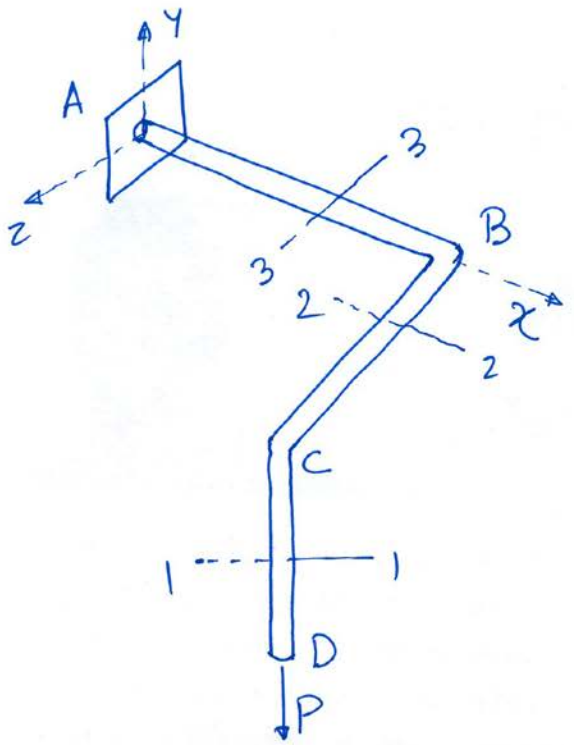
Solution for Q4



We know

$$U = \frac{F_A^2 L}{2AE}, \quad A = \text{x-sectional area}$$

$$\frac{\partial U}{\partial Q} = \frac{F_A L}{AE} \cdot \frac{\partial F_A}{\partial Q}$$

$$U = \int \frac{M_b^2}{2EI}\, dx$$

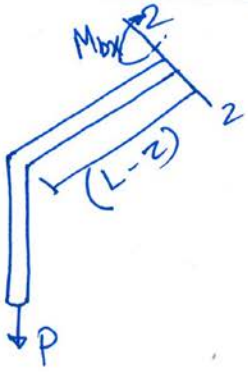
$$\frac{\partial U}{\partial Q} = \int \frac{M_b}{EI} \frac{\partial M_b}{\partial Q}\, dx$$

For CD (Sec 1-1)

Axial force $F_A = P$

$$\frac{\partial F_A}{\partial P} = 1$$

For BC (Sec 2-2)



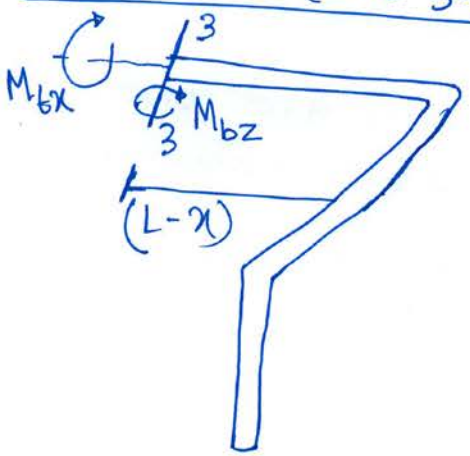
$$-M_{bx} + P(L-z) = 0$$

$$\therefore M_{bx}\Big|_{2-2} = P(L-z)$$

$$\frac{\partial M_{bx}|_{2-2}}{\partial P} = (L-z)$$

Solution for Q4

For AB (Sec 3-3)



$$M_{tx}\big|_{3-3} = PL, \quad \frac{\partial M_{tx}}{\partial P}\Big|_{3-3} = L$$

$$M_{bz}\big|_{3-3} = -P(L-x), \quad \frac{\partial M_{bz}}{\partial P}\Big|_{3-3} = -(L-x)$$

∴ Total strain energy of the system

$$U = U_{AB} + U_{BC} + U_{CD}$$

$$\therefore \delta_D^V = \frac{\partial U}{\partial P} = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P}$$

$$= \left[\int_0^L \frac{M_{bz}|_{3-3}}{EI} \cdot \frac{\partial M_{bz}|_{3-3}}{\partial P} dx + \int_0^L \frac{M_{tx}|_{3-3}}{GI_P} \cdot \frac{\partial M_{tx}|_{3-3}}{\partial P} dx\right]$$

$$+ \left[\int_0^L \frac{M_{bx}|_{2-2}}{EI} \cdot \frac{\partial M_{bx}}{\partial P} dz\right] + \left[\frac{F_A L}{AE} \cdot \frac{\partial F_A}{\partial P}\right]$$

$$= \left[\int_0^L \frac{P(L-x)(L-x)}{EI} dx + \int_0^L \frac{PL \cdot L}{GI_P} dx\right]$$

$$+ \int_0^L \frac{P(L-z)(L-z)}{EI} dz + \frac{PL}{AE} \cdot 1$$

$$= \frac{P}{EI} \cdot \frac{L^3}{3} + \frac{PL^3}{GI_P} + \frac{P}{EI} \cdot \frac{L^3}{3} + \frac{PL}{AE}$$

$$= PL\left[\frac{2L^2}{3EI} + \frac{L^2}{GI_P} + \frac{1}{AE}\right)$$

$$= PL\left[\frac{2L^2}{3E} \cdot \frac{64}{\pi d^4} + \frac{L^2}{G} \cdot \frac{32}{\pi d^4} + \frac{4}{AEd^2}\right] = PL\left[\frac{128L^2}{3\pi E d^4} + \frac{32L^2}{\pi G d^4} + \frac{4}{AE}\right]$$

$$= \frac{PL}{\pi d^2}\left[\frac{128L^2}{2Ed^2} + \frac{32L^2}{Gd^2} + \frac{4}{E}\right]$$

Ans.

Grading Scheme

3 marks (1 mark each):
FBD for AB
FBD for BC
FBD for CD

5 marks
Torsion moment and its derivative w.r.t P in AB
Bending moment and its derivative w.r.t P in AB

2 marks:
Bending moment and its derivative w.r.t P in BC

2 marks:
Axial force and its derivative w.r.t P in CD

3 marks:
Total energy of the system adding individual terms of AB, BC, CD

2 marks: Izz, Ipolar

3 marks: simplification of displacement at D and statement of the final answer