# **Homework-9 Solutions**

### Q 8-19

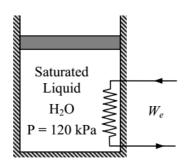
*Analysis* The exergy of the supplied heat, in the rate form, is the amount of power that would be produced by a reversible heat engine,

$$\eta_{\text{th,max}} = \eta_{\text{th,rev}} = 1 - \frac{T_0}{T_H} = 1 - \frac{298 \text{ K}}{1500 \text{ K}} = 0.8013$$
Exergy =  $\dot{W}_{\text{max,out}} = \dot{W}_{\text{rev,out}} = \eta_{\text{th,rev}} \dot{Q}_{\text{in}}$ 
=  $(0.8013)(150,000/3600 \text{ kJ/s})$ 
=  $33.4 \text{ kW}$ 

## Q 8-36

Assumptions 1 The kinetic and potential energy changes are negligible. 2 The cylinder is well-insulated and thus heat transfer is negligible. 3 The thermal energy stored in the cylinder itself is negligible. 4 The compression or expansion process is quasi-equilibrium.

Analysis (a) From the steam tables (Tables A-4 through A-6),



$$u_1 = u_{f@120 \text{ kPa}} = 439.27 \text{ kJ/kg}$$

$$P_1 = 120 \text{ kPa} \begin{cases} \mathbf{v}_1 = \mathbf{v}_{f@120 \text{ kPa}} = 0.001047 \text{ m}^3/\text{kg} \\ h_1 = h_{f@120 \text{ kPa}} = 439.36 \text{ kJ/kg} \end{cases}$$
sat. liquid
$$s_1 = s_{f@120 \text{ kPa}} = 1.3609 \text{ kJ/kg} \cdot \text{K}$$

The mass of the steam is

$$m = \frac{V}{V_1} = \frac{0.008 \text{ m}^3}{0.001047 \text{ m}^3/\text{kg}} = 7.639 \text{ kg}$$

We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. The energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{\text{in}} - E_{\text{out}}}_{\text{Net energy transfer}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{\text{e,in}} - W_{\text{b,out}} = \Delta U$$

$$W_{\text{e,in}} = m(h_2 - h_1)$$

since  $\Delta U + W_b = \Delta H$  during a constant pressure quasi-equilibrium process. Solving for  $h_2$ ,

$$h_2 = h_1 + \frac{W_{e,in}}{m} = 439.36 + \frac{1400 \text{ kJ}}{7.639 \text{ kg}} = 622.63 \text{ kJ/kg}$$

Thus,

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{622.63 - 439.36}{2243.7} = 0.08168$$

$$P_2 = 120 \text{ kPa}$$

$$h_2 = 622.63 \text{ kJ/kg}$$

$$\begin{cases} s_2 = s_f + x_2 s_{fg} = 1.3609 + 0.08168 \times 5.93687 = 1.8459 \text{ kJ/kg} \cdot \text{K} \\ u_2 = u_f + x_2 u_{fg} = 439.24 + 0.08168 \times 2072.4 = 608.52 \text{ kJ/kg} \end{cases}$$

$$\mathbf{v}_2 = \mathbf{v}_f + x_2 \mathbf{v}_{fg} = 0.001047 + 0.08168 \times (1.4285 - 0.001047) = 0.1176 \text{ m}^3/\text{kg}$$

The reversible work input, which represents the minimum work input  $W_{\text{rev,in}}$  in this case can be determined from the exergy balance by setting the exergy destruction equal to zero,

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net exergy transfer by heat, work, and mass}} - \underbrace{X_{\text{destroyed}}^{70 \text{ (reversibe)}}}_{\text{Exergy destruction}} = \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}} \rightarrow W_{\text{rev,in}} = X_2 - X_1$$

Substituting the closed system exergy relation, the reversible work input during this process is determined to be

$$W_{\text{rev,in}} = -m[(u_1 - u_2) - T_0(s_1 - s_2) + P_0(\mathbf{v}_1 - \mathbf{v}_2)]$$

$$= -(7.639 \text{ kg})\{(439.27 - 608.52) \text{ kJ/kg} - (298 \text{ K})(1.3609 - 1.8459) \text{ kJ/kg} \cdot \text{K} + (100 \text{ kPa})(0.001047 - 0.1176)\text{m}^3 / \text{kg}[1 \text{ kJ/1 kPa} \cdot \text{m}^3]\}$$

$$= 278 \text{ kJ}$$

(b) The exergy destruction (or irreversibility) associated with this process can be determined from its definition  $X_{\text{destroyed}} = T_0 S_{\text{gen}}$  where the entropy generation is determined from an entropy balance on the cylinder, which is an insulated closed system,

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$S_{\text{gen}} = \Delta S_{\text{system}} = m(s_2 - s_1)$$

Substituting,

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} = m T_0 (s_2 - s_1) = (298 \text{ K})(7.639 \text{ kg})(1.8459 - 1.3609) \text{kJ/kg} \cdot \text{K} = 1104 \text{ kJ}$$

Assumptions 1 Air is an ideal gas with constant specific heats. 2 The kinetic and potential energies are negligible.

