

# High Precision Actuators

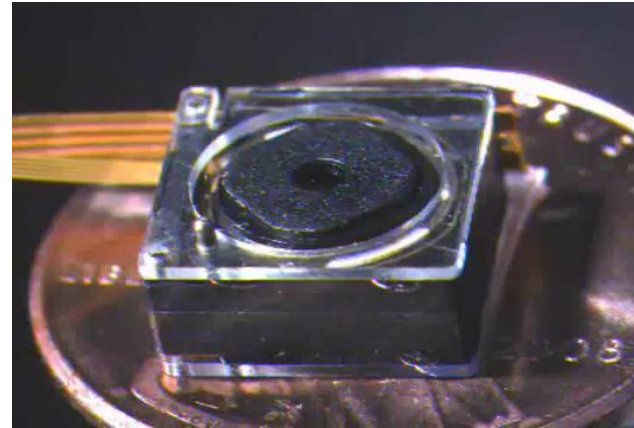
# Contents

- High Bandwidth Low Strain(HBLS) Smart Actuators
- Multilayered Piezoelectric Materials
  - Series and Parallel modeling for Mechanical systems
  - Series and Parallel modeling for Electrical systems
  - Piezoelectric Actuator Modeling
- Advantages of Multilayered PZT Actuators
- How multilayers are developed?

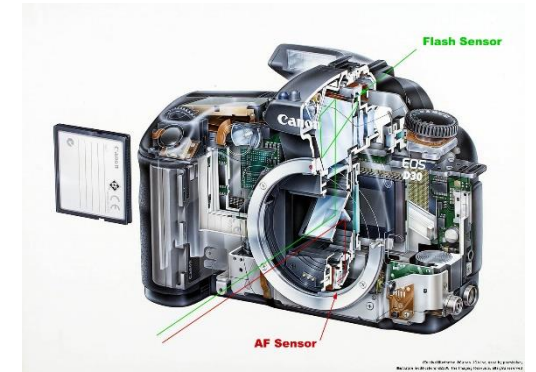
# Common Smart Actuators

## Thin Disk Buzzer

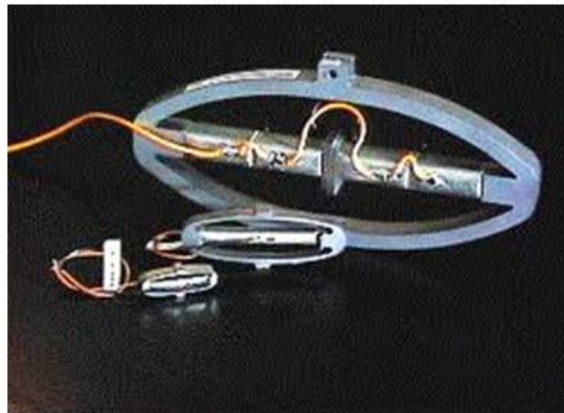
Metal disk with piezoelectric disk attached, used in a buzzer.



