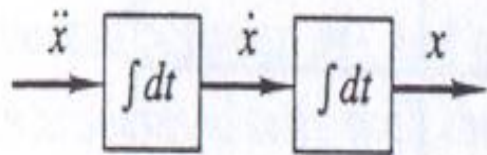


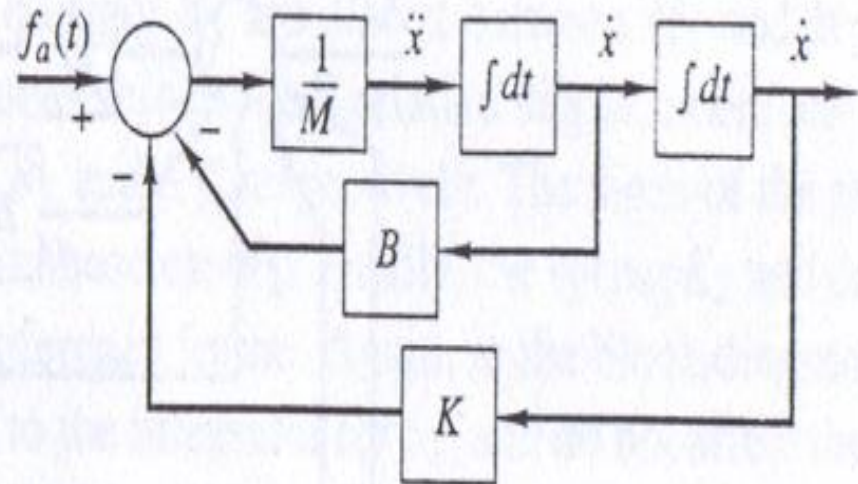
Electro-Mechanical Systems and their Representations

Block Diagram for a SDOF SYSTEM



(a) ↑

Two Integrators

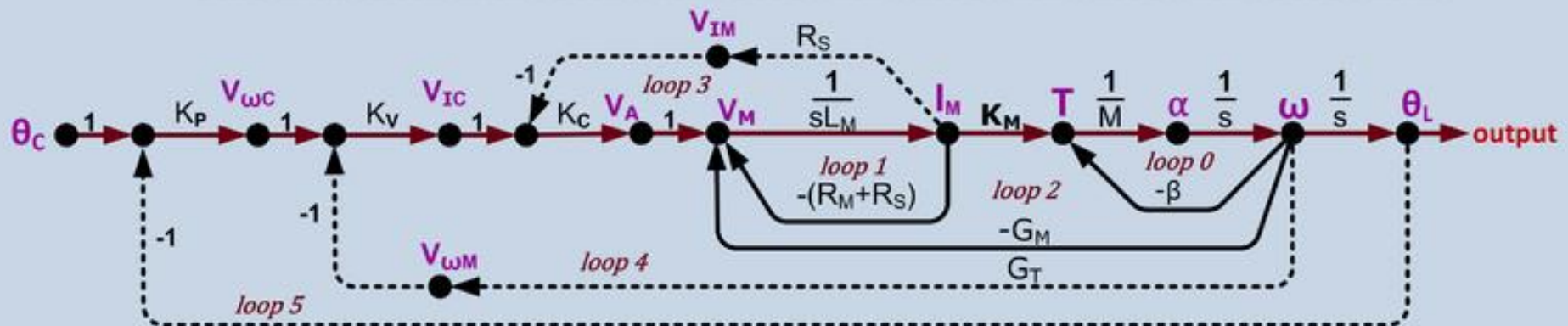
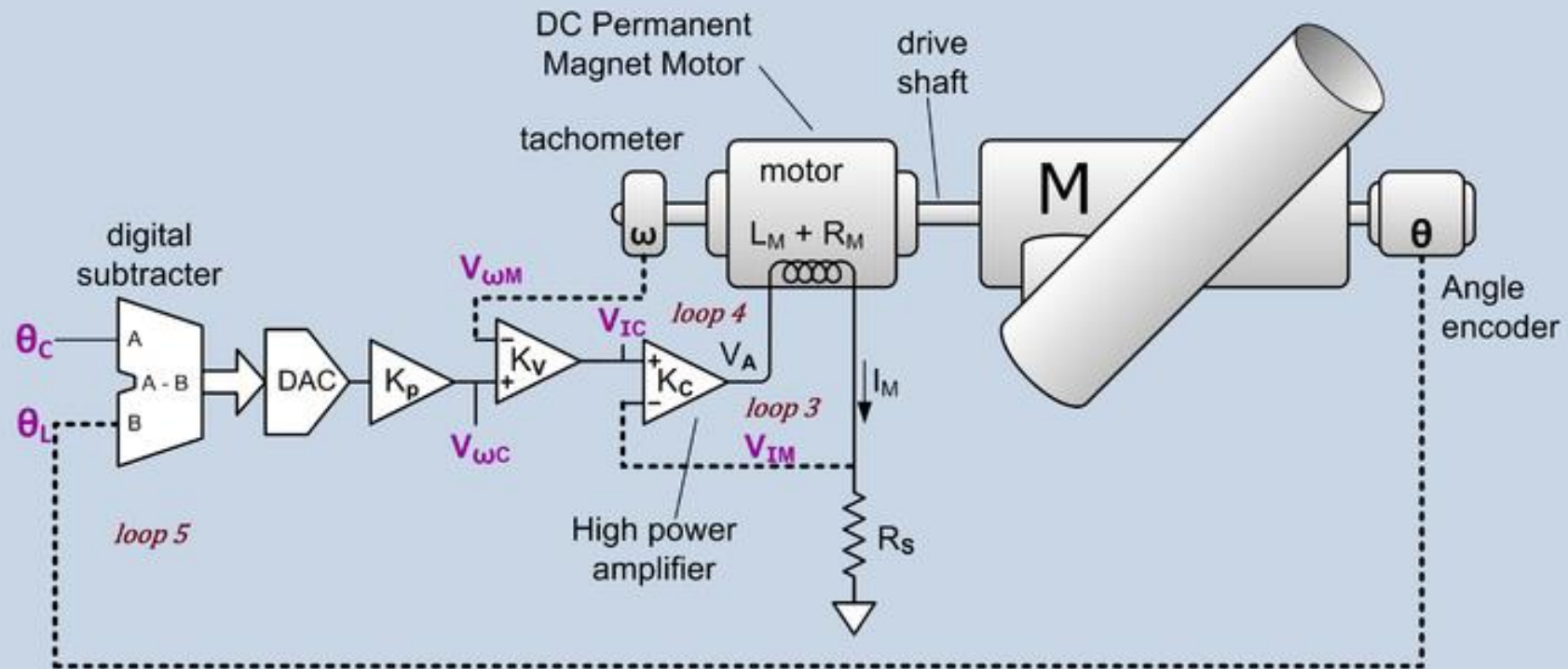


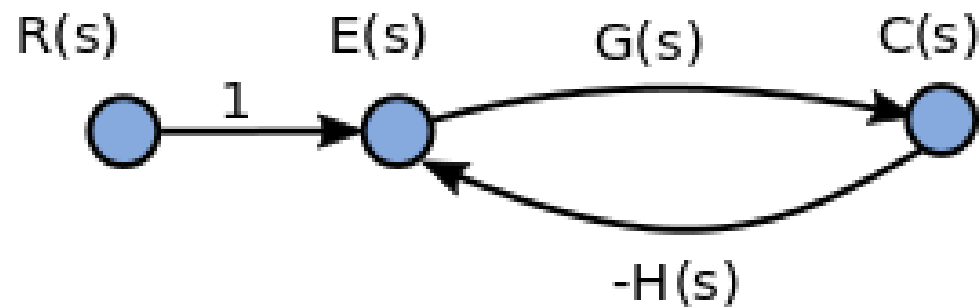
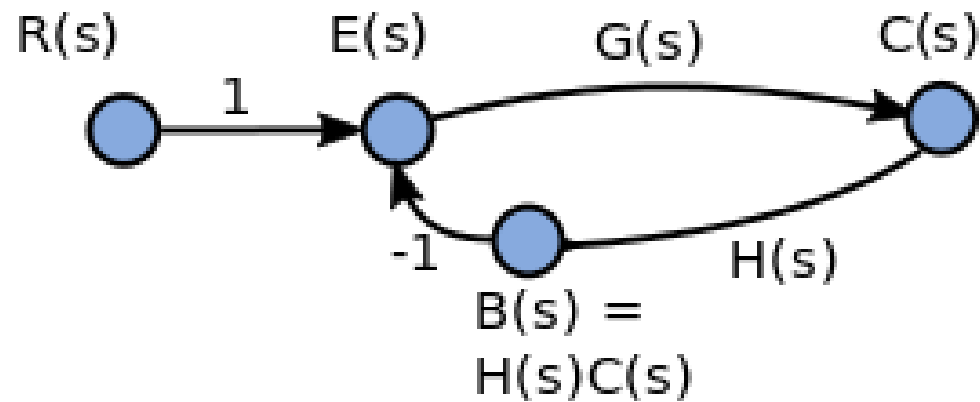
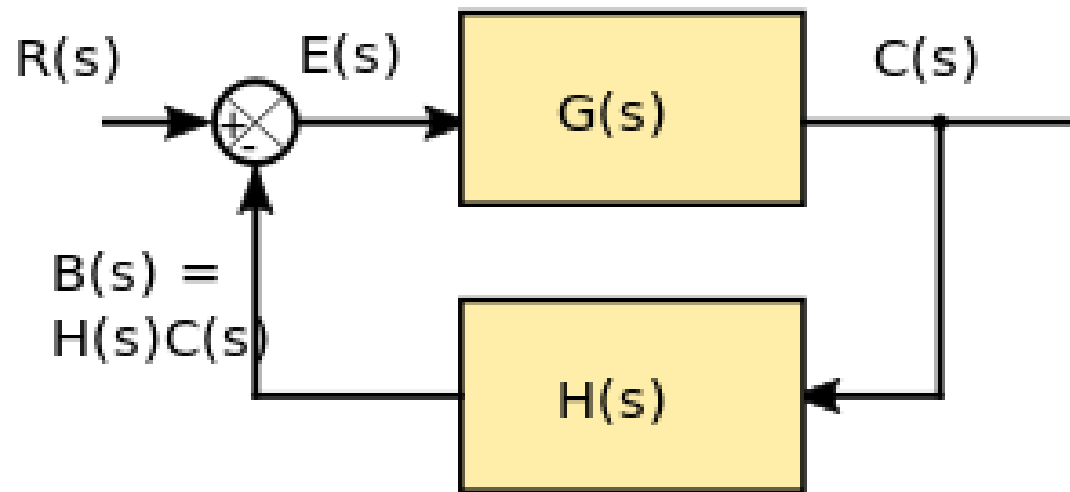
(b) ↑

