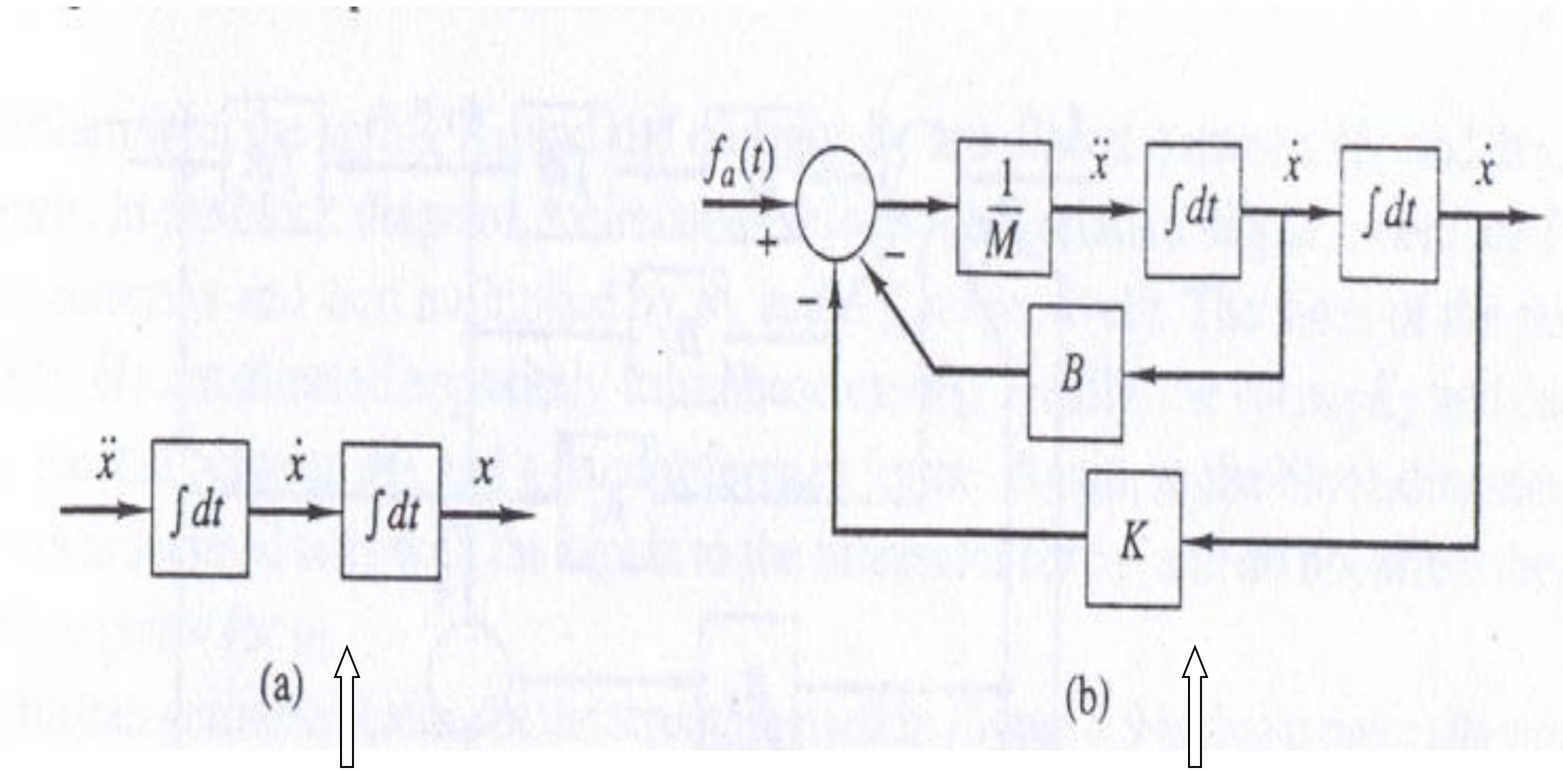
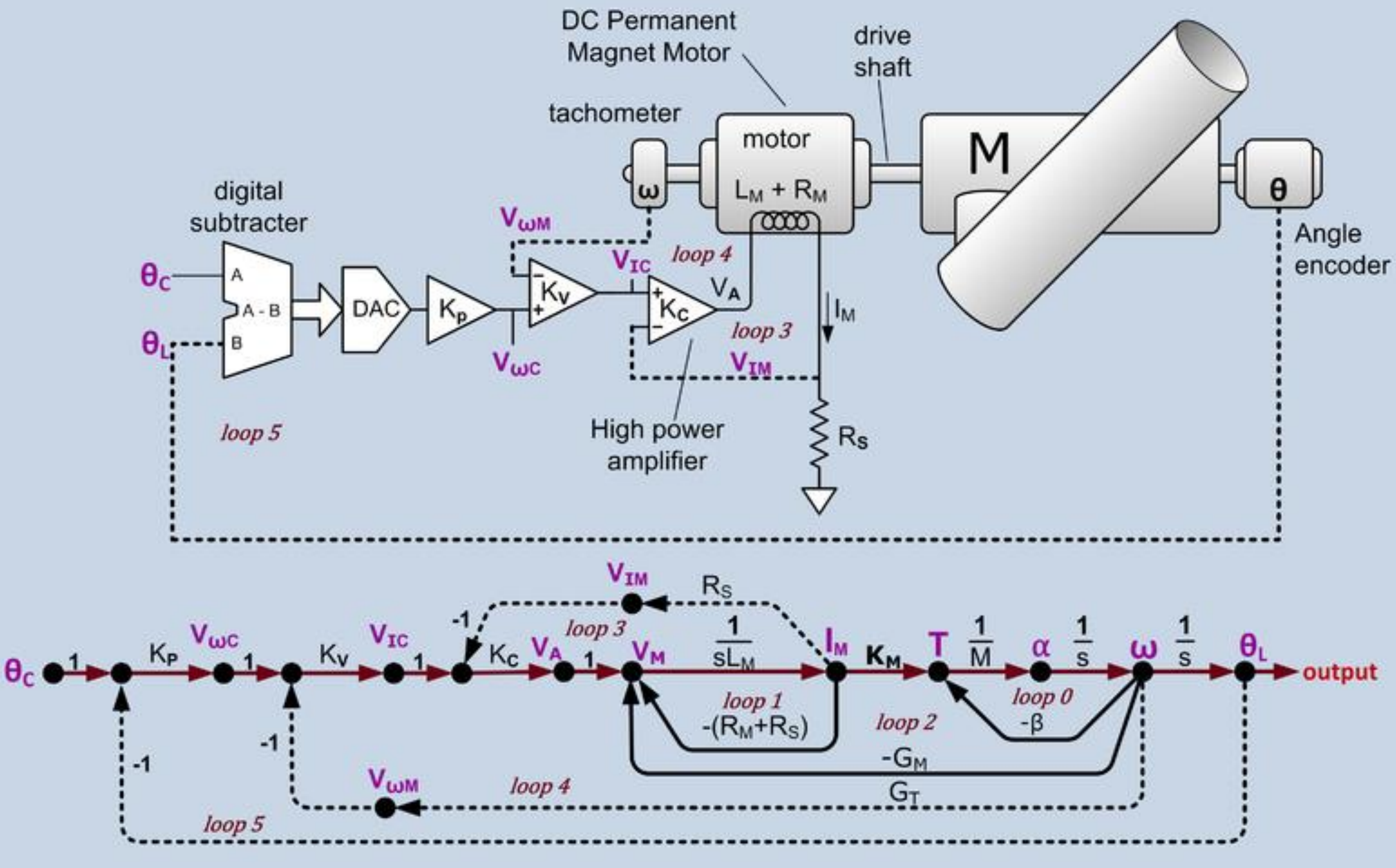
Electro-Mechanical Systems and their Representations

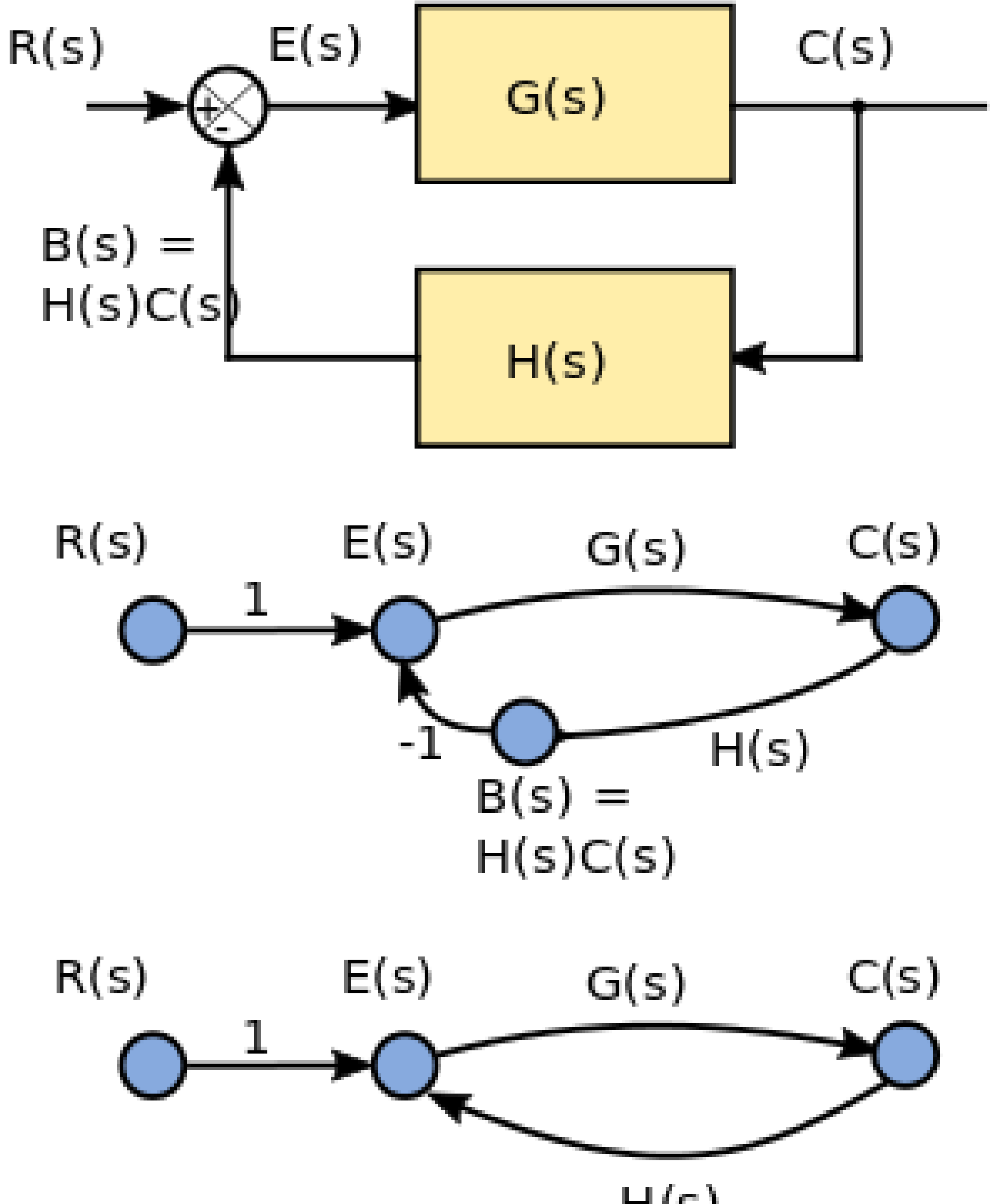
Block Diagram for a SDOF SYSTEM



Two Integrators **Complete Diagram**

Angular Position Servo and Signal Flow Graph: Source Wikipedia




R(s)
E(s)
G(s)
C(s)
+
-
B(s) =
H(s)C(s)
H(s)
R(s)
E(s)
G(s)
C(s)
1
-1
H(s)
B(s) =
H(s)C(s)
R(s)
E(s)
G(s)
C(s)
1
-H(s)

Elements of Signal-Flow Graph

- ***Node***. A node is a point representing a variable of' signal.

