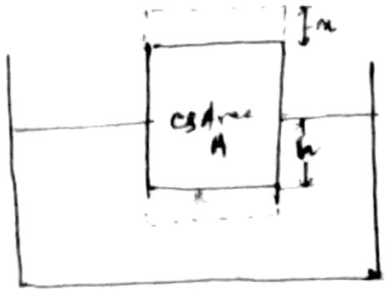
Assignment ① Solutions (ME 354A) ①

1)



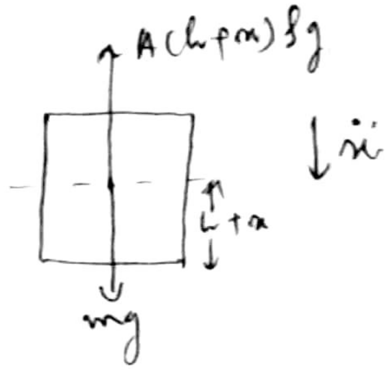
Mass density of liquid $= \rho$

Mass of buoy $= m$.

At equilibrium $mg = Ah\rho g$

'm' is depressed by a distance x

Using Newton's 2nd Law



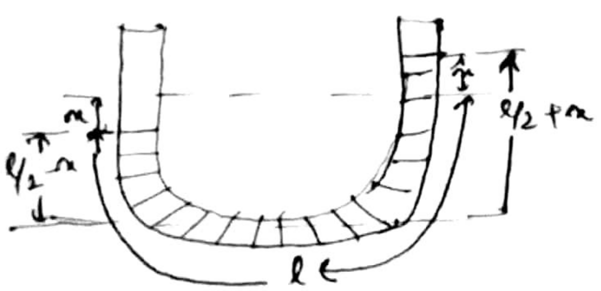
$-mg + A(h+x)\rho g = -m\ddot{x}$

or $Ax\rho g + m\ddot{x} = 0$.

or $\ddot{x} + \frac{A\rho g}{m} x = 0 \rightarrow$ equation of motion.

so $\omega_n = \sqrt{\frac{A\rho g}{m}} \rightarrow$ Natural frequency

②



Total length of Liquid column $= L$

Using Newton's Second Law

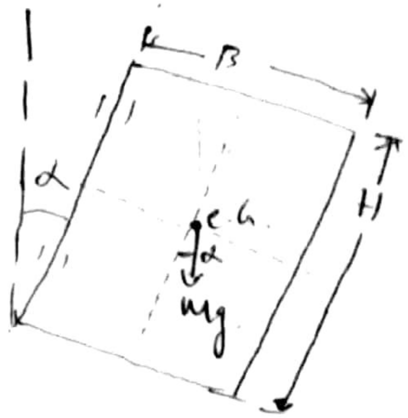
$\rho A L \ddot{x} = \rho g (\frac{l}{2} - x) A - \rho g (\frac{l}{2} + x) A$

$\Rightarrow \ddot{x} L = -g 2x$

$\Rightarrow \ddot{x} + \frac{2g}{L} x = 0 \longrightarrow$ Equation of motion

$\omega_n = \sqrt{\frac{2g}{l}} \longrightarrow$ Natural frequency

(3)



Using Angular Momentum Balance

$$I\ddot{\theta} = -Mg\cos\alpha \cdot \sin\theta \cdot B/2.$$

$$\Rightarrow \frac{M(H^2+B^2)}{12}\ddot{\theta} + Mg\cos\alpha\sin\theta\,\frac{B}{2} = 0$$

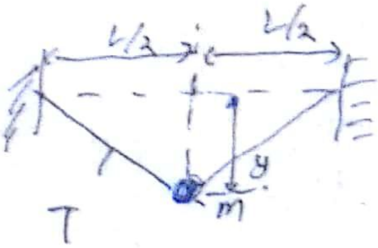
$$\Rightarrow \ddot{\theta} + \frac{6gB\cos\alpha}{(H^2+B^2)}\sin\theta = 0$$

for small oscillations $\sin\theta \approx \theta$

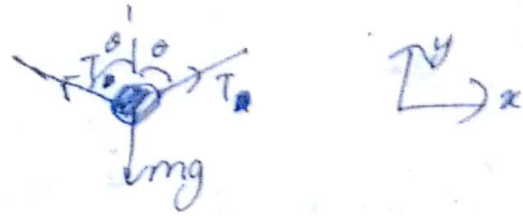
$$\Rightarrow \ddot{\theta} + \frac{6gB\cos\alpha}{H^2+B^2}\theta = 0 \longrightarrow \text{Equation of motion}$$

$$\omega_n = \left(\frac{6gB\cos\alpha}{H^2+B^2}\right)^{1/2} \longrightarrow \text{Natural frequency}$$

Q4)



