ME361 – Manufacturing Science Technology

Oblique Cutting

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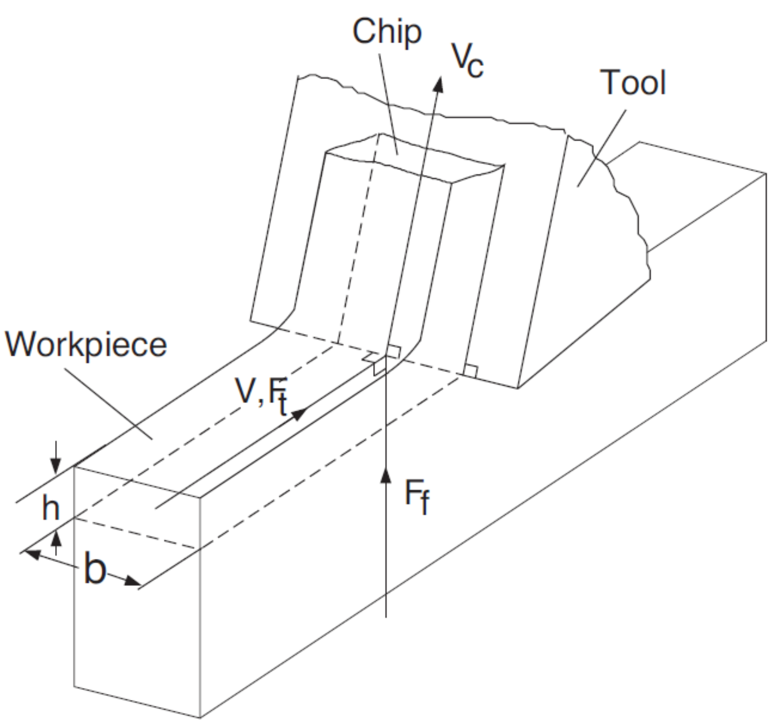
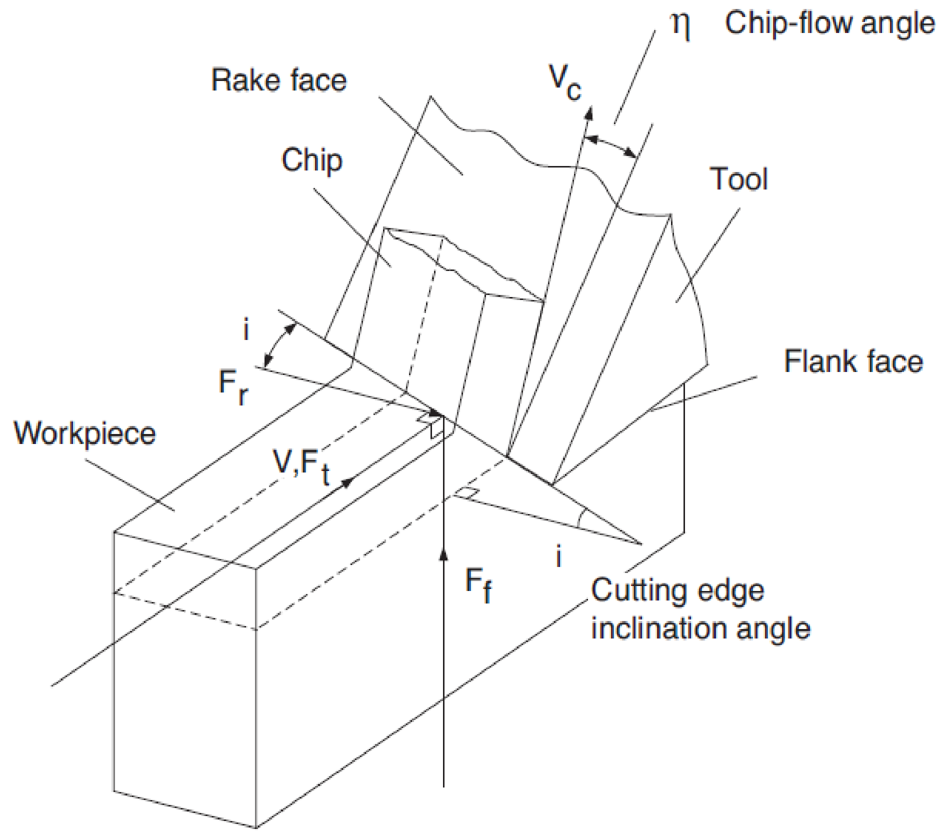
Orthogonal and oblique cutting geometry



