# Design of Controller using Root Locus Method

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# The Lecture Contains

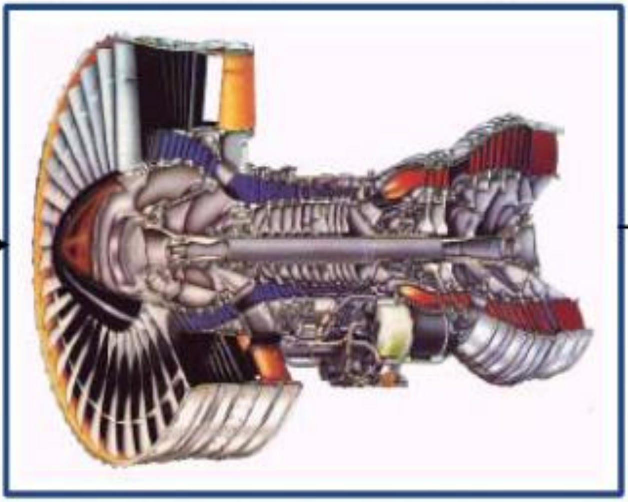
- Parameters of Engine Control
- Cascaded Compensators
- Lag Compensator

# Parameters for Engine Control

•Fuel Flow

•Stator Vane Angle

•Nozzle Area



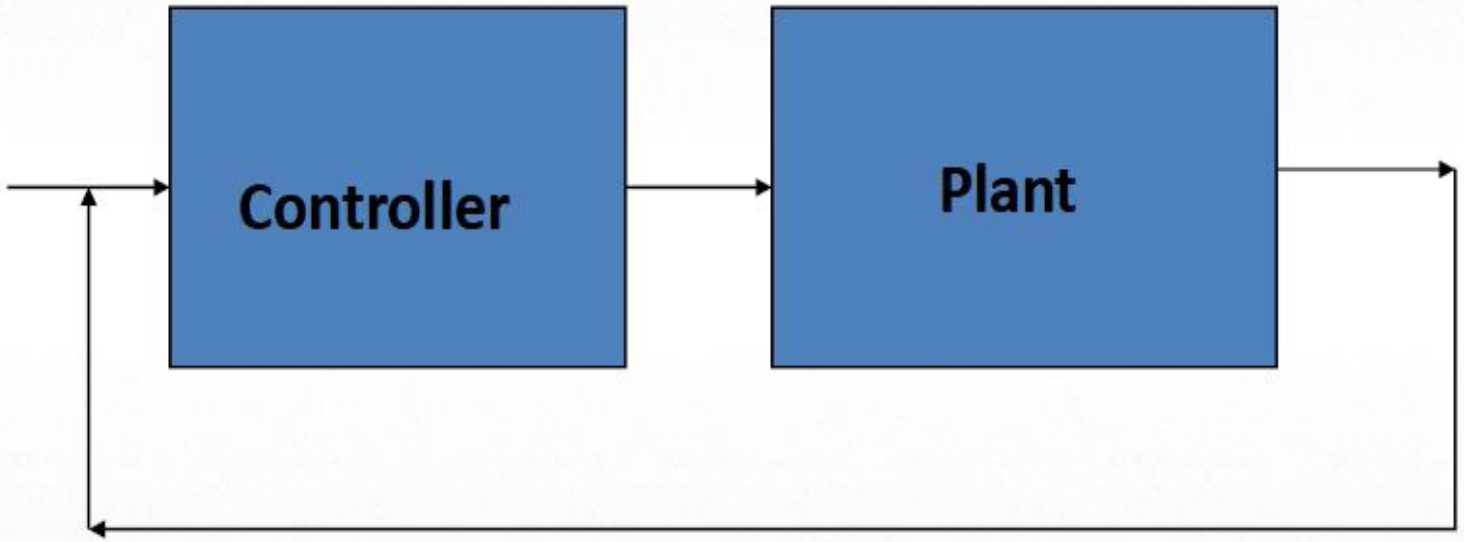
•Thrust

•Metal Temperatures

•Flows

# Cascaded Controller

Cascaded Controllers/Compensators are provided in the forward loop itself as shown below.




Types of Compensators: Lag Compensator, Lead Compensator, Lead-Lag Compensator, Notch Filter

Well defined relation between primary and secondary variables, faster inner loop, disturbances and uncertainties in the inner loop

# Lag Filter/Compensation

- Introduce a set of Pole and Zero, Overall there is a reduction of phase in the system. You may remember that addition of a pole reduces the phase (phase lag) and a zero increases the phase (phase lead). In this case we introduce a dominant pole and a non-dominant zero to the system.

- Improves Steady State Performance if the Pole is very near to the origin and the zero is little away in the left side from the pole.

