ME361 – Manufacturing Science Technology

Orthogonal Cutting

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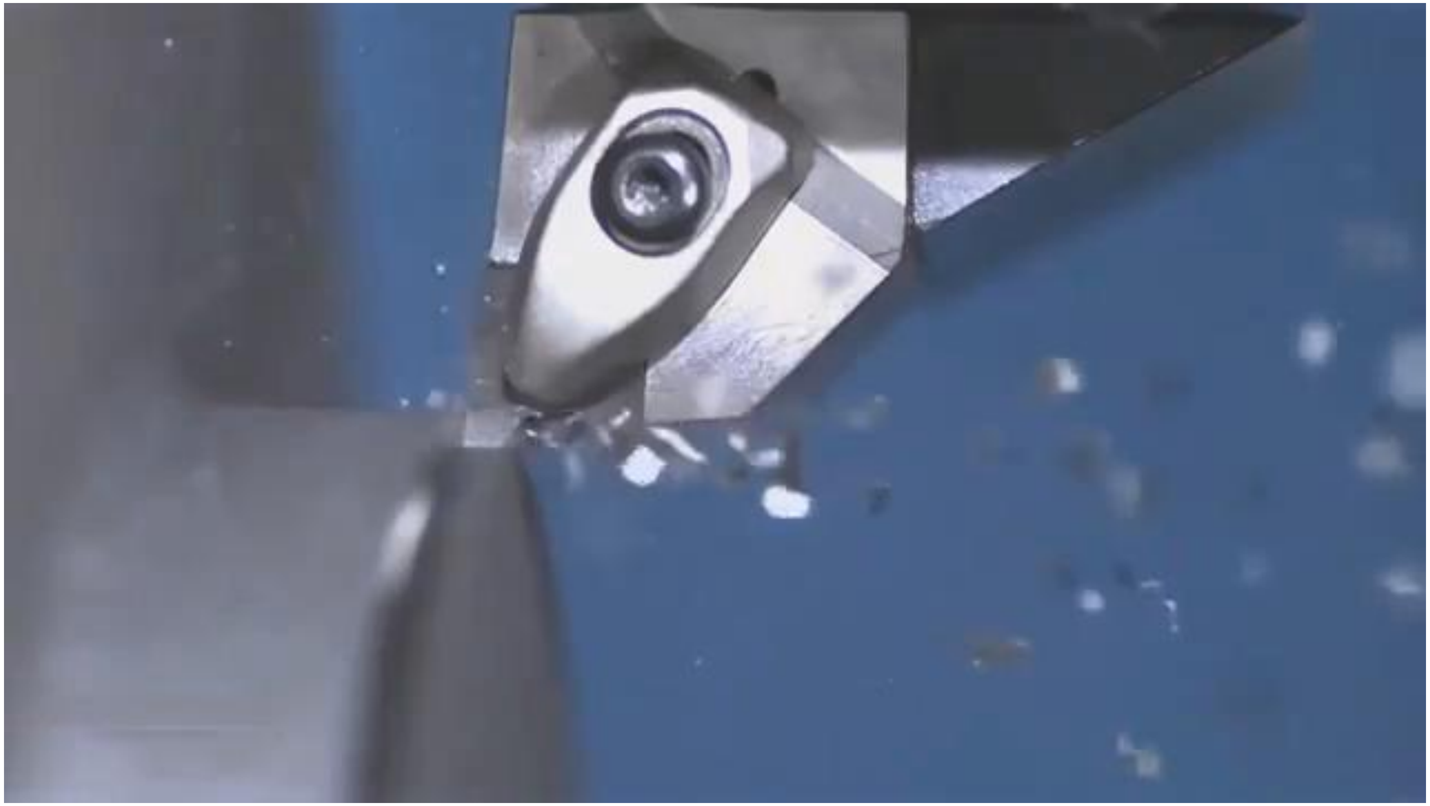
Machining processes

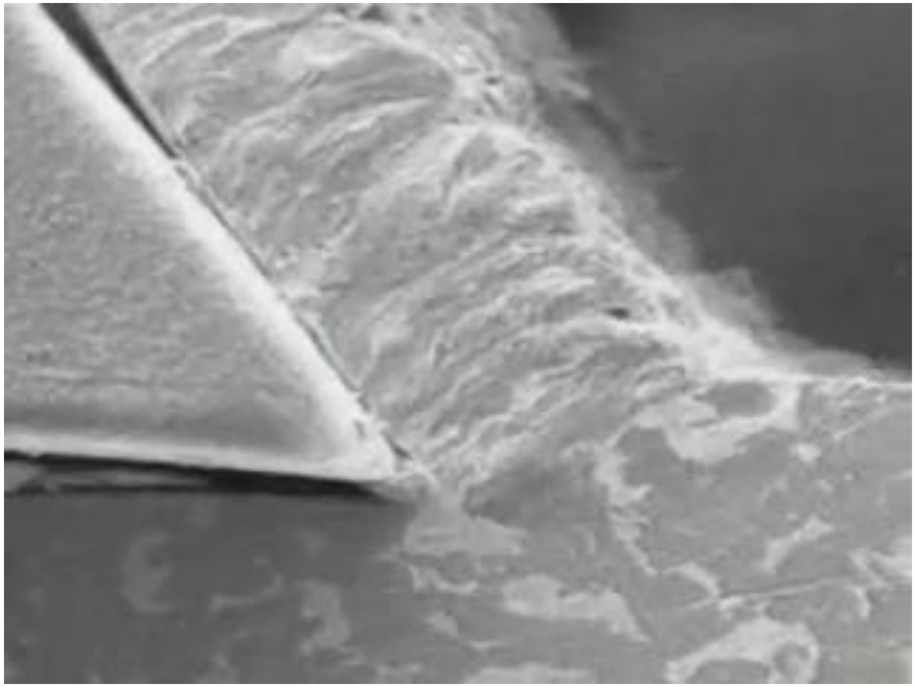


Most machining operations are geometrically complex and 3D

https://www.youtube.com/watch?v=JoVQAn7Suto

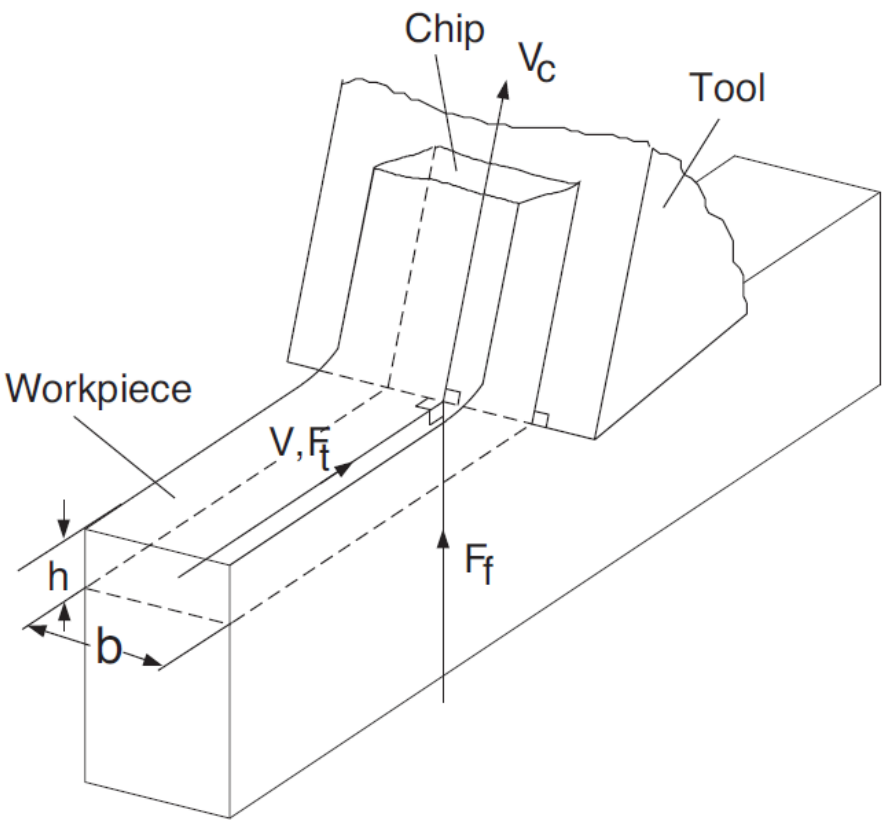
Mechanics of cutting

Simple 2D orthogonal cutting can help explain the general mechanics of metal removal