Digital Single Lens reflector Camera



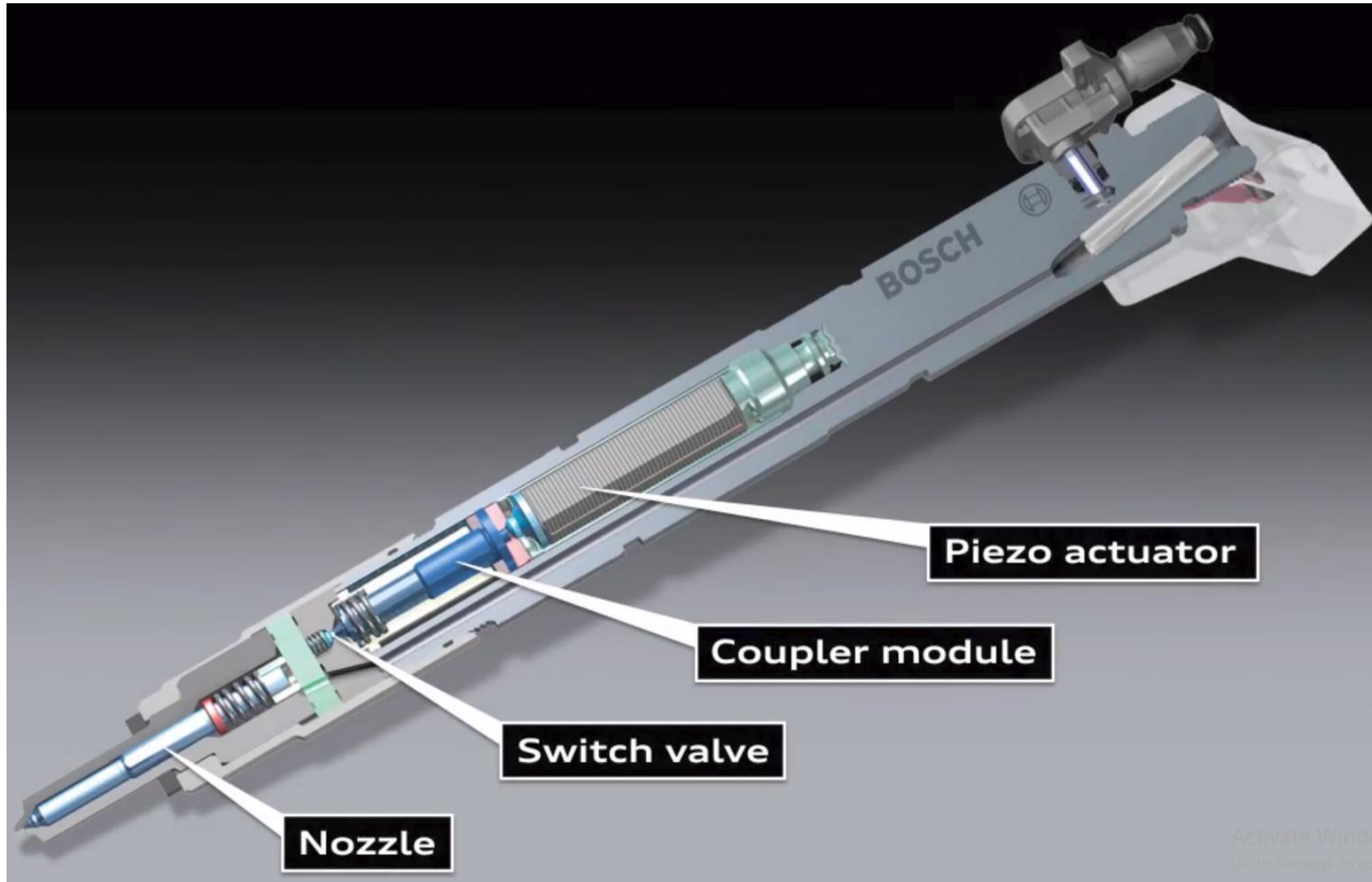
## Amplified Piezo Actuator



Epson Printer

# Introduction

- Current trend in **Automotive Electronics** is to use actuators for functions which require faster, more powerful and highly precise motion.
  - Eg: Piezo Fuel Actuator, Piezoelectric Pressure Sensors, Piezo Level Sensors and Air Transducers
- Simple Unimorph/Bimorph/Discs are not popular in the industrial scale due to lack of efficiency, displacement and safety.
- Initiated application of Piezoelectric Actuators and Rheological Fluids for the control of Fuel Injection and motion control.



Cut-out of a [Piezoelectric Fuel Injector](#) (BOSCH)

## Comparison of Different Actuators

Type	Device	Accuracy	Response
Pneumatic	Motor	Degrees	10 secs
Hydraulic	Motor	Degrees	1 sec
Electro-magnet	Stepper	10 $\mu\text{m}$	0.1 sec
Piezoelectric	Actuator	0.01 $\mu\text{m}$	0.0001 sec
Magnetostrictive	Actuator	0.01 $\mu\text{m}$	0.001 sec
Piezoelectric	Ultrasonic	minutes	0.001 sec

## Issues with PZT

- A relatively high electric field is necessary to develop an equivalent system with associated high voltage requirement to generate appreciable or useful strain.
- Example: PZT block of height 1mm, apply 1KV field,  $d_{33}=600\text{m/pV}$ , strain  $6 \times 10^{-4}$  and corresponding displacement is the same magnitude in mm.

