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Introduction to Transport Phenomena

by

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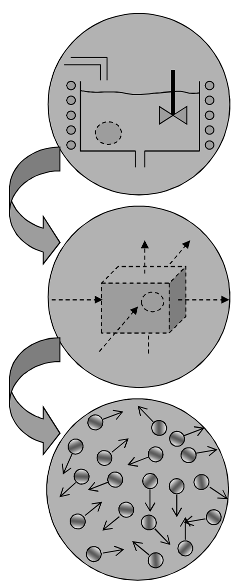
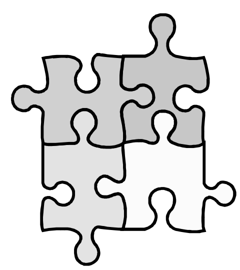


In this lecture…

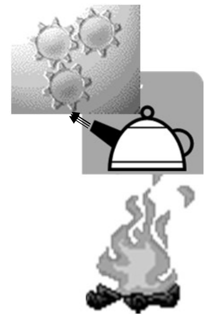
- Introduction to transport phenomena
- Types of species
  - momentum, energy and mass
- Molecular transport of species
- Microscopic transport of species
- Macroscopic transport of species
- Inter-relation between these levels
- Construction of the rate equations



Introduction

In engineering systems we encounter transport of species

Momentum – Energy – Mass



Transport of species is brought by various "forces"/ " driving potentials"

$$\text{(Net Change)} \propto \frac{\text{(Potential to change)}}{\text{(Resistance to change)}}$$

The relative magnitude of <u>one potential</u> with respect to <u>some other potential</u> will decide which change will dominate

Transport phenomena vs Thermodynamics

<u>Thermodynamics</u>

Global accounting of momentum, energy and mass

Feasibility of events under equilibrium conditions

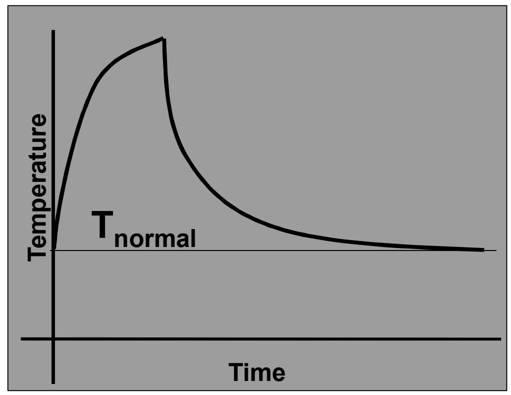
No consideration of the <u>mechanisms of species transport</u>

Does not tell us how to compute the <u>rate of species transport</u>

$$\eta = \frac{output}{input}$$

!! Transport of species is inherently a non-equilibrium process !!

Common experience: burning a finger



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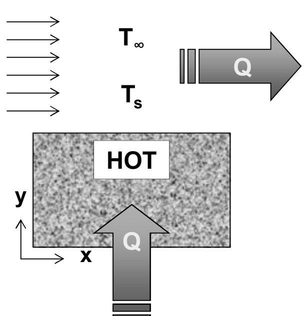
The Convection problem for interface transport

Typically we encounter solid-fluid boundaries

Net transport is linearly dependent on difference of driving potential at the interface



Newton's law of cooling

$$q'' = \frac{Q}{A_{cs}} = h \cdot \left(T_s - T_\infty\right) = -k_f \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