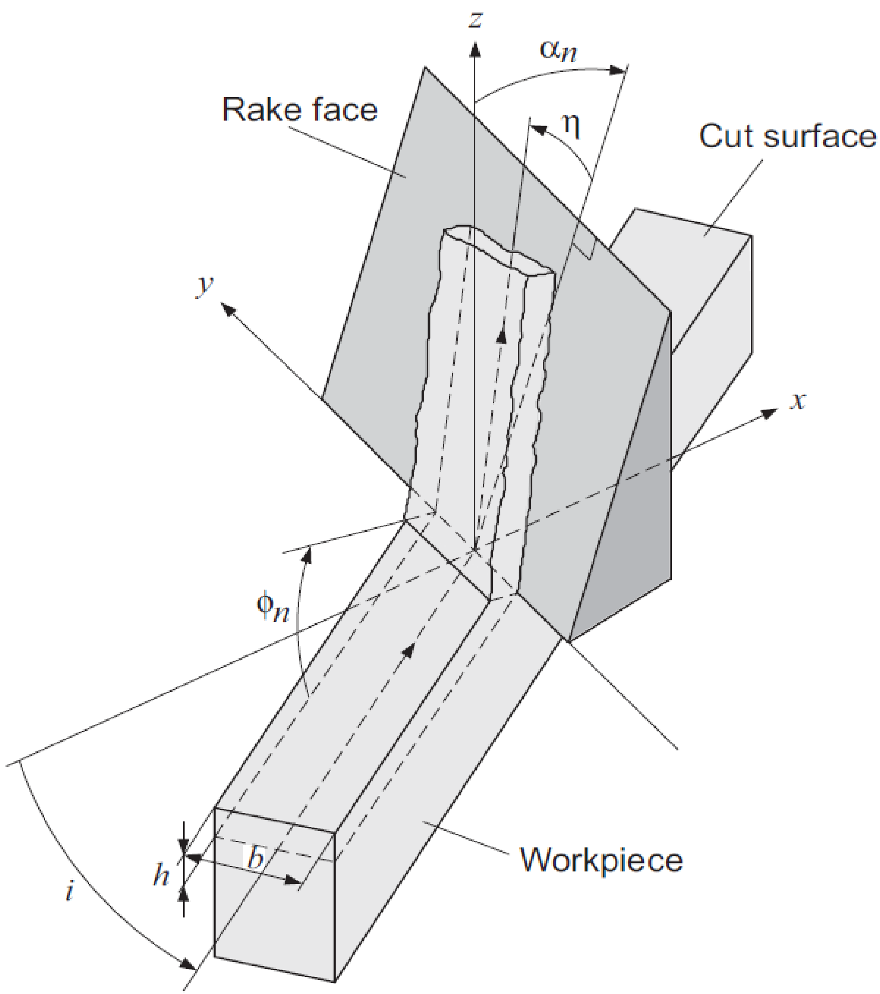
Cutting velocity is perpendicular to cutting edge



Cutting velocity is inclined at an acute angle i to the cutting edge

Altintas, Mfg. Automation

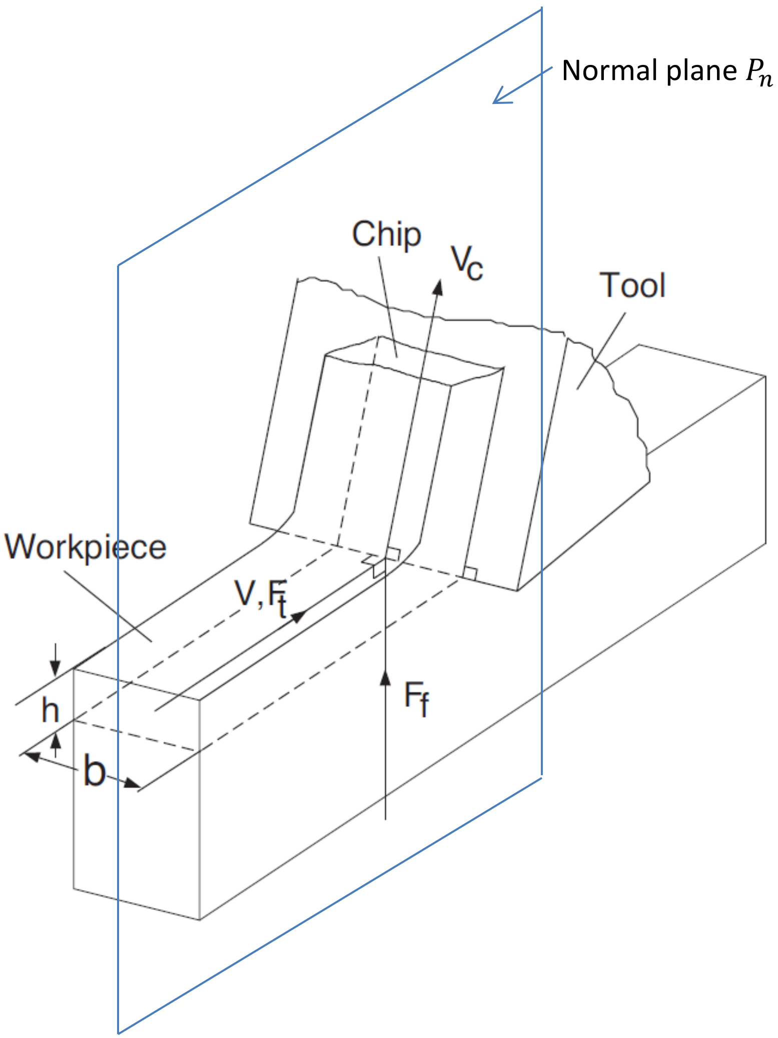
Oblique cutting geometry



Considerations:

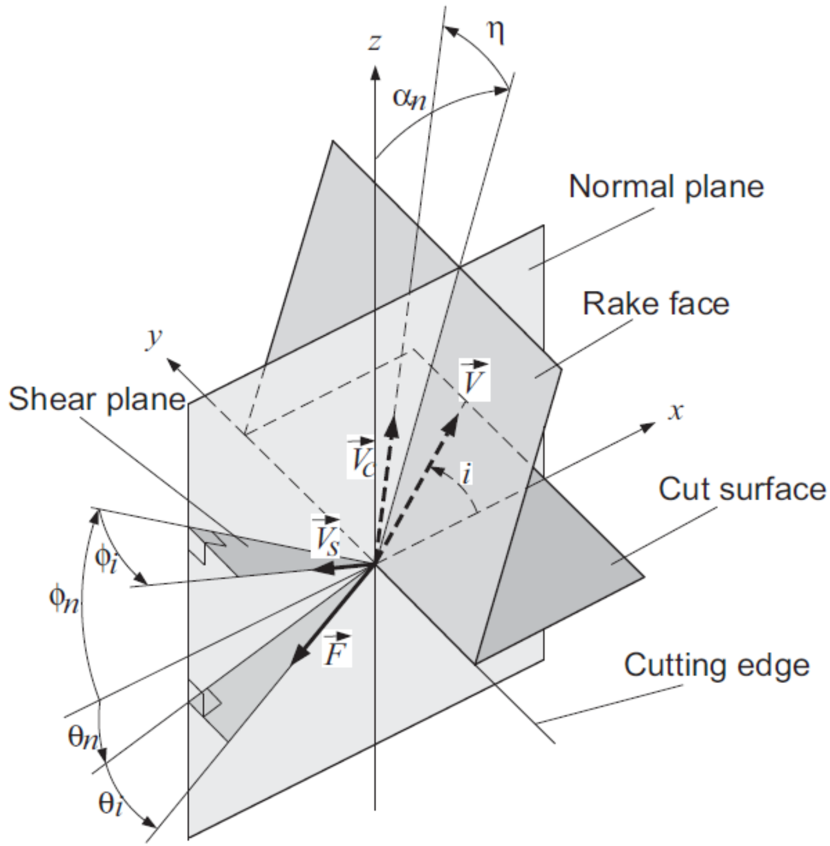
1. Cutting edge is perfectly sharp and no edge rubbing
2. Continuous chip with no built up-edge
3. Tool subjected to 3D system of cutting forces because of angle of obliquity, i
4. Non-plane strain deformation (treated however as a modified plane strain problem)
5. Uniform stress distribution on shear plane

Recalling orthogonal cutting



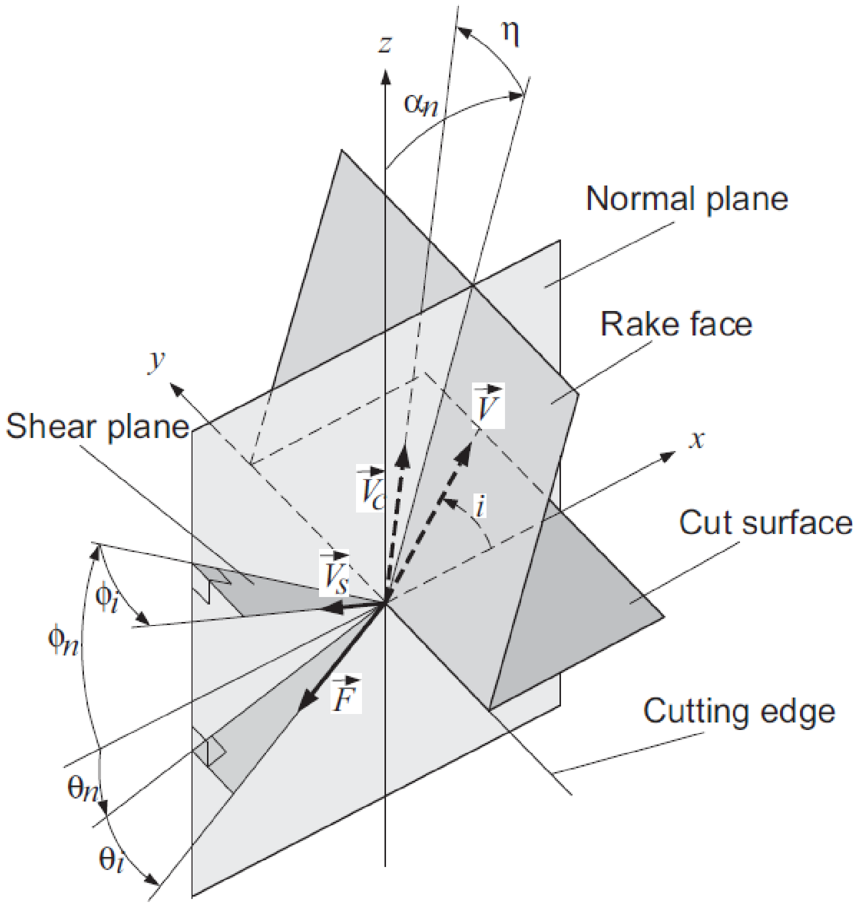
- Plane normal to the cutting edge, and aligned with the cutting velocity V is the normal plane P_n
- Because of plane strain deformation (and no side spread), shearing and chip motion are identical on all normal planes ∥ to V and ⊥ to the cutting edge
- Hence, all velocities V, V_s, and V_c are all ⊥ to the cutting edge and lie in the velocity plane P_v ∥ to or coincident with P_n
- Resultant force F_c, along with other forces acting on the shear and chip-rake face contact zone, also lie in the normal plane P_n
- No cutting force ⊥ to P_n, and edge forces = 0

Oblique cutting geometry



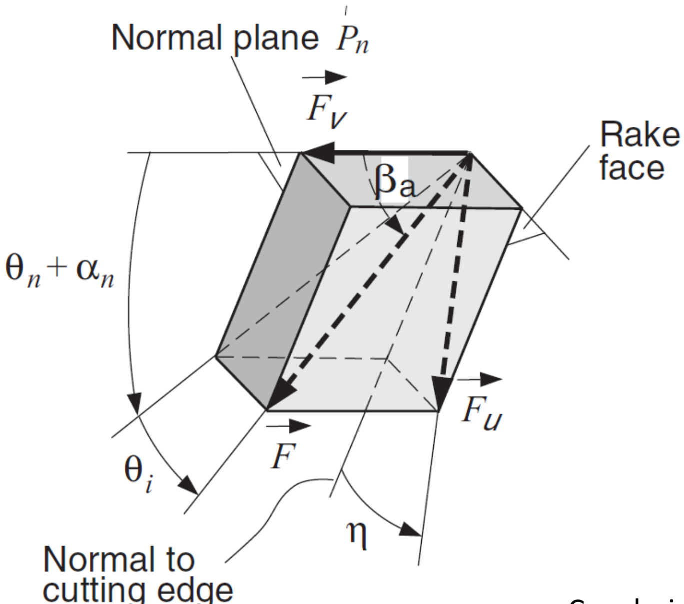
- Cutting velocity is inclined at an acute angle i to the cutting edge, hence direction of shear, friction, chip flow, and resultant force vectors have components in all there Cartesian coordinates (x, y, z)
- Plane normal to the cutting edge, and inclined at an acute angle i with the cutting velocity V is the normal plane P_n
- x is $\perp$ to the cutting edge, but lies on cut surface y is aligned with cutting edge z is $\perp$ to xy plane
- Important planes are shear plane, rake face, cut surface xy normal plane xz (or P_n), and the velocity plane P_v

