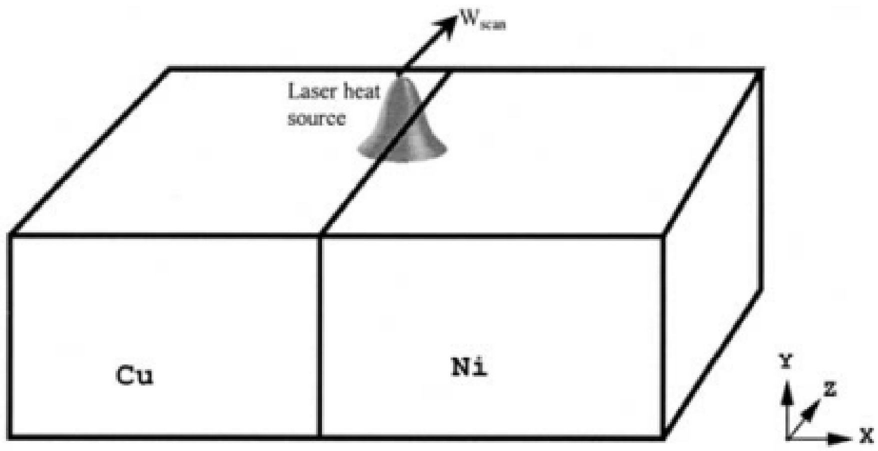
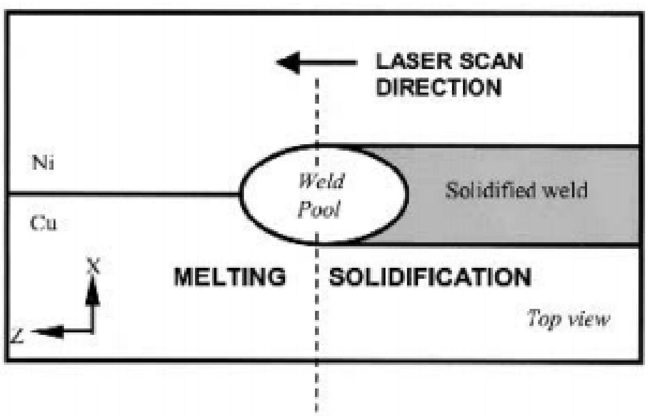
ME361A
Lecture 19

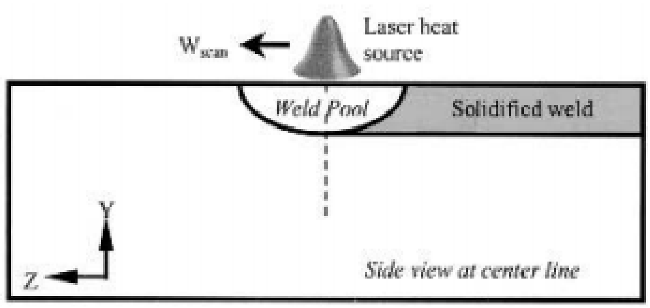
Computational Modeling of Welding

Dr. Arvind Kumar



Schematic of laser welding setup.





Schematic illustrating simultaneous melting and solidification during continuous welding.

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad [1]$$

Momentum:

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho uu) + \frac{\partial}{\partial y}(\rho vu) + \frac{\partial}{\partial z}(\rho wu) = \frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial u}{\partial z}\right) - \frac{\partial p}{\partial x} + S_x \qquad [2]$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(\rho uv) + \frac{\partial}{\partial y}(\rho vv) + \frac{\partial}{\partial z}(\rho wv) = \frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial v}{\partial z}\right) - \frac{\partial p}{\partial y} + S_y \qquad [3]$$

$$\frac{\partial}{\partial t}(\rho w) + \frac{\partial}{\partial x}(\rho uw) + \frac{\partial}{\partial y}(\rho vw) + \frac{\partial}{\partial z}(\rho ww) = \frac{\partial}{\partial x}\left(\mu\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(\mu\frac{\partial w}{\partial z}\right) - \frac{\partial p}{\partial z} + S_z \qquad [4]$$

Energy:

$$\frac{\partial}{\partial t}(\rho H) + \frac{\partial}{\partial x}(\rho u H) + \frac{\partial}{\partial y}(\rho v H) + \frac{\partial}{\partial z}(\rho w H) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + S_h \qquad [5]$$

Mass fraction:

$$\frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(uC) + \frac{\partial}{\partial y}(vC) + \frac{\partial}{\partial z}(wC) = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C}{\partial z}\right) + S_C \qquad [6]$$

In the preceding equations, t is time; ρ is density; u, v, and w are velocities along x, y, and z directions, respectively; μ is viscosity; p is pressure; k is thermal conductivity; H is enthalpy; C is species concentration; and D is species diffusivity.

A. *Spot Welding*

The source terms in Eqs. [1] through [6] according to the enthalpy-porosity formulation are expressed as follows:

$$S_x = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)u \quad [7]$$

$$S_y = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)v + \rho g[\beta_T(T - T_r) - \beta_C(C - C_r)] \quad [8]$$

$$S_z = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)w \quad [9]$$

$$S_C = 0 \quad [10]$$

where K is permeability; b is a small number to avoid division by zero; g is acceleration due to gravity; β_T is compressibility; β_C is the fractional difference in the densities of the two metals; T_r and C_r are reference temperature ($T_{\mathrm{melt,Ni}}$) and composition (0) for buoyancy, respectively; and ε is liquid fraction.

The first terms in Eq. [7] through [9] represent the porous medium like resistance in the mushy region at the solid-liquid interface. In the fully liquid region, the value of ε = 1, making the porous medium resistance terms zero. On the other hand, in the fully solid region, ε = 0, thus forcing the porous medium resistance terms in Eqs. [7] through [9] to be very large. This large negative source term offers a high flow resistance, making the velocities in the entire solid region effectively zero.

Since the velocity in the weld pool is large, the mushy zone
is expected to be very thin:

$$S_h = -\frac{\partial}{\partial t}(\rho \Delta H) \qquad [11]$$

The enthalpy, H, of a material can be expressed as

$$H = h_s + \Delta H \qquad [12]$$

$$h_s = cT \qquad [13]$$

where h_s is the sensible heat, ΔH is the latent heat content, and c is the specific heat.

Final form of the energy equation:

$$\frac{\partial}{\partial t}(\rho c T) + \nabla \cdot (\rho c T) = \nabla \cdot (k \nabla T) - \frac{\partial}{\partial t}(\rho \Delta H) \quad [16]$$

where ΔH is the latent heat content of a control volume, and ε is the liquid fraction calculated as $\Delta H/L$, with L being the latent heat of melting for the corresponding metal.

During solidification of a spot weld, the enthalpy, H, used in Eq. [15] is defined in the liquid state at any location using mixture theory as given in the following equation:

$$H = H_{\mathrm{Cu}} C + H_{\mathrm{Ni}} (1 - C) \quad [17]$$

where H_{Cu} and H_{Ni} are the enthalpy values of copper and nickel, respectively, at a given temperature. The melting point of any alloy is approximated as follows:

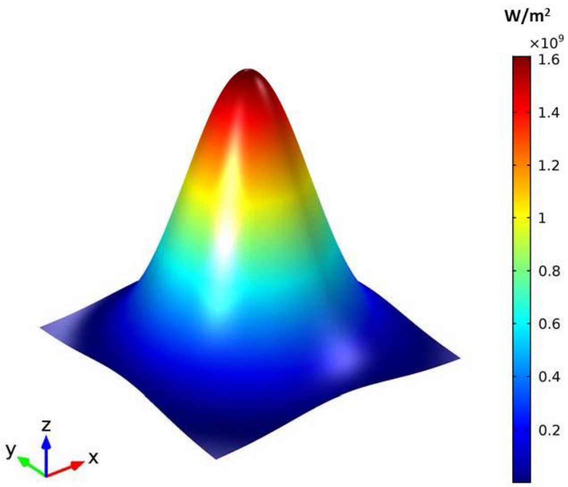
$$T_{\mathrm{melt}} = T_{\mathrm{melt,Cu}} C + T_{\mathrm{melt,Ni}} (1 - C) \quad [18]$$

Boundary and initial conditions

At time $t = 0$, the entire domain is in the solid state at room temperature. At time $t > 0$, the following boundary conditions are applied.

