

1) Given,

$$P_1 = 3.5 \text{ bar}$$

$$P_b = 1 \text{ bar}$$

$$T_1 = 405^\circ\text{C} = 678 \text{ K}$$

$$\dot{m} = 18 \text{ kg/s}$$

$$C_d = 0.99, \eta_n = 0.94, v_1 = 0 \text{ m/s}, \gamma = 1.33$$

$$C_p = 1.113 \text{ kJ/kg}$$

Now,

$$C_d = \frac{\dot{m}}{\dot{m}_{is}} \Rightarrow \dot{m}_{is} = \frac{1}{\frac{0.99}{18}} = 18.182 \text{ kg/s}$$

$$v_1 = \frac{R T_1}{P_1} = \frac{276 \times 678}{3.5 \times 10^5} = 0.53 \text{ m}^3/\text{kg}$$

$$\frac{P_2}{P_1} = \frac{1}{3.5} = 0.286$$

$$\frac{P^*}{P_1} = \left(\frac{2}{n+1} \right)^{\frac{n}{n-1}} = \left(\frac{2}{2.33} \right)^{\frac{1.33}{0.33}} = 0.54$$

$$\Rightarrow P^* = 1.89 > P_b$$

$\therefore Ma = 1$ at the throat

and $P_t = P^*$ at throat

$$\frac{\dot{m}_{is}}{A_t} = \sqrt{\frac{2n}{n-1} \frac{P_1}{v_1} \left[\left(\frac{P_t}{P_1} \right)^{\frac{2}{n}} - \left(\frac{P_t}{P_1} \right)^{\frac{n+1}{n}} \right] \eta_n}$$

$$18.182 = 529.95$$

$$A_t$$

$$\Rightarrow A_t = 0.034 \text{ m}^2$$

For exit area

$$\frac{\dot{m}}{A_{ex}} = \sqrt{\frac{2n}{n-1} \frac{P_1}{\rho_1} \left[\left(\frac{P_b}{P_1} \right)^{\frac{2n}{n-1}} - \left(\frac{P_b}{P_1} \right)^{\frac{n+1}{n-1}} \right] \eta_N}$$

$$18.182 = \frac{\dot{m}}{A_{ex}} = 450.96 \Rightarrow A_{ex} = 0.04 \text{ m}^2$$

Given, $P_1 = 7 \text{ bar}$, $T_1 = 473 \text{ K}$, $P_b = P_2 = 3 \text{ bar}$

$$\dot{m} = 0.1 \text{ kg/s}$$

$$v_1 = 0.3 \text{ m}^3/\text{kg}, s_1 = 6.887 \text{ kJ/k kg} = s_2, h_1 = 2844.77$$

$$P_2 = 3 \text{ bar}$$

$$s_g = 6.99 \text{ kJ/k kg}, s_{fg} = 5.319 \text{ kJ/k kg}, s_f = 1.671 \text{ kJ/k kg}$$

$$s_2 = s_f + x s_{fg}$$

$$\Rightarrow 6.887 = 1.671 + x(5.319)$$

$$x = 0.98$$

$$\begin{aligned} \Rightarrow h_2 &= h_f + x h_{fg} = 561.16 + 0.98 \times 2163.47 \\ &= 2681.36 \text{ kJ/k kg} \end{aligned}$$

$$T_2 = T_{\text{sat}} @ 3 \text{ bar}$$

$$= 133.54^\circ \text{C} = 406.54 \text{ K}$$

$$\begin{aligned} v_2 &= v_f + x v_{fg} = 0.001073 + 0.98 \times 0.6046 \\ &= 0.5936 \text{ m}^3/\text{kg} \end{aligned}$$

$$V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(2844.77 - 2681.36) \times 10^3}$$

$$v_0 = 331.68 \text{ m/s}$$

$$\dot{m} = \frac{A_0 v_0}{v_0} \quad A_0 = \frac{0.1203926}{331.68} = 1.028 \times 10^{-4} \text{ m}^2$$

isothermal

$$p v^{1.3} = \text{constant}$$

$$p_1 v_1^{1.3} = p_2 v_2^{1.3}$$

$$\Rightarrow v_2 = 0.376 \text{ m/s}$$

$$\frac{\dot{m}}{A_0} = \sqrt{\frac{2n}{n-1} \left(\frac{p_1}{p_2} \right)^{\frac{1}{n}} \left[\left(\frac{p_2}{p_1} \right)^{\frac{2}{n}} - \left(\frac{p_2}{p_1} \right)^{\frac{n+1}{n}} \right]}$$

