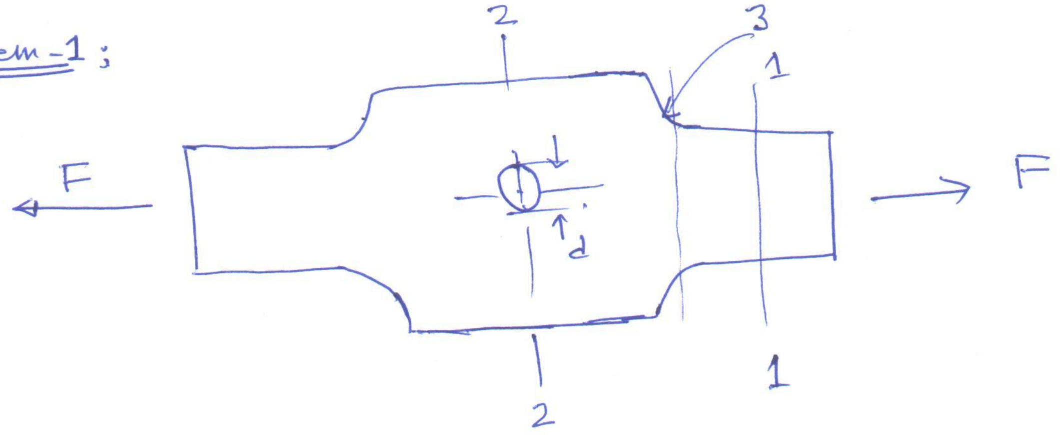
Tutorial -2: Solution - January 23

Problem -1;



Nominal stresses: (1) in section 1-1: $\sigma_{01} = \frac{F}{ah}$

(2) in section 2-2: $\sigma_{02} = \frac{F}{(b-d)h}$

Stress- Concentration Factor:

For section - 1-1: (Fig A-15-5)

$$\frac{r}{a} = \frac{6}{60} = \frac{1}{10} = 0.1.$$

$$\frac{b}{a} = \frac{100}{60} = \frac{5}{3} = 1.67 \quad \text{(in figure curve is for } \cancel{1.50} = \tfrac{D}{d} = 1.50)$$

$\Rightarrow$ So extrapolate

$$\boxed{K_{t1} \simeq 2.3}$$

For section 2-2: (Fig A-15-1)

$$\frac{d}{b} = \frac{20}{100} = 0.2 \Rightarrow \boxed{K_{t2} = 2.5}$$

(a) **Critical location:** Stress at 3-3:

$$\sigma_1 = K_{t1} \cdot \sigma_{01} = 2.3 \times \frac{1}{a} \cdot \frac{F}{h}$$

$$= \frac{2.3 \times 10^3}{60} \cdot \frac{F}{h}$$

$$\Rightarrow \boxed{\sigma_1 = 38.33 \; \frac{F}{h}}$$

Stress at 2-2:

$$\sigma_2 = K_{t2} \cdot \sigma_{02} = 2.5 \cdot \frac{F}{(b-d)} \cdot \frac{1}{h}$$

$$\Rightarrow \boxed{\sigma_2 = \frac{2.5}{80 \times 10^{-3}} \cdot \frac{F}{h} = 31.25 \; \frac{F}{h}} < \sigma_1$$

$\Rightarrow$ 3-3 is the critical location for stress analysis

(b) Factor of safety against yielding = 3 :

$\Rightarrow F_{max} = 80\,kN.$

$\Rightarrow \sigma_{max}\big|_{allowable} = \frac{\sigma_1}{\cancel{\;}} = \frac{38.33 \cancel{\times 10^{3}} \times 80\times10^{3}}{h}$

$\Rightarrow \boxed{\sigma_{max}\big|_{allowable} = \frac{3\times1022.13}{h} \times 10^{3}\ Pa}$

Material Properties : AISI-1018 CR Steel

$S_y = 370\,MPa$ (A-20)

$E = 207\ \cancel{MPa}\ GPa$ (A-5).

$S_{ut} = 440\,MPa.$ (A-20).