At the top surface of the work piece, a heat flux with a Gaussian distribution is applied, as given by

$$q''(r) = \frac{\eta Q}{\pi r_q^2} \exp\left(-\frac{r^2}{r_q^2}\right) \qquad [19]$$



where η is efficiency of absorption of laser, Q is laser power, and r_q is radius of the laser beam. No mass transfer is considered at the top surface. All the sides are subjected to convective and radiative heat losses. At the flat free surface of the liquid, shear force due to surface tension (Marangoni force) is expressed as

$$\mu \frac{\partial u}{\partial y}\bigg|_{y=h} = \frac{\partial \sigma}{\partial x}\bigg|_{y=h} \qquad [20]$$

$$\mu \frac{\partial w}{\partial y}\bigg|_{y=h} = \frac{\partial \sigma}{\partial z}\bigg|_{y=h} \qquad [21]$$

B. *Continuous Welding*

The reference frame used for the simulation of continuous welding is a moving frame fixed to the laser. The laser moves along the *z*-axis with a speed w_{scan}. If (*x, y, z*') is the stationary coordinate system and (*x, y, z*) is the system in reference to the laser, then $z' = z + w_{scan}t$. The governing equations in the moving reference frame would remain the same as in Eqs. [2] through [6], except for additional source terms corresponding to the coordinate transformation.

$$S_x = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)u - \frac{\partial}{\partial z}(\rho u w_{\mathrm{scan}}) \qquad [22]$$

$$S_y = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)v + \rho g[\beta_T (T - T_r) - \beta_C (C - C_r)] - \frac{\partial}{\partial z}(\rho v w_{\mathrm{scan}}) \qquad [23]$$

$$S_z = -\left(\frac{K(1-\varepsilon)^2}{\varepsilon^3 + b}\right)w - \frac{\partial}{\partial z}(\rho w w_{\mathrm{scan}}) \qquad [24]$$

$$S_h = -\frac{\partial}{\partial t}(\rho \Delta H) - \frac{\partial}{\partial z}(\rho c T w_{\text{scan}}) - \frac{\partial}{\partial z}(\rho w_{\text{scan}} \Delta H) \quad [25]$$

$$S_C = -\frac{\partial}{\partial z}(C w_{\text{scan}}) \quad [26]$$

Initial and boundary conditions

The initial and boundary conditions remain the same as in spot welding except for the additional species mass flux conditions at the phase change interface.

Melting front:

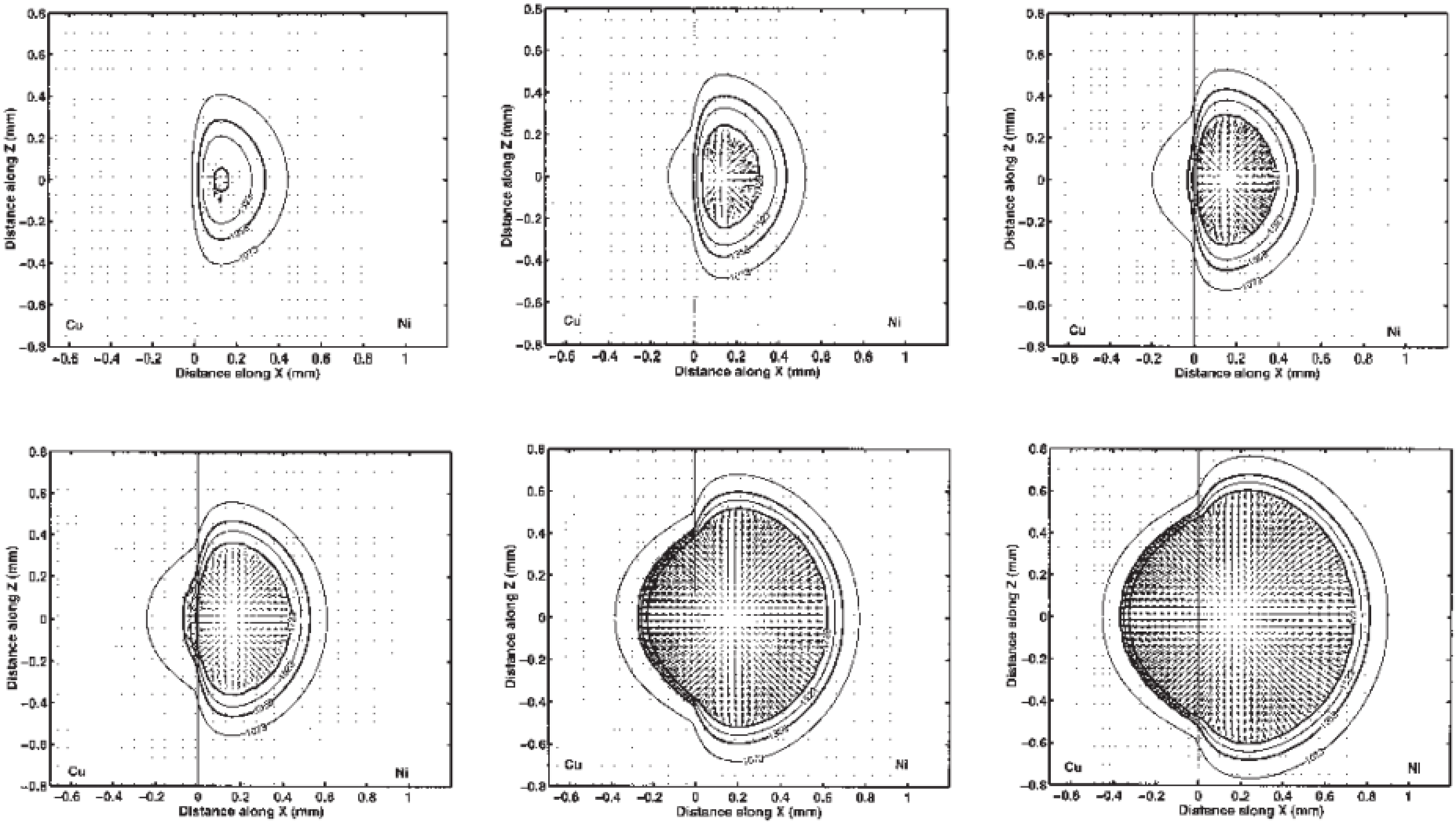
$$v_n C_l = -D \frac{\partial C_l}{\partial y} \quad [27]$$

Solidification front:

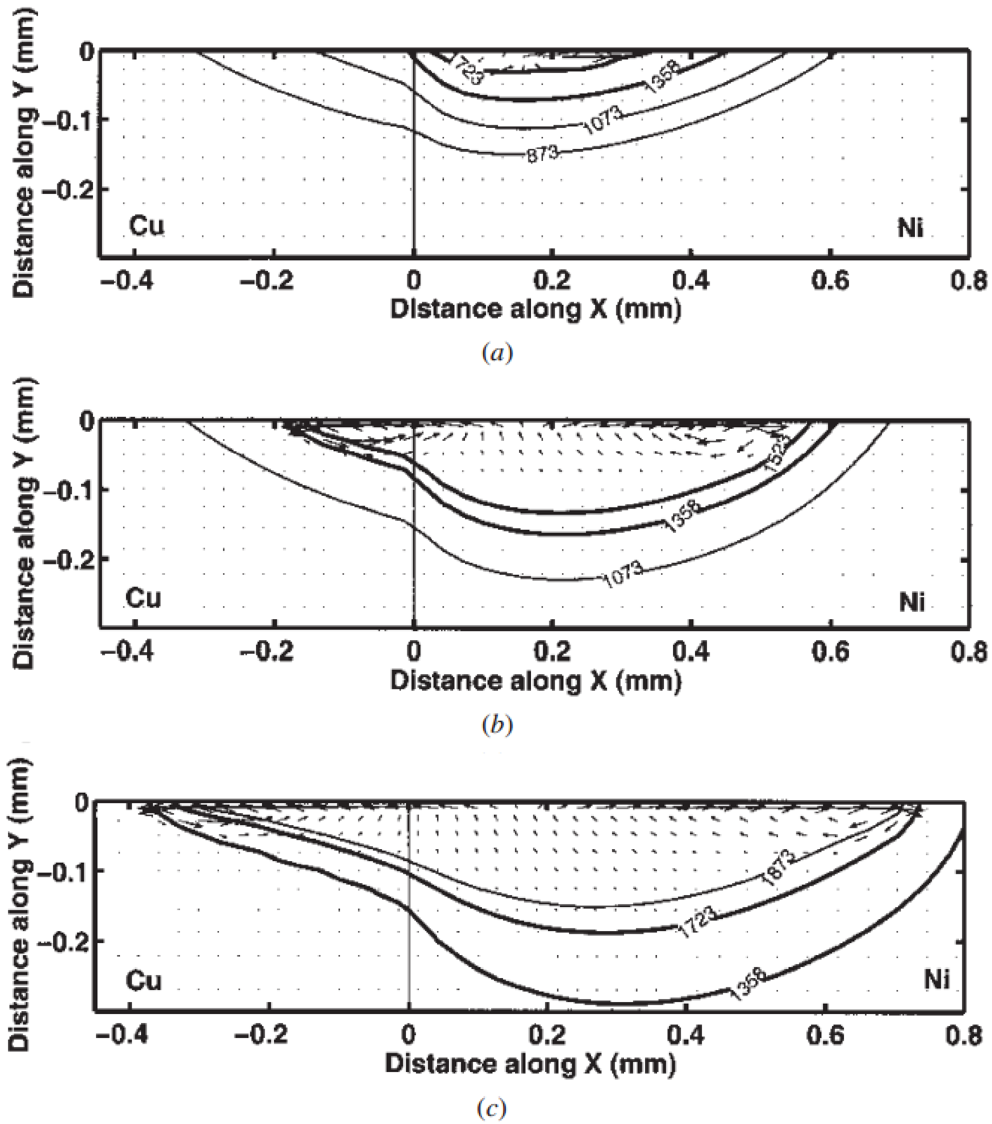
$$(1 - k_p) v_n C_l = -D \frac{\partial C_l}{\partial y} \quad [28]$$

