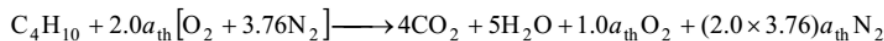# Homework-13 Solutions

Q 15-20

Solution:

Assumptions **1** Combustion is complete. **2** The combustion products contain CO_2, H_2O, O_2, and N_2 only.
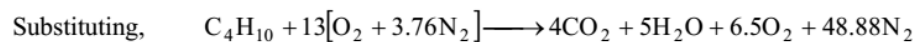
Properties The molar masses of C, H_2, O_2, and air are 12 kg/kmol, 2 kg/kmol, 32 kg/kmol, and 29 kg/kmol, respectively (Table A-1).

Analysis The combustion equation in this case can be written as

$$C_4H_{10} + 2.0a_{th}[O_2 + 3.76N_2] \longrightarrow 4CO_2 + 5H_2O + 1.0a_{th}O_2 + (2.0 \times 3.76)a_{th}N_2$$

where a_{th} is the stoichiometric coefficient for air. We have automatically accounted for the 100% excess air by using the factor $2.0a_{th}$ instead of a_{th} for air. The stoichiometric amount of oxygen ($a_{th}O_2$) will be used to oxidize the fuel, and the remaining excess amount ($1.0a_{th}O_2$) will appear in the products as free oxygen. The coefficient a_{th} is determined from the O_2 balance,

O_2 balance: $\quad 2.0a_{th} = 4 + 2.5 + 1.0a_{th} \longrightarrow a_{th} = 6.5$

Substituting, $\quad C_4H_{10} + 13[O_2 + 3.76N_2] \longrightarrow 4CO_2 + 5H_2O + 6.5O_2 + 48.88N_2$

The mole fractions of the products are

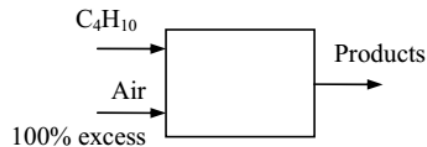$$N_m = 4 + 5 + 6.5 + 48.88 = 64.38 \text{ kmol}$$

$$y_{CO2} = \frac{N_{CO2}}{N_m} = \frac{4 \text{ kmol}}{64.38 \text{ kmol}} = \textbf{0.0621}$$

$$y_{H2O} = \frac{N_{H2O}}{N_m} = \frac{5 \text{ kmol}}{64.38 \text{ kmol}} = \textbf{0.0777}$$

$$y_{O2} = \frac{N_{O2}}{N_m} = \frac{6.5 \text{ kmol}}{64.38 \text{ kmol}} = \textbf{0.1010}$$

$$y_{N2} = \frac{N_{N2}}{N_m} = \frac{48.88 \text{ kmol}}{64.38 \text{ kmol}} = \textbf{0.7592}$$

The mass of carbon dioxide in the products per unit mass of fuel burned is

$$\frac{m_{CO2}}{m_{C4H10}} = \frac{(4 \times 44) \text{ kg}}{(1 \times 58) \text{ kg}} = \textbf{3.034 kg } CO_2\textbf{/kg } C_4H_{10}$$

The air-fuel ratio is determined by taking the ratio of the mass of the air to the mass of the fuel,

$$AF = \frac{m_{air}}{m_{fuel}} = \frac{(13 \times 4.76 \text{ kmol})(29 \text{ kg/kmol})}{(1 \text{ kmol})(58 \text{ kg/kmol})} = \textbf{30.94 kg air/kg fuel}$$

Q 15-28

Solution:

Assumptions **1** Combustion is complete. **2** The combustion products contain CO_2, H_2O, and N_2 only.

Properties The molar masses of C, H_2, O_2 and air are 12 kg/kmol, 2 kg/kmol, 32 kg/kmol, and 29 kg/kmol, respectively (Table A-1).