https://www.youtube.com/watch?v=mRuSYQ5Npek&t=21s

Orthogonal cutting geometry

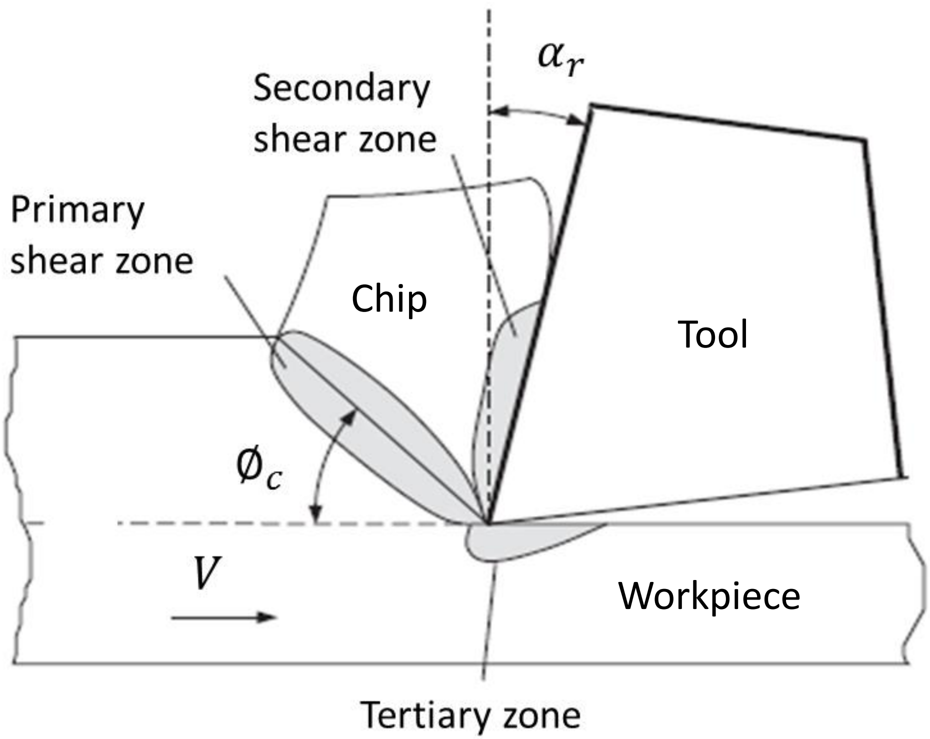


Assumptions:

1. Cutting edge is perfectly sharp
2. Uncut chip thickness is constant and << than width
3. Width of tool > width of workpiece
4. Continuous chip with no built up-edge
5. Uniform cutting along edge
6. 2D plane strain deformation
7. No side spreading of material
8. Uniform stress distribution on shear plane
9. Forces primarily in directions of velocity and uncut chip thickness

Orthogonal cutting geometry

Simple 2D orthogonal cutting can help explain the general mechanics of metal removal

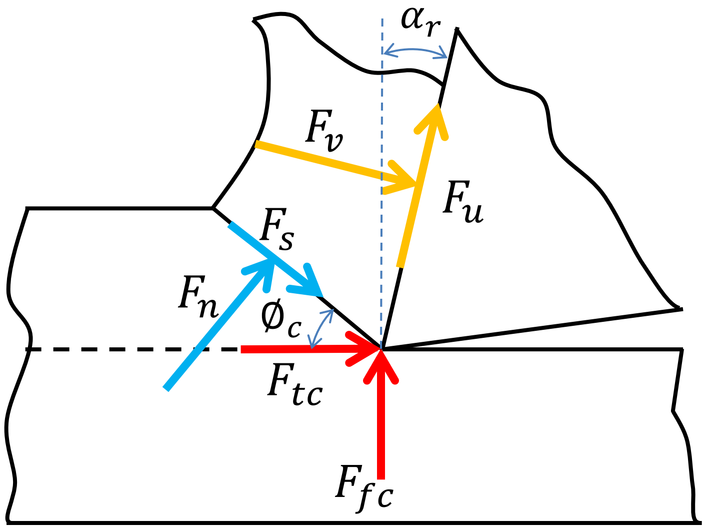


Primary shear zone:
Material ahead of tool is sheared to form a chip

Secondary shear zone:
Sheared material (chip) partially deforms and moves along the rake face

Tertiary zone:
Flank of tool rubs the newly machined surface

Primary shearing zone



α_r- rake angle;
ϕ_c - shear angle
F_{tc} - tangential force;
F_{fc} - feed force
F_s - force acting along the shear plane
F_n - normal force acting on the shear plane
F_v - force acting on the rake face
F_u - frictional force

Force components in primary shear zone

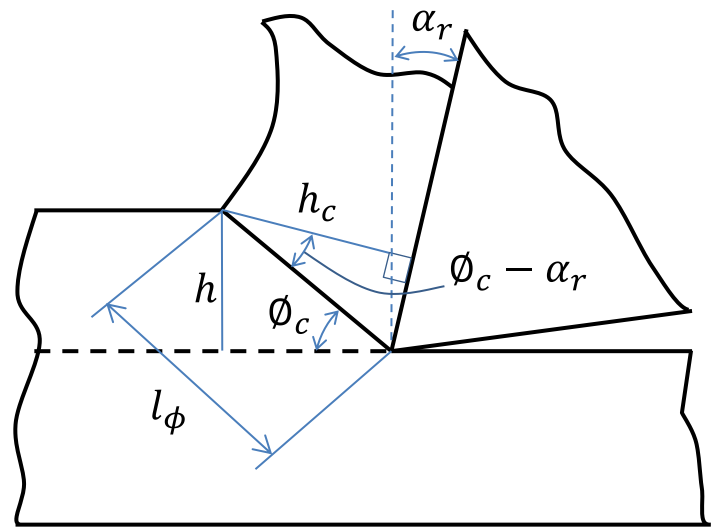
$$F_s = F_{tc} \cos\phi_c - F_{fc} \sin\phi_c \, ;$$
$$F_n = F_{tc} \sin\phi_c + F_{fc} \cos\phi_c \, ;$$

$$\begin{Bmatrix} F_s \\ F_n \end{Bmatrix} = \begin{bmatrix} \cos\phi_c & -\sin\phi_c \\ \sin\phi_c & \cos\phi_c \end{bmatrix} \begin{Bmatrix} F_{tc} \\ F_{fc} \end{Bmatrix}$$

Chip ratios – basic characteristics

$$\boxed{\sin\phi_c = \frac{h}{l_\phi}} \quad \& \quad \boxed{\cos(\phi_c - \alpha_r) = \frac{h_c}{l_\phi}}$$

$$\boxed{h = l_\phi \sin\phi_c} \quad \& \quad \boxed{h_c = l_\phi \cos(\phi_c - \alpha_r)}$$



Chip thickness ratio

$$\boxed{r_c = \frac{h}{h_c} = \frac{\sin\phi_c}{\cos(\phi_c - \alpha_r)}}$$

From mass (volume flow rate) conservation, chip thickness ratio ≅ chip length ratio

$$\boxed{r_c = \frac{h}{h_c} = \frac{l_c}{l} = r_l}$$

No side spreading assumption, $r_w = 1$

Shear angle – from chip ratios

Inaccuracies in shear angle prediction from models are overcome by experimental identification

Recalling chip ratios

$$r_c = \frac{h}{h_c} = \frac{\sin\phi_c}{\cos(\phi_c - \alpha_r)}$$

Rearranging

$$\phi_c = \tan^{-1}\frac{r_c\cos\alpha_r}{1 - r_c\sin\alpha_r}$$

Obtaining r_c?