# How can we Maximize the Displacement

Consider a multilayered piezoelectric stack, neglecting elastic deformation, total displacement available from a ' $n$ ' layered stack will be:

$$\Delta = \left( l * d * \frac{V}{(l/n)} \right) = d * V * n$$

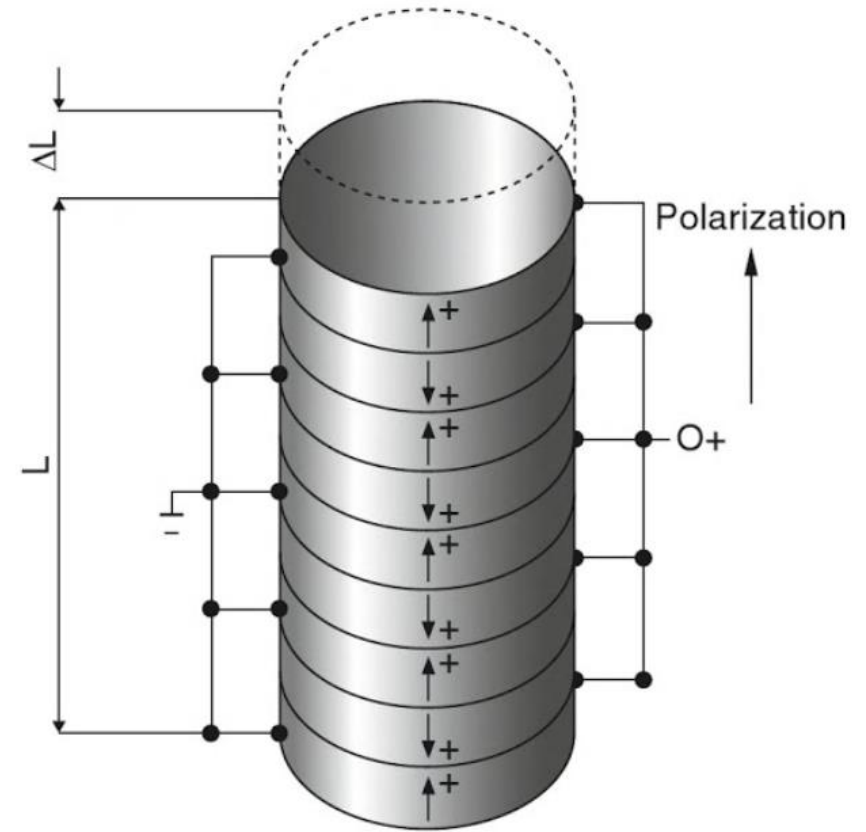
where,  $l$  = length

$n$  = number of layers

subjected to a voltage  $V$

$d$  = piezoelectric coefficient

Total displacement is directly proportional to the number of layers  $n$ !





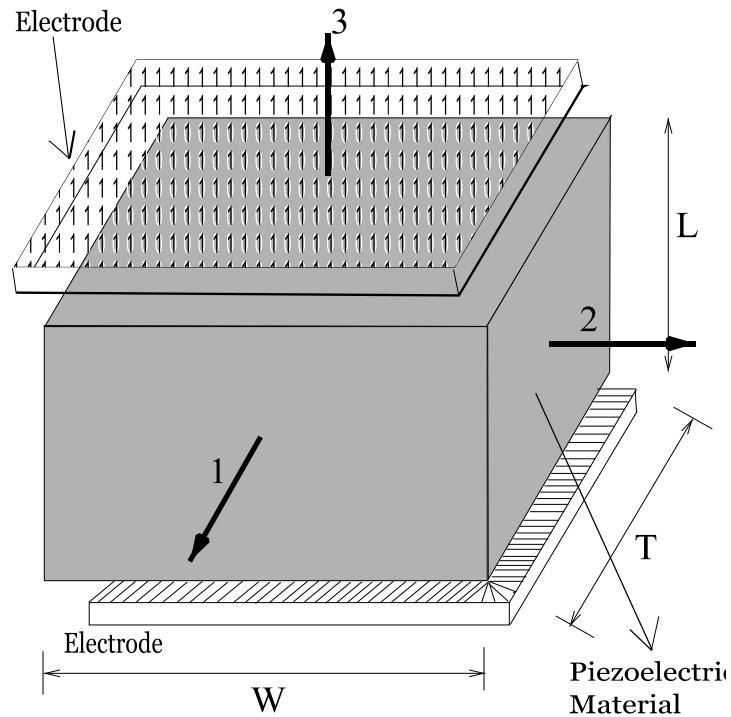
## Multilayer Actuators

- Typical layer thickness is about 50 $\mu$ m
- Typical strain available 0.1%
- Hence, for a 100 mm stack actuator with 2000 piezoelectric layers and an applied voltage of about 100V, the displacement will be:  $10 \times 10^{-9} \times 100 \times 2000 = 200 \mu\text{m}$
- Blocking force = 100 kgf
- Lifetime =  $10^{11}$  cycles

