ME401A: Energy Systems II

Assignment - 2

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Q1 Given, axial flow compressor.
overall pressure ratio = 4.0
mass flow rate, $\dot{m}$ = 3 Kg/s.
polytropic efficiency, η_{p_c} = 88%.
stagnation pressure rise per stage ≤ 25 K.
Absolute velocity approaching last rotor = 165 m/s
Angle of absolute velocity = 20° from axial dir^n
work done factor, λ = 0.83
Mean diameter of last stage rotor = 18 cm
Ambient cond^n : 1.01 bar and 288 K.

To find:
no. of stages required?
pressure ratios of first and last stages?
rotational speed and the length of last stage rotor blade at inlet to the stage?

Assumption: equal temperature rise in all stages & velocity diagram is symmetrical.

$$T_{o_2} - T_{o_1} = N \Delta T_{o_s}$$

ΔT_{o_s} : stagnation temperature rise per stage
T_{o_2} : Total temperature at outlet (last stage)
T_{o_1} : Total temperature at inlet (first stage)

$$\Rightarrow \frac{T_{o_2}}{T_{o_1}} = 1 + \frac{N \Delta T_{o_s}}{T_{o_1}}$$

$$\Rightarrow \left(\frac{p_{o_2}}{p_{o_1}}\right)^{\frac{\gamma-1}{\eta_{p_c}\gamma}} = 1 + \left(\frac{N \Delta T_{o_s}}{T_{o_1}}\right)$$

$$\Rightarrow \frac{\left[(4)^{\frac{(1.4-1)}{(0.88)(1.4)}}\right] - 1}{N} \times 288 = \Delta T_{o_s} \leq 25$$

$$\Rightarrow N \geq \frac{(0.568) \times 288}{25} \Rightarrow N \geq 6.55$$

$$\therefore \boxed{N = 7}$$

Ans: no. of stages required = 7.

Now, let's find stagnation pressure rise per stage

$$T_{0_2} - T_{0_1} = N(\Delta T_0)_{stage}$$

$$\Rightarrow \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma \eta_{pc}}} = 1 + \frac{N \Delta T_{0s}}{T_{01}}$$

$$\Rightarrow (4)^{\frac{0.4}{(1.4)(0.88)}} = 1 + \frac{7 \Delta T_{0s}}{288}$$

$$\Rightarrow \Delta T_{0s} = 23.39 \text{ K}$$

For, first stage,

$$T_{01} = T_{amb} = 288 \text{ K}.$$

1 refers to inlet of first stage and 2 refers to outlet.

$$T_{0_2} = T_{0_1} + \Delta T_{0_s} = 288 + 23.39 = 311.39 \text{ K}$$

$$\therefore \text{pressure ratio, } \left(\frac{p_{02}}{p_{01}}\right)_{\text{first stage}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma \eta_{pc}}{1-\gamma}}$$

$$= \left(\frac{311.39}{288}\right)^{\frac{(1.4)(0.88)}{0.4}}$$

$$\boxed{\left(\frac{p_{02}}{p_{01}}\right)_{\text{first stage}} = 1.272} \quad \underline{\text{Ans}}$$

Similarly for last stage:

$$(T_{02})_{\text{last stage}} = (T_{01})_{\text{first stage}} + N(\Delta T_0)_{stage}$$

$$= 288 + (7)(23.39)$$

$$= 451.73$$

and

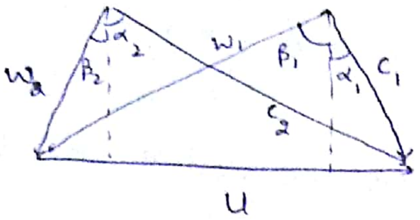
$$(T_{01})_{\text{last stage}} = 451.73 - 23.39$$

$$= 428.34$$

$$\therefore \text{pressure ratio, } \left(\frac{p_{02}}{p_{01}}\right)_{\text{last stage}} = \left(\frac{451.73}{428.34}\right)^{\frac{(1.4)(0.88)}{(0.4)}}$$

$$\boxed{\left(\frac{p_{02}}{p_{01}}\right)_{\text{last stage}} = 1.178} \quad \underline{\text{Ans}}$$

velocity diagram for the last stage



~~Given, $\dot{m}$ = 3 kg/s~~

~~$\rho Q = 3$~~

~~$\Rightarrow Q = 3$~~

Given, $C_1 = 165$ m/s

$\alpha_1 = 20^\circ$

$\Rightarrow \beta_2 = 20^\circ$ (since velocity diagram is symmetrical)

