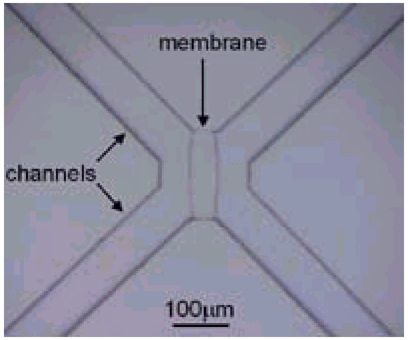
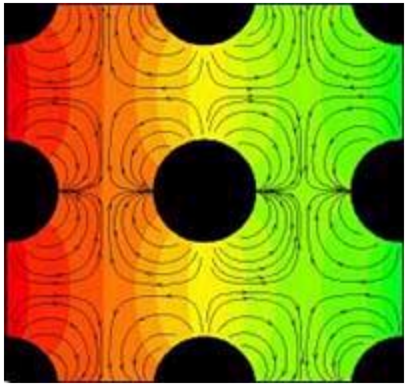
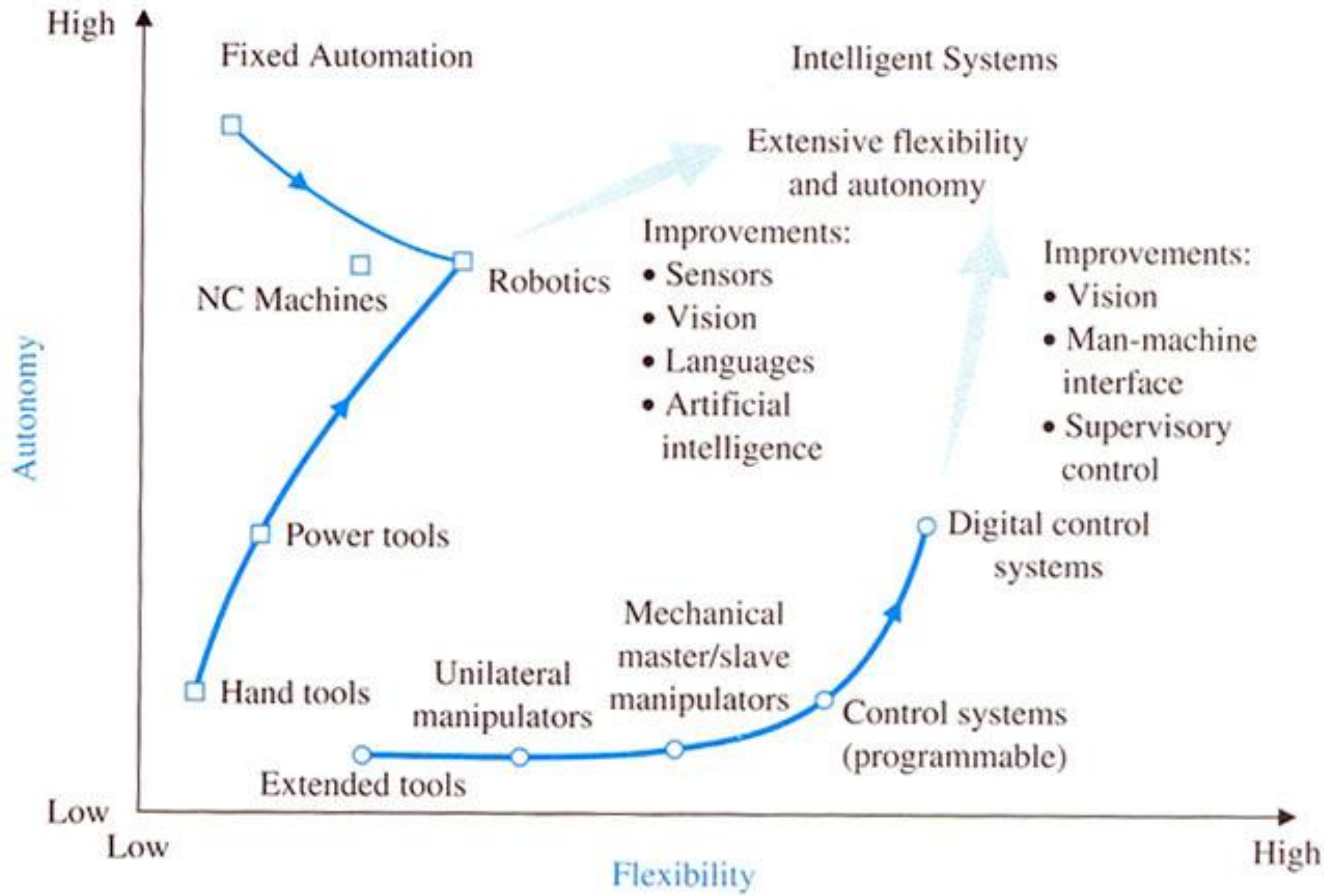