# Constitutive Equations of a Stack Actuator connected with a Gripper

$D_3$  is the Electric field Displacement

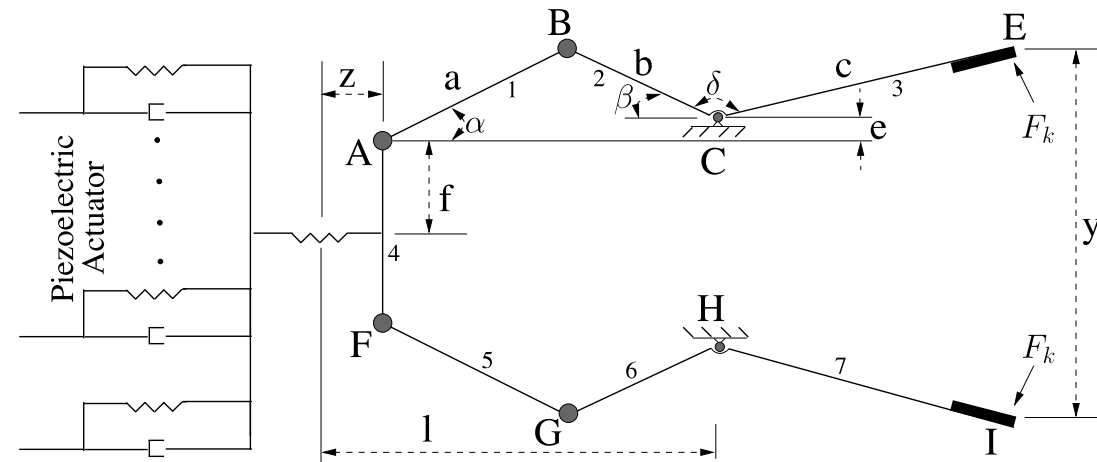
$S_3$  is the Strain at a point



$$D_3 = d_{33}\sigma_3 + \epsilon_{33}E_3.,$$

$$S_3 = \frac{1}{E_p} \sigma_3 - d_{33} E_3.$$

where considering plane stress assumption,  $\mathbf{s}_{33}^E = \frac{1}{E_p}$ ,  $E_p$  being the modulus of elasticity of PZ material.



# Modified Relationship in terms of Force and Voltage

$L$  : length along actuation direction (direction 3)

$A$  : normal cross-sectional area

$F$  : Force of actuation

$V$  : applied voltage

$$\epsilon = \frac{\Delta L}{L}, \quad \sigma = \frac{F}{A}, \quad E = \frac{V}{L}.$$

$$D_3 = d_{33} \left( \frac{F}{A} \right) + \epsilon_{33} \left( \frac{V}{L} \right).$$

$$\Delta L = \left( \frac{1}{E_p} \right) \left( \frac{F}{A} \right) L - d_{33} V.$$

# Force Delivered by the Piezo-stack to the Gripper

$$F_{st} = k_{con}(z - \Delta L_{st}),$$

$$\Delta L_{st} = z - \frac{F_{st}}{k_{con}}.$$

$$k_{con} = \frac{AE}{L_o}$$

where,  $A$  : cross-sectional area of the actuator assembly

$E$  : Young's Modulus of assembly connector material

$L_o$  : initial length of the connector

# Mechanical Series – Electrical Series

When both the electrical and mechanical parts of PZ actuator have been modeled as series assemblies of capacitors and springs respectively, then the total applied external voltage across actuator stack,  $V_{st}$  can be given as:

$$V_{st} = nV,$$

The force delivered by PZ stack actuator,  $F_{st}$  and the net displacement of the stack,  $\Delta L_{st}$  is given by:

$$F_{st} = F,$$

$$\Delta L_{st} = n\Delta L,$$

where,  $V$  : voltage across each element,

$F$  : force delivered by each element

$\Delta L$  : displacement of each element

# Mechanical Series – Electrical Series

Displacement of Each Stack Element:

$$\Delta L_{st} = \left( \frac{1}{E_p} \right) \left( \frac{F_{st}}{A} \right) nL - d_{33} V_{st}.$$

Force Provided by the Stack Element:

$$F_{st} = \frac{z + d_{33} V_{st}}{\left( \frac{1}{k_{con}} + \frac{nL}{E_p A} \right)}$$

