Subrata Sarkar
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Brief Syllabus.

- Turbomachinary
- Similarity and dimensional Analysis.
- Cascade Theory
- Axial and Radial flow machines.
- Gas and Steam power Cycles.
- Steam Turbine & Power Plant.
- Water Turbine & Pump.
- Miss Topics.

Books.

- Fluid Mechanics and Thermodynamics of Turbomachinary Thermo
  S, L, Dixon
- Gas Turbine Theory, H. Cohen.
  GFC Rogers.

Mid - Sem.	-	25 %	Closed Book/Notes.
Announced quizs	-	20 %	
Final Exam.	-	40 %	
Assignments.	-	10 %	
Attendance	-	5 %	

Final Marks - 100 x 0·8 + 20 (LAB)

01/08/18

Turbo → Rotating Device

↓

Impeller → Heart

↓

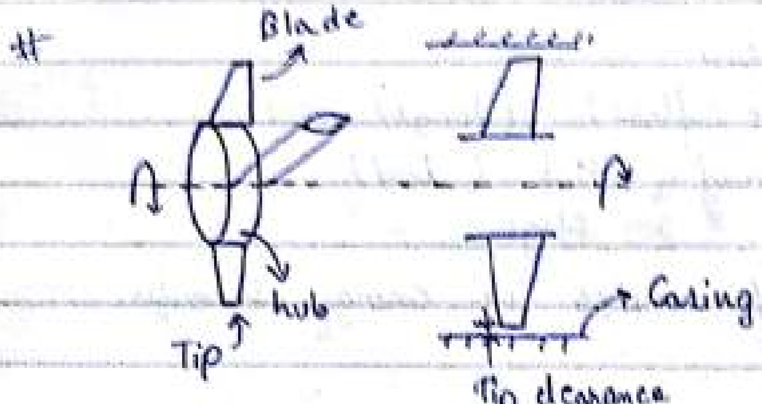
Change of Angular Momentum

Turbomachines — Axial flow impeller

Radial flow

Mixed flow.

#



Axial Flow.

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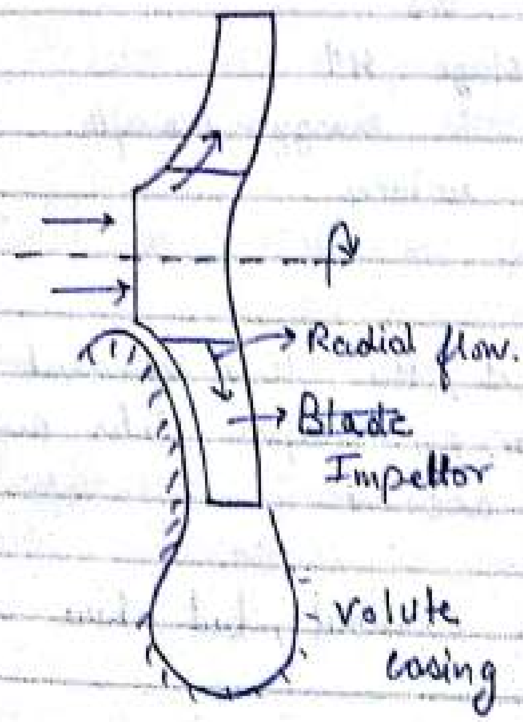
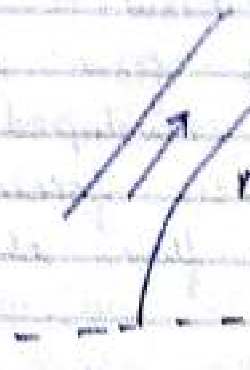




Mixed flow.

Fluid passes through impellor, change in angular momentum occurs, and thus results in torque, which gives power.

Based on kind of impellor, we have axial, radial or mixed flow.

Axial → axial entering and axial exist.

Radial → · axial in and radial out (impeller radial exit)

· encased in a volute casing

Mixed → mixture of ~~mix~~ radial + axial.

Axial Flow :-

- handles large flow.
- i.e. Q↑ (volume flow) (large)
- but heat developed per stage → H↓ (low).
- aerodynamic lift helps in energy transfer.

Radial Flow :-

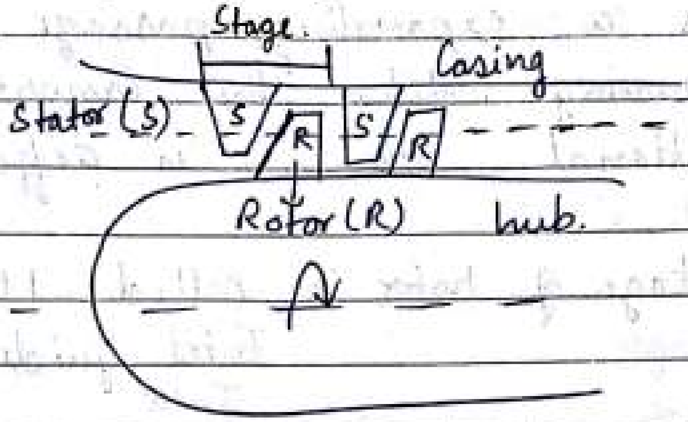
- Volume flow is low Q↓
- Heat developed per stage H↑
- Coriolisis forces helps in energy transfer.
- The flow channels rotates.

In case of radial flows, the flow channels, rotates with the impellor, so simple Euler and other analysis, do not account.

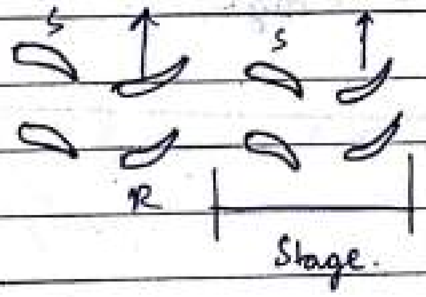
~~Here~~ instead of flow in channel, but here we

analysis flow in a passage to which additional inertia is imparted, so to general analysis, we add this additional inertia.

The kind of fluid that are handled by these impellors are gas, steam and water (incompressible)



Cross-section at the circumpheric[ial] line.



The flow in Rotor will experience torque and power. ~~and~~ as the angular M changes.

In Stator, H changes, but there will be no ~~no~~ power. as there is no rotation.

Flow in stator is stationary, but in rotor, it is rotatory, so at every intersection, there is transition from stationary to rotorary

Compressor generally compress air, (a compressible medium). So we talk in terms of P, ρ.

Pump works with incompressible fluid and hence named Hidal machines.

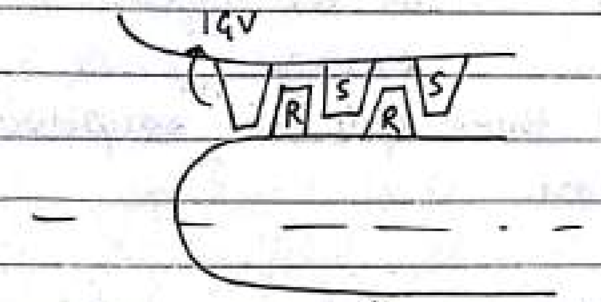
⇒ Fan handling air at low speed (Mach No less than 3) is like a pump.

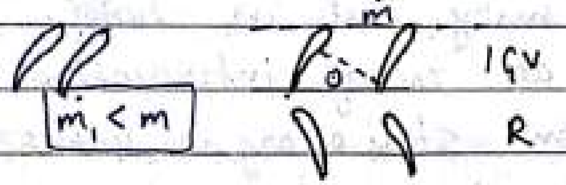
⇒ Turbine has a expanding passage, with Pressure decreasing. unlike the Compressor. The constructional structure is different.

The first stage of rotor is called IGV
~~Fatt~~ Inlet Guide ~~Vane~~ Vane.

Compressor :	R + S = Stage.
Turbine. :	S + R = Stage

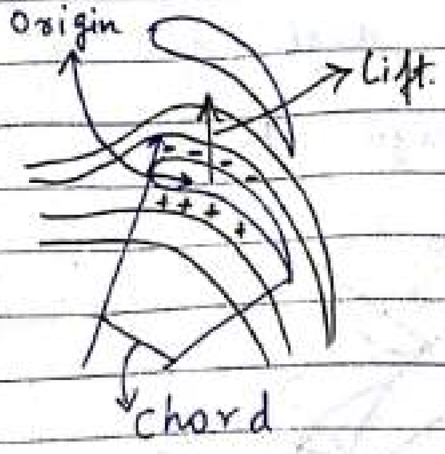
02/08/18





o = Throat
m = ~~The~~ Throat × circumferical area.

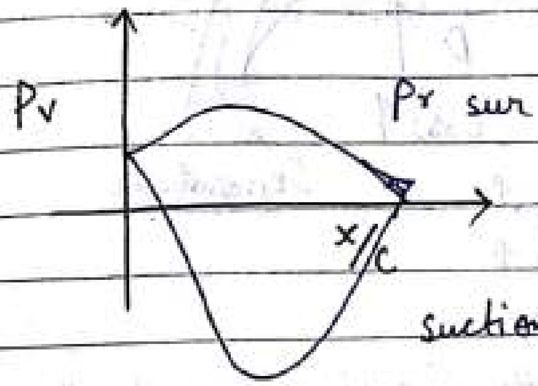
⇒ The mass flow rate to the machine can be controlled by the IGV, by changing the throat area, by changing the angle



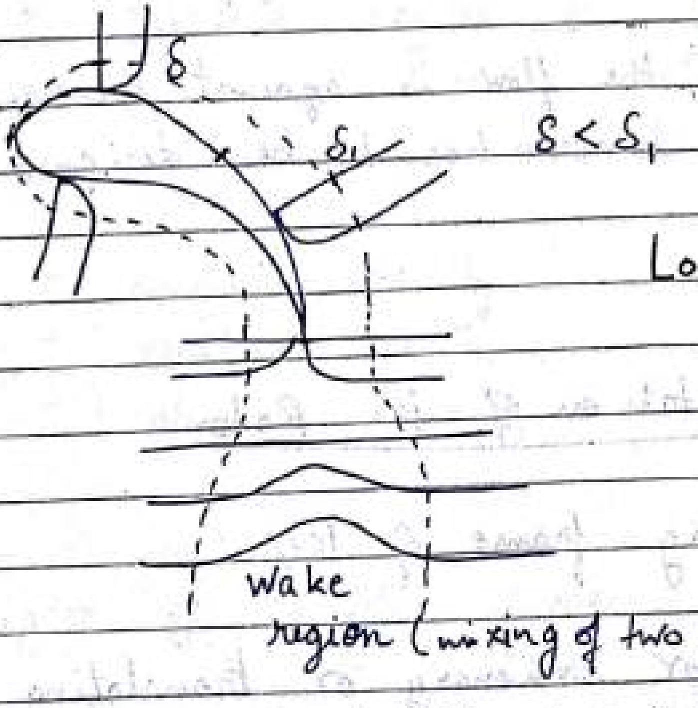
--- Suction surface

+++ Pressure surface.

Due to different acceleration and deceleration, there is creation of different pressure on the two surfaces, which results in the driving force.



- On the suction surface, there is motion, against adverse pr. gradient, thus we have separation of boundary layer.



$\delta < \delta_1$

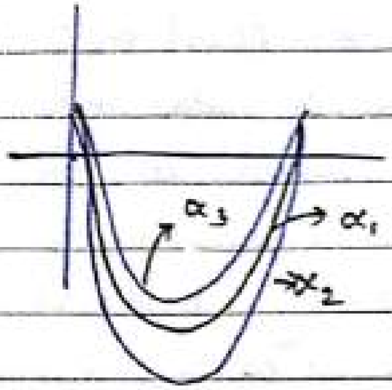
Loss → due to boundary layer, called profile drag, and also, drag due to wake forming at end.

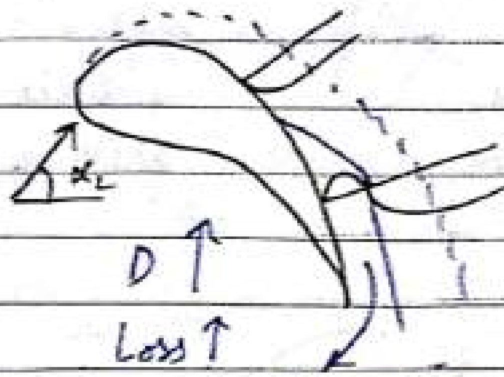
So, we have Lift(L) and drag (D), and for an aerofoil,

$$\boxed{\frac{D}{L} << 1}$$

The flow also depends on the angle of attack. so, for

$\alpha_2 > \alpha_1 \rightarrow$ Design inlet α.
$\alpha_3 < \alpha_1$





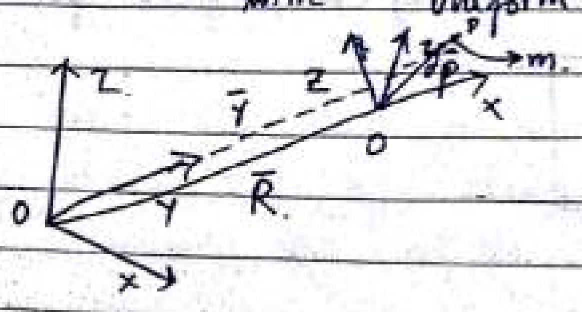
Due to separation of BL, Drag ↑ and Loss ↑.

The BL, also grows on the casing and the hub.
In a compressor, the flow is against adverse pr. gradient and hence has to be designed carefully.

⇒ <u>Effect of the Rotation of the Rotor.</u>

Rotating or moving frame of Ref.

$P = mf$, valid for stationary or translating with uniform speed frame.



Ω is the angular ~~velocity~~ speed

$$\bar{r} = \bar{R} + \bar{\rho}$$
$$\dot{\bar{r}} = \dot{\bar{R}} + \dot{\bar{\rho}} \quad \text{[Differente it].}$$
$$\ddot{\bar{r}} =$$

Now, as D is not moving respect to O, So. we can ~~tt~~ write.

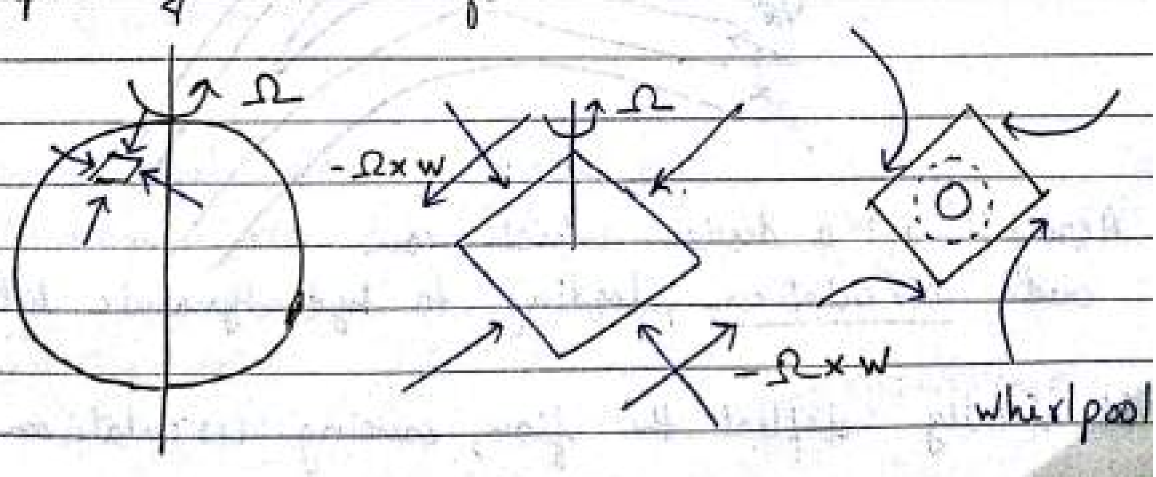
$$m\bar{f} = P - m^2(\bar{\Omega} \times \bar{w}) - m\bar{\Omega} \times (\bar{\Omega} \times \bar{\rho})$$

Coriolis force Centripetal.

$\bar{W}$ is the sense of velocity or relative velocity.
$\bar{f}$ = relative accelesation.
[wrt relative frame of ref. which is rotationg with Ω.]

Ω - angular velocity
W - relative velocity
V - absolute velocity wrt absolute frame of ref.

Examples of Coriolis forces.



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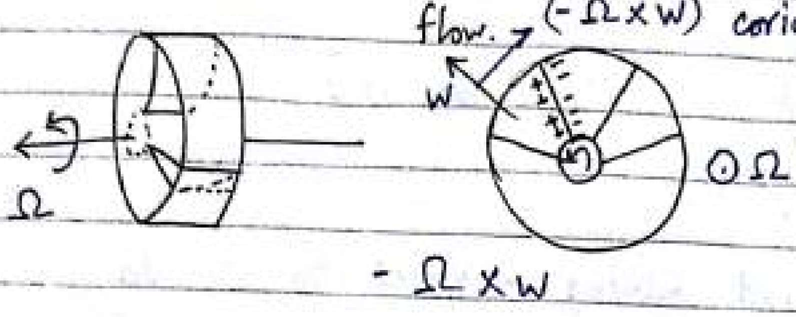
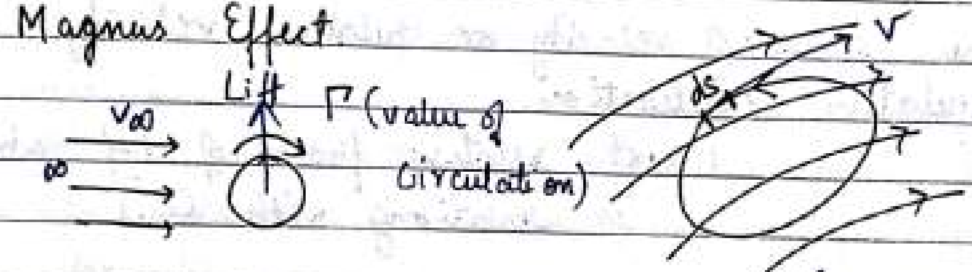
Turbomachines (fully radial flow m/c).



- For a fully radial m/c, energy transfer occurs because of coriolis component.
- No hydrodynamic/aerodynamic lift.
-

Magnus Effect.

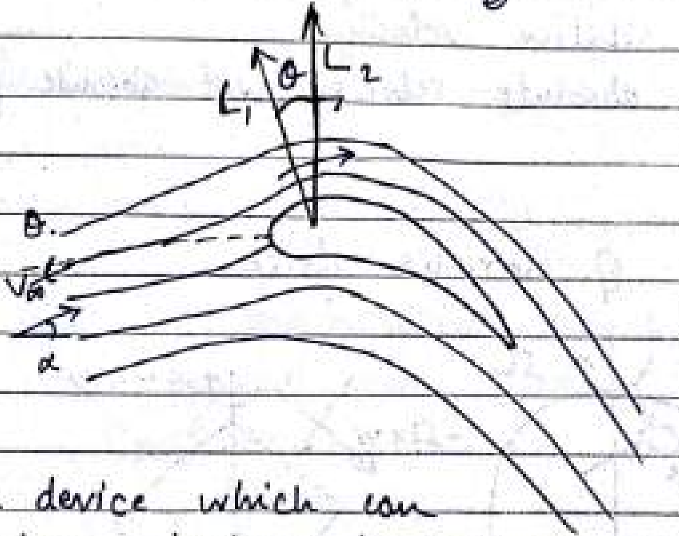


$$\boxed{L = \rho \Gamma V_\infty}$$

$L \rightarrow m^2/s.$

$$\oint \bar{V} . d\bar{s} = \Gamma$$

Aerofoil.



- Aerofoil is a device which can create circulation, leading to hydrodynamic lift.
- It locally deflect the flow, causing circulation.

Scanned by CamScanner

- The lift is always $\perp$ to the approaching velocity.
- In axial flow m/c, all energy transfer occurs due to aerodynamic lift.

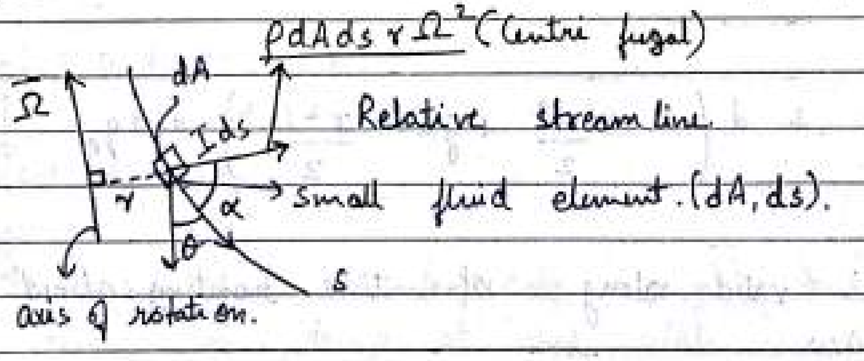
- In mixed flow, total energy transfer is partly due to lift and partly by coriolis force.

$\Rightarrow F = m\,\vec{\Omega} \times (\vec{\Omega} \times \vec{\rho})$ ~~Centrifugal~~ petal. force.

Centri fugal force.

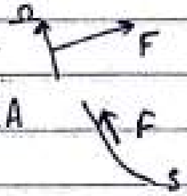
$|F| = m r |\Omega|^2$

Energy eq^n in rotating frame of ref. for viscous flow.

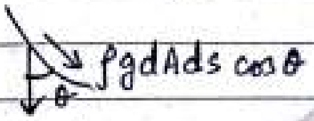


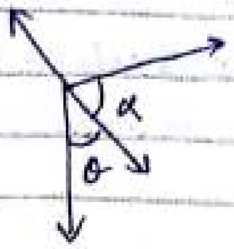
Centri fugal force : $\rho\, dA\, ds\, r\,\Omega^2$

Pressure force : $\left(\frac{\partial p}{\partial s}\, ds\right) dA$



Gravity force : $\rho g\, dA\, ds$





Components in direc^n of flow.

Centrifugal : $\rho\, dA\, ds\, r \Omega^2 \cos\alpha$. ↘
Gravity : $\rho g\, dA\, ds \cos\theta$. ↘
Pressure : $\left(\frac{\partial p}{\partial s} ds\right) dA$. ↖

Viscous : F ↖.

- Coriolis force would not have any component along the flow.

Energy Eq. $\frac{dW}{dt}$. $\frac{DW}{Dt}$

$$\rho\, dA\, ds \left(\frac{\partial W}{\partial t} + W\frac{\partial W}{\partial s}\right) = -\left(\frac{\partial p}{\partial s} ds\right) dA + \rho g\, dA\, ds \cos\theta$$
$$+ \rho\, dA\, ds\, r\Omega^2 \cos\alpha - F$$

Energy Eq.

$$\frac{\partial W}{\partial t} ds + d\left(h + \frac{W^2}{2} + gz - \frac{r^2\Omega^2}{2}\right) = dq_e$$

This is valid along a streamline rotating about an axis.

If $[dq_e = 0]$, there is absense of external energy.

Energy Eq^n

$$dq_e = \frac{\partial w}{\partial t} ds + d\left(h + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}\right)$$

Steady flow : $\frac{\partial w}{\partial t} = 0.$

Hence $d\left(h + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}\right) = 0. = h_{o_r} = C_p T_{o_r}$

Hence along the streamline, Rel. total Temp.

$$\underbrace{h + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}}_{\text{relative enthalpy}} = C$$

In absence of external energy, the relative enthalpy remains const, even if there are viscous losses and dissipations.

$$\underset{\text{Total enthalpy}}{h_0} = h + \frac{v^2}{2} + gz$$

If a flow is isentropically brought to stop, then the temp at that state is the total Temperature.

$$\frac{\partial w}{\partial t} ds + d\left(\frac{p}{\rho} + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}\right) = \delta W_e - \delta W_f$$

If there is steady state $\frac{\partial w}{\partial t} = 0$.

$$d\Bigg[\overbrace{\frac{p}{\rho} + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}}^{\text{Total relative pressure } \frac{p_o}{\rho}}\Bigg] = \underbrace{\delta W_e}_{\text{External Energy}} - \underbrace{\delta W_f}_{\text{Losses.}}$$

In absence of external energy and absence of dissipation,

$$\frac{p_{o_r}}{\rho} = \frac{p}{\rho} + \frac{w^2}{2} + gz - \frac{r^2\Omega^2}{2}$$

$$p_{o_r} = p + \frac{\rho w^2}{2} + \rho g z - \frac{\rho r^2\Omega^2}{2}$$

⇒ In case of relative flow, in presence of dissipation,

Total relative enthalpy $h_{o_r} = c$
p_{o_r} decreases along the streamline.

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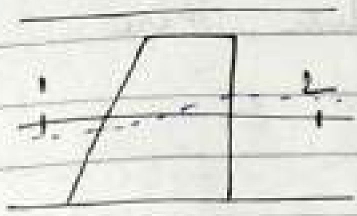
Axial flow machines



$$h_1 + \frac{w_1^2}{2} + \cancel{g z_1} - \cancel{\frac{r_1^2 \Omega^2}{2}} = h_2 + \frac{w_2^2}{2} + \cancel{g z_2} - \cancel{\frac{r_2^2 \Omega_2^2}{2}}$$

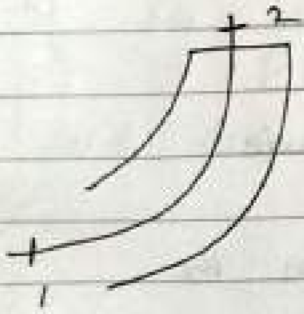
gz not significant.

$= \frac{r^2 \Omega^2}{2}$ cancels each other as streamline remain at equal r from center in axial flow machines.

$$\boxed{h_1 - h_2 = \frac{w_2^2 - w_1^2}{2}}$$

Either acc. or deacceleration is cause for energy transfer.

Radial flow machines



$$h_1 + \frac{w_1^2}{2} + \cancel{g z_1} - \frac{r_1^2 \Omega_1^2}{2} = h_2 + \frac{w_2^2}{2} + \cancel{g z_2} - \frac{r_2 \Omega_2^2}{2}$$

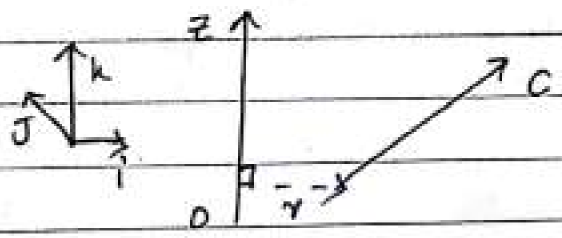
Thus, the radial component is also significant in the energy transfer in the radial flow.

In axial flow, either acc

Equation for Torque.

There is change of angular momentum, there is torque.

Considering a steady system
- a CV enclosed where the velocity remains const.
- fluid is entering the control volume.
- const Ω.



$c = ic_r + jc_\theta + kc_z$

$\bar{r} = ir + 0j + 0k$

$$\vec{r} \times \vec{c} = \begin{vmatrix} i & j & k \\ r & 0 & 0 \\ c_r & c_\theta & c_z \end{vmatrix}$$

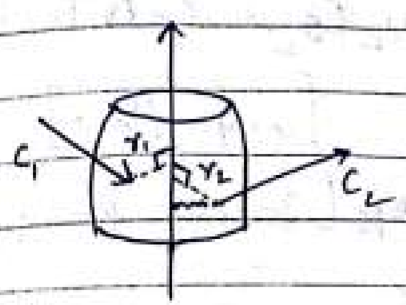
$$\vec{r} \times \vec{c} = 0\hat{i} - c_z r\hat{j} + rc_\theta\hat{k}$$

Represent moment of c about an axis.

⇒ The j component $-c_z r$ exerts force on the hinges holding the machine.

⇒ Whereas k component rc_θ is responsible for energy transfer.

Euler Eq^n for Turbomachinery



Moment of momentum leaving the C.V. at B = $\dot{m} r_2 C_{\theta_2}$

at A = $\dot{m} r_1 C_{\theta_1}$

Moment of momentum flux out of C.V.

$$= \dot{m}(r_2 C_{\theta_2} - r_1 C_{\theta_1})$$

Torque on the impeller = $\dot{m}(r_1 C_{\theta_1} - r_2 C_{\theta_2})$

↳ positive for turbine. (gives work)

~~-ve for compressor. (work given)~~

Torque ~~for~~ per unit mass flow rate

$$= (r_2 C_{\theta_2} - r_1 C_{\theta_1})$$

If Ω is the angular speed.

