Q 7) $$\frac{d^3y}{dt^3} + \frac{3d^2y}{dt^2} + 5\frac{dy}{dt} + y = \frac{d^3x}{dt^3} + \frac{4d^2x}{dt^2} + \frac{6dx}{dt} + 8x. \quad \rightarrow (i)$$

To find $Y(s)/X(s)$.

- Assumptions: initial condition are zero.

Taking laplace transform of all terms of equation (i)

$$s^3Y(s) + 3s^2Y(s) + 5sY(s) + Y(s) + \underbrace{\text{initial cond}^n \text{ involving } y(t)}_{\rightarrow 0}$$
$$= s^3X(s) + 4s^2X(s) + 6sX(s) + 8X(s) + \underbrace{\text{initial cond}^n \text{ involving } x(t)}_{\rightarrow 0}$$

$$\Rightarrow Y(s)\left(s^3 + 3s^2 + 5s + 1\right) = X(s)\left(s^3 + 4s^2 + 6s + 8\right)$$

$$\Rightarrow \boxed{\frac{Y(s)}{X(s)} = \frac{s^3 + 4s^2 + 6s + 8}{s^3 + 3s^2 + 5s + 1}}$$

Q8 Assumption initial cond^n are zero.

Q8) (a) $$\frac{X(s)}{F(s)} = \frac{7}{s^2 + 5s + 10}$$

$$\Rightarrow (s^2 + 5s + 10)X(s) = 7F(s)$$

$$\Rightarrow s^2X(s) + 5sX(s) + 10X(s) = 7F(s) \quad \rightarrow (i)$$

Taking inverse laplace transform of each term of eq^n (i)

$$\Rightarrow \left(\frac{d^2x(t)}{dt^2} + \frac{5dx(t)}{dt} + 10x(t) = 7f(t).\right) \rightarrow \text{D.E.}$$

(b) $$\frac{X(s)}{F(s)} = \frac{15}{s^2+21s+110}$$

$$\Rightarrow X(s)(s^2+21s+110) = 15F(s)$$

$$\Rightarrow s^2X(s) + 21sX(s) + 110X(s) = 15F(s) \rightarrow (i)$$

Taking inverse laplace transform of each term of eq^n (i)

$$\Rightarrow \left(\frac{d^2x(t)}{dt^2} + \frac{21dx(t)}{dt} + 110x(t) = 15f(t)\right) \rightarrow D.E.$$

(c) $$\frac{X(s)}{F(s)} = \frac{s+3}{s^3+11s^2+12s+18}$$

$$\Rightarrow (s^3+11s^2+12s+18)X(s) = (s+3)F(s)$$

$$\Rightarrow s^3X(s) + 11s^2X(s) + 12sX(s) + 18X(s) = sF(s) + 3F(s)$$

Taking inverse laplace transform

$$\Rightarrow \left(\frac{d^3x(t)}{dt^3} + \frac{11d^2x(t)}{dt^2} + \frac{12dx(t)}{d(t)} + 18x(t) = \frac{df(t)}{dt} + 3f(t)\right)$$

$\downarrow$

D.E.

Q 9)

$$R(s) \longrightarrow \boxed{\frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5}} \longrightarrow C(s)$$

Assumption initial condⁿ are zero.

$$\frac{C(s)}{R(s)} = \frac{s^5+2s^4+4s^3+s^2+4}{s^6+7s^5+3s^4+2s^3+s^2+5}$$

$$R(s)\left(s^5+2s^4+4s^3+s^2+4\right) = C(s)\left(s^6+7s^5+3s^4+2s^3+s^2+5\right)$$