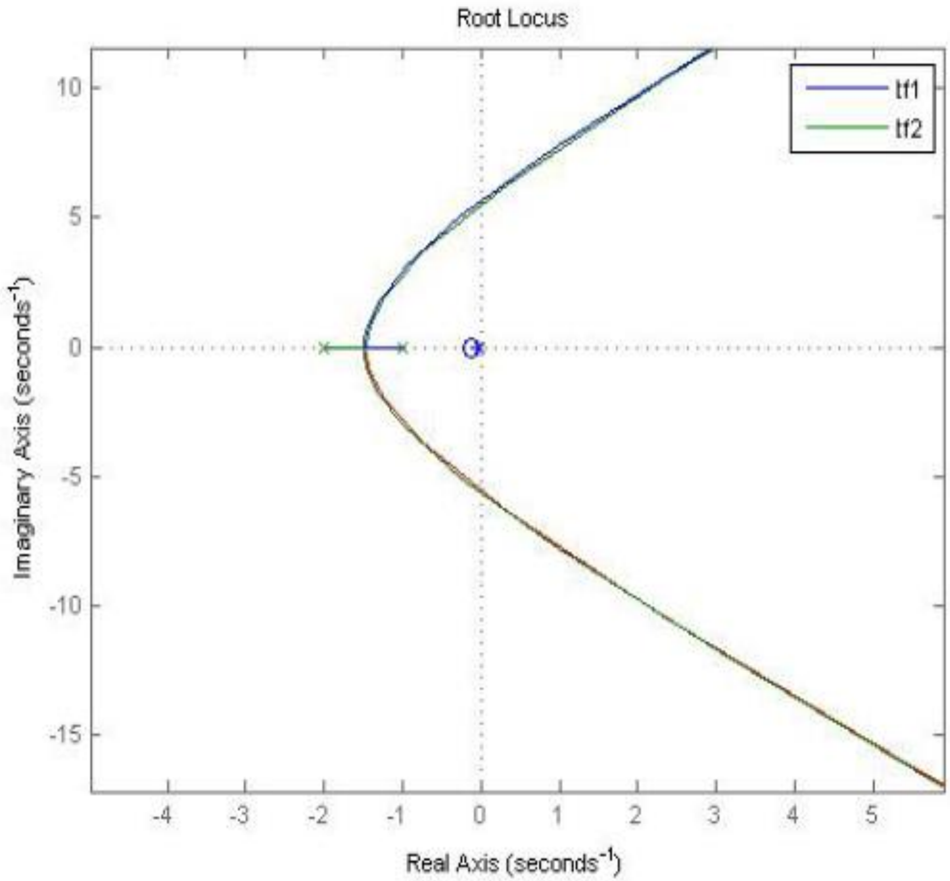
- $K_0$ (uncompensated) = Kz1z2…/$p_1p_2$….

- K (compensated) = $K_o\, z_c/p_c$

# Design of a Lag filter

- Consider a Plant with open loop poles at -1,-2 and -10, Find $K_p$, Improve steady state error by a factor of 10 and get the closed loop poles. Damping ratio should be around 0.174.
- $K_p = 8.23$ (Dominant Pair Gain/$p_1p_2p_3$)
- $e_\alpha = 1/1+K_p = .108$
- Desired $e_\alpha = .0108$, $K_p = 91.59$
- $z_c/p_c = 91.59/8.23 = 11.13$, Use $p_c = 0.01$,
- $z_c = 11.13 * P_c = 0.111$
- The root-loci of the uncompensated and the compensated plants follows:

# Root Loci for the two systems

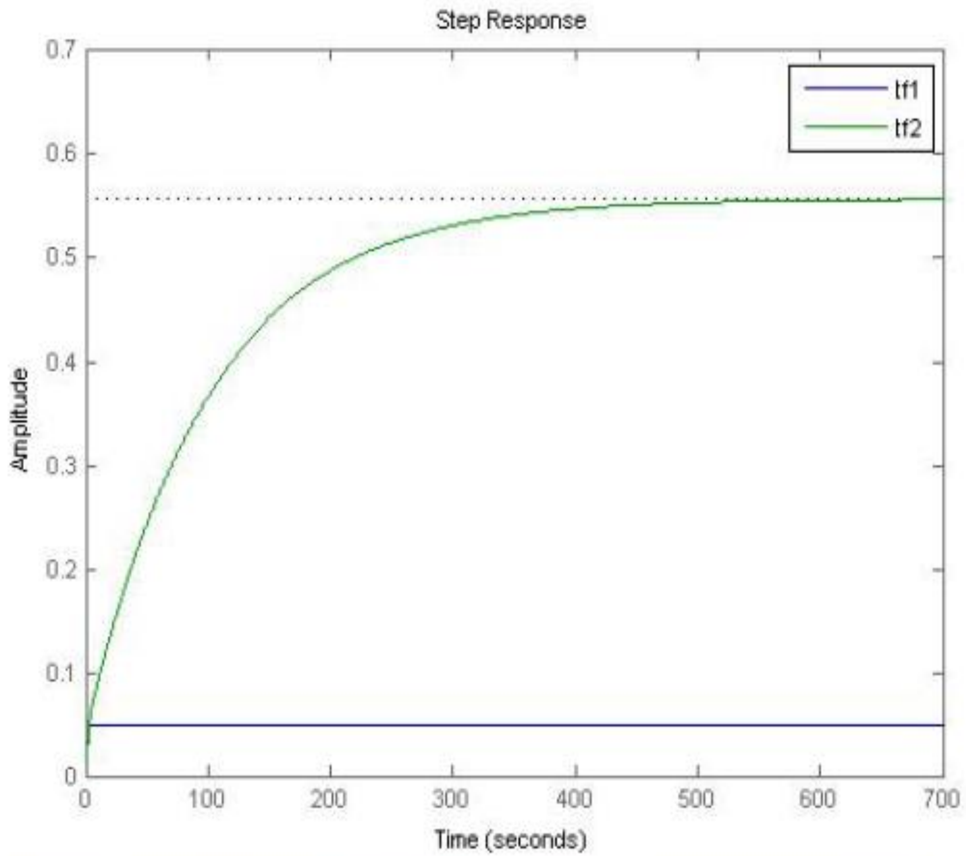



$$tf_1 = \frac{1}{(s+1)(s+2)(s+10)}$$

$$tf_2 = \frac{(s+0.11)}{(s+0.01)(s+1)(s+2)(s+10)}$$

There's almost no effect of compensator on the transient behavior and stability of the system

# Steady State Response of the two systems




$$tf_1 = \frac{1}{(s+1)(s+2)(s+10)}$$

$$tf_2 = \frac{(s+0.11)}{(s+0.01)(s+1)(s+2)(s+10)}$$

You may have observed that the steady state response has increased more than five times.

