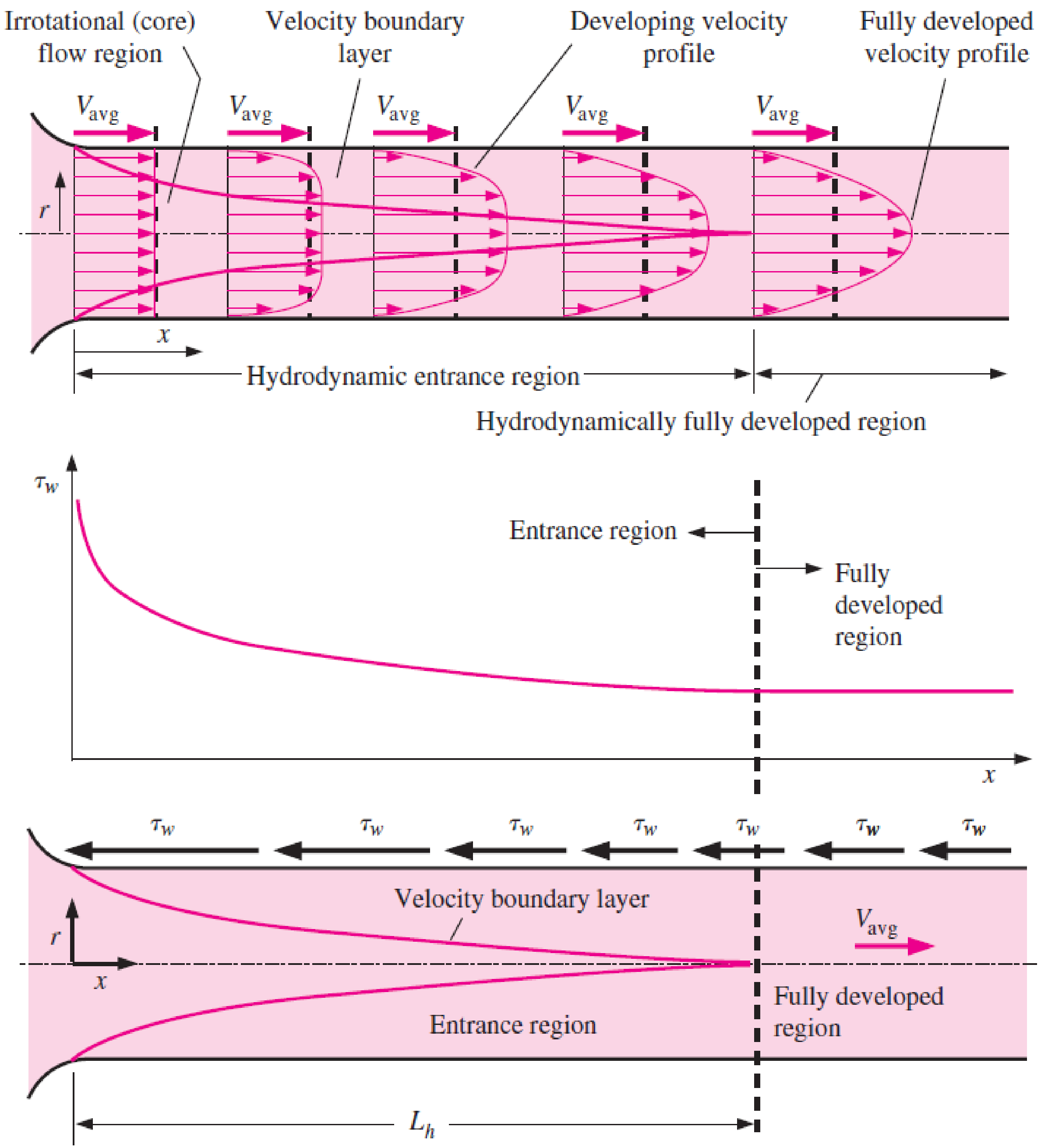
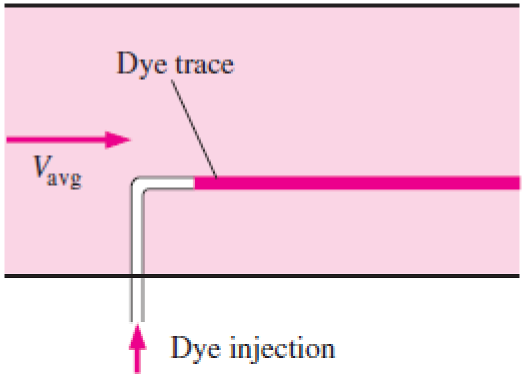
Pipe Flow

