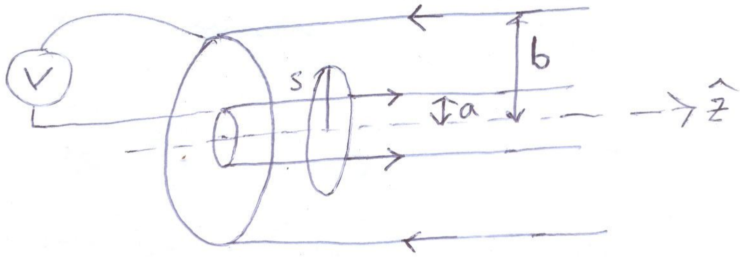
1. Consider an Amperian loop betn the two cylinders (i.e. $a<s<b$)

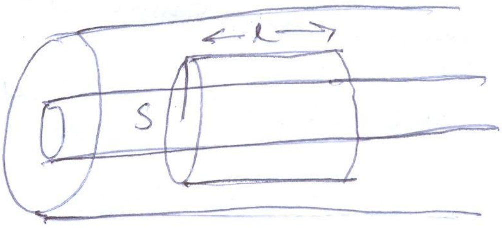



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl.}$$

$$\Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\varphi}$$

If $\lambda$ = charge density per unit length on the inner cylinder

Consider a Gaussian cylindrical surface of radius $s$ and length $l$




$$E \cdot 2\pi s l = \frac{\lambda l}{\epsilon_0}$$

$$\Rightarrow \vec{E} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s}$$

Potential difference betn the two cylinders $= V = \int \vec{E}\cdot d\vec{l}$

$$\Rightarrow V = \frac{\lambda}{2\pi\epsilon_0}\int_{s=a}^{b}\frac{1}{s}ds = \frac{\lambda}{2\pi\epsilon_0}\ln(b/a)$$

$$\Rightarrow \lambda = \frac{2\pi\epsilon_0 V}{\ln(b/a)}$$

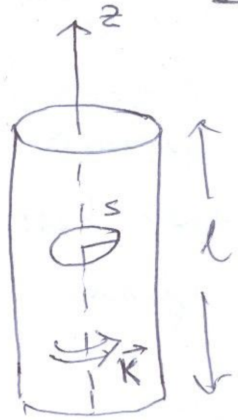
$$\Rightarrow \vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{1}{\cancel{\mu_0}}\frac{\cancel{\mu_0} I}{2\pi s}\cdot\frac{\cancel{2\pi\epsilon_0} V}{\cancel{2\pi\epsilon_0} s\ln(b/a)}(\hat{s}\times\hat{\varphi})$$

$$= \frac{VI}{2\pi s^2\ln(b/a)}\hat{z}$$

Power transported $P = \oint \vec{S}\cdot d\vec{a} = \frac{VI}{2\pi\ln(b/a)}\int_a^b \frac{1}{s^2}(2\pi s\,ds)$

$$= \frac{VI}{2\pi\ln(b/a)}\cdot 2\pi\ln(b/a) = VI$$

2. $\vec{B} = \mu_0 K \hat{z}$ inside
$= 0$ outside the solenoid.




$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial K}{\partial t} \hat{z}$

From symmetry, $\vec{E}$ is independent of $\varphi$ & $z$

B is in $\hat{z}$ direction $\Rightarrow \vec{E}$ will be in $\hat{\varphi}$ direction.

$\Rightarrow$ We can write $\vec{E} = E(s)\hat{\varphi}$

consider an Amperian loop of radius $s$ $(s<R)$

$$\oint \vec{E}\cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{a}$$

$$\Rightarrow 2\pi s E = -\frac{\partial B}{\partial t}\cdot \pi s^2$$

$$\Rightarrow E = -\frac{\mu_0}{2} s \frac{\partial K}{\partial t}$$

$$\Rightarrow \vec{E} = -\frac{\mu_0}{2} s \frac{\partial K}{\partial t}\hat{\varphi}$$

Another way to calculate $\vec{E}$:

$\vec{E} = E(s)\hat{\varphi}$

$\nabla \times \vec{E} = \frac{1}{s}\frac{\partial}{\partial s}(sE)\hat{z}$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial K}{\partial t}\hat{z}$

$\Rightarrow -\mu_0 \frac{\partial K}{\partial t} = \frac{1}{s}\frac{\partial}{\partial s}(sE)$