Non-uniform chip thickness after machining makes measuring h_c difficult

instead

From mass (volume flow rate) conservation, chip thickness ratio ≅ chip length ratio

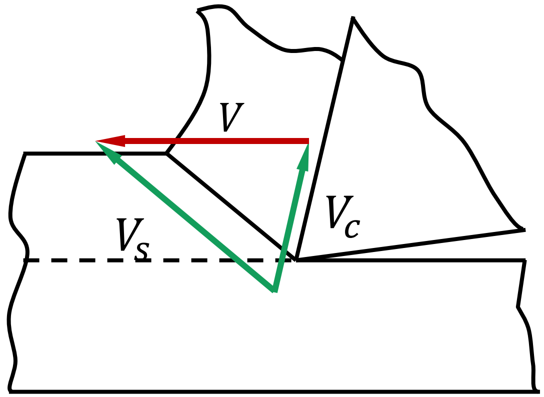
$$r_c = \frac{h}{h_c} = \frac{l_c}{l} = r_l$$

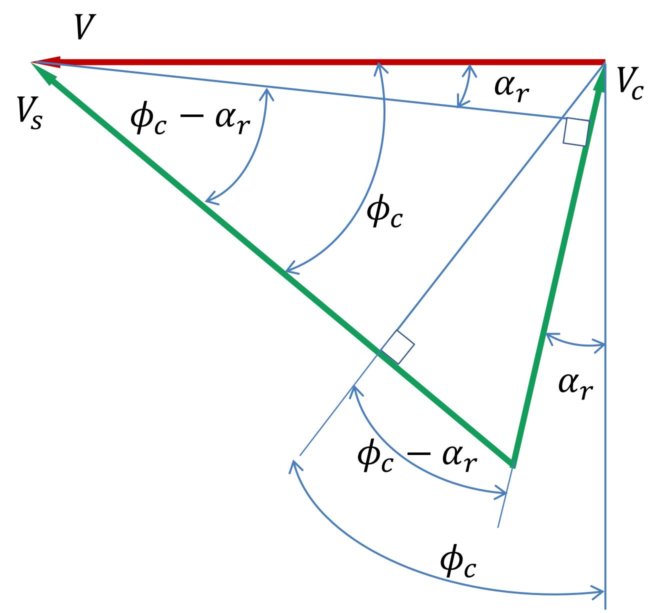
Measure l_c by scribing the un-machined surface with l

Alternatively

Without scribing, h_c is computed using ρ, measured weight of chip and measured l_c

Velocity relations





Chip velocity, V_c (acknowledging conservation of volume-flow rate):

$$\boxed{\frac{V_c}{V} = \frac{l_c/\Delta t}{l/\Delta t} = r_l \Rightarrow V_c = r_l V}$$

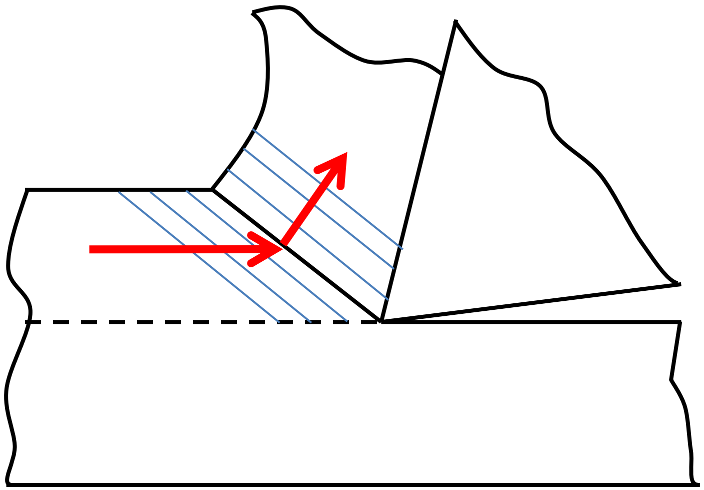
$$\boxed{r_c = r_l = \frac{\sin\phi_c}{\cos(\phi_c - \alpha_r)}}$$

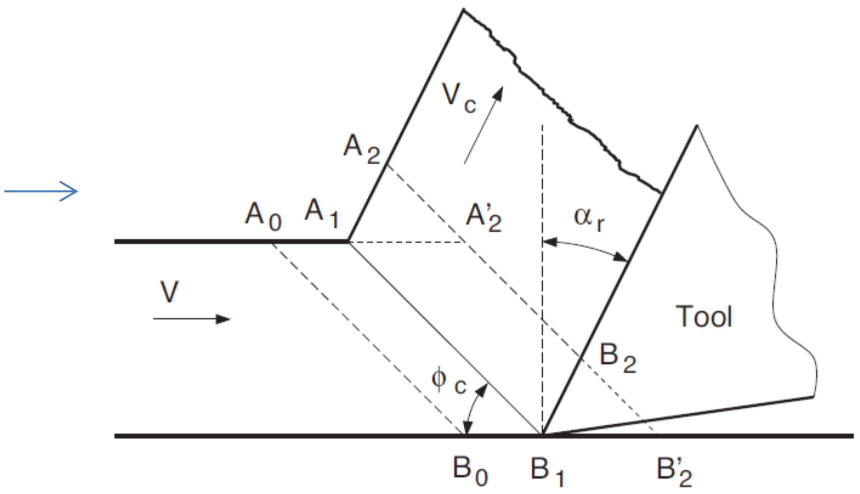
$$\boxed{V_c = \frac{\sin\phi_c}{\cos(\phi_c - \alpha_r)} V}$$

Shear velocity, V_s is vector sum of V_c and V

$$\boxed{V_s = V\frac{\cos\alpha_r}{\cos(\phi_c - \alpha_r)}}$$

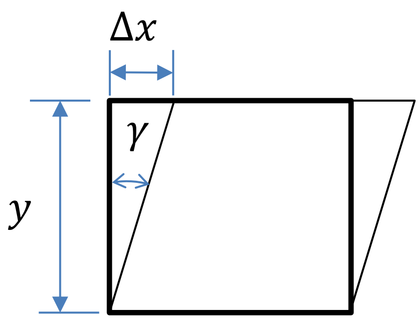
Shear strain and material movement

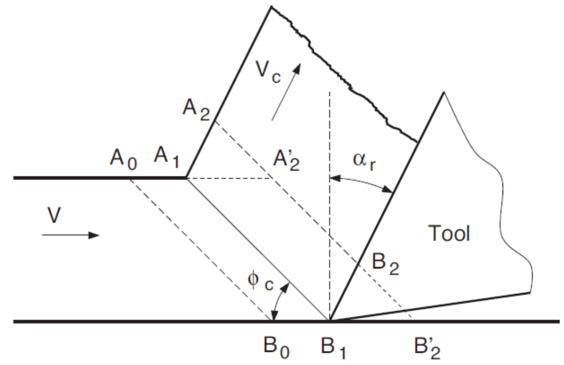


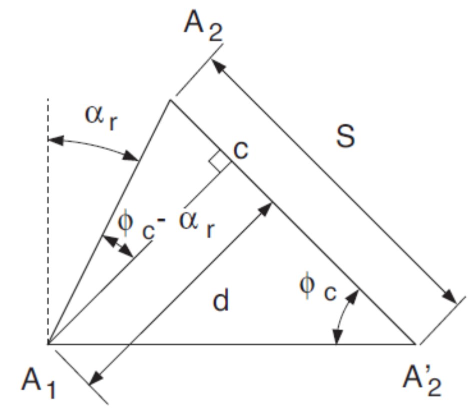


- Undeformed chip section $A_0B_oA_1B_1$moves with velocity V
- When one element traverses the shear plane in time Δt:
  - point A_1 moves to point A_2, point A_0 moves to point A_1
  - point B_1 moves to point B_2, point B_0 moves to point B_1
- Undeformed chip section $A_0B_oA_1B_1$hence becomes deformed chip with section $A_1B_1A_2B_2$
- Hence chip is shifted from expected position of $B_2{}'\ A_2{}'$ to B_2A_2 because of shearing