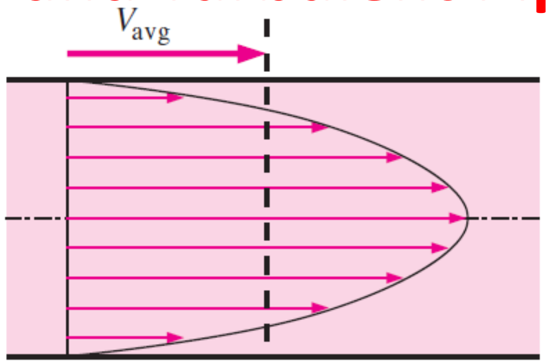


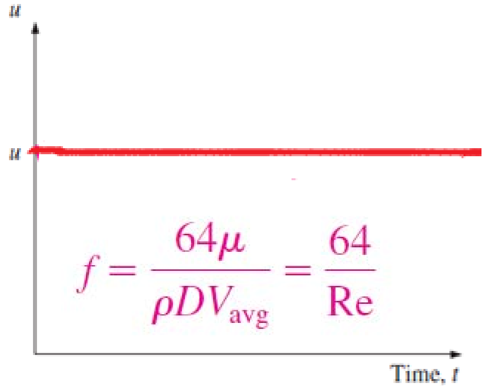
Irrotational (core) flow region
Velocity boundary layer
Developing velocity profile
Fully developed velocity profile
Vavg
r
x
Hydrodynamic entrance region
Hydrodynamically fully developed region
τw
Entrance region
Fully developed region
x
τw
Velocity boundary layer
Vavg
r
x
Entrance region
Fully developed region
Lh

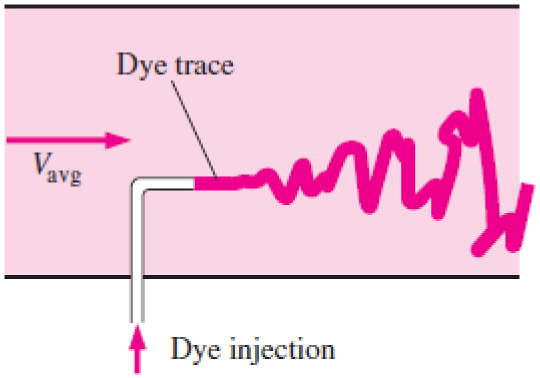
Laminar and Turbulent Pipe Flow



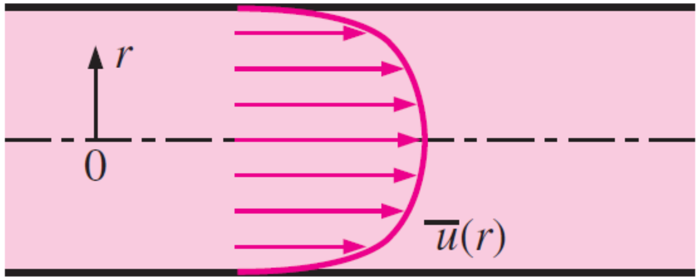
(*a*) Laminar flow

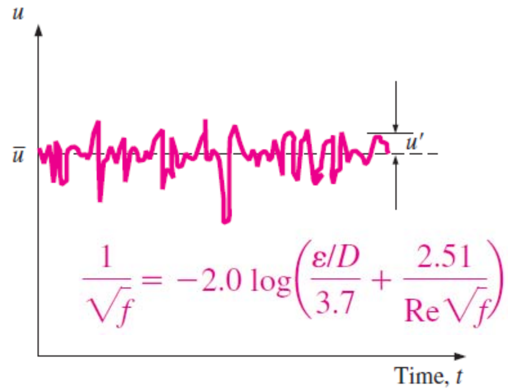




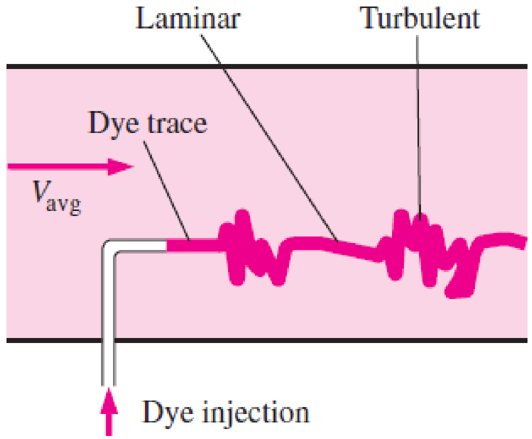


(*b*) Turbulent flow



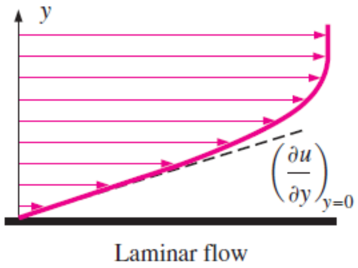


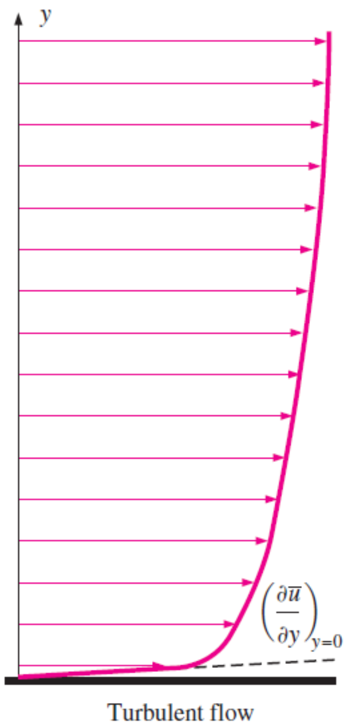
$\text{Re} \lesssim 2300$	laminar flow
$2300 \lesssim \text{Re} \lesssim 4000$	transitional flow
$\text{Re} \gtrsim 4000$	turbulent flow

