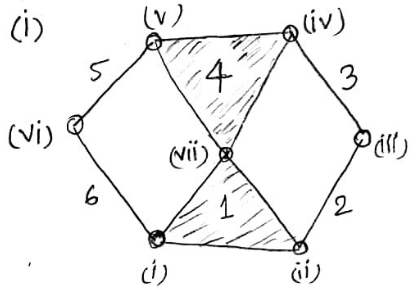
Quiz 1

Q. 1. (i)

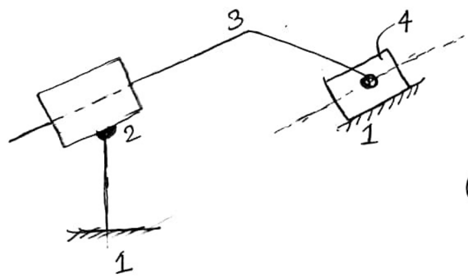


No. of links = 6
(link 1 and link 4 are solid triangles)

No. of Joints = 7

$F = 3(n-1) - 2j$
$= 3(6-1) - 2\times 7$
$F = 1$ Ans

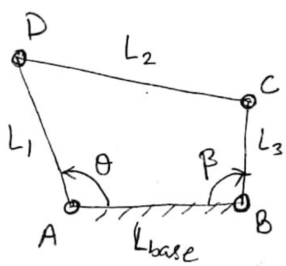
(ii)



No. of links = 4
$F_e = 0$, $F = 1$ (but redundant)

(No marks deducted for reporting either 1 DOF or 0 DOF)

Q.2.

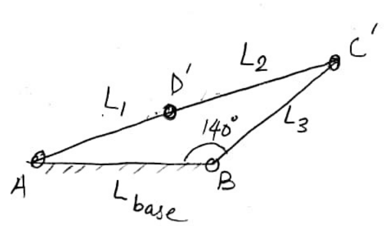


$L_{base} = 0.91\,m$

$L_1 = 0.81\,m$

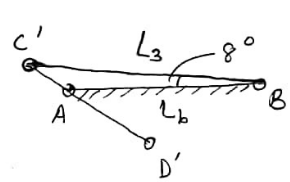
$\beta_{min} = 8°$, $\beta_{max} = 140°$

when $\beta = 140°$



$L_3^2 + L_b^2 - (L_1+L_2)^2 - 2L_3L_b\cos 140° = 0$ —(1)

when $\beta = 8°$



$L_3^2 + L_b^2 - (L_2-L_1)^2 - 2L_3L_b\cos 8° = 0$ —(2)

To solve these two equations by Newton-Raphson method

Define;

$$f_1 = l_3^2 + l_6^2 - (l_1 + l_2)^2 - 2 l_3 l_6 \cos 140^\circ$$

$$f_2 = l_3^2 + l_6^2 - (l_2 - l_1)^2 - 2 l_3 l_6 \cos 8^\circ$$

$$\begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_{new} = \begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_{old} - J^{-1} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}_{old}$$

where, $J = \begin{bmatrix} \frac{\partial f_1}{\partial l_2} & \frac{\partial f_1}{\partial l_3} \\ \frac{\partial f_2}{\partial l_2} & \frac{\partial f_2}{\partial l_3} \end{bmatrix}$

$$J = \begin{bmatrix} -2(l_1 + l_2) & 2 l_3 - 2 l_6 \cos 140^\circ \\ -2(l_2 - l_1) & 2 l_3 - 2 l_6 \cos 8^\circ \end{bmatrix}$$

Observe that the data is (deliberately) very close to the assignment data. Answers from there can be used as initial guess here.

$$\begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_1 = \begin{Bmatrix} 0.94 \\ 0.95 \end{Bmatrix} - \begin{bmatrix} -3.5 & 3.294 \\ -0.26 & 0.097 \end{bmatrix}^{-1} \begin{Bmatrix} -0.0074 \\ 0.0015 \end{Bmatrix}$$

$$\begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_1 = \begin{Bmatrix} 0.9512 \\ 0.9641 \end{Bmatrix}$$

$$\begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_2 = \begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_1 - [J]^{-1} \begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix}_1$$

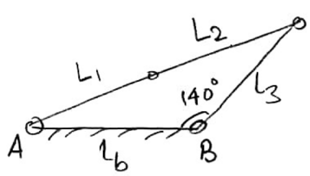
$$= \begin{Bmatrix} 0.9512 \\ 0.9641 \end{Bmatrix} - \begin{bmatrix} -3.522 & 3.322 \\ -0.282 & 0.125 \end{bmatrix}^{-1} \begin{Bmatrix} -8.755 \times 10^{-5} \\ 6.562 \times 10^{-5} \end{Bmatrix}$$

$$\begin{Bmatrix} l_2 \\ l_3 \end{Bmatrix}_2 = \begin{Bmatrix} 0.9517 \\ 0.9646 \end{Bmatrix}$$

so, $\left.\begin{array}{l} l_2 = 0.951\text{ m} \\ l_3 = 0.96\text{ m} \end{array}\right\}$ **Ans.**

Q.3 This is similar to previous problem (Q.2), now we have two linear equations for two unknown, which can be easily solved.

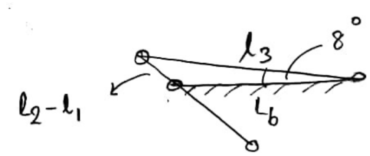
Given: $L_3 = 0.8$ and $l_b = 0.91$



$$\rightarrow l_1 + l_2 = \sqrt{l_3^2 + l_b^2 - 2 l_3 l_b \cos 140^\circ}$$

$$(l_1 + l_2) = \sqrt{0.8^2 + 0.91^2 - 2 \times 0.8 \times 0.91 \times \cos 140^\circ}$$

$$l_1 + l_2 = 1.6073 \quad \text{—(1)}$$



$$\rightarrow (l_2 - l_1) = \sqrt{l_3^2 + l_b^2 - 2 l_3 l_b \cos 8^\circ}$$

$$= \sqrt{0.8^2 + 0.91^2 - 2 \times 0.8 \times 0.91 \times \cos 8^\circ}$$

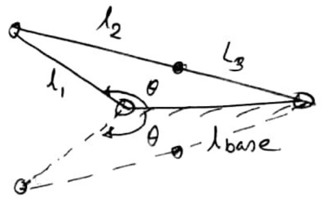
$$(l_2 - l_1) = 0.1621 \quad \text{—(2)}$$

From (1) & (2),

$l_1 = 0.7226$ m

$l_2 = 0.8847$ m

Q.4. Given: $l_b = 1\,m$, $l_1 = 0.7\,m$, $l_2 = 0.72\,m$, $l_3 = 0.6\,m$



(Dashed portion is the another extreme position of link l_1)

$$\cos\theta = \frac{l_1^2 + l_b^2 - (l_2 + l_3)^2}{2 l_1 l_b}$$

$$\cos\theta = \frac{0.7^2 + 1^2 - (0.72 + 0.6)^2}{2 \times 0.7 \times 1}$$

$$\theta = \cos^{-1}(-0.18028)$$

$$\theta = 100.386^\circ$$

$$\text{Total Deflection} = 2\theta$$
$$= 2 \times 100.386^\circ$$
$$= 200.77^\circ \text{ Ans}$$