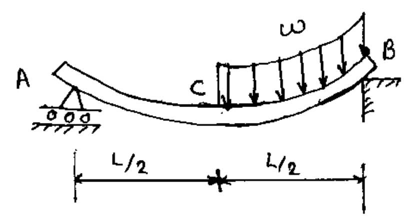
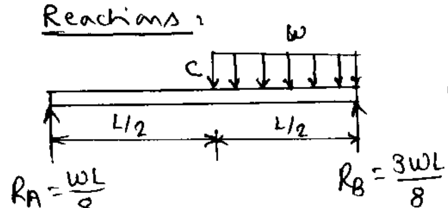
Solution to H/W and Practice Problems for Chapter 8

Solution to problem 8.2



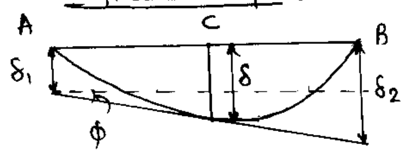
(a)

Reactions:



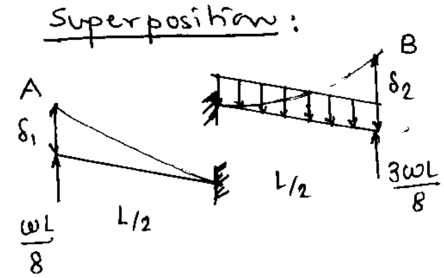
(b)

Deflected shape:



(c)

Superposition:



(d)

- Let δ_1 and δ_2 be the distances of the end-points A and B from the tangent at the midpoint C (Fig. (c)). Then, they can be found as the deflections of the 2 cantilever beams shown in Fig. (d):

$$\delta_1 = \frac{\frac{wL}{8}\left(\frac{L}{2}\right)^3}{3EI} = \frac{wL^4}{192EI}.$$

$$\delta_2 = \frac{\frac{3wL}{8}\left(\frac{L}{2}\right)^3}{3EI} - \frac{w\left(\frac{L}{2}\right)^4}{8EI}$$

$$= \frac{wL^4}{64EI} - \frac{wL^4}{128EI} = \frac{wL^4}{128EI}$$

- Then, the slope of the tangent at C is:

$$\phi = \frac{\delta_2 - \delta_1}{L}.$$

$$= \frac{1}{L}\left[\frac{wL^4}{128EI} - \frac{wL^4}{192EI}\right] = \frac{wL^3}{384EI}$$

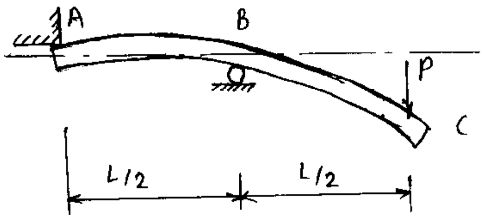
- Finally, the deflection of midpoint C is:

Central deflection: $\delta = \delta_1 + \phi\frac{L}{2}.$

$$= \frac{wL^4}{192EI} + \frac{wL^3}{384EI}\cdot\frac{L}{2}$$

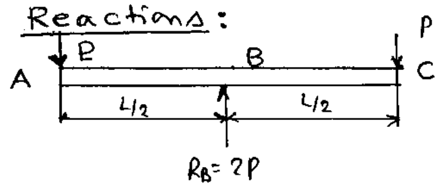
$$= \frac{5wL^4}{768EI}.$$

Solution to problem 8.3



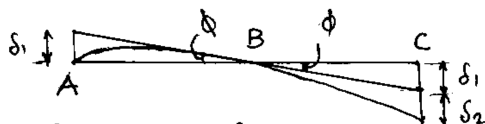
(a)

Reactions:



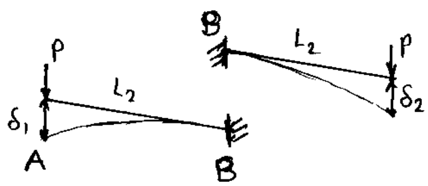
(b)

Deflected shape:



(c)

Superposition:



(d)

- Let δ_1 be the distance of point A from the tangent at point B. Further, let δ_2 be the distance of the deflected position of point C from the tangent at point B. (Fig. (c)). Then, δ_1 and δ_2 can be found as the deflections of the 2 cantilevers shown in Fig. (d).

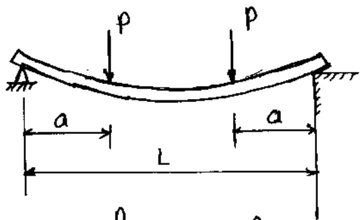
- Actual deflection at C $= \delta_1 + \delta_2$

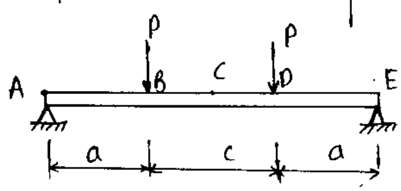
$$= 2\,\delta_2$$

$$= \frac{2\,P\,(L/2)^3}{3EI}$$

$$= \frac{PL^3}{12\,EI}.$$

Solution to problem 8.4



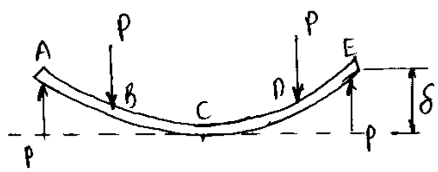


To find: central deflection.

note: • due to symmetry reactions at A and E are P each.

• The slope at C is zero.

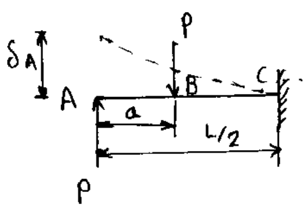
• Therefore, the deflected shape is symmetric about point C.



* consider ~~either~~ the beam ABC.

• Since the slope at C is zero, the point C can be considered fixed.