Taking inverse laplace transform

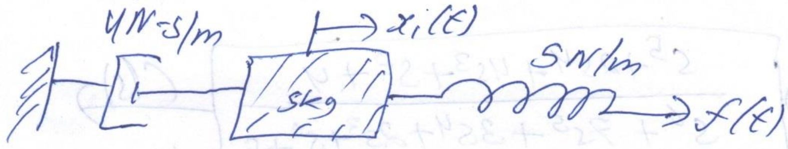
$$\frac{d^5r(t)}{dt^5} + \frac{2d^4r(t)}{dt^4} + \frac{4d^3r(t)}{dt^3} + \frac{d^2r(t)}{dt^2} + 4r(t)$$

$$= \frac{d^6c(t)}{dt^6} + \frac{7d^5c(t)}{dt^5} + \frac{3d^4c(t)}{dt^4} + \frac{d^2c(t)}{dt^2} + 5\,c(t) + \frac{2d^3c(t)}{dt^3}$$

$$\Rightarrow \boxed{\frac{d^5r}{dt^5} + \frac{2d^4r}{dt^4} + \frac{4d^3r}{dt^3} + \frac{d^2r}{dt^2} + r = \frac{d^6c}{dt^6} + \frac{7d^5c}{dt^5} + \frac{3d^4c}{dt^4} + \frac{2d^3c}{dt^3} + \frac{d^2c}{dt^2} + 5c}$$

D.E.

Q23)

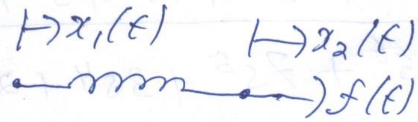


To find $G(s) = X_1(s)/F(s)$; $c = 4N\text{-}s/m$

$m = 5kg$

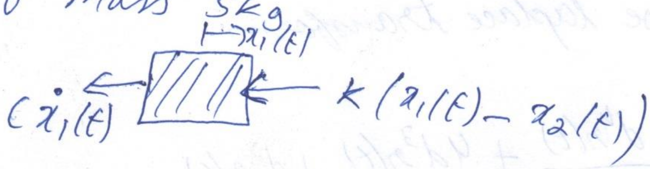
$k = 5N/m$

FBD of spring



$$f(t) = k(x_2(t) - x_1(t)) \rightarrow (1)$$

FBD of mass 5kg



$$k(x_2(t) - x_1(t)) - c\dot{x}_1(t) = m\ddot{x}_1(t)$$

$$\Rightarrow \quad f(t) = m\ddot{x}_1(t) + c\dot{x}_1(t) \quad \{\text{from } (1)\}$$

Taking laplace transform

$$F(s) = ms^2 X_1(s) + cs X_1(s)$$

$$\Rightarrow \quad G(s) = \frac{X_1(s)}{F(s)} = \frac{1}{ms^2 + cs}$$

$$\Rightarrow \quad \boxed{G(s) = \frac{1}{5s^2 + 4s}}$$

Chapter-2

24/ equations of motion.