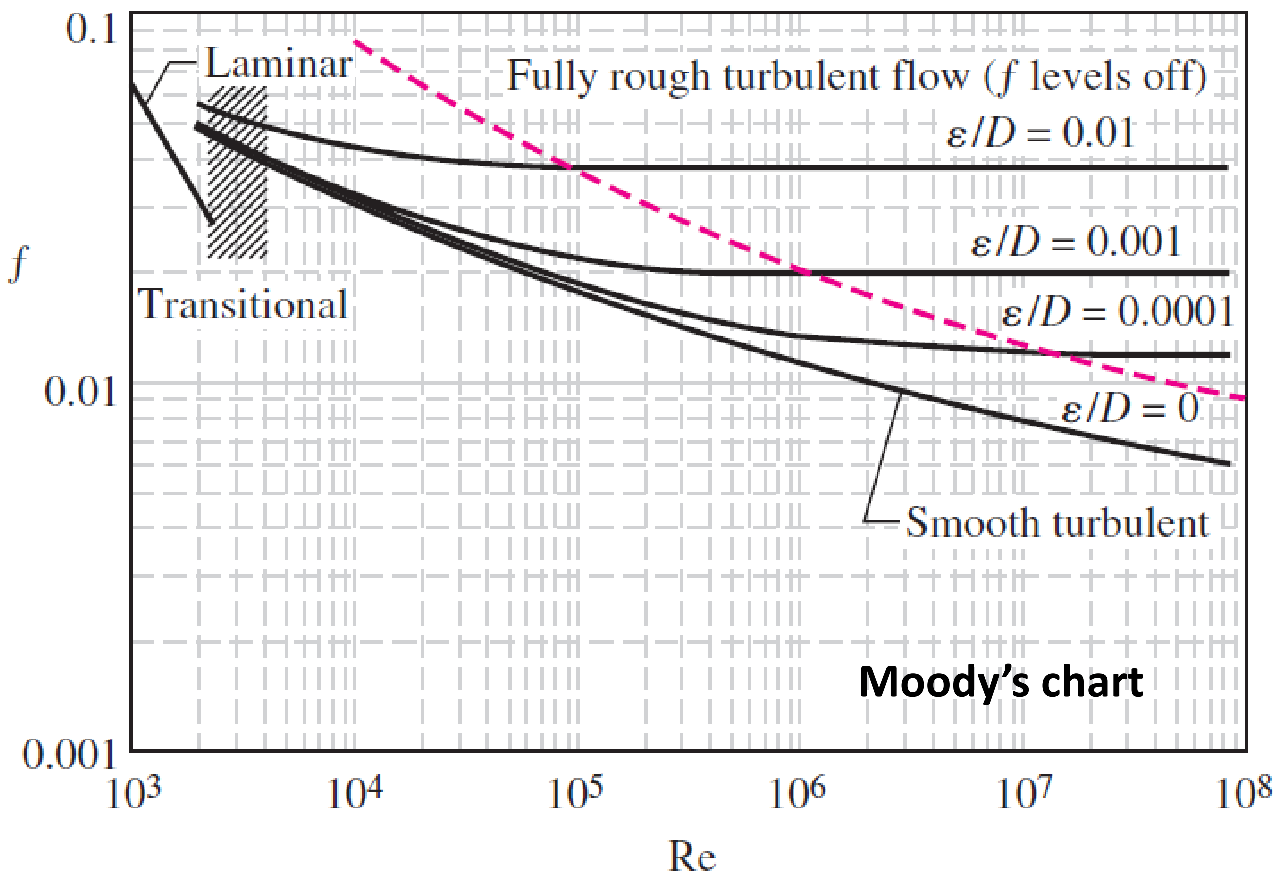


Transitional

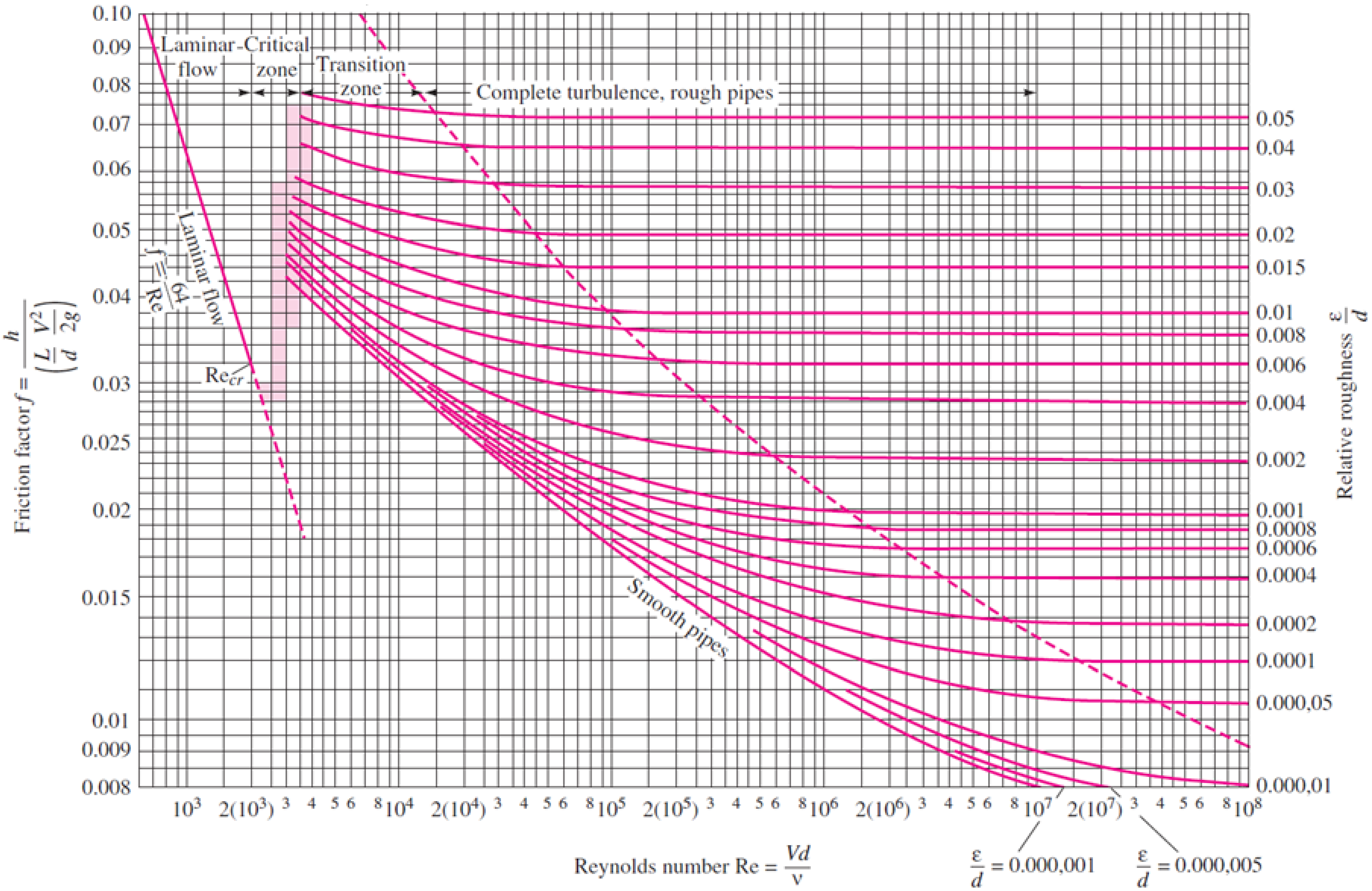
Moody's Chart







At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number.



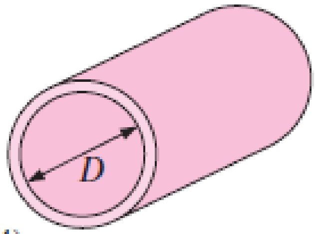
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
Laminar flow $f = \frac{64}{Re}$
Re_{cr}
Smooth pipes
Friction factor $f = \frac{h}{\left(\frac{L}{d}\,\frac{V^2}{2g}\right)}$
Relative roughness $\frac{\varepsilon}{d}$
Reynolds number $Re = \frac{Vd}{\nu}$
$\frac{\varepsilon}{d}$ = 0.000,001
$\frac{\varepsilon}{d}$ = 0.000,005
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000,05
0.000,01
10^3 $2(10^3)$ 3 4 5 6 8 10^4 $2(10^4)$ 3 4 5 6 8 10^5 $2(10^5)$ 3 4 5 6 8 10^6 $2(10^6)$ 3 4 5 6 8 10^7 $2(10^7)$ 3 4 5 6 8 10^8

Flow through Non-Circular Duct

Reynolds number $\mathrm{Re} = \dfrac{\rho V_{av} D_h}{\nu}$ Hydraulic diameter $D_h = \dfrac{4A_c}{P}$

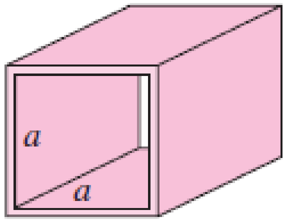
where A_c is the cross-sectional area of the pipe and P is its wetted perimeter.

Circular tube:



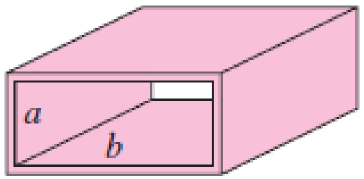
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Channel: ??????