Complete Diagram

Angular Position Servo and Signal Flow Graph

Graph: Source Wikipedia



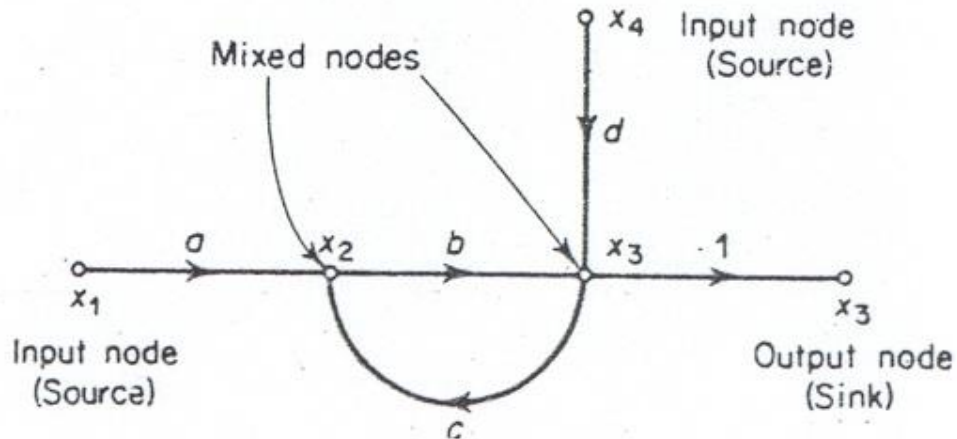


Elements of Signal-Flow Graph

- **Node**. A node is a point representing a variable of signal.
- **Transmittance**. a real gain or complex gain between two nodes.
Such gains can be expressed in terms of the transfer function between two nodes.
- **Branch**. a directed line segment joining two nodes. The gain of a branch is a transmittance.
- **Input node or source**. An input node or source is a node that has only outgoing branches.
This corresponds to an independent variable.
- **Output node or sink**. An output node or sink is the node that has only incoming branches.
This corresponds to a dependent variable.

- **Mixed node.** both incoming and outgoing branches.
- **Path.** A path is a traversal of connected branches in the direction of the branch arrows. If no node is crossed more than once, the path is **open**. If the path ends at the same node from which it began and does not cross any other node more than once, it is **closed**.
- Loop.** A loop is a closed path.
- **Loop gain.** The loop gain is the product of the branch transmittances of a loop.
- **Nontouching loops.** Loops are nontouching if they do not possess any common nodes.
- **Forward path.** A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.
- **Forward path gain.** A forward path gain is the product of the branch transmittances of a forward path.

Signal Flow Graph



- Signals travel along branches only in the direction of the arrows.
- A signal travelling along any branch is multiplied by the transmission of that branch.
- The value of any node variable is the sum of all signals entering the node.
- The value of any node variable is transmitted on all branches leaving that node.

Signal-graph basic rules

- The value of a node with one incoming branch and gain 'a' is. $x_2 = ax_1$
- The total transmittance of cascaded branches is equal to the product of the branch transmittances. Cascaded branches can thus be combined into a single branch by multiplying the transmittances
- Parallel branches may, be combined by adding the transmittances
- Mixed nodes and loops may be eliminated to calculate the complete transfer function, for example, a loop may be eliminated at junction 2 in the last figure by noting that

$$x = bx_2 \quad , \quad x_2 = ax_1 + cx_3$$

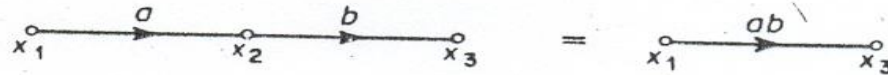
Signal Flow Graphs and Simplifications

(a)



1'

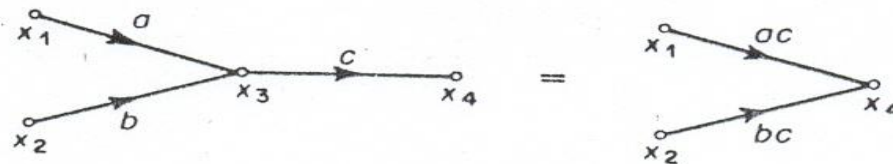
(b)



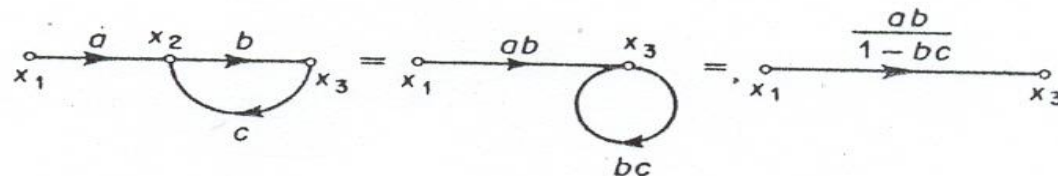
(c)

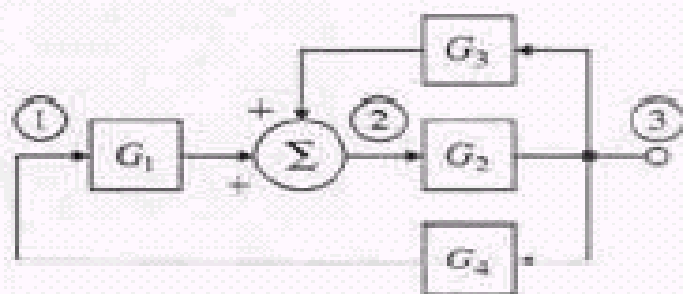


(d)

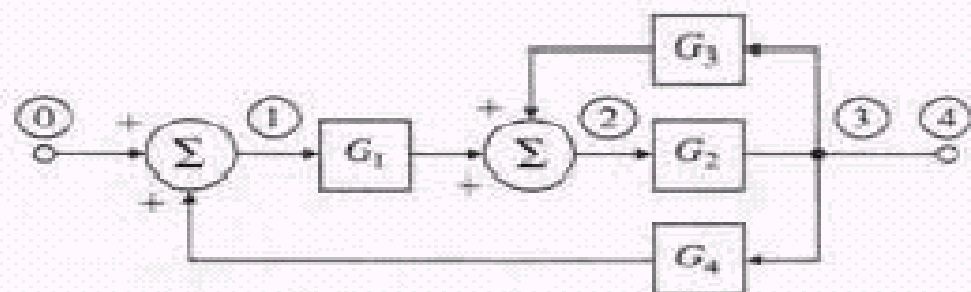
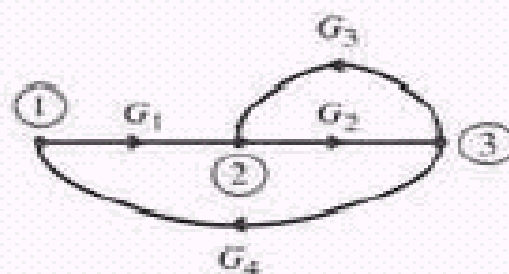


(e)

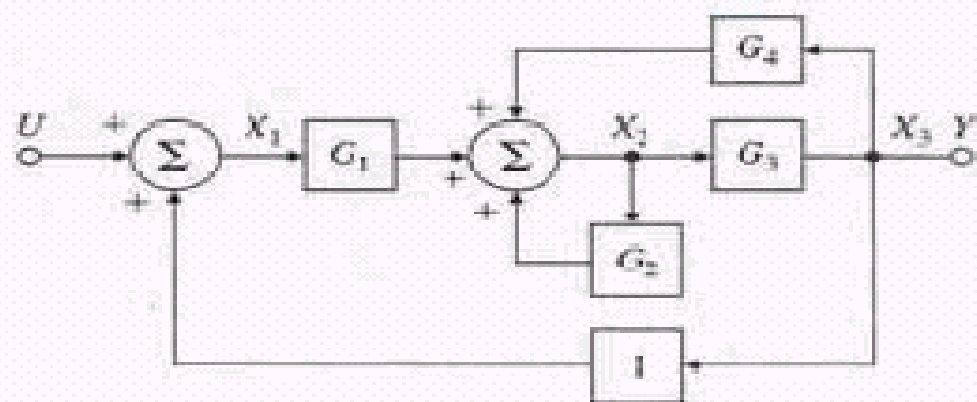
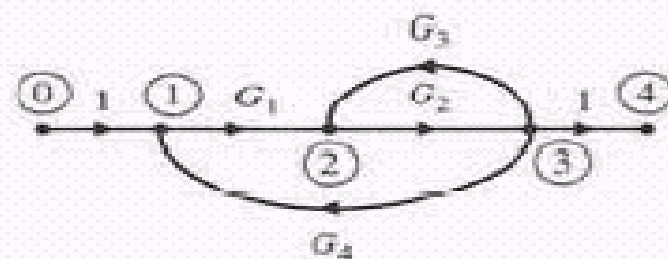




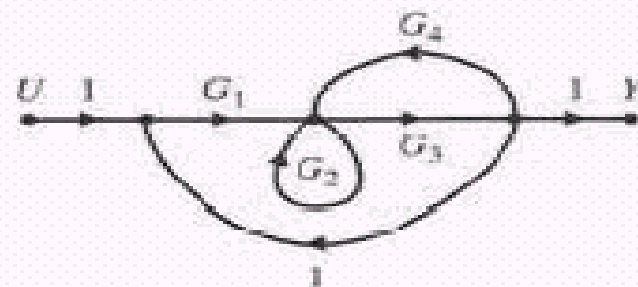
(a)



(b)



(c)



For a complex signal flow graph, evaluation of the transfer function based on first principles is quite cumbersome. An algorithmic way of evaluating the same based on Graph Theory is known as **Mason's rule**.

