

Solution for Q2

Solid steel bar, dia = 50 mm

Polar moment of inertia, $J_B = \frac{\pi \times 50^4}{32} \text{ mm}^4$ → 2

Hollow steel tube, $J_T = \frac{\pi \times (75^4 - 60^4)}{32} \text{ mm}^4$ → 2

Torque $M_t = 2 \text{ kN}\cdot\text{m}$

Compatibility, $\phi_B = \phi_T$ → 1 (a)

Equilibrium, $M_B + M_T = M_t$ → 1 (b)

$$\begin{aligned} J_B &= 6.14 \times 10^5 \text{ mm}^4 \\ J_T &= 1.83 \times 10^6 \text{ mm}^4 \end{aligned}$$

Eqn. (a) gives

$$\frac{M_B L}{G J_B} = \frac{M_T L}{G J_T} \rightarrow 2$$

$$\therefore M_B = M_T \cdot \frac{J_B}{J_T} = \frac{(50)^4}{(75^4 - 60^4)} M_T = 0.33 M_T$$

Therefore from eqn. (b) $1.33 M_T = 2$

$$\therefore \begin{aligned} M_T &= 1.5 \text{ kN}\cdot\text{m} \\ M_B &= 0.5 \text{ kN}\cdot\text{m} \end{aligned} \rightarrow 2$$

resses: in bar

$$\tau_{\theta z} = \frac{0.5 \times \frac{25}{1000}}{J_B} \times (10^3)^4 = 2.037 \times 10^4 \text{ kN/m}^2 \rightarrow 2$$

in tube, $\tau_{\theta z} = \frac{1.5 \times \frac{37.5}{1000}}{J_T} \times (10^3)^4 = 3.067 \times 10^4 \text{ kN/m}^2 \rightarrow 2$

ngle of rotation of plate

$$\phi = \frac{0.5 \times 10^3 \times 0.65}{80 \times 10^9 \times 6.14 \times 10^{-7}} = 6.616 \times 10^{-3} \text{ rad} \rightarrow 2$$

rsional stiffness

$$K_\theta = \frac{M_t}{\phi} = \frac{2 \times 10^3}{6.616 \times 10^{-3}} = 3.02 \times 10^5 \text{ N}\cdot\text{m/rad} \rightarrow 2$$