$\Rightarrow \sigma_{max|all.} < S_y/3$

$\Rightarrow \frac{3066.4\times10^{3}}{h} < \frac{370\times10^{6}}{3} \Rightarrow \boxed{h > 24.86\times10^{-3}\,m}$

Choose a $\boxed{h = 25\,mm}$

(c) For infinite life :

Endurance limit : $S_e' = 0.5\,S_{ut} = 220\,MPa.$

Factors : surface factor $K_a = a.\,S_{ut}^{\,b}$

~~Surface~~

$= 4.51 \times (440)^{-0.265}$

(a, b from table 6-2)

$\Rightarrow \boxed{K_a = 0.899}$

Size factor $\boxed{K_b = 1}$ (axial loading)

Loading Factor $\boxed{K_c = 0.85}$ (axial loading).

Temp. Factor $\boxed{K_d = 1}$

This factor if is different if a diff. reliability value is chosen

Reliability Factor $\boxed{K_e = 1}$ assume 50% reliability (Table - 6-5)

Miscellaneous Factor $\boxed{K_f = 1}$

$\Rightarrow S_e = K_a K_b K_c K_d K_e K_f .\ S_e'$

$= 0.899 \times 1 \times 0.85 \times 1 \times 1 \times 1 .\ S_e'$

$\Rightarrow \boxed{S_e = 168\,MPa}$

~~For infinite life~~

For Fatigue use fatigue factor of safety:

Notch Sensitivity : $\boxed{q = 0.8}$ Figure 6-20 for $S_{ut} \simeq 0.4\,GPa$, $r > 4$ mm (extrapolate / interpolate)

$$\Rightarrow K_f = 1 + q(K_t - 1) = 1 + 0.8(2.3 - 1)$$

$$\Rightarrow \boxed{K_f = 2.04}$$

$$\Rightarrow \boxed{\sigma_1 = 33.997 \cdot \frac{F}{h}}$$

Factor of safety = 2.5

$$\Rightarrow \sigma_{1max|all} \leq \frac{S_e}{2.5}$$

$$\Rightarrow \frac{33.997 \times 80 \times 10^3}{h} \leq \frac{168 \times 10^6}{2.5}$$

$$\Rightarrow \boxed{h \geq 40.47 \times 10^{-3}}$$

$$\Rightarrow \boxed{h = 45\,mm}$$ (*) ← standard size From Table A-17

(*) Give full marks if students choose 41 mm

(d) For Finite Life : $N = 10^4$ cycles ⇒ high cycle fatigue.

$$S_f = a \cdot N^b.$$

$$a = \frac{(f \cdot S_{ut})^2}{S_e} = \frac{(0.9 \times 440)^2}{168}$$ ($f = 0.9$ for $S_{ut} < 490$ MPa Fig - 6-18)

$$= 933.43\,MPa$$

$$b = -\frac{1}{3} \log\left(\frac{f \cdot S_{ut}}{S_e}\right) = -0.124$$

$$\Rightarrow \text{For } 10^4 \text{ cycles : } S_f = 933.43 \times (10^4)^{-0.124}$$

$$\boxed{S_f = 297.91\,MPa}$$

New load level : $\frac{33.997\,F}{h} \leq \frac{S_f}{2.5} \Rightarrow F \leq \frac{297.91 \times 10^6 \times 45 \times 10^{-3}}{2.5 \times 33.997}$

$$\Rightarrow \boxed{F \leq 157.73\,KN}$$

(e) Fluctuating Loading:

$$F_{max} = 80\,kN$$
$$F_{min} = -20\,kN$$

$$\Rightarrow \boxed{\begin{array}{l} F_m = 30\,kN \\ F_a = 50\,kN \end{array}} \leftarrow \text{Below yielding load}$$

$$\Rightarrow \sigma_m = K_f * \underbrace{\left(\frac{F_m}{ah}\right)}_{\sigma_{mo}} = \frac{2.04 \times 30\times10^3}{60\times10^{-3} \times 45\times10^{-3}}$$

$$\boxed{\sigma_m = 22.67\,MPa}$$

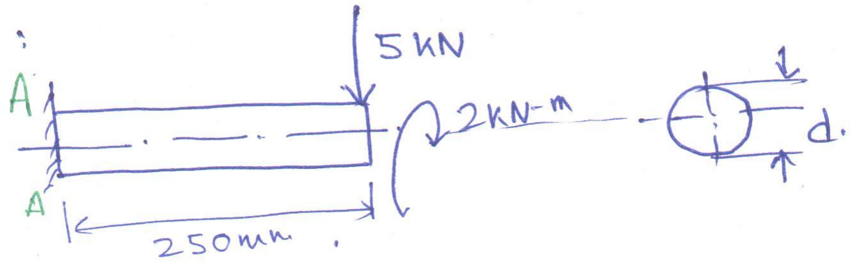
$$\sigma_a = K_f \cdot \frac{F_a}{ah} \Rightarrow \boxed{\sigma_a = 37.78\,MPa}$$

