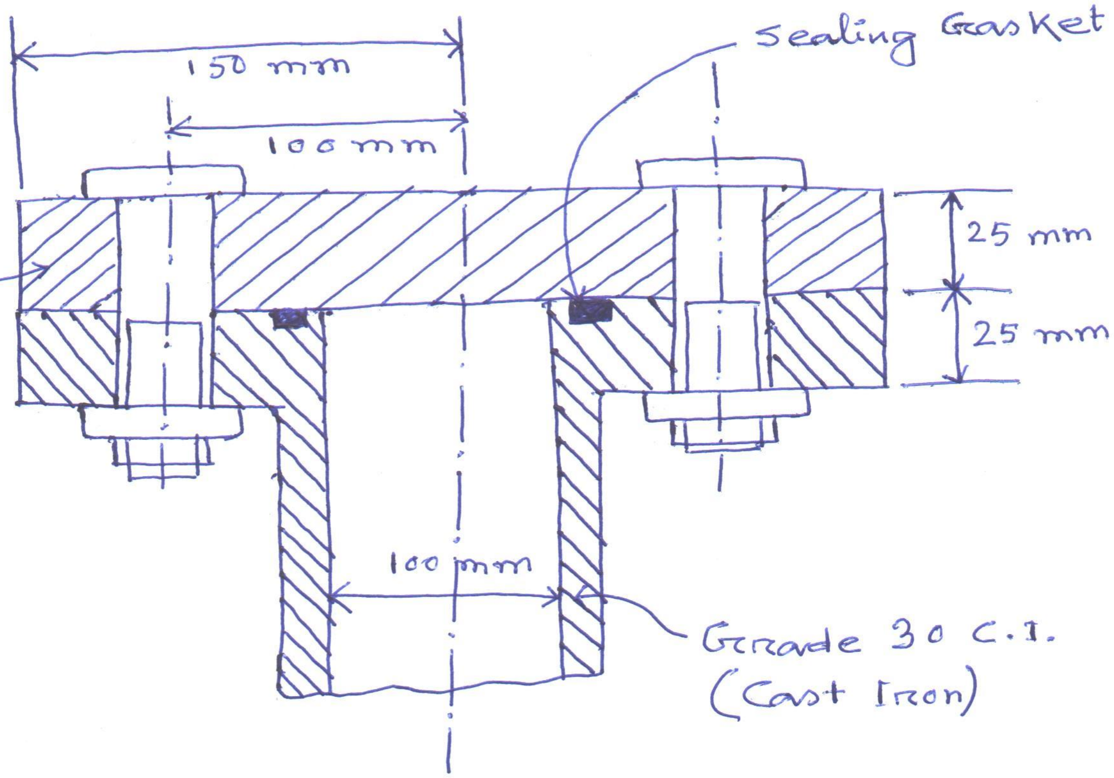
ME 351

Tutorial - 3 Solution

Problem - 1



ISO 9.8
M12 X 1.75
10 bolts

1. Bolt length:

Nut Height, $H = 10.8$ mm (Table A 31)

Grip Length, $l = 50$ mm (No washer)

$L \geq l + H = 60.8$ mm

Table A-17 suggests: $L = 80$ mm. (too large)

$\boxed{\text{Choose } L = 65 \text{ mm}}$ ← Should be available

Threaded length: $L_T = 2d + 6 = 30$ mm
Unthreaded length: $l_d = L - L_T = 35$ mm
Threaded portion in grip: $l_t = l - l_d = 15$ mm

2. Stiffnesses:

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = 424.71 \text{ MN/m}$$

$A_d = \frac{\pi d^2}{4} = 113.1$ mm^2
$A_t = 84.3$ mm^2 (Table 8-1)
$E = 207$ GPa (Table A5)

