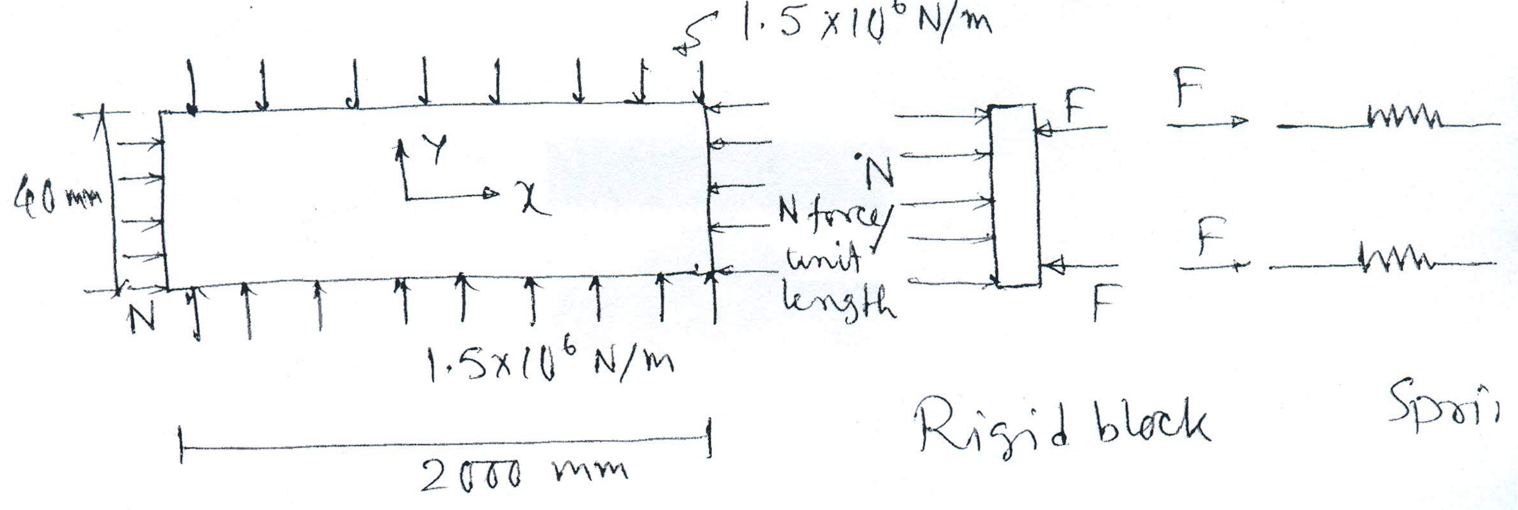
Solution of Problem # 3

FBDs of different parts



Plate

Equilibrium of rigid block

$$2F = N \times (40\times10^{-3}) = N \times 4\times10^{-2}$$

$$\therefore\ F = (2N\times10^{-2}) \text{ Newton} \quad \cdots (1)$$

Geometric compatibility

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$$\Delta L - \delta = 1\times10^{-3}$$

Where ΔL = Plate extension in x-direction

δ = spring compression

Now for plate: $\sigma_x = -\dfrac{N}{t}$ [$\because$ N is force/unit length & t = Plate thickness]

$$= -\frac{N}{2.5\times10^{-3}}$$

5

$$= -4N\times10^{2} \quad N/m^2$$

$$\sigma_y = -\frac{1.5\times10^6}{t} = -\frac{1.5\times10^6}{2.5\times10^{-3}} = -6\times10^{8}\ N/m$$

$$\therefore \epsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right]$$

$$= \frac{1}{2\times10^{11}}\left[-4N\times10^{2} + 0.33(6\times10^{8})\right]$$

$$= \frac{1}{10^9}\left[-2N + (0.99\times10^{6})\right]$$

$$= \left[-2N + (0.99\times10^{6})\right]\times10^{-9} \text{ — } 4$$

Now the plate extension in x-directio

$$\Delta L = (\epsilon_x \cdot L) =$$

$$= \left[-2N + (0.99\times10^{6})\right]\times10^{-9}\times(2000\times10^{-3})$$

$$= \left[-4N + 1.98\times10^{6}\right]\times10^{-9} \text{ m} \quad \cdots (2)$$

Spring compression

$$\delta = \frac{F}{k} = \frac{F}{5\times10^{6}} = 2F\times10^{-7} \quad \cdots (3)$$

Now we know,

$$\Delta L - \delta = 1\times10^{-3} \quad \cdots (4)$$

Substitute (2) & (3) in (4)

$$(-4N + 1.98\times10^{6})\times10^{-9} - 2F\times10^{-7} = 10^{-3}$$

$$\text{or} \quad -4N\times10^{-6} - 2F\times10^{-4} = -0.98 \quad \cdots (5)$$

$$\therefore \epsilon_x = \frac{1}{E}\left[\sigma_x - \nu \sigma_y\right]$$

$$= \frac{1}{2\times 10^{11}}\left[-4N\times 10^{2} + 0.33\,(6\times 10^{8})\right]$$

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Solving eqs. (1) & (5), we get

$$-4N \times 10^{-6} - 2(2N \times 10^{-2}) \times 10^{-4} = -0.98$$

or $-4N - 4N = -0.98 \times 10^{6}$

or $N = 0.1225 \times 10^{6}$

$$\boxed{N = 1.225 \times 10^{5} \text{ N}} \quad \text{—— } 2$$

Therefore $F = 2N \times 10^{-2}$ N

$$\boxed{F = 2.45 \times 10^{3} \text{ N}} \quad \text{—— } 4$$

1 No units/wrong units ⟹ zero marks for that part

2 $\epsilon_x = \frac{1}{E}(\sigma_x - \nu(\sigma_y + \sigma_z))$

If proceeded with $\sigma_x = 0$ then <u>no marks</u> since assumed $\sigma_x = 0 \Rightarrow F_x = 0 \Longrightarrow$ no force in springs

e.g. $\sigma_x = 0$, $\epsilon_x = \cdots$ then $\Delta x = L\epsilon_x = 1.98$ mm

$\delta_s = 1.98 - 1$ mm $= 0.98$ mm, $F = k\delta_s$

This is wrong since assumption: no force in x-direction.