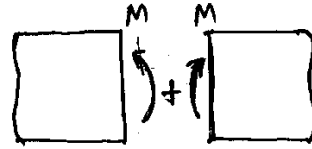
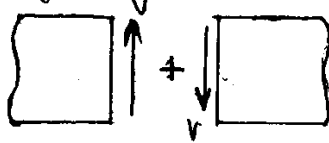


①

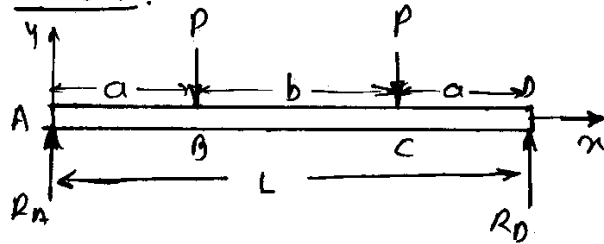
# Solutions to Homework & Practice Problems of CHAPTER 3

## Solution to problem 3.5

Sign convention :



F.B.D.:



$$\sum F_y = 0 \Rightarrow$$

$$R_A - P - P + R_D = 0$$

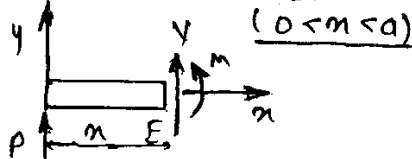
$$\sum M_A = 0 \Rightarrow$$

$$-P \cdot a - P(a+b) + R_D \cdot L = 0.$$

Above equations give

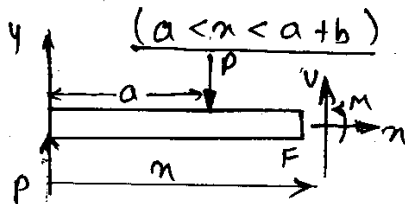
$$R_A = P, \quad R_D = P.$$

Expressions For V & M :



$$\sum F_y = 0 \Rightarrow V = -P$$

$$\sum M_E = 0 \Rightarrow M = P \cdot x.$$

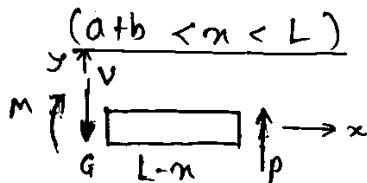


$$\sum F_y = 0 \Rightarrow V + P - P = 0$$

$$\Rightarrow V = 0$$

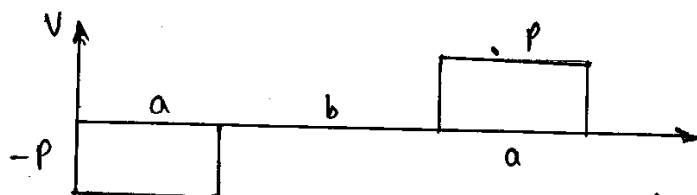
$$\sum M_F = 0 \Rightarrow M = -P(a+x) + P \cdot x$$

$$M = P \cdot a.$$

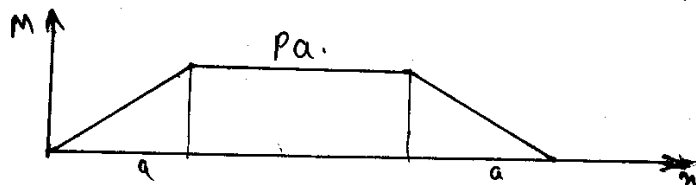


$$\sum F_y = 0 \Rightarrow V = +P$$

$$\sum M_G = 0 \Rightarrow M = P(L-x).$$



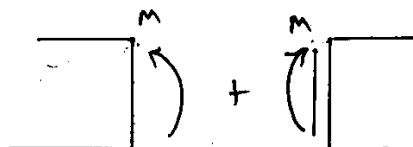
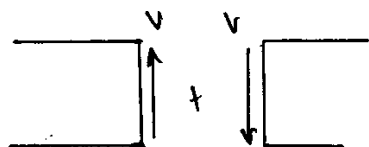
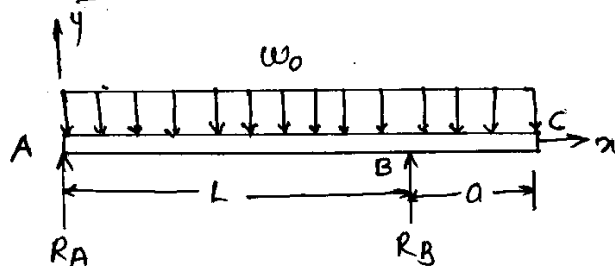
shear force diagram



Bending Moment diagram.

Solution to problem 3.7

Sign convention:

F.B.D :

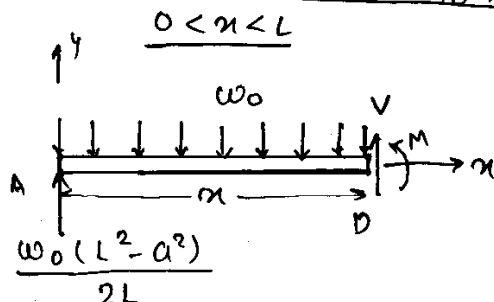
$$\sum F_y = 0 \Rightarrow$$

$$R_A + R_B - w_0(L+a) = 0$$

$$\sum M_A = 0 \Rightarrow$$

$$-\frac{1}{2}w_0(L+a)^2 + R_B L$$

$$\therefore R_A = \frac{w_0(L^2 - a^2)}{2L}, \quad R_B = \frac{w_0(L+a)^2}{2L}$$

Expressions for V and M:

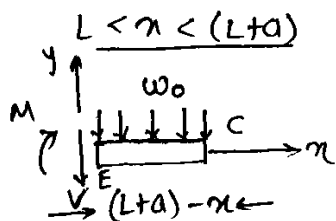
$$\sum F_y = 0 \Rightarrow$$

$$V + \frac{w_0(L^2 - a^2)}{2L} - w_0 x = 0$$

$$\Rightarrow V = -\frac{w_0(L^2 - a^2)}{2L} + w_0 x$$

$$\sum M_D = 0 \Rightarrow M - \frac{w_0(L^2 - a^2)}{2L} \cdot x + \frac{1}{2}w_0 x^2 = 0$$

$$\Rightarrow M = \frac{w_0(L^2 - a^2)}{2L} \cdot x - \frac{1}{2}w_0 x^2$$

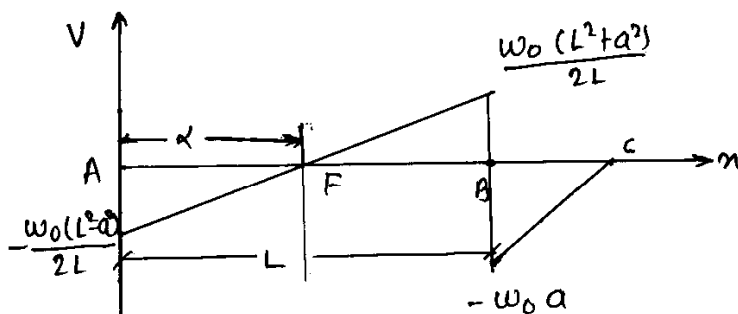


$$\sum F_y = 0 \Rightarrow V + w_0[(L+a) - x] = 0$$

$$\Rightarrow V = -w_0[(L+a) - x]$$

$$\sum M_E = 0 \Rightarrow M = -\frac{1}{2}w_0[(L+a) - x]^2$$

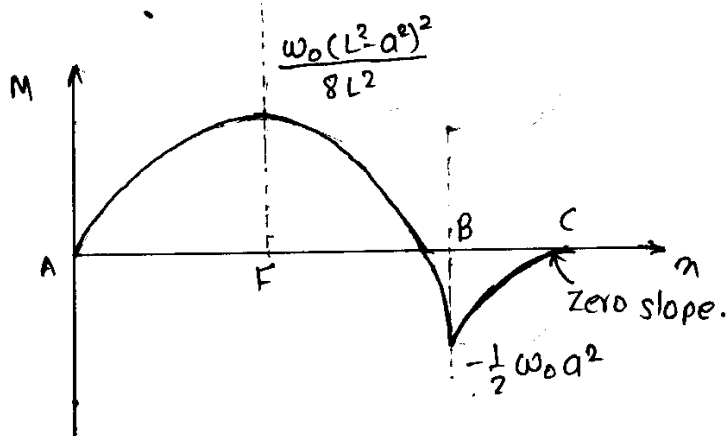
(Problem 3.7 contd.).