- ***Transmittance.*** a real gain or complex gain between two nodes.
  Such gains can be expressed in terms of the transfer function between two nodes.

- ***Branch***. a directed line segment joining two nodes. The gain of a branch is a transmittance.

- ***Input mode or source***. An input node or source is a node that has only outgoing branches.
  This corresponds to an independent variable.
- ***Output node or sink***. An output node or sink is the node that has only incoming branches.
  This corresponds to a dependent variable.

- ***Mixed node***. both incoming and outgoing branches.

- ***Path***. A path is a traversal of connected branches in the direction of the branch arrows. If no node is crossed more than once, the path is **open**. If the path ends at the same node from which it began and does not cross any other node more than once, it is **closed**.

Loop. A loop is a closed path.

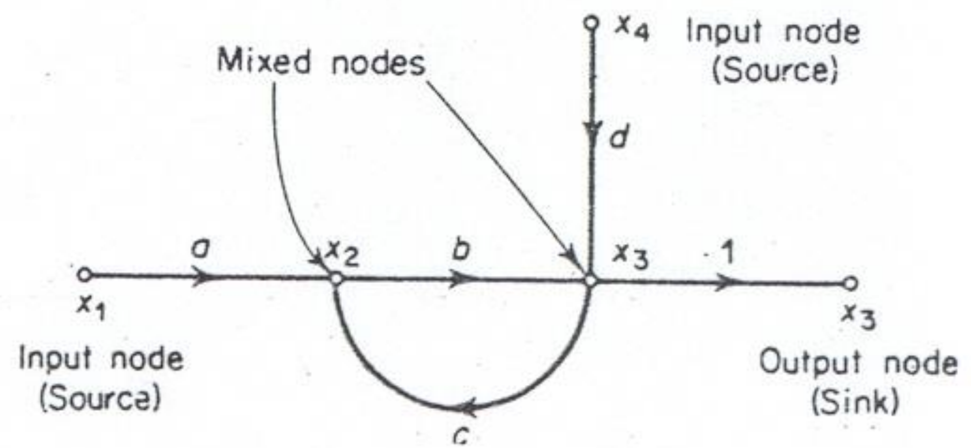
- ***Loop gain***. The loop gain is the product of the branch transmittances of a loop.

- ***Nontouching loops***. Loops are nontouching if they do not possess any common nodes.

- ***Forward path***. A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.

- ***Forward path gain***. A forward path gain is the product of the branch transmittances of a forward path.

Signal Flow Graph



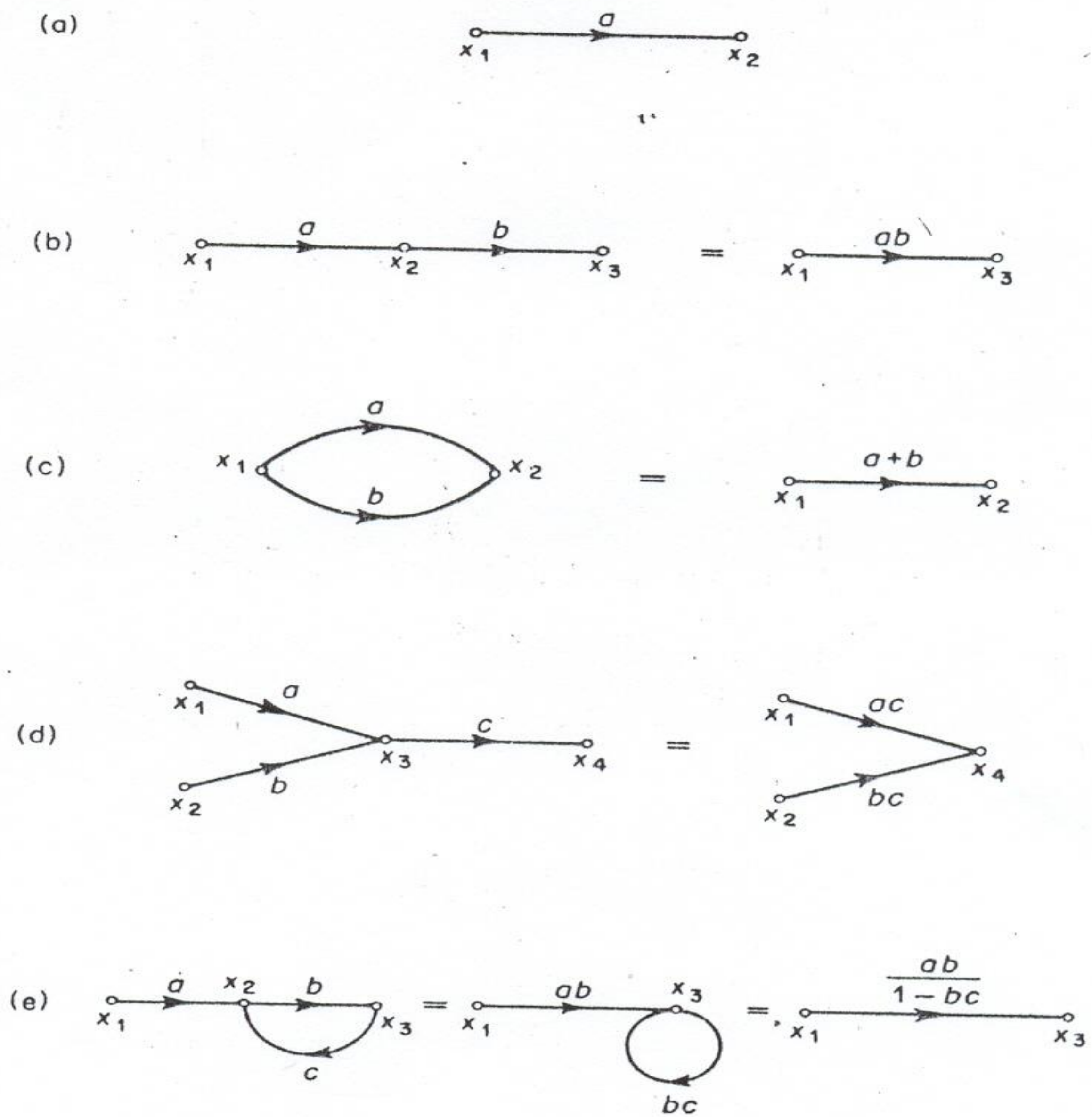
- Signals travel along branches only in the direction of the arrows.
- A signal travelling along any branch is multiplied by the transmission of that branch.
- The value of any node variable is the sum of all signals entering the node.
- The value of any node variable is transmitted on all branches leaving that node.

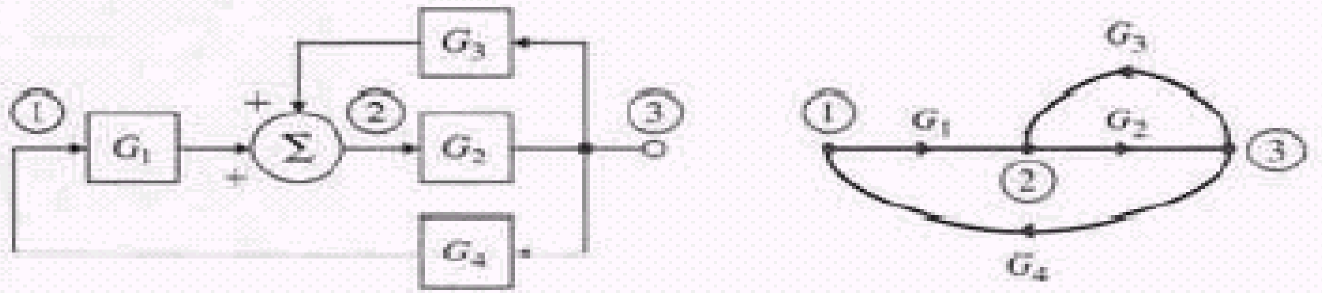
Signal-graph basic rules

- The value of a node with one incoming branch and gain ‘a’ is. $x_2 = ax_1$
- The total transmittance of cascaded branches is equal to the product of the branch transmittances. Cascaded branches can thus be combined into a single branch by multiplying the transmittances
- Parallel branches may, be combined by adding the transmittances
- Mixed nodes and loops may be eliminated to calculate the complete transfer function, for example, a loop may be eliminated at junction 2 in the last figure by noting that