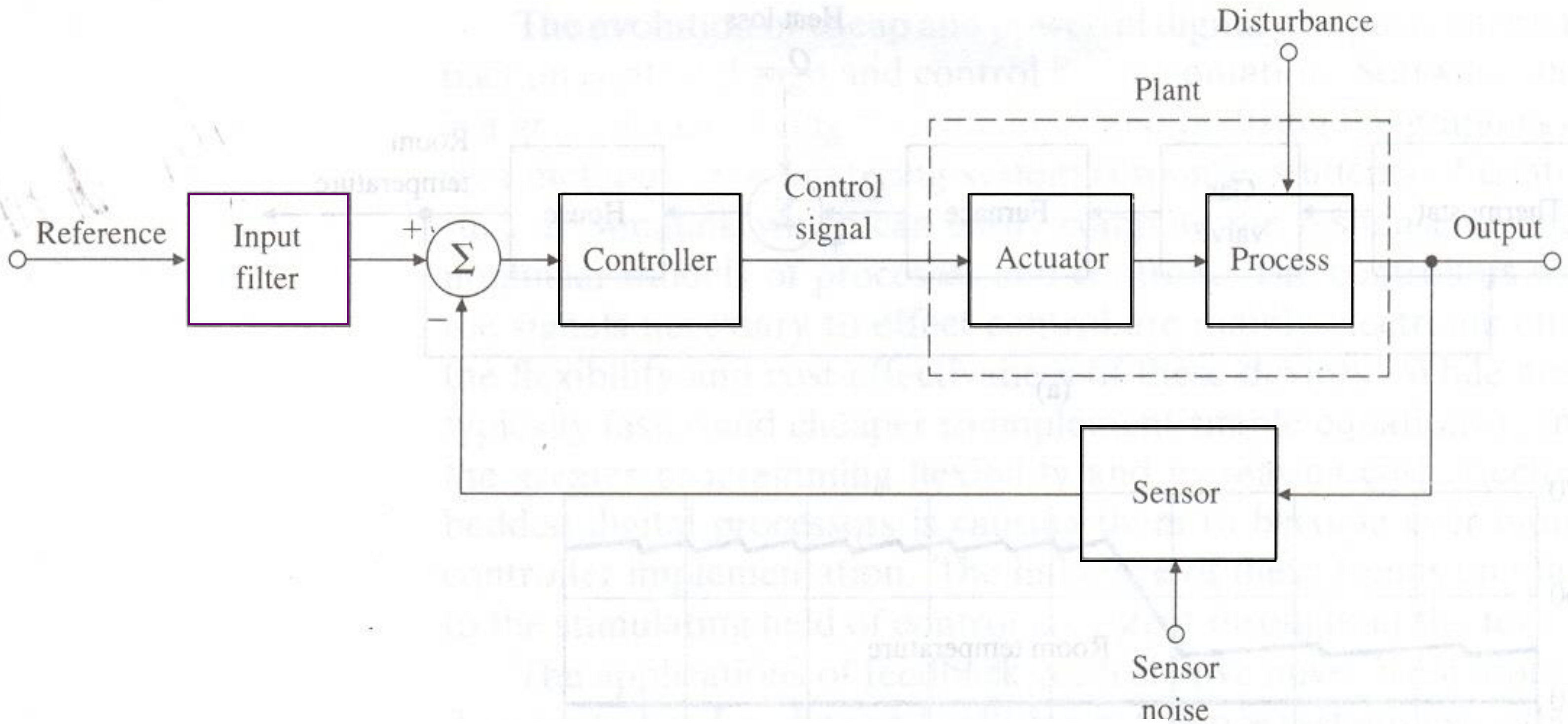


Basic System Modelling

SMART MATERIALS & STRUCTURES LABORATORY
IIT-KANPUR



Future evolution of control systems and robotics.



Closed Loop Control of a Plant

Modelling a Dynamical System

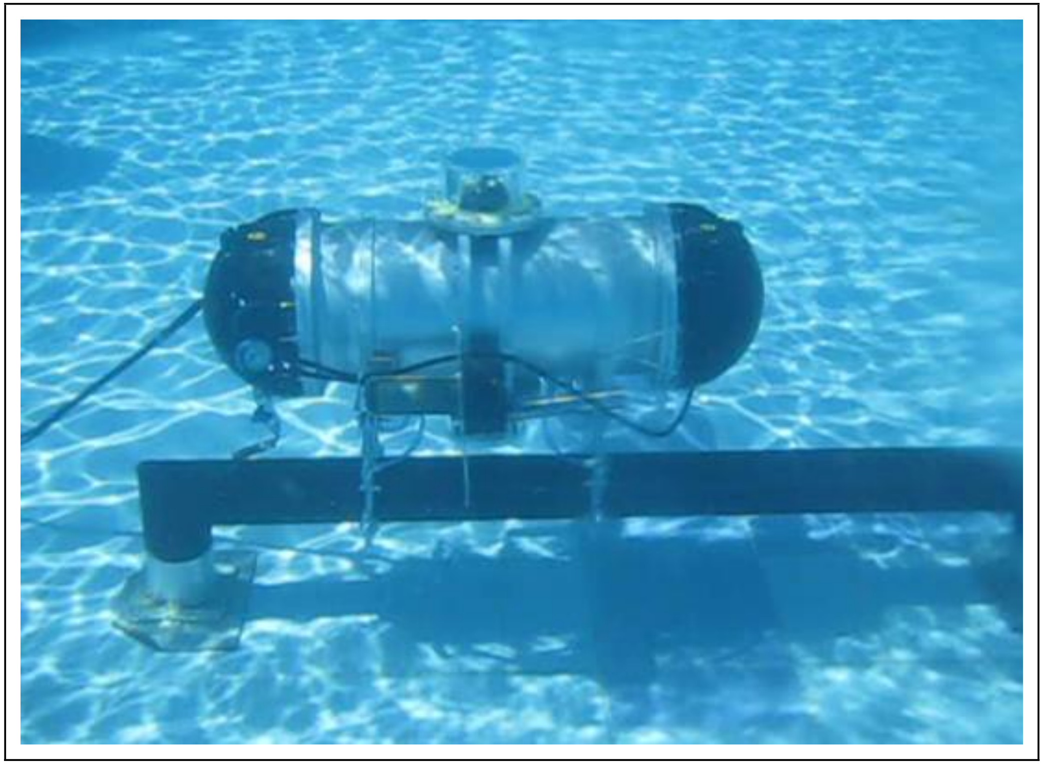
*Models are a mathematical **representations** of system dynamics*

