Chapter 2 - part 1
Fundamental concepts and techniques

Time value of money

means: €1 now worth more than €1 later

is expressed in *risk free interest rate*

price for postponing/advancing consumption

does not include risk premium

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Two reasons why money now has higher value than money later:

Time preference or 'human impatience'

- people prefer present to future consumption
- more than just impatience (food, house)

Productive investment opportunities

- increase consumption later by giving up consumption now

Consequence of time value of money:

Amounts on different points in time cannot be directly compared

- cannot say that €100 now is worth less (or more) than €108 next year

amounts have to be moved through time to same point, adjusting for time value, called:

- *compounding* if moved forward in time
- *discounting* if moved backward in time

Interest is *compounded* when it

- is added to principal sum
- starts earning interest (interest on interest)

Simple example: yearly interest rate 10%, compounded yearly

- deposit €100 in a bank
- after 1 year, 10% is added to your account $\Rightarrow$ €110
- second year, interest over €110 is €11 $\Rightarrow$ €121, etc.

Formula for future value, FV, after T years is

$$FV_T = PV(1+r)^T$$

PV is present value, r is interest rate.

Same principle applies to *discounting*, moving money back in time

Future value of €100 at time T

has value of 100/1.1=€90.90 at T-1

which has value of 90.90/1.1=€82.60 at T-2, etc.

In formula, simply move interest rate factor to other side:

$$PV = \frac{FV_T}{(1+r)^T}$$

Can also re-write formula for the interest rate:

$$r = \sqrt[T]{\frac{FV_T}{PV}} - 1$$

is geometric average rate, $<$ than arithmetic if r fluctuates

Compounding periods not necessarily same as interest periods

e.g. corporate bonds often pay interest 2$\times$ per year

even though interest is annual rate

10% bond pays 5% every half year

bondholders earn interest on interest in second half year

effective annual rate is $1.05^2 = 1.1025$ or 10.25%

if compounded quarterly $1.025^4 = 1.1038$ or 10.38%

Future value formula with variable compounding frequency, n, is:

$$FV_T = PV\left(1 + \frac{r}{n}\right)^{Tn}$$

If compounding frequency $n \to \infty$

compounding periods become infinitesimal

compounding becomes continuous

Future value formula found by multiplying Tn by r/r and splitting in n/r and rT :

$$FV_T = PV \left[\left(1 + \frac{r}{n} \right)^{n/r} \right]^{rT}$$

Defining $c = n/r$

$$FV_T = PV \left[\left(1 + \frac{1}{c} \right)^{c} \right]^{rT}$$

As $c \to \infty$, $\left(1+\frac{1}{c}\right)^c \to e = 2.7183..$, base of natural logarithms

$$\lim_{c\to\infty}\left(1+\frac{1}{c}\right)^c = e = 2.71828....$$

Formulae then become:

$$FV_T = PVe^{rT} \quad \text{and} \quad PV = FV_T e^{-rT}$$

re-writing for the interest rate gives $FV_T/PV = e^{rT}$
Taking logarithms:

$$\ln\frac{FV_T}{PV} = \ln e^{rT} = rT$$

These logarithmic rates of return are frequently used in continuous time finance (option pricing)

Advantages of continuously compounded log-returns:

easily calculated from e.g. daily stock prices $S_0, S_1, S_2, etc.$

additive over time:

$$\ln\left(\frac{S_1}{S_0} \times \frac{S_2}{S_1}\right) = \ln\frac{S_1}{S_0} + \ln\frac{S_2}{S_1} = \ln e^{r_1} + \ln e^{r_2} = r_1 + r_2$$

week-return sum of day-returns

But *not* additive across investments:

- logarithmic transformation not linear
- log of a sum $\neq$ sum of logs

Discretely compounded returns $\frac{S_1 - S_0}{S_0}, \frac{S_2 - S_1}{S_1}$:

easily aggregated across investments

- weighted returns are additive
- for example, two stocks A and B
- return A, $r_A = 10\%$, return B, $r_B = 20\%$
- equally weighted portfolio of A and B gives

$$\frac{1}{2} \times 10 + \frac{1}{2} \times 20 = 15$$

But: not additive over time:

