**IIT Kanpur, Mechanical Engineering**

ME 354A, Vibration and Control

<u>Handout for Experiment 1a</u>

*Determination of the effective radius of gyration of an irregular body through torsional oscillation of a trifilar suspension*

**Overview:**

The *radius of gyration* of an irregular body can be determined using a trifilar suspension. Figure (1) shows such a system and figure (2) shows a schematic diagram of the same setup. It consists of a disc supported by three inextensible light strings, each of length $L$. These strings are attached to the base disc symmetrically at a distance $r$ from the centre $O$. Let the *radius of gyration* and mass of the base disc to be $k_d$ and $m_d$ respectively and the same for an irregular body be $k_c$ and $m_c$.


Figure 1: Actual experimental setup

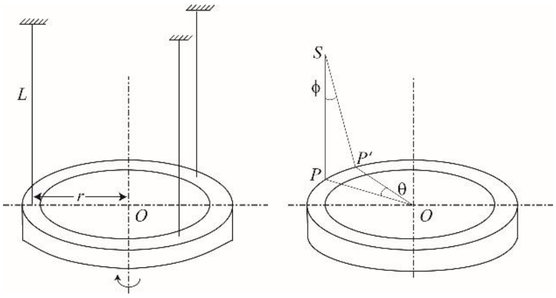


Figure 2: Schematic of the experimental setup

The equation of motion for the small amplitude torsional oscillation of the system is given by (try to derive)

$$\ddot{\theta} + \left(\frac{m_d + m_c}{m_d k_d^2 + m_c k_c^2}\right)\left(\frac{g r^2}{L}\right)\theta = 0 \tag{1}$$

Therefore, the time period of the oscillation is

$$\tau_n = 2\pi\sqrt{\left(\frac{m_d k_d^2 + m_c k_c^2}{m_d + m_c}\right)\left(\frac{L}{g r^2}\right)} \tag{2}$$

Hence, the unknown *radius of gyration* can be calculated from the following relation

$$k_c = \sqrt{\frac{1}{m_c}\left[\frac{\tau_n^2 g r^2 (m_d + m_c)}{4\pi^2 L} - m_d k_d^2\right]} \tag{3}$$

Note the possible error due to cancellation of the term under square root.

**Procedure:**

1. Set the base disc into torsional oscillation about its vertical axis through its centre. Measure the time taken for 10 oscillations, determine the average time period of oscillation with estimated error bounds.
2. Put the composite disc over the base disc and measure the average time period of oscillation of the combination.
3. Now in place of the composite disc put a book on the base disc and repeat the step (2).

4. Measure the dimensions of the base disc, composite disc and the book.

**Laboratory report:**

1. Attach your observations (datasheet).
2. Calculate mass of the base disc, composite disc and the book. Hence calculate the radius of gyration using the relation given in equation (3). Show all the calculations.
3. Using standard formula for moment of inertia calculate the radius of gyration and compare with the results obtained in the previous step. Show all the calculations.
4. Fill up the following table.

| Case | Time period | Radius of gyration | |
|---|---|---|---|
| | | Using experimental data | Using standard formula for moment of inertia |
| Base disc | | | |
| Composite disc | | | |
| Book | | | |

5. Any other observation and comments.
6. Your suggestion to improve the lab experiment and experience.

**Note:**

1. There will be a viva before the start of the experiment to check your preparation. It will have marks along with your lab report and attendance.
2. While performing the experiment take a photograph of the group along with the TA and the setup, email it to the concerned TA and attach a printout (black and white is accepted) with the report.
3. Any observed datasheet must contain signature of the concerned TA. Any one student may record the datasheet during the experiment. He/she will submit the original signed sheet with the report and the rest must submit a photocopy of the same.

**IIT Kanpur, Mechanical Engineering**

ME 354A, Vibration and Control

Handout for Experiment 1b

*Beat phenomenon of a coupled pendulum*

**Overview:**

An interesting phenomenon is observed when two natural frequencies of a two degree of freedom system are close in value. This is physically illustrated by this experiment by taking two near identical pendulums connected by a spring as shown in figure (1) and (2).

The simple mathematical model as shown in figure (2) assumes a rod of negligible mass in comparison to concentrated bob mass.

Figure1: Actual experimental setup

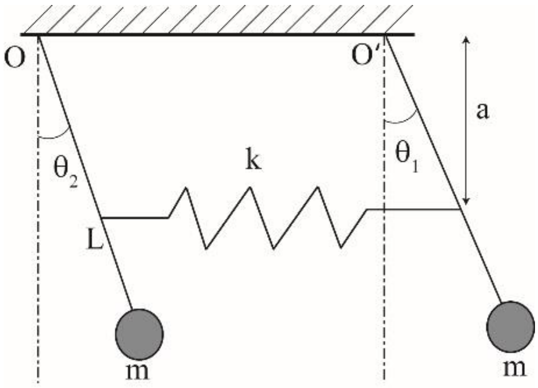


Figure 2: Schematic of coupled pendulum

For small angular displacements the equations of motion for the system under consideration are given by

$$\begin{bmatrix} mL^2 & 0 \\ 0 & mL^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} mgL + ka^2 & -ka^2 \\ -ka^2 & mgL + ka^2 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{1}$$

Corresponding natural frequencies are

$$\omega_1 = \sqrt{\frac{g}{L}}, \omega_2 = \sqrt{\frac{g}{L} + 2\frac{k}{m}\frac{a^2}{L^2}} \tag{2}$$

Respective modes are defined by the following amplitude ratios (see figure 3)

$$\frac{\Theta_{21}}{\Theta_{11}} = 1, \frac{\Theta_{22}}{\Theta_{12}} = -1 \tag{3}$$

So, in the first natural mode both the pendulums move like a single pendulum with the spring unstretched, which can also be concluded from the fact that the first natural frequency of the system is that of the simple pendulum. On the other hand, in the second natural mode, the two pendulums are 180°out of phase. The two modes are shown in figure (3).

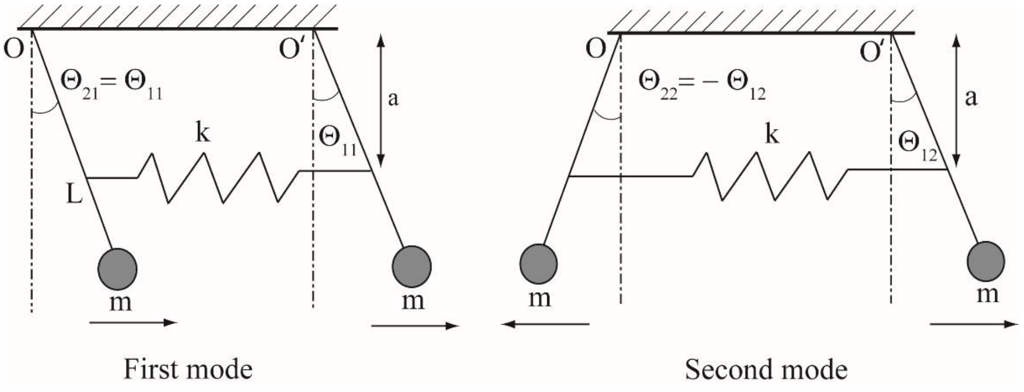


Figure 3: Mode shapes

Choosing $\Theta_{11} = \Theta_{12} = 1$ and initial conditions to be $\theta_1(0) = \theta_0, \theta_2(0) = 0, \dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$, the solution for equation system (1) becomes

$$\begin{aligned} \theta_1(t) &= \frac{1}{2}\theta_0 \cos\omega_1 t + \frac{1}{2}\theta_0 \cos\omega_2 t = \theta_0 \cos\frac{\omega_2 - \omega_1}{2} t \cos\frac{\omega_2 + \omega_1}{2} t \\ \theta_2(t) &= \frac{1}{2}\theta_0 \cos\omega_1 t - \frac{1}{2}\theta_0 \cos\omega_2 t = \theta_0 \sin\frac{\omega_2 - \omega_1}{2} t \sin\frac{\omega_2 + \omega_1}{2} t \end{aligned} \tag{4}$$

For $ka^2 \ll mgL$, i.e. weak coupling, $\omega_2$ can be written as $\omega_2 = \sqrt{\frac{g}{L}}(1+\epsilon)$, where $\epsilon = \frac{k}{mg}\frac{a^2}{L}$. For such case, the above solution becomes

$$\begin{aligned} \theta_1(t) &\cong \theta_0 \cos\left(\frac{1}{2}\omega_B t\right)\cos(\omega_{avg} t) \\ \theta_2(t) &\cong \theta_0 \sin\left(\frac{1}{2}\omega_B t\right)\sin(\omega_{avg} t) \end{aligned} \tag{5}$$

Where, $\frac{\omega_B}{2} = \frac{\omega_2 - \omega_1}{2} \cong \frac{1}{2}\frac{k}{m}\frac{a^2}{\sqrt{gL^3}}$ and $\omega_{avg} = \frac{\omega_2 + \omega_1}{2} \cong \sqrt{\frac{g}{L}} + \frac{1}{2}\frac{k}{m}\frac{a^2}{\sqrt{gL^3}}$. Hence, $\theta_1(t)$ and $\theta_2(t)$ can be regarded as amplitude modulated harmonic functions with frequency $\omega_{avg}$ and amplitudes varying slowly according to $\theta_0 \cos\left(\frac{1}{2}\omega_B t\right)$ and $\theta_0 \sin\left(\frac{1}{2}\omega_B t\right)$ respectively. This is known as the beat phenomenon, and the frequency of modulation $\omega_B$, which in this particular case is equal to $\frac{ka^2}{m\sqrt{gL^2}}$ is called the *beat frequency*. Experimentally verify that such a solution exists, and show it to the TA. Interpret it in your report as beats.

**Experimental data:**

$$L = 460 \text{ mm}, m_{bob} = 483.2 \text{ gm}, m_{rod} = 161.3 \text{ gm}$$

Consider the equivalent mass of the rod for all calculations (by theory to be covered in class, increase $m_{bob}$ by $\frac{m_{rod}}{2}$ and subsequently ignore rod mass).

**Procedure:**

1. For a small value of $a$ (approximately $\frac{L}{5}$), determine experimentally the two natural frequencies of oscillation $\omega_1$ and $\omega_2$, for first and second modes from measurement of average time period of free oscillations.

