Failure under variable loading

- Mechanical components are subjected to loads which vary with time
  - Shafts, connecting rod, push rods etc.
- The stresses fluctuate between some level and continue for long periods
- Parts subjected to such variable loading has been observed to fail after some number of load cycles even when the stresses are well below that required for yielding or failure- **Fatigue failure**
- Fatigue failure is **sudden** in nature
- Parts have to be designed to avoid fatigue failure during their expected service life

Mechanism: Stage 1

- One or more micro-cracks initiate at sites of stress concentration
  - Holes, section changes, scratches, tool marks
  - Material defects like voids or inclusions
- The cracks form due to cyclic plastic deformation at a microscopic level: Strain hardening leads to loss of ductility and crack formation.
- These cracks are too small to be detected

Mechanism: Stage 2

- The micro cracks after many load cycles grow into a major macro-crack
- This crack extends in size with every load cycle
- This leads to formation of striations on the fracture surface
- The fracture surfaces also have parallel markings, **beach marks** which clearly converge to the point of initiation of the crack

Mechanism: Stage 3

- After many cycles, the crack reaches a size large enough that the remaining cross-section cannot withstand the load
- This leads to sudden fracture often in a brittle manner

➢ **Design for variable loading:** Given the loads and the expected life (number of load cycles) design the part such that it will not fail by
  - o Fatigue failure before its expected life
  - o Yielding or rupture at the first load cycle

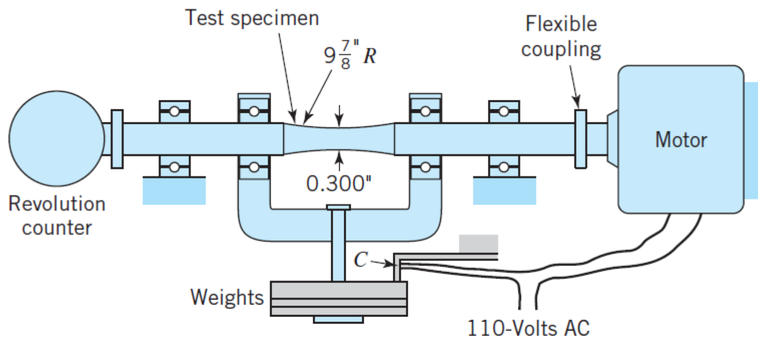


FIGURE 8.3
R. R. Moore rotating-beam fatigue-testing machine.

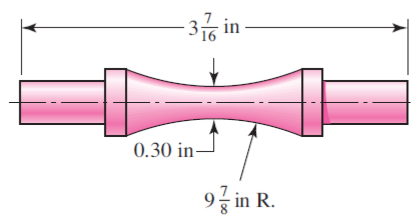
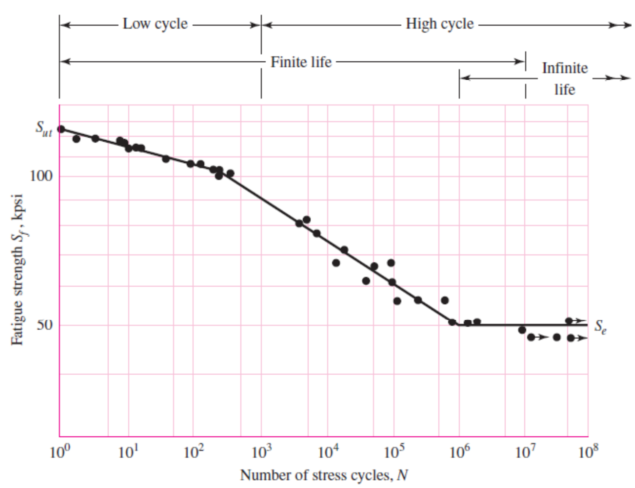
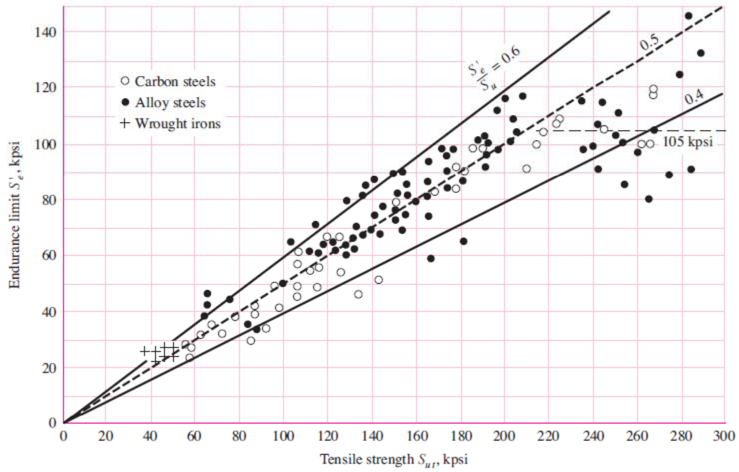