- y is very small compared to L/2
- $2y/L << 1$
- FBD of mass m



- $\Sigma F_x = 0 \Rightarrow T\sin\theta - T\sin\theta = 0$
- $\Sigma F_y = 0 \Rightarrow mg - 2T\cos\theta = +m\ddot{y}$

$$\left[\cos\theta = \frac{y}{\sqrt{y^2 + L^2/4}} = \frac{2y}{L\sqrt{\left(\frac{2y}{L}\right)^2 + 1}} \approx \frac{2y}{L}\right]$$

So. $$mg - 2T\left(\frac{2y}{L}\right) = +m\ddot{y}$$

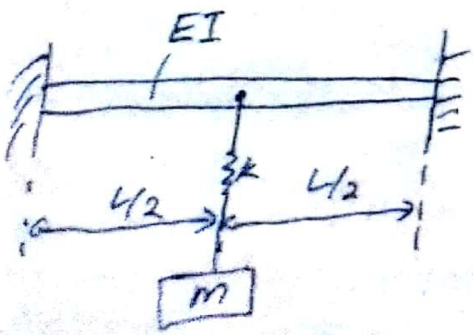
{neglect gravity}

$$\Rightarrow m\ddot{y} + \frac{4T}{L}y = 0$$

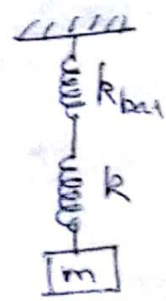
$$\Rightarrow \ddot{y} + \omega_n^2 y = 0$$

$$\Rightarrow \omega_n^2 = \frac{4T}{mL} \Rightarrow \omega_n = 2\sqrt{\frac{T}{mL}}$$

Q5)



$\Rightarrow$



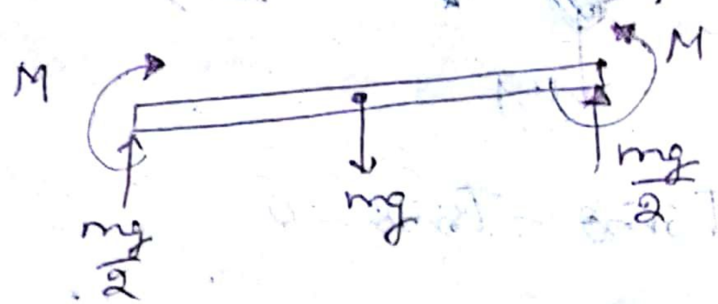
Modelling: as series combination of two spring k & k_{bar}.

$\therefore \omega_n = \sqrt{\frac{k_{eff}}{m}}$

$\frac{1}{k_{eff}} = \frac{1}{k_{bar}} + \frac{1}{k}$

Eq^n of motion will be $\boxed{m\ddot{x} + k_{eff}\, x = 0}$

<u>To find K_{eff}</u>



for <u>$x < l/2$</u>

$$EI\frac{d^2\omega}{dx^2} = M + \frac{mg}{2}x \quad -(1)$$

for <u>$x > l/2$</u>

$$EI\frac{d^2\omega}{dx^2} = M + \frac{mg}{2}(L - x) \quad -(2)$$

Apply B.C.s $\omega\big|_{x=0} = 0$ & $\frac{d\omega}{dx}\Big|_{x=0} = 0$ & $\frac{d\omega}{dx}\Big|_{x=\frac{l}{2}} = 0$

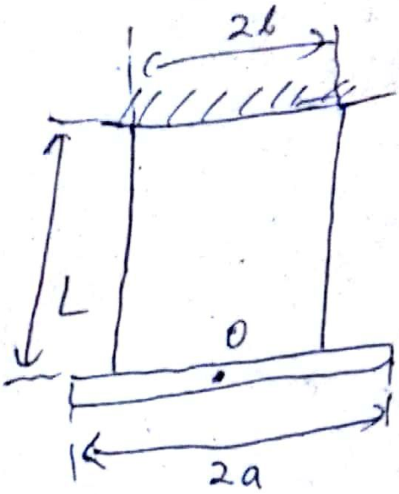
Solve eq^n (1) & (2) using B.C.s

find $\omega\big|_{L/2}$

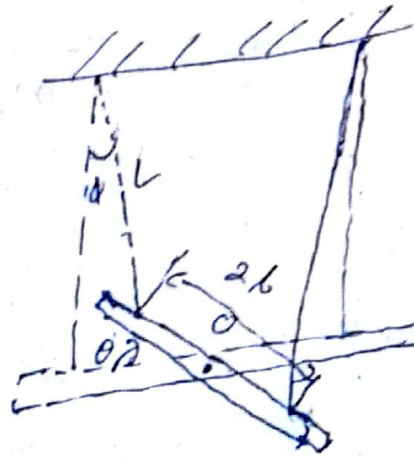
then, $k_{bar} = \frac{mg}{\omega|_{L/2}} = \frac{192EI}{l^3}$