- 5% over 10 years is

$$1.05^{10} = 1.629$$

or 62.9%, not 50%

Annuities and perpetuities

Cash flows (payments and receipts) often come in series

called annuity (yearly) and perpetuity (for ever)

use mathematical series properties to calculate value

e.g. series of n payments of amount A:

$$PV = \frac{A}{1+r} + \frac{A}{(1+r)^2} + ... + \frac{A}{(1+r)^n}$$

Annuities not frequently used in finance

look them up in the book if needed

One exception: *Gordon growth model*

present value of perpetuity

perpetuity = annuity with infinite number of payments

Formula easily derived (see book):

$$PV = \frac{A}{r}$$

Formula for perpetuity with growth rate g is:

$$PV = \frac{A}{r-g}$$

assumes $r > g$

Gordon growth model:

- often used for its simplicity
- also in exam questions (easy for students)
- usually applied such that number for A is given

Gordon growth model:

Example: stock price as discounted dividends

A stock is expected to pay €10 in dividends 1 year from now
dividends are expected to continue forever
and to grow with the inflation rate of 2%
investors expect a 10% return on the stock
Value of the stock is:

$$\frac{10}{.1 - .02} = €125$$

Accounting numbers frequently used in finance, but:

market values are used when possible

book values used when necessary

Book values come from accounting system, records:

flows of goods/money through firms
what came in and what went out

effects on assets and liabilities equity
what the firm owns and where the money came from

Accounting has its own principles and practices
rules also cover large firms, extreme cases, exceptions

Look at some accounting statements (called: financial statements)

Income statement ZX co.

year ended 31 December:	2011	2012
Sales	250	300
− Cost of goods sold	175	200
Gross profit	75	100
− Cost (personal, depreciation, other)	35	50
Operating income	40	50
+ Financial revenue (interest received)	3	4
− Interest paid and other financial cost	13	14
Profit before taxes	30	40
− Income taxes	9	12
± Income/loss from discontinued operations	-	-
Net profit	21	28

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Balance sheet ZX co. at 31 December 2012

Assets			Liabilities & equity		
plant, property	250		issued capital	50	
− accum. depr.	−110		retained earnings	150	
financial assets	45		Total equity		200
intangible assets	30				
Total fixed assets		215	long bank loans	75	
			Total long debt		75
cash, bank	40				
accounts receivable	50		accounts payable	50	
other	35		other short debt	15	
Tot. current assets		125	Total current liab.		65
Total assets		340	Tot. liab. & equity		340

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values are 'frozen' when entered into the books

- market values change continuously
- can drift far away from book value (for long-lived assets)

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- market position, trade marks
- value CocaCola $>$ replacement value tangible assets

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Accounting provides accountability

- backward looking ('what happened')
- includes arbitrary allocations of costs & profits over time (depreciation)
- includes elements not relevant for decisions (irreversible investments or 'sunk costs')

Illustrate use with example, technology project ZXco

technical and economic viability demonstrated in large test, cost €15 million

now considering commercial launch

Management set following parameters:

Cost of capital for project is 25%

- includes time value of money and expected inflation
- plus risk premium estimated from similar projects
- thus defined, it is *opportunity cost of capital*

corporate tax rate is 30%

Company's staff made following estimates:

Project details, amounts in €10^6 :

- will generate sales in 3 years, 250, 500 and 250
- sales start 1 year after investment
- 50% work will be outsourced
- operating costs are 35, 65 and 30
- requires investment now of 180, plus 15 paid for test
- investment depreciated in equal parts: $(180+15)/3=65$
- required working capital 10 now and 20, 35 after 1,2 years
- working capital liquidated last year

This gives following pro-forma income statement and balance sheet

	year	0	1	2	3
	Income statement				
1	Sales	-	250	500	250
2	Cost of goods sold	-	125	250	125
3	Gross profit (1-2)	-	125	250	125
4	Operating expenses	-	35	65	30
5	Depreciation	-	65	65	65
6	Profit before taxes (3-4-5)	-	25	120	30
7	Tax @ 30%	-	7.5	36	9
8	Net profit (6-7)	-	17.5	84	21

	year	0	1	2	3
	Balance sheet				
9	Investment (gross)	195	195	195	195
10	Accumulated depreciation	-	65	130	195
11	Book value inv. year end (9-10)	195	130	65	0
12	Net working capital	10	20	35	0
13	Book value proj. year end (11+12)	205	150	100	0
14	Book value proj. year begin	0	205	150	100
	Book return on investment (8/14)		.085	.560	.210