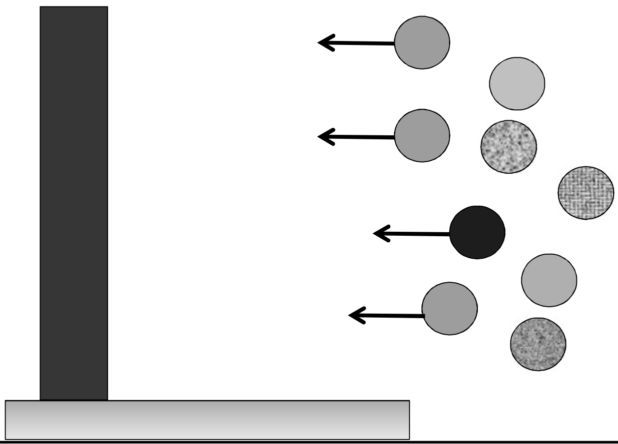
Similarly for mass transfer

$$N''_A = \frac{N}{A_{cs}} = h_m \cdot \left(C_{A,s} - C_{A,\infty}\right) = -D_{AB} \left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

Heat/Mass transfer coefficients are 'fudge' factors

Thermophyscial properties on which transport depends

Hot wall



Balls of different materials

Say,

Copper

Wood

Diamond

Cork

Potatoes

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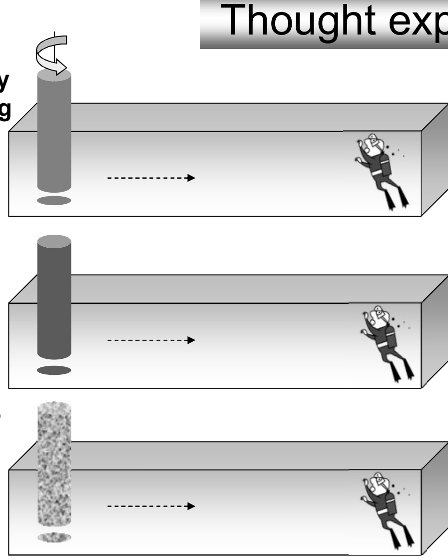
Thought experiment

Slowly turning

HOT

SALT



TRANSPORT OF

Momentum

Energy

Mass

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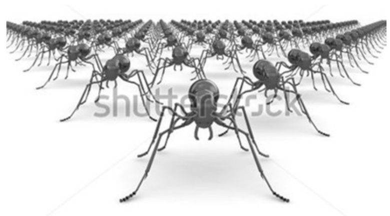
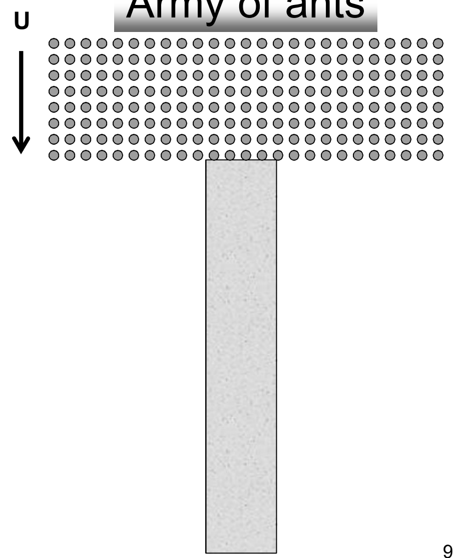
Army of ants





Transport phenomenon – holistic view

- **Length scales are different on the three levels**
- **Information must pass from molecular level to macroscopic level**

- **From <u>molecular description</u> we get viscosity thermal conductivity and diffusivity**
- **From <u>microscopic description</u> we find velocity, temperature and concentration,**
- **From <u>macroscopic description</u> we get bulk interactions and overall efficiency**

Species conservation laws are applicable at all levels

Conservation of momentum

Conservation of energy

Conservation of mass

Transport phenomenon – holistic view

Net Species Transport = $\sum$ **(Molecular Transport + Convective Transport)**

- **Momentum Transport** = Molecular Momentum Transport + Convective Momentum Transport
- **Energy Transport** = Molecular Energy Transport + Convective Energy Transport
- **Mass Transport** = Molecular Mass Transport + Convective Mass Transport

Diffusion terms **Advection terms**

Rate equations

How to solve problems?