2. Observe the beat phenomenon and record the beat frequency by starting free oscillations with the following initial conditions

| | | | |
|---|---|---|---|
| 1) | $\theta_1 = 4°, \theta_2 = 0, \dot{\theta}_1 = 0, \dot{\theta}_2 = 0$ | 2) | $\theta_1 = -4°, \theta_2 = 0, \dot{\theta}_1 = 0, \dot{\theta}_2 = 0$ |
| 3) | $\theta_1 = 0, \theta_2 = 4°, \dot{\theta}_1 = 0, \dot{\theta}_2 = 0$ | 4) | $\theta_1 = 0, \theta_2 = -4°, \dot{\theta}_1 = 0, \dot{\theta}_2 = 0$ |

3. Repeat the steps (1) and (2) with various values of $a$.

**Laboratory report:**

1. Attach your observations (datasheet).
2. Interpret beat phenomenon and calculate beat frequencies from $\omega_B \approx \omega_2 - \omega_1$ and compare with the experimentally observed values.
3. Estimate the stiffness of the spring.
4. Fill up the following table.

| No. | $\frac{a}{L}$ | Frequency (rad/s) | | | | Remarks |
|---|---|---|---|---|---|---|
| | | $\omega_1$ | $\omega_2$ | $\omega_2 - \omega_1$ | $\omega_B$ (experimentally observed) | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

5. Any other observation and comments.
6. Your suggestion to improve the lab experiment and experience.

**Note:**

1. There will be a viva before the start of the experiment to check your preparation. It will have marks along with your lab report and attendance.
2. While performing the experiment take a photograph of the group along with the TA and the setup, email it to the concerned TA and attach a printout (black and white is accepted) with the report.
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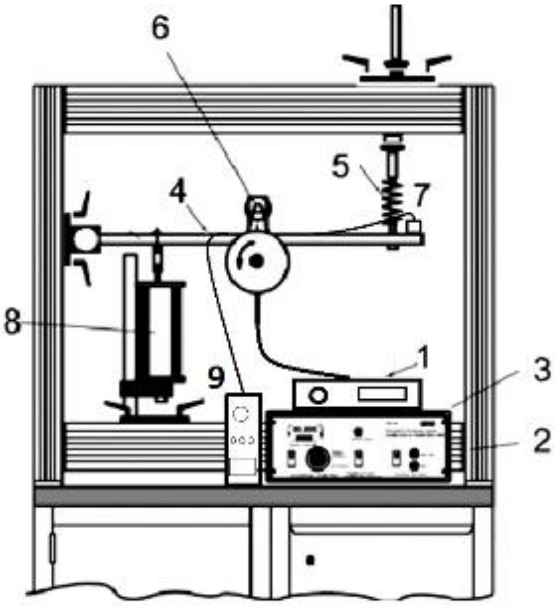
## IIT KANPUR, Mechanical Engineering

ME 354A, Vibration and Control

<u>Handout for Experiment 2</u>

### Free and forced and free vibration of a rigid beam-spring-damper system

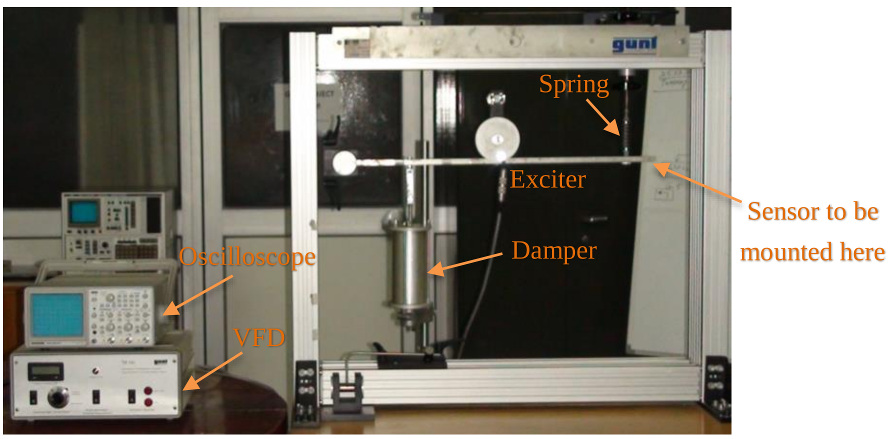



**Objective:**

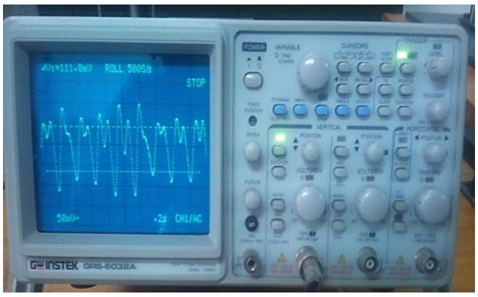
The aim of this experiment is to study the free and forced vibration response of a rigid beam with different values of spring stiffness and damping coefficients.

**Overview:**

The system is a “rigid” hinged beam, with a single degree of freedom, oscillating about its hinge. A spring and a damper are attached to the beam at two different locations. The free vibrations are generated by displacing the beam from its equilibrium position and the forced vibrations are generated by an electrical motor-driven imbalance exciter. The exciter frequency can be set precisely using a VFD (variable frequency drive) with digital display. An oil damper is used for vibration damping.




**Experimental setup**



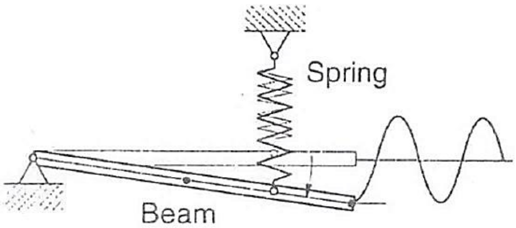
**Charge amplifier** **Stroboscope** **Oscilloscope** **VFD (variable frequency drive)**

**Specification:**

| No. | Name | Specification | Remarks |
|---|---|---|---|
| 1 | VFD (variable frequency drive) | Frequency range < 15 Hz | 230 V-50 Hz supply |
| 2 | Frame | Material | Aluminium |
| 3 | Oscilloscope | Make: Instek | Model: GRS 6032A |
| 4 | Beam | Pinned-free beam | L= 70 mm, B= 25 mm, H= 12 mm, M= 1.6 Kg |
| 5 | Spring | Stiffness (k) | 3 settings: 0.75, 1.5, 3 N/mm |
| 6 | Imbalance exciter | Mass*eccentricity (m*e) | 100 g-cm |
| 7 | Piezoelectric sensor | Gives charge output | Sensitivity: 0.886 pC/(m/sec$^2$) |
| 8 | Damper system | Variable damping | 5-15 N-s/m |
| 9 | Charge amplifier | Amplification | 100 times |

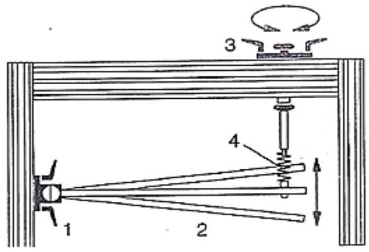
# Free vibrations

The system is initially deflected from its equilibrium position and then released. Theoretically the system will oscillate about this position with fixed amplitude but it oscillates with ***linearly decaying*** amplitude (to be discussed in class). The response is observed on the oscilloscope. This process is repeated for three different spring settings.




**Setup of the spring:**

- Attach adjustable spring mount (3) as described in the adjoining figure.
- Insert spring (4) required between threaded stem of spring mount and beam and secure
- Horizontally align beam over spring mount




**Procedure of setting up different springs**

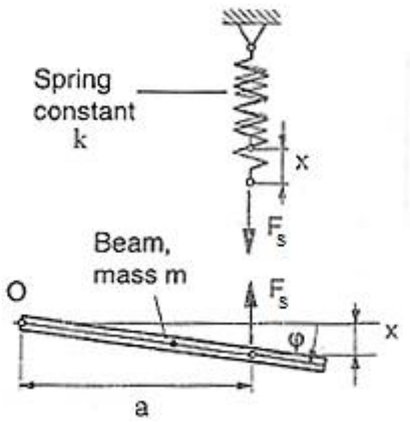
## Equations of motion: Undamped oscillation

From moment balance about the fulcrum O, we have

$$\Sigma M_0 = J_0\ddot{\phi} = -F_s a \tag{1}$$

The spring force $F_s$ results from the deflection $x$ and the spring constant $k$. For a small angle $\phi$, the spring extension is $\phi * a$, where $a$ is the lever arm.

$$F_s = kx = k\phi a \tag{2}$$




The mass moment of inertia of the beam about the fulcrum point is

$$J_0 = \frac{mL^2}{3} \tag{3}$$

From Equations (1-3) the equation of motion is given by following homogeneous differential equation

$$\ddot{\phi} + \frac{3ka^2}{mL^2}\phi = 0 \tag{4}$$

The system vibrates with the natural frequency $f$ and the time period T given by

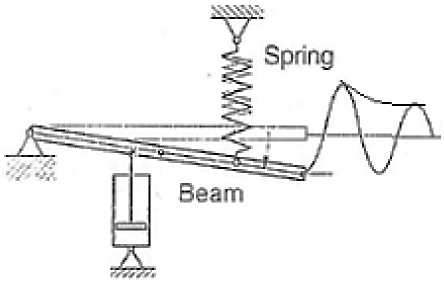
$$\omega_0^2 = \frac{3ka^2}{mL^2}, f = \frac{1}{2\pi}\sqrt{\frac{3ka^2}{mL^2}}, T = 2\pi\sqrt{\frac{mL^2}{3ka^2}} \tag{5}$$

The natural frequency calculated theoretically for various spring constants is compared with the natural frequency obtained from the experiment. The mass of beam $m = 1.680$ kg and the length of beam $L =$ 730 mm. Theoretically the oscillations in an un-damped system will not decay but in the actual system we observe a ***linearly decaying*** response. The linear decline in amplitude over time is typical of dry friction effects in oscillating systems which would be discussed in the class later. Time taken for 5 – 10 oscillations is extracted from the oscilloscope and the corresponding frequencies are compared in the table to the theoretical results.

| Frequencies of un-damped oscillation | | | | |
|---|---|---|---|---|
| Experiment no. | Spring no., constant $k$ in N/mm | Experiment | | Theoretical frequency (Hz) |
| | | Time period (sec) | Frequency (Hz) | |
| 1 | 0.75 | 0.2 | 5 | 5.18 |
| 2 | 1.50 | 0.142 | 7.04 | 7.38 |
| 3 | 3.00 | 0.0952 | 10.50 | 10.38 |

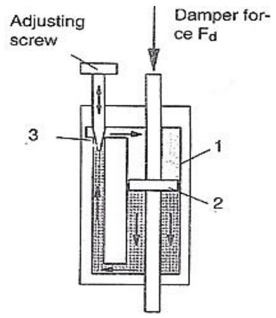
# Free damped vibrations

Similar to free un-damped vibration case the system is deflected from its equilibrium position. System oscillates about its mean position with ***exponentially decaying*** amplitude. The response is observed in the oscilloscope. This process is repeated for three different damping settings.




**Setup of the damper:**

- Attach the damper between threaded stem of the beam as instructed by teaching assistant.
- Adjust the screw to get different values of the damping coefficient.