# Structure of Cascaded Lag Compensation

- A Cascaded Lag Compensation may be symbolically represented as

$$G_c(s) = \frac{K_c(s+a)}{s+b}$$

- Where, $K_c$ , *a* and *b* are positive constants and *a>b*. A proportional integral (PI) compensator is a special case of this system, where *b=0*.

- The error constant of an uncompensated system [ $G_p$ (s) , K] may be written as:

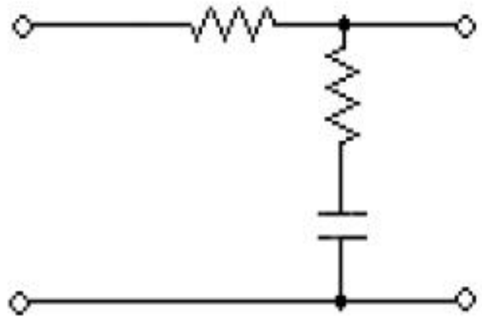
$$K_u = K \lim_{s \to 0} s^i G_p(s)$$

- The error constant gets modified after the introduction of $G_c$ (s) such that:

$$K_{comp} = K_c K \left(\frac{a}{b}\right) \lim_{s \to 0} s^i G_p(s)$$

# Practical Realization of a Lag Compensator

The figure below shows the practical realization of a Lag Compensator with the help of resistors and capacitor. In terms of mechanical elements we can realize the same by using dashpot and springs. Just replace t the resistors by dashpots and capacitor by spring.



# Assignment

- A Plant is having the following form

$$G_p(s) = \frac{1}{s(s+2)(s+4)}, K = 5$$

- Design a suitable lag compensator such that the error constant is improved by a factor of 10.

# Design of a Lead Compensator

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# The Lecture Contains

- Standard Forms of a Lead Controller
- Design of a Lead Filter
- Effect of Lead Compensation
- Practical Realization
- Assignment

# What is a Lead Compensator?

- Lead Compensator is similar in structure as a Lag Compensator, the transfer function could be written as:

$$K_{lead}(s) = \frac{K_c\,(s+a)}{s+b}$$

- However, in this case the zero is dominating over the pole and hence *b>a*. Remember, if a pole or zero is closer to the origin it becomes dominating. Now since zero gives a positive phase and a pole negative phase; hence, in this configuration the compensator provides a neat positive phase and thus the name 'lead' is justified.

- Lead Compensator improves the stability of a system by shifting the root locus towards the left of the origin. Thus while a lag compensator improves the steady state response sacrificing the stability, lead improves the stability of the system.

# Design of a Lead filter

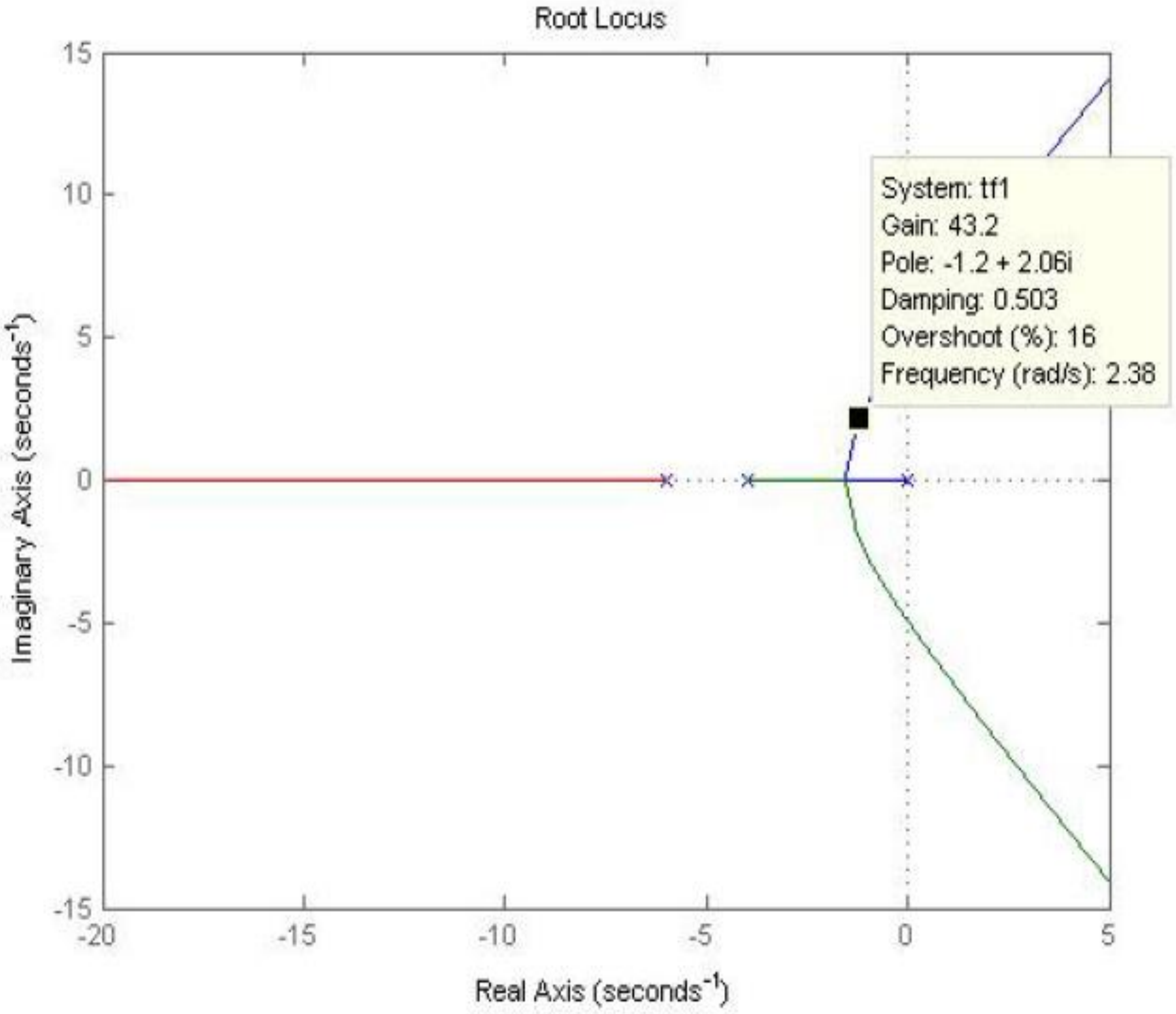
- Consider a Plant with open loop poles at 0,-4 and