$$x = bx_2 \quad , \quad x_2 = ax_1 + cx_3$$

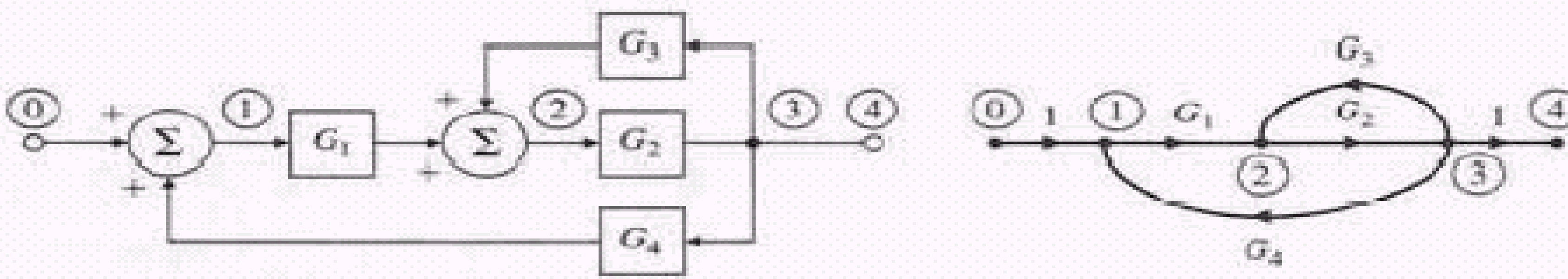
Signal Flow Graphs and Simplifications





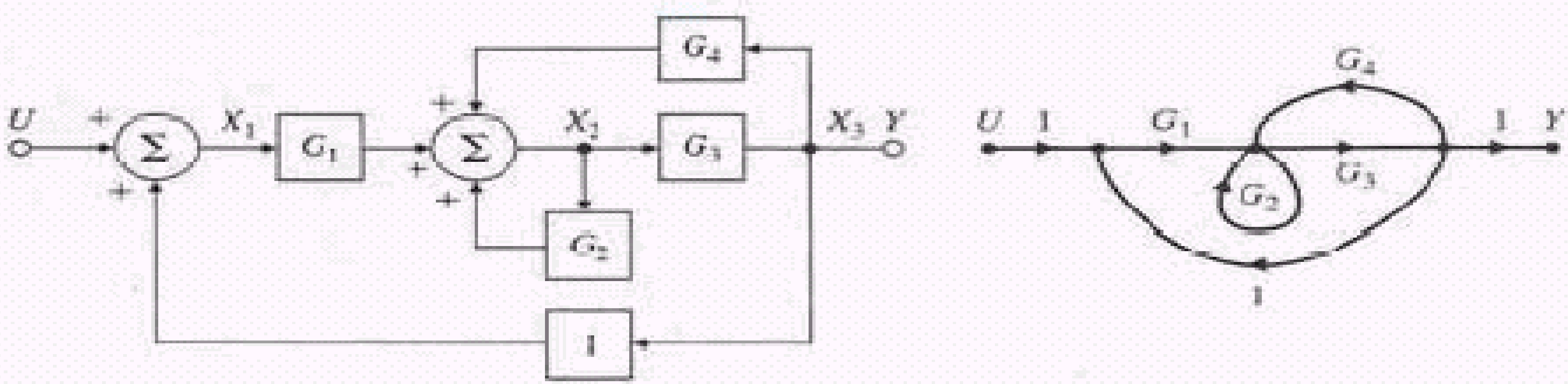
①
②
③
G_1
G_2
G_3
G_4
Σ
+
+

(a)



⓪
①
②
③
④
G_1
G_2
G_3
G_4
1
1
Σ
+
+
+
+

(b)



U
X_1
X_2
X_3
Y
G_1
G_2
G_3
G_4
1
Σ
+
+
+
+
+

(c)

For a complex signal flow graph, evaluation of the transfer function based on first principles is quite cumbersome. An algorithmic way of evaluating the same based on Graph Theory is known as **Mason's rule**.

Mason's rule – Key points

- **Forward Path Gain – Product of branch Gains found by traversing a path from the input to output node in the direction of signal flow**
- **Non-touching Loops – Loops that do not have any nodes in common**
- **Non-touching Loop Gain – The product of loop gains from non-touching loops**

Mason's Rule

The transfer function, $C(s)/R(s)$, of a system represented by a signal-flow graph is

$$\boxed{G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}} \tag{5.28}$$

where

k = number of forward paths

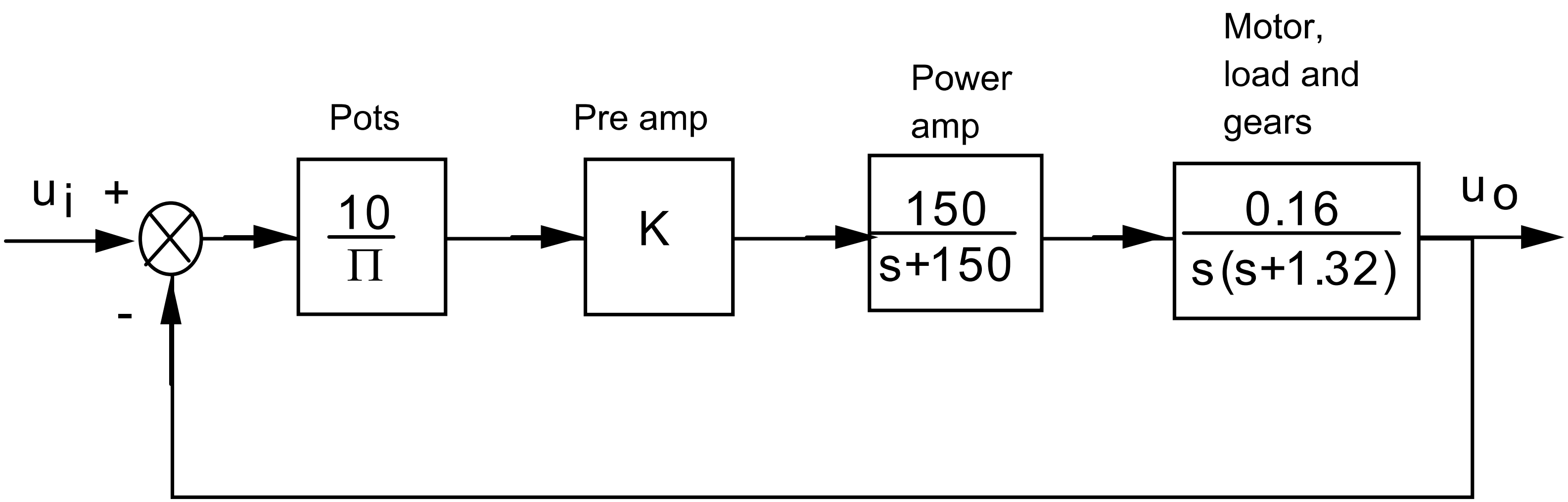
T_k = the kth forward-path gain

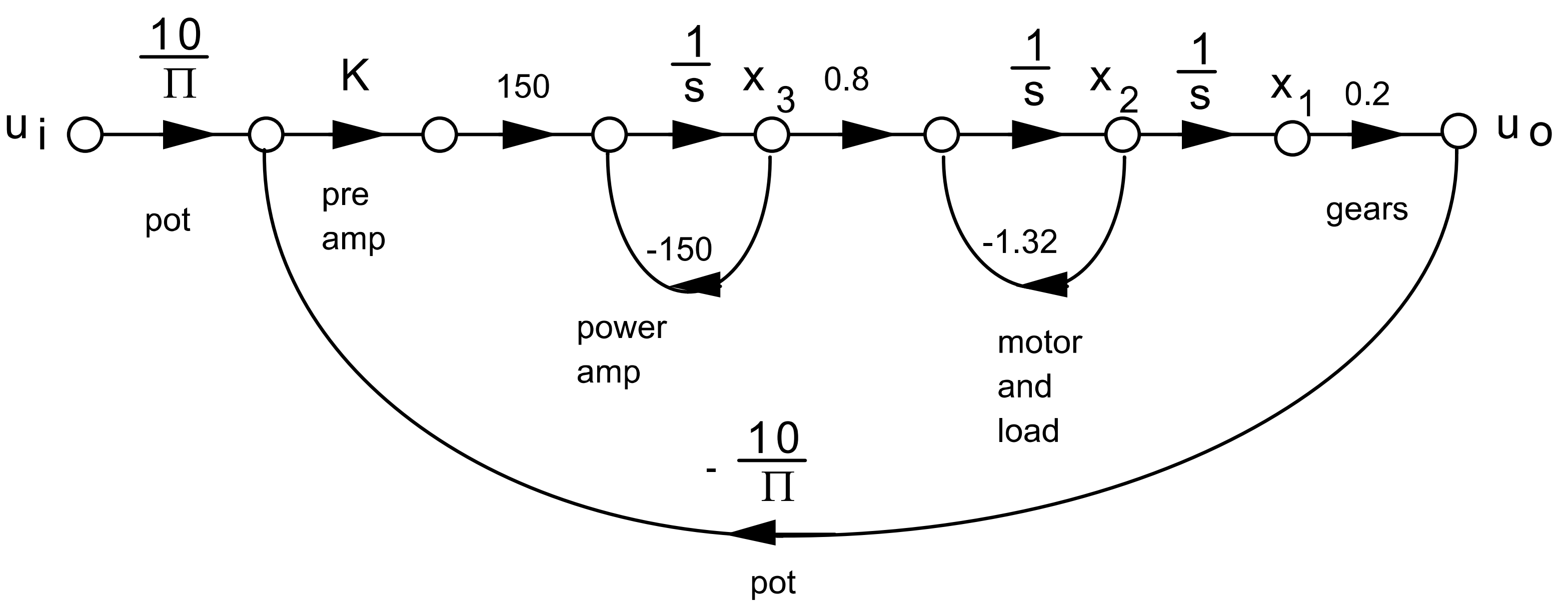
$\Delta = 1 - \Sigma$ loop gains $+ \Sigma$ nontouching-loop gains taken two at a time $- \Sigma$ nontouching-loop gains taken three at a time $+ \Sigma$ nontouching-loop gains taken four at a time $- \ldots$

$\Delta_k = \Delta - \Sigma$ loop gain terms in Δ that touch the kth forward path. In other words, Δ_k is formed by eliminating from Δ those loop gains that touch the kth forward path.