Oblique cutting geometry



- Assume that mechanics of oblique cutting in the normal plane P_n are equivalent to orthogonal cutting. Hence project velocities and forces into normal plane
- Angle between shear and xy plane is the normal shear angle - ϕ_n
- Shear velocity lies on shear plane but makes an oblique shear angle - ϕ_i with the vector normal to the cutting edge on the normal plane
- Sheared chip moves over rake face with a chip flow angle of - η measured from a vector on the rake face but normal to cutting edge
- Angle between z axis and normal vector on rake face is the normal rake angle - α_n

Force diagram – oblique cutting



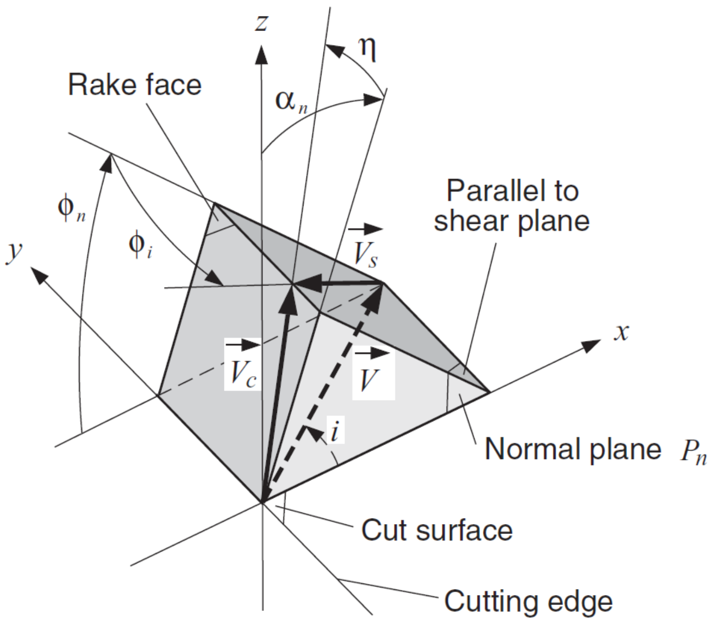
- Friction force on the rake face $\vec{F}_u$ and normal force to the rake $\vec{F}_v$ form a resultant cutting force $\vec{F}_c$ with a friction angle β_a
- This resultant $\vec{F}_c$ has an acute projection angle of θ_i with the normal plane P_n, which in turn has an in-plane angle of $\theta_n + \alpha_n$ with the normal force $\vec{F}_v$
- Where θ_n is the angle between the x axis and the projection of $\vec{F}_c$ on P_n

Can derive the following geometric relations

$$\boxed{F_u = F_c \sin\beta_a = F\frac{\sin\theta_i}{\sin\eta} \rightarrow \sin\theta_i = \sin\beta_a \sin\eta}$$

$$\boxed{F_u = F_v \tan\beta_a = F_v\frac{\tan(\theta_n+\alpha_n)}{\cos\eta} \rightarrow \tan(\theta_n+\alpha_n) = \tan\beta_a \cos\eta}$$

Velocity diagram – oblique cutting



Defining each velocity vector by its Cartesian components:

$$\vec{V} = (V\cos i\,, V\sin i\,, 0),$$
$$\vec{V}_c = (V_c\cos\eta\sin\alpha_n\,, V_c\sin\eta\,, V_c\cos\eta\cos\alpha_n),$$
$$\vec{V}_s = (-V_s\cos\phi_i\cos\phi_n\,, -V_s\sin\phi_i\,, V_s\cos\phi_i\sin\phi_n)$$

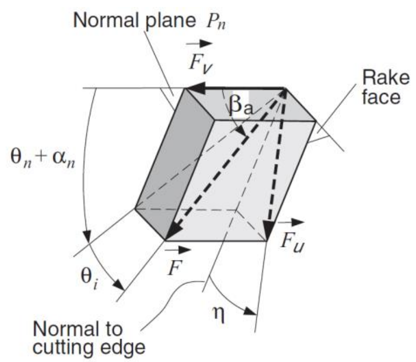
Eliminate V, V_c, and V_s

$$\vec{V}_s = \vec{V}_c - \vec{V}$$

rearranging, following geometric relation between shear and chip flow directions

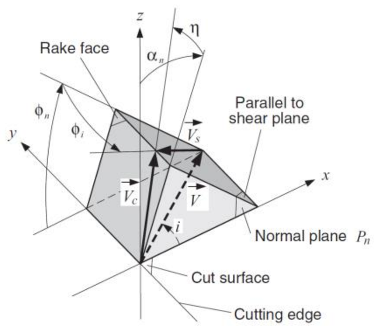
$$\tan\eta = \frac{\tan i\cos(\phi_n - \alpha_n) - \cos\alpha_n\tan\phi_i}{\sin\phi_n}$$

Solving for oblique cutting parameters



$$\boxed{\sin\theta_i = \sin\beta_a \sin\eta}$$

$$\boxed{\tan(\theta_n + \alpha_n) = \tan\beta_a \cos\eta}$$



$$\boxed{\tan\eta = \frac{\tan i \cos(\phi_n - \alpha_n) - \cos\alpha_n \tan\phi_i}{\sin\phi_n}}$$

Five unknown oblique cutting parameters that define the directions of resultant force (θ_n, θ_i), shear velocity (ϕ_n, ϕ_i), and chip flow (η).

