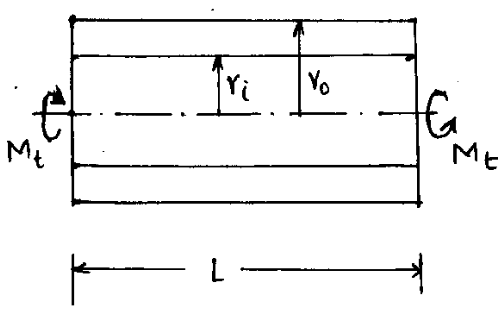
Solutions to H/W and Practice Problems of Chapter 6

Solution to problem 6.2 :



Given:

$L = 2.5\ m \qquad M_t = 2.5 \times 10^4\ N.m$

$\phi = \phi_L - \phi_0 = 2^\circ = \frac{\pi}{90}$ rad

$(\tau_{\theta z})_{max} = 8.2 \times 10^7\ N/m^2$

$G = 8.2 \times 10^{10}\ N/m^2$.

To find: r_i, r_o.

- Note:

$$\phi = \frac{M_t L}{G I_{zz}} \quad — \quad 1)$$

$$(\tau_{\theta z})_{max} = \frac{M_t r_o}{I_{zz}} \quad — \quad 2)$$

- From equations 1) and 2)

$$\frac{\phi}{(\tau_{\theta z})_{max}} = \frac{M_t L}{G I_{zz}} \times \frac{I_{zz}}{M_t r_o} = \frac{L}{G r_o}$$

$$\therefore \ r_o = \frac{L}{G} \frac{(\tau_{\theta z})_{max}}{\phi} \quad — \quad 3)$$

Substitute the values of $L, G, (\tau_{\theta z})_{max}$ & ϕ. we get

$$r_o = \frac{2.5}{8.2 \times 10^{10}} \cdot \frac{8.2 \times 10^7}{\pi/90}$$

or $\underline{r_o = 7.2 \times 10^{-2}\ m}$ — 4).

- Equation 1) can be rearranged as

$$I_{zz} = \frac{M_t L}{G \phi} \quad — \quad 5)$$

Substituting the values we get

$$I_{zz} = \frac{2.5 \times 10^4 \times 2.5}{8.2 \times 10^{10} \times \pi/90} = 2.18 \times 10^{-5}\ m^4 \quad - 6)$$

(problem 6.2 contd.)

Now
$$I_{zz} = \frac{\pi\,(r_o^4 - r_i^4)}{2}$$

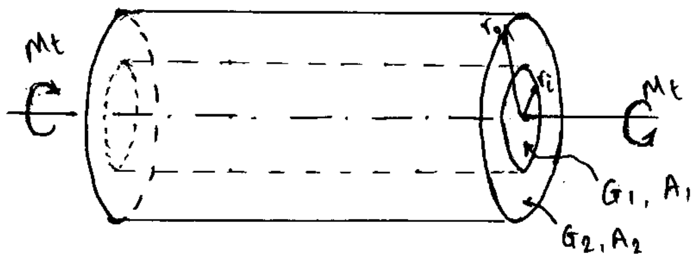
$$\therefore\; r_i^4 = -\frac{2 I_{zz}}{\pi} + r_o^4$$

$$(4)\,\&\,(6) \Rightarrow r_i^4 = -\frac{2 \times 2.18 \times 10^{-5}}{\pi} + (7.2 \times 10^{-2})^4$$

$$\Rightarrow \quad \underline{r_i = 6 \times 10^{-2}\ \text{m}.}$$

Note: After finding r_o, equation (2) also can be used to find r_i.

Solution to problem 6.3 :



- Deformation & Stresses :

Since geometry pattern is same, deformation pattern is same.

$$\therefore \epsilon_r = \epsilon_\theta = \epsilon_z = \gamma_{r\theta} = \gamma_{rz} = 0$$

$$\gamma_{\theta z} = r \frac{d\phi}{dz}.$$

and $\sigma_r = \sigma_\theta = \sigma_z = \tau_{r\theta} = \tau_{rz} = 0$

$$\left.\begin{aligned} \tau_{\theta z} &= G_1 r \frac{d\phi}{dz} \quad 0<r<r_i \\ &= G_2 r \frac{d\phi}{dz} \quad r_i<r<r_o \end{aligned}\right| \quad -(1)$$

- Boundary condition :

$$M_t = \int_{A_1} r(\tau_{\theta z})_1 \, dA + \int_{A_2} r(\tau_{\theta z})_2 \, dA.$$

$$= \int_{A_1} r G_1 r \frac{d\phi}{dz} dA + \int_{A_2} r G_2 r \frac{d\phi}{dz} dA.$$

$$= (G_1 I_{zz_1} + G_2 I_{zz_2}) \frac{d\phi}{dz}. \quad — (2)$$

where, $I_{zz_1} = \dfrac{\pi r_i^4}{2}$

$$I_{zz_2} = \frac{\pi (r_o^4 - r_i^4)}{2}$$

Rearranging eqn (2) we get

$$\frac{d\phi}{dz} = \frac{M_t}{G_1 I_{zz_1} + G_2 I_{zz_2}} \quad - (3)$$

Integrating equation (3)

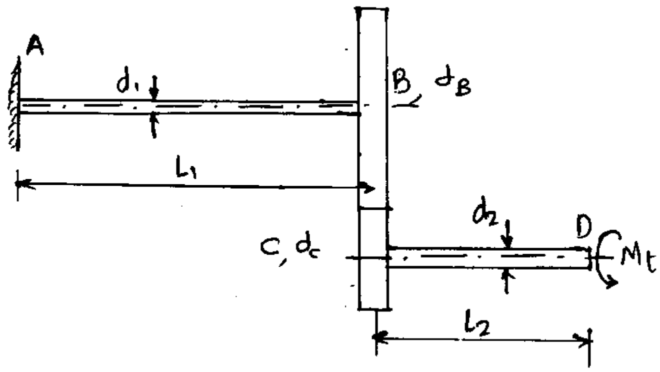
$$\boxed{\phi = \frac{M_t L}{G_1 I_{zz_1} + G_2 I_{zz_2}}}, \quad (\phi = \phi_L - \phi_o).$$

(Problem 6.3 contd.)

equations ① and ② $\Longrightarrow$

$$\boxed{\begin{aligned} \tau_{\theta z} &= G_1 r \frac{M_t}{G_1 I_{zz_1} + G_2 I_{zz_2}} \qquad && 0 < r < r_i \\ &= G_2 r \frac{M_t}{G_1 I_{zz_1} + G_2 I_{zz_2}} \qquad && r_i < r < r_o \, . \end{aligned}}$$

