
Classification of PDE and Canonical Form

MSO 203B

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1. Classify the following PDE's based on its Discriminant:

a) $x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} + xu_x + yu_y = 0$.

b) $x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} + xu_x + yu_y = u$

c) $u_{xx} + x^2 u_{yy} = 0$

d) $u_{tt} + x^2 u_{xt} = \sin(x)$

2. Reduce the equation

$$x^2 u_{xx} - 2xy u_{xy} + y^2 u_{yy} = 0$$

to its canonical form.

3. Find the solution of the equation

$$u_{xx} - 2 \sin x u_{xy} - \cos^2 x u_{yy} - \cos x u_y = 0$$

which satisfies $u(0, y) = f(y)$ and $u_x(0, y) = g(y)$.

4. Reduce the equation

$$u_{xx} + (1 + y^2)^2 u_{yy} - 2y(1 + y^2) u_y = 0$$

to its canonical form.

5. Prove the existence of infinitely many solutions to the problem

$$u_{tt} - 4u_{xx} = 0$$