$P = T.\omega$.

Work done on shaft / unit mass flow rate.

$$= \Omega(r_1 C_{\theta_1} - r_2 C_{\theta_2})$$

$$= U_1 C_{\theta_1} - U_2 C_{\theta_2}$$

$U = r\Omega$ peripheral blade speed.

For pump or compressor = $(r_2 C_{\theta_2} - r_1 C_{\theta_1})\dot{m}$

$$\boxed{W/\dot{m} = U_2 C_{\theta_2} - U_1 C_{\theta_1}}$$

For a turbine.

absolute velocity.

$$W/\text{unit mass flow rate} = u_1 C_{\theta_1} - u_2 C_{\theta_2} = C_p \Delta T_0$$

[Comp flow]

$$W/\text{unit volume} = \rho [u_1 C_{\theta_1} - u_2 C_{\theta_2}] \quad \text{[Incomp flow]}$$

$$\rho [u_1 C_{\theta_1} - u_2 C_{\theta_2}] = \Delta p_0 \quad \text{[Ideal flow no loss]}$$

$$\frac{\rho [u_1 C_{\theta_1} - u_2 C_{\theta_2}]}{\eta_H} = \Delta p_0 \quad \text{[actual flow]}$$

$$W/\text{unit wt flow} = \frac{u_1 C_{\theta_1} - u_2 C_{\theta_2}}{g} = \Delta H \quad \text{(ideal)}$$

Incomp and comp flow.

$$\Rightarrow \frac{u_1 C_{\theta_1} - u_2 C_{\theta_2}}{\eta_H g} = \Delta H \quad \text{(actual)}$$

Hydraulic efficiency.

for a turbine

Check?

$$\frac{P_N}{P} = \eta_c \qquad \frac{P}{P_N} = \eta_t$$

$$\boxed{P_N = \rho g H Q.}$$

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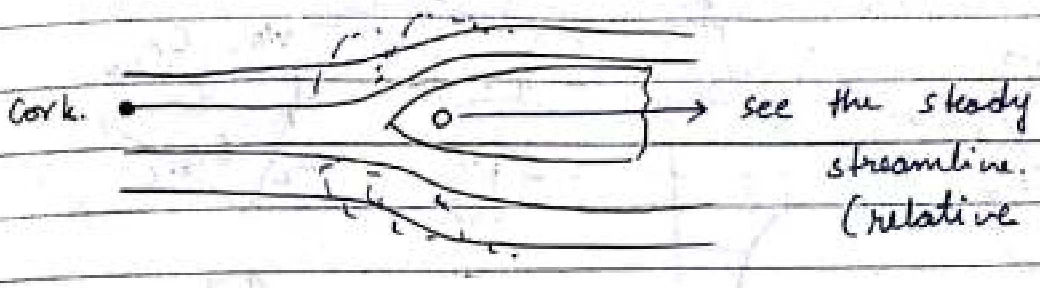
Velocity Δ^s for turbomachinary

Absolute velocity vs Relative velocity.



o (absolute flow) unsteady.

So, if we are tying our frame with the blade, then the flow ~~with~~ would be steady streamline while when ~~we~~ our frame is not attached to blade, that would be the absolute flow of particle.

Velocity Δ^s give us the vectorial relationship of all the velocity component, whether relative or ~~ab~~ absolute from any point either on impeller or outside the blade.

$\overrightarrow{\text{Relative vel.}} + \overrightarrow{\text{Blade speed}} = \overrightarrow{\text{absolute vel.}}$

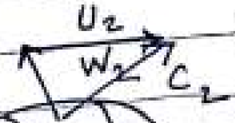
designated by (W) (U) (C)

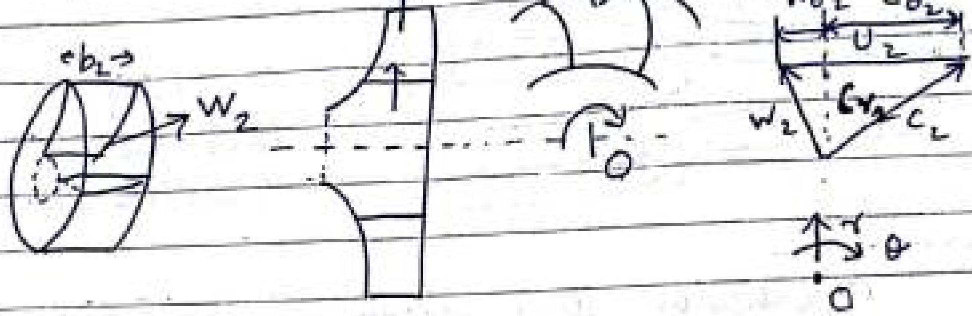
Subscript ⇒ 1 - inlet

2 - outlet

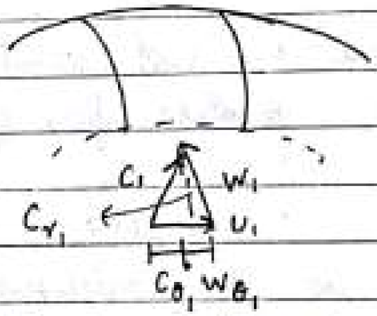
Radial flow ~~compressor~~.

1) Compressor or pump.





W_2 exit velocity will flow the exit blade angle.



Any component in direcⁿ of rotation is +ve, and opposite can be taken as −ve.

$\Rightarrow$ $\boxed{W_2 < W_1}$ for compressor or pump. (deaccelerate) pressure ↑.

$C_{r_1} = C_{m_1}$

Sometime we design such that there is no inlet swirl, to increase the η.

$$C_{r_2} = \frac{Q}{K_2 2\pi r_2 \times b_2} \qquad K_2 > 1$$

used to account for some blockage due BL development on the blades.

$$C_{r_1} = \frac{Q}{K_1 \times 2\pi r_1 b_1} \qquad K_1 \approx 1$$

no inlet swirl.

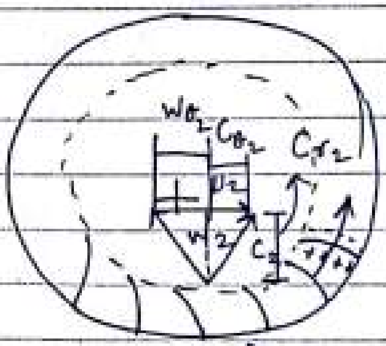
$$H = \frac{u_2 C_{\theta_2} - u_1 \cancel{C_{\theta_1}}}{g} \qquad \text{(Ideal)}$$

Any turbine is a inlet flow machine for stability and balance.

2) Turbine

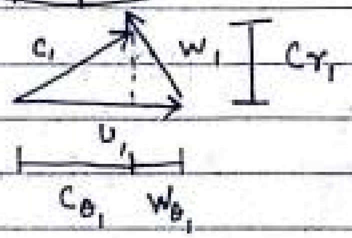
Inward flow machine.
flow accelerate as pressure drop.

$\boxed{W_2 > W_1}$



if exit swirl is made zero.

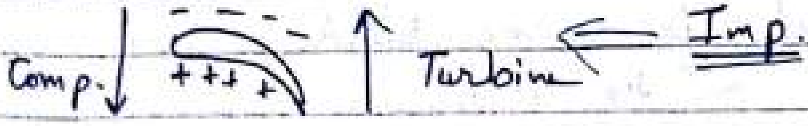
$$W = u_1 C_{\theta_1} - u_2 \cancel{C_{\theta_2}}$$



Some swirling component is needed to increase the η of the duct tube used for collecting the exit water.

C_{θ_2} - exit swirl.

⇒ The spining of compressor is opposite of turbine i.e. suction side to pressure side.



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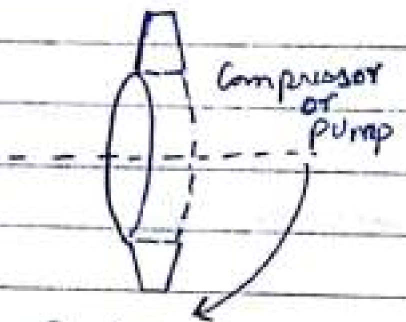
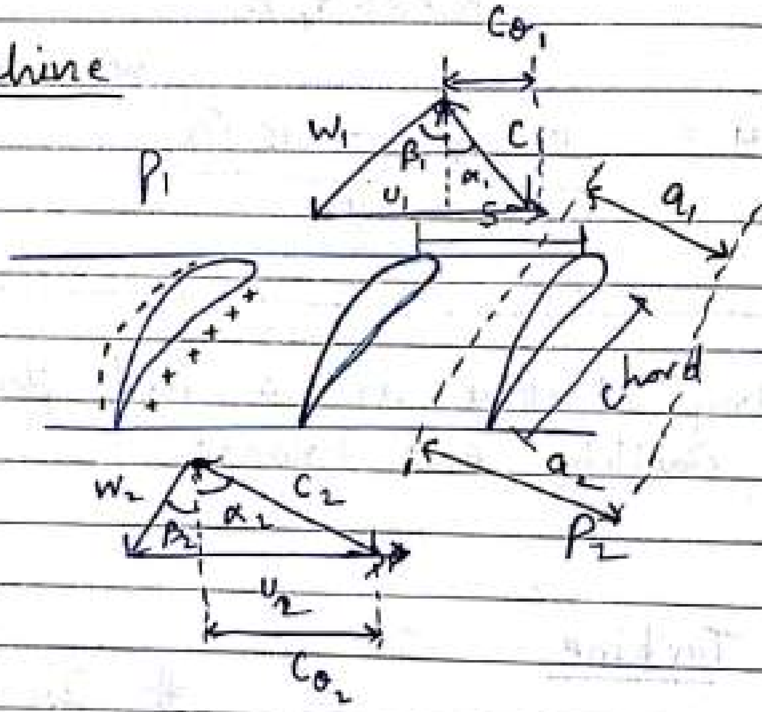
Axial flow machine





$P_2 > P_1$

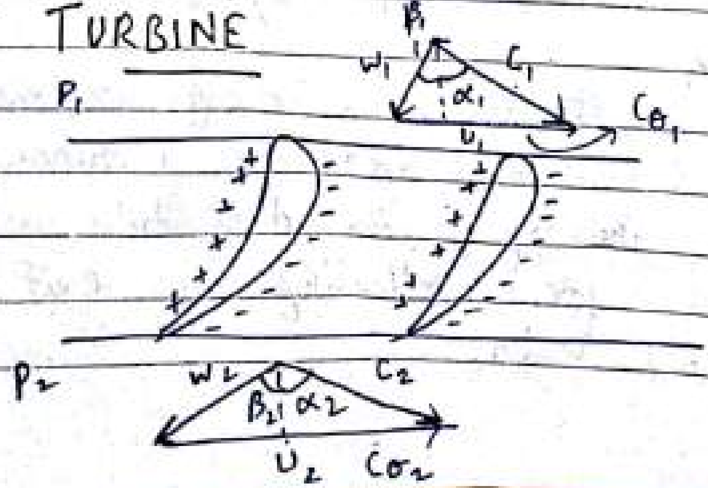
$u_1 = u_2 = U$

$a_2 > a_1$

$$\Delta H = \frac{\eta_H}{g}\left(u_2 C_{\theta_2} - u_1 C_{\theta_1}\right)$$

$$\Delta p_u = \eta_H \rho \left(u_2 C_{\theta_2} - u_1 C_{\theta_1}\right)$$

TURBINE



$P_2 < P_1$

$W_1 < W_2$

$a_2 < a_1$

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* ΔH is the enthalpy, or pressure head change of fluid, whereas $(u_2 C_{\theta_2} - u_1 C_{\theta_1})$ is the power generated on inputed at shaft.

$$\Delta H = \frac{u_1 C_{\theta_1} - u_2 C_{\theta_2}}{\eta_H g} \quad \Bigg| \quad \Delta p_0 = \frac{\rho (u_1 C_{\theta_1} - u_2 C_{\theta_2})}{\eta_H}$$

Similarity Analysis

Similarity is how we can extrapolate the analysis of a prototype from a model.

- Behaviour of machines of similar kind can be related by applying

1. Having done the model test, how we can extrapolate it the prototype.
2. Describing the behaviour of prototype irrespective of size.

Following conditions are to be satisfied:
1. Two machines must be geometrically similar.
2. The flow pattern b/w two machine has to be same.

If these two points are satisfied, then the two machines, they are called kinematically similar.

3. The Re is similar for the machines is same then it dynamic similar.

If all three points are ✓, then they are called Homologous machines.

Model and Prototype.

Incompressible flow.

Head $gH = f_1\left(Q, N, D, \rho, \mu, \frac{l}{D}\right)$

Efficiency, $\eta = f_2(\ldots\ldots)$

Power $P = f_3(\ldots\ldots)$

Dimensional analysis.

$\Psi = \frac{gH}{(ND)^2}$ Non dimensional head.

$= f\left(\frac{Q}{ND^3} \rightarrow \phi, \underbrace{\frac{\rho ND^2}{\mu}}_{\text{Reynold's Number}}, \frac{l}{D}\right)$

$\eta = f(\ldots\ldots)$

$\hat{\text{Power}} = \frac{P}{\rho N^3 D^5} = f(\ldots\ldots)$

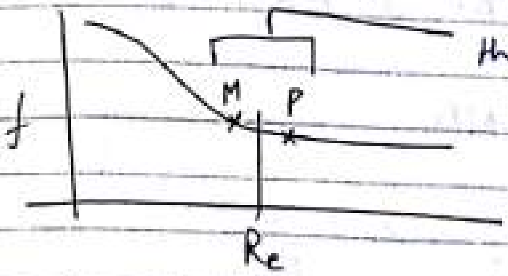
* If the machine is geometrically similar and dynamically similar, then $\frac{Q}{ND^3}$ is denoted by ϕ.

$\psi = \frac{gH}{(ND)^2}$, $\phi = \frac{Q}{ND^3}$, $\hat{P} = \frac{P}{\rho N^3 D^5}$, η

$$\boxed{\hat{P} = \phi\psi/\eta}$$

$\psi, \eta, \hat{P} = f(\phi) \rightarrow$ Flow coeff

$\psi \rightarrow$ Head Rise Coeff.



The f_m and f_p is almost same, and thus, we make the models of size 20 inches.

$u \propto ND$, $D^2 \propto A$, $\frac{Q}{D^2} \propto C_z$

So, $\phi = \frac{C_z}{u}$ u, at meridonal radius.

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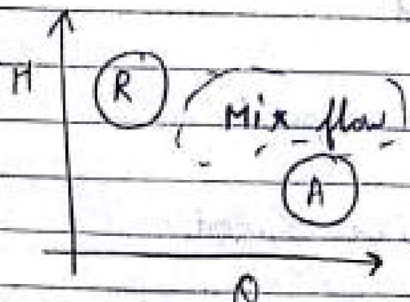
Specific Speed.



Specific Speed is a ~~single~~ non-dimensional parameter that characterize a similar kind of machine.

Similar m/c $\longrightarrow$ Same specific speed.

	SI	FPS	(Industries)
H	m	ft	ft
Q	m^3/s	ft^3/s	Gallon per minute GPM
N	rad/s	RPM	RPM

We found that

$H \propto N^2D^2$, $Q \propto ND^3 \Rightarrow Q \propto D^2\sqrt{H}$

$P \propto N^3D^5 \Rightarrow P \propto D^2 H^{3/2}$

Now, u (blade speed) $\propto ND$.

$u \propto \sqrt{H}$

$\propto \sqrt{gH} \Rightarrow \dfrac{u}{\sqrt{2gH}}$

$\propto \sqrt{2gH}$

$\xrightarrow{v}$ Prototype $\qquad \xrightarrow{u}$ Model.

$\xrightarrow{v/\sqrt{2gH}} \; \equiv \; \xrightarrow{u/\sqrt{2gh}}$

When velocity is divided by $\sqrt{2gH}$, the values for P and M become equal.

$\dfrac{V}{\sqrt{2gH}} = v$ Specific Blade Speed.

$\dfrac{C}{\sqrt{2gH}} = c$ Specific absolute Speed.

$\dfrac{W}{\sqrt{2gH}} = w$ Specific Relative Speed.

From $H \propto ND \Rightarrow$ we conclude $\dfrac{gH}{N^2D^2} = \psi$

$\Rightarrow \boxed{\dfrac{H}{u^2/2g} = \psi}$ (Head Rise Coefficient).

N_s Radial < Mixed < Axial.

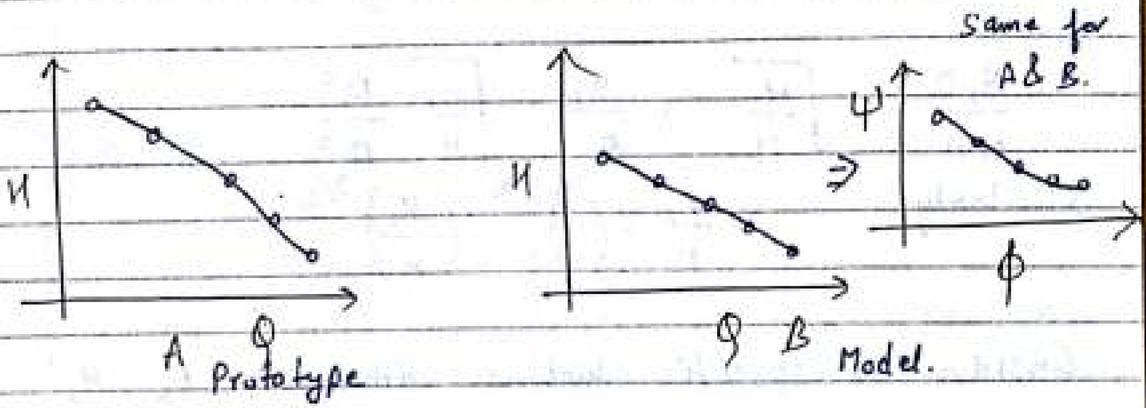
$$\psi = \frac{H}{u^2/2g} = \frac{\Delta P}{\frac{1}{2}\rho H^2} \text{ or } \frac{C_p \Delta T}{u^2/2}$$

thus $\Rightarrow$ $H = \dfrac{C_p \Delta T}{g}$

Similarly,

$$\phi = \frac{Q}{ND^3} = \frac{Q/A}{u} = \frac{C_z}{u} = \phi$$

Meridonial (Axial velocity)



Now, $N_s =$ (Specific) $\dfrac{\text{rpm}\sqrt{\text{GPM}}}{(ft)^{3/2}} = \dfrac{N\sqrt{Q}}{(H)^{3/2}}$

$$\boxed{N_s = \frac{N\sqrt{Q}}{(gH)^{3/4}}}$$

$N_s = \dfrac{N\sqrt{Q}}{(gH)^{3/4}}$ (dimensionless)

Radial flow – 500 - 2000 rpm
Mixed flow – 2000 - 10000 rpm.
Axial flow – 10000 - 15000. rpm.

Specific Speed.

* ~~a single parameter~~

- is the speed of the machine that produce 1 vol flow rate under unit Head..
- is the speed of the machine that produce unit power under unit head.

Lets consider two machines.

$N_1, Q_1, P_1, \ldots$ $\qquad$ $N, Q, P.$

$$\frac{N_1 D_1}{N D} = \sqrt{\frac{H_1}{H}}, \quad \frac{Q_1}{Q} = \sqrt{\frac{H_1}{H}} \frac{D_1^2}{D^2}$$

Similarly, $\frac{P_1}{P} = \left(\frac{D_1}{D}\right)^2 \left(\frac{H_1}{H}\right)^{3/2}.$

Consider a specific turbine with N_s, D_s, H_s
$Q_s, P_s.$

Thus for specific turbine.

$$\frac{P}{P_s} = \frac{H^{5/2}}{N^2} \times \frac{N_s^2}{H_s^{5/2}} \Rightarrow N_s^2 = \frac{P}{P_s} \times N^2 \times \left(\frac{H_s}{H}\right)^{5/2}$$

For $H_s = 1$, $P_s = 1$ $\Rightarrow$ $\boxed{N_s = \frac{N\sqrt{P/\rho}}{H^{5/4}}}$ in terms of $P, H.$

Similarly for analysis with Q and $H.$

$\boxed{N_s = \frac{N\sqrt{Q}}{H^{3/4}}}$ in terms of Q, and $H.$

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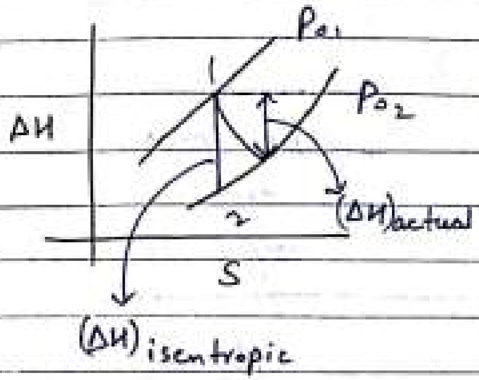
$N_s = \dfrac{N\sqrt{Q}}{H^{3/4}}$ not dimensionless.

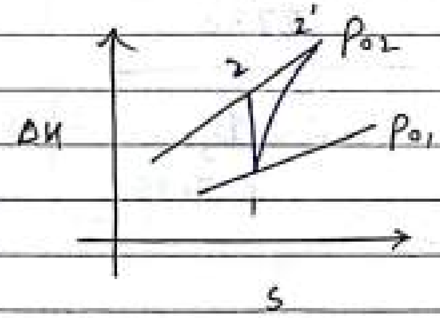
$n_s = \dfrac{N\sqrt{Q}}{(gH)^{3/4}}$ non dimensional, also $N_s = \dfrac{N\sqrt{P/\rho}}{(gH)^{5/4}}$

* Dimensionless quantities denoted by small letters.



For a turbine.

$P_{01} > P_{02}$



For a compressor.

$1-2 \Rightarrow (\Delta H)$ isentropic $\qquad P_{01} < P_{02}$

$1-2' \Rightarrow (\Delta H)_{actual}$

Compressible flow

$n_s = \dfrac{N\sqrt{Q}}{(gH)^{3/4}} = \dfrac{N\sqrt{Q}}{(C_p \Delta T)^{3/4}}$ Check

Sp. Diameter

$Q_s = 1 \qquad H_s = 1$

Thus, for specific turbine, $\dfrac{Q_s}{Q} = \left(\dfrac{D_s}{D}\right)^2 \sqrt{\dfrac{H_s}{H}}$

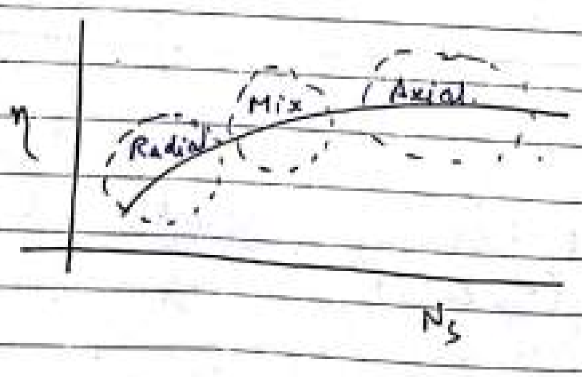
H - Head

Hence, $D_s = \sqrt{\frac{Q_s}{Q}}\ D\left(\frac{H}{H_s}\right)^{1/4}$

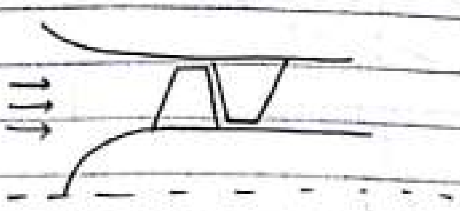
$\Rightarrow Q_s = 1,\ H_s = 1$

$$\boxed{D_s = \frac{D(H)^{1/4}}{\sqrt{Q}}} \Rightarrow d_s = \frac{D(gH)^{1/4}}{\sqrt{Q}}$$

Q	ft^3/sec	m^3/s	gallon p.m
H	ft	m	ft
N	rpm.	rpm	rpm
n_s	N_s	N_s	N_s
1	128.8	52.9	2730
d_s	D_s	D_s	D_s
1	0.42	0.565	0.0198



Compressible flow.



Δh_{0s} (isentropic total enthalpy change) will fall or rise as we extract or give energy to flow.

$\Delta h_{0s}, \eta, P.$

$$\Delta h_s, \eta, P = f(\mu, N, D, \dot{m}, \rho_0, \underbrace{a_0}_{\text{speed of sound}}, \underbrace{\gamma}_{= C_p/C_v})$$

Non - dimensionalising.

$$\frac{\Delta h_{0s}}{N^2 D^2}, \eta, \frac{P}{\rho_{0_1} N^3 D^5} = f\left(\frac{\dot{m}}{\rho_{0_1} N D^3}, \underbrace{\frac{\rho_{0_1} N D^2}{\mu}}_{Re}, \frac{ND}{a_{0_1}}, \gamma\right)$$

* $\boxed{\text{Blade Mach no.} = \frac{ND}{a_{0_1}}}$ $\quad ND \propto a_{0_1}$

* Flow coefficient $\phi = \dfrac{\dot{m}}{\rho_{0_1} N D^3}$

$a_{0_1} = \sqrt{\gamma R T_{0_1}}$

$C_p = \dfrac{\gamma R}{\gamma - 1}$

$\dfrac{P_{0_1}}{\rho_{0_1}} = R T_{0_1}$

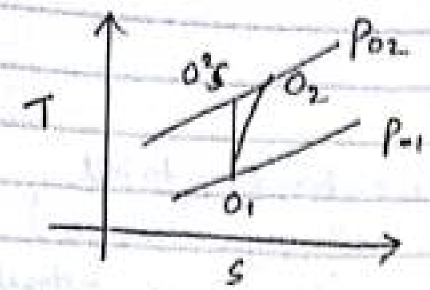
$$= \frac{\dot{m}}{\rho_{0_1} a_{0_1} D^2}$$

$$= \frac{\dot{m}}{\frac{P_{0_1}}{R T_{0_1}} \times \sqrt{\gamma R T_{0_1}}\, D^2}$$

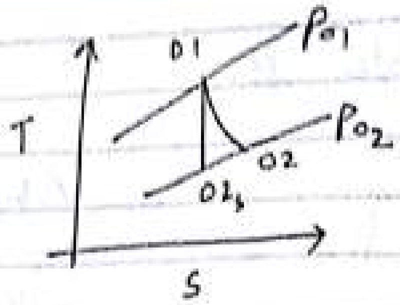
$$= \boxed{\frac{\dot{m} \sqrt{R T_{0_1}}}{P_{0_1} \sqrt{\gamma}\, D^2} = \phi}$$

ϕ is put in terms of inlet variable as we know the inlet parameters

h - enthalpy
H - Head



Compressor



Turbine.

Change in enthalpy $\Delta h_{0_s} = C_p (T_{02_s} - T_{01})$

$$\Delta h_{0_s} = C_p T_{01} \left(\frac{T_{02_s}}{T_{01}} - 1 \right)$$

$$\boxed{\frac{T_{02_s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}} \qquad \boxed{\Delta h_{0_s} = C_p T_{01} \left(\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1 \right)}$$

thus.

$$\Delta h_{0s} = f\left(\frac{P_{02}}{P_{01}}\right)$$

Non D Head

$$\psi = \boxed{\frac{\Delta h_{0s}}{N^2 D^2}} \equiv \frac{\Delta h_{0s}}{(a_{01})^2} = \frac{\Delta h_{0s}}{\gamma R T_{01}} = f\left(\frac{P_{02}}{P_{01}}\right)$$

* for compressible flow, non-dimensional enthalpy is depended on ratio of pressure.

* For a machine of given size, and handling only gas, what would be the characteristics.

Delete ⇒ D, γ, Re.

If the features of flow doesn't change then Re would be independent.