Mason's rule – Key points

- **Forward Path Gain** – Product of branch Gains found by traversing a path from the input to output node in the direction of signal flow
- **Non-touching Loops** – Loops that do not have any nodes in common
- **Non-touching Loop Gain** – The product of loop gains from non-touching loops

Mason's Rule

$$G(s) = \frac{\sum_{i=1}^k T_i \Delta_i}{\Delta}$$

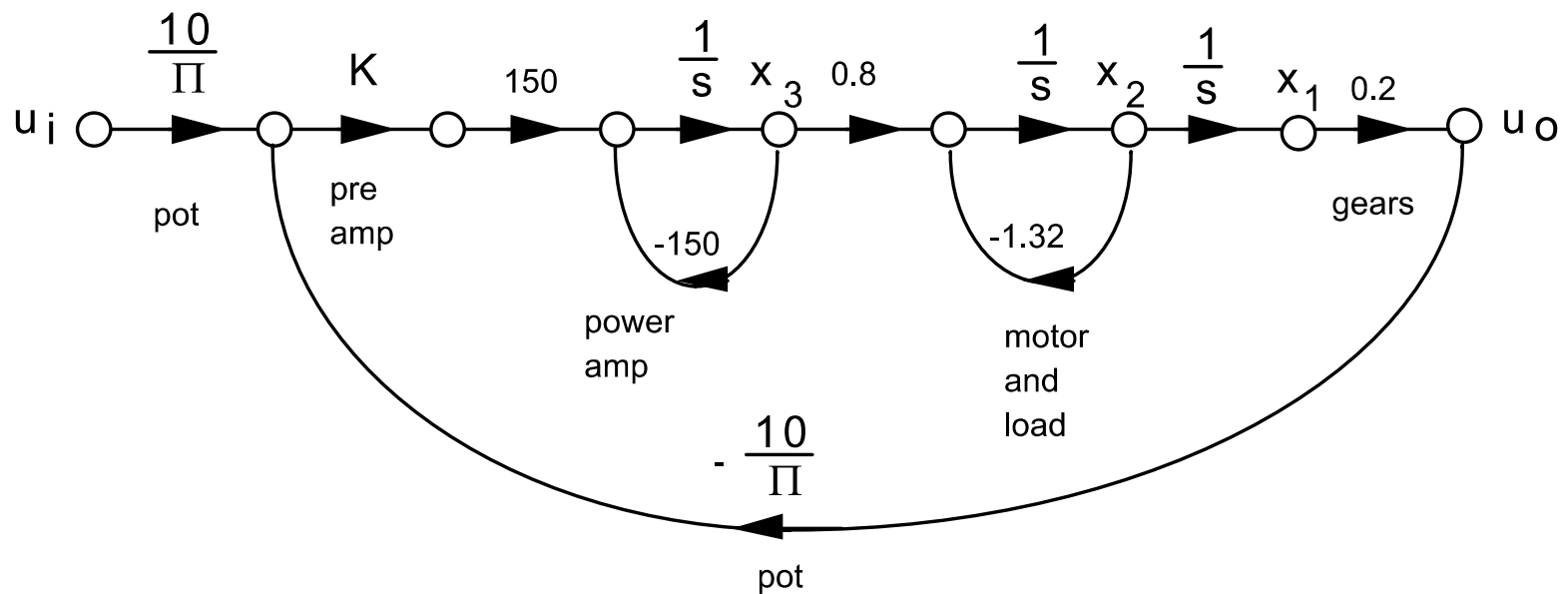
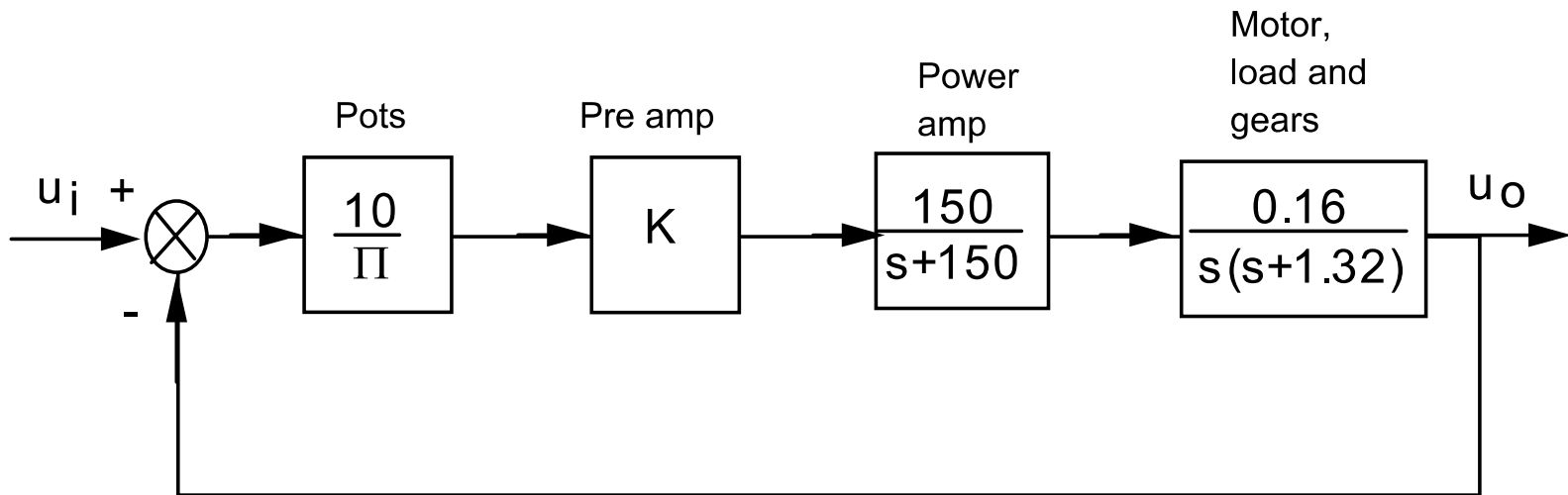
k = number of forward path gain

T_i = the i -th forward path gain

$\Delta = 1 - \Sigma$ loop gains + Σ non-touching loop gains taken two at a time + Σ non-touching loop gains taken three at a time etc.

$\Delta_i = \Delta - \Sigma$ loop gain terms in Δ that touch the i -th forward path

A Servo Motor Problem



$$\text{c. } T_1 = \left(\frac{10}{\pi}\right)(K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2) = \frac{76.39}{s^3}$$

$$G_{L1} = \frac{-150}{s}; G_{L2} = \frac{-1.32}{s}; G_{L3} = (K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2)\left(\frac{-10}{\pi}\right) = \frac{-76.39K}{s^3}$$

Nontouching loops:

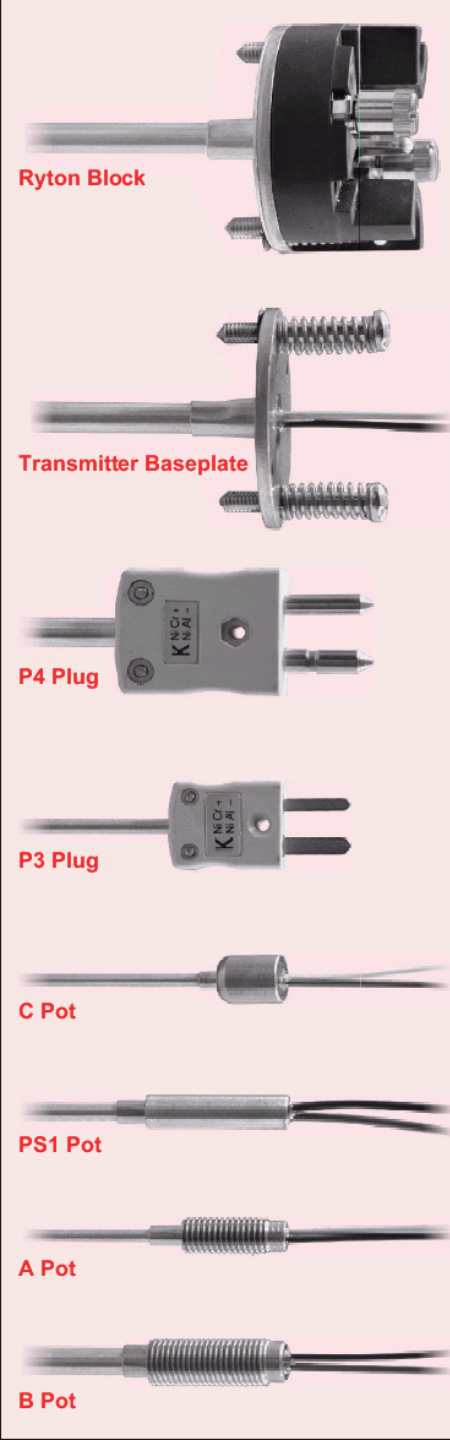
$$G_{L1}G_{L2} = \frac{198}{s^2}$$

$$\Delta = 1 - [G_{L1} + G_{L2} + G_{L3}] + [G_{L1}G_{L2}] = 1 + \frac{150}{s} + \frac{1.32}{s} + \frac{76.39K}{s^3} + \frac{198}{s^2}$$