**Procedure of adjusting damper settings**

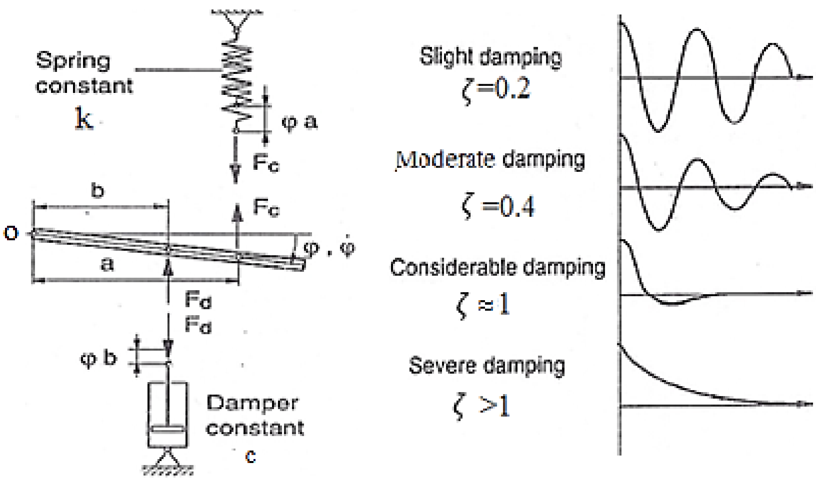
## Equations of motion: Damped oscillation

Again taking the moment balance about point O we have

$$\Sigma M_0 = J_0\ddot{\phi} = -F_s a - F_d b \tag{6}$$

For small angles the damper force $F_d$ is given by

$$F_d = c\dot{x} = c\dot{\phi}b \tag{7}$$




The resultant equation of motion is given by the following homogeneous differential equation

$$\ddot{\phi} + \frac{cb^2}{J_0}\dot{\phi} + \frac{ka^2}{J_0}\phi = 0 \tag{8}$$

The solution produces decaying harmonic oscillations

$$\phi(t) = \frac{\omega_0}{\omega_d}\hat{\phi}e^{-\zeta\omega_0 t}\sin(\omega_d t + \psi) \tag{9}$$

With natural frequency

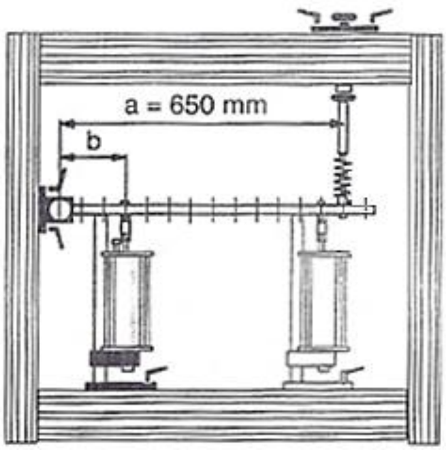
$$\omega_d = \omega_0\sqrt{1-\zeta^2}, \qquad \omega_0 = \sqrt{\frac{ka^2}{J_0}} \tag{10}$$

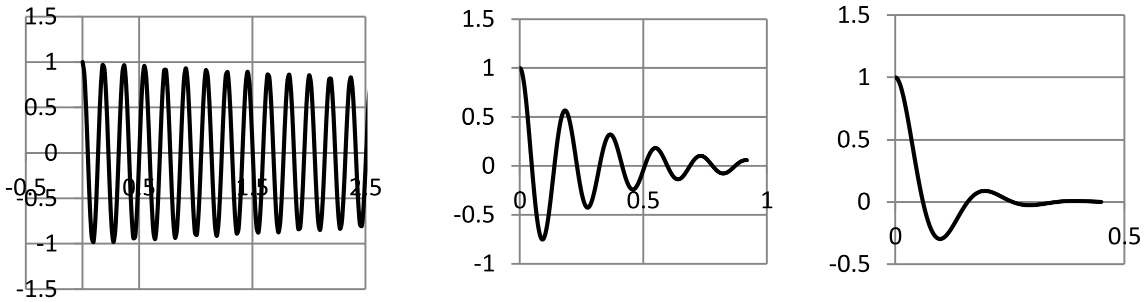
And the damping ratio $\zeta$ is

$$\zeta = \frac{c}{2J_0\omega_0} \tag{11}$$

The oscillations in a viscously damped system decays exponentially. For different settings of the adjusting screw, the time taken for 5 – 10 oscillations is extracted from the oscilloscope. From this the natural frequency $\omega_0$ is calculated. Next the value of the damping ratio $\zeta$ is estimated using the logarithmic decrement. Finally the value of damping coefficient is estimated using Equation (11).

| No. | Screw setting | Time period (sec) | Frequency $\omega_0$ (rad) | $\zeta = \frac{\log(x1/x2)}{2\pi}$ | Damping $c$ |
|---|---|---|---|---|---|
| 1 | 1 turns (slight damping) | | | | |
| 2 | 3 turns (moderate damping) | | | | |
| 3 | 5 turns (high damping) | | | | |




**Time series responses for various values of damping**

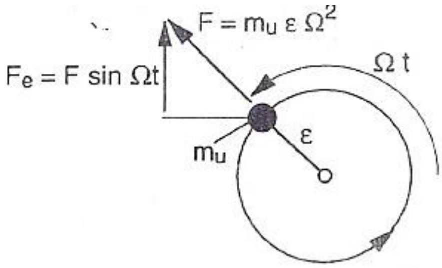
## Forced vibrations

In forced vibrations, an oscillatory system is excited by external means. After transients decay, oscillations occur at the excitation frequency. If excitation and natural frequency coincide, this is referred to as resonance. The oscillation becomes very pronounced at resonance.

## Equations of motion: Forced vibrations

The exciter is attached to the centre of the beam at $L/2$. The exciter produces a force given by

$$F_e = m_u \varepsilon \Omega^2 \sin \Omega t \tag{12}$$




**Rotating unbalanced mass as source of harmonic excitation**

The moment equilibrium about the fulcrum point O of the beam gives

$$J_0\ddot{\phi} + cb^2\dot{\phi} + ka^2\phi = \frac{m_u \varepsilon L}{2}\Omega^2 \sin \Omega t \tag{13}$$

Or with $y = \phi L$

$$\ddot{y} + 2\zeta\omega_0\dot{y} + \omega_0^2 y = \frac{m_u}{J_0}\frac{L^2}{2}\,\varepsilon\,\Omega^2 \sin \Omega t \tag{14}$$

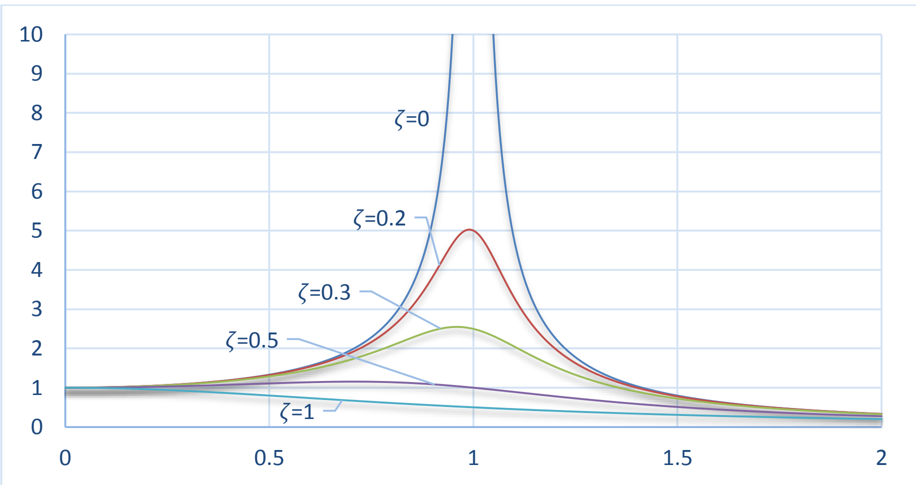
Once the transients decay over the course of time, the solution to this inhomogeneous differential equation results in the forced oscillations given by

$$y(t) = B\frac{\Omega^2}{\sqrt{\left[1-\left(\frac{\Omega}{\omega_0}\right)^2\right]^2 + \left[2D\frac{\Omega}{\omega_0}\right]^2}}\sin(\Omega t + \psi) \tag{15}$$

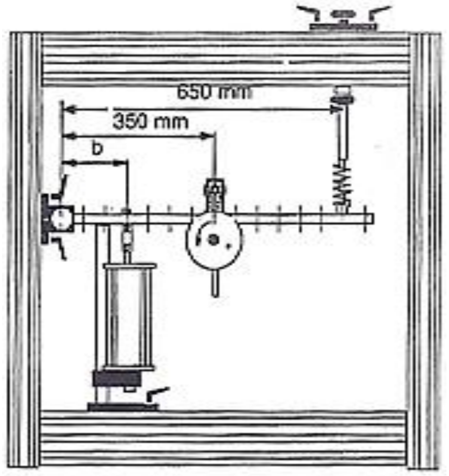
where,

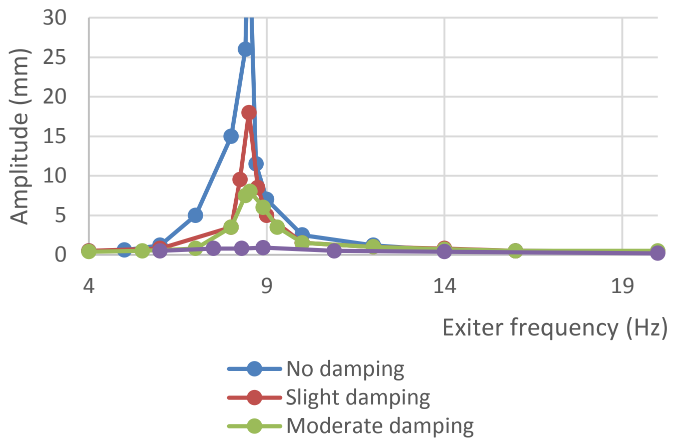
$$B = \frac{m_u \varepsilon L^2}{2J_0}$$

The profile of the oscillation response as a function of the ratio $\frac{\Omega}{\omega_0}$ is plotted in the figure below




**Amplitude ratio vs. frequency ratio for several values of damping ratio**




The experiment is set-up as shown in the figure above. The deflections are plotted for the various damper settings. 1 Hz increments are appropriate in the range between 6 and 10 Hz, whereas 0.2 – 0.5 Hz increments are to be used in the immediate vicinity of the resonance level. Frequency and amplitude is recorded for various damper settings. The measured values are plotted on a graph as shown above.

Resonance curve for damper setting no. 1: No damping, $k = 3.0$ N/mm, $a = 650$ mm

| Frequency | 4 | 5 | 6 | 7 | 8 | 8.4 | 8.5 | 8.7 | 9 | 10 | 12 | 14 | 16 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Amplitude | 0.5 | 0.6 | 1.2 | 5 | 15 | 26 | 42 | 11.5 | 7 | 2.5 | 1.2 | 0.6 | 0.5 | 0.4 |

Resonance curve for damper setting no. 2: Slight damping, $k = 3.0$ N/mm, $a = 650$ mm

| Frequency | 4 | 6 | 8 | 8.25 | 8.5 | 8.75 | 9 | 10 | 12 | 14 | 16 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Amplitude | 0.5 | 0.8 | 3.5 | 9.5 | 18 | 8.5 | 5 | 1.5 | 1.0 | 0.8 | 0.5 | 0.4 |

Resonance curve for damper setting no. 3: Moderate damping, $k = 3.0$ N/mm, $a = 650$ mm

| Frequency | 4 | 5.5 | 7 | 8 | 8.4 | 8.53 | 8.9 | 9.3 | 10 | 12 | 14 | 16 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Amplitude | 0.4 | 0.5 | 0.8 | 3.5 | 7.5 | 8.0 | 6.0 | 3.5 | 1.5 | 1.0 | 0.7 | 0.5 | 0.5 |

Resonance curve for damper setting no. 4: Considerable damping, $k = 3.0$ N/mm, $a = 650$ mm

| Frequency | 6 | 7.5 | 8.3 | 8.9 | 10.9 | 14 | 20 |
|---|---|---|---|---|---|---|---|
| Amplitude | 0.5 | 0.8 | 0.8 | 0.9 | 0.5 | 0.4 | 0.2 |

**IIT Kanpur, Mechanical Engineering,**

ME 354A, Vibration and Control

Handout for Experiment 4

*Vibration transmissibility of a single degree of freedom system*

**Overview**

This experiment focuses on the response of a single degree of freedom system put on a base, which is excited with a constant amplitude sinusoidal displacement through a motor. The mass moves in one direction only (vertical). The acceleration of the mass is measured using a piezo sensor. The piezo sensor is connected to a charge amplifier and then to an oscilloscope, which displays the motion of the mass in terms of voltage. This voltage can be converted to displacement in *mm*.