• Using superposition, deflection at A is:



$$(\delta_A)_{\uparrow P} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI} \quad \underline{\text{upward.}}$$

$$(\delta_A)_{\downarrow P} = \frac{P(L/2-a)^2\left[3\frac{L}{2} - (L/2-a)\right]}{6EI} \quad \underline{\text{downward}}$$

$$= \frac{P(L^3 - 3L^2a + 4a^3)}{24EI}$$

$$\text{Net upward displacement } (\delta_A) = \frac{PL^3}{24EI} - \frac{P(L^3 - 3L^2a + 4a^3)}{24EI}$$

$$= \frac{Pa}{24EI}(3L^2 - 4a^2).$$

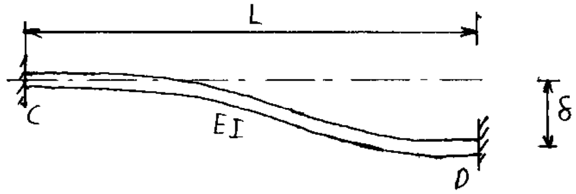
* Note: δ (central deflection of original beam)

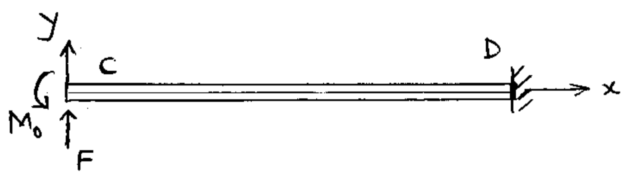
$$= \delta_A$$

$$= \frac{Pa(3L^2-4a^2)}{24EI}$$

Solution to problem 8.8

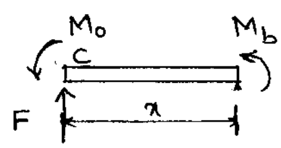
- Statically indeterminate problem
- Since, all the beams deform in the same manner, consider one of the beams, lebelled





- **Reactions:**

Let the reaction at D consist of a force F and moment M_0.



- Bending Moment:

$$EI\frac{d^2v}{dx^2} = M_b = F.x - M_0.$$

- Integrating,

$$EI\frac{dv}{dx} = \frac{Fx^2}{2} - M_0 x + c_1$$

$$EI\,v = \frac{Fx^3}{6} - \frac{M_0 x^2}{2} + c_1 x + c_2.$$

- Boundary conditions: at $x=0$, $v=0$, $\frac{dv}{dx}=0$.

at $x=L$, $v=+\delta$, $\frac{dv}{dx}=0$. (Compatibility Conditions)

These B.C.s give $c_1 = c_2 = 0$. $F = \frac{12EI\delta}{L^3}$, $M_0 = \frac{6EI\delta}{L^2}$.

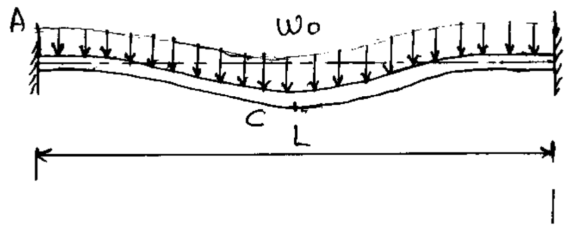
- Max. BM:

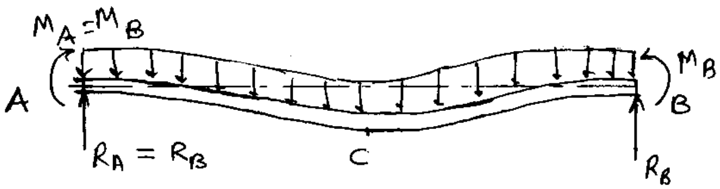
$$M_b = \frac{12EI\delta}{L^3}x - \frac{6EI\delta}{L^2}.$$

$$\therefore |M_b|_{max} \text{ at } x=0, x=L = \frac{6EI\delta}{L^2}.$$

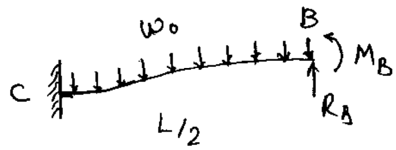
Solution to problem 8.11

- Statically indeterminate problem
- Because of symmetry,
  - * $R_A = R_B$,
    $M_A = M_B$.
  - * Further, slope at C is zero.





- Consider the beam CB. The point C can be considered fixed



* Superposition:

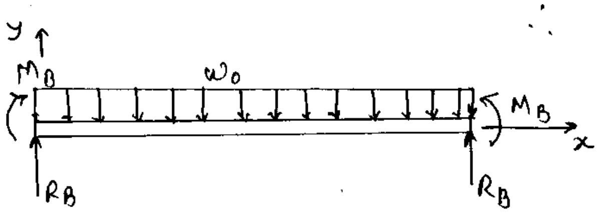
$$\phi_B = \frac{w_0 (L/2)^3}{6EI} - \frac{R_B (L/2)^2}{2EI} - \frac{M_B (L/2)}{EI}$$

$$= \frac{w_0 L^3}{48EI} - \frac{R_B L^2}{8EI} - \frac{M_0 L}{2EI}$$

$$= \frac{L}{48EI}\left[w_0 L^2 - 6 R_B L - 24 M_B\right].$$

* Compatibility: $\phi_B = 0$

$$\therefore \; w_0 L^2 - 6 R_B L - 24 M_B = 0 \qquad \text{— (1)}.$$



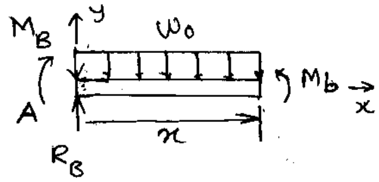
- Equilibrium of the whole beam

$$R_B = \frac{w_0 L}{2} \qquad \text{— (2)}.$$

(1) and (2) $\Longrightarrow$ $w_0 L^2 - 6\left(\frac{w_0 L}{2}\right) L - 24 M_B = 0$.

$$\Longrightarrow \; M_B = -\frac{w_0 L^2}{12}.$$

* To find maximum Bending moment:



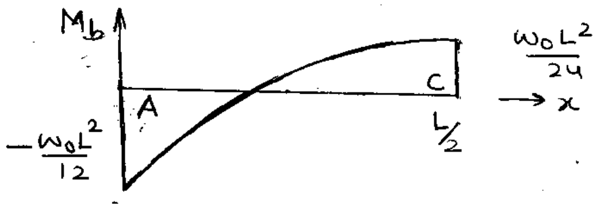
Bending moment at x:

$$M_b = R_B \cdot x + M_B - w_0 \frac{x^2}{2}.$$

$$= \frac{w_0 L x}{2} - \frac{w_0 L^2}{12} - w_0 \frac{x^2}{2}.$$

(problem 8.11 contd.)

