ME-451 ASSIGNMENT-2

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Roll NO:- 150162
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Q1) Given,

Overall Pressure Ratio $= 4.0$

$\dot{m} = 3\,kg/s$

$\eta_p = 88\%$

(ΔT_0) Stagnation pr. rise $< 25K$

i) $N = ?$ and pr. ratio for first & last stage

we know,

$$\frac{p_{02}}{p_{01}} = \left(1 + N\frac{\Delta T_0}{T_{01}}\right)^{\frac{\eta_p \gamma}{\gamma - 1}} \quad \text{—(1)}$$

using the above relation, and taking $T_0 = 288\,K$ (ambient)

$$4.0 = \left(1 + N \times \frac{25}{288}\right)^{\frac{0.88 \times 1.4}{0.4}} \Leftrightarrow \underline{N = 6.54}$$

$\because$ N represents no. of stages & $\therefore$ should be an integer ≥ 6.54

Ans → $\boxed{\therefore N = 7}$; $\therefore$ actual $\Delta T_0 = 23.37K$ (from (1), for $N=7$)

$$T_{02} = T_{01} + \Delta T$$
$$= 311.37\,K$$

$\therefore$ Pressure ratio for first stage $\left(\frac{P_{02}}{P_{01}}\right) = \left(\frac{311.37}{288}\right)^{\frac{1.4 \times 0.88}{0.4}}$

$\boxed{= 1.27}$ ← Ans

$\therefore$ Pressure ratio for last stage $(T_0)_{in} = 288 + 6(23.37) = 428.22\,K$

$(T_{02})_{out} = 288 + 7(23.37) = 451.59\,K$

$$\therefore \left(\frac{p_{02}}{p_{01}}\right) = \left(\frac{451.6}{428.2}\right)^{\frac{1.4 \times 0.88}{0.4}}$$

$\boxed{= 1.178}$ ← Ans

(ii) Given, λ (work factor) $= 0.83$, $\Delta T = 25K$

also, $c_p = 1.005$ kJ/kgK & $C_z = 165$ m/s

$$\psi = \frac{c_p \Delta T}{u^2} = \frac{\lambda C_z (\tan\beta_1 - \tan\beta_2)}{u} = \lambda[1 - \phi(\tan\beta_2 + \tan\alpha_1)] \quad \{\because \alpha_1 = \beta_2\}$$

then, $\frac{c_p \Delta T}{\lambda} = u^2 - 2uc_z \tan 20°$ $\left(\because \frac{c_p \Delta T}{u^2} = (1 - \frac{2c_z \tan 20°}{u})\lambda\right)$

$$\Rightarrow u^2 - 2uc_z \tan 20° - \frac{c_p \Delta T}{\lambda} = 0$$

$0.83u^2 - 93.68u - 23475.16 = 0$ } Now solving the eq^n

$\boxed{u = 233.23 \text{ m/s}} = \omega r$

Now, $r = 0.09$m ; $\omega \Rightarrow \frac{2712.4 \times 60}{2\pi} = 413.7$ radian/s

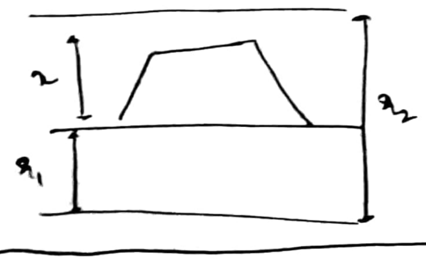
$\therefore \dot{m} = \rho c_z \frac{\pi}{4}(d_2^2 - d_1^2)$ with $d_2 = 18$cm

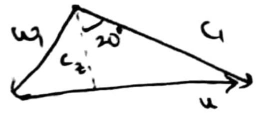
where, $\rho \Rightarrow \frac{P}{RT} = \frac{1.01 \times 10^5}{287 \times 288} = 1.18$ kg/m^3

$$d_2^2 - d_1^2 = \frac{4\dot{m}}{\rho c_z \pi} = \frac{4 \times 3}{1.18 \times 165 \times 3.14} = 0.021 \text{ m}^2$$

$\therefore \boxed{d_1 = 10.7 \text{ cm}}$

Now, length of rotor $= r_2 - r_1$
$= 9 - 5.35$
$= 3.65$ cm





$c_z = c_1 \cos 20°$
$= 165 \cos 20°$
$\boxed{c_z = 155 \text{ m/s}}$

Q2) Given, N = 10 ; Axial flow compressor $\frac{p_{02}}{p_{01}} = 5:1$

$\eta_s = 87\%$; $T_{in} = 288\,K$; identical stages

also, $R = 1/2$, $C_z = 170\,m/s$, $u_1 = 210\,m/s$, $\lambda = 1$; $\beta_1, \beta_2 = ?$

$$\eta_{overall} \Rightarrow \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_{02} - T_{01}} = \eta_s = 0.87$$

$$\underbrace{T_{02} - T_{01}}_{10\ \text{stages}} = \frac{288\left[(5)^{\frac{0.4}{1.4}} - 1\right]}{0.87} = 193.3\,K$$

$$\boxed{\therefore \Delta T\ (\text{single stage}) = 19.3\,K}$$

Now,

$$\Delta T_s = \frac{\lambda u C_z}{C_p}(\tan\beta_1 - \tan\beta_2)$$

$$(\tan\beta_1 - \tan\beta_2) = \frac{19.3 \times 1.005}{(210) \times (170)} = 0.54$$