Figure 6–10

An *S-N* diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel, normalized; S_{ut} = 116 kpsi; maximum S_{ut} = 125 kpsi. *(Data from NACA Tech. Note 3866, December 1966.)*





$$S'_e = \begin{cases} 0.5 S_{ut}\ ; & S_{ut} \le 1400\,\text{MPa} \\ 700\,\text{MPa}\ ; & S_{ut} > 1400\,\text{MPa} \end{cases}$$

Surface factor

$$k_a = a S_{ut}^b$$

Surface Finish	Factor a S_{ut}, MPa	Exponent b
Ground	1.58	−0.085
Machined or cold-drawn	4.51	−0.265
Hot-rolled	57.7	−0.718
As-forged	272.	−0.995

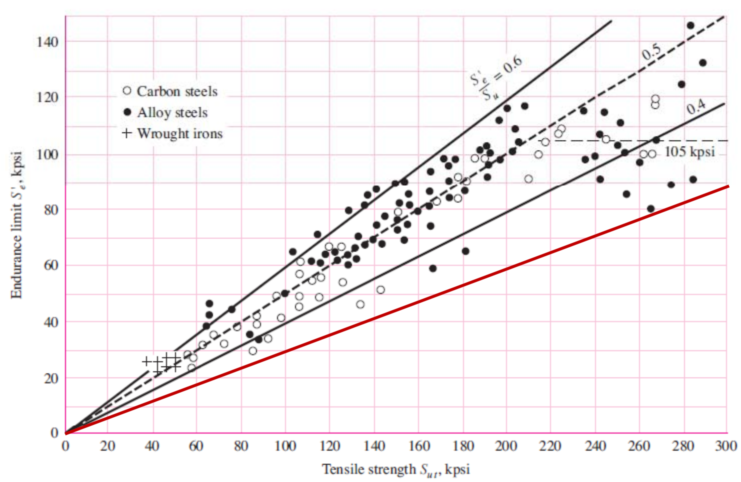
Size factor

For Bending and Torsion

$$k_b = \begin{cases} \left(\dfrac{d}{7.62}\right)^{-0.107} = 1.243 d^{-0.107};\ 2.79 \le d \le 51\text{ mm} \\ 1.51 d^{-0.157};\ 51 < d \le 254\text{ mm} \end{cases}$$

For axial loading, $k_b = 1$

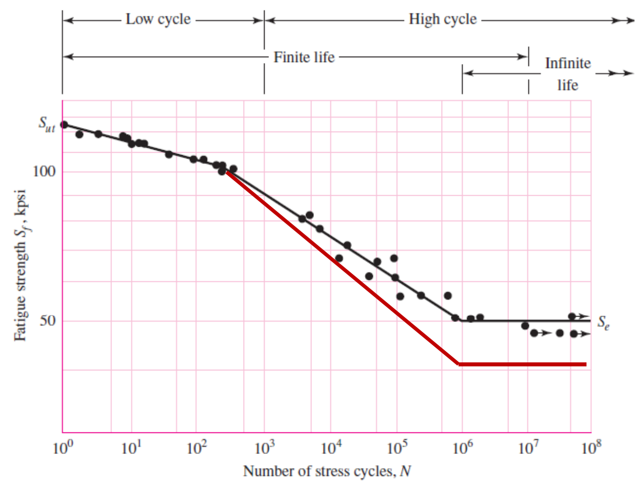
Reliability, %	Reliability Factor k_e
50	1.000
90	0.897
95	0.868
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659
99.9999	0.620



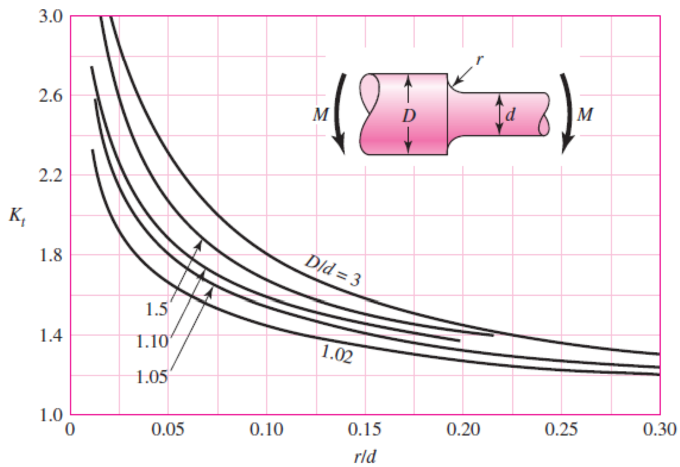
$$S'_e = \begin{cases} 0.5S_{ut}\ ;\ \ S_{ut} \le 1400\,\text{MPa} \\ 700\,\text{MPa}\ ;\ S_{ut} > 1400\,\text{MPa} \end{cases}$$

