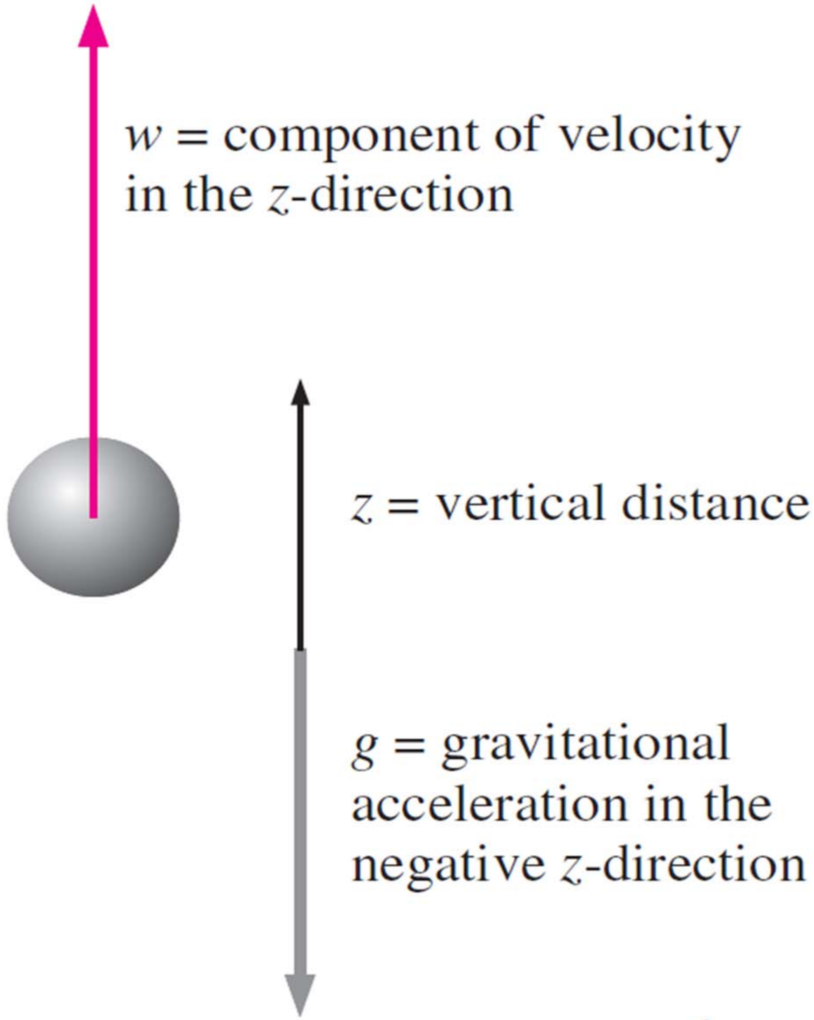
Advantages of Non-dimensionalization



Equation of motion: $$\frac{d^2z}{dt^2} = -g$$

The initial location of the object is z_0 and its initial velocity is w_0 in the z-direction.

Dimensional result: $$z = z_0 + w_0 t - \frac{1}{2} g t^2$$

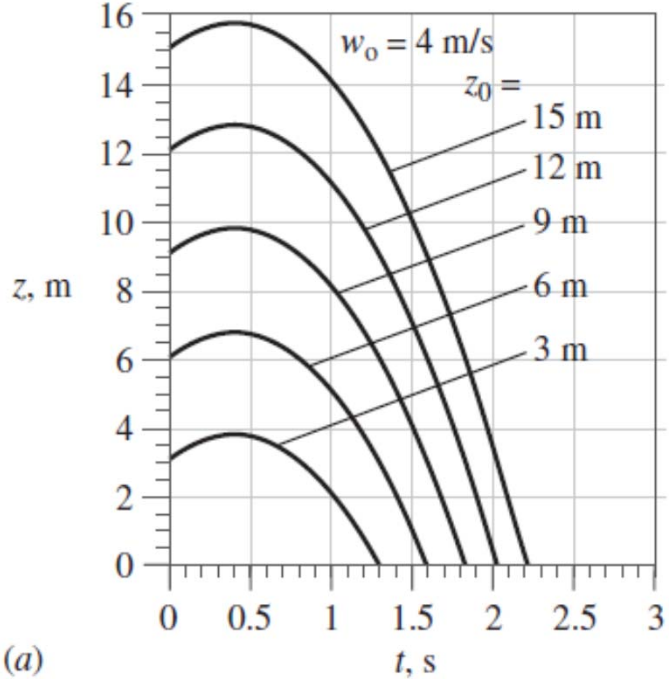
Nondimensionalized variables: $$z^* = \frac{z}{z_0} \qquad t^* = \frac{w_0 t}{z_0}$$