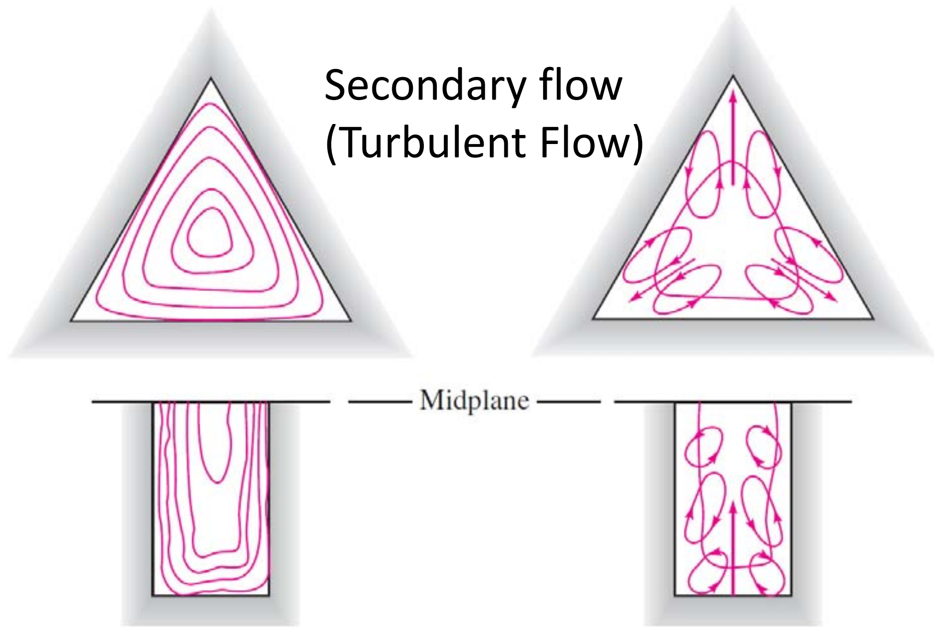
Head loss: **$D = D_h$**

$$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{\text{avg}}^2}{2g}$$

$$f = F\left(\frac{VD_h}{\nu}, \frac{\epsilon}{D_h}\right)$$

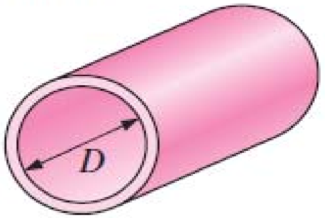
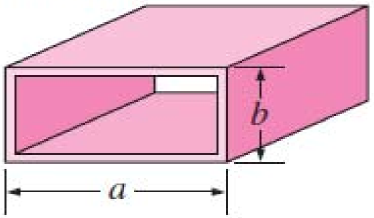
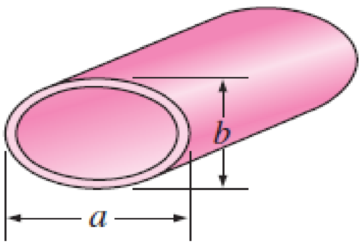
$V = V_{avg}$

Secondary flow
(Turbulent Flow)



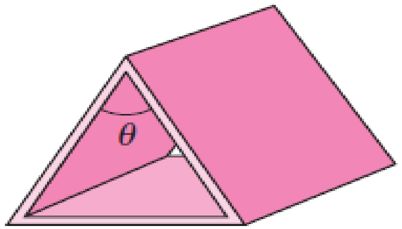
Major Losses

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\mathrm{Re} = V_{avg}\, D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re

Major Losses

Isosceles triangle



θ	
10°	50.80/Re
30°	52.28/Re
60°	53.32/Re
90°	52.60/Re
120°	50.96/Re

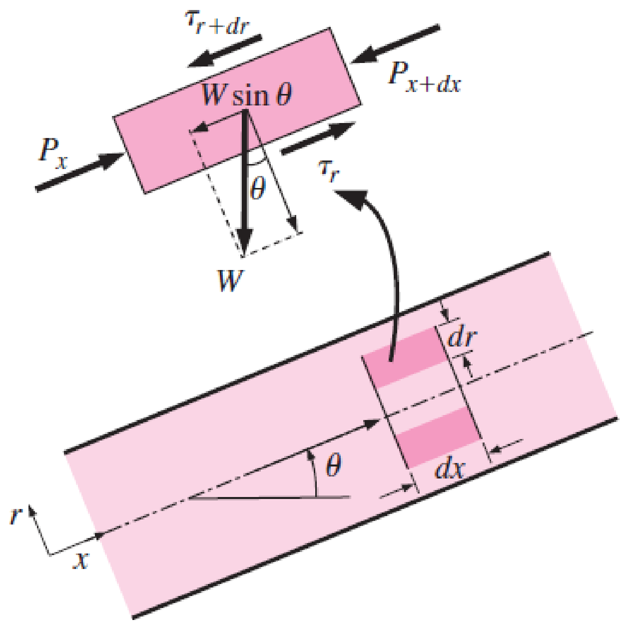
The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe:

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L$$

Inclined Pipes

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin\theta) D^2}{32 \mu L}$$



Head Loss

Head loss: (major) $$h_L = \frac{\Delta P_L}{\rho g} = f \frac{L}{D} \frac{V_{avg}^2}{2g}$$

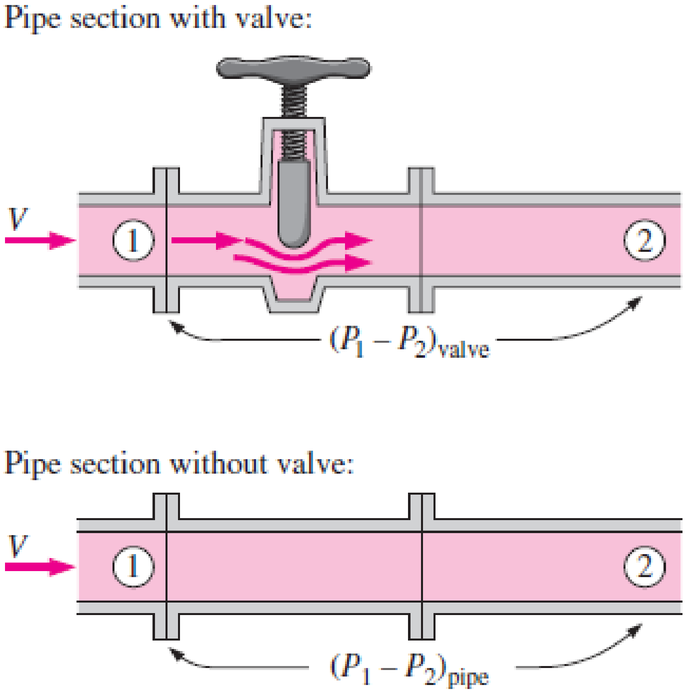
Major losses are due to frictional losses in the pipe