Accounting representation gives no clear decision criterion
Accept project or not?

book return $<$ CoC in 2 of 3 years

could use their weighted averages:

$$\frac{205 \times .085 + 150 \times .56 + 100 \times .21}{205 + 150 + 100} = 0.269 > CoC \Rightarrow \text{Accept?}$$

heavily influenced by depreciation

ignores time & risk: later returns less valuable

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Financial representation provides proper decision framework:

uses only data relevant for decision

uses cash flows as they occur, no arbitrary divisions over time

Financial representation makes 3 changes:

Replaces depreciation by cash outflow of investment

- depreciation spreads costs over time to give yearly profits
- not necessary for decision
- note: time pattern of cash flows is relevant

Includes changes in net working capital

- is cash outflow (and investment) too
- sometimes 50% of investment, or more
- liquidated last year: becomes cash inflow

Removes part of investment irrelevant for decision

- €15 for test already paid
- cannot be undone: sunk costs

Gives following cash flow statement

	year	0	1	2	3
	Cash flow statement				
1	Net profit	-	17.5	84	21
2	Depreciation	-	65	65	65
3	Change in net working capital	-10	-10	-15	35
4	Cash flow from operations (1+2+3)	-10	72.5	134	121
5	Cash flow from investment	-180			
6	Total cash flow (4+5)	-190	72.5	134	121
7	PV cash inflows @ 25%	205.7			
	Net present value NPV (6+7)	15.7			

Cash flows moved to same point in time (now):

by discounting expected future values

at opportunity cost of capital of 25%

Subtracting the investment gives the project's *Net Present Value* (NPV):

$$\frac{72.5}{1.25} + \frac{134}{1.25^2} + \frac{121}{1.25^3} = 205.7 - 190 = 15.7 = NPV$$

Decision rule:

ZXco should go ahead with project if NPV>0

then project adds to the value of the company

NPV is correct investment criterion, leads to value maximizing decisions (theoretical foundation later)

Some other aspects:

Project may generate more than cash flows:

- intangible assets like reputation and growth opportunities
- can be very valuable
- discussed in real options analysis

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Project is analysed as if all equity financed

no interest or debt repayments

most projects partly financed with debt

usual to analyse in this way

- does not mix investment and financing decision
- financing effect usually in discount rate, not cash flows

Other investment criteria than NPV also used in practice
not as good as NPV:

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internal rate of return = discount rate that makes NPV=0

- found by solving

$$-190 + \frac{72.5}{(1+r)} + \frac{134}{(1+r)^2} + \frac{121}{(1+r)^3} = 0$$

which gives r=.3 or 30%

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$$-190 + \frac{72.5}{(1+r)} + \frac{134}{(1+r)^2} + \frac{121}{(1+r)^3} = 0$$

  which gives r=.3 or 30%
- leads to correct decisions if used with rule:
  invest if IRR > CoC
- but only for 'normal' cash flow patterns:
  first investment, then positive cash flows

Economic depreciation

not necessary for investment decision (don't need profit per year, just cash flows)

can be calculated anyway:

- difference in project value from year to year, e.g.
- now (t=0) value cash inflows is 205.7
- 1 year later (t=1) 72.5 is realized, value remaining cash flows is:

$$\frac{134}{1.25} + \frac{121}{1.25^2} = 184.6$$

- difference $205.7 - 184.6 = 21.1$ is economic depreciation
- economic profit is $72.5 - 21.1 = 51.4$
- return is $51.4/205.7 = 0.25$ or 25%

Calculations summarized in table:

Economic depreciation and return

	year	0	1	2	3
1	Cash inflows from project		72.5	134	121
2	PV cash inflows, year end	205.7	184.6	96.8	0
3	PV cash inflows, year begin	0	205.7	184.6	96.8
4	Economic depreciation (2-3)	-	-21.1	-87.8	-96.8
5	Profit from project (1+4)	-	51.4	46.2	24.2
6	Return on investment (5/3)		.25	.25	.25

Economic depreciation

changes from year to year

depending on how much of project is realized

but return is constant

Economic depreciation

- changes from year to year
- depending on how much of project is realized
- but return is constant

In accounting representation

- depreciation is arbitrarily set as a constant
- so that return jumps up and down
- makes second year exceptionally good (bonuses?)

