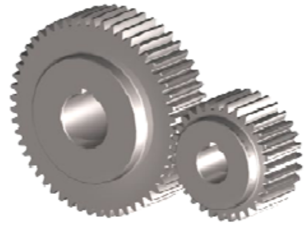
Topics
❖ Introduction to design of systems and machine elements, Modes of failure
❖ Yield criteria: Tresca, von-Mises, Mohr and modified Mohr, Stress concentration
❖ Failure by instability: Euler and Johnson Columns
❖ Fatigue failure: SN-diagram, Modification factors, Fluctuating loading, Modified Goodman, Combined loading
❖ Helical compression springs, Leaf springs
❖ Design of bolted joints and welded joints
❖ Rolling contact bearings
❖ **Spur and Helical gears**
❖ Shafts
❖ Brakes and Clutches
❖ Probabilistic approach to design
❖ Introduction to use of techniques like FEM for design

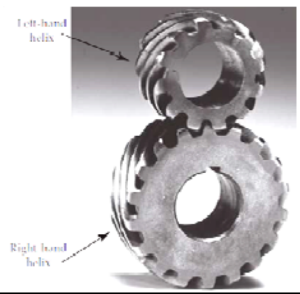
Gears

- Toothed cylindrical or conical wheels used for power transmission with or without speed reduction

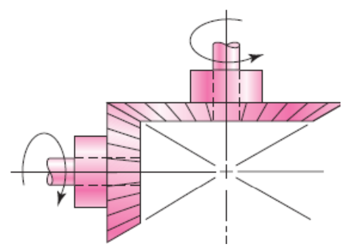
Spur gears
- Teeth are parallel to the axis of rotation
- Used for transmitting power between parallel shafts

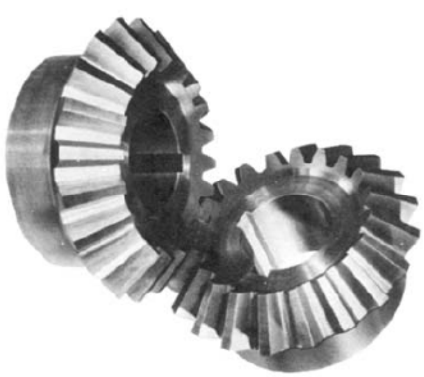


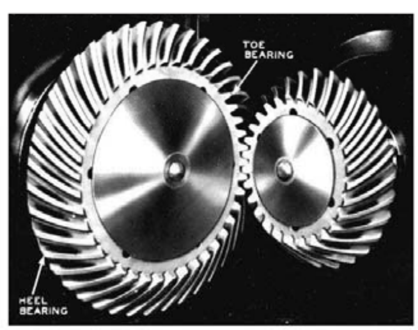
Helical gears
- Teeth are inclined to the axis of rotation
- Relatively quieter in operation
- Can be used for transmitting power between non-parallel shafts also



Bevel gears
- Teeth are formed on a conical surface
- Teeth can be straight or spiral
- Used for transmitting power between perpendicular intersecting shafts







Hypoid gears

- Similar to bevel gear but shafts can be offset, perpendicular to each other

Worm and Worm wheel

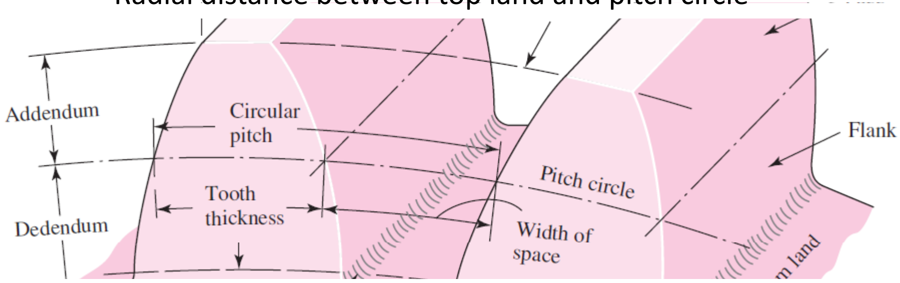
- Shafts perpendicular, non intersecting
- Large speed reduction possible

Pitch circle

- Theoretical circle upon which all calculations are made
- Its diameter is called pitch circle diameter – d
- Pitch circles of two mating gears are tangent to each other

Addendum, *a*

- Radial distance between top land and pitch circle

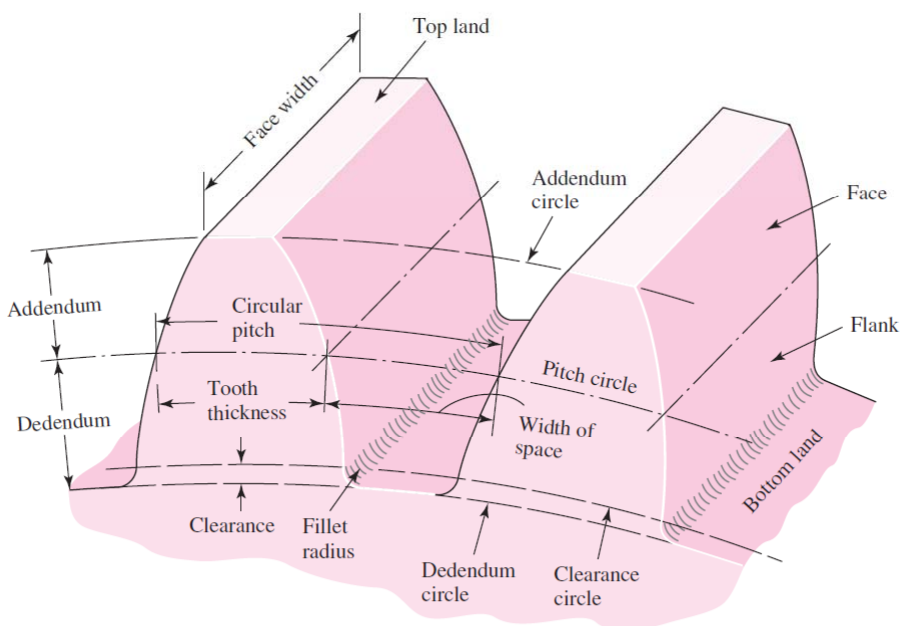


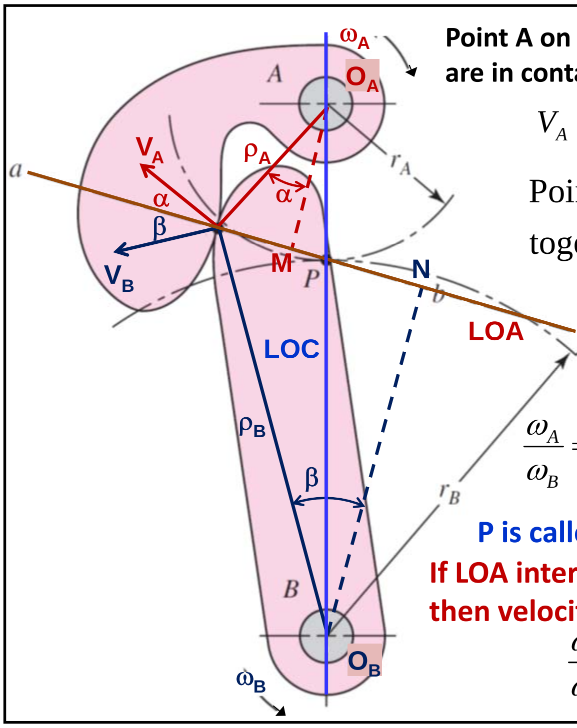
Circular pitch (*p*)

Dedendum, *b*

- Radial distance between bottom land and pitch circle
- *p* = tooth thickness + width of space (measured on PC)
- *p* = πd/N, N is number of teeth

- Pinion: the smaller of the two mating gears
- Gear: the larger among the two mating gears
- Idler: used for changing direction
- **Module**
  - $m = d/N$ (standard values given in data book)
- Clearance circle: The circle which is tangent to the addendum circle of mating gear, $c = b-a$
- Whole teeth depth, $h_t = a + b$
- Backlash
  - Amount by which the width of the tooth space exceeds the thickness of the engaging tooth, both measured on PC
- *a* and *b* have definite proportion with *m* in a standard gear





Point A on body A and Point B on body B are in contact at the instant shown.