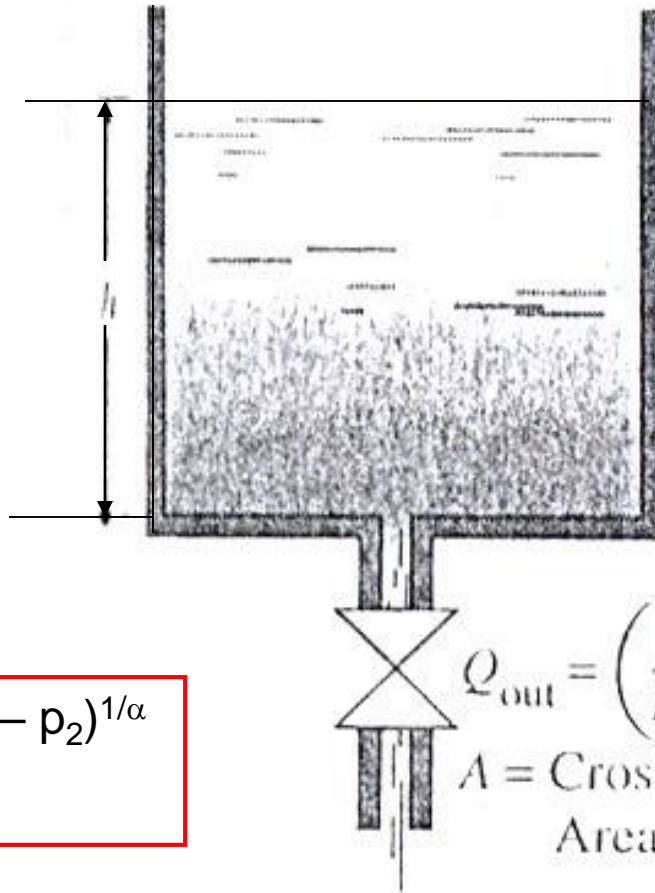
$$\Delta_1 = 1$$

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{76.39K}{s^3 + 151.32s^2 + 198s + 76.39K}$$

Dynamic Response of First Order Systems



Leaking Tank: A First Order System



$$\text{MFR} = 1/R(p_1 - p_2)^{1/\alpha}$$

$$\alpha = 1 \text{ for } \text{Re} < 1000$$

Incompressible Fluid

$$d/dt (A h(t)) = -Q_{\text{out}} = -(1/R)h(t)$$

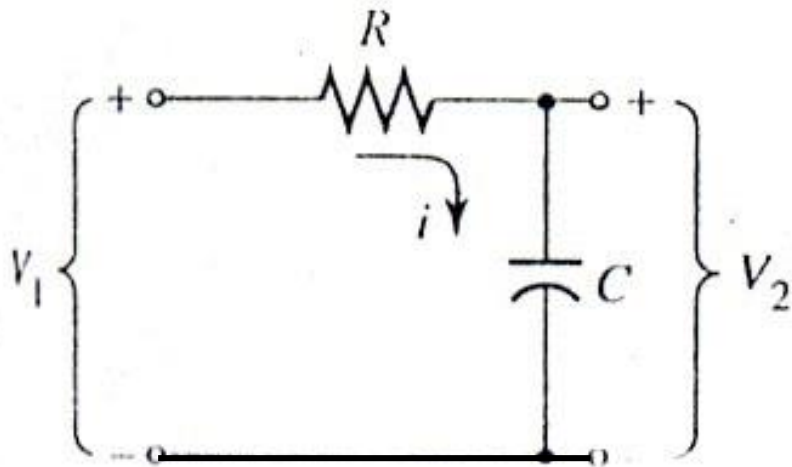
$$d/dt (h) = (-1/AR) h$$

$$X=h, X_o = h_o, d/dt (X) = A X$$

$$Q_{\text{out}} = \left(\frac{1}{R} \right) h$$

A = Cross-sectional
Area of Tank

A Low-Pass RC Filter

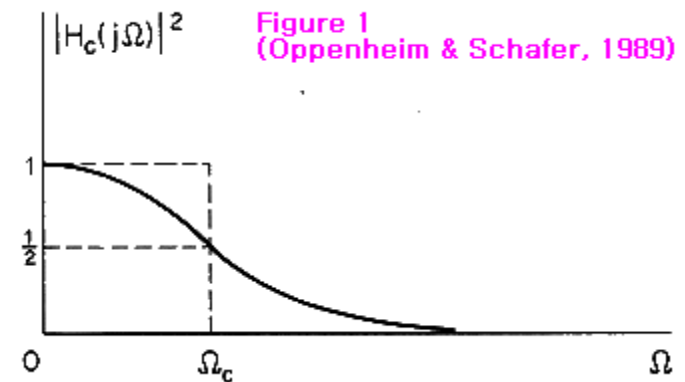


Low-pass RC Filter

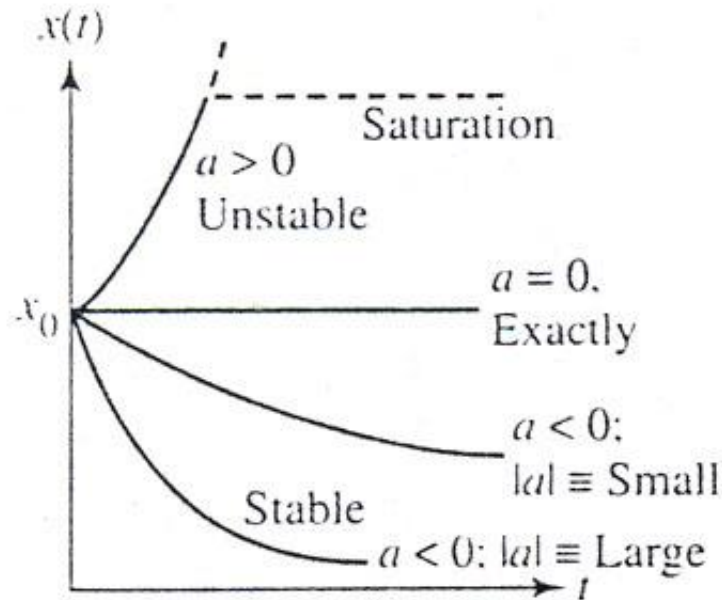
$$\frac{d}{dt}(V_2) = \frac{1}{RC}(V_1 - V_2)$$

$$V_1 = 0$$

$$\frac{d}{dt}(V_2) = (-1/RC)V_2$$



Free-Response of a First Order System



Graph of e^{at} for Ranges of a

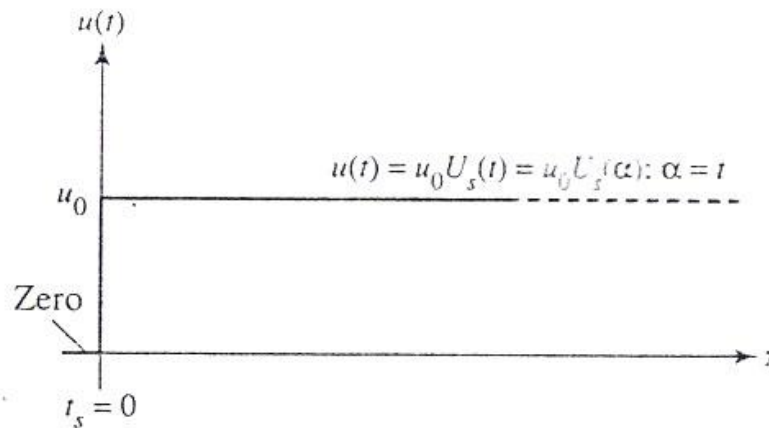
$$\mathbf{x(t) = e^{at} x_0}$$

$a = 0$, Open circuit condition

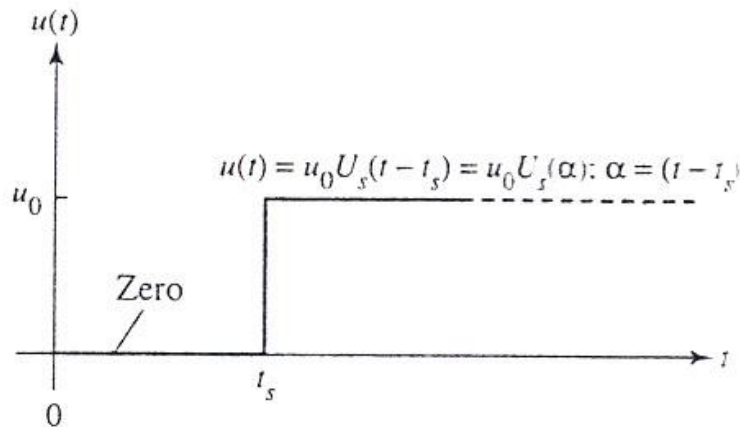
$T = -1/a$ time constant, time taken to reach $1/e$ of the initial value

Forced Excitation (Unit Step)

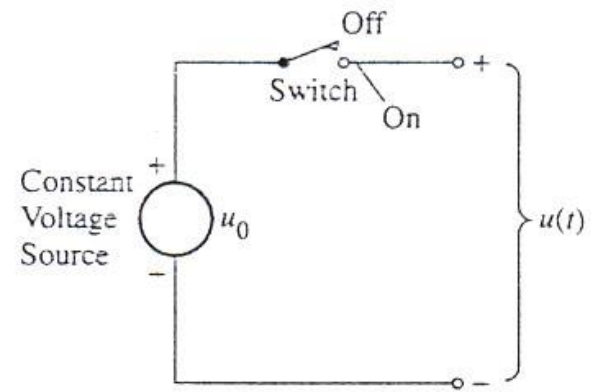
Step Input Function and a Switch as Function Generator



(a)



(b)