Example project re-used later in marked efficiency

Utility and risk aversion

Finance studies people's choices among risky future values
Choices express the *preferences* people have:

prefer A to B: $A \succ B$

prefer bundle 1 to 2: $B1 \succ B2$

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Preferences based on what alternatives 'mean' to people

Economic concept for that is *utility,*

preferences are described by utility:

if A is preferred to B

then utility of A, U(A), is larger than utility of B, U(B)

Is also true the other way around:

if utility of A is larger than utility of B

then A is preferred to B

$$A \succ B \iff U(A) > U(B)$$

Is also true the other way around:

if utility of A is larger than utility of B

then A is preferred to B

$$A \succ B \Longleftrightarrow U(A) > U(B)$$

Utility is individual and situation-dependent:

greedy $\Leftrightarrow$ generous people

rich $\Leftrightarrow$ poor people

old $\Leftrightarrow$ young people

at home $\Leftrightarrow$ on the job $\Leftrightarrow$ holiday

To make a structured analysis possible, we make three very simple and general assumptions:

- People are greedy: they prefer more of a good to less
- Each additional unit gives less utility than its predecessor: the first APPLE tastes better than the next, etc.
- Peoples' preferences are well-behaved, e.g.:
  - asymmetric: $a \succ b \Rightarrow b \not\succ a$
  - transitive: $a \succ b$ and $b \succ c \Rightarrow a \succ c$

These simple assumptions have important consequences

Third assumption means:

preferences can be expressed in *utility function*

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First and second assumptions mean:

utility function is concave:

- strictly increasing (positive marginal utility or positive first derivative)
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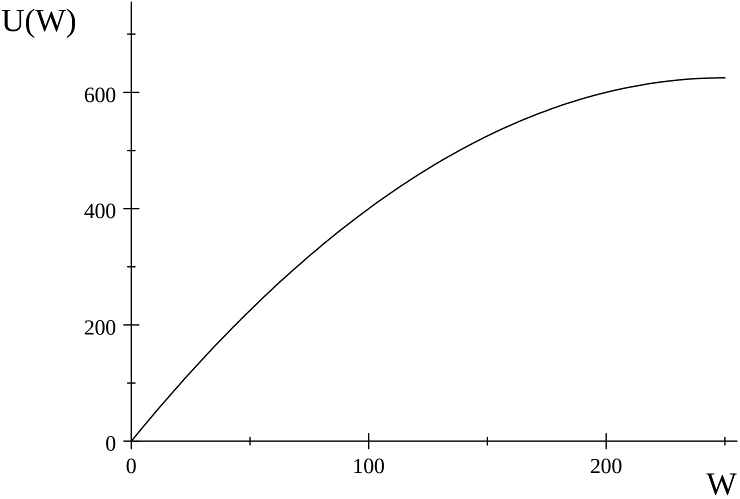
Well known utility functions are:

logarithmic utility function: $U(W) = \ln W$

quadratic utility function: $U(W) = \alpha + \beta W - \gamma W^2$



A typical utility function ($U = 5W - .01W^2$)

W typically stands for wealth but can also mean apples, beer, bundle32, etc.

Note that these utility functions are not so well behaved:

- logarithmic utility function:
  $U(W) = \ln W$ requires W to be positive
- quadratic utility function:
  $U(W) = \alpha + \beta W - \gamma W^2$ is only increasing over a certain range of values for W (up to the 'bliss point' $W = \frac{1}{2}\beta/\gamma$)

Financial markets often facilitate choices independent of utility functions, as we shall see, but we use them every now and then.

