_b = 180\,mm$

Bolt spacing - s $\quad 3d < s < 6d$, $d = 10\,mm$

$s = \frac{\pi D_b}{N}$, N - number of studs

$\frac{\pi \times 180}{N} = 6d = 60$, $N = 9.42$ take $\underline{N = 10}$ — $N = 10$.

Total force on the bolts $= \frac{\pi D_s^2}{4} \times p = \frac{\pi \times 144^2}{4} \times 5\,N/mm^2$

$= 81430\,N$

Load per bolt $P = \frac{81430}{10} = \underline{8143\,N}$ — $P = 8143\,N$

Bolt stiffness

Grip length $l = 400 + 20 + 20 = 440\,mm$

$k_b = \frac{AE}{l} = \frac{\pi \times 10^2 \times 200 \times 10^3}{4 \times 440} = 35700\,N/mm$

Joint stiffness

we will assume the end plates to be rigid compared to the cylinder

$$k_m = \frac{A_{cyl} E_{cyl}}{l_{cyl}} = \frac{\pi(150^2 - 138^2) \times 200 \times 10^3}{4 \times 400} = 1357.2 \times 10^3 \text{ N/mm}$$

Joint stiffness coefficient

$$C = \frac{10 k_b}{10 k_b + k_m} = 0.2083$$

$C = 0.2083$.

$$F_i = 0.75 S_p A_t = 13485 \text{ N}, \quad \sigma_i = \frac{F_i}{A_t} = 232.5 \text{ MPa}$$

$$\sigma_b = \sigma_i + \frac{n_d C P}{A_t} = 0.75 \times 310 + \frac{2 \times 0.2083 \times 8143}{58} = 291 \text{ MPa}$$

$$291 \text{ MPa} < 310 \text{ MPa } (S_p)$$

$$n = \frac{S_p A_t - F_i}{CP} = \underline{2.65 > 2 \quad O.K}$$

$n = 2.65$
against proof

Joint separation

$$P_0 = \frac{F_i}{(1 - C)} = 17033 \text{ N}$$

$$n_0 = \frac{P_0}{P} = \underline{2.09}$$

$n_0 = 2.09$

Sealing pressure $= \dfrac{-4 F_m \times N}{\pi(150^2 - 138^2)}$

$$F_m = (1 - C)P - F_i = -7038.2 \text{ N}$$

Sealing pressure $= 26 \text{ MPa} > 5 \text{ MPa}$

Preloading the assembly

$$T = 0.2 F_i d = 0.2 \times 13485 \times 10 = 26970 \text{ N-mm}$$

$$\tau = \frac{16T}{\pi d^3} = 137.35 \text{ MPa}$$

use von-Mises $\sigma' = \sqrt{\sigma_i^2 + 3\tau^2} = 332.7 \text{ MPa} < 340 \text{ MPa}$

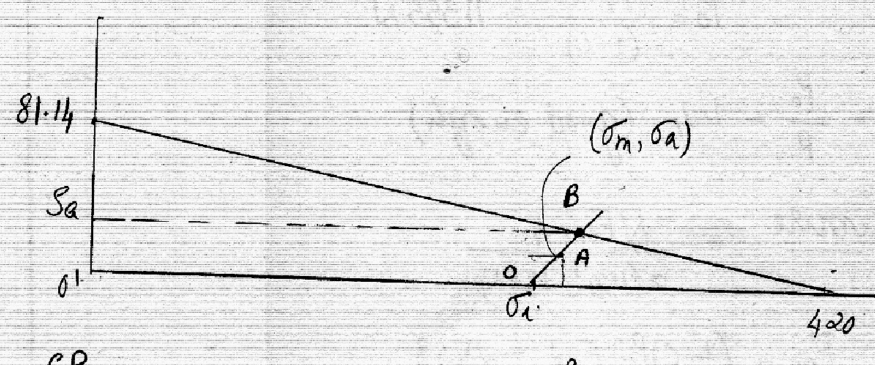
There is very little margin, so T has to be reduced by lubrication