$\dot{m}\,\Delta H_{1s}$

The term $\dfrac{P}{\rho_{01} N^3 D^5} = \hat{P} = \dfrac{\dot{m}\, C_p \Delta T_0}{\{\rho_{01} \times D^2 \times (ND)\}(ND)^2}$

$$\hat{P} = \frac{C_p \Delta T_0}{(a_{01})^2} = \frac{\Delta T_0}{T_{01}}$$

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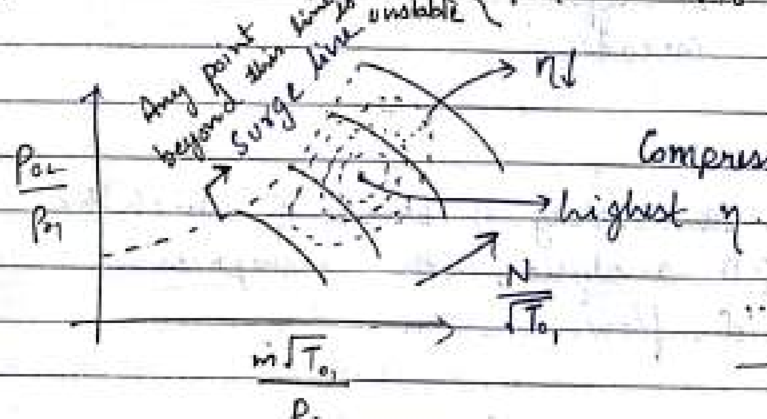
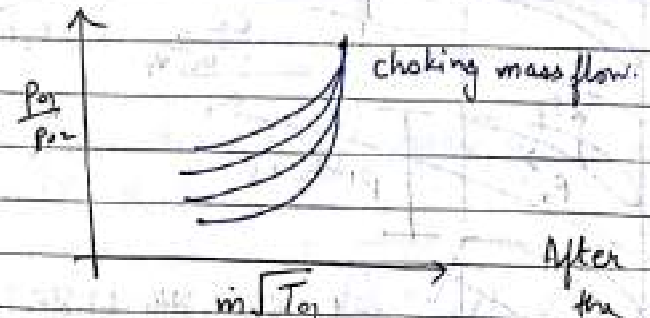
$$\frac{p_{02}}{p_{01}},\ \eta,\ \frac{\Delta T_0}{\Delta T_1} = f\left(\frac{\dot{m}\sqrt{T_{01}}}{p_{01}},\ \frac{N}{\sqrt{T_0}}\right).$$



Compressor.

...... η lines

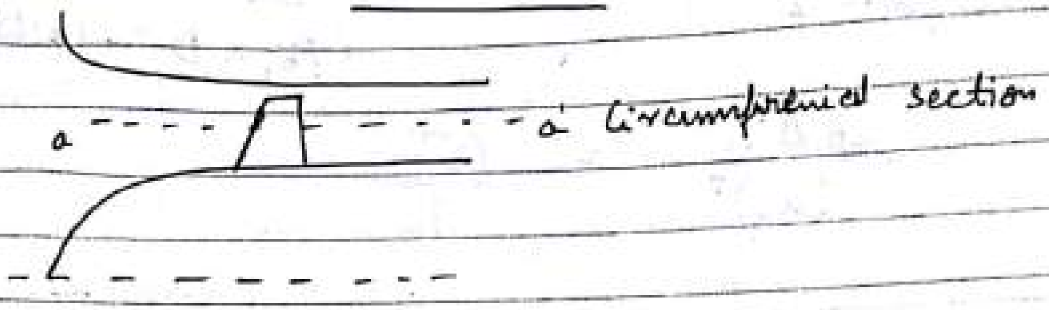
— relation b/w $\frac{p_{02}}{p_{01}}$ Vs $\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$



Turbine

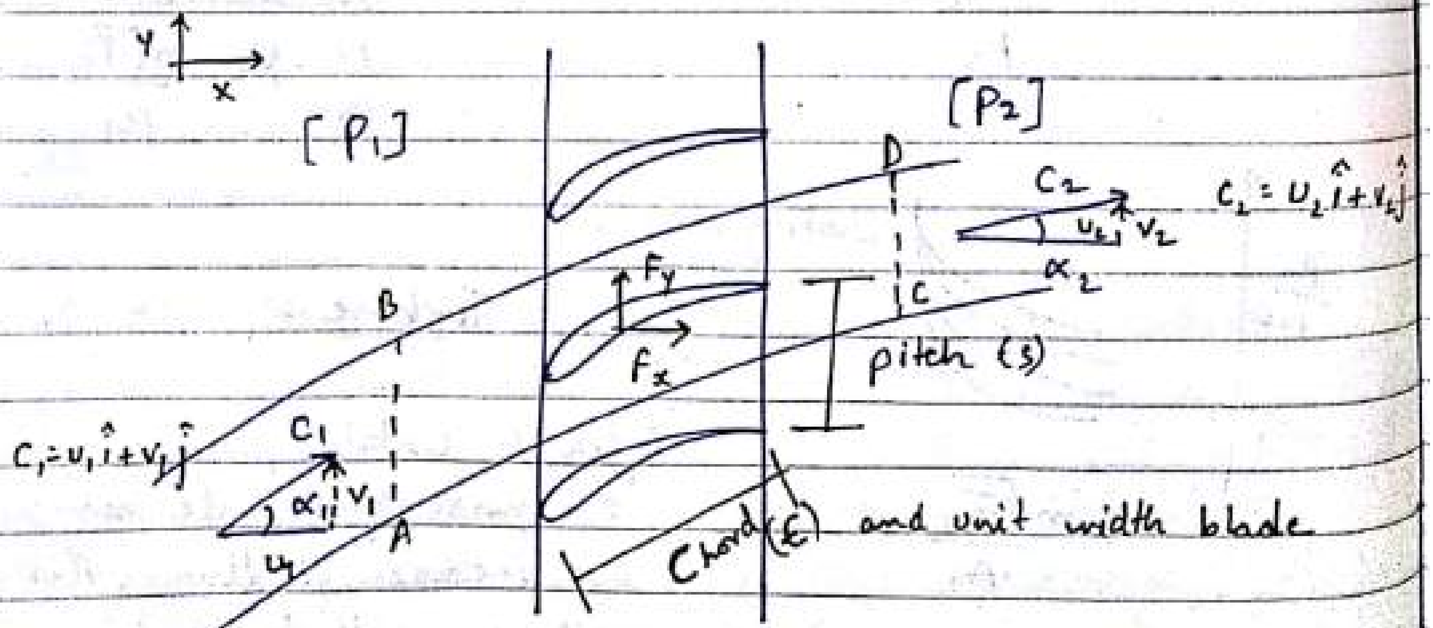
After a certain point, the mass flow rate can not increase further. This point is called choking point.

CASCADE



⇒ So, we first analyse the cascaded blades via a stationary frame in 2D. We would learn about the affects of aerodynamics on this cascade.

⇒ Cascade is an array of blades, of which we do the 2D analysis, with incompressible and invisid flow.



- We take the absolute velocity as there is stationary frame.

ABCD is the control volume.

Now,

$\rho c_1 \cos\alpha_1 = \rho c_2 \cos\alpha_2 \Rightarrow \boxed{u_1 = u_2 = u}$ the velocity along the mass flow do not change.

X-momentum

$$p_1 s - p_2 s - F_x = \rho u s (u_2 - u_1) = 0.$$

$$\boxed{F_x = s(p_1 - p_2)}$$

$$= s\frac{1}{2}\rho\left[c_2^2 - c_1^2\right] = s\frac{1}{2}\rho\left[u_2^2 + v_2^2 - u_1^2 - v_1^2\right]$$

$$\boxed{F_x = \frac{1}{2}\rho s\left[v_2 + v_1\right]\left[v_2 - v_1\right]} \quad \text{—— (1).}$$

Circulation. $\Gamma = \oint \bar{v} \cdot d\bar{s}$

$$= \oint_{ABCD} \bar{v} \cdot d\bar{s} = \boxed{v_1 s - v_2 s = \Gamma}$$

- Thus there is a clockwise circulation of value $(v_1 - v_2) s$.

$\Rightarrow$ Now in (1), $\boxed{F_x = -\frac{1}{2}\rho\Gamma(v_1 + v_2)}$ —— (2).

Y- momentum

Force on fluid would be in -y direcⁿ.

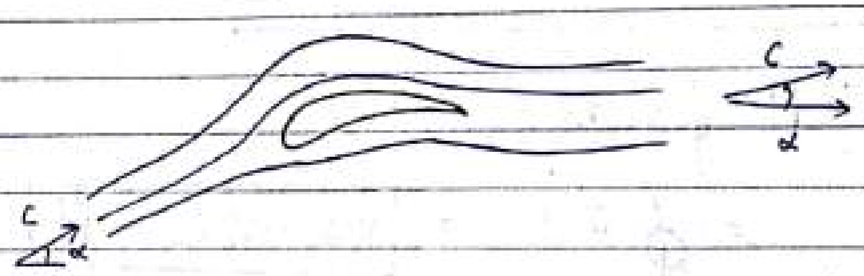
$$-F_y = \rho U S (v_2 - v_1)$$

$$\boxed{+F_y = \rho U \Gamma} \quad \text{--- (3)}$$

So, the resultant force on the blade will be

$$\boxed{L = \sqrt{F_x^2 + F_y^2} = \rho \Gamma \left[\left(\frac{v_1 + v_2}{2} \right)^2 + U^2 \right]^{1/2}}$$

Isolated Blade



- c and α is same for both entering and exiting fluid, but there is local circulation on the blade.

$\Gamma = s(v_1 - v_2)$, for a single blade, the Γ would be zero, as $v_1 = v_2$, but Γ is ~~also~~ always finite.

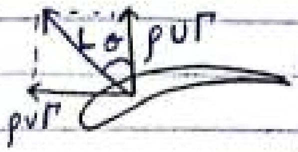
$s \uparrow \infty$

$(v_1 - v_2) \rightarrow 0$

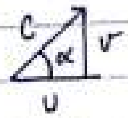
$\Downarrow$

$\Gamma =$ const

$$F = \underset{m/s}{\rho\, \overset{kg/m^3}{v}}\, \Gamma \rightarrow \frac{m^2}{s}$$



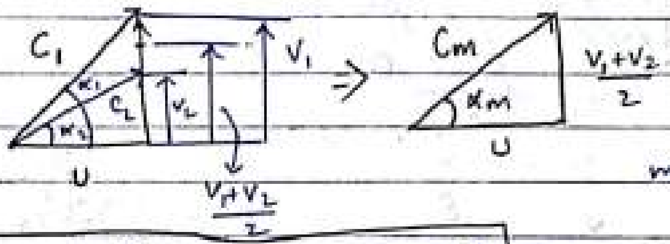
considering
$v_1 = v_2 = v$
$F_x = \rho v \Gamma$



$$\tan\alpha = \frac{v}{u} = \frac{F_x}{F_y} = \tan\theta \Rightarrow \boxed{\alpha = \theta}$$

Kutta-Joukowsky relation
Says that if there is a circulation, with a approaching flow, there will be a lift.

* Going back to the cascade
Velocity triangles are:



C_m and α_m
vectorial
mean speed and angle

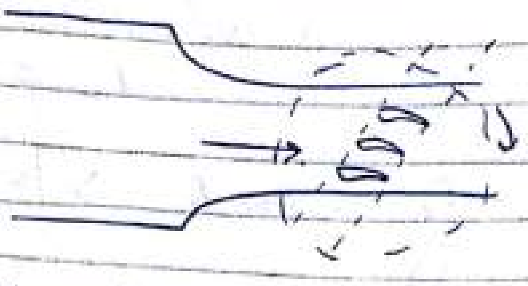
$$\boxed{\tan\alpha_m = \frac{1}{2}\left[\tan\alpha_1 + \tan\alpha_2\right]}$$

Thus, $L = \rho\Gamma\left[\left(\frac{V_1+V_2}{2}\right)^2 + U^2\right]^{1/2} = \boxed{\rho\Gamma C_m = L}$

So, $\boxed{\theta = \alpha_m \quad , \quad L = \rho\Gamma C_m.}$

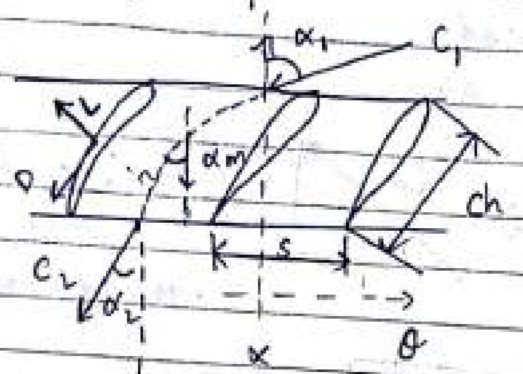
The lift will always be perpendicular to the approaching velocity.

Cascade tunnel.



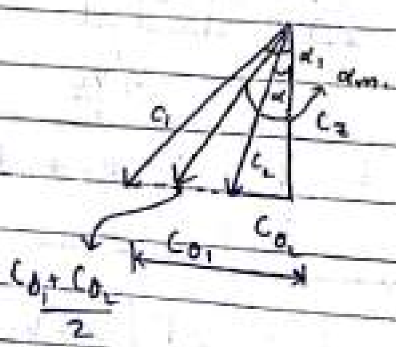
In development of a highly efficient axial flow machine, 2D analysis of a cascade is important.

2D analysis of cascade. (incompressible, inviscid flow)



Compressor cascade

$c_2 < c_1$



$$\Gamma = s(C_{\theta_1} - C_{\theta_2})$$
$$L = \rho C_m \Gamma$$
$$= \rho C_m s (C_{\theta_1} - C_{\theta_2})$$

Further, $C_{\theta_1} = C_2 \tan\alpha_1$ | $C_{\theta_2} = C_2 \tan\alpha_2$

$$\Rightarrow L = \rho C_m s C_2 [\tan\alpha_1 - \tan\alpha_2]$$

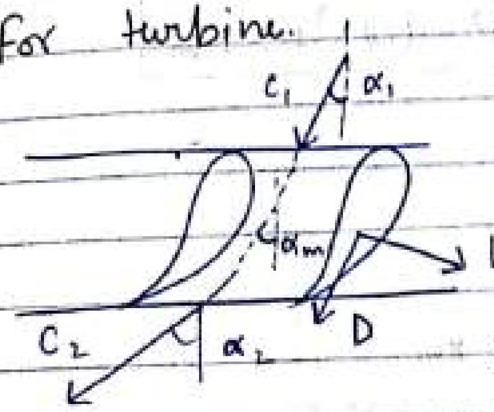
C_L (Coefficient of Lift) $= \dfrac{L}{\frac{1}{2}\rho C_m^2 (ch)}$

$$C_L = \frac{\rho C_m s \,(C_{\theta_1} - C_{\theta_2})}{\frac{1}{2}\rho C_m^2 (ch)}$$

$$\boxed{C_L = 2\left(\frac{s}{ch}\right) \times \left[\tan\alpha_1 - \tan\alpha_2\right] \cos\alpha_m}$$

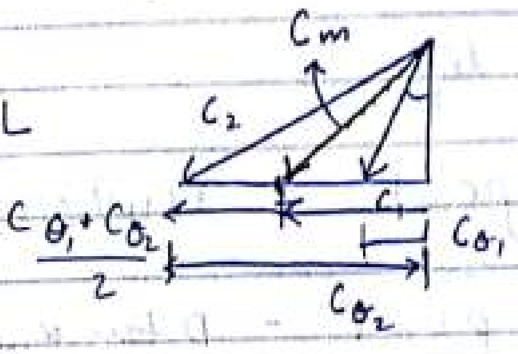
for a compressor.

For turbine.



$L = \rho \Gamma C_m$

$\Gamma = s[C_{\theta_2} - C_{\theta_1}]$

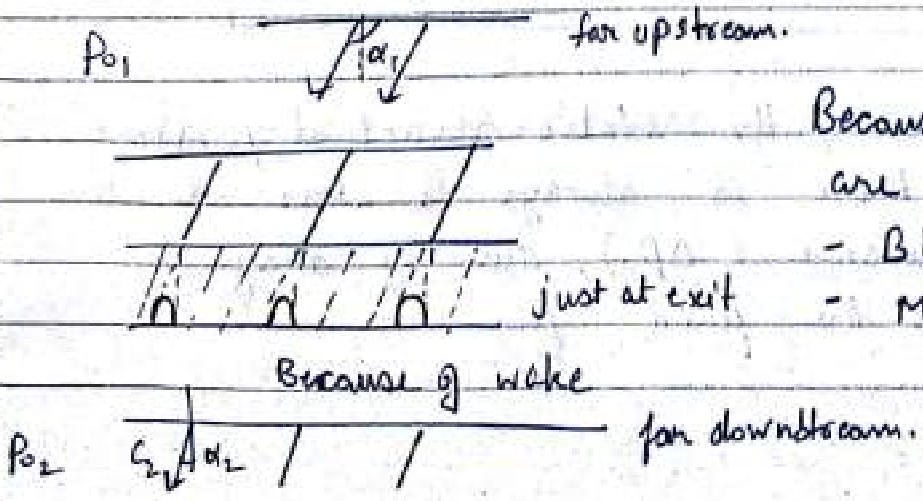


$L = \rho C_m s \,(C_{\theta_2} - C_{\theta_1})$

$$\boxed{C_L = 2\left(\frac{s}{ch}\right) \times \left[\tan\alpha_2 - \tan\alpha_1\right] \cos\alpha_m}$$

for turbine.

Effect of viscous flow



Because of viscosity, there are losses.

- BL formation.
- Mixing of wake downstream.

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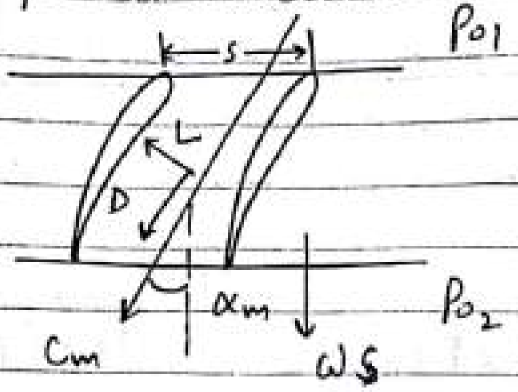
Viscous Effect

Compressor Cascade.



$\omega = P_{01} - P_{02}$

= total pressure loss

$D = (\omega s)(\cos\alpha_m)$

$$\boxed{D = (\omega s)\cos\alpha_m}$$

Effective lift

$$L = \rho C_m \Gamma - (s\omega)\sin\alpha_m.$$

$$\boxed{L = \rho C_m \Gamma - D\tan\alpha_m.}$$

Ex-7

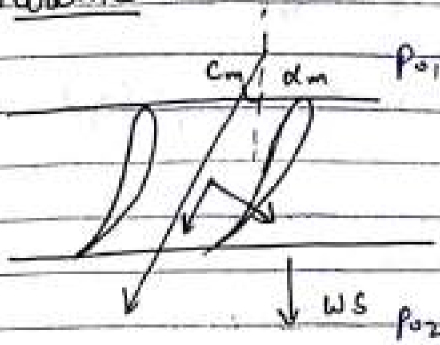
Actual Lift Coefficient =

$$\cancel{C_L = 2\frac{s}{c}(\tan\alpha_1 - \tan\alpha_2) - C_D\tan\alpha_m}$$

$$\boxed{C_L = 2\frac{s}{Ch}(\tan\alpha_1 - \tan\alpha_2)\cos\alpha_m - C_D\tan\alpha_m.}$$

⇒ In a compressor, the static or actual pressure increases, but there is always a loss in stagnation pressure (ΔP_0), due to drag acting ~~on~~ on the fluid flow.

⇒ Turbine



$\omega = P_{01} - P_{02}$

$\boxed{D = \omega s \cos\alpha_m}$

$L = \rho C_m \Gamma + \omega s \sin\alpha_m$

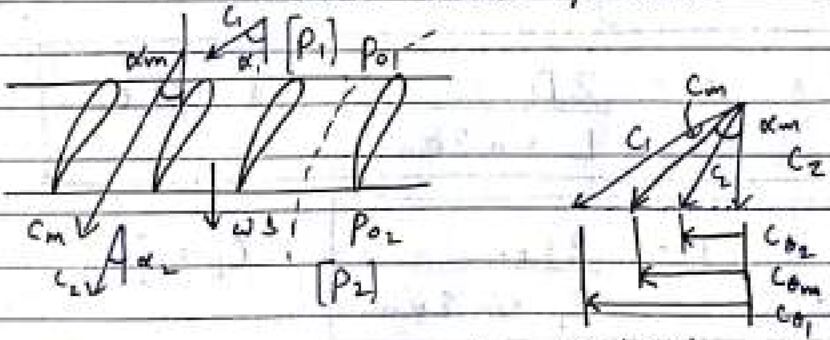
$\boxed{L = \rho C_m \Gamma + D \tan\alpha_m}$

$$\boxed{C_L = 2\frac{s}{Ch}\left(\tan\alpha_2 - \tan\alpha_1\right)\cos\alpha_m + C_D \tan\alpha_m}$$

Blade Efficiency (η_b)

1) Compressor cascade.

$$\eta_{b_c} = \frac{\text{Actual static pressure rise.}}{\text{Ideal static pressure rise.}}$$



$C_z = \frac{\dot{m}}{\rho A}$

α_m vectorial mean angle.

Consider a streamline passing away from the blades, we can use Bernoulli's Eq^n.

So,

$$\boxed{\eta_{b_c} = \frac{\frac{1}{2}\rho\left[c_1^2 - c_2^2\right] - \omega}{\frac{1}{2}\rho\left[c_1^2 - c_2^2\right]}}$$

$$C_1^2 = C_z^2 + C_{\theta_1}^2 \quad , \quad C_2^2 = C_z^2 + C_{\theta_2}^2$$

So,

$$C_1^2 - C_2^2 = 2\, C_m \sin\alpha_m \cdot (C_{\theta_1} - C_{\theta_2})$$

$$\cancel{\eta_{bc} = \frac{1}{2}\rho \Gamma C}$$

$$\eta_{bc} = \frac{\frac{1}{2}\rho \times 2 C_m \sin\alpha_m \times \frac{\Gamma}{s} - \omega}{\frac{1}{\cancel{2}}\rho \times \cancel{2} C_m \sin\alpha_m \times \frac{\Gamma}{s}}$$

$$= 1 - \frac{\omega s}{\Gamma \rho C_m \sin\alpha_m}$$

$$\boxed{\eta_{bc} = 1 - \frac{\Delta p_0}{\Gamma \rho C_m \sin\alpha_m}}$$

or. $$\eta_{bc} = 1 - \frac{\omega s \cos\alpha_m}{\rho \Gamma C_m \sin\alpha_m \cos\alpha_m}$$

$$\boxed{\eta_{bc} = 1 - \frac{2D}{L \sin 2\alpha_m} \quad , \quad L = \rho \Gamma C_m}$$

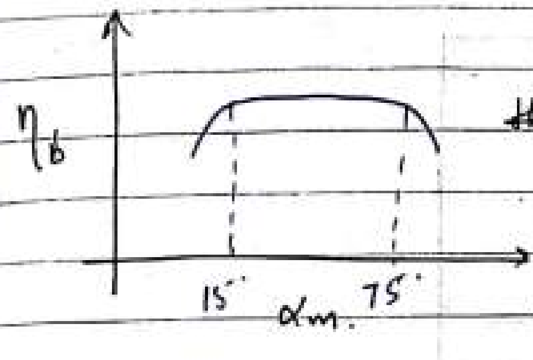
$$\boxed{\eta_{bc} = 1 - \frac{2 C_D}{C_L \sin 2\alpha_m}} \qquad C_L = ?$$

If we take very good blade, $\frac{C_D}{C_L} << 1$

and thus the η_{bc} would be around 96 to 98 %.

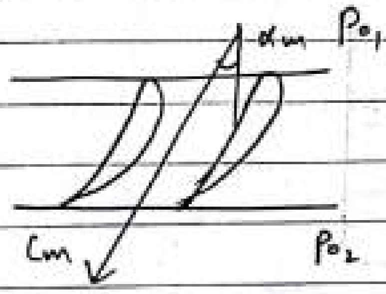
Blade effⁿ becomes max^m, $\frac{d\eta_b}{d\alpha_m} = 0$

$\therefore \cos 2\alpha_m = 0$, $\Rightarrow$ $\boxed{\alpha_m = 45^\circ}$



~~theoret~~ experimentally observed that η_b is max for a great range of α_m.

$\Rightarrow$ Turbine



$$\eta_{b_t} = \frac{\text{Ideal static pr. } (\Delta p_s) \text{ drop}}{\text{Actual static pr. drop}}$$

$$\eta_{b_t} = \frac{\frac{1}{2}\rho(c_2^2 - c_1^2)}{\frac{1}{2}\rho(c_2^2 - c_1^2) + \omega}$$

$$\eta_{b_t} = \frac{1}{1 + \frac{\omega}{\frac{1}{2}\rho(c_{\theta_2} - c_{\theta_1})(c_{\theta_2} + c_{\theta_1})}}$$

$$\boxed{\eta_{b_t} = \frac{1}{1 + \frac{2C_D}{C_L \sin 2\alpha_m}}}$$

For a well designed blade $\frac{C_D}{C_L} << 1$

$$\eta_b = \left[1 + \frac{2C_D}{C_L \sin 2\alpha_m}\right]^{-1}$$

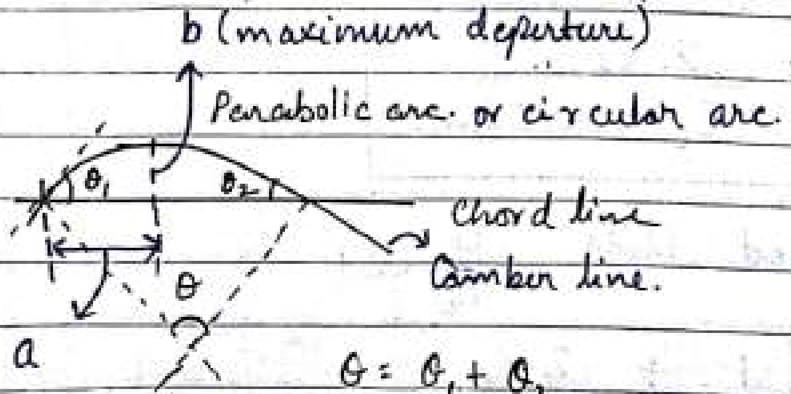
$$\boxed{\eta_{b_t} \simeq 1 - \frac{2C_D}{C_L \sin 2\alpha_m}}$$

Derive the actual Blade Efficiency of turbine and compressor from fundamental considering effect of drag in UH.

$$\boxed{\eta_{b_C} = \frac{1 - \frac{C_D}{C_L}\cot\alpha_m}{1 + \frac{C_D}{C_L}\tan\alpha_m} \qquad \eta_{b_t} = \frac{1 - \frac{C_D}{C_L}\tan\alpha_m}{1 + \frac{C_D}{C_L}\cot\alpha_m}}$$

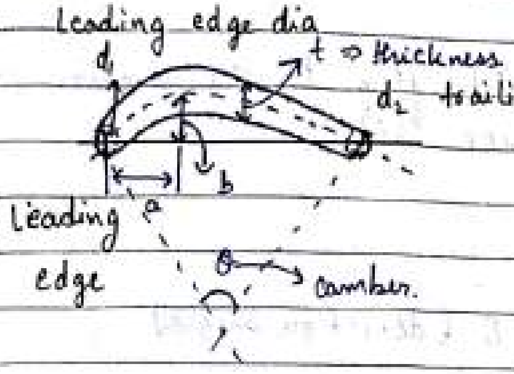
30/08/18

Cascade Nomenclature



$\theta = \theta_1 + \theta_2$

Camber.

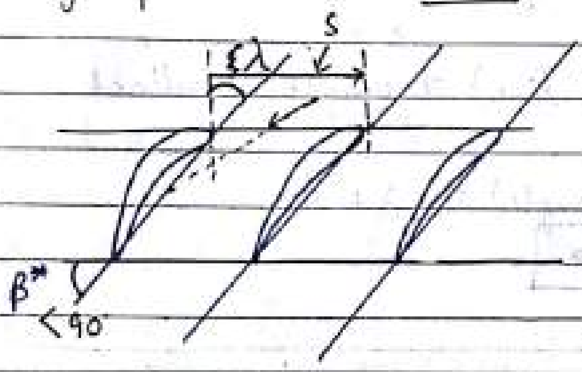


$\boxed{\text{Solidity} - c/s}$

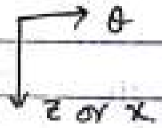
So first we decide pitch and then the stagger angle.