-6, even if this is a $3^{rd}$ order system, the third pole is quite away.

Objective: Get 16% overshoot with three-fold reduction in settling time.

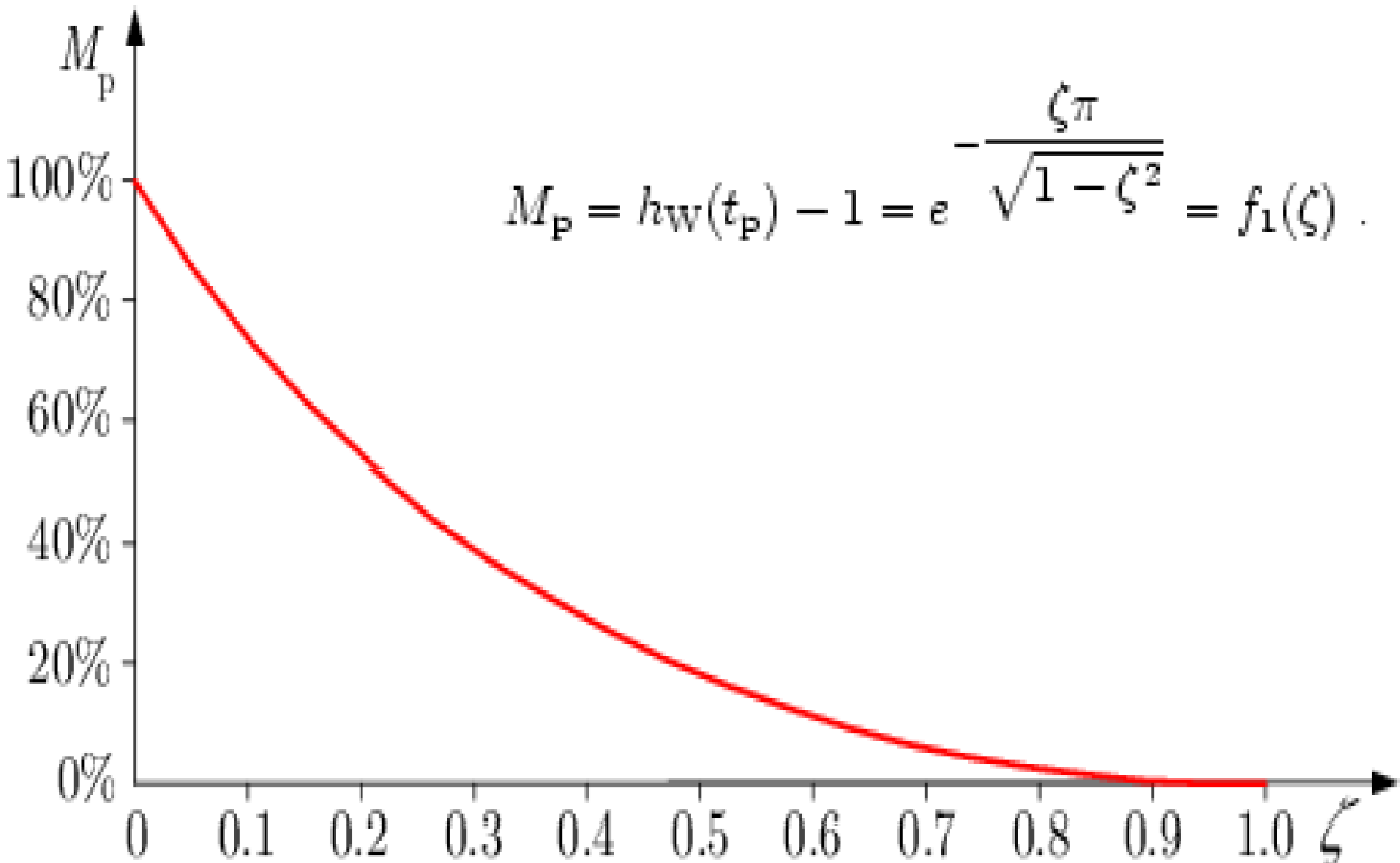
- $\zeta$=.504 (for 16% OS) (see the chart in the next page)
- Poles at -1.20 +- j2.06, K=43.20
- The uncompensated system is shown in the next slide.

