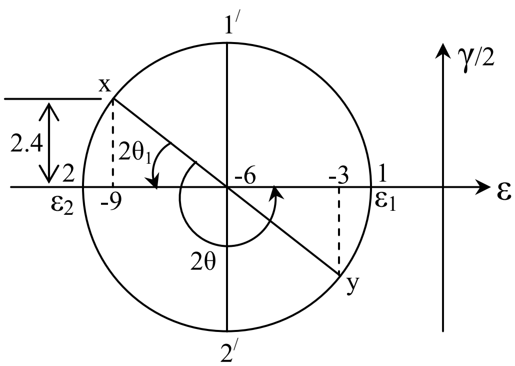
ESO 202A/204: MECHANICS OF SOLIDS (2016-17 II Semester) Assignment No. 5- SOLUTIONS

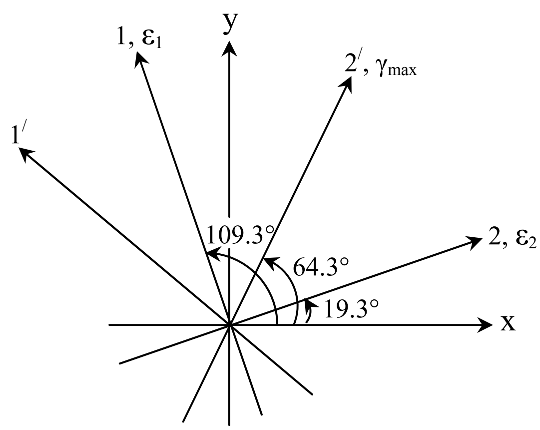
5.1 $\sigma_{xx} = -100MPa \qquad \sigma_{yy} = -50MPa \qquad \tau_{xy} = -20MPa \qquad \nu = 0.2 \qquad E = 100GPa$

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\sigma_{yy}\right] = \frac{-100 - 0.2(-50)}{10^5} = -9\times10^{-4}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\sigma_{xx}\right] = \frac{-50 - 0.2(-100)}{10^5} = -3\times10^{-4}$$

$$\gamma_{xy} = \tau_{xy}/G = \tau_{xy}\frac{2(1+\nu)}{E} = -\frac{20\times2\times1.2}{10^5} = -4.8\times10^{-4}$$





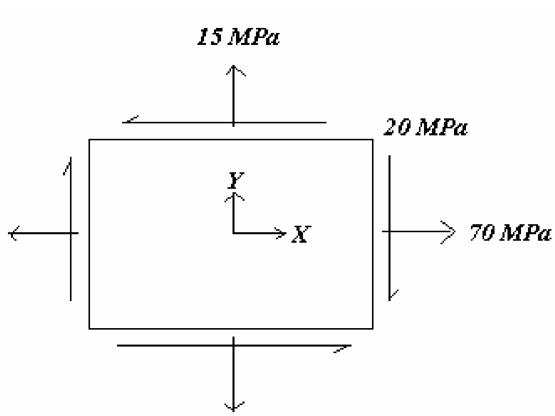
$$\left.\begin{aligned} \varepsilon_1 &= (-6+3.84)10^{-4} = -2.16\times10^{-4} \\ \varepsilon_2 &= (-6-3.84)10^{-4} = -9.84\times10^{-4} \\ \gamma_{max} &\quad 2\times3.84\times10^{-4} = 7.68\times10^{-4} \end{aligned}\right\}$$

$$\tan 2\theta_1 = \frac{2.4}{3} = 0.8 \Rightarrow 2\theta_1 = 38.6° \Rightarrow \theta_1 = 19.3°$$

$$2\theta = 218.6° \Rightarrow \theta = 109.3°$$

5.2

The plate is in the state of plane stress,



For any point in the plate

$$\sigma_1 = \frac{15+70}{2} + \sqrt{\left(\frac{70-15}{2}\right)^2 + 20^2} = 76.5MPa$$

$$\sigma_2 = \frac{15+70}{2} - \sqrt{\left(\frac{70-15}{2}\right)^2 + 20^2} = 8.5MPa$$