Our aim is to study the transmissibility of the vibrations from the base to the mass. So, we record the vibration amplitude of the mass for different frequencies of base excitation.

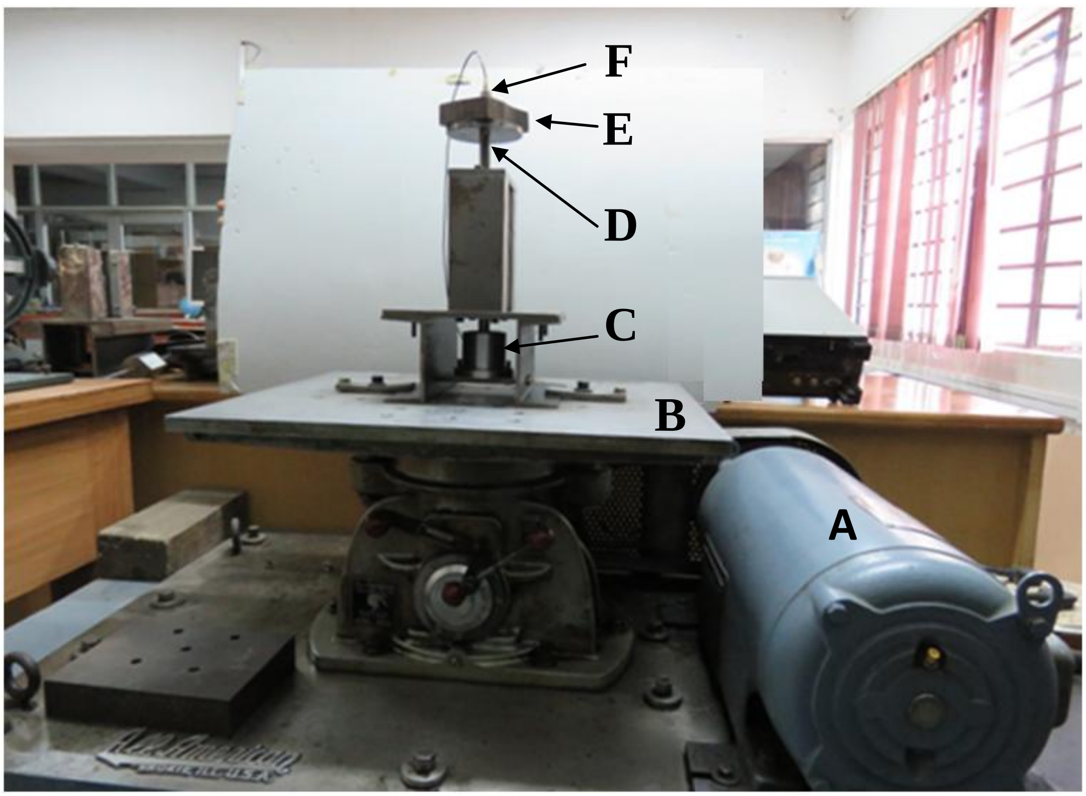



Figure 1: Experiment setup. (A) Motor (B) Base (C) Viscous damper (D) Primary mass system (E) Added extra mass (F) Piezo sensor.

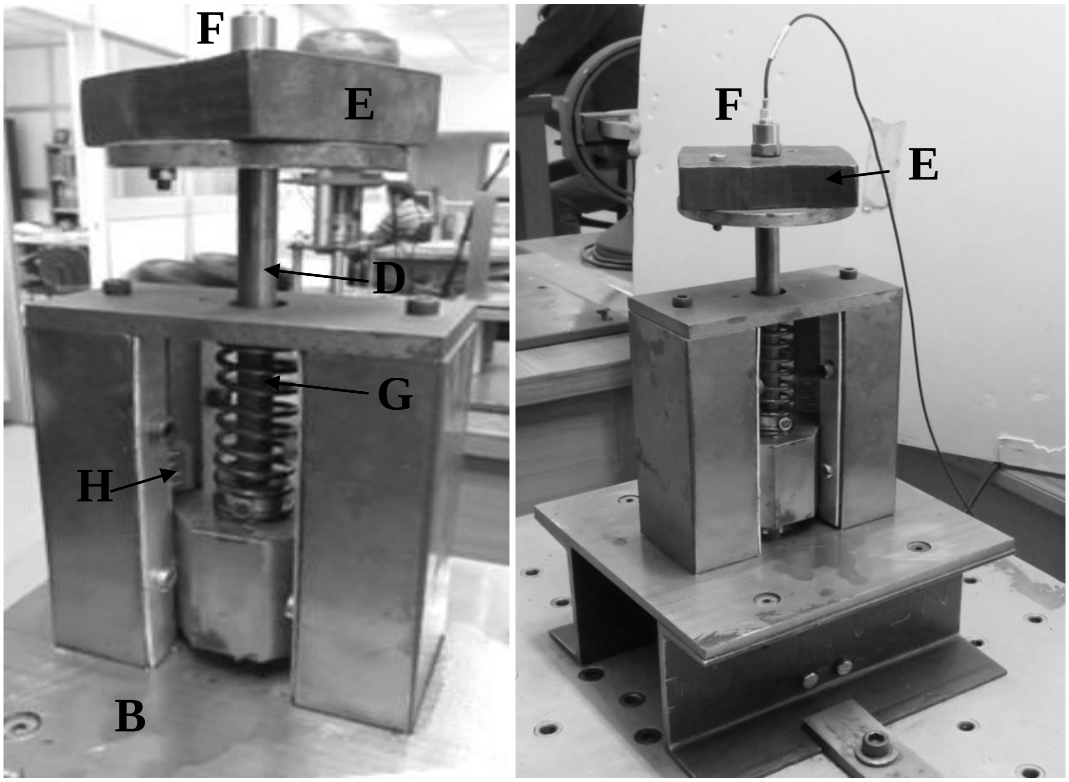


Figure 2: Experiment setup. (B) Base (D) Primary system (E) Added extra mass (F) Piezo sensor (G) Spring (H) Linear bearing.

**Experimental procedure:**

1. The base displacement amplitude is about 0.25″ (6.35 mm) peak to peak. Verify the amplitude by attaching the sensor to the base. You can convert the signal on the oscilloscope (which is in voltage) to *mm* by multiplying with a suitable factor (next page). See how to adjust the gain on the charge amplifier as instructed by the TA.

2. Put the piezo sensor on the mass '*m*' and measure the displacement for the frequency range of 5 to a maximum of 35 Hz (do not go beyond 35 Hz in any case). Take about 20 readings. Be careful to take only small increments and to let system settle down for a while after each increment. Take several readings close to the resonant frequency.

3. Attach the known extra mass '*M*' (weight of the extra mass is given on next page) over '*m*'. Repeat step 2 for the combined system within the same frequency range.

4. Remove the viscous fluid from the damper pot through suction syringe (this removes the damping). Repeat step 2 for the combined system as mentioned in step 3.

5. Fill in the frequency-amplitude data for each step in the observation tables given below.

Values you will need

Weight of the extra mass = 1.1 Kg.
Displacement conversion from oscilloscope: 100 mV = 1mm

**Observation tables:**

SDOF System with damper and without extra mass '*M*'

| **Data points** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | | | | | | | | | | | | | | | |
| **Amplitude** | | | | | | | | | | | | | | | |

SDOF System with damper and with extra mass '*M*'

| **Data points** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | | | | | | | | | | | | | | | |
| **Amplitude** | | | | | | | | | | | | | | | |

SDOF System without damper and with extra mass '*M*'

| **Data points** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** | **11** | **12** | **13** | **14** | **15** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Frequency** | | | | | | | | | | | | | | | |
| **Amplitude** | | | | | | | | | | | | | | | |

**Laboratory report:**

Plot the transmissibility (ratio of the amplitude of the mass to the amplitude of the base) for step 2, step 3 and step 4 in procedure. Discuss the results and compare these plots. Determine original mass ($m$) using the transmissibility equation (see theory below).

**Theory:**

Let $y$ be the harmonic displacement of the base and measure the inertial displacement $x$ of the mass $m$.

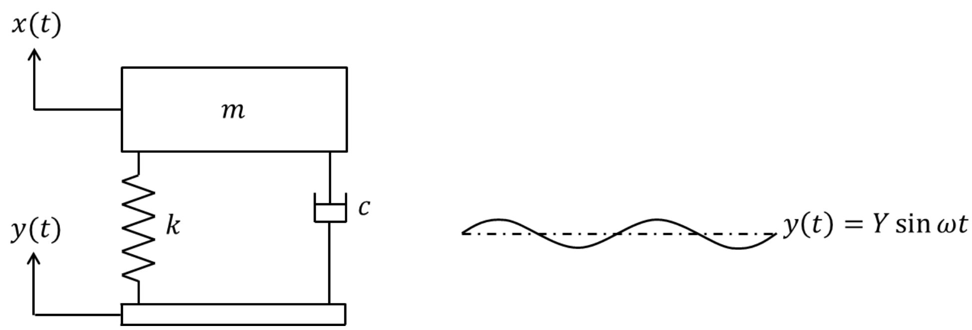



Figure 1: Spring mass dashpot system with base excitation

The equation of motion is (measured from mean position to eliminate gravity)

$$m\ddot{x} = -c(\dot{x} - \dot{y}) - k(x - y) \tag{1}$$

Let $(x - y) = z$

$$m\ddot{z} + c\dot{z} + kz = -m\ddot{y} = mY\omega^2 \sin wt \tag{2}$$

The solution is

$$z = Z \sin(\omega t - \Phi)$$
$$Z = \frac{mY\omega^2}{\sqrt{(k - m\omega^2)^2 + c\omega^2}} \tag{3}$$

with

$$\tan \Phi = \frac{c\omega}{k - m\omega^2} \tag{4}$$

Since the inertial motion '$x$' of the mass is desired, we find $x = y + z$

Using the exponential form of harmonic motion gives

$$y = Ye^{i\omega t} \tag{5}$$

$$z = Ze^{i(\omega t - \Phi)} = Ze^{-i\Phi}e^{i\omega t} \tag{6}$$

$$x = Xe^{i(\omega t - \psi)} = Xe^{-i\psi}e^{i\omega t} \tag{7}$$

From above equations and equation of motion, we obtain

$$Ze^{-i\Phi} = \frac{mY\omega^2}{k - m\omega^2 + ic\omega} \tag{8}$$

and

$$x = Ze^{-i\Phi} + Ye^{i\omega t} = \left(\frac{k + ic\omega}{k - m\omega^2 + ic\omega}\right)Ye^{i\omega t} \tag{9}$$

The steady state amplitude and phase is:

$$\left|\frac{X}{Y}\right| = \sqrt{\frac{k^2 + (c\omega)^2}{(k - m\omega^2)^2 + (c\omega)^2}} \tag{10}$$

and

$$\tan\psi = \frac{mc\omega^3}{k(k - m\omega^2) + (c\omega)^2} \tag{11}$$

You are not measuring the phase. To what extent does the amplitude versus frequency curve match theory?

