

Correction in Maximum Principle

MSO203B

November 4, 2016

Correction to the Argument

If we define, $v(x, y) = u(x, y) + \epsilon(x^2 + y^2)$ and (x_0, y_0) is a point of maxima of u may not imply it is a point of maxima of v . Since, grad of u at (x_0, y_0) i.e $\nabla u(x_0, y_0) = 0$ implies

$$\nabla v(x_0, y_0) = \nabla u(x_0, y_0) + 2\epsilon z_0 = 2\epsilon z_0 \neq 0 \text{ where } z_0 = (x_0, y_0) \text{ unless } z_0 = (0, 0).$$

So modify $v(x, y) = u(x, y) + \epsilon((x - x_0)^2 + (y - y_0)^2)$

Hence one has, $\nabla v(x, y) = \nabla u(x, y) + 2\epsilon(z - z_0)$ where $z = (x, y)$.

So, if $\nabla v(x_0, y_0) = 0$ then $\nabla u(x_0, y_0) = 0$ which imply the maximum of v will be attained at the maximum of u .