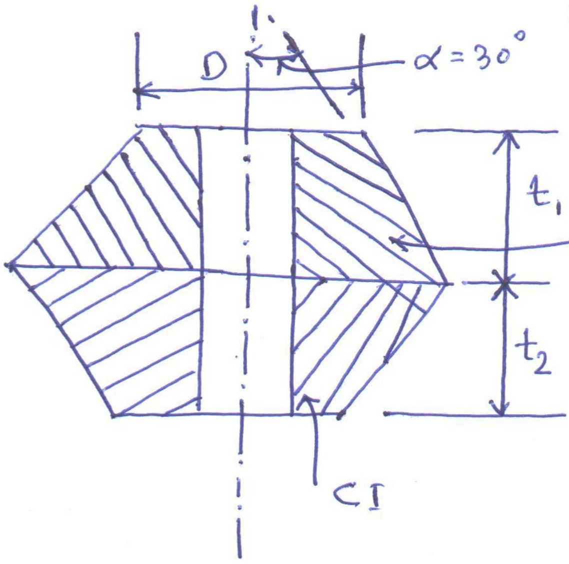
⇒ $\boxed{\text{Bolt stiffness, } k_b = 424.71 \text{ MN/m}}$

Cylinder head: $t = 25$ mm, $d = 12$ mm, $D = 1.5d = 18$ mm

$\alpha = 30°$

$$K_1 = \frac{0.5774\, d \times \pi}{\ln\left(\frac{1.155t + D - d}{1.155t + D + d} \cdot \frac{D+d}{D-d}\right)} \quad \text{(Equation 8-20)}$$

$$\Rightarrow \boxed{K_1 = 4149.84 \text{ MN/m}}$$



Pressure - Cone

Cast Iron: $t = 25$ mm, $d = 12$ mm

$D = 18$ mm, $\alpha = 30°$

$E = 100$ GPa (Table 8-8)

$$\Rightarrow \boxed{K_2 = 2004.76 \text{ MN/m}}$$

Member Stiffness:

$$K_m = \left(\frac{1}{K_1} + \frac{1}{K_2}\right)^{-1} = 1351.74 \text{ MN/m}$$

Stiffness Constant: $\boxed{C = \frac{K_b}{K_m + K_b} = 0.239}$

Proof load: $F_i = 0.75\, S_p A_t$ (Assume reusable bolt)

$$\Rightarrow \boxed{\begin{aligned} F_i &= 0.75 \times (84.3)\, 650 \text{ N} \\ &= 41.1 \text{ KN} \end{aligned}}$$

Table 8-11
ISO 9.8, M12
$S_p = 650$ MPa
$S_{ut} = 900$ MPa

(a) Static loading: Pressure in the cylinder:

$P_g = 10$ MPa, Cyl. dia = 100 mm

$\Rightarrow$ Total external load, $P_{total} = 10 A_c = 10 \times \frac{\pi \times (100)^2}{4}$ N

$$\Rightarrow P_{total} = 78.54 \text{ KN}$$

External load/bolt: $\boxed{P = \frac{P_{total}}{N} = 7.854 \text{ KN}}$

Yielding factor of safety: $\boxed{n_p = \frac{S_p A_t}{CP + F_i} = 1.274}$

Loading Factor (Overload factor of safety)

$$\boxed{n_L = \frac{S_p A_t - F_i}{CP} = 7.296}$$

Seperation Factor of Safety: $\boxed{n_o = \frac{F_i}{P(1-C)} = 6.876}$

Summary
$S_p = 650$ MPa
$A_t = 84.3\ mm^2$
$F_i = 41.1$ KN
$C = 0.239$
$P = 7.854$ KN

Note: Check for Gasket Sealing:

$$\frac{\pi D_b}{N d} = \frac{\pi \times 200}{10 \times 12} = 5.236 \text{ (OK)}$$

From Table 8-1: M12 X 1.75 is a coarse thread. If fine thread was chosen the A_t would increase making the joint stronger.

(b) **Fatigue Loading:** $p_g\big|_{max} = 20$ MPa

$p_g\big|_{min} = 10$ MPa

$$\Rightarrow \boxed{\begin{aligned} P_{max} &= 15.708 \text{ KN} \\ P_{min} &= 7.854 \text{ KN} \end{aligned}}$$

$$\boxed{\begin{aligned} F_{b\,max} &= C P_{max} + F_i = 44.854 \text{ KN} \\ F_{b\,min} &= C P_{min} + F_i = 42.977 \text{ KN} \end{aligned}}$$

Alternating Stress: $\sigma_a = \frac{C(P_{max} - P_{min})}{2A_t} = \frac{0.239 \times 7.854 \times 10^3}{2 \times 84.3 \times 10^{-6}}$ Pa