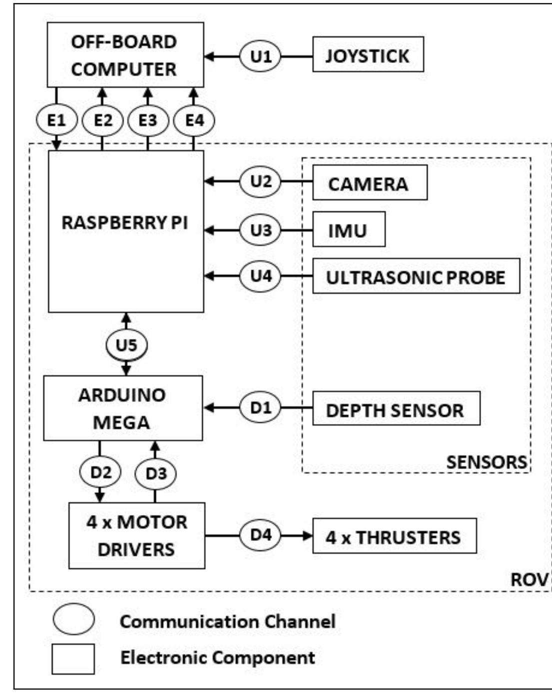
Models allow the dynamics to be simulated and analyzed, without having to build the system

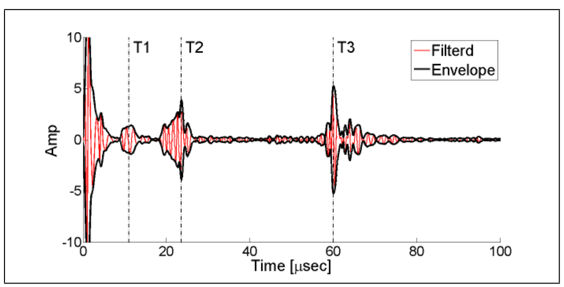
Models are never exact, but they can be predictive
Models can be used in ways that the system can’t perform
Certain types of analysis (eg, parametric variations) can’t easily be done on the actual system

Models can be run much more quickly
The model you use depends on the questions you want to answer

<u>*A single system may have many models*</u>




OFF-BOARD COMPUTER
JOYSTICK
U1
E1
E2
E3
E4
RASPBERRY PI
U2
CAMERA
U3
IMU
U4
ULTRASONIC PROBE
U5
ARDUINO MEGA
D1
DEPTH SENSOR
SENSORS
D2
D3
4 x MOTOR DRIVERS
D4
4 x THRUSTERS
ROV
Communication Channel
Electronic Component


T1
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Amp
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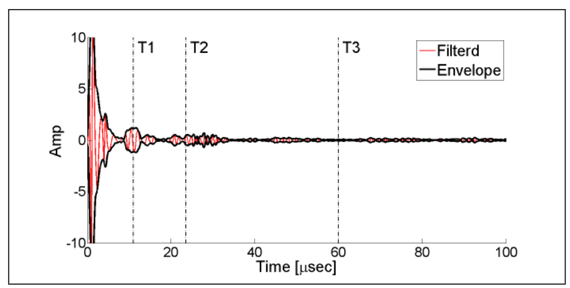
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T2
T3
Filterd
Envelope
Amp
Time [μsec]

A system is modelled either

•by defining the input-output characteristics or

•by selecting a set of state variables

Inputs describe the external excitation of the dynamics. Inputs are extrinsic to the system dynamics (externally specified). Constant inputs are often considered to be parameters

Outputs are variables that are to be calculated or measured

Input-output relation is often governed by a generalised ODE of the form

$$a_n y^{(n)} + \ldots.. + a_2 y^{(2)} + a_1 y^{(1)} + a_o y = b_m u^{(m)} + \ldots. + b_1 u^{(1)} + b_o u(t)$$