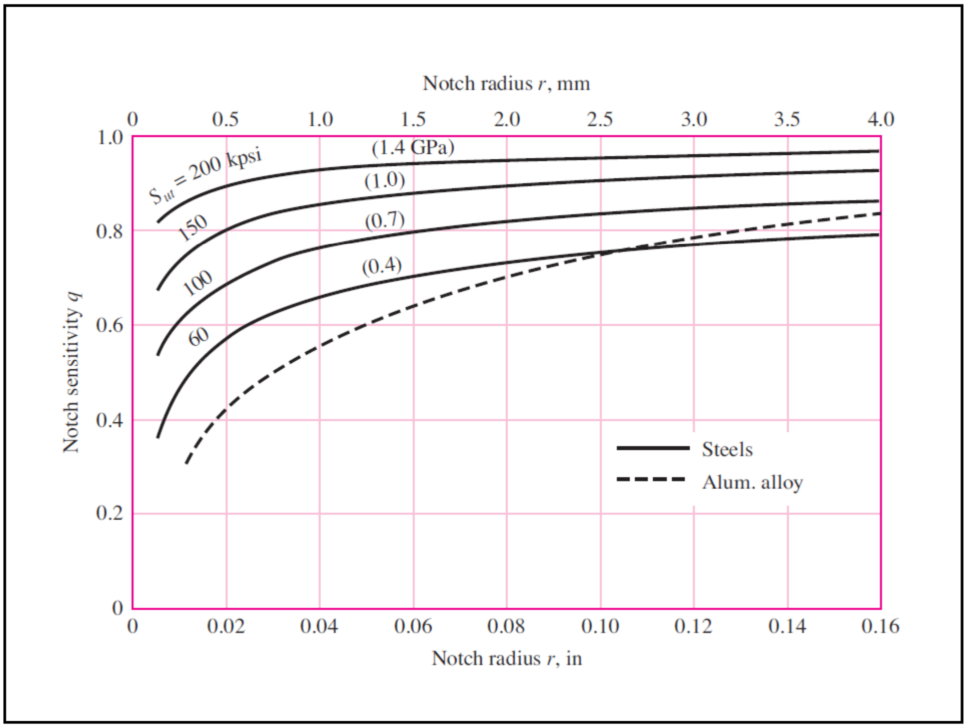
Figure 6–10

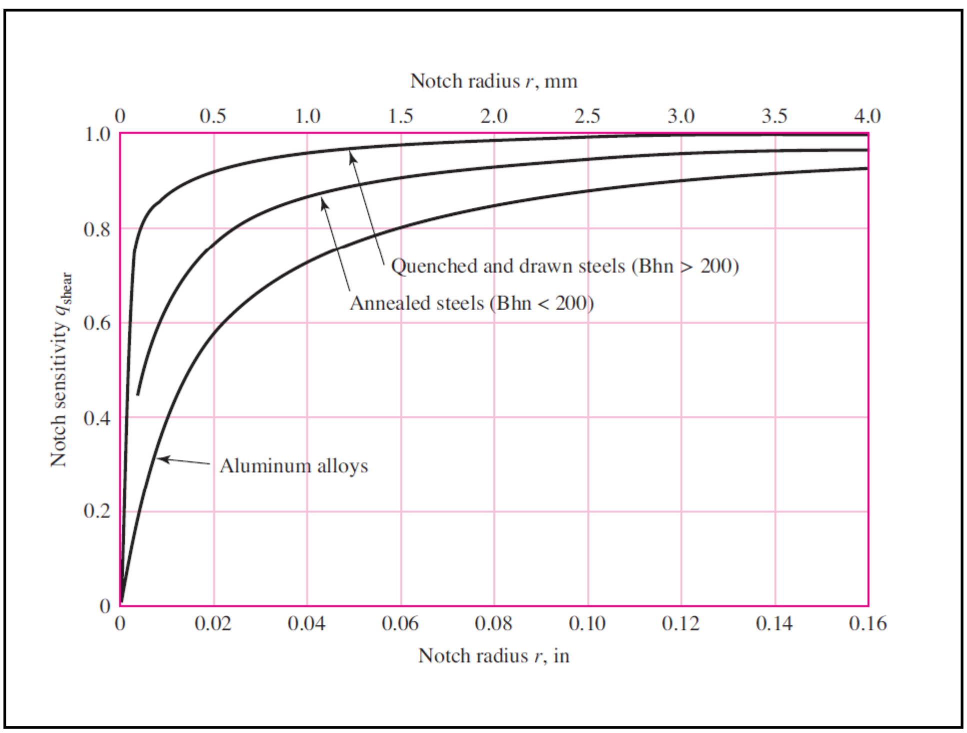
An *S-N* diagram plotted from the results of completely reversed axial fatigue tests. Material: UNS G41300 steel, normalized; $S_{ut} = 116$ kpsi; maximum $S_{ut} = 125$ kpsi. *(Data from NACA Tech. Note 3866, December 1966.)*