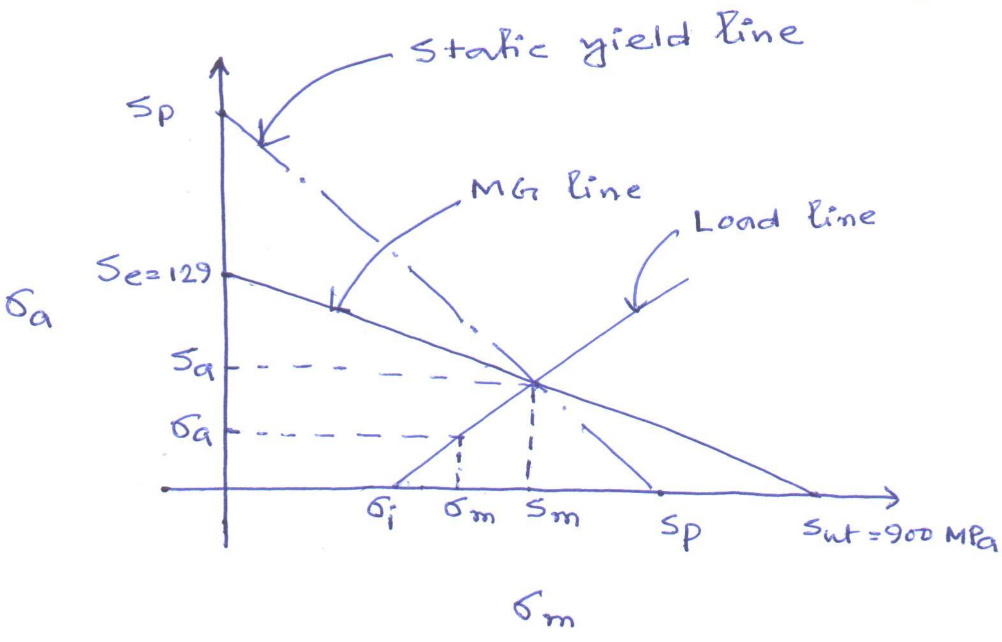
$$\Rightarrow \boxed{\sigma_a = 11.13 \text{ MPa}}$$

Mid-range Stress: $\sigma_m = \frac{C(P_{max} + P_{min})}{2A_t} + \frac{F_i}{A_t}$

$= (33.40 + 487.54)$ MPa

$$\Rightarrow \boxed{\sigma_m = 520.94 \text{ MPa}}$$

$$\boxed{\sigma_i = 487.54 \text{ MPa}}$$



Fully Corrected Endurance limit: $S_e = 140$ MPa (Table 8-17)

$$S_a = \frac{S_e \cdot \sigma_a (S_{ut} - \sigma_i)}{S_{ut}\sigma_a + S_e(\sigma_m - \sigma_i)} = 43.74 \text{ MPa}$$

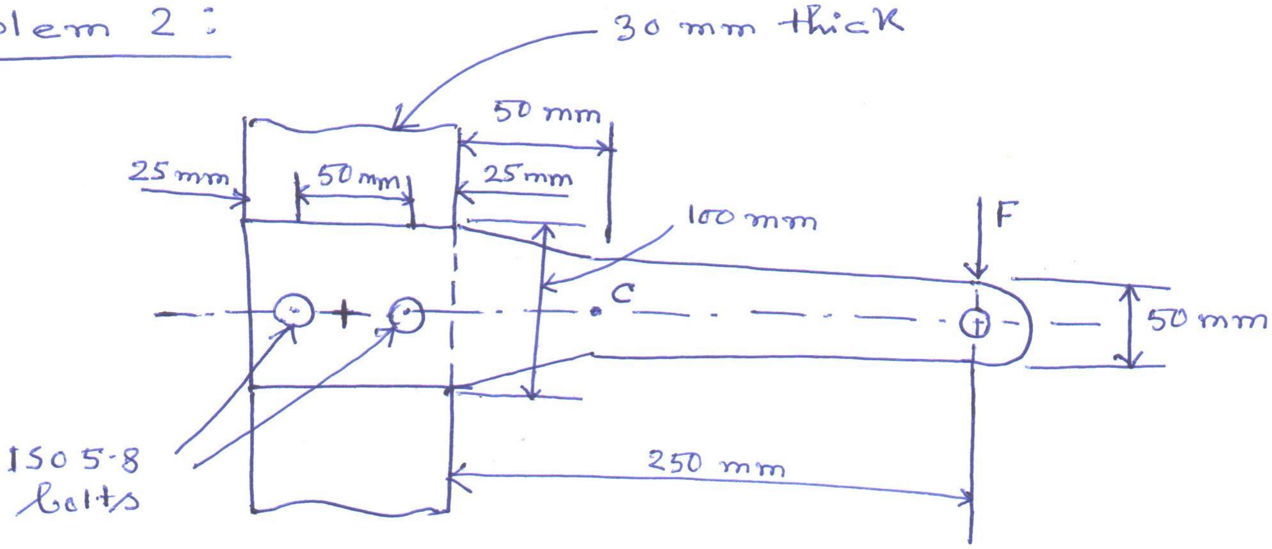
Fatigue Factor of Safety: $\boxed{n_f = \frac{S_a}{\sigma_a} = 3.93}$