$\Rightarrow sE = -\frac{\mu_0}{2} s^2 \frac{\partial K}{\partial t} + \underset{0}{\underset{\parallel}{\text{Constant}}}$

as $E=0$ if $\frac{\partial K}{\partial t}=0$

$\Rightarrow \vec{E} = -\frac{\mu_0}{2} s \frac{\partial K}{\partial t}\hat{\varphi}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} = -\frac{1}{\cancel{\mu_0}}(\cancel{\mu_0}K)\frac{\mu_0}{2} s\frac{\partial K}{\partial t}(\hat{\varphi}\times\hat{z})$$

$$= -\frac{\mu_0}{2} sK\frac{\partial K}{\partial t}\hat{s}$$

Energy flux coming in $P = -\oint \vec{S}\cdot d\vec{a}\Big|_{s=R}$

$$= \frac{\mu_0}{2} RK\frac{\partial K}{\partial t}\cdot 2\pi R l$$ over a length of $l$

$$= \frac{\mu_0}{2}\pi R^2 l \frac{\partial}{\partial t}(K^2)$$

Total current $I = Kl$
$\Rightarrow K = I/l$

$$= \frac{\mu_0}{2}\frac{\pi R^2 \cancel{l}}{l\cancel{l}}\cdot\frac{\partial I^2}{\partial t^2}$$

Self inductance of the solenoid $L = \frac{\Phi}{I} = \frac{\mu_0 \cancel{K}\cdot \pi R^2}{(\cancel{K}l)} = \frac{\mu_0 \pi R^2}{l}$

$\therefore P = \frac{1}{2}L\frac{\partial I^2}{\partial t} = \frac{\partial}{\partial t}\left(\frac{1}{2}LI^2\right)$ = rate of change in magnetic energy stored in the solenoid.

3.

a) $\vec{E} = \sqrt{2}E_0\left(\frac{\hat{y}+\hat{z}}{\sqrt{2}}\right)\sin(kx-\omega t)$

linearly parized along $\frac{\hat{y}+\hat{z}}{\sqrt{2}}$ direction

propagating along $+x$ direction.

Amplitude $= \sqrt{2}E_0$.

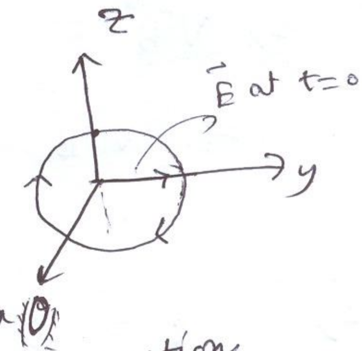
b) $E_y = E_0\cos(kx-\omega t)$

$E_z = E_0\sin(kx-\omega t)$

$\Rightarrow \frac{E_y^2}{E_0^2} + \frac{E_z^2}{E_0^2} = 1$

at $x=0$

| $\omega t$ | $E_y$ | $E_z$ |
|---|---|---|
| 0 | 0 | $E_0$ |
| $\pi/4$ | $E_0/\sqrt{2}$ | $-E_0/\sqrt{2}$ |
| $\frac{\pi}{2}$ | 0 | $-E_0$ |
| ⋮ | ⋮ | ⋮ |




Right circularly polarized

as E-field vector rotates in clockwise direction

as it propagates (observed from x axis).

(c) $\vec{E} = Re\left[E_0\left(\hat{x}+(1+i)\hat{y}\right)e^{-i(kz-\omega t)}\right]$

$E_x = Re\left[E_0 e^{-i(kz-\omega t)}\right] = E_0\cos(kz-\omega t)$

$E_y = Re\left[E_0((1+i))e^{-i(kz-\omega t)}\right] = Re\left[\sqrt{2}E_0\left(\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}\right)e^{-i(kz-\omega t)}\right]$

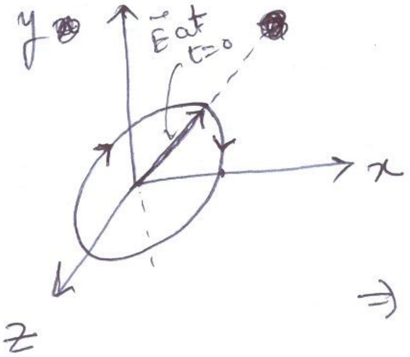
$= Re\left[\sqrt{2}E_0 e^{i\pi/4}e^{-i(kz-\omega t)}\right]$

$= \sqrt{2}E_0\cos(kz-\omega t-\pi/4)$

$\Rightarrow$ wave propaging in $+z$-dirn.

