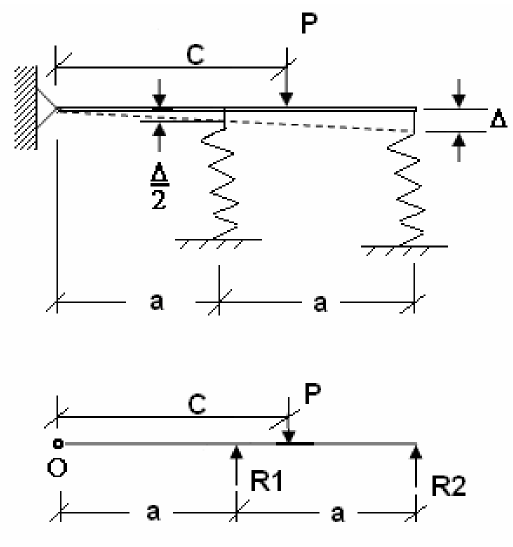
ESO-202A/204, Mechanics of Solids (2016-17 II semester)

Solution of Assignment No.-2

2.1



From the F.B.D., taking

$\sum M_O = 0$, we get,

$P.c = R_1.a + R_2.2a$ ……….. (1)

Here, $k = \dfrac{R_1}{\frac{\Delta}{2}}$ and also, $k = \dfrac{R_2}{\Delta}$

Hence, $R_1 = \dfrac{R_2}{2}$

From eqn. (1) we get,

$R_2 = \dfrac{2.P.c}{5.a}$ and $R_1 = \dfrac{P.C}{5.a}$ …….(2)

Deflection under load P = $\dfrac{\Delta.c}{2a}$

Hence, effective stiffness = $\dfrac{P}{\frac{\Delta.c}{2a}} = \dfrac{20k}{9}$ or, $\dfrac{P}{\frac{20.k}{9}} = \dfrac{\Delta.c}{2.a}$ or, $\dfrac{c}{a} = \dfrac{9P}{10k\Delta}$ ……….. (3)

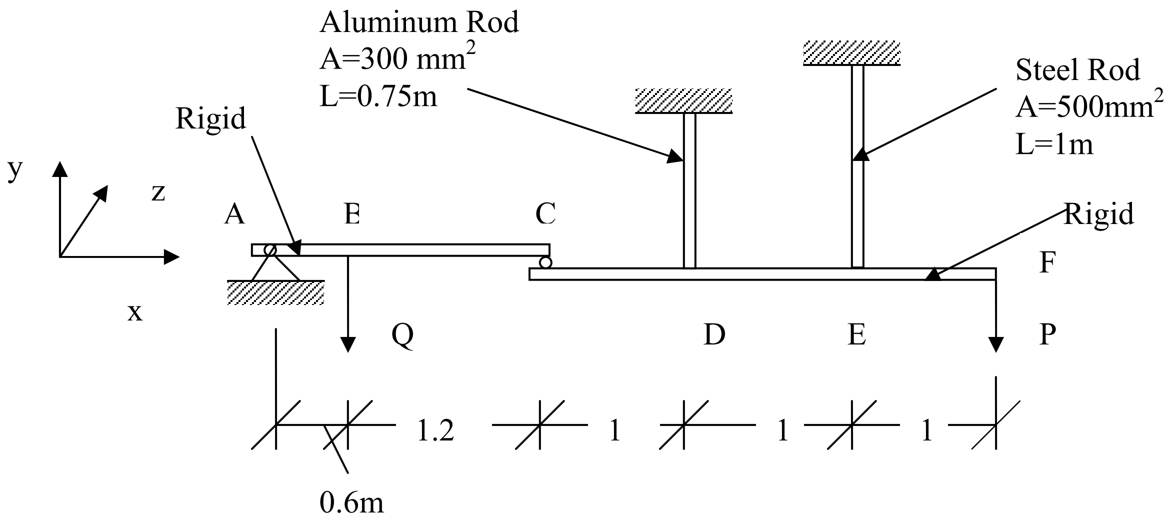
But, $\Delta = \dfrac{R_2}{k} = \dfrac{2P.C}{5.a.k}$ [From eqn. (2)] ………… (4)