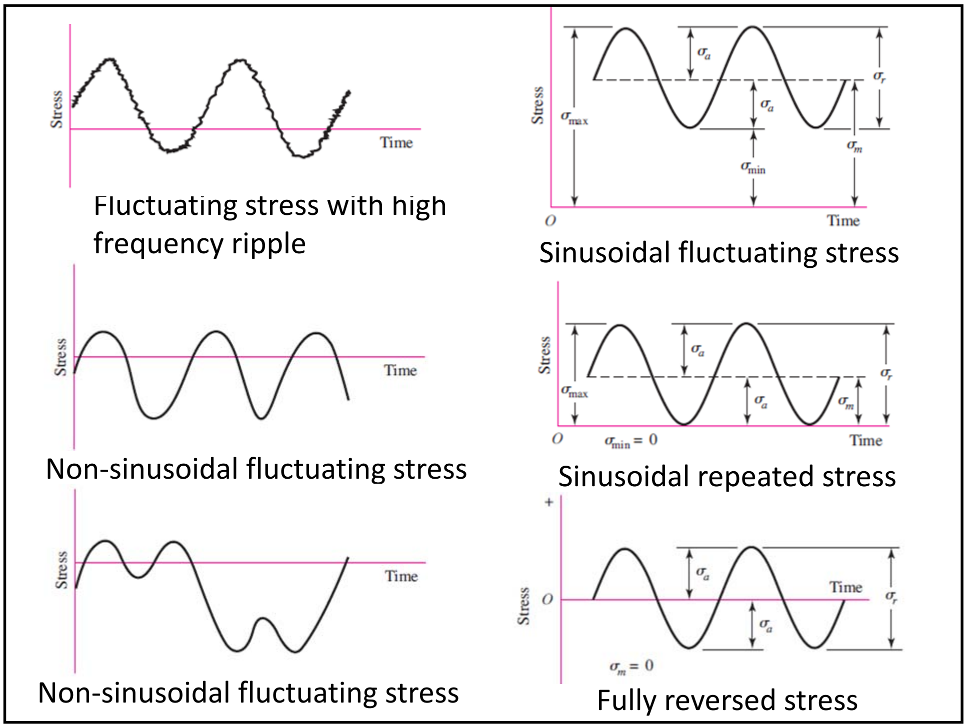


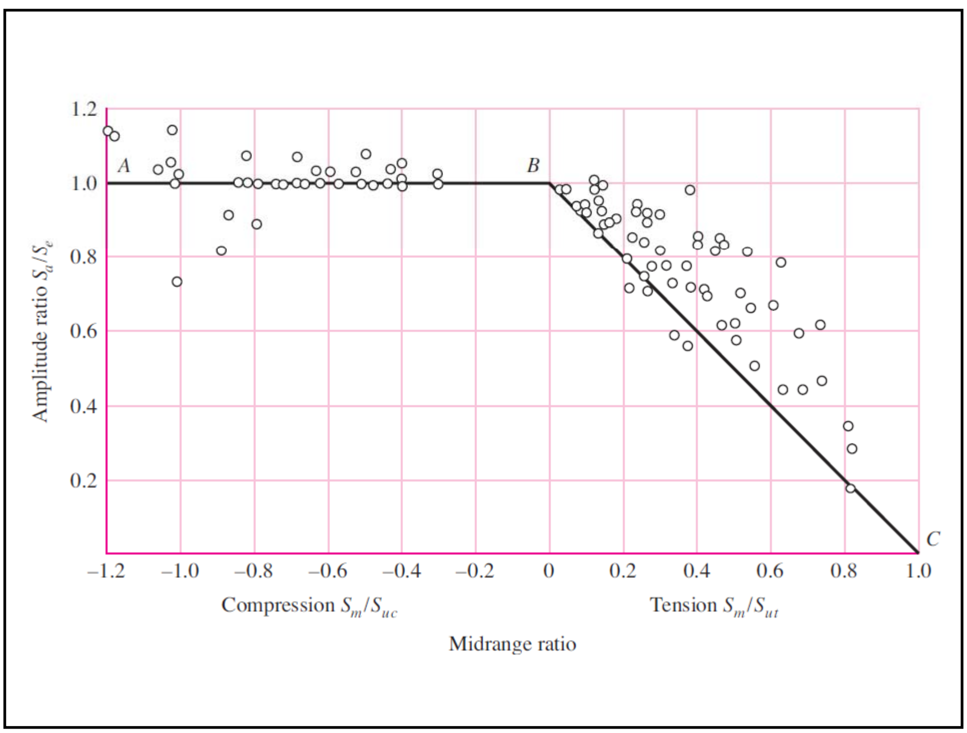
Grooved round bar in bending. $\sigma_0 = Mc/I$, where $c = d/2$ and $I = \pi d^4/64$.