Notice the alternating signs for the components of Δ

A Servo Motor Problem





c. $T_1 = \left(\frac{10}{\pi}\right)(K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2) = \frac{76.39}{s^3}$

$G_{L1} = \frac{-150}{s}$; $G_{L2} = \frac{-1.32}{s}$; $G_{L3} = (K)(150)\left(\frac{1}{s}\right)(0.8)\left(\frac{1}{s}\right)\left(\frac{1}{s}\right)(0.2)\left(\frac{-10}{\pi}\right) = \frac{-76.39K}{s^3}$

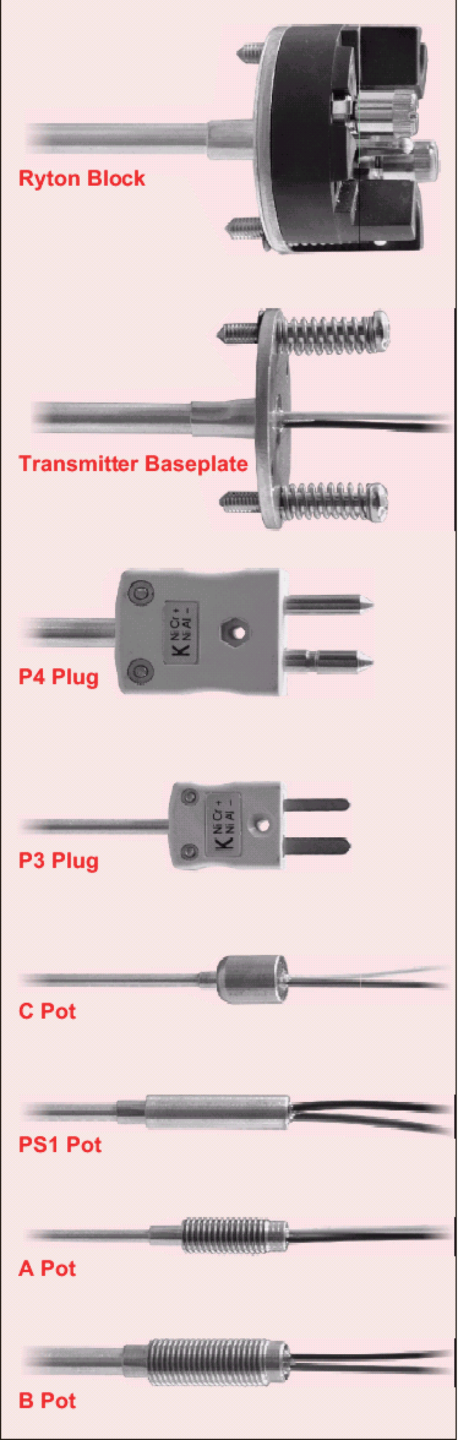
Nontouching loops:

$$G_{L1}G_{L2} = \frac{198}{s^2}$$

$$\Delta = 1 - [G_{L1} + G_{L2} + G_{L3}] + [G_{L1}G_{L2}] = 1 + \frac{150}{s} + \frac{1.32}{s} + \frac{76.39K}{s^3} + \frac{198}{s^2}$$

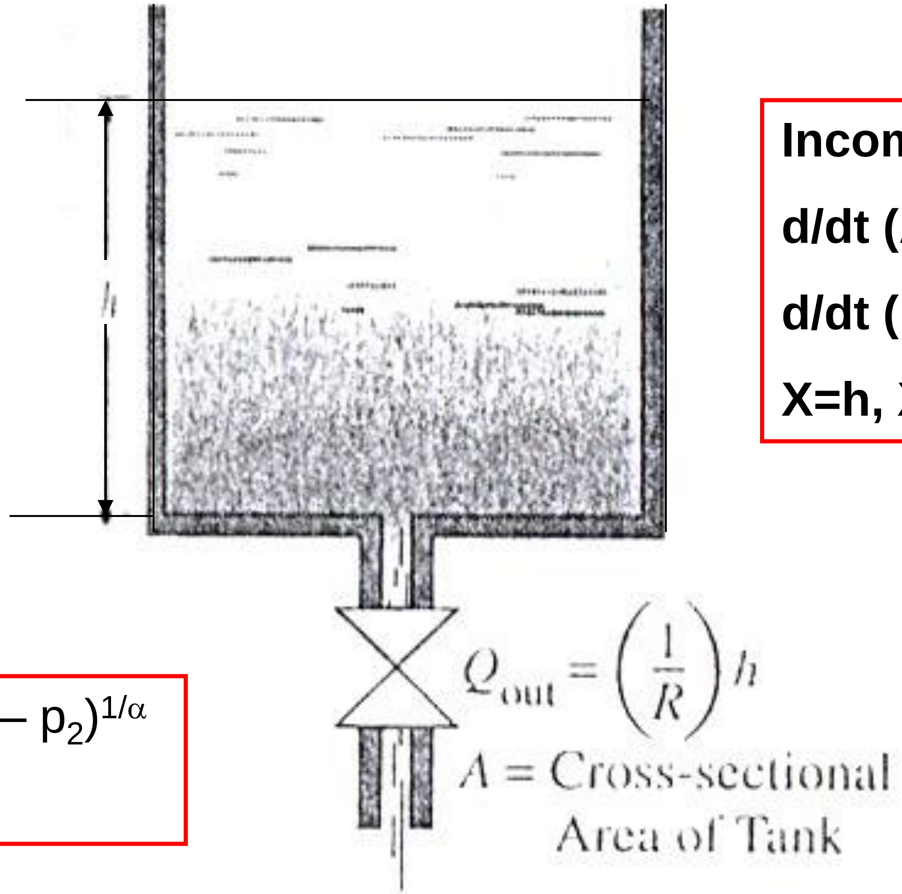
$$\Delta_1 = 1$$

$$T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{76.39K}{s^3+151.32s^2+198s+76.39K}$$