Similarity of triangles

$$\frac{\alpha}{L-\alpha} = \frac{\frac{\omega_0(L^2-a^2)}{2L}}{\frac{\omega_0(L^2+a^2)}{2L}} \Rightarrow$$

$$\alpha = \frac{L^2-a^2}{2L}$$

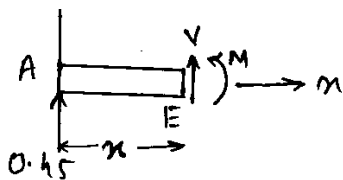


$M_F$  = area of V-diagram upto F

$$= \frac{1}{2} \alpha \left[ \omega_0 \frac{(L^2-a^2)}{2L} \right]$$

$$= \frac{\omega_0(L^2-a^2)}{8L^2}$$

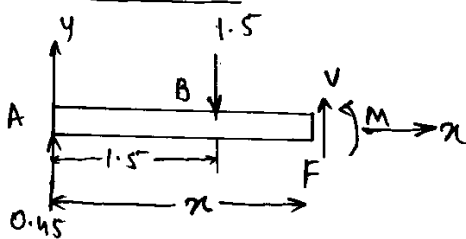
④



$$\sum F_y = 0 \Rightarrow V = -0.45$$

$$\sum M_E = 0 \Rightarrow M = 0.45x$$

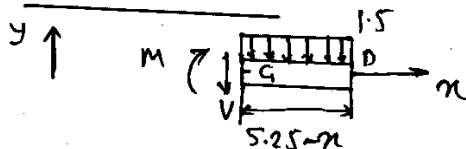
$$1.5 < x < 3.75$$



$$\sum F_y = 0 \Rightarrow V = 1.05 \text{ kN}$$

$$\sum M_F = 0 \Rightarrow M = -1.05x + 2.25$$

$$3.75 < x < 5.25$$

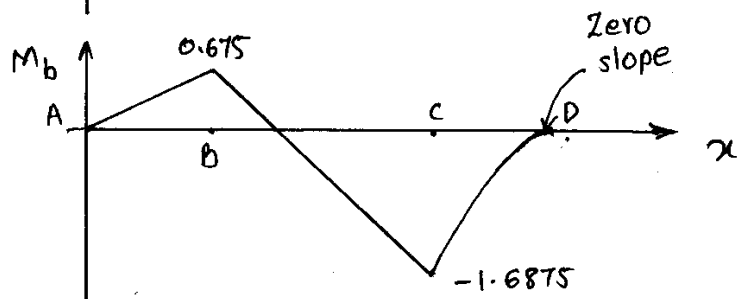
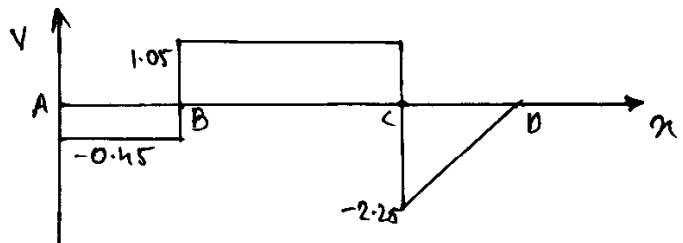


$$\sum F_y = 0 \Rightarrow V = -1.5x + 7.875$$

$$\sum M_G = 0 \Rightarrow M = -\frac{1}{2}(5.25-x)^2 \times 1.5$$

Shear Force and Bending

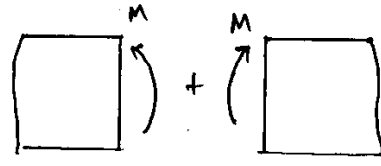
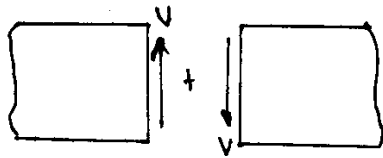
Moment diagrams:



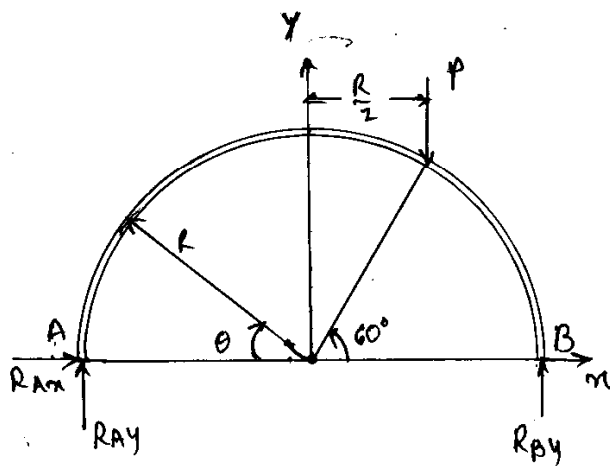
⑤

# Solution to problem 3.11

Sign convention:



F.B.D



No moments at A, B ;  
No horizontal force at B

$$\sum (M_z)_A = 0 \Rightarrow$$

$$R_{By} \cdot (2R) - P \left( \frac{3R}{2} \right) = 0$$

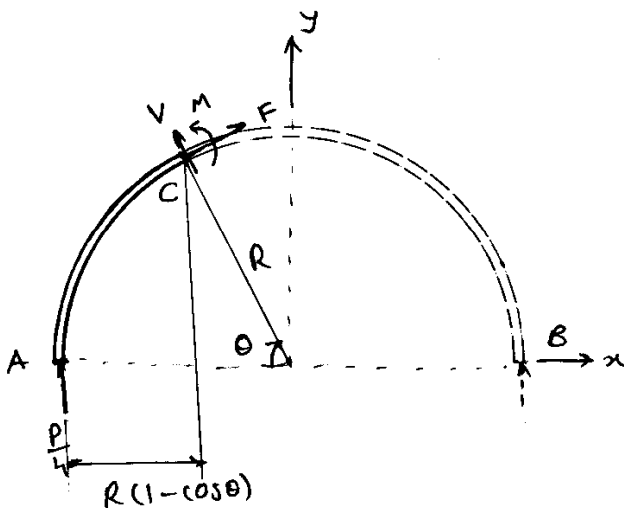
$$\Rightarrow R_{By} = \frac{3P}{4}$$

$$\sum F_x = 0 \Rightarrow R_{Ax} = 0$$

$$\sum F_y = 0 \Rightarrow$$

$$R_{Ay} + R_{By} - P = 0 \Rightarrow R_{Ay} = \frac{P}{4}$$

Free body for  $\theta < 120^\circ$ .



$$\sum (M_z)_C = 0 \Rightarrow$$

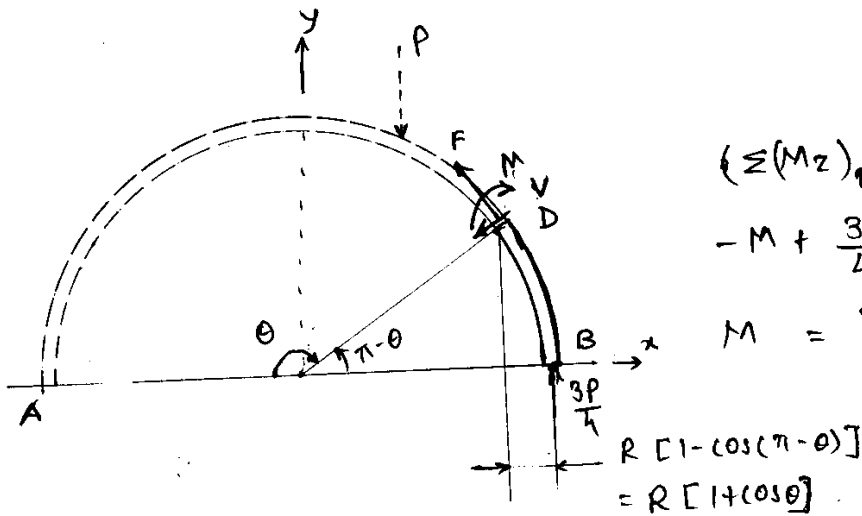
$$M - \frac{P}{4} [R(1 - \cos \theta)] = 0$$

$$\Rightarrow M = \frac{PR}{4} (1 - \cos \theta), \theta < 120^\circ$$

6

(Problem 3.11 contd.)

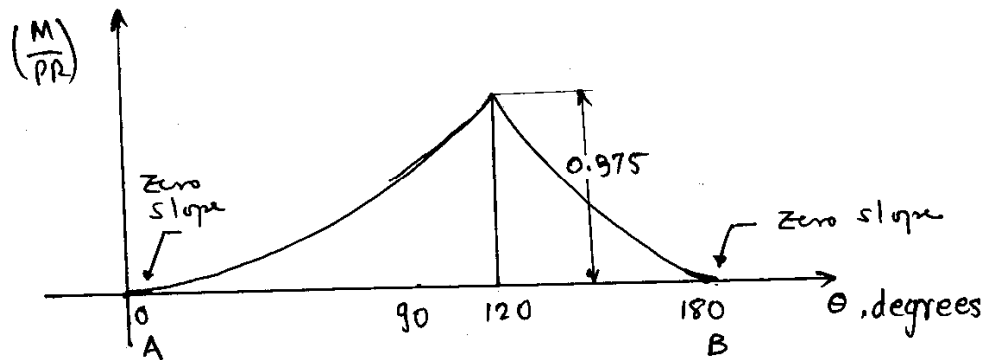
Free body:  $\theta > 120^\circ$ .