$R = \frac{1}{2}$

$\Rightarrow \frac{C_z}{2u}(\tan\beta_1 + \tan\beta_2) = \frac{1}{2}$

$\Rightarrow u = C_z(\tan\beta_1 + \tan\beta_2)$

$= (165\cos 20^\circ)(\tan\beta_1 + \tan\beta_2)$

$= (155.05)(\tan\beta_1 + \tan\beta_2)$

Also, $\Delta T_{0s} = \frac{\lambda u C_z}{C_p}(\tan\beta_1 - \tan\beta_2)$

$\Rightarrow u(\tan\beta_1 - \tan\beta_2) = \frac{(23.42)(1005)}{(0.83)(155.05)}$

$\Rightarrow \tan^2\beta_1 - \tan^2\beta_2 = \frac{(23.42)(1005)}{(0.83)(155.05)^2}$

$\Rightarrow \tan^2\beta_1 = \tan^2 20^\circ + 1.1796$

$\tan^2\beta_1 = 1.3121$

$\Rightarrow \beta_1 = 48.88^\circ$

Now, $u = (155.05)(\tan 48.88^\circ + \tan 20^\circ)$

$= 234.04$ m/s

$U = \pi D N$ (rev/s) $\Rightarrow N = \frac{u}{\pi D}$

$= \frac{234.04}{\pi\,(18/100)}$

$\Rightarrow \boxed{N = 413.87 \text{ rev/s}}$ Ans

For last stage, total temp at inlet to last stage

$T_{1st} = T_{01} - \frac{C_1^2}{2C_p}$

static temp at inlet to last stage $= 428.34 - \frac{165^2}{2(1005)}$

$= 414.8$ K.

Total pressure at inlet to last stage, $p_{01} = \frac{4}{1.178} = 3.4$

Now, $p_1^{1-\gamma}\, T_1^{\gamma} = p_{01}^{1-\gamma}\, T_{01}^{1-\gamma}$ $\Rightarrow p_1 = p_{01}\left(\frac{T_{01}}{T_1}\right)^{\frac{1-\gamma}{\gamma}}$

$= (3.4)\left(\frac{428.34}{414.8}\right)^{\frac{-0.4}{1.4}}$

$= 3.37$ bar

$\rho = \frac{p_1}{RT_1} = \frac{3.37\times10^5}{287\times414.8} = 2.83$ kg/m³

$\dot{m} = A C_z \rho = C_z \rho \pi D_m h$

$\Rightarrow h = \frac{3\times100}{(155.05)(2.83)\pi(18)} = 1.21$ cm Ans

Q2 Given, axial flow compressor.

No. of stages = 10

overall pressure ratio = 5 : 1

overall isoentropic efficiency = 87 % ≈ 0.87

Temperature of air at inlet = 15°C = 288K.

$R = \frac{1}{2}$. ⇒ velocity diagram is symmetrical

Blade speed, $u = 210$ m/s

axial velocity, C_z = const = 170 m/s.

work done factor, $\lambda = 1$.

To find, blade angles, β_1 & β_2 = ?

overall isoentropic efficiency of compressor is given by,

$$\eta_c = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_{p_c}}} - 1}$$

$$\Rightarrow (5)^{\frac{0.4}{1.4}} - 1 = (0.87)\left((5)^{\frac{0.4}{(1.4)(\eta_{p_c})}} - 1\right)$$

$$\Rightarrow 1.67 = 5^{\left(\frac{0.286}{\eta_{p_c}}\right)}$$

$$\Rightarrow \ln(1.67) = \left(\frac{0.286}{\eta_{p_c}}\right) \ln 5$$

$$\Rightarrow \eta_{p_c} = 0.898 \simeq 89.8\ \%$$

Now, $R = \frac{1}{2} = \phi \tan\beta_m$

Q3

$$\Rightarrow \frac{C_z}{2u}(\tan\beta_1 + \tan\beta_2) = \frac{1}{2}$$

$$\Rightarrow \tan\beta_1 + \tan\beta_2 = \frac{u}{C_z} = \frac{210}{170}$$

$$\Rightarrow \tan\beta_1 + \tan\beta_2 = 1.24 \quad \text{— (1)}$$

$$T_{o_2} - T_{o_1} = N \Delta T_{os}$$

$$\Rightarrow \frac{T_{o_2}}{T_{o_1}} = 1 + \frac{N \Delta T_{os}}{T_{o_1}}$$

$$\Rightarrow \left(\frac{p_{o_2}}{p_{o_1}}\right)^{\frac{\gamma-1}{\gamma \eta_{pc}}} = 1 + \frac{N \Delta T_{os}}{T_{o_1}}$$