$(s^2+s+1)X_1(s)-(s+1)X_2(s)=F(s)$

$-(s+1)X_1(s)+(s^2+s+1)X_2(s)=0$

Solving for $X_2(s)$

$$X_2(s)=\frac{(s+1)F(s)}{s^2(s^2+2s+2)}$$

From which

$$\frac{X_2(s)}{F(s)}=\frac{(s+1)}{s^2(s^2+2s+2)}$$

26/ $(s^2+6s+9)X_1(s)-(3s+5)X_2(s)=0.$

$-(3s+5)X_1(s)+(2s^2+5s+5)X_2(s)=F(s)$

solving for $X_1(s)$

$$X_1(s)=\frac{(3s+5)F(s)}{2s^4+17s^3+44s^2+45s+20}$$

Thus $G(s)=X_1(s)/F(s)$

$$=\frac{(3s+5)}{2s^4+17s^3+44s^2+45s+20}.$$

28/

(a) $(4s^2+8s+5)X_1(s)-8sX_2(s)-5X_3(s)=F(s)$

$-8sX_1(s)+(4s^2+16s)X_2(s)-4sX_3(s)=0.$

$-5X_1(s)-4sX_2(s)+(4s+5)X_3(s)=0.$

~~Solu~~ solving for $X_3(s)$

$$\frac{X_3(s)}{F(s)}=\frac{13s+20}{4s(4s^3+25s^2+43s+15)}$$

(b) $(8s^2+4s+16)X_1(s)-(4s+1)X_2(s)-15X_3(s)=0.$

$-(4s+1)X_1(s)+(3s^2+20s+1)X_2(s)-16sX_3(s)=F(s)$

$-15X_1(s)-16sX_2(s)+(16s+15)X_3(s)=0$

Solving for $X_3(s)$

$$\frac{X_3(s)}{F(s)}=\frac{128s^3+64s^2+316s+15}{384s^5+1064s^4+3476s^3+165s^2}$$

$\alpha = 2\zeta\omega_n$ ...

chapter-5

Ques-14

$T(s) = \dfrac{K}{s(s+30)}$; $T(s) = \dfrac{G(s)}{1+G(s)} = \dfrac{K}{s^2+30s+K}$

Therefore $2\zeta\omega_n = 30$;

Thus $\zeta = 15/\omega_n = 0.5912$ (i.e. 100% overshoot)

Hence $\omega_n = 25.37 = \sqrt{K}$

so $K = 643.6$.

25.

a. Writing the equation of motion yields, $(5s^2 + 5s + 28)X(s) = F(s)$

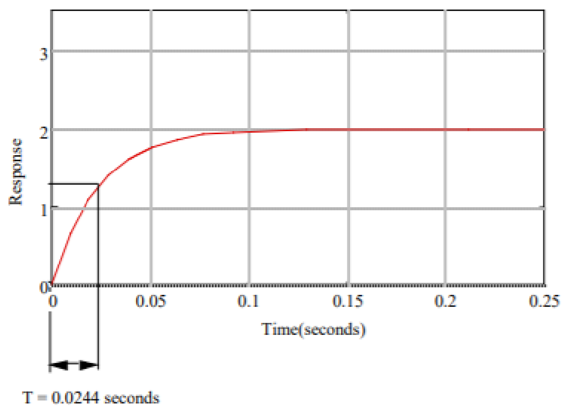
Solving for the transfer function,

$$\frac{X(s)}{F(s)} = \frac{1/5}{s^2 + s + \dfrac{28}{5}}$$

b. $\omega_n{}^2 = 28/5$ r/s, $2\zeta\omega_n = 1$. Therefore $\zeta = 0.211$, $\omega_n = 2.37$. $T_s = \dfrac{4}{\zeta\omega_n} = 8.01$ s; $T_P = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} =$ 1.36 s; $\%OS = e^{-\zeta\pi / \sqrt{1-\zeta^2}} \times 100 = 50.7$ %; $\omega_n T_r = (1.76\zeta^3 - 0.417\zeta^2 + 1.039\zeta + 1)$; therefore, $T_r = 0.514$ s.

29.

a. Measuring the time constant from the graph, T = 0.0244 seconds.

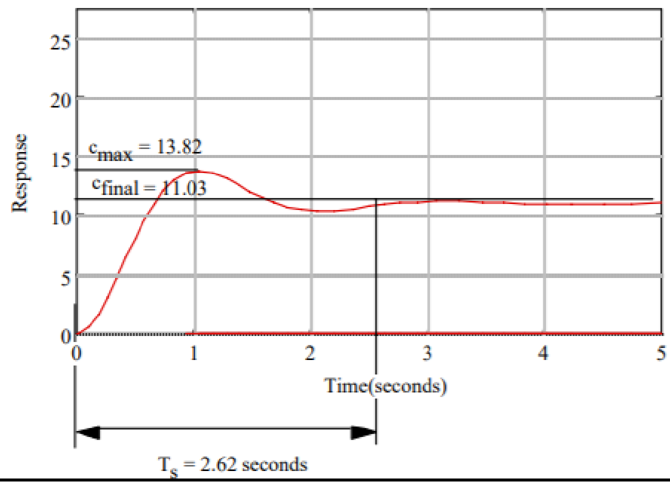


Estimating a first-order system, $G(s) = \frac{K}{s+a}$. But, $a = 1/T = 40.984$, and $\frac{K}{a} = 2$. Hence, K = 81.967.

