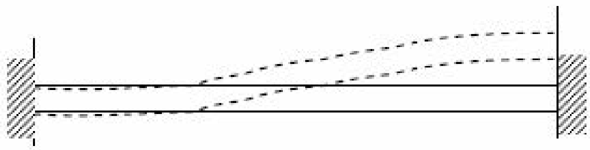
ESO 202A/204: Mechanics of Solids
(2016-17 II semester)
Solution of Assignment No. 10

10.1



$$q = 0 \Rightarrow EI\frac{d^4v}{dx^4} = 0$$

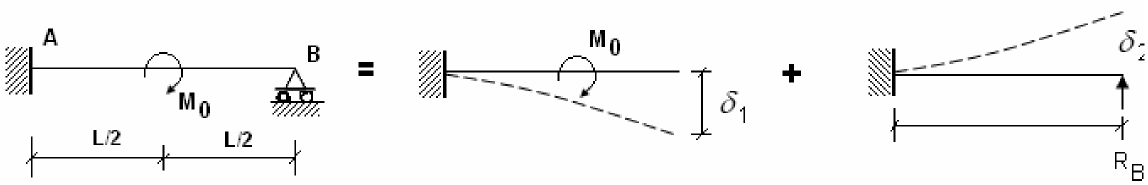
$$\Rightarrow EIv = \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

At x = 0, v = 0 and $\frac{dv}{dx} = 0 \Rightarrow C_3 = C_4 = 0$

At x = L, v = Δ and $\frac{dv}{dx} = 0 \Rightarrow C_1 = -\frac{12EI\Delta}{L^3}$ and $C_2 = \frac{6EI\Delta}{L^2}$

Thus, v = $\Delta(3\frac{x^2}{L^2} - 2\frac{x^3}{L^3})$

10.2



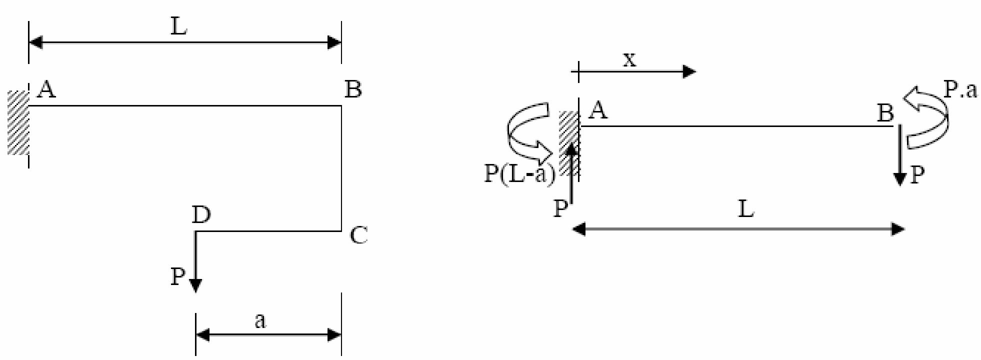
$$\delta_2 = \frac{R_BL^3}{3EI} \qquad \delta_1 = \frac{M_0(L/2)^2}{2EI} + \frac{M_0(L/2)}{EI}\frac{L}{2} = \frac{M_0L^2}{8EI} + \frac{M_0L^2}{4EI} = \frac{3M_0L^2}{8EI}$$

Geometric constraint: $\delta_1 - \delta_2 = 0$

$$\Longrightarrow \quad \frac{R_BL^3}{3EI} = \frac{3M_0L^2}{8EI}$$

$$\Longrightarrow \quad R_B = \frac{9M_0}{8L}$$

10.3



Slope at B due to force P = $\theta_P = \frac{PL^2}{2EI}(Clockwise)$

Deflection at B due to force P = $\Delta_P = \frac{PL^3}{3EI}(Downward)$

Slope at B due to moment P.a = $\theta_{M=P.a} = \frac{PaL}{2EI}(anti-clockwise)$

Deflection at B due to moment P.a = $\Delta_{M=P.a} = \frac{PaL^2}{2EI}(upward)$

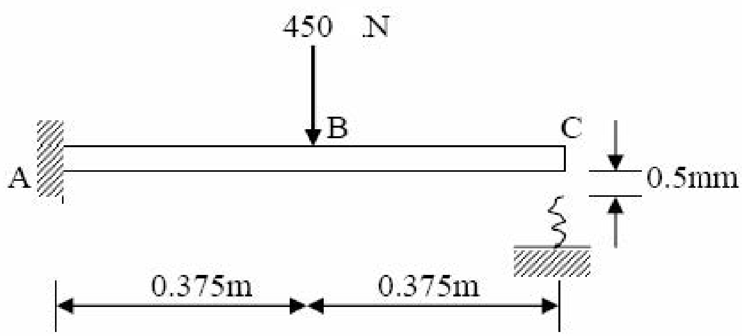
(i) For no net deflection at B,

$\Delta_P = \Delta_{M=P.a}$ gives, a/L =2/3

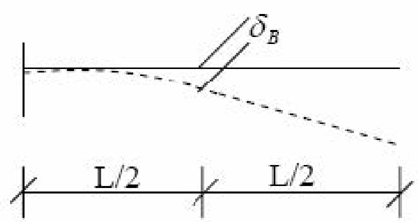
(ii) For no net angular rotation at B,

$\theta_P = \theta_{M=P.a}$ gives, a/L =1/2

10.4



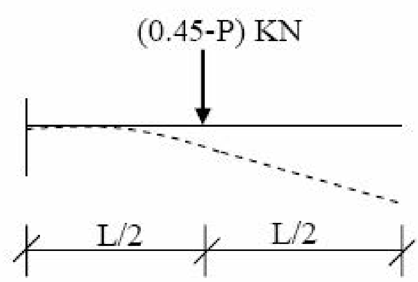
End C behaves as a free end up to a deflection of 0.5mm.