Solution to problem 6.6 :



Given:

$d_1 = 25$ mm

$d_2 = 10$ mm

$d_B = 150$ mm

$d_c = 50$ mm

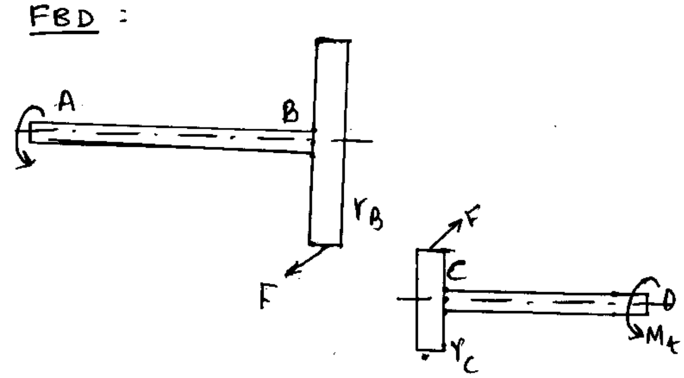
$L_1 = 1$ m , $L_2 = 0.67$ m

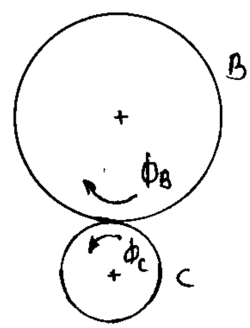
$M_t = 10$ N·m

$G = 8 \times 10^{10}$ N/m²

To find : ϕ_D

* FBD :





ϕ_B : CW

ϕ_c : CCW

Equilibrium: $F = \dfrac{M_t}{r_c}$ — 1)

* Force-deformation:

$$\phi_B - \cancel{\phi_A}^{\,0} = -\frac{(F \cdot r_b) L_1}{G\,(I_{zz})_1} \quad (M_t = -F r_B) \quad \text{— 2)} \quad (I_{zz})_1 = \frac{\pi d_1^4}{32}.$$

$$\phi_D - \phi_c = \frac{M_t L_2}{G\,(I_{zz})_2}, \quad \text{— 3)} \quad (I_{zz})_2 = \frac{\pi d_2^4}{32}.$$

* Compatibility

$$r_B |\phi_B| = r_c \phi_c$$

$$\therefore \phi_c = \frac{r_B}{r_c} |\phi_B| \quad \text{— 4)}$$

* Solution:

1), 2), 3) and 4) give

$$\phi_D = \phi_c + \frac{M_t L_2}{G (I_{zz})_2}.$$

(Problem 6.6 contd.)

$$\therefore \phi_D = \frac{r_B}{r_c}|\phi_B| + \frac{M_t L_2}{G(I_{zz})_2}$$

$$= \frac{r_B}{r_c}\frac{(F r_B)L_1}{G(I_{zz})_1} + \frac{M_t L_2}{G(I_{zz})_2}.$$

$$= \frac{r_B}{r_c}\frac{M_t}{r_c}\frac{r_B L_1}{G(I_{zz})_1} + \frac{M_t L_2}{G(I_{zz})_2}$$

$$= \frac{M_t L_2}{G(I_{zz})_2}\left[1 + \left(\frac{r_B}{r_c}\right)^2 \frac{L_1}{L_2}\frac{(I_{zz})_2}{(I_{zz})_1}\right]$$

$$= \frac{M_t L_2}{G(I_{zz})_2}\left[1 + \left(\frac{d_B}{d_c}\right)^2 \frac{L_1}{L_2}\left(\frac{d_2}{d_1}\right)^4\right] \quad \text{--- 5)}$$

* Numerical values:

Substituting the values of $M_t, L_1, L_2, G, (I_{zz})_2, d_1, d_2, d_B, d_c$ we get:

$$\phi_D = \frac{10 \times 0.67}{8\times10^{10}\ \frac{\pi}{32}\ 10^4 \times 10^{-12}}\left[1 + \left(\frac{150}{50}\right)^2 \frac{1}{0.67}\left(\frac{10}{25}\right)^4\right]$$

or $\phi_D = 0.1146$ rad

$= 6.57°$.

Solution to problem 6.7 :

- Refer to solution of problem 6.6.

Shaft CD. :

$$(\tau_{\theta z})_{max} = \frac{16 M_{t2}}{\pi d_2^3}, \qquad (M_t)_2 = M_t .$$

Shaft AB :

$$(\tau_{\theta z})_{max} = \frac{16 M_{t_1}}{\pi d_1^3}, \qquad (M_t)_1 = F r_B = M_t \frac{r_B}{r_C} = 3 M_t$$

$$\therefore \frac{(\tau_{\theta z})_{max,\,CD}}{(\tau_{\theta z})_{max,\,AB}} = \frac{M_{t_2}}{M_{t_1}} \frac{d_1^3}{d_2^3} = \frac{1}{3}\left(\frac{25}{10}\right)^3$$

$$= 5.208.$$

$\therefore (\tau_{\theta z})_{max,\,CD}$ is the largest stress.

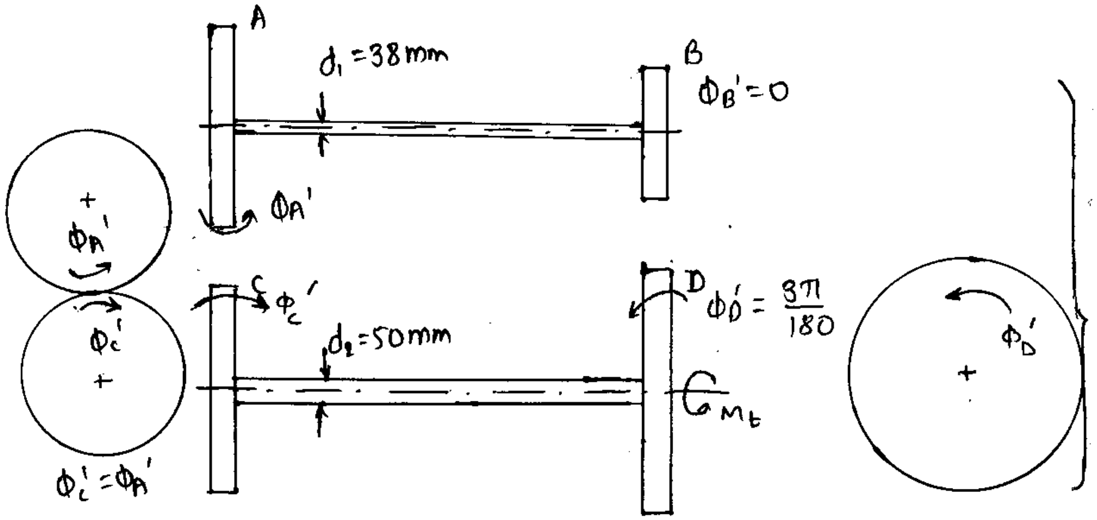
- $(\tau_{\theta z})_{max,\,CD} = 275\ MN/m^2.$

