ESO 201A: Thermodynamics

2016-2017-I semester

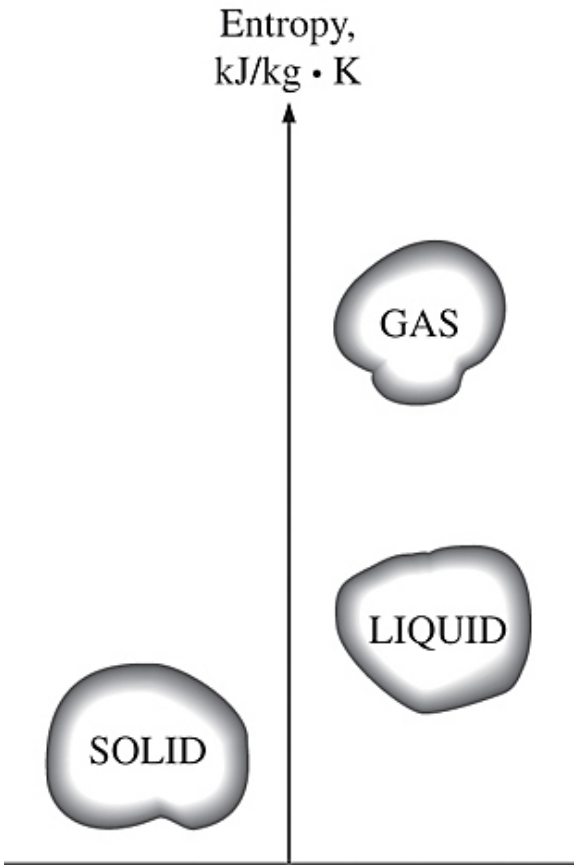
Entropy: part 3

Dr. Jayant K. Singh
Department of Chemical Engineering
Faculty Building 469,
Telephone: 512-259-6141
E-Mail: jayantks@iitk.ac.in
home.iitk.ac.in/~jayantks/ESO201/index.html

Learning objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

What is Entropy?

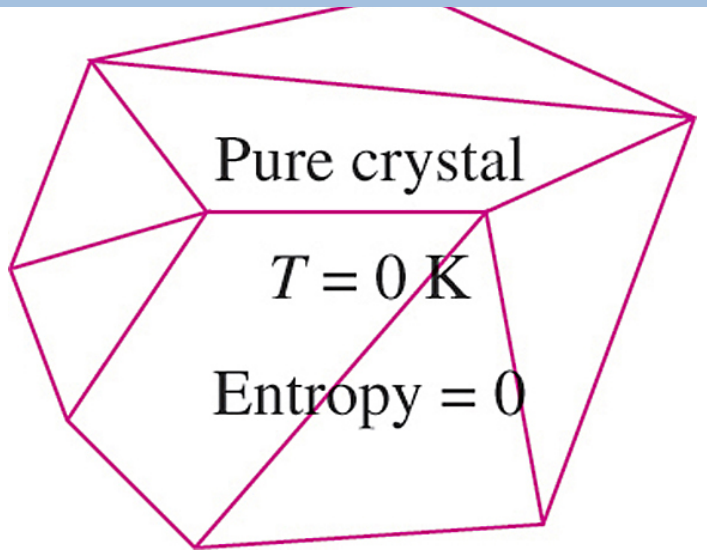


The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.

Boltzmann relation

$$S = k \ln \Omega$$

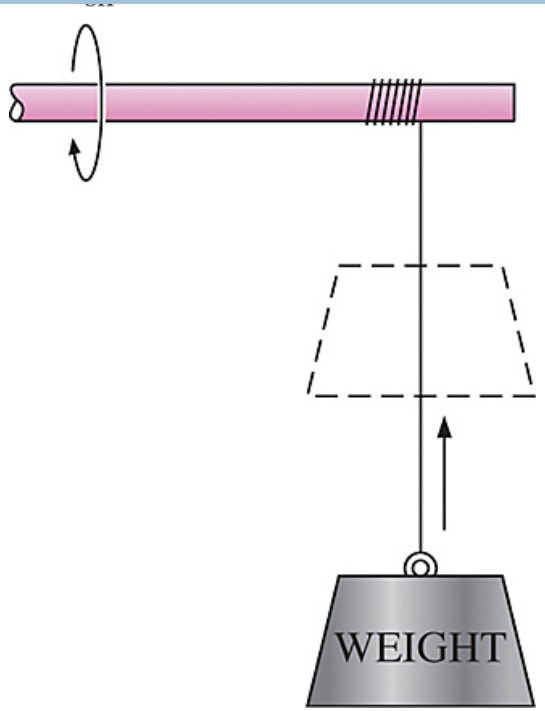
$$k = 1.3806 \times 10^{-23}\ \text{J/K}$$



A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero (**the third law of thermodynamics**).