Loss coefficient: $$K_L = \frac{h_L}{V^2/(2g)}$$

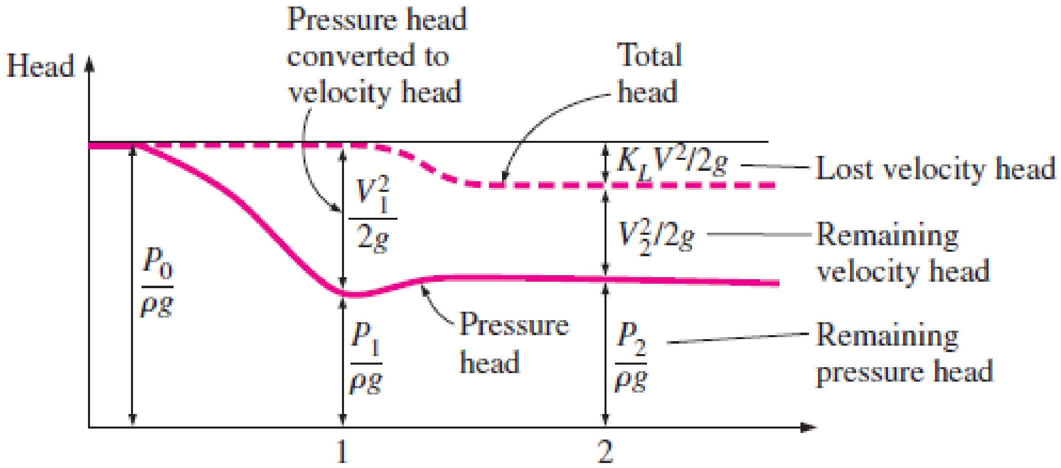
Minor loss: $$h_L = K_L \frac{V^2}{2g}$$

Minor losses are usually expressed in terms of the **loss coefficient** K_L
Minor losses are due to flow irregularities and mixing such as

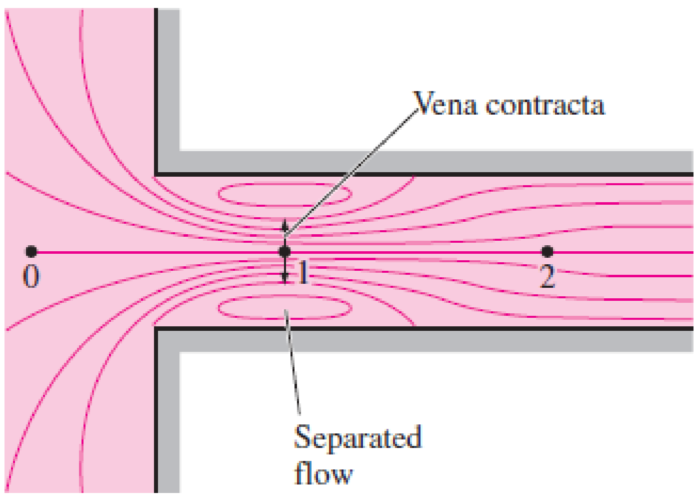
1. Pipe entrance or exit.
2. Sudden expansion or contraction.
3. Bends, elbows, tees, and other fittings.
4. Valves, open or partially closed.
5. Gradual expansions or contractions.



Pipe section with valve:
V
1
2
$(P_1 - P_2)_{valve}$
Pipe section without valve:
V
1
2
$(P_1 - P_2)_{pipe}$



Head
Pressure head converted to velocity head
Total head
$K_L V^2/2g$
Lost velocity head
$\frac{V_1^2}{2g}$
$V_2^2/2g$
Remaining velocity head
$\frac{P_0}{\rho g}$
$\frac{P_1}{\rho g}$
Pressure head
$\frac{P_2}{\rho g}$
Remaining pressure head
1
2



Vena contracta
0
1
2
Separated flow

Total Head Loss

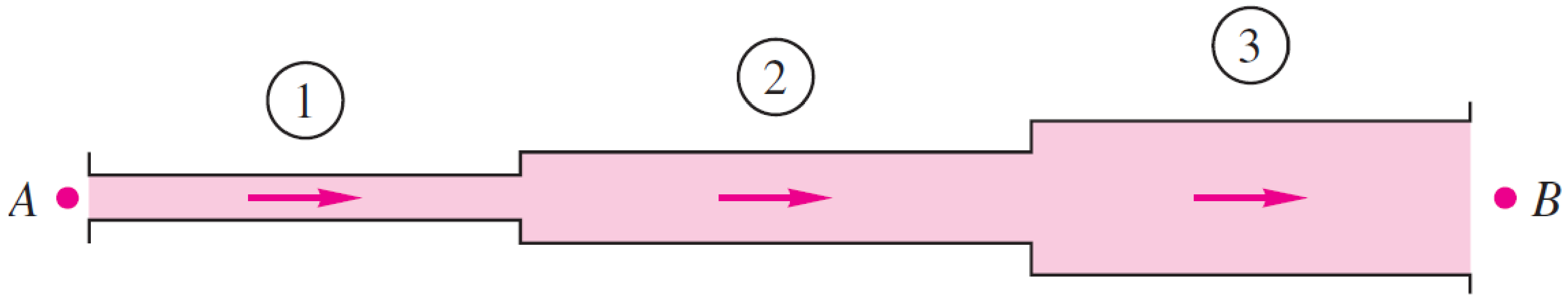
Total head loss (general):