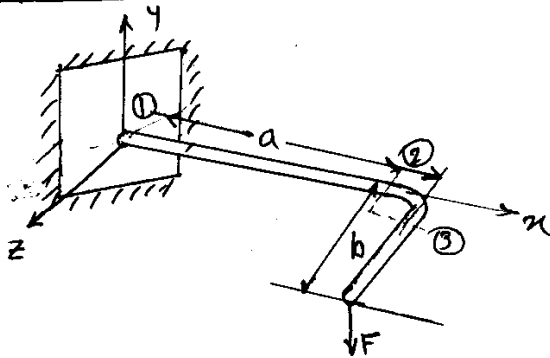
$$(\sum M_z)_D = 0 \Rightarrow$$

$$-M + \frac{3P}{4}R(1 + \cos \theta) = 0.$$

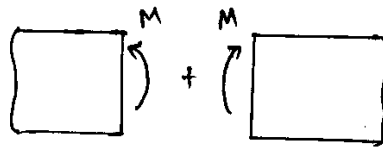
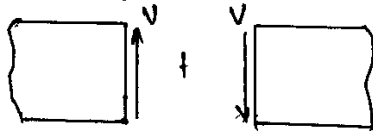
$$M = \frac{3PR}{4}(1 + \cos \theta), \theta > 120^\circ.$$



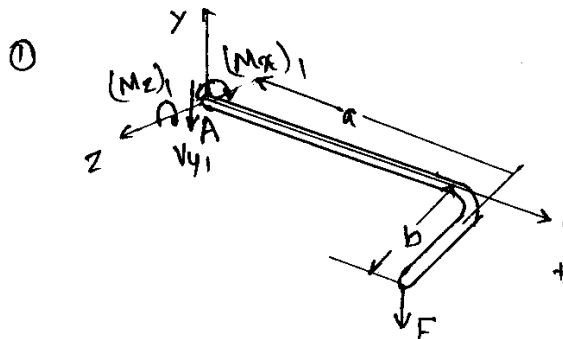
Solution to problem 3.13



Sign Convention :



FBD (Section 1) :



$\sum F_x = 0$ ,  $\sum F_z = 0$ ,  $\sum M_y = 0$   
are satisfied identically.

$$+\uparrow \sum F_y = 0 \Rightarrow$$

$$-V_{y1} - F = 0 \Rightarrow V_{y1} = -F. \quad (1)$$

$$+\circlearrowleft \sum M_{xA} = 0 \Rightarrow$$

$$-(M_x)_1 + Fb = 0 \Rightarrow (M_x)_1 = Fb. \quad (2)$$

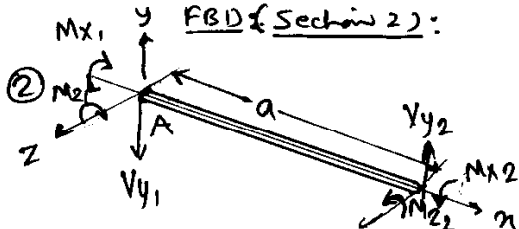
$$+\circlearrowleft \sum M_{zA} = 0 \Rightarrow$$

$$-(M_z)_1 - Fa = 0 \Rightarrow (M_z)_1 = -Fa. \quad (3)$$

note that expressions equations ①, ②, ③ are for finding shearforce ( $V_{y1}$ ), twisting moment ( $(M_x)_1$ ) and Bending moment ( $(M_z)_1$ ).

at section 1.

FBD (Section 2) :



$\sum F_x = 0$ ,  $\sum F_y = 0$ ,  $\sum M_y = 0$  are  
identically satisfied.

$$\text{Shear} \equiv +\uparrow \sum F_y = 0 \Rightarrow -V_{y1} + V_{y2} = 0 \Rightarrow V_{y2} = V_{y1} = -F$$

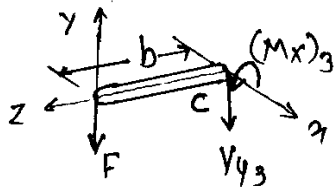
$$\text{Twisting} \equiv +\circlearrowleft \sum M_{xA} = 0 \Rightarrow -(M_x)_1 + (M_x)_2 = 0 \Rightarrow (M_x)_2 = (M_x)_1 = Fb$$

$$\text{Bending} \equiv +\circlearrowleft \sum M_{zA} = 0 \Rightarrow -(M_z)_1 + (V_{y2} \times a) + (M_z)_2 = 0 \Rightarrow$$

$$(M_z)_2 = (M_z)_1 - (V_y)_2 a = -Fa - (-F)a = 0.$$

(Problem 3.13 (contd.)).

FBD: (Section 3):



$\sum F_m = 0$ ,  $\sum F_z = 0$ ,  $\sum M_y = 0$ ,  $\sum M_z = 0$   
are satisfied identically.

Shear:  $+\uparrow \sum F_y = 0 \Rightarrow -F - V_{y3} = 0$

$$\Rightarrow V_{y3} = -F$$

Bending:  $+\circlearrowleft \sum M_{xc} = 0 \Rightarrow$

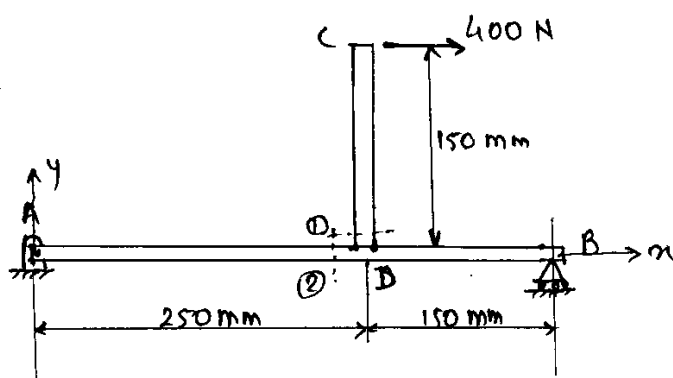
$$F \times b - (M_x)_3 = 0$$

$$\Rightarrow (M_x)_3 = Fb.$$



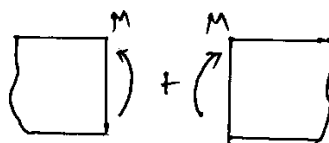
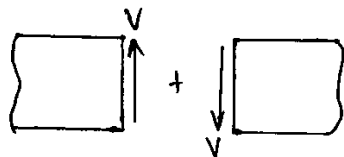
9

# Solution to problem 3.14.

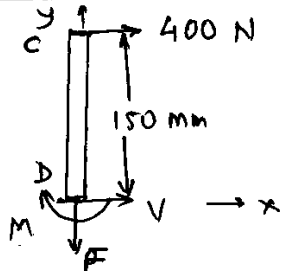


To find:  
internal forces and  
moments at sections ①②

Sign convention:



Section 1:



$$\sum F_y = 0 \Rightarrow F = 0$$

$$\sum F_x = 0 \Rightarrow V + 400 = 0$$

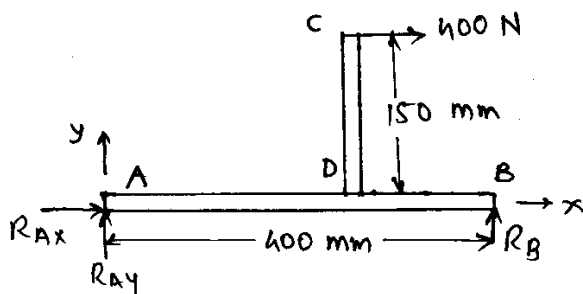
$$\Rightarrow V = -400 \text{ N.}$$

$$\sum M_D = 0 \Rightarrow M + 400 \times 0.15 = 0$$

$$\Rightarrow M = -60 \text{ Nm.}$$

Section 2:

Reactions:



$$\sum F_x = 0 \Rightarrow$$

$$R_{Ax} + 400 = 0$$

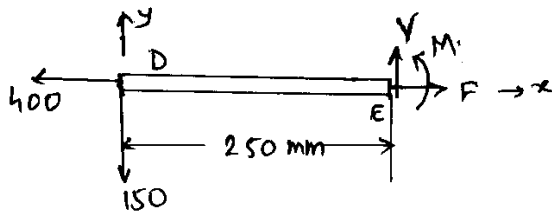
$$\Rightarrow R_{Ax} = -400 \text{ N}$$

$$\sum M_B = 0 \Rightarrow$$

$$400 \times 0.15 + R_{Ay} \times 0.4 = 0$$

$$\Rightarrow R_{Ay} = -150 \text{ N.}$$

(Problem 3.14 contd.)