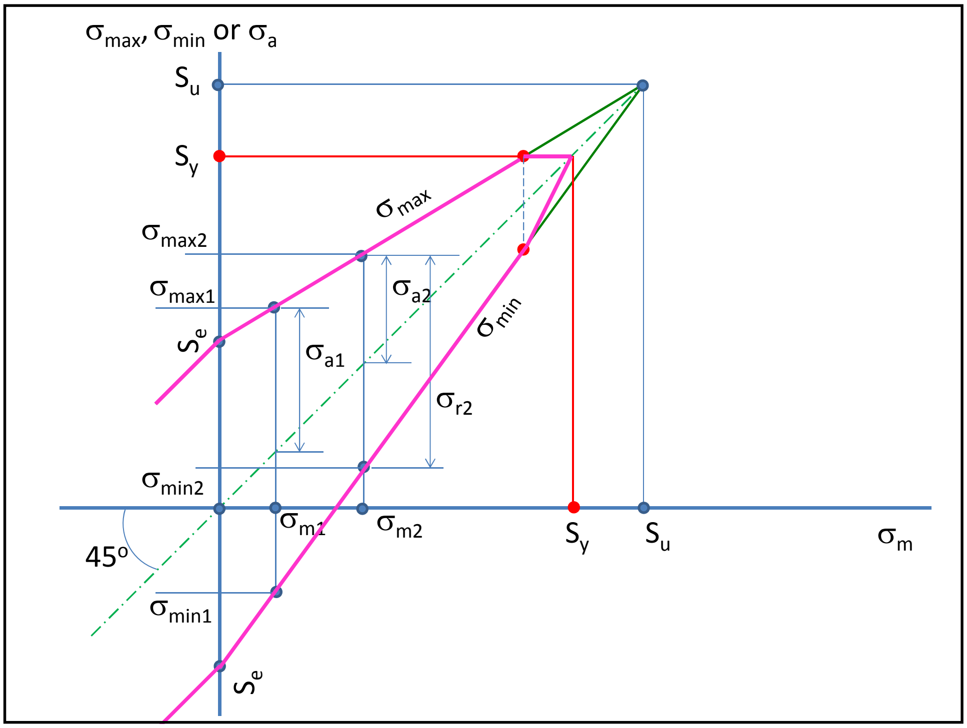



Notch radius r, mm
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1.0
0.8
0.6
0.4
0.2
0
S_{ut} = 200 kpsi
(1.4 GPa)
150
(1.0)
100
(0.7)
60
(0.4)
Steels
Alum. alloy
Notch sensitivity q
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
Notch radius r, in