$$(\tau_{\theta z})_{max,\,CD} = \frac{16 M_{t_2}}{\pi d_2^3}$$

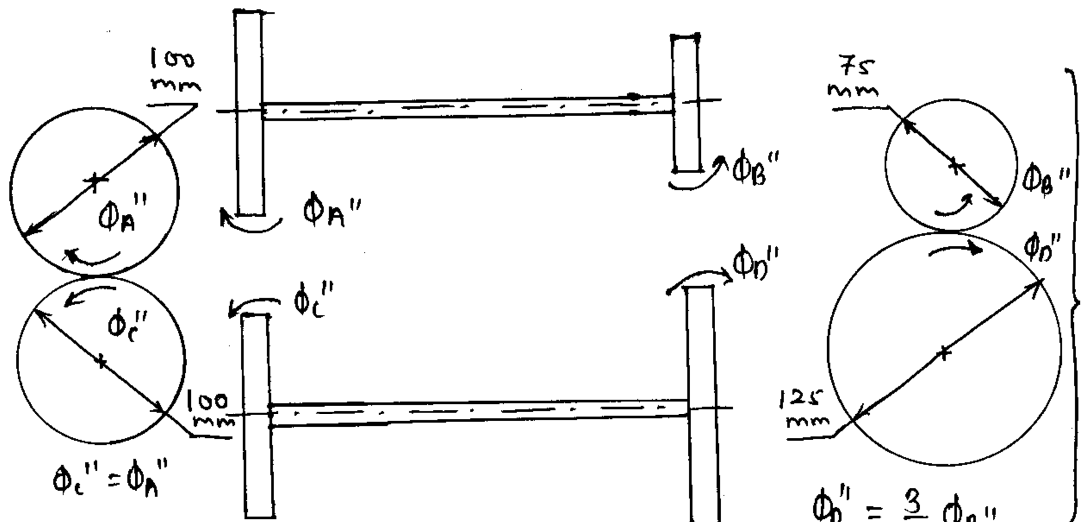
$$\therefore 275 \times 10^6 = \frac{16 M_{t_2}}{\pi \times (10)^3 \times 10^{-9}}.$$

$$\therefore M_{t_2} = 54 \quad N.m$$

Solution to problem 6.9 :



When D is rotated through 3° ccw (ϕ_c' assumed cw)



$\phi_D'' = \frac{3}{5}\phi_B''$
$= 0.6\,\phi_B''$

After assembly.

[ϕ_D'' & ϕ_c'' assumed opposite to ϕ_D' & ϕ_c']

Total rotation (ccw) : $\phi_D' - \phi_D''$ at D , $\phi_c'' - \phi_c'$ at C

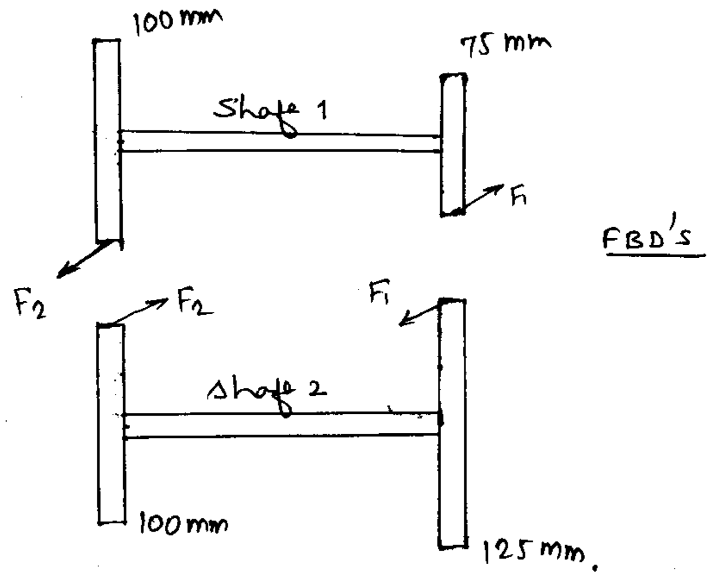
ϕ_B'' at B , $\phi_A' - \phi_A''$ at A.

Twist (ccw) : in AB : $\phi_B'' - (\phi_A' - \phi_A'') = \phi_B'' + (\phi_A'' - \phi_A')$

in CD : $(\phi_D' - \phi_D'') - (\phi_c'' - \phi_c') = (\phi_D' - 0.6\,\phi_B'') - (\phi_A'' - \phi_A')$

(problem 6.9 contd.)

Equilibrium (After assembly):



Equilibrium of shaft 1:

$$F_1\left(\frac{75\times10^{-3}}{2}\right) - F_2\left(\frac{100\times10^{-3}}{2}\right) = 0.$$

$$3F_1 - 4F_2 = 0. \quad \text{—— (1)}$$

Equilibrium of shaft 2:

$$F_1\left(\frac{125\times10^{-3}}{2}\right) - F_2\left(\frac{100\times10^{-3}}{2}\right) = 0.$$

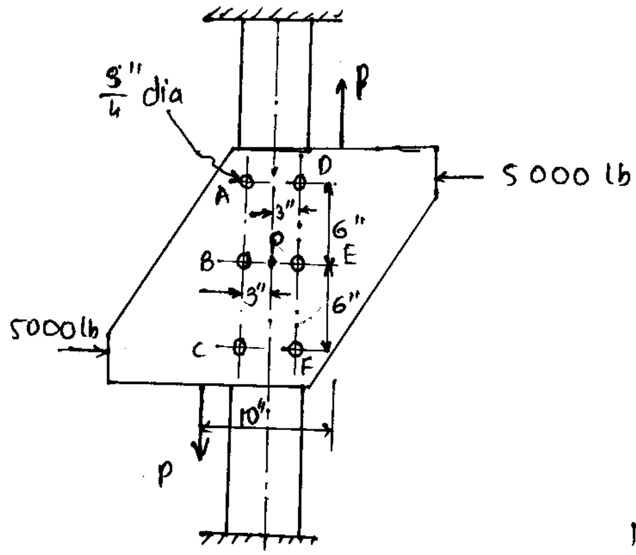
$$\therefore\ 5F_1 - 4F_2 = 0 \quad \text{—— (2)}$$

Equations (1) and (2) $\Rightarrow$ $F_1 = F_2 = 0.$

$\therefore M_{t_1} = M_{t_2} = 0.$

$\therefore$ stresses are zero.