One way to solve problems is to solve the (coupled)
MOMENTUM, ENERGY AND MASS TRANSPORT RATE EQUATIONS

Only simple problems have analytical solutions

Others need CFD/CHT for solving
the complex partial differential equations

The other way is "more" practical and also very useful

Use experimental data and dimensional analysis to formulate semi empirical/empirical equations

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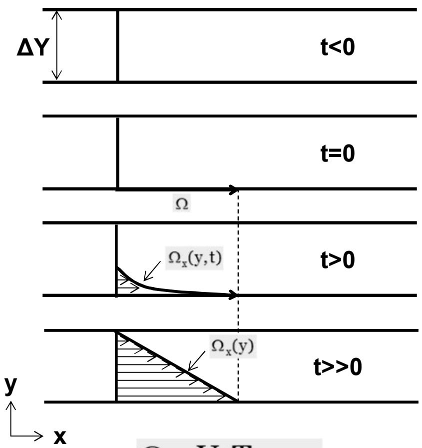
Molecular transport of species



$\Omega \sim V, T \text{ or } \omega$

Momentum ►► Newton' Law of viscosity

Energy ►► Fourier's Law of conductivity

Mass ►► Fick's Law of diffusion

Basic forms of equation
(One dimensional/incompressible)

$$\frac{F}{A} = \tau = -(\mu)\frac{\Delta V}{\Delta Y}$$

$$\frac{Q}{A} = q = -(k)\frac{\Delta T}{\Delta Y}$$

$$\frac{N_{Ay}}{A} = j_{Ay} = -(\rho D_{AB})\frac{\Delta \omega_{Ay}}{\Delta Y}$$

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Molecular transport of species

Momentum diffusivity = μ / ρ also called ν

Thermal diffusivity = $k / \rho C_p$ also called α

Mass diffusivity = D_{AB}

!! ALL HAVE THE SAME DIMENSIONS !!
$(\text{length})^2$ / (time)

Taking any two at a time to form non-dimensional numbers

The Prandtl Number = $\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu C_p}{k}$

The Schmidt Number = $\mathrm{Sc} = \dfrac{\nu}{D_{AB}} = \dfrac{\mu}{\rho D_{AB}}$

The Lewis Number = $\mathrm{Le} = \dfrac{\alpha}{D_{AB}} = \dfrac{k}{\rho C_p D_{AB}}$

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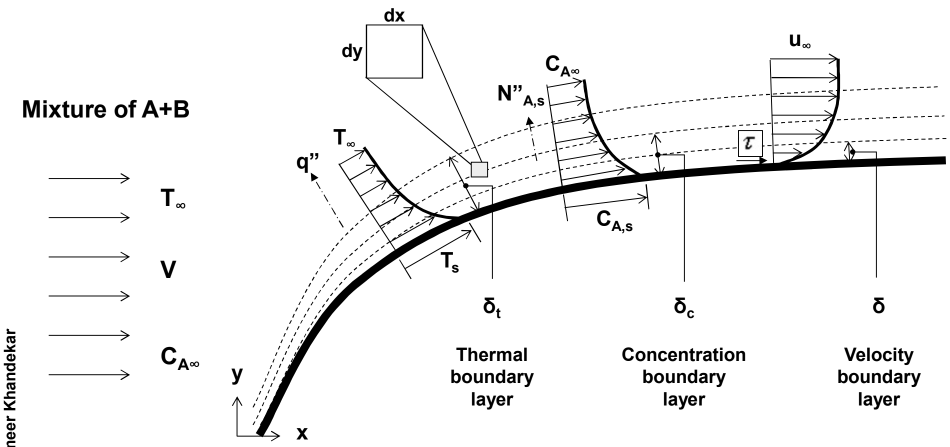
The concept of boundary layer



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Implications of the boundary layers

Velocity boundary layer	**► velocity gradients**	**►shear stress**
Thermal boundary layer	**►temperature gradients**	**►heat flux**
Concentration boundary layer	**►concentration gradients**	**►molar flux**

For practical engineering design and applications the direct implications are:

Momentum transfer	► ► Friction Factor (non-dimensional shear stress)
Energy transfer	► ► Nusselt Number (heat transfer coefficient)
Mass Transfer	► ► Sherwood Number (mass transfer coefficient)

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Scaling Laws

Connecting the molecular level to microscopic level

$$\frac{\delta}{\delta_t} \approx \Pr^n \approx \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}}$$

If Pr~1	►► $\delta \sim \delta_t$	(gases)
If Pr>>1	►► $\delta_t \ll \delta$	(Oils)
If Pr<<1	►► $\delta_t \gg \delta$	(liquid metals)

Similarly

$$\frac{\delta}{\delta_c} \approx Sc^n \approx \frac{\text{Momentum diffusivity}}{\text{Mass diffusivity}}$$

$$\frac{\delta}{\delta_t} \approx Le^n \approx \frac{\text{Thermal diffusivity}}{\text{Mass diffusivity}}$$

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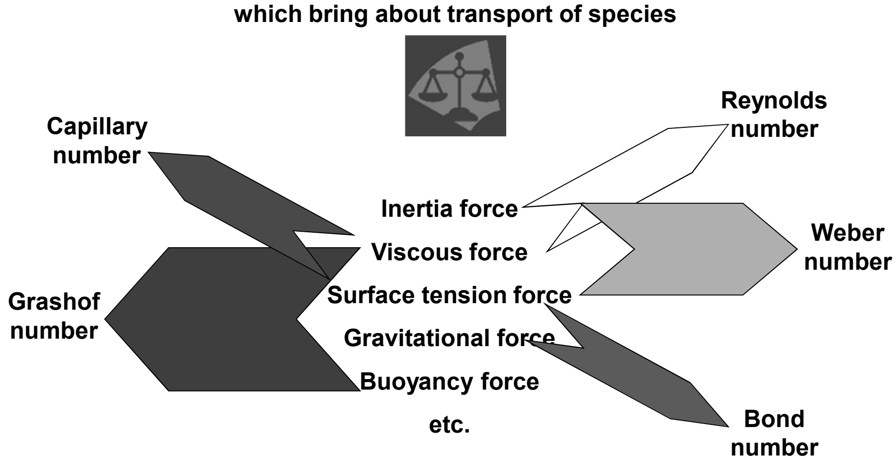
Scaling of transport mechanisms

Other "forces" OR "potentials" OR "effects"

which bring about transport of species



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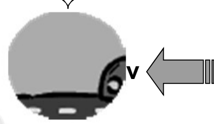
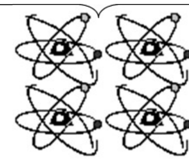
Rate Equations

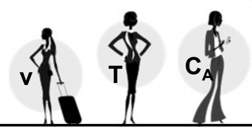
$$\rho\left(\frac{\partial v_i}{\partial t}+v_x\frac{\partial v_i}{\partial x}+v_y\frac{\partial v_i}{\partial y}+v_z\frac{\partial v_i}{\partial z}\right)=\mu\left(\frac{\partial^2 v_i}{\partial x^2}+\frac{\partial^2 v_i}{\partial y^2}+\frac{\partial^2 v_i}{\partial z^2}\right)-\frac{\partial p}{\partial x}\qquad \Big| i=x,y,z$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+v_x\frac{\partial T}{\partial x}+v_y\frac{\partial T}{\partial y}+v_z\frac{\partial T}{\partial z}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial C_A}{\partial t}+v_x\frac{\partial C_A}{\partial x}+v_y\frac{\partial C_A}{\partial y}+v_z\frac{\partial C_A}{\partial z}\right)=\rho D_{AB}\left(\frac{\partial^2 C_A}{\partial x^2}+\frac{\partial^2 C_A}{\partial y^2}+\frac{\partial^2 C_A}{\partial z^2}\right)$$