$$\delta_C = \delta_B + \text{(slope at B).(L/2)}$$
$$= -\frac{P(L/2)^3}{3EI} - \frac{P(L/2)^2}{2EI}.\frac{L}{2}$$
$$= -\frac{5PL^3}{48EI}$$

Equating $\delta_C = -0.5mm$, we have P = 0.34133 KN

From superposition principle,



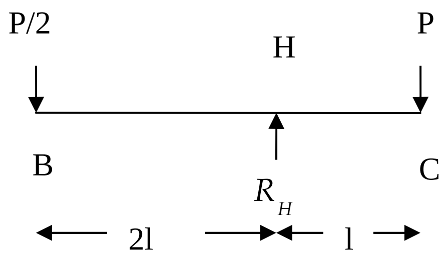
$$-EI\Delta = -\frac{5.(0.45-P)L^3}{48} + \frac{k.\Delta.L^3}{3}$$

$\Rightarrow \Delta = -1.687 \times 10^{-5} m$

$\Rightarrow$ force in the spring = k. Δ = 0.0304 KN.

10.5

Free body diagram of Beam BC



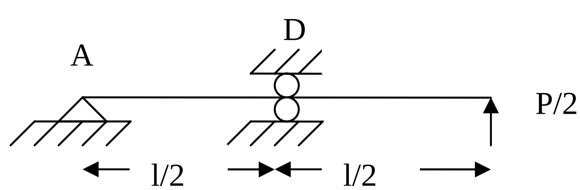
Force at B,

Force equilibrium, at H.

$\Rightarrow$ Force at B = P/2.

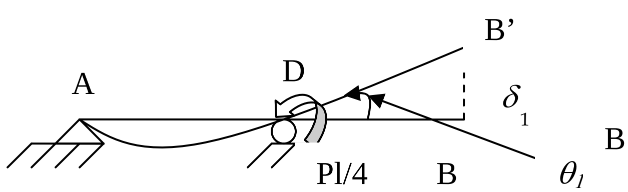
$\Rightarrow$ Force on the beam AB at B is P/2 ($\uparrow$)

Deflection at B due to P/2 ($\uparrow$) acting at B on the beam AB.



From the above diagram,

$M_D = \frac{Pl}{4}$. The effect of P/2, on the beam AB, is as shown below.



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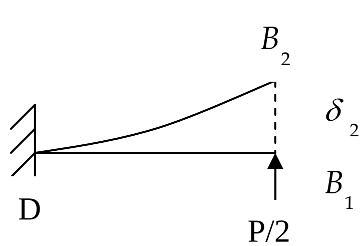


$$\theta_1 = \frac{Pl}{4} \cdot \frac{l}{2} \cdot \frac{1}{3EI}$$

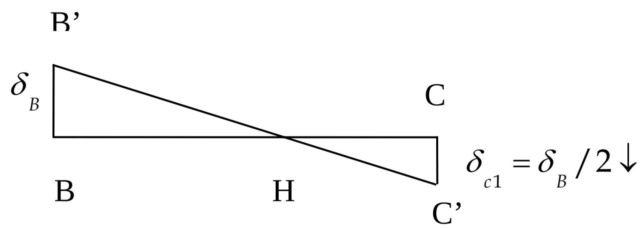
$$\delta_1 = \frac{l}{2} . \theta_1 = \frac{Pl^3}{48EI} \uparrow$$

$$\delta_2 = \frac{\frac{P}{2}(\frac{l}{2})^3}{3 \; EI} \uparrow$$

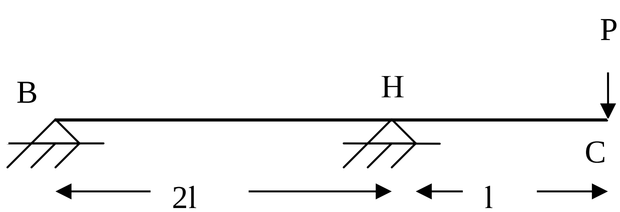
$$\delta_B = \delta_1 + \delta_2 = \frac{Pl^3}{24EI}$$

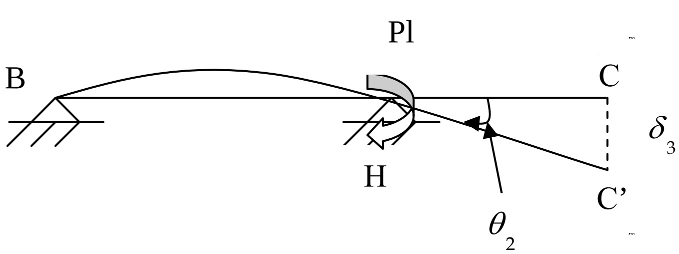
If the beam BC was rigid, the deflection at C

Through symmetry one can find $\delta_{c1} = \frac{Pl^3}{48EI}$



Superimposing bending deflection of BC on B'C'





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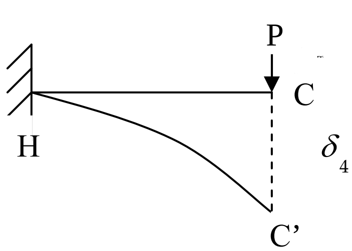

$$\theta_2 = \frac{Pl.2l}{3EI}$$

$$\delta_3 = l\theta_2 = \frac{2Pl^3}{3EI}\downarrow$$

$$\delta_4 = \frac{Pl^3}{3EI}\downarrow$$

so,

$$\delta_c = \delta_{c1} + \delta_3 + \delta_4 = \frac{49}{48}.\frac{Pl^3}{EI}$$