$$\Rightarrow (5)^{\frac{0.4}{(1.4)(0.898)}} = 1 + \frac{(10)\Delta T_{os}}{288}$$

$$\Rightarrow \Delta T_{os} = 19.26 \text{ K.}$$

Now, $\Delta T_{os} = \frac{\lambda U C_z}{C_p}(\tan\beta_1 - \tan\beta_2)$

$$19.26 = \frac{(1)(210)(170)}{1005}(\tan\beta_1 - \tan\beta_2)$$ { taking C_p at 288 K $= 1005 \, J/kg\,K$

$$\Rightarrow \tan\beta_1 - \tan\beta_2 = 0.542 \quad (2)$$

From (1) & (2).

$$2\tan\beta_1 = 1.782$$

$$\Rightarrow \boxed{\beta_1 = 41.7^\circ}$$ <u>Ans</u>

and, $2\tan\beta_2 = 0.698$

$$\boxed{\beta_2 = 19.2^\circ}$$ <u>Ans</u>

<u>Q3</u>: Given, axial flow compressor stage.

axial velocity = 150 m/s.

NO. inlet guide vanes.

tip diameter, D_t = 60 cm.

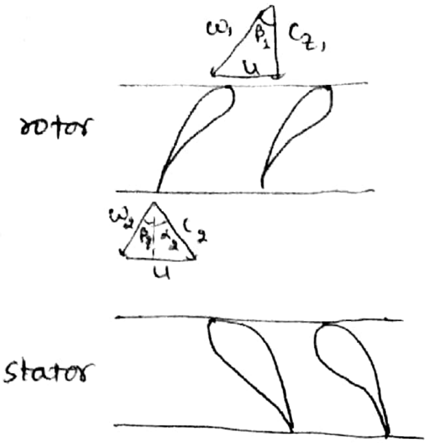
hub diameter, D_h = 50 cm

rps, N = 100.

Air turned through 30° as it passes through rotor.

(13) (6) (7)

(i) velocity diagram



Since no IGV, $C_{\theta_1} = 0$

So, $C_1 = C_{z_1}$

Now, $u = \pi \left(\frac{D_t + D_h}{2} \right) N$

$= \pi \left(\frac{0.6 + 0.5}{2} \right) (100)$

$= 172.79$ m/s.

$\tan \beta_1 = \frac{u}{C_{z_1}}$

$= \frac{172.79}{150}$

$\Rightarrow \beta_1 = 49.04^\circ$

So, $\beta_2 = \beta_1 - 30^\circ = 19.04^\circ$

Now, $\tan \alpha_2 = \frac{u - C_z \tan \beta_2}{C_z}$

$= \frac{(172.79) - (150)(\tan 19.04)}{150}$

$\Rightarrow \alpha_2 = 38.9^\circ$

$\cos \alpha_2 = \frac{C_z}{C_2}$

$\Rightarrow C_2 = \frac{C_z}{\cos \alpha_2}$

$= \frac{150}{\cos(38.9^\circ)}$

$= 192.74$ m/s.

Now, $\dot{m} = \rho_1 A C_{z_1}$

Given, $T_{0_1} = 288$ K and $p_{0_1} = 1.0132$ bar.

$T_{0_1} = T_1 + \frac{C_1^2}{2C_p}$

$\Rightarrow T_1 = T_{0_1} - \frac{C_1^2}{2C_p}$

$= 288 - \frac{(150)^2}{2(1005)}$ $\quad (\because C_1 = C_{z_1})$

$T_1 = 276.81$ K

$p^{1-\gamma} T^{\gamma} = \text{constant}$

$\Rightarrow p_{0_1}^{1-\gamma} T_{0_1}^{\gamma} = p_1^{1-\gamma} T_1^{\gamma} \Rightarrow p_1 = p_{0_1} \left(\frac{T_{0_1}}{T_1} \right)^{\frac{\gamma}{1-\gamma}}$

$= (1.0132) \left(\frac{288}{276.81} \right)^{\frac{1.4}{-0.4}}$

$= 0.882$ bar

$\therefore \rho_1 = \frac{p_1}{RT_1}$

$= \frac{(0.882) \times 10^5}{(287)(276.81)} = 1.11$ kg/m^3.