$$V_A = \rho_A \omega_A; \quad V_B = \rho_B \omega_B$$

Points A and B are moving together along LOA

$$V_A \cos\alpha = V_B \cos\beta$$

$$\rho_A \omega_A \cos\alpha = \rho_B \omega_B \cos\beta$$

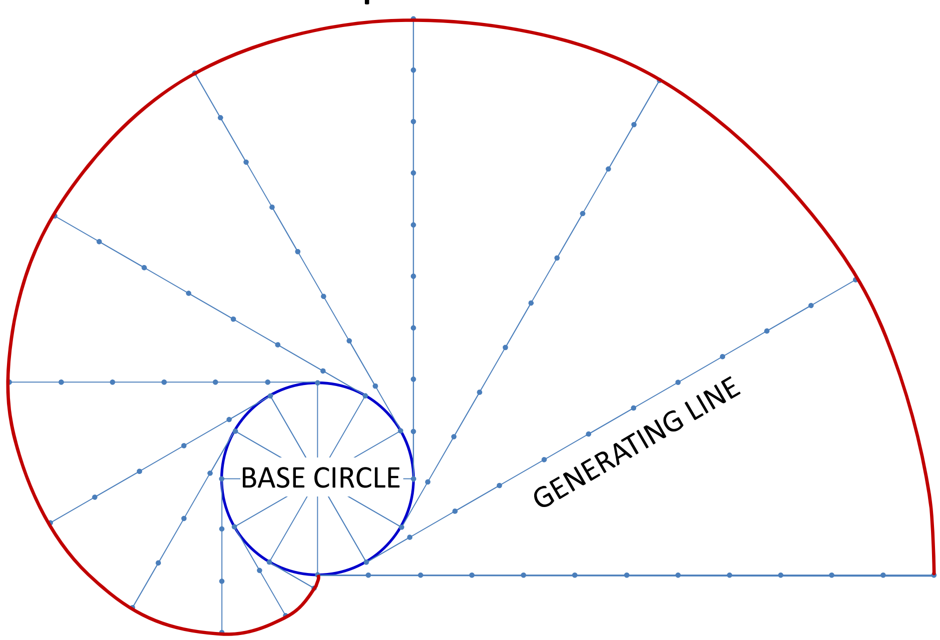
$$\frac{\omega_A}{\omega_B} = \frac{\rho_B \cos\beta}{\rho_A \cos\alpha} = \frac{O_B N}{O_A M} = \frac{O_B P}{O_A P}$$

P is called the pitch point

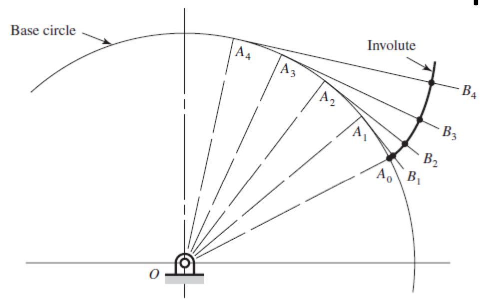
If LOA intersects LOC at the same point, then velocity ratio remains constant

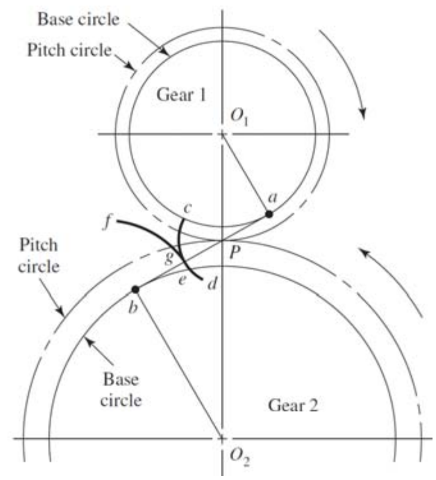
$$\frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}$$

Tooth profile- Involute



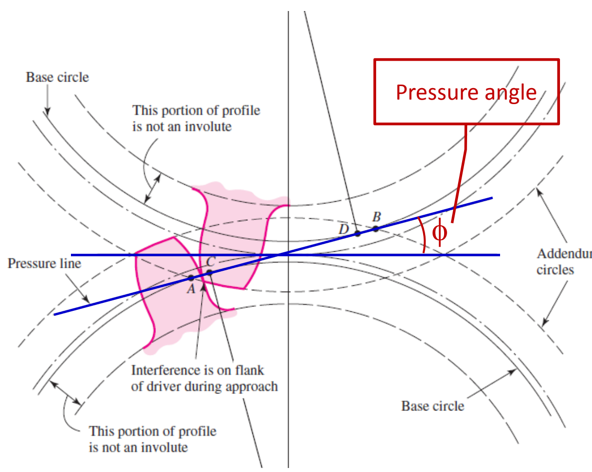
Involute profile

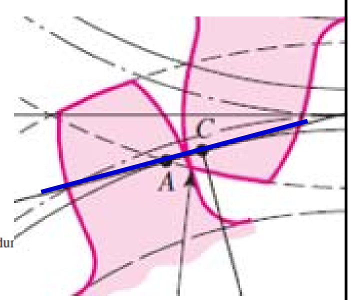




The circle on which the involute is generated is called **Base circle**

- LOA is the common tangent to the base circle of the two gears
- LOA is also the normal to the tooth profiles at the contact point



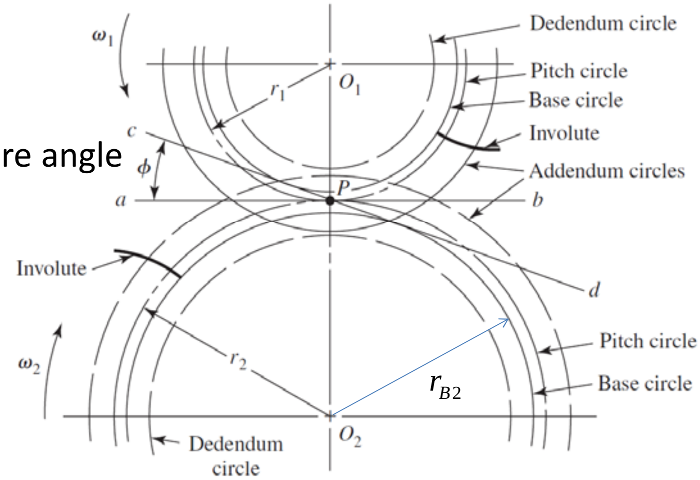


- **Module**
  - $m = d/N$ (standard values given in data book)
- *a* and *b* have definite proportion with *m* in a standard gear
  - For 20^o pressure angle, a = m and b = 1.25m

ϕ is the pressure angle

$$r_{B_1} = r_1 \cos\phi$$

$$r_{B_2} = r_2 \cos\phi$$



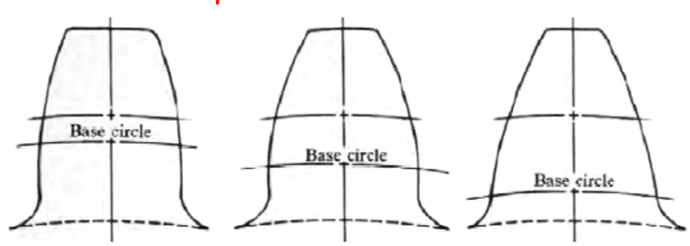
- When two gears are in mesh, their pitch circles roll on each other without slipping

$$\left|\frac{\omega_1}{\omega_2}\right| = \frac{r_2}{r_1} = \frac{d_2}{d_1} = \frac{mN_2}{mN_1} = \frac{N_2}{N_1}$$