$$\therefore k_{eff} = \frac{1}{\frac{1}{k} + \frac{l^3}{192EI}}$$

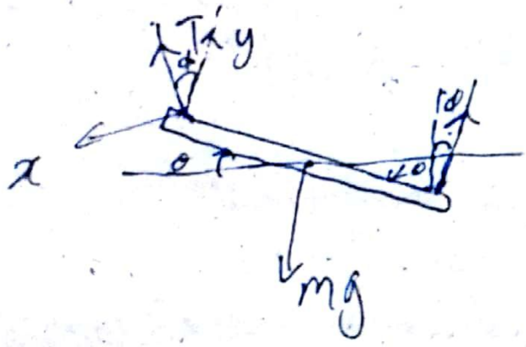
Q6)



after small twist about O



FBD of rod



From geometry

$\phi L = b\theta$

$\Rightarrow \phi = \frac{b}{L}\theta$

$\Sigma F_x = 0$

$\Sigma F_y = 0 \Rightarrow 2T\cos\phi - mg = 0$

$\Rightarrow 2T = mg$

[as θ is small $\Rightarrow \phi$ is also small $\Rightarrow \cos\phi \sim 1$]

$\Sigma M/_O = I\ddot{\theta}$

$\Rightarrow -2Tb\sin\phi = I\ddot{\theta} \Rightarrow 2Tb\sin\left(\frac{b\theta}{L}\right) + \frac{1}{3}ma^2\ddot{\theta} = 0$

as $\frac{b\theta}{L}$ is very small $\sin\left(\frac{b\theta}{L}\right) \sim \frac{b\theta}{L}$

$\Rightarrow 2T\frac{b^2\theta}{L} + \frac{1}{3}ma^2\ddot{\theta} = 0$

$\Rightarrow \ddot{\theta} + \omega_n^2\theta = 0$

6

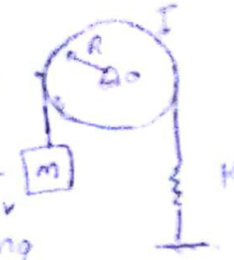
$$\omega_n^2 = \left(\frac{6 T b^2}{m L a^2} \right)$$

$$\omega_n = \sqrt{\frac{6 T b^2}{m L a^2}}$$

Since $2T = mg$

$$\boxed{\omega_n = \sqrt{\frac{3 g b^2}{L a^2}}}$$

Ans



Ex

Total energy of system :)

$x = R\theta$

KE of man + Pulley + Energy stored in spring

$$E = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2 + \frac{1}{2} K x^2$$

We know, $\omega = \dot{\theta}$, $V = R\omega$, $x = R\theta$

$$E = \frac{1}{2} m R^2 \dot{\theta}^2 + \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} K R^2 \theta^2$$

Now, $\frac{dE}{dt} = 0$

$$\frac{dE}{dt} = mR^2 \dot{\theta} \ddot{\theta} + I \dot{\theta} \ddot{\theta} + K R^2 \theta \dot{\theta} = 0$$

$$\Rightarrow (mR^2 + I) \ddot{\theta} + KR^2 \theta = 0$$

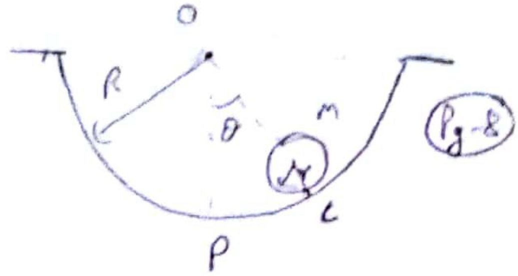
$$\omega = \sqrt{\frac{KR^2}{mR^2 + I}} \Rightarrow$$

$K = 4.3782 \times 10^5$ N/m, $I = 67.79$ N m s^2,

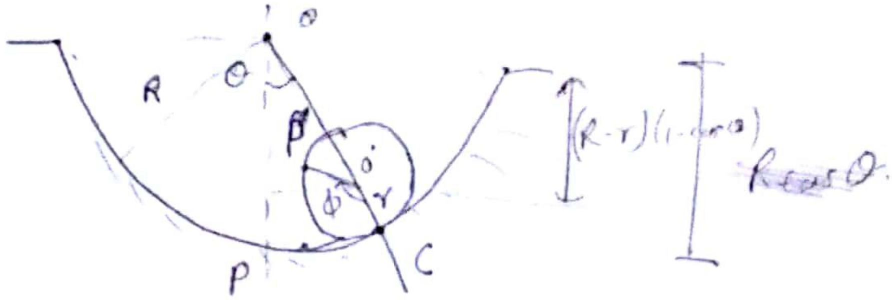
$m = 437.82$ kg, $R = 0.51$m.

$$\omega = \sqrt{\frac{113876.982}{183.667}} \Rightarrow 24 \text{ rad/s}$$

8 →



(Pg-8)