Three equations – what to do? $\longrightarrow$ Empirical, model based?

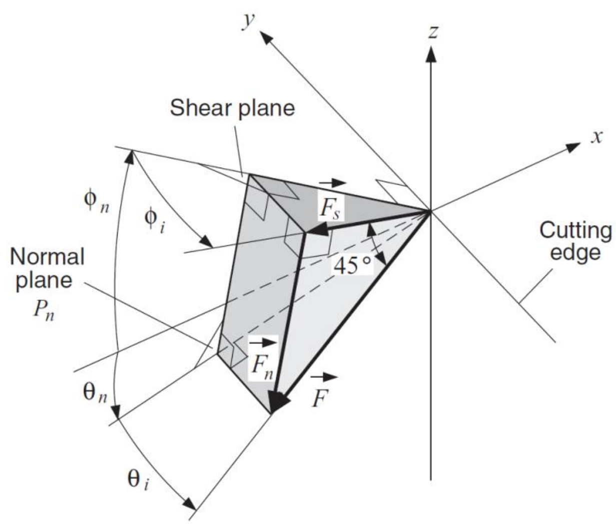
Minimum energy principle

Recall – Merchant's approach to predict shear angle by applying the principle of minimum energy principle to orthogonal cutting

Power consumed during cutting: $\boxed{P_{tc} = VF_{tc}}$ → $\boxed{\dfrac{dP_{tc}}{d\phi_c} = 0 \rightarrow \phi_c = \dfrac{\pi}{4} - \dfrac{\beta_a - \alpha_r}{2}}$

Extend the same principle here for oblique cutting

First need shear force (primary power consumption is during shearing)



Represent shear force as a projection of resultant $\vec{F_c}$ in the direction of shear:

$$\boxed{F_s = F_c[\cos(\theta_n + \phi_n)\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i]}$$

Minimum energy principle

Represent shear force as a projection of resultant $\vec{F}_c$ in the direction of shear:

$$F_s = F_c[\cos(\theta_n + \phi_n)\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i]$$

or

Representing shear force as a product of shear stress and shear plane area

$$F_s = \tau_s A_s = \tau_s \frac{b}{\cos i}\frac{h}{\sin\phi_n}$$

$$F_c = \frac{\tau_s bh}{[\cos(\theta_n + \phi_n)\cos\theta_i \cos\phi_i + \sin\theta_i \sin\phi_i]\cos i \sin\phi_n} \quad \text{(a)}$$

Power consumed during cutting:

$$P_{tc} = VF_{tc}$$

In terms of F_c

$$P_{tc} = F_c(\cos\theta_i \cos\theta_n \cos i + \sin\theta_i \sin i)V \quad \text{(b)}$$

Substitute (a) into (b)

$$P_t{}' = \frac{P_{tc}}{V\tau_s bh} = \frac{\cos\theta_n + \tan\theta_i \tan i}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Minimum energy principle

Power $P_t{}'$: $\boxed{P_t{}' = \frac{P_{tc}}{V\tau_s bh} = \frac{\cos\theta_n + \tan\theta_i \tan i}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}}$

Minimum energy principle dictates that the cutting power must be minimum for a unique shear angle solution. Since τ_s, b, h, and V is constant, and since the direction of shear is characterized by ϕ_n and ϕ_i, we have:

$$\boxed{\frac{\partial P_t{}'}{\partial \phi_n} = 0; \frac{\partial P_t{}'}{\partial \phi_i} = 0} \qquad (4\text{-}5)$$

Which gives us two additional equations, in addition to the earlier three to solve for five unknowns