# Mechanical Parallel – Electrical Series

$$\begin{aligned} V_{st} &= nV, \\ F_{st} &= nF, \\ \Delta L_{st} &= \Delta L, \end{aligned} \quad \Delta L_{st} = \left( \frac{1}{E_p} \right) \left( \frac{F_{st}}{nA} \right) L - d_{33} \frac{V_{st}}{n}$$

$$F_{st} = \alpha_2 z + \beta_2 V_{st}$$

$$\text{where } \alpha_2 = \frac{1}{\left( \frac{1}{k_{con}} + \frac{L}{E_p n A} \right)}, \quad \beta_2 = \frac{d_{33}}{n} \alpha_2$$

# Mechanical Series – Electrical Parallel

$$V_{st} = V,$$

$$F_{st} = F,$$

$$\Delta L_{st} = n\Delta L,$$

$$\Delta L_{st} = \left( \frac{1}{E_p} \right) \left( \frac{F_{st}}{A} \right) nL - nd_{33}V_{st}.$$

$$V_{st} = V,$$

$$F_{st} = nF,$$

$$\Delta L_{st} = \Delta L,$$

$$F_{st} = \alpha_3 z + \beta_3 V_{st}$$

$$where \quad \alpha_3 = \frac{1}{\left( \frac{1}{k_{con}} + \frac{nL}{E_p A} \right)}, \quad \beta_3 = nd_{33}\alpha_3$$



# Mechanical Parallel – Electrical Parallel

$$V_{st} = V,$$

$$F_{st} = nF,$$

$$\Delta L_{st} = \Delta L,$$

$$\Delta L_{st} = \left( \frac{1}{E_p} \right) \left( \frac{F_{st}}{A} \right) nL - nd_{33}V_{st}.$$

$$F_{st} = \alpha_3 z + \beta_3 V_{st}$$

$$\text{where } \alpha_3 = \frac{1}{\left( \frac{1}{k_{con}} + \frac{nL}{E_p A} \right)}, \quad \beta_3 = nd_{33}\alpha_3$$

# Mechanical Series – Electrical Parallel

$$V_{st} = V,$$

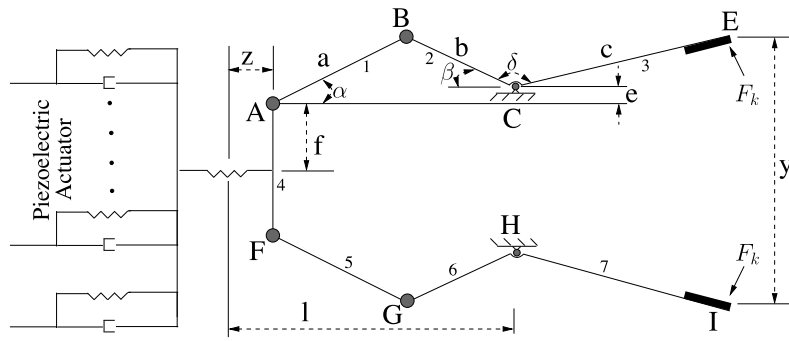
$$F_{st} = nF,$$

$$\Delta L_{st} = \Delta L,$$

$$\Delta L_{st} = \left( \frac{1}{E_p} \right) \left( \frac{F_{st}}{nA} \right) L - d_{33} V_{st}.$$

$$F_{st} = \alpha_4 z + \beta_4 V_{st}$$

$$\text{where } \alpha_4 = \frac{1}{\left( \frac{1}{k_{con}} + \frac{L}{E_p nA} \right)}, \quad \beta_4 = d_{33} \alpha_4$$



Electrical System

Series

Parallel

Series

$$F_{st} = \frac{z + d_{33}V_{st}}{\left(\frac{1}{k_{con}} + \frac{nL}{E_pA}\right)}$$

$$F_{st} = \frac{z + d_{33}nV_{st}}{\left(\frac{1}{k_{con}} + \frac{nL}{E_pA}\right)}$$

Mechanical

System

Parallel

$$F_{st} = \frac{z + \frac{d_{33}V_{st}}{n}}{\left(\frac{1}{k_{con}} + \frac{L}{E_pnA}\right)}$$

$$F_{st} = \frac{z + d_{33}V_{st}}{\left(\frac{1}{k_{con}} + \frac{L}{E_pnA}\right)}$$