Static Yielding Criteria: $\boxed{n_p = \frac{S_p}{(\sigma_m + \sigma_a)} = 1.22}$ ✓

Upper Bound on Preload: $(1-c) S_{ut} A_t = 57.737$ KN

Since $F_i < 57.737$ KN, the joint will be okay.

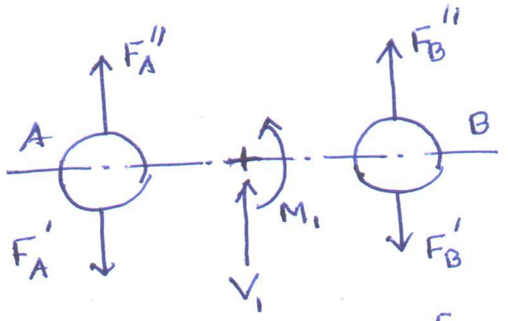
Problem 2:



Information Given: Thickness of cantilever beam, $t = 10\ mm$

Bolts used: M12 X 1.75, Factor of Safety: 2.5

Bolt System:



$M_1 = 0.3\ F$
$V_1 = F$

Primary Shear: $F_A' = F_B' = \frac{F}{2}$

Secondary Shear: $(F_A'' + F_B'') \times 0.025 = \overbrace{0.3F}^{M_1}$

$F_A'' = F_B''$ ($r_A = r_B = 0.025\ m$)

$\Rightarrow \boxed{F_A'' = F_B'' = 6F}$

Resultant Shear: On Bolt A: $F_A = F_A'' - F_A'$

$\Rightarrow \boxed{F_A = 5.5\ F}$

On Bolt B: $\boxed{F_B = F_B'' + F_B' = 6.5\ F}$

$\Rightarrow$ Design will be based on the load on bolt B.

Bolt Material Grad: ISO 5.8 $\Rightarrow S_y = 420$ MPa (8-11)

Cantilever Material: AISI 1020 Cold Drawn: $S_y = 390$ MPa (A-20)

(a) Ignore bending of bolts

1. Shearing of bolts: Shoulder cross-section

$$A_s = \frac{\pi (12)^2}{4} = 113.1 \text{ mm}^2$$

$$S_{sy} = 0.577(420) = 242.34 \text{ MPa}$$

$$\tau_{max} = \frac{F_B}{A_s} = \frac{S_{sy}}{n} \Rightarrow F_B\Big|_{max} = 10.963 \text{ KN}$$

$$\Rightarrow \boxed{F_{max} = 1.687 \text{ KN}}$$

2. Bearing on bolt: $A_{bearing} = t \cdot d = 10 \times 12 = 120 \text{ mm}^2$

$$\sigma_{bearing} = \frac{F_B}{A_{bearing}} \leq \frac{S_y}{n} \Rightarrow F_B \leq 20.16 \text{ KN.}$$