Where $y^{(n)} = d^n y/dt^n$, $u^{(m)} = d^{(m)} u/dt^m$

Choice of inputs and outputs depends on point of view

Inputs: what factors are external to the model that you are building

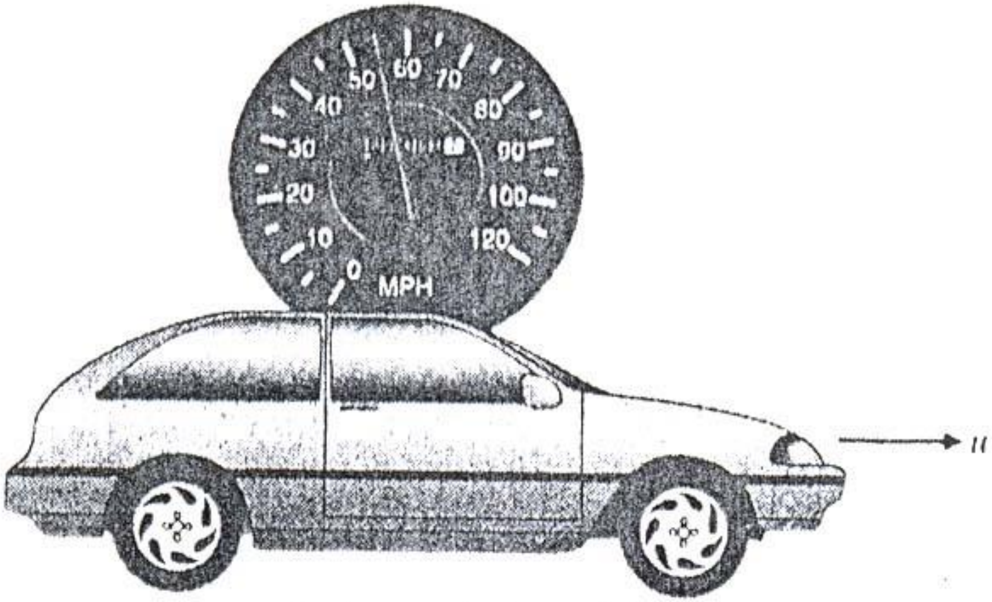
Inputs in one model might be outputs of another model (eg, the output of a cruise controller provides the input to the vehicle model)

Outputs: what physical variables (often states) can you measure

Choice of outputs depends on what you can sense and what parts of the component model interact with other component models

States: Chosen such that their values at a reference time and the corresponding inputs are known. May or may not include Outputs.

A Cruise Control Model



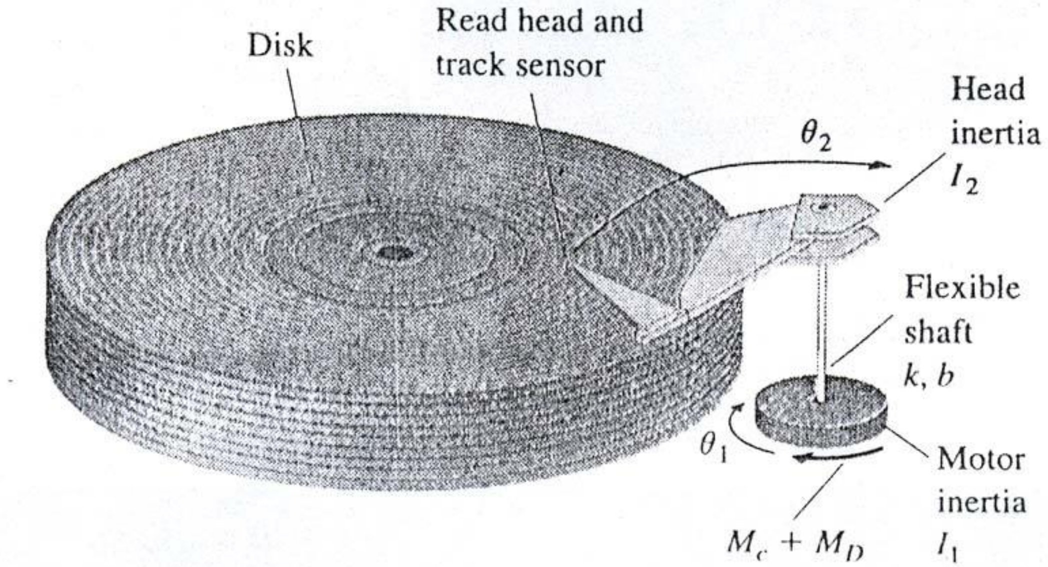
Cruise-control model

EOM?

Transfer function?

Disk Reading

Disk read/write head schematic for modeling



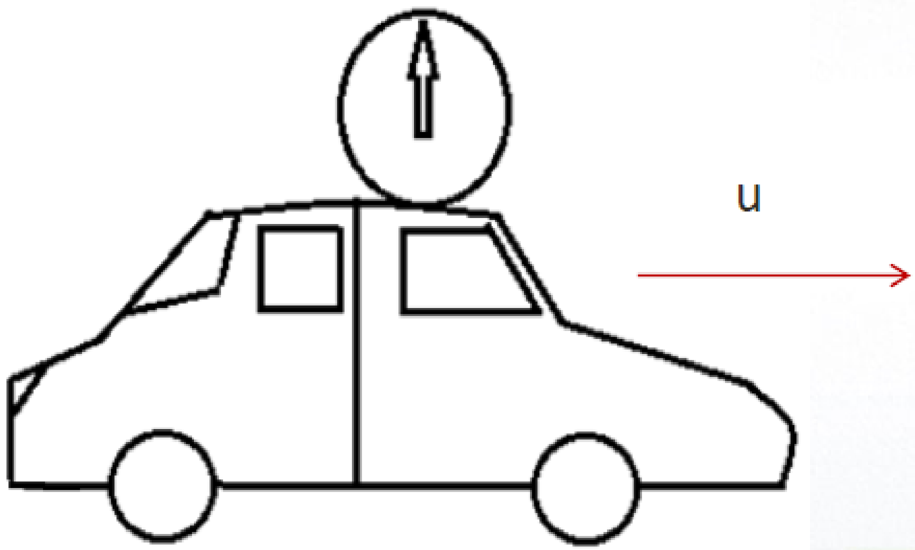
Non-collocated sensing

Input M_c

Output Θ_2

Let us consider first the problems mooted in the last lecture

Problem 1:



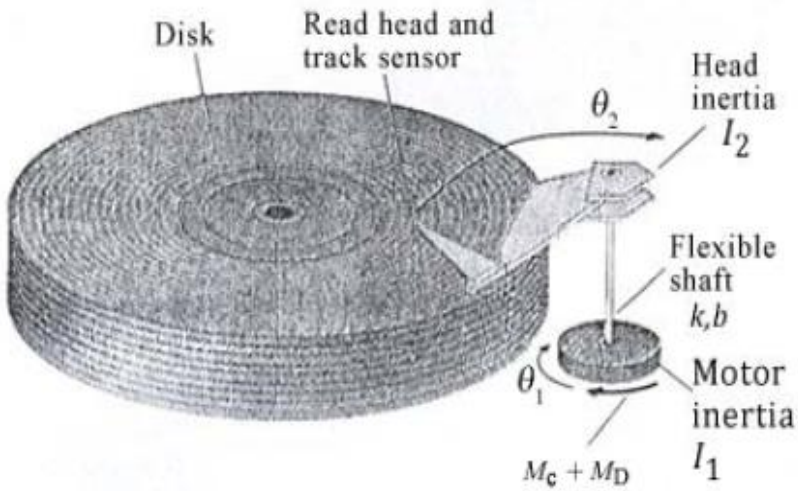
From the cruise control perspective: we can have a simplified model for the automobile system.

Assuming *M* to be the mass of the car and *u* to be the velocity, the governing EOM could be written as:

$$M\dot{u} + Du = F(t)$$

Where, *D* is the Drag /Resistance Coefficient and *F(t)* is the cruising force.

Problem 2:



Input M_c
Output θ_2

Again, a simplified model could be developed by neglecting the disturbance, the EOM will be (*C* – shaft damping coefficient and *k* shaft stiffness)

$$I_2 \ddot{\theta}_2 + C(\dot{\theta}_2 - \dot{\theta}_1) + k(\theta_2 - \theta_1) = M_c(t)$$

$$I_1 \ddot{\theta}_1 + C(\dot{\theta}_1 - \dot{\theta}_2) + k(\theta_1 - \theta_2) = 0$$

System Transfer Functions

- The last two examples relate to the description of the systems in Time Domain.
- By Laplace transformation one can convert the equations into frequency domain.
- Assuming zero initial condition, the first mathematical model could be expressed as :

$$sMU(s)+BU(s)=\bar{F}(s)$$

$$\frac{U(s)}{\bar{F}(s)}=\frac{1}{sM+B}$$

- Similarly, the second mathematical model could be expressed as:

$$\frac{\theta_2(s)}{\bar{M}_c}=\frac{1}{s^2 I_2+sC+k}$$

The RHS of both the equations are known as Transfer Function that describes the relationship between output and input. Example 1, pertains to a first order transfer function, while Example 2 presents a second order transfer function. Such relationship could be graphically represented for better perception and this technique will be discussed in this lecture.

State-Space Modelling

The *state* of a model of a dynamic system is a set of independent physical quantities, the specification of which (in the absence of excitation) completely determines the future evolution of the system

Dynamics describes how the state evolves. The *dynamics* of a model is an update rule for the system state that describes how the state evolves, as a function on the current state and any external inputs

When we talk about electro-mechanical systems modeled by differential equations, such as masses and springs, electric circuits or satellites (rigid bodies) rotating in space, we can attach some additional intuition: the variables in the state should be adequate to specify the **energy** of the system.

For example, take a ball free-falling to earth: we can specify the position of the ball by specifying the height (h) above the ground, but we also need to include the velocity of the ball (dh/dt) to specify the total energy ($E = 1/2*m*(dh/dt)^2 + mgh$). Therefore, the state of the ball is (h,dh/dt).

State Space Modelling of a Single Degree of Freedom System

- **Consider a SDOF system (with mass M, stiffness K and Damping constant C) such that the equation of motion corresponding to force excitation is given by:**

$$M\ddot{x}+C\dot{x}+Kx=F(t)$$

- **The following pair of states or their linear combinations could be considered for the modelling:**

$$\begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}, \begin{Bmatrix} x \\ \ddot{x} \end{Bmatrix}, \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix}$$

The EOM in State Space Form

- **Consider for example, the position and velocity as the state coordinates.**
- **The state vector could be written as:**

$$X = \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix}$$

- **Based on these states, the EOM could be rewritten as:**

$$\frac{d}{dt}\begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix}\begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix}(f/m)$$

$$\dot{X} = AX + BU$$

$$A = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix},\ B = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix},\ U = f/m$$

Output of a State-space System

- Many a times states of a system are not directly measurable and hence are not of direct interest. For example, if you consider, state-space representation of a finite element model pertaining to a Spacecraft. The number of states could be as high as three to four thousand! However, one cannot have so many sensors to measure all the states. In such cases, we fix a feasible number of outputs that are observable/measurable.
- Suppose for a system of *n*-states there are *r* outputs that are measurable. Then the output vector Y(t) of size *r* could be represented as a linear combination of input to the system and the states as follows:

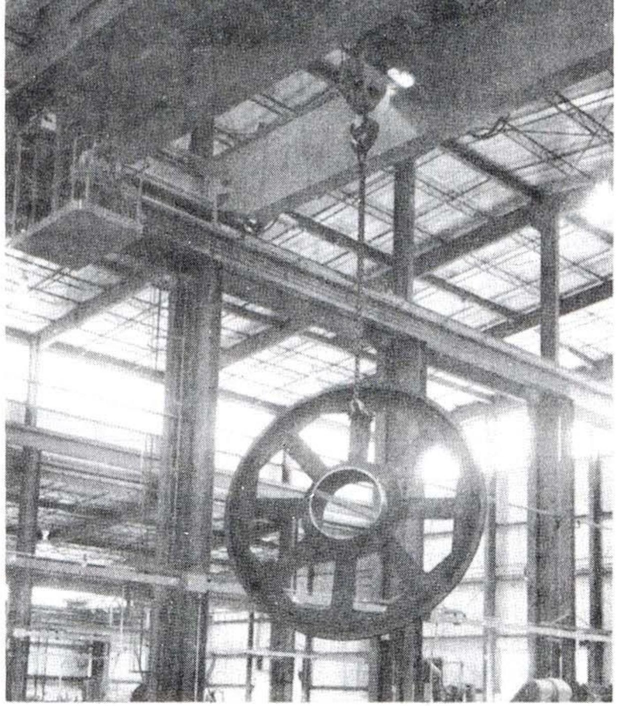
$$Y(t) = \begin{Bmatrix} y_1(t) \\ y_2(t) \\ . \\ . \\ y_r(t) \end{Bmatrix} = C\,X(t) + D\,U(t)$$

Where C &D are constants for an LTIV system. For majority of dynamic systems it is observed that D = 0, meaning outputs are not directly affected by the system inputs.

Time Domain Solution for a Vector State Equation

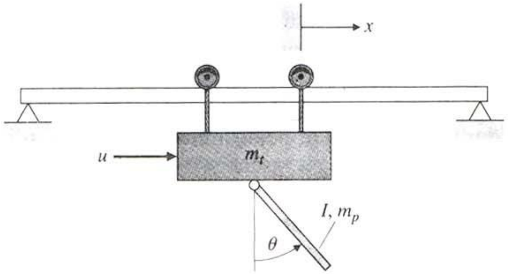
- $X(t) = e^{At} X_o + \int^t_o e^{A(t-\tau)} B\, U(t)\, dt$
- $e^{At} = I + At + (At)^2/2! + (At)^3/3!$
- $X(s) = (sI-A)^{-1} B\, U(s)$
- Find out the eigen values and eigen vectors of sI – A, Obtain the transformation matrix and convert the state matrix into diagonal form
- Solve using a Discrete Time -Model

load (*Photo courtesy of Harnischfeger Corporation, Milwaukee, Wisconsin*)



Can you model?

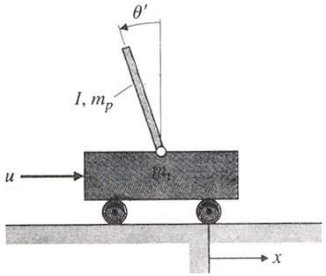
Schematic of the crane with hanging load



$\Theta = 0$

and

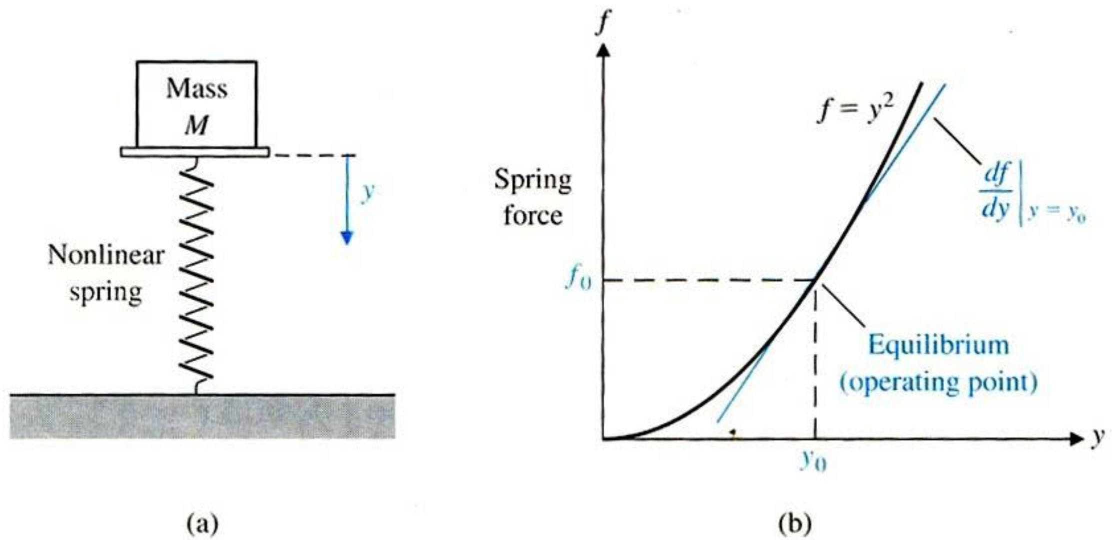
Inverted pendulum



$\Theta = \pi$

Nonlinearity

(a) A mass sitting on a nonlinear spring. (b) The spring force versus *y*.