Fatigue failure

$S_e = \dfrac{0.85 \times S_e'}{K_f}$, $S_e' = 0.5 S_{ut} = 210\ MPa$,

$S_e = \dfrac{0.85 \times 210}{2.2} = \underline{81.14\ MPa}$

$S_e' = 210\ MPa$
$K_f = 2.2$
$k_e = 1$
$\underline{S_e = 81.14\ MPa}$



$\sigma_m = \dfrac{F_i}{A_t} + \dfrac{CP}{2A_t} = 232.5 + 14.62 = 247.1\ MPa$

$\sigma_a = \dfrac{CP}{2A_t} = 14.62\ MPa$

$n_f = \dfrac{OB}{OA} = \dfrac{S_a}{\sigma_a}$

$\dfrac{\sigma_i + S_a}{S_{ut}} + \dfrac{S_a}{S_e} = 1$, $S_a\left(\dfrac{1}{S_e} + \dfrac{1}{S_{ut}}\right) = 1 - \dfrac{\sigma_i}{S_{ut}}$

$S_a = 30.35$, $\underline{n_f = 2.08}$

$n_f = 2.08$

Final part

when $P = P_0$, $F_m = 0$ and joint will leak

$P_0 = 17033 N$, Pressure in the cylinder $= \dfrac{4 N P_0}{\pi \times 144^2} = 10.5\ MPa$

The cylinder is designed with n_d of 4 i.e it can go up to 20 MPa pressure – safe

when $F_m = 0$, $\sigma_b = \sigma_i + \dfrac{P}{A_t} = 232.5 + \dfrac{17033}{58} = 526.2\ MPa$

$\sigma_b > S_{ut}$ hence bolts will fail first –

Not a good situation as it does not give any warning. whereas a leak will prevent over pressurizing

Take $F_i = 0.5 S_p A_t = 8990\,N$.

Bolt failure

$$n = 5.3 > 2.0$$

Joint separation

$$P_0 = \frac{F_i}{(1-C)} = 11355\,N$$

$$n_0 = \frac{P_0}{P} = 1.4 \text{ (good enough)}$$

Sealing pressure $= \dfrac{-NF_m \times 4}{\pi(150^2 - 138^2)}$

$$F_m = (1-C)P - F_i = -2543$$

Sealing pressure $= 9.4\,MPa > 5\,MPa$

Pressure at joint separation

$$p = \frac{4P_0 \times 10}{\pi \times 144^2} = \underline{6.97\,MPa}$$

Bolt stress at joint separation

$$\sigma_b = \sigma_i + \frac{P_0}{A_t} = 155 + \frac{11355}{58} = 351\,MPa < S_{ut}$$

For this case at most the bolt will yield a little and due to leakage pressure will drop.

Fatigue failure

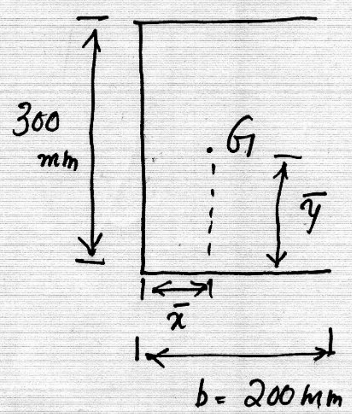
$$\sigma_m = 155 + 14.62 = 169.62\,MPa$$

$$\sigma_a = 14.62\,MPa$$

$$S_a = 43\,MPa$$

$$n_f = \frac{43}{14.62} = \underline{2.93}$$

Torsional loading - use table 9-1



$$\bar{x} = \frac{b^2}{2b+d} = 57.14\,mm$$

$$\bar{y} = \frac{d}{2} = 150\,mm$$

$$A_t = 0.707h(2b+d) = h \times 494.9\,mm^2$$

$$= 494.9h\ mm^2$$

$$J_u = \frac{8b^3 + 6bd^2 + d^3}{12} - \frac{b^4}{2b+d} = 14297619.\ mm^3$$

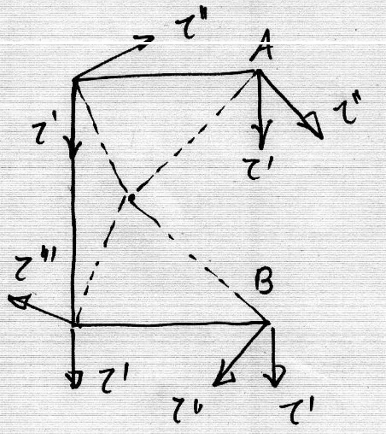
$$J = 0.707h\,J_u = 10108416.7\,h\ mm^4$$

$$\text{Torque} = F(1000 + 200 - 57.14) \quad (\text{about } G)$$

$$= 50 \times 10^3 \times 1142.86\ N\text{-}mm$$

Primary shear $\tau' = \frac{F}{A_t} = \frac{50 \times 10^3}{494.9h} = \frac{101.03}{h}$

Secondary shear $\tau'' = \frac{Tr}{J} = \frac{50 \times 10^3 \times 1142.86 \times r}{10108416.7} = \frac{5.653r}{h}$



A and B critical points. (equally)

Take A.

