Department of Physics, IIT-Kanpur
PHY103A/N, 2014-15, Sem-II

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1. Evaluate curl of the vector function $\mathbf{A} = A_0\hat{\phi}/s$ everywhere. Here, A_0 is a constant and ϕ and s are cylindrical coordinates. [**6** marks]
   In cylindrical coordinates,

$$\nabla \times \mathbf{V} = \Big[\frac{1}{s}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\Big]\,\hat{s} + \Big[\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\Big]\,\hat{\phi} + \frac{1}{s}\Big[\frac{\partial (sV_\phi)}{\partial s} - \frac{\partial V_s}{\partial \phi}\Big]\,\hat{z}.$$

   Answer: $\nabla \times \mathbf{A} = 0$ except at $s = 0$. The value of $\nabla \times \mathbf{A}$ at $s = 0$ can be evaluated using Stokes' theorem:

$$\int_S (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \oint_C \mathbf{A} \cdot d\mathbf{r}.$$

   The value of the line integral for the given vector is

$$\oint_C \mathbf{A} \cdot d\mathbf{r} = A_0 \int_0^{2\pi} \frac{\hat{\phi}}{s} \cdot s d\phi\, \hat{\phi} = A_0 \int_0^{2\pi} d\phi = A_0 2\pi.$$

   Note that the line integral does not depend on the radius of the circle. In order to satisfy the Stokes' theorem, we have to take

$$\nabla \times \mathbf{A} = A_0 2\pi \delta^2(\mathbf{s})\,\hat{z} = A_0 2\pi \delta(x)\delta(y)\,\hat{z}.$$

   To make the surface integral non-zero, the direction of $\nabla \times \mathbf{B}$ has to be parallel to the z axis.

2. The electrostatic potential due to some charge configuration is given by

$$V(x, y, z) = -\frac{V_0}{a^4}(x^2yz + xy^2z + xyz^2),$$

   where V_0 and a are constants. Calculate charge density in the xy plane and at the point $P(a, a, a)$. [**6**]

   Answer: The electric field is

$$\mathbf{E} = \frac{V_0}{a^4}\Big[\hat{i}\,(2xyz + y^2z + yz^2) + \hat{j}\,(x^2z + 2xyz + xz^2) + \hat{k}\,(x^2y + xy^2 + 2xyz)\Big].$$

   The charge density

$$\rho(x, y, z) = \epsilon_0 \nabla \cdot \mathbf{E} = \frac{2V_0\epsilon_0}{a^4}(xy + yz + zx).$$

   The charge densities

$$\rho(x, y, z = 0) = \frac{2V_0\epsilon_0}{a^4}xy, \qquad \rho(a, a, a) = \frac{6V_0\epsilon_0}{a^2}.$$

3. A thick metallic shell of inner radius a and outer radius b has a charge Q on it. A point charge q is kept at the center of the shell. Calculate charge on each surface of the shell. Also, calculate electric field and potential everywhere. [**7**]

Answer: The problem has a spherical symmetry. The induced charge densities on the inner and outer walls of the shell must be uniform. The total charge induced on the cavity wall is $q_{\text{ind}} = -q$. The total charge on the outer wall is $Q + q$.

The electric fields in various regions:

$$\mathbf{E}(r \geq b) = \frac{(Q+q)}{4\pi\epsilon_0 r^2}\hat{r}, \quad \mathbf{E}(a < r < b) = 0, \quad \mathbf{E}(r \leq a) = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}.$$

The potentials in various regimes:

$$V(r \geq b) = \frac{(Q+q)}{4\pi\epsilon_0 r} \quad V(a < r < b) = \frac{(Q+q)}{4\pi\epsilon_0 b}.$$

The potential at $r < a$ is

$$V(r) = \frac{1}{4\pi\epsilon_0}\Big[\frac{Q+q}{b} - \frac{q}{a} + \frac{q}{r}\Big].$$

4. Consider an infinitely-long cylinder of radius R carrying a non-uniform volume charge density $\rho = ks^2$, with k being a constant. Using Gauss's law, find the electric field everywhere. [**6**]
Answer:

$$\mathbf{E}(s \leq R) = \frac{ks^3}{4\epsilon_0}\hat{s}, \quad \mathbf{E}(s \geq R) = \frac{kR^4}{4\epsilon_0\, s}\hat{s}.$$