Shear strain and shear strain rate







$$\tan\gamma = \frac{\Delta x}{y}$$ Small angles → $$\gamma = \frac{\Delta x}{y}$$

$$\gamma = \frac{\Delta s}{\Delta d} = \frac{A_2 A_2{}'}{A_1 C} = \frac{A_2{}' C}{A_1 C} + \frac{C A_2}{A_1 C} = \cot\phi_c + \tan(\phi_c - \alpha_r)$$

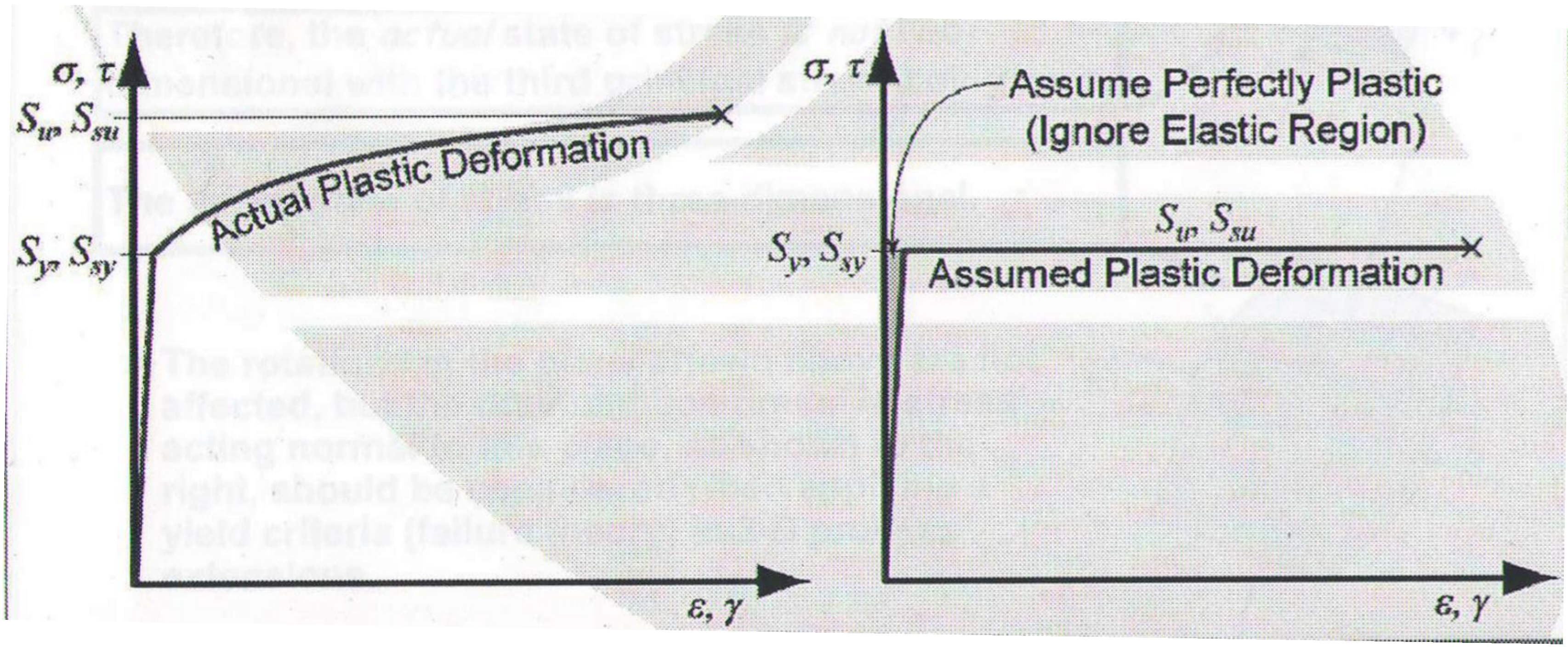
Shear strain rate

Assuming shear zone increment is Δs and the thickness of the shear deformation zone is Δd

$$\dot{\gamma} = \frac{\gamma}{\Delta t}$$ → $$\gamma = \frac{\Delta s}{\Delta d}$$ $$V_s = \frac{\Delta s}{\Delta t}$$ $$\dot{\gamma} = \frac{V_s}{\Delta d} = V\frac{\cos\alpha_r}{\Delta d\cos(\phi_c - \alpha_r)}$$

$\Delta d \rightarrow 0 \rightarrow$ thin shear plane $\rightarrow$ very large strain rates

Primary shearing zone



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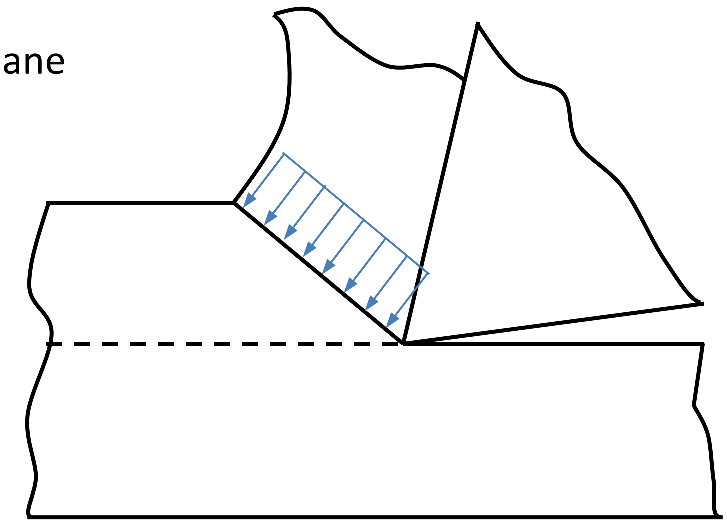
Primary shearing zone

Assuming uniform shear stress distribution on the shear plane and that all shear plane material is plastically shearing

$$\tau_s = \frac{F_s}{A_s}$$

$$F_s = F_{tc} \cos \phi_c - F_{fc} \sin \phi_c$$

$$A_s = b \frac{h}{\sin \phi_c}$$



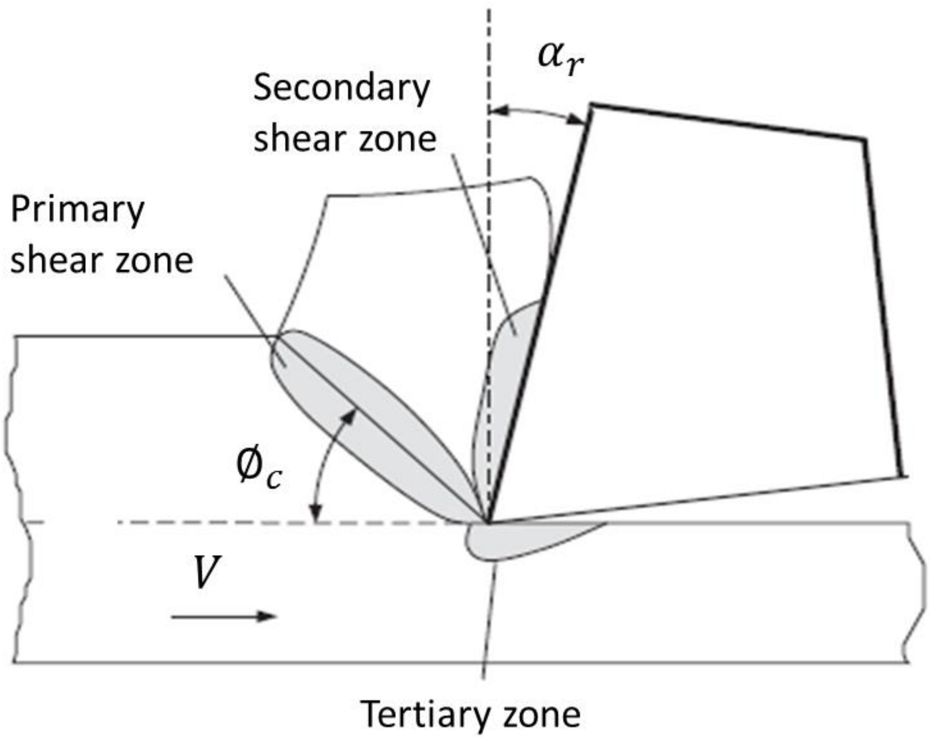
Normal stress on the shear plane

$$\sigma_s = \frac{F_n}{A_s}$$

$$F_n = F_{tc} \sin \phi_c + F_{fc} \cos \phi_c$$

A_s - shear plane area
b – width of cut (depth of cut in turning)
h - uncut chip thickness
ϕ_c - shear angle
F_s - force acting along the shear plane
F_n - normal force acting on the shear plane