* λ - Stagger
s/c - pitch/chord ratio.

Array of blades Cascade. (Compressor)

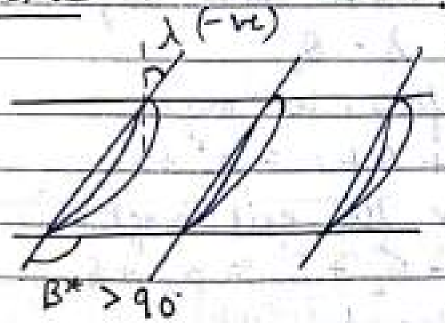


+ve λ for compressor.
-ve λ for turbine

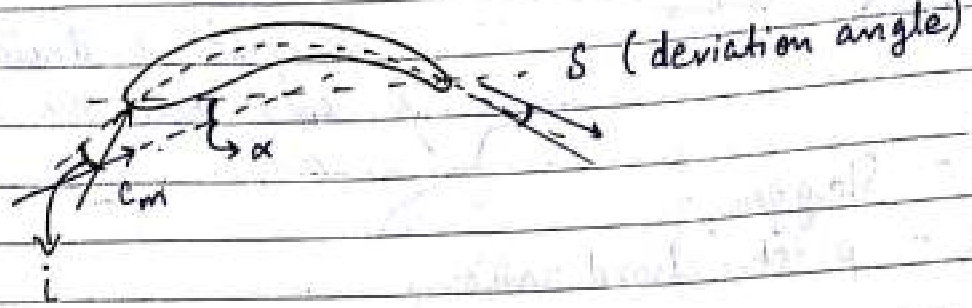


A replacement to λ.
Stagger $\beta^* < 90°$ compressor.
Stagger. $\beta^* > 90°$ turbine.

Turbine



* Angle made by the inlet flow to the camber line, is called incidence. (i)



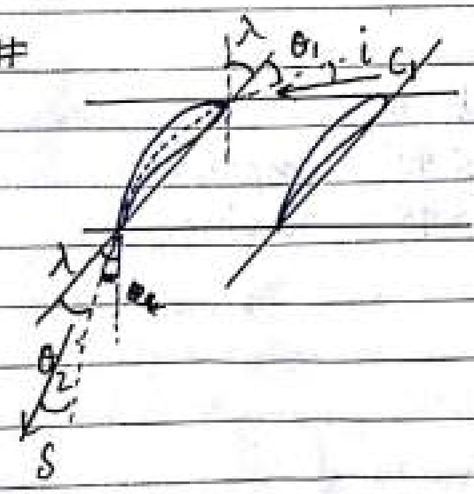
So, α_1 (inlet flow angle) =
α_2 (outlet flow angle) =

α ($\angle$ C_m and chord line) $\Rightarrow$ angle of attack.

α_m (vectorial mean angle) = $\lambda + \alpha$

$$\boxed{\alpha_m = \lambda + \alpha}$$

#



α_1' = blade inlet angle
$= \lambda + \theta_1$

α_2' = blade outlet angle
$= \lambda - \theta_2$

α_1 = air flow inlet angle.
$= \lambda + \theta_1 + i = \alpha_1' + i$

α_2 = air flow exit angle
$= \lambda - \theta_2 + \delta = \alpha_2' + \delta$.

Deflection is the angle the flow has turned.

$$\epsilon = \alpha_1 - \alpha_2 = \alpha_1' - \alpha_2' + i - \delta.$$

$$\boxed{\epsilon = \theta + i - \delta}$$ $[\theta = \theta_1 + \theta_2]$

* The ϵ by which the flow turns is completely decided by the designed angle θ.

For a circular arc

$\theta_1 = \theta_2 = \dfrac{\theta}{2}$	$\lambda = \dfrac{\alpha_1' + \alpha_2'}{2}$	All angles

Parabolic arc.

$$\lambda + \tan^{-1}\frac{4b}{4a-c} = \alpha_1'$$

$$\lambda - \tan^{-1}\frac{4b}{4a-c} = \alpha_2'$$

$\theta \rightarrow$ camber angle.

$\alpha_1' \rightarrow$ blade inlet angle

$\alpha_2' \rightarrow$ blade exit angle.

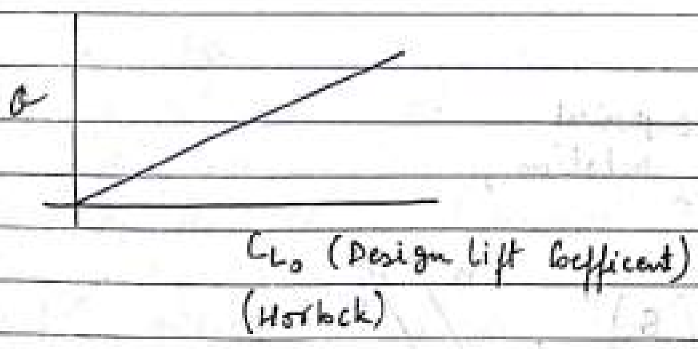
$i \rightarrow$

British Nomenclature (C series)

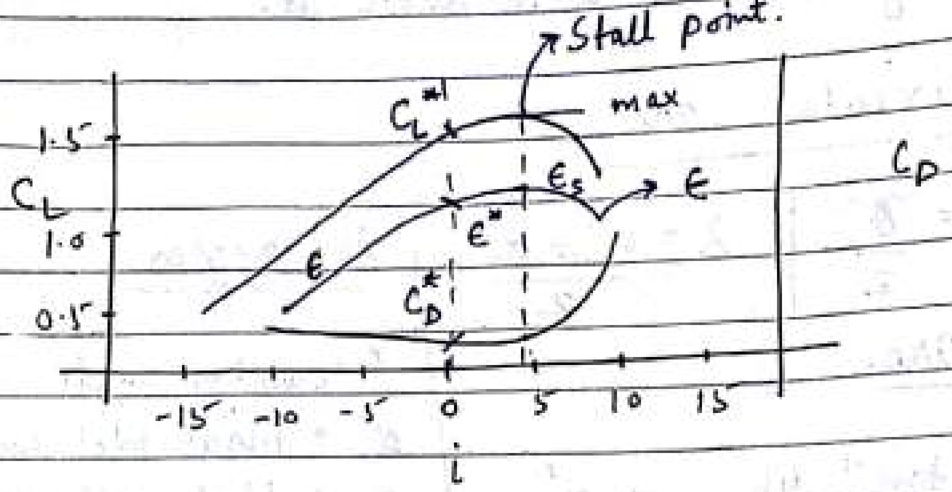
12 C4/35 P 30

$\frac{t}{c} = 12\%$; C4 $\rightarrow$ Base table ; 35 $\rightarrow$ camber angle (in °) ; P 30 $\rightarrow$ $a = 30\%$ of c

04/09/18



Compressor Cascade Performance.



$i = 0$, condition for which aerofoil is designed.

- At the nominal condition, $i = 0$, so it is incident at the angle designed, and values at nominal condition is denoted with (*).

- As $i \uparrow$ the lift increases, but also at the same time the factor of B.L separation enters and L decreases.

- The point of max L is called <u>Stall point</u>.

$$\boxed{\epsilon^* = 0.8\,\epsilon_s}$$

Howell gives a empirical relation,

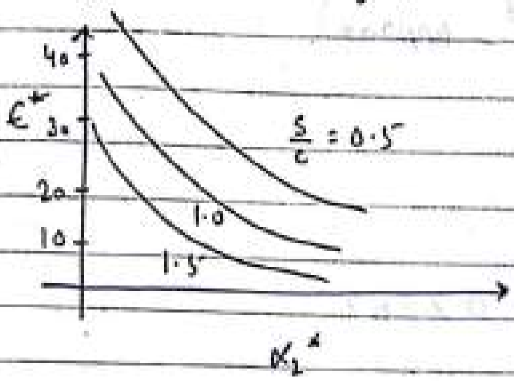
$$\delta^* = m\theta\left(\frac{s}{c}\right)^n, \quad n = \frac{1}{2}, \; m \neq 0.$$

$$\delta^* = m\theta\left(\frac{s}{c}\right)^{n}$$

$n = 1/2$ for a compressor cascade.

$m = 0.23\left(\frac{2a}{c}\right) + \alpha_2^*/500.$, {α_2^*, (air flow exit angle)*

- He gave another graphical relation.



Q. A compressor cascade has $s/c = 1.0$, blade inlet and exit angle are $50°$ and $20°$ respectively. If the ~~cascade~~ blade camber line is a circular path, and the cascade is designed to work on Howell condition. Find (i) incidence (ii) C_L^* at design point, (iii) deflection.

$s/c = 1.0.$
$\alpha_1' = 50°$
$\alpha_2' = 20°$
$\theta_1 = \theta_2 = \theta/2$ (circular camber)

- 1st approxia $\delta^* = 0.$
Let's consider $\alpha_2^* = \alpha_2' = 20°$

Using Howell's condition.

$\delta^* = m\theta\sqrt{s/c}$, $\theta = \alpha_1' - \alpha_2'$
$= 30^\circ$

$m = 0.23\left(\frac{2a}{c}\right) + \frac{\alpha_2^*}{500}$ $= \cancel{0.2}\ 0.27$

$\delta_1^* = 8.1^\circ$ (2^{nd} approx)

$\alpha_2^* = \alpha_2' + \delta^* = 28.1^\circ$

For a better approx.

$\alpha_2^* = 28.1^\circ$

$m = 0.23 + \frac{28.1}{500} = 0.2862$

$\delta^* = 0.2862 \times 30 = 8.6^\circ$

So,

$\alpha_2^* = \alpha_2' + \delta^* = 28.6^\circ$

So, our final value, $\alpha_2^* = 28.6^\circ$, $\delta^* = 8.6^\circ$

From chart, $\epsilon^* = 21^\circ$.

$\epsilon^* = \alpha_1^* - \alpha_2^*$

$\Rightarrow \alpha_1^* = 21 + 28.6$

$\alpha_1^* = 49.6^\circ$

Now,

$\alpha_1^* = \alpha_1' + i^*$

$i^* = \alpha_1^* - \alpha_1'$

$= 49.6 - 50$

$\boxed{i^* = -0.4^\circ}$

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- C_L^*, (C_L at nominal condition)

: Considering $C_D = 0$

$$C_L^* = 2\left(\frac{s}{ch}\right)\left(\tan\alpha_1^* - \tan\alpha_2^*\right)\cos\alpha_m^*$$

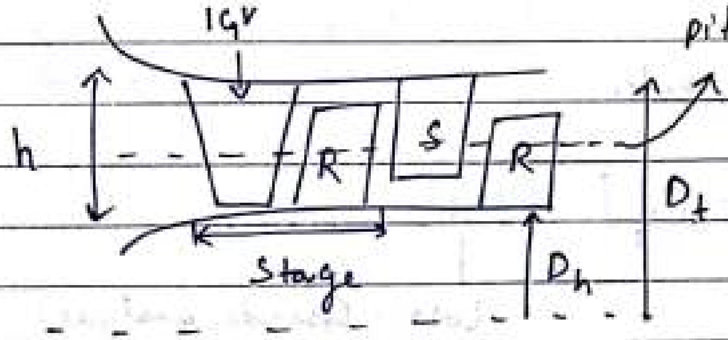
$$\theta_1 \quad \alpha_m^* = \tan^{-1}\left[\frac{1}{2}\left[\tan\alpha_1^* + \tan\alpha_2^*\right]\right]$$

$$\boxed{C_L^* = 0.95}$$

03/09/18

AXIAL FLOW MACHINES

(Incompressible flow)



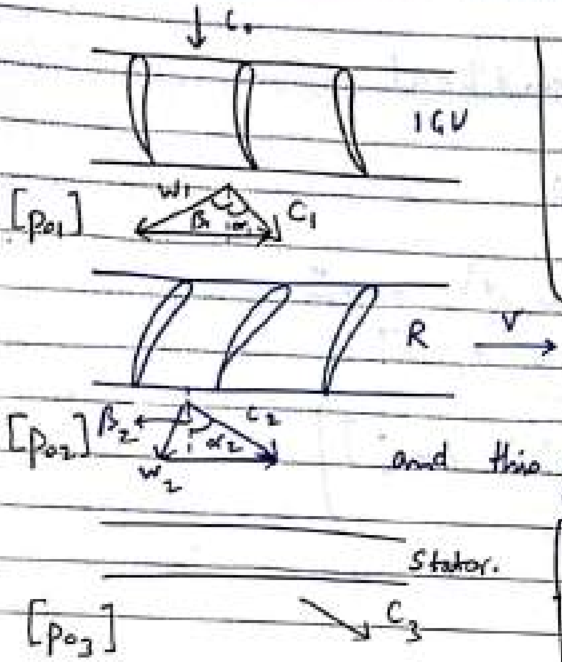
D_h - half radius

D_t - tip radius.

Pitch line Design at $\frac{D_h + D_t}{2}$

If $\frac{D_h}{D_t} \uparrow$, $h \downarrow$, (h - height).

and pitch line design is valid as variation between D_h and D_t is very low.



and this goes further.

Compressor

If $\alpha_1 = \alpha_3$
& $c_3 = c_1$
then only one set of blades can be designed, so that for each stage same operation is required.
This system is called Repeating stage.

Incompressible flow

Euler eq^n for the rotor

$$\underbrace{p_{0_2} - p_{0_1}}_{\text{total } \Delta p} = \rho U [C_{\theta_2} - C_{\theta_1}] - \omega.$$

Loss (Cascade analysis)

$\omega = p_{01_r} - p_{02_r}$
(relative pressure)

Static Pressure (Bernoulli's Eq^n)

$$p_2 - p_1 = \frac{1}{2}\rho [w_1^2 - w_2] - \omega.$$

Work Coefficient

$$\boxed{\psi = \frac{\text{Ideal } \Delta p_0}{\rho U^2}}$$

$$\Psi = \frac{\text{Ideal total pressure drop}}{\rho U^2} \text{ [Across the rotor]}$$

$$= \frac{\rho U [C_{\theta_2} - C_{\theta_1}]}{\rho U^2} = \frac{\Delta C_\theta}{U} = \frac{W_{\theta_1} - W_{\theta_2}}{U}$$

$$\boxed{\Psi = \frac{C_{\theta_2} - C_{\theta_1}}{U} = \frac{W_{\theta_1} - W_{\theta_2}}{U}}$$

Total Pr. rise coefficient

Actual total pressure rise across the rotor divided by ρU^2

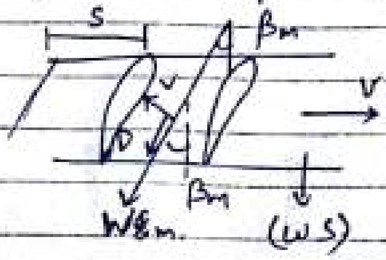
$$\boxed{\psi = \frac{\text{Actual } \Delta p_0}{\rho U^2}} \Rightarrow \psi = \eta_H \frac{\rho u [\Delta C_\theta]}{\rho U^2}$$

$$\Rightarrow \boxed{\psi = \eta_H \Psi}$$

η_H → Hydraulic Efficiency.

flow Coefficient $= \boxed{\phi = \frac{C_z}{U}}$

Hydraulic Efficiency (η_H)



Shaft work $= \rho u [C_{\theta_2} - C_{\theta_1}]$

$$\boxed{\eta_H = \frac{\text{Actual } \Delta p_0}{\text{Shaft work}}}$$

For compressor. (only for rotor)

$$\eta_H = \frac{\rho U [C_{\theta_2} - C_{\theta_1}] - w}{\rho U [C_{\theta_2} - C_{\theta_1}]}$$

$$= 1 - \frac{s w \cos\beta_m \times C_2}{s \rho U [C_{\theta_2} - C_{\theta_1}] \cos\beta_m \times C_2}$$

$$= 1 - \frac{C_2}{U} \frac{s w \cos\beta_m}{(\rho C_2 s)(C_{\theta_2} - C_{\theta_1}) \cos\beta_m}$$

$$= 1 - \frac{\phi D}{(L \cancel{\cos}\beta_m - D \sin\beta_m) \cos\beta_m}$$

$$\boxed{\eta_H = 1 - \frac{\phi (1 + \tan^2\beta_m)}{\left(C_L/C_D + \tan\beta_m\right)}}$$

$C_L/C_D \uparrow, \ \eta_H \uparrow$

For $\alpha_m = 45°$, $\frac{C_L}{C_D} = 20$, $\phi = 0.5$

$\eta_H = 0.95$

~~For turbine~~

$$\cancel{\eta_H = \frac{\text{Actual total pressure drop}}{\cancel{\Delta P} \text{ Shaft work}}}$$

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Turbine

$$\eta_H = \frac{\text{ideal pr drop (shaft work).}}{\text{Actual pr. drop.}}$$

$$= \frac{\rho U [C_{\theta_2} - C_{\theta_1}]}{\rho U [C_{\theta_2} - C_{\theta_1}] + w.}$$

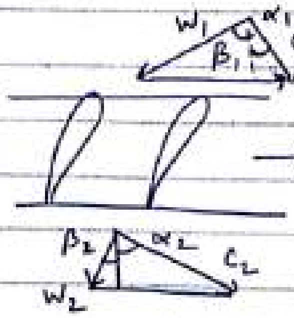
$$\boxed{(\eta_H)_T = \frac{1}{1 + \frac{\phi(1+\tan^2\beta_m)}{\left(\frac{C_L}{C_D} + \tan\beta_m\right)}}}$$

Degree of Reaction.

$$R = \frac{\text{ideal static pr rise across the rotor.}}{\text{ideal }\cancel{\text{static}}\text{ total pr rise across the rotor.}}$$

⇒ incompressible, inviscid flow.

Rotor.



$\rightarrow$

$$\boxed{R = \frac{\frac{1}{2}\rho[W_1^2 - W_2^2]}{\rho U[C_{\theta_2} - C_{\theta_1}]}}$$

$W_1^2 = W_{\theta_1}^2 + C_z^2$, $W_2^2 = W_{\theta_2}^2 + C_z^2$

So,

$$R = \frac{1}{2U} \frac{(W_{\theta_1} + W_{\theta_2})(W_{\theta_1} - W_{\theta_2})}{(C_{\theta_1} - C_{\theta_2})}$$

$$\boxed{C_{\theta_1} - C_{\theta_2} = W_{\theta_1} - W_{\theta_2}}$$

$$\boxed{R = \phi \tan\beta_m = \frac{w_{\theta_1} + w_{\theta_2}}{2u}}$$

So, finally we get.

$$\boxed{R = \frac{w_{\theta_1} + w_{\theta_2}}{2u}} \quad \text{(i)} \qquad \boxed{R = \phi \tan\beta_m} \quad \text{(ii)}$$

where, $\phi = \frac{C_z}{U}$,

$\tan\beta_m = \frac{1}{2}\left[\tan\beta_1 + \tan\beta_2\right]$

Further,

$$\phi = \frac{C_z}{U} = \frac{C_z}{w_{\theta_1} + C_{\theta_1}} = \frac{1}{\tan\beta_1 + \tan\alpha_1} \quad \text{(iii)}$$

Similarly, $\phi = \frac{1}{\tan\beta_2 + \tan\alpha_2}$ ... (iv)

Now,

If $R = 1/2$

$$\frac{1}{\cancel{2}} = \phi\left[\frac{\tan\beta_1 + \tan\beta_2}{\cancel{2}}\right]$$

$$\boxed{\frac{1}{\phi} = \tan\beta_1 + \tan\beta_2} \quad \text{... (v)}$$

from (iii) and (v)

$$\frac{1}{\phi} = \tan\beta_1 + \tan\beta_2 = \tan\beta_1 + \tan\alpha_1$$

$\boxed{\alpha_1 = \beta_2}$ Similarly. $\boxed{\alpha_2 = \beta_1}$

$\boxed{\alpha_1 = \beta_2}$; $\boxed{\alpha_2 = \beta_1}$ when $R = \frac{1}{2}$

When $\boxed{R = 1/2}$

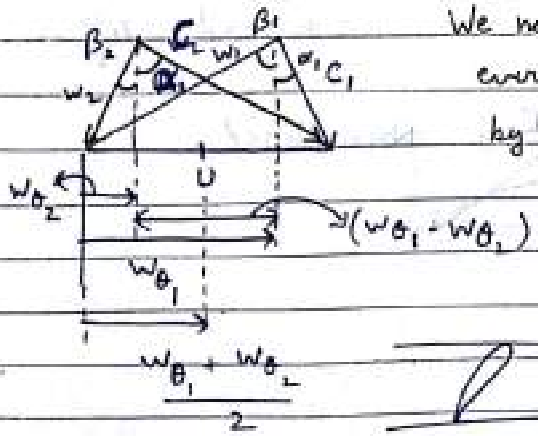
$\Rightarrow$ So the velocity triangles are symmetrical when the degree of rxn is $1/2$. That means kinematics of flow in stator and rotor are same. i.e. BL growing on rotor and stator are same. So the ~~design~~ flow for both are same

We get huge practical benefit.

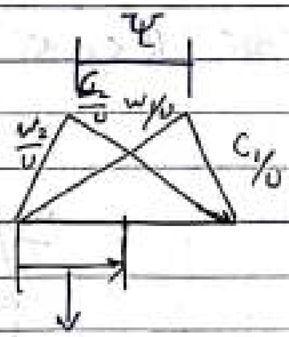
- Design for blade ~~and~~ & of rotor and stator is same.
- Manufacturing cost same
- At time of setup only the stagger angle of stator and rotor has to be ~~different~~ same

$$R = \frac{W_{\theta_1} + W_{\theta_2}}{2U} \quad ; \quad \Psi = \frac{C_{\theta_2} - C_{\theta_1}}{U} = \frac{W_{\theta_1} - W_{\theta_2}}{U}$$

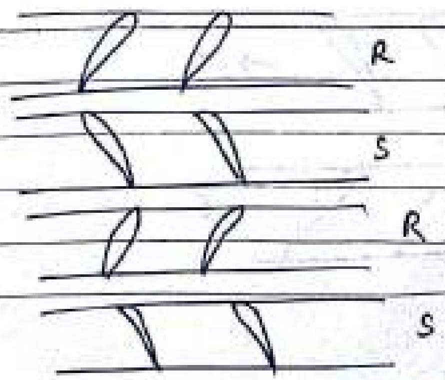
$$\phi = \frac{C_z}{U} = \frac{W_z}{U}.$$



We normalize every velocity by dividing by U.



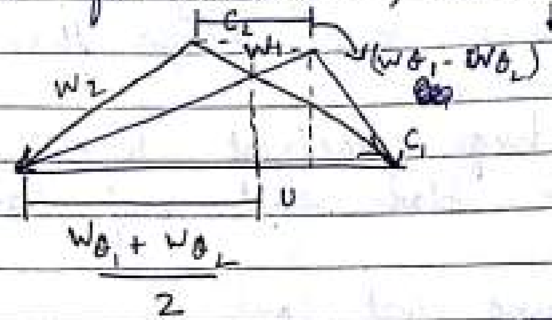
$$R = \frac{W_{\theta_1} + W_{\theta_2}}{2U}.$$



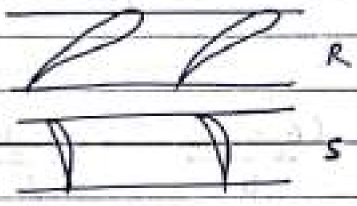
$\boxed{\lambda_R = \lambda_S}$

From experimental data, it is seen that Rotor can withstand higher pressure diff than stator, so $R > 0.5$ is preferred.

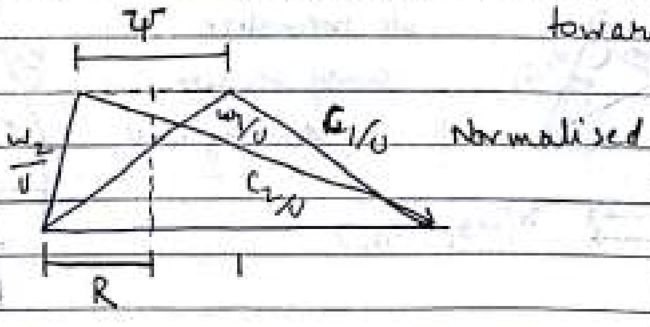
So, velocity Δ for $R > 0.5$, the mid point has to shift right

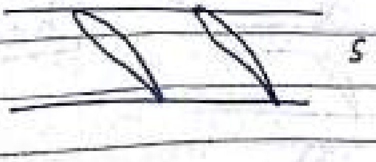


Rotor stagger(λ) > Stator stagger λ $\Rightarrow$ $\boxed{\lambda_R > \lambda_S}$



$R < 0.5 \Rightarrow \lambda_S > \lambda_R$. Mid point has to move towards left.





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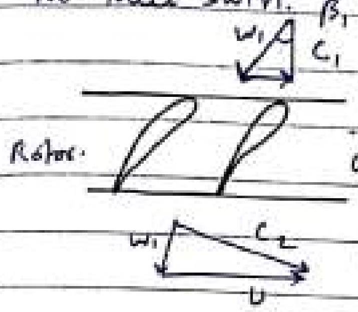
Assignment - 3

Draw the velocity triangle

R = 0 => Impulse machine

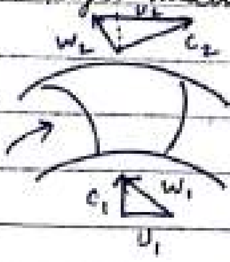
Off design Performance

No inlet swirl.



$U_1 = U_2 = U$

Centrifugal machine.



$U_1 \neq U_2$

$$\psi^* = \frac{\Delta p_0}{\rho U^2}$$

$$\psi^* = \frac{\Delta p_0}{\rho U^2}$$

$$\Delta p_0 = \rho U (c_{\theta_2}^*) \quad (U = U_2)$$
$$= \rho U [U - w_{\theta_2}^*]^*$$

$$\psi^* = 1 - \frac{c_2^*}{U}\phi \tan\beta_2^* = \boxed{1 - \phi^* \tan\beta_2^* = \psi^*}$$

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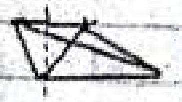
Assignment - 3

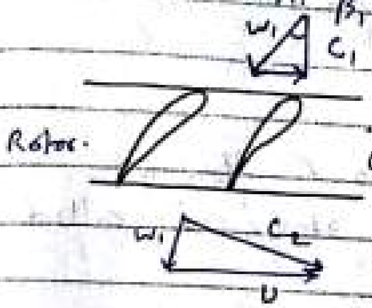
→ draw the velocity triangle

$R=0 \Rightarrow$ Impulse machine



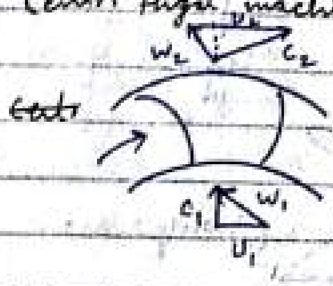
Off design Performance [when the operating point is not for which m/c is designed]

No inlet swirl.



$U_1 = U_2 = U$

Centrifugal machine.



$U_1 \neq U_2$

$$\psi^* = \frac{\Delta p_0}{\rho U^2}$$

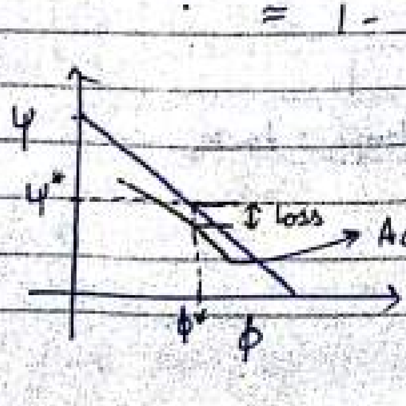
$$\psi^* = \frac{\Delta p_0}{\rho U^2}$$

$$\Delta p_0 = \rho U (C_{\theta_2}^*) \quad (U = U_2)$$
$$= \rho U [U - W_{\theta_2}^*]^*$$

$$\psi^* = 1 - \frac{C_x^*}{U}\phi \tan\beta_2^* = \boxed{1 - \phi^* \tan\beta_2^* = \psi^*}$$

$\Rightarrow \beta_2$ does not change with mass flow rate and is only guided by the blade.

