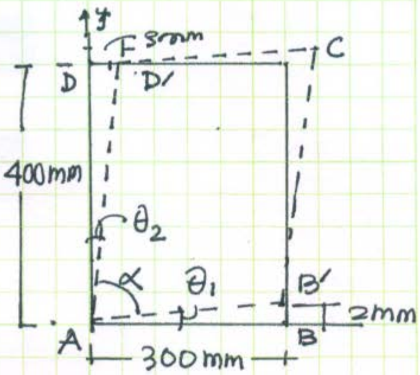
ESO 202A/204: Mechanics of Solids (2016-17 II Semester)
Assignment No. – 4 (Solution)

PROB#1



(a) $\theta_1 \simeq \tan\theta_1 = \frac{2}{300} = 0.0067$ rad

$\theta_2 \approx \tan\theta_2 = \frac{3}{400} = 0.0075$ rad

$\gamma_{xy} = \theta_1 + \theta_2$

$= 0.0067 + 0.0075 = \underline{0.0142 \text{ rad}}$ ANS

(b) $AD' = \sqrt{(400)^2 + (3)^2} = 400.01125$ mm

$\theta_2 = \tan^{-1}\left(\frac{3}{400}\right) = 0.42971°$

$AB' = \sqrt{(300)^2 + (2)^2} = 300.00667$ mm

$\theta_1 = \tan^{-1}\left(\frac{2}{300}\right) = 0.381966°$

$\therefore \alpha = 90° - 0.42971° - 0.381966° = 89.18832°$

$D'B' = \sqrt{(400.01125)^2 + (300.00667)^2 - 2(400.01125)(300.00667)\cos(89.18832°)}$

$= 496.6014$ mm

$DB = \sqrt{(400)^2 + (300)^2} = 500$ mm

$\therefore \varepsilon_{DB} = \frac{496.6014 - 500}{500} = \underline{-0.00680 \text{ mm/mm}}$ ANS

$\varepsilon_{AD} = \frac{AD' - AD}{AD} = \frac{400.01125 - 400}{400} = \underline{0.0281 \times 10^{-3} \text{ mm/mm}}$ ANS

PROB #2

IN-PLANE PRINCIPAL STRAINS:

$\epsilon_x = -300\times10^{-6}$, $\epsilon_y = 0$ and $\gamma_{xy} = 150\times10^{-6}$

$$\epsilon_{1,2} = \frac{\epsilon_x+\epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= \left[\frac{-300+0}{2} \pm \sqrt{\left(\frac{300-0}{2}\right)^2 + (150)^2}\right]\times10^{-6} = (-150 \pm 167.71)\times10^{-6}$$

$\therefore \epsilon_1 = 17.7\times10^{-6}$ $\quad \epsilon_2 = -318\times10^{-6}$ **ANS**

ORIENTATION OF PRINCIPAL STRAINS:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\epsilon_x-\epsilon_y} = \frac{150\times10^{-6}}{(-300-0)\times10^{-6}} = -\frac{1}{2}$$

$\therefore \theta_p = -13.28^\circ$ and 76.72°. **ANS**

Sub. $\theta_p = -13.28^\circ$ in strain-transformation eqn.

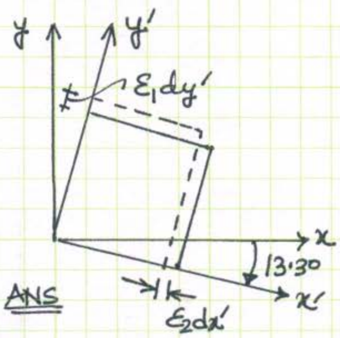
$$\epsilon_{x'} = \frac{\epsilon_x+\epsilon_y}{2} + \frac{\epsilon_x-\epsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= \left[\frac{-300+0}{2} + \frac{-300-0}{2}\cos(26.57^\circ) + \frac{150}{2}\sin(26.57^\circ)\right]\times10^{-6}$$

$$= -318\times10^{-6} = \epsilon_2$$

Thus, $(\theta_p)_1 = 76.7^\circ$ and $(\theta_p)_2 = -13.3^\circ$

Deformed element is as shown in fig.