Orthogonal cutting geometry

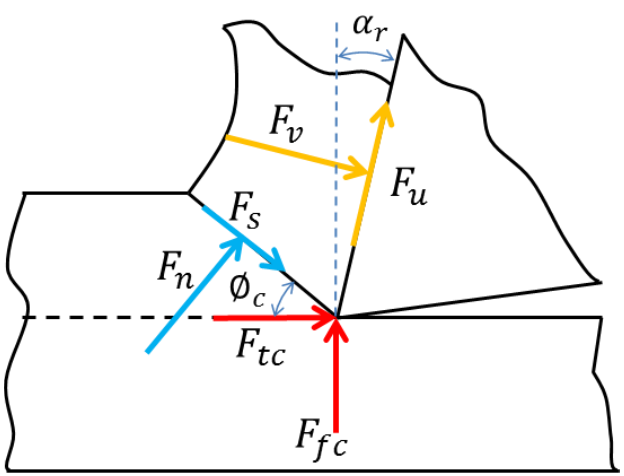


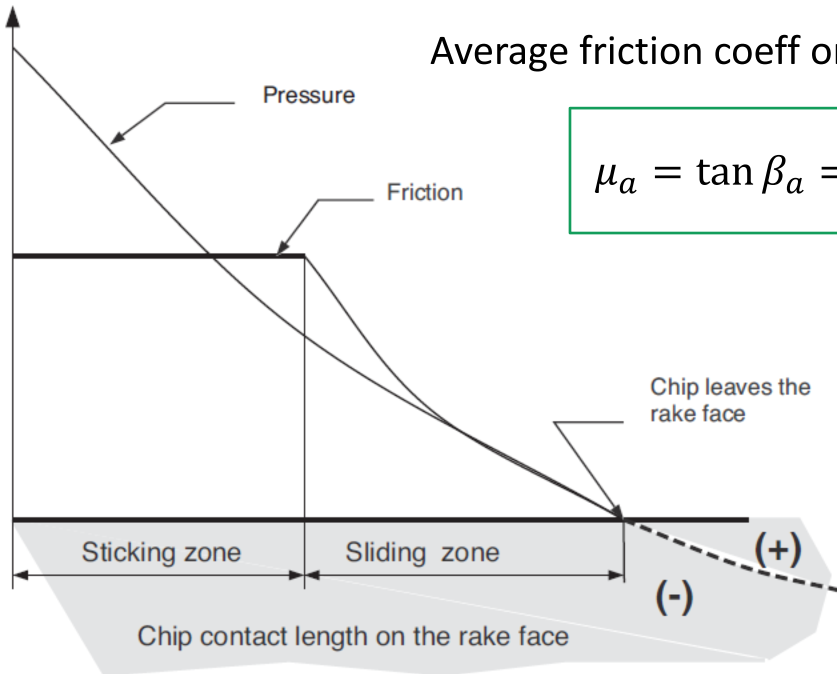
Primary shear zone:
Material ahead of tool is sheared to form a chip

Secondary shear zone:
Sheared material (chip) partially deforms and moves along the rake face

Tertiary zone:
Flank of tool rubs the newly machined surface

Secondary shear zone





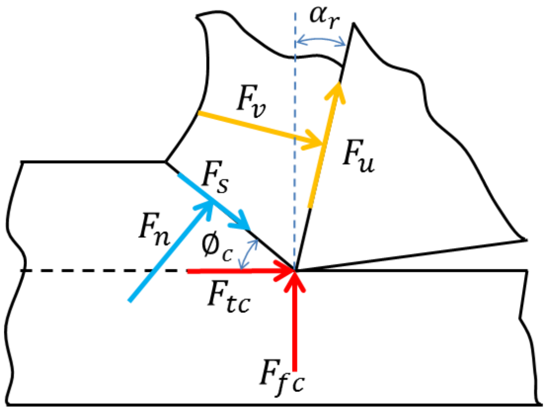
Average friction coeff on rake face:

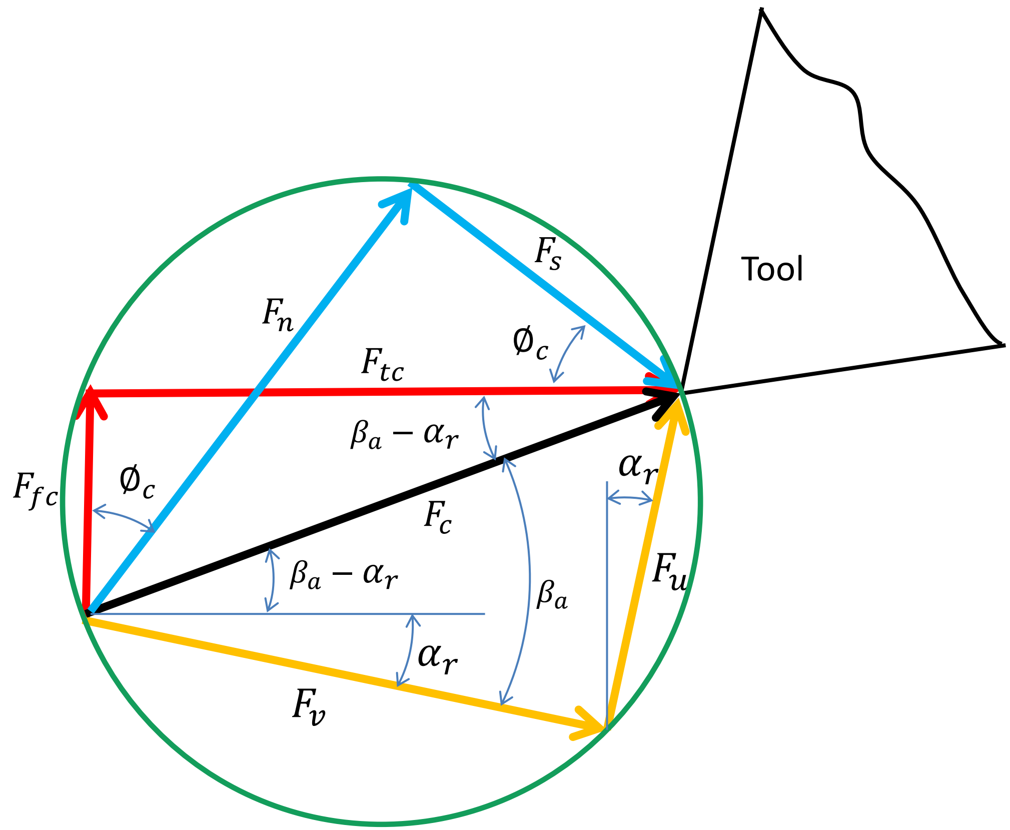
$$\boxed{\mu_a = \tan\beta_a = \frac{F_u}{F_v}}$$

