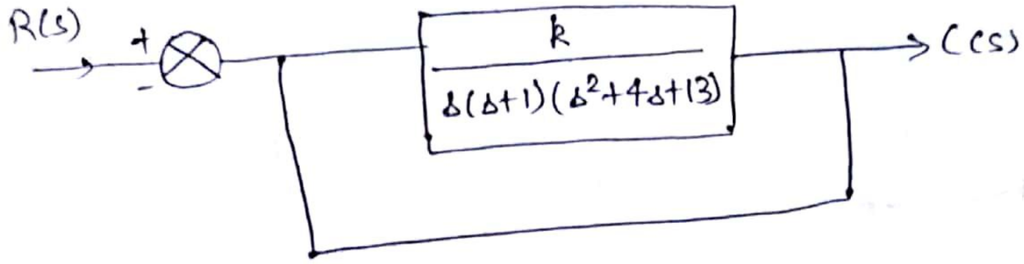
Quiz - 6 Solution

Find / Draw the root locus for the following :-



$$G_c(s) = \frac{k}{s(s+1)(s^2+4s+13)+k}$$

(1) Number of branches = # of closed loop poles.

Open loop Poles are $s = 0, -1, \frac{-4 \pm \sqrt{16-52}}{2}$

$s = 0, -1, -2 \pm 3i$

$\therefore$ # of branches = 4

(2) It has to be symmetric about σ-axis.

(3) Real axis Intercept.

$$\sigma_a = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

$$= \frac{0+(-1)+(-2+3i)+(-2-3i) - 0}{4-0}$$

$$= \frac{-5}{4} = -1.25$$

Angle $\theta_a = \frac{(2k+1)\pi}{\# \text{ finite poles} - \# \text{ finite zeros}} = \frac{(2k+1)\pi}{4}$ where $k = 0, 1, 2, 3 ..$

$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

As $s \to \infty$, func. $\to 0 \Rightarrow \exists$ a zero at ∞

This function also has zeros at ∞. (# = 4)

To find intercept of root locus on $j\omega$ axis.

Closed Loop transfer function

$$G_c(s) = \frac{k}{s^4+5s^3+17s^2+13s+k}$$

Consider the func.

$s^4+5s^3+17s^2+13s+k$

put $s = j\omega$ to find intercept

$(j\omega)^4+5(j\omega)^3+17(j\omega)^2+13(j\omega)+k=0$

$\omega^4 - 5j\omega^3 - 17\omega^2 + 13j\omega + k = 0$ (2 eq^n's, 2 variables ω, k)

Compare the real and imaginary parts

$\omega^4 - 17\omega^2 + k = 0$

$13\omega - 5\omega^3 = 0$

$\omega(13-5\omega^2) = 0$

$\omega = \sqrt{\frac{13}{5}} = \sqrt{2.6} = \boxed{1.61245}$

Also, $(\sqrt{2.6})^4 - 17(\sqrt{2.6})^2 + k = 0$

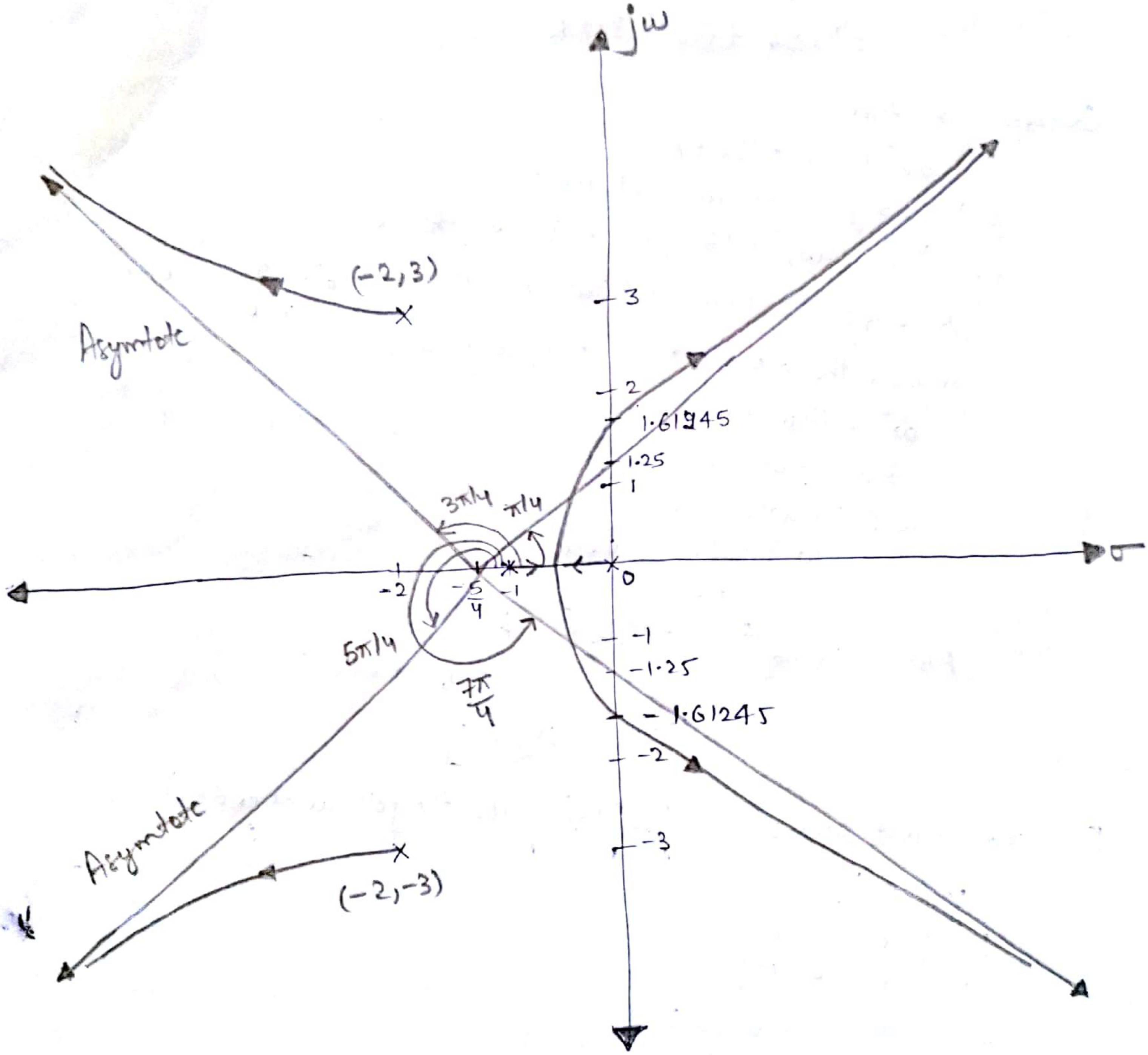
$k = 17\times2.6 - 2.6\times2.6$

$\boxed{k = 37.44}$

Can plot root locus in Matlab (Use the following code.)

```
num = 1;
D1 = [1 1 0];
D2 = [1 4 13];
den = conv(D1, D2);
G = tf(num, den);
rlocus(G);
```

Root Locus



4 - branches