In particular, Eq. 10 (and figure 3) shows that transmissibility is 1 when the forcing frequency is √2 times the natural frequency. Can you use this to estimate $k$ and $m$ of the original system (without added mass)? If you change the damping (with oil and without oil), does the transmissibility curve intersect at "transmissibility = 1" as per theory?

Write your thoughts on these questions and on the experiment. (Evaluative discussion.)

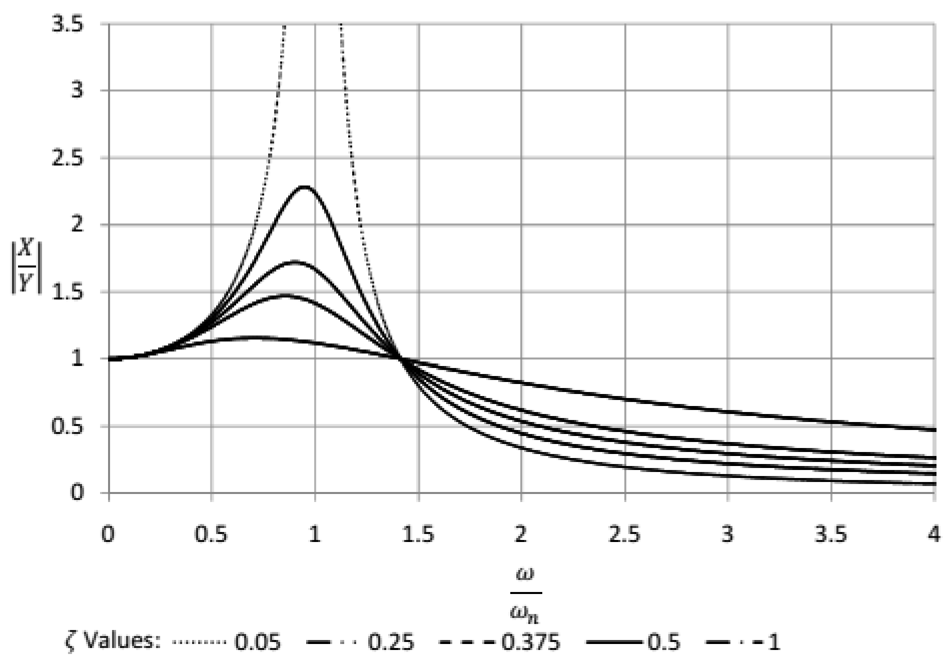



Figure 3: Amplitude ratio vs. frequency ratio

# IIT Kanpur, Mechanical Engineering,

ME 354A, Vibration and Control
Handout for Experiment 4
Tuned vibration absorber

**Overview**

A single degree of freedom mass supported by a spring, acted upon by a harmonic force, is studied widely. When the excitation frequency approaches the natural frequency of the spring mass system, the amplitude grows large (called resonance).

When a smaller mass (secondary) is spring-mounted on the forced mass (primary), and the excitation frequency approaches the natural frequency of the secondary spring-mass system, then the primary system has a near zero amplitude (see “theory” below). We will experimentally study the validity of this theoretical result. We will verify if, as we excite the combined system at this chosen frequency, the primary mass stands almost still and only the secondary mass moves. We will also see if we can detect two resonance peaks (corresponding to two degrees of freedom).

**Experimental setup:**

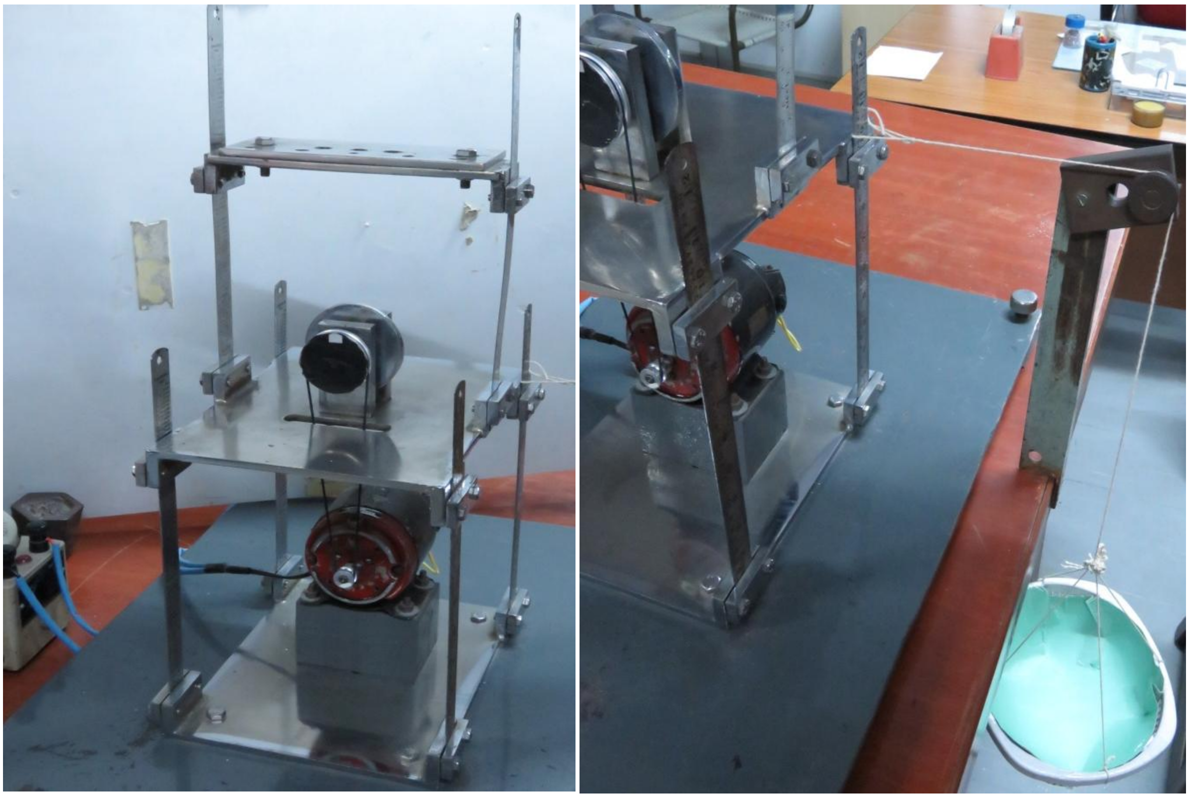


Figure 1: Experiment setup. Left: the primary (bottom) and the secondary system (top). Here the steel rulers act as springs, and the platforms will move in the lateral direction. Right: another view. Weights are put inside the hanging basket to estimate the stiffness of the ruler.

**Overview (continued):**

You will excite the primary system at 3-4 different frequencies and note its response amplitudes. You will characterize the secondary system's spring stiffness and choose its strip length to tune it to each of those 3-4 frequencies, and in each case you will verify if the secondary manages to reduce response amplitude of the primary, as per theory. You will also seek two resonant frequencies of the 2DOF system in each case.

Note that the system (both the primary and the secondary) being supported by two strips ensures that the mass vibrates laterally without tilting, which would have happened if a single strip was used.

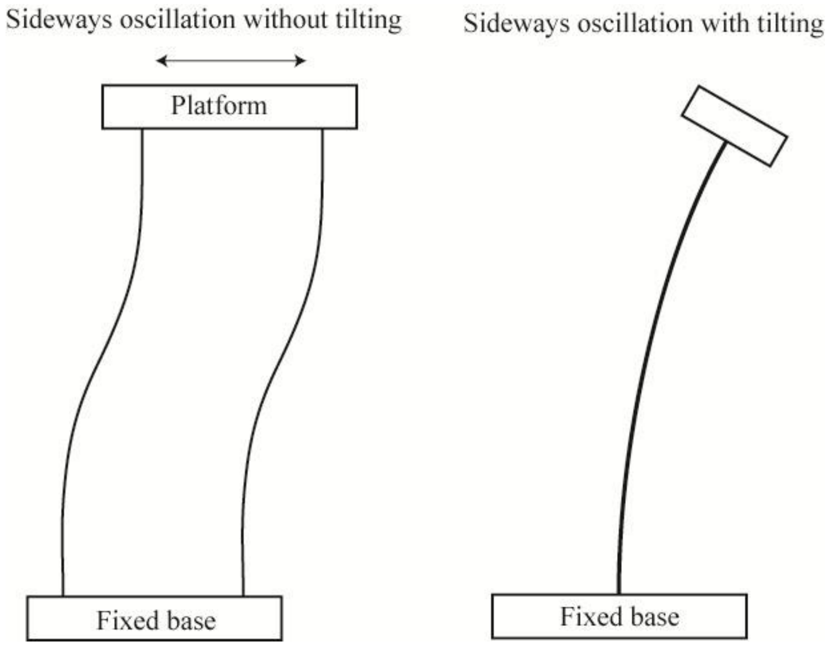



Figure 2: Platform supported by two strips allows lateral vibration without tilting (tilting would require one of the strips to either lengthen, shorten, or buckle). Two strips make the single degree of freedom model appropriate. With one strip, there would be lateral displacement as well as tilting, requiring a two degree of freedom model.

**Procedure:**

1. Primary system: At a given length ($l_1$, given to you by the TA) of the strips of the primary system, roughly measure oscillation amplitude against frequency (report it along with estimated error bounds). Measure amplitudes using the stroboscope at 4 different frequencies of excitation (as instructed by the TA; *be careful of moving parts*). After each change in forcing frequency, wait until transients decay before measuring amplitude. Plot the frequency-amplitude data and estimate the resonance frequency.

2. Secondary system: Remove the primary system and mount the secondary system. Measure the lateral deflection of the secondary system using the provided system of

string, pulley, hanging weights and a dial gauge, at 3 different strip lengths. Verify the force-deflection relationship (stiffness inversely proportional to cube of length?).

3. The mass of the secondary system is given. For the specific frequency values from step 1 (one by one), estimate $k_2$ (and thus $l_2$) such that that specific frequency is the natural frequency of the secondary system.

4. Attach the secondary system on top of the primary. Excite the combined system at the frequency value chosen in step 3. Note the reduction in amplitude of the primary system. Change the excitation frequency and observe the large amplitude in-phase and out of phase motions of the primary and the secondary mass. *Estimate the resonance frequencies for these two motions. Write them in your data sheet and get the TA's signature.* Compute the same later from theory, to enter in your lab report.

**5.** Repeat steps 3 and 4 for each of the frequencies of step 1.

<u>Values you will be needing</u>

$m_1 = 4.8$ kg, $m_2 = 1.75$ kg.