## Examples of commercially available PZ actuators with characteristic properties

Characteristic property	Commercial PZ actuator examples
High mechanical compliance	P-601, P-602, P-603, P-604 PiezoMove series
High mechanical stiffness	P-882 P-888 PICMA
Low response time	P-885.55, P-885.95, P-888.55 encapsulated PICMA
High robustness to electrical disturbances	N-470, N-470.V, E-870 PiezoMike

## Governing Criteria

- The PZT actuators showing higher mechanical compliance should be modelled as a series assembly of springs; those with higher stiffness can be modelled with a parallel assembly of springs.
- In the electrical domain, PZ actuators having faster response time should be modelled with a series assembly of the capacitors; those showing higher robustness to electrical disturbances can be modelled with a parallel circuit

## Other important properties

- The resonating frequency of a fixed-free multilayer actuator is given by:

$$f_n = \frac{1}{2l \sqrt{\rho S_{33}^D}}$$

- Where,  $\rho$  is the density and  $S_{33}$  denotes the compliance modulus
- For example, one 1 cm sample will have resonating frequency about 100 kHz.

# Advantages of Multilayered Piezoelectric Actuators

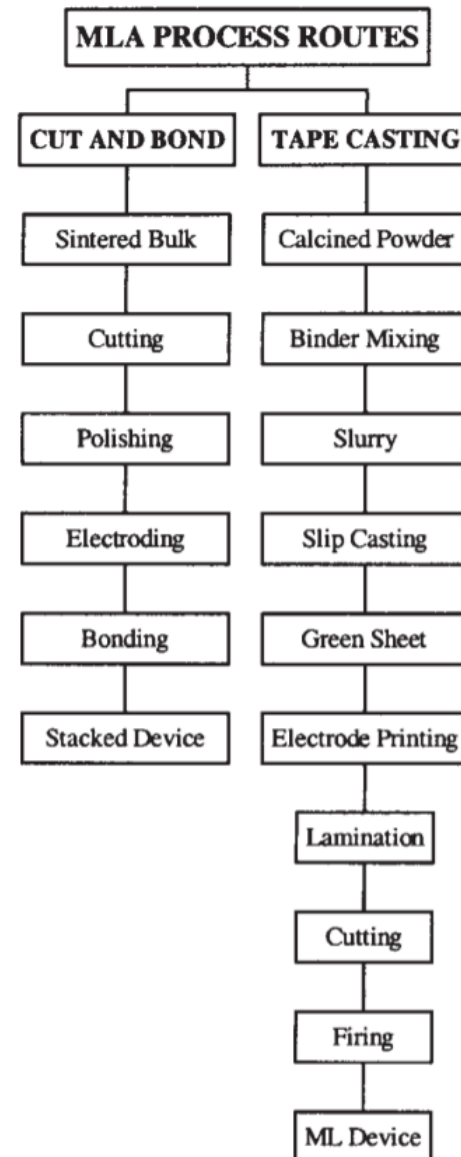
- Requires less voltage
- Produces larger deformation/displacement
- Safer to use
- High Life Cycle
- Lighter and More Compact
- Concurrent engineering – advantages from the development of multi-layered capacitors

## How Multilayers are developed?

- Two common techniques – Cut and Bond and Tape-Casting.
- In cut and bond technique PZT wafers are cut (typical thickness 0.2mm) and bond with intermittent metal foils. Major draw back is that this is a labor intensive process.
- In tape-casting method, ceramic green sheets are printed with electrodes and cofired. There are various ways of electrically connecting such layers.