Dynamic Response of First Order Systems

Leaking Tank: A First Order System



Incompressible Fluid

$d/dt\,(A\,h(t)) = -Q_{out} = -(1/R)h(t)$

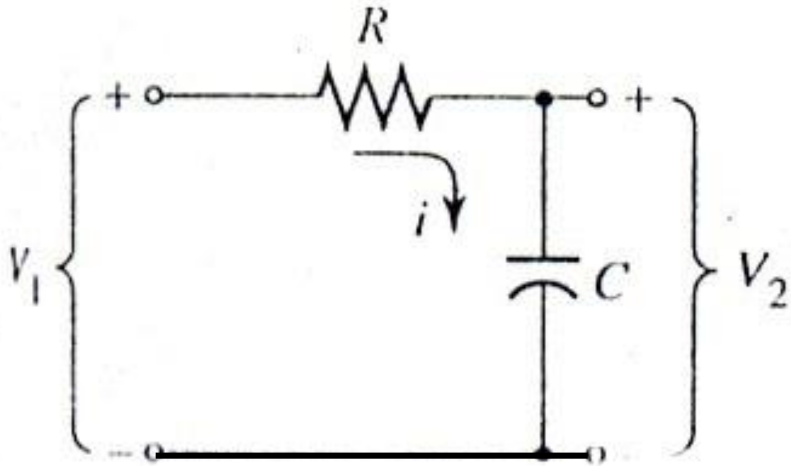
$d/dt\,(h) = (-1/AR)\,h$

$X=h,\ X_o = h_o,\ \ d/dt\,(X) = A\,X$

$MFR = 1/R(p_1 - p_2)^{1/\alpha}$

$\alpha = 1$ for Re <1000

A Low-Pass RC Filter

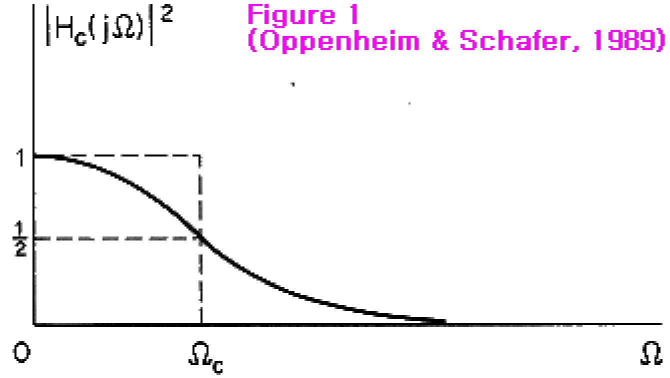


Low-pass *RC* Filter

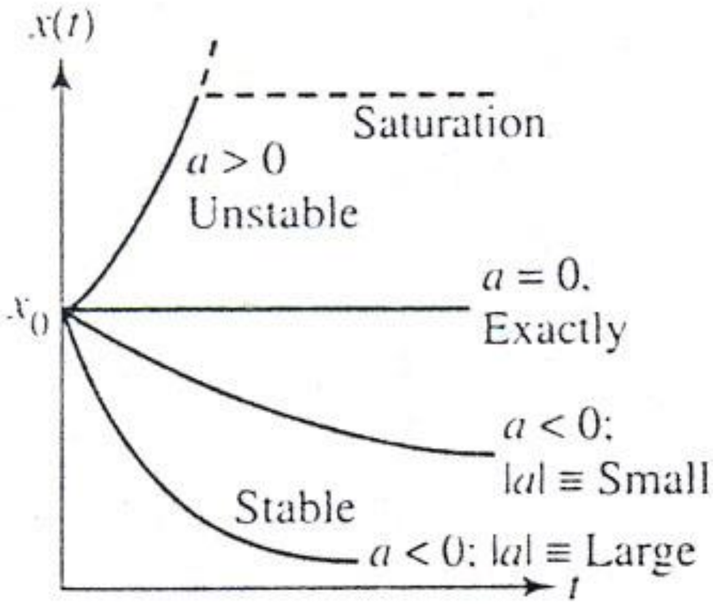
$\frac{d}{dt}(V_2) = 1/RC\,(V_1 - V_2)$

$V_1 = 0$

$\frac{d}{dt}(V_2) = (-1/RC)\,V_2$



Free-Response of a First Order System



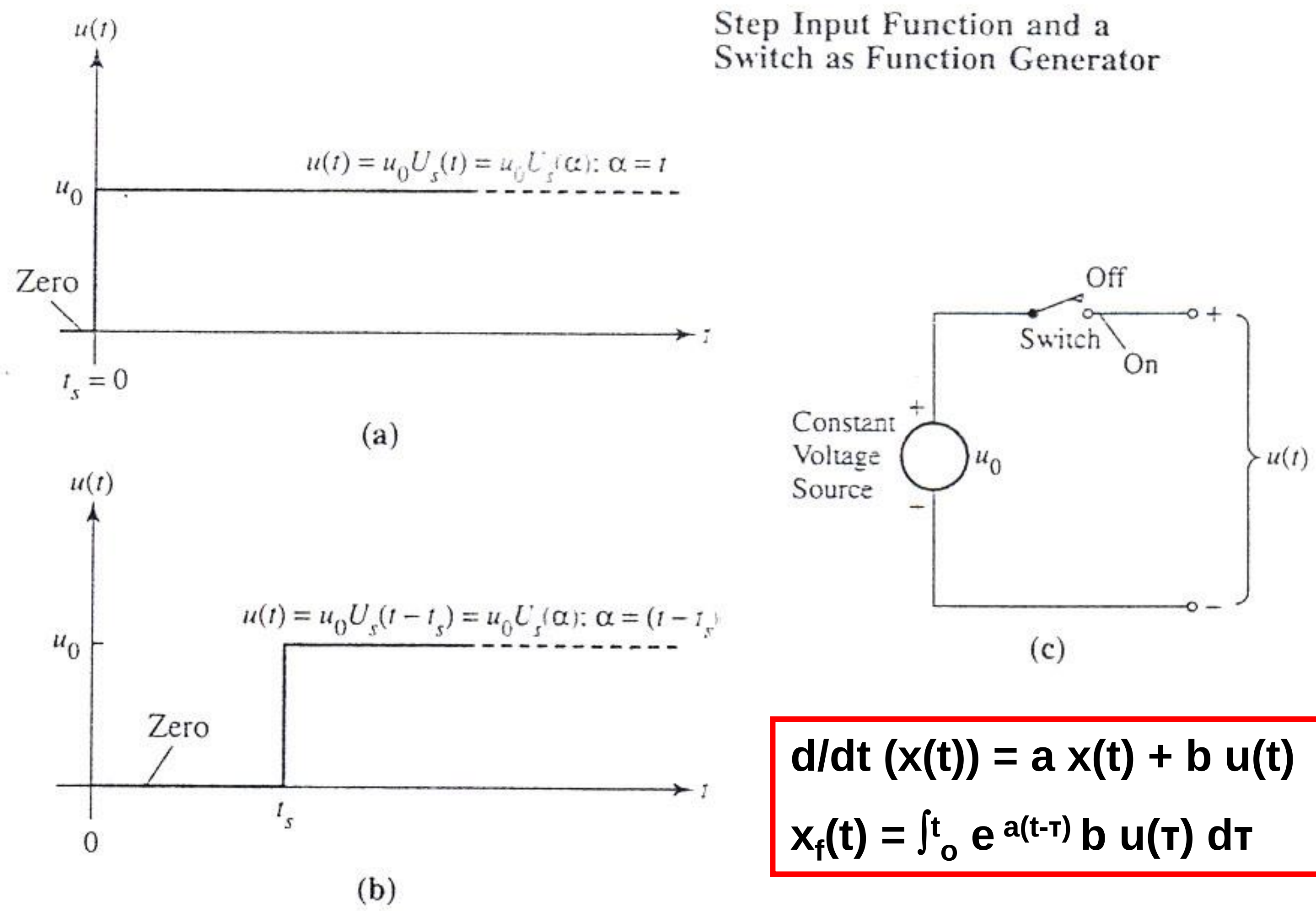
Graph of e^{at} for Ranges of a

$x(t) = e^{at} x_o$

a= 0, Open circuit condition

T= - 1/a time constant, time taken to reach 1/e of the initial value

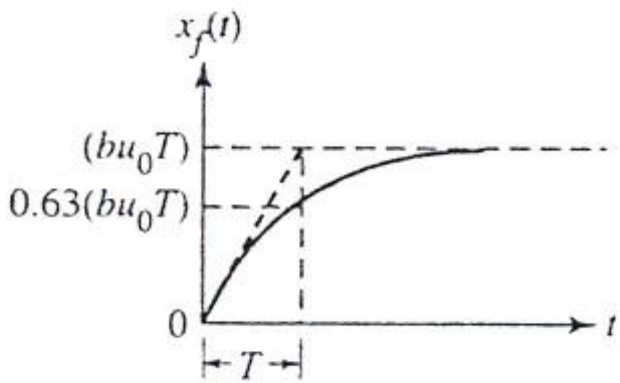
Forced Excitation (Unit Step)



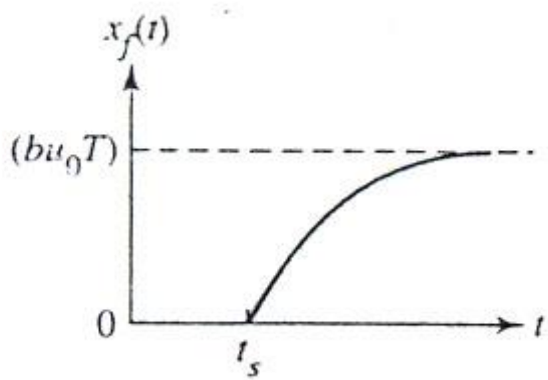
$d/dt\,(x(t)) = a\,x(t) + b\,u(t)$

$x_f(t) = \int_o^t e^{a(t-\tau)}\, b\, u(\tau)\, d\tau$

Forced Response (Unit Step)



(a) $u(t) = u_0 U_s(t)$

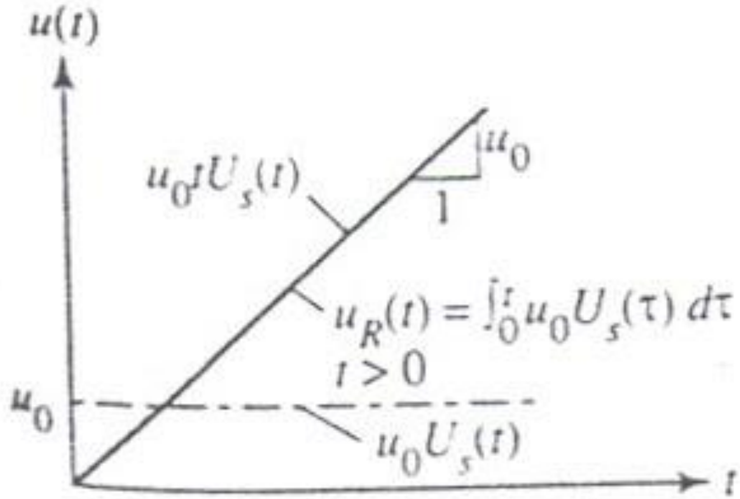


(b) $u(t) = u_0 U_s(t - t_s)$

Sketch of a Forced Step Response

$$x_f(t) = (bu_0T)[1-e^{-t/T}]\ U_s(t)$$

Forced Excitation (Ramp Input)

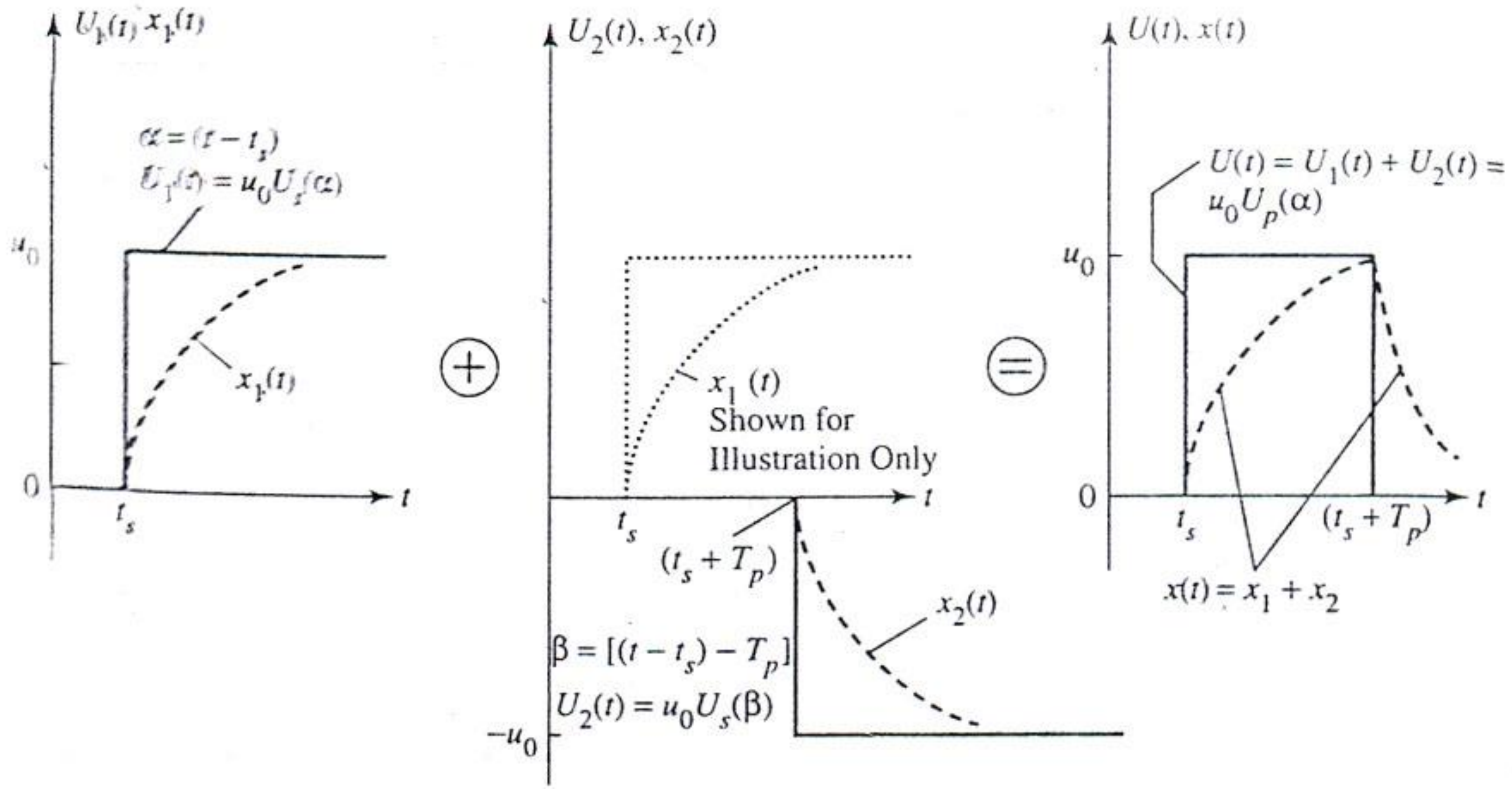


Ramp Input Function

$$\boxed{x_f(t) = bu_oT^2(e^{-t/T} + t/T - 1)}$$

Forced Response (Pulse Input)?

