Tutorial 5 – Solution

(a) General Constraints: $4 \le C \le 12$

$3 \le N_a \le 15$

$\zeta \ge 0.15$

$n_s \ge 1.2$

Problem Specific Constraints: $L_o \le 48\,mm$

$L_s \le 31.5\,mm$

$n_1 \ge 1.5$

(b) Choose wire diameter $d = 2\,mm$

Free length $L_o = 48\,mm$.

$\Rightarrow$ Maximum deflection (at service load $F_{max} = 45N$)

$y_{max} = (48 - 37.5) = 10.5\,mm$

$\Rightarrow$ Spring rate $\boxed{K = \frac{F_{max}}{y_{max}} = 4.286}$

Sizes: Screw Thread M10 x 1.25 g Clearance = 1.25 mm

$\Rightarrow$

$ID = 10 + 1.25 = 11.25\,mm$

$OD = ID + 2d = 15.25\,mm$

$D = ID + d = 13.25\,mm$

$C = D/d = 6.625\,mm$

Shear Modulus: $G = 79.3\,MPa$

(Table-10-5: d between 1.8–3 mm $\Rightarrow$ d ~ 0.064—0.125 in)

$\Rightarrow \boxed{N_a = \frac{d^4 G}{8 K D^3} = 15.9 \text{ turns}}$ (Not okay)

$N_t = N_a + 2 = 17.9$ turns (Table 10-1)

(for squared and ground ends)

Solid height: $\boxed{L_s = d N_t = 35.8\,mm}$ (Table 10-1) ← (not okay)

$\Rightarrow y_s = 48 - 35.8 = 12.2\,mm$

(deflection at solid length)

$\Rightarrow F_s = K y_s = 52.289\,N$

$\Rightarrow \boxed{\zeta = \frac{F_s}{F_{max}} - 1 = 0.1627015}$ not okay

[redacted] is outside linear operating region)

continue with further calculation

$$K_B = \frac{4C+2}{4C-3} = 1.213$$

$$\Rightarrow \tau_s = K_B\left(\frac{8F_s D}{\pi d^3}\right) = 267.51 \text{ MPa}$$

Table 10-4: $A = 1783 \text{ MPa}\cdot(\text{mm})^m$, $m = 0.19$
relative cost = 1 (for A227-HD)

$$\Rightarrow S_{sy} = 0.45 S_{ut} = 0.45 \frac{A}{d^m}$$

$$\Rightarrow \boxed{S_{sy} = 703 \text{ MPa}}$$

$$\Rightarrow \boxed{n_s = \frac{S_{sy}}{\tau_s} = 2.63 > 1.2} \text{ (okay)}$$

$$\tau_{max} = K_B\left(\frac{8F_{max}D}{\pi d^3}\right) = 230 \text{ MPa}$$

$$\Rightarrow \boxed{n_1 = \frac{S_{sy}}{\tau_{max}} = 3.054 > 1.5} \text{ (okay)}$$

$$\boxed{f_{om} = -\frac{\pi^2 d^2 D N_t}{4} = -2341}$$

Since we do not satisfy all the condition, especially we are outside linear operation region. Need to reiterate.

d	1.8 mm	2 mm	2.2 mm
ID	11.25	11.25	11.25
OD	14.85	15.25	15.65
D	13.05	13.25	13.45
C	7.25	6.625	6.11
N_a	10.9	15.9	22.2
L_o	48	48	48
N_t	12.9	17.9	24.2
L_s	23.22	35.8	53.24 > L_o (not okay)
ζ	1.36	0.139	
n_s	0.994	2.63	
n_1	2.3	3.054	
fom	−1345	−2341	

Note: We do not need to iterate further. Observation

1. For d = 2 mm: $\zeta = 0.139 < 0.15$
   ⇒ outside linear regime
   Na = 15.9 may still be tolerated.
2. d = 1.8 mm: $n_s = 0.994 < 1$
   This is not at all desirable
3. d = 2.2 and higher:
   $L_s > L_o$ = free length
   Can not be manufactured.