Disorganized energy does not create much useful effect, no matter how large it is.

What is Entropy?



In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.

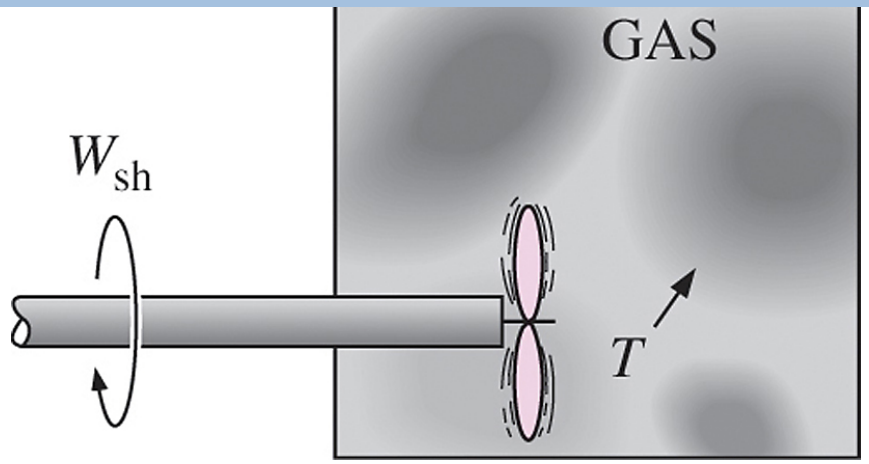
Rotating shaft- energy of the molecules are organized as molecules rotate in the same direction together.

Useful to perform tasks such as raising a weight or generating electricity.

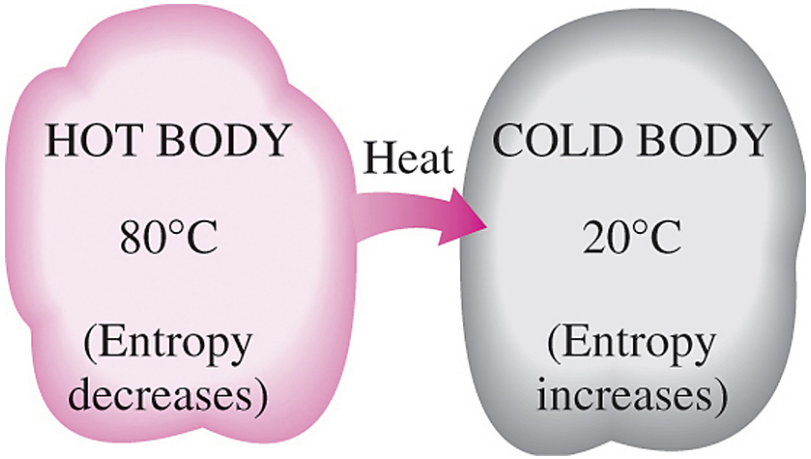
There is no entropy transfer associated with energy transfer or work.

What is Entropy?

Quantity of energy is preserved
Quality is bound to decrease
which is accompanied by an
increase in entropy.



The paddle-wheel work done on a gas.
Organizing form of energy gets converted to disorganized form.



During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

Increases in the level of disorder (entropy) of the gas, and thus energy is degraded during this process.

The Tds relation

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev,out}} = dU$$

$$\delta Q_{\text{int rev}} = T\,dS$$

the first *T ds*, or *Gibbs equation*

$$\delta W_{\text{int rev,out}} = P\,dV$$

$$T\,dS = dU + P\,dV \qquad \text{(kJ)}$$

$$T\,ds = du + P\,dv \qquad \text{(kJ/kg)}$$

$$\left.\begin{aligned} h &= u + Pv \\ dh &= du + P\,dv + v\,dP \\ T\,ds &= du + P\,dv \end{aligned}\right\} \quad T\,ds = dh - v\,dP$$

the second *T ds equation*

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

Differential changes in entropy in terms of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

Since $dv \cong 0$ for liquids and solids

$$ds = \frac{du}{T} = \frac{c\,dT}{T}$$

since $c_p = c_v = c$ and $du = c\,dT$

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

Liquids, solids: $$s_2 - s_1 = \int_1^2 c(T)\,\frac{dT}{T} \cong c_{avg} \ln \frac{T_2}{T_1} \qquad (\text{kJ/kg} \cdot \text{K})$$

For and isentropic process of an incompressible substance

Isentropic: $$s_2 - s_1 = c_{avg} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

THE ENTROPY CHANGE OF IDEAL GASES

From the first *T ds* relation

$$ds = \frac{du}{T} + \frac{P\,dv}{T}$$

$$ds = c_v \frac{dT}{T} + R\frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}$$

