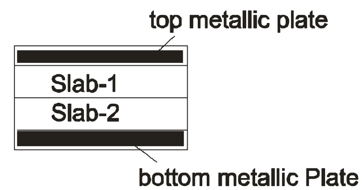
1. A short cylinder of radius R and length L carries a uniform polarization P parallel to its axis. Find the electric field at the center.

[Ans. $\frac{PL}{2\varepsilon_0}\left[\frac{1}{L}-\frac{1}{\sqrt{L^2+4R^2}}\right]$]

2. The space between the plates of a parallel plate capacitor is completely filled with two slabs of linear dielectric materials (figure). Each slab has thickness a so the total distance between the plates is $2a$. Slab -1 has a dielectric constant 2 and slab-2 has a dielectric constant 1.5. The free charge density on the top plate (on the side of slab-1) is σ and that on the bottom plate is $-\sigma$.



(a) Find $\mathbf{D}, \mathbf{P}$ and $\mathbf{E}$ in each slab.

(b) Find the potential difference between the plates.

(c) Find the location and amount of all bound charges.

(d) From the charges, calculate the field $\mathbf{E}$ in each slab and see that you get back the values found in part (a).

[Ans. (a) $\mathbf{D}=\sigma\hat{n}$ everywhere, $\mathbf{E}-\frac{\sigma}{2\varepsilon_0}\hat{n}$ in slab-1 and $\frac{2\sigma}{3\varepsilon_0}\hat{n}$ in slab-2, $\mathbf{P}=\frac{\sigma}{2}\hat{n}$ in slab-1 and $\frac{\sigma}{3}\hat{n}$ in slab-2, $\hat{n}$ is the unit vector in downward direction.]

3. Consider the situation shown in the figure. All the surfaces of the metal plates and the dielectric slabs are large. $+\sigma$ and $-\sigma$ give the surface charge densities on the plates. Show separately plots of D, E and P as a function of z.

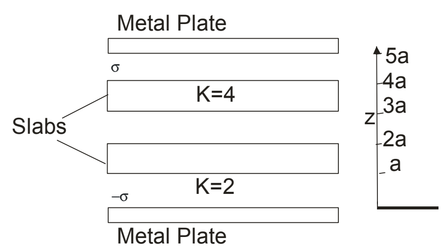


Figure 9 E.3

4. A water molecule has dipole moment of 6.0×10^{-30} Cm.
Suppose the molecules have their dipoles aligned parallel to each other in a drop (radius 0.2 mm) of water. Find the maximum electric field just outside the surface of the drop.

[**Ans.** $1.5\times10^{10} V/m$]

5. A positive point charge q is placed at the center of a spherical shell made of a material with dielectric constant *1.5* and inner and outer radii a and b. Find the fields $\mathbf{D}, \mathbf{P}$ and $\mathbf{E}$ everywhere.

6. A long conducting cylinder of radius R carries a uniform charge density □. It is surrounded by a coaxial dielectric shell of inner and outer radii a and b, the dielectric constant of the shell being K. Find (a) $\mathbf{D}$ in the dielectric, (b) the electric field everywhere, (c) the bound charge densities appearing on the shell.

[**Ans.** $(c) -\frac{(K-1)R\sigma}{Ka}$ on the inner surface and $\frac{(K-1)R\sigma}{Kb}$ on the outer surface]

7. An infinite circular cylinder of dielectric constant K is placed in vacuum having a transverse uniform electric field $\mathbf{E}_0$. Assuming the polarization to be uniform, find the field inside.

[**Ans.** $\dfrac{2E_0}{K+1}$]

8. A thin, long rod of dielectric constant K is placed in an otherwise uniform external electric field $\mathbf{E}_0$ along the axis of the rod. Find the $\mathbf{D}$ vector inside the rod, away from the ends.

[**Ans.** $\varepsilon_0 K\mathbf{E}_0$]

9. A thin disk of radius R, thickness $d(<< R)$ and dielectric constant K is placed in an otherwise uniform electric field $\mathbf{E}_0$ along the axis of the disk. Find the $\mathbf{D}$ vector inside the disk, away from the edges. [**Ans:** $\varepsilon_0\mathbf{E}_0$]

10. Suppose the field inside a large piece of dielectric of irregular shape is placed in an external electric field. The resultant electric field inside the dielectric is $\mathbf{E}_0$ everywhere and the polarization is $\mathbf{P}$, so that the displacement is $\mathbf{D}_0 = \varepsilon_0 E_0 + \mathbf{P}$. A small spherical cavity is carved out of the material. Assume that the polarization remains uniform even when the cavities are carved out in the dielectric. Find the field in the cavity in terms of $\mathbf{E}_0$ and $\mathbf{P}$. Also find the displacement in the cavity in terms of $\mathbf{D}_0$ and $\mathbf{P}$.

[**Ans.** $\mathbf{E}_0 + \dfrac{\mathbf{P}}{3\varepsilon_0}, \mathbf{D}_0 - \dfrac{2}{3}\mathbf{P}$]

11. Show that the electric field lines "refract" at the interface between two dielectrics obeying $\dfrac{\tan\theta_2}{\tan\theta_1} = \dfrac{K_2}{K_1}$ where θ denotes the angle of the electric field with the normal and K denotes the dielectric constant.