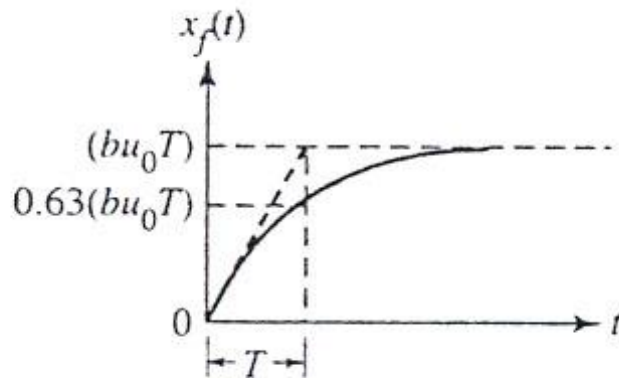


(c)

$$\frac{d}{dt} (x(t)) = a x(t) + b u(t)$$

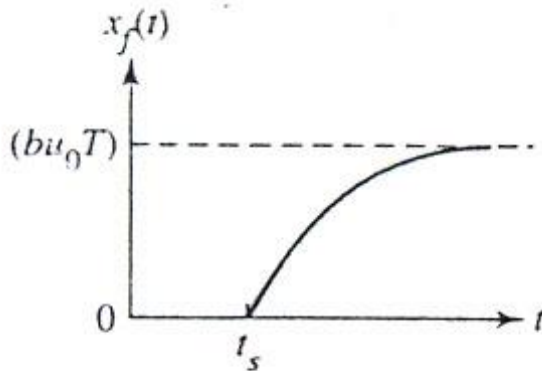
$$x_f(t) = \int_0^t e^{a(t-\tau)} b u(\tau) d\tau$$

Forced Response (Unit Step)



(a) $u(t) = u_0 U_s(t)$

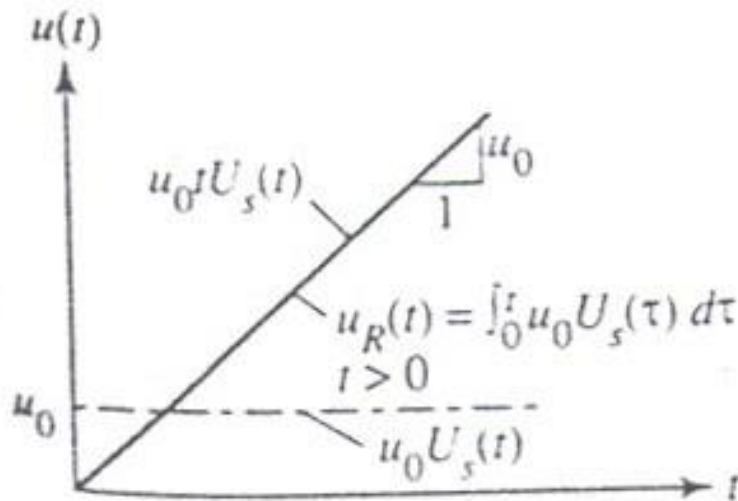
$$x_f(t) = (bu_0 T)[1 - e^{-t/T}] U_s(t)$$



(b) $u(t) = u_0 U_s(t - t_s)$

Sketch of a Forced
Step Response

Forced Excitation (Ramp Input)

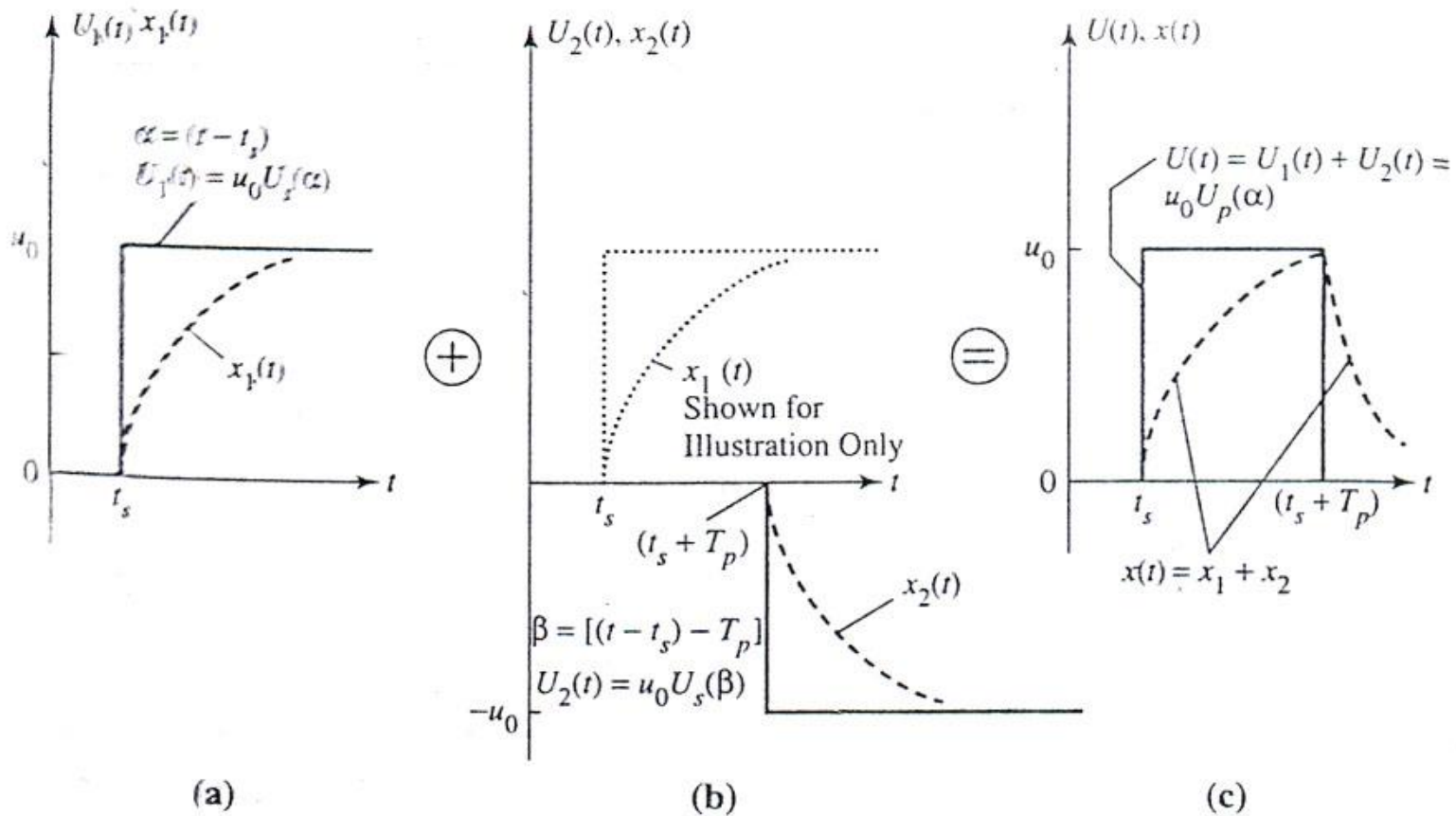


Ramp Input Function

$$x_f(t) = bu_0 T^2 (e^{-t/T} + t/T - 1)$$

Forced Response (Pulse Input)?

Pulse Response as Sum of Step Responses



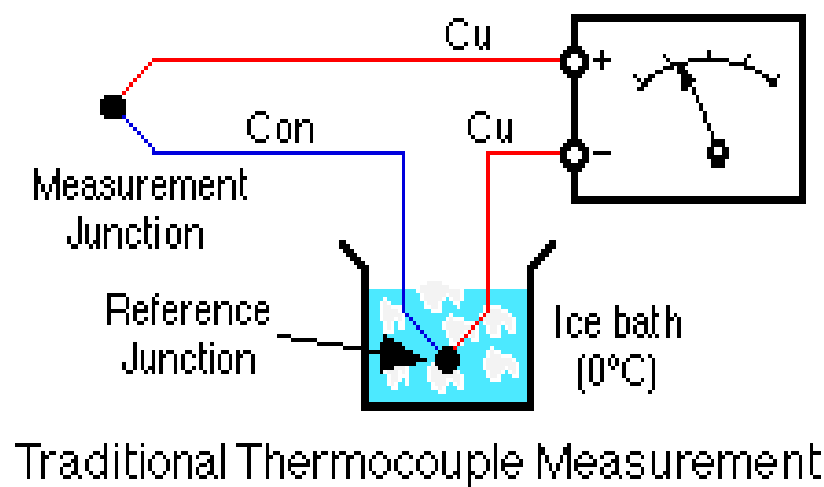
First Order Systems

A first order system has a differential equation of the form

$$\tau \frac{dy}{dt} + y = kr$$

$$\mathbf{Y(s) = G(s) R(s) = k \left(\frac{1/\tau}{s + 1/\tau} \right)}$$

$$y = k (1 - e^{-t/\tau})$$



Example:

A thermocouple which has a transfer function linking its voltage output V and temperature input of T as

$$G(s) = \frac{30 \times 10^{-6}}{10s + 1} V/^{\circ}C$$

Determine the response of the system when it is suddenly immersed in a water bath at $100^{\circ}C$

The output as an ' s ' function is

$$V(s) = G(s) \text{ input } (s)$$

The output as an '**s**' function is

$$V(\mathbf{s}) = G(\mathbf{s}) * \text{input}(\mathbf{s})$$

The sudden immersion of the thermometer gives a step input of size 100° C

and so the input as an **s** function as 100/**s**. Thus

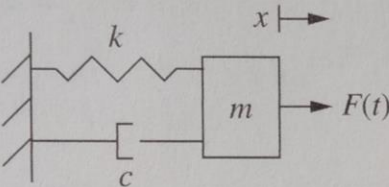
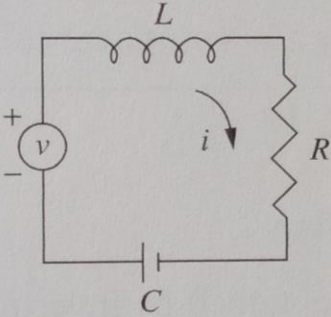
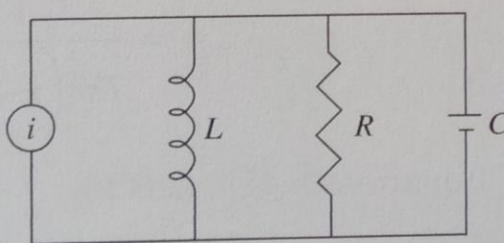
$$\mathbf{V} = \frac{30 \times 10^{-6}}{10s + 1} \times \frac{100}{s} \qquad \frac{30 \times 10^{-4} \times 0.1}{s(s + 0.1)}$$