v_n is the projection of the traverse speed onto the normal to the solid-liquid boundary, C_l is the species concentration in the liquid, and D is the diffusivity and k_p is the partition coefficient.

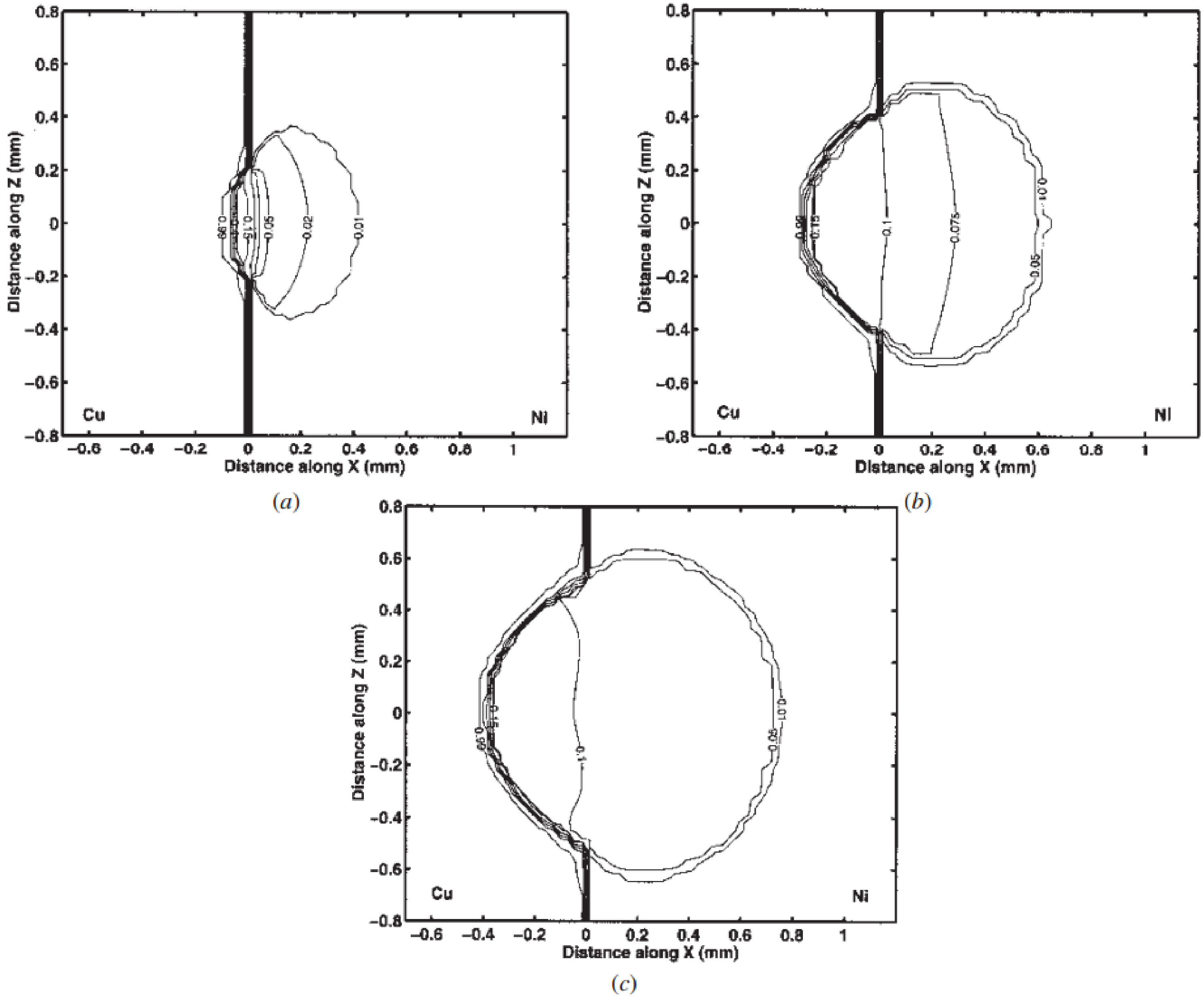
Results for Spot Welding:



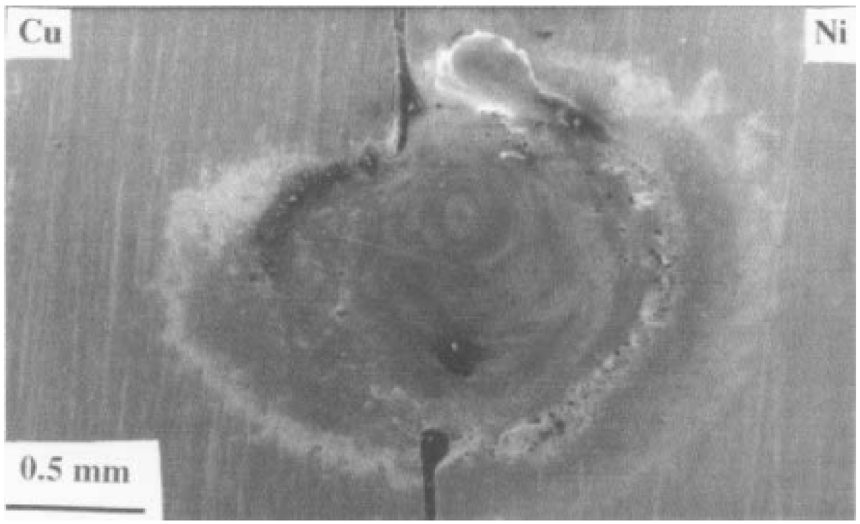
Temperature contours and velocity profile of the top view of weld pool after (a) 1 ms, (b) 1.8 ms, (c) 2.6 ms, and (d) 3.4 ms, (e) 11 ms, and (f) 25 ms.