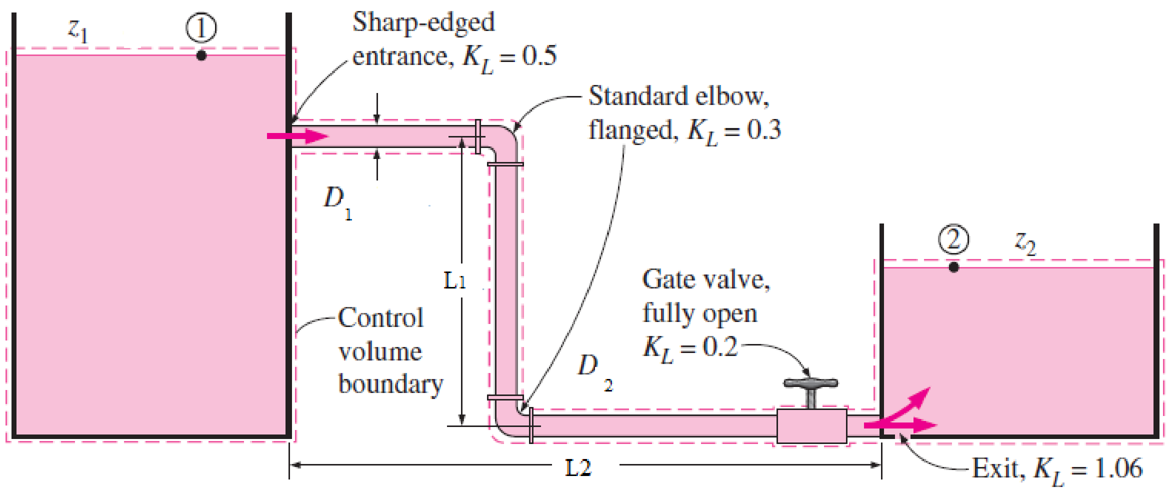
$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}}$$

$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,j}\frac{V_j^2}{2g}$$

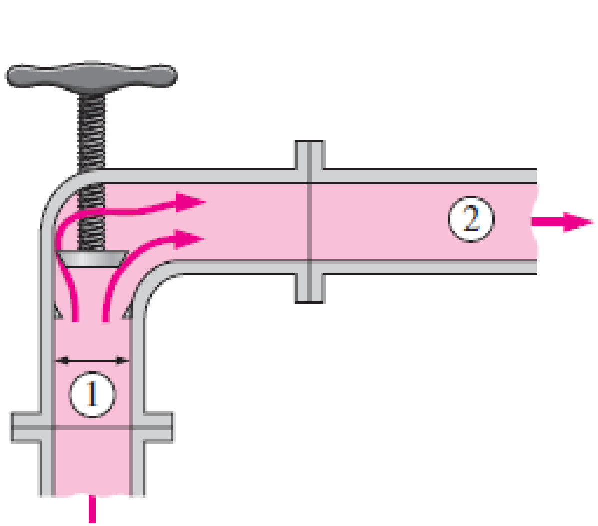
Total head loss (D = constant):

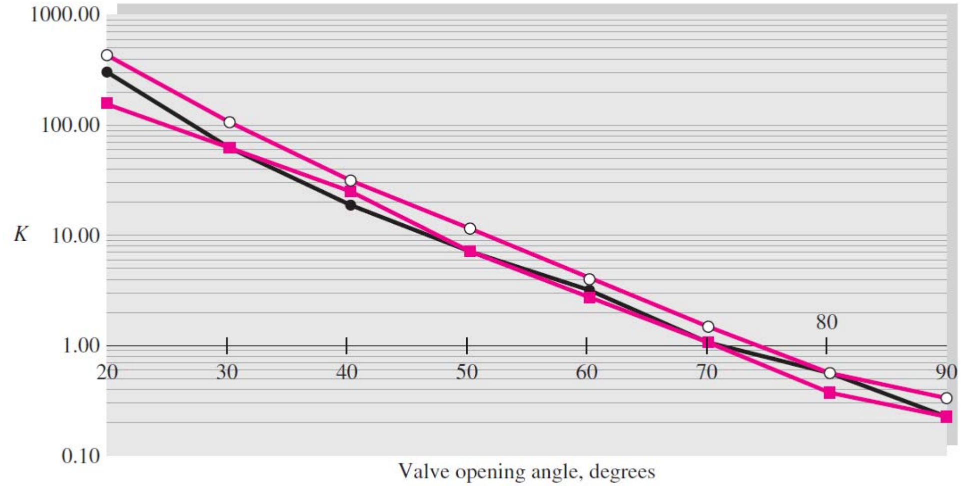
$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