Thus,

$$G(s) = \frac{81.967}{s+40.984}$$

b. Measuring the percent overshoot and settling time from the graph: %OS = (13.82-11.03)/11.03 = 25.3%,



and T_s = 2.62 seconds. Estimating a second-order system, we use Eq. (4.39) to find $\zeta = 0.4$, and Eq. (4.42) to find $\omega_n = 3.82$. Thus, $G(s) = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2}$. Since $C_{final} = 11.03$, $\frac{K}{\omega_n^2} = 11.03$. Hence,

$K = 160.95$. Substituting all values,

$$G(s) = \frac{160.95}{s^2+3.056s+14.59}$$

c. From the graph, %OS = 40%. Using Eq. (4.39), $\zeta = 0.28$. Also from the graph,

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 4$. Substituting $\zeta = 0.28$, we find $\omega_n = 0.818$.

Thus,

$$G(s) = \frac{K}{s^2+2\zeta\omega_n s+\omega_n^2} = \frac{0.669}{s^2+0.458s+0.669}.$$

.

33.

a.

(1) $$C_{a1}(s) = \frac{1}{s^2+3s+36} = \frac{\frac{1}{\sqrt{33.75}}\sqrt{33.75}}{(s+1.5)^2+33.75} = \frac{0.17213\sqrt{33.75}}{(s+1.5)^2+33.75} = \frac{0.17213\cdot 5.8095}{(s+1.5)^2+33.75}$$

Taking the inverse Laplace transform

$$C_{a1}(t) = 0.17213\, e^{-1.5t} \sin 5.8095t$$

(2) $$C_{a2}(s)s = \frac{2}{s\left(s^2+3s+36\right)} = \frac{1}{18}\frac{1}{s} - \frac{\frac{1}{18}s+\frac{1}{6}}{s^2+3s+36} =$$

$$\frac{1}{18}\frac{1}{s} - \frac{\frac{1}{18}\left(s+\frac{3}{2}\right) + \frac{0.083333}{\sqrt{33.75}}\sqrt{33.75}}{\left(s+\frac{3}{2}\right)^2+33.75}$$

$$= 0.055556\,\frac{1}{s} - \frac{0.055556\left(s+\frac{3}{2}\right)+0.014344\sqrt{33.75}}{\left(s+\frac{3}{2}\right)^2+33.75}$$

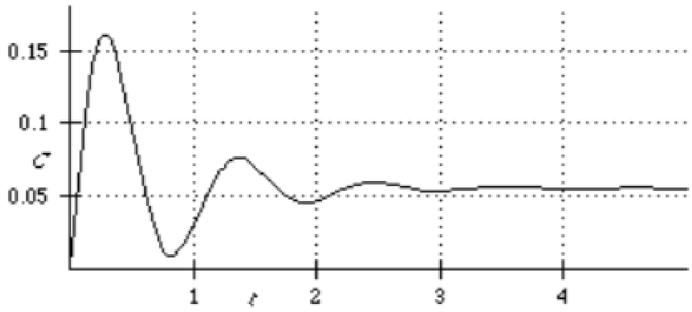
Taking the inverse Laplace transform

$$C_{a2}(t) = 0.055556 - e^{-1.5t}\,(0.055556 \cos 5.809t + 0.014344 \sin 5.809t)$$

The total response is found as follows:

$$C_{at}(t) = C_{a1}(t) + C_{a2}(t) = 0.055556 - e^{-1.5t}\,(0.055556 \cos 5.809t - 0.157786 \sin 5.809t)$$

Plotting the total response:



b.