↓

Recalling the earlier three

$$\boxed{\sin\theta_i = \sin\beta_a \sin\eta} \qquad (1)$$

$$\boxed{\tan(\theta_n + \alpha_n) = \tan\beta_a \cos\eta} \qquad (2)$$

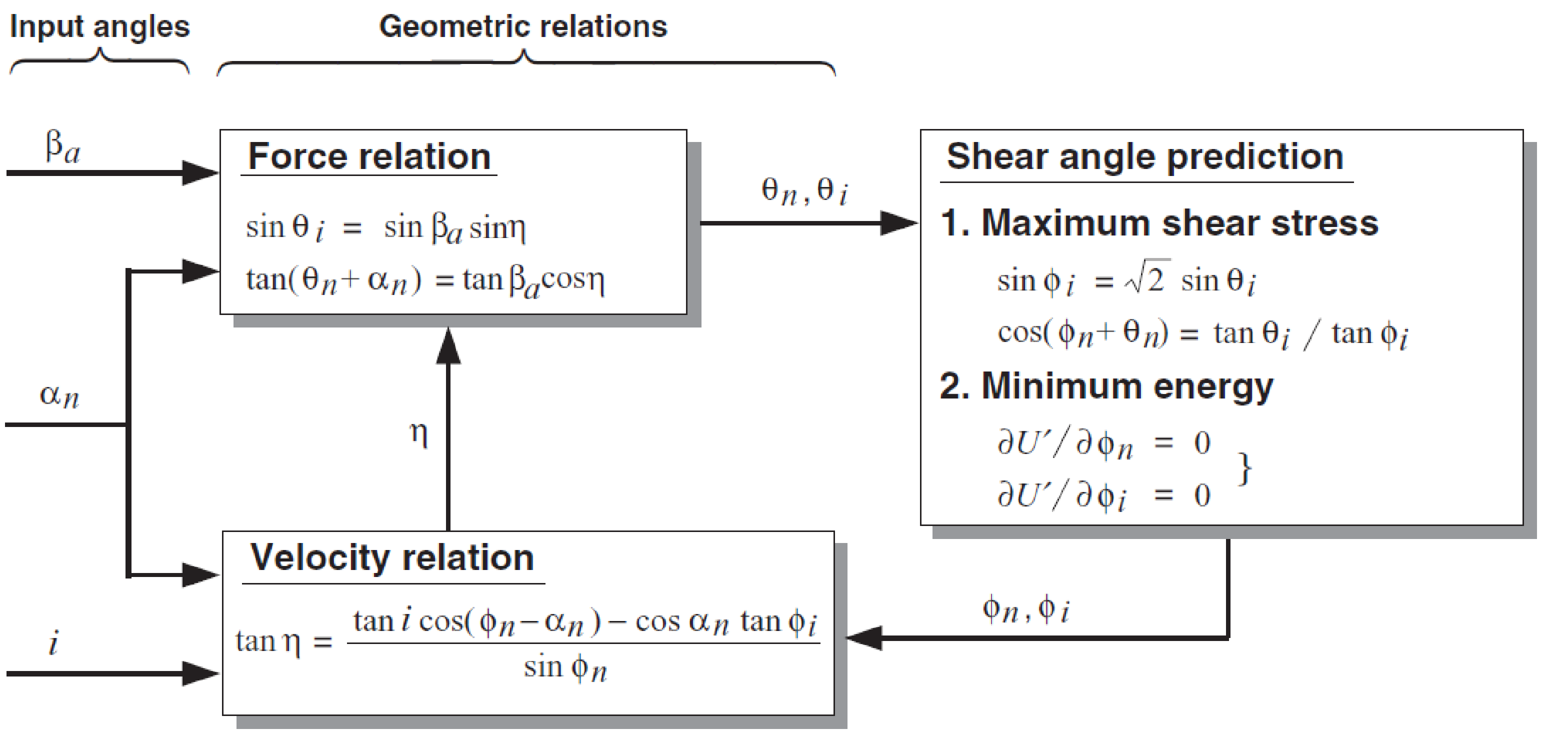
$$\boxed{\tan\eta = \frac{\tan i \cos(\phi_n - \alpha_n) - \cos\alpha_n \tan\phi_i}{\sin\phi_n}} \qquad (3)$$

Solving for oblique cutting parameters

Solving five equations analytically for five unknowns is not trivial, hence numerical iteration techniques are instead used

↓

Start with $\eta = i$ – Stabler's rule



Altintas, Mfg. Automation

Empirical solutions to oblique cutting parameters

Three equations five unknowns $(\phi_n, \phi_i, \eta, \theta_i, \theta_n)$

$$\boxed{\sin\theta_i = \sin\beta_a \sin\eta} \quad (1)$$

$$\boxed{\tan(\theta_n + \alpha_n) = \tan\beta_a \cos\eta} \quad (2)$$

$$\boxed{\tan\eta = \frac{\tan i \cos(\phi_n - \alpha_n) - \cos\alpha_n \tan\phi_i}{\sin\phi_n}} \quad (3)$$

Assume:

1. Shear velocity is collinear with shear force – from Stabler's work
2. Chip length ratio in oblique is same as in orthogonal – from experiments

Combine Eq. (1) to (3):

$$\boxed{\tan(\phi_n + \beta_n) = \frac{\cos\alpha_n \tan i}{\tan\eta - \sin\alpha_n \tan i}} \quad (1a) \qquad \boxed{\beta_n = \theta_n + \alpha_n}$$

$$\boxed{\tan\beta_n = \tan\beta_a \cos\eta} \quad (2a)$$

$$\boxed{\phi_n = \tan^{-1}\left(\frac{r_c(\cos\eta\,/\cos i)\cos\alpha_n}{1 - r_c(\cos\eta\,/\cos i)\sin\alpha_n}\right)} \quad (3a)$$

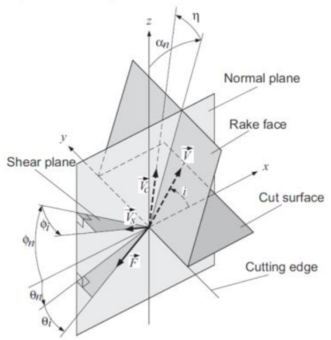
Solve Eq. (1a) – (3a) numerically to get η, ϕ_n and β_n.

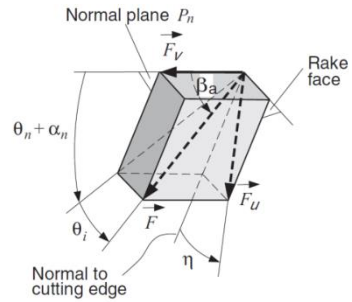
Or assume $\eta = i$ (Stabler's rule) for analytical solution

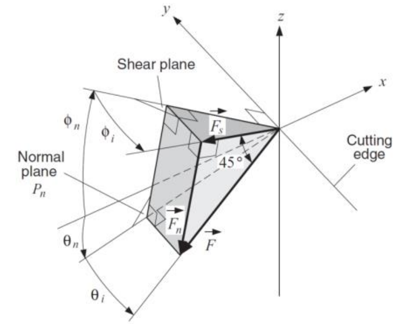
Armarego & Whitfield

Prediction of cutting forces

Cutting force components are derived as projections of resultant cutting force F_c after subtracting the edge components F_e from measured resultant F