Arc CP = Arc CP'

$r\phi = R\theta$

Translational vel of centre of disc

$\Rightarrow (R-r)\dot{\theta}$

Rotational vel of cylinder $\Rightarrow (\dot{\phi} - \dot{\theta})$

$K.E = (K.E)_{Tr} + (KE)_{rot}$

$\Rightarrow \frac{1}{2} M[(R-r)\dot{\theta}]^2 + \frac{1}{2} I_0(\dot{\phi} - \dot{\theta})^2$

where,

I_0 = MI of disc about it axis

$P.E = Mg(R-r)(1-\cos\theta)$

$T.E = KE + PE = \text{const}$

$\frac{1}{2} M[(R-r)\dot{\theta}]^2 + \frac{1}{2} I_0(\dot{\phi} - \dot{\theta})^2 + Mg(R-r)(1-\cos\theta) = Co$

Replacing $\dot{\phi}$ by $\frac{R\dot{\theta}}{r}$, $I_o = \frac{Mr^2}{2}$ (solid disk) (9)

$$\frac{1}{2}M[(R-r)^2\dot{\theta}^2] + \frac{1}{2}\frac{Mr^2}{2}\left[\frac{R\dot{\theta}}{r} - \dot{\theta}\right]^2 + Mg(R-r)(1-\cos\theta) = \text{Const}$$

$$\frac{1}{2}M[(R-r)^2\dot{\theta}^2] + \frac{1}{4}M[R-r^2]\dot{\theta}^2 + Mg(R-r)(1-\cos\theta) = \text{Const}$$

$$\cancel{\frac{3}{4}M(R-r)}$$

$$E = \frac{3}{4}\frac{(R-r)\dot{\theta}^2}{g} + (1-\cos\theta) = \text{Const}.$$

$\rightarrow \frac{dE}{dt}$

$$\Rightarrow \frac{3}{2}\frac{(R-r)}{g}\dot{\theta}\ddot{\theta} + \sin\theta\,\dot{\theta} = 0$$

$$\Rightarrow \frac{3}{2}\frac{(R-r)}{g}\ddot{\theta} + \sin\theta = 0$$

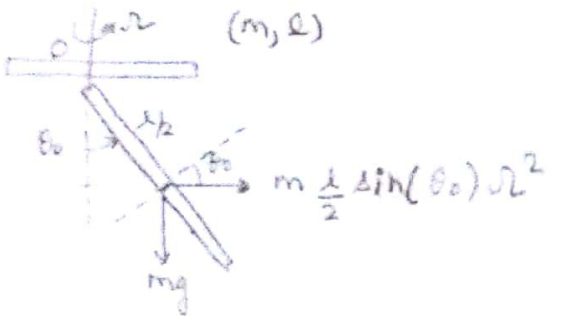
↳ Equation of Motion for large ∠'

- For Small ∠' about $\theta = 0$

$\sin\theta \approx \theta$

$$\frac{3}{2}\frac{(R-r)}{g}\ddot{\theta} + \theta = 0 \quad \Rightarrow \boxed{w_n = \sqrt{\frac{3}{2}\left(\frac{R-r}{g}\right)}} \text{ Ans}$$

Equation of Harmonic Motion $\rightarrow$ $\ddot{\theta} + w_n^2\theta = 0$



a) For equilibrium position

Net force on system is $\Sigma F_{ext} = 0$

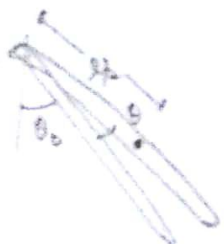
$$\therefore \; m\frac{l}{2}\cos\theta_0\,\Omega^2\,\sin\theta_0 = mg\sin\theta_0$$

$$\cos\theta_0 = \frac{2g}{\Omega^2 l}$$

$$\boxed{\theta_0 = \cos^{-1}\left(\frac{2g}{l\Omega^2}\right)}$$

b) Taking moment about point O

$$\Sigma T_{ext} = J\ddot{\theta}$$



θ_1 being small

System displaced from equilibrium position.

$$m\frac{l}{2}\sin(\theta_0+\theta_1)\,\Omega^2\cdot\frac{l}{2}\cos(\theta_0+\theta_1) - mg\frac{l}{2}\sin(\theta_0+\theta_1) = \frac{ml^2}{3}\ddot{\theta}_1$$

$$\frac{l}{4}\Omega^2\sin(\theta_0+\theta_1)\cos(\theta_0+\theta_1) - \frac{g}{2}\sin(\theta_0+\theta_1) = \frac{l\ddot{\theta}_1}{3}$$

$$\boxed{\ddot{\theta}_1 + \frac{3g}{2l}\sin(\theta_0+\theta_1) - \frac{3\,\Omega^2}{4}\sin(\theta_0+\theta_1)\cos(\theta_0+\theta_1) = 0}$$