Standard pressure angles

20°, 25°, 14.5°

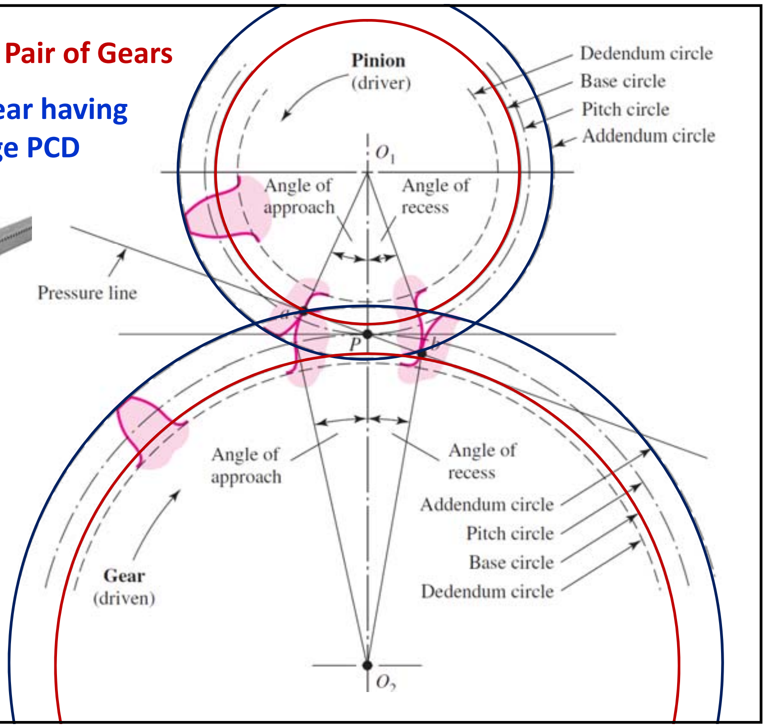
- The involute profile starts at the base circle and extends up to the addendum circle
- The gear tooth (in some cases) extends beyond the base circle up to the dedendum circle to have clearance
- The portion of tooth below the base circle is not an involute and does not provide conjugate action
- Contact in this portion should be avoided

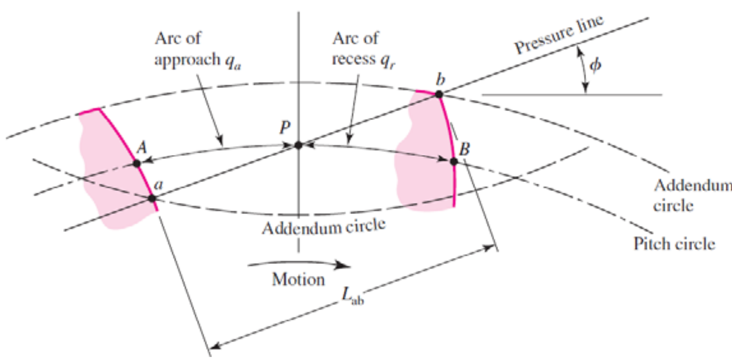


14.5°   20°   25°

Meshing of a Pair of Gears

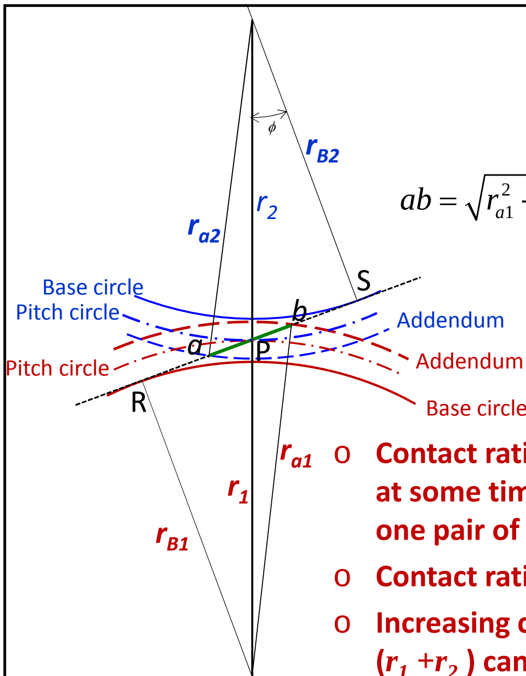
Rack: Spur gear having infinitely large PCD





$$L_{ab} = \sqrt{r_{a1}^2 - r_{B1}^2} + \sqrt{r_{a2}^2 - r_{B2}^2} - (r_1 + r_2)\sin\phi$$

Contact ratio: $m_c = \dfrac{L_{ab}}{p_b}$; $p_b = p\cos\phi$ is called base pitch



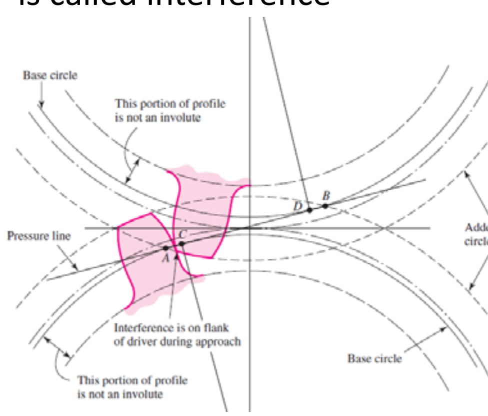
$$ab = Rb - RP + Sa - SP$$

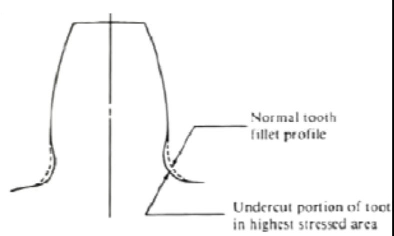
$$ab = \sqrt{r_{a1}^2 - r_{B1}^2} - r_1 \sin\phi + \sqrt{r_{a2}^2 - r_{B2}^2} - r_2 \sin\phi$$

- **Contact ratio of greater than 1 implies that at some time during meshing more than one pair of teeth isin contact**
- **Contact ratio of < 1.2 not recommended**
- **Increasing center to center distance ($r_1 + r_2$) can decrease contact ratio**

Interference

- Contact of tooth portions that are not involute (conjugate) is called interference





- In a gear manufactured by generation process, undercutting happens and this eliminates interference
- Undercutting weakens the tooth

Avoiding interference: spur gears

- The smallest number of teeth on a spur pinion and gear which can exist without interference is

$$N_p = \frac{2k}{(1+2m_G)\sin^2\phi}\left\{m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right\};\ m_G = \frac{N_G}{N_p}$$

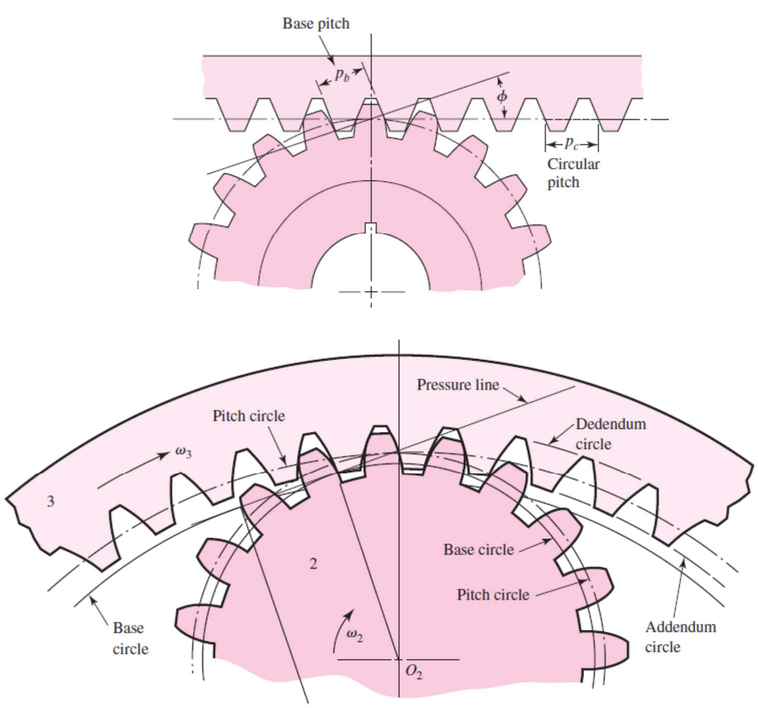
- The largest gear for a specified pinion to avoid interference

$$N_G = \frac{N_p^2 \sin^2\phi - 4k^2}{4k - 2N_p \sin^2\phi};\ k = \begin{cases} 1 \text{ for full depth} \\ 0.8 \text{ for stub depth} \end{cases}$$