⇒ Need to iterate again.

(c) One more parameter that we can play with is L_o, satisfying $L_o < 48$ mm.

⇒ Choose $L_o = 45$ mm.

⇒ $y_{max} = (45 - 37.5) = 7.5$ mm

⇒ $k = \frac{45}{7.5} = 6$ N/mm $= 6000$ N/m

⇒ For $L_o = 45$ mm

d	1.8mm	2mm.	2.2mm
ID	11.25mm	11.25mm	11.25
OD	14.85	15.25	15.65
D	13.05	13.25	13.45
C	7.25	6.625	6.11
N_a	7.8	11.36	15.9
L_o	45	45	45
N_t	9.8	13.36	17.9
L_s	19.04	26.72	39.38 > 37.5 = workpiece thickness (not feasible)
ζ	3.528	1.437	
n_s	0.594	1.288	
n_1	2.3	3.054	
fom	−1022.4	−17.47	

⇒ Only the wire diameter $d = 2$ mm satisfy all the criteria.

⇒ This should be chosen

(d) Buckling: $L_o < \frac{2.63D}{\alpha} = 5.26D = 69.7$mm (Table 10-2 : $\alpha = 0.5$) (safe)

Critical frequency: $f = \frac{1}{2}\sqrt{\frac{kg}{W}}$, $W = \frac{\gamma \pi^2 d^2 D N_a}{4} = 0.113$ N

(Table-A5) $\gamma = 76$ kN/m³ ⇒ $f = 361$ Hertz.

since, operating frequency is 5 Hz, the spring is safe.

Fatigue factor of safety:

Zimmerli Data (unpeened):

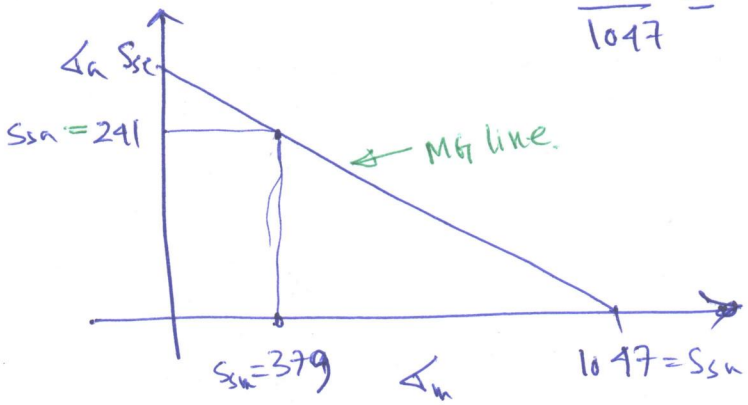
$S_{sa} = 241\ MPa, \quad S_{sm} = 379\ MPa$

Table 10-4: $S_{ut} = \frac{A}{d_m} = 1563\ MPa$

$\Rightarrow S_{sn} = 0.67 S_{ut} = 1047\ MPa.$

$\Rightarrow$ Using Goodman Criterion

$$\frac{S_{se}}{1047} = \frac{S_{sa}}{(1047 - S_{sm})} = \frac{241}{(1047-379)}$$



$\Rightarrow \boxed{S_{se} = 377.7\ MPa}$

$$\Rightarrow n_f = \frac{1}{\frac{\tau_a}{S_{sa}} + \frac{\tau_m}{S_{sn}}}$$

$\tau_{max} = 230.00\ MPa \Rightarrow \tau_a = \tau_m = 115\ MPa$

$\tau_{min} = 0\ MPa$

$\Rightarrow \boxed{n_f = 2.414}$

Using Sines Criteria: $S_{se} = S_{sa} = 241\ MPa.$

$$\Rightarrow \boxed{n_f = \frac{S_{sa}}{\tau_a} = 2.10}$$