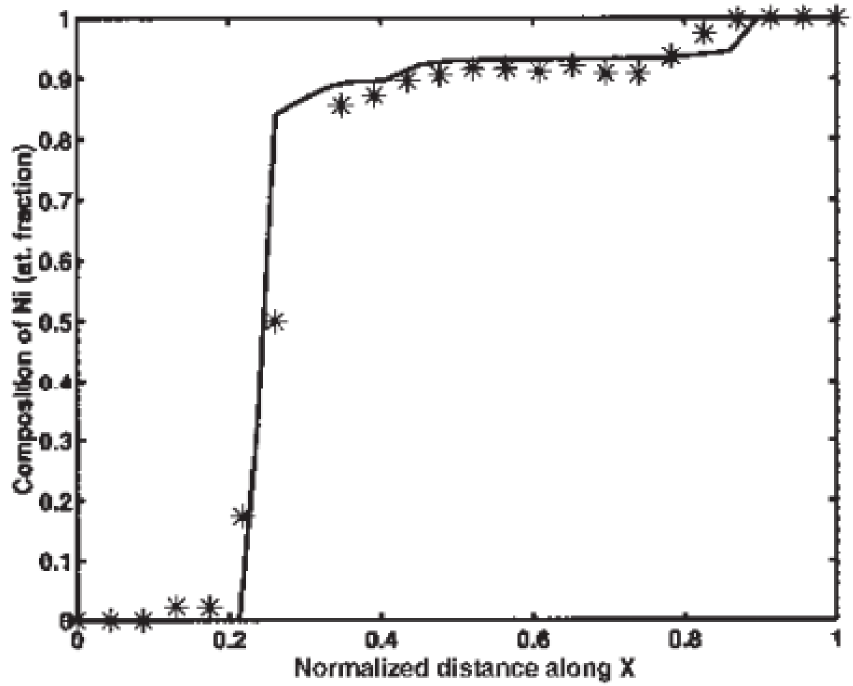
Contours of temperature and velocity profile of the cross-sectional view of the weld pool after (*a*) 3.4 ms, (*b*) 11 ms, and (*c*) 25 ms.



Composition contours and velocity profile of the top view of weld pool after (*a*) 3.4 ms, (*b*) 11 ms, and (*c*) 25 ms. Contour labels are in Cu weight fraction



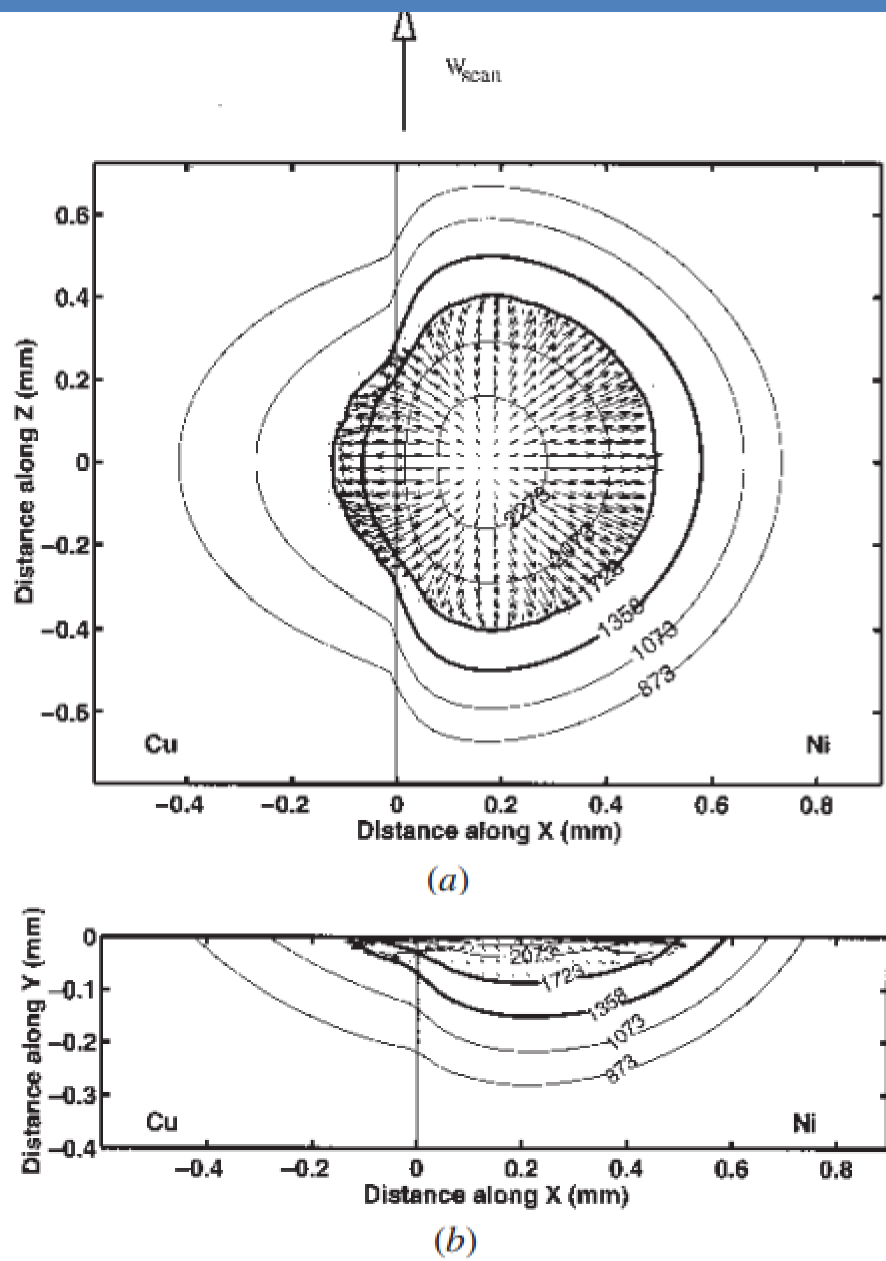
(*a*)



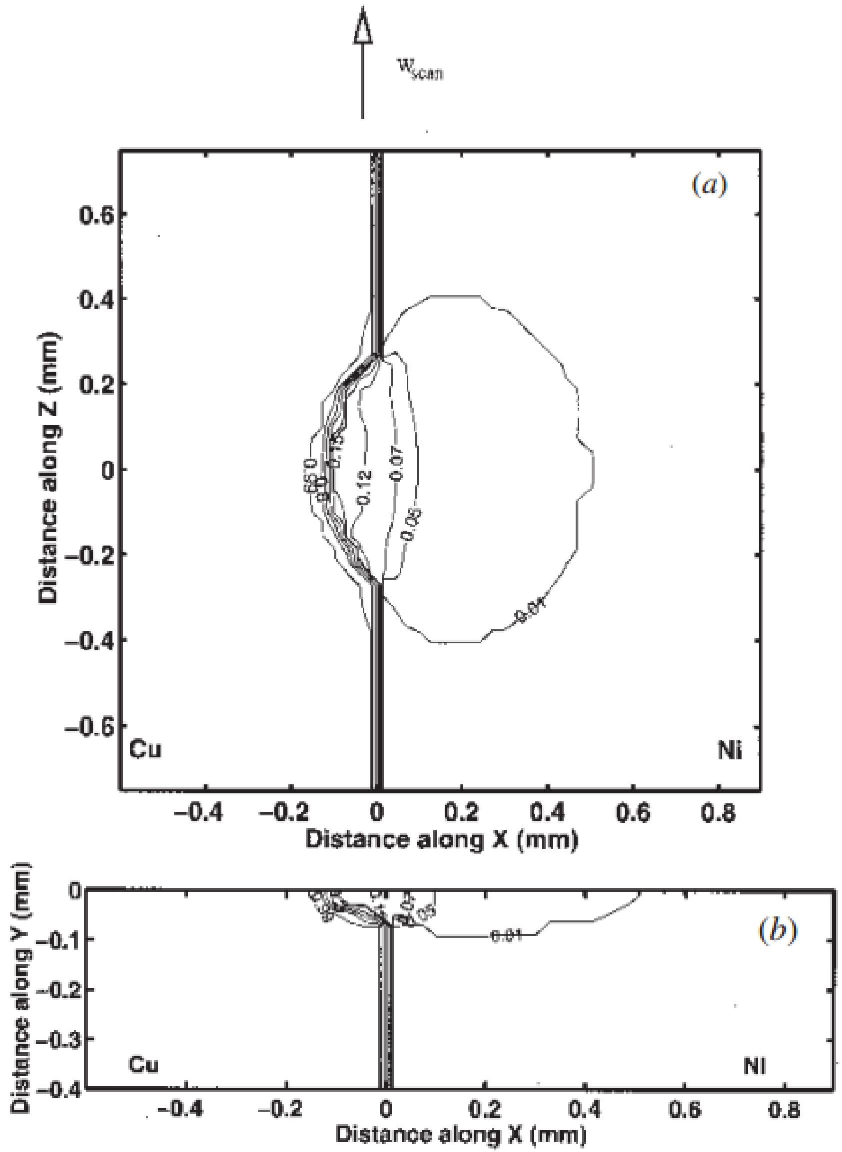
(*b*)