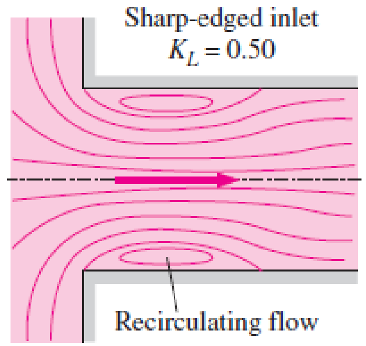




Losses due to Valves and Expansion/Contraction of Flow

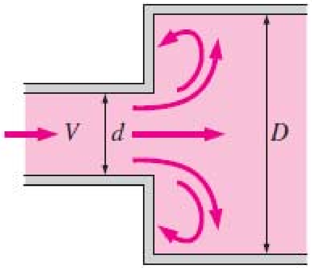


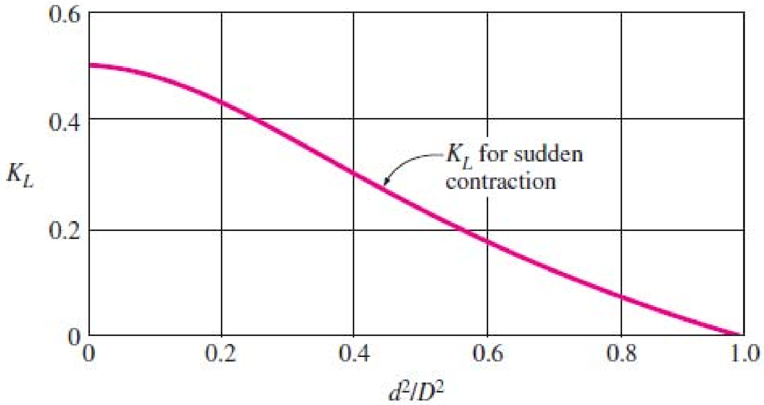




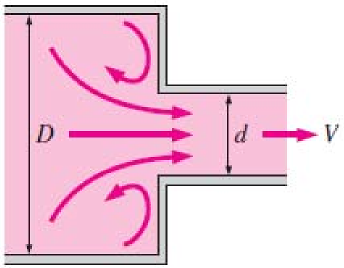
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

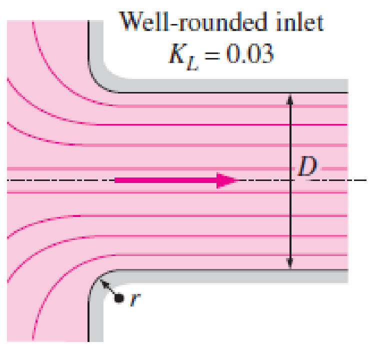
Sudden expansion: $K_L = \left(1 - \frac{d^2}{D^2}\right)^2$





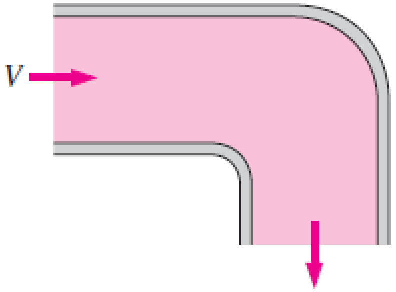
Sudden contraction: See chart.



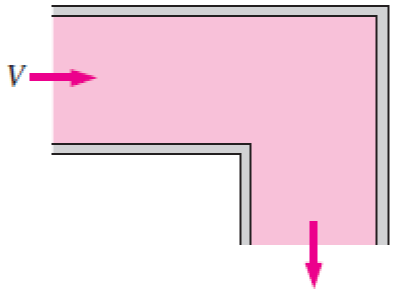


Bends and Branches

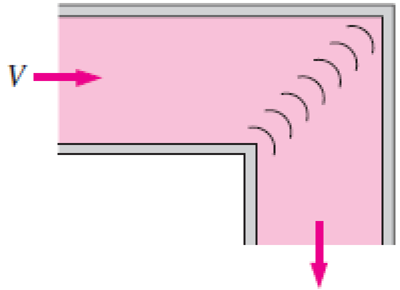
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



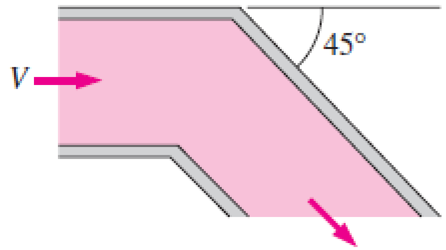
90° miter bend
(without vanes): $K_L = 1.1$



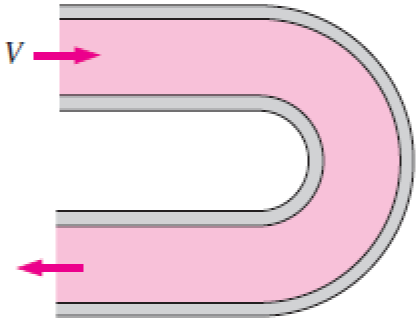
90° miter bend
(with vanes): $K_L = 0.2$



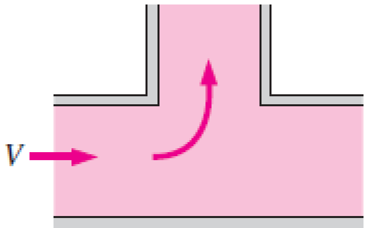
45° threaded elbow:
$K_L = 0.4$



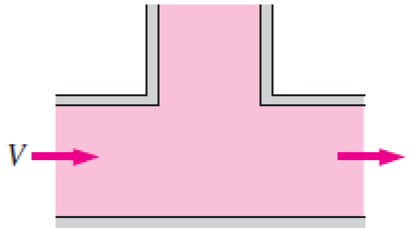
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$