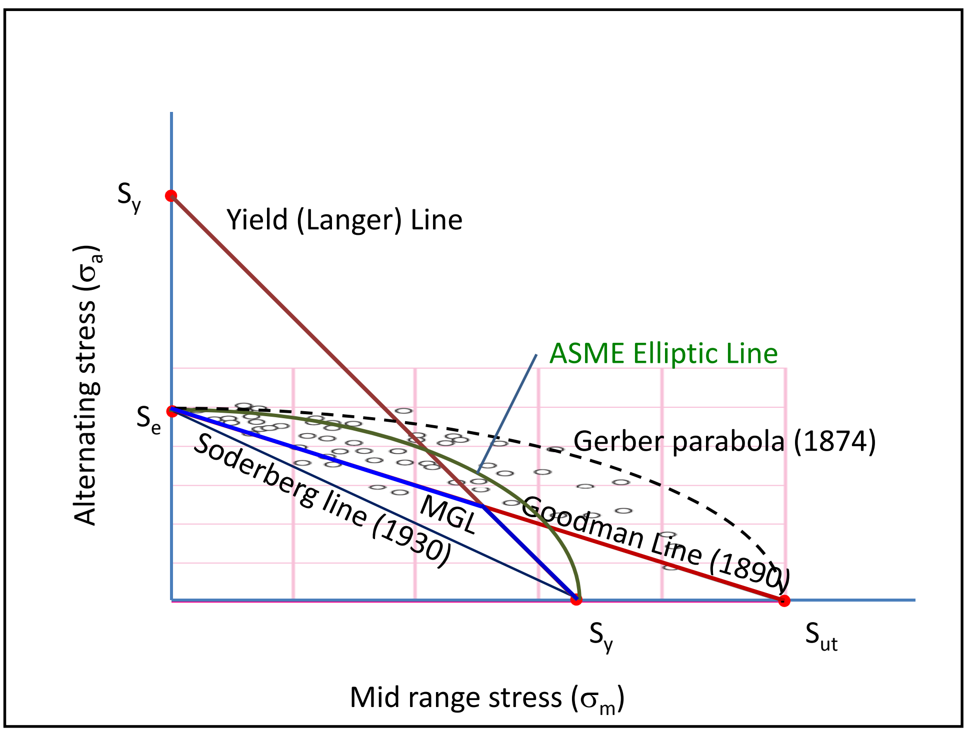

Notch radius r, mm
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
1.0
0.8
0.6
0.4
0.2
0
Quenched and drawn steels (Bhn > 200)
Annealed steels (Bhn < 200)
Aluminum alloys
Notch sensitivity q_{shear}
0.02
0.04
0.06
0.08
0.10
0.12
0.14
0.16
Notch radius r, in


Stress
Time
Fluctuating stress with high frequency ripple
Stress
σ_a
σ_r
σ_{max}
σ_m
σ_{min}
O
Time
Sinusoidal fluctuating stress
Stress
Time
Non-sinusoidal fluctuating stress
Stress
σ_{max}
$\sigma_{min} = 0$
Time
Sinusoidal repeated stress
Stress
Time
Non-sinusoidal fluctuating stress
+
Stress
O
$\sigma_m = 0$
Time
Fully reversed stress


Amplitude ratio S_a/S_e
1.2
1.0
0.8
0.6
0.4
0.2
A
B
C
−1.2
−1.0
−0.8
−0.6
−0.4
−0.2
0
0.2
0.4
0.6
0.8
1.0
Compression S_m/S_{uc}
Tension S_m/S_{ut}
Midrange ratio



σ_{max}, σ_{min} or σ_a
S_u
S_y
σ max
σ_{max2}
σ_{max1}
S_e
σ_{a2}
σ_{a1}
σ min
σ_{r2}
σ_{min2}
σ_{m1}
σ_{m2}
S_y
S_u
σ_m
45^o
σ_{min1}
S_e



S_y
Yield (Langer) Line
ASME Elliptic Line
Alternating stress (σ_a)
S_e
Gerber parabola (1874)
Soderberg line (1930)
MGL
Goodman Line (1890)
S_y
S_{ut}
Mid range stress (σ_m)

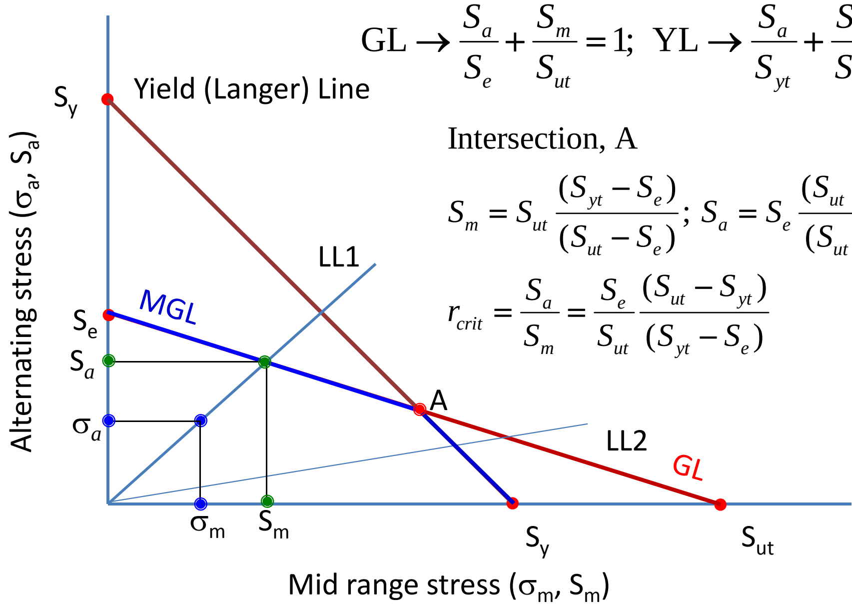
For infinite life

$$\text{GL} \rightarrow \frac{S_a}{S_e} + \frac{S_m}{S_{ut}} = 1; \quad \text{YL} \rightarrow \frac{S_a}{S_{yt}} + \frac{S_m}{S_{yt}} = 1$$

Intersection, A

$$S_m = S_{ut}\frac{(S_{yt} - S_e)}{(S_{ut} - S_e)}; \; S_a = S_e\frac{(S_{ut} - S_{yt})}{(S_{ut} - S_e)}$$

$$r_{crit} = \frac{S_a}{S_m} = \frac{S_e}{S_{ut}}\frac{(S_{ut} - S_{yt})}{(S_{yt} - S_e)}$$



