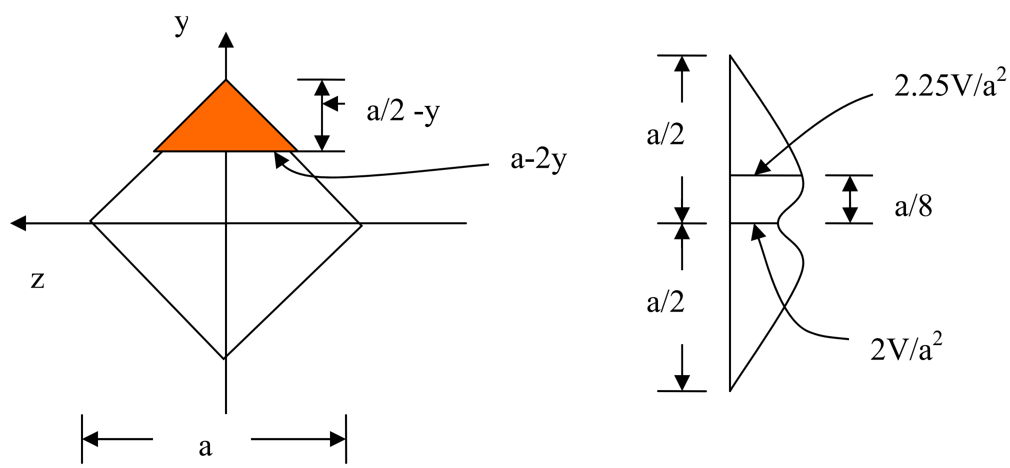
ESO 202A/204: Mechanics of Solids (2016 -17 II semester) Solution of Assignment No. 9

9.1



Shear stress

$I_{zz}=a^4/48$

Area of the shaded portion = $\frac{1}{2}$ (a-2y) (a/2 – y) = $\frac{1}{4}(a-2y)^2$

CG of the area from NA= y+$\frac{1}{3}$ (a/2 – y)= (1/6) (a+4y)

Therefore Q = $\frac{1}{4}(a-2y)^2.\frac{1}{6}(a+4y)$, b= (a-2y)

$$\tau_{xy} = \frac{VQ}{bI_{zz}} = \frac{2V(a-2y)(a+4y)}{a^4}$$

For τ_{xy} to be minimum $\frac{d\tau_{xy}}{dy} = 0 \Rightarrow y = a/8$

$\tau_{xy,max}$= 2.25 V/a^2

$\tau_{xy,NA}$= 2.0V/a^2

9.2

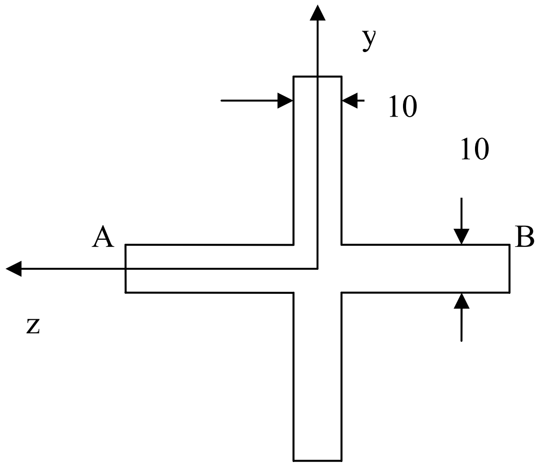
$$I_{zz} = \frac{10x100^3}{12} + 90x\frac{10^3}{12} = 840000\,\text{mm}^4$$

At level AB,

Q= 10x45(22.5+5) = 12350mm^3

b= 10 mm, 100mm

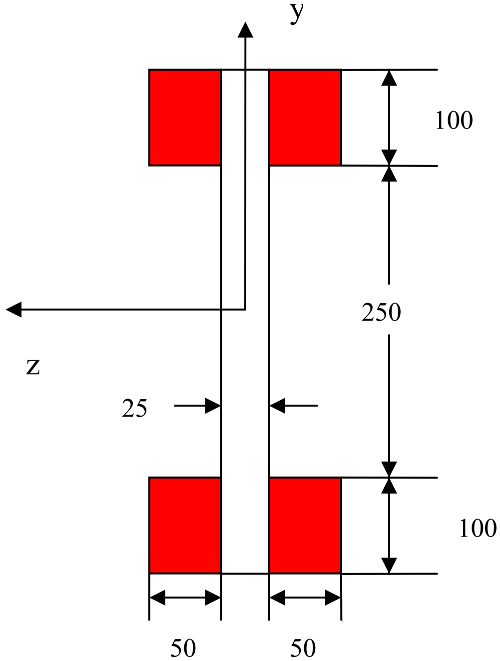
$$\tau_{xy} = \frac{10000x12350}{bx840000} = 14.7\text{ MPa and }1.47\text{ MPa}$$



At the neutral axis:
b=100mm
Q=12350+100x5x2.5=13600mm^3

$\tau_{xy}= \dfrac{10000x13600}{100x840000} = 1.62$ MPa

9.3



$$I_{zz} = 4\left[\frac{50x100^3}{12} + 50x100x175^2\right] + \frac{25x450^3}{12} = 8186.7x10^5\,\text{mm}^4$$

(i) for bending M max $= \dfrac{\sigma_{xx\max} I_{zz}}{y} = \dfrac{10x8186.7x10^5}{225}$ Nmm

= 36.4kNm

(ii) for shear in wood (Web portion)

Q= 125x100x175+125x25x62.5=2385000mm^3

$$V=\frac{\tau_{xy} I_{zz} b}{Q}=\frac{0.8x8186.7x10^5 x25}{2385000}=6870\,\text{N}$$