The fraction element of the form a/s(s+a) and so the output as a function of time is :

$$V = 30 \times 10^{-4} \left(1 - e^{-0.1t} \right)$$

Various Basic Second Order Systems

TABLE 6.4 SECOND-ORDER SYSTEMS

Name	Schematic	Differential Equation	Natural Frequency	Damped Ratio
Mass-spring-viscous damper		$m\ddot{x} + c\dot{x} + kx = F(t)$	$\sqrt{\frac{k}{m}}$	$\frac{c}{2\sqrt{mk}}$
Series LRC circuit		$L \frac{di}{dt} + Ri + \frac{1}{C} \int_0^t i dt = v(t)$	$\sqrt{\frac{1}{LC}}$	$\frac{R}{2} \sqrt{\frac{C}{L}}$
Parallel LRC circuit		$C \frac{dv}{dt} + \frac{v}{R} + \frac{1}{L} \int_0^t v dt = i(t)$	$\frac{1}{\sqrt{LC}}$	$\frac{1}{2R} \sqrt{\frac{L}{C}}$

Standard Forms of a 2nd order system

In terms of the time constant T

$$\frac{\theta_o}{\theta_i}(s) = \frac{k}{T^2 s^2 + 2 T \delta s + 1}$$

In terms of the natural frequency ω_n

$$\frac{\theta_o}{\theta_i}(s) = \frac{\omega_n^2 k}{s^2 + 2 \delta \omega_n s + \omega_n^2}$$

In terms of the poles (polynomial)

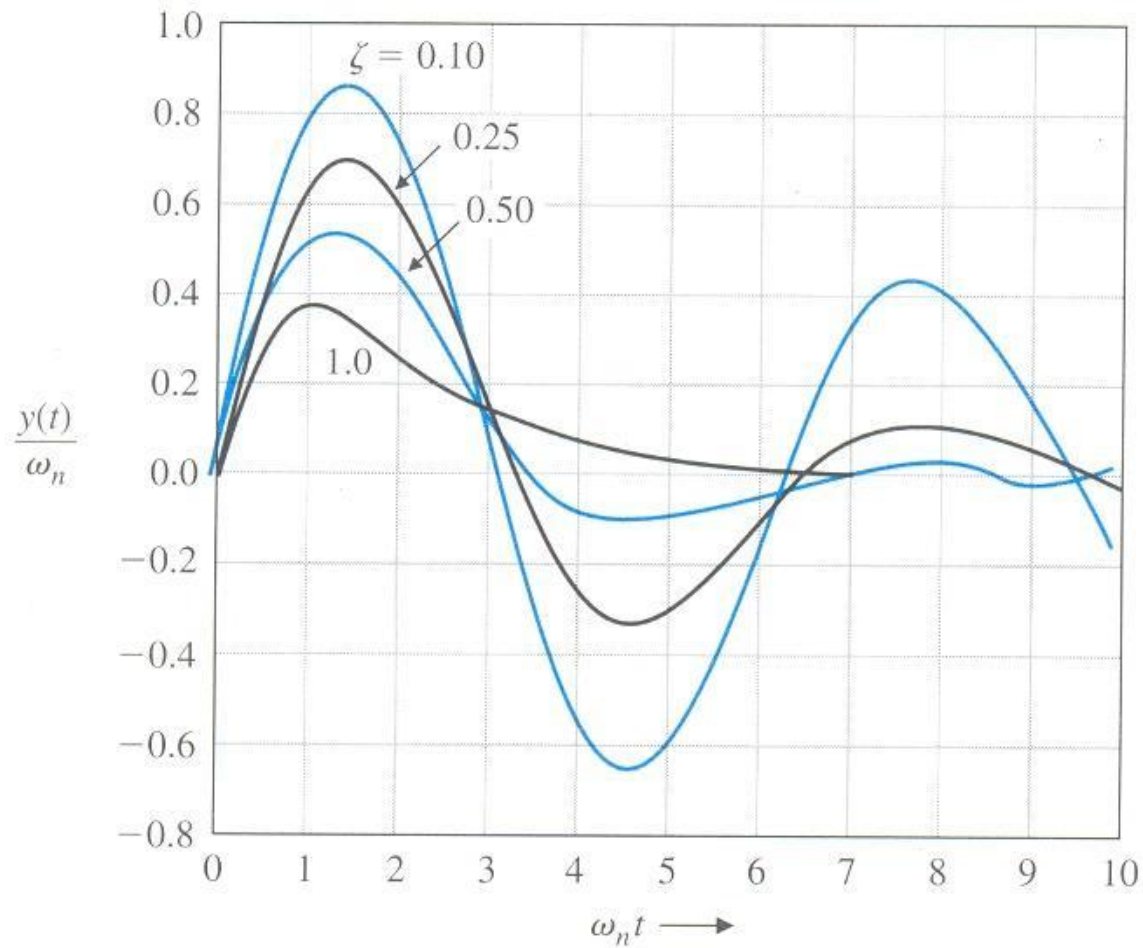
$$\frac{\theta_o}{\theta_i}(s) = \frac{\omega_n^2 k}{(s - p_1)(s - p_2)}$$

In terms of two parameters σ and ω_r

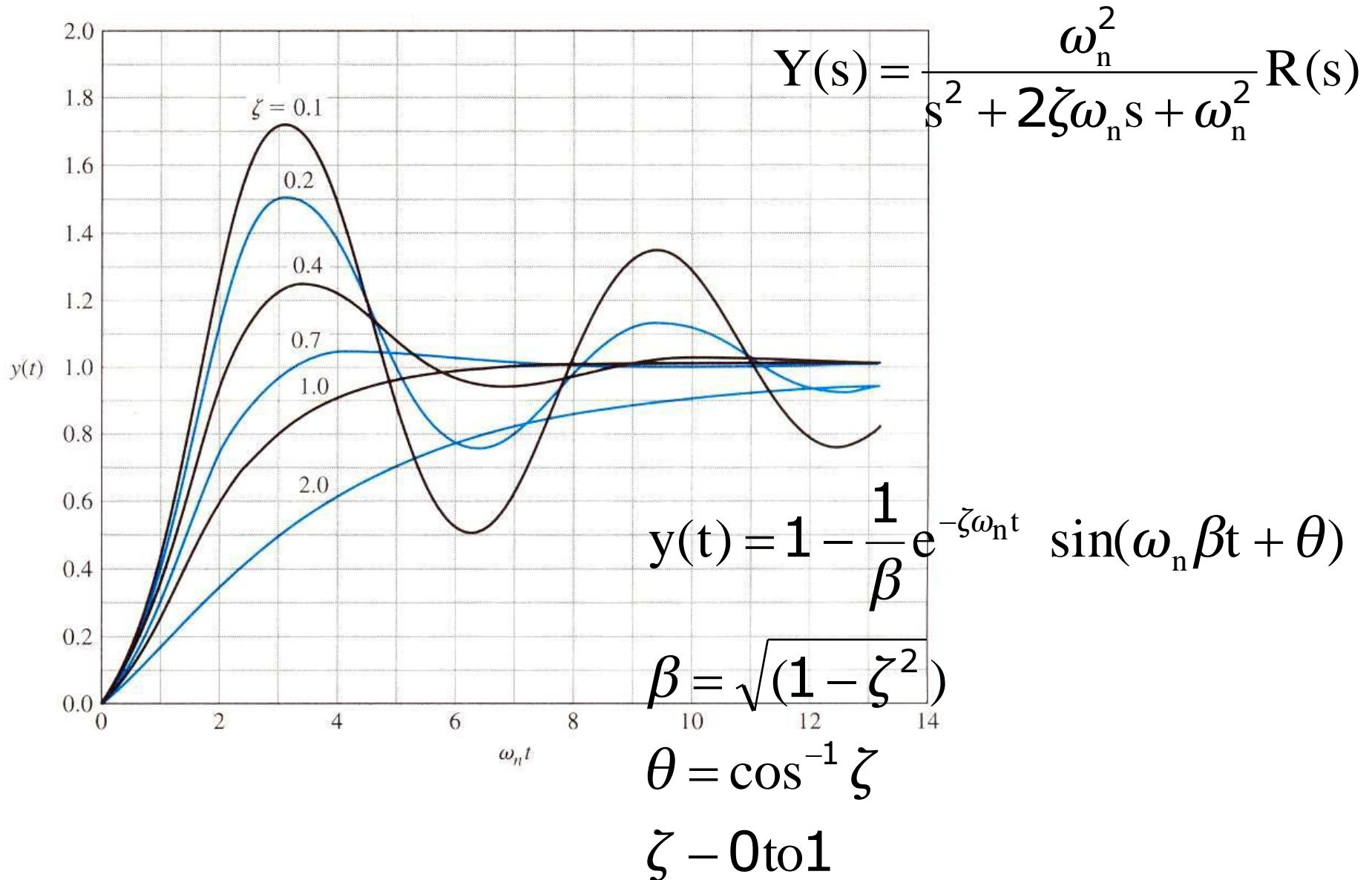
$$\frac{\theta_o}{\theta_i}(s) = \frac{\omega_n^2 k}{(s + \sigma)^2 + \omega_r^2} = \frac{\omega_r^2 + \sigma \omega_r^2}{(s + \sigma)^2 + \omega_r^2}$$

Remember that δ is the damping ratio (**ζ in other texts**)