**Experimental observation:**

Amplitude vs frequency for the primary system

| **Primary system for length $l_1$ =** | | | | | |
|---|---|---|---|---|---|
| **S. No.** | **1** | **2** | **3** | **4** | **5** |
| **Frequency (Hz)** | | | | | |
| **Amplitude (m)** | | | | | |

Estimate stiffness ($k_2$) of the secondary system for three different strip lengths ($l_2$)

| **Secondary system** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **length $l_2$ =** | | | **length $l_2$ =** | | | **length $l_2$ =** | | |
| | **Weight (kg)** | **Force (N)** | **Deflection (m)** | **Weight (kg)** | **Force (N)** | **Deflection (m)** | **Weight (kg)** | **Force (N)** | **Deflection (m)** |
| **1** | | | | | | | | | |
| **2** | | | | | | | | | |
| **3** | | | | | | | | | |

Estimate stiffness ($k_2$) and strip length ($l_2$) needed for frequency values used in step 1 (one by one).

| | **Frequency** | **Stiffness ($k_2$)** | **Length ($l_2$)** |
|---|---|---|---|
| **1** | | | |
| **2** | | | |
| **3** | | | |
| **4** | | | |

**Theory**

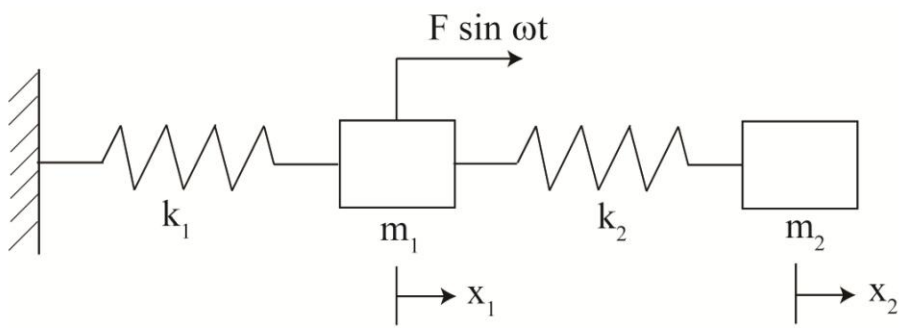



Figure 2: Schematic of a vibration absorber. Excitation is applied to the primary mass.

The primary system (mass $m_1$, stiffness $k_1$) supports the secondary system (mass $m_2$, stiffness $k_2$) and is excited by force $F\,sin\,\omega t$. The equations of motion for the two degree of freedom system are

$$\ddot{x_2} = -\frac{k_2}{m_2}(x_2 - x_1)$$

$$\ddot{x_1} = -\frac{k_1}{m_1}x_1 + \frac{k_2}{m_1}(x_2 - x_1) + F\sin\omega t. \tag{1}$$

Assume $\omega = \sqrt{\frac{k_2}{m_2}}$, and assume the steady state solution to be

$$x_1 = \mathrm{X}_1 \sin\omega\mathrm{t}, \qquad x_2 = \mathrm{X}_2 \sin\omega\mathrm{t}. \tag{2}$$

From the above equations, we get

$$-\omega^2\mathrm{X}_2 = -\omega^2\mathrm{X}_2 + \omega^2\mathrm{X}_1$$

$$-\omega^2 \mathrm{X}_1 = \quad -\frac{k_1}{m_1}\mathrm{X}_1 + \frac{k_2}{m_1}(\mathrm{X}_2 - \mathrm{X}_1) + \mathrm{F}, \tag{3}$$

which gives

$$X_1 = 0,$$

$$X_2 = -\frac{m_1}{k_2}F. \tag{4}$$

Thus, we see that at this special $\omega$, the response of the primary system is zero (in the undamped case). Note that $\omega_1$ need not be special. The secondary mass has finite amplitude of vibration, while the primary mass stands still.

Present an evaluative discussion of the extent to which experiment matches theory.

# IIT Kanpur, Mechanical Engineering

ME354A, Vibration and Control

Handout for Experiment 5

*Speed Measurement of a DC Motor Using Various Sensors*

Electrical motors are broadly classified into two categories: (a) Direct Current (DC) motors, and (b) Alternative Current (AC) motors. In most cases, an electric motor consists of a stator (stationary field) and a rotor (rotating field or armature). Electrical motors work through the interaction of magnetic flux and electric current to produce rotational speed and torque. You can read more on electrical motors from any book on electrical machines, e.g., *Electric Motors and Drives: Fundamentals, Types and Applications*, Newnes, Elsevier, 2006 by Austin Hughes (available online).

In this experiment, we will measure angular speed of a DC motor using following six techniques:

1. Magnetic pick up
2. Hall effect sensor
3. Inductive sensor
4. Photo reflective method
5. Photo interruptive method
6. Stroboscope method

Magentic pick up, Hall effect sensor and Inductive sensor are based on magnetism, whereas Photo reflective method, Photo interruptive method and Stroboscope method are based on optics. You are encouraged to read in detail about the above six topics. A small viva at the start of the experiment will check if you have done minimum background reading.

Figure 1 shows the experimental setup.

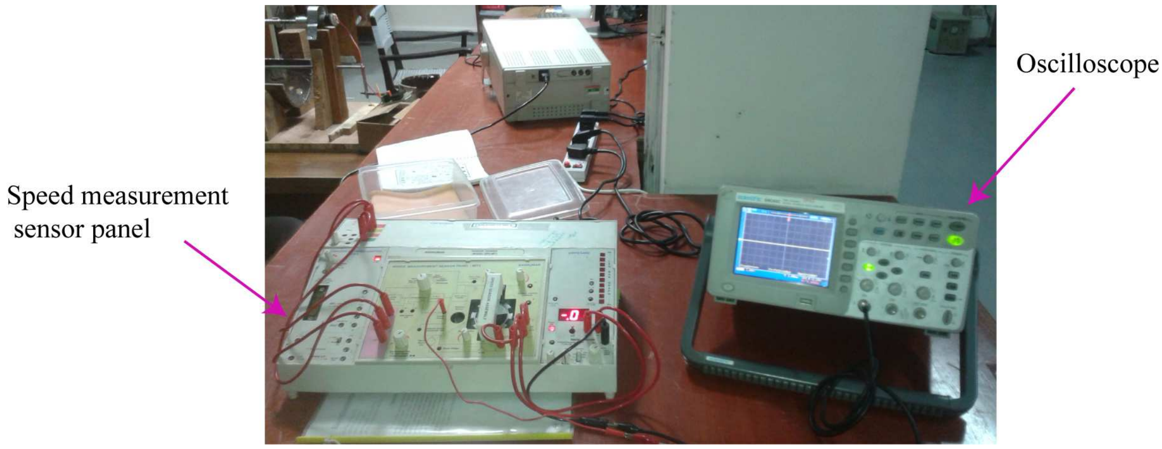



Figure 1: Experimental setup. The motor is inside, and drives the slotted disk.

Figure 2 schematically shows the speed measurement sensor panel. Note the wheel with 8 slots in the figure. The slotted wheel is fitted to the motor shaft. The slotted wheel causes alteration in the magnetic field as well interruption in the optical path. This results in periodic generation of voltage. The periodic signals are amplified and converted into pulses.

For an eight teeth rotating wheel, we get 8 times the motor shaft frequency for the first ***five*** methods mentioned above. Thus, we divide the frequency by 8 to get motor shaft frequency. For the ***sixth*** method, we need to divide by 2 (why?).

### Motor specifications

12V DC/ 600 MW, permanent magnet, rated speed 0 to 4000 RPM.

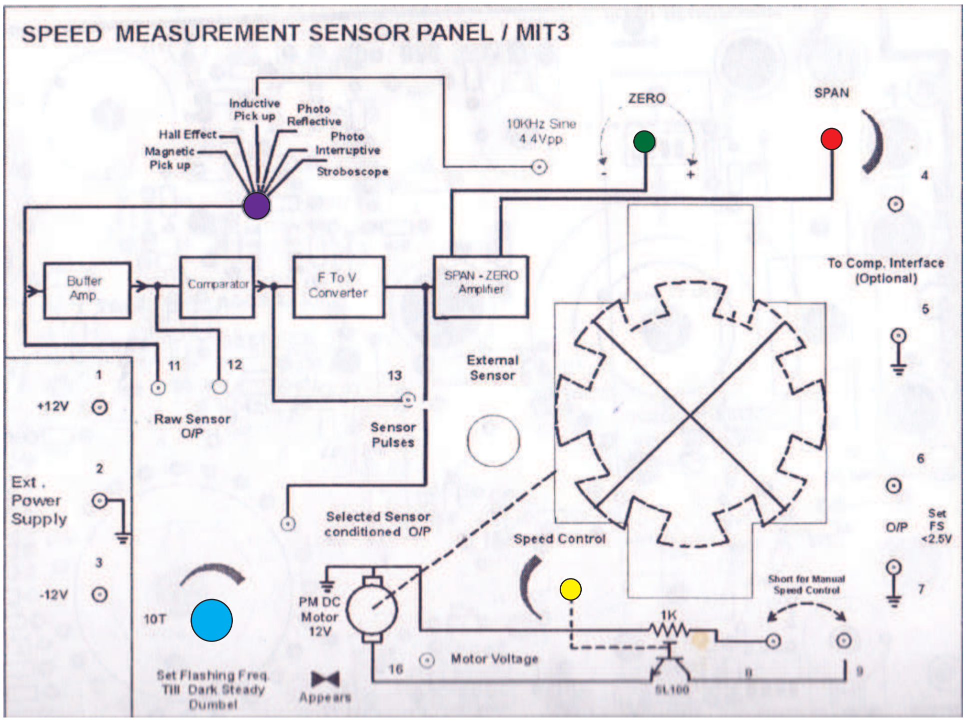



Figure 2: Speed measurement sensor panel. Figure is taken from User's guide (Anshuman Tech Pvt. Ltd.)

### Signal conditioning

The panel consists of all six sensors for all six methods mentioned above. We can activate any of the sensors by turning the knob to the desired position. See the purple marker in Figures 2 and 3. The panel also contains a motor speed control potentiometer. See the yellow marker in Figures 2 and 3. All sensors output is finally converted to DC voltage. Speed measurement range is adjusted by zero set and span adjustment potentiometer on board. See the green and red markers in Figures 2 and 3.

### Span and zero adjustment procedure

This is the first step of the experiment.

It is desired that output will be zero when motor speed is zero, and output will be slightly under 2 Volts (say 1.95 Volts) at maximum speed (say 4100 RPM). To achieve this, set span knob (red) near maximum, then set motor speed (yellow) to zero, then adjust the zero knob (green) to obtain zero output. Now change speed to maximum and readjust the span knob to obtain an output of 1.95 Volts. Again change motor speed to zero, and see if output is zero. Adjust zero knob if necessary. Repeat as needed to obtain desired zero point and span.