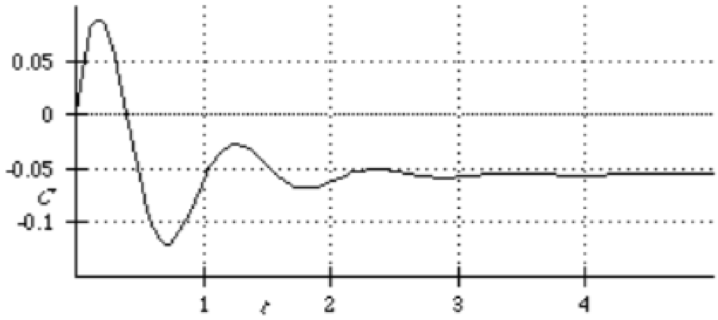
(1) Same as (1) from part (a), or $C_{b1}(t) = C_{a1}(t)$

(2) Same as the negative of (2) of part (a), or $C_{b2}(t) = -C_{a2}(t)$

The total response is

$C_{bt}(t) = C_{b1}(t) + C_{b2}(t) = C_{a1}(t) - C_{a2}(t) = -0.055556 + e^{-1.5t}\,(0.055556 \cos 5.809t + 0.186474 \sin$

$5.809t)$



Notice the nonminimum phase behavior for $C_{bt}(t)$.

53. Consider the un-shifted Laplace transform of the output

$$Y(s)=\frac{2.5(1+0.172s)(1+0.008s)}{s(1+0.07s)^2(1+0.05s)^2}=\frac{280.82(s+5.814)(s+125)}{s(s+14.286)^2(s+20)^2}$$

$$=\frac{A}{s}+\frac{B}{(s+14.286)^2}+\frac{C}{(s+14.286)}+\frac{D}{(s+20)^2}+\frac{E}{(s+20)}$$

$$A=\frac{280.82(s+5.814)(s+125)}{(s+14.286)^2(s+20)^2}\Big|_{s=0}=2.5$$

$$B=\frac{280.82(s+5.814)(s+125)}{s(s+20)^2}\Big|_{s=-14.286}=564.7$$

$$C=\frac{d}{ds}\frac{280.82(s+5.814)(s+125)}{s(s+20)^2}\Bigg|_{s=-14.286}=\frac{d}{ds}\frac{280.82s^2+36735.2s+204085.94}{s^3+40s^2+400s}\Big|_{s=-14.286}$$

$$=\frac{(s^3+40s^2+400s)(561.64s+36735.2)-(280.82s^2+36735.2s+204085.94)(3s^2+80s+400)}{(s^3+40s^2+400s)^2}\Big|_{s=}$$

$$=-219.7$$

$$D=\frac{280.82(s+5.814)(s+125)}{s(s+14.286)^2}\Big|_{s=-20}=640.57$$

$$E=\frac{d}{ds}\frac{280.82(s+5.814)(s+125)}{s(s+14.286)^2}\Bigg|_{s=-20}=\frac{d}{ds}\frac{280.82s^2+36735.2s+204085.94}{s^3+28.572s^2+204.09s}\Big|_{s=-20}$$

$$=\frac{(s^3+28.572s^2+204.09s)(561.64s+36735.2)-(280.82s^2+36735.2s+204085.94)(3s^2+57.144s+204.09)}{(s^3+28.572s^2+204.09s)^2}\Big|_{s=-20}$$

$$=217.18$$

thus

$$Y(s)=\frac{2.5}{s}+\frac{564.7}{(s+14.286)^2}-\frac{219.7}{(s+14.286)}+\frac{640.57}{(s+20)^2}+\frac{217.18}{(s+20)}$$

Obtaining the inverse Laplace transform of the latter and delaying the equation in time domain we get

$$y(t)=[2.5+564.7(t-0.008)e^{-14.286(t-0.008)}-219.7e^{-14.286(t-0.008)}+640.57(t-0.008)e^{-20(t-0.008)}$$
$$+217.18e^{-20(t-0.008)}]u(t-0.008)$$

Chapter Q5

Solution

Please visit section 6.2 and 6.3 for general rules about creating Routh Table. Remember, we can also multiply entire table by some positive constant without changing values of rows below.