So, putting eqn.(4) in eqn.(3) we get,

$$\frac{c}{a} = \frac{9}{4}.\frac{a}{c}$$

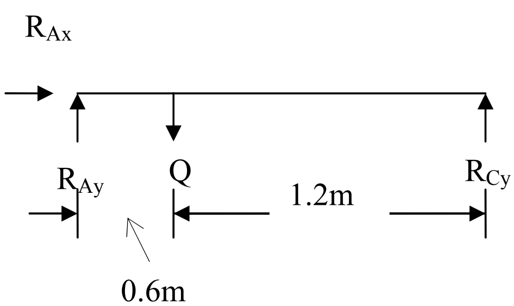
Or, $\dfrac{c}{a} = \dfrac{3}{2}$

2.2

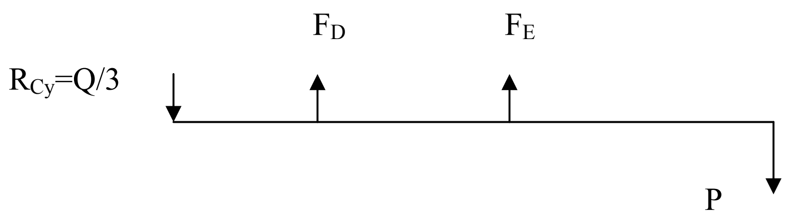


Before P and Q are applied, both rigid bars are level. First P is applied and then Q. Find Q in terms of P if the rigid bars CF is to be level after two loads are applied.

Force equation



$$R_{Ax} = 0,$$
$$R_{Ay} + R_{Cy} - Q = 0$$
$$\sum M_{Az} = 0, \textit{gives}$$
$$R_{Cy}\, x1.8 - Qx\, 0.6 = 0$$
$$R_{Cy} = \frac{Q}{3}$$



$$\sum F_y = 0, \textit{gives} ----> \frac{Q}{3} + P = F_D + F_E \text{................}(1)$$
$$\sum M_{CZ} = 0, \textit{gives} ---> F_D x1 + F_E x2 = 3P \text{................}(2)$$

Geometric Compatibility:
For CF to remain horizontal, extensions in both rods should be same, i.e. $\delta_D = \delta_E$

Force-deformation:

$$\delta_D = \frac{F_D l_A}{A_A E_A}, \delta_E = \frac{F_E l_S}{A_S E_S}$$

$$\delta_D = \delta_E, gives$$

$$\frac{F_D l_A}{A_A E_A} = \frac{F_E l_S}{A_S E_S} ------> F_D = F_E.\frac{l_S}{l_A}\frac{A_A}{A_S}.\frac{E_A}{E_S} = F_E \frac{1}{0.75} x \frac{300}{500} x \frac{0.75x10^5}{2.0x10^5}$$

$$or, F_D = 0.3F_E..................(3)$$

(3) & (2)

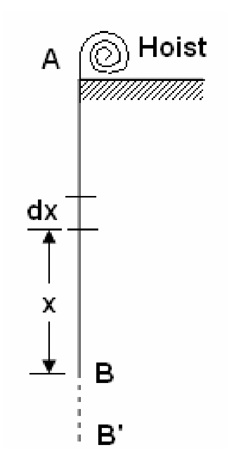
$$Hence, 0.3F_E + 2F_E = 3P ----> F_E = \frac{3}{2.3}P ----> F_D = \frac{9}{23}P$$

$$From(1)$$

$$\frac{Q}{3} + P = \frac{9}{23}P + \frac{30}{23}P$$

$$Hence, Q = \frac{48}{23}P$$

2.3



Let m_0 = mass of rope / unit length
Due to the self wt. the rope extends
Tension at B = 0; at A = m_0 g L, where L is the length of the rope
At section x, T(x) = m_o g x
Hence, Extension of the rope element of length dx due to T(x) is

$$d\delta = \frac{T(x)dx}{AE}; \delta = \int_0^L \frac{T(x).dx}{AE} = \frac{m_0 g L^2}{2(AE)_{rope}}$$

Spring constant of rope =5.345x10^7 i.e. with 1m length of rope one needs 5.345x10^7 N to extend by 1m.
Thus $(AE)_{rope}$ = 5.345x10^7

Hence, $\delta = \dfrac{23.38x1824x1824}{2x5.345x10^7} = 0.7276\,\text{m}$

So, B' is at a distance 1824+0.7276 = 1824.73 m. from the ground level. The miners can reach the rope if the rope can be further extended by, say, about 5m (The difference of 0.27m can be made up by raising hands overhead. To generate 5m extension one needs to hang a weight W at the lower end where,

$$W = (AE)_{rope} x \frac{5}{1824} = 146518N$$

With this added weight, the maximum tension at A is $m_0gL + W$ = 189163 N

Hence, Rope is subjected to force / unit area = $\dfrac{189163}{\frac{\pi}{4}x(25.4)^2} = 373.3N/mm^2$

So, we must have a steel rope which can withstand this force.