$$\Rightarrow \frac{0.1}{A_0} = 987.33$$

$$\Rightarrow A_0 = 1.012 \times 10^{-4} \text{ m}^2$$

now,

$$\left(\frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2}$$

$$\Rightarrow T_2 = 388.99 \text{ K}$$

$$\text{degree of undercooling} = 406.54 - 388.98 = 17.55 \text{ K}$$

$$p_{\text{sat}} @ 388.98 \text{ K} = 1.75 \text{ bar}$$

$$\text{degree of supersaturation} = \frac{3}{1.75} = 1.71$$

$$A_c = \pi \times (0.05)^2 = 1.96 \times 10^{-5} \text{ m}^2$$

$$p_1 = 10 \text{ bar} \quad p_2 = 1.5 \text{ bar}$$

$$v_1 = 0.1945 \text{ m/s}$$

$$\frac{p^*}{p_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1.5}{10} \right)^{\frac{1.35}{0.135}} = 0.533$$

$$P^* = 5.77 > P_2$$

→ nozzle is choked at throat

$$\frac{\dot{m}}{A_1} = \sqrt{\frac{2n}{n-1} \frac{P_1}{\rho_1} \left[\left(\frac{P^*}{P_1} \right)^{2/n} - \left(\frac{P^*}{P_1} \right)^{n+1/n} \right]}$$

$$\rightarrow \frac{\dot{m}}{1.96 \times 10^{-5}} = 1441.9 \quad \Rightarrow \dot{m} = 0.028 \text{ kg/s}$$

Now, $\eta_N = 0.9$

$$\frac{\dot{m}}{A_2} = \sqrt{\frac{2n}{n-1} \frac{P_1}{\rho_1} \left[\left(\frac{P_2}{P_1} \right)^{2/n} - \left(\frac{P_2}{P_1} \right)^{n+1/n} \right]} \times 0.9$$

$$\frac{0.028}{A_2} = 745.2$$

$$\Rightarrow A_2 = 3.757 \times 10^{-5} \text{ m}^2$$

$$\Rightarrow \frac{\pi d^2}{4} = 3.757 \times 10^{-5}$$

$$\boxed{d = 0.69 \text{ cm}}$$

$C_1 = 1000 \text{ m/s}, \quad \alpha_1 = 20^\circ, \quad v = 400 \text{ m/s}$

$$P_1 = P_2$$

$$w_1^2 = C_1^2 + v^2 - 2C_1 v \cos 20^\circ$$

$$w_1^2 = 408245.903$$

$$\Rightarrow w_1 = 638.94 \text{ m/s}$$

$$w_1 \sin \beta_1 = C_1 \sin \alpha_1$$

$$\sin \beta_1 = 0.535, \quad \beta_1 = 32.34^\circ$$

$$\beta_2 = \beta_1 = 32.34^\circ$$

a) Neglecting friction effects, $w_2 = w_1 = 638.94 \text{ m/s}$

$$\Delta C_\theta = \Delta w_{\theta_1} = w_1 \cos \beta_1 + w_2 \cos \beta_2$$

$$= 2w_1 \cos \beta_1 = 2 \times 638.94 \times \cos 32.34$$

$$\Delta C_\theta = 1079.67 \text{ m/s}$$

Tangential force = $\dot{m} \Delta C_\theta = 0.75 \times 1079.67$

$$= 809.75 \text{ N}$$

$$P = \dot{m} U \Delta C_\theta$$

$$P = 809.75 \times 400 = 323.89 \text{ kW}$$

$$\Delta C_z = 0 \text{ (}\therefore \text{ symmetric blades)}$$

$$\Rightarrow \text{axial thrust} = 0$$

$$\eta_b = \frac{U \Delta C_\theta}{C_{p2}} = \frac{400 \times 1079.67}{1000 \times 0.75} = 86.37\%$$

b) $k = 0.8$

$$w_2 = kw_1 = 511.15 \text{ m/s}$$

$$\Delta C_\theta = \Delta w_\theta = w_1 \cos \beta_1 + w_2 \cos \beta_2$$

$$= (638.94 + 511.15) \cos 32.34$$

$$\Delta C_\theta = 971.69 \text{ m/s}$$

$$P = \dot{m} U \Delta C_\theta = 291.51 \text{ kW}$$

$$\therefore R_z = \dot{m} \Delta C_z$$

$$= 0.75 \times (w_1 \sin \beta_1 - w_2 \sin \beta_2)$$

$$R_z = 51.07 \text{ N}$$

$$\eta_b = \frac{U \Delta C_\theta}{C_{p2}} = \frac{400 \times 971.69}{1000 \times 0.75} = 77.74\%$$