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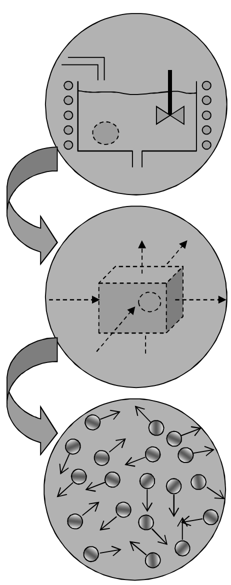
Summary and Conclusions

- **A holistic view of transport phenomena was provided**
- **“Thermodynamics” and “Transport Phenomena” are complimentary**
- **Momentum, energy and mass transport are guided by very similar laws**
- **Conservation of species is applicable in all cases**
- **Information at all levels: molecular, microscopic and macroscopic is needed**
- **Understanding can be improved by a unified study of specie transport**
- **Thermal/fluid management and design of thermal/fluid systems is indeed vital for the success of critical technologies and systems**

End of Lecture



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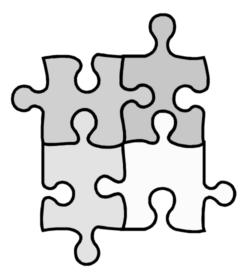
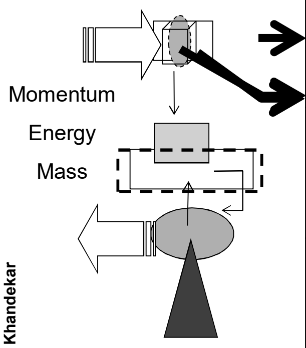
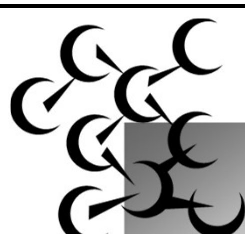


Transport phenomenon – holistic view

SYSTEM



	Momentum transport	Energy transport	Mass transport
Macroscopic transport/ description	**Macroscopic balances (isothermal)**	**Macroscopic balances (non-isothermal)**	**Macroscopic balances (mixtures)**
Microscopic transport/ description	**Friction factor (empirical)**	**Heat transfer coefficient (empirical)**	**Mass transfer coefficient (empirical)**
Molecular transport/ description	**Dynamic Viscosity (momentum flux tensor)**	**Thermal Conductivity (Heat flux vector)**	**Mass Diffusivity (Mass flux vector)**



Molecular transport

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Microscopic transport

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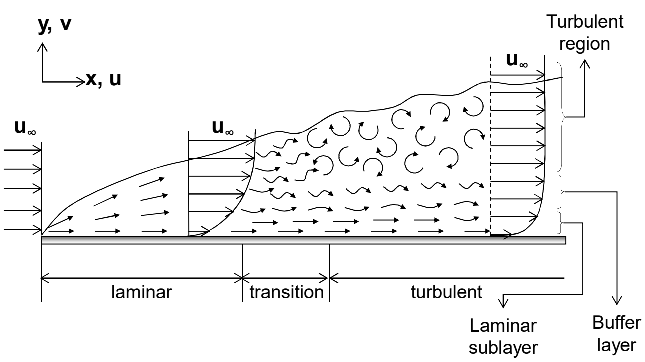
Microscopic transport parameters



Reynolds Number

$$\frac{\text{Inertia force}}{\text{Viscous force}}$$

$$\mathrm{Re} = \frac{(\rho \cdot V^2)}{(\mu \cdot V / L)} = \frac{\rho \cdot V \cdot L}{\mu}$$

Criterion for flow transition

LAMINAR ►TURBULENT

Laminar region ►Highly ordered flow ►streamlines

Turbulent region ► highly irregular, 3D ►enhanced species transport ► increased boundary layer thickness ►mixing►flatter velocity profile

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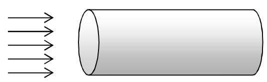
Microscopic transport parameters

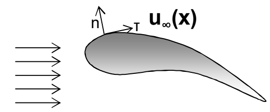
In engineering practice we usually encounter two types of flow

u_m
(mean velocity)



Internal flows:
flow confined in a conduit, pipe, duct, etc. of constant cross section

u
(upstream velocity)



External flows:
Flow past a flat plate, submerged bodies aerofoils, bluff bodies, etc.