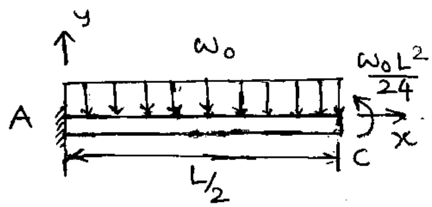
B.M.D.



From the BM Diagram,

$$\left|(M_b)_{max}\right| = \left|M_b\right|_{x=0}\Big| = \frac{w_0 L^2}{12}.$$

* To find maximum deflection.



shear force = 0 at C due to symmetry

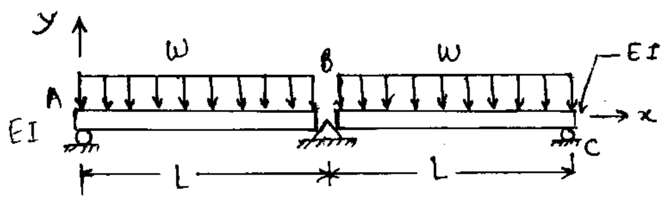
Superposition :

$$\delta_{max} = \delta\big|_C = \delta\big|_C \text{ due to } w_0 + \delta\big|_C \text{ due to } M_0 = \frac{w_0 L^2}{24}$$

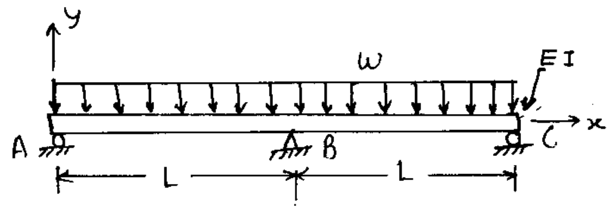
$$= \frac{w_0 (L/2)^4}{8EI} - \frac{(w_0 L^2/24)\, L}{2EI}$$

$$= \frac{wL^4}{384EI} \quad \text{(downward)}$$

Solution to problem 8.16



(a)



(b)

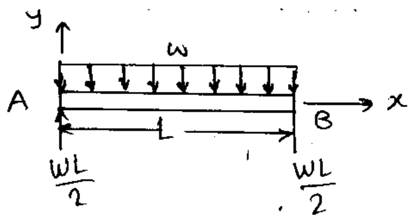
Statically indeterminate problem

To compare: i) Reactions at central support
ii) Max M_b.
iii) Max. deflection

} in the two cases.

a) AB and BC are 2 different beams. Consider beam AB

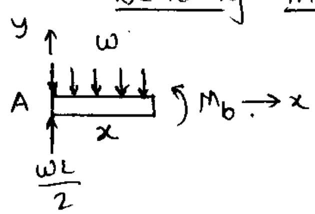
(i) Reactions :



$\therefore$ Total reaction at centre $= 2\frac{wL}{2}$

$= wL$.

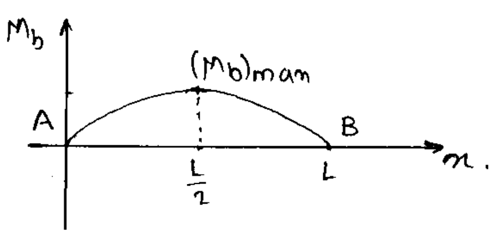
(ii) Bending moment :



$$M_b = \frac{wLx}{2} - \frac{wx^2}{2}.$$

$$\frac{dM_b}{dx} = 0 \Rightarrow \frac{wL}{2} - \frac{1}{2}w\,2x = 0.$$

$$\Rightarrow x = \frac{L}{2}.$$



$$\therefore (M_b)_{max} = M_b\big|_{x=L/2}$$

$$= \frac{wL}{2}\left(\frac{L}{2}\right) - \frac{w}{2}\left(\frac{L}{2}\right)^2$$

$$= \frac{wL^2}{8}.$$

Bending Moment Diagram is same for beam BC.

$$\therefore (M_b)_{max} = M_b\big|_{x=3L/2} = \frac{wL^2}{8} \text{ for BC}$$

(problem 8.16 contd.)

(iii) Deflection:

$$EI\frac{d^2v}{dx} = M_b = \frac{wL}{2}x - \frac{1}{2}wx^2$$

$$EI\frac{dv}{dx} = \frac{wL}{2}\frac{x^2}{2} - \frac{1}{2}w\frac{x^3}{3} + C_1$$

$$EI\,v = \frac{wL}{4}\frac{x^3}{3} - \frac{1}{6}wx^4/4 + C_1x + C_2.$$

$v=0$ at $x=0 \Rightarrow C_2 = 0.$

$v=0$ at $x=L \Rightarrow \frac{wL^4}{12} - \frac{1}{6}\frac{wL^4}{4} + C_1L = 0.$

$$\Rightarrow C_1 = -\frac{wL^3}{24}.$$

$$\therefore v = \frac{w}{EI}\left[\frac{Lx^3}{12} - \frac{x^4}{24} - \frac{L^3x}{24}\right].$$

$$= \frac{w}{24EI}\left[2Lx^3 - x^4 - L^3x\right].$$

$$\frac{dv}{dx} = 0 \Rightarrow \frac{w}{24EI}\left[6Lx^2 - 4x^3 - L^3\right] = 0.$$

$$\Rightarrow x = 1.366L, -0.366L, 0.5L$$

In the above solution $x = 0.5$ is only ~~cor~~ feasible solution.

$$\therefore \underline{v_{max}} = v|_{x=\frac{L}{2}} = \frac{w}{24EI}\left[2L\left(\frac{L^3}{8}\right) - \frac{L^4}{16} - wL^3\left(\frac{L}{2}\right)\right]$$

$$= \frac{wL^4}{24EI}\left[\frac{1}{4} - \frac{1}{16} - \frac{1}{2}\right].$$

$$= -\frac{5wL^4}{384EI}.$$

Deflection expression is same for beam BC.

$$\therefore \underline{v_{max}} = v|_{x=\frac{3L}{2}} = -\frac{5wL^4}{384EI} \quad \text{for BC}$$

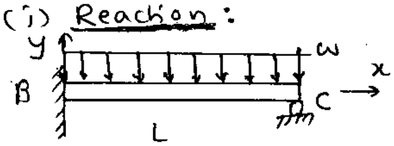
(problem 8.16 contd.)

(b) Continuous beam.