$$\Rightarrow \boxed{F_{max} = 3.10 \text{ KN}}$$

3. Bearing of cantilever: $\frac{F_B}{A_{bearing}} \leq \frac{S_y}{n} = \frac{390}{2.5}$ MPa

$$\Rightarrow F_B \leq 18.72 \text{ KN}$$

$$\Rightarrow \boxed{F_{max} = 2.88 \text{ KN}}$$

4. Bearing of cantilever: At B

$$I = \frac{1}{12} \times 10 \times (100^3 - 12^3) \text{ mm}^4$$

$$= 8.319 \times 10^5 \text{ mm}^4$$

$$C = 50 \text{ mm}$$

$$\sigma_{max} = \frac{M_B \cdot C}{I} = \frac{0.275 F \times 0.05}{8.319 \times 10^5 \times 10^{-12}} \leq \frac{S_y}{n} = \frac{390 \times 10^6}{2.5}$$

$$\Rightarrow F \leq \frac{390}{2.5} \times \frac{8.319}{0.275 \times 0.05} \times 10^{-1}$$

$$\Rightarrow \boxed{F_{max} = 9.438 \text{ KN}}$$

At C: $I = 1.042 \times 10^5 \text{ mm}^4$, $c = 0.025 \text{ m} = 25 \text{ mm}$

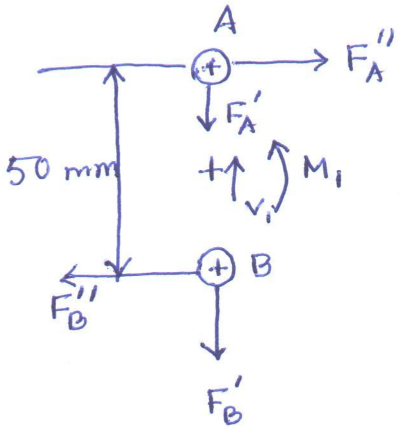
$$M_c = 0.2 F$$

$$\Rightarrow \boxed{F_{max} = 3.251 \text{ KN}}$$

$\Rightarrow$ Critical load is $F = 1 \cdot 687$ KN based on shearing of bolts.

ⓑ If the bolts were arranged vertically

$$F_A'' = F_B'' = 6F$$



$\Rightarrow$ Resultant Shear

$$F_A = F_B = \sqrt{\frac{F^2}{4} + (6F)^2} = 6 \cdot 02 F$$

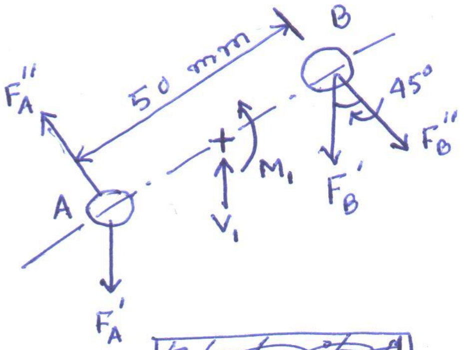
$\Rightarrow$ Maximum load on both bolts is less than that of case A.

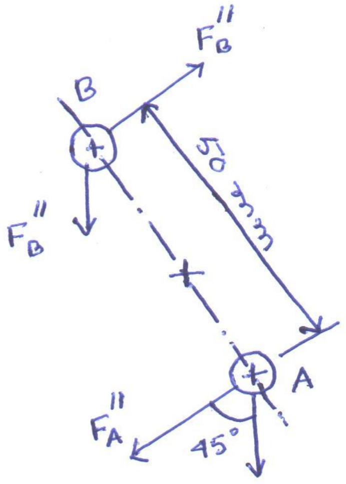
$\Rightarrow$ This arrangement is better.

Critical load: $\boxed{F_{max} = 1 \cdot 821 \text{ KN}}$

ⓒ If bolts were arranged diagonally, at same distance.

Two Scenarios





Scenario - 1	Scenario - 2
• Maximum load on bolt B	• Maximum load on bolt A
• Without doing any further calculation	
$\underbrace{6 \cdot 02 F}_{(b)} < F_B\big\rvert_{max} < \underbrace{6 \cdot 5 F}_{(a)}$	$\underbrace{6 \cdot 02 F}_{(b)} < F_A\big\rvert_{max} < \underbrace{6 \cdot 5 F}_{(a)}$

$\Rightarrow$ Arrangement (b) - vertical - is better as it can withstand larger load.