**Properties** The gas constant of air is  $R = 0.287 \text{ kPa.m}^3/\text{kg.K}$  (Table A-1). The specific heats of air at the average temperature of (298+423)/2=360 K are  $c_p = 1.009 \text{ kJ/kg·K}$  and  $c_v = 0.722 \text{ kJ/kg·K}$  (Table A-2).

**Analysis** (a) We realize that  $X_1 = \Phi_1 = \mathbf{0}$  since air initially is at the dead state. The mass of air is

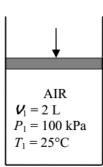
$$m = \frac{P_1 V_1}{RT_1} = \frac{(100 \text{ kPa})(0.002 \text{ m}^3)}{(0.287 \text{ kPa} \cdot \text{m}^3 / \text{kg} \cdot \text{K})(298 \text{ K})} = 0.00234 \text{ kg}$$

Also,

$$\frac{P_2 \mathbf{V}_2}{T_2} = \frac{P_1 \mathbf{V}_1}{T_1} \longrightarrow \mathbf{V}_2 = \frac{P_1 T_2}{P_2 T_1} \mathbf{V}_1 = \frac{(100 \text{ kPa})(423 \text{ K})}{(600 \text{ kPa})(298 \text{ K})} (2 \text{ L}) = 0.473 \text{ L}$$

and

$$\begin{split} s_2 - s_0 &= c_{p,\text{avg}} \ln \frac{T_2}{T_0} - R \ln \frac{P_2}{P_0} \\ &= (1.009 \, \text{kJ/kg} \cdot \text{K}) \ln \frac{423 \, \text{K}}{298 \, \text{K}} - (0.287 \, \text{kJ/kg} \cdot \text{K}) \ln \frac{600 \, \text{kPa}}{100 \, \text{kPa}} \\ &= -0.1608 \, \text{kJ/kg} \cdot \text{K} \end{split}$$



Thus, the exergy of air at the final state is

$$X_{2} = \Phi_{2} = m \left[ c_{\mathbf{v},\text{avg}} (T_{2} - T_{0}) - T_{0} (s_{2} - s_{0}) \right] + P_{0} (\mathbf{V}_{2} - \mathbf{V}_{0})$$

$$= (0.00234 \text{ kg}) \left[ (0.722 \text{ kJ/kg} \cdot \text{K}) (423 - 298) \text{K} - (298 \text{ K}) (-0.1608 \text{ kJ/kg} \cdot \text{K}) \right]$$

$$+ (100 \text{ kPa}) (0.000473 - 0.002) \text{m}^{3} \left[ \text{kJ/m}^{3} \cdot \text{kPa} \right]$$

$$= \mathbf{0.171 \text{ kJ}}$$

(b) The minimum work input is the reversible work input, which can be determined from the exergy balance by setting the exergy destruction equal to zero,

$$\underbrace{X_{\text{in}} - X_{\text{out}}}_{\text{Net exergy transfer by heat, work, and mass}} - \underbrace{X_{\text{destroyed}}}_{\text{Exergy destruction}} = \underbrace{\Delta X_{\text{system}}}_{\text{Change in exergy}}$$

$$W_{\text{rev,in}} = X_2 - X_1$$

$$= 0.171 - 0 = \mathbf{0.171 kJ}$$

(c) The second-law efficiency of this process is

$$\eta_{\rm II} = \frac{W_{\rm rev,in}}{W_{\rm u,in}} = \frac{0.171 \,\text{kJ}}{1.2 \,\text{kJ}} = 14.3\%$$

Assumptions: 1 Both the water and the iron block are incompressible substances with constant specific heats at room temperature. 2 The system is stationary and thus the kinetic and potential energies are negligible. 3 The tank is well insulated and thus there is no heat transfer. **Properties:** The density and specific heat of water at 25°C are  $\rho = 997 \text{ kg/m3}$  and  $c_p = 4.18 \text{ kJ/kg.°F}$ . The specific heat of iron at room temperature (the only value available in the tables) is cp = 0.45 kJ/kg.°C (Table A-3).