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mass flow rate, $\dot{m} = \rho_1 A_1 C_{z1} = \rho \frac{\pi}{4}(D_t^2 - D_h^2) C_z$

$= (1.11)\frac{\pi}{4}((0.6)^2 - (0.5)^2)(150)$

$\boxed{\dot{m} = 14.385 \text{ Kg/s}}$ Ans

(iv) Degree of reaction,

$R = \phi \tan\beta_m$

$= \frac{C_z}{2u}(\tan\beta_1 + \tan\beta_2)$

$= \frac{150}{2(172.79)}(\tan(49.04^\circ) + \tan(19.04^\circ))$

$\boxed{R = 0.65}$ Ans

(iii) Power, $P = \dot{m}(C_p)\Delta T_0$

$= \dot{m}\lambda u C_z(\tan\beta_1 - \tan\beta_2)$

$= (14.385)(1)(172.79)(150)(\tan 49.04^\circ - \tan 19.04^\circ)$

$=$

Q4 Given, axial flow air compressor,

$R = 50\% = \frac{1}{2}$

Blade inlet & outlet angle, $\beta_1 = 45^\circ$, $\beta_2 = 10^\circ$.

Pressure ratio $= 6:1$

overall isoentropic efficiency, $\eta_c = 0.85$

$T_1 = 37\,C$.

Blade speed & axial velocity are constant.

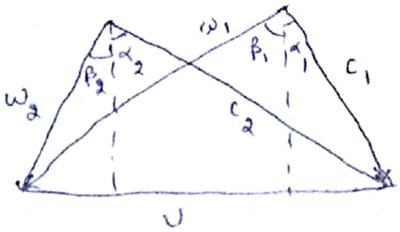
Blade speed, $u = 200$ m/s.

To find, no. of stages, N

when (1) $\lambda = 1$

(2) $\lambda = 0.87$

$$\eta_c = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_{p_c}}} - 1}$$



$$\Rightarrow \quad 0.85 = \frac{(6)^{\frac{0.4}{1.4}} - 1}{(6)^{\frac{0.4}{(1.4)\eta_{p_c}}} - 1}$$

$$\Rightarrow \quad 6^{\frac{0.286}{\eta_{p_c}}} = 1.786$$

$$\Rightarrow \quad \frac{0.286}{\eta_{p_c}} \ln 6 = \ln 1.786$$

$$\Rightarrow \quad \eta_{p_c} = 0.884 \simeq 88.4\%$$

Now, $R = \frac{1}{2} \Rightarrow \frac{c_z}{2u}(\tan\beta_1 + \tan\beta_2) = \frac{1}{2}$

$$\Rightarrow \tan\beta_1 + \tan\beta_2 = \frac{u}{c_z}$$

$$\Rightarrow \tan 45^\circ + \tan 10^\circ = \frac{200}{c_z}$$

$$\Rightarrow c_z = 170.02 \text{ m/s}.$$

~~$c_z \tan\beta_2 +$~~

$\alpha_2 = \beta_1 = 45^\circ$

$\alpha_1 = \beta_2 = 10^\circ$

$\cos\alpha_2 = \frac{c_z}{c_2}$

$\Rightarrow c_2 = \frac{170.02}{\cos(45^\circ)}$

$= 240.44$ m/s

$c_1 = \frac{c_z}{\cos\alpha_1}$

$= \frac{170.02}{\cos 10^\circ}$

$= 172.64$ m/s

$T_{01} = T_1 + \frac{c_1^2}{2c_p}$

$= 310 + \frac{(172.64)^2}{2(1005)}$

$= 324.82$

Now, $\Delta T_0 = \frac{\lambda u c_z}{c_p}(\tan\beta_1 - \tan\beta_2)$

for $\lambda = 1$, $\Delta T_0 = \frac{(1)(200)(170.02)}{(1005)}(\tan 45^\circ - \tan 10^\circ)$