Analysis For 100 kg of fuel mixture, the mole numbers are

$$N_{CH4} = \frac{mf_{CH4}}{M_{CH4}} = \frac{60\,kg}{16\,kg/kmol} = 3.75\,kmol$$

$$N_{C2H6O} = \frac{mf_{C2H6O}}{M_{C2H6O}} = \frac{40\,kg}{46\,kg/kmol} = 0.8696\,kmol$$

60% CH_4
40% C_2H_6O

Air

100% theoretical

Products
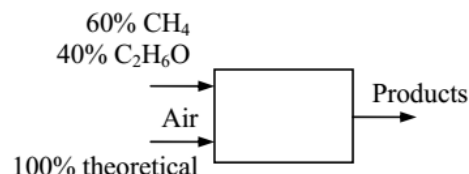
The mole fraction of methane and ethanol in the fuel mixture are

$$x = \frac{N_{CH4}}{N_{CH4}+N_{C2H6O}} = \frac{3.75\,kmol}{(3.75+0.8696)\,kmol} = 0.8118$$

$$y = \frac{N_{C2H6O}}{N_{CH4}+N_{C2H6O}} = \frac{0.8696\,kmol}{(3.75+0.8696)\,kmol} = 0.1882$$

The combustion equation in this case can be written as

$$x\,CH_4 + y\,C_2H_6O + a_{th}\left[O_2 + 3.76N_2\right] \longrightarrow B\,CO_2 + D\,H_2O + F\,N_2$$
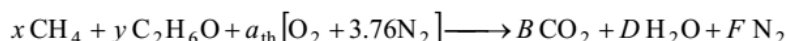
where a_{th} is the stoichiometric coefficient for air. The coefficient a_{th} and other coefficients are to be determined from the mass balances

Carbon balance:	$x + 2y = B$
Hydrogen balance:	$4x + 6y = 2D$
Oxygen balance:	$2a_{th} + y = 2B + D$
Nitrogen balance:	$3.76a_{th} = F$

Substituting x and y values into the equations and solving, we find the coefficients as

$x = 0.8118$	$B = 1.188$
$y = 0.1882$	$D = 2.188$
$a_{th} = 2.188$	$F = 8.228$

Then, we write the balanced reaction equation as
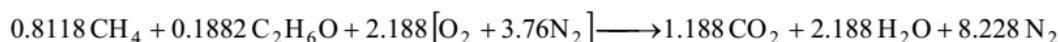
$$0.8118\,CH_4 + 0.1882\,C_2H_6O + 2.188\left[O_2 + 3.76N_2\right] \longrightarrow 1.188\,CO_2 + 2.188\,H_2O + 8.228\,N_2$$

The air-fuel ratio is determined by taking the ratio of the mass of the air to the mass of the fuel,

$$AF = \frac{m_{air}}{m_{fuel}} = \frac{(2.188 \times 4.76\,kmol)(29\,kg/kmol)}{(0.8118\,kmol)(12+4\times1)kg/kmol + (0.1882\,kmol)(2\times12+6\times1+16)kg/kmol}$$
$$= 13.94\,kg\,air/kg\,fuel$$

Then, the required flow rate of air becomes

$$\dot{m}_{air} = AF\dot{m}_{fuel} = (13.94)(10\,kg/s) = \textbf{139.4 kg/s}$$

Q 15-60

Solution:

Assumptions **1** Steady operating conditions exist. **2** Air and combustion gases are ideal gases. **3** Kinetic and potential energies are negligible. **4** Combustion is complete.
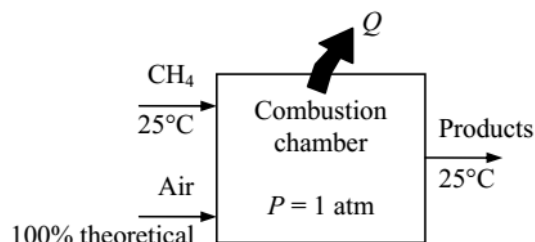
Analysis The fuel is burned completely with the stoichiometric amount of air, and thus the products will contain only H_2O, CO_2 and N_2, but no free O_2. Considering 1 kmol of fuel, the theoretical combustion equation can be written as