*Analysis* We take the entire contents of the tank, water + iron block, as the system, which is a closed system. The energy balance for this system can be expressed as

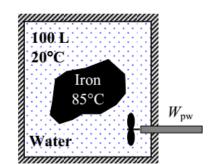
$$E_{\text{in}} - E_{\text{out}} = \underbrace{\Delta E_{\text{system}}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{\text{system}}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$W_{\text{pw,in}} = \Delta U = \Delta U_{\text{iron}} + \Delta U_{\text{water}}$$

$$W_{\text{pw,in}} = [mc(T_2 - T_1)]_{\text{iron}} + [mc(T_2 - T_1)]_{\text{water}}$$

where

$$m_{\text{water}} = \rho \mathbf{V} = (997 \text{ kg/m}^3)(0.1 \text{ m}^3) = 99.7 \text{ kg}$$
  
 $W_{\text{pw}} = \dot{W}_{\text{pw,in}} \Delta t = (0.2 \text{ kJ/s})(20 \times 60 \text{ s}) = 240 \text{ kJ}$ 



Substituting,

240 kJ = 
$$m_{\text{iron}}$$
 (0.45 kJ/kg·°C)(24-85)°C + (99.7 kg)(4.18 kJ/kg·°C)(24-20)°C  $m_{\text{iron}}$  = **52.0 kg**

(b) The exergy destruction (or irreversibility) can be determined from its definition  $X_{\text{destroyed}} = T_0 S_{\text{gen}}$  where the entropy generation is determined from an entropy balance on the system, which is an insulated closed system,

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$
 
$$\underbrace{Change}_{\text{in entropy}}$$
 
$$\underbrace{S_{\text{gen}} = \Delta S_{\text{system}}}_{\text{system}} = \Delta S_{\text{iron}} + \Delta S_{\text{water}}$$

where

$$\Delta S_{\text{iron}} = mc_{\text{avg}} \ln \left( \frac{T_2}{T_1} \right) = (52.0 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \ln \left( \frac{297 \text{ K}}{358 \text{ K}} \right) = -4.371 \text{ kJ/K}$$

$$\Delta S_{\text{water}} = mc_{\text{avg}} \ln \left( \frac{T_2}{T_1} \right) = (99.7 \text{ kg})(4.18 \text{ kJ/kg} \cdot \text{K}) \ln \left( \frac{297 \text{ K}}{293 \text{ K}} \right) = 5.651 \text{ kJ/K}$$

Substituting,

$$X_{\text{destroyed}} = T_0 S_{\text{gen}} = (293 \text{ K})(-4.371 + 5.651) \text{ kJ/K} = 375.0 \text{ kJ}$$

#### Q 8-61

#### **Solution:**

Assumptions 1 This is a steady-flow process since there is no change with time. 2 Potential energy change is negligible. 3 The temperature of the surroundings is given to be 25°C. **Properties** From the steam tables (Tables A-4 through A-6)

$$P_1 = 6 \text{ MPa}$$
  $h_1 = 3658.8 \text{ kJ/kg}$   
 $T_1 = 600 \text{ °C}$   $s_1 = 7.1693 \text{ kJ/kg} \cdot \text{K}$ 

$$P_2 = 50 \text{ kPa}$$
  $h_2 = 2682.4 \text{ kJ/kg}$   
 $T_2 = 100^{\circ}\text{C}$   $s_2 = 7.6953 \text{ kJ/kg} \cdot \text{K}$ 

**Analysis** (b) There is only one inlet and one exit, and thus  $\dot{m}_1 = \dot{m}_2 = \dot{m}$ . We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\frac{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}{\text{Rate of net energy transfer}} = \underbrace{\Delta \dot{E}_{\text{system}}}^{70 \text{ (steady)}} = 0$$
Rate of net energy transfer by heat, work, and mass by heat, work, and heat

The reversible (or maximum) power output is determined from the rate form of the exergy balance applied on the turbine and setting the exergy destruction term equal to zero,

$$\frac{\dot{X}_{\rm in} - \dot{X}_{\rm out}}{\underset{\rm Rate of \ net \ exergy}{\rm rester} \ transfer} - \dot{X}_{\rm destroyed}^{\rm 700 \ (reversibe)} = \underbrace{\Delta \dot{X}_{\rm system}^{\rm 700 \ (steady)}}_{\rm Rate of \ change} = 0$$

$$\dot{X}_{\rm in} = \dot{X}_{\rm out}$$

$$\dot{m} \psi_1 = \dot{W}_{\rm rev,out} + \dot{m} \psi_2$$

$$\dot{W}_{\rm rev,out} = \dot{m} (\psi_1 - \psi_2) = \dot{m} [(h_1 - h_2) - T_0 (s_1 - s_2) - \Delta ke - \Delta pe^{\rm 70}]$$