At $z=0$

| $\omega t$ | $E_x$ | $E_y$ |
|---|---|---|
| 0 | $E_0$ | $E_0$ |
| $\pi/4$ | $\frac{E_0}{\sqrt{2}}$ | 0 |
| | | $-E_0$ |
| $\pi/2$ | 0 | ⋮ |
| ⋮ | ⋮ | |




E-field vector rotates in clockwise direction (observed from +z axis)

$\Rightarrow$ Right handed elliptic polarization.

4. $\vec{B} = B_0 \cos\beta x \cos\omega t \; \hat{z}$

$B_0 = 2\times10^{-6}\,T, \quad \omega = 2.4\times10^{12}\,s^{-1}$

a) $\dfrac{\partial^2 \vec{B}}{\partial x^2} = -\beta^2 \vec{B}$

$\dfrac{\partial^2 \vec{B}}{\partial t^2} = -\omega^2 \vec{B}$

$\Rightarrow \nabla^2 \vec{B} = -\beta^2 \vec{B} = \dfrac{\beta^2}{\omega^2} \dfrac{\partial^2 \vec{B}}{\partial t^2}$

$\Rightarrow \boxed{\nabla^2 \vec{B} = \dfrac{1}{c^2} \dfrac{\partial^2 \vec{B}}{\partial t^2}}$ where $c^2 = \dfrac{\omega^2}{\beta^2}$.

$\rightarrow$ wave eqn in vacuum.

b) $c = \dfrac{\omega}{\beta} \Rightarrow \beta = \dfrac{\omega}{c} = \dfrac{2.4\times10^{+12}\,s^{-1}}{3\times10^{8}\,m/s} = 0.8\times10^{4}\,m^{-1}$

c) $\bar{\nabla}\times\bar{B} = \cancelto{0}{\mu_0 \vec{J}_f} + \mu_0\epsilon_0 \dfrac{\partial \bar{E}}{\partial t}$

$\bar{\nabla}\times\bar{B} = \hat{y}\,\beta B_0 \sin\beta x \cos\omega t$

$\Rightarrow \mu_0\epsilon_0 \dfrac{\partial \bar{E}}{\partial t} = \hat{y}\,\beta B_0 \sin\beta x \cos\omega t$

$\Rightarrow \vec{E} = -\hat{y}\,\dfrac{\beta}{\mu_0\epsilon_0\omega} B_0 \sin\beta x \sin\omega t$

$\left\{ \dfrac{1}{\mu_0\epsilon_0} = c^2, \quad \dfrac{\omega}{\beta} = c \right.$

$= -\hat{y}\,\dfrac{c^{\cancel{2}}}{\cancel{c}} B_0 \sin\beta x \sin\omega t$.

$= -\hat{y}\, B_0 c \sin\beta x \sin\omega t$

d) $\vec{S} = \dfrac{1}{\mu_0} \vec{E}\times\vec{B} = \dfrac{B_0^2 c}{\mu_0} \sin\beta x \cos\beta x \sin\omega t \cos\omega t \,(-\hat{y}\times\hat{z})$

$= \dfrac{1}{\mu_0} B_0^2 c \,\dfrac{1}{4} \sin(2\beta x) \sin(2\omega t)\,(-\hat{x})$

At any time $t$, the energy flows in $-\hat{x}$ direction.

But for $\sin 2\beta x = 0$ there is no energy flow

$$\Downarrow$$

$$2\beta x = n\pi \Rightarrow \beta x = \frac{n\pi}{2} \quad (n = 0, 1, 2, \cdots) \Rightarrow [\text{nodes}]$$

At any other point $x$, energy flows in $-\hat{x}$ direction
for $\sin(2\omega t)$ ~~...~~ $> 0$
and in $+\hat{x}$ direction for ~~...~~. $\sin(2\omega t) < 0$

$$\langle \vec{S} \rangle = \frac{1}{T}\int_0^T \sin 2\omega t \; \frac{B_0^2 c}{4\mu_0} \sin 2\beta x \, (-\hat{x}) \, dt$$

$$= 0$$

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* $\vec{B} = B_0 \cos\beta x \cos\omega t \; \hat{z}$

$$= \frac{B_0}{2}\left[\cos(\beta x + \omega t) + \cos(\beta x - \omega t)\right] \hat{z}$$

$$= \frac{B_0}{2}\left[\cos(-\beta x - \omega t) + \cos(\beta x - \omega t)\right] \hat{x}$$

$$\Downarrow$$

Superposition of two waves with same amplitude $(B_0/2)$ and frequency but one is propagating in $-x$ direction and the other in $+x$ direction. $\Rightarrow$

<u>Standing wave.</u>