# ASSIGNMENT IV MSO 202 A

IDENTITY THEOREM, ZEROS OF A HOLOMORPHIC FUNCTION, POLES

Exercises 0.1-0.4 rely on the Identity Theorem: If $F$ is an entire function such that there exists a convergent sequence $\{z_n\}$ such that $F(z_n) = 0$ then $F(z) = 0$ for all $z \in \mathbb{C}$.

**Exercise 0.1 :** Let $f, g$ be entire functions such that $f(z) = g(z)$ for all $z = x \in (0, 1)$. Show that $f(z) = g(z)$ for all $z \in \mathbb{C}$.

**Exercise 0.2 :** Does there exist an entire function $f$ such that $f(z) = |z|^3$ for all $z = x + iy$, $x \in (-1, 1)$ ?

**Exercise 0.3 :** Assuming $\sin(2x) = 2\sin(x)\cos(x)$ for all $x \in \mathbb{R}$, show that $\sin(2z) = 2\sin(z)\cos(z)$ for all $z \in \mathbb{C}$.

**Remark 0.4 :** One can prove similarly that

$$\cos(z) = \frac{e^{iz} + e^{-i\pi z}}{2} \tag{0.1}$$

**Exercise 0.5 :** Let $f$ be a non-zero entire function. Show that for any $R > 0$, $f$ can have finitely many zeros in the closed disc $\overline{\mathbb{D}_R(0)}$ centred 0 and of radius $R$.

**Exercise 0.6 :** Show that all zeros of $\cos(\frac{\pi}{2}z)$ are at odd integers. Show further that all zeros are simple.

**Exercise 0.7 :** Show that all poles of $\tan(\frac{\pi}{2}z)$ are at odd integers. Show further that all poles are simple.