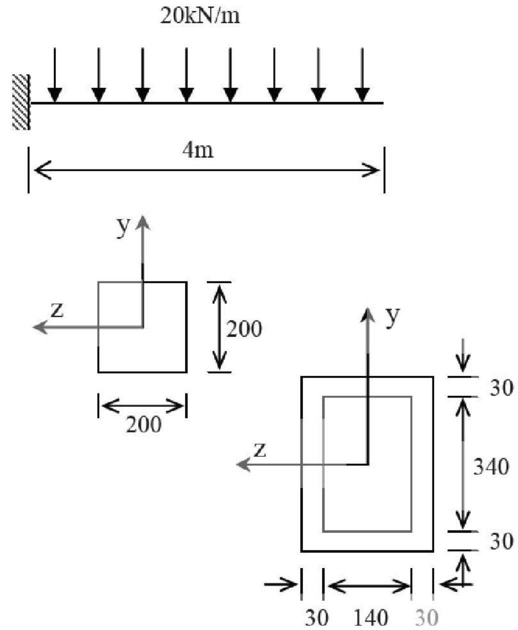
ESO 202A/204: Mechanics of Solids (2016 -17 II semester) Solution of Assignment No. 8

8.1 Maximum BM $= -\dfrac{q_0 l^2}{2}$ = -160kNm



a. Solid square section (200mm square)

$$I_{zz} = \frac{200\times 200^3}{12}\,\text{mm}^4$$

$$\sigma_{xx}\Big|_{\max} = -\frac{-160\times 10^3\times 10^3}{200^4/12}\times(\pm 100) = 120\,\text{MPa}$$

b. Hollow box section

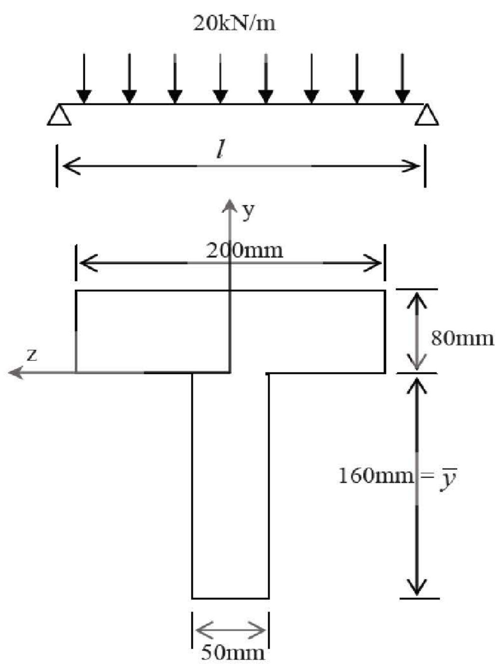
$$I_{zz} = \left(\frac{200\times 400^3}{12} - \frac{140\times 340^3}{12}\right)\text{mm}^4$$

$$= 6.08\times 10^8\,\text{mm}^4$$

$$\sigma_{xx}\Big|_{\max} = \frac{160\times 10^6}{6.08\times 10^8}\times 200 = 52.63\,\text{MPa}$$

8.2

Let the maximum span be l m.



$$\text{BM} = \frac{q_0 l^2}{8} = \frac{20 l^2}{8} = 2.5 l^2\,\text{kNm}$$

Let NA lie at $\bar{y}$ from the bottom fibre.

$$\bar{y} = \frac{200\times 80\times 200 + 160\times 50\times 80}{200\times 80 + 160\times 50}$$

$= 160$mm (i.e. at the junction of web and flange)

$$I_{zz} = \frac{200\times 80^3}{12} + 200\times 80\times 40^2 + \frac{1}{3}\times 50\times 160^3$$

$$= 10240\times 10^4\,mm^4$$

$$\left|\sigma_{xx}\big|_{\max}\right| = \frac{2.5 l^2\times 10^6}{10240\times 10^4}\times 160 = 140\ \text{(given)}$$

$$\therefore l^2 = 35.84 \Rightarrow l = 5.98\,\text{m}$$

8.3

Since the supports are located in the <u>optimum</u> position, the maximum bending moment in the shelf must be minimum:

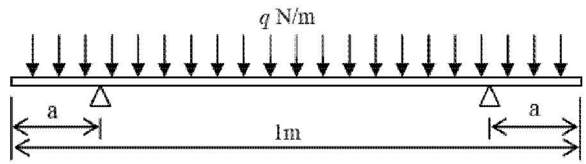
i.e. $\dfrac{qa^2}{2} = \dfrac{ql}{2}\left(\dfrac{l}{2} - a\right) - \dfrac{ql^2}{8}$

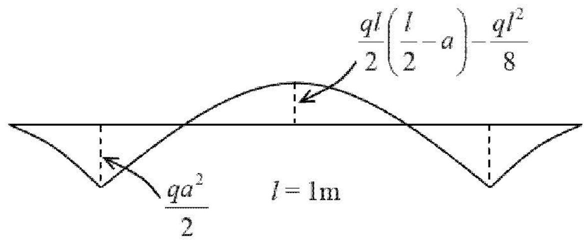
$\Rightarrow a = \dfrac{l}{2}\left(\sqrt{2} - 1\right)$