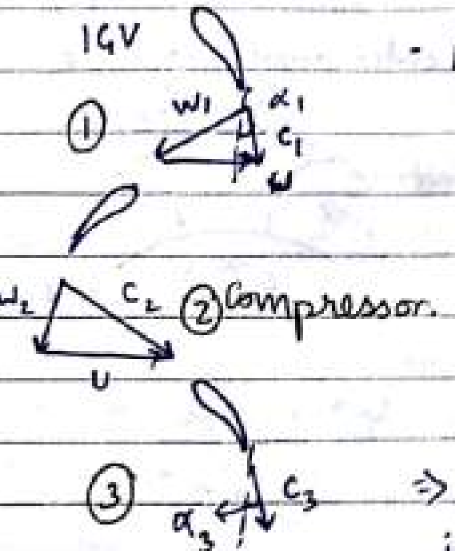
So, $\psi = 1 - \phi \tan\beta_2^*$
$= 1 - \phi C$



Actual $\psi - \phi$ Here loss decreases with ϕ.

Axial flow m/c (Compressible flow).

- Enthalpy changes when we deal with compressible flow.
  (Along the pitch streamline)



- Based on the design, we can make

$$C_3 = C_1$$
$$\alpha_1 = \alpha_3$$

Then the kinematics of flow is repeated and the stage is called <u>Repeating stage.</u>

$\Rightarrow$ Now as there is no work input at the stator, thus we analyse the rotor.

$$W = U(C_{\theta_2} - C_{\theta_1}) = U(W_{\theta_1} - W_{\theta_2})$$

$$h_{0_2} - h_{0_1} = U(W_{\theta_1} - W_{\theta_2})$$

Assuming C_p const, $C_p(T_{0_2} - T_{0_1}) = UC_z(\tan\beta_1 - \tan\beta_2)$

Stage loading, $\Psi = \dfrac{C_p \Delta T_0}{U^2} = \dfrac{C_z}{U}(\tan\beta_1 - \tan\beta_2)$

$$\Rightarrow \boxed{\Psi = \phi(\tan\beta_1 - \tan\beta_2)}$$

$$\phi = \frac{C_z}{U} = \frac{1}{\tan\beta_1 + \tan\alpha_1} = \frac{1}{\tan\beta_2 + \tan\alpha_2}$$

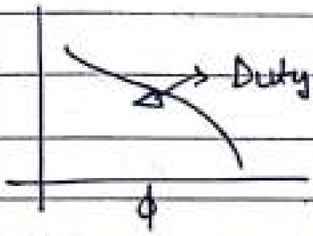
Scanned by CamScanner

β_1 is inlet relative angle, and we only know the entry ~~exit~~ angle of the blade so we try to express ψ in α's and not β's. and similarly at exit we know β_2 but not α_2.

so, $\psi = \phi (\tan\beta_1 - \tan\beta_2)$

$$\Rightarrow \boxed{\psi = 1 - \phi (\tan\beta_2 + \tan\alpha_1)}$$

Again similar to Incom. flow, we have a dependence of ψ on ϕ.



For a $\boxed{\text{repeating State}}$, $c_1 = c_3$.
- there is no work input in stator.
- so, total p rise across rotor is equal to that of the stage.

Hence, $T_{o_3} - T_{o_1} = T_{o_2} - T_{o_1} \Rightarrow \boxed{\underbrace{\Delta T_{o_S}}_{\text{Stage}} = \underbrace{\Delta T_{o_R}}_{\text{Rotor.}}}$

S - Stage

s : Stator.

$$\boxed{C_p (T_{o_2} - T_{o_1}) = U C_z (\tan\beta_1 - \tan\beta_2)}$$

For $\boxed{\text{repeating stage,}}$

total = static

$\Delta T_o = \Delta T_s$ (as the kinematics are equal at 1 and 3.

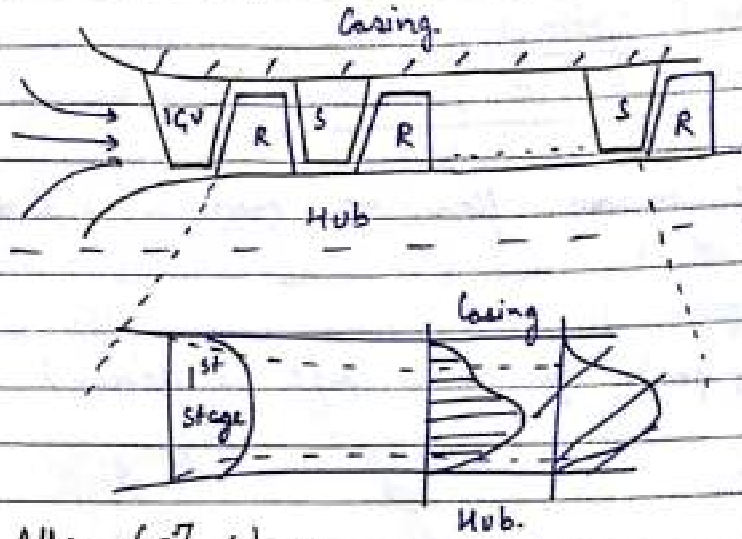
as the initial and final velocity is same.

Static

to account for the 3-dimensional effect.

$$\Delta T_0 = \Delta T_s = \lambda \frac{U C_z}{C_p}\left[\tan\beta_1 - \tan\beta_2\right]$$

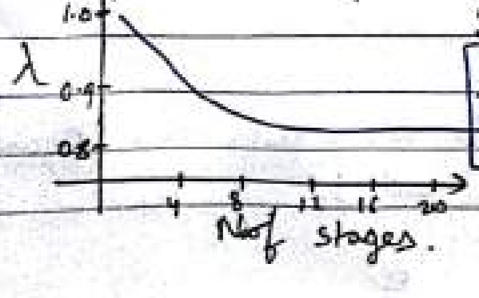
<u>Three - dimensional Effect</u>



After 6-7 stage, the work capacity decreases in the middle due to acc of flow, but there is compensation at the hub and casing boundary as the mass flow rate decreases. But the compensation is not sufficient. And there is overall decrease in the work capacity.

- Thus due to three-dimensional effect, work capacity ψ decreases.

λ (work done factor) =



So,

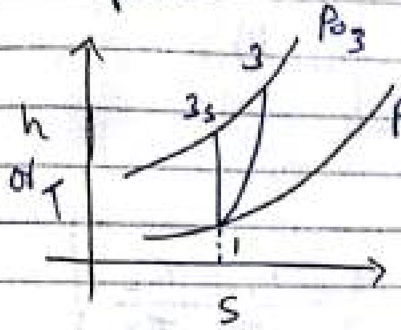
$$\boxed{\Delta T_0 = \lambda \frac{U C_z}{C_p}\left[\tan\beta_1 - \tan\beta_2\right]}$$

From Euler work, we get ΔT_0 and from use that to get ΔP_0.

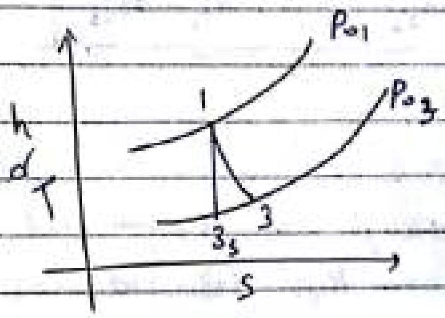
Stage Isentropic Efficiency

- For compressible flow, we see ΔT_0 across stage and not ΔP_0.

- For incompressible flow, we can get ΔT_0 from momentum or energy eq^n but not for compressible flow.



$$\boxed{\eta_{s_c} = \frac{T_{o_{3_s}} - T_{o_1}}{T_{o_3} - T_{o_1}} = \frac{h_{o_{3_s}} - h_{o_1}}{h_{o_3} - h_{o_1}}}$$



$$\boxed{\eta_{s_T} = \frac{T_{o_1} - T_{o_3}}{T_{o_1} - T_{o_{3_s}}}}$$

Now,

$$\eta_{s_c} = \frac{T_{o_1}\left[\left(\frac{T_{o_{3_s}}}{T_{o_1}}\right) - 1\right]}{T_{o_3} - T_{o_1}} = \frac{T_{o_1}\left[\left(\frac{P_{o_3}}{P_{o_1}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{o_3} - T_{o_1}}$$

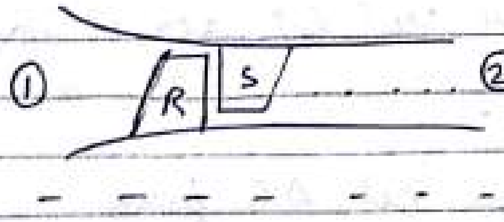
$$\boxed{\frac{P_{o_3}}{P_{o_1}} = \left[1 + \frac{\eta_s \Delta T_0}{T_{o_1}}\right]^{\frac{\gamma}{\gamma-1}}}$$

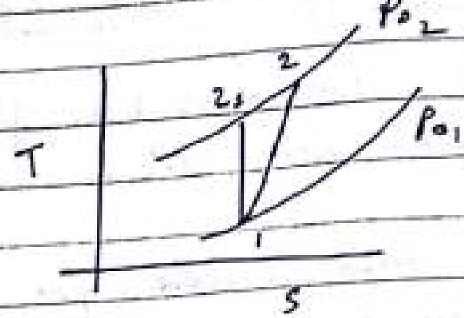
$$\Delta T_0 = T_3 - T_1 = \frac{\lambda U C_a}{C_p}\left[\tan\beta_1 - \tan\beta_2\right]$$

T_{o_1} → Changes for every stage.

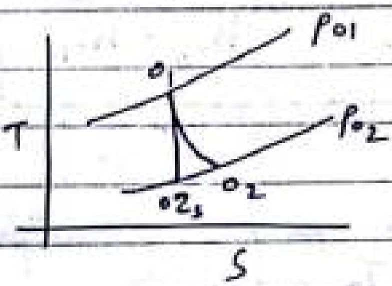
Overall

Compressor or turbine effn.





$$\boxed{\eta_c = \frac{h_{o_{2s}} - h_{o_1}}{h_{o_2} - h_{o_1}} = \frac{T_{o_{2s}} - T_{o_1}}{T_{o_2} - T_{o_1}}} \quad (1)$$



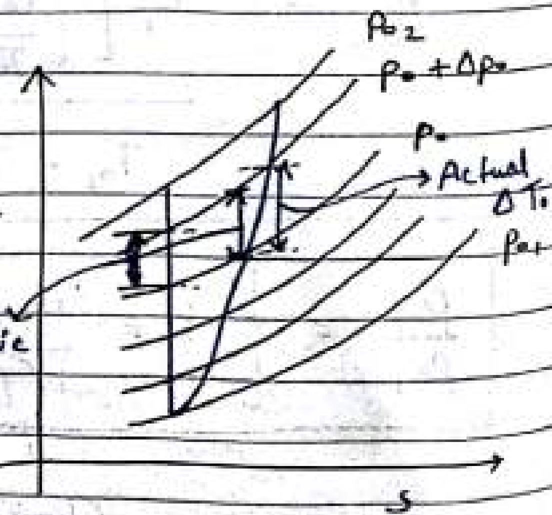
$$\boxed{\eta_T = \frac{h_{o_1} - h_{o_2}}{h_{o_1} - h_{o_{2s}}} = \frac{T_{o_1} - T_{o_2}}{T_{o_1} - T_{o_{2s}}}}$$

\# For each stage, the temp change is same but, not the pressure differennce as the latter depends on inlet total Temperature.

\# Small Stage Effn.

$$\boxed{\eta_s = \frac{\Delta T_{0s}}{\Delta T_0}\bigg|_{stage}}$$

⇒ If the stages are similar designed, thus the ~~to~~ small stage effn will remain const and loss would be invariant across stages.



Compressor and const η_s
⇓

⇒ $\Delta T_0 = \frac{\Delta T_{0s}}{\eta_s}$ ⇒ $T_{o_2} - T_{o_1} = \sum \frac{\Delta T_{os}}{\eta_s}$

⇒ $\boxed{T_{o_2} - T_{o_1} = \frac{1}{\eta_s} \sum \Delta T_{os}}$ ② [for similar stage design. η_s = const]

⇒ From (1) and ②

$$\frac{\eta_s}{\eta_c} = \frac{\sum \Delta T_{os}}{T_{o_{2s}} - T_{o_1}} > 1$$ as pressure lines are divergent.

$\boxed{\eta_c < \eta_s}$ ⇒ preheating.

⇒ because of passing through one stage, the work requirement for next stage increases, thus the overall $\eta_c < \eta_s$.

for turbine $\boxed{\eta_T > \eta_s}$ → reheating.

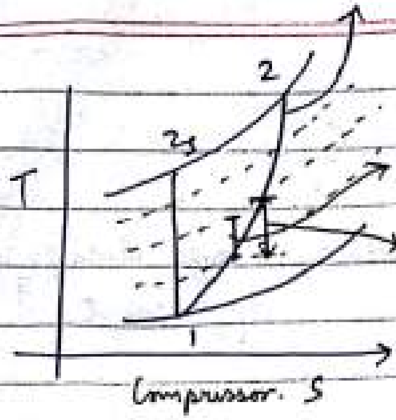
At each stage some heat is produced due to frictional losses which increases the work extracted from the next stage.

Polytropic Effn.

We assume very-very small stages (elemental small stages). Efficiency of this stages is called - ~~The~~ the polytropic effn.

$$T ds = dh - dP/\rho$$

Between 1-2 we assume a polytropic process of index n.



dT_{0s} (elemental isentropic rise in the elemental stage)

dT_0

So, $pv^n = c$.

$p^{1-n} T^n = c$.

$$\Rightarrow (1-n)\frac{dp}{p} + n\frac{dT}{T} = 0. \Rightarrow \boxed{dT = \left(\frac{n-1}{n}\right)\frac{dp}{p} \cdot T}$$

So, for total quantity of $\Delta T_0 = \left(\frac{n-1}{n}\right)\frac{dp_0}{p_0} \cdot T_0$

For isentropic $\Delta T_s = \left(\frac{\gamma-1}{\gamma}\right)\frac{dp_0}{p_0} \cdot T_0$

$$\Rightarrow \eta_{p_c} = \frac{dT_{0s}}{dT_0} = \frac{(\gamma-1)/\gamma}{(n-1)/n}$$

Hence, $\boxed{\frac{n-1}{n} = \frac{\gamma-1}{\gamma\,\eta_{p_c}}}$ $n > \gamma$

For turbine $\boxed{\frac{n-1}{n} = \eta_{p_T}\frac{\gamma-1}{\gamma}}$ $n < \gamma$

If nothing is given $\eta_{p_T} = \eta_s$ (small stage effu)

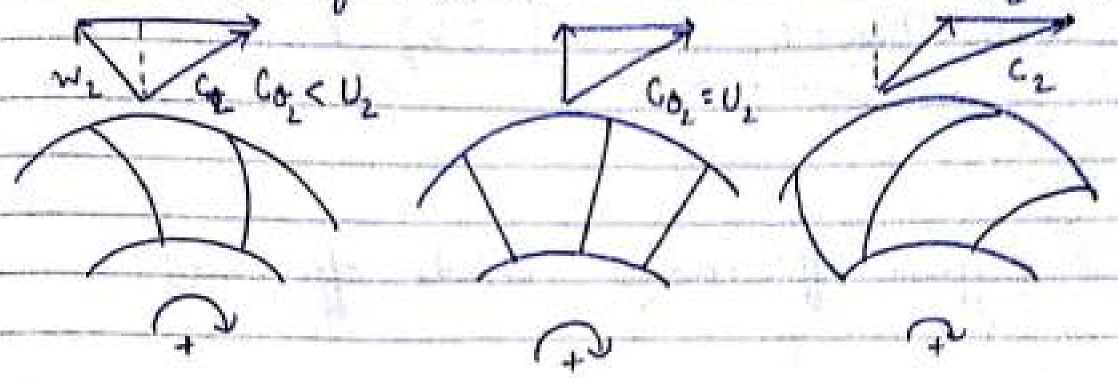
for very large no of stages.

MID SEM

13/09/18

Tutorial - 1 (Assignment 1)

Q4



U_2 is same for each situation.

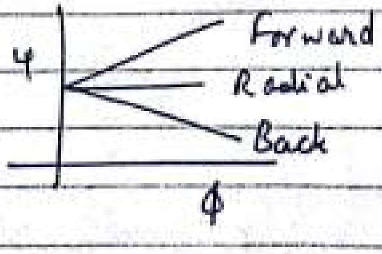
$(C_{\theta_2})_B > (C_{\theta_2})_{str.} > (C_{\theta_2})_f$

Work produced $W_f > W_s > W_B$.

Backword is prefered as the noise level is high in case of forward.
But if we have mud in flow then, forward is preferred as it throw away the sludge from the rotor.

$$gH = u_2 C_{\theta_2} - u_1 C_{\theta_1}$$

$$\frac{gH}{U_2^2} = \boxed{\Psi = 1 - \phi \tan\beta_2} \qquad \phi = \frac{C_2}{U}$$



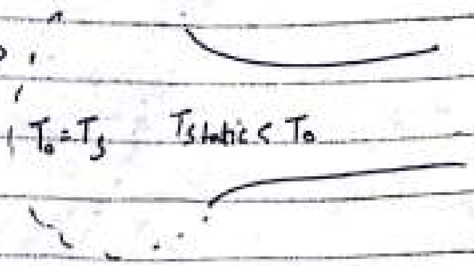
9) $$n_s = \frac{N Q^{1/2}}{(\Delta h_0)^{3/4}} = \frac{N \left(\frac{m}{\rho}\right)^{1/2}}{(\Delta h_0)^{3/4}}$$

$$\Delta h_0 = C_p (T_{02} - T_{01})$$
$$= C_p T_{01} \left[\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$T_{01} = 298\,K$, $\frac{P_{02}}{P_{01}} = 2.$

$\cancel{m} C_p T = \frac{1}{2} \cancel{m} V^2$

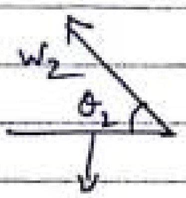
$\Rightarrow$



$\frac{V^2}{2 C_p} = T$

$\Rightarrow$ Diffuser efficiency = Blade efficiency

12. Vane angle



β_2 = Vane angle = 35°

$\alpha_1 = 0$.

25/09/18

$$\eta_c = \frac{T_{0s} - T_{01}}{T_{02} - T_{01}}$$

Overall efficiency

$$\eta_c = \frac{\frac{T_{0s}}{T_{01}} - 1}{\frac{T_{02}}{T_{01}} - 1} = \frac{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{n-1}{n}} - 1}$$

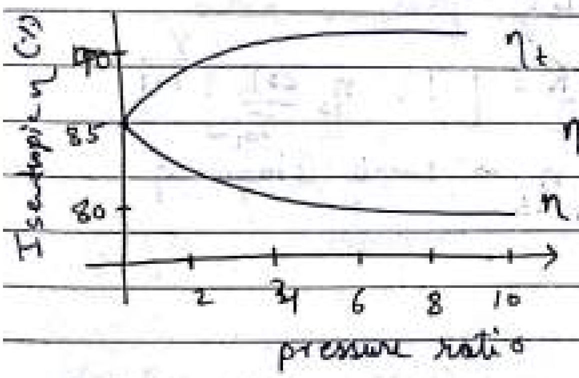
relation of n and γ

$$\frac{n-1}{n} = \frac{\gamma-1}{\gamma \eta_{p_c}}$$

$$\boxed{\eta_c = \frac{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma \eta_{p_c}}} - 1}}$$

for turbine

$$\boxed{\eta_t = \frac{1 - \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma \eta_{p_t}}}}{1 - \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}}}$$



We know that,

$$\eta_{p_c} = \frac{dT_{0s}}{dT_0} \Rightarrow \eta_{p_c} \frac{dT_0}{T_0} = \frac{\gamma-1}{\gamma} \frac{dp}{p}$$

$$\boxed{\eta_{p_c} = \frac{\ln\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}}{\ln\left(\frac{T_{02}}{T_{01}}\right)}}$$

So, from the total pressure and the temperature at ends, we can get polytropic η.

ΔT_{0s} - for a stage.

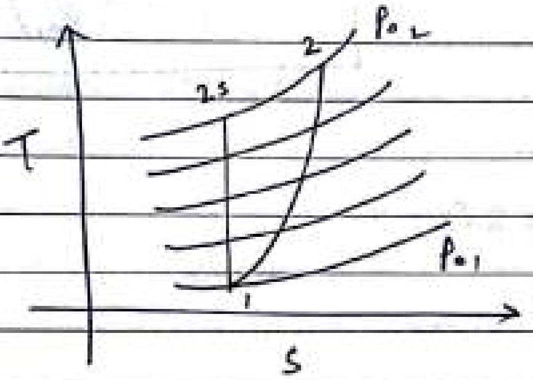
So,

$$T_{o_2} - T_{o_1} = N \Delta T_{0s}$$

$R_s = \left(\frac{P_{o_3}}{P_{o_1}}\right)_{stage}$ → P_{o_3} end of stage. P_{o_1} entering the stage.

⇒ $\boxed{\frac{P_{o_2}}{P_{o_1}} \neq R_s^N}$ as at every stage, pressure ratio depends on the inlet temperature of the stage.

Overall Pressure ratio of a multistage compressor



Stage pressure ratio.

$$R = \left[1 + \eta_s \frac{\Delta T_0}{T_{o_1}}\right]^{\frac{\gamma}{\gamma-1}}$$

η_s → small stage η.

$$R = \frac{p_{o_2}}{p_{o_1}} \neq R_s^N$$

But

$T_{o_2} - T_{o_1} = N \Delta T_{o_s}$ (i) [If all stages are similar in design.]

Further,

$\frac{n}{n-1} = \eta_{P_c} \frac{\gamma}{\gamma-1} \simeq \eta_s \frac{\gamma}{\gamma-1}$ [For practical purposes $\eta_p \simeq \eta_s$.]

From (i) $\frac{T_{o_2}}{T_{o_1}} = 1 + \frac{N \Delta T_0}{T_{o_1}}$,

$\rho = \frac{P_0}{RT_0}$ Static ~~Total~~ pressure. ~~Total~~ Static temperature

$Ma = \frac{C}{\sqrt{\gamma R T_1}}$ → Static Temperature

$\frac{T_1}{T_{01}} = \left(\frac{P_1}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ Imp.

$$\frac{p_{02}}{p_{01}} = \left(1 + \frac{N\Delta T_0}{T_{01}}\right)^{\frac{n}{n-1}}$$ → Total Temperature

$$\boxed{\frac{p_{02}}{p_{01}} = \left(1 + \frac{N\Delta T_0}{T_{01}}\right)^{\eta_s \frac{\gamma}{\gamma-1}}}$$

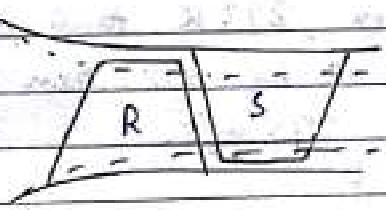
$\frac{p_{02}}{p_{01}}$ is for the overall compressor.

where, $\Delta T_{0s} = \Delta T_s = \lambda \frac{U C_z}{C_p}\left[\tan\beta_1 - \tan\beta_2\right]$

Degree of Reaction.

$$R = \frac{\Delta T_{static\ in\ rotor}}{\Delta T_{static\ in\ stage}} = \phi \tan\beta_m.$$

Losses



⇒ BL will develop on the hub, casing and the surface of the blade.

⇒ Because of the tip clearance, there will be vortices, leading loss.

⇒ All these factors will lead to certain losses in turbomachinary.

1) Profile Loss

Due to BL growing on blade surface.

- Can be ~~defined~~ found from Cascade test.

2) Annulus loss.

BL developing on the hub and casing.

3) Secondary flow loss.

All the other forms of losses.

$$\boxed{C_D = C_{D_p} + C_{D_a} + C_{D_s}} \rightarrow \boxed{C_{D_s} = 0.018\, C_L^2}$$

C_{D_p} → Cascade test.

C_{D_a} → $\boxed{C_{D_a} = 0.02 \frac{S}{H}}$ [S pitch, H height of blade]

25/09/18

- In multi stage axial compressor, compressing air, inlet T 293 K to a pressure ratio of 5:1. Each stage is to be 50% rxn and the blade mean speed is 275 m/s.

  Flow coefficient = 0.5 = ϕ

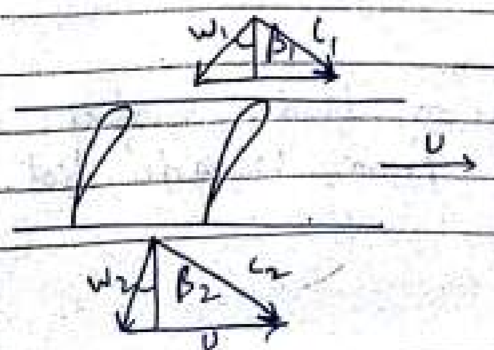
  Stage loading factor = 0.3 = ψ

  The ~~st~~ data is const for all stages.

  Determine flow angles, no of stages required, if η_s is 88.8 %.

  Determine overall η (isentropic).

  Given C_p = 1.005 kJ/kg K, γ = 1.4.



$$\Psi = \phi\,(\tan\beta_1 - \tan\beta_2) \quad \text{——(1)}$$

$$R = \frac{\phi}{2}\,(\tan\beta_1 + \tan\beta_2) \quad \text{——(2)}$$

$$\tan\beta_1 = \left(R + \tfrac{\Psi}{2}\right)\Big/\phi \qquad \tan\beta_2 = \left(R - \tfrac{\Psi}{2}\right)\Big/\phi.$$

$= \cancel{0.65 \times 0.5}$ $\qquad \beta_1 = \alpha_2 = 52.43°$

$\beta_2 = \alpha_1 = 35°$

$$\Psi = \frac{C_p \Delta T_0}{U^2} \Rightarrow \Delta T_0 = 22.975°$$

$\Delta T_0 = T_{01} + N\Delta T_0$ ← per stage

$$\boxed{\Delta T_{0s} = \frac{U C_a}{C_p}\,(\tan\beta_1 - \tan\beta_2)}$$

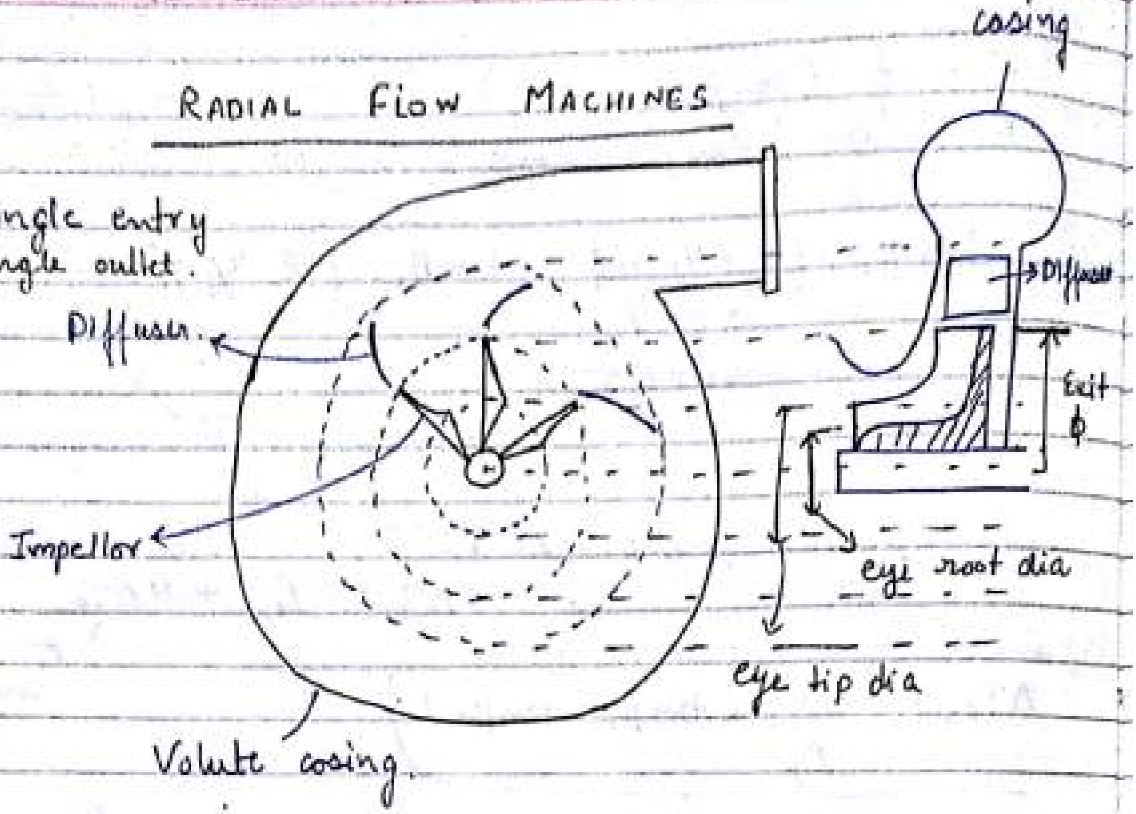
$$\frac{T_{02}}{T_{01}} = 1 + \frac{N\Delta T_0}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\eta_p \gamma}}$$

$$N \simeq \frac{T_{01}}{\Delta T_0}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\eta_s \gamma}} - 1\right]$$

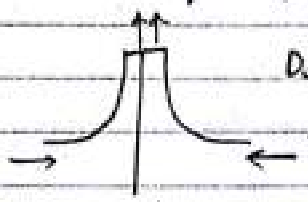
$$\eta_t = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\eta_s \gamma}} - 1} = 86.3\%$$

RADIAL FLOW MACHINES

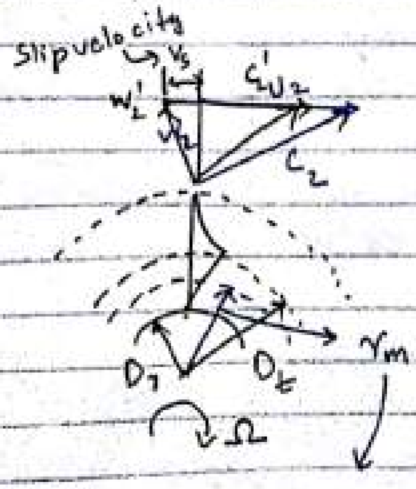
Single entry
Single outlet.