Combined stresses

Calculte the mid range component of stress due to axial load ($\sigma_{\text{m-axial}}$), bending, ($\sigma_{\text{m-bening}}$) and torsion (τ_{m})

Calculte the amplitude component of stress due to axial load ($\sigma_{\text{a-axial}}$), bending, ($\sigma_{\text{a-bening}}$) and torsion (τ_{a})

Determine $K_{t-axial}$, $K_{t-bending}$ and $K_{s-torsion}$

Determine q and q_s and calculate $K_{f-axial}$, $K_{f-bending}$ and $K_{fs-torsion}$

Increase the mid range component and amplitude component of each stress by multiplying with respective fatigue SCF

Calculate the von-Mises stress for mid-range component,

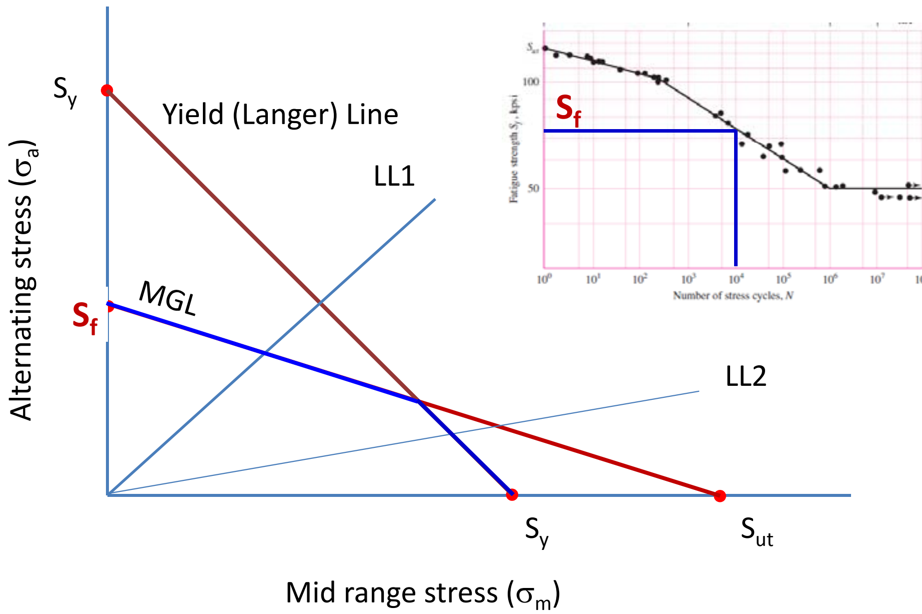
$$\sigma'_{\text{m}} = \left(\left(\sigma_{\text{m-axial}} \times K_{f-axial} + \sigma_{\text{m-bening}} \times K_{f-bending}\right)^2 + 3\left(\tau_{\text{m}} \times K_{fs-torsion}\right)^2\right)^{1/2}$$

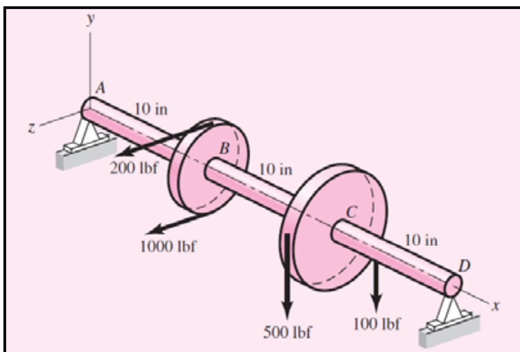
Calculate the von-Mises stress for mid-range component

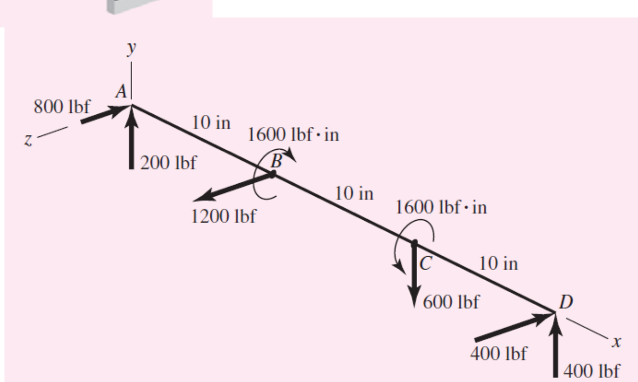
$$\sigma'_{\text{a}} = \left(\left(\sigma_{\text{a-axial}} \times K_{f-axial} + \sigma_{\text{a-bening}} \times K_{f-bending}\right)^2 + 3\left(\tau_{\text{a}} \times K_{fs-torsion}\right)^2\right)^{1/2}$$