Solution to problem 6.12 :



- The set of rivets may be considered as a non circular shaft with an externally applied couple, M. From symmetry, any rotation must occur with P as the centre.
- Tangential shear stress, assumed constant across rivet, proportional to r, distance from P ∴ shear stress $= Br$

Moment of this stress about P

$= \left(\frac{\pi}{4} d^2 \cdot r\right)(Br)$ d = dia. of rivet. B = constant

- Equilibrium :

∴ Total moment about P, $\frac{\pi}{4} B d^2 \Sigma r^2 = M$

$$\therefore \quad B = \frac{M}{\frac{\pi d^2}{4} \Sigma r^2} \qquad \text{(Thus, } B \propto M\text{)}$$

In this problem, $M = 12 \times 5000 = 60{,}000$ lb.in.

$d = \frac{3}{4}''$

$$\Sigma r^2 = (AP)^2 + (BP)^2 + (CP)^2 + (DP)^2 + (EP)^2 + (FP)^2$$

$$= 2 \times BP^2 + 4 \times AP^2 = (2 \times 9 + 4 \times 45)$$

$$= 198.$$

$\left[\begin{array}{l} CP = FP = DP \\ \quad = AP \\ EP = BP \end{array}\right.$

$$\therefore \quad B = \frac{60000}{\frac{\pi}{4}\left(\frac{3}{4}\right)^2 \times 198} = 685.9 \text{ lb/in}^3$$

- Shear stress in A, C, D, F $= B \times \sqrt{45} = 4600$ psi.

  Shear stress in B, E $= B \times 3 = 2058$ psi.
- Shear stress will reach 10,000 psi first in A, C, D, F.

(problem 6.12 contd.)

Stress is linearly related to external moment.

$\therefore$ extra external moment permissible:

4600 psi is produced .... 60,000 lb.in

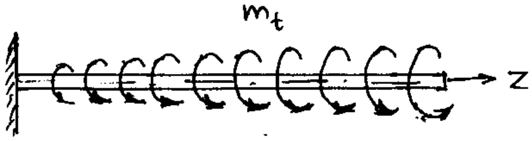
$\therefore$ (10000 - 4600) psi is produced by $\dfrac{5400 \times 60000}{4600}$

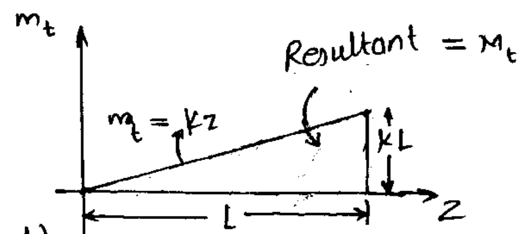
additional (extra) external moment

$$\therefore \quad P \times 10 = \frac{5400 \times 60{,}000}{4600}$$

$$P = 7043.5 \text{ lb.}$$

Solution to problem 6.14 :



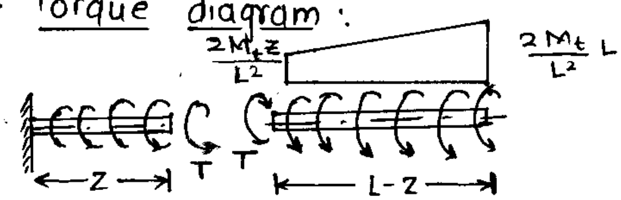


- Assume : $m_t = kz$ (k = constant)

Then, $M_t = \int_0^L m_t\, dz = \frac{1}{2}(kL)L \Rightarrow$

$$k = \frac{2M_t}{L^2} \qquad \therefore\ m_t = \frac{2M_t}{L^2} z .$$

- Torque diagram :

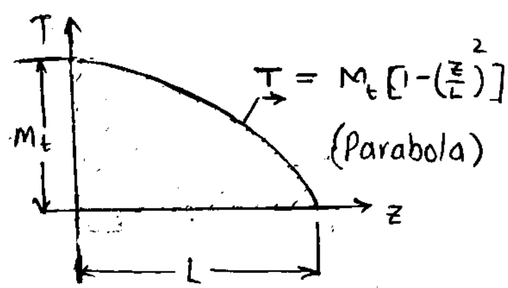


$$+\circlearrowleft\ \Sigma M_z = 0 \Rightarrow -T + \frac{1}{2}\left[\frac{2M_t}{L^2} z + \frac{2M_t}{L^2} L\right](L-z) = 0$$

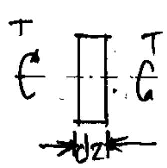
$$\Rightarrow T = \frac{M_t}{L^2}(z+L)(L-z)$$

$$= \frac{M_t}{L^2}(L^2 - z^2)$$

$$= M_t\left[1 - \left(\frac{z}{L}\right)^2\right]$$



Twist :



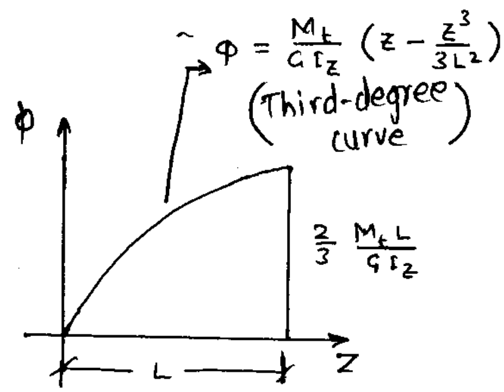
$$d\phi = \frac{T}{GI_z} dz$$

$$= \frac{M_t}{GI_z}\left[1 - \left(\frac{z}{L}\right)^2\right] dz$$

$$\therefore\ \phi = \int_0^z \frac{M_t}{GI_z}\left(1 - \frac{z^2}{L^2}\right) dz + C$$

$$\therefore\ \phi = \frac{M_t}{GI_z}\left(z - \frac{z^3}{3L^2}\right) + C$$

$\phi = 0$ at $z = 0 \Rightarrow C = 0$.