$$\sum F_x = 0 \Rightarrow$$

$$F - 400 = 0$$

$$F = 400 \text{ N.}$$

$$\sum F_y = 0 \Rightarrow$$

$$V - 150 = 0$$

$$\Rightarrow V = 150 \text{ N.}$$

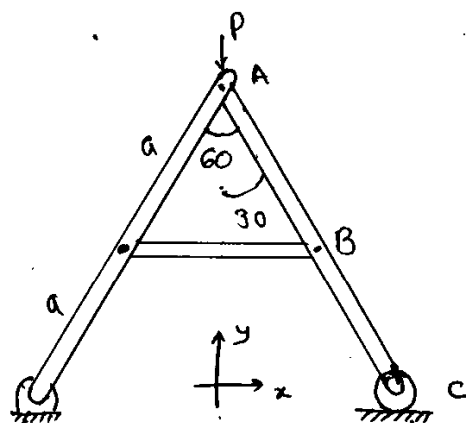
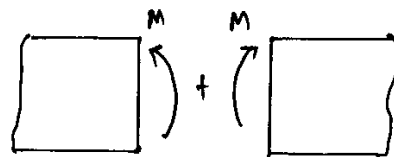
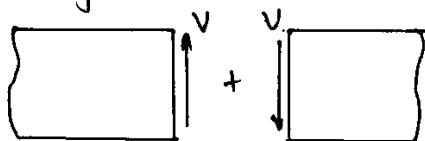
$$\sum M_E = 0 \Rightarrow$$

$$M + 150 \times 0.25 = 0$$

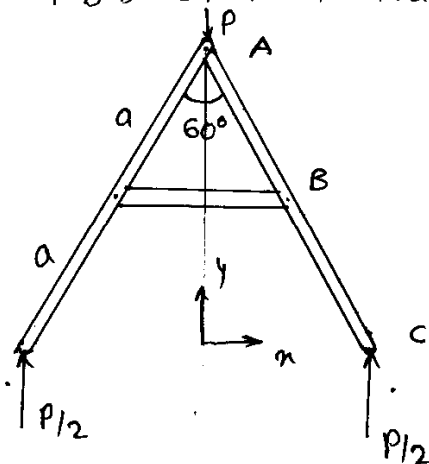
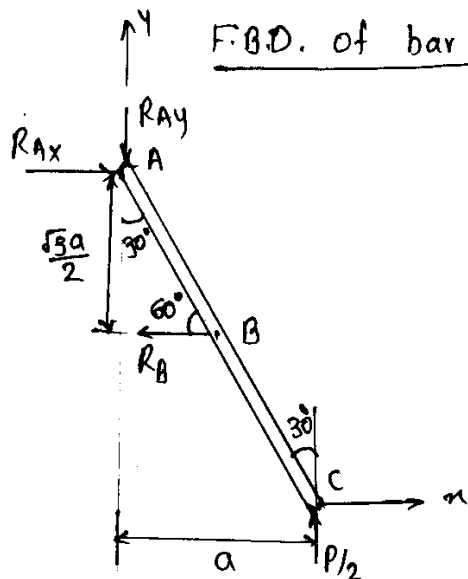
$$\Rightarrow M = -37.5 \text{ N}\cdot\text{m.}$$

Solution to problem 3.15.

Sign convention:



F.B.D. of whole frame.

F.B.D. of bar AC :

$$\sum M_A = 0 \Rightarrow$$

$$\frac{P}{2} \cdot a - R_B \frac{\sqrt{3}}{2} a = 0$$

$$\Rightarrow R_B = \frac{P}{\sqrt{3}} = \frac{\sqrt{3}P}{3}$$

$$\sum F_x = 0 \Rightarrow$$

$$R_{Ax} - R_B = 0 \Rightarrow$$

$$R_{Ax} = R_B = \frac{\sqrt{3}P}{3}$$

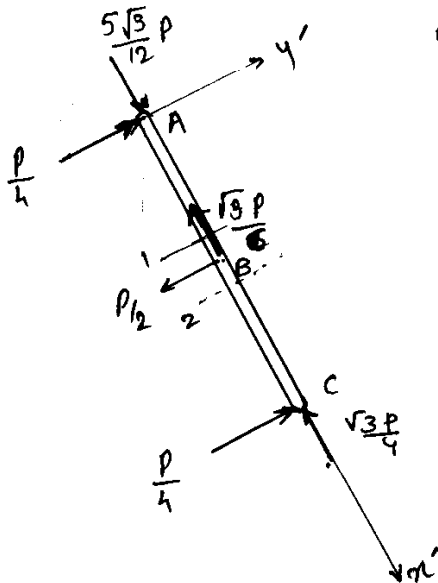
$$\sum F_y = 0 \Rightarrow$$

$$R_{Ay} - \frac{P}{2} = 0.$$

$$R_{Ay} = \frac{P}{2}$$

(Problem 3.15 contd.)

loading diagram:

A:  $R_{Ax}'$  (along the rod)

$$= R_{Ay} \cos 30 + R_{Ax} \cos 60$$

$$= \frac{P}{2} \frac{\sqrt{3}}{2} + \frac{\sqrt{3}P}{3} \frac{1}{2} = \frac{5\sqrt{3}P}{12}$$

 $R_{Ay}'$  (perpendicular to the rod)

$$= -R_{Ay} \sin 30 + R_{Ax} \sin 60$$

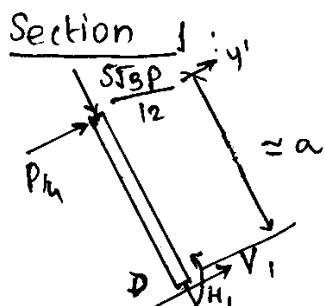
$$= -\frac{P}{2} \cdot \frac{1}{2} + \frac{\sqrt{3}P}{3} \cdot \frac{\sqrt{3}}{2} = \frac{P}{4}$$

B:  $R_{Bx}'$  (along the rod):

$$= -R_B \cos 60 = -\frac{\sqrt{3}P}{3} \cdot \frac{1}{2} = -\frac{\sqrt{3}P}{6}$$

 $R_{By}'$  (perpendicular to the rod)

$$= -R_B \sin 60 = -\frac{\sqrt{3}P}{3} \cdot \frac{\sqrt{3}}{2} = -\frac{P}{2}$$

C:  $R_{Cx}'$  (along the rod) =  $-\frac{P}{2} \cos 30 = -\frac{\sqrt{3}P}{4}$  $R_{Cy}'$  (perpendicular to the rod) =  $\frac{P}{2} \sin 30 = \frac{P}{4}$ 

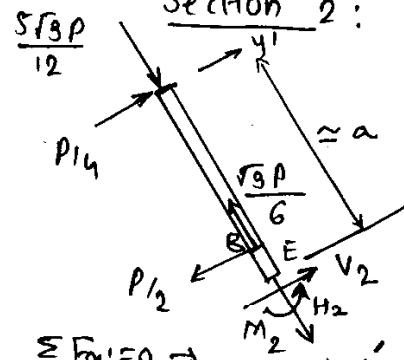
$$\sum F_{x'} = 0 \Rightarrow$$

$$H_1 = -\frac{5\sqrt{3}}{12}P$$

$$\sum F_{y'} = 0 \Rightarrow V_1 = -\frac{P}{4}$$

$$\sum M_D = 0 \Rightarrow M_1 = \frac{P \cdot a}{4}$$

Section 2:



The distance BE is negligible

$$\sum F_{x'} = 0 \Rightarrow$$

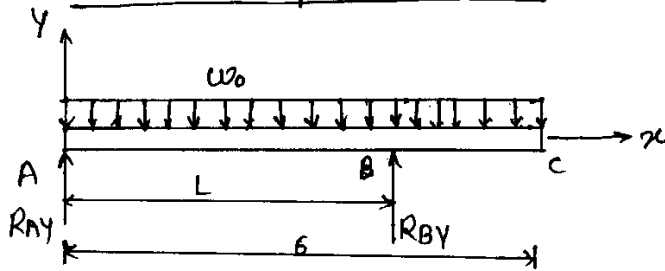
$$H_2 = \frac{\sqrt{3}P}{6} - \frac{5\sqrt{3}P}{12} = -\frac{\sqrt{3}P}{4}$$

$$\sum F_{y'} = 0 \Rightarrow$$

$$V_2 = P/2 - P/4 = P/4$$

$$\sum M_E = 0 \Rightarrow M_2 = \frac{P \cdot a}{4}$$

### Solution to problem 3.17



$$+\uparrow \Sigma F_y = 0 \Rightarrow$$

$$R_{Ay} + R_{By} - w_0 \times 6 = 0.$$

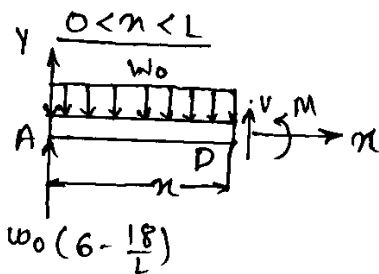
$$+\circlearrowleft \Sigma M_A = 0 \Rightarrow$$

$$R_{By} \times L - \frac{1}{2} w_0 \times 6^2 = 0.$$

From above equations:

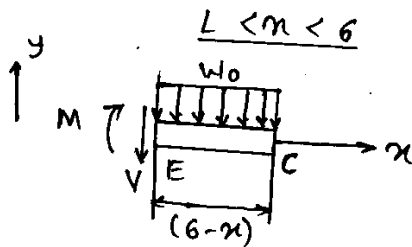
$$R_{By} = \frac{18 w_0}{L}.$$

$$R_{Ay} = w_0 \left( 6 - \frac{18}{L} \right).$$



$$\Sigma M_D = 0 \Rightarrow$$

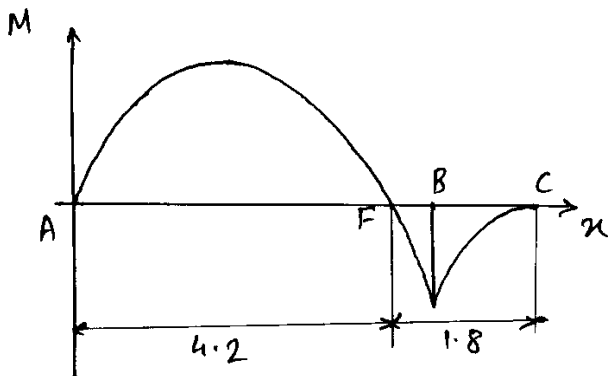
$$M = w_0 \left( 6 - \frac{18}{L} \right) \cdot x - \frac{1}{2} w_0 x^2.$$



$$\Sigma M_C = 0 \Rightarrow$$

$$M = -\frac{1}{2} w_0 (6-x)^2.$$

### Bending moment diagram.



The bending moment is zero at point F. (between points A & B)

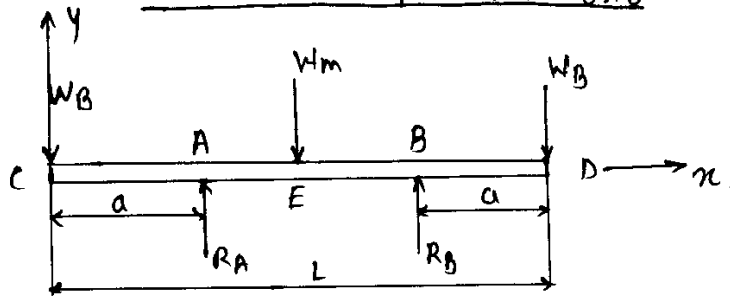
Bending moment equation for AB gives

$$w_0 \left( 6 - \frac{18}{L} \right) x - \frac{1}{2} w_0 x^2 \Big|_{x=4.2} = 0$$

$$w_0 \left( 6 - \frac{18}{L} \right) \cdot 4.2 - \frac{1}{2} w_0 \cdot 4.2^2 = 0.$$

$$\Rightarrow \underline{L = 4.615 \text{ m.}}$$

Solution to problem 3.18



a) Apparently he is not minimizing the bending moment by putting the bricks at the end. The bending moment would be less if he puts the bricks right above the support.

$$b) \uparrow \sum F_y = 0 \Rightarrow R_A + R_B - W_M - 2W_B = 0.$$

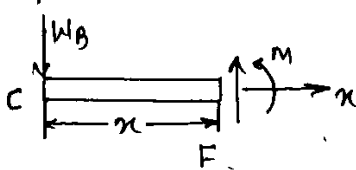
$$+ \circlearrowleft \sum M_A = 0 \Rightarrow W_B \cdot a - W_M \cdot \left(\frac{L}{2} - a\right) + R_B \cdot (L - 2a) - W_B \cdot (L - a) = 0.$$

from above equations :

$$R_B = W_B + \frac{W_M}{2}$$

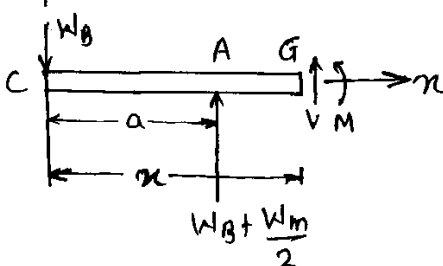
$$R_A = W_B + \frac{W_M}{2}.$$

$$y \uparrow \quad 0 < x < a$$



$$\sum M_F = 0 \Rightarrow M = -W_B \cdot x.$$

$$y \uparrow \quad a < x < \frac{L}{2}$$



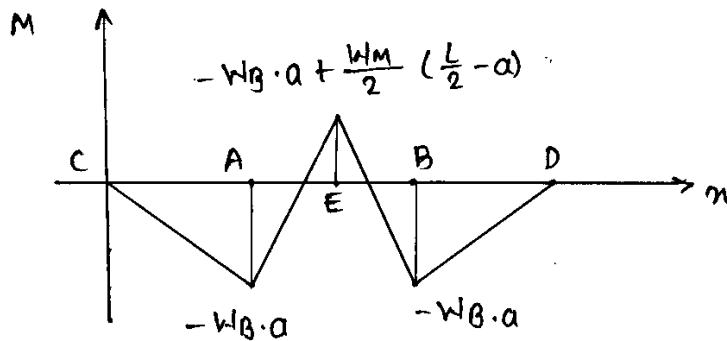
$$\sum M_G = 0 \Rightarrow$$

$$M = -W_B x + \left(W_B + \frac{W_M}{2}\right) \cdot (x - a)$$

$$= \frac{W_M}{2} \cdot x - \left(\frac{W_M}{2} + W_B\right) \cdot a$$

Symmetry will give <sup>the</sup> same expressions for  $M$  in  $EB$  and  $BD$ .

(problem 9.18 contd.)

Bending moment diagram:

$$\begin{aligned}
 M|_{x=\frac{L}{2}} &= \\
 \frac{W_m}{2} \cdot \frac{L}{2} - \left( \frac{W_m}{2} + W_B \right) \cdot a \\
 &= -W_B \cdot a + \frac{W_m}{2} \left( \frac{L}{2} - a \right). \\
 &\text{(Assumed positive)}
 \end{aligned}$$

To minimize the maximum Bending Moment:

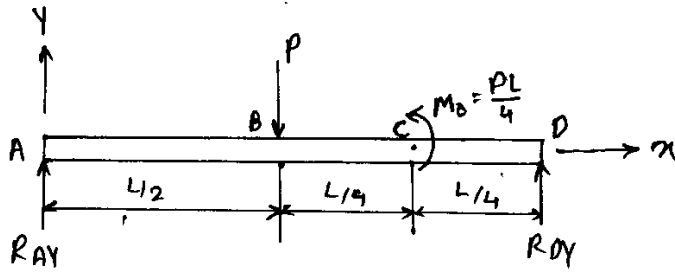
magnitude of M at A = magnitude of M at E.

$$\therefore W_B \cdot a = -W_B \cdot a + \frac{W_m}{2} \left( \frac{L}{2} - a \right).$$

$$\therefore 2W_B \cdot a = \frac{W_m}{2} \left( \frac{L}{2} - a \right)$$

$$\therefore W_B = \frac{W_m}{4a} \left( \frac{L}{2} - a \right).$$

Solution to problem 3.22



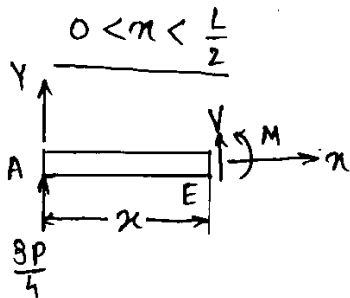
$$+\uparrow \sum F_y = 0 \Rightarrow R_{AY} - P + R_{DY} = 0.$$

$$+\circlearrowleft \sum M_A = 0 \Rightarrow -P\left(\frac{L}{2}\right) + M_o + R_{DY}(L) = 0.$$

$\Rightarrow$

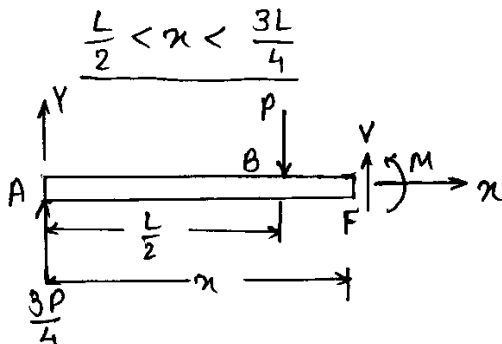
$$\therefore R_{AY} = \frac{3P}{4}$$

$$R_{DY} = \frac{P}{4}$$



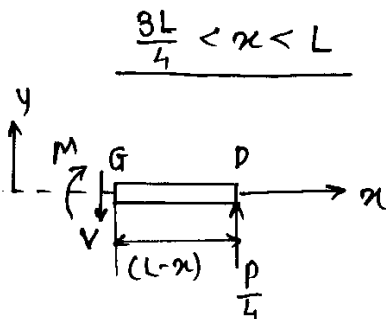
$$\sum F_y = 0 \Rightarrow V = -\frac{3P}{4}$$