Substituting,

$$\dot{W}_{\text{rev,out}} = \dot{W}_{\text{out}} - \dot{m}T_0(s_1 - s_2)$$
= 5000 kW - (5.156 kg/s)(298 K)(7.1693 - 7.6953) kJ/kg·K = **5808 kW**

(b) The second-law efficiency of a turbine is the ratio of the actual work output to the reversible work,

$$\eta_{\rm II} = \frac{\dot{W}_{\rm out}}{\dot{W}_{\rm rev,out}} = \frac{5 \,\text{MW}}{5.808 \,\text{MW}} = 86.1\%$$

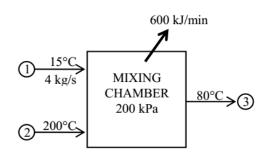
**Discussion** Note that 13.9% percent of the work potential of the steam is wasted as it flows through the turbine during this process.

Assumptions 1 This is a steady-flow process since there is no change with time. 2 Kinetic and potential energy changes are negligible. 3 There are no work interactions.

**Properties** Noting that  $T < T_{\text{sat }@200 \text{ kPa}} = 120.23^{\circ}\text{C}$ , the cold water and the exit mixture streams exist as a compressed liquid, which can be approximated as a saturated liquid at the given temperature. From Tables A-4 through A-6,

$$\begin{array}{c} P_1 = 200 \text{ kPa} \\ T_1 = 15^{\circ}\text{C} \end{array} \begin{array}{c} h_1 \cong h_{f@15^{\circ}\text{C}} = 62.98 \text{ kJ/kg} \\ s_1 \cong s_{f@15^{\circ}\text{C}} = 0.22447 \text{ kJ/kg} \cdot \text{K} \end{array}$$
 
$$\begin{array}{c} P_2 = 200 \text{ kPa} \\ T_2 = 200^{\circ}\text{C} \end{array} \begin{array}{c} h_2 = 2870.4 \text{ kJ/kg} \\ s_2 = 7.5081 \text{ kJ/kg} \cdot \text{K} \end{array}$$
 
$$\begin{array}{c} P_3 = 200 \text{ kPa} \\ T_3 = 80^{\circ}\text{C} \end{array} \begin{array}{c} h_3 \cong h_{f@80^{\circ}\text{C}} = 335.02 \text{ kJ/kg} \\ s_3 \cong s_{f@80^{\circ}\text{C}} = 1.0756 \text{ kJ/kg} \cdot \text{K} \end{array}$$

**Analysis** (a) We take the mixing chamber as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as



Mass balance:

$$\dot{m}_{\rm in} - \dot{m}_{\rm out} = \Delta \dot{m}_{\rm system} \stackrel{\mbox{\scriptsize $\phi$}0 \mbox{ (steady)}}{=} 0 \longrightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\frac{\dot{E}_{\rm in} - \dot{E}_{\rm out}}{\dot{E}_{\rm in} - \dot{E}_{\rm out}} = \underbrace{\Delta \dot{E}_{\rm system}^{70 \text{ (steady)}}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{\rm system}^{70 \text{ (steady)}}}_{\text{Rate of change in internal, kinetic, potential, etc. energies}} = 0$$

$$\dot{E}_{\rm in} = \dot{E}_{\rm out}$$

$$\dot{m}h_1 + \dot{m}_2h_2 = \dot{Q}_{\rm out} + \dot{m}_3h_3$$

Combining the two relations gives  $\dot{Q}_{\text{out}} = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3 = \dot{m}_1 (h_1 - h_3) + \dot{m}_2 (h_2 - h_3)$ 

Solving for  $\dot{m}_2$  and substituting, the mass flow rate of the superheated steam is determined to be

$$\dot{m}_2 = \frac{\dot{Q}_{\text{out}} - \dot{m}_1 \left( h_1 - h_3 \right)}{h_2 - h_3} = \frac{(600/60 \text{ kJ/s}) - \left( 4 \text{ kg/s} \right) \left( 62.98 - 335.02 \right) \text{kJ/kg}}{\left( 2870.4 - 335.02 \right) \text{kJ/kg}} = \textbf{0.429 kg/s}$$