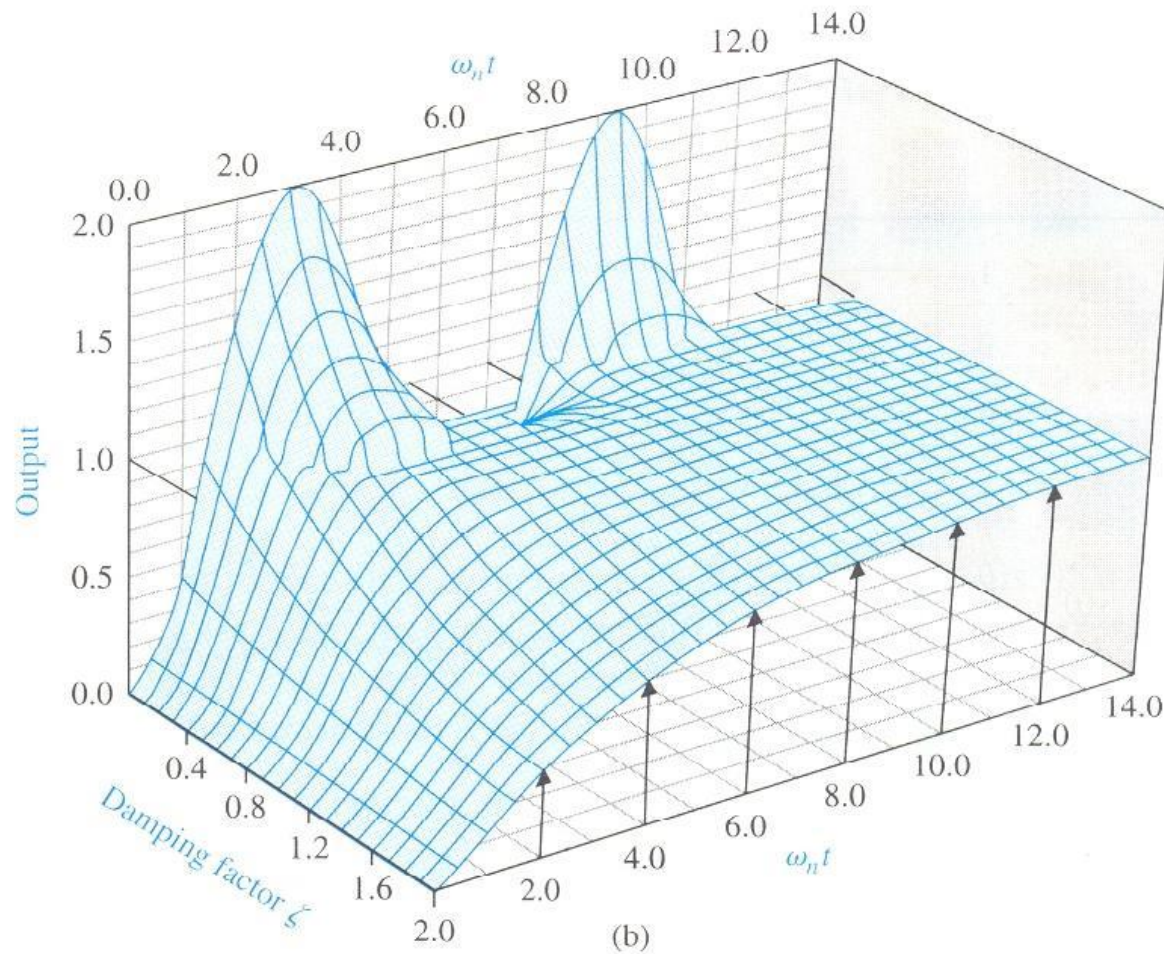
Unit Impulse Response



Transient response of a second order system



A 3-D representation of unit step response

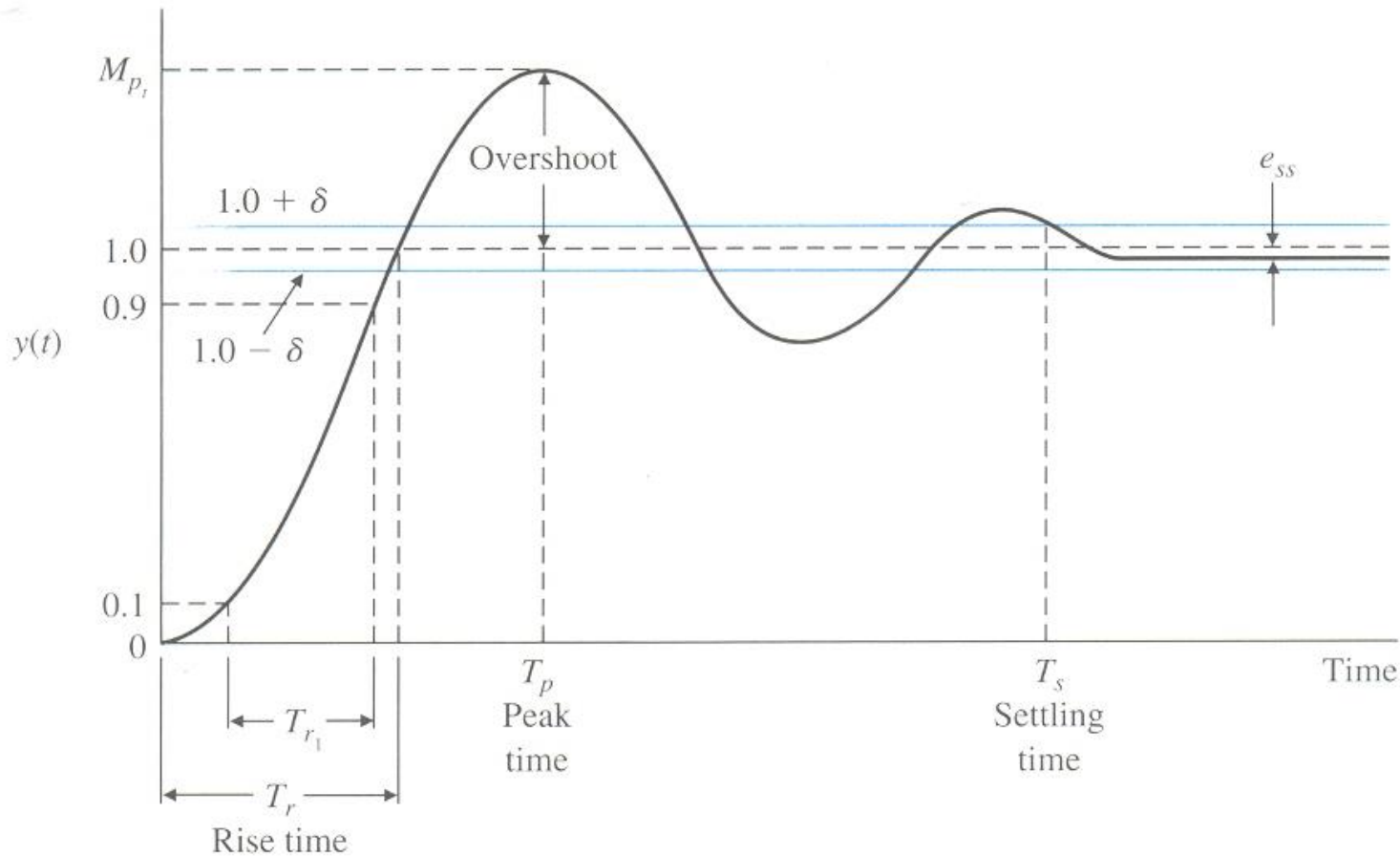


Unit step response

$$\text{PO} = \frac{M_p - f_v}{f_v} \times 100\%$$

$$e^{-\zeta\omega_n T_s} < 0.02$$

$$T_s = \frac{4}{\zeta\omega_n}$$

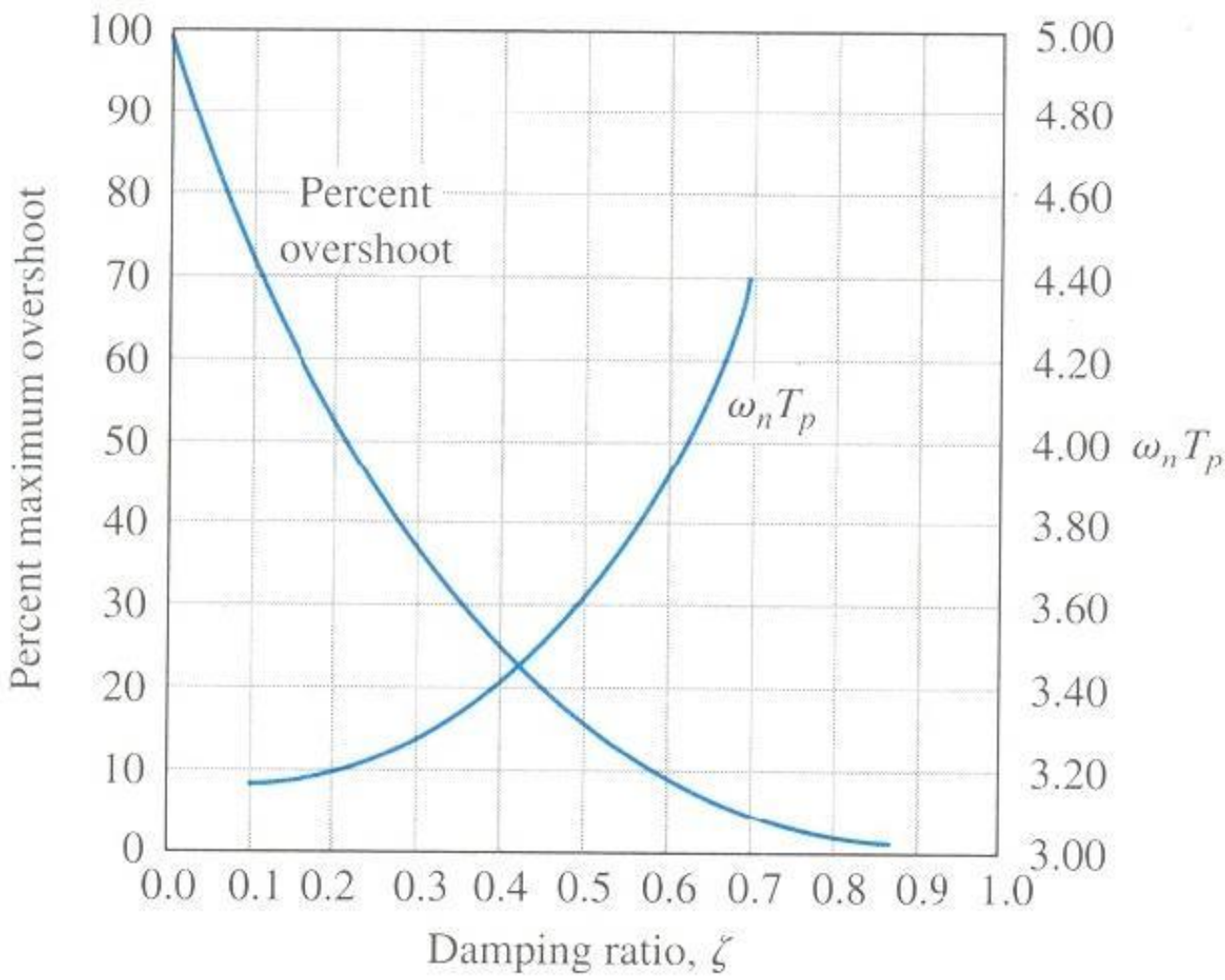


PO: A Measure of Closeness of response

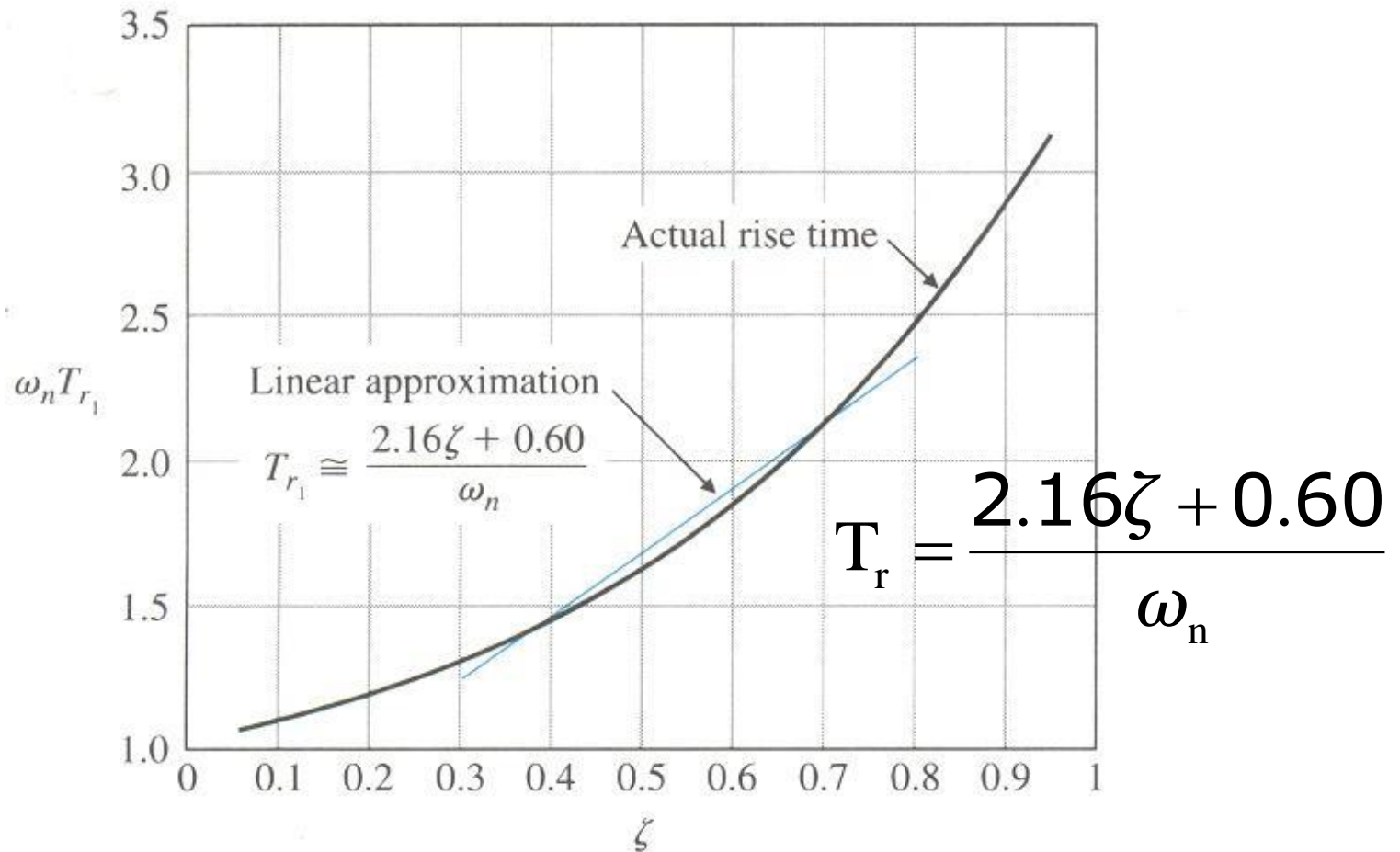
$$PO = 100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$$

