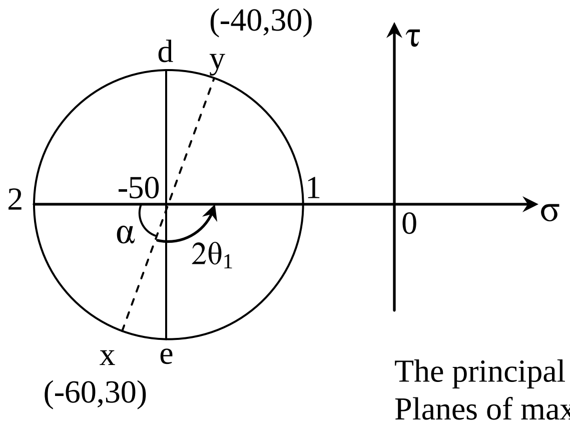
ESO 202A/204: MECHANICS OF SOLIDS (2016-17 II Semester)
Assignment No. 3 Answer Sheet

3.1



radius = $\sqrt{30^2 + 10^2} = 31.6\,\text{MPa}$

$\sigma_d = \sigma_e = -50$ MPa

$\sigma_1 = -50 + 31.6 = -18.4$ MPa

$\sigma_2 = -50 - 31.6 = -81.6$ MPa

$\tan\alpha = {}^{30}\!/_{10} \Rightarrow \alpha = 71°33'$

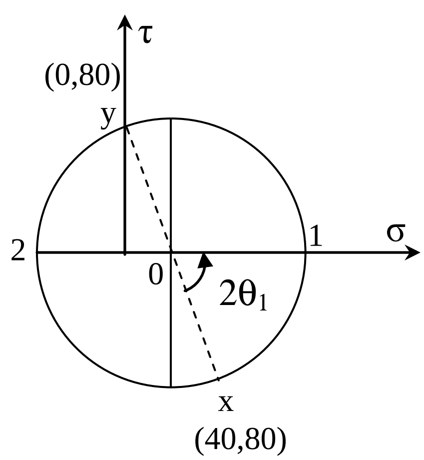
$2\theta_1 = 108°27' \Rightarrow \theta_1 = 54°13.5'$

Maximum shearing stress = 31.6 MPa

The principal planes are $54°13.5'$ and $144°13.5'$

Planes of maximum shear are $9°13.5'$ and $99°13.5'$

3.2



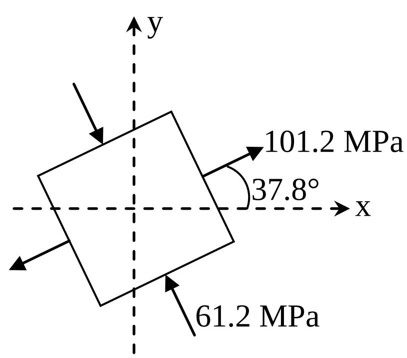
radius = $\sqrt{20^2 + 80^2} = 81.2$ MPa

$\sigma_1 = 20 + 81.2 = 101.2$ MPa

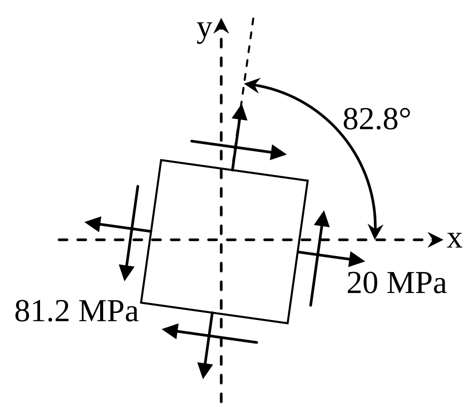
$\sigma_2 = 20 - 81.2 = -61.2$ MPa

$\tau_{max} = 81.2$ MPa

$\tan 2\theta_1 = {}^{80}\!/_{20} = 4 \Rightarrow 2\theta_1 = 75.6° \Rightarrow \theta_1 = 37.8°$

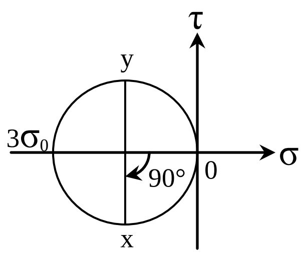


Element oriented along Principal axes



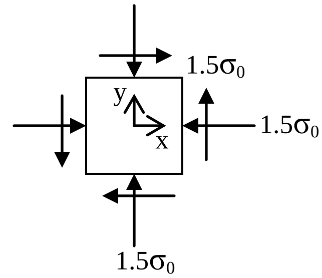
Element oriented along maximum shear axes

3.3 First resolve the second part along x-y directions and superimpose on the first part.

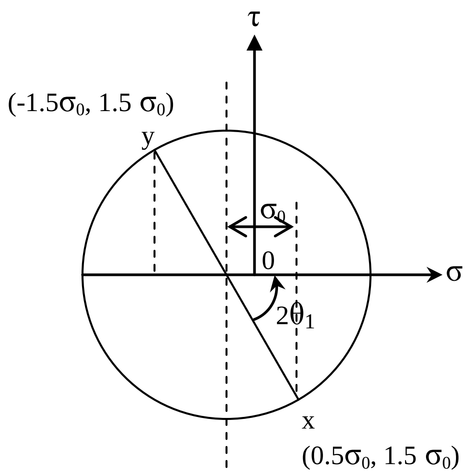


$\sigma_{xx} = -1.5\sigma_0$
$\sigma_{yy} = -1.5\ \sigma_0$
$\tau_{xy} = 1.5\sigma_0$

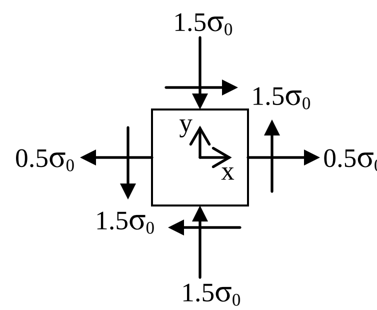
$\Rightarrow$



$\Downarrow$



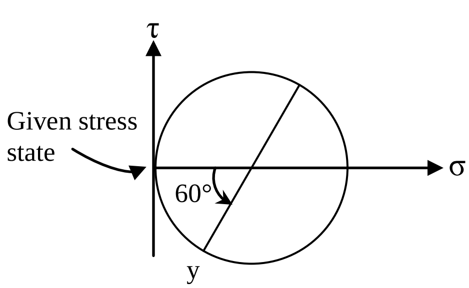
$\Leftarrow$



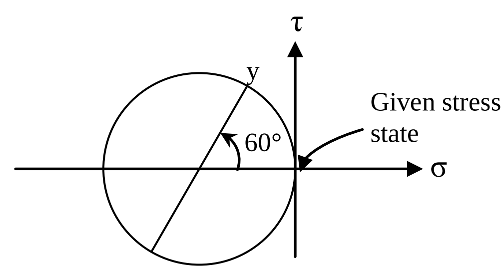
Principal Direction:

$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{1.5\sigma_