Now,

$$R = \frac{1}{2}\phi(\tan\beta_1 + \tan\beta_2) \Rightarrow \boxed{\frac{1}{\phi} = \tan\beta_1 + \tan\beta_2 = \frac{u}{C_z} = 1.23}$$

from above two equations,

$$\boxed{\beta_1 = 41.67^\circ} \quad \& \quad \boxed{\beta_2 = 19^\circ}$$

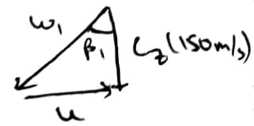
Q3) Given, $p_{in} = 1.0132$ bar , $C_z = 150\,m/s$ (no IGV)

$d_t = 60$ cm , $d_{hub} = 50$ cm , $N = 100$ rps

For blade speed (u) $\Rightarrow$ $u = 2\pi N r_{mean}$

$= 2\pi(100)(0.55)$ m/s

$= 172.78$ m/s

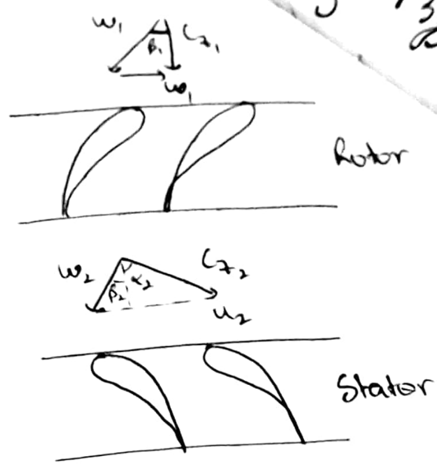


then, $w_1 = \sqrt{C_z^2 + u^2} = 228.8$ m/s

$\tan\beta_1 = u/C_z \Rightarrow \beta_1 = 49.036^\circ$

for the complete design,

since air is deflected by 30°



$\beta_2 = \beta_1 - 30° \Rightarrow \boxed{\beta_2 = 19.036°}$

$$w_2 = \frac{C_z}{\cos(19.036)} = 158.67 \text{ m/s}$$

Now, $c_2 = \sqrt{C_z^2 + (u_2 - w_2 \sin\beta_2)^2}$

$= 192.74$ m/s

also, $\alpha_2 = \cos^{-1}\dfrac{150}{192.74} \Rightarrow \boxed{\alpha_2 = 38.4°}$

$\dot{m} = \rho A C_z$ & $T_1 \Rightarrow T_0 - \dfrac{c^2}{2c_p} = 276.8$ K

with, $P = P_0\left(\dfrac{T}{T_0}\right)^{\gamma/\gamma-1} \Rightarrow \boxed{P_1 = 0.8816 \text{ bar}}$

$\rho = \dfrac{P_1}{RT} = 1.110$ kg/m³ $\Rightarrow \boxed{\dot{m} = 14.38 \text{ kg/s}}$

Power $\Rightarrow \dot{m}(u_2 C_{\theta_2} - u_1 C_{\theta_1}^{0}) = 14.38 \times 172.78 \times 192.74 \times \sin 38.9°$

$= 300.72$ kW

$$R \Rightarrow \frac{w_{\theta_1} + w_{\theta_2}}{2u} = \frac{u + w_2 \sin\beta_2}{2u}$$

$\boxed{\therefore R = 0.65}$

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Given, axial flow compressors ; $R = 1/2$; $\beta_1 = 45°$, $\beta_2 = 10°$

Q4) $T_{01} = 310\ K$; $\frac{p_{02}}{p_{01}} = 6$ & $\eta_s = 0.85$

Assume, $u = 200 m/s$

$$\eta_s = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\cancel{300}\ T_{01}\left[\left(\frac{p_{02}}{p_{01}}\right)^{\gamma-1/\gamma} - 1\right]}{T_{02} - T_{01}} = \frac{310\left[6^{\frac{0.4}{1.4}} - 1\right]}{\cancel{0.85}\ T_{02} - T_{01}}$$

$$\Rightarrow \boxed{T_{02} - T_{01} = 243.8 K}$$ for all stages

Now, to find C_z $\beta_1 = 45°$; $\beta_2 = 10°$; $\tan\beta_m \Rightarrow \frac{\tan\beta_1 + \tan\beta_2}{2} = 0.58$

Now, $R = \phi \tan\beta_m = 1/2$

$$\phi = \frac{C_z}{u} = \frac{1}{2\tan\beta_m} \Rightarrow \boxed{C_z = \frac{100}{0.588} = 170.06 m/s}$$