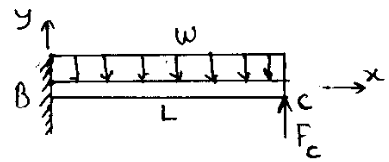
Slope and deflection at B ~~is~~ are zero.

Consider ~~either~~ the part ~~AB or~~ BC. It can be considered as a cantilever beam.

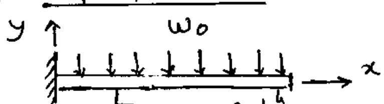
(i) Reaction:



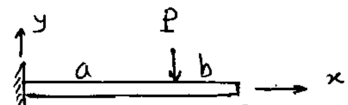
Replace the support at C by a force ⇒



* Superposition:



$$\delta_{max} = \frac{w_o L^4}{8EI}$$



$$\delta_{max} = \frac{Pa^2(3L-a)}{6EI}.$$

i) deflection due to w:

$w_o \rightarrow w.$

$$\delta_w = \frac{wL^4}{8EI} \quad \text{(downward)}$$

ii) deflection due to F_c

$a \rightarrow L, \; P \rightarrow -F_c$

$$\delta_F = \frac{FL^3}{3EI} \quad \text{(upward).}$$

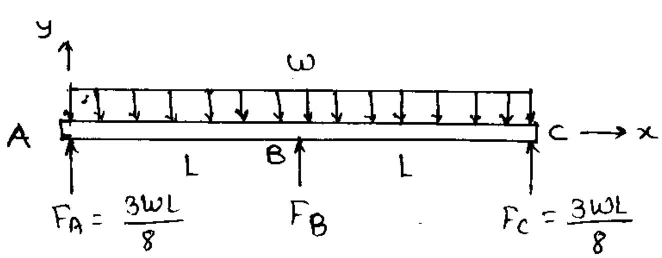
iii) Net downward deflection:

$$\delta_c = \frac{wL^4}{8EI} - \frac{F_c L^3}{3EI}.$$

* compatibility:

$$\delta_c = 0 \Rightarrow \frac{wL}{8} - \frac{F_c}{3} = 0 \Rightarrow F_c = \frac{3}{8} wL.$$

Reaction at the centre. Consider the whole beam



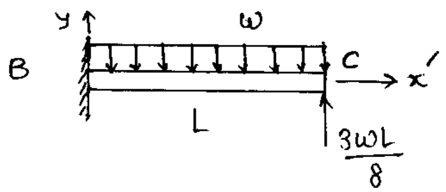
By symmetry, $F_A = \frac{3}{8} wL$

$$\therefore F_B = 2wL - 2\left(\frac{3}{8} wL\right)$$

$$= \frac{5wL}{4}.$$

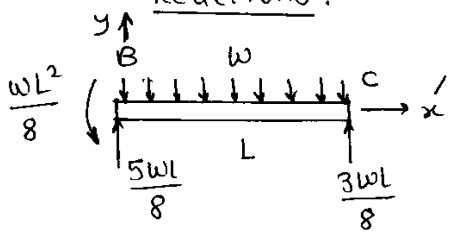
(problem 8.16 contd.)

(ii) Bending Moment: Again consider part BC

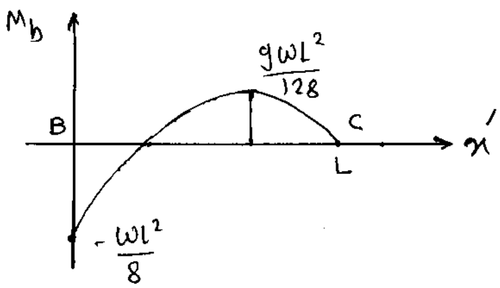


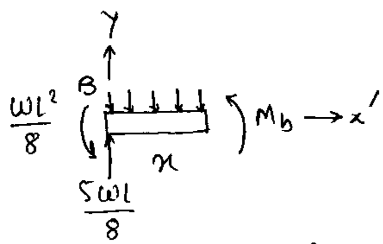
(x is measured from point A)

* Reactions:



* Bending Moment:





$$M_b = -\frac{wL^2}{8} + \frac{5wL}{8}x' - \frac{1}{2}wx'^2.$$

$$\frac{dM_b}{dx'} = 0 \Rightarrow x' = \frac{5L}{8}.$$

$$M_b\Big|_{x'=\frac{5L}{8}} = -\frac{wL^2}{8} + \frac{5wL}{8}\left(\frac{5L}{8}\right) - \frac{1}{2}w\left(\frac{5L}{8}\right)^2$$

$$= \frac{9}{128}wL^2.$$

But this value is less than $\left|M_b|_{x'=0}\right| = \frac{wL^2}{8}$.

$$\therefore \underline{|(M_b)_{max}|} = \frac{wL^2}{8}.$$

This occurs at point B

(problem 8.16 contd.)

(iii) Deflection:

$$EI\frac{d^2v}{dx'^2} = -\frac{wL^2}{8} + \frac{5wL}{8}x' - \frac{1}{2}wx'^2.$$

$$\therefore EI\frac{dv}{dx'} = -\frac{wL^2}{8}x' + \frac{5wL}{8}\frac{x'^2}{2} - \frac{1}{2}w\frac{x'^3}{3} + C_1$$

$$EI\,v = -\frac{wL^2}{8}\frac{x'^2}{2} + \frac{5wL}{16}\frac{x'^3}{3} - \frac{1}{6}w\frac{x'^4}{4} + C_1x' + C_2.$$

$\frac{dv}{dx'} = 0$ at $x' = 0$ $\Rightarrow$ $C_1 = 0$.