$= 27.87^\circ$ (Assuming constant specific heat)

for $\lambda = 0.87$ $\Delta T_0 = 24.25^\circ$

Now, $\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_p}} = 1 + \frac{N\,\Delta T_0}{T_{01}}$

$\underline{\lambda = 1}$, $(6)^{\frac{0.4}{(1.4)(0.884)}} = 1 + \frac{N(27.87)}{324.82} \Rightarrow 9.14 = N$

$\therefore \boxed{N = 9 \text{ stages.}}$

$\underline{\lambda = 0.87}$ $(6)^{\frac{0.4}{(1.4)(0.884)}} = 1 + N\frac{24.25}{324.82} \Rightarrow N = 10.5$

$\Rightarrow \boxed{N = 10 \text{ stages.}}$

Ans

$\varphi = c_z A_{oLL} = c.(1-A ...)$

Q5 Given, axial flow compressor,

total head pressure ratio : 4

overall total head isentropic efficiency : 85%.

Total head inlet temperature : 290 K.

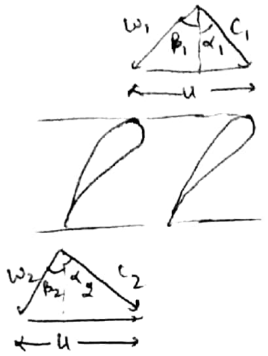
$\beta_1 = 45°$, $\beta_2 = 10°$

mean blade speed U_{mean} & axial velocity c_z are constant.

$U_{mean} = 220$ m/s ; $\lambda = 0.86$

To find, $N = ?$

$Ma_{inlet} = ?$



Given, stages are symmetrical

$\Rightarrow R = \frac{1}{2}$

and $\alpha_1 = \beta_2$; $\alpha_2 = \beta_1$

$$\eta_c = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_p}} - 1}$$

$$\Rightarrow 0.85 = \frac{(4)^{\frac{0.4}{1.4}} - 1}{(4)^{\frac{0.4}{1.4\eta_p}} - 1}$$

$$\Rightarrow (4)^{\frac{0.286}{\eta_{p_c}}} = 1.57$$

$$\Rightarrow (0.286/\eta_{p_c}) \ln 4 = \ln 1.57$$

$$\Rightarrow \eta_{p_c} = 0.879 \simeq 87.9\%$$

Now, $$(\Delta T_0)_{stage} = \frac{\lambda u c_z}{c_p}(\tan\beta_1 - \tan\beta_2)$$

$$= \frac{(0.86)(220)(187.02)}{1005}(\tan 45° - \tan 10°)$$

$$= 29\ \text{K}$$

$$R = \frac{1}{2} = \frac{c_z}{2u}(\tan\beta_1 + \tan\beta_2) \Rightarrow c_z = \frac{u}{\tan\beta_1 + \tan\beta_2}$$

$$= \frac{220}{\tan 45° + \tan 10°} = 187.02\ \text{m/s}$$

Total temp. rise

Now, $T_{02} - T_{01} = N(\Delta T_0)_{stage}$

$$\Rightarrow \frac{T_{02}}{T_{01}} = 1 + \frac{N(\Delta T_0)_{stage}}{T_{01}} \Rightarrow \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma\eta_{p_c}}} = 1 + N\frac{(\Delta T_0)_{stage}}{T_{01}}$$

$$\Rightarrow (4)^{\frac{0.4}{(1.4)(0.879)}} = 1 + N \frac{29}{290}$$

$$\Rightarrow N = 5.69 \simeq 6.$$

$\therefore \boxed{\text{no. of stages } N = 6}$ Ans

Now, mach number at inlet, $= \frac{w_1}{\sqrt{\gamma R T_1}}$

$$T_1 = T_{o_1} - \frac{c_1^2}{2C_p}$$

$$= T_{o_1} - \frac{\left(C_7/\cos\alpha_1\right)^2}{2\,C_p}$$

$$= 290 - \left[\left(\frac{187.02}{\cos 10^\circ}\right)^2 / 2(1005)\right]$$

$$= 272.06\ K.$$

$$\therefore Ma_{inlet} = \frac{C_7/\cos\beta_1}{\sqrt{\gamma R T_1}}$$

$$= \frac{187.02/(\cos 45^\circ)}{\sqrt{1.4 \times 284.6 \times 272.02}} = \frac{264.49}{329.24}$$