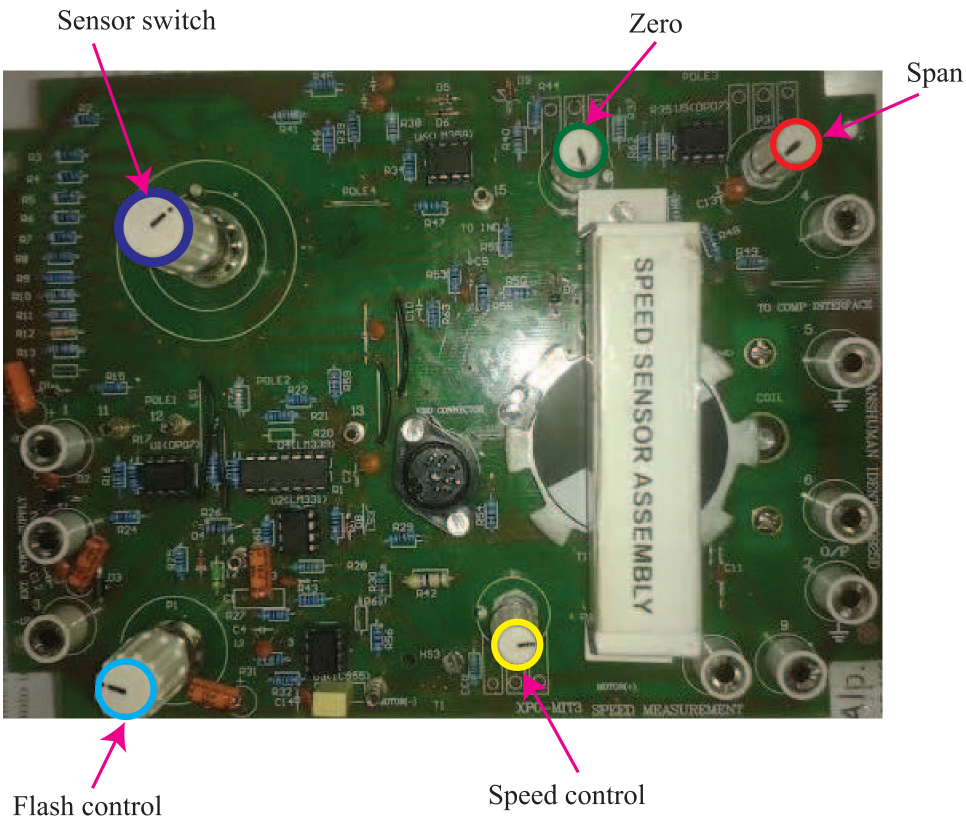



Figure 3: Speed measurement sensor panel (MIT3): circuit board.

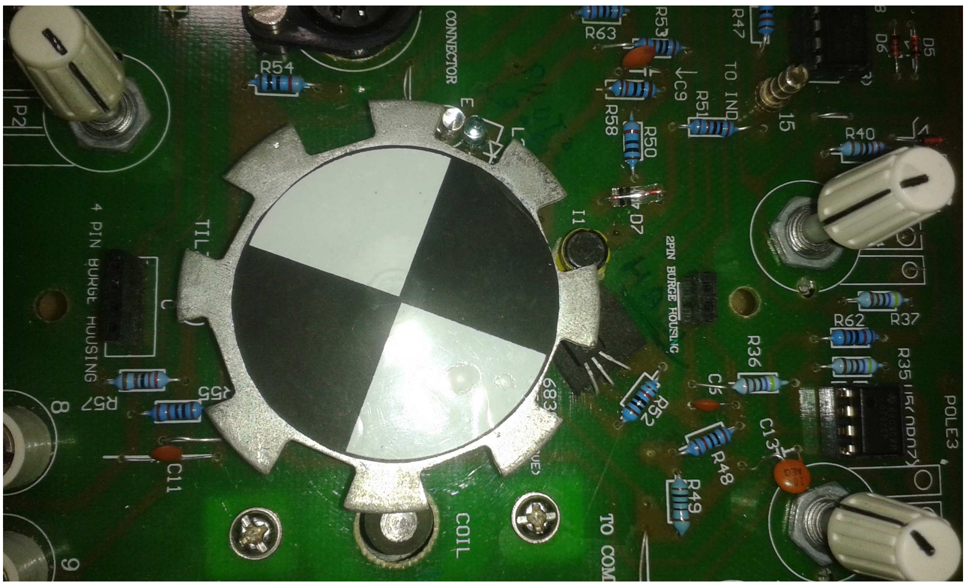



Figure 4: Enlarged view of the slotted wheel.

# 1 Magnetic pick up

### Overview

A magnetic pick up is a coil placed in magnetic field. Variation in magnetic field induces a voltage in the coil. Magnetic flux produced by magnet can be altered by moving any external magnetic material into the magnetic field which will induce voltage in the coil placed in it. This voltage from magnetic pick up is proportional to the rate of change of flux.

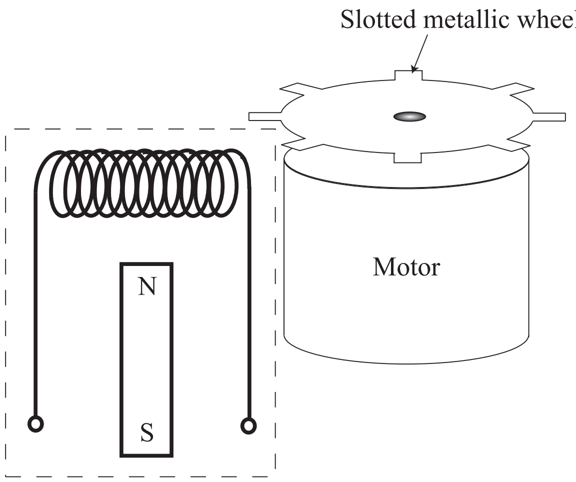



Figure 5: Magnetic pick up circuit

Every time a tooth of the wheel passes over the magnet it will cut magnetic lines and hence induce a voltage. We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. Since there are eight teeth, we have to divide the frequency by eight to get the frequency of the shaft rotation.

See more on Magnetic pickups on: www.ece.rochester.edu/courses/ECE140/resources/Guitar-Project/Mag_pickups.pdf

# 2 Hall effect method

### Overview

In 1879 E. H. Hall observed that when an electrical current passes through a sample in a magnetic field, a potential is developed across the material in a direction perpendicular to both the current and the magnetic field. This effect is known as Hall effect.

In the experimental setup, a permanent magnet is mounted just below the slotted wheel. Permanent magnet produces magnetic field around Hall effect sensor. On rotation of the motor, when a tooth of the wheel comes in between magnet and the Hall sensor, a voltage pulse is generated due to the Hall effect. We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. Since there are eight teeth, we have to divide the frequency by eight to get the frequency of the shaft rotation.

See more on Hall effect and Hall sensor from wikipedia documents: https://en.wikipedia.org/wiki/Hall_effect, https://en.wikipedia.org/wiki/Hall_effect_sensor.

# 3 Inductive method

### Overview

When a magnetic field is disturbed for example in presence of a magnetic material, the inductance (or inductive reactance) of the magnetic field changes. In the experimental setup, an inductance coil is installed below the slotted wheel. Permanent magnet establishes a magnetic field around the inductance coil. When the motor rotates, the slotted wheel connected to the motor shaft disturbs the magnetic field due to presence of teeth. This causes change in the inductance. Hence voltage across coil changes each time the magnetic field is disturbed. We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. Since there are eight teeth, we have to divide the frequency by eight to get the frequency of the shaft rotation.

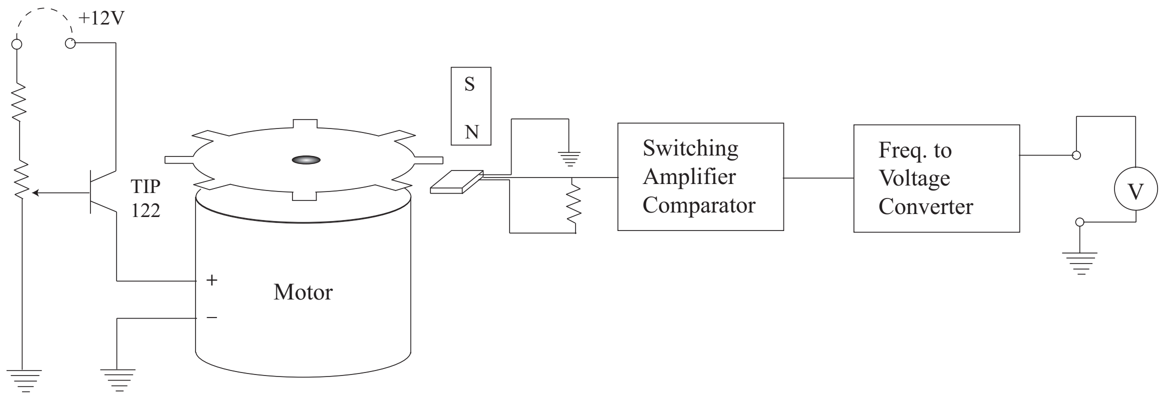



Figure 6: Hall effect sensor method

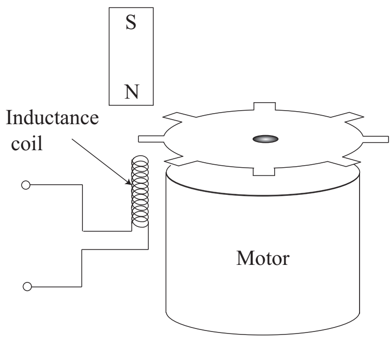



Figure 7: Inductive sensor method

See more on inductive sensor on: http://www.fargocontrols.com/sensors/inductive_op.html, https://www.omega.com/manuals/manualpdf/M4594.pdf.

# 4 Photo reflector sensor method

### Overview

In this method, the slotted wheel connected to the motor shaft is used as reflector for infrared signal. An infrared transmitter and a detector is installed below the teeth area of the wheel. The infrared transmitter transmits light which gets reflected by the teeth of the slotted wheel. This reflected light is detected by the photo transistor whose output made available for speed measurement. A schematic of photo reflector sensor is shown in Figure 8. When slotted wheel rotates, each tooth of the wheel passes over the transmitter receiver pair and every time receiver detects light gives a voltage output. We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. Since there are eight teeth, we have to divide the frequency by eight to get the frequency of the shaft rotation.

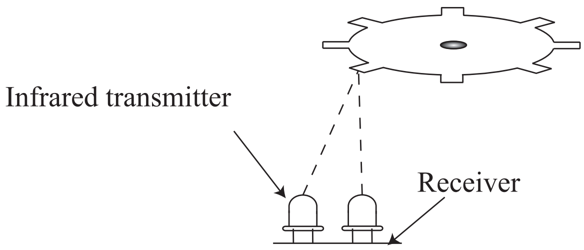



Figure 8: Photo reflector for speed measurement

See more on photo electric sensor on: http://www.softnoze.com/downloads/Sensor\%20Basics\%204.pdf

# 5 Photo interruptive method

### Overview

The method works as follows: A transmitter is mounted above the teeth area of the slotted wheel and a receiver is mounted below the teeth area. Figure 9 shows a schematic of Photo interruptive sensor. The transmitter transmits light rays and the receiver detects the light. On rotation of the wheel, the light rays get interrupted every time a tooth of the wheel comes in the path. Each interruption between transmitter and receiver causes change in output voltage. We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. Since there are eight teeth, we have to divide the frequency by eight to get the frequency of the shaft rotation.

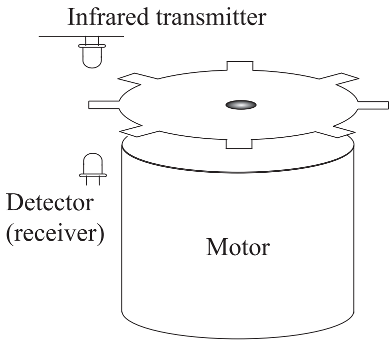



Figure 9: Photo interruptive sensor method

See more on photo interruptive sensor on: `http://www.digikey.com/en/articles/techzone/2013/apr/ir`.