Force components in secondary shear zone

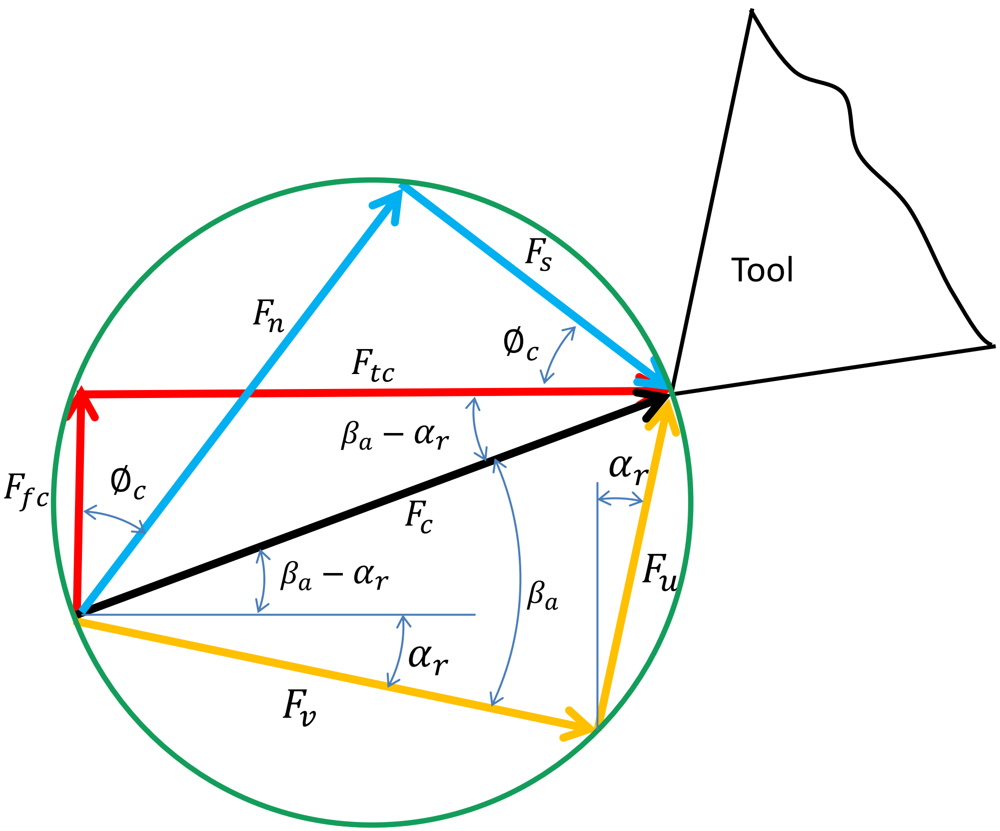
$$\boxed{\begin{aligned} F_v &= F_{tc}\cos\alpha_r - F_{fc}\sin\alpha_r\,; \\ F_u &= F_{tc}\sin\alpha_r + F_{fc}\cos\alpha_r\,; \end{aligned}} \rightarrow \boxed{\begin{Bmatrix} F_v \\ F_u \end{Bmatrix} = \begin{bmatrix} \cos\alpha_r & -\sin\alpha_r \\ \sin\alpha_r & \cos\alpha_r \end{bmatrix} \begin{Bmatrix} F_{tc} \\ F_{fc} \end{Bmatrix}}$$

Force circle diagram





Force circle diagram



Force components in primary shear zone

$$F_s = F_{tc} \cos \phi_c - F_{fc} \sin \phi_c \,;$$
$$F_n = F_{tc} \sin \phi_c + F_{fc} \cos \phi_c \,;$$

Force components in primary shear zone using the force circle diagram

$$F_s = F_c \cos(\phi_c + \beta_a - \alpha_r) \,;$$
$$F_n = F_c \sin(\phi_c + \beta_a - \alpha_r) \,;$$

Force components in secondary shear zone using the force circle diagram

$$F_v = ? \,; F_u = ?$$

Total power consumed in cutting

Total power consumed in cutting is sum of energy spent in shear and friction zones

$$P_{tc} = P_s + P_u$$

$$P_{tc} = F_s V_s + F_u V_c$$

$$F_s = F_{tc} \cos\phi_c - F_{fc} \sin\phi_c$$

$$F_u = F_{tc} \sin\alpha_r + F_{fc} \cos\alpha_r$$

$$V_s = V \frac{\cos\alpha_r}{\cos(\phi_c - \alpha_r)}$$

$$V_c = V \frac{\sin\phi_c}{\cos(\phi_c - \alpha_r)}$$

ϕ_c - shear angle?
F_s - force acting along the shear plane?
F_u - frictional force?

Force prediction for power consumption

Total power consumed in cutting is sum of energy spent in shear and friction zones

$$\boxed{P_{tc} = F_s V_s + F_u V_c}$$

Primary chip generation mechanism is shearing

$$\boxed{P_s = F_s V_s}$$

$$\boxed{F_s = F_{tc}\cos\phi_c - F_{fc}\sin\phi_c}$$

$$\boxed{F_s = F_c\cos(\phi_c + \beta_a - \alpha_r)}$$

$$\boxed{V_s = V\frac{\cos\alpha_r}{\cos(\phi_c - \alpha_r)}}$$

Shear force can also be expressed as

$$\boxed{F_s = \tau_s A_s = \tau_s\, b\frac{h}{\sin\phi_c}}$$

Resultant cutting force can be now be expressed in terms of shear stress, friction and shear angles, width of cut, and feed rate as follows:

$$\boxed{F_c = \frac{F_s}{\cos(\phi_c + \beta_a - \alpha_r)} = \tau_s\, bh\frac{1}{\sin\phi_c\cos(\phi_c + \beta_a - \alpha_r)}}$$

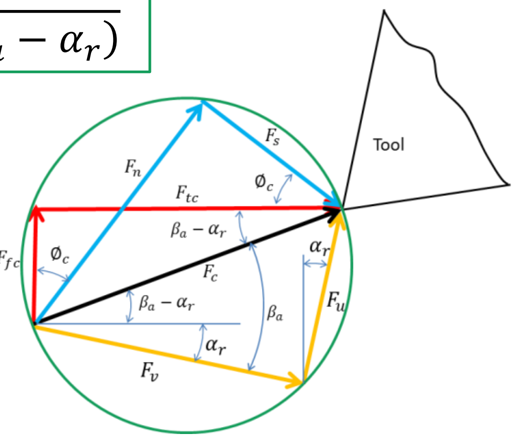
Know everything but for ϕ_c

Shear angle – from Merchant's energy principle

Resultant cutting force can be now be expressed in terms of shear stress, friction and shear angles, width of cut, and feed rate as follows:

$$\boxed{F_c = \frac{F_s}{\cos(\phi_c + \beta_a - \alpha_r)} = \tau_s\, bh \frac{1}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

Recalling the FCD



$$\boxed{F_{tc} = F_c \cos(\beta_a - \alpha_r) = \tau_s\, bh \frac{\cos(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

Power consumed during cutting: $\boxed{P_{tc} = VF_{tc}}$

Nature always take the past of least resistance, so during cutting ϕ_c takes a value such that least amount of energy is consumed, i.e. since τ_s, b, h, and β_a and α_r are given and do not change, power consumed becomes:

$$\boxed{P_{tc}(\phi_c) = \frac{constant}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

Shear angle – from Merchant's energy principle

Nature always take the past of least resistance, so during cutting , ϕ_c takes a value such that least amount of energy is consumed, i.e. since τ_s, b, h, and β_a and α_r are given and do not change, power consumed becomes:

$$\boxed{P_{tc}(\phi_c) = \frac{constant}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