$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{20\times2}{70-15}\right) = 18.01^0 \text{....}(clockwise)$$

$$\theta_2 = 90^0 - 18.01^0 = 71.99^0 \text{....}(anticlockwise)$$

$$\Rightarrow \varepsilon_1 = \frac{1}{E}[\sigma_1 - \nu\sigma_2] = 3.6975\times10^{-4}$$

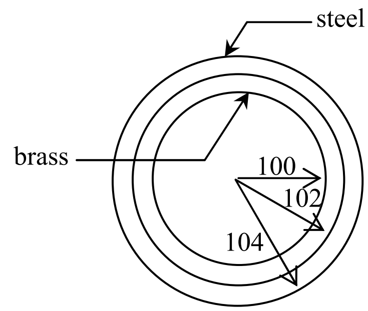
$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \nu\sigma_1] = -7.225\times10^{-5}$$

Diameter of the circle along the principle directions 1,2 will remain at 90^0 even after deformations.
$\Rightarrow \varepsilon_1$ and ε_2 will give the dimensions a and b of the major and minor axes of the ellipse.

$$\Rightarrow \text{a}=300*(1+3.6975\times10^{-4}) = 300.111mm$$

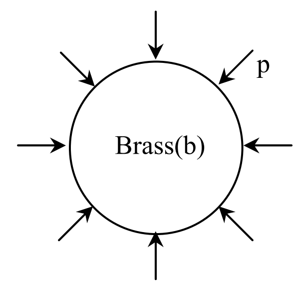
$$\text{b}=300*(1\text{-}7.225\times10^{-5}) = 299.978mm$$

Major axis is oriented at 18.01^0 clockwise to the X-axis while the minor axis is at 71.99^0 anti-clockwise.



5.3 Assume $\sigma_{yy} \approx 0$ in both hoops.
If it is assumed that the hoops are free to slide in the z-direction, $\sigma_{zz} = 0$.
Thus only $\sigma_{\theta\theta} \neq 0$.
Compatibility; $\varepsilon_{\theta\theta}\,|_{brass} = \varepsilon_{\theta\theta}\,|_{steel}$

Now, $\varepsilon_{\theta\theta} = \dfrac{\sigma_{\theta\theta}}{E} + \alpha\Delta T$; $\sigma_{\theta\theta} = \dfrac{pr}{t}$ (for internal pressure)
where p is the contact pressure between steel and brass hoops.
Thus,

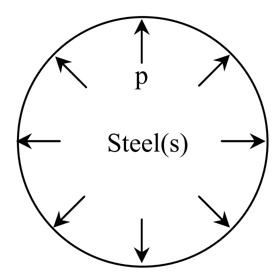


$$\frac{-pr_b}{E_b t_b} + \alpha_b \Delta T = \frac{+pr_s}{E_s t_s} + \alpha_s \Delta T$$

$$\Rightarrow p\left[\left(\frac{r}{Et}\right)_{steel} + \left(\frac{r}{Et}\right)_{brass}\right] = (\alpha_b - \alpha_s)\Delta T$$

$$\Rightarrow p\left[\frac{102}{2\times10^5\times2} + \frac{100}{10^5\times2}\right] = (20-10)10^{-6}\times75$$

$$\Rightarrow p = {}^{75}\!/_{75.5} \approx 1MPa$$



$$\therefore\ \sigma_{\theta\theta} = \left.\frac{pr}{t}\right|_b = \frac{1\times100}{2} = 50MPa$$

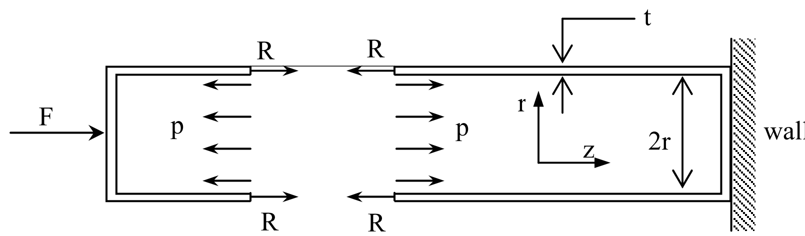
$$\sigma_{\theta\theta} = \left.\frac{pr}{t}\right|_s = \frac{1\times102}{2} = 51MPa$$