# Design of a Lead filter




$$tf_1 = \frac{1}{s(s+4)(s+6)}$$

# Damping Factor Vs. Overshoot

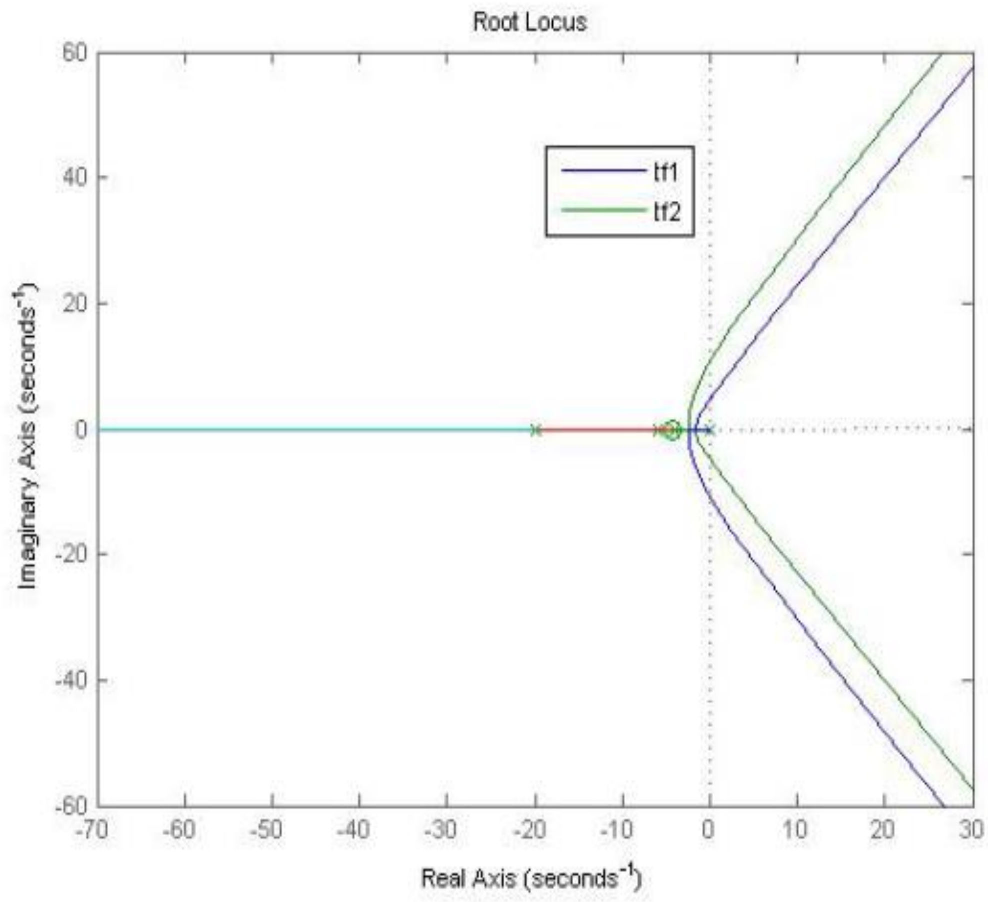


# Design of a Lead filter

- As per design specification: at the current location
- $T_s = 4/\zeta\omega = 4/1.20 = 3.33$
- Recheck it as a 2nd order system
- Desired $T_s = 1.107$ s, corresponding $\sigma = 4/T_s = 3.613$
- With same damping, $\omega_d = 3.613 \tan(180-120.26) = 6.193$
- Hence, the desired poles are at -3.613 +- j6.193
- Plot and get the phase at the point as $-275.6^o$
- Hence, angle of the compensator zero is $95.6^o$, Use graphical plot to get the real part of zero.
- The compensator zero could be obtained as -4.22
- The compensator pole may be chosen at -20.

# Root Loci of the Compensated and uncompensated system

- The plot clearly shows that the introduction of the lead compensator has shifted the plot towards left and increased the stability of the system.