Force exerted by fluid on the object

(a) Forces present even if the fluid is stationary (Buoyancy)

(b) Forces due to fluid motion (frictional drag and form drag)

The motion of fluid is due to kinetic energy and we are interested to know how much of this kinetic energy is manifested as pressure drop/shear stress

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Microscopic transport parameters

Internal flows = Darcy friction factor $f = \dfrac{-(dp/dx)D}{\frac{1}{2}\rho \cdot u_m^2}$

External flows = Coefficient of friction $C_f = \dfrac{\tau_s}{\frac{1}{2}\rho \cdot u_\infty^2} = \dfrac{\text{(Drag force per unit area)}}{\frac{1}{2}\rho \cdot u^2}$

(note the difference between basis velocities for kinetic energy scaling)

We can see that the functional form of these equations are in the form:

$$\boxed{f \text{ or } C_f = \frac{2}{\mathrm{Re}} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} \approx \mathrm{F^{\underline{n}}(Re)}}$$

!! The significance of this result should not be overlooked !!

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Microscopic transport parameters

Now coming back to the definition of heat and mass transfer coefficients

$$h \cdot \left(T_s - T_\infty\right) = -k_f \cdot \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$h_m \cdot \left(C_{A,s} - C_{A,\infty}\right) = -D_{AB} \left.\frac{\partial C_A}{\partial y}\right|_{y=0}$$

Again, it can be shown that the functional form of these equations are in the form:

$$h = \frac{k_f}{L} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} \qquad OR \qquad Nu = \frac{hL}{k_f} = \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

$$h_m = \frac{D_{AB}}{L} \left.\frac{\partial C_A^*}{\partial y^*}\right|_{y^*=0} \qquad OR \qquad Sh = \frac{h_m L}{D_{AB}} = \left.\frac{\partial C_{AB}^*}{\partial y^*}\right|_{y^*=0}$$

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Microscopic transport parameters

FINAL PICTURE WHICH EMERGES

$$C_f \approx \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

$$Nu \approx \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

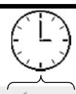
$$Sh \approx \left.\frac{\partial C_{AB}^*}{\partial y^*}\right|_{y^*=0}$$

The Nusselt Number is to the thermal boundary layer what the friction factor is to the velocity boundary layer

The Sherwood Number is to the concentration boundary layer what the Nusselt Number is to the thermal boundary layer

All three quantities manifest the non-dimensional gradients at the interface of the particular specie they respectively represent

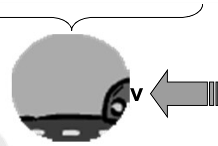
Rate Equations

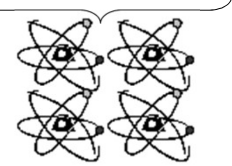


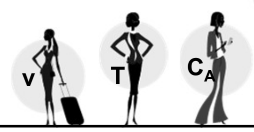
$$\rho \left(\frac{\partial v_i}{\partial t} + v_x \frac{\partial v_i}{\partial x} + v_y \frac{\partial v_i}{\partial y} + v_z \frac{\partial v_i}{\partial z} \right) = \mu \left(\frac{\partial^2 v_i}{\partial x^2} + \frac{\partial^2 v_i}{\partial y^2} + \frac{\partial^2 v_i}{\partial z^2} \right) - \frac{\partial p}{\partial x} \qquad \Big| i = x,y,z$$

$$\rho C_p \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

$$\rho \left(\frac{\partial C_A}{\partial t} + v_x \frac{\partial C_A}{\partial x} + v_y \frac{\partial C_A}{\partial y} + v_z \frac{\partial C_A}{\partial z} \right) = \rho D_{AB} \left(\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} \right)$$







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