From the second *T ds* relation

$$ds = \frac{dh}{T} - \frac{v\,dP}{T}$$

$$dh = c_p\,dT \qquad v = RT/P$$

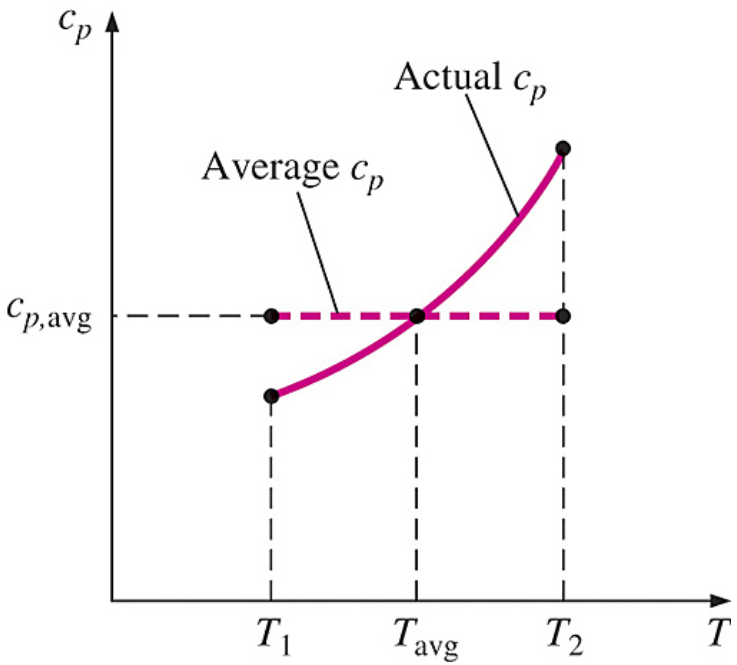
$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}$$

$$Pv = RT$$
$$du = C_v\,dT$$
$$dh = C_p\,dT$$

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1} \rightarrow s_2 - s_1 = c_{v,\text{avg}}\ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}$$

$$s_2 - s_1 = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1} \rightarrow s_2 - s_1 = c_{p,\text{avg}}\ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} \quad (\text{kJ/kg}\cdot\text{K})$$



Entropy change of an ideal gas on a unit–mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,\text{avg}}\ln\frac{T_2}{T_1} + R_u\ln\frac{v_2}{v_1} \quad (\text{kJ/kmol}\cdot\text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,\text{avg}}\ln\frac{T_2}{T_1} - R_u\ln\frac{P_2}{P_1} \quad (\text{kJ/kmol}\cdot\text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function s° as

$$s^\circ = \int_0^T c_p(T)\,\frac{dT}{T}$$

$$\int_1^2 c_p(T)\,\frac{dT}{T} = s_2^\circ - s_1^\circ$$

T, K	s°, kJ/kg • K
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

(Table A-17)

On a unit–mass

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R\ln\frac{P_2}{P_1} \qquad (\text{kJ/kg}\cdot\text{K})$$

On a unit–mole

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u\ln\frac{P_2}{P_1} \qquad (\text{kJ/kmol}\cdot\text{K})$$

The entropy of an ideal gas depends on both T and P. The function s represents only the temperature-dependent part of entropy.

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,\text{avg}} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_2}{v_1}$$

$$\ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2}\right)^{R/c_v}$$

$R = c_p - c_v$, $k = c_p/c_v$

and thus $R/c_v = k - 1$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \left(\frac{P_2}{P_1}\right)_{s=\text{const.}} = \left(\frac{v_1}{v_2}\right)^{k}$$

$$\left(\frac{T_2}{T_1}\right)_{s=\text{const.}} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{v_1}{v_2}\right)^{k-1}$$

VALID FOR
- *ideal gas
- *isentropic process
- *constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

$$Tv^{k-1} = \text{constant}$$

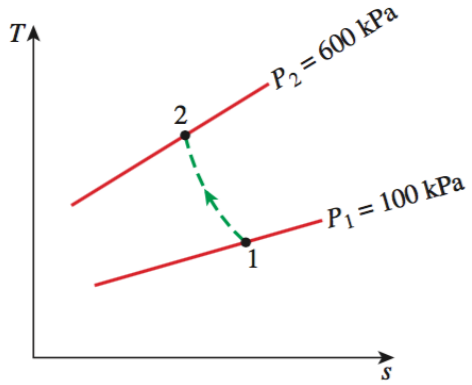
$$TP^{(1-k)/k} = \text{constant}$$

$$Pv^{k} = \text{constant}$$

Example

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (a) property values from the air table and (b) average specific heats

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$



$$s_2 - s_1 = c_{p,\text{avg}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Next lecture

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.