Factor of safety (Modified Goodman):

$$n_f = \frac{1}{\left(\sigma_a/S_e + \frac{\sigma_m}{S_{ut}}\right)} = \frac{1}{\frac{37.78}{168} + \frac{22.67}{440}}$$

$$\Rightarrow \boxed{n_f = 3.61}$$

Problem-2 :



Bending moment is maximum @ localion A-A.

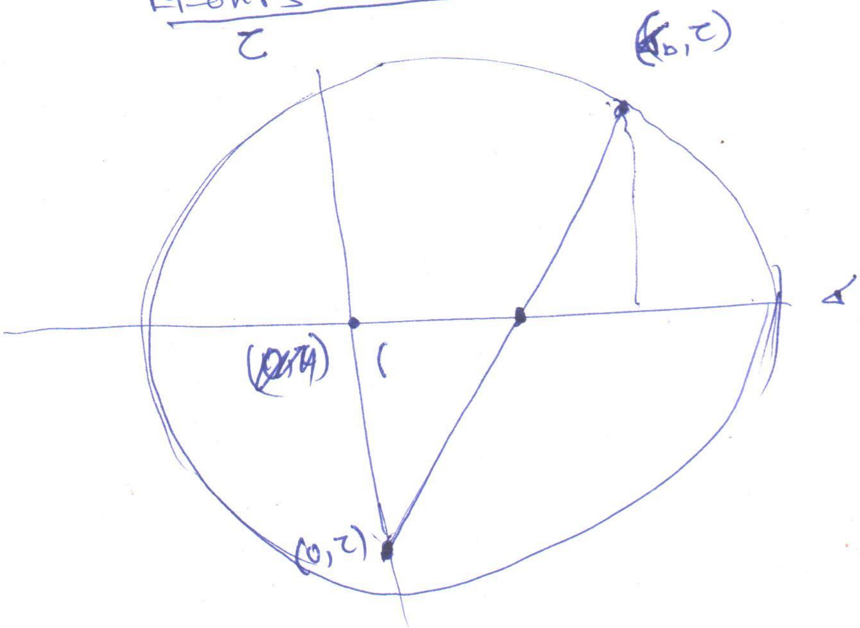
$$M_{max} = 5\,kN \times 250 \times 10^{-3}\,m = 1.25\,kN\text{-}m$$

Torque, $T = 2\,kN\text{-}m$.

Maximum Stress due to bending: $\sigma_b = \frac{32M}{\pi d^3} = \frac{12.73}{d^3} \times 10^3\,Pa$

Shear stress due to torsion: $\tau = \frac{16T}{\pi d^3} = \frac{10.19}{d^3} \times 10^3\,Pa$

Mohr's circle:



Radius: $\sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$

$$= \frac{12.01}{d^3} \times 10^3\,Pa.$$

$$\Rightarrow \sigma_1 = \frac{\sigma_b}{2} + \text{Radius} = \frac{24.74}{d^3} \times 10^3\,Pa$$

$$\sigma_2 = \frac{\sigma_b}{2} - \text{Radius} = -\frac{0.72}{d^3} \times 10^3\,Pa.$$

~~Assu~~ Material: AISI-1020 hot rolled: $S_y = 210\,MPa$.

(a) Tresca: $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{25.46}{2d^3}\,MPa. = \frac{12.73}{d^3} \times 10^3\,Pa.$

$$\leq \frac{S_y}{2 \times 2.5} = \frac{210 \times 10^6}{5}\,Pa$$

$$\Rightarrow d \geq 67.17\,mm.$$

(b) Von-mises: $\sigma' = \sqrt{\sigma_b^2 + 3\tau^2} \leq \frac{S_y}{2.5} \Rightarrow d \geq 63.74\,mm$

(c) Mohr-Coulomb: Since $S_{yc} = S_{yt}$: same as Tresca.

$\Rightarrow$ Final dia: $\boxed{d = 70\,mm}$ Standard Size.