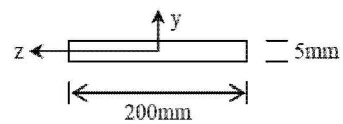
$\text{BM} = \dfrac{q}{2}\left(\dfrac{\sqrt{2}-1}{2}\right)^2 l^2 = 0.0214\, q \text{ Nm}$

$I_{ZZ} = \dfrac{200 \times 5^3}{12}, \ \sigma_{\max} = 10\,\text{MPa}$

$\Rightarrow 10 = \dfrac{.0214 \times q \times 10^3}{200 \times 5^3/12} \times 2.5 \Rightarrow q = 389\,\text{N/m}$







8.4

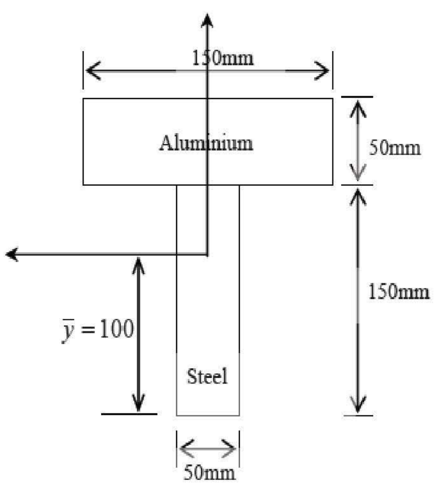
$E_{Al} = 70\text{GPa} \qquad E_S = 210\text{GPa} \qquad \text{BM} = 100\text{kNm}$

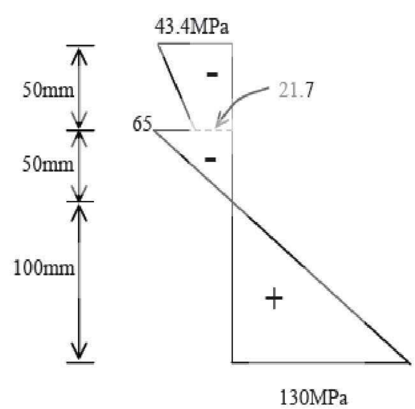
$$\bar{y} = \frac{2.1\times10^5 \times 150 \times 50 \times 75 + 0.7\times10^5 \times 150 \times 50 \times 175}{2.1\times10^5 \times 150 \times 50 + 0.7\times10^5 \times 150 \times 50} = 100mm$$

$$EI_{ZZ} = 2.1\times10^5\left(\frac{50\times150^3}{12} + 50\times150\times25^2\right) + 0.7\times10^5\left(\frac{150\times50^3}{12} + 50\times150\times75^2\right) = 16.18\times10^{12}\ Nmm^2$$

$$\sigma_{xx} = -\frac{M.y}{EI_{ZZ}} = \frac{100\times10^6 y}{16.18\times10^{12}} = -0.62\times10^{-5} y; \qquad -100 \le y \le 100$$

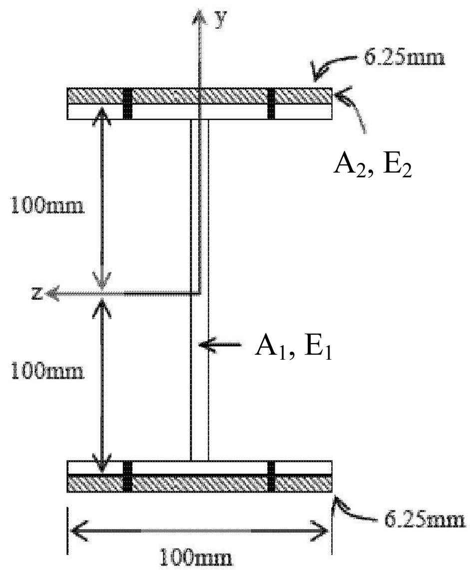
$$\sigma_{XX} = -2.1\times10^5 \times 0.62\times10^{-5} y = -1.3y; \qquad -100 \le y \le 50$$
$$= -0.7\times10^5 \times 0.62\times10^{-5} y = -0.434y; \quad 50 \le y \le 100$$





σ_{xx}

8.5



Although it is a nonhomogeneous section, the neutral axis does not get shifted because the reinforcing plates are symmetrically attached.