# PD Vs Lead Compensator

$$G(s) = \frac{1}{s(s+1)}$$

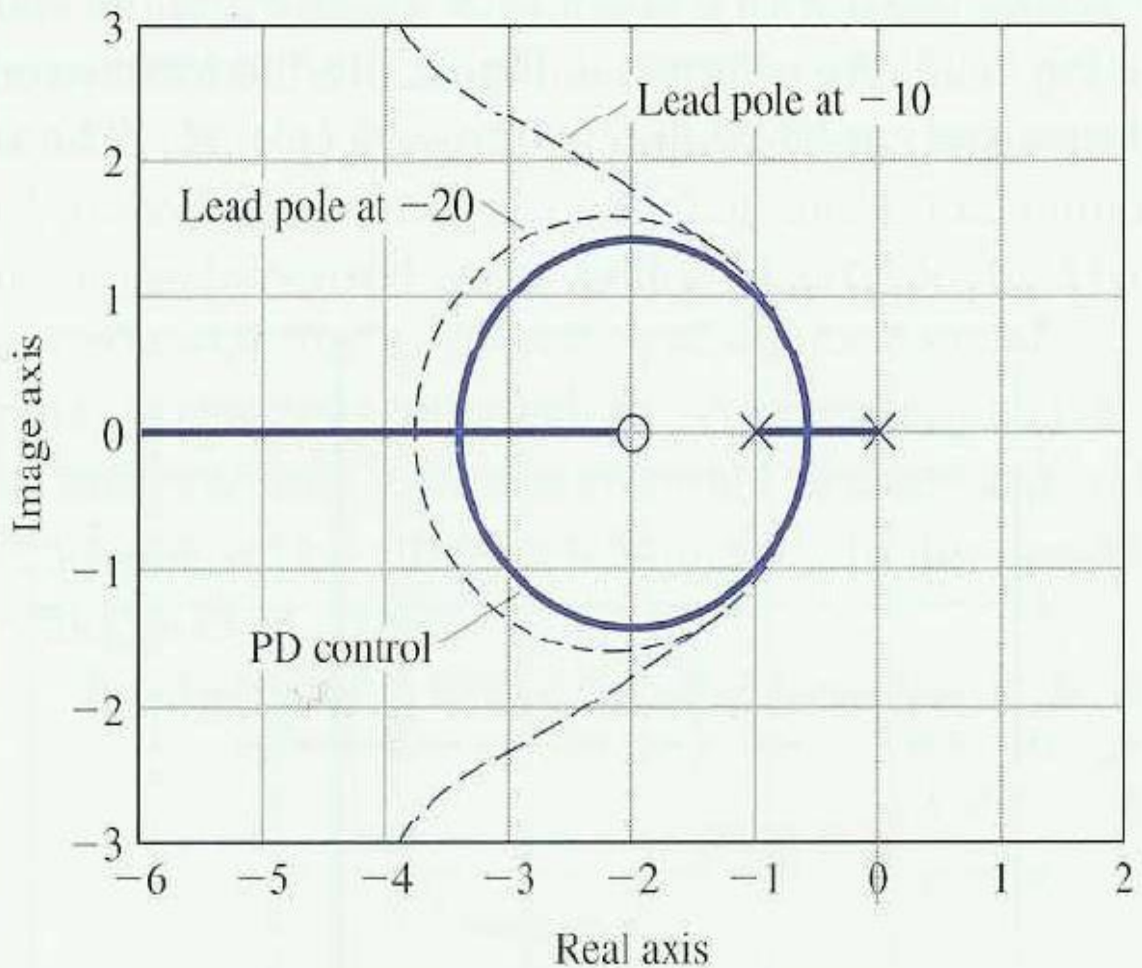
PD - control

$$D(s) = K(s+2)$$

Lead compensation

$$D(s) = K\frac{s+2}{s+p},$$

$$p = \begin{cases} 5z = 10 \\ 10z = 20 \end{cases}$$



# Assignment

Consider a unity feedback system as

$$G(s) = \frac{2}{s(s+20)(s+40)}$$

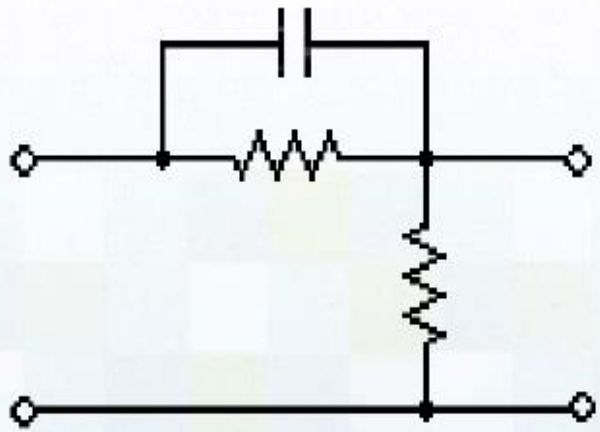
Find out the lead compensator such that

(a) the maximum overshoot  allowed is about 20%

(b) the settling time improves by a factor of five.

# Practical Realization of a Lead Compensator

The figure below shows the practical realization of a Lead Compensator with the help of resistors and capacitor. In terms of mechanical elements we can realize the same by using dashpot and springs. Just replace the resistors by dashpots and capacitor by spring.



# Lead-Lag Compensator and Notch Filter

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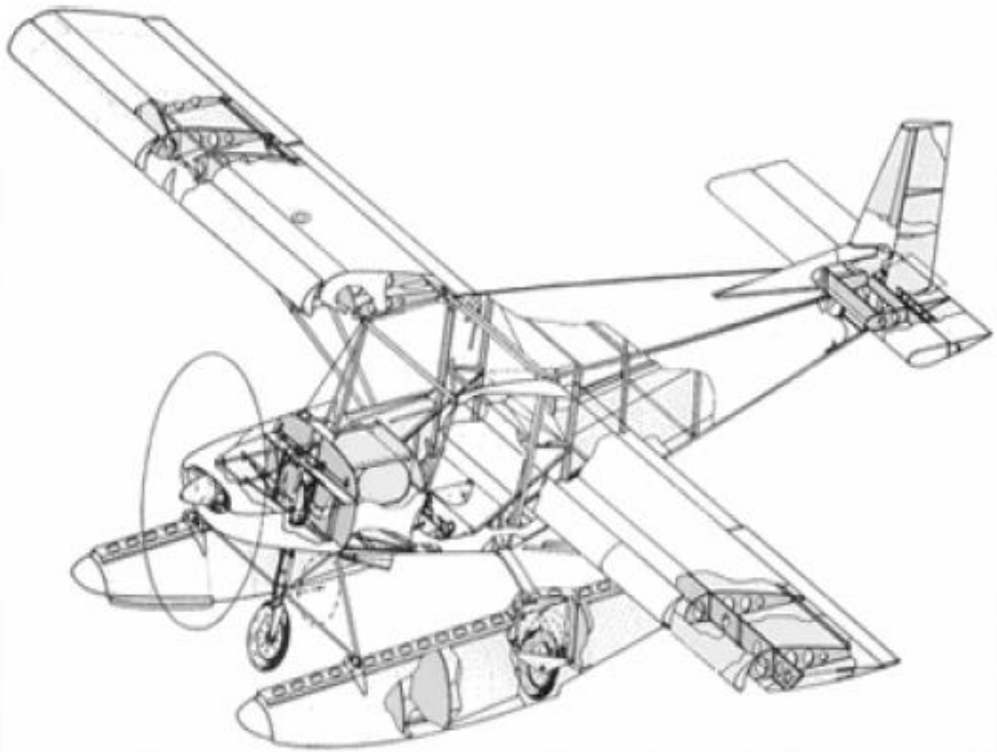
# This Lecture Contains