- For rack and pinion

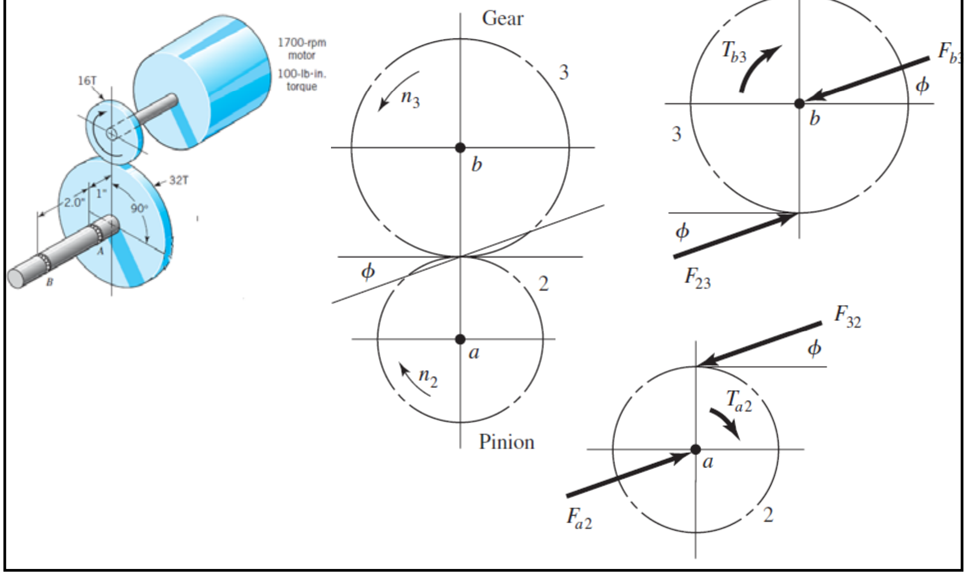
$$N_p = \frac{2k}{\sin^2\phi}$$

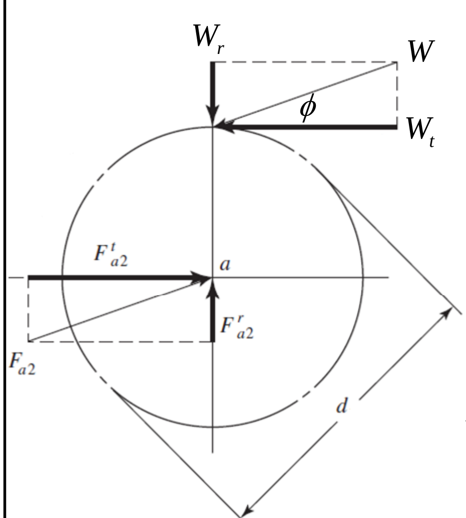
For 20º pressure angle and k=1		
N_P	Maximum N_G	m_G
17	1309	77
16	101	6.3
13	16	1.23



Spur gear force analysis

- The force exerted between a pair of meshing teeth can be at the pitch point into two components





Power, $H = T\omega$

$$H = W_t \frac{\pi d n}{60000}$$

H in kW

W_t in kN

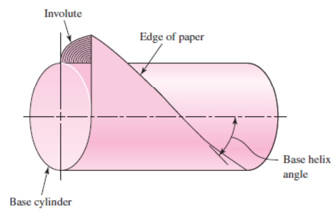
n in revolutions/minute

d in mm

$$W^t = W\cos\phi;\ W^r = W\sin\phi$$

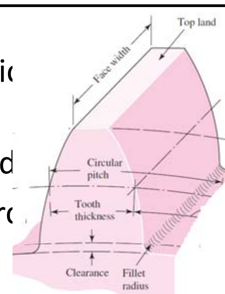
Helical gears

- Shape of the tooth is an involute helicoid

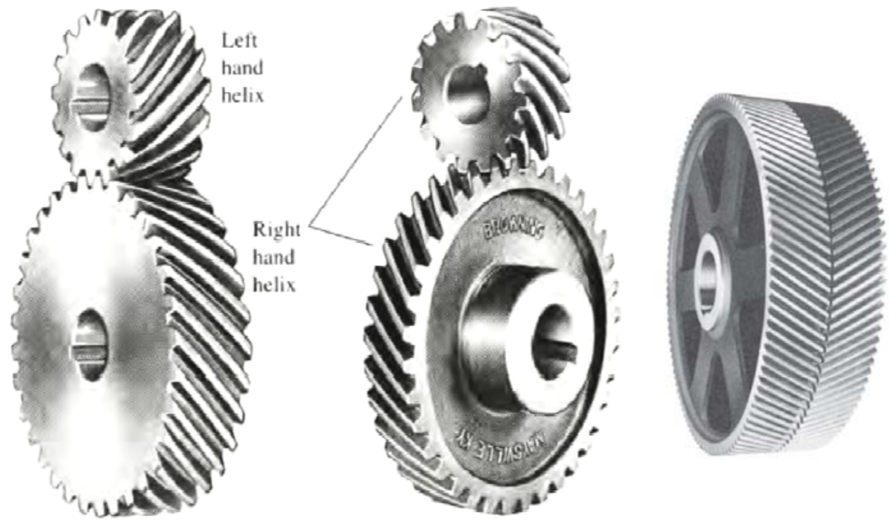


- Parallel shafts, pinion and gear has left and right hand teeth
- Perpendicular shafts, both gear and pinion has same type
- **Left handed helix**: teeth appear to lean to the left when viewed along the axis of rotation
- **Right handed helix** : teeth appear to lean to the left when viewed along the axis of rotation

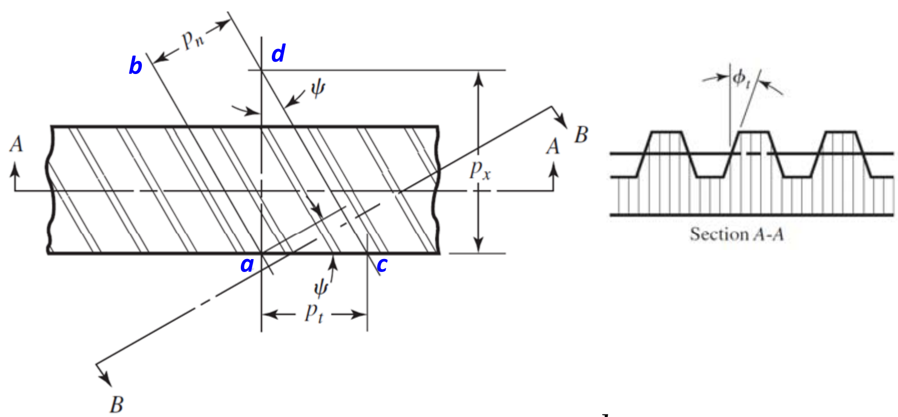
- In spur gears the contact is over a line whic face of the gear
- Contact is simultaneous along the face wid
- The line contact starts from addendum circ

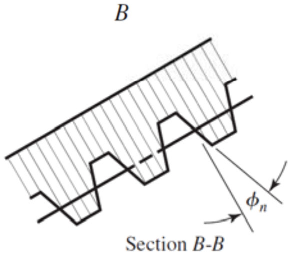


- In a helical gear, the initial contact is at a point and then it extends into a line as the teeth come into engagement
- The line contact extends diagonally across the tooth face going from tip to the root
- Engagement is therefore more gradual for a pair of tooth with smooth load transfer
- Contact ratio is not a concern, rather sufficient face width should be provided
- Generates an axial load on the shaft



(a) Helical gears with parallel shafts (b) Crossed helical gears, shafts at right angle