- All the work is done in the impellor, and the diffuser only converts KE of flow to PE of flow.



Double entry
single outlet.

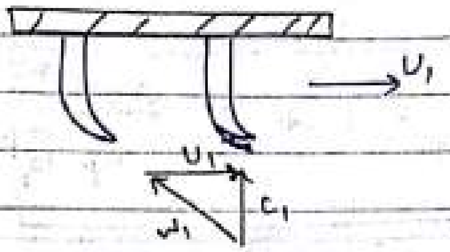


- In the ideal case, w_2 is $\perp$ to v_2 at exit.

- But due to the inertia of flow, the vector w_2 is moving backwards, i.e $\beta_2 > 90$. This is known as slip.

mean eye radius or dia. So, $(C_{\theta_2})_a < (C_{\theta_2})_i$

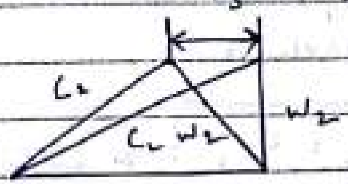
Impellor from top.



$\boxed{\text{Thus } C_{\theta_1} = 0}$

\# Slip factor = $\boxed{\sigma = \dfrac{U_2 C_{\theta_2} \text{ (with } v_s)}{U_2 C_{\theta_2} \text{ (without } v_s)}}$

$v_s \Rightarrow$ Slip velocity.



Theoretical work done

$= U_2 C_{\theta_2} - U_1 \cancel{C_{\theta_1}}^{0}$

Without slip $C_{\theta_2} = U_2 \Rightarrow \dot{W} = U_2^2$

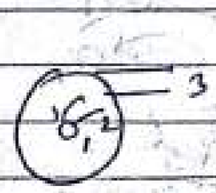
With slip Actual work done : $\dot{W} = \sigma U_2^2$

- Because of BL development we have losses, so we multiple a factor ρ in W.

Euler Work $W = \rho \sigma U_2^2$ $\quad \rho > 1$

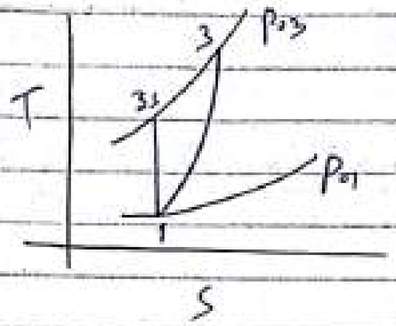
↳ power input factor.

- The pressure ratio.



- Work is done only in impeller, and no work is done in the diffuser or volute casing.

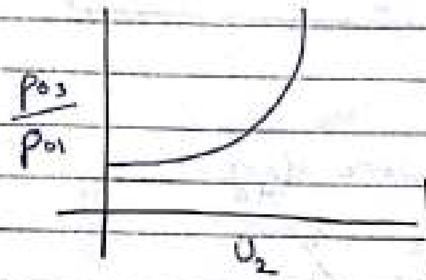
$T_{o_2} - T_{o_1} = \dfrac{\rho \sigma U^2}{C_p} \Rightarrow \boxed{T_{o_3} - T_{o_1} = \dfrac{\rho \sigma U^2}{C_p}}$



$$\eta_c = \frac{T_{o_{3s}} - T_{o_1}}{T_{o_3} - T_{o_1}}$$

$$\boxed{\frac{T_{o_{3s}}}{T_{o_1}} = 1 + \eta_c \frac{p \sigma U^2}{C_p T_{o_1}}}$$

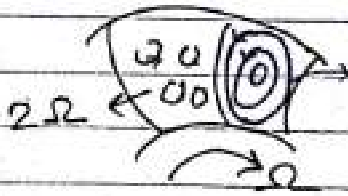
Thus, $\boxed{\left(\frac{p_{o3}}{p_{o1}}\right) = \left(\frac{T_{o_{3s}}}{T_{o_1}}\right)^{\frac{\gamma}{\gamma-1}} = \left[1 + \eta_c \frac{p \sigma U^2}{C_p T_{o_1}}\right]^{\gamma/\gamma-1}}$



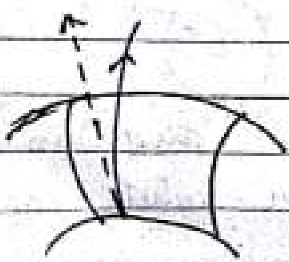
Pr. ratio depends on the U of the machine.

$\boxed{(\text{pr. ratio})_{max} = 5:1}$ to avoid issues related to high U_2

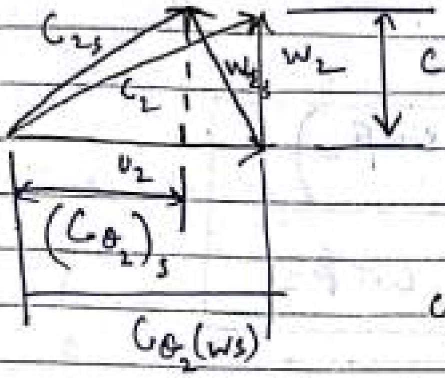
Relative Eddy



There is formation relative eddy in the flow due to local smaller eddies.



- relative streamline without effect of eddy.
- Now due to eddy, the flow lags behind and this is the cause of the slip occuring.



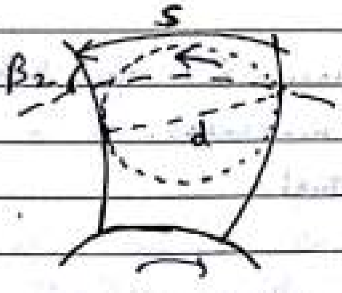
Without slip.

$$C_{\theta_2} = U_2 - C_r \cot\beta_2$$

with slip, $C_{\theta_2} = U_2 - C_r \cot\beta_2 - v_s$.

$$\boxed{\sigma = 1 - \frac{v_s}{U_2 - C_r \cot\beta_2}}$$

~~Stro~~ Stodola Relation for slip.



The eddy is approximated by a solid cylinder. with.

$$\boxed{d = s \sin\beta_2}$$

and.

$$\boxed{v_s = \frac{1}{2}\,\Omega\, s \sin\beta_2}$$

$$\Rightarrow \quad v_s = \frac{1}{2}\,\Omega \times \frac{2\pi r_2}{N}\,\sin\beta_2$$

$$\boxed{v_s = \frac{\pi U_2}{N}\,\sin\beta_2}$$

Slip factor.

$$\boxed{\sigma = 1 - \frac{\left(\frac{\pi U_2}{N} \sin\beta_2\right)}{U_2 - C_r \cot\beta_2}}$$

$$\boxed{\sigma = 1 - \frac{\frac{\pi}{N} \sin\beta_2}{1 - \phi_2 \cot\beta_2}}$$

⇒ For a fully radial flow machine.

$\beta_2 = 90°$ ⇒ $\sigma = 1 - \frac{\pi}{N}$

⇒ If no of blades is greater than 8, this relation is correct for the radial machines.

V_3(stodola) = V_3(actual).

Stanitz's Correlation

$$\boxed{\sigma = 1 - \frac{0.63\ \pi/N}{1 - \phi_2 \cot\beta_2}}$$

Compressibility effect

(Read chapter from FM White).

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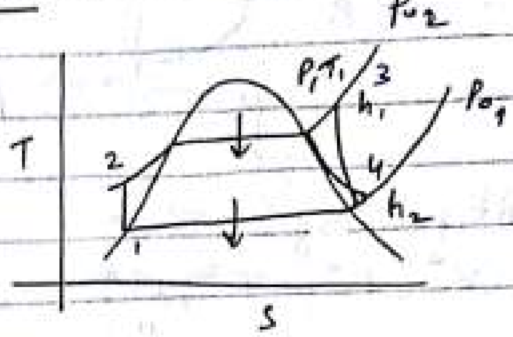
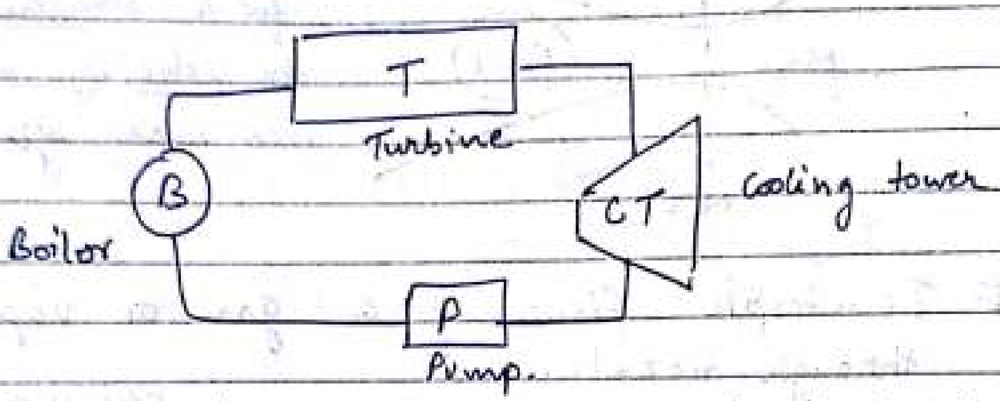
STEAM TURBINE

Rankine Cycle.



1-2. Pump
2-3 Boiler
3-4 nozzle
4-1 Condenser.

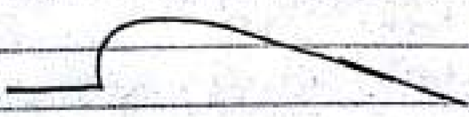


⇒ The major amount of water is required in the cooling tower.

⇒ η can be enhanced by increasing T or P of boiler and decreasing P or T of condenser.

Steam Turbine is nothing but a assembly of nozzles and rotating drum.

- We need a convergent or divergent nozzle to produce a supersonic steam.



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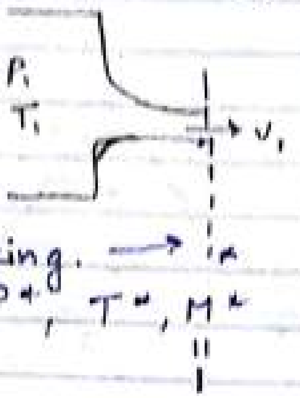
(ideal gas)

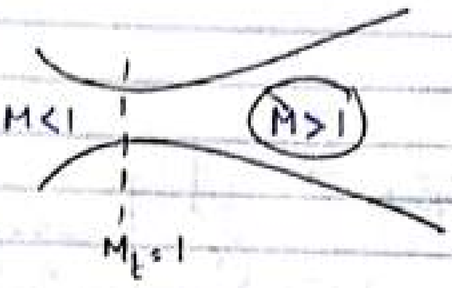


$M = \frac{v_1}{\sqrt{\gamma R T_1}}$

If the local M no, is 1, the velocity at the throat of nozzle does not increase and the nozzle chokes.

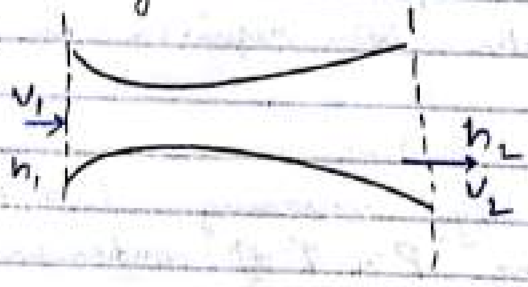
At choking. → *
$V^*, P^*, T^*, M^* = 1$

#



- After the throat of nozzle, for a compressible flow, the velocity increases continuously after the throat.

Isentropic Flow of a gas or vapour through nozzle.

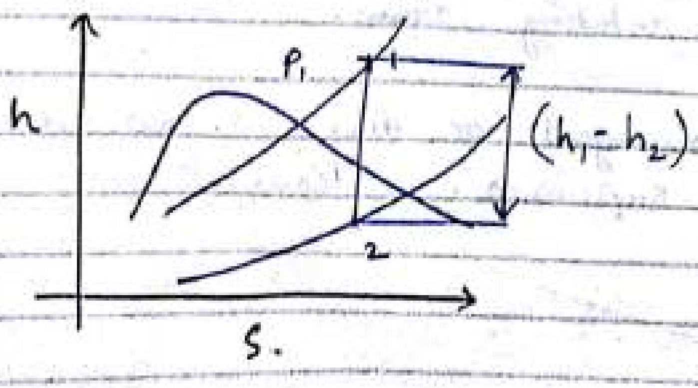


neglected.

$$h_1 + \cancel{\frac{v_1^2}{2}} = h_2 + \frac{v_2^2}{2}$$

$$\Rightarrow \boxed{V_2 = \sqrt{2\underbrace{(h_1 - h_2)}_{\text{Enthalpy drop in nozzle.}}}}$$

⇒ Using Mollier's diagram.



Pinpointing h_1 and h_2 using T and P to get the enthalpy drop.

flow through the nozzle and super saturated ~~flow~~ Expansion.

Calculating Δh without Mollier diagram.

$$h = u + pv \Rightarrow dh = \underbrace{du + pdv} + vdp$$

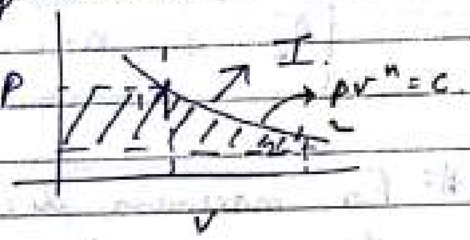
$$dh = dQ + vdp.$$

For adiabatic process, $dQ = 0$.

$$\boxed{dh = \frac{dP}{\rho}} \Rightarrow \boxed{h_2 - h_1 = +\int_1^2 \frac{dP}{\rho}} = I$$

$\Rightarrow$ So we have corelated Δh to P and V which are easily measurable quantities.

So in PV diagram.



$p \propto \frac{1}{v}$

$$h_1 - h_2 = \ominus \int v\,dp = \cancel{\frac{p_1v_1 - p_2v_2}{n}}$$

$$= p_1v_1 + \frac{p_1v_1 - p_2v_2}{n-1} - p_2v_2$$

$$\boxed{\int v\,dp = \frac{n}{n-1}\left(p_1v_1 - p_2v_2\right)}$$

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$$\boxed{h_1 - h_2 = \frac{n}{n-1} p_1v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$$

and $V_2 = \sqrt{\frac{2n}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}$

and $n = \gamma$ for ideal gas.

We try to eliminate v_2 as the rest are either design parameter or the inlet parameter.

Applying Continuity Equations:

$$\dot m = \rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\dot m = \frac{A_2 V_2}{v_2}$$

Hence $$\frac{\dot m}{A_2} = \frac{V_2}{v_2} = \frac{1}{v_2}\left[\sqrt{2\frac{n}{n-1}p_1 v_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]}\right]$$

Also, $p_1 v_1^n = p_2 v_2^n \Rightarrow v_2 = \left(\frac{p_1}{p_2}\right)^{1/n} v_1$

So,

$$\boxed{\frac{\dot m}{A_2} = \sqrt{2\frac{n}{n-1}\frac{p_1}{v_1}\left[\left(\frac{p_2}{p_1}\right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1}\right)^{\frac{n+1}{n}}\right]}}$$

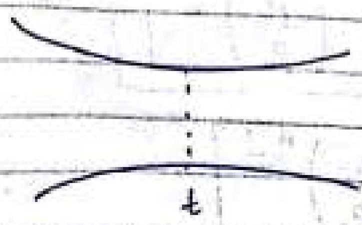
For maximum discharge.

Let $\frac{p_2}{p_1} = y$.

For max discharge. $\frac{d}{dy}\left[y^{2/n} - y^{n+1/n}\right] = 0$

$$\Rightarrow \boxed{y_m = \left(\frac{2}{n+1}\right)^{n/n-1}}$$

- The pressure ratio for which discharge is max is called critical pr ratio.
- Thus,



$$\frac{p_2}{p_1} = \left(\frac{p_2}{p_1}\right)^* = \frac{p_{throat}}{p_i} = \left(\frac{2}{n+1}\right)^{\frac{n}{n+1}}$$

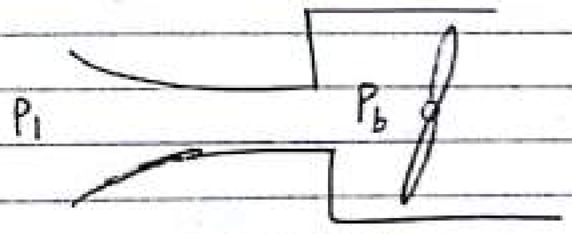
Discharge is maximum when the ~~flo~~ pressure at throat is equal to critical pressure, and this condition is said to be choked.

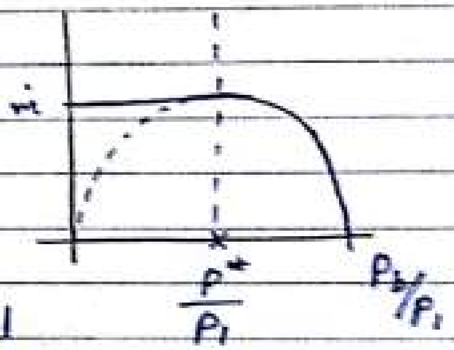
The value of n.

$n = \gamma = 1.4$ for diatomic gas.
$n = \cancel{\gamma} = 1.3$ for superheated steam.
$n = 1.135$ for dry saturated steam.
$n = 1.135 + 0.1x$ for wet steam.
x is dryness fraction.

⇒ For $n = 1.3$.
$P_t = 0.546\, P_1$
$n = 1.4$, $P_2 = 0.528\, P_1$

#





- - - - theoretical
—— Actual
p^* critical pressure

After $\frac{p^*}{P_1}$, $\dot{m}$ do not decrease.

Considering and putting the value of critical pr. in the V_2 eqn.

$$V_2 = \sqrt{\frac{2n}{n-1}\left(P_1V_1 - P_2V_2\right)} = \sqrt{\frac{2n}{n-1}P_2V_2\left[\left(\frac{P_2}{P_1}\right)^{\frac{1-n}{n}} - 1\right]}$$

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n-1}$$

So,

$$V_2 = \sqrt{2\frac{n}{n-1} P_2 v_2 \left[\frac{n+1}{2} - 1\right]}$$

$\Rightarrow$

$$V_2 = \sqrt{n P_2 v_2}$$

For perfect gas,

$$\boxed{V_2 = \sqrt{\gamma R T_2} = \text{sonic velocity}}$$

That is at critical pressure, locally there is sonic velocity.

At the max. discharge, the local flow velocity is equal to the sonic velocity. Hence we look at the throat.

$$\frac{p_2}{p_1} = \left(\frac{2}{n+1}\right)^{n/n-1}$$

So,

$$V_2 = \sqrt{2\frac{n}{n-1} p_2 v_2 \left[\frac{n+1}{2} - 1\right]}$$

$\Rightarrow$

$$V_2 = \sqrt{n p_2 v_2}$$

For perfect gas,

$$\boxed{V_2 = \sqrt{\gamma R T_2} \ : \ \text{sonic velocity}}$$

That is at critical pressure, locally there is sonic velocity.

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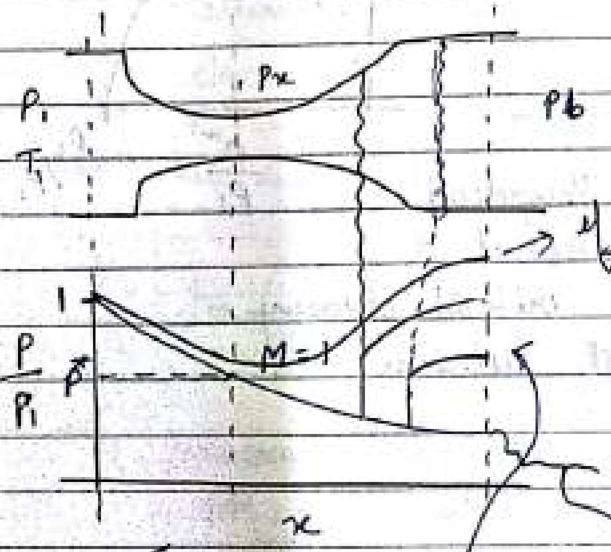


design p.
$p_e > p_b$

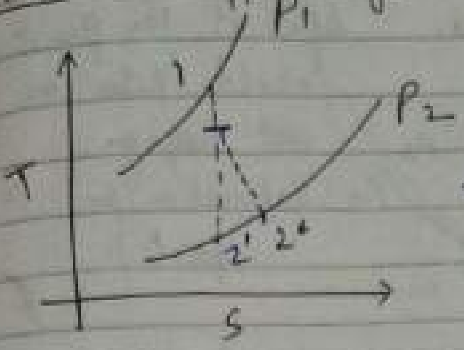
If we increase the back pressure, a reverse flow will occur and act as a diffuser and further lead to a shock.

\# $p_e < p_b$ over expansion.

\# $p_e > p_b$ under expansion.

\# p_b is the pressure of the exit region.

Nozzle Efficiency



Main losses occur in the divergent part of the nozzle.

Thus,

$$\eta_N = \frac{h_1 - h_2}{h_1 - h_2'}$$

$$\boxed{\eta_N = \frac{V_2^2 - V_1^2}{V_{2'}^2 - V_1^2} \approx \left(\frac{V_2}{V_2'}\right)^2}$$

Coefficient of velocity $= \boxed{C_v = \sqrt{\eta_N} = \frac{V_2}{V_2'}}$

$$V_2 = \sqrt{2(h_1 - h_2')\eta_N}$$

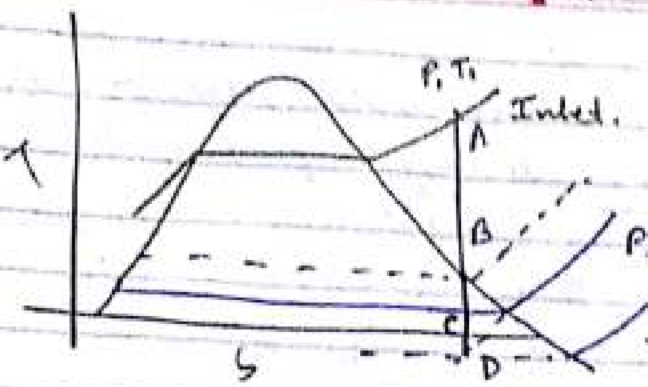
Coefficient of discharge $\boxed{C_d = \frac{\dot{m}_2}{\dot{m}_2'}}$

Super Saturated Expansion.

Super saturated Expansion of steam through a nozzle.

- There is always some difference b/w the calculated and observed mass flow rate of nozzle because of Super Saturated Expansion.

  - Expansion of gas or steam is a very rapid process, So, the expanded gas will always in thermal equil^m.



After B, the fluid would enter the wet region.

Now Condensation is a slow process, being a surface phenomenon.

- Because of slow condensation, the steam would stay in a meta stable form and do not change into liquid.

- This lead to super heated state expansion. and thus extend to D. and lead to further lower Temperture.

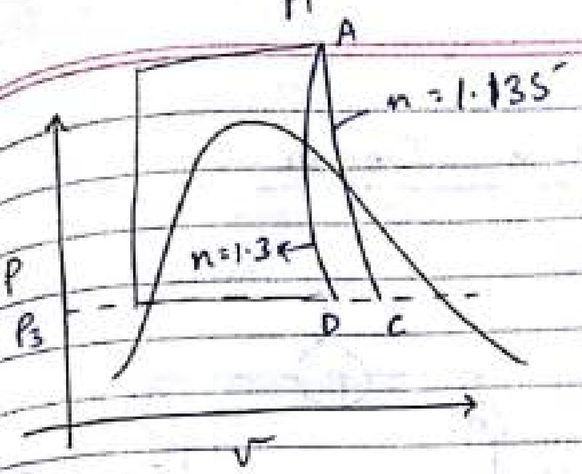
ABC → Normal Expansion.
ABD → SS Expansion

$$\boxed{\text{Degree of Undercooling} = T_C - T_D}$$

$$\boxed{\text{Degree of supersaturation} = P_3/P_4}$$

P_4 is the hypothetical pressure at D. The actual parameters at D are (T_D, P_3).

Scanned by CamScanner



The process ABC will become $Pv^n = C$.

$n = 1.135$ wet

$n = 1.3$ SS.

$v_{ss} < v_{normal}$.

$v_D < v_C$

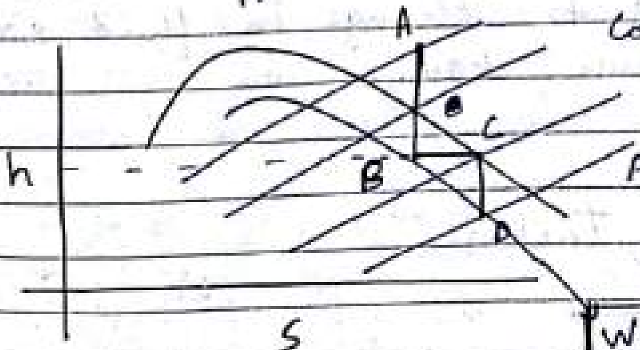
Now, as $\dot{m} = \frac{A\bar{V}}{v}$ $\rightarrow$ $v_C > v_D$

For super saturated expansion $\boxed{v\downarrow}$, however ratio of $\boxed{\frac{V}{v}\uparrow}$

thus, ~~so~~ in $\dot{m} = A\frac{\bar{V}}{v}\uparrow$ for SS.

So, $\dot{m}$ increases for SS.

⇒ Now, there is a limit for this SS exp. to occur, but after sometime condension occurs on the dust particles present leading to a different temperature



Const pressure line.

ABC

P_4 BC occurs at const enthalpy.

$\boxed{\text{Wilson line}}$ tells the limit of SS exp.