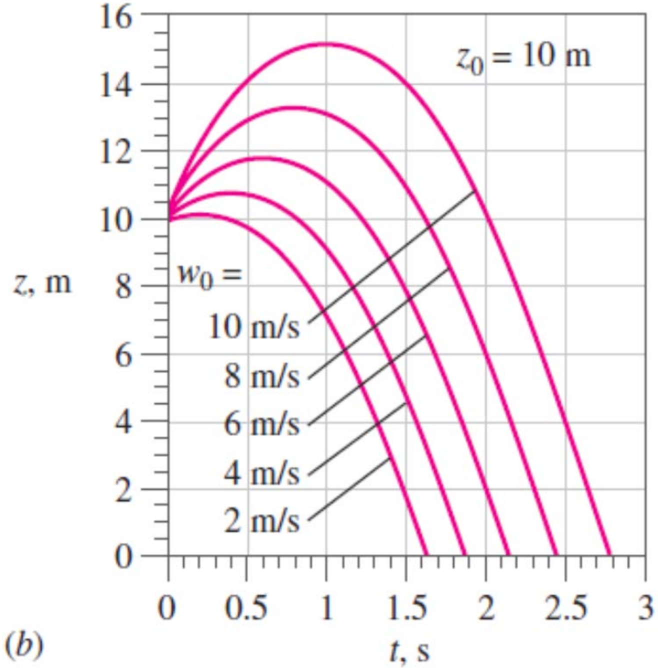
$$\frac{d^2 z}{dt^2} = \frac{d^2(z_0 z^*)}{d(z_0 t^*/w_0)^2} = \frac{w_0^2}{z_0}\frac{d^2 z^*}{dt^{*2}} = -g \qquad \rightarrow \qquad \frac{w_0^2}{g z_0}\frac{d^2 z^*}{dt^{*2}} = -1$$

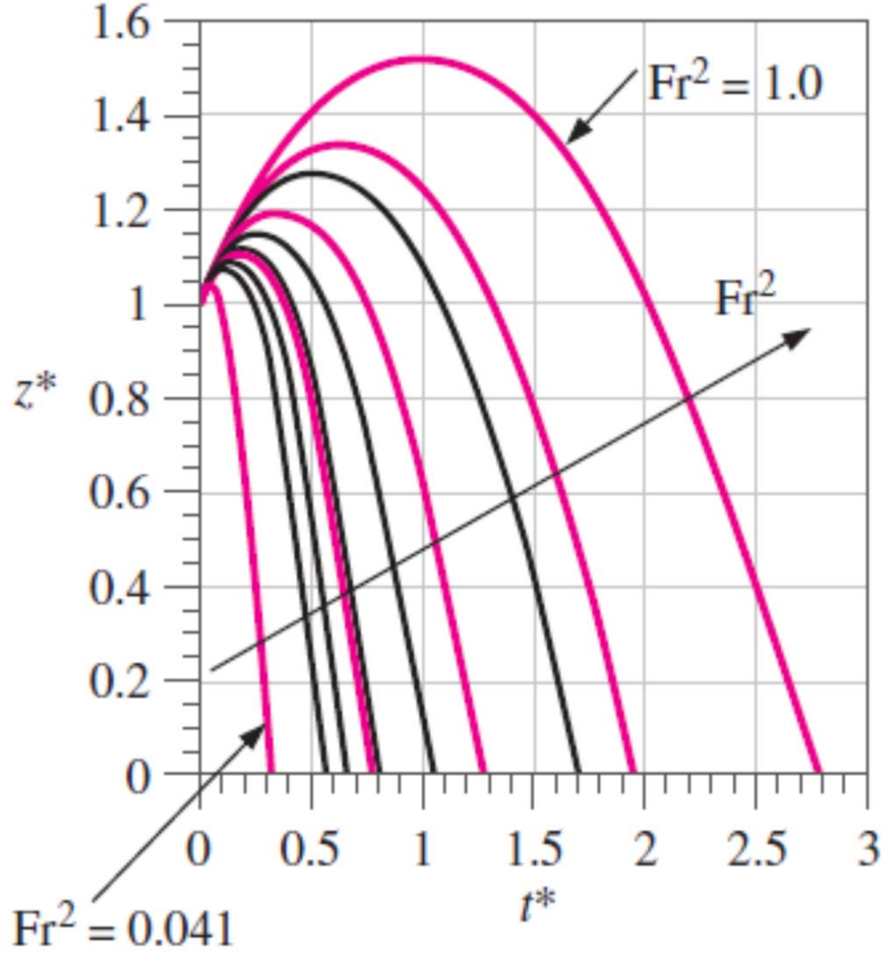
$$\mathrm{Fr} = \frac{w_0}{\sqrt{g z_0}}$$