Now,

$$\ddot{\theta}_1 + \frac{3g}{2l}\sin(\theta_0+\theta_1) - \frac{3\,\Omega^2}{8}\sin 2(\theta_0+\theta_1) = 0$$

$$\ddot{\theta}_1 + \frac{3g}{2l}\left[\sin\theta_0\cos\theta_1 + \cos\theta_0\sin\theta_1\right] - \frac{3\,\Omega^2}{8}\sin 2\theta_0\cos 2\theta_1 + \sin 2\theta_1\cos 2\theta_0 = 0$$

for small θ_1, $\cos\theta_1 \approx 1$, $\sin\theta_1 = \theta_1$

$$\ddot{\theta}_1 + \frac{3g}{2l}\left[\sin\theta_0 + \theta_1\cos\theta_0\right] - \frac{3\Omega^2}{8}\left[\sin 2\theta_0 + 2\theta_1\cos 2\theta_0\right] = 0$$

Since $\cos\theta_0 = \dfrac{2g}{l\Omega^2}$ $\qquad \cos 2\theta_0 = 2\cos^2\theta_0 - 1$

$$\sin\theta_0 = \frac{\sqrt{(l\Omega^2)^2 - 4g^2}}{l\Omega^2} \qquad = \left(2 \cdot \frac{4g^2}{l^2\Omega^4} - 1\right)$$

$$= \frac{8g^2}{l^2\Omega^4} - 1$$

$$\ddot{\theta}_1 + \frac{3g}{2l}\left[\theta_1 \cdot \frac{2g}{l\Omega^2}\right] + \frac{3g}{2l}\sin\theta_0 - \frac{3}{8}\Omega^2 \cdot 2\theta_1\left[\frac{8g^2}{l^2\Omega^4} - 1\right] - \frac{3l\Omega^2}{8} \cdot 2\sin\theta_0 \frac{2g}{l\Omega^2} = 0$$

$$\ddot{\theta}_1 + \frac{3g^2}{l^2\Omega^2}\theta_1 + \frac{3g}{2l}\sin\theta_0 - \frac{6g^2\theta_1}{l^2\Omega^2} + \frac{3}{4}\Omega^2\theta_1 - \frac{3g\sin\theta_0}{2l} = 0$$

$$\ddot{\theta}_1 - \frac{3g^2}{l^2\Omega^2}\theta_1 + \frac{3}{4}\Omega^2\theta_1 = 0$$

$$\boxed{\ddot{\theta}_1 + \left(\frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2}\right)\theta_1 = 0}$$ Required differential eq^n

(c) For θ_1 to be harmonic

$$\frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2} > 0$$

$$\Omega^4 > \frac{4g^2}{l^2} \Rightarrow \Omega^2 > \frac{2g}{l} \Rightarrow \boxed{\Omega > \sqrt{\frac{2g}{l}}}$$

(d) $$\omega_n^2 = \frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2}$$

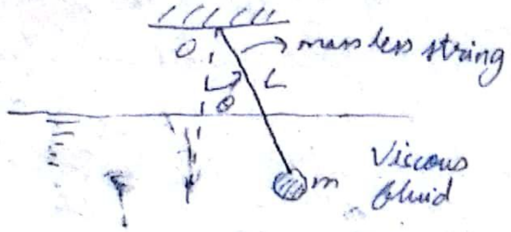
$$\boxed{\omega_n = \sqrt{\frac{3}{4}\Omega^2 - \frac{3g^2}{l^2\Omega^2}}}$$

(e) for very large Ω $\qquad \frac{3}{4}\Omega^2 \gg \frac{3g^2}{l^2\Omega^2}$

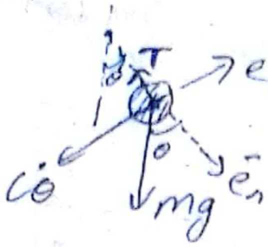
$$\therefore \omega_n = \sqrt{\frac{3}{4}\Omega^2} = \frac{\sqrt{3}}{2}\Omega$$

Natural frequency does not depend on gravity values.

Q10



FBD of mass m



$\sum \vec{F}_{\hat{e_n}} = 0$

$mg\cos\theta = T$ (T is tension in string)

$\sum \vec{F}_{\hat{e_\theta}} = m\vec{a}_\theta$ $\quad (\vec{a}_\theta = l\ddot{\theta}\,(\hat{k}))$

$\Rightarrow -mg\sin\theta - c\dot{\theta} = ml\ddot{\theta}$

$\Rightarrow ml\ddot{\theta} + c\dot{\theta} + mg\sin\theta = 0$

For small oscillations. $\sin\theta \sim \theta$

So $ml\ddot{\theta} + c\dot{\theta} + mg\theta = 0$

$\Rightarrow \ddot{\theta} + \frac{c}{ml}\dot{\theta} + \frac{g}{l}\theta = 0$

$\omega_n^2 = \frac{g}{l} \Rightarrow \boxed{\omega_n = \sqrt{\frac{g}{l}}} \Rightarrow \boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{g}{l}}}$