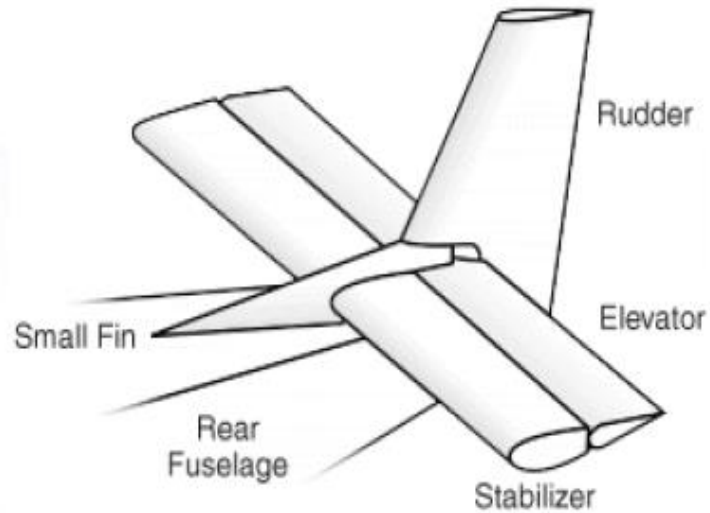
- Design of a Lead-Lag Filter
- Example
- Notch Compensation
- Assignment

# Design of a Lead – Lag filter

- Evaluate the performance of uncompensated system and get the desired pole location.
- Design the Lead Compensator to meet the transient response
- Simulate and Redesign to meet the performance
- Evaluate steady state performance and obtain the lag compensator
- Simulate and Iterate to check all the performances

$$G_0(s) = \frac{k_0(s+1)}{s(s+2)(s^2+12s+40)}.$$

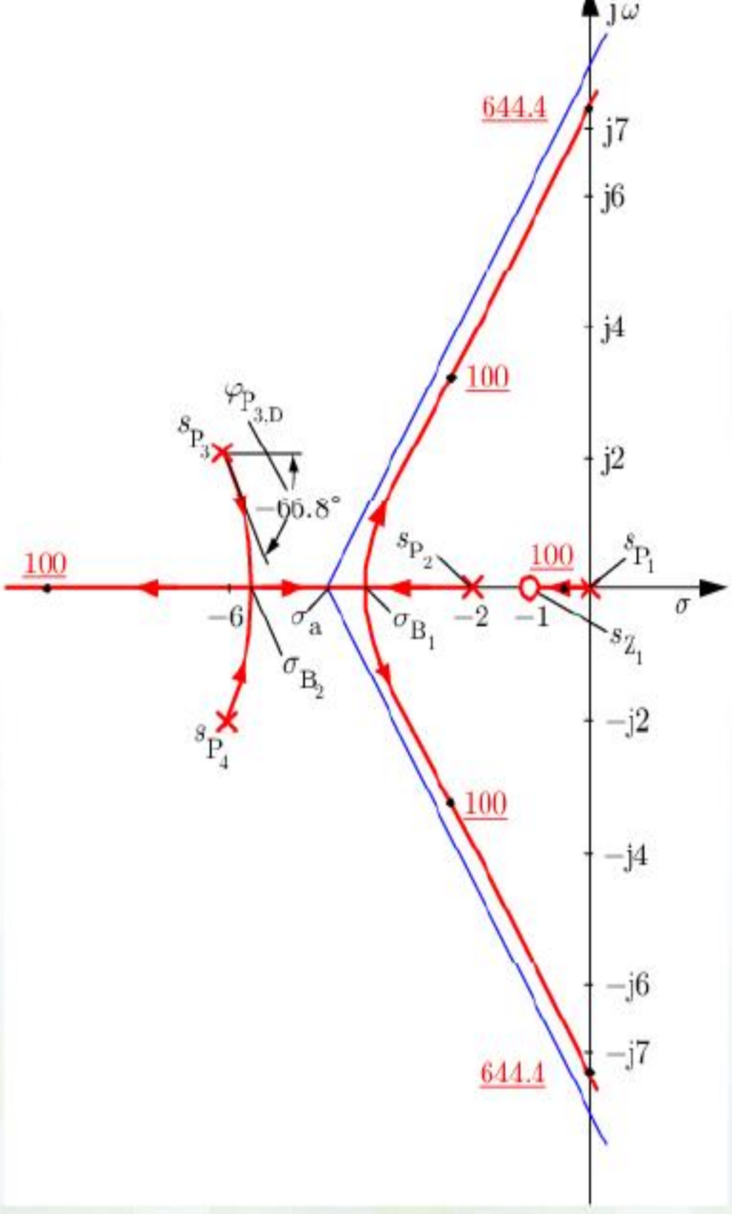





For the above transfer function (typical of an air-craft elevation control), Design a Lead-Lag filter for 5 times reduction in steady state error and 3 times reduction in settling time. Assume 20% overshoot as the operating point.

Root Locus of the uncompensated system

$$G_0(s) = \frac{k_0(s+1)}{s(s+2)(s^2+12s+40)} \, .$$



# Notch compensation

- **Example system: (1/s(s+1))**
  - *Consider a system with a lead-lag controller*

$$D(s) = 127\frac{s+5.4}{s+20}\frac{s+0.03}{s+0.01}$$

  - *Now, suppose the real system has a rather un-damped oscillation about 50 rad/sec.*
  - *So, we include this oscillation in the model*

