Practice Problem Set - 5

ME-231A

P1. A two-dimensional steady flow in a viscous liquid is described by the equation:

$$u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} + \frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Use a length scale, L, and a velocity scale, V_o, to non-dimensionalize this equation. Obtain the dimensionless groups that characterize this flow.

P2. The wall shear stress, τ_w, in a boundary layer depends on distance from the leading edge of the body, x, the density, ρ, and viscosity, μ, of the fluid, and the freestream speed of the flow, U. Obtain the dimensionless groups and express the functional relationship among them.

P3. The boundary-layer thickness, δ, on a smooth flat plate in an incompressible flow without pressure gradients depends on the freestream speed, U, the fluid density, ρ, the fluid viscosity, μ, and the distance from the leading edge of the plate, x. Express these variables in dimensionless form.

P4. If an object is light enough it can be supported on the surface of a fluid by surface tension. Tests are to be done to investigate this phenomenon. The weight, W, supportable in this way depends on the object's perimeter, p, and the fluid's density, ρ, surface tension σ, and gravity, g. Determine the dimensionless parameters that characterize this problem.

P5. The mean velocity, $\overline{u}$, for turbulent flow in a pipe or a boundary layer may be correlated using the wall shear stress, τ_w, distance from the wall, y, and the fluid properties, ρ and μ. Use dimensional analysis to find one dimensionless parameter containing u and one containing y that are suitable for organizing experimental data. Show that the result may be written as

$$\frac{\overline{u}}{u_*} = f\left(\frac{y u_*}{\nu}\right)$$

Where $u_* = (\tau_w/\rho)^{1/2}$ is the friction velocity.

P6. In a fluid mechanics laboratory experiment a tank of water, with diameter D, is drained from initial level h_o. The smoothly rounded drain hole has diameter d. Assume the mass flow rate from the tank is a function of h, D, d, g, ρ, and μ, where g is the acceleration of gravity and ρ and μ are fluid properties. Measured data are to be correlated in dimensionless form. Determine the number of dimensionless parameters that will result. Specify the number of repeating parameters that must be selected to determine the dimensionless parameters. Obtain the Π parameter that contains the viscosity.

P7. In a fan-assisted convection oven, the heat transfer rate to a roast, $\dot{Q}$ (energy per unit time), is thought to depend on the specific heat of air, c_p, temperature difference, Θ, a length scale, L, air density, ρ, air viscosity, μ, and air speed, V. How many basic dimensions are included in these variables? Determine the number of Π parameters needed to characterize the oven. Evaluate the Π parameters.

P8. The thrust of a marine propeller is to be measured during "open-water" tests at a variety of angular speeds and forward speeds ("speeds of advance"). The thrust, F_T, is thought to depend on water density, ρ, propeller diameter, D, speed of advance, V, acceleration of gravity, g, angular speed, ω, pressure in the liquid, p, and liquid viscosity, μ. Develop a set of dimensionless parameters to characterize the performance of the propeller. (One of the resulting parameters, gD/V^2, is known as the Froude speed of advance.)

P9. The power, P, required to drive a propeller is known to depend on the following variables: freestream speed, V, propeller diameter, D, angular speed, ω, fluid viscosity, μ, fluid density, ρ, and speed of sound in the fluid, c. How many dimensionless groups are required to characterize this situation? Obtain these dimensionless groups.

P10. The mass burning rate of flammable gas $\dot{m}$ is a function of the thickness of the flame δ, the gas density ρ, the thermal diffusivity α, and the mass diffusivity D. Using dimensional analysis, determine the functional form of this dependence in terms of dimensionless parameters. Note that α and D has the dimensions L^2/t.

P11. At very low speeds, the drag on an object is independent of fluid density. Thus the drag force, F, on a small sphere is a function only of speed, V, fluid viscosity, μ, and sphere diameter, D. Use dimensional analysis to determine how the drag force F depends on the speed V.

P12. A model differential equation, for chemical reaction dynamics in a plug reactor, is as follows:

$$u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} - kC - \frac{\partial C}{\partial t}$$

where u is the velocity, D is a diffusion coefficient, k is a reaction rate, x is distance along the reactor, and C is the (dimensionless) concentration of a given chemical in the reactor. (a) Determine the appropriate dimensions of D and k. (b) Using a characteristic length scale L and average velocity V as parameters, rewrite this equation in dimensionless form and comment on any Pi groups appearing.

P13. A 1/25 scale model of a submarine is being tested in a wind tunnel in which p = 200 kPa and T = 300 K. If the prototype speed is 30km/hr, what should be the free-stream velocity in the wind tunnel? What is the drag ratio? Assume that the submarine would not operate near the free surface of the ocean.

P14. In turbulent flow past a flat surface, the velocity u near the wall varies approximately logarithmically with distance y from the wall and also depends upon viscosity μ, density ρ, and wall shear stress τ_w. For a certain airflow at 20°C and 1 atm, τ_w = 0.8 Pa and u = 15 m/s at y = 3.6 mm. Use this information to estimate the velocity u at y = 6 mm.

P15. The pressure drop in a venturi meter varies only with the fluid density, pipe approach velocity, and diameter ratio of the meter. A model venturi meter tested in water at 20°C shows a

5-kPa drop when the approach velocity is 4 m/s. A geometrically similar prototype meter is used to measure gasoline at 20°C and a flow rate of 9 m^3/min. If the prototype pressure gage is most accurate at 15 kPa, what should the upstream pipe diameter be?

P16. The drag of a sonar transducer is to be predicted, based on wind tunnel test data. The prototype, a 1-ft diameter sphere, is to be towed at 5 knots (nautical miles per hour) in seawater at 40°F. The model is 6 in. in diameter. Determine the required test speed in air. If the drag of the model at these test conditions is 0.60 lbf, estimate the drag of the prototype.

P17. A one-twelfth-scale model of an airplane is to be tested at 20°C in a pressurized wind tunnel. The prototype is to fly at 240 m/s at 10-km standard altitude. What should the tunnel pressure be in atm to scale both the Mach number and the Reynolds number accurately?

P18. If a vertical wall at temperature T_w is surrounded by a fluid at temperature T_o, a natural convection boundary layer flow will form. For laminar flow, the momentum equation is

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho\beta\left(T-T_o\right)g+\mu\frac{\partial^2 u}{\partial y^2}$$

to be solved, along with continuity and energy, for (u, v, T) with appropriate boundary conditions. The quantity β is the thermal expansion coefficient of the fluid. Use ρ, g, L, and (T_w-T_o) to non-dimensionalize this equation. Note that there is no "free-stream" velocity in this type of flow.