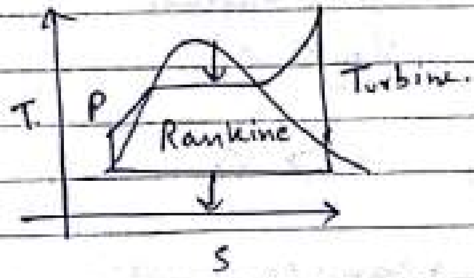
AB - SS exp.

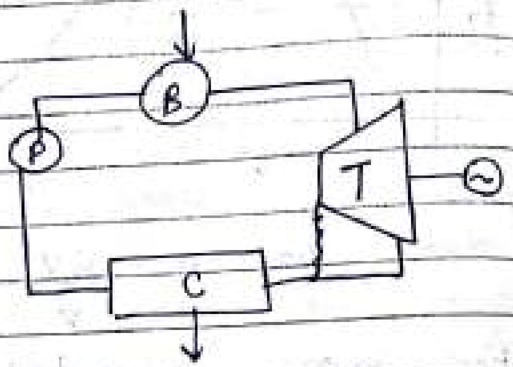
BC - Condensation (sudden)

CD - Normal exp

Axial Flow Steam Turbine

Impulse Turbine
Reaction Turbine.



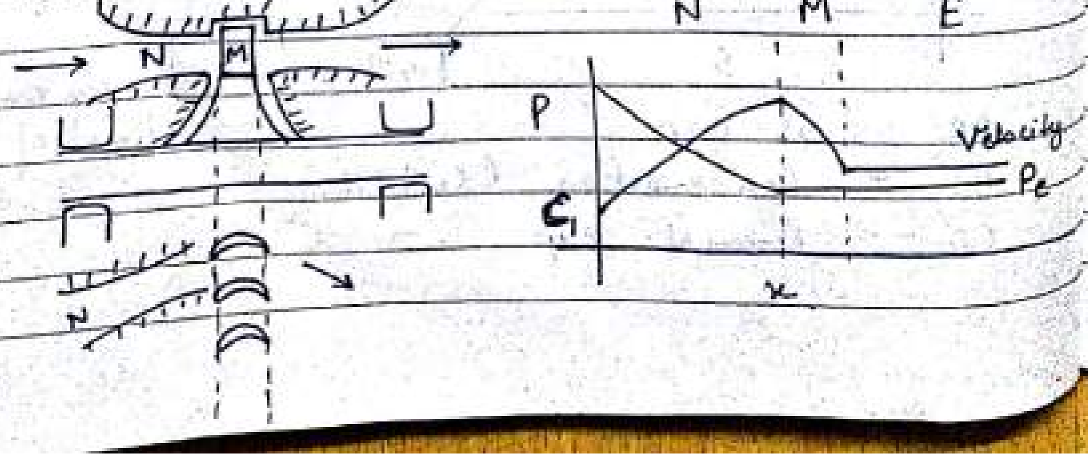


In a power plant, turbine plays an important role of producing work from the pressurised steam in the Rankine Cycle.

⇒ Impulse turbine nozzle + rotating blade.
The pressure remains const in the section of rotating blade, while in nozzle ~~in~~ the pressure changes.

⇒ Reaction Turbine.
work done due to change in fluid direction and the pressure change.

Simple Impulse turbine.

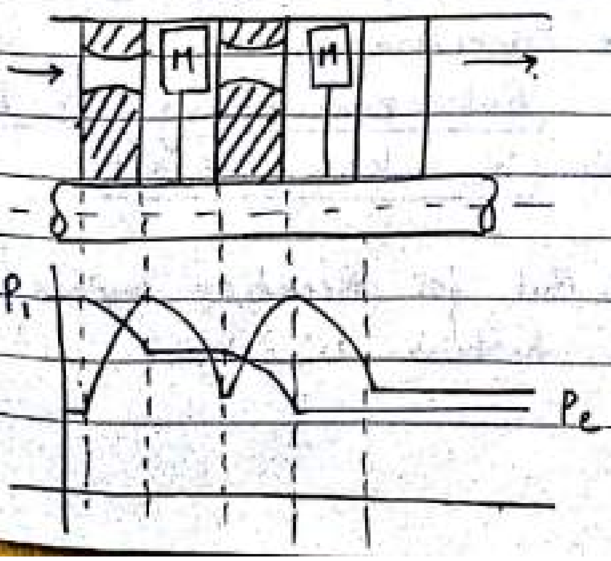


- The construction of Simple impulse turbine is simple as there is only one set of blade.

- In the nozzle, the pressure drop is high, and due to this the pressure remains const, so no back flow or leakage, but the exit velocity is high and due to this a lot of KE is lost to atmosphere.
Thus the η of simple turbine is low.

- The method of utilizing the high KE lost to atmosphere can be through two methods.
1. Pressure compounding
2. Velocity "
3. Combination of 1 and 2

Pressure Compounding - Instead of expanding total pressure drop in one nozzle, we can have no of impulse turbine in series. In reality we only put no of turbines to divide the total pr. drop.



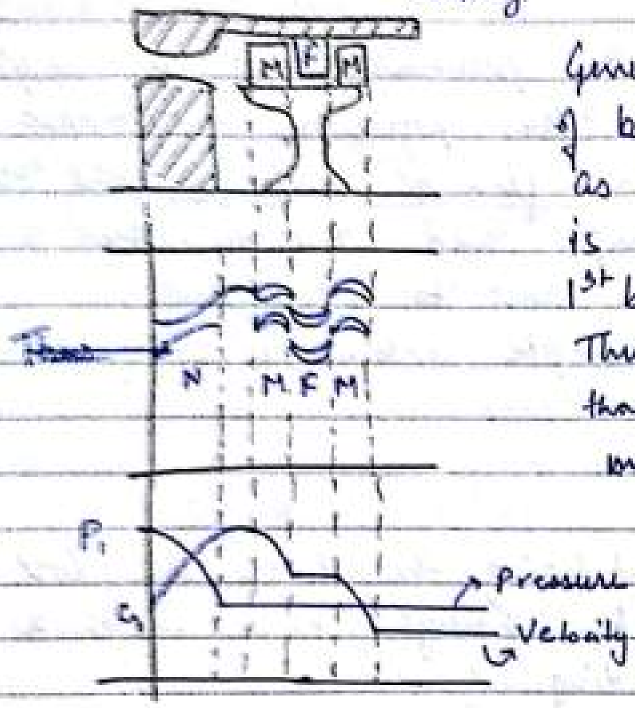
- As we see that there is pressure difference so there may be leakage.
- across the nozzle.
- Also there may be back flow in the blade which is prevented by tip clearance

Quiz on 29 axial flow compressor. or gas turbine.

End Sem
16 Nov 7-9

⇒ Velocity Compounding

In this we have no of ~~rotating~~ moving blades mounted on a ~~v~~ disc. rotating.

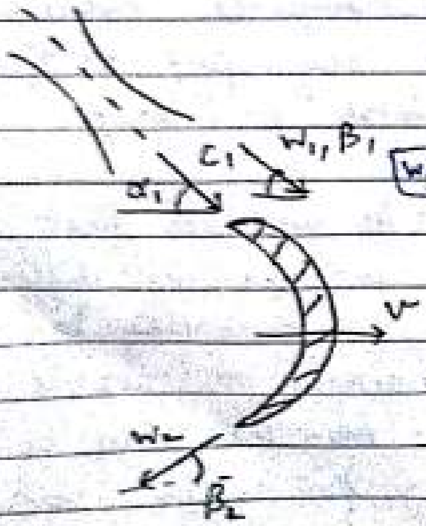


Generally only 2 set of blades are preferred, as power in 2^{nd} blade is a fraction of that in 1^{st} blade.

Thus if we have more than 2, we don't have much change.

Thus, the high KE developed by nozzle is used by two blades and thus exit KE is low

Impulse Turbine

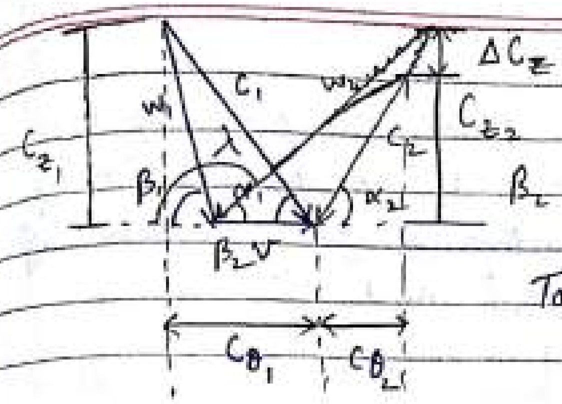


As P remains const in the blade, there will be no $\boxed{w_1 = w_2}$ = acceleration in w.

But considering friction

$$\boxed{W_2 = kW_1, \; k < 1}$$

⇒ But for Reaction impulse turbine $W_2 > W_1$



$\boxed{\text{angle clockwise is +ve.}}$ from tangent.

$\beta_2 = 180 - \lambda$.

Tangetial force on blade $= \dot{m}(C_{\theta_1} - C_{\theta_2})$ (turbine)

Any velocity measured opposite to the direction of V, is taken -ve.

So, $\boxed{F_\theta = \dot{m}(C_{\theta_1} - C_{\theta_2}) = \dot{m}\underbrace{(C_{\theta_1} + C_{\theta_2})}_{\text{Following sign convention.}}}$

\# Axial force => $\boxed{F_z = \dot{m}\Delta C_z}$

\# Work done $= \dot{m}\,\Delta C_\theta \times u$

\# Blade efficiency or diagram efficiency

consider only the blade $= \dfrac{\text{Work done}}{\text{Energy input}} = \boxed{\dfrac{2\Delta C_\theta u}{C_1^2} = \eta_b}$

\# Stage efficiency (consider nozzle, blade)

$\eta_s = \dfrac{\text{Work done/unit mass}}{\text{isentropic enthalpy drop (occuring in nozzle)}}$

$= \dfrac{\Delta C_\theta u}{\Delta h_{is}} = \dfrac{2\Delta C_\theta u}{C_1^2} \times \dfrac{C_1^2/2}{\Delta h_{is}}$

$$\boxed{\eta_s = \eta_b \times \eta_N}$$

- U is the speed of the rotor. In an impulse turbine c_1 is high, and U is a fraction of c_1. So, $c_1 \uparrow$, $U \uparrow$. Further if the Pressure of injection is high, $c_1 \uparrow$ and $U \uparrow$.

So, we define.

$$\boxed{\rho = \frac{u}{c_1}}$$ (Non dimensional Blade Speed)

Optimum non-dimen^n ρ of impulse turbine for max η.

For a given system, β_1, β_2, c_1 known.

$$U \Delta C_\theta = U(C_{\theta_1} + C_{\theta_2}) = U(W_{\theta_1} + W_{\theta_2})$$

$$\Delta C_\theta = C_{\theta_1} + C_{\theta_2} = W_{\theta_1} + W_{\theta_2}$$
$$= W_1 \cos\beta_1 + W_2 \cos\beta_2$$

$$= W_1 \cos\beta_1 \left[1 + \frac{W_2}{W_1} \frac{\cos\beta_2}{\cos\beta_1} \right]$$

For a certain design $\frac{\cos\beta_2}{\cos\beta_1} = c$ [generally $c = 1$]

Also, $w_2 = k w_1$

So,

$$\Delta C_\theta = w_1 \cos\beta_1 [1 + kc]$$
$$= [c_1 \cos\alpha_1 - u](1 + kc).$$

We know, $\eta_b = \frac{2\Delta C_\theta U}{C_1^2}$

$\eta_b = \frac{2u(C_1 \cos\alpha_1 - u)}{C_1^2}(1+kC)$ $\left[\rho = \frac{u}{C_1}\right]$

$$\boxed{\eta_b = 2\rho \times (\cos\alpha_1 - \rho)(1+kC)}$$

- For a given design $\eta_b = f(\rho)$, other parameters are const.

⇒ For max^m efficiency

$\frac{d\eta_b}{d\rho} = 2\cos\alpha_1 - 4\rho \Rightarrow \boxed{\rho = \frac{\cos\alpha_1}{2}}$

For $\rho = \frac{\cos\alpha_1}{2}$, $\boxed{(\eta_b)_{max} = \frac{\cos^2\alpha_1(1+kC)}{2}}$

Now for an ideal system $k = 1$
and for a symmetric vane $\beta_1 = \beta_2$
$\Rightarrow C = 1$.

$\boxed{(\eta_b)_{max} = \cos^2\alpha_1}$ $\alpha_1\downarrow, \eta\uparrow$

But as $P\downarrow$ for $\alpha_1\downarrow$ so, an compromise is made.

$\boxed{\alpha_1 : 15° < \alpha < 25°}$

Velocity Compounding

- It can be proved that ρ_{opt} of optimum is

$$\boxed{\rho_{opt} = \frac{\cos^2\alpha_1}{2n}}$$

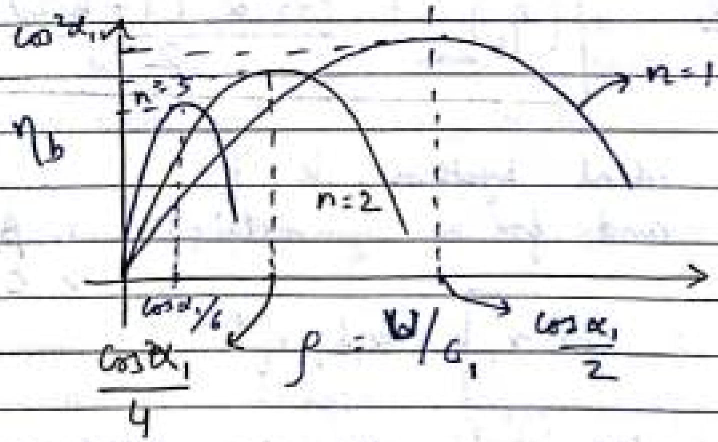
$K = 1$

$C = 1$

where n is equal to no of moving blade, and there is no friction and blades are symmetrical $\beta_1 = \beta_2$.

- For all optimum conditions, we assume minimum energy is lost in the exit flow, so α_2

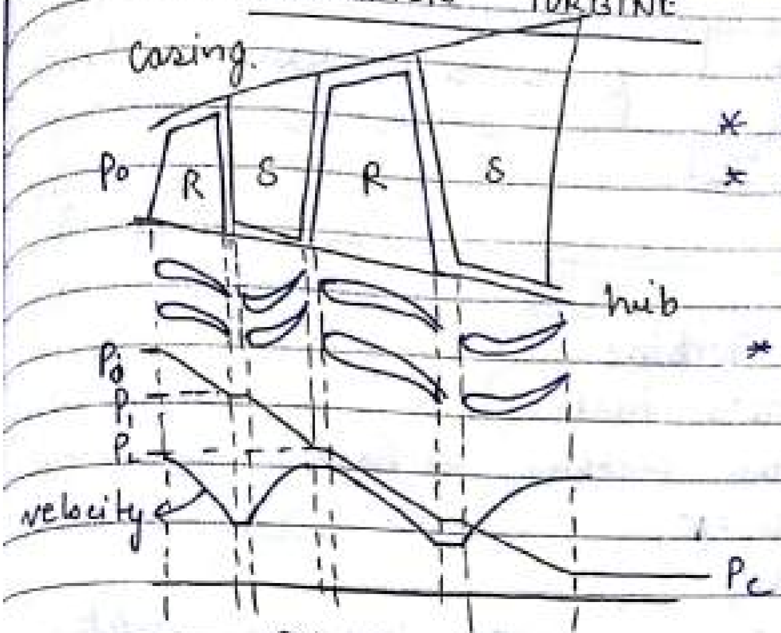
$\alpha_2 = 90°$, C_2 is minimum.



As, optimum speed ↓ for n ↑ but at the same time η ↓. So we don't go further $n = 3$.

11/10/18

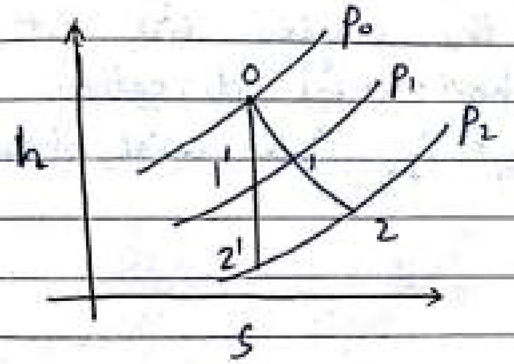
REACTION TURBINE



* P_r drop continuity.
* Serious leakage problem as P_r drop continuously.
* Thus need good surface finish of each blade.

P↓ ⟶ d↓ ⟶

V↑ ⟶



1-2 : enthalpy drop in the stator.

0-1 : enthalpy drop in the rotor.

⇒ When $R = 0.5$, the design for the rotor and stator blades are same and thus are cheaper.

Degree of Reaction $= R = \dfrac{(\Delta h_R)\text{ isentropic}}{(\Delta h_{stage})\text{ isentropic}}$

$$= \frac{h_0 - h_1'}{h_0 - h_2'}$$

⇒ If blade eff. of rotor and stage or stator is same, Then,

$$\boxed{R = \frac{h_0 - h_1}{h_0 - h_2}}$$ for a 50% rxn machine.

50% Reaction Turbine.

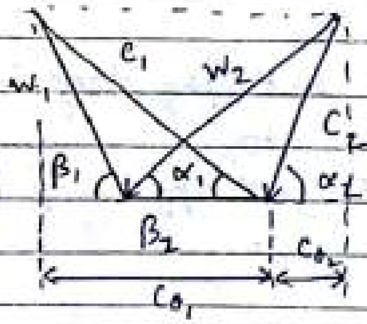
→ (Parson's turbine)

* For a 50% rxn machine, we have

$\beta_1 = \alpha_2$ | $W_1 = C_2$

$\beta_2 = \alpha_1$ | $W_2 = C_1$

⇒ Exit velocity Δ is mirror image of inlet velocity Δ.



All the angles are wrt to horizontal direction.

⇒ $C_{Z_1} = C_{Z_2}$ ⇒ No axial thrust.

Energy Input per unit mass =

= Energy of jet coming in + Pressure energy (due to expn.)

$$= \frac{C_1^2}{2} + \frac{1}{2}(W_2^2 - W_1^2) \qquad (W_2 > W_1)$$

$$E_{in} = \frac{C_1^2}{2} + \frac{1}{2}(C_1^2 - W_1^2) \qquad (W_2 = C_1)$$

Now, for inlet Δ = $W_1^2 = C_1^2 + U^2 - 2C_1 U \cos\alpha_1$

Scanned by CamScanner

Substituting w_1^2 in E_{in}

$$E_{in} = \frac{c_1^2}{2} + \frac{1}{2}\left[-u^2 + 2c_1 u \cos\alpha_1\right]$$

$$\boxed{E_{in} = \frac{c_1^2 - u^2 + 2c_1 u \cos\alpha_1}{2}}$$

Work done per unit mass

$= u \Delta c_\theta \Rightarrow \dot{W}/m = u(c_{\theta_1} + c_{\theta_2})$

From the symmetrical velocity triangle.

$c_{\theta_1} + c_{\theta_2} = c_{\theta_1} + (c_{\theta_1} - u)$

as Projection of c_2 and w_1 is equal.

$$\boxed{\Delta c_\theta = 2c_{\theta_1} - u = 2c_1 \cos\alpha_1 - u.}$$

So,

$\dot{W}/m = u\Delta c_\theta = u[2c_1 \cos\alpha_1 - u]$

Thus,

Blade efficiency $= \boxed{\eta_b = \frac{2u[2c_1\cos\alpha_1 - u]}{c_1^2 - u^2 + 2c_1 u \cos\alpha_1}}$

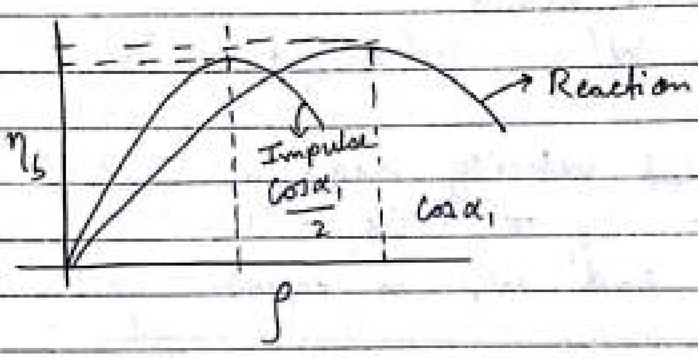
$\rho = \frac{u}{c_1}$, so,

$$\boxed{\eta_b = \frac{2\rho[2\cos\alpha_1 - \rho]}{1 - \rho^2 + 2\rho\cos\alpha_1}} = f(\rho) \text{ for a given design}$$

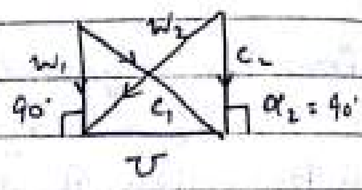
• Now for max η_b.

$$\frac{d\eta_b}{d\rho} = 0. \Rightarrow \rho_{opt} = \left(\frac{U}{C_1}\right)_{opt} = \boxed{\cos\alpha_1 = \rho_{opt}}$$

$$\boxed{(\eta_b)_{max}\Big|_{\rho_{opt}} = \frac{2\cos^2\alpha_1}{1+\cos^2\alpha_1}}$$



velocity Δ



Further, at the $\eta_b)_{max}$, C_2 would become axial and correspondingly $\boxed{\alpha_2 = \beta_1 = 90°}$

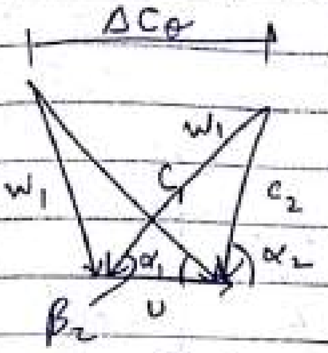
Governing mechanism to maintain turbine steam mass flow rate.

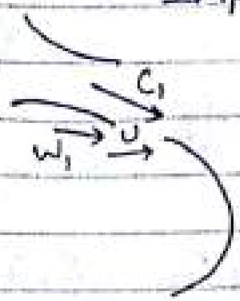
25/10/18

Assignment - 3

Impulse turbine

Q5.





$\alpha_1 = 20°$

$\beta_2 = 30°$

$U = 130$ m/s. $\quad C_1 = 330$ m/s.

$K_p = 0.8 \quad \eta_N = 0.85$

$\Rightarrow$ In inlet vel Δ

$w_1 \sin\beta_1 = C_1 \sin\alpha_1 \quad$ (i)

$w_1 \cos\beta_1 = C_1 \cos\alpha_1 - U \quad$ (ii)

$\tan\beta_1 = \dfrac{C_1 \sin\alpha_1}{C_1 \cos\alpha_1 - U} = \boxed{\beta_1 = 32.075°}$

$w_1 = 330 \times \dfrac{\sin 20°}{\sin 32.075°}$

$w_1 = \cancel{133.19\ m/s}\ 212.54$ m/s.

$w_2 = K w_1 \Rightarrow K = $ friction.

$w_2 = 170$ m/s.

$\Delta C_\theta = w_1 \cos\beta_1 + w_2 \cos\beta_2$

$= 327.35$ m/s.

$W/\text{kg steam} = U(C_{\theta_2} - C_{\theta_1}) = 130 \times 327.35$

$= 42.55$ kJ/kg

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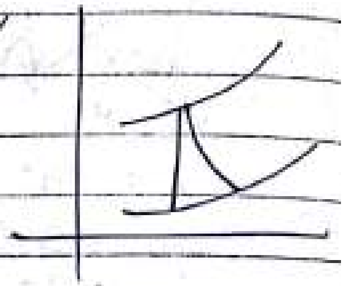
Blade efficiency $= \frac{u \Delta C_\theta}{C_1^2/2}$

$= 78.15\%$

$\eta_s = \eta_N \times \eta_B$

$= 66.4\%$



$\eta_s = \frac{(T_{02} - T_{01})}{(T_{02})_s - (T_{01})}$

$T_{02} = T_{01} - N \Delta T_{01}$

$\Delta T_{01} = \frac{pow}{C_p} =$

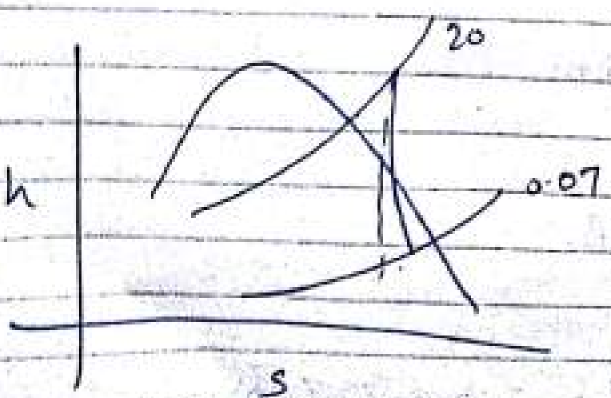
$\left(\frac{P_2}{P_1}\right)^{\gamma/\gamma-1} = \frac{T_2}{T_1} = \cancel{T_2 =}$

$T_2 = 200.68\,°C$.

$T_2 = T_1 - \eta_s[(\Delta T_0)_s - (T_{01})]$

$= 523 - 0.664 \times (523 - 200.68)$

$T_2 = 308.97.$



For 20 bar and 250°C

h_1, s_1

For 0.07 bar.

h_f

h_{fg}

s_f

s_{fg}

$s_2 = s_1 = s_f + x\, s_{fg}$

get x.

$h_2 = h_f + x\, h_{fg}$.

Now, $\eta_t = \eta_s \times$ reheat factor

$\eta_t = 0.664 \times 1.06$

$= 70.41\%$.

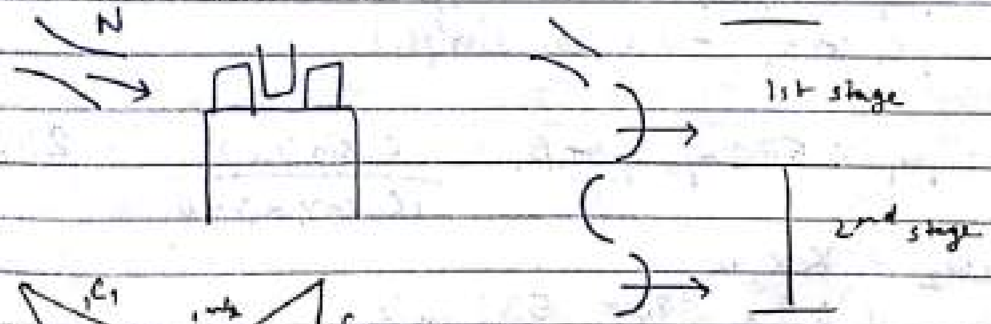
So,

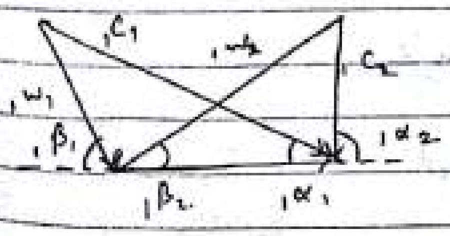
$\eta_t = \dfrac{h_1 - h_2}{h_1 - h_{2s}} \Rightarrow 612.57 = h_1 - h_2$

$N = \dfrac{h_1 - h_2}{\Delta h_{stage}} = 15$

---

6)



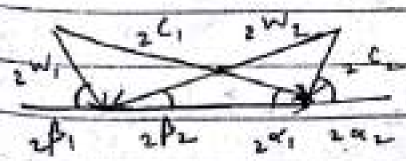


${}_iC_j$

i = stage

j = 1 (inlet)

2 (exit)



∴ Blade efficiency $= \eta_b = \dfrac{u \Delta C_\theta}{C_1^2/2}$, $\Delta C_\theta = \Delta C_{\theta_1} + \Delta C_{\theta_2}$

Calculation of 2^{nd} stage.

${}_2C_1 = k \times {}_1C_2 = 0.9 \times 423.13$ m/s $= 380$ m/s.

Similar calculations for 2^{nd} stage rotor as for 1^{st}.