For any G(s) given as

$$G(s) = \frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}$$

, the basic Routh table can be generated as The same can be checked from table 6.2 in your book.

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$-\frac{(a_4 a_1 - a_2 a_3}{a_3} = b_1$	$-\frac{(0 - a_3 a_0}{a_3} = b_2$	0
s^1	$-\frac{(a_3 b_2 - a_1 b_1}{b_1} = c_1$	0	0
s^0	$-\frac{(0 - c_1 b_2}{c_1} = d_1$	0	0

Table 1: Routh Table

Following this, the Routh table for problem is as described below

s^4	1	8	15	0
s^3	4	20	0	0
s^2	3	15	0	0
s^1	6	0	0	0
s^0	15	0	0	0

Table 2: Routh Table

Again as clear from the section 6.3, above table can be easily analysed for poles in LHP, RHP and on $j\omega$ axis. The same is as follows:

$$Even(2) = 2j\omega \tag{1}$$

$$Rest(2) = 2\text{Left Half plane} \tag{2}$$

$$Total = 2j\omega\text{axis plane, 2 Left half plane} \tag{3}$$

Chapter Q9

Again, check section 6.2 for creating Routh table, same can be obtained as follows So, it is clear form here that the

s^4	1	35	264
s^3	10	50	0
s^2	30	264	0
s^1	-38	0	0
s^0	264	0	0

Table 3: Routh Table

system has 2 poles in Right Half Plane and 2 in Left half plane. Also, first column has a sign change, stability of system can be checked from the same.

Chapter Q12

From the question, G(s) is given as

$$G(s) = \frac{K(s+2)}{s(s-1)(s+3)}$$

Using the same, the characteristic equation for the system can be written as

$$1 + K\frac{(s+2)}{s(s-1)(s+3)} = 0$$

The same can be further simplified as

$$\begin{aligned} s(s-1)(s+3) + K(s+2) &= 0 \\ s^3 - 2s^2 + (K-3)s + 2K &= 0 \end{aligned}$$

Using the last equation, we can generate Routh table as follows

s^3	1	K-3
s^2	2	2K
s^1	-3	0
1	2K	0

Table 4: Routh Table

Now, As can be seen from the above table, first column will always have a sign change, no matter what we choose value of K.

So, there is no value of K that stabilize the system

Chapter Q46

As in question, G(s) is given as

$$G(s) = \frac{1.3s^7 + 90.5s^6 + 1970s^5 + 15000s^4 + 3120s^3 - 41300s^2 - 5000s - 1840}{s^8 + 103s^7 + 1180s^6 + 4040s^5 + 2150s^4 - 8960s^3 - 10600s^2 - 1550s - 415}$$

Again following the same procedure, the Routh Table can be represented as

s^8	1	1800	2150	-10600	-415
s^7	103	4040	-8960	-15500	0
s^6	1140.7767	2236.99	-10584.95	-415	0
s^5	3838.023	-8004.29	-1512.53	0	0
s^4	4616.11	-10135.38	-415	0	0
s^3	422.685	-1167.48	0	0	0
s^2	2614.575	-415	0	0	0
s^1	-1100.39	0	0	0	0
s^0	-415	0	0	0	0

Table 5: Routh Table

a) From the above table, it can be clearly seen that system has 1 Right half plane, 7 left half plane and zero $j\omega$ axis
b) Now, since column has one sign change or simply 1 Right Half plane, this system defined by G(s) is not stable