# How Multilayers are developed?

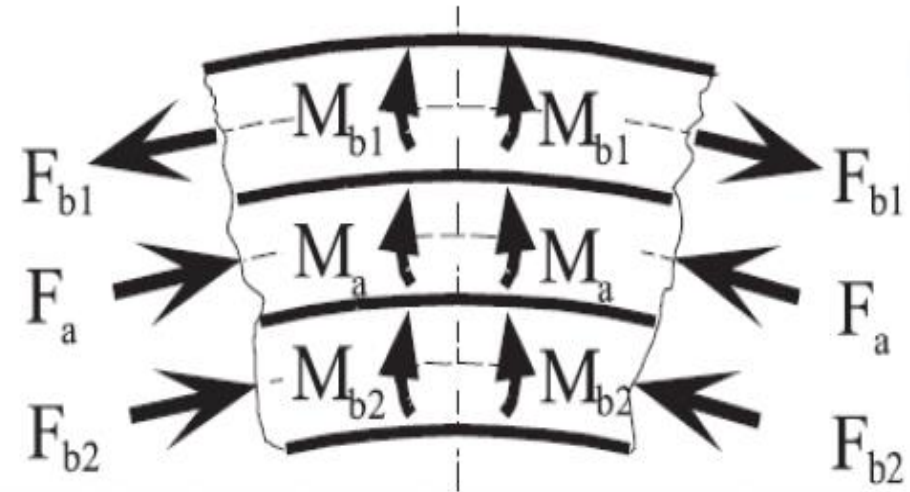
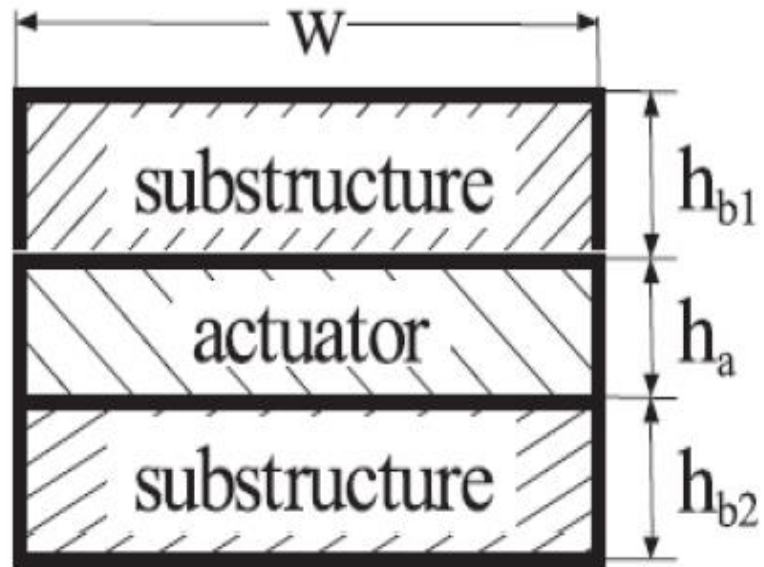


# Governing Equation for Embedded Actuation

# Introduction

- In this lecture, we will consider one of the high precision actuators – Unimorph actuator and obtain the governing equation of the actuator. The major assumptions for the system are as follows:
  - Formulation is based on Euler-Bernoulli Beam Theory
  - Ideal Bonding of Layers are assumed
  - Linear Strain Distribution and Continuity of strain across the layers are assumed
  - Small Curvature of the Unimorph is considered

# An Unimorph System



## Actuator Governing Equation

Consider force and moment equilibrium of the system:

$$F_{b1} = F_a + F_{b2},$$

$$M_{b1} + M_a + M_{b2} = F_{b1} \left( \frac{h_a + h_{b1}}{2} + \frac{h_a + h_{b2}}{2} \frac{F_{b2}}{F_{b1}} \right),$$

$$\frac{F_{b1}}{E_b A_{b1}} + \frac{M_{b1} h_{b1}}{2 E_b I_{b1}} = - \frac{F_a}{E_a A_a} - \frac{M_a h_a}{2 E_a I_a} + \varepsilon_{a,0},$$

$$- \frac{F_a}{E_a A_a} + \frac{M_a h_a}{2 E_a I_a} + \varepsilon_{a,0} = - \frac{F_{b2}}{E_b A_{b2}} - \frac{M_{b2} h_{b2}}{2 E_b I_{b2}},$$

## Actuator Governing Equation contd..

For the active layer, the strain may be expressed as:

$$\varepsilon_{a,0} = d_{31} \frac{V}{h_a}$$

Where,  $V$  is the applied voltage,  $h_a$  the thickness of the active layer and  $d_{31}$  is the electro-mechanical coupling coefficient.

Assuming equal curvature for the composite system:

$$\frac{E_a I_a}{M_a} = \frac{E_b I_{b1}}{M_{b1}} = \frac{E_b I_{b2}}{M_{b2}}.$$

## Actuator Governing Equation contd..

Maximum Moment Generated by the system may be expressed as:

$$M_f = M_{b1} + M_a + M_{b2} = C_f \varepsilon_{a,0}$$

Where,

$$C_f = \frac{C_{f1}}{E_a C_{f2} + E_b C_{f3}},$$

$$C_{f1} = 28A_a E_a E_b [A_{b1}(h_a + h_{b1}) - A_{b2}(h_a + h_{b2})] C_{f4},$$

$$C_{f2} = A_a \left\{ A_{b1} E_b (h_a^2 + 15h_a h_{b1} + 14h_{b1}^2) + 14 \left[ A_{b2} E_b (h_a + h_{b2})^2 + 4C_{f4} \right] \right\},$$