$v = 0$ at $x' = 0$ $\Rightarrow$ $C_2 = 0$.

$$\therefore v = \frac{w}{48EI}\left[-3L^2x'^2 + 5Lx'^3 - 2x'^4\right].$$

$$\frac{dv}{dx'} = 0 \Rightarrow \frac{wx}{48}\left[-6L^2 + 15Lx' - 8x'^2\right] = 0.$$

$$\Rightarrow x' = 0, \quad 8x'^2 - 15Lx' + 6L^2 = 0.$$

$$\Rightarrow x' = 0,\ 0.58L,\ 1.3L.$$

$x' = 0.58L$ is only possible solution.

$$\underline{v_{max}} = v|_{x'=0.58L} = \frac{w}{48EI}\left[-3L^2(0.58L)^2 + 5L(0.58L)^3 - 2(0.58L)^4\right]$$

$$= \frac{wL^4}{48EI}\left[-1.0092 + 0.976 - 0.226\right].$$

$$= -\frac{0.259\,wL^4}{48EI}.$$

$$= -\frac{2.07\,wL^4}{384EI}.$$

(This is at distance, 1.58L from A since x' is measured from B)

In part AB, v_{max} occurs at a distance 0.58L from B

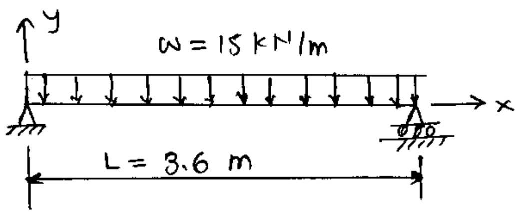
$\therefore$ $\underline{v_{max}} = -\frac{2.07\,wL^4}{384EI}$ at 0.42L from A for $\underline{AB}$

(Problem 8.16 contd.)

Comparison :

	Case (a)	Case (b)
i) Central Reaction →	wL	$\frac{5wL}{4}$
ii) Max M_b →	$\frac{wL^2}{8}$	$\frac{wL^2}{8}$
location	$\frac{L}{2}, \frac{3L}{2}$	at B (L)
iii) Max θ (downward) →	$\frac{5wL^4}{384EI}$	$\frac{2.07wL^4}{384EI}$
location	$\frac{L}{2}, \frac{3L}{2}$	$0.42L, 1.58L$.

Solution to problem 8.28 :



Given :

- $E = 7\ GN/m^2$,
- $\dfrac{\delta_{max}}{L} = \dfrac{1}{360}$.

To find : I_{min}

From the results of problem 8.16 (a),

$$\delta_{max} = \frac{5wL^4}{384\,EI} \Rightarrow \frac{\delta_{max}}{L} = \frac{5wL^3}{384\,EI}$$

$$\frac{5wL^3}{384\,EI} = \frac{1}{360}.$$

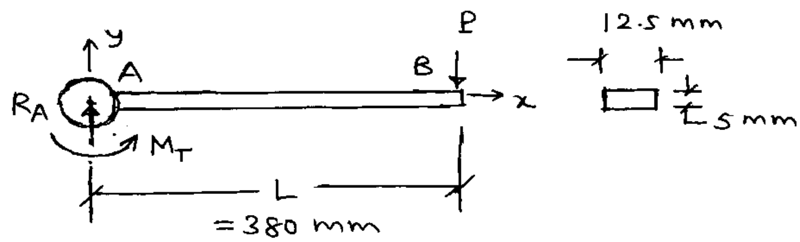
$$\Rightarrow \frac{5 \times 15\times10^3 \times 3.6^3}{384 \times 7\times10^9\ I} = \frac{1}{360}.$$

$$\Rightarrow I = \frac{360 \times 5 \times 15\times10^3 \times 3.6^3}{384 \times 7 \times 10^9}$$

$$= 4.686 \times 10^{-4}\ m^4.$$

This is the minimum value of I (i.e. I_{min})

Solution to problem 8.31



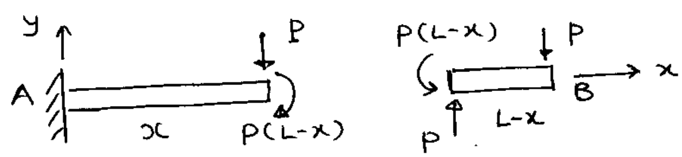
Given:
- E (steel) = 210 GN/m^2
- scale at $x = 300$ mm

- Equilibrium $\Rightarrow$ $R_A = P$

  $M_T = PL$ or $P = \frac{M_T}{L}$

- $I = \frac{1}{12}(12.5)(5)^3 = 130.2 \text{ mm}^4$

- Deflection at x:



* Superposition:

$$\delta_x = \delta_x \text{ due to } P + \delta_x \text{ due to } P(x-L)$$

$$= \frac{Px^3}{3EI} + \frac{[P(L-x)]\,x^2}{2EI}$$

$$= \frac{Px^2}{6EI}[2x + 3(L-x)]$$

$$= \frac{Px^2(3L-x)}{6EI}$$

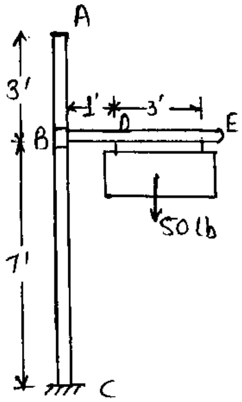
$$= \frac{M_T\, x^2(3L-x)}{6LEI} \quad \left(\text{Since } P = \frac{M_T}{L}\right)$$

* At the scale,

$$\delta\Big|_{x=300\text{ mm}} = \frac{M_T(300)^2[3\times380 - 300]}{6(380)\times210\times10^3\times130.2}$$

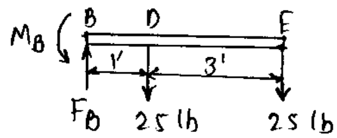
$$= 1.21\times10^{-3}\, M_T$$

Solution to problem 8.32



- Equilibrium :

Free body diagram of BE

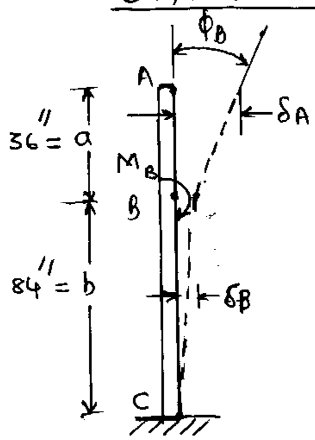


Reactions at B :

* $F_B = 50$ lb
* $M_B = 1500$ lb.in

Note that the effect of the axial force F_B ~~F_B contribute~~ on the bending moment (in the signpost) is negligible.