$$\sum M_E = 0 \Rightarrow M = \frac{3P}{4}x.$$



$$\sum F_y = 0 \Rightarrow V = \frac{P}{4}$$

$$\sum M_F = 0 \Rightarrow M = \frac{PL}{2} - \frac{Px}{4}.$$

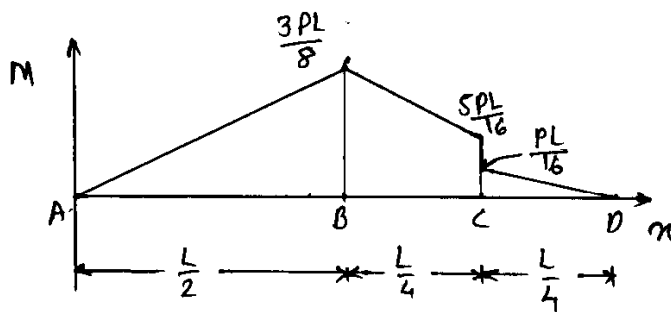
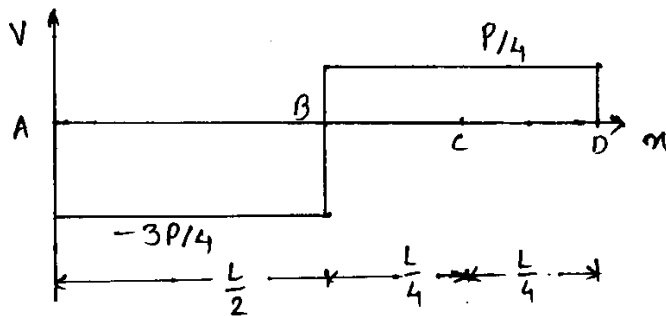


$$\sum F_y = 0 \Rightarrow V = \frac{P}{4}$$

$$\sum M_G = 0 \Rightarrow M = \frac{P}{4}(L-x).$$



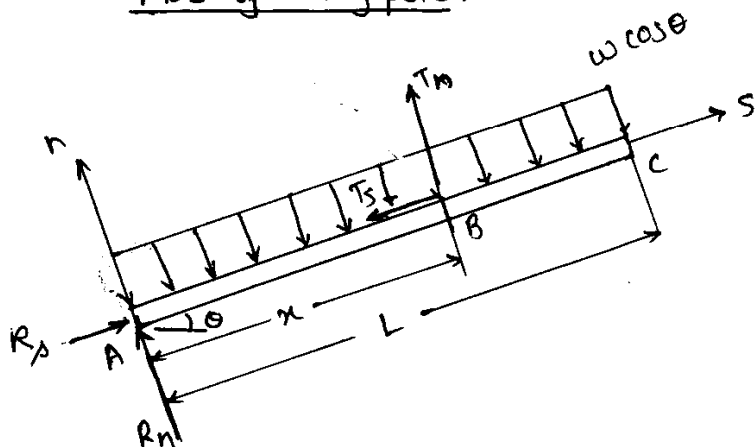
(Problem 3.22 contd.)

Shear force and Bending moment diagrams:

Note: At point c, there is a concentrated moment of magnitude  $\frac{PL}{4}$  in ccw direction. ~~And~~ <sup>Therefore</sup>, there is a jump in the bending moment diagram of the same magnitude in the downward direction.

# Solution to problem 3.26

FBD of Flagpole:



(The component  $w \sin \theta$  is not shown)

$$L = 75'$$

If the flagpole doesn't bend to an appreciable extent, then bending moment can only be caused by forces normal to flagpole. These are: i)  $R_n$ , ii)  $T_n$ , 3) Normal component of  $w = w \cos \theta$ .

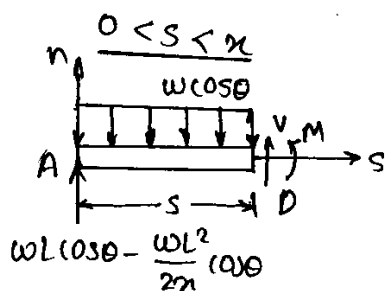
$$\sum (M_z)_A = 0 \Rightarrow T_n = \frac{wL^2}{2\pi} \cos \theta.$$

$$\sum F_n = 0 \Rightarrow R_n - w \cos \theta \cdot L + T_n = 0.$$

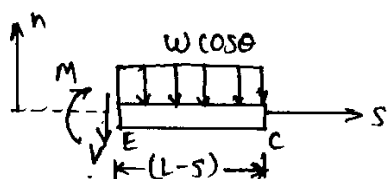
$$\Rightarrow R_n = wL \cos \theta - \frac{wL^2}{2\pi} \cos \theta.$$

$$\sum F_n = 0 \Rightarrow V = -wL \cos \theta + \frac{wL^2}{2\pi} \cos \theta + w \cos \theta \cdot s \quad \text{--- (1)}$$

$$\sum M_D = 0 \Rightarrow M = (wL \cos \theta - \frac{wL^2}{2\pi} \cos \theta) \cdot s - w \cos \theta \cdot \frac{s^2}{2} \quad \text{--- (2)}$$



$x < B < L$



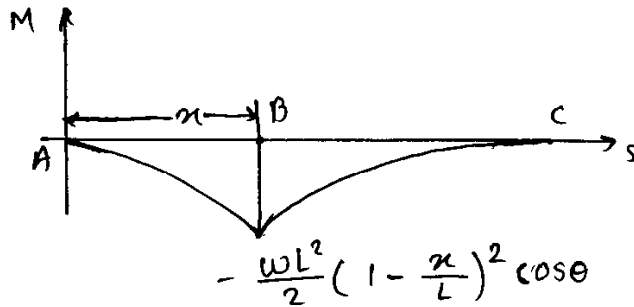
$$\sum F_n = 0 \Rightarrow V = -w \cos \theta (L-s) \quad \text{--- (3)}$$

$$\sum M_E = 0 \Rightarrow M = -\frac{1}{2} w \cos \theta (L-s)^2 \quad \text{--- (4)}$$

(problem 3.26 contd.)

(17)

Bending moment diagram for  $x < \frac{L}{2}$  (Case 1):

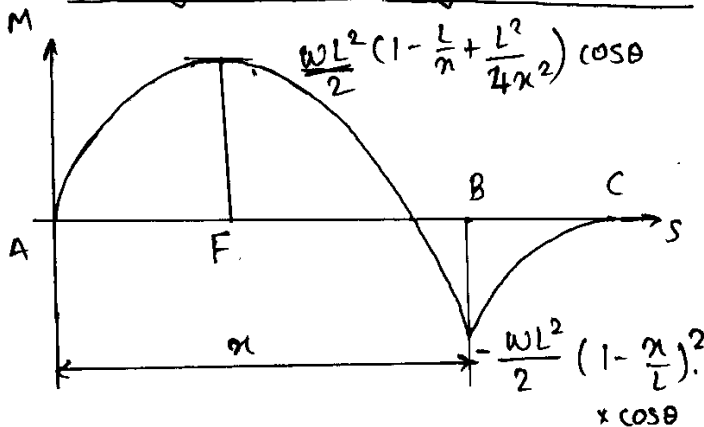


$$M|_x = WLx \cos \theta - wLx \cos \theta \frac{x^2}{2} - \frac{WL^2}{2x} \cdot x \cos \theta \quad (\text{eq. 2})$$

$$= -\frac{WL^2}{2} \cos \theta \left(1 - \frac{x}{L}\right)^2$$

Maximum bending moment becomes minimum when  $x = L$   
But it is outside the range.  $\therefore$  This is not an acceptable solution

Bending Moment diagram for  $x > \frac{L}{2}$  (Case 2):



At F,  $s = L - \frac{L^2}{2x}$  from  
the condition  $V|_F = 0$ .  
(eq. 1)

$$M|_{s=L-\frac{L^2}{2x}} = \left( WL - \frac{WL^2}{2x} \right) \left( 1 - \frac{L^2}{2x} \right) \cos \theta$$

$$= \frac{WL^2}{2} \left( L - \frac{L^2}{2x} \right)^2 \cos \theta$$

$$\therefore M|_{s=L-\frac{L^2}{2x}} = \frac{WL^2}{2} \left( 1 - \frac{L}{x} + \frac{L^2}{4x^2} \right) \cos \theta \quad (\text{eq. 2})$$

maximum

To minimize the bending moment:

magnitude of M at F = magnitude of M at B.

$$\therefore \frac{WL^2}{2} \left( 1 - \frac{L}{x} + \frac{L^2}{4x^2} \right) \cos \theta = \frac{WL^2}{2} \left( 1 - \frac{x}{L} \right)^2 \cos \theta$$

$$\Rightarrow \frac{x^4}{L^4} - \frac{2x^3}{L^3} + \frac{x}{L} - \frac{1}{4} = 0$$