Nondimensionalized equation of motion: $$\frac{d^2 z^*}{dt^{*2}} = -\frac{1}{\mathrm{Fr}^2}$$

Nondimensional result: $$z^* = 1 + t^* - \frac{1}{2\mathrm{Fr}^2} t^{*2}$$







A complete data set for three parameters with five levels of each parameter would require 5^3=125 experiments. Non-dimensionalization reduces the number of parameters from three to one - a total of only 5^1=5 experiments are required for the same resolution.

Name	Definition	Ratio of Significance
Archimedes number	$\mathrm{Ar} = \frac{\rho_s g L^3}{\mu^2}(\rho_s - \rho)$	$\frac{\text{Gravitational force}}{\text{Viscous force}}$
Aspect ratio	$\mathrm{AR} = \frac{L}{W}$ or $\frac{L}{D}$	$\frac{\text{Length}}{\text{Width}}$ or $\frac{\text{Length}}{\text{Diameter}}$
Biot number	$\mathrm{Bi} = \frac{hL}{k}$	$\frac{\text{Surface thermal resistance}}{\text{Internal thermal resistance}}$
Bond number	$\mathrm{Bo} = \frac{g(\rho_f - \rho_v)L^2}{\sigma_s}$	$\frac{\text{Gravitational force}}{\text{Surface tension force}}$
Cavitation number	$\mathrm{Ca}\ (\text{sometimes } \sigma_c) = \frac{P - P_v}{\rho V^2}$ $\left(\text{sometimes } \frac{2(P - P_v)}{\rho V^2}\right)$	$\frac{\text{Pressure} - \text{Vapor pressure}}{\text{Inertial pressure}}$
Darcy friction factor	$f = \frac{8\tau_w}{\rho V^2}$	$\frac{\text{Wall friction force}}{\text{Inertial force}}$
Drag coefficient	$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$	$\frac{\text{Drag force}}{\text{Dynamic force}}$

Eckert number	$\mathrm{Ec} = \dfrac{V^2}{c_p T}$	$\dfrac{\text{Kinetic energy}}{\text{Enthalpy}}$
Euler number	$\mathrm{Eu} = \dfrac{\Delta P}{\rho V^2} \left(\text{sometimes } \dfrac{\Delta P}{\frac{1}{2}\rho V^2}\right)$	$\dfrac{\text{Pressure difference}}{\text{Dynamic pressure}}$
Fanning friction factor	$C_f = \dfrac{2\tau_w}{\rho V^2}$	$\dfrac{\text{Wall friction force}}{\text{Inertial force}}$
Fourier number	$\mathrm{Fo}\ (\text{sometimes } \tau) = \dfrac{\alpha t}{L^2}$	$\dfrac{\text{Physical time}}{\text{Thermal diffusion time}}$
Froude number	$\mathrm{Fr} = \dfrac{V}{\sqrt{gL}} \left(\text{sometimes } \dfrac{V^2}{gL}\right)$	$\dfrac{\text{Inertial force}}{\text{Gravitational force}}$
Grashof number	$\mathrm{Gr} = \dfrac{g\beta\|\Delta T\|L^3\rho^2}{\mu^2}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$
Jakob number	$\mathrm{Ja} = \dfrac{c_p(T - T_{\mathrm{sat}})}{h_{fg}}$	$\dfrac{\text{Sensible energy}}{\text{Latent energy}}$
Knudsen number	$\mathrm{Kn} = \dfrac{\lambda}{L}$	$\dfrac{\text{Mean free path length}}{\text{Characteristic length}}$
Lewis number	$\mathrm{Le} = \dfrac{k}{\rho c_p D_{AB}} = \dfrac{\alpha}{D_{AB}}$	$\dfrac{\text{Thermal diffusion}}{\text{Species diffusion}}$
Lift coefficient	$C_L = \dfrac{F_L}{\frac{1}{2}\rho V^2 A}$	$\dfrac{\text{Lift force}}{\text{Dynamic force}}$