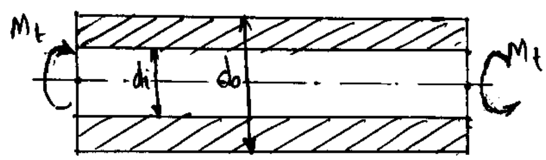
problem 6. 14 contd.

$$\phi\big|_{z=L} = \frac{M_t}{GI_z}\left(L - \frac{L^3}{3L} \right)$$

$$= \frac{2}{3}\frac{M_t L}{GI_z} .$$

$$= \frac{1}{GI_z}\left[\text{Area under } T-z \text{ diagram upto } z=L \right]$$

Solution to problem 6.17 :



Given: $d_o = 2 d_i$.

To find: P (Power) as a function of rpm, $(\tau_{\theta z})_{max}$ and d_o

$$(\tau_{\theta z})_{max} = \frac{M_t\, d_o/2}{I_{zz}} = \frac{M_t\, d_o/2}{\frac{\pi (d_o^4 - d_i^4)}{32}} = \frac{16\, M_t\, d_o}{\pi (d_o^4 - d_o^4/16)}$$

$$= \frac{256\, M_t}{15\, \pi\, d_o^3}.$$

$$\therefore M_t = \frac{15 \pi\, d_o^3 (\tau_{\theta z})_{max}}{256}.$$

$$\underset{\text{(Power)}}{P} = M_t \frac{N}{60} \times 2\pi \qquad (N \equiv rpm)$$

$$= \frac{2\pi M_t N}{60}$$

$$= \frac{\pi N}{30} \times \frac{15 \pi d_o^3 (\tau_{\theta z})_{max}}{256}$$

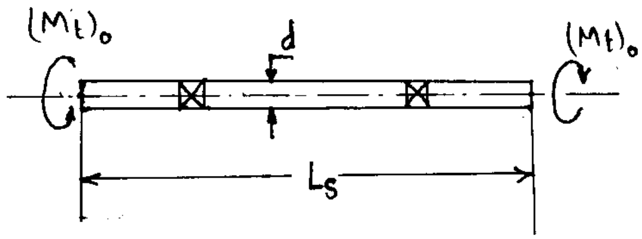
$$= \frac{\pi^2 d_o^3 N (\tau_{\theta z})_{max}}{512}$$

Solution to problem 6.23:

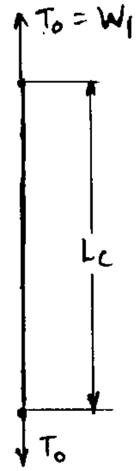
- Equilibrium:

FBD Before people step in:

$W_2 > W_1$

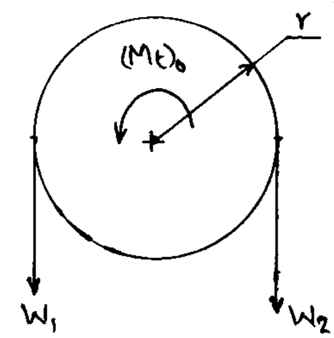


steel shaft



steel cable

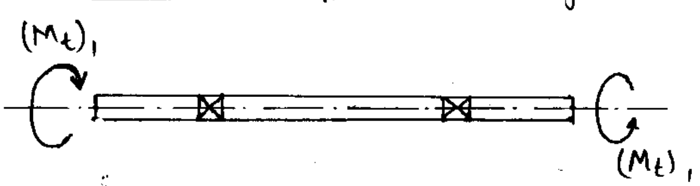
Area $= A_c$

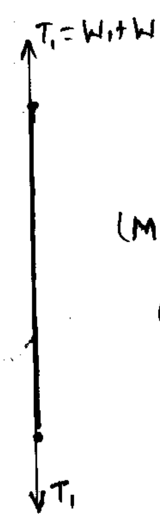


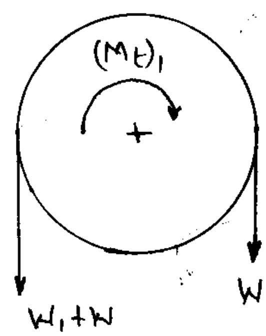
$(M_t)_o = (W_2 - W_1)r$

(CW on shaft)

FBD after people of wight W step in:







$(M_t)_1 = (W_1 + W - W_2)r$

(CCW on shaft.

Additional torque on the shaft, $\Delta M_t = Wr$ —— 1)

(CCW on shaft)

(Problem 6.23 contd.)

Additional tension in the cable, $\Delta T = W$ — 2)

- Force - deformation relations :

(for additional deformation only).

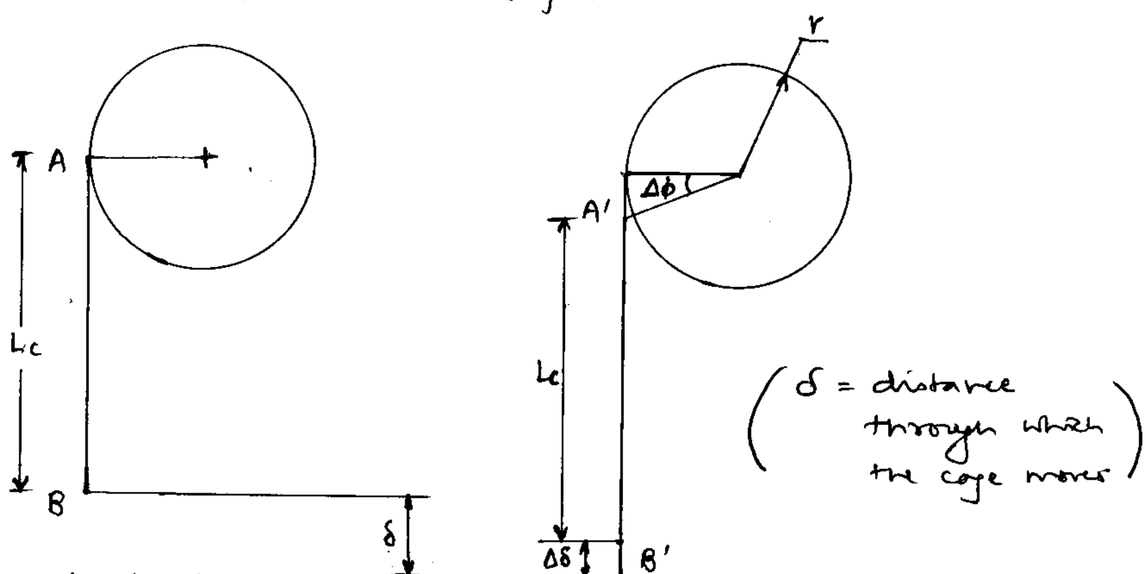
Additional twist, $\Delta\phi = \dfrac{\Delta M_t L_s}{G (I_z)_s}$

$$\Delta\phi = \frac{32 \Delta M_t L_s}{G \pi d^4} \qquad \text{— 3)}$$

Additional increase in length of cable $= \Delta\delta = \dfrac{\Delta T L_c}{E A_c}$ — 4)

- Compatibility :

Additional displacement of cage :



Before people step in

After people step in.

$$\delta = \Delta\delta + r\,\Delta\phi \qquad \text{— 5)}$$

equations 1) – 5) $\Longrightarrow$

$$\delta = \frac{W L_c}{E A_c} + \frac{32 (W r) L_s}{G \pi d^4}\, r \qquad \text{— (6)}$$

(problem 6.23 contd.)