$$= 0.8$$

$\therefore \boxed{Ma_{inlet} = 0.8}$ Ans

Q7 Given, $Q = \frac{1000}{60}\ m^3/s = 16.67\ m^3/s.$

$T_{o_1} = 288\ K$, $P_{o_1} = 0.9$ bar

Projected area of blades $= 19.25\ cm^2$

and blade length $= 6.75 cm$

blade ring mean diameter $= 60$ cm

speed, $N = 6000$ rpm

$C_L = 0.6$, $C_D = 0.05$ at zero angle of incidence

on each blade ring, 50 blades which occupy 10% of axial area of flow.

To find, pressure rise per blade ring and power input per stage.

$$Q = C_z A_{eff} = C_z (1-0.1)\pi D_m h$$

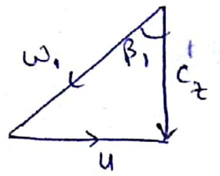
$$\cancel{C_z \pi D_m h}$$

$$16.67 = C_z (1-0.1)\pi\left(\frac{60}{100}\right)\left(\frac{6.75}{100}\right)$$

$$\Rightarrow C_z = \frac{16.67 \times 10^4}{(0.9)\pi(60)(6.75)} = 145.58 \text{ m/s}$$

Blade velocity $u = \dfrac{\pi D_m N}{60} = \dfrac{\pi(0.60)(6000)}{60}$

$$= 188.5 \text{ m/s}$$



$$\tan\beta_1 = \frac{188.5}{145.58} = 1.295 \Rightarrow \beta_1 = 52.33^\circ$$

$$\therefore w = \sqrt{u^2 + C_z^2}$$

$$= \sqrt{(188.5)^2 + (145.58)^2}$$

$$= 238.17 \text{ m/s.}$$

Now, $C_L = \dfrac{L}{\frac{1}{2}\rho w^2 A_c}$

where $\rho = \dfrac{p_1}{RT_1} = \dfrac{0.9 \times 10^5}{287 \times 288} = 1.09 \text{ kg/m}^3$

$$\therefore L = (C_L)\left(\frac{1}{2}\rho w^2 A_c\right)$$

$$= (0.6)\left(\frac{1}{2}\right)(1.09)(238.17)^2\left(\frac{19.25}{10000}\right)$$

$$= 35.71 \text{ N.}$$

Similarly, $D = C_D\left(\frac{1}{2}\rho w^2 A_c\right)$

$$= (0.05)\left(\frac{1}{2}\right)(1.09)(238.17)^2\left(\frac{19.25}{10000}\right)$$

$$= 2.98 \text{ N.}$$

Now, Power input per stage

$$= (L\cos\beta_1 + D\sin\beta_1)\,u \cdot n$$

where n: no. of blades on each blade ring

$$= \left[35.71\cos(52.33^\circ) + (2.98) \times \sin(52.33^\circ)\right] \times (188.5) \times 50$$

$$= 227.9 \text{ kW.}$$

$$P = \dot{m} C_p (T_2 - T_1)$$

$$= \dot{m} C_p T_1 \left(\frac{T_2}{T_1} - 1 \right)$$

$$P = \dot{m} C_p T_1 \left(\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$P = \rho Q C_p T_1 \left(\left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right)$$

$$\Rightarrow \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = \frac{P}{\rho Q C_p T_1} + 1$$

$$= \frac{227.9 \times 10^3}{(1.09)(16.67) \times (1005)(288)} + 1$$

$$\left(\frac{p_2}{p_1} \right)^{0.286} = 1.043$$

$$\frac{p_2}{p_1} = 1.16$$

$\therefore$ $p_2 = (1.16)(0.9) = 1.044$ bar

$\therefore$ pressure rise $= 1.044 - 0.9$

$= 0.144$ bar. Ans

Q8: Given, axial flow compressor stage

Blade root velocity $= 150$ m/s

Blade mean velocity $= 200$ m/s

Blade tip velocity $= 250$ m/s

Stagnation temperature rise $= 20$ K. } constant from root to tip.

axial velocity $= 150$ m/s.

$\lambda = 0.93$.

At mean radius, $R = 1/2$.