$\Rightarrow 2\xi\omega_n = \frac{c}{ml}$

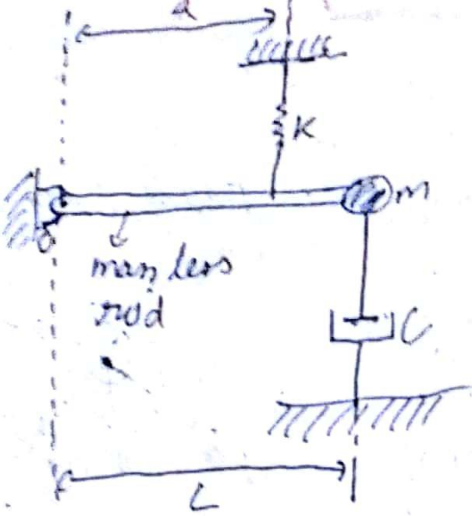
$\Rightarrow \xi = \frac{c}{2ml\omega_n} = \frac{c}{2ml\sqrt{\frac{g}{l}}}$

$\Rightarrow \boxed{\xi = \frac{c}{2m\sqrt{gl}}}$

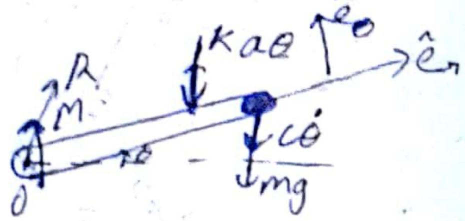
$\omega_d = \omega_n\sqrt{1-\xi^2} = \sqrt{\frac{g}{l}}\sqrt{1-\frac{c^2}{4m^2gl}}$

$\boxed{f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l}}\sqrt{1-\frac{c^2}{4m^2gl}}}$

Q11)



FBD of system rod & mass bar small amplitude θ



$$\sum \vec{F}_{\hat{e}_n} = ma_n = 0$$

$$\sum \vec{F}_{\hat{e}_\theta} = ma_\theta = mL\ddot{\theta}$$

$$\sum \vec{M}_{/O} = mL^2\ddot{\theta}$$

$$\Rightarrow -mgL - c\dot{\theta}L - ka^2\theta = mL^2\ddot{\theta}$$

$$\Rightarrow mL^2\ddot{\theta} + c\dot{\theta}L + ka^2\theta + mgL = 0$$

Here neglecting Force due to gravity.

$$\Rightarrow mL^2\ddot{\theta} + c\dot{\theta}L + ka^2\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{c}{mL}\dot{\theta} + \frac{ka^2}{mL^2}\theta = 0$$

$$w_n = \sqrt{\frac{ka^2}{mL^2}} = \sqrt{\frac{7.0051\times10^5 \times \cancel{1.27\times1.27}}{1751.27 \times \frac{2.54}{2}\times\frac{2.54}{2}}}\ \text{rad/s}^2$$

$$\boxed{w_n = 10.00\ \text{rad/s}^2}$$

$$\frac{c}{mL} = 2\zeta w_n = \frac{3502.54}{1751.27\times2.54}$$

$$\Rightarrow \zeta = \frac{3502.54}{2\times1751.27\times2.54\times10} = 0.039$$

$$\boxed{\zeta \simeq 0.04}$$

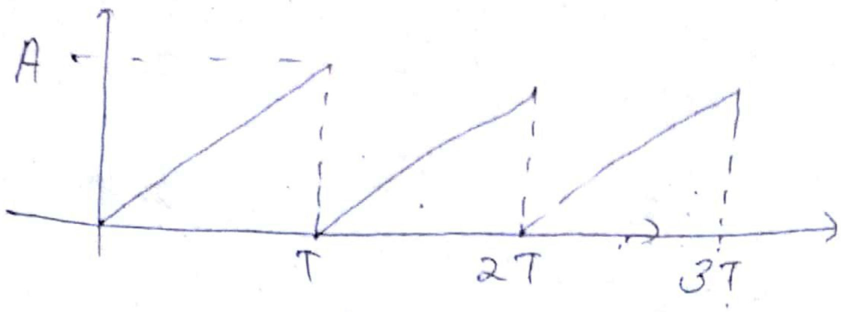
$\omega_d = \omega_n \sqrt{1-\xi^2}$

$\boxed{\omega_d = 9.99 \text{ rad/s}^2}$

$C_c = mL = 1751.27 \times 2.54 \text{ Ns/m}$

$\boxed{C_c \doteq 4448.23 \text{ Ns/m}}$ → Critical damping

Q12)