$$CH_4 + a_{th}(O_2 + 3.76N_2) \longrightarrow CO_2 + 2H_2O + 3.76a_{th}N_2$$

where a_{th} is determined from the O_2 balance,

$$a_{th} = 1 + 1 = 2$$

Substituting,

$$CH_4 + 2(O_2 + 3.76N_2) \longrightarrow CO_2 + 2H_2O + 5.64N_2$$

The heat transfer for this combustion process is determined from the energy balance $E_{in} - E_{out} = \Delta E_{system}$ applied on the combustion chamber with $W = 0$. It reduces to

$$-Q_{out} = \sum N_P(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ)_P - \sum N_R(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ)_R = \sum N_P \bar{h}_{f,P}^\circ - \sum N_R \bar{h}_{f,R}^\circ$$

since both the reactants and the products are at 25°C and both the air and the combustion gases can be treated as ideal gases. From the tables,

Substance	$\bar{h}_f^\circ$ kJ/kmol
CH_4	-74,850
O_2	0
N_2	0
H_2O (ℓ)	-285,830
CO_2	-393,520

Thus,

$$-Q_{out} = (1)(-393,520) + (2)(-285,830) + 0 - (1)(-74,850) - 0 - 0 = -890,330 \text{ kJ / kmol } CH_4$$

or

$$Q_{out} = \textbf{890,330 kJ / kmol } CH_4$$

If combustion is achieved with 100% excess air, the answer would still be the same since it would enter and leave at 25°C, and absorb no energy.

Q 15-62

Solution:

Assumptions **1** Combustion is complete. **2** The combustion products contain CO_2, CO, H_2O, SO_2, and N_2. **3** Combustion gases are ideal gases.

Properties The molar masses of C, H_2, N_2, O_2, S, and air are 12, 2, 28, 32, 32, and 29 kg/kmol, respectively (Table A-1).

Analysis We consider 100 kg of coal for simplicity. Noting that the mass percentages in this case correspond to the masses of the constituents, the mole numbers of the constituent of the coal are determined to be

$$N_C = \frac{m_C}{M_C} = \frac{39.25 \text{ kg}}{12 \text{ kg/kmol}} = 3.271 \text{ kmol}$$

$$N_{H2} = \frac{m_{H2}}{M_{H2}} = \frac{6.93 \text{ kg}}{2 \text{ kg/kmol}} = 3.465 \text{ kmol}$$

$$N_{O2} = \frac{m_{O2}}{M_{O2}} = \frac{41.11 \text{ kg}}{32 \text{ kg/kmol}} = 1.285 \text{ kmol}$$

$$N_{N2} = \frac{m_{N2}}{M_{N2}} = \frac{0.72 \text{ kg}}{28 \text{ kg/kmol}} = 0.0257 \text{ kmol}$$

$$N_S = \frac{m_S}{M_S} = \frac{0.79 \text{ kg}}{32 \text{ kg/kmol}} = 0.0247 \text{ kmol}$$
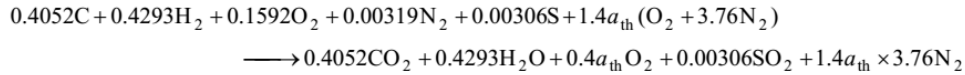
```
39.25% C
6.93% H₂
41.11% O₂
0.72% N₂
0.79% S
11.20% ash
(by mass)
```