$Y=g(x_0) + dg/dx|_{x=x_0} (x-x_0)$

When the principle of superposition and homogeneity gets violated

Linearizable for mechanical & electrical elements

Linearity: Zero at origin, law of addition and multiplication

Find the state-space and T.F. representations

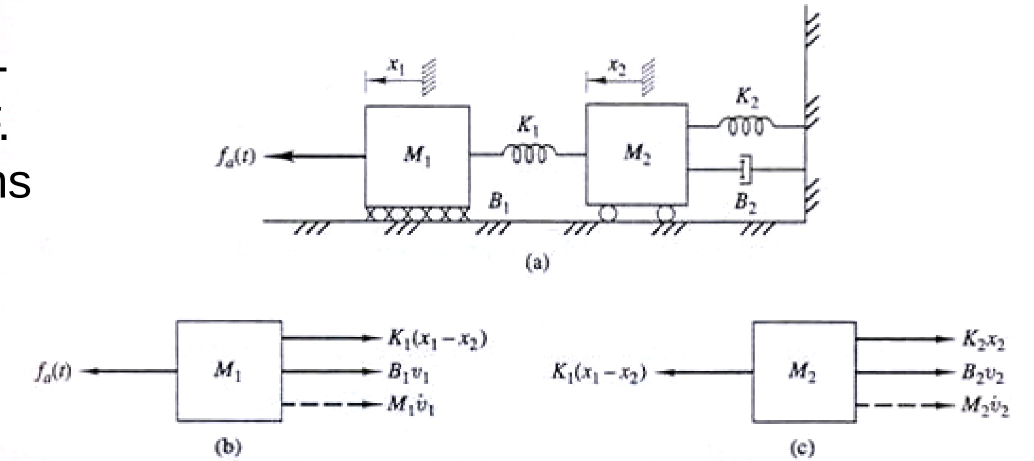
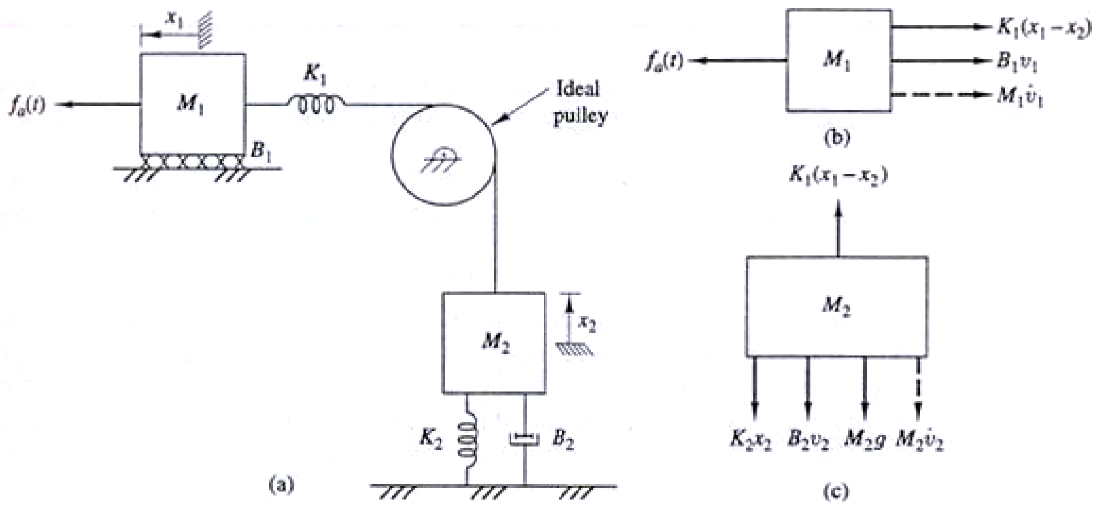
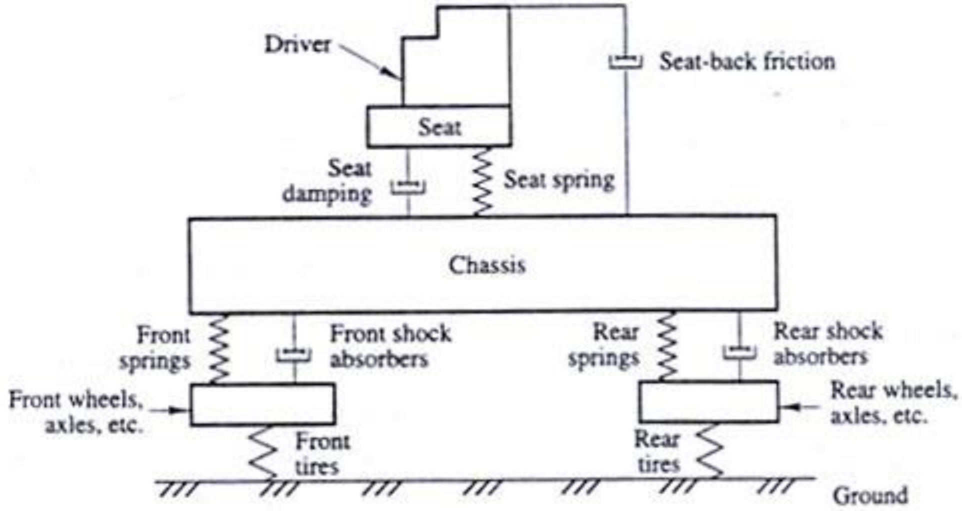
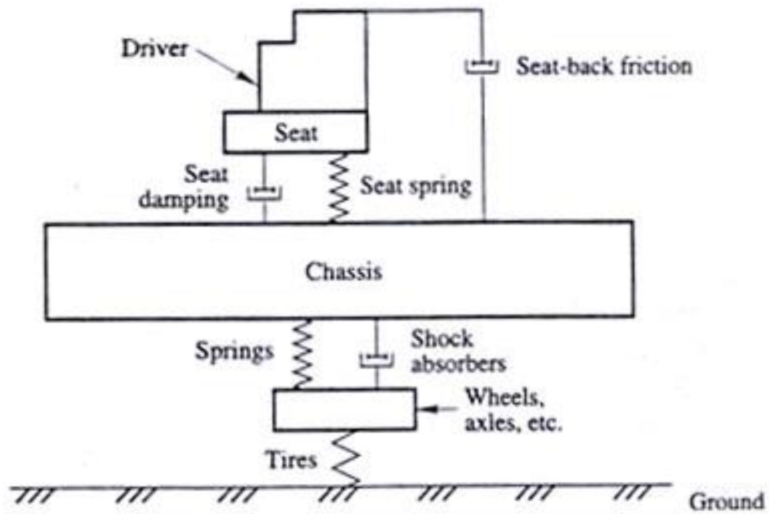