$f(t) = \frac{A}{T} t$ for $t \in [0, T)$

$f(t+nT) = f(t)$

Fourier series expansion

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + \sum b_n \sin\left(\frac{n\pi t}{L}\right)$$

here $L = T/2$

$$a_0 = \frac{1}{L}\int_0^{2L} f(t)\,dt$$

$$a_n = \frac{1}{L}\int_0^{2L} f(t)\cos\left(\frac{n\pi t}{T}\right)dt$$

$$b_n = \frac{1}{L}\int_0^{2L} f(t)\sin\left(\frac{n\pi t}{T}\right)dt$$

$$a_0 = \frac{2}{T}\int_0^T \frac{At}{T}\,dt = \frac{A}{T^2} t^2\Big|_0^T$$

$\Rightarrow \boxed{a_0 = A}$

$\Rightarrow \boxed{a_0 = A}$

$a_n = \frac{2}{T}\int_0^T \frac{At}{T}\cos\left(\frac{2n\pi t}{T}\right)dt$

$\Rightarrow a_n = \frac{2A}{T^2}\int_0^T t\cos\left(\frac{2n\pi t}{T}\right)dt$

$\Rightarrow a_n = \frac{A}{2(n\pi)^2}\int_0^T \left(\frac{2n\pi t}{T}\right)\cos\left(\frac{2n\pi t}{T}\right)d\left(\frac{2n\pi t}{T}\right)$

$\Rightarrow a_n = \frac{A}{2(n\pi)^2}\int_0^T \left(\frac{2n\pi t}{T}\right)\cos\left(\frac{2n\pi t}{T}\right)d\left(\frac{2n\pi t}{T}\right)$

Substituting $y = \frac{2n\pi t}{T}$

$\Rightarrow a_n = \frac{A}{2(n\pi)^2}\int_0^{2n\pi} y\cos y\,dy$

applying method of by part integration

$\Rightarrow a_n = \frac{A}{2(n\pi)^2}\left[\cancel{y\sin y\Big|_0^{2n\pi}}^{\,0} - \int_0^{2n\pi}\sin y\,dy\right]$

$\Rightarrow a_n = \frac{A}{2(n\pi)^2}\left[\cancel{\cos y\Big|_0^{2n\pi}}^{\,0}\right]$

$\Rightarrow \boxed{a_n = 0}$

[So Cosine series of given function is 0]

$$b_n = \frac{2}{T}\int_0^T \frac{At}{T} \sin\left(\frac{2n\pi t}{T}\right) dt$$

$$\Rightarrow b_n = \frac{2A}{2(n\pi)^2}\int_0^T \left(\frac{2n\pi t}{T}\right)\left(\frac{2n\pi}{T}\right)\sin\left(\frac{2n\pi t}{T}\right) dt$$

$$\Rightarrow b_n = \frac{A}{2(n\pi)^2}\int_0^T \left(\frac{2n\pi t}{T}\right)\sin\left(\frac{2n\pi t}{T}\right) d\left(\frac{2n\pi t}{T}\right)$$

$\Rightarrow$ ~~b_n~~ Substituting $y = \frac{2n\pi t}{T}$

$$\Rightarrow b_n = \frac{A}{2(n\pi)^2}\int_0^{2n\pi} y \sin y \, dy$$

applying method of by part integration

$$\Rightarrow b_n = \frac{A}{2(n\pi)^2}\left[-y\cos y\Big|_0^{2n\pi} + \int_0^{2n\pi}\cos y \, dy\right]$$

$$\Rightarrow b_n = \frac{A}{2(n\pi)^2}\left[-2n\pi\cos(2n\pi) + 0 - \cancel{\sin y\Big|_0^{2n\pi}}^{\,0}\right]$$

$$\Rightarrow \boxed{b_n = \frac{-A}{(n\pi)}}$$

Sine series of given function is $= -\sum_{n=1}^{\infty} \frac{A}{n\pi}\sin\left(\frac{2n\pi t}{T}\right)$

So for $A = 2$ & $T = 1$

~~$f(t)$~~ $\boxed{a_0 = 2;\ a_n = 0,\ b_n = \frac{-2}{n\pi}}$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi t}{T}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi t}{T}\right)$$

$$\Rightarrow f(t) = 1 - \sum_{n=1}^{\infty} \frac{2}{\pi n} \sin(2n\pi t)$$

$$\Rightarrow \boxed{f(t) = 1 - \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(2n\pi t)}$$

(13) $f(t) = 5\sin(10t - 2.5)$

$= 5(\sin(10t)\cos(2.5) - \cos(10t)\sin(2.5))$

$f(t) = -4.0057\sin(10t) - 2.9924\cos(10t)$

(14) $y'' + 3y' + 2y = -5\sin(t) + 5\cos(t)$ — ①

$y(0) = 5 \; ; \; y'(0) = -3$

we know $\mathcal{L}(y^n(t)) = s^n \mathcal{L}(y(t)) - s^{n-1}y(0) - s^{n-2}y'(0) \dots y^{n-1}(0)$