- Deflected shape of ABC :



* Part AB bends as a straight line since $M_b = 0$ in AB
* Method of superposition.

$$\delta_A = \delta_B + AB \times \phi_B .$$
$$= \delta_B + a\,\phi_B .$$

$$\delta_B = \frac{M_B b^2}{2EI} \quad , \quad \phi_B = \frac{M_B b}{EI}$$

$$\therefore \ \delta_A = \frac{M_B\, b}{EI}\left(b/2 + a\right)$$

- $E = 30\times10^6$ psi (steel)
- Outer diameter of the c/s $= 4''$
  inner diameter of the c/s $= 3.5''$

$\therefore \ I = \frac{\pi}{64}(4^4 - 3.5^4)$

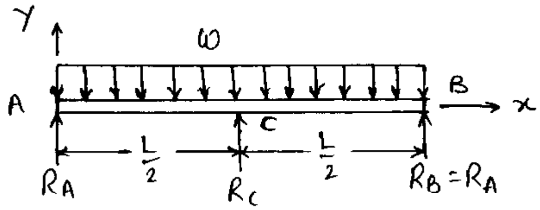
$= 5.2 \text{ in}^4$

* Numerical value :

$$\delta_A = \frac{1500 \times 84}{30\times10^6 \times 5.2}\left(\frac{84}{2} + 36\right)$$

$$= 0.063''$$

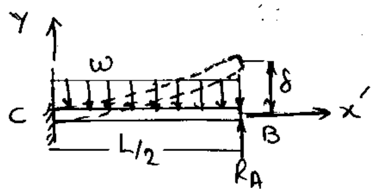
Solution to problem 8.33



(a) FBD of the whole beam

- Statically indeterminate problem
- Symmetry $\Rightarrow$
  - * Reactions $R_B = R_A$
  - * slope at $C = 0$
- consider part CB as a cantilever beam (Fig (b))

(b)



$\delta = \delta$ due to ω + δ due to R_A.

$$= -\frac{\omega (L/2)^4}{8\,EI} + \frac{R_A (L/2)^3}{3\,EI} \quad \text{— ①.}$$

This is the deflection of the original beam at C

- Compatibility for the Original Beam :

$$R_C = \text{buoyancy force} = \gamma A \delta \quad \text{— ②.}$$

① & ② $\Longrightarrow$

$$\frac{R_C}{\gamma A} = \frac{R_A L^3}{24\,EI} - \frac{\omega L^4}{128\,EI}.$$

$$\therefore \frac{48\,EI\,R_C}{\gamma A L^3} - 2R_A = -\frac{3\omega L}{8} \quad \text{— ③.}$$

- Equilibrium of the Original Beam (Fig (a))

$$R_C + 2R_A = \omega L \quad \text{— ④.}$$

- Solution

③ & ④ $\Rightarrow \left(1 + \frac{48\,EI}{\gamma A L^3}\right) R_C = \frac{5\omega L}{8}$

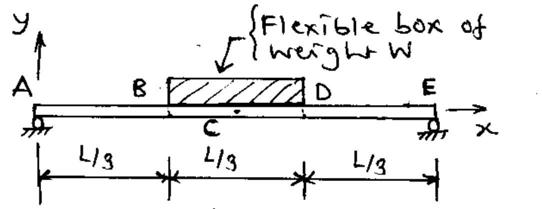
$$\therefore R_C = \frac{5\omega L}{8} \cdot \frac{1}{\left(1 + \frac{48\,EI}{\gamma A L^3}\right)} \quad \text{— ⑤.}$$

② and ⑤ $\Rightarrow$

$$\delta = \frac{R_C}{\gamma A} = \frac{5\omega L}{8} \frac{1}{\gamma A \left(1 + \frac{48\,EI}{\gamma A L^3}\right)}$$

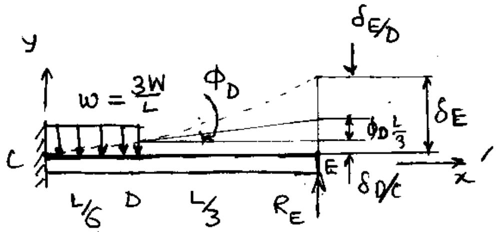
$$= \frac{5\omega L^4}{384\,EI \left(1 + \frac{\gamma A L^3}{48 EI}\right)}.$$

Solution to problem 8.35



(a)

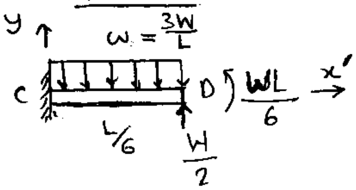
- Statically indeterminate problem.
- Assume: The force exerted by the block is UDL with intensity $w = \frac{W}{L/3}$
- Symmetry $\Rightarrow$
  - * $R_A = R_E = W/2$
  - * slope at point C is zero.
- Consider CDE as a cantilever beam fixed at C. (Fig. (b))
  - * δ_E (Deflection at E) can be found by superposition by considering the parts CD and DE as cantilever beams (Figures (c) & (d))



(b)

* Superposition $\Rightarrow$ $\delta_E = \delta_{D/C} + \phi_D \frac{L}{3} + \delta_{E/D}$ —— (1)

Part CD



(c)

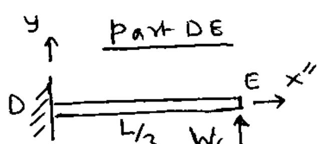
$$\delta_{D/C} = -\frac{w\left(\frac{L}{6}\right)^4}{8EI} + \frac{W}{2}\frac{(L/6)^3}{3EI} + \frac{WL}{6}\frac{(L/6)^2}{2EI}$$

$$= \frac{29\,WL^3}{10368\,EI} \quad \text{—— (2)}$$

$$\phi_D = -\frac{w\,(L/6)^3}{6EI} + \frac{W}{2}\frac{(L/6)^2}{2EI} + \frac{WL}{6}\frac{(L/6)}{EI}$$

$$= \frac{42\,WL^2}{1296\,EI} \quad \text{—— (3)}$$

Part DE



(d)

$$\delta_{E/D} = \frac{\frac{W}{2}\left(\frac{L}{3}\right)^3}{3EI} = \frac{WL^3}{162\,EI} \quad \text{—— (4)}$$

$$(1)\text{–}(4) \Rightarrow \delta_E = \frac{WL^3}{EI}\left(\frac{1}{162} + \frac{29}{10368} + \frac{14}{1296}\right) = \frac{205\,WL^3}{10368\,EI}.$$