For moment equilibrium,

$$M = -\int_A \sigma_{xx} y dA = \frac{E_1}{\rho}\int_{A1} y^2 dA + \frac{E_2}{\rho}\int_{A2} y^2 dA$$

$$\Rightarrow M = \frac{E_1}{\rho}(I_{ZZ})_1 + \frac{E_2}{\rho}(I_{ZZ})_2$$

Stiffness is defined as $\dfrac{M}{d\phi/ds} = M \times \rho$

$$= E_1(I_{ZZ})_1 + E_2(I_{ZZ})_2 = k_{(stiffened)}$$

$$\frac{k_{(stiffened)}}{k_{(unstiffened)}} = \frac{E_1(I_{ZZ})_1 + E_2(I_{ZZ})_2}{E_1(I_{ZZ})_1} = 1 + \frac{E_2(I_{ZZ})_2}{E_1(I_{ZZ})_1}$$

Now, $(I_{ZZ})_2 = 2\left[100 \times 6.25^3/12 + 100 \times 6.25 \times 103.125\right] mm^4 = 13.3 \times 10^6 mm^4$

$\therefore$ Stiffness ratio $= 1 + \dfrac{2.0 \times 10^5}{0.7 \times 10^5} \times \dfrac{13.3 \times 10^6}{23.7 \times 10^6} = 2.6$

$$\frac{\sigma_{xx\max}|_{Al}}{\sigma_{xx\max}|_{St}} = \frac{E_1(y_{\max})_1}{E_2(y_{\max})_2} = \frac{0.7 \times 10^5 \times 100}{2 \times 10^5 \times 106.25} = 0.329$$

8.6

(i) $M_z = (Ph/4)$, $M_y = -(Pb/4)$, $I_{zz} = (bh^3/12)$, $I_{yy} = (hb^3/12)$

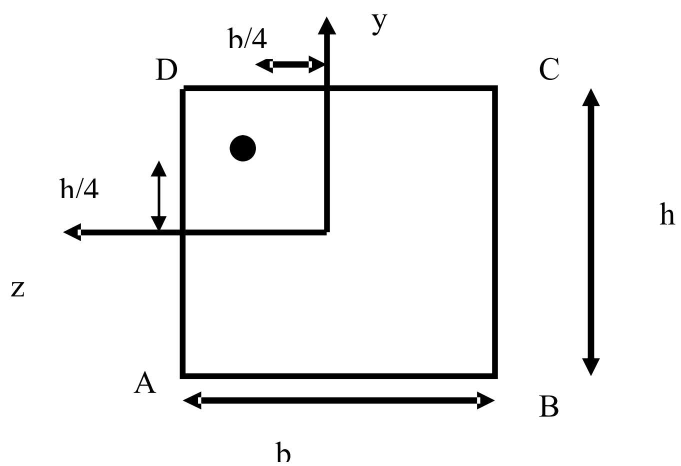
Stress σ_x at C due to

(a) Direct axial force P : $\sigma_x^a = P/bh$ (-) (1)

(b) M_z: $\sigma_x^b = \left\{(Ph/4)/(bh^3/12)\right\}.h/2 = (3/2).P/bh$ (+) (2)

(c) M_y: $\sigma_x^c = \left\{-(Pb/4)/(hb^3/12)\right\}.-b/2 = (3/2).P/bh$ (+) (3)

Total stress σ_x at C = (1) + (2) + (3) = (2P/bh) (+)



(ii) Let P pass through a point in (y,z).
Then,
$M_z = Py$, $M_y = -Pz$

Then, σ_x at C $= -(P/bh) + \left\{(Py)/(bh^3/12)\right\}.(h/2) + \left\{-(Pz)/(hb^3/12)\right\}.(-b/2)$

$= (P/bh).\left\{-1 + (6y/h) + (6z/b)\right\}$

For σ_x at C ≤ 0 (no tension),

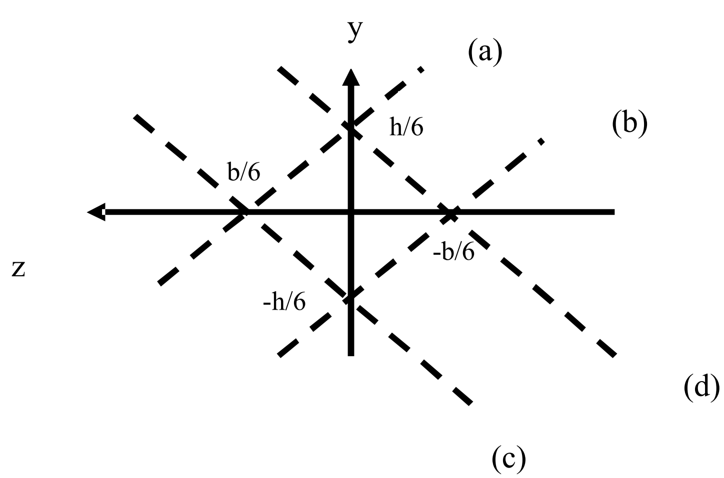
$(y/h) + (z/b) \leq (1/6)$ (a)

Similarly,

σ_x at A $= (P/bh).\left\{-1 - (6y/h) - (6z/b)\right\} \leq 0$ implies, $(y/h) + (z/b) \geq -(1/6)$ (b)

σ_x at B $= (P/bh).\left\{-1 - (6y/h) + (6z/b)\right\} \leq 0$ implies, $(z/b) - (y/h) \leq (1/6)$ (c)

σ_x at D $= (P/bh).\left\{-1 + (6y/h) - (6z/b)\right\} \leq 0$ implies, $(z/b) - (y/h) \geq -(1/6)$ (d)



P should be within the dotted lines. Note that lines (a),(b),(c)and (d) are for 'equal to' sign.