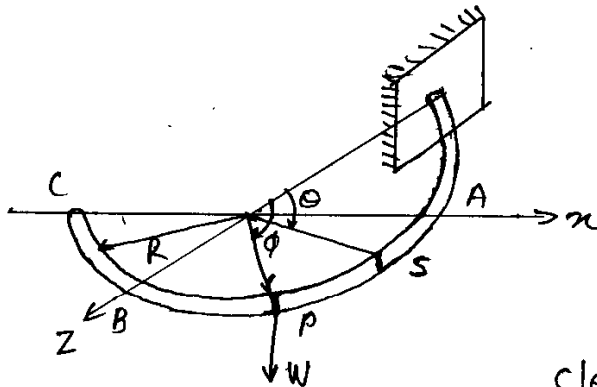
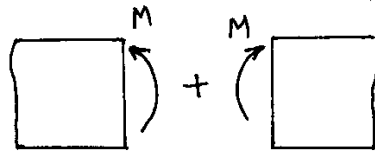
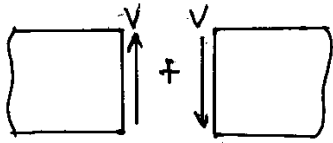
$$\Rightarrow \frac{x}{L} = 1.7071, -0.707, 0.707, 0.293$$

As  $x$  is between  $L/2$  to  $L$

$$x = L \times 0.707 = 53.03'$$

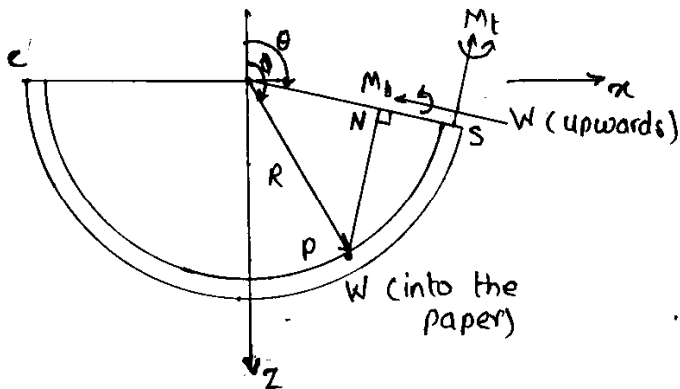
Solution to problem 3.27

Sign conventions:



Consider the general case where a load  $w$  is applied at a point  $P$  (angle  $\phi$ ) and we wish to find the bending and twisting moments for any point  $S$  at angle  $\theta$ .

Clearly, for  $\theta > \phi$ ,  $M_b = M_t = 0$ .

Projection of free body:For  $\theta \leq \phi$ 

(from adjacent fig.)

$$M_b = W(PN)$$

$$M_t = W(NS)$$

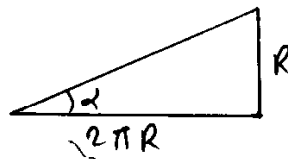
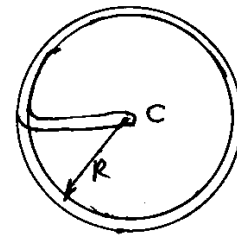
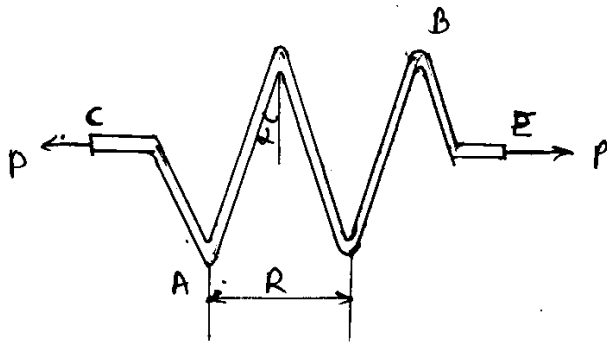
$$PN = R \sin(\phi - \theta)$$

$$NS = R[1 - \cos(\phi - \theta)]$$

$$\therefore \left. \begin{aligned} M_b &= WR \sin(\phi - \theta) \\ M_t &= WR[1 - \cos(\phi - \theta)] \end{aligned} \right\} \theta \leq \phi$$

1. load at A,  $\phi = 90^\circ$ :  $M_b = WR \cos \theta$ ;  $(M_b)_{\max} = WR$  at  $\theta = 0^\circ$   
 $M_t = WR(1 - \sin \theta)$ ;  $(M_t)_{\max} = WR$  at  $\theta = 0^\circ$ .
2. Load at B,  $\phi = 180^\circ$ :  $M_b = WR \sin \theta$ ;  $(M_b)_{\max} = WR$  at  $\theta = 90^\circ$   
 $M_t = WR[1 + \cos \theta]$ ;  $(M_t)_{\max} = 2WR$  at  $\theta = 0^\circ$ .
3. Load at C,  $\phi = 270^\circ$ :  $M_b = -WR \cos \theta$ ;  $(M_b)_{\max} = WR$  at  $\theta = 0^\circ, 180^\circ$   
 $M_t = WR[1 + \sin \theta]$ ;  $(M_t)_{\max} = 2WR$  at  $\theta = 90^\circ$ .

Solution to problem 3.2g

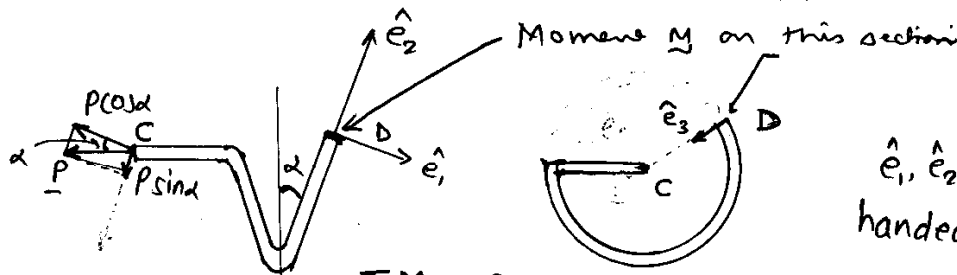


$$\tan \alpha = \frac{1}{2\pi}$$

$$\tan^2 \alpha + 1 = \frac{1}{\cos^2 \alpha}$$

$$\therefore \frac{1}{4\pi^2} + 1 = \frac{1}{\cos^2 \alpha}$$

$$\cos \alpha = \frac{2\pi}{\sqrt{1+4\pi^2}}, \quad \sin \alpha = \frac{1}{\sqrt{1+4\pi^2}}$$



$\hat{e}_1, \hat{e}_2, \hat{e}_3$  is right handed triad.

$$\sum \underline{M}_D = 0 \Rightarrow \underline{M} - \underline{D}_C \times \underline{P} \Rightarrow \underline{M} = \underline{D}_C \times \underline{P}$$

$$\therefore \underline{M} = R \hat{e}_3 \times (-P \cos \alpha \hat{e}_1 - P \sin \alpha \hat{e}_2)$$

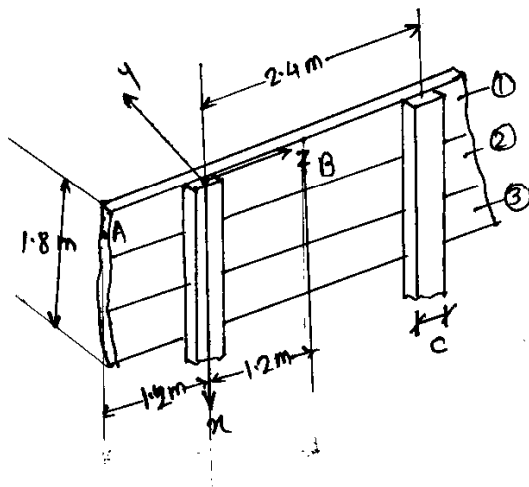
$$= -PR \cos \alpha \hat{e}_2 + PR \sin \alpha \hat{e}_1$$

This expression is valid at every point from A to B.

$M_t$  (twisting Moment) = coefficient of  $\hat{e}_2 = -PR \cos \alpha$

$M_b$  (Bending Moment) = coefficient of  $\hat{e}_1 = PR \sin \alpha$ .

# Solution to problem 3.34



The slices in the planks are symmetrical about the  $n$ -axis. Thus there can be no shear ~~stresses~~ <sup>force</sup>  $V_{zy}$  at A & B in plank ① and similar points in the other planks.

Now consider the  $y$ -direction equilibrium of the plank. The  $y$ -direction forces are:

- i) Linearly distributed water pressure.
- ii) Force exerted by the upright (unknown distribution)

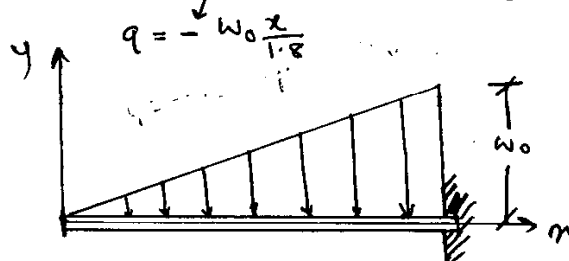
By writing  $\sum F_y = 0$  and  $\sum M_z = 0$  we can find magnitude and point of application of the resultant force between upright and plank. The distribution cannot be determined by simple statics. However, since we need to know the distribution to calculate  $M$  and  $V$  in uprights, we will assume that it is the same as water pressure distribution.