- This is same as δ_C (deflection of the midpoint) of the original beam

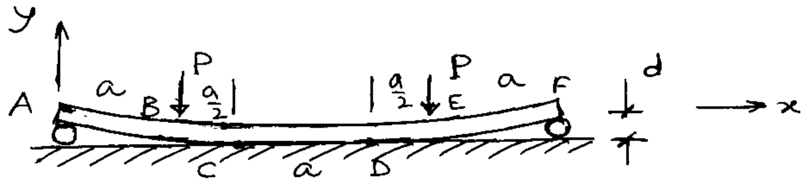
$$(\delta_c)_{exact} = \frac{205\,WL^3}{10368\,EI} \quad \text{—— (a)}$$

- Deflection δ_c by replacing box by a concentrated force W at centre:

$$(\delta_c)_{approx} = \frac{WL^3}{48EI} \quad \text{—— (b)}$$

$$(a)\ \&\ (b) \Rightarrow \frac{(\delta_c)_{approx}}{(\delta_c)_{exact}} = \frac{10368}{48 \times 205} = 1.05$$

Solution to Problem 8-36



• Determination of the reactions for the part CD:

In part CD, $\frac{1}{\rho} = 0 \Rightarrow M_b = 0$ for $\frac{3a}{2} < x < \frac{5a}{2}$

$(M_b = EI/\rho)$

$\Rightarrow \frac{dM_b}{dx} = 0$

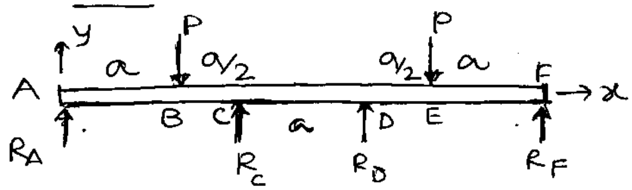
$(V = -\frac{dM_b}{dx}) \quad \Rightarrow V = 0$ for $\frac{3a}{2} < x < \frac{5a}{2}$

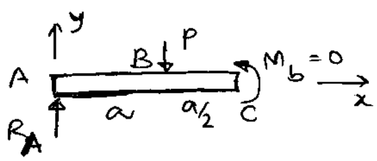
$\Rightarrow \frac{dV}{dx} = 0$

$(q = -\frac{dV}{dx}) \quad \Rightarrow q = 0$

$\therefore$ No distributed force between C and D. Only the point forces at C and D

• FBD:





* Symmetry $\Rightarrow$

• $R_A = R_F, \quad R_C = R_D$ —— (1)

* Equilibrium:

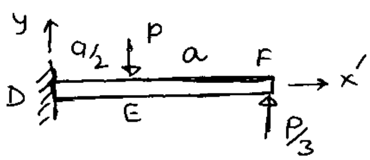
$\Sigma F_y = 0 \Rightarrow 2R_A + 2R_C = P$ — (2)

* Bending Moment at C = 0

$\Sigma M_c = 0 \Rightarrow \frac{3a}{2} R_A - \frac{a}{2} P = 0$ — (3)

$(1)-(3) \Rightarrow R_A = R_F = P/3, \quad R_C = R_D = 2P/3$

• Since the deflection and slope at D are zero, consider the part DEF as a cantilever beam fixed at D



By superposition,

$\delta_F = \delta_F$ due to $P + \delta_F$ due to $P/3$

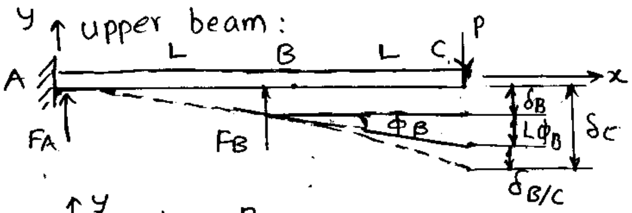
$$= -\frac{P(a/2)^2[3(\frac{3a}{2}) - a/2]}{6EI} + \frac{P/3\,(3a/2)^3}{3EI}$$

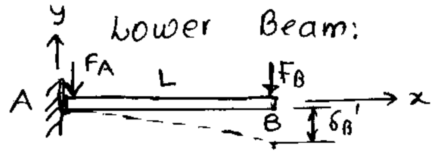
$$= \frac{5Pa^3}{24EI}$$

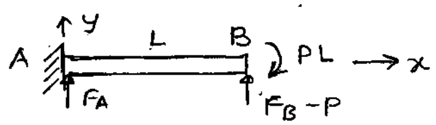
• $\delta_F = d \Rightarrow \frac{5Pa^3}{24EI} = d \Rightarrow P = \frac{24EId}{5a^3}$

Solution to problem 8.37

- Since the beams touch each other only at the points A ($x=0$) and B ($x=L$), the interactions at A and B and the deflection curves are as follows.







*** Deflection:**

* upper beam:

$$\left.\begin{aligned}\delta_B &= -\frac{(F_B - P)L^3}{3EI} + \frac{PL \cdot L^2}{2EI} \\ &= \frac{L^3}{6EI}(5P - 2F_B)\end{aligned}\right\} \text{(downward)}$$

* Lower beam:

$$\delta_B' = \frac{F_B L^3}{3EI} \text{ (downward)}$$

* Geometric compatibility: $\delta_B = \delta_B'$

$$\therefore \frac{L^3}{6EI}(5P - 2F_B) = \frac{F_B L^3}{3EI}.$$

$$\Rightarrow F_B = \frac{5}{4}P.$$

* Deflection at C for upper beam: $\delta_C = \delta_B + L\phi_B + \delta_{B/C}$

$$\therefore \delta_C = \frac{L^3}{6EI}\left(5P - 2\cdot\frac{5}{4}P\right) + L\cdot\left[\frac{PL\cdot L}{EI} - \frac{(\frac{5P}{4} - P)L^2}{2EI}\right] + \frac{PL^3}{3EI}$$

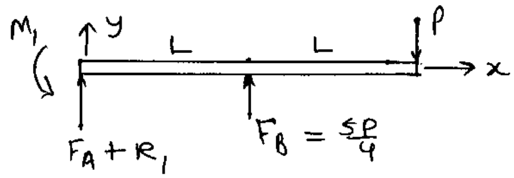
$$\text{or } \delta_C = \frac{13PL^3}{8EI}.$$

- Deflection curves:

* F.B.D.s and Reactions

(problem 8.37 contd.)