2.4

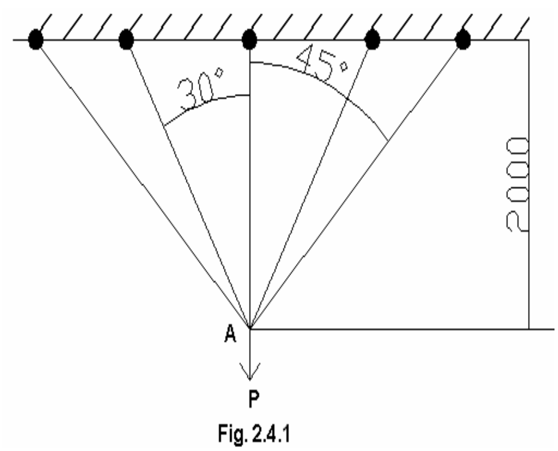


Fig. 2.4.1

Due to symmetry, displacement of A, say δ, will be vertical. [Fig.2.4.1]

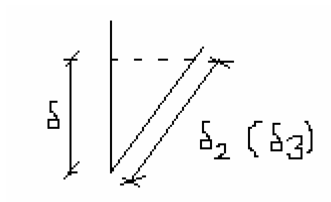


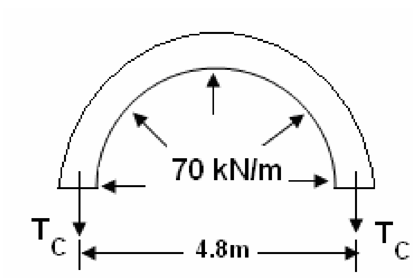
Fig.2.4.2

Let δ_2 be the deflection of bars inclined at 30^0 and δ_3 be that of bars at 45^0.
$\delta_2 = \delta.\cos 30^0 = (\sqrt{3}/2).\ \delta$
$\delta_3 = \delta.\cos 45^0 = \delta/\sqrt{2}$

Vertical equilibrium of forces at joint A gives:
$AE\ \delta/2000 + (2AE\ [(\sqrt{3}/2).\ \delta]/(2000/\cos 30^0))\cos 30^0 + (2AE\ [(\delta/\sqrt{2})]/(2000/\cos 45^0))\cos 45^0 = 2000$ [in KN]

Here from, we have δ = 13.3 mm.

2.5



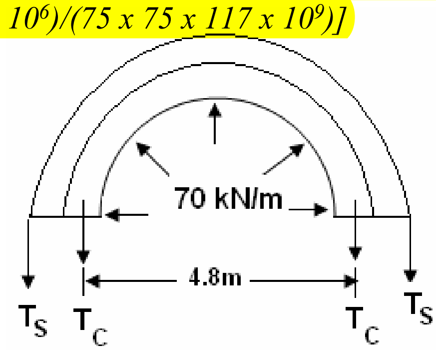
First, we should find out if the two rings come into contact or not.
From F.B.D. on the left,
$2T_C = 70x4.8$ or $T_C = 168$ kN

$$\delta_{C(along\_the\_circumference)} = \frac{T_C L_C}{(AE)_C}$$

Hence, increase along the dia., $\delta_D = \dfrac{1}{\pi}\dfrac{T_C L_C}{(AE)_C} = 1.225\ mm$

*$1.225\ mm = (1/\pi)*[(168 \times 10^3 \times \pi \times 4.8 \times 10^6)/(75 \times 75 \times 117 \times 10^9)]$

But the zero current clearance is 0.5mm.
Hence, the ring will come in contact.
When the rings are in contact, from F.B.D., $2(T_C+T_S) = 70x4.8$
Or, $T_C+T_S = 168$ kN ……… (1)



Clearly then,

$$(\delta_D)_C = 1 + (\delta_D)_S \, mm \text{..........(2)}$$

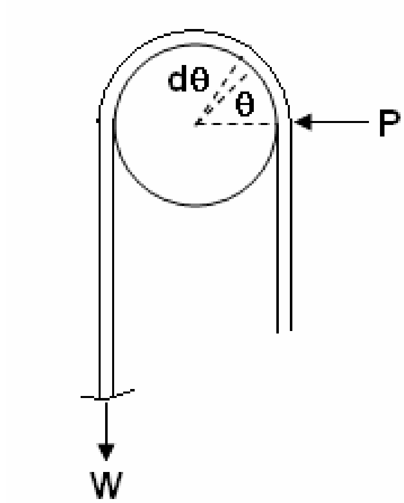
But,

$$(\delta_D)_C = \frac{T_C L_C}{\pi (AE)_C} \; and \; (\delta_D)_S = \frac{T_S L_S}{\pi (AE)_S} \text{.................(3)}$$

Using (3) in (2) and the resulting equation in (1), we get,

$T_C = 157$ kN

2.6



Refer Lecture notes:

$\frac{dT}{d\theta} = f.T$ Where T is the tension at angle θ.