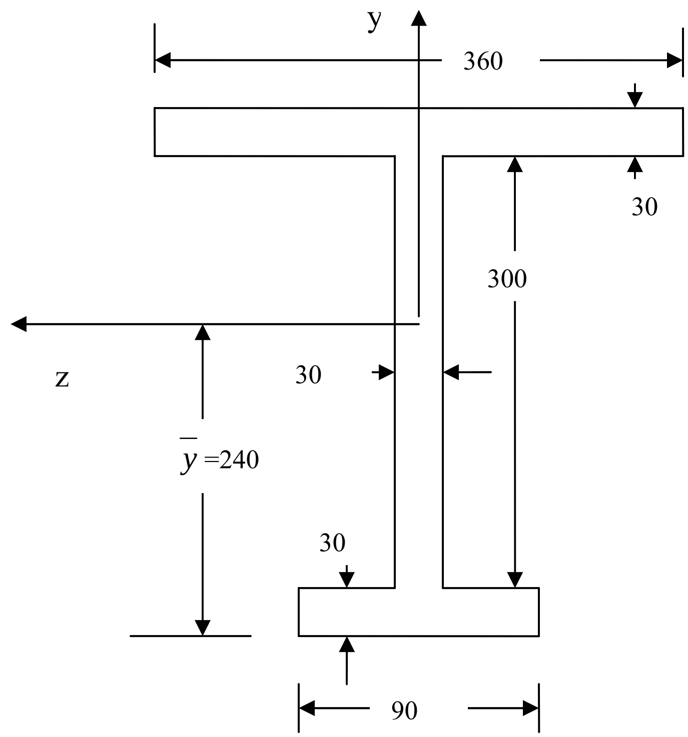
(iii) for shear in glue joint b=100mm Q=50x100x175=875000mm^3

$$V=\frac{0.4x8186.7x10^5 x100}{875000}=37500\,\text{N}$$

Thus, maximum allowable shear force=6870N

Maximum allowable bending moment=36.4kNm

9.4



$$\bar{y}=\frac{360x30x345 \pm 300x30x180+90x30x15}{360x30+300x30+90x30}=240\,\text{mm}$$

$$I_{zz}=\frac{360x30^3}{12}+360x30x105^2+\frac{30x300^3}{12}+30x300x60^2+\frac{90x30^3}{12}+90x30x225^2=3.56x10^8$$

I_{zz}= 3.56x10^8 mm^4

(i)= Top flange and web junction:

Q= 360x30x105=1134000mm^3

b=360mm,30mm

$$\tau_{xy}=\frac{120x10^3 x1134000}{bx3.56x10^8}=1.05\text{MPa and }12.7\text{MPa}$$

(ii)Neutral Axis:

Q=1134000+30x90x45=1255500mm^3

$$\tau_{xy}=\frac{120x10^3 x1255500}{30x3.56x10^8}=14.1\,\text{MPa}$$

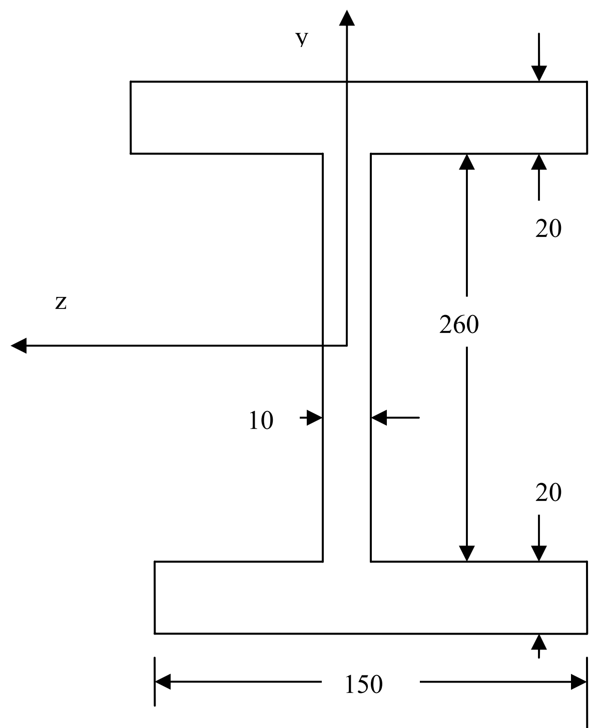
(iii) Bottom flange and web junction:

$Q=30x90x225=607500mm^3$

$b=90mm,30mm$

$$\tau_{xy}=\frac{120x10^3x607500}{bx3.56x10^8}=2.27\text{MPa and }6.812\text{MPa}$$

9.5



M=96kNm V=32kN

$$I_{zz}=\frac{150x300^3}{12}-\frac{140x260^3}{12}=13250x10^4\text{ mm}^4$$

(i) At y=150mm, $\tau_{xy}=0$

$$\sigma_{xx}=-\frac{96x10^6x150}{13250x10^4}=-108.6MPa$$

Since shear stress is zero, these are principal stresses.

(ii) At NA: $\sigma_{xx}=0$;

$Q=150x20x140+10x130x65=504500mm^3$

$b=10mm$

therefore, $\tau_{xy}=\dfrac{32x10^3x504500}{10x13250x10^4}=12.2\text{ MPa}$

(iii) At 50mm from NA:

$$\sigma_{xx}=\frac{50}{150}(-108.6)=-36.2\text{ Mpa}$$

$Q=150x20x140+80x10x(40+50)=492000mm^3$

$$\tau_{xy}=\frac{32x10^3x492000}{10x13250x10^4}=11.9\text{ MPa}$$