$P_{tc}(\phi_c)$ will be a minimum when the denominator is a maximum, hence differentiate denominator w.r.t ϕ_c and equate it to zero:

$$\boxed{\cos\phi_c \cos(\phi_c + \beta_a - \alpha_r) - \sin\phi_c \sin(\phi_c + \beta_a - \alpha_r) = 0}$$

$$\boxed{\cos(2\phi_c + \beta_a - \alpha_r) = 0}$$

$$\boxed{2\phi_c + \beta_a - \alpha_r = \frac{\pi}{2}} \longrightarrow \boxed{\phi_c = \frac{\pi}{4} - \frac{\beta_a - \alpha_r}{2}}$$

Force prediction

Shear angle using Merchant's minimum energy principle

$$\boxed{\phi_c = \frac{\pi}{4} - \frac{\beta_a - \alpha_r}{2}}$$

Force prediction

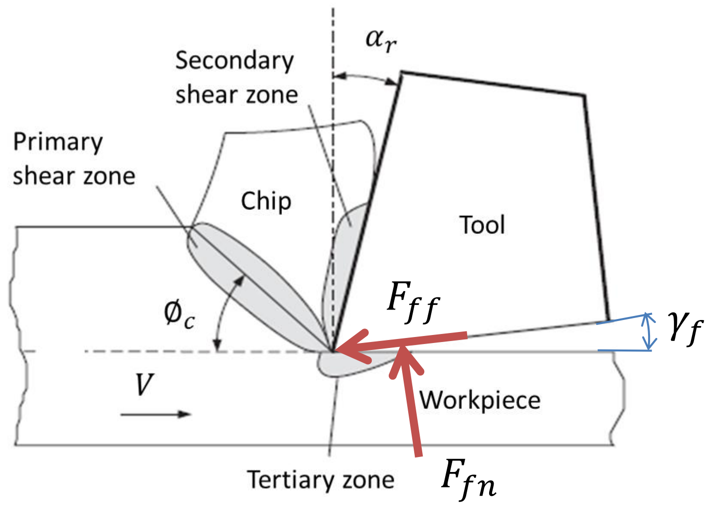
$$\boxed{F_{tc} = F_c \cos(\beta_a - \alpha_r) = \tau_s\, bh \frac{\cos(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

Power consumed

$$\boxed{P_{tc} = VF_{tc}}$$

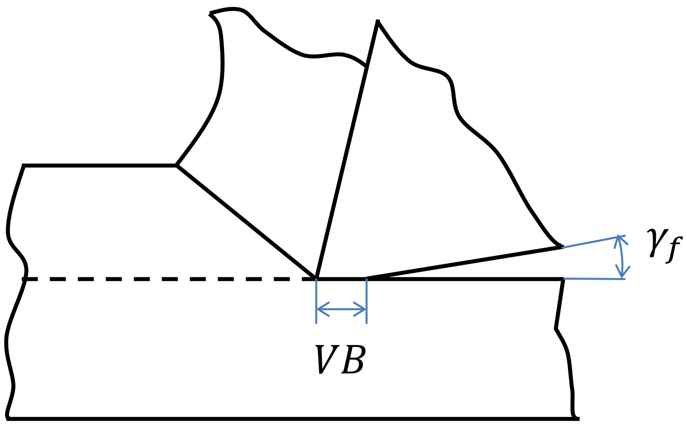
- Shear angle prediction with these and other such models are not very accurate
- But, they provide important relationships between tool geometry and shear angle – which is important for tool design, i.e. the rake angle must increase without compromising strength of the cutting edge
- Also important is the relationship between friction coeff. and shear angle, it gives a sense of friction characteristics between workpiece and tool, and suggests that for easier cutting, friction should be reduced
- Importantly, force and power consumed decrease with increase in shear angle

Tertiary deformation zone



- Contact mechanics between flank face and finished surface depends on tool wear, preparation of cutting edge and friction characteristics of tool and workpiece
- Assume total friction force on flank face is F_{ff}
- And force normal to flank face in F_{fn}
- Assume pressure (σ_f) on the flank face to be uniform (a gross oversimplification)

Tertiary deformation zone



Normal force on the flank face

$$\boxed{F_{fn} = \sigma_f VBb}$$

Because the tool rubs on the finished surface, there is friction

$$\boxed{\mu_f = F_{ff}/F_{fn}}$$

Resolving contact forces into tangential and feed directions →

$$\boxed{\begin{aligned} F_{te} &= F_{fn}\sin\gamma_f + F_{ff}\cos\gamma_f\,; \\ F_{fe} &= F_{fn}\cos\gamma_f + F_{ff}\sin\gamma_f; \end{aligned}}$$

Any measured forces will include forces due to shearing and to the tertiary deformation process, i.e. rubbing/ploughing at the flank of the cutting edge →

$$\boxed{\begin{aligned} F_t &= F_{tc} + F_{te}; \\ F_f &= F_{fc} + F_{fe}; \end{aligned}}$$

All cutting force expressions presented up until now were only for shearing, but in reality edge forces also exist, hence edge forces must be subtracted from measured tangential and feed forces before applying laws of orthogonal cutting mechanics

Mechanistic modeling for shear angle estimation

Recalling how shear forces are expressed

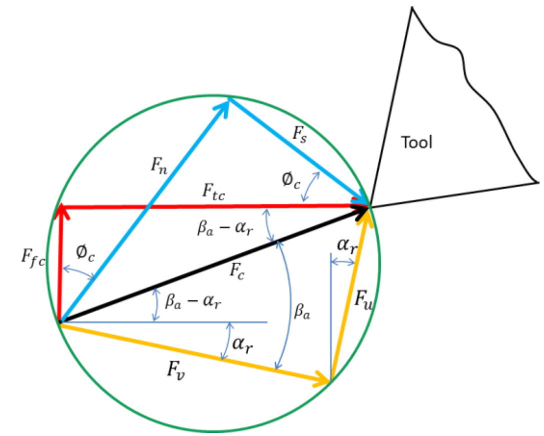
$$F_s = \tau_s A_s = \tau_s\, b \frac{h}{\sin \phi_c}$$

Resultant cutting force is expressed in terms of shear stress, friction and shear angles, width of cut, and feed rate as follows:

$$F_c = \tau_s bh \frac{1}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)} \quad \text{(a)}$$

Expressing tangential and feed forces in terms of resultant (from FCD)

$$F_{tc} = F_c \cos(\beta_a - \alpha_r)\,; \qquad F_{fc} = F_c \sin(\beta_a - \alpha_r)\,; \quad \text{(b)}$$



Substituting (a) into (b)

$$F_{tc} = bh\left[\tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]$$

$$F_{fc} = bh\left[\tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin \phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]$$

Mechanistic modeling for shear angle estimation

Measured main cutting forces as function of tool geometry and cutting conditions (uncut chip thickness, with of cut), and process-material-dependent terms ($\tau_s, \beta_a, \alpha_r, \phi_c$)

$$\boxed{F_{tc} = \mathrm{bh}\left[\tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]}$$

$$\boxed{F_{fc} = \mathrm{bh}\left[\tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]}$$

In metal cutting literature, we define tangential and feed force cutting force coefficients as

$$\boxed{K_{tc} = \tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

$$\boxed{K_{fc} = \tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}}$$

Cutting force coefficients are a function of shear stress, friction angle, shear angle tool geometry, i.e. rake angle. Of these only know tool geometry beforehand. Moreover, shear angle models are inaccurate, and friction too is less understood. Hence these are often calibrated from experiments.