$$C_{f3} = A_{b1} \{ A_{b2} E_b (h_{b2} - h_{b1}) [15h_a + 14(h_{b1} + h_{b2})] \} + 56(A_{b1} - A_{b2}) C_{f4},$$

$$C_{f4} = E_a I_a + E_b (I_{b1} + I_{b2}),$$



# Actuator Transfer Function

The Actuator transfer function for an applied voltage  $V(s)$  may now be expressed as:

$$\frac{M(s)}{V(s)} = \frac{C_f d_{31}}{h_a}$$

This shows that the Active Moment-Applied Voltage relationship is of 0-th order. However, if you consider the applied voltage to be proportional to the velocity of the system then the relationship will become

$$\frac{M(s)}{X(s)} = \frac{C_f C d_{31} s}{h_a}$$

Where,  $C$  is the control gain.



## Application in a closed loop system

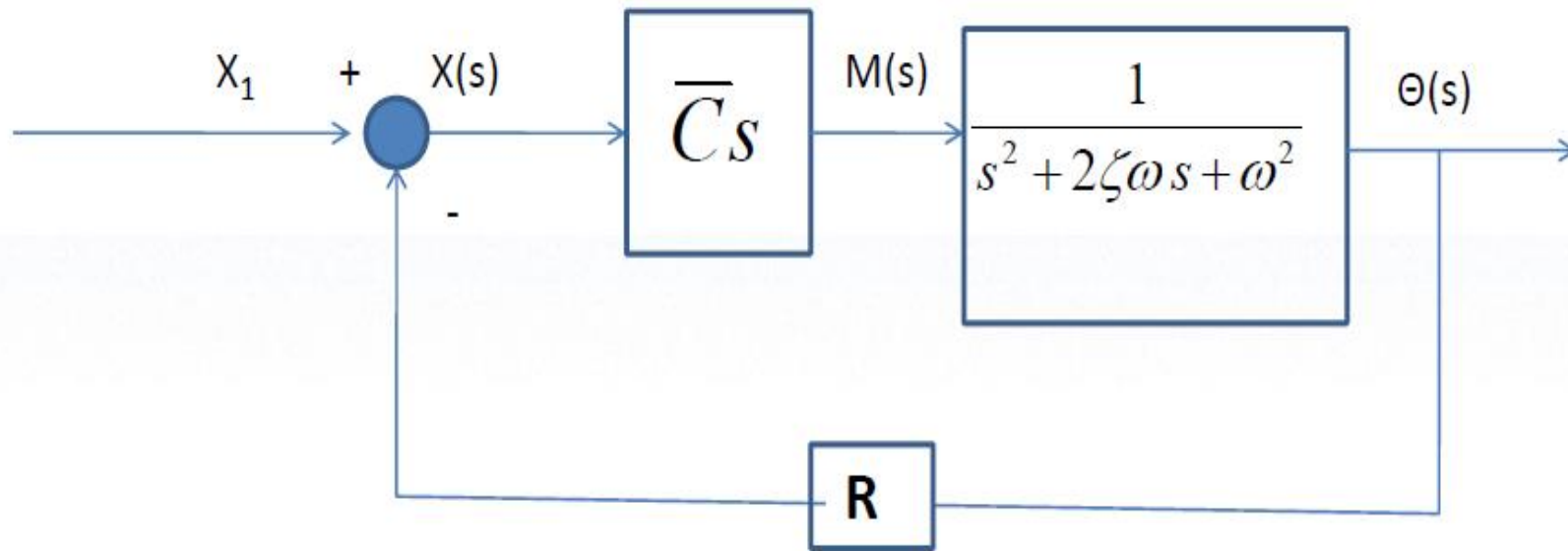
Consider a second order system which is actuated by the moment actuator such that:

$$\frac{\theta(s)}{M(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Also, consider, the relationship between the angular displacement of the system and the displacement of the actuator to be linear and it is sensed such that

$$X(s) = R \theta(s)$$

Let us consider the system to be in closed loop condition



The closed loop transfer function is given by:

$$\frac{\overline{C}Rs}{s^2 + (2\zeta\omega + R)s + \omega^2}$$

You may notice that the actuator has contributed to the system by adding damping and a zero at the origin, which may be used to enhance stability.