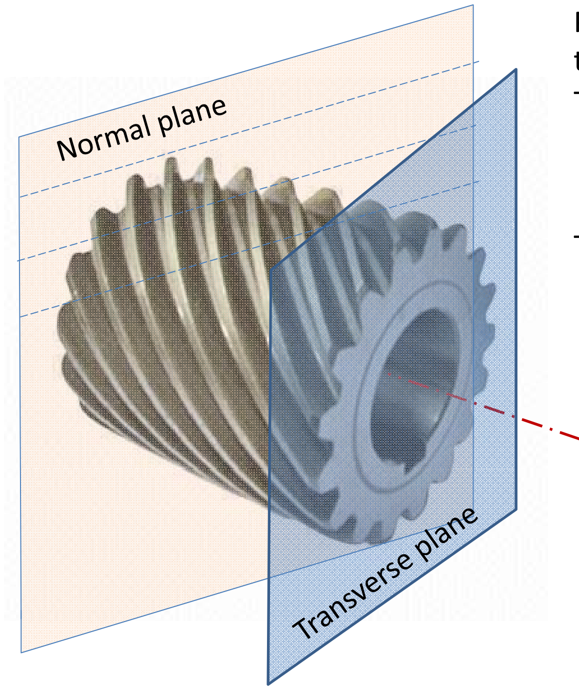




distance ac; $p_t = \dfrac{\pi d}{N}$; transverse pitch

distance ae; $p_n = p_t \cos\psi$; normal pitch

distance ad; $p_x = \dfrac{p_t}{\tan\psi}$; axial pitch



PCD (d) is measured in the transverse plane

Transverse circular pitch

$$p_t = \frac{\pi d}{N}$$

Transverse module

$$m_t = \frac{d}{N}$$

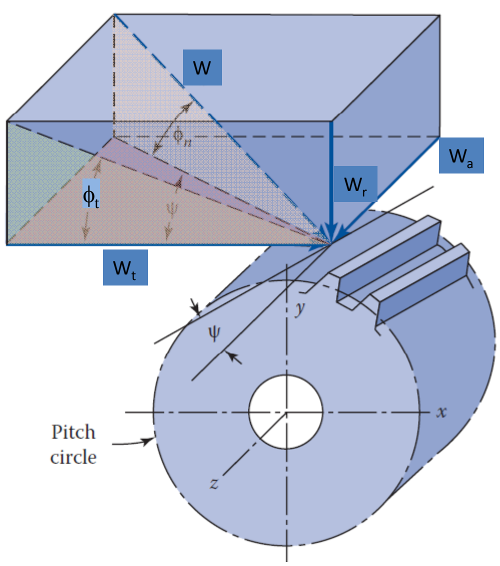
Normal circular pitch

$$p_n = p_t \cos\psi$$

Normal module

$$m_n = m_t \cos\psi$$

Helix angle is measured on the cylindrical pitch surface



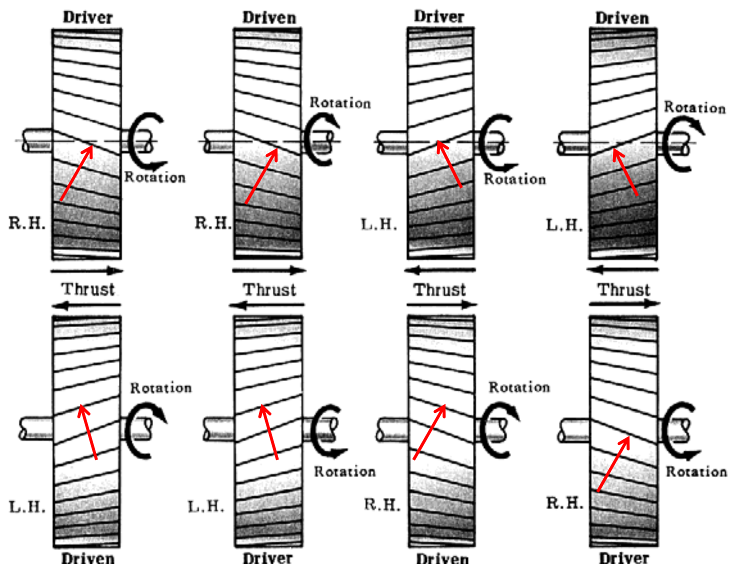
Normal pressure angle ϕ_n

Transverse pressure angle

$$\phi_t = \tan^{-1}\left\{\frac{\tan\phi_n}{\cos\psi}\right\}$$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

$$W_r = W\sin\phi_n$$
$$W_t = W\cos\phi_n\cos\psi$$
$$W_a = W\cos\phi_n\sin\psi$$



Avoiding interference: helical gears

- The smallest number of teeth on a helical pinion and helical gear which can exist without interference is

$$N_p = \frac{2k\cos\psi}{(1+2m_G)\sin^2\phi_t}\left\{m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi_t}\right\};\ m_G = \frac{N_G}{N_p}$$

- The largest gear for a specified pinion to avoid interference

$$N_G = \frac{N_p^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_p\sin^2\phi_t};\ k = \begin{cases}1 \text{ for full depth}\\ 0.8 \text{ for stub depth}\end{cases}$$

- For rack and pinion

$$N_p = \frac{2k\cos\psi}{\sin^2\phi_t}$$

Commonly used standard tooth systems for spur gear

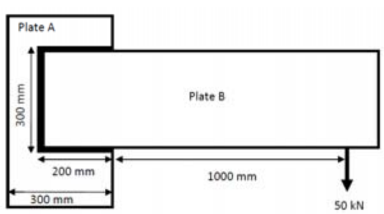
Tooth System	Pressure Angle ϕ, deg	Addendum a	Dedendum b
Full depth	20	$1m$	$1.25m$
			$1.35m$
	$22\frac{1}{2}$	$1m$	$1.25m$
			$1.35m$
	25	$1m$	$1.25m$
			$1.35m$
Stub	20	$0.8m$	$1m$

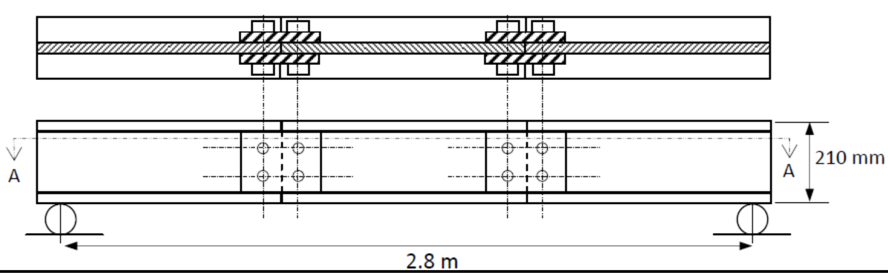
Commonly used standard tooth systems for helical gear

$$addendum,\ a = m_n$$
$$dedendum,\ b = 1.25m_n$$
$$PCD:\ d = \frac{N_p m_n}{\cos\psi},\ \ D = \frac{N_G m_n}{\cos\psi}$$

$$Base\ diameter:$$
$$d\cos\phi_t\ \ for\ \ pinion$$
$$D\cos\phi_t\ \ for\ \ gear$$

Modules	This is the normal module m_N for Helical gears
Preferred	1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50
Next Choice	1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45





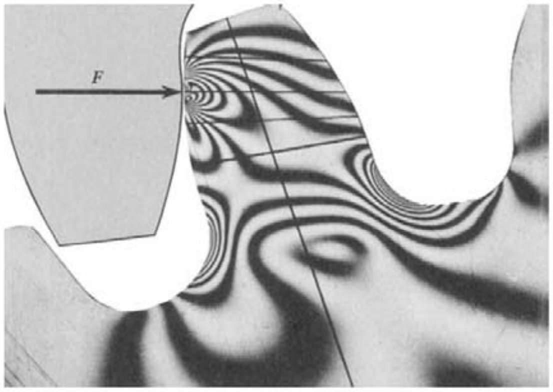


FIGURE 15.19 Photoelastic pattern of stresses in a spur gear tooth. (From T. J. Dolan and E. L. Broghammer, "A Study of Stresses in Gear Tooth Fillets," Proc. 14th Eastern Photoelasticity Conf., PE December 1941.)