(*a*) Top view of the spot weld pool processed with the same conditions as those of computation.
(*b*) Composition of the solidified spot weld along a line across the weld near the top. The asterisks superimposed are the experimental compositions of nickel taken from a similar spot weld

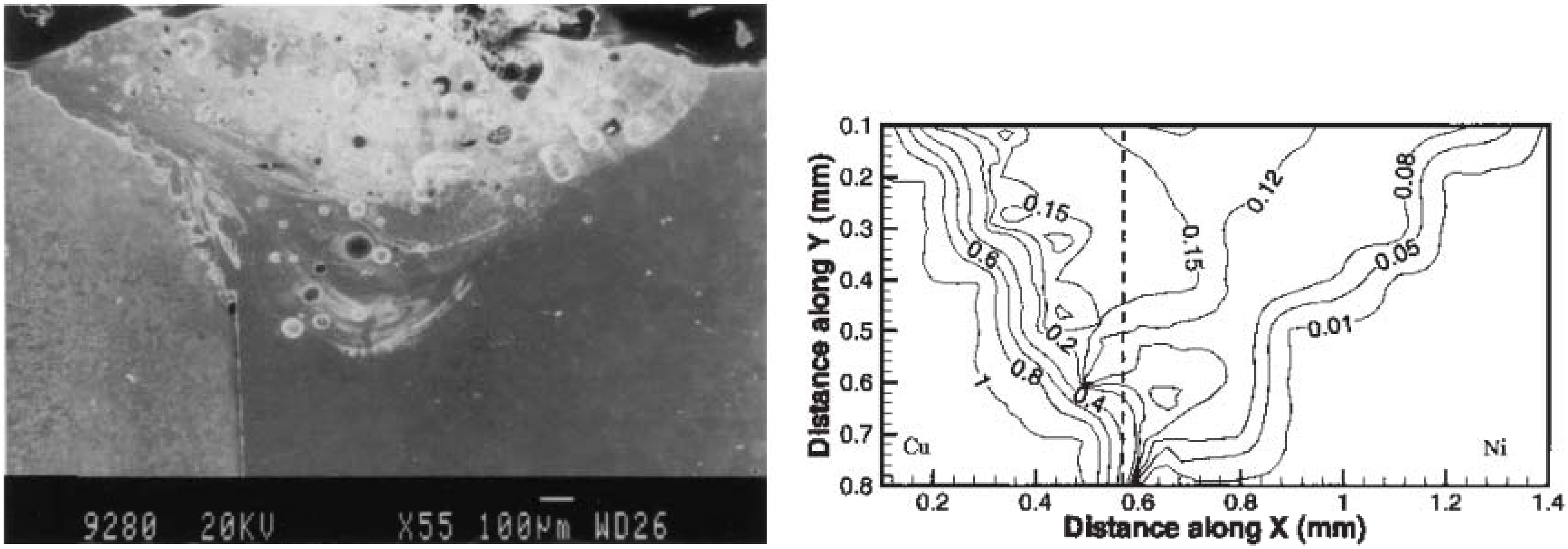
Results for Continous Welding:



Temperature contours and velocity profile in (*a*) top view and (*b*) transverse section of continuous weld at a snapshot after 7 ms.



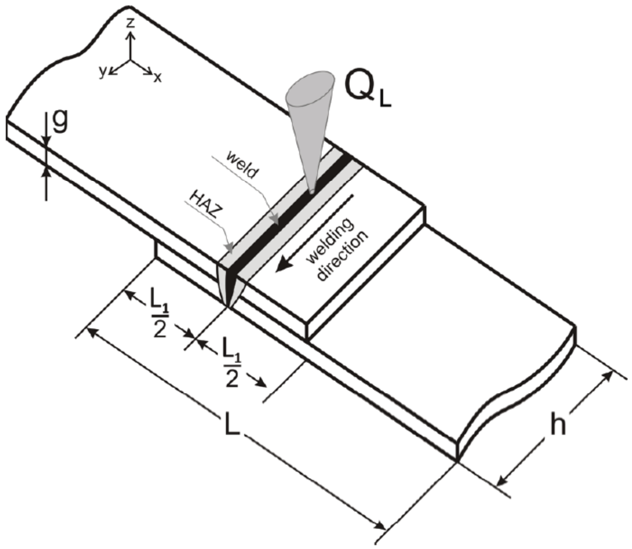
Composition contours in (*a*) top view and (*b*) transverse view at a snapshot after 7 ms. Contour labels are in Cu weight fraction.

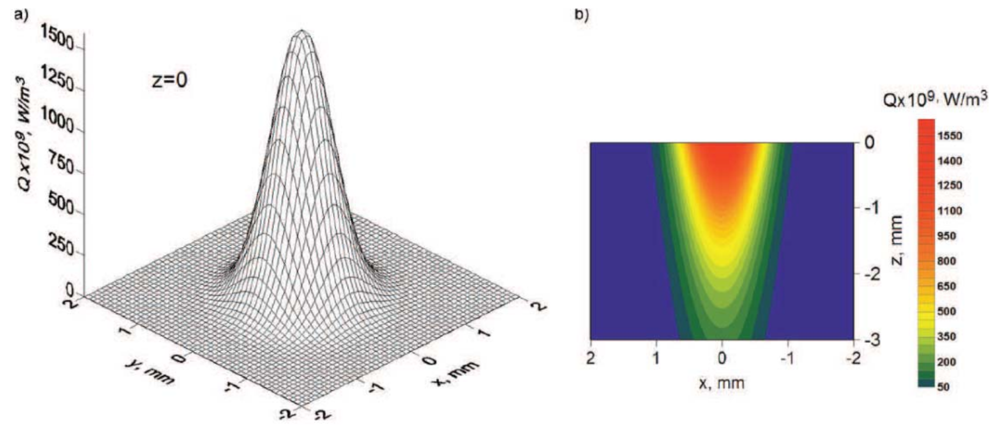


(a) (b)

(a) Cross-sectional view of the continuous laser weld processed with the same conditions as those of the computation. (b) Composition map in the whole of the weld pool with copper on the left and nickel on the right for the weld pool shown in (a). Contour labels are in Cu weight fraction

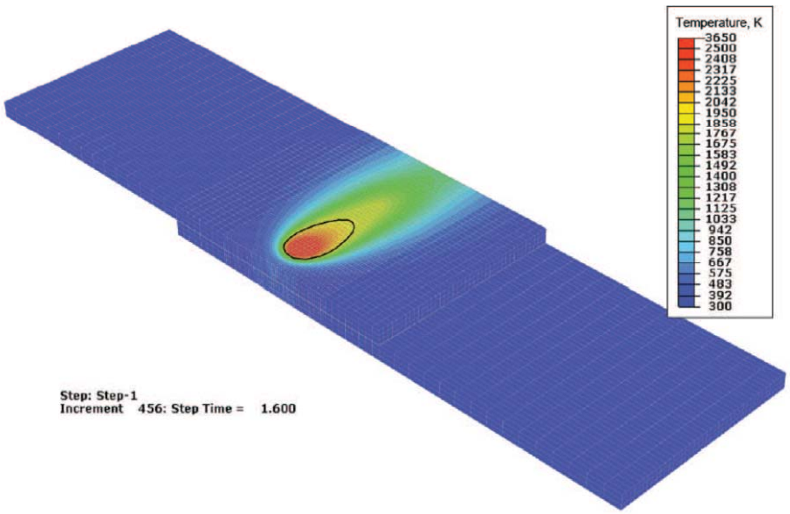
Computer simulation of temperature field in laser beam welded lap joint