$$G(s) = \frac{1}{s(s+1)}\frac{2500}{(s^2+s+2500)}$$

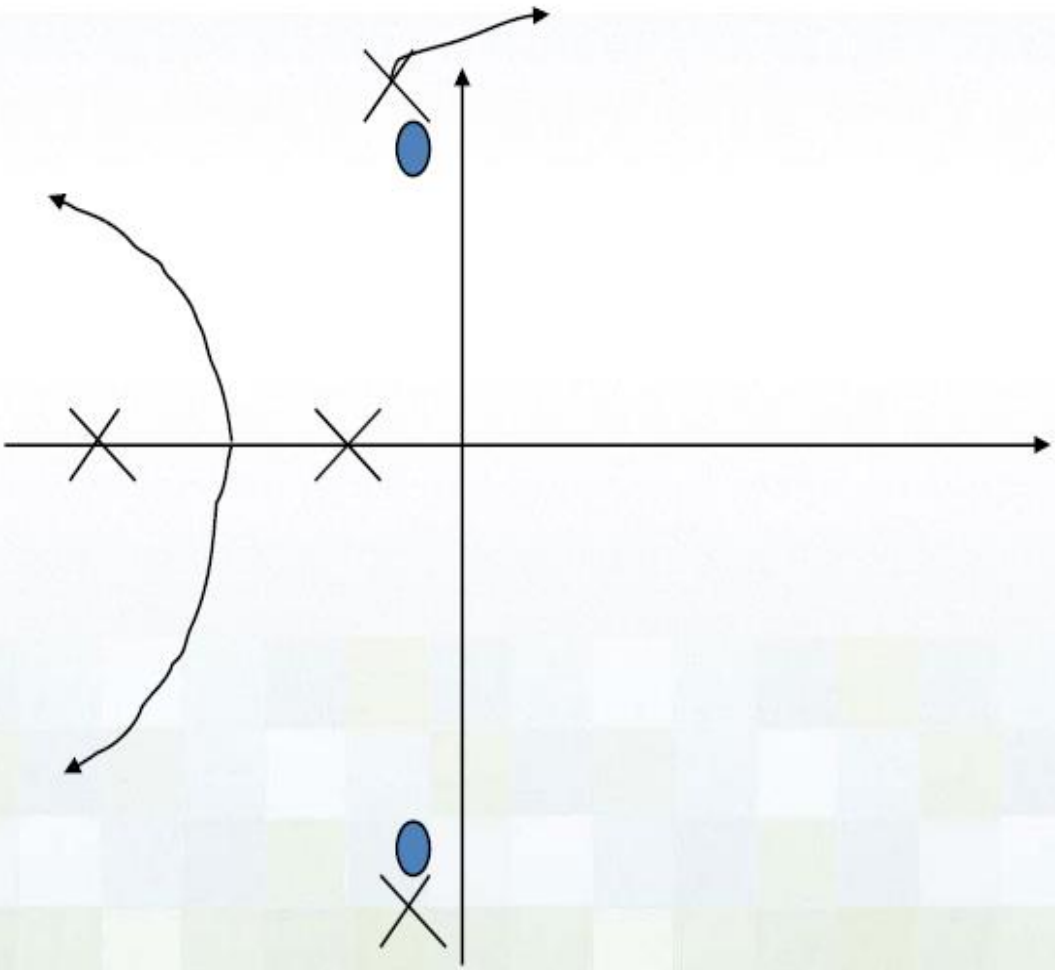
  - *Can we use the original controller ?*

# Notch compensation

- Aim: Remove or dampen the oscillations
- Possibilities
  - *Gain stabilization*
    - Reduce the gain at high frequencies
    - Thus, insert poles above the bandwidth but below the oscillation frequency – might not be feasible
  - *Phase stabilization (notch compensation)*
    - A zero near the oscillation frequency
    - A zero increases the phase
    - Possible transfer function

$$D_n(s) = \frac{s^2 + 2\zeta\omega_0 s + \omega_0^2}{(s+\omega_0)^2}$$

# Design of a notch filter



Use zeroes close to low damping ratio poles

## How to build a transfer function in MATLAB?

- **Suppose we consider the following equation:**

$$Form1: G(s) = \frac{K(s+z_1)}{(s+p_1)(s+p_2)}$$

$$Form2: G(s) = \frac{Ks+Kz_1}{s^2+(p_1+p_2)s+p_1p_2} = \frac{s+k_2}{s^2+as+b}$$

- **Now, we can develop this transfer function in MATLAB by the following ways:**
- **Form 1: tf1=zpk([z1],[p1,p2],k)**
- **Form 2: tf1=tf([1,k2],[1,a,b])**
- **You can draw the root-locus by the simple command: rlocus(tf1)**

# Some more useful commands

- For finding the response of the system – you can use the following commands as and when required:

- For step response: step(tf1)
- For impulse response: impulse(tf1)
- For other responses: lsim(tf1,U,T), where T is the time vector and U is the corresponding excitation vector.
- You can use nyquist(tf1) for obtaining the nyquist plot of the system and freqresp(tf1) for obtaining the frequency response.

# Use of MATLAB for ROOT LOCUS DESIGN

```
num=[1.151, 0.1774]; % Defines the numerator
den=[1 0.739, 0.921, 0]; % Defines the denominator
Wn=0.9; zeta=0.52; % Defines the frequency and damping
rlocus (num,den) % Root-locus plot Command
sgrid (zeta,Wn) %Grid for the Root-Locus
axis ([-1 0 -2.5 2.5])%Set the axes sizes
[K, poles]=rlocfind (num,den) de=0.2; %Obtain the gain and
closed loop poles
[numc,denc]=cloop (K*num,den,-1);%The closed loop transfer
function
step (de*numc,denc)%step response of the   system
```

# Assignment

- Consider a unity feedback system with a plant transfer function:

$$G(s) = \frac{(s+6)}{(s+2)(s+4)(s+7)(s+8)}$$

- Input the transfer function in MATLAB and sketch the root locus.
- Find out the co-ordinates of the dominant poles for damping coefficient 0.707.
- Find the corresponding Gain.
- Find out the validity of assuming the system to be of second-order.