MAX. IN-PLANE SHEAR STRAIN:

$$\frac{\gamma_{max}^{in\text{-}plane}}{2} = \sqrt{\left(\frac{\epsilon_x-\epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 2\sqrt{\left(\frac{-300-0}{2}\right)^2 + \left(\frac{150}{2}\right)}\times10^{-6} = 335\times10^{-6} \quad \textbf{ANS}$$

Orientation:

$$\tan 2\theta_s = -\left(\frac{\epsilon_x-\epsilon_y}{\gamma_{xy}}\right) = \left[-\frac{-300-0}{150}\right] = 2$$

$\Rightarrow \theta_s = 31.7^\circ$ and 121.7° **ANS**

The algebraic sign for $\gamma_{max}^{in\text{-}plane}$ when $\theta = \theta_s = 31.7^\circ$ can be obtained by using.

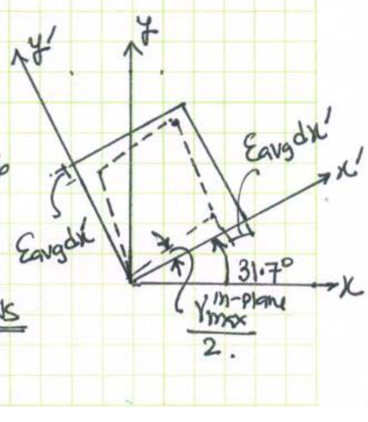
$$\frac{\gamma_{x'y'}}{2} = -\left(\frac{\epsilon_x-\epsilon_y}{2}\right)\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$= -[(-300-0)\sin 63.4^\circ + 150\cos 63.4^\circ]\times10^{-6}$$

$$= 335\times10^{-6} \Rightarrow \text{+ve strain}$$

Avg. normal strain:

$$\epsilon_{avg} = \frac{\epsilon_x+\epsilon_y}{2} = \frac{-300+0}{2}\times10^{-6} = -150\times10^{-6} \quad \textbf{ANS}$$



Deformed element for this state of strain is shown in fig.

PROB#3

Given:

$\varepsilon_x = 120 \times 10^{-6}$

$\varepsilon_y = -180 \times 10^{-6}$

$\gamma_{xy} = 150 \times 10^{-6}$

∴ Co-ordinates of reference points on Mohr's circle are:

A(120, 75), B(-180, 75) and C(-30, 0)

Radius of circle:

$R = CA = \sqrt{(120+30)^2 + 75^2} \times 10^{-6} = 167.71 \times 10^{-6}$

Principal strain corresponding to points D & E are.

$\varepsilon_1 = (-30 + 167.71) \times 10^{-6} = 138 \times 10^{-6}$

$\varepsilon_2 = (-30 - 167.71) \times 10^{-6} = -198 \times 10^{-6}$

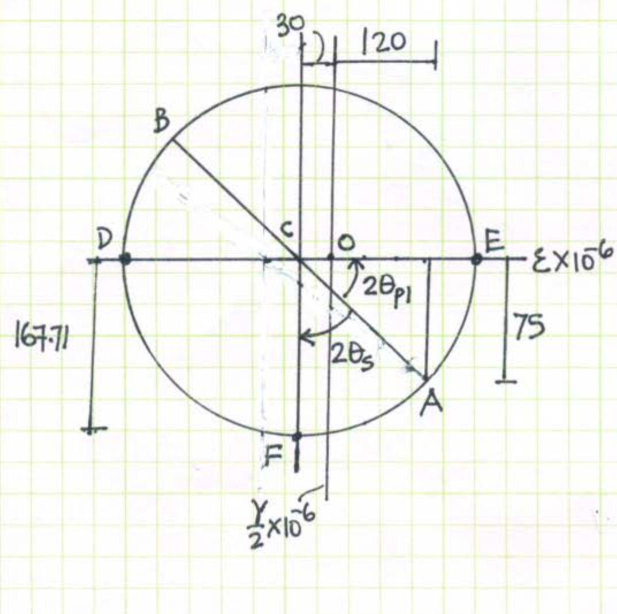
Orientation; $\tan 2\theta_p = \left(\frac{75}{30+120}\right) \Rightarrow \theta_p = 13.3°$ (counterclockwise)