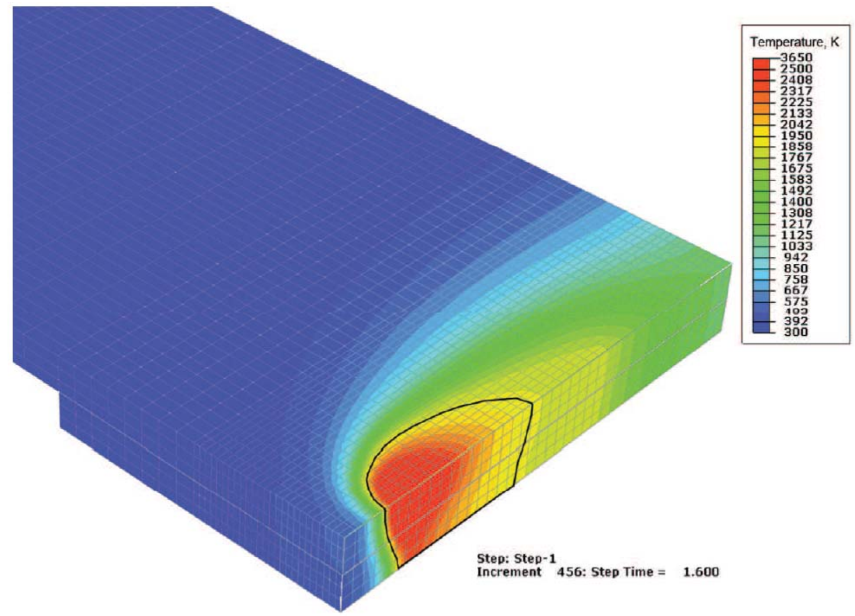


Parameter	Value
Welding speed	$v = 1$ cm/s
Laser beam power	$Q_L = 2.2$ kW
Beam radius	$r_0 = 1$ mm
Penetration depth	$s = 4$ mm

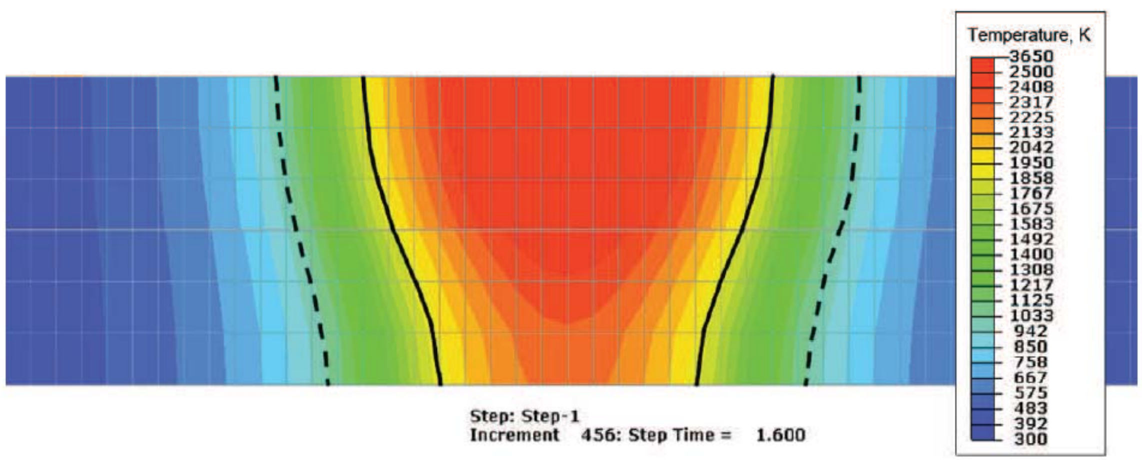
Results



Temperature, K
3650
2500
2408
2317
2225
2133
2042
1950
1858
1767
1675
1583
1492
1400
1308
1217
1125
1033
942
850
758
667
575
483
392
300
Step: Step-1
Increment 456: Step Time = 1.600



Temperature, K
3650
2500
2408
2317
2225
2133
2042
1950
1858
1767
1675
1583
1492
1400
1308
1217
1125
1033
942
850
758
667
575
392
300
Step: Step-1
Increment 456: Step Time = 1.600



Temperature, K
3650
2500
2408
2317
2225
2133
2042
1950
1858
1767
1675
1583
1492
1400
1308
1217
1125
1033
942
850
758
667
575
483
392
300
Step: Step-1
Increment 456: Step Time = 1.600

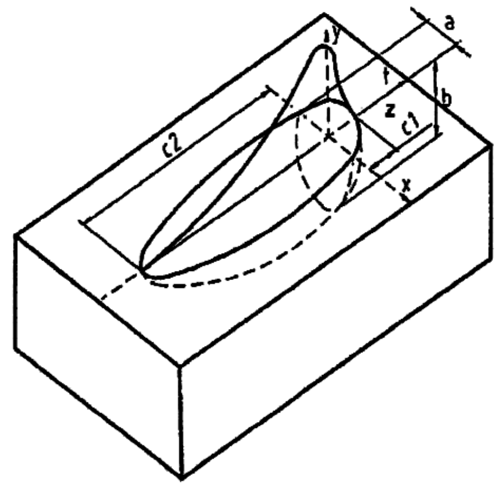
Finite element simulation of laser spot welding

3 D Double Ellipsoid Volumetric Heat Source

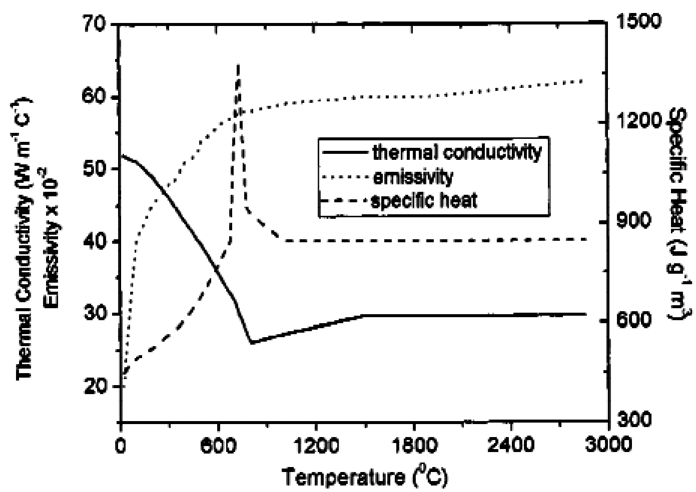
The heat input is explicitly given by

$$q(x,y,z,t)=\frac{6\sqrt{3}f_{1,2}Q}{abc_{1,2}\pi\sqrt{\pi}}\exp\left(-3\frac{x^2}{a^2}\right)\exp\left(-3\frac{y^2}{b^2}\right)\times\exp\left\{-3\frac{[z-v(\tau-t)]^2}{a^2}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

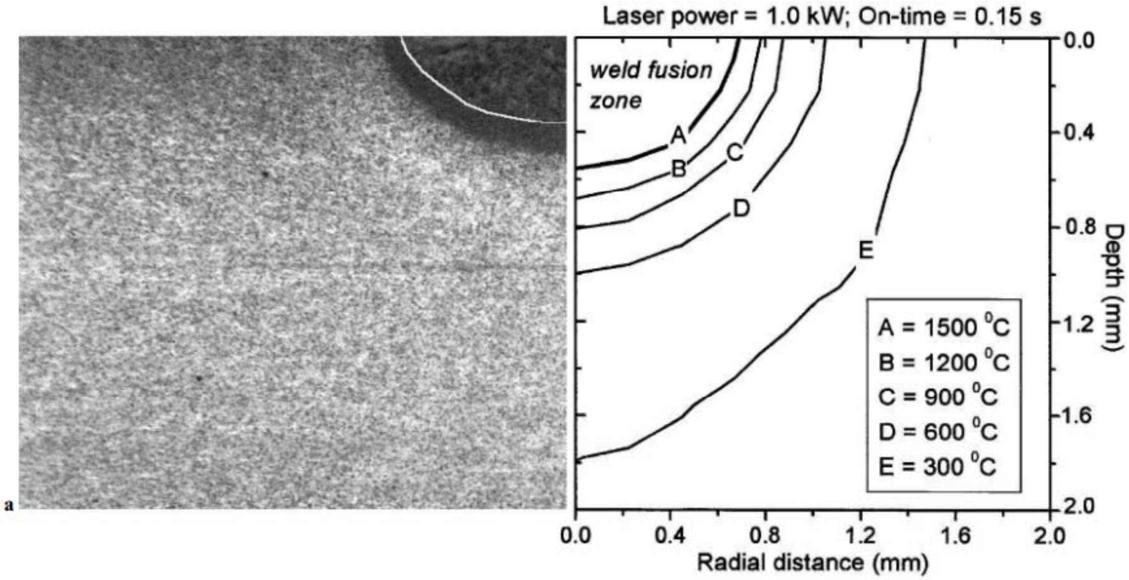
where Q is the source intensity due to the heat source (i.e.



