Department of Physics, IIT-Kanpur

Instructors: TKG & DpC PHY103A (2014-15, II-SEM) Problem Set 3

1. A model for the electrostatic potential of an atom, due to the nucleus (charge $+Ze$) and electrons, is the so-called "screened Coulomb potential" is given by

$$V(r) = A\frac{e^{-\lambda r}}{r},$$

where $A = Ze/(4\pi\epsilon_0)$, $1/\lambda$ is an effective atomic radius and $r = |\mathbf{r}|$. Find the electric field $\mathbf{E}(\mathbf{r})$, charge density $\rho(\mathbf{r})$ and total charge Q. Sketch $\rho(\mathbf{r})$.

2. A point charge q is at the midpoint of the axis of a cylinder of radius R and height L. Calculate the electric flux through the cylindrical curved surface.

3. A hollow spherical shell has charge density $\rho = k/r^2$, in the region $a < r < b$. Calculate electric field everywhere and sketch it. Find the potential at the center using infinity as the reference point.

4. Two identical spheres carrying uniform volume charge densities $-\rho$ and $+\rho$, respectively, are placed so that they partially overlap. Calculate the electric field in the overlap region in terms of $\mathbf{d}$, the vector from the center of the positively charged sphere to the center of the other sphere.

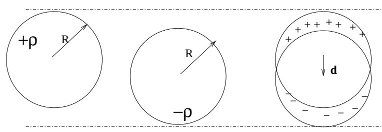

Figure 1:

Exercises

1. The gradient operator in spherical coordinates is given by

$$\boldsymbol{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\frac{1}{r}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}.$$

Calculate $\boldsymbol{\nabla}\cdot(\boldsymbol{\nabla} f)$, where $f = f(r,\theta,\phi)$ is any scaler function. **Note: The partial derivatives $\frac{\partial}{\partial\theta}$ and $\frac{\partial}{\partial\phi}$ in the left-hand $\boldsymbol{\nabla}$ operator act on the unit vectors $\hat{r},\hat{\theta},\hat{\phi}$ of right-hand $\boldsymbol{\nabla}$ operator**.

2. Write down charge density using Dirac delta function for the following configurations: a) The charge Q is uniformly distributed over a spherical shell of radius R. b) The charge Q is uniformly distributed on a ring of radius R, which is lying on the xy plane with its center at the origin. (write in both cylindrical and spherical coordinates)

3. Sketch the following function
$$g(x) = \lim_{\epsilon\to 0} \frac{1}{\sqrt{\pi}\epsilon} e^{-x^2/\epsilon^2}.$$
Show that $\int_{-\infty}^{\infty} g(x)dx = 1$. Can you consider it as a Dirac delta function?

4. Plot the following function:
$$f(x) = \lim_{t\to 0^+} \frac{1}{e^{(x-b)/t}+1},$$
where b is a real constant. Express $f(x)$ in terms of the unit step function $\theta(x)$. [**Hint: first evaluate the function at any point in the two different regions: $x < b, x > b$ and at the point $x = b$. Take the limit after evaluating the function**.]

5. Consider 6 charges at the corners of a regular hexagon. The distance from the origin to any of the charges is a. What is the electric field at any point $\mathbf{r}$ in the xy plane. What is the electric field on the x axis? What is the electric field on the x axis far from the origin, accurate through order $1/x^4$? What is the electric field at the origin? If the 6-th charge is removed, what is the electric field at the origin for the remaining 5 charges?

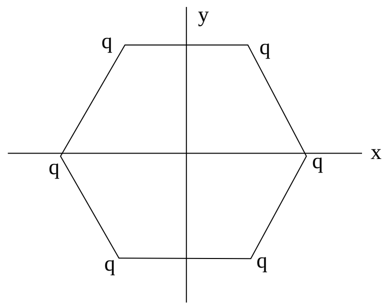

Figure 2:

6. A particle of mass m and charge $-Q$ is constrained to move along the axis of a ring (in the xy plane) of radius R. The ring carries a uniform line charge density $+\lambda$ along its length. Initially the particle is in the plane of the ring where the force on it is zero. Show that the period of oscillations of the particle when it is slightly displaced $z \ll R$ from its equilibrium position is given by $T = 2\pi\sqrt{2\epsilon_0 m R^2/(\lambda Q)}$.

7. Find the electric field everywhere using Gausss law for the following charge configurations: (i) a spherical shell of radius R carrying a uniform surface charge density, (ii) an infinitely-long cylinder of radius R carrying a uniform volume charge density, and (iii) an infinite plane slab of thickness d with a uniform volume charge density ρ.

8. Consider a cylindrical surface of radius R and length L is centered on the origin with its axis along the z-axis. It carries a uniform surface charge density σ on its curved surface (not on the flat surfaces). Calculate the potential at the point $z > L/2$ on the z axis.