$\therefore \mathcal{L}(y'(t)) = sY(s) - y(0)$ $\{ Y(s) = \mathcal{L}(y(t)) \}$

$\mathcal{L}(y''(t)) = s^2 Y(s) - sy(0) - y'(0)$

Taking Laplace transform of eq ①

$(s^2Y(s) - 5s + 3) + 3(sY(s) - 5) + 2Y(s) = \mathcal{L}(5\cos t - 5\sin t)$

$= 5\left[\frac{s}{s^2+1} - \frac{1}{s^2+1}\right]$

$Y(s).(s^2+3s+2) - 5s - 12 = \frac{5(s-1)}{s^2+1}$

$Y(s).(s^2+3s+2) = \frac{5s - 5 + (5s+12)(s^2+1)}{s^2+1} = \frac{5s - 5 + 5s^3 + 5s + 12s^2 + 12}{s^2+1}$

$Y(s) = \frac{5s^3 + 12s^2 + 10s + 7}{(s^2+1)(s^2+3s+2)} = \frac{5s^3+12s^2+10s+7}{(s^2+1)(s+1)(s+2)}$

Now taking inverse laplace

$y(t) = \mathcal{L}^{-1}\left(\frac{5s^3+12s^2+10s+7}{(s^2+1)(s+1)(s+2)}\right) = \mathcal{L}^{-1}(f(s))$

$f(s) = \frac{A}{s+1} + \frac{B}{s+2} + \frac{Cs+D}{s^2+1}$

after routine partial fraction technique

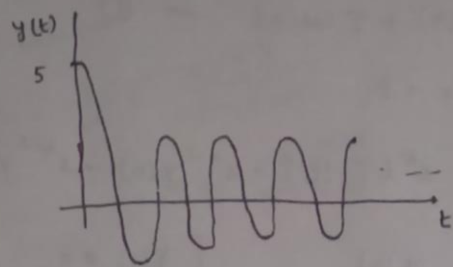
$f(s) = \frac{2}{s+1} + \frac{1}{s+2} + \frac{2s}{s^2+1} + \frac{1}{s^2+1}$

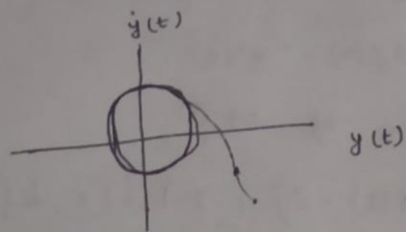
Taking inverse laplace transform

$\boxed{y(t) = 2e^{-t} + 1.e^{-2t} + 2\cos(t) + \sin(t)}$

$y(t) = 2e^{-t} + e^{-2t} + 2\cos t + \sin t$

After a few seconds, the exponents will die out and only the periodic solution will be significant and hence contribute
15 to the behaviour of $y(t)$.





The solution will form a limit cycle of

a radius = $\sqrt{2^2+1^2} = \sqrt{5}$.

(15) a) $$F(s) = \frac{5s^2 + 8s - 5}{s^2(s^2 + 2s + 5)}$$

partial fraction :

$$F(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 2s + 5}$$

$$F(s) = \frac{2}{s} - \frac{1}{s^2} + \frac{-2(s)}{s^2 + 2s + 5} + \frac{2}{s^2 + 2s + 5}$$

Taking inverse laplace

$$f(t) = 2 - t - 2e^{-t}(\cos(2t) - \sin(2t))$$

b) $$F(s) = \frac{s(1 + e^{-1.5s} + e^{-2.2s}) + e^{-1.5s}}{s(s+2)}$$

$$F(s) = \frac{1}{s+2} + \frac{e^{-1.5s}}{s+2} + \frac{e^{-2.2s}}{s+2} + \frac{e^{-1.5s}}{(s)(s+2)} \quad - (1)$$

$\mathcal{L}^{-1}$ (1)

$$f(t) = e^{-2t} + e^{-2(t-1.5)}\,\delta(t-1.5) + e^{-2(t-2.2)}\,\delta(t-2.2)$$

$$+ \mathcal{L}^{-1}\left(\frac{1}{2}\left(\frac{e^{-1.5s}}{s} - \frac{e^{-1.5s}}{s+2}\right)\right)$$

$$f(t) = e^{-2t} + e^{-2(t-1.5)}\,\delta(t-1.5) + e^{-2(t-2.2)}\,\delta(t-2.2)$$

$$+ \frac{1}{2}\,1.\,\delta(t-1.5) - \frac{1}{2}e^{-2(t-1.5)}\,\delta(t-1.5)$$

{using identity $\mathcal{L}^{-1}(e^{-as}F(s)) = f(t-a)$ }