Temp. drop for single stage, $\Delta T = \frac{\lambda u C_z}{C_p}(\tan\beta_1 - \tan\beta_2)$

$\therefore$ for $\lambda = 1$; $\Delta T = 27.87 K$ & for $\lambda = 0.87$, $\Delta T = 24.24K$

$N_1 \Rightarrow \frac{243.8}{27.87} = 8.74$ | $N_2 \Rightarrow \frac{243.8}{24.24} = 10.05$

which gives us $N_1 = 9$ stages ; $N_2 = 10$ stages

Q5) Given, head pr. ratio $= 4$; $\beta_1 = 45°$ & $\beta_2 = 10°$

isentropic eff. $= 85\%$; $R = 1/2$

head inlet temp $= 290\ K$; u & C_z are constant

also, $u = 220 m/s$ & $\lambda = 0.86$; $N = ?$

$$\eta_c = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma\eta_p}} - 1}$$

where, $\eta_c = 0.85$, $\frac{P_2}{P_1} = 4$, $\gamma = 1.4$

& η_p = polytropic efficiency

$$\Rightarrow \eta_c = \frac{(4)^{\frac{0.4}{1.4}} - 1}{4^{\frac{0.4}{1.4\eta_p}} - 1} = 0.85$$

$$\frac{0.485}{0.85} = 4^{\left(\frac{0.4}{1.4\eta_p}\right)} - 1 \Rightarrow \boxed{\eta_p = 0.866 \text{ or } \eta_p = 86.6\%}$$

Rotor & stator blades are symmetric ($\alpha_1 = \beta_2$, $\beta_1 = \alpha_2$)

$$R = 0.5 \Rightarrow \frac{C_z}{2u}(\tan\beta_1 + \tan\beta_2)$$

$$\frac{220}{(\tan 45^\circ + \tan 10^\circ)} = C_z \quad \Big| \quad \boxed{C_z = 187.02 \text{ m/s}}$$

Q7) $P_1 = 0.9$ bar

$T_1 = 288$ K

$c_1 = c_z$ $\alpha_1 = 0$

Density $(\rho) \Rightarrow \frac{P_1}{RT_1} = \frac{0.9 \times 10^5}{287 \times 288} = 1.09$ kg/m³

Given,

$\dot{Q} = 1000$ m³/min ; Area of aerofoil $= 19.25$ cm²

$(\text{Length})_{\text{blade}} = 6.75$ cm ; $N = 6000$ rpm ; $D_m = 60$ cm

∴ no. of blades = 50 (16% volume) ; $C_L = 0.6$ & $C_D = 0.05$

$u \Rightarrow \frac{\pi D_m N}{60}$

$\Rightarrow 188.5$ m/s

$c_z \Rightarrow \dot{Q}/A$

$\Rightarrow \frac{1000 \times 10^4}{60(1.01) \times 60 \times 6.75} \Rightarrow \boxed{C_z = 145.6 \text{ m/s}}$

Scanned by CamScanner

$\tan \beta_1 \Rightarrow \frac{u}{c_z} = \frac{188.5}{145.6} = 1.295 \Rightarrow \boxed{\beta_1 = 52.3^\circ}$

$$\boxed{w_1 = \sqrt{145.6^2 + 188.5^2} = 238.15 \text{ m/s}}$$

Lift $\Rightarrow \frac{C_L \rho w_1^2 A_c}{2} = \frac{0.6 \times 1.09 \times (238.15)^2 \times 19.25 \times 10^{-4}}{2} = 35.7 \text{ N}$

Drag $\Rightarrow \frac{C_D \rho w_1^2 A_c}{2} = \frac{0.05 \times 1.09 \times (238.15)^2 \times 19.25 \times 10^{-4}}{2} = 2.98 \text{ N}$

Now,

Power input per stage $\Rightarrow (L \cos \beta_1 + D \sin \beta_1)\, u N$

$\Rightarrow [35.7 \cos(52.35) + 2.9 \sin(52.35)](188.5 \times 50 \times 10^{-3})$

$$\boxed{\Rightarrow 227.85 \text{ kW}}$$

$\dot{m} = \dot{Q} \rho = \frac{9000 \times 1.09}{60} = 18.167 \text{ kg/s}$

Now,

$\frac{P}{\dot{m}} = \frac{c_p T_1}{\eta_c} \left[r^{\frac{\gamma-1}{\gamma}} - 1 \right]$

$\Rightarrow r^{\frac{0.4}{1.4}} \Rightarrow \frac{P}{\dot{m}} \times \frac{\eta_c}{c_p T_1} + 1 \Rightarrow \left[\frac{227.85 \times 1}{18.33 \times 1.005 \times 288} - 1 \right]$

or $\boxed{r = 1.16 = \frac{P_2}{P_1}}$ $\boxed{\therefore \Delta P_{stage} = 1.14 \text{ bar}}$ Ans