# SOLUTION
## Tutorial-6

**Given:** Gear ratio $m_G = \frac{N_G}{N_P} = 4$

Pressure angle $\phi = 20^\circ$.

(a) $\Rightarrow$ To avoid interference: No. of teeth in pinion :- $N_P$

$$N_P \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2+(1+2m)\sin^2\phi}\right)$$

$$= 15.44$$

($k = 1$ full depth teeth)

$\Rightarrow$ Choose $\boxed{N_P = 16}$ $\Rightarrow$ $\boxed{N_G = m_G N_P = 64}$

Module:

Center to Center distance:

$$C = \left(\frac{N_P + N_G}{2}\right).m = 200 \text{ mm}$$

$$\Rightarrow \boxed{m = 5\text{mm}}$$

(b) Diameters: $\boxed{\begin{aligned} d_P &= 80\text{mm} \\ d_G &= 320\text{mm} \end{aligned}}$

Power rating: $H = 25$ kW

$\Rightarrow$ Transmitted load: $W_t = \frac{60000H}{\pi d n} = \frac{60000 \times 25}{\pi \times 80 \times 2000}$ kN

$$\Rightarrow \boxed{W_t = 2.984\text{ kN} = 2984\text{ N}}$$

Pitch line velocity $\boxed{V = \frac{\pi d n}{60} \times 10^{-3} \text{ m/s} = 8.378 \text{ m/s.}}$

## Factors for Bending stress and Strength calculation

Overload Factor: $\boxed{K_o = 2.0}$ (for Light shock power source and Heavy shock driven machine)

Surface factor: $K_s = 0.843\,(b m \sqrt{Y})^{0.0535}$

facewidth $b = 50$mm, $m = 5$mm,

$\boxed{\begin{aligned} Y_P &= 0.296 \quad \text{(pinion)} \\ Y_G &= 0.425 \quad \text{(gear)} \end{aligned}}$

(Table 14-2 and linear interpolation)

$\Rightarrow \boxed{\begin{aligned} (K_s)_P &= 1.0964 \quad \text{(pinion)} \\ (K_s)_G &= 1.1071 \quad \text{(gear)} \end{aligned}}$

## Dynamic Factor

Equation 14-27

$$K_v = \left( \frac{A + 200V}{A} \right)^B \quad : \quad A = 50 + 56(1-B)$$

$$B = 0.25(12 - Q_v)^{2/3}$$

($Q_v$ ← Quality no.)

$\Rightarrow$ $B = 0.3969$

$A = 83.776$

$\Rightarrow$ $\boxed{K_v = 1.171}$

## Load distribution factor

$$K_H = 1 + C_{mc}(C_{pf} C_{pm} + C_{ma} C_e) \quad \text{(Eq. 14-30)}$$

$C_{mc} = 1$ (uncrowned teeth)

$$C_{pf} = \frac{b}{10d} - 0.0375 + \frac{0.0125\, b}{25.4} \quad \text{(Eq. 14-32)}$$

$\Rightarrow$ $\boxed{(C_{pf})_P = 0.0496 \text{ (pinion)}}$

$\boxed{(C_{pf})_G = 0.0371 \text{ (gear)}}$ ← $\frac{b}{10d} = 0.0156 \leq 0.05$

$\Rightarrow \frac{b}{10d} = 0.05$ is chosen

$S_1/S = 0$ (See Fig 14-10)

$\Rightarrow$ $\boxed{C_{pm} = 1}$

$C_e = 1$, $C_{ma} \cong 0.1$ (curve-3 Fig 14-11, $b = 50mm = 1.97$ in)

$\Rightarrow$ $(K_H)_P = 1 + 1(0.0496 * 1 + 0.1)$

$\Rightarrow$ $\boxed{(K_H)_P = 1.1496 \text{ (pinion)}}$

$\boxed{(K_H)_G = 1.1371 \text{ (gear)}}$

Rim thickness-to-tooth height ratio $= 1.5$

$\Rightarrow$ $\boxed{K_B = 1}$ (Figure 14-16)

**Geometry Factor:** $\boxed{(Y_J)_P = 0.27 \text{ (pinion)}}$ (Fig 14-6)

$\boxed{(Y_J)_G = 0.41 \text{ (gear)}}$

**Stress Cycle factor:** $Y_N = 1.3558 \times N^{-0.0178}$ (Fig 14-14)

$\Rightarrow$ $\boxed{(Y_N)_P = 0.977 \text{ (pinion, } N = 10^8 \text{ cycles)}}$

$\boxed{(Y_N)_G = 1.009 \text{ (gear, } N = \frac{10^8}{4} \text{ cycles)}}$

Temperature factor: $\boxed{Y_\theta = 1}$

Reliability factor :- $\boxed{Y_Z = 1}$ (Reliability = 0.99) (Table 14-10)

**Bending strength :**

$S_t = 0.7255\, H_B + 153.63$ (Nitriding Steel 2.5% Chrome, Grade 2) Figure 14-4

$\boxed{\begin{aligned}(S_t)_P &= 335\ MPa \quad (\text{Pinion},\ HB=250)\\ (S_t)_G &= 298.73\ MPa \quad (\text{Gear},\ HB=200)\end{aligned}}$

**Bending failure**

Pinion : $\sigma = \dfrac{W_t}{bm} K_o K_v K_s \dfrac{K_H K_B}{Y_J}$

$= \dfrac{2989}{50\times5} \times 2 \times 1.171 \times 1.0964 \times \dfrac{1.1496 \times 1}{0.27}$