Peak Time: A Measure of swiftness

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}$$

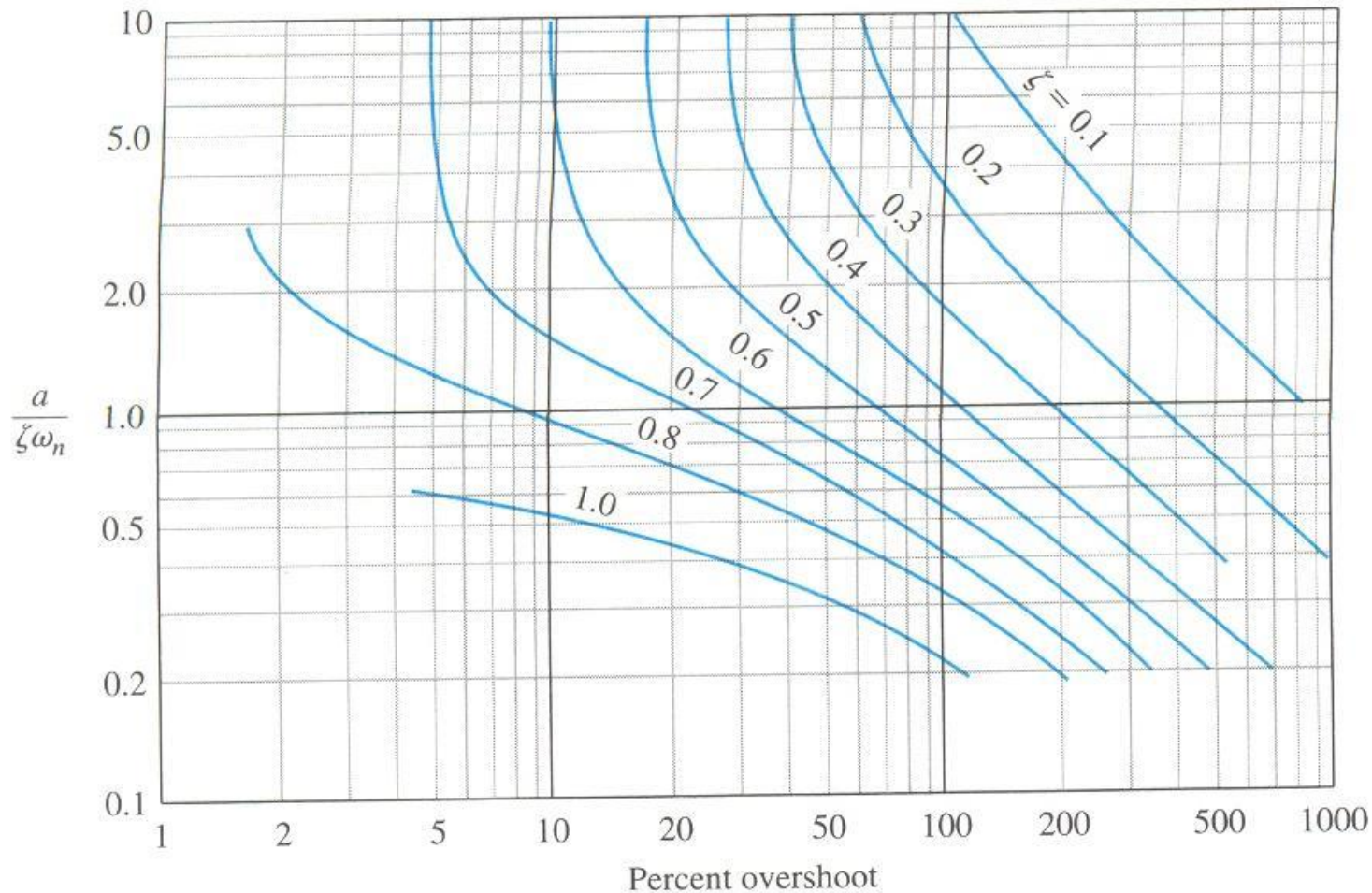


Normalised Rise-time



Effect of additional zero

$$T(s) = \frac{(\omega_n^2 / a)(s + a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



System with Additional Pole and Zero

$$T(s) = \frac{(\omega_n^2 / a)(s + a)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(1 + \tau s)}$$

Find out the effect of 'a' and 'T' on the system response corresponding to a step input

THANK YOU