The mole number of the mixture and the mole fractions are

$$N_m = 3.271 + 3.465 + 1.285 + 0.0257 + 0.0247 = 8.071 \text{ kmol}$$

$$y_C = \frac{N_C}{N_m} = \frac{3.271 \text{ kmol}}{8.071 \text{ kmol}} = 0.4052$$

$$y_{H2} = \frac{N_{H2}}{N_m} = \frac{3.465 \text{ kmol}}{8.071 \text{ kmol}} = 0.4293$$

$$y_{O2} = \frac{N_{O2}}{N_m} = \frac{1.285 \text{ kmol}}{8.071 \text{ kmol}} = 0.1592$$

$$y_{N2} = \frac{N_{N2}}{N_m} = \frac{0.0257 \text{ kmol}}{8.071 \text{ kmol}} = 0.00319$$

$$y_S = \frac{N_S}{N_m} = \frac{0.0247 \text{ kmol}}{8.071 \text{ kmol}} = 0.00306$$
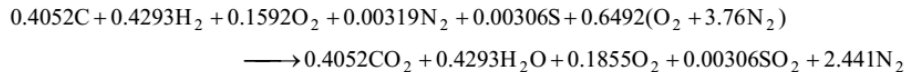
Ash consists of the non-combustible matter in coal. Therefore, the mass of ash content that enters the combustion chamber is equal to the mass content that leaves. Disregarding this non-reacting component for simplicity, the combustion equation may be written as

$$0.4052C + 0.4293H_2 + 0.1592O_2 + 0.00319N_2 + 0.00306S + 1.4a_{th}(O_2 + 3.76N_2)$$
$$\longrightarrow 0.4052CO_2 + 0.4293H_2O + 0.4a_{th}O_2 + 0.00306SO_2 + 1.4a_{th} \times 3.76N_2$$

According to the O_2 mass balance,

$$0.1592 + 1.4a_{th} = 0.4052 + 0.5 \times 0.4293 + 0.4a_{th} + 0.00306 \longrightarrow a_{th} = 0.4637$$

Substituting,

$$0.4052C + 0.4293H_2 + 0.1592O_2 + 0.00319N_2 + 0.00306S + 0.6492(O_2 + 3.76N_2)$$
$$\longrightarrow 0.4052CO_2 + 0.4293H_2O + 0.1855O_2 + 0.00306SO_2 + 2.441N_2$$

The heat transfer for this combustion process is determined from the energy balance $E_{in} - E_{out} = \Delta E_{system}$ applied on the combustion chamber with $W = 0$. It reduces to

$$-Q_{out} = \sum N_P \left(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ\right)_P - \sum N_R \left(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ\right)_R$$
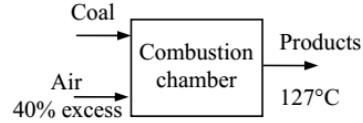
Assuming the air and the combustion products to be ideal gases, we have $h = h(T)$. From the tables,

Substance	$\bar{h}_f^\circ$ kJ/kmol	$\bar{h}_{298\ K}$ kJ/kmol	$\bar{h}_{400\ K}$ kJ/kmol
O_2	0	8682	11,711
N_2	0	8669	11,640
H_2O (g)	-241,820	9904	13,356
CO_2	-393,520	9364	13,372
SO_2	-297,100	-	-

The enthalpy change of sulfur dioxide between the standard temperature and the product temperature using constant specific heat assumption is

$$\Delta\bar{h}_{SO2} = c_p\Delta T = (41.7\ \text{kJ/kmol} \cdot \text{K})(127-25)\text{K} = 4253\ \text{kJ/kmol}$$

Substituting into the energy balance relation,

$$-\bar{Q}_{out} = (0.4052)(-393,520+13,372-9364)+(0.4293)(-241,820+13,356-9904)$$
$$+(0.1855)(0+11,711-8682)+(2.441)(0+11,640-8669)+(0.00306)(-297,100+4253)-0$$
$$= -253,244\ \text{kJ/kmol C}_8\text{H}_{18}$$

or $\quad \bar{Q}_{out} = 253,244$ kJ/kmol fuel

Then the heat transfer per kg of fuel is

$$Q_{out} = \frac{\bar{Q}_{out}}{M_{fuel}} = \frac{253,244\ \text{kJ/kmol fuel}}{(0.4052\times12+0.4293\times2+0.1592\times32+0.00319\times28+0.00306\times32)\ \text{kg/kmol}}$$
$$= \frac{253,244\ \text{kJ/kmol fuel}}{11.00\ \text{kg/kmol}}$$
$$= \textbf{23,020 kJ/kg coal}$$

Q 15-79

Solution:

Assumptions **1** Steady operating conditions exist. **2** Air and combustion gases are ideal gases. **3** Kinetic and potential energies are negligible. **4** There are no work interactions. **5** The combustion chamber is adiabatic.