Pulse Response as Sum of
Step Responses



(a) (b) (c)

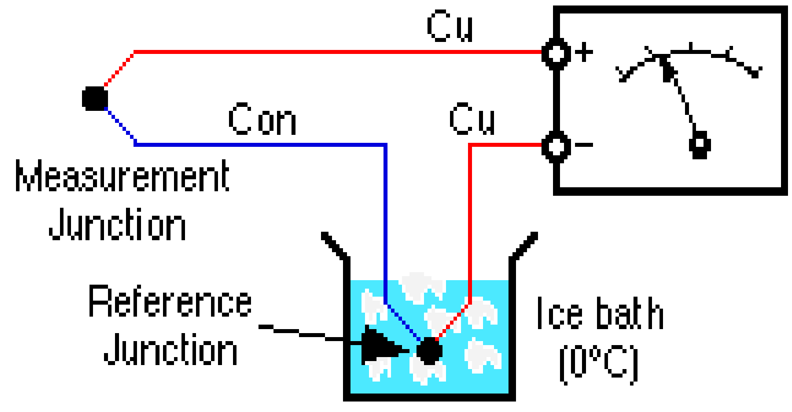
First Order Systems

A first order system has a differential equation of the form

$$\tau \frac{dy}{dt} + y = kr$$

$$\mathbf{Y(s) = G(s)\ R(s) = k} \left(\frac{1/\tau}{s + 1/\tau} \right)$$

$$y = k\,(1 - e^{-t/\tau})$$



Traditional Thermocouple Measurement

Example:

A thermocouple which has a transfer function linking its voltage output *V* and temperature input of T as

$$\mathrm{G}(\boldsymbol{s}) = \frac{30 \times 10^{-6}}{10s + 1} V/{}^{0}C$$

Determine the response of the system when it is suddenly immersed in a water bath at 100º C

The output as an ‘***s***’ function is

$V(\boldsymbol{s})$ = G($\boldsymbol{s}$)   input ($\boldsymbol{s}$)

The output as an '*s'* function is

$$V(s) = G(s) * \text{input}(s)$$

The sudden immersion of the thermometer gives a step input of size 100º C

and so the input as an ***s*** function as 100/***s.*** Thus

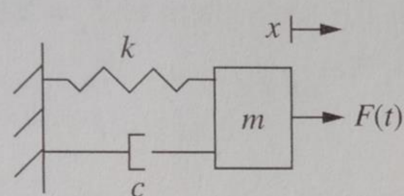
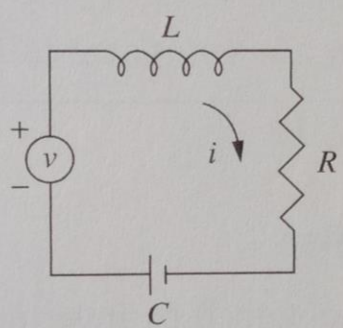
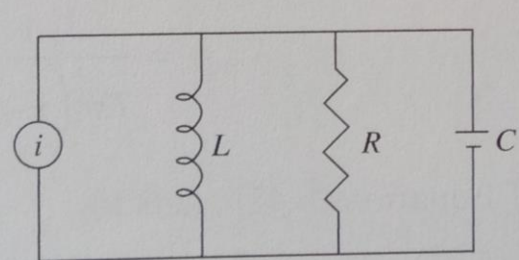
$$\mathbf{V} = \frac{30\times 10^{-6}}{10s+1}\times\frac{100}{s} \qquad \frac{30\times 10^{-4}\times 0.1}{s(s+0.1)}$$

The fraction element of the form a/s(s+a) and so the output as a function of time is :

$$V = 30 \times 10^{-4}\left(1 - e^{-0.1t}\right)$$

Various Basic Second Order Systems

TABLE 6.4 SECOND-ORDER SYSTEMS

Name	Schematic	Differential Equation	Natural Frequency	Damped Ratio
Mass-spring-viscous damper		$m\ddot{x} + c\dot{x} + kx = F(t)$	$\sqrt{\frac{k}{m}}$	$\frac{c}{2\sqrt{mk}}$
Series LRC circuit		$L\frac{di}{dt} + Ri + \frac{1}{C}\int_0^t idt = v(t)$	$\sqrt{\frac{1}{LC}}$	$\frac{R}{2}\sqrt{\frac{C}{L}}$
Parallel LRC circuit		$C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int_0^t vdt = i(t)$	$\frac{1}{\sqrt{LC}}$	$\frac{1}{2R}\sqrt{\frac{L}{C}}$

Standard Forms of a 2nd order system

In terms of the time constant T $$\frac{\theta_o}{\theta i}(s) = \frac{k}{T^2 s^2 + 2T\delta s + 1}$$

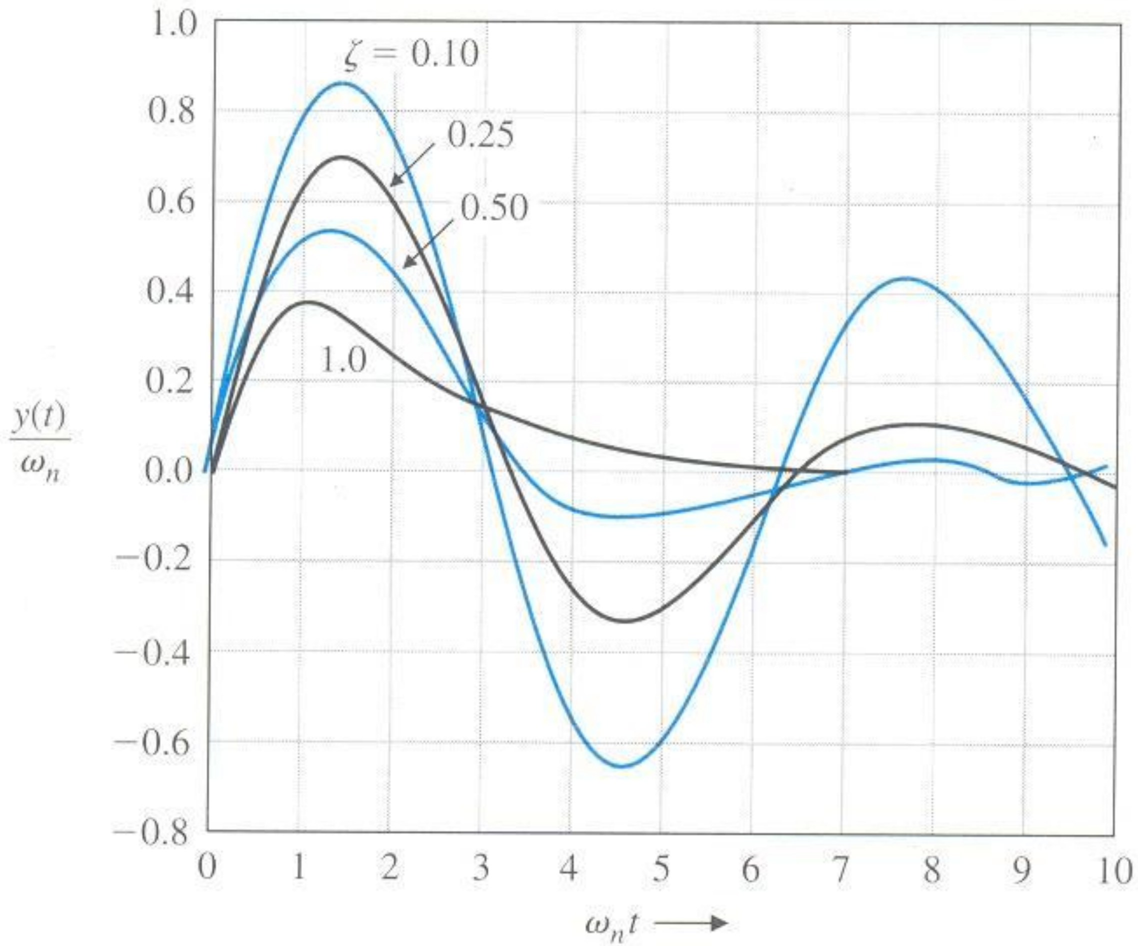
In terms of the natural frequency ω_n $$\frac{\theta_o}{\theta i}(s) = \frac{\omega_n^2 k}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

In terms of the poles (polynomial) $$\frac{\theta_o}{\theta i}(s) = \frac{\omega_n^2 k}{(s - p_1)(s - p_2)}$$

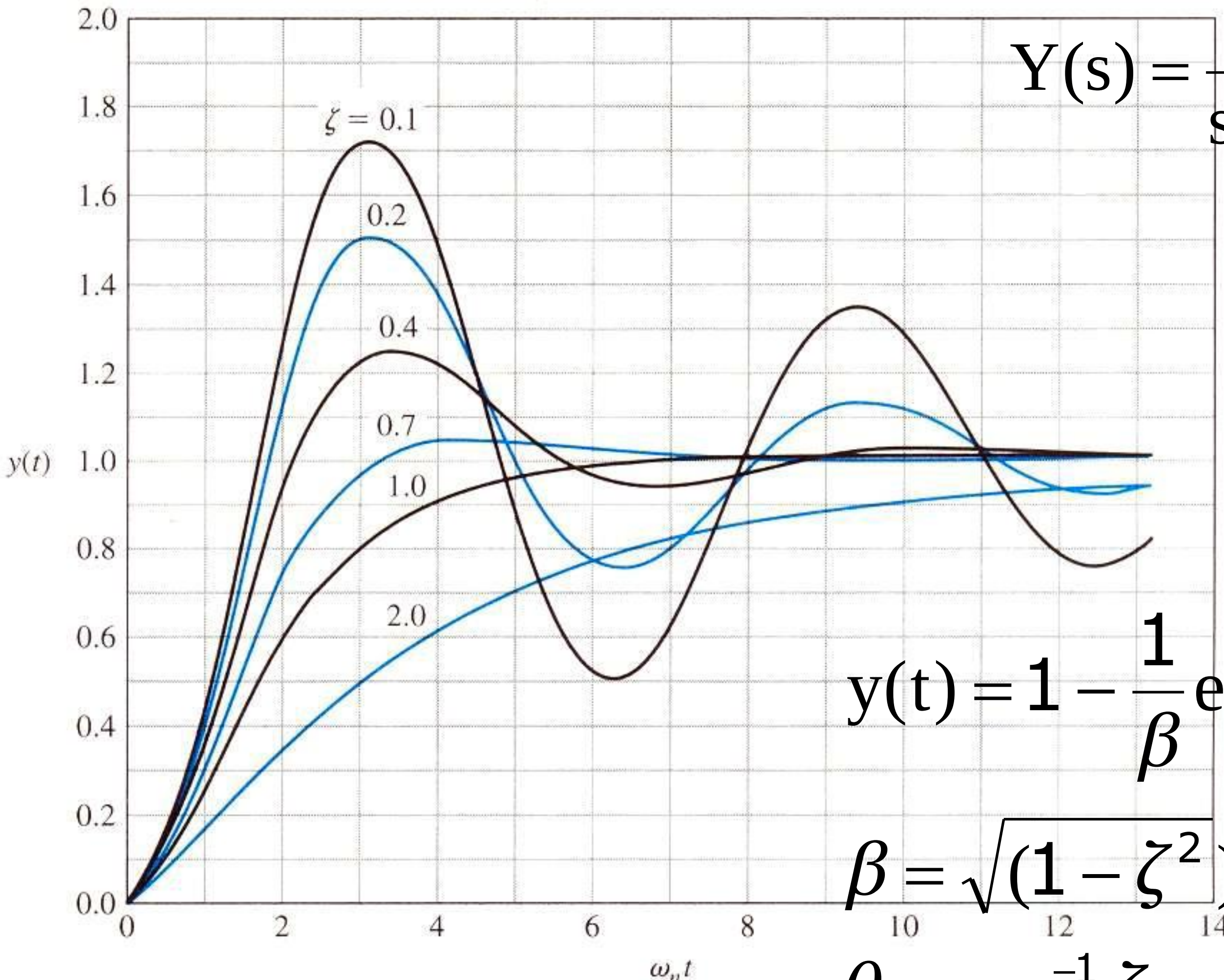
In terms of two parameters σ and ω_r $$\frac{\theta_o}{\theta i}(s) = \frac{\omega_n^2 k}{(s+\sigma)^2 + \omega_r^2} = \frac{\omega_r^2 + \sigma\omega_r^2}{(s+\sigma)^2 + \omega_r^2}$$