Mechanistic modeling

$$F_t = F_{tc} + F_{te};$$
$$F_f = F_{fc} + F_{fe};$$

$$F_{tc} = bh\left[\tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]$$

$$F_{fc} = bh\left[\tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}\right]$$

$$F_t = K_{tc}bh + K_{te}b;$$
$$F_f = K_{fc}bh + K_{fe}b;$$

$$K_{tc} = \tau_s \frac{\cos(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}$$

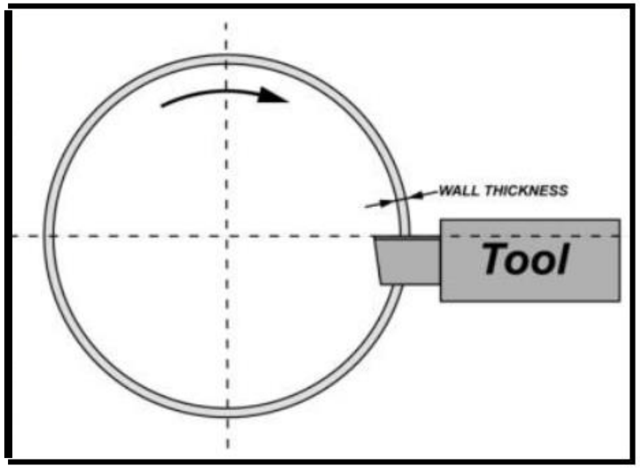
$$K_{fc} = \tau_s \frac{\sin(\beta_a - \alpha_r)}{\sin\phi_c \cos(\phi_c + \beta_a - \alpha_r)}$$

Experimental identification of cutting coeffs.

Step 1
Select material of interest

Step 2
Setup tube on a lathe

Step 3
Turning tool with zero inclination angle, flat rake face, and no chip breaker



Step 4
Select speeds most often used in your application

Step 5
Measure cutting forces using dynamometer

Step 6
Repeat tests for a range of feed rates

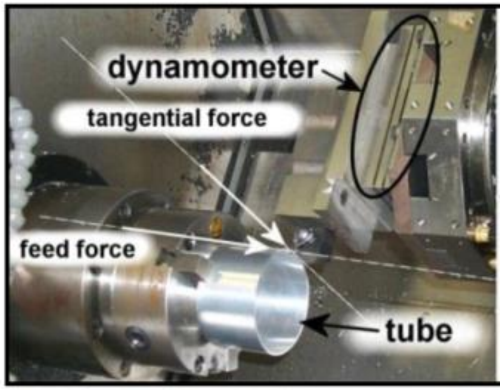
Repeat tests for different speeds and tool geometries as necessary

Run cutting coefficient identification algorithm

Orthogonal cutting coefficients

Advantages:

- Experiments and measurements are not very difficult
- Identified coefficients can be transformed using the orthogonal to oblique transformation methods
- This enables prediction of cutting coefficients for oblique cutting using experiments from orthogonal cutting



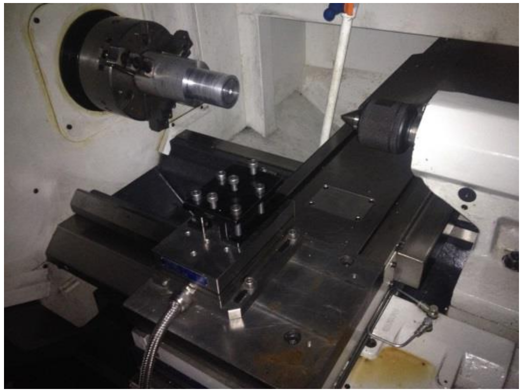
Source: CutPro Guide, MAL Inc.

Disadvantages:

- Need to specially prepare tubes and tools
- Need to conduct quite a many tests for complete characterization of speeds, feeds and geometry
- Not directly applicable to tools with proper geometry and chip breakers used in many applications

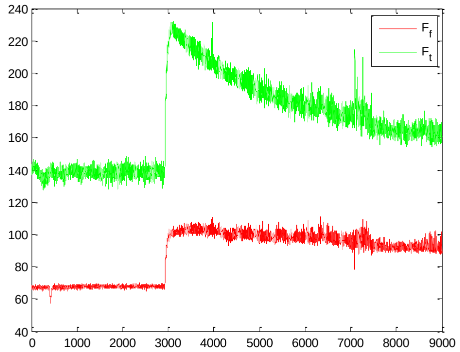
Orthogonal cutting tests for identification

Conduct a dedicated series of tests at different feed rates and identify coefficients directly for the tool-workpiece-cutting parameter combination of interest

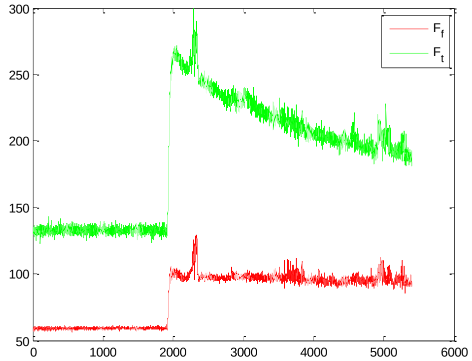


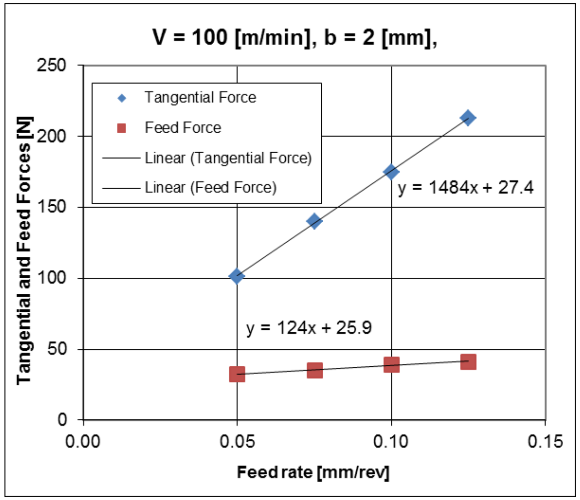
Test No:	Feed Rate h [mm/rev]	Measured Ftc [N]	Measured Ffc [N]
1	0.050	101	32
2	0.075	140	35
3	0.100	175	39
4	0.125	213	41

Test 1



Test 4





Orthogonal cutting tests for identification

Forces are composed of shearing and edge rubbing force components

$$F_t = F_{tc} + F_{te}; \quad F_f = F_{fc} + F_{fe};$$

or

$$F_t = K_{tc}bh + K_{te}b; \quad F_f = K_{fc}bh + K_{fe}b;$$

Consider only shearing components (since edge forces do not contribute to cutting):

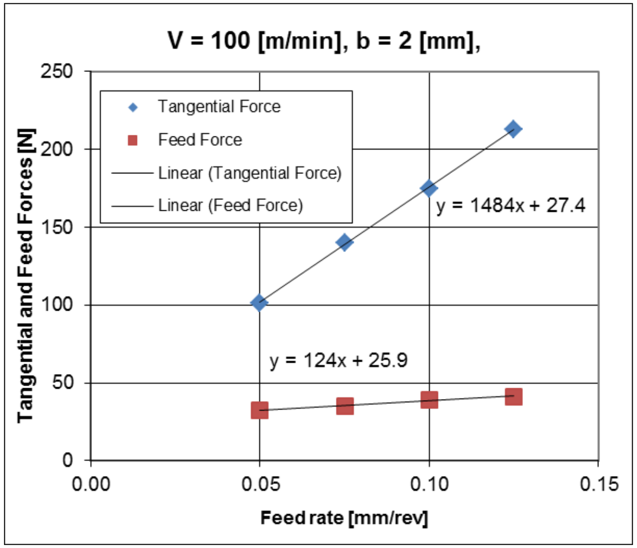
$$F_{tc} = K_{tc}bh; F_{fc} = K_{fc}bh;$$

Hence the cutting coefficients are:

Prediction

$$K_{tc} = \frac{F_{tc}}{bh}; K_{fc} = \frac{F_{fc}}{bh};$$

$$K_{te} = \frac{F_{te}}{b}; K_{fe} = \frac{F_{fe}}{b};$$



Slopes from graph: $K_{tc}bh$ and $K_{fc}bh$

Slopes from graph: 1484 and 124

Units: $\frac{N}{mm^2}$

$$K_{tc} = 712; \quad K_{fc} = 62;$$

$$K_{te} = 14; \quad K_{fe} = 13;$$

Units: $\frac{N}{mm}$