Hence, $T = C.e^{f\theta}$ after integrating, C is constant.

At, $\theta = \pi$, T = W gives,
$W = C.e^{\pi f}$
Or, $C = W.e^{-\pi f}$

Hence, $T = W.e^{f(\theta-\pi)}$
At, $\theta = 0$, $T = T_0 = W^{-f\pi}$

In order to prevent weight from dropping,
We require $T_0 = f.P$

Hence, $P = \frac{W}{f} e^{-f\pi}$

2.7

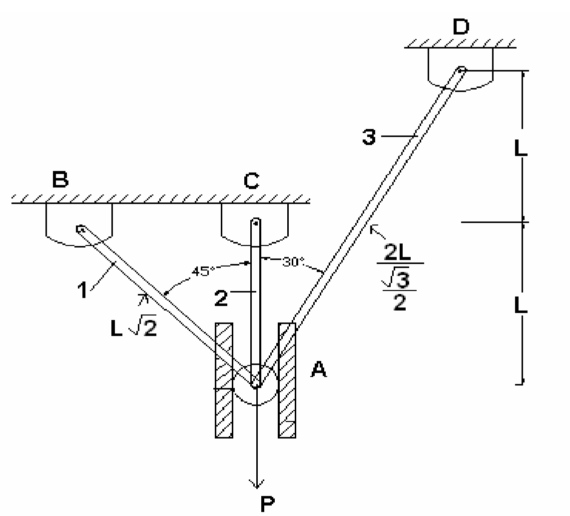


Fig. 2.7.1

Force Balance:

$F_1 \cos 45^0 + F_2 + F_3 \cos 30^0 = P$
$H - F_1 \sin 45^0 + F_3 \sin 30^0 = 0$

Or,

$F_1/\sqrt{2} + F_2 + (\sqrt{3})/2\ .F_3 = P$ ………. (1)
$H = F_1/\sqrt{2} - F_3/2$ ………. (2)

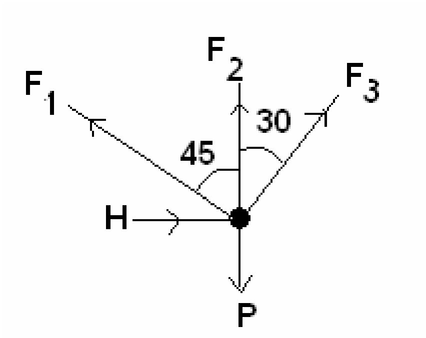


Fig.2.7.2

Compatibility:

$\delta_1 = \delta_2/\sqrt{2}$, $\delta_3 = \sqrt{3}.\delta_2/2$ ………. (3)

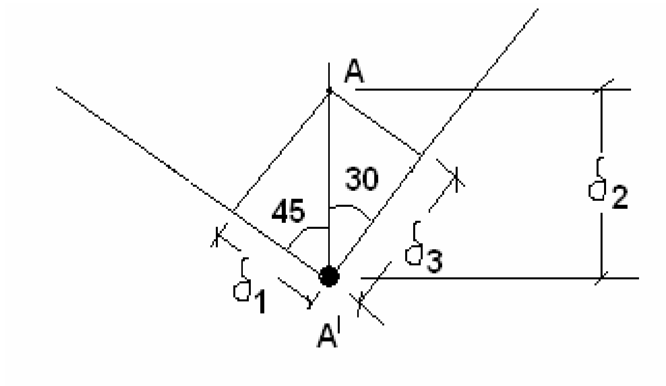


Fig.2.7.3

Force- Deflection:

$\delta_1 = (F_1 L\sqrt{2})/ AE$
$\delta_2 = (F_2 L)/AE$
$\delta_3 = (4F_3 L)/ (\sqrt{3}\ AE)$ ……….. (4)

Solving:

Sub (4) in (3)

$\sqrt{2}F_1 = F_2/\sqrt{2} \longrightarrow F1 = F_2/2$
$4\ F_3/\sqrt{3} = \sqrt{3}\ F_2/2 \longrightarrow F_3 = 3\ F_2/8$

Sub in (1)

$F_2/(2\sqrt{2}) + F_2 + (3\sqrt{3}\ F_2)/16 = P$
$F_2 = P/1.678 = 0.596P$
$F_1 = 0.298P$
$F3 = 0.223P$

Sub in (2)

$H = (0.298/\sqrt{2} - 0.223/2)\ P$
$= 0.0992P$