Expressing force components as a $f(\tau_s, \phi_n, \phi_i, \theta_i, \theta_n)$:

Force in direction of cutting speed

$$F_{tc} = F_c(\cos\theta_i \cos\theta_n \cos\theta_i + \sin\theta_i \sin i) = \frac{\tau_s bh(\cos\theta_n + \tan\theta_i \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of thrust

$$F_{fc} = F_c \cos\theta_i \sin\theta_n = \frac{\tau_s bh \sin\theta_n}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of normal

$$F_{rc} = F_c(\sin\theta_i \cos\theta_i - \cos\theta_i \cos\theta_n \sin i) = \frac{\tau_s bh(\tan\theta_i - \cos\theta_n \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Prediction of cutting forces

Expressing force components as a $f(\tau_s, \phi_n, \phi_i, \theta_i, \theta_n)$:

Force in direction of cutting speed

$$F_{tc} = \frac{\tau_s bh(\cos\theta_n + \tan\theta_i \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of thrust

$$F_{fc} = \frac{\tau_s bh \sin\theta_n}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of normal

$$F_{rc} = \frac{\tau_s bh(\tan\theta_i - \cos\theta_n \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Rewriting forces in the convenient form of:

$$F_t = K_{tc} bh + K_{te} b;$$
$$F_f = K_{fc} bh + K_{fe} b;$$
$$F_r = K_{rc} bh + K_{re} b;$$

$$K_{tc} = \frac{\tau_s(\cos\theta_n + \tan\theta_i \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

$$K_{fc} = \frac{\tau_s \sin\theta_n}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

$$K_{rc} = \frac{\tau_s bh(\tan\theta_i - \cos\theta_n \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Prediction of cutting forces – Armarego's model

Expressing force components as a $f(\tau_s, \phi_n, \phi_i, \theta_i, \theta_n)$:

Force in direction of cutting speed

$$F_{tc} = \frac{\tau_s bh(\cos\theta_n + \tan\theta_i \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of thrust

$$F_{fc} = \frac{\tau_s bh \sin\theta_n}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Force in direction of normal

$$F_{rc} = \frac{\tau_s bh(\tan\theta_i - \cos\theta_n \tan i)}{[\cos(\theta_n + \phi_n)\cos\phi_i + \tan\theta_i \sin\phi_i]\sin\phi_n}$$

Using Armarego's classical oblique cutting model, forces are transformed in terms of $f(\tau_s, \beta_n, \phi_n \alpha_n, \eta)$

$$F_t = bh\left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n - \alpha_n) + \tan i \tan\eta \sin\beta_n}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

$$F_f = bh\left[\frac{\tau_s}{\sin\phi_n \cos i}\frac{\sin(\beta_n - \alpha_n)}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

$$F_r = bh\left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n - \alpha_n)\cos(\beta_n - \alpha_n)\tan i - \tan\eta \sin\beta_n}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

Armarego's force model

Oblique cutting – force coefficients

Using Armarego's classical oblique cutting model, forces are transformed as

$$F_t = bh\left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n-\alpha_n)+\tan i\tan\eta\sin\beta_n}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

$$F_f = bh\left[\frac{\tau_s}{\sin\phi_n\cos i}\frac{\sin(\beta_n-\alpha_n)}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

$$F_r = bh\left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n-\alpha_n)\cos(\beta_n-\alpha_n)\tan i-\tan\eta\sin\beta_n}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

$$F_t = K_{tc}bh + K_{te}b;$$
$$F_f = K_{fc}bh + K_{fe}b;$$
$$F_r = K_{rc}bh + K_{re}b;$$

$$K_{tc} = \left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n-\alpha_n)+\tan i\tan\eta\sin\beta_n}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

$$K_{fc} = \left[\frac{\tau_s}{\sin\phi_n\cos i}\frac{\sin(\beta_n-\alpha_n)}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

$$K_{rc} = \left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n-\alpha_n)\cos(\beta_n-\alpha_n)\tan i-\tan\eta\sin\beta_n}{\sqrt{cos^2(\phi_n+\beta_n-\alpha_n)+tan^2\eta sin^2\beta_n}}\right]$$

Armarego's force model

Practical approach – for force prediction

- Evaluate shear angle ϕ_c, friction angle β_a, and shear stress τ_s from orthogonal cutting tests
- Assume orthogonal shear angle equals normal shear angle, i.e. $\phi_c \equiv \phi_n$
- Assume normal rake angle equals rake angle in orthogonal cutting, i.e. $\alpha_r \equiv \alpha_n$
- Assume chip flow angle equals oblique angle (Stabler's rule), i.e. $\eta \equiv i$
- Assume friction angle in orthognal cutting is same as in oblique, i.e. $\beta_a \equiv \beta_n$
- Assume shear stress remains the same in orthogonal and oblique cutting

Evaluate cutting force coefficients

$$K_{fc} = \left[\frac{\tau_s}{\sin\phi_n \cos i}\frac{\sin(\beta_n - \alpha_n)}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

$$K_{tc} = \left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n - \alpha_n) + \tan i \tan\eta \sin\beta_n}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

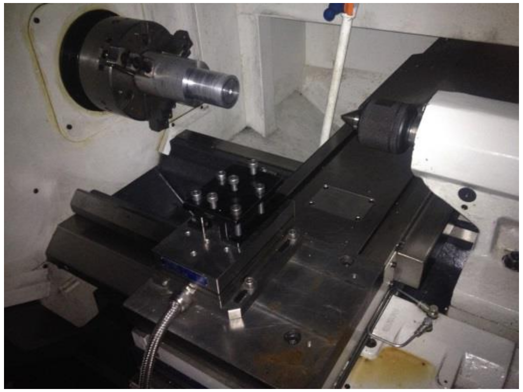
$$K_{rc} = \left[\frac{\tau_s}{\sin\phi_n}\frac{\cos(\beta_n - \alpha_n)\cos(\beta_n - \alpha_n)\tan i - \tan\eta \sin\beta_n}{\sqrt{cos^2(\phi_n + \beta_n - \alpha_n) + tan^2\eta sin^2\beta_n}}\right]$$