$\Rightarrow \boxed{(\sigma)_P = 130.5\ MPa}$

Allowable stress: $\sigma_{all} = \dfrac{S_t}{S_F} \cdot \dfrac{Y_N}{Y_\theta Y_Z}$

$\Rightarrow (\sigma_{all})_{pinion} = \dfrac{335 \times 0.977}{S_F \times 1 \times 1} = \dfrac{327.3}{(S_F)_P}\ MPa.$

$\Rightarrow$ Factor of Safety : $\boxed{(S_F)_P = 2.508}$

Gear : $(\sigma)_G = \dfrac{2984}{50\times5} \times 2 \times 1.171 \times 1.1071 \times \dfrac{1.1371 \times 1}{0.41}$

$\Rightarrow \boxed{(\sigma)_G = 85.83\ MPa}$

Allowable stress $(\sigma_{all})_{Gear} = \dfrac{298.73}{S_F} \times \dfrac{1.001}{1\times1} = \dfrac{299}{S_F}\ MPa$

Factor of Safety : $\boxed{(S_F)_G = 3.484}$

$\Rightarrow \boxed{(S_F)_P = 2.508,\ (S_F)_G = 3.484}$

## Factors for pitting-resistance & strength:

Elasticity coefficient: $Z_E = 191\ \sqrt{MPa}$ (Table 14-8)

Surface Condition Factor: $Z_R = 1$

Geometry factor: $Z_I = \dfrac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \dfrac{m_G}{m_G+1}$ (Equation 14-23)

$\phi_t = \phi = 20°$, $m_N = 1$, (spur gear)
$m_G = 4$

$$\Rightarrow \boxed{Z_I = 0.1285}$$

Stress cycle factor: $Z_N = 1.4488\, N^{-0.023}$ (Fig 14-15)

$$\Rightarrow \boxed{\begin{aligned}(Z_N)_P &= 0.948 \quad \text{(pinion)}\\ (Z_N)_G &= 0.979 \quad \text{(gear)}\end{aligned}}$$

Hardness Ratio factor:

$$1.7 \geq \frac{H_{BP}}{H_{BG}} = \frac{250}{200} = 1.25 \geq 1.2$$

$$\Rightarrow A' = \left(8.98\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29\right)\times 10^{-3} \quad \text{(Eq. 14-36)}$$
$$= 2.935 \times 10^{-3}$$

$$\Rightarrow (Z_W)_G = 1.0 + A'(m_G - 1) = 1.009. \quad \text{(Eq. 14-36)}$$

$$\Rightarrow \boxed{\begin{aligned}(Z_W)_P &= 1. \quad \text{(pinion)}\\ (Z_W)_G &= 1.009 \quad \text{(gear)}\end{aligned}}$$

Contact strength: $\boxed{S_c = 1350\ MPa}$ for both pinion and gear.

Pitting Failure.

Pinion: $$(\sigma_c)_P = Z_E \left\{ W_t K_o K_v K_s \cdot \frac{K_H}{d_P \cdot b} \frac{Z_R}{Z_I} \right\}^{1/2}$$

$$= 191 \left\{ 2984 \times 2 \times 1.171 \times 1.0964 \times \frac{1.1496}{80 \times 50} \times \frac{1}{0.1285} \right\}^{1/2}$$

$$\Rightarrow \boxed{(\sigma_c)_P = 790.68\ MPa}$$

Allowable contact stress: $\sigma_{c,all} = \dfrac{S_c}{S_H} \dfrac{Z_N Z_W}{Y_\theta Y_Z}$

$$\Rightarrow \boxed{(\sigma_{c,all})_P = \frac{1350 \times 0.948 \times 1}{S_H \quad 1 \times 1} = \frac{1279.8}{S_H}\ MPa}$$

Factor of Safety: $\boxed{(S_H)_P = 1.619}$

Gear: $(\sigma_c)_G = 191 \left\{ 2984 \times 2 \times 1.171 \times 1.1071 \times \frac{1.1371}{50 \times 80} \times \frac{1}{0.1285} \right\}$

$\boxed{(\sigma_c)_G = 790.2 \text{ MPa}}$

Allowable contact stress

$$\boxed{(\sigma_{c,all})_G = \frac{1350 \times 0.979 \times 1.009}{S_H \times 1 \times 1} = \frac{1333.59}{S_H} \text{ MPa}}$$

Factor of Safety: $\boxed{(S_H)_G = 1.688}$

$\Rightarrow$ $\boxed{(S_H)_P = 1.619, \quad (S_H)_G = 1.688}$

(c) Likely Failure Mode:

Pinion: $\boxed{(S_F)_P = 2.508, \quad (S_H)_P^2 = 2.621}$

Gear: $\boxed{(S_F)_G = 3.484, \quad (S_H)_G^2 = 2.849}$

$\Rightarrow$ Most threat to failure is from bending of pinion tooth.

(d) Power to failure:

$(\sigma_{all})_P = (\sigma)_P$ for pinion. with $(S_F)_P = 1$.

$\Rightarrow$ $\sigma_P = 327.3 \text{ MPa}$.

$\Rightarrow$ $\boxed{W_t = 7.484 \text{ kN}}$ at failure

$\Rightarrow$ $\boxed{H = 62.7 \text{ kW}}$ power rating at failure

This is 151% overload