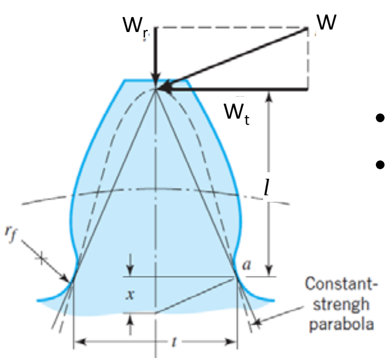
Tooth failure modes

- Bending stress at the root of the tooth
  - Yielding
  - Fatigue failure
- Contact stress at point of contact
  - Pitting or surface fatigue failure

Bending stress

Analysis by Wilfred Lewis in 1892

- Treat the tooth as a cantilever of span l, depth t and width F



$$\sigma = \frac{6W_t l}{Ft^2}$$

- Both l and t depends on tooth profile
- From similar triangles

$$\frac{t^2}{4l} = x, \quad \sigma = \frac{W_t p}{F\,\frac{2}{3}xp} = \frac{W_t}{FpY}$$

p is circular pitch

$$Y = \frac{2x}{3p}; \text{ is called Lewis form factor}$$

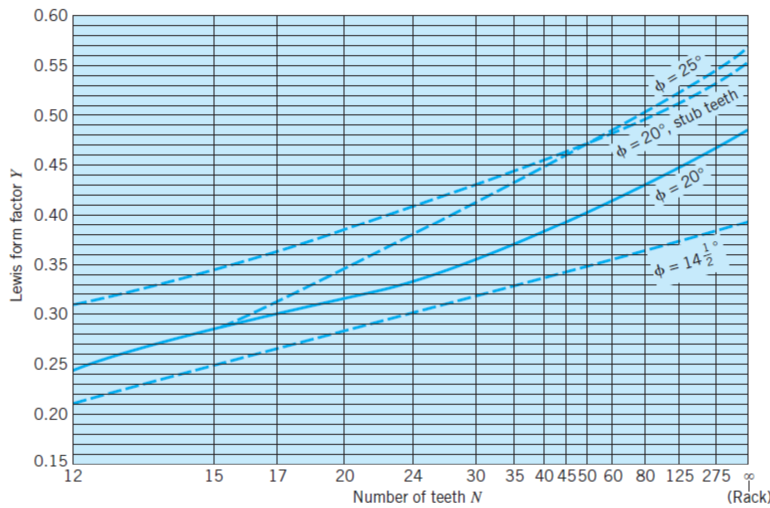


FIGURE 15.21
Values of Lewis form factor Y for standard spur gears (load applied at tip of the tooth).

- Y increases with increasing number of teeth or increasing pressure angle

- The worst condition is taken; i.e. the load applied at tip of the tooth
- Only one pair of teeth is assumed to engage at a time
- Load sharing (when contact ratio is > 1) is not considered
- Stress concentration at the root is not considered
- All of the above depends on tooth geometry, number of teeth etc.
- Examination of a run-in tooth indicates that heaviest loads occur at the middle of the tooth (load shearing)
- Hence the form factor is modified further to account for these effects

AGMA Design Equations

Tooth bending

$$\sigma = W_t K_O K_V K_S \frac{1}{Fm_t} \frac{K_H K_B}{Y_J} \quad \text{(SI units)}$$

F is face width, m_t is transverse module

K_O is overload factor, K_V is dynamic factor

K_S is size factor, K_H is load distrubution factor

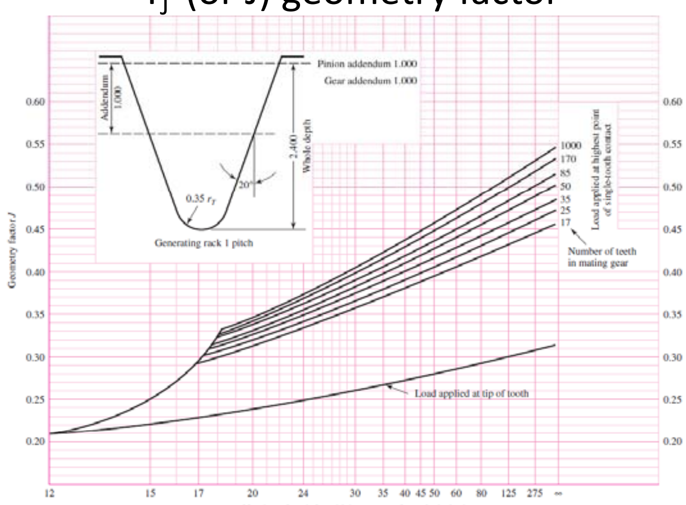
K_B is rim thickness factor

Y_J is geometry factor for bending strength

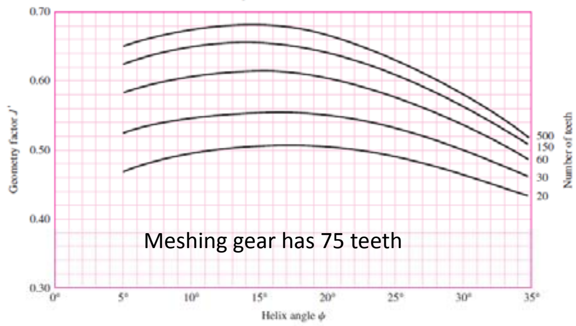
(includes the stress concentration factor K_F)

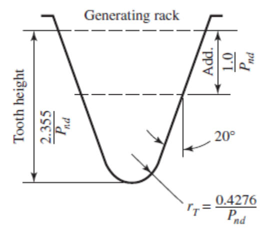
F is usually 3 to 5 times circular pitch p

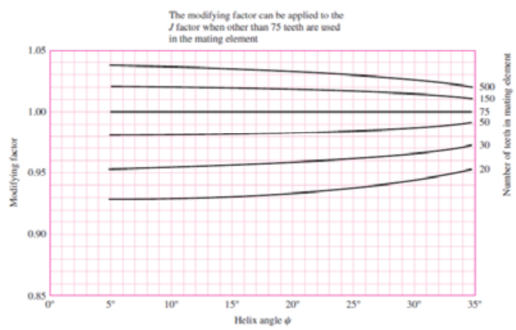
Y_J (or J) geometry factor



- Captures the effect of tooth form on bending stresses
- Will be different for pinion and gear







For helical gear, determine Y_J (J) from the first figure and multiply that with the modifying factor from the second figure

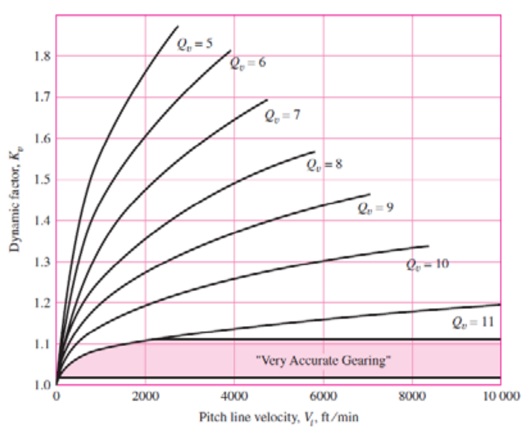
K_O – overload factor

- Accounts for all external loads in excess of W^t

	Driven Machine		
Power source	Uniform	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.75
Light shock	1.25	1.50	2.00
Medium shock	1.50	1.75	2.25

K_V – dynamic factor

- This accounts for inaccuracies in manufacturing and meshing of a gear teeth due to
  - Vibration during machining
  - Dynamic unbalance of rotating members
  - Wear and permanent deformation of tooth
  - Shaft misalignment due to linear and angular deflection
  - Tooth friction
- All of these causes transmission error (departure from constant angular velocity ratio)
- These lead to an increase in load on the tooth
- K_V depends on pitch line velocity



$$K_V = \left\{ \frac{A + \sqrt{200V}}{A} \right\}^B \text{ ; V in m/sec}$$

$$A = 50 + 56(1 - B)$$

$$B = 0.25(12 - Q_V)^{2/3}$$

- These curves are from test data on large number of gears
- Q_V is AGMA quality number
  - 3 to 7 : Commercial quality
  - 8 to 12: Precision quality
- K_V increases as V increases or Q_V decreases

K_S – Size factor

- Accounts for non-uniformity in material properties due to size

Module, m (mm)	K_s
≤ 5	1.00
6	1.05
8	1.15
12	1.25
20	1.40

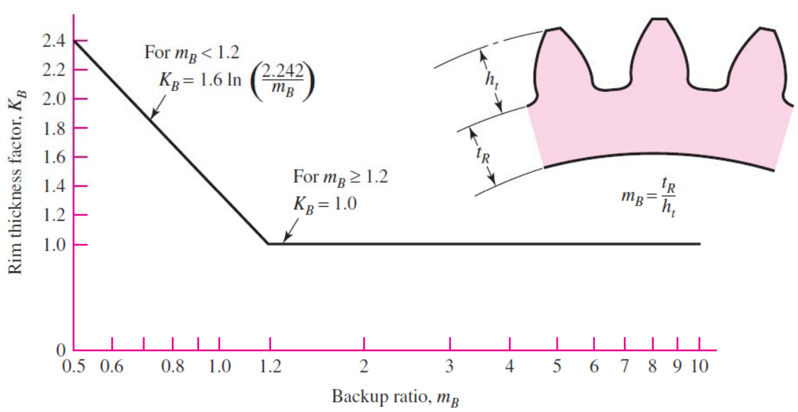
K_H – Load distribution factor

- Accounts for non-uniformity of load across the line of contact

	Face width (mm)			
Characteristics of Support	up to 50	150	225	> 400
Accurate mountings, small bearing clearances, minimum deflection, precision gears	1.3	1.4	1.5	1.8
Less rigid mountings, less accurate gears, contact across the full face	1.6	1.7	1.8	2.2
Accuracy and mounting such that less than full-face contact exists		Over 2.2		