Thus if  $q$  = load per meter of upright then

$$q = c \times p \quad \text{where } p \text{ is water pressure} = \gamma x, \quad c = \text{width of upright}$$

$$\therefore q = \omega_0 \frac{x}{1.8} \quad \text{where } \omega_0 \equiv \text{constant} = c\gamma (1.8)$$

Negative sign because  $q$  is along  $-y$ -axis



$$\begin{aligned} & \text{mean pressure over} \\ & 1.8 \text{ m depth} \\ & = \text{pressure at } 0.9 \text{ m} \\ & = 1000 \times 9.81 \times 0.9 \\ & = 8.83 \times 10^3 \text{ N/m}^2 \end{aligned}$$

Total force on  $1.8 \times 2.4$  section of planks

$$= 1.8 \times 2.4 \times 8.83 \times 10^3 = 38.14 \text{ kN}$$

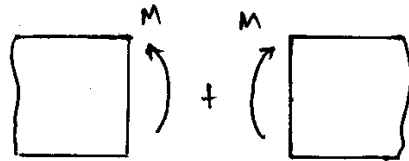
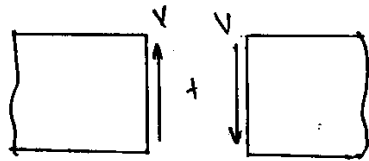
(Problem - 3.34 contd.)

(23)

∴ Total load on upright  $\Rightarrow \int_0^{1.8} w_0 \left( \frac{x}{1.8} \right) dx = 38.14 \text{ kN}.$

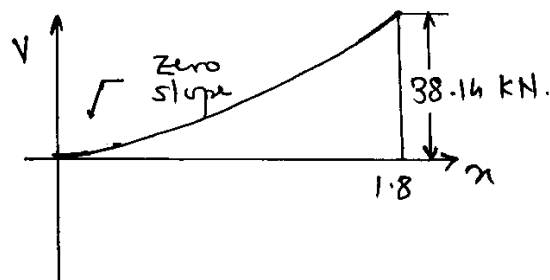
$\Rightarrow 0.9 w_0 = 38.14 \quad \Rightarrow w_0 = 42.38 \text{ kN/m}.$

Sign convention:  $\Rightarrow q(x) = - \frac{42.38}{1.8} x = -23.54 \times 10^3 x$



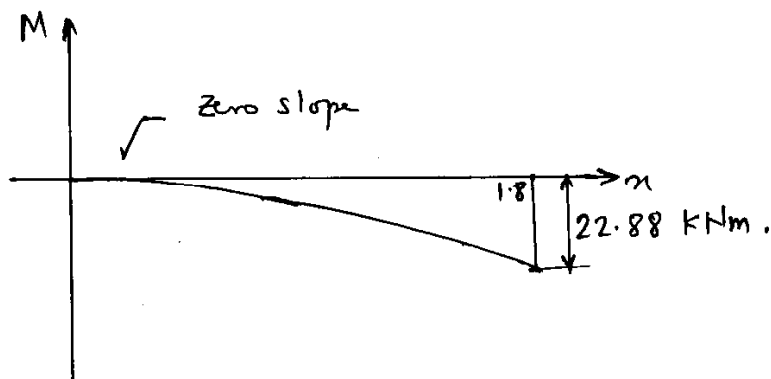
Shear force distribution:

$$V(x) = - \int_0^x q(x) dx = \frac{23.54 \times 10^3}{2} x^2$$



Bending moment distribution

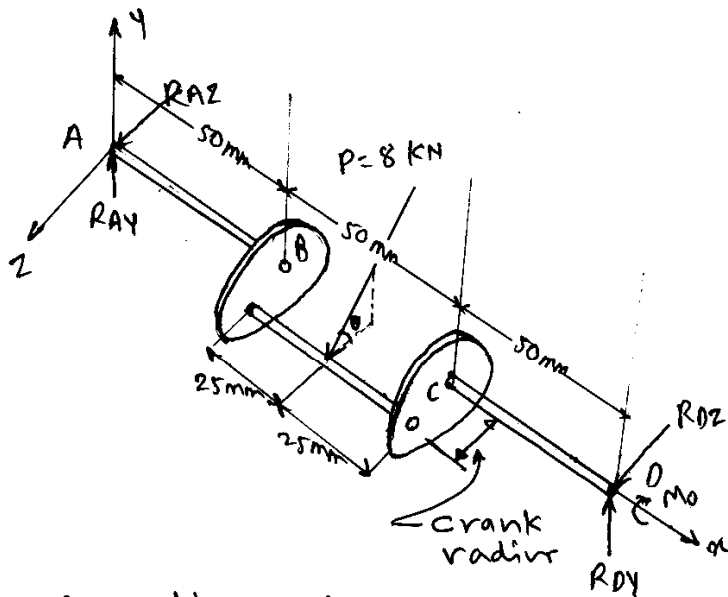
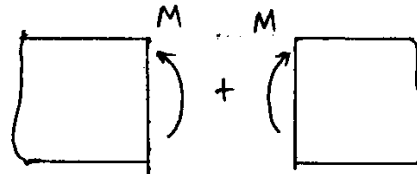
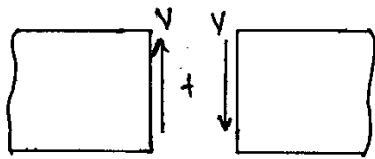
$$M(x) = - \int_0^x V(x) dx = - \frac{23.54 \times 10^3}{6} x^3$$



Solution to problem 3.40 :

(24)

Sign convention:



$$\begin{aligned} \text{Crank radius} &= \frac{1}{2} \times \text{stroke} \\ &= \frac{75}{2} \text{ mm} \end{aligned}$$

Assume that bearings do not transmit moments to shaft.

connecting rod length = 125 mm.

$$\cos \theta = \frac{75/2}{125} \therefore \theta = 72.54^\circ \text{ (In one particular position)}$$

Overall equilibrium:

$$\begin{aligned} \sum M_x = 0 &\Rightarrow M_0 - P \cos \theta \times 0 - P \sin \theta \times \frac{75}{2} = 0 \\ \therefore M_0 &= P \sin \theta \times \frac{75}{2} = 8 \times \sin 72.54^\circ \times \frac{75}{2} \\ &= 286.2 \text{ N}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} \sum (M_y)_A = 0 &\Rightarrow -R_{Dz} \times 150 - P \cos \theta \times 75 = 0 \\ \Rightarrow R_{Dz} &= -1.2 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum (M_z)_A = 0 &\Rightarrow R_{Dy} \times 150 - P \sin \theta \times 75 = 0 \\ \Rightarrow R_{Dy} &= 3.8 \text{ kN} \end{aligned}$$

By symmetry:

$$R_{Az} = R_{Dz} = -1.2 \text{ kN}$$

$$R_{Ay} = R_{Dy} = 3.8 \text{ kN}$$

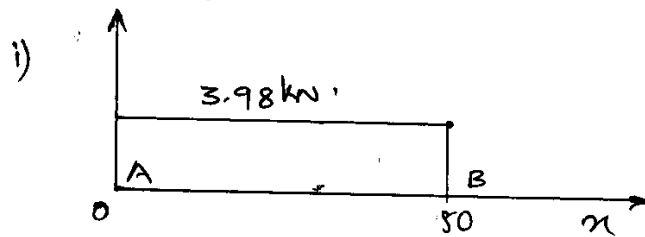


(Problem 3.40 contd.)

(25)

It is only necessary to sketch shear force & Bending moment diagram for AB. For CD, <sup>they are</sup> identical except that the sign of V is reversed.

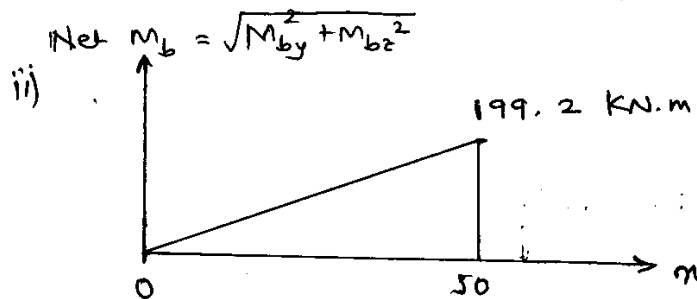
$$\text{Net } V = \sqrt{V_y^2 + V_z^2}$$



Shear force for AB, CD.

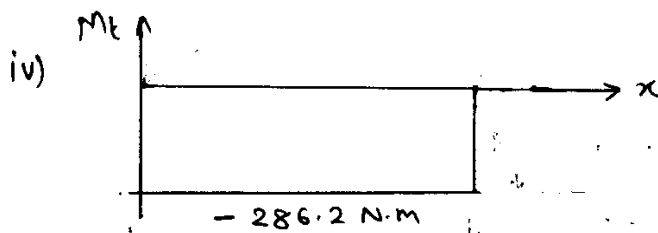
Net reaction at A:

$$\begin{aligned} R_A &= \sqrt{R_{Ay}^2 + R_{Az}^2} \\ &= \sqrt{(3.8)^2 + (-1.2)^2} \\ &= 3.98 \text{ kN} \end{aligned}$$



Bending moment for AB, CD

iii) Twisting moment for AB = 0.



Twisting moment for CD.

Sign convention for twisting moment