2 Double ellipsoidal representation of heat source

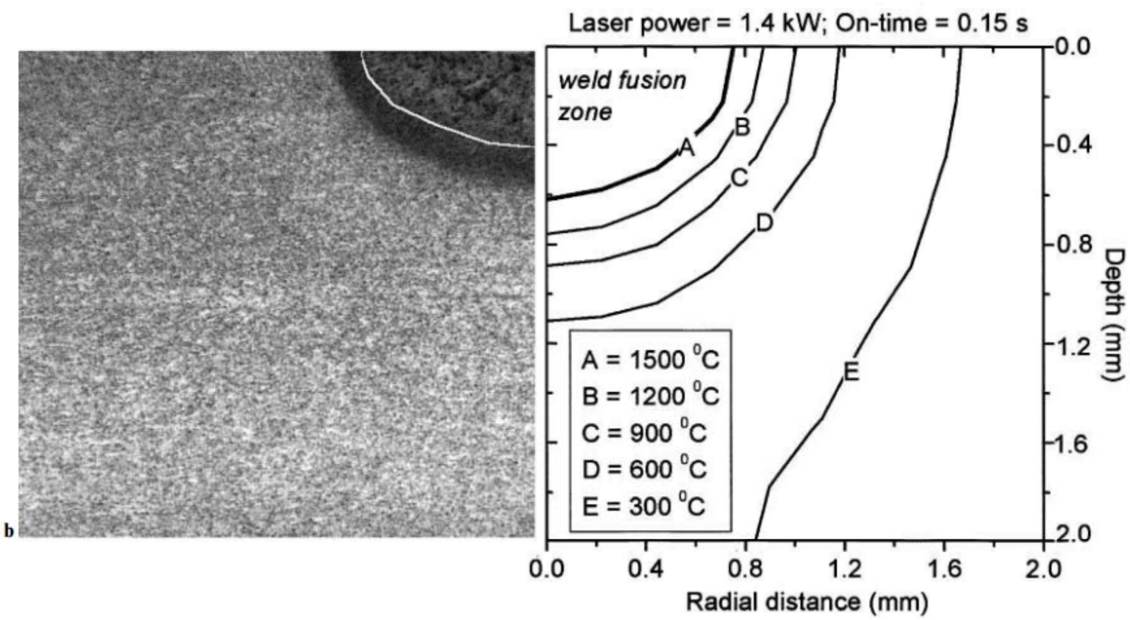


Results:



Laser power = 1.0 kW; On-time = 0.15 s
weld fusion zone
A
B
C
D
E
A = 1500 ^{0}C
B = 1200 ^{0}C
C = 900 ^{0}C
D = 600 ^{0}C
E = 300 ^{0}C
0.0
0.4
0.8
1.2
1.6
2.0
Radial distance (mm)
Depth (mm)