Remember that δ is the damping ratio (**ζ in other texts)**

Unit Impulse Response



Transient response of a second order system



$$Y(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} R(s)$$

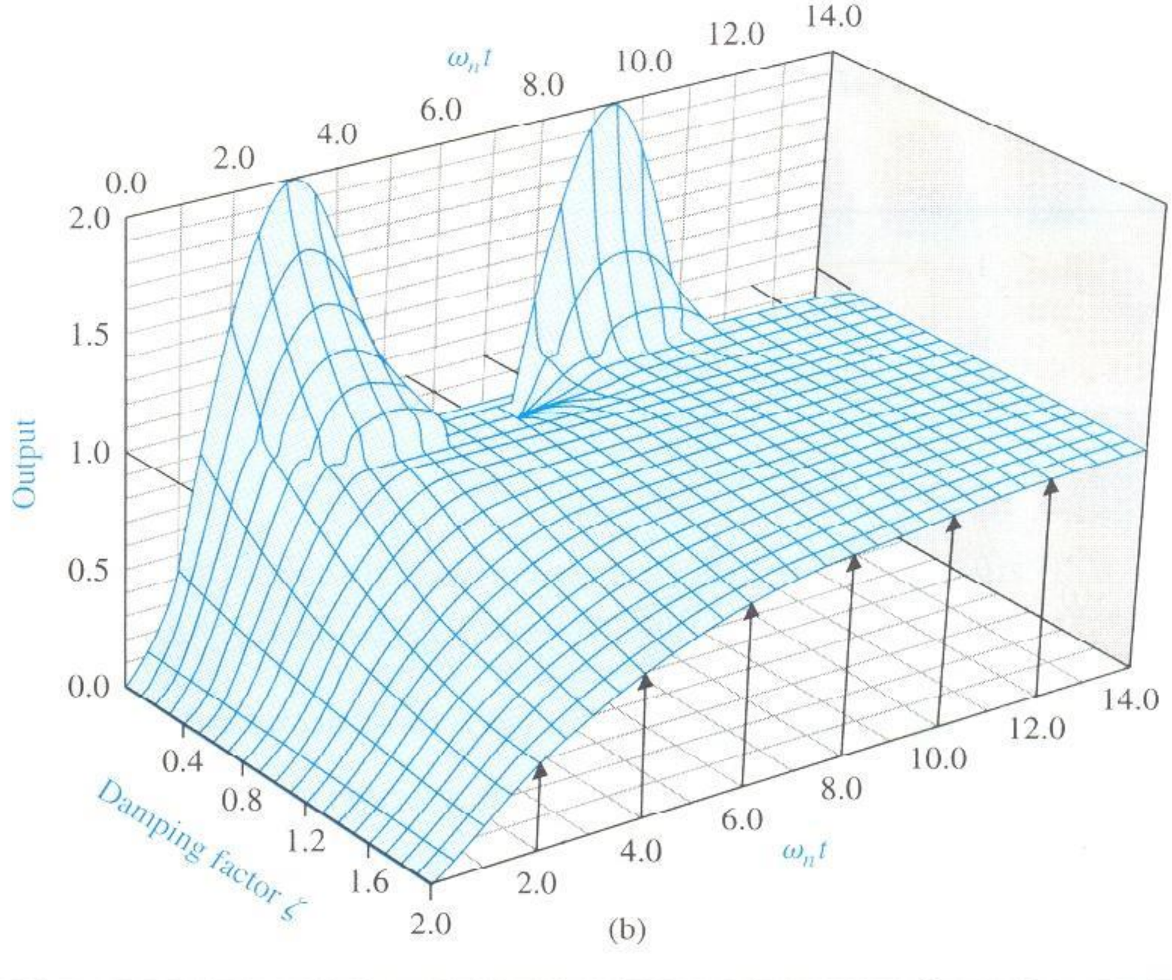
$$y(t) = 1 - \frac{1}{\beta} e^{-\zeta\omega_n t} \sin(\omega_n \beta t + \theta)$$

$$\beta = \sqrt{(1-\zeta^2)}$$

$$\theta = \cos^{-1}\zeta$$

$$\zeta - 0 \text{to} 1$$

A 3-D representation of unit step response



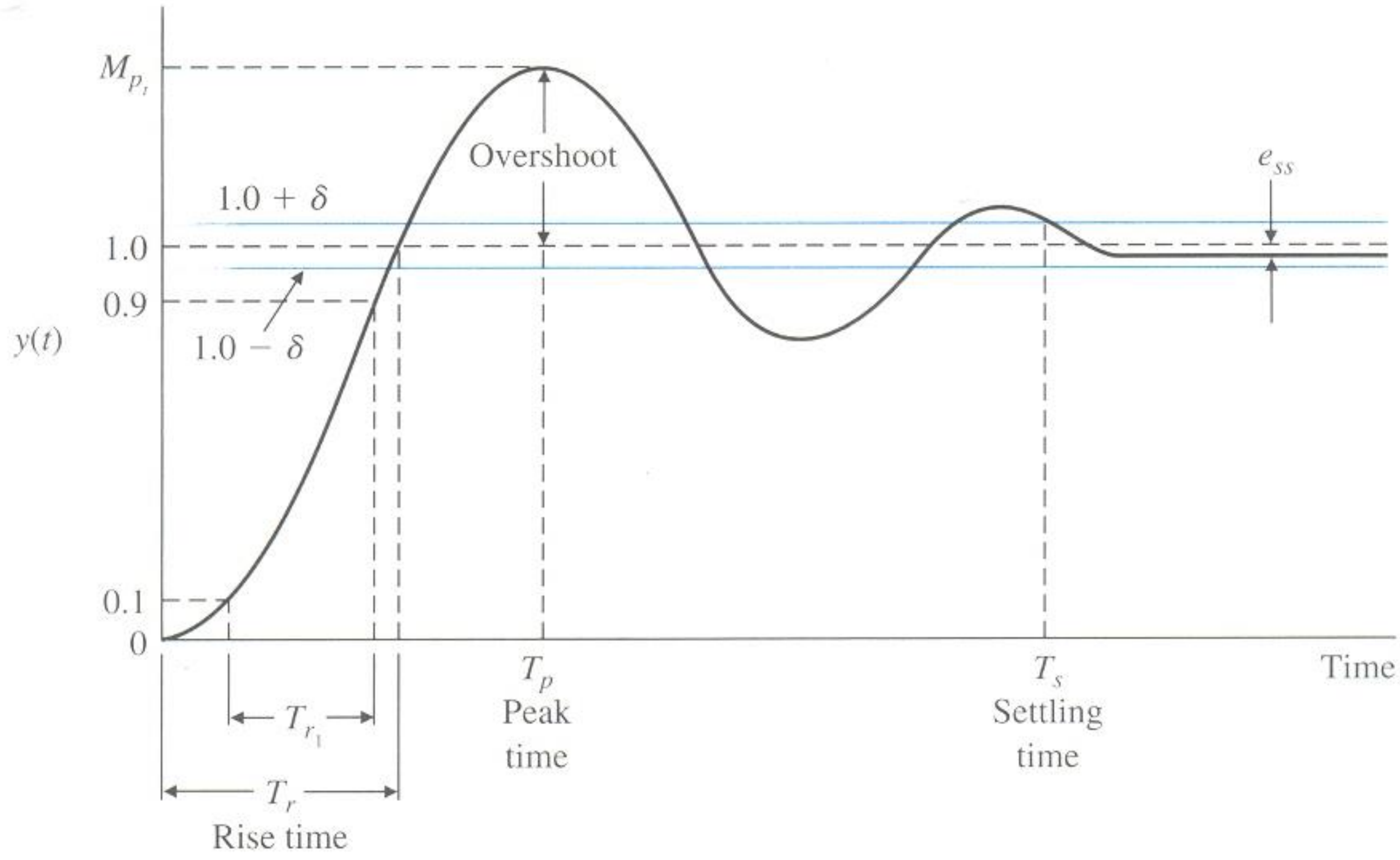
(b)