Also, 
$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 4 + 0.429 = 4.429 \text{ kg/s}$$

(b) The exergy destroyed during a process can be determined from an exergy balance or directly from its definition  $X_{\text{destroyed}} = T_0 S_{\text{gen}}$  where the entropy generation  $S_{\text{gen}}$  is determined from an entropy balance on an *extended* system that includes the mixing chamber and its immediate surroundings. It gives

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change of entropy}} = 0$$

$$\underbrace{\dot{R}_{\text{ate of change of entropy}}}_{\text{Rate of entropy}} = \dot{M}_{1}s_{1} + \dot{M}_{2}s_{2} - \dot{M}_{3}s_{3} - \frac{\dot{Q}_{\text{out}}}{T_{\text{b,surr}}} + \dot{S}_{\text{gen}} = 0 \quad \Rightarrow \dot{S}_{\text{gen}} = \dot{M}_{3}s_{3} - \dot{M}_{1}s_{1} - \dot{M}_{2}s_{2} + \frac{\dot{Q}_{\text{out}}}{T_{0}}$$

Substituting, the exergy destruction is determined to be

$$\dot{X}_{\text{destroyed}} = T_0 \dot{S}_{\text{gen}} = T_0 \left( \dot{m}_3 s_3 - \dot{m}_2 s_2 - \dot{m}_1 s_1 + \frac{\dot{Q}_{out}}{T_{b,surr}} \right) 
= (298 \text{ K})(4.429 \times 1.0756 - 0.429 \times 7.5081 - 4 \times 0.22447 + 10 / 298) \text{kW/K} 
= 202 \text{ kW}$$

Assumptions: 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 Heat loss from the chamber is negligible.

Analysis (a) The properties of water are (Tables A-4 through A-6)

$$T_1 = 15^{\circ}\text{C}$$
  $h_1 = h_0 = 62.98 \text{ kJ/kg}$   
 $x_1 = 0$   $s_1 = s_0 = 0.22447 \text{ kJ/kg.K}$   
 $T_3 = 45^{\circ}\text{C}$   $h_3 = 188.44 \text{ kJ/kg}$   
 $x_1 = 0$   $s_3 = 0.63862 \text{ kJ/kg.K}$ 

Water
15°C
4.6 kg/s

Mixing
chamber

Mixture
45°C

An energy balance on the chamber gives

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2) h_3$$

$$(4.6 \text{ kg/s})(62.98 \text{ kJ/kg}) + (0.23 \text{ kg/s}) h_2 = (4.6 + 0.23 \text{ kg/s})(188.44 \text{ kJ/kg})$$

$$h_2 = 2697.5 \text{ kJ/kg}$$

The remaining properties of the saturated steam are

$$h_2 = 2697.5 \text{ kJ/kg}$$
  $T_2 = 114.3 ^{\circ}\text{C}$   
 $x_2 = 1$   $s_2 = 7.1907 \text{ kJ/kg.K}$ 

(b) The specific exergy of each stream is

$$\psi_1 = 0$$

$$\psi_2 = h_2 - h_0 - T_0(s_2 - s_0)$$

$$= (2697.5 - 62.98) \text{kJ/kg} - (15 + 273 \text{ K})(7.1907 - 0.22447) \text{kJ/kg.K} = 628.28 \text{ kJ/kg}$$

$$\psi_3 = h_3 - h_0 - T_0(s_3 - s_0)$$

$$= (188.44 - 62.98) \text{kJ/kg} - (15 + 273 \text{ K})(0.63862 - 0.22447) \text{kJ/kg.K} = 6.18 \text{ kJ/kg}$$

The exergy destruction is determined from an exergy balance on the chamber to be

$$\dot{X}_{\text{dest}} = \dot{m}_1 \psi_1 + \dot{m}_2 \psi_2 - (\dot{m}_1 + \dot{m}_2) \psi_3 
= 0 + (0.23 \text{ kg/s})(628.28 \text{ kJ/kg}) - (4.6 + 0.23 \text{ kg/s})(6.18 \text{ kJ/kg}) 
= 114.7 kW$$

(c) The second-law efficiency for this mixing process may be determined from

$$\eta_{\rm II} = \frac{(\dot{m}_1 + \dot{m}_2)\psi_3}{\dot{m}_1\psi_1 + \dot{m}_2\psi_2} = \frac{(4.6 + 0.23 \text{ kg/s})(6.18 \text{ kJ/kg})}{0 + (0.23 \text{ kg/s})(628.28 \text{ kJ/kg})} = \mathbf{0.207}$$