Values: $\delta = 0.2''$, $L_c = 200' = 2400''$, $L_s = 5' = 60''$

$r = 3' = 36''$ $A_c = \frac{1}{2}\ in^2$ $d = 4''$

$E = 30 \times 10^6$ psi $G = 12 \times 10^6$ psi.

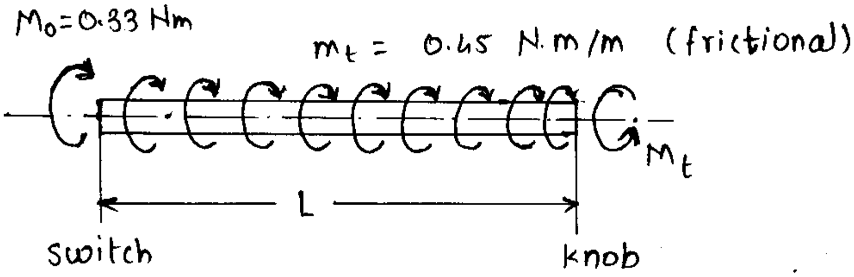
Substituting above values in eqn 6) we get

$$0.2 = \left[\frac{2400}{30 \times 10^6 \times \frac{1}{2}} + \frac{32\ (36)^2\ 60}{12 \times 10^6 \times \pi\ (4)^4} \right] W.$$

$$\Rightarrow \quad 0.2 = 4.18 \times 10^{-4}\ W$$

$$\therefore W = 478.6\ lb.$$

Solution to problem 6.24 :



- Equilibrium: $M_t = M_o + m_t L$

maximum torque is at the knob. Therefore maximum stress will be there.

$$(\tau_{\theta z})_{max} = \frac{16\, M_t}{\pi d^3} = \frac{16\ (M_o + m_t L)}{\pi d^3} .$$

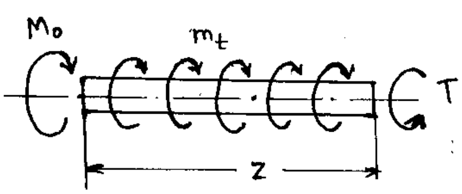
$$\therefore\ M_o + m_t L = \frac{\pi d^3\ (\tau_{\theta z})_{max}}{16}$$

Values : $d = 3$ mm , $(\tau_{\theta z})_{max} = 280$ MPa .

$$L = \frac{1}{m_t}\left[\frac{\pi d^3\ (\tau_{\theta z})_{max}}{16} - M_o\right]$$

$$= \frac{1}{0.45}\left[\frac{\pi \times 3^3 \times 10^{-9} \times 280 \times 10^6}{16} - 0.33\right]$$

$$= 2.565 \text{ m} .$$

- 

Equilibrium:

$$T = M_o + m_t z$$

(problem 6.24 contd.)

$$\frac{d\phi}{dz} = \frac{T\,dz}{GI_z} = \frac{(M_0 + m_t z)}{GI_z}\,dz$$

$\therefore$ Twist at the knob end is given by

$$\phi = \int_0^L \frac{(M_0 + m_t z)}{GI_z}\,dz$$

$$= \frac{1}{GI_z}\left(M_0 L + m_t \frac{L^2}{2}\right)$$

$$= \frac{32}{G\pi d^4}\left(M_0 L + m_t L^2/2\right)$$

Taking $G = 80$ GPa.

$$\phi = \frac{32}{80\times10^9 \times \pi \times 3^4 \times 10^{-12}}\left[0.33 \times 2.565 + 0.45\left(\frac{(2.565)^2}{2}\right)\right]$$

$$= 3.658 \text{ radians}$$

The shaft is turned in one direction, first, and then in the other direction.

$\therefore$ Play at the knob end $= 2\phi$
$= 7.316$ radians.

Solution to problem 6.28:

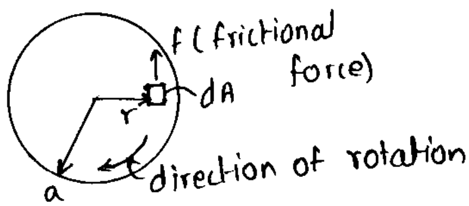
- **Assume**: Let the wheel reaction W be uniformly distributed over the circular contact area with intensity p.

$$\therefore p\pi a^2 = W \Rightarrow a = \sqrt{\frac{W}{p\pi}} = \sqrt{\frac{1000}{30\times\pi}} = 3.26 \text{ in}$$

$\left[\begin{array}{l} a = \text{radius of contact area} \\ W = 1000 \text{ lb}, \ p = 30 \text{ psi} \end{array}\right.$

- Estimation of Torque on Steering Column:

Bottom View



$\mu = 0.6$

Frictional force on area dA

$$f = \mu p \, dA.$$

Twisting moment due to f

$$dM_t = fr = (\mu p \, dA) r$$

Twisting moment of one tyre.

$$M_t = \int_A dM_t = \int_A \mu r p \, dA$$

$$= \int_0^a \int_0^{2\pi} \mu r p \, r \, dr \, d\theta.$$

$$= \mu p \int_0^a \int_0^{2\pi} r^2 dr \, d\theta.$$

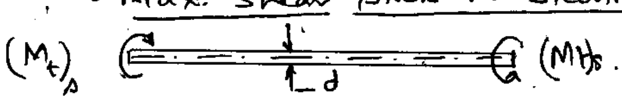
$$= \frac{2\pi}{3} \mu p a^3.$$

Twisting moment of 2 tyres $= 2 \times \frac{2\pi}{3} \mu p a^3 = \frac{4}{3} \pi \mu p a^3.$

$(M_t)_s$ = torque on steering column

$= \frac{1}{20} \times$ Torque on tyres.