Observation Table

| Sl. No. | Output Voltage | Motor speed in RPM (f/8) x 60 | | | | |
|---|---|---|---|---|---|---|
| | | Method 1 | Method 2 | Method 3 | Method 4 | Method 5 |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |
| | | | | | | |

# 6 Stroboscopic method

### Overview

Stroboscope is basically a lamp, flashing at precise intervals. Flashing rate is controllable and calibrated. When this light is directed on a vibrating or rotating object, a steady pattern may be visible if the frequency of vibration/ rotation of the object coincides with the frequency of the flashing.

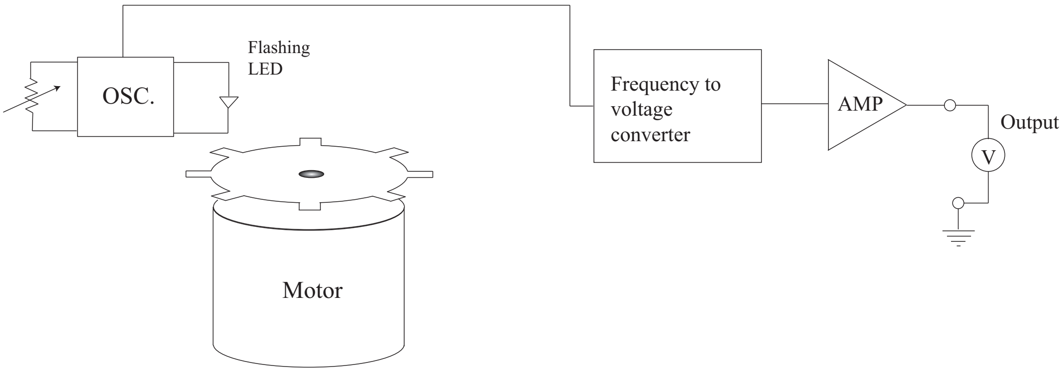



Figure 10: Stroboscope arrangement for speed measurement

When the stroboscope frequency equals twice the motor spin frequency, you will see a dumbbell pattern.

We observe the voltage pulses in the oscilloscope and get the frequency of the output signal. We divide the frequency by two to get the frequency of the shaft rotation.

See more on Stroboscope and Stroboscopic effect from wikipedia documents: `https://en.wikipedia.org/wiki/Stroboscope`, `https://en.wikipedia.org/wiki/Stroboscopic_effect`.

Observation Table

| Sl. No. | Output Voltage | Motor speed in RPM (f/2) x 60 |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |

## Simpler experiment to measure speed of a DC motor

The above experiment is done using a instrument made by a commercial manufacturer (Anshuman Tech. Pvt. Ltd.). In this experiment, we will measure speed of a DC motor by a hands on experiment. Figure 11 schematically shows the experimental setup.

We will make a simple circuit to run a DC motor. We will use a permanent magnet, and a copper wire wound on a iron nail to form coil. A bolt is attached to the motor shaft with axis of the bolt perpendicular to the shaft axis. The bolt will rotate as the motor runs. We will hold the permanent magnet along with the copper coil near the path of rotation of the bolt. When the bolt will come close to the coil, it will disturb the induced magnetic field. This will result in generation of a voltage. We will see these voltage pulses on the oscilloscope. From the output signal, we will measure the frequency of the shaft rotation.

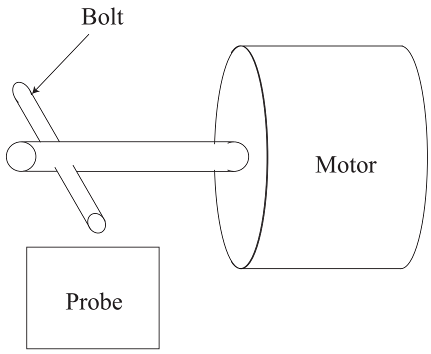



Figure 11: Schematic of the experimental setup.

**IIT Kanpur, Mechanical Engineering**

ME 354A, Vibration and Control

SPEED TORQUE CHARACTERISTICS OF DC SERVOMOTOR
(Experiment 6-A)

# Overview

The speed of a DC servo motor can be controlled by changing both the field current and the armature current. In this experiment, we determine the speed torque characteristics by applying external load to the rotor of the DC motor for two different settings of field current. The external load is applied by means of a frictional belt wrapped around flywheel connected with the rotor shaft.

The relevant equations for the experiment are

$$T_m = K_m \phi I_a \tag{1}$$

$$T_m = \frac{K_m \phi}{R_a} \left(V_a - K_e \phi \omega_m\right) \tag{2}$$

where $T_m$ is the motor torque, $\phi$ is the magnetic flux (as a result of the field current set by the experimenter), $I_a$ is the armature current, $R_a$ is the armature resistance, $e_b$ is the back emf, $\omega_m$ is the angular speed, $K_m$ is a proportionality constant known as motor torque constant and $K_e$ is a proportionality constant known as back emf constant.

The objectives of the experiment are following–

1. Verify the relationships given in Eqs. (1)-(2) from the experimental data.
2. Comment on the frictional torque developed in rotor
3. Plot $I_a$ vs. $T_m$ and $\omega_m$ vs. $T_m$ curve
4. It is evident from Eq. (1) that the magnetic field strength ($\phi$) is proportional to the slope of the $I_a$ vs. $T_m$ line. Fit straight lines to the experimental data and obtain the slopes for two (or three) values of the field current. The ratio of two such fitted slopes gives the ratio of the fluxes (say $\frac{\phi_1}{\phi_2}$) developed for two different values of field current.
5. It can be seen from Eq. (2) that the slope of $\omega_m$ vs. $T_m$ line is proportional to $\phi^2$. Calculate the slope of each line obtained from each setting of field current. Then, determine the ratio $\frac{\phi_1}{\phi_2}$ and compare with the earlier value.

# Experimental Analysis

## Observations

**Case I**
Armature voltage $V_a =$
Field current $I_f = 0.25\text{A}$

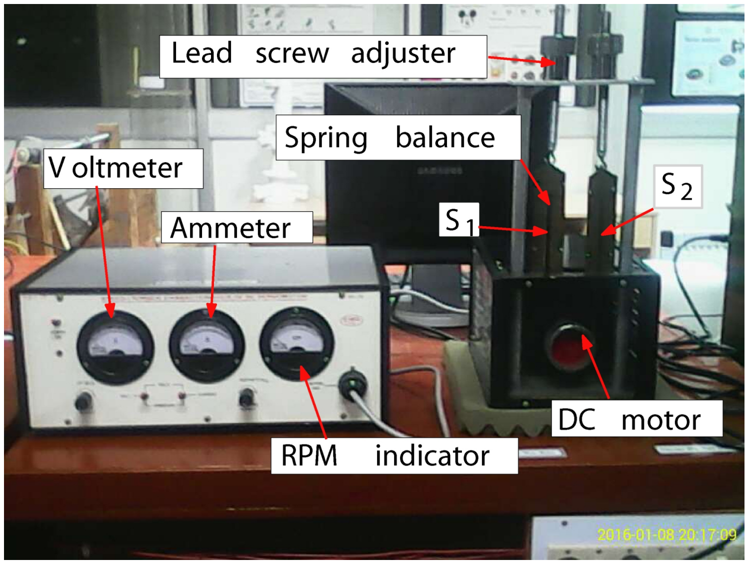



Figure 1: Experimental setup

Table 1: Case 1

| Sr. no. | $S_1$ (g) | $S_2$ (g) | $N$ (rpm) | $I_a$ (amp) | $T$ (N) |
|---|---|---|---|---|---|
| 1 | | | 1500 | | |
| 2 | | | 1400 | | |
| 3 | | | 1200 | | |
| 4 | | | 1000 | | |
| 5 | | | 800 | | |
| 6 | | | 600 | | |
| 7 | | | 400 | | |
| 8 | | | 0 | | |

**Case II**
Armature voltage $V_a =$
Field current $I_f = 0.5$A

## Calculation

Armature resistance $(R_a) = \frac{V_a}{I_a}$

Torque $(T) = (S_1 - S_2) \times r$ where r is the radius of pulley = 2.5 cm

Table 2: Case 2

| Sr. no. | $S_1$ (g) | $S_2$ (g) | $N$ (rpm) | $I_a$ (amp) | $T$ (N) |
|---|---|---|---|---|---|
| 1 | | | 3000 | | |
| 2 | | | 2500 | | |
| 3 | | | 2000 | | |
| 4 | | | 1500 | | |
| 5 | | | 1000 | | |
| 6 | | | 500 | | |
| 7 | | | 0 | | |

MEASUREMENT OF LINEAR DISPLACEMENT BY POTENTIOMETER
(Experiment 6-B)

## Overview

The objective of the experiment is to measure linear displacement using a potentiometer. The displacement of the slider changes the resistance of the potentiometer. As a result the voltageacross the potentiometer terminals changes. This voltage signal is then amplified and displayed through a digital panel. The generated signal can be calibrated using the offset controlling knob and gain adjusting knob available with the setup.

The objective of this experiment is to verify the linearity between the actual displacement and the displacement sensed by the potentiometer.

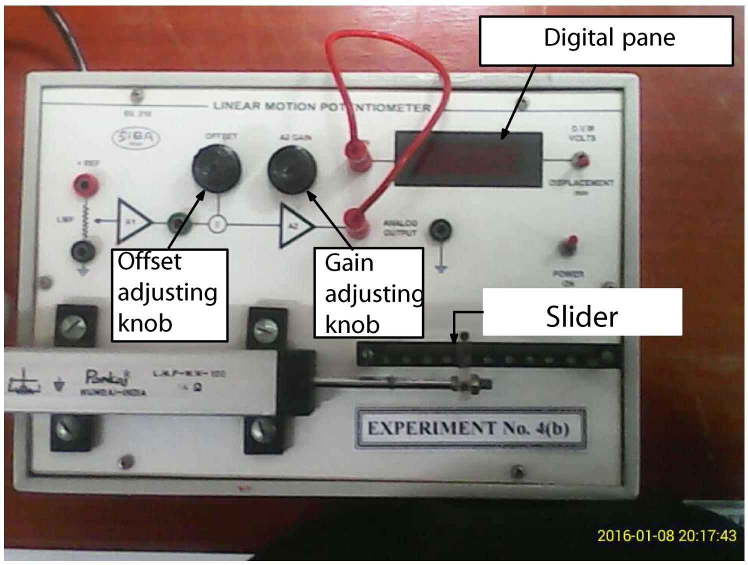



Figure 2: Experimental setup

## Experimental procedure

1. Calibration of the system :

   (a) Adjust the slider to 10 mm (1cm) line. Adjust offset control to read 10.0 mm at the Digital voltage meter (DVM).

   (b) Now bring the slider to 90 mm line. Adjust gain control to read 90.0 mm in digital voltage meter (DVM).

   (c) Repeat step 1 and 2, two to three times, until further adjustments are not needed.

2. Calibrated Reading:

   Displace the slider with an increment of 10 mm. Note down the corresponding displacement and voltage displayed in digital panel.

# 1 Observation and calculations:

1. Plot a graph between the displacement on DVM and output volts for calibrated reading.
2. Verify the linearity of the system from the V-d graph.

## Observation table

| Sl. No. | D (mm.) | V(volt) | d (mm) |
|---|---|---|---|
| | | | |
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