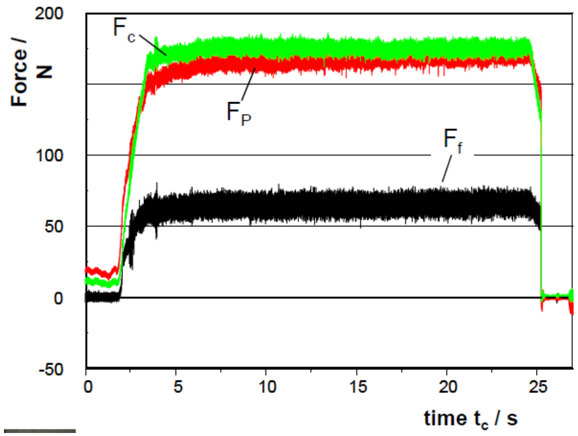
Predict cutting forces

$$F_t = K_{tc}bh + K_{te}b;$$
$$F_f = K_{fc}bh + K_{fe}b;$$
$$F_r = K_{rc}bh + K_{re}b;$$

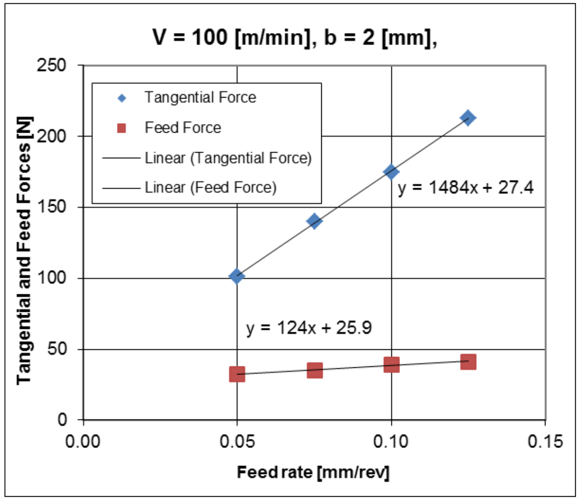
Yet another practical approach

Conduct a dedicated series of tests at different feed rates and identify coefficients directly for the tool-workpiece-cutting parameter combination of interest





Test No:	Feed Rate h [mm/rev]	Measured Ftc [N]	Measured Ffc [N]
1	0.050	101	32
2	0.075	140	35
3	0.100	175	39
4	0.125	213	41



Yet another practical approach

Forces are composed of shearing and edge rubbing force components

$$F_t = F_{tc} + F_{te}; \quad F_f = F_{fc} + F_{fe};$$

or

$$F_t = K_{tc}bh + K_{te}b; \quad F_f = K_{fc}bh + K_{fe}b;$$

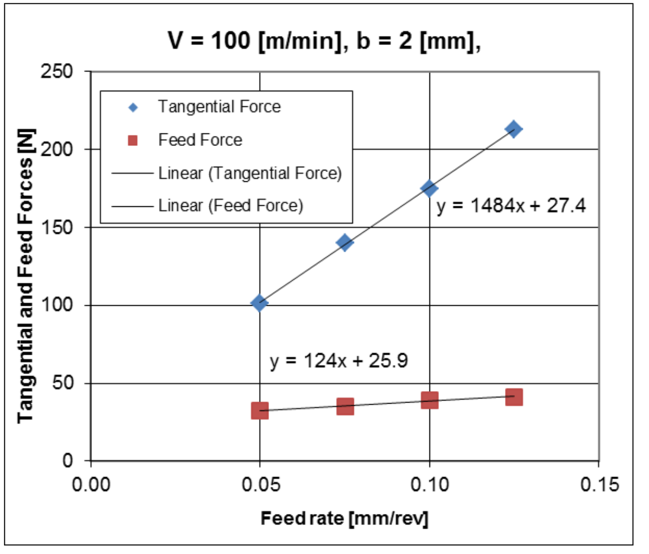
Prediction

Consider only shearing components (since edge forces do not contribute to cutting):

$$F_{tc} = K_{tc}bh; \; F_{fc} = K_{fc}bh;$$

Hence the cutting coefficients are:

$$K_{tc} = \frac{F_{tc}}{bh}; \; K_{fc} = \frac{F_{fc}}{bh}; \qquad K_{te} = \frac{F_{te}}{b}; \; K_{fe} = \frac{F_{fe}}{b};$$



Force models

linear approximation:

$$F_i = A \cdot b \cdot h + B \cdot b$$

- result of a curve fit
- first part is based on the shear plane theory
- very easy function

- not very precise
- calculations are not sufficiently verified (method is not commonly used)

researchers:

- **Schlesinger (1931)**
- Pohl (1934)
- Klein (1938)
- Richter (1954)
- Hucks (1956)
- Thomson (1962)
- Altintas (1998)

potential approximation:

$$F_i = ki_{1.1} \cdot b \cdot h^{(1-m)}$$

- result of a curve fit
- calculation of the cutting force is statistically verified
- very precise

- no theoretical basis
- calculation of the other force components is not sufficiently verified

researchers:

- **Taylor (1883/1902)**
- Fischer (1897)
- Friedrich (1909)
- Hippler (1923)
- **Salomon(1924)**
- Kronenberg (1927)
- Klopstock (1932)
- **Kienzle (1952)**

WZL Aachen