Unit step response

$$PO = \frac{M_p - fv}{fv} x100\%$$

$$e^{-\zeta\omega_n T_s} < 0.02$$

$$T_s = \frac{4}{\zeta\omega_n}$$

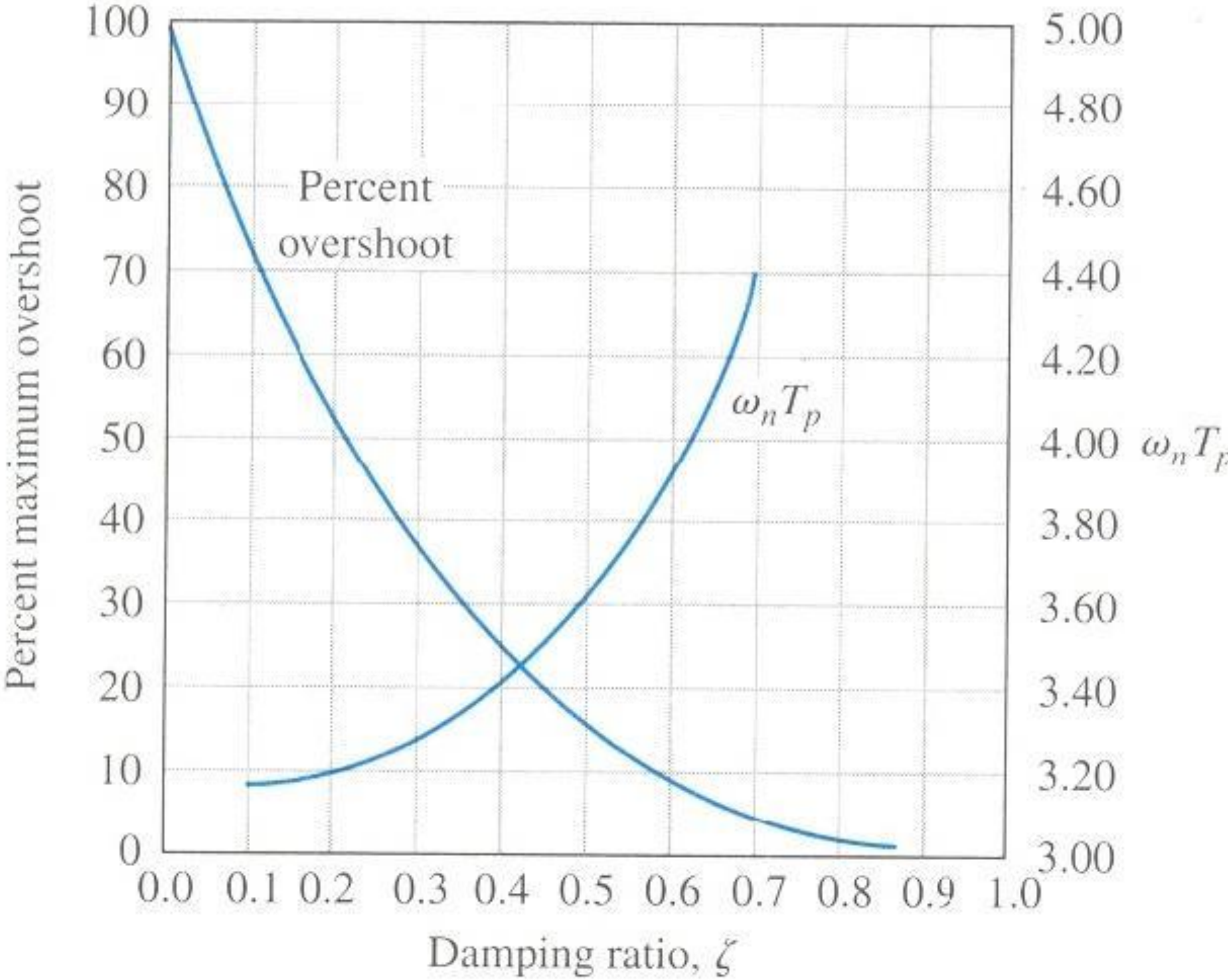


PO: A Measure of Closeness of response

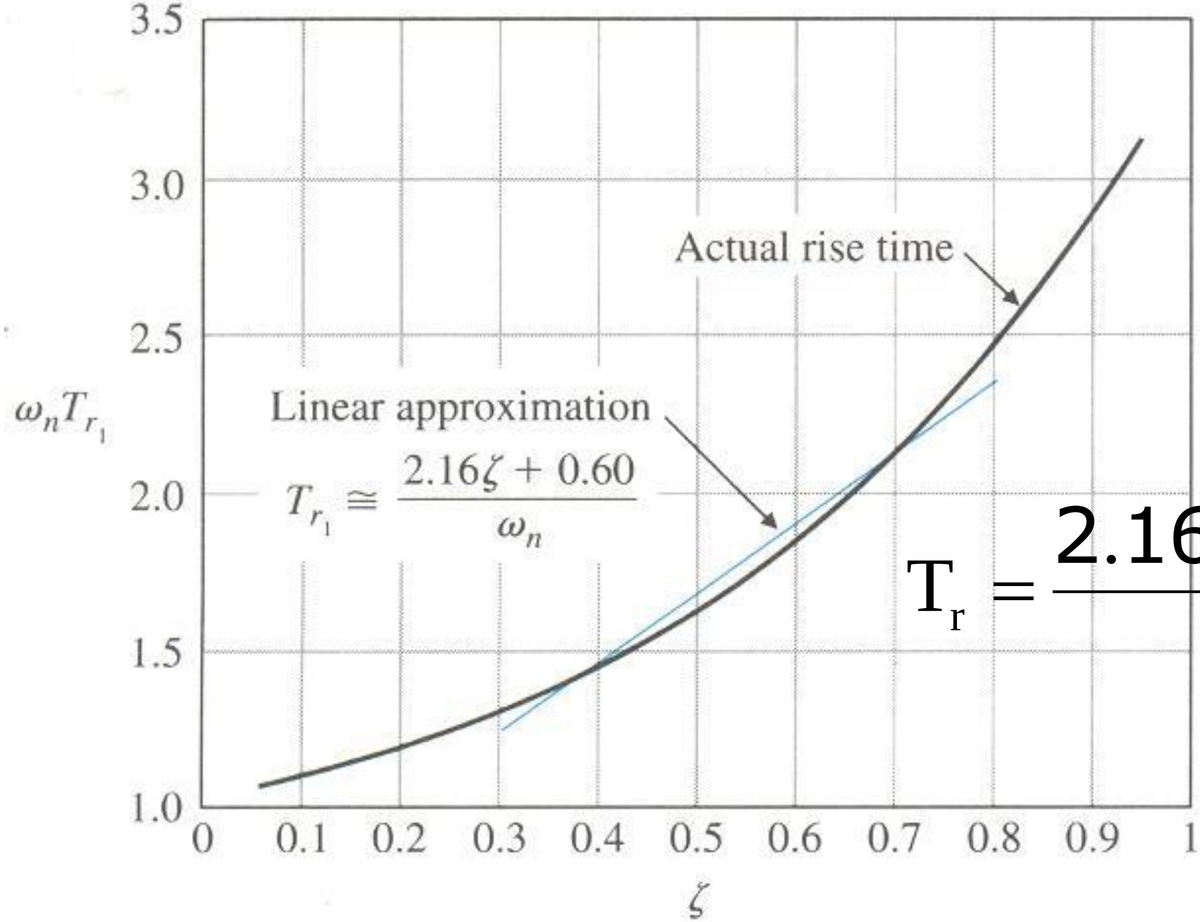
$$PO = 100e^{-\zeta\pi/(\sqrt{(1-\zeta^2)})}$$

Peak Time: A Measure of swiftness

$$T_p = \frac{\pi}{\omega_n\sqrt{(1-\zeta^2)}}$$



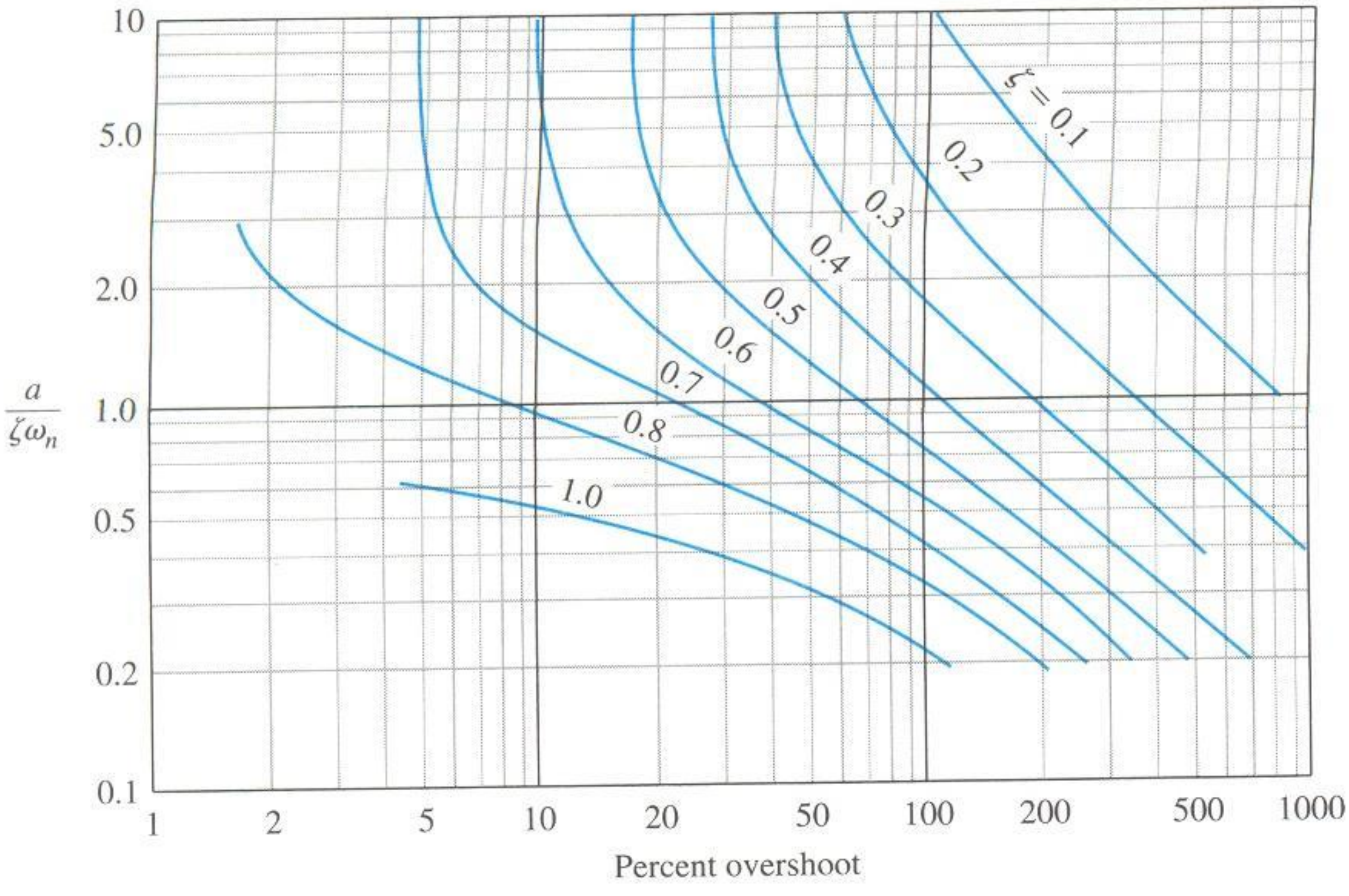
Normalised Rise-time



$$T_r = \frac{2.16\zeta + 0.60}{\omega_n}$$

Effect of additional zero

$$T(s) = \frac{(\omega_n^2/a)(s+a)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



System with Additional Pole and Zero

$$T(s) = \frac{(\omega_n^2 / a)(s + a)}{(s^2 + 2\zeta\omega_n s + \omega_n^2)(1 + \tau s)}$$

Find out the effect of 'a' and 'T' on the system response corresponding to a step input

THANK YOU