5.4 Let F be the force exerted by the rigid wall on the cylinder.

$\sum F_Z = 0 \Rightarrow F + R = p\pi r^2$

$\Rightarrow R = p\pi r^2 - F$

Axial stress, $\sigma_{ZZ} = \dfrac{R}{2\pi rt} = \dfrac{p\pi r^2 - F}{2\pi rt}$



Circumferential stress, $\sigma_{\theta\theta} = \dfrac{pr}{t}$

Radial stress, $\sigma_{rr} = 0$

The geometry demands that $\varepsilon_{ZZ} = 0$

Now, $\varepsilon_{ZZ} = \dfrac{1}{E}\left[\sigma_{ZZ} - \nu\left(\sigma_{\theta\theta} + \sigma_{rr}\right)\right] = 0$

$$\Rightarrow \sigma_{ZZ} = \nu\sigma_{\theta\theta}$$

$$\Rightarrow \frac{p\pi r^2 - F}{2\pi rt} = \nu\frac{pr}{t}$$

$$\Rightarrow F = p\pi r^2(1 - 2\nu)$$

5.5 $\varepsilon_{ZZ} = 0 \qquad \varepsilon_{rr} \neq 0 \qquad \varepsilon_{\theta\theta} \neq 0$

Also, the cross-section does not change its shape.

Hence, $\gamma_{r\theta} = \gamma_{\theta z} = \gamma_{rz} = 0 \Rightarrow$ a case of plane strain in r-θ plane.

All shear stresses are zero. Assume 'end-effects' to be 'localized' $\Rightarrow$ 'free' radial and tangential expansion.

$$\therefore \sigma_{\theta\theta} = \sigma_{rr} = 0; \quad \sigma_{zz} \neq 0 \Rightarrow \text{ also a plane-stress situation.}$$

Apply generalized Hooke's Law

$$\varepsilon_{zz} = \varepsilon_{zz}^e + \varepsilon_{zz}^t = \varepsilon_{zz}^e + \alpha\Delta T = 0$$

But $\varepsilon_{zz}^e = \dfrac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{\theta\theta} + \sigma_{rr}\right)\right] = \dfrac{\sigma_{zz}}{E}$

$$\therefore \sigma_{zz} = -\alpha\Delta TE = 12\times10^{-6}\times35\times2\times10^5 = 84MPa$$

Similarly, $\varepsilon_{rr} = \varepsilon_{zz}^e + \varepsilon_{zz}^t = -\dfrac{\nu\sigma_{zz}}{E} + \alpha\Delta T$

$$= -\frac{0.3\times84}{2\times10^5} - 12\times10^{-6}\times35 = -5.46\times10^{-4}$$

Since $\varepsilon_{\theta\theta} = \varepsilon_{rr}$, $\quad \varepsilon_{\theta\theta} = -5.46\times10^{-4}$

5.6

$$P_{St\max} = A_{St}Y_{St} = \frac{\pi}{4}(170^2 - 150^2) \times \frac{600}{10^6} = 3.016MN$$

$$P_{Al\max} = A_{Al}Y_{Al} = \frac{\pi}{4}(100^2) \times \frac{400}{10^6} = 3.142MN$$

For equilibrium, $P_{St} = P_{Al} \Rightarrow$ max. load for elastic behaviour = 3.016 *MN*
Let δ_T be the total deflection for P = 1.5(3.016) = 4.524 *MN*

$$\delta_T = \delta_{St} + \delta_{Al}$$

$$\sigma_{St} = \frac{P}{A_{St}} = 900MPa$$

$$\sigma_{Al} = \frac{P}{A_{Al}} = 576MPa$$

$$\delta_{St} = \delta_{St\,elastic} + \delta_{St\,plastic} = \frac{Y_{St} \bullet l_{St}}{E_{St}} + \frac{(\sigma_{St} - Y_{St})l_{St}}{(\frac{d\sigma}{d\varepsilon})_{St,plastic}} = 9.4mm$$

$$\delta_{Al} = \delta_{Al\,elastic} + \delta_{Al\,plastic} = \frac{Y_{Al} \bullet l_{Al}}{E_{Al}} + \frac{(\sigma_{Al} - Y_{Al})l_{Al}}{(\frac{d\sigma}{d\varepsilon})_{Al,plastic}} = 8.64mm$$

$$\delta_T = \delta_{St} + \delta_{Al} \approx 18mm$$