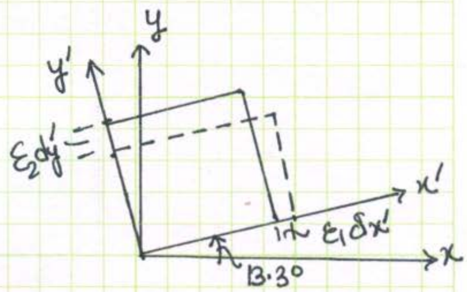
Point F represent ε_{avg} and $\gamma_{max}^{in\text{-}plane}$, thus

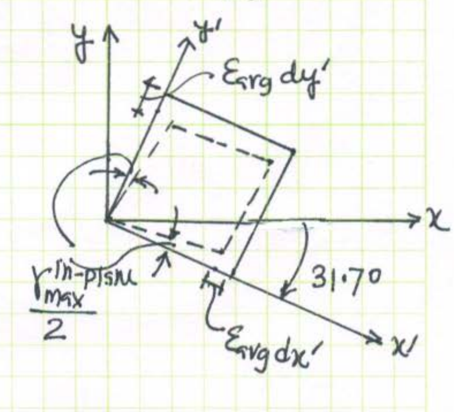
$\frac{\gamma_{max}^{in\text{-}plane}}{2} = R = 167.71 \times 10^{-6} \Rightarrow \gamma_{max}^{in\text{-}plane} = 335.4 \times 10^{-6}$

Also $\tan 2\theta_s = \frac{120-(-30)}{75} \Rightarrow \theta_s = \frac{\tan^{-1} 2}{2} = 31.72°$

$\varepsilon_{avg} = \frac{138 - 198}{2} = -30 \times 10^{-6}$







PROB # 4

Clearly: $\theta_A = 60^\circ$, $\theta_B = 120^\circ$ & $\theta_C = 180^\circ$

$\therefore \varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy} \sin\theta_a \cos\theta_a$

$200\times10^{-6} = \varepsilon_x \cos^2 60^\circ + \varepsilon_y \sin^2 60^\circ + \gamma_{xy} \sin 60^\circ \cos 60^\circ$

$\Rightarrow 0.25\varepsilon_x + 0.75\varepsilon_y + 0.433\gamma_{xy} = 200\times10^{-6}$ (1)

$\|$ly $-450\times10^{-6} = \varepsilon_x \cos^2 120^\circ + \varepsilon_y \sin^2 120^\circ + \gamma_{xy} \sin 120^\circ \cos 120^\circ$

$\Rightarrow 0.25\varepsilon_x + 0.75\varepsilon_y - 0.433\gamma_{xy} = -450\times10^{-6}$ (2)

$250\times10^{-6} = \varepsilon_x \cos^2 180^\circ + \varepsilon_y \sin^2 180^\circ + \gamma_{xy} \sin 180^\circ \cos 180^\circ$

$\Rightarrow \varepsilon_x = 250\times10^{-6}$ (3)

Sub. ε_x in (1) and (2), and on solving,

$\varepsilon_y = -250\times10^{-6}$ and $\gamma_{xy} = 750.56\times10^{-6}$

Now $\varepsilon_{avg} = \dfrac{\varepsilon_x + \varepsilon_y}{2} = \dfrac{250-250}{2} = 0$ ANS

Hence, the co-ordinates of reference points on Mohr's circle are.

A(250, 375.28), B(−250, 375.28) & C(0, 0)

Rad. of circle

$R = CA = \sqrt{(250-0)^2 + 375.28^2} \times10^{-6} = 450.92\times10^{-6}$

Principal strain corresponding to points E & D are

$\varepsilon_1 = 451\times10^{-6}$ and $\varepsilon_2 = -451\times10^{-6}$ ANS

Referring to geometry of circle: $\tan(2\theta_p)_1 = \dfrac{375.28}{250} = 1.5011$

$\Rightarrow \theta_{p_1} = 28.2^\circ$ (counterclockwise) ANS

Point F represent ε_{avg} and $\gamma_{max}^{in\text{-}plane}$, thus

$\dfrac{\gamma_{max}^{in\text{-}plane}}{2} = R = 450.92\times10^{-6} \Rightarrow \gamma_{max}^{in\text{-}plane} = 902\times10^{-6}$

Also $\tan 2\theta_s = \dfrac{250}{375.28} \Rightarrow \theta_s = 16.8^\circ$ (clockwise).

$\varepsilon_{avg} = 0$