Analysis Under steady-flow conditions the energy balance $E_{in} - E_{out} = \Delta E_{system}$ applied on the combustion chamber with $Q = W = 0$ reduces to

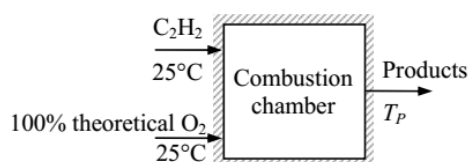$$\sum N_P \left(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ\right)_P = \sum N_R \left(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ\right)_R \longrightarrow \sum N_P \left(\bar{h}_f^\circ + \bar{h} - \bar{h}^\circ\right)_P = \sum N_R \bar{h}_{f,R}^\circ$$

since all the reactants are at the standard reference temperature of 25°C. Then, for the stoichiometric oxygen

$$C_2H_2 + 2.5O_2 \longrightarrow 2CO_2 + 1H_2O$$

From the tables,

Substance	$\bar{h}_f^\circ$ kJ/kmol	$\bar{h}_{298K}$ kJ/kmol
C_2H_2 (g)	226,730	---
O_2	0	8682
N_2	0	8669
H_2O (g)	-241,820	9904
CO_2	-393,520	9364

Thus,

$$(2)\left(-393,520 + \bar{h}_{CO2} - 9364\right) + (1)\left(-241,820 + \bar{h}_{H2O} - 9904\right) = (1)(226,730) + 0 + 0$$

It yields

$$2\bar{h}_{CO2} + 1\bar{h}_{H2O} = 1,284,220 \text{ kJ}$$

The adiabatic flame temperature is obtained from a trial and error solution. A first guess is obtained by dividing the right-hand side of the equation by the total number of moles, which yields 2,284,220/(2+1) = 428,074 kJ/kmol. The ideal gas tables do not list enthalpy values this high. Therefore, we cannot use the tables to estimate the adiabatic flame temperature In Table A-2b, the highest available value of specific heat is c_p = 1.234 kJ/kg·K for CO_2 at 1000 K. The specific heat of water vapor is c_p = 1.8723 kJ/kg·K (Table A-2a). Using these specific heat values,

$$(2)\left(-393,520 + \bar{c}_p\Delta T\right) + (1)\left(-241,820 + \bar{c}_p\Delta T\right) = (1)(226,730) + 0 + 0$$

where $\Delta T = (T_{af} - 25)°C$. The specific heats on a molar base are

$$\bar{c}_{p,CO2} = \bar{c}_p M = (1.234 \text{ kJ/kg} \cdot \text{K})(44 \text{ kg/kmol}) = 54.3 \text{ kJ/kmol} \cdot \text{K}$$
$$\bar{c}_{p,H2O} = \bar{c}_p M = (1.8723 \text{ kJ/kg} \cdot \text{K})(18 \text{ kg/kmol}) = 33.7 \text{ kJ/kmol} \cdot \text{K}$$

Substituting,

$$(2)\left(-393,520 + 54.3\Delta T\right) + (1)\left(-241,820 + 33.7\Delta T\right) = 226,730$$
$$(2 \times 54.3)\Delta T + 33.7\Delta T = 1,255,590$$
$$\Delta T = \frac{1,255,590 \text{ kJ/kmol}}{(2 \times 54.3 + 33.7) \text{ kJ/kmol} \cdot \text{K}} = 8824 \text{ K}$$

Then the adiabatic flame temperature is estimated as

$$T_{af} = \Delta T + 25 = 8824 + 25 = \textbf{8849°C}$$