Name	Definition	Ratio of Significance
Mach number	Ma (sometimes M) $= \dfrac{V}{c}$	$\dfrac{\text{Flow speed}}{\text{Speed of sound}}$
Nusselt number	$\mathrm{Nu} = \dfrac{Lh}{k}$	$\dfrac{\text{Convection heat transfer}}{\text{Conduction heat transfer}}$
Peclet number	$\mathrm{Pe} = \dfrac{\rho L V c_p}{k} = \dfrac{LV}{\alpha}$	$\dfrac{\text{Bulk heat transfer}}{\text{Conduction heat transfer}}$
Power number	$N_P = \dfrac{\dot{W}}{\rho D^5 \omega^3}$	$\dfrac{\text{Power}}{\text{Rotational inertia}}$
Prandtl number	$\mathrm{Pr} = \dfrac{\nu}{\alpha} = \dfrac{\mu c_p}{k}$	$\dfrac{\text{Viscous diffusion}}{\text{Thermal diffusion}}$
Pressure coefficient	$C_p = \dfrac{P - P_\infty}{\frac{1}{2}\rho V^2}$	$\dfrac{\text{Static pressure difference}}{\text{Dynamic pressure}}$
Rayleigh number	$\mathrm{Ra} = \dfrac{g\beta \lvert\Delta T\rvert L^3 \rho^2 c_p}{k\mu}$	$\dfrac{\text{Buoyancy force}}{\text{Viscous force}}$
Reynolds number	$\mathrm{Re} = \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu}$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$
Richardson number	$\mathrm{Ri} = \dfrac{L^5 g\, \Delta\rho}{\rho \dot{V}^2}$	$\dfrac{\text{Buoyancy force}}{\text{Inertial force}}$

Schmidt number	$\mathrm{Sc} = \dfrac{\mu}{\rho D_{AB}} = \dfrac{\nu}{D_{AB}}$	$\dfrac{\text{Viscous diffusion}}{\text{Species diffusion}}$
Sherwood number	$\mathrm{Sh} = \dfrac{VL}{D_{AB}}$	$\dfrac{\text{Overall mass diffusion}}{\text{Species diffusion}}$
Specific heat ratio	$k \text{ (sometimes } \gamma) = \dfrac{c_p}{c_v}$	$\dfrac{\text{Enthalpy}}{\text{Internal energy}}$
Stanton number	$\mathrm{St} = \dfrac{h}{\rho c_p V}$	$\dfrac{\text{Heat transfer}}{\text{Thermal capacity}}$
Stokes number	$\mathrm{Stk} \text{ (sometimes St)} = \dfrac{\rho_p D_p^2 V}{18 \mu L}$	$\dfrac{\text{Particle relaxation time}}{\text{Characteristic flow time}}$
Strouhal number	$\mathrm{St} \text{ (sometimes S or Sr)} = \dfrac{fL}{V}$	$\dfrac{\text{Characteristic flow time}}{\text{Period of oscillation}}$
Weber number	$\mathrm{We} = \dfrac{\rho V^2 L}{\sigma_s}$	$\dfrac{\text{Inertial force}}{\text{Surface tension force}}$

* A is a characteristic area, D is a characteristic diameter, f is a characteristic frequency (Hz), L is a characteristic length, t is a characteristic time, T is a characteristic (absolute) temperature, V is a characteristic velocity, W is a characteristic width, $\dot{W}$ is a characteristic power, ω is a characteristic angular velocity (rad/s). Other parameters and fluid properties in these Π's include: c = speed of sound, c_p, c_v = specific heats, D_p = particle diameter, D_{AB} = species diffusion coefficient, h = convective heat transfer coefficient, h_{fg} = latent heat of evaporation, k = thermal conductivity, P = pressure, T_{sat} = saturation temperature, $\dot{V}$ = volume flow rate, α = thermal diffusivity, β = coefficient of thermal expansion, λ = mean free path length, μ = viscosity, ν = kinematic viscosity, ρ = fluid density, ρ_f = liquid density, ρ_p = particle density, ρ_s = solid density, ρ_v = vapor density, σ_s = surface tension, and τ_w = shear stress along a wall.