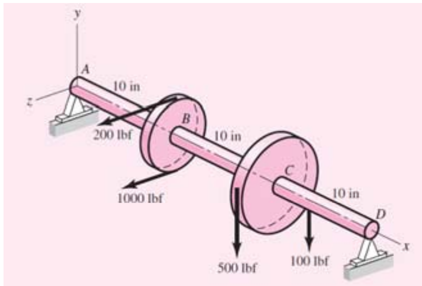
Use these in MGL

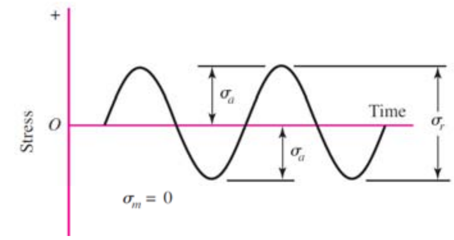
For finite life

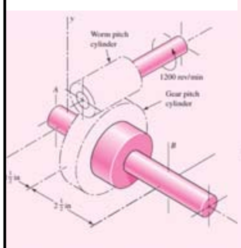


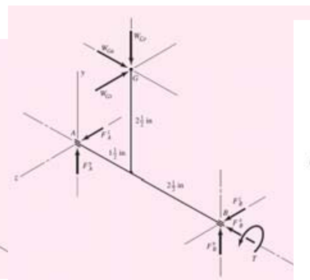


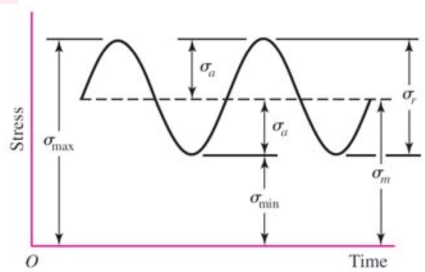


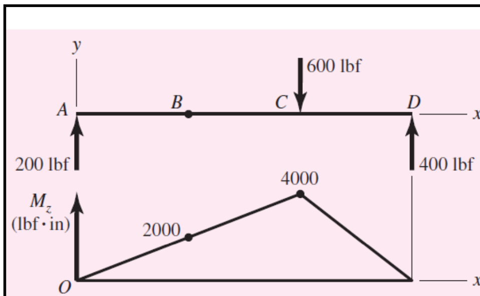

y
A
10 in
z
200 lbf
B
10 in
1000 lbf
C
10 in
D
x
500 lbf
100 lbf

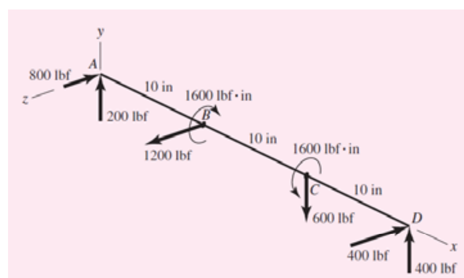

+
Stress
O
σ_a
σ_a
Time
σ_r
$\sigma_m = 0$

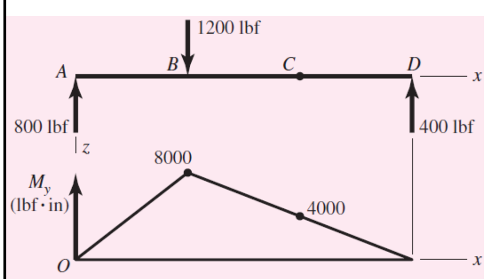

Worm pitch cylinder
1200 rev/min
Gear pitch cylinder

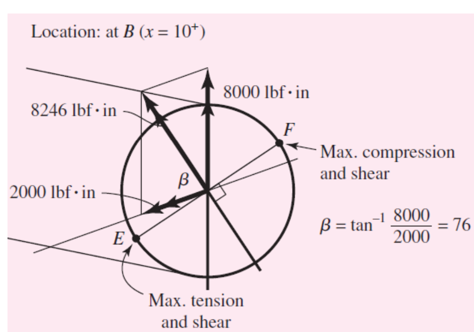



Stress
σ_a
σ_r
σ_a
σ_{max}
σ_m
σ_{min}
O
Time


y
600 lbf
A
B
C
D
x
200 lbf
400 lbf
4000
M_z
(lbf · in)
2000
O
x


y
800 lbf
A
z
10 in
1600 lbf · in
200 lbf
B
1200 lbf
10 in
1600 lbf · in
C
10 in
600 lbf
D
x
400 lbf
400 lbf


1200 lbf
A
B
C
D
x
800 lbf
400 lbf
z
8000
M_y
(lbf · in)
4000
O
x


Location: at B ($x = 10^+$)
8000 lbf · in
8246 lbf · in
F
Max. compression and shear
β
2000 lbf · in
$\beta = \tan^{-1} \frac{8000}{2000} = 76$
E
Max. tension and shear