Figure 2.18 (a) Translational system for Example 2.7. (b), (c) Free-body diagrams.

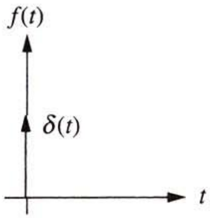
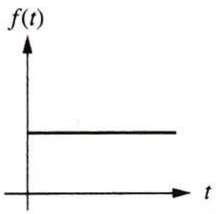
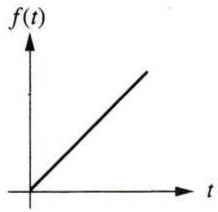
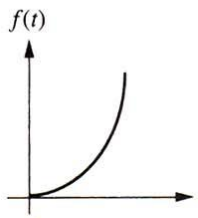
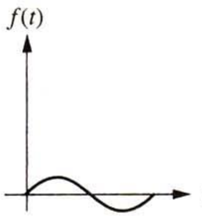



Driver
Seat-back friction
Seat
Seat damping
Seat spring
Chassis
Front springs
Front shock absorbers
Rear springs
Rear shock absorbers
Front wheels, axles, etc.
Rear wheels, axles, etc.
Front tires
Rear tires
Ground

(a)


Driver
Seat-back friction
Seat
Seat damping
Seat spring
Chassis
Springs
Shock absorbers
Wheels, axles, etc.
Tires
Ground

Test waveforms used in control systems

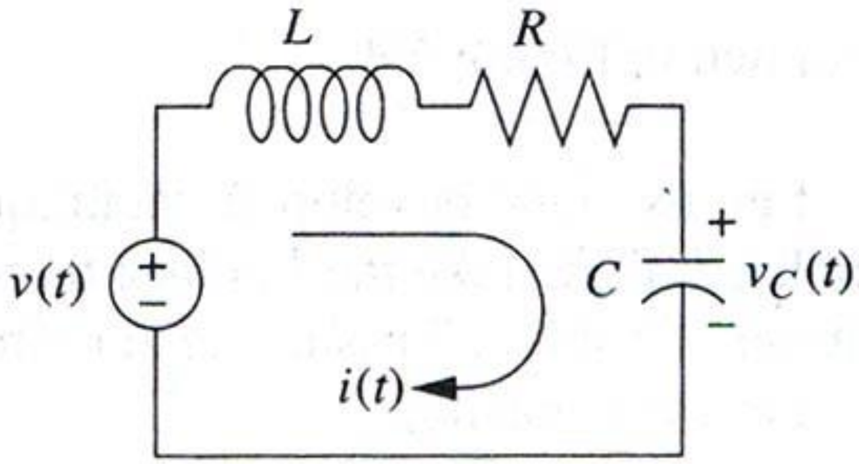
Input	Function	Description	Sketch	Use
Impulse	$\delta(t)$	$\delta(t) = \infty$ for $0- < t < 0+$ $= 0$ elsewhere $\int_{0-}^{0+} \delta(t)\, dt = 1$		Transient response Modeling
Step	$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient response Steady-state error
Ramp	$tu(t)$	$tu(t) = t$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Parabola	$\frac{1}{2}t^2u(t)$	$\frac{1}{2}t^2u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-state error
Sinusoid	$\sin \omega t$			Transient response Modeling Steady-state error

Modelling of Electrical Elements

- **Kirchoff's Current Law: The algebraic sum of currents leaving a junction or node equals the algebraic sum of currents entering the node.**

- **Kirchoff's Voltage Law: The algebraic sum of all voltages taken around a closed path in a circuit is zero.**

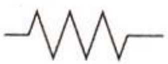
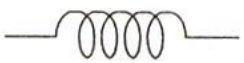
Modelling of Electrical System



RLC network

Impedance based representation

Voltage-current, voltage-charge, and impedance relationships for capacitors, resistors, and inductors

Component	Voltage-current	Current-voltage	Voltage-charge	Impedance $Z(s) = V(s)/I(s)$	Admittance $Y(s) = I(s)/V(s)$
 Capacitor	$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$v(t) = \frac{1}{C}q(t)$	$\frac{1}{Cs}$	Cs
 Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	$v(t) = R\frac{dq(t)}{dt}$	R	$\frac{1}{R} = G$
 Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau$	$v(t) = L\frac{d^2q(t)}{dt^2}$	Ls	$\frac{1}{Ls}$

Note: The following set of symbols and units is used throughout this book: $v(t)$ = V (volts), $i(t)$ = A (amps), $q(t)$ = Q (coulombs), C = F (farads), R = Ω (ohms), G = ℧ (mhos), L = H (henries).

THANK YOU