K_B – Rim thickness factor

- The bending stress calculation assumes that the tooth is like a cantilever with root of it fixed.
- Large gears have a rim and if the rim is not sufficiently thick, then it can induce root flexibility



Allowable bending stress

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{Y_\theta Y_Z}$$

S_t – Bending strength (N/mm^2)

S_F – AGMA safety factor

Y_N – Stress cycle factor for bending

Y_θ – Temperature factor = 1

Y_Z – Reliability factor (table 14-10)

Reliability	K_R (Y_Z)
0.9999	1.50
0.999	1.25
0.99	1.00
0.90	0.85
0.50	0.70

- Gear teeth are subjected to repeated loading
- **The teeth in an idler gear will undergo two way bending, hence use *0.75S_t* in above equation**

Gear materials

- Can be made of steel, bronze, gray cast iron, even plastics
- Strength and pitting resistance are important factors
- Through hardened steels
  - 180 < H_B < 400 recommended
  - e.g AISI 3140, 4140, 4340, 6150, 8150 etc. are good
  - % elongation of 12% of more is desired
- Case hardened steels
  - H_B > 400
  - Induction hardening, Flame hardening, Ntriding etc. produce a hard surface layer and tough core
  - The hardening depth has to be ensured

Repeatedly Applied Bending strength: S_t

10^7 cycles and 0.99 reliability

Through-Hardened Steels ($175 < H_B < 400$)

$S_t = 0.533H_B + 88.3$ MPa –grade 1
$S_t = 0.703H_B + 113$ MPa –grade 2

Grade 1 is basic standard
Grade 2 requires high degree of microstructure control

Nitrided – through hardened steels (($270 < H_B < 340$) (ANSI 4140, 4340)

$S_t = 0.568H_B + 83.8$ MPa –grade 1
$S_t = 0.749H_B + 110$ MPa –grade 2

Nitrided steel (Nitralloy) ($270 < H_B < 340$)

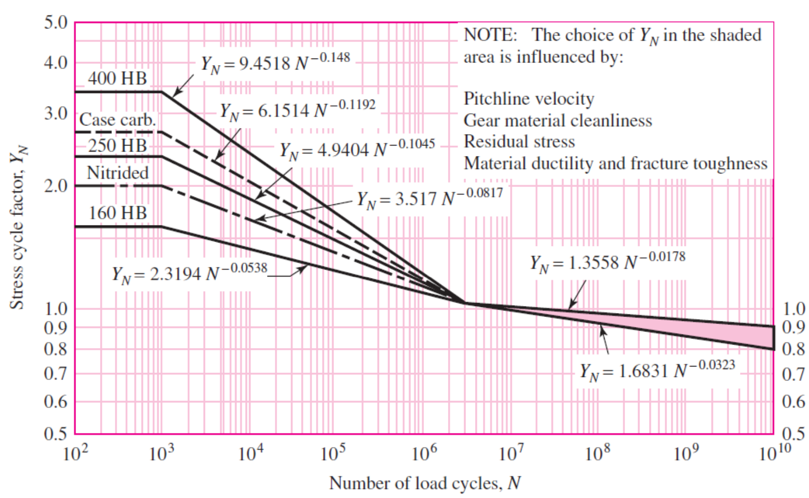
$S_t = 0.594H_B + 87.7$ MPa –grade 1
$S_t = 0.784H_B + 114.8$ MPa –grade 2

Nitriding produces a very thin hard case: Avoid nitrided steel if overloading and shock loads are anticipated

Nitrided- 2.5% Chrome steel (($300 < H_B < 340$)

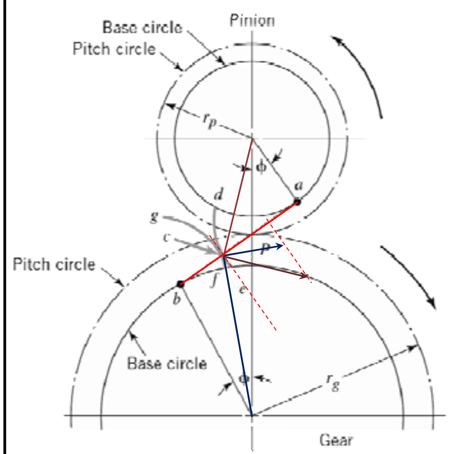
$S_t = 0.7255H_B + 63.9$ MPa –grade 1
$S_t = 0.7255H_B + 153.6$ MPa –grade 2
$S_t = 0.7255H_B + 201.9$ MPa –grade 3

- Y_N is used when life is not 10^7 cycles
- In a gear set, the number of load cycles for pinion and gear are different. So this factor will be different for pinion and gear



Surface durability

- In a gear teeth pair, perfect rolling happens only when the contact is at the pitch circle (P)
- At other contact points, there is rolling with sliding

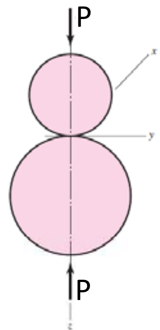


- Relative velocity along the direction perpendicular to LOA is not zero
- The sliding velocity reverses direction as the contact crosses the pitch point
- The magnitude of relative velocity is also not constant

Surface durability

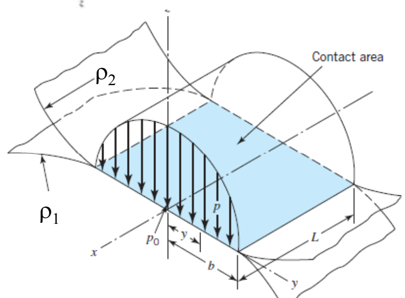
- The sliding causes
  - Abrasive wear due to trapped foreign particles
- Scoring
  - Higher velocity in the absence of proper lubrication (elasto-hydrodynamic) results in high friction
  - The high **contact pressure** and high friction lead to localized heating
  - This heating results in local welding of surface particles and their subsequent tearing
  - Lubrication and cooling can minimize this
- Pitting and spalling
  - Surface and subsurface fatigue failure under **contact stress**

Hertz contact stress



- Consider two cylinders in contact under load as shown
- The contact is over a narrow rectangular patch of width 2*b* and length L

$$b = \left\{ \frac{4P}{\pi L} \frac{\left[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2 \right]}{1/\rho_1 + 1/\rho_2} \right\}^{1/2}$$



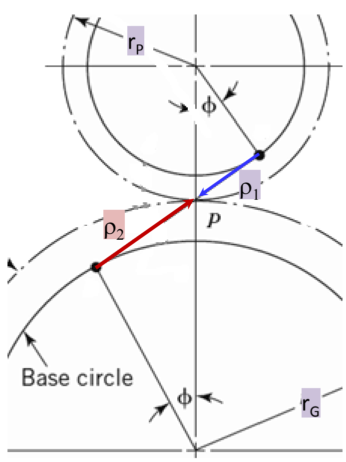
(b)
Two parallel cylinders

$$p_0 = \frac{2P}{\pi b L} \quad = Z_E \sqrt{\frac{P}{L}\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)}$$

$$Z_E = \sqrt{\frac{1}{\pi \left[\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right]}}$$