${}_2\beta_1 = 37.9°$ ${}_2w_1 = 280$ m/s.

${}_2\alpha_2 = 58.98°$ ${}_2C_2 = 169.5$ m/s.

${}_2w_2 = k \times {}_2w_1 = 252$ m/s

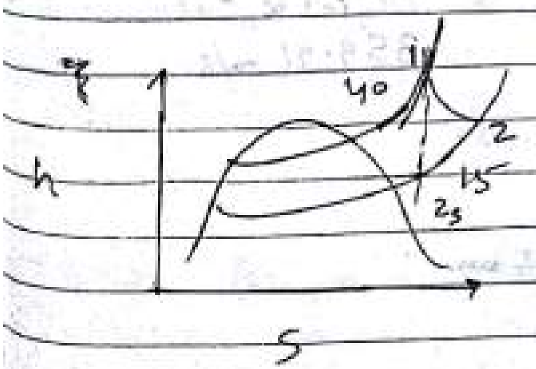
We have,

$\Delta C_{\theta_1} = {}_1w_1 \cos({}_1\beta_1) + {}_1w_2 \cos({}_1\beta_2) = 1044.5$ m/s

for 2^{nd} stage,

$\Delta C_{\theta_2} = 428.86$ m/s.

$$\eta_b = \frac{(1044.5 + 428.86) \times 117.8}{\frac{{}_1C_1^2}{2}(700)} = \cancel{78.9\%} \; 70.6$$



$\eta_s = \dfrac{\text{Work done / unit mass}}{\text{isentropic enthalpy drop across the stage.}}$

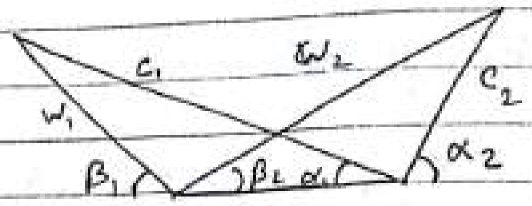
$(\Delta h)_{1s} = h_1 - h_{2s}$

$$\eta_s = \frac{U \Delta C_\theta}{(\Delta h)_{1s}} = 66\%$$

7) $R = 0.5$

$U/V_1 = 0.7$

$\beta_1 = 60^\circ = \alpha_2$



$P_1 = 2$ bar

$T_1 = \cancel{150}\ 130^\circ C$

$D_m = 0.7 m$

$h = 0.05 m$

$N = 6000$ rpm.

$U = \frac{\pi D N}{60} = 220 m/s.$

$C_1 = \frac{220 \times}{0.7} = 314.3 m/s.$

Velocity Δ

$w_1 \sin\beta_1 = c_1 \sin\alpha_1$

$w_1 \cos\beta_1 = c_1 \cos\alpha_1 - U$

$c_1^2 = w_1^2 + U^2 - 2Uw_1 \cos(180 - \beta_1) - 3.$

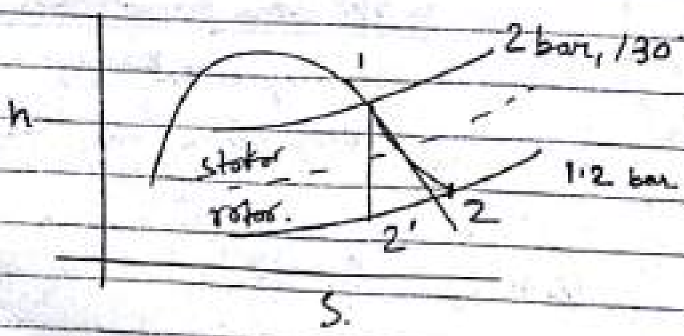
$w_1 = 140 m/s.$

$\beta_2 = \alpha_1 = 22.7^\circ$

$c_1 = w_2$

$w_1 = c_2$

$\Delta C_\theta = w_1 \cos\beta_1 + w_2 \cos\beta_2 = 2C_1 \cos\alpha_1 - U$
$= 359.91 m/s.$



@ 2.0 bar, 130°C $h = 2730$ kJ/kg

@ 1.2 bar, Isentropic $h = 2635.5$ kJ/kg

$\eta = \dfrac{\Delta C_\theta \, U}{h_1 - h_{2s}} = 83.8\%$

$P = \dot m \, U \Delta C_\theta$

$\dot m = A_1 C_z \, , \, v_{2s}$

$A = \pi D_m h = 0.10996 \, m^2$

$C_z = w_1 \sin\beta_1 = 121.24 \, m/s$

$\Delta h|_{stator} = \Delta h_{is}|_{rotor} = \dfrac{\Delta h}{2}$

$h_1 - (h_{2s})_{stator} = \dfrac{\Delta h}{2} \Rightarrow h_{2s} = 2682.8 \, kJ/kg$

$\hookrightarrow p_2 = 1.5 \, bar, \; v_{s_2} = 1.15 \, m^3/kg$

$\dot m_{th} = \dfrac{A C_z}{v_{s_2}} = 11.6 \, kg/s$

$\dot m_{ac} = (1 - 0.06) \times 11.6 = 10.9 \, kg/s$

$P = \dot m \, U \Delta C_\theta = 862.834 \, kJ$

$h_1 - h_2 = \dot W \Rightarrow h_2 = 2650.84 \, kJ/kg$

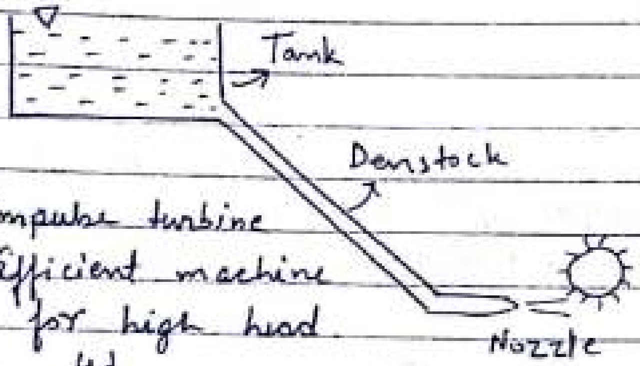
$h_2' = h_2 + \dfrac{\Delta h}{2} = 2690.4 \, kJ/kg$

Static & total enthalphy.

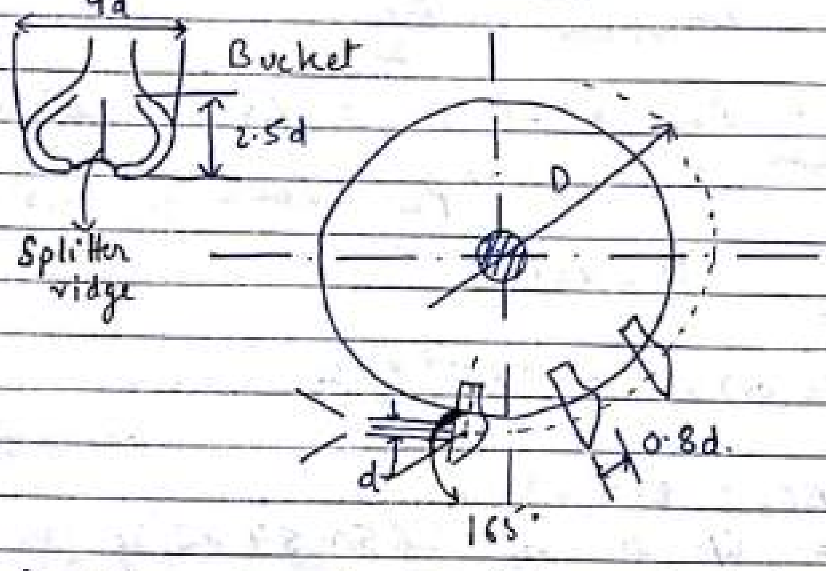
$h = h_s + \dfrac{v^2}{2}$	rotor inlet $\mid h_{2s} = 2680.6 \, kJ/kg$ ($h_2 - \dfrac{w_1^2}{2}$)
$h_s = h - \dfrac{v^2}{2}$	rotor exit $h_{2s} = h_2 - \dfrac{C_2^2}{2} = 2641.0 \, kJ/kg$

06/11/18

Hydraulic Machines



* Impulse turbine
* Efficient machine for high head.

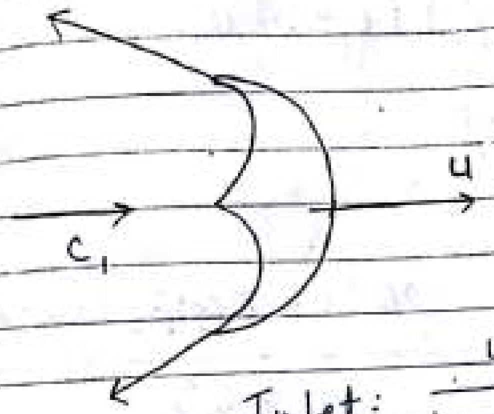


⇒ Specific speed of Pelton wheel is very low due to high head and low mass flow rate.
$$N = \frac{N\sqrt{Q}}{(gH)^{3/4}}$$

⇒ The water jet is deflected into two parts, where the water is deflected by 165°

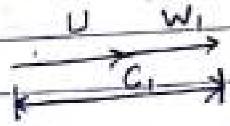
Max deflection of 180° might have give max power, but then the two jets would have collided among each other

D (diameter of wheel)
Large D - very slow speed.
Small D - distances b/w buckets is small and would hinder the function.

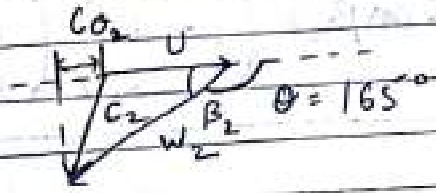


jet, $C_1 = C_v\sqrt{2gH}$

coffecient of velocity

Inlet: 

$\boxed{u + w_1 = C_1}$

Exit: 

C_{θ_2} is -ve as it is in opposite direcn to u.

$\Rightarrow W_2 = kW_1$, $k > 1$ (friction).

Now, Energy delivered/unit mass flow rate

$\dot{W} = u_1 C_{\theta_1} - u_2 C_{\theta_2}$ (Turbine work)

$= u(C_{\theta_1} - C_{\theta_2})$

$C_{\theta_1} = w_1 + u$

$C_{\theta_2} = -(w_2 \cos\beta_2 - u)$

So, $\dot{W} = u(w_1 + u - -(w_2\cos\beta_2 - u))$

$= u(w_1 + \cancel{u} + w_2\cos\beta_2 - \cancel{u})$

$= u(w_1 + w_2\cos\beta_2)$

$\dot{W}/\dot{m} = u w_1 (1 + k\cos\beta_2)$

So, $\boxed{\dot{W}/m = uw_1 (1 + k\cos\beta_2)}$

Power = $\dot{m}W$.

$= \rho Q [1 + k\cos\beta_2] (c_1 - u) u.$

Now, KE of jet = $\frac{1}{2}\rho Q c_1^2$

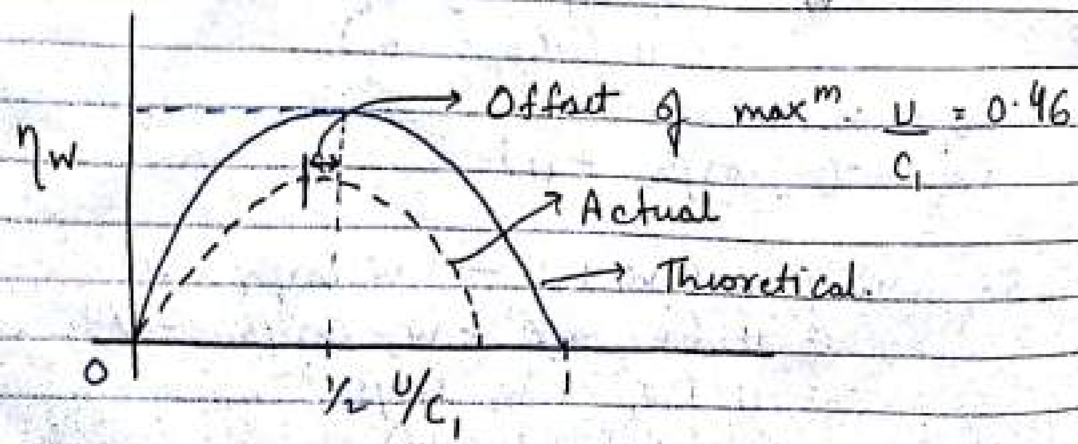
So,

Wheel efficiency $\eta_w = \dfrac{\rho Q [1 + k\cos\beta_2](c_1 - u)u}{\rho Q\, c_1^2/2}$

$$\boxed{\eta_w = 2[1 + k\cos\beta_2]\left(1 - \frac{u}{c_1}\right)\left(\frac{u}{c_1}\right)}$$

For max^m η_w,

$\dfrac{d\eta_w}{d\left(\frac{u}{c_1}\right)} = 0$ at $\dfrac{u}{c_1} = \dfrac{1}{2}$



Some corelations and small charts, if we follow these correlations, we can get the most efficient design of wheel.

⇒ $\boxed{\frac{D}{d} = \frac{206}{N_s}}$, N_s specific speed.

~~206~~, follow the units for N_s or 206 would change.

$\boxed{N_s = \frac{N\sqrt{P}}{H^{5/4}} \quad , \quad N - rpm,\ P - kW,\ H - m.}$

⇒ $\frac{D}{d} \rightarrow$ 6 - 26. approx

$\frac{D}{d} \rightarrow$ 12 - 16, strict guideline for a well designed wheel.

⇒ $\boxed{\phi = \frac{u}{\sqrt{2gH}}}$ ⇒ Speed factor.

⇒

N_s (m, kW, rpm)	ϕ
7.62	0.47
11.42	0.46
15.24	0.45
19.05	0.44
22.86	0.433
26.65	0.425

- The number of jets should be limited to 4 - 6. (Not more than 6)

⇒ A small impulse wheel is to be used to drive a generator of 60 Hz. ~~So a 5r~~ power supply, the head is 100m, discharge is 40 lt/s, determine the dia of wheel, and speed of wheel, given C_v of 0.98, and assuming η_w is 80%.

$Q = 40\ lt/s = 0.04\ m^3/s$

$H = 100\ m$

$\gamma = \rho g = 9806$

$$P = \eta \rho g Q H = 0.8 \times \frac{9806}{1000} \times 0.04 \times 100\ kW$$

$$P = 31.38\ kW$$

Trial

$$N_s = 15 = \frac{N\sqrt{P}}{H^{5/4}} \Rightarrow N_s = \frac{N_s H^{5/4}}{\sqrt{P}}$$

$N = 847\ rpm$

For a generator of 60 Hz.

$$\text{Speed} = \frac{3600}{N}$$

$$\text{poles } (N) = 5 \Rightarrow rpm = \frac{3600}{5} = 720$$

$$N = 4 \Rightarrow rpm = \frac{3600}{4} = 900$$

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Now, the design rpm would be $N = 900$ rpm.

Thus, the corrected specific speed (N_s)

$$N_s = \frac{N\sqrt{P}}{H^{5/4}} = 15.94$$

$N_s = 15.94$ $\qquad \phi = 0.448$

Hence,

$$U = \phi\sqrt{2gH} = 19.84 \text{ m/s}$$

$$U = \frac{\pi D N}{60}$$

$$\Rightarrow D = \frac{19.84 \times 60}{\pi \times 900} = 421 \text{ mm}$$

velocity of jet.

$$C_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 100}$$

$$C_1 = 43.3 \text{ m/s}$$

Now,

$$\frac{\pi}{4} d^2 C_1 = Q \Rightarrow d = \sqrt{\frac{4Q}{C_1}}$$

$$\Rightarrow d = 34.3 \text{ mm}$$

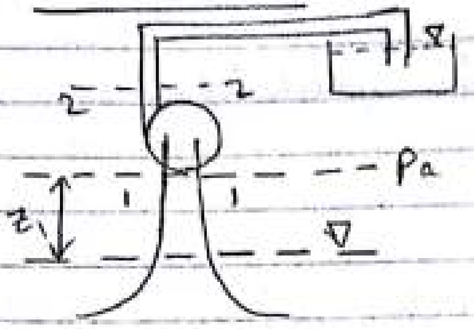
Dia ratio, $\frac{D}{d} = \frac{\cancel{34.3}\ 421}{34.3} = 12.27$

$$\frac{D}{d} = \frac{206}{N_s} = 12.92$$

Thus, the wheel dia $= 421$ mm.

speed $= 900$ rpm.

Cavitation



Cavitation.

At 1-1, Pressure is less than atm. pressure.

- Many ~~at~~ a times pr. at suction is below the vopour pressure, which results in formation of bubbles. Now these bubbles would go into impellor, where pressure increases, causing the bubble to collapse. This causes ~~int~~ impiching that erodes the blade material.

$$\Delta H = \left(\frac{p_2}{\rho g} + \frac{c_2^2}{2g} + Z_2\right) - \left(\frac{p_1}{\rho g} + \frac{c_1^2}{2g} + Z_1\right)$$

$$\text{Inlet} = \frac{p_1}{\rho g} + \frac{c_1^2}{2g} + Z_1 = \frac{p_a}{\rho g} - h_f$$

loss in the suction pipe.

For a given design,

$$\frac{c_1^2}{2g} = \sigma_c H.$$

[c_1 should be proportional to H]

Putting $c_1^2/2g$ value in ~~ΔH~~ Inlet condition.

Considering p_1 to be allowable minimum pressure below, which cavitation would start.

$p_1 = p_{min}$.

$$\boxed{\sigma_c = \frac{\left[\frac{p_a - p_{min}}{\rho g} - z_1 - h_f\right]}{H}}$$

- For cavitation not to occur, p_{min} must greater than p_v (vapour pressure). Hence, we define the cavitation parameter, σ_c st. we replace p_{min} by p_v

Cavitation parameter
$$\boxed{\sigma_c = \frac{\left[\frac{(p_a - p_v)}{\rho g} - z_1 - h_f\right]}{H}} \qquad (1)$$

$\boxed{\sigma_{operation} < \sigma_c}$, For cavitation not to occur, in any installation.

needs to be satisfied.

- The numerator of eq 1 is known as Net positive suction head. (NPSH)
In order that, the σ to be as large as possible, ~~just one~~ z_1 must be as small as practical. However, in some installation, it may be necessary to set the pump below the reservoir level. (z_1 becoming -ve) to get higher NPSH value.

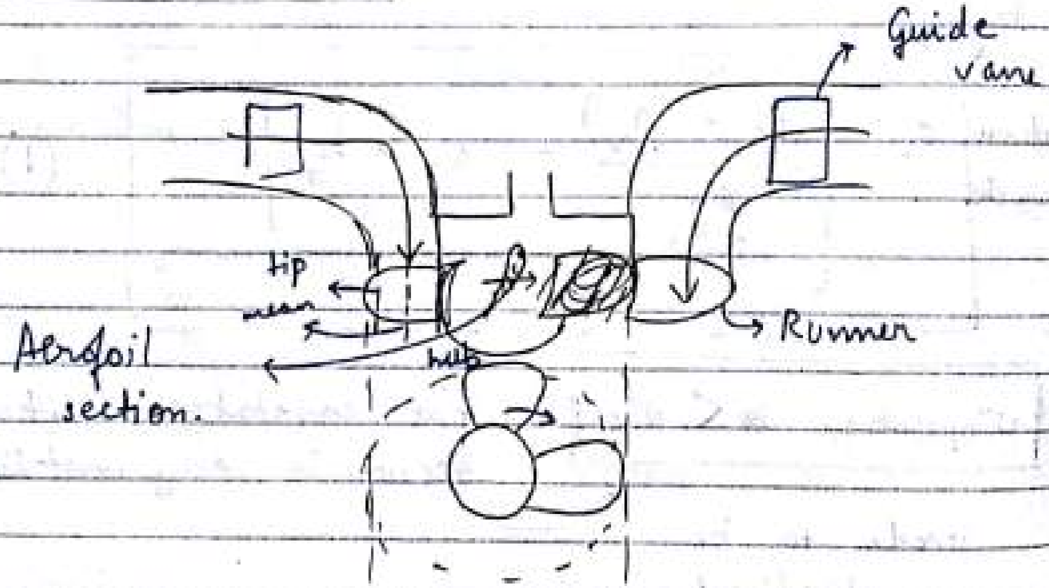
Scanned by CamScanner

- In turbine, we have exit cavitation, instead of inlet cavitation in the pump.

13/11/18

Pelton - Impulse m/c
Francis - Mixed flow m/c
Kaplan - Axial flow m/c.

KAPLAN Turbine



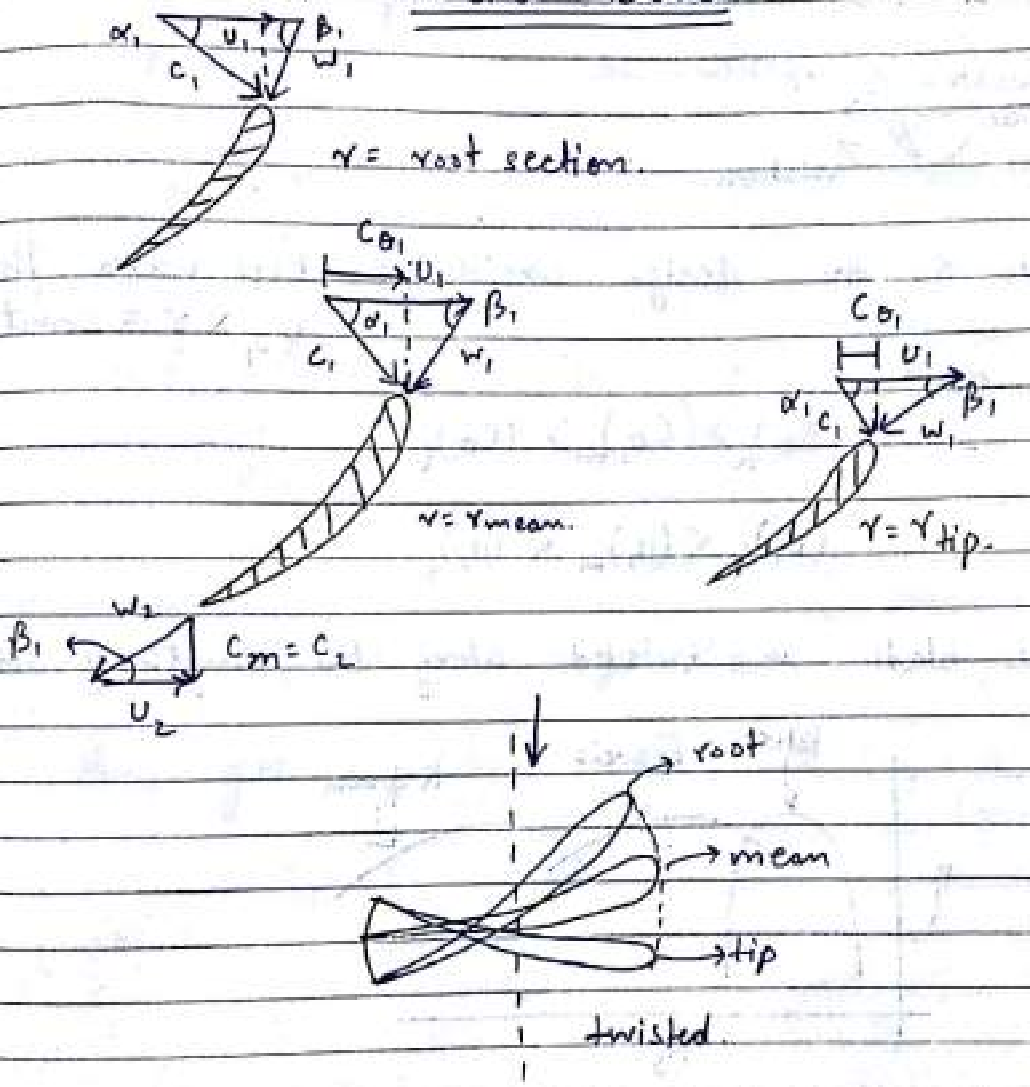
No of runner blade
4 or 6. looks like ship propellor.

Design of blade follows

Free vortex design. Hence $\boxed{\begin{aligned} r C_{\theta_1} &= \text{const.} \\ C_{\theta_1} &= \frac{1}{r} \end{aligned}}$

Again $u \propto r$

⇒ Result in twisted blade.



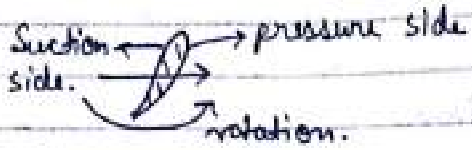
⇒ High speed m/c. so more efficient for low head and high flow rate.

H↓ $\quad N_s = \frac{N\sqrt{Q}}{H^{3/4}}$ ↑

Q↑

⇒ For a certain dia of m/c Q has to be high.

⇒ Guide vane controls a the flow. with a runner with aerofoil cross-section.



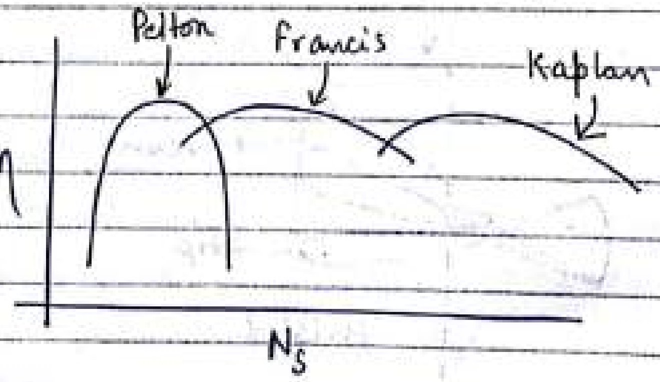
⇒ We do the design considering Free vortex flow.
$C_{\theta_1} \times r = \text{const.}$

thus,

$$(C_{\theta_1})_h > (C_{\theta_1})_m > (C_{\theta_1})_t$$

$$(U_1)_h < (U_1)_m < (U_1)_t$$

The blade is twisted along the length of blade



Q. A Kaplan turbine designed with a power specific speed $\Omega_{sp} = \frac{\Omega \sqrt{P/\rho}}{(gH)^{5/4}} = 3.3$ rad. A runner tip
↓
safe factor.

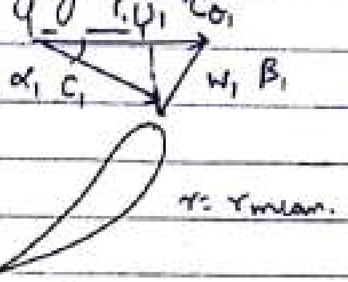
4.5, and hub = 2m. operates with a net head of 25 m and a shaft speed of 150 rev/s. Let flow at exit is axial. $\eta_H = 90\%$. Determine P and Q.

→ Relative angles at inlet and exit considering design to be free vortex.

Given $N = 150$ rpm. $\Rightarrow \Omega$

$\Omega_{sp} = \dfrac{\Omega \cdot \sqrt{P/\rho}}{(gH)^{5/4}}$ find $P = \_ \_ \_ \_$

$\eta_H = \dfrac{P}{\rho g H Q} \Rightarrow Q = \_ \_ \_ \_$



D_t, $D_h \Rightarrow D_m = \_ \_ \_ \_$

$U_m = \dfrac{\pi D_m \times N}{60}$

$A = \_ \_ \_ \_$

and then.

$C_z A = Q$ to get $C_z = \_ \_ \_ \_$

Now, $gH = \dfrac{U_1 C_{\theta_1} - U_2 \cancel{C_{\theta_2}}^{\,0}}{\eta_H}$ as $C_{\theta_2} = 0$ (axial)

$C_{\theta_1} = \_ \_ \_ \_$

So,

$C_1 = \sqrt{C_{\theta_1}^2 + C_z^2} = C_1$ and α_1

and then β_1, W_1

and also β_2

Now, for root and tip radius.

$(r_1 C_{\theta_1})_{mean} = (r_1 C_{\theta_1})_{root} = (r_1 C_{\theta_1})_{tip}$

Syllabus.

* Fundamentals
* Axial flow m/c } Compressor
  Radial flow m/c } gas turbine
* Nozzle
* Steam Turbine
* Hydraulics m/c