- Max. shear stress in steering column:



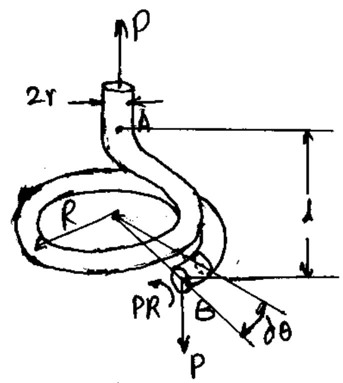
steering column

$$(\tau_{\theta z})_{max} = \frac{16 (M_t)_s}{\pi d^3} = \frac{16 \times \frac{1}{20} \times \frac{4}{3} \pi \mu p a^3}{\pi d^3}$$

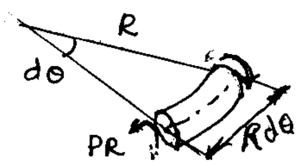
$\left\{\begin{array}{l} \mu = 0.6 \\ p = 30 \text{ psi} \\ a = 3.26'' \\ d = 3/4'' \end{array}\right.$

$$= 1576.8 \ \text{lb/in}^2$$

Solution to problem 6.30:



- Cut the spring at some point B, making the cut perpendicular to the wire. If the pitch is small, it can be assumed that the plane of cut contains the axis of the spring, and for equilibrium we require, at B, a force P (parallel to the axis) and a moment PR.



- The moment PR causes twisting of an element of the wire of length $Rd\theta$ through an angle

$$d\phi = \frac{PR\,(Rd\theta)}{GI_z} = \frac{2}{\pi}\frac{PR^2}{Gr^4}\,d\theta.$$

- Now consider the lower part of the spring (not shown) to remain fixed and determine the effect at A due solely to the twist angle $d\phi$ (A rotates ~~about~~ relative to B), in plane of the section at B. The upward component is

$$d\delta = R\,d\phi.$$

Integrating the upward component over the whole spring,

$$\delta = \int_0^l d\delta = \int_0^{2\pi n} R\,d\phi \qquad (n = \text{no. of coils})$$

$$= \int_0^{2\pi n} R\,\frac{2}{\pi}\cdot\frac{PR^2}{Gr^4}\,d\theta$$

$$= \frac{2}{\pi}\frac{PR^3}{Gr^4}(2\pi n)$$

$$= \frac{4PR^3n}{Gr^4}$$

Solution to problem 6.34:

- The only fixed requirement is spring constant $_{(k)}$ = 125 lb/in.

$*$ Load = P, Twist angle, $\phi = \dfrac{(Px)L}{G I_z} = \dfrac{PxL}{G \frac{\pi}{32} d^4}$. $\left[\begin{array}{l} M_t = Px \\ G = 12\times10^6 \text{ psi} \\ \text{for steel} \end{array}\right.$

$*$ deflection at load $= \phi x$.

$*$ Then, the spring constant is given by

$$\therefore k = \frac{P}{\phi x} = \frac{G\pi d^4}{32 x^2 L} \Rightarrow \frac{d^4}{x^2 L} = \frac{32 k}{G\pi}.$$

$$\therefore \frac{d^4}{x^2 L} = \frac{32 \times 125}{12 \times 10^6 \pi} = \frac{1}{3000\pi} \quad \text{—— 1)}$$

- The load is composed of static and dynamic parts:
  - Static : 1000 lb
  - dynamic : $(\pm 6'' \times 125 \text{ lb/in}) = \pm 750$ lb. (due to deflection of wheel)
  - max. load $_{(P)}$ = 1000 + 750 = 1750 lb.

$*$ $(M_t)_{max} = 1750\, x$ lb.in

$*$ $$(\tau_{\theta z})_{max} = \frac{16 (M_t)_{max}}{\pi d^3} = \frac{16(1750\, x)}{\pi d^3} = 8900 \frac{x}{d^3}.$$

$*$ We require

$$(\tau_{\theta z})_{max} = 8900 \cdot \frac{x}{d^3} \leq 50{,}000$$

$$\text{or} \quad \frac{x}{d^3} \leq 5.62 \quad \text{—— 2)}$$

(problem 6.34 contd.)

- Equation 1) and 2) can be satisfied by many combinations of x, L & d.

* One possiblity is obtained by taking for x and L their maximum possible values of 30'' and 120''.

then from 1), $d^4 = \frac{900 \times 120}{3000\pi} = 11.4$ and $d = 1.84''$

Equation 2) is satified as

$$\frac{x}{d^3} = \frac{30}{6.2} = 4.835 < 5.62.$$

Then, $(\tau_{oz})_{max} = \frac{4.835}{5.62} \times 50000 = 43,020$ psi.

* Alternatively, 1) can be written as $Ld^2 = \frac{3000\pi}{x^2/d^6}$.

and substituting 2), $Ld^2 \geq \frac{3000\pi}{(5.62)^2}$.

Ld^2 is proportional to the volume of the shaft and cost ~~design~~ is made minimum by setting

$$Ld^2 = \frac{3000\pi}{(5.62)^2}.$$

Taking $L = 120''$ as a further design condition, we get

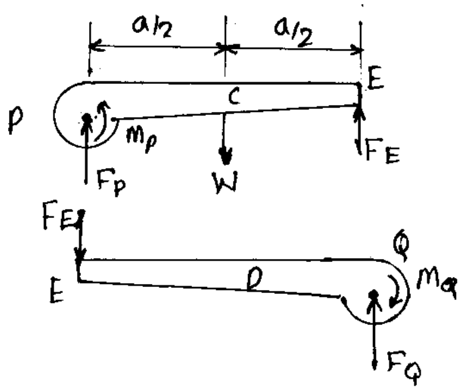
$d = 1.57''$, $x = 21.4''$.

Taking $x = 30''$ as another alternative, we get

$d = 1.75''$ and $L = 98.0''$.

note: In both cases $(\tau_{oz})_{max} = 50,000$ psi.

Solution to problem 6.35:



Consider free bodies of PE and EQ:

For equilibrium,

$$M_P = W\frac{a}{2} - aF_E \quad \text{— 1)}$$

$$M_Q = F_E \cdot a \quad \text{— 2)}$$

Compatibility :

Displacement at E of C = Displacement at E of D.