Contact stresses in meshing gear teeth

- Pitting damage is mostly observed around the pitch line
  - Sliding velocity is very small because of which hydrodynamic action is absent (no oil film)
- Therefore the contact stresses are evaluated at the pitch point



$$\rho_1 = r_P \sin\phi;\ \rho_2 = r_G \sin\phi;\ P = \frac{W_t}{\cos\phi}$$

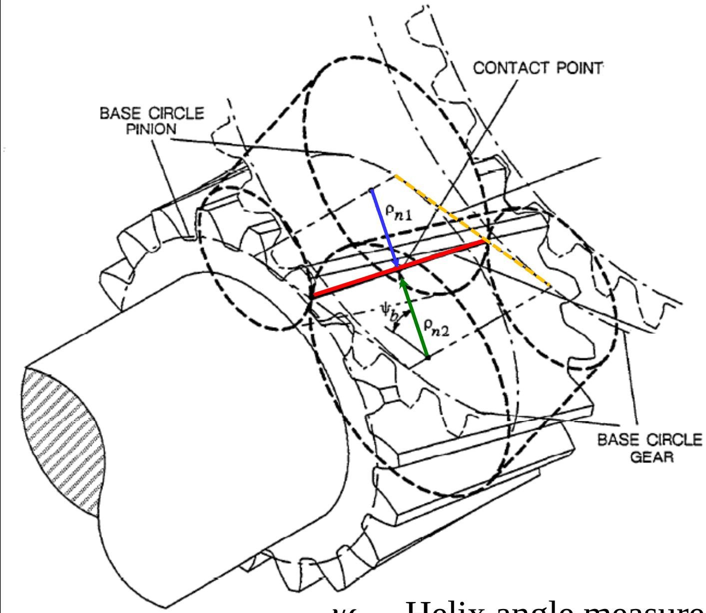
$$\sigma_c = Z_E \sqrt{\frac{W_t}{F d_P Z_I}}$$

$$Z_I = \frac{\sin\phi\cos\phi}{2} \frac{m_G}{m_G \pm 1}$$

+ for external gear set; - for internal gear set

Z_I is the geometry factor for surface strength

Z_I for helical gear



$$\rho_{n1} = \frac{r_P \sin\phi_t}{\cos\psi_b}$$

$$\rho_{n2} = \frac{r_G \sin\phi_t}{\cos\psi_b}$$

ψ_b – Helix angle measured on base circle

Z_I for helical gear

- In a helical gear, the contact load acts over a larger distance than the face width due to helix angle
- There is load sharing

$$Z_I = \frac{\sin\phi_t \cos\phi_t}{2m_N} \frac{m_G}{m_G \pm 1}$$

$m_N = \dfrac{p_n \cos\phi_n}{0.95Z}$ is load sharing ratio

p_n is normal circular pitch, $p_n = p_t \cos\psi$, $p_t = \dfrac{\pi d}{N}$

Z is length of line of action in transverse plane

$$Z = \sqrt{r_{aP}^2 - r_{bP}^2} + \sqrt{r_{aG}^2 - r_{bG}^2} - (r_P + r_G)\sin\phi_t$$

base circle radius, $r_b = r\cos\phi_t$; addendum circle radius, $r_a = r + a$

AGMA Design Equations

Tooth pitting

$$\sigma_c = Z_E \sqrt{W_t K_O K_V K_S \frac{K_H}{d_P F} \frac{Z_R}{Z_I}} \quad \text{(SI units)}$$

F is face width, d_P is PCD of pinion

Z_I is geometry factor for pitting resistance

Z_R is surface condition factor for accounting effect of surface finish (=1)

$$Z_E = \sqrt{\frac{1}{\pi\left[\frac{1-\nu_P^2}{E_P} + \frac{1-\nu_G^2}{E_G}\right]}}$$ is called the elastic coeffficient

Allowable contact stress

$$\sigma_{c,all} = \frac{S_c}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$

S_C – Contact strength (N/mm^2), S_H – AGMA safety factor

Z_N – Stress cycle factor for contact strength

Y_θ – Temperature factor = 1

Y_Z – Reliability factor (table 14-10)

Z_W – Hardness ratio factor (=1 for pinion)

Repeatedly applied contact strength, S_c

10^7 cycles and 0.99 reliability

Through-Hardened Steels ($175 < H_B < 400$)

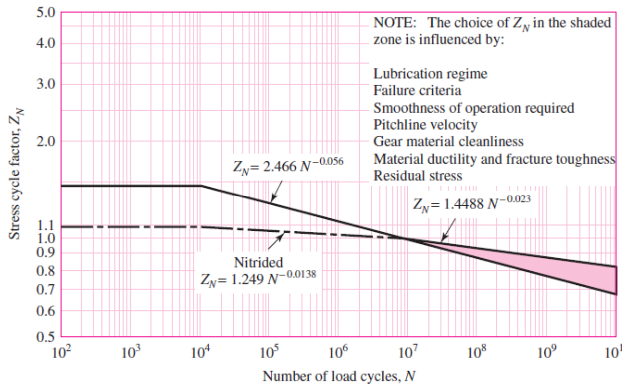
$S_c = 2.22H_B + 200$ MPa –grade 1
$S_c = 2.41H_B + 237$ MPa –grade 2

Nitrided Nitralloy 135M

$S_c = 1172$ MPa –grade 1
$S_c = 1261$ MPa –grade 2
$S_c = 1344$ Mpa –grade 3

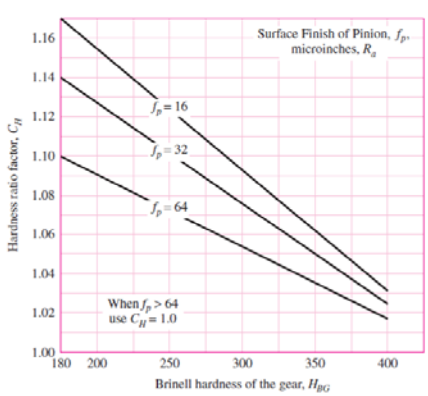
Nitrided 2.5% chrome ste

$S_c = 1068$ MPa –grade 1
$S_c = 1186$ MPa –grade 2
$S_c = 1303$ MPa –grade 3

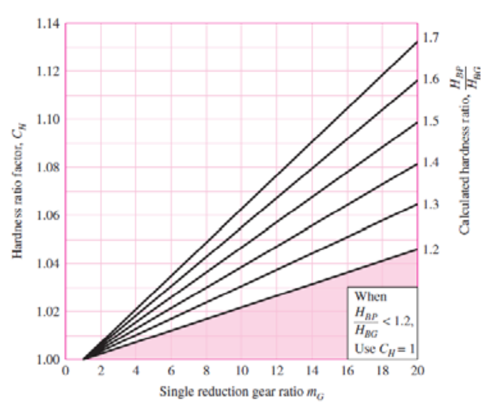


Hardness ratio factor, Z_W

- Good gear design practice suggests
  - Making pinion teeth more harder than gear teeth
  - Gear teeth are smoothened and work hardened during service and this improves pitting resistance



Both pinion and gear are through hardened
Surface hardened pinion of Rc > 48



Pinion is surface hardened
and gear is through hardened

Safety factors S_F and S_H

$$\sigma_{all} = \frac{S_t}{S_F}\frac{Y_N}{Y_\theta Y_Z}; \quad \sigma = W_t K_O K_V K_S \frac{1}{Fm_t}\frac{K_H K_B}{Y_J}$$

$$S_F = \frac{S_t}{\sigma}\frac{Y_N}{Y_\theta Y_Z}; \text{ stress } (\sigma) \text{ scales linearly with } W^t$$

$$\sigma_{c,all} = \frac{S_c}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z}; \quad \sigma_c = Z_E\sqrt{W_t K_O K_V K_S \frac{K_H}{d_P F}\frac{Z_R}{Z_I}}$$

$$S_H = \frac{S_c}{\sigma_c}\frac{Z_N Z_W}{Y_\theta Y_Z}; \text{ stress } (\sigma_c) \text{ scales with } \sqrt{W^t}$$

- If a gear is designed such that $S_F = S_H = 2$, in which mode (bending fatigue or surface fatigue) it will fail?
- Which failure mode is more catasrophic?

Safety factor for pinion and gear

- The safety factors for pinion and gear need not be the same
  - $(Y_J)_P \neq (Y_J)_G$, $(Y_N)_P \neq (Y_N)_G$, $(Z_N)_P \neq (Z_N)_G$
- What are the implications?
  - One of them will fail first (the one with lower safety factor)
  - Replacing one element in a set is not good as the other element (already weaker) can fail resulting in shut down
- Matching the factor of safety of pinion and gear is therefore desirable

$$\boxed{(S_F)_P = (S_F)_G \rightarrow (S_t)_G = (S_t)_P \frac{(Y_N)_P}{(Y_N)_G}\frac{(Y_J)_P}{(Y_J)_G}} \quad \boxed{(S_H)_P = (S_H)_G \rightarrow (S_c)_G = (S_c)_P \frac{(Z_N)_P}{(Z_N)_G}\frac{1}{Z_W}}$$

- Can be achieved by
  - Having different hardness for pinion and gear
  - Using different materials for pinion and gear