Upper beam

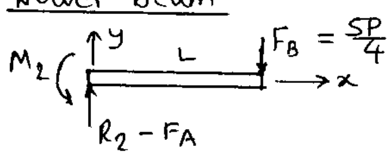


Equilibrium $\Rightarrow$

$$F_A + R_1 = -P/4$$

$$M_1 = \frac{3PL}{4}$$

Lower beam



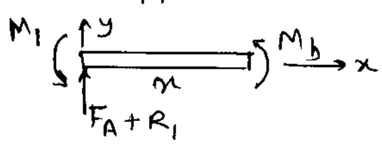
Equilibrium $\Rightarrow$

$$R_2 - F_A = \frac{5P}{4}$$

$$M_2 = \frac{5PL}{4}$$

* Bending Moments & Boundary Conditions

Upper Beam



$$M_b = (F_A + R_1)x - M_1$$

$$= -P/4\,(x + 3L)$$

$$\therefore\ EI \frac{d^2 v_1}{dx^2} = -\frac{P}{4}(x + 3L).$$

$$EI \frac{dv_1}{dx} = -\frac{P}{4}\left(\frac{x^2}{2} + 3Lx\right) + C_1$$

$$EI\, v_1 = -\frac{P}{4}\left(\frac{x^3}{6} + \frac{3}{2}Lx^2\right) + C_1 x + C_2.$$

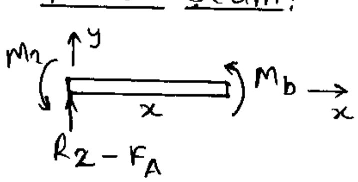
Boundary conditions

at $x = 0$ $\quad v_1 = 0 \Rightarrow C_2 = 0$

$\frac{dv_1}{dx} = 0 \Rightarrow C_1 = 0.$

$$\therefore\ v_1 = -\frac{P}{4EI}\left(\frac{x^3}{6} + \frac{3}{2}Lx^2\right) \quad \text{——— (1)}$$

Lower beam:



$$M_b = (R_2 - F_A)x - M_2$$

$$= \frac{5P}{4}(x - L)$$

(problem 8.37 contd.)

$$\therefore EI \frac{d^2 v_2}{dx^2} = \frac{5P}{4}(x - L)$$

$$\therefore EI \frac{dv_2}{dx} = \frac{5P}{4}\left(\frac{x^2}{2} - Lx\right) + C_1$$

$$EI\, v_2 = \frac{5P}{4}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) + C_1 x + C_2 .$$

Boundary conditions:

at $x = 0$: $v_2 = 0 \Rightarrow C_2 = 0$

$\frac{dv_2}{dx} = 0 \Rightarrow C_1 = 0.$

$$\therefore v_2 = \frac{5P}{4EI}\left(\frac{x^3}{6} - \frac{Lx^2}{2}\right) \quad \text{—— (2)}$$

* Comparison

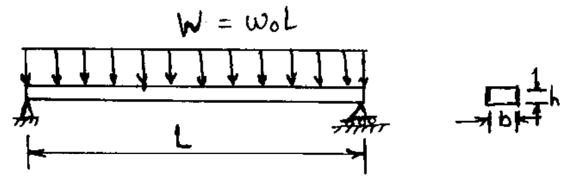
(1) & (2) $\Rightarrow v_1 - v_2 = \frac{P}{4EI}\left(Lx^2 - x^3\right) > 0$ for $0 < x < L$ (v_1 is less negative)

$= 0$ for $x = L$.

This means the beams touch only at B.

Hence the assumption is correct.

Solution to problem 8.68



Given:

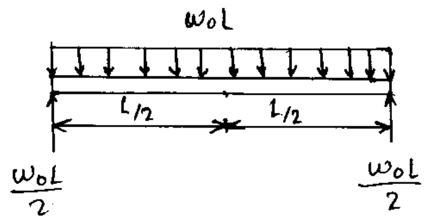
- beam with minimum beam weight W_b to be designed.
- $W_b << W$
- L and h fixed.; material (i.e. σ_{max}, E, and weight density γ) to be varied.

To show: a) strength based design: $\left(\frac{W}{W_b}\right)_{max} \Rightarrow \left(\frac{\sigma_{max}}{\gamma}\right)_{max}$.

b) rigidity based design: $\left(\frac{W}{W_b}\right)_{max} \Rightarrow \left(\frac{E}{\gamma}\right)_{max}$.

(δ_{max} is specified & reaches before the stress limit)

- FBD & Reactions



- a) $(M_b)_{max} = \frac{w_0 L}{2}\left(\frac{L}{2}\right) - \left(\frac{w_0 L}{2}\right)\frac{L}{4} = \frac{w_0 L^2}{8}$,

(W_b neglected while calculating σ_{max})

$$\sigma_{max} = \frac{(M_b)_{max}\, h/2}{\frac{bh^3}{12}} = \frac{\frac{w_0 L^2}{8}\cdot\frac{h}{2}}{bh^3/12} = \frac{3}{4}\frac{WL}{bh^2}.$$

$$\therefore W = \frac{4bh^2}{3L}\sigma_{max}.$$

$$W_b = (Lbh)\gamma$$

$$\therefore \frac{W}{W_b} = \frac{4bh^2}{3L}\cdot\frac{1}{Lbh}\cdot\frac{\sigma_{max}}{\gamma}$$

$$= \left(\frac{4h}{3L^2}\right)\frac{\sigma_{max}}{\gamma} \qquad \left(\frac{4h}{3L^2} \text{ is fixed}\right)$$

$\therefore$ max $\left(\frac{W}{W_b}\right)$ is achieved by choosing the material with largest $\frac{\sigma_{max}}{\gamma}$ ratio.

(problem 8.68 contd.)

b) $$\delta_{max} = \frac{5 w_o L^4}{384 EI} = \frac{5 W L^3}{384 E \left(\frac{bh^3}{12}\right)} = \frac{5 W L^3}{32 E b h^3}.$$ (Wb neglected while calculating δ_{max})

$$\therefore W = \frac{32 E b h^3}{5 L^3} \delta_{max}.$$

$$W_b = (Lbh)\gamma$$

$$\therefore \frac{W}{W_b} = \frac{32 E b h^3 \delta_{max}}{5 L^3} \cdot \frac{1}{L b h \gamma}.$$

$$= \left(\frac{32}{5} \frac{\delta_{max} h^2}{L^4}\right) \frac{E}{\gamma}.$$

note that δ_{max}, h, L are fixed.

$\therefore$ max $\frac{W}{W_b}$ is achieved by choosing the material with the largest $\frac{E}{\gamma}$ ratio.