$\therefore$ ϕ_P (rotation of shaft at P) = ϕ_Q (rotation of shaft at Q) —(3)

Torque-Twist Relations: $\phi_P = \dfrac{32 M_P L}{G\pi d_A^4}$ —(4) , $\phi_Q = \dfrac{32 M_Q L}{G\pi d_B^4}$ —(5)

d_A = diameter of shaft A
d_B = diameter of shaft B
L = length of shafts A & B
G = shear modulus of shafts A & B

Solution: (3),(4),(5) $\Rightarrow$ $\dfrac{32 M_P L}{G\pi d_A^4} = \dfrac{32 M_Q L}{G\pi d_B^4}$

$$\Rightarrow \frac{M_P}{d_A^4} = \frac{M_Q}{d_B^4}. \quad \text{— 6)}$$

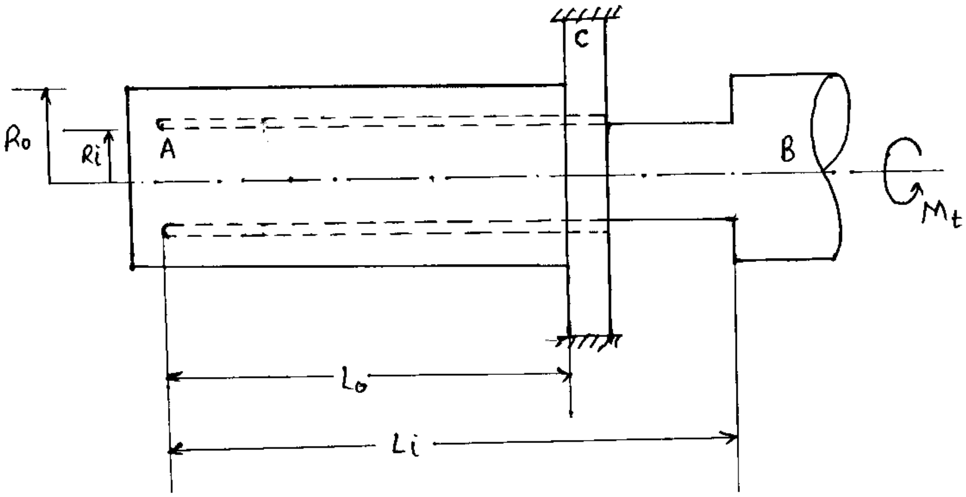
From 1), 2) and 6),

$$M_P = \frac{Wa}{2\left[1+\left(\frac{d_B}{d_A}\right)^4\right]}, \quad M_Q = \frac{Wa}{2\left[1+\left(\frac{d_A}{d_B}\right)^4\right]}$$

$$F_E = \frac{W}{2\left[1+\left(\frac{d_A}{d_B}\right)^4\right]}$$

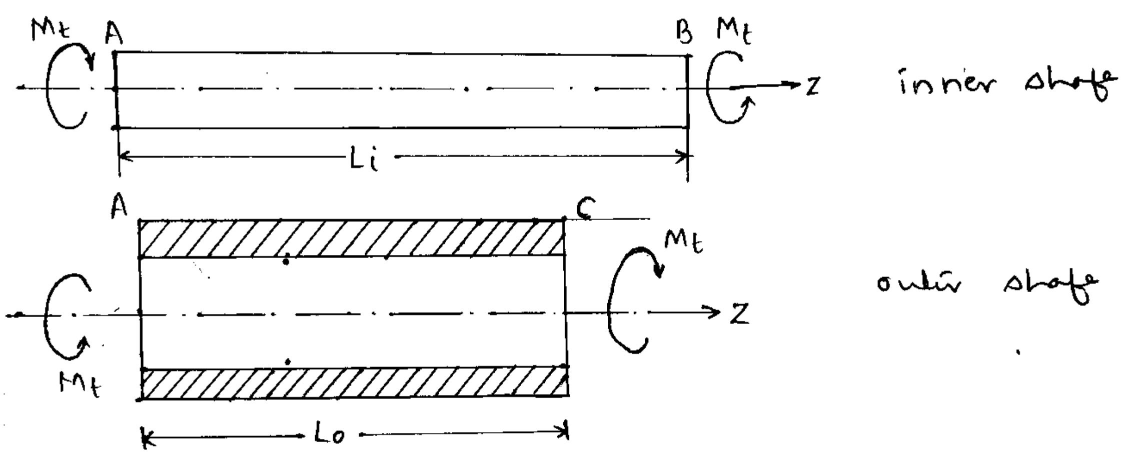
$$\phi_P = \phi_Q = \frac{32 WaL}{2\left[1+\left(\frac{d_B}{d_A}\right)^4\right] G\pi d_A^4} = \frac{16 WaL}{\pi(d_A^4+d_B^4)G}$$

Solution to problem 6.43:



a) Equilibrium:

FBD :



- Torque - Twist relations :

Inner shaft : $\phi_B - \phi_A = \dfrac{M_t L_i}{G (I_z)_i}$ — 1) $\quad (I_z)_i = \dfrac{\pi R_i^4}{2}$

Outer shaft : $(\cancel{\phi_C}^{\,0} - \phi_A) = -\dfrac{M_t L_o}{G (I_z)_o}$ — 2) $\quad (I_z)_o = \dfrac{\pi (R_o^4 - R_i^4)}{2}$

(Twisting moment is negative).

- Compatibility :

Rotations at end A of inner and outer shafts are same.
(already incorporated in equations 1) and 2))

(problem 6.43 contd.)

- Spring constant:

Equations 1) and 2) $\Rightarrow$

$$\phi_B = \phi_A + \frac{M_t L_i}{G (I_z)_i}$$

$$= \frac{M_t L_o}{G (I_z)_o} + \frac{M_t L_i}{G (I_z)_i}$$

$$= \frac{M_t}{G}\left(\frac{L_o}{(I_z)_o} + \frac{L_i}{(I_z)_i}\right)$$

$$\text{Torsional spring constant} = \frac{M_t}{\phi_B}$$

$$= \frac{G}{\frac{L_o}{(I_z)_o} + \frac{L_i}{(I_z)_i}}$$

b)

Inner shaft: $(\tau_{\theta z})^i_{max} = \dfrac{M_t R_i}{(I_z)_i}$

Outer shaft: $(\tau_{\theta z})^o_{max} = \dfrac{M_t R_o}{(I_z)_o}$.

Both the shafts yield simultaneously.

$$\therefore (\tau_{\theta z})^i_{max} = (\tau_{\theta z})^o_{max}.$$

$$\therefore \frac{R_i}{(I_z)_i} = \frac{R_o}{(I_z)_o}$$

$$\therefore \frac{2R_i}{\pi R_i^4} = \frac{2R_o}{\pi (R_o^4 - R_i^4)} \Rightarrow \frac{R_o^4 - R_i^4}{R_i^4} = \frac{R_o}{R_i}.$$

$$\therefore \left(\frac{R_o}{R_i}\right)^4 - \frac{R_o}{R_i} - 1 = 0.$$

$$\frac{R_o}{R_i} \simeq 1.2207$$