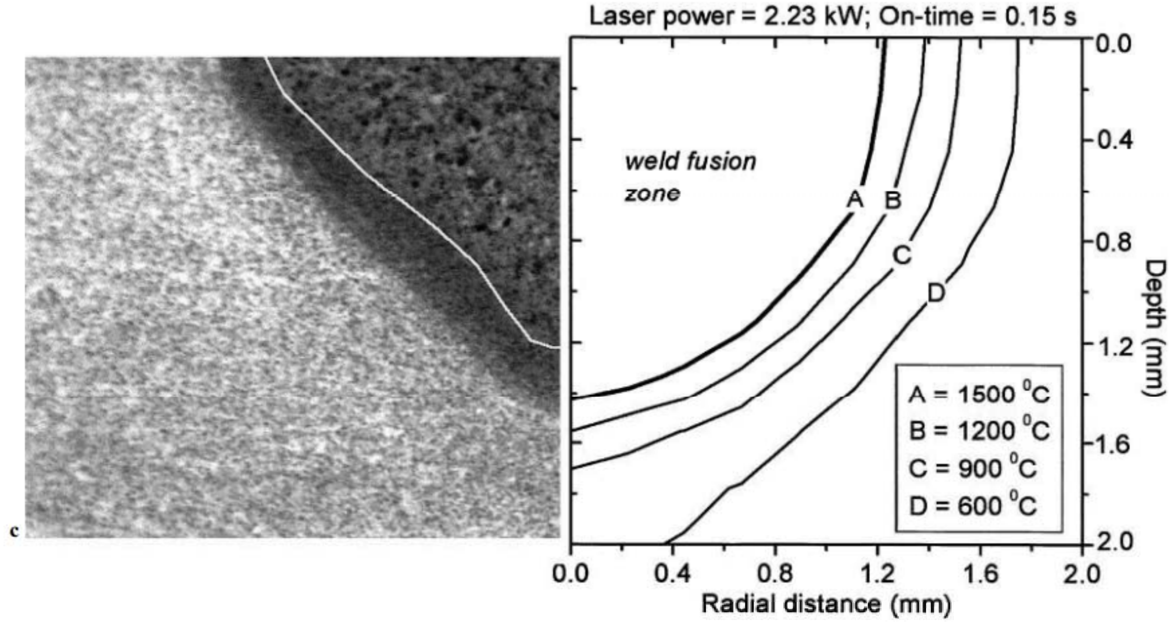
a

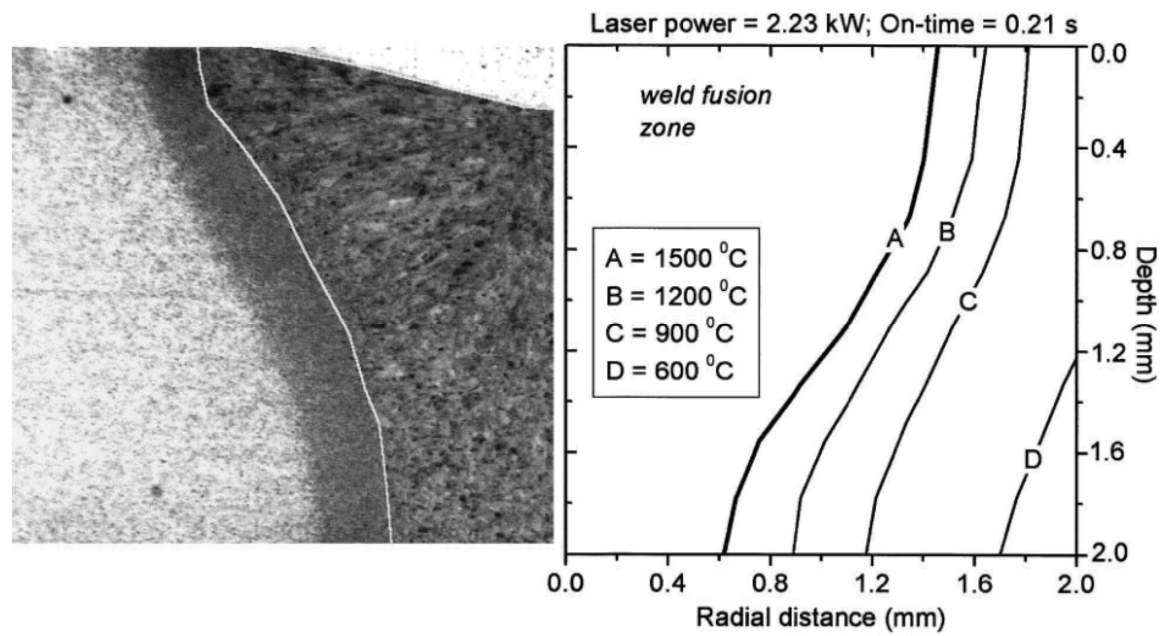


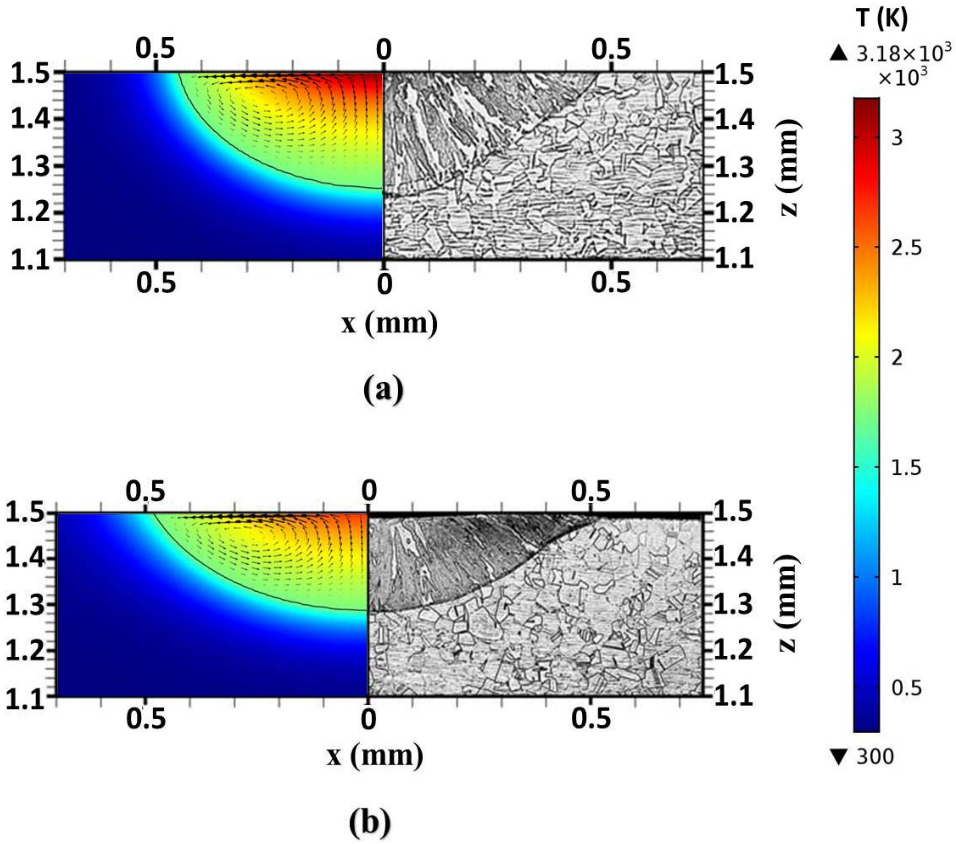
Laser power = 1.4 kW; On-time = 0.15 s
weld fusion zone
A
B
C
D
E
A = 1500 ^{0}C
B = 1200 ^{0}C
C = 900 ^{0}C
D = 600 ^{0}C
E = 300 ^{0}C
0.0
0.4
0.8
1.2
1.6
2.0
Radial distance (mm)
Depth (mm)

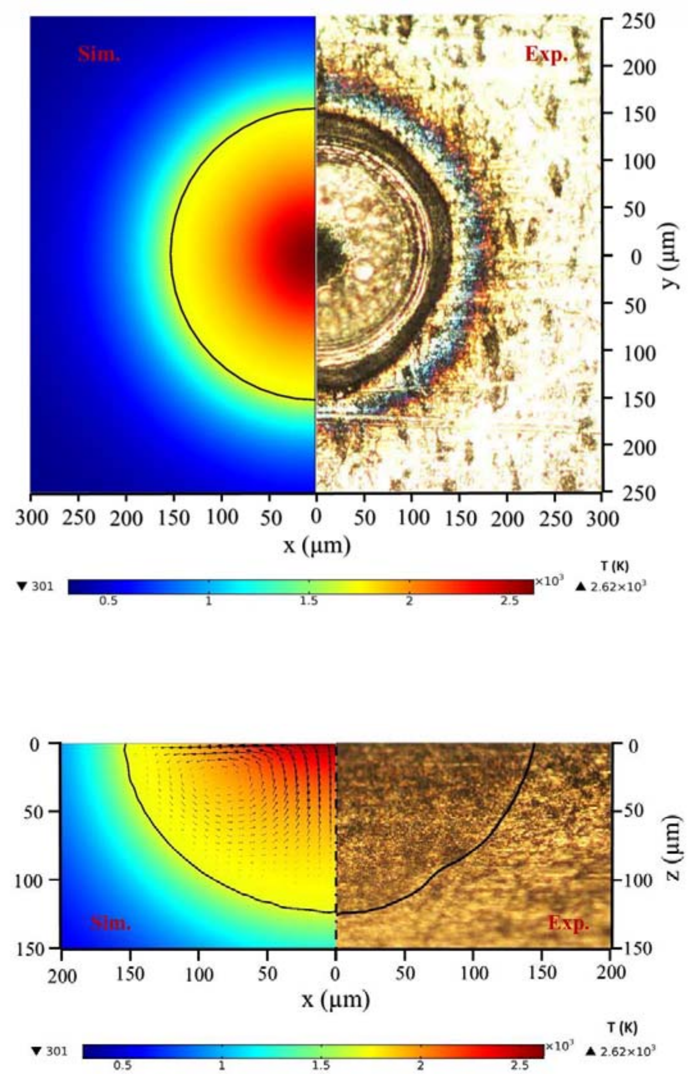
b

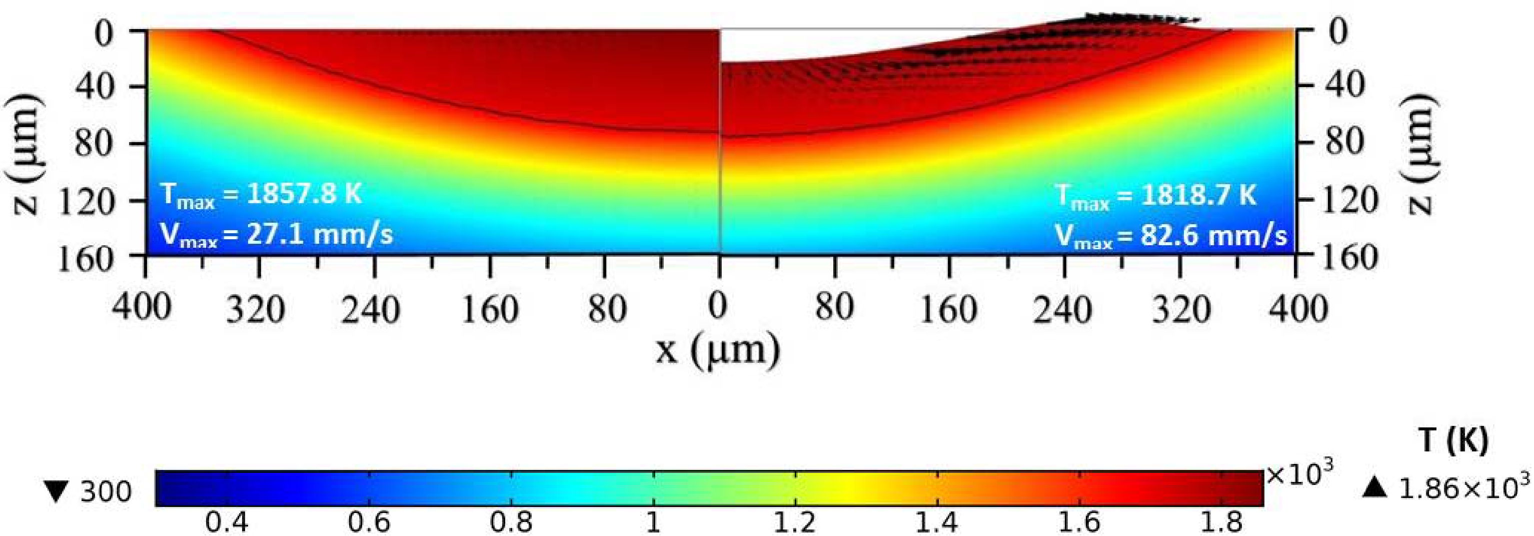
Results:


Laser power = 2.23 kW; On-time = 0.15 s
weld fusion zone
A
B
C
D
A = 1500 °C
B = 1200 °C
C = 900 °C
D = 600 °C
0.0
0.4
0.8
1.2
1.6
2.0
Radial distance (mm)
Depth (mm)
c


Laser power = 2.23 kW; On-time = 0.21 s
weld fusion zone
A = 1500 °C
B = 1200 °C
C = 900 °C
D = 600 °C
A
B
C
D
0.0
0.4
0.8
1.2
1.6
2.0
Radial distance (mm)
Depth (mm)


T (K)
▲ 3.18×10^3
×10^3
3
2.5
2
1.5
1
0.5
▼ 300
x (mm)
z (mm)
(a)
(b)


Sim.
Exp.
x (μm)
y (μm)
z (μm)
T (K)
▼ 301
▲ 2.62×10^3
×10^3



T_{max} = 1857.8 K
V_{max} = 27.1 mm/s
T_{max} = 1818.7 K
V_{max} = 82.6 mm/s
z (μm)
x (μm)
0
40
80
120
160
400
320
240
160
80
0
80
160
240
320
400
T (K)
▼ 300
▲ 1.86×10^3
×10^3
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8