To find, stage air angles at root, mean & tip and R at root & tip.

At mean radius, $R = \frac{C_z}{2u}(\tan\beta_1 + \tan\beta_2) = \frac{1}{2}$

$$\Rightarrow \tan\beta_1 + \tan\beta_2 = \frac{200}{150} = 1.33 \quad (1)$$

$$\Delta T_{0_{stage}} = \frac{\lambda u C_z}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$\Rightarrow \tan\beta_1 - \tan\beta_2 = \frac{(20)(1005)}{(0.93)(200)(150)}$$

$$\tan\beta_1 - \tan\beta_2 = 0.72 \quad (2)$$

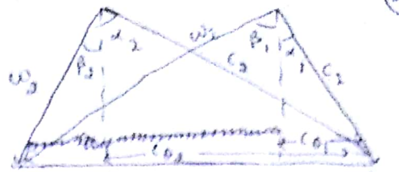
From (1) & (2).

$2\tan\beta_1 = 2.05$

$\Rightarrow \boxed{\beta_1 = 45.71^\circ}$ Ans

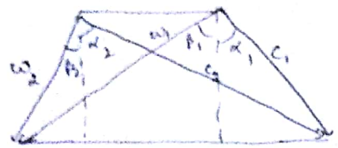
& $2\tan\beta_2 = 0.61$

$\Rightarrow \boxed{\beta_2 = 16.96^\circ}$ Ans



velocity diagram at mean radius

At Blade tip



velocity diagram at blade tip.

work done on air,

$[U(C_{\theta_2}-C_{\theta_1})]_{mean\ radius} = [(U(C_{\theta_2}-C_{\theta_1})]_{tip}$ — (3)

At mean radius,

$C_{\theta_2} = U - C_2\tan\beta_2$

$= 200 - (150)\tan(16.96)$

$= 154.25$

& $C_{\theta_1} = C_2\tan\alpha_1$

$= (150)\tan(16.96)$

$= 45.75$

Putting these value in eq^n (3).

$200(154.25 - 45.75) = (250)(\Delta C_\theta)_{tip}$

$\Rightarrow (\Delta C_\theta)_{tip} = 86.8$ m/s

Free-vortex design,

$C_\theta \cdot r = \text{const}$.

$\Rightarrow (C_{\theta_1} D)_{tip} = (C_{\theta_1} D)_{mean}$

$(C_{\theta_1})_{tip} = C_{\theta_1,mean}\dfrac{D_{mean}}{D_{tip}}$

$= C_{\theta_1,mean}\dfrac{U_{mean}}{U_{tip}} = (45.75)\dfrac{200}{250}$

$= 36.6$ m/s.

$\tan\alpha_1 = \dfrac{C_{\theta_1}}{C_2} = \dfrac{36.6}{150} \Rightarrow \alpha_1 = 13.7^\circ$

$\Rightarrow C_{\theta_2 tip} = 86.8 + 36.6 = 123.4$ m/s.

$\tan\beta_1 = \dfrac{U - C_{\theta_1}}{C_2} = \dfrac{250 - 36.6}{150} = 1.48$

$\Rightarrow \beta_1 = 54.05^\circ$

$$\tan\beta_2 = \frac{U - C_{\theta_2}}{C_z}$$

$$\Rightarrow \tan\beta_2 = \frac{250 - 123.4}{150}$$

$$\Rightarrow \beta_2 = 40.16^\circ.$$

At blade root

- $\left[u\left(C_{\theta_2} - C_{\theta_1}\right)\right]_{root} = \left[u\left(C_{\theta_2} - C_{\theta_1}\right)\right]_{mean}$

$$\Rightarrow 150\,\Delta C_{\theta_{root}} = 200\,(154.25 - 45.75)$$

$$\Delta C_{\theta_{root}} = 144.67 \text{ m/s}.$$

Also, $C_{\theta_1 root} = C_{\theta_1 mean}\dfrac{U_{mean}}{U_{root}}$

$$= 45.75\,\frac{200}{150}$$

$$= 61 \text{ m/s}.$$

$\therefore C_{\theta_2 root} = C_{\theta_1 root} + 144.67$

$$= 61 + 144.67$$

$$= 205.67 \text{ m/s}.$$

$$\tan\beta_2 = \frac{U - C_{\theta_2}}{C_z} =$$

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