$\tau'_x = 0 \qquad \tau'_y = +\frac{101.03}{h}$

$\tau''_x = \frac{T\bar{y}}{J} = \frac{5.653 \times 150}{h} = \frac{847.95}{h}$

$\tau''_y = \frac{T(200-\bar{x})}{J} = \frac{807.6}{h}$

Resultant

$$\tau = \left\{ (\tau_x' + \tau_x'')^2 + (\tau_y' + \tau_y'')^2 \right\}^{1/2}$$

$$\tau = \left\{ 847.95^2 + (101.03 + 807.6)^2 \right\}^{1/2} \times \frac{1}{h} = \frac{1242.8}{h}.$$

For electrode E80 $S_{ut} = 551\,MPa$, $S_{yt} = 462\,MPa$

(table 9-3).

$$S_{sy} = 0.577\, S_y = 0.577 \times 462 = 266.574\ MPa$$

$$\frac{S_{sy}}{n_f} = \tau \Rightarrow \frac{0.577 \times 462}{1.5} = \frac{1242.8}{h}, \quad h = 6.993\ mm$$

Leg size $h = 7\ mm$

Stress in the plate

$$I = \frac{10 \times 300^3}{12}, \quad M = 50 \times 10^3 \times 1000$$

$$\sigma = \frac{Md/2}{I} = \frac{M \times 150}{I} = \frac{50 \times 10^6 \times 150 \times 12}{10 \times 300^3} = 333.33\,MPa$$

For 1030 CD, $S_y = 440\,MPa$, $n_f = \frac{S_y}{\sigma} = 1.32$ (o.k)

Q. 2

Combined reliability for two bearings A and B is $R_A R_B = 0.9$, $R_A = R_B = \sqrt{0.9} = 0.949$

L_{10} and C_{10} are based on 0.9 reliability.

$$C_{10} = F_D \left(\frac{L_D}{6.84\, L_{10}}\right)^{1/a} \left(\frac{1}{\left\{\ln\left(\frac{1}{R}\right)\right\}^{\frac{1}{1.17a}}}\right)$$

$a = 3$ for ball bearing and $a = 10/3$ for roller bearing

Consider roller bearing

$F_D = 6000 \times 1.2 = 7200\,N$

$L_{10} = 10^6$ revolutions

$L_D = 1000 \times 60 \times 400 = 24 \times 10^6$ revolutions

$$C_{10} = 7200 \times \left(\frac{24 \times 10^6}{6.84 \times 10^6}\right)^{1/a} \times \left(\frac{1}{\left\{\ln\left(\frac{1}{0.949}\right)\right\}^{1/(1.17 \times 10/3)}}\right)$$

$C_{10} = 7200 \times 1.4573 \times 2.1271 = 22.32\,kN$

Select 02 Series with bore of 30 mm from table (11-3)

Deep-Groove ball bearing.

$F_a = 1000\,N$, $F_r = 4000\,N$, $V = 1$ (Inner race rotating)

$\frac{F_a}{V F_r} = 0.25$ Start with $X_2 = 0.56$, $Y_2 = 1.5$

$F_e = 0.56 \times 4000 + 1.5 \times 1000 = 3740\,N$

$F_D = 1.2 F_e = 4488\,N$

(Note $a = 3$).

$C_{10} = 4488 \times 1.5196 \times 2.3131$

Take 02 series bore 30 (table 11-2)
deep groove ball bearing

$C_{10} = 19.5\,kN, \quad C_0 = 10\,kN$

$$\frac{F_a}{C_0} = \frac{1000}{10000} = 0.1$$

From table 11-1

$$\frac{F_a}{C_0} = 0.084 \qquad e = 0.28$$

$$\frac{F_a}{C_0} = 0.11 \qquad e = 0.3$$

For our case $\frac{F_a}{F_r} = 0.25 < e$ so

$X_1 = 1, \; Y_1 = 0$ applies

$F_e = F_r = 4000\,N$

$F_D = 1.2 \times 4000 = 4800\,N$

$C_{10} = 4800 \times 1.5196 \times 2.3131 = 16.87\,kN < 19.5\,kN$

So chosen bearing is fine