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Financial markets often facilitate choices independent of utility functions.

Why would it be an advantage to eliminate utility functions from the analysis?

From utility functions we derive 2 other important concepts:

Indifferences curves:

- combinations of choices that give same utility
- instruments in rational decision making process
- their shape and location determine economic choices:
- 'map' all indifference curves on all possible choices and choose alternative on highest indifference curve

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Risk aversion:

- risk is a negative quality, something to be avoided
- (most) people require a reward to accept risk
- follows from concave utility functions

To construct an indifference curve:

plot utility as function of 2 $W's$ (wealth now, wealth next period or apples, pears, etc.)

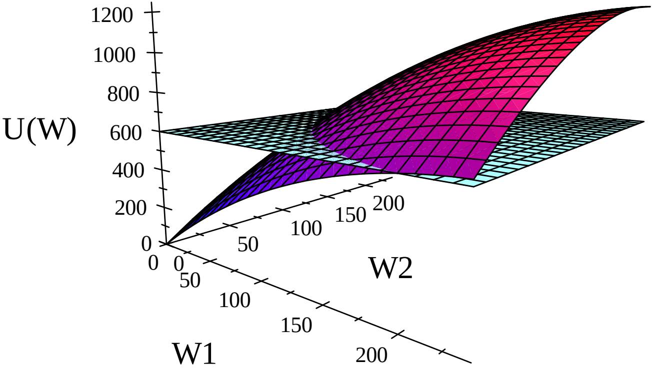
example:

$$U = 5W_1 - .01W_1^2 + 5W_2 + .01W_2^2$$

Indifference curve is collection of points with same value of U, e.g.

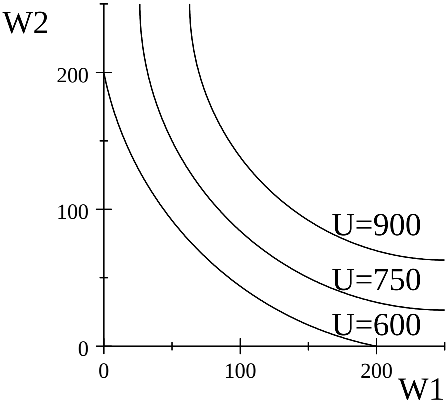
$$5W_1 - .01W_1^2 + 5W_2 + .01W_2^2 = 600$$

Graphically, indifference curve is where utility surface intersects fixed value plane:



2 dimensional utility function and the U=600 plane

Seen from 'above' in W1-W2 plane indifference curves have their familiar shape, utility increases away from origin:



Indifference curves

Shape of indifference curves reflects:

Decreasing marginal utility (2nd simple assumption)

- the more units you already have of something, the less utility an additional unit of that something gives you

Means in indifference curve context:

- the more units you have of something, the more units you are willing to give up to get 1 unit of something else
- if you have 10 apples and no pears you would give 3 apples for a pear and the other way around

Individual preferences expressed in the way the curves are 'tilted' towards one of the axes

- one person with 10 apples and no pears would give 3 apples for a pear, another person only 2

Risk aversion

Look again at the utility function $U(W) = 5W - .01W^2$

The utility of $100W$ is
$U(100) = 500 - .01 \times 100^2 = 400$

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$U(100) = 500 - .01 \times 100^2 = 400$

What if this 100 is not certain

- but e.g. the expectation of 50 and 150
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Risk aversion

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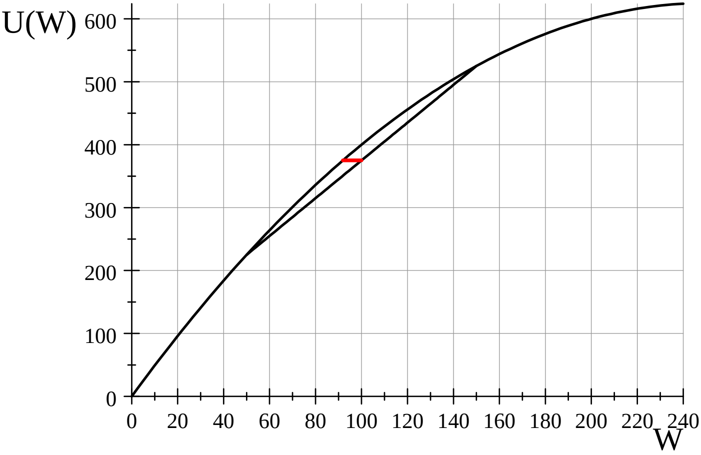
We can calculate 2 things:

$U[E(W)]$ utility of expected wealth
is on the curved utility function

$E(U[W])$ expected utility of wealth
is a straight line interpolation (prob. weighted) between points on the curved utility function

Difference between the 2 reflects risk aversion



Utility function $U(W) = 5W - .01W^2$ and an uncertain value of (W)

Filling in the numbers:

Quadratic utility function gives:

- $U(50) = 250 - .01 \times 50^2 = 225$
- $U(150) = 750 - .01 \times 150^2 = 525$
- so that $E(U[W]) = (225 + 525)/2 = 375$

Lower than 400 we calculated for $U(100)$

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Risk aversion follows from concave utility functions:

If W 100$\rightarrow$ 150, U(W) 400$\rightarrow$ 525, increase 125

If W 100$\rightarrow$ 50, U(W) 400$\rightarrow$ 225, decrease 175

We now try some different values: 25 and 175:

same expectation, larger risk

$U(25) = 125 - .01 \times 25^2 = 118.75$

$U(175) = 875 - .01 \times 175^2 = 568.75$

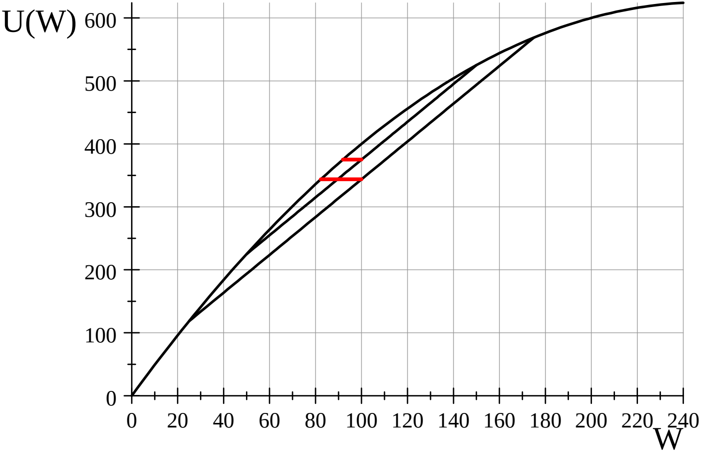
so that $E(U[W]) = (118.75 + 568.75)/2 = 343.75$

$U = 343.75$ corresponds to certain $W = 82.3$
the required risk premium is 17.7

Required risk premium increases with risk

also increases with curvature of utility function

- used in *risk aversion coefficients*



Utility function $U(W) = 5W - .01W^2$ and 2 uncertain values of (W)

Now we do one more calculation:

combine equal proportions of the 2 uncertain values (50 - 150) and (25 - 175)

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- (50+175)/2=112.5
- (150+25)/2=87.5

$U(87.5) = 437.5 - .01 \times 87.5^2 = 361.19$

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By combining risky choices risk almost disappears!

Some more observations:

Risk reduction (or *diversification*) effect depends on correlation characteristics:

- No diversification effect if the two uncertain values are *positively* correlated (both high or both low)
- More generally: less than perfectly positive correlation ($\rho < 1$) gives diversification effect

Risk is easily (partly) eliminated by diversification

- in financial context, combining stocks, bonds, etc. is very easy
- *must* make risk premia of individual choices of limited value
- would give others (the market) opportunity to re-combine and profit, called arbitrage

We draw 3 general conclusions:

- The higher the risk, the more premium people require to accept the risk (the more E(W) has to increase to give same utility)
- Combining choices (projects, stocks, investments) can give reduced risk because of correlation characteristics, so that the combination has lower risk than any of individual choices
- Risk premium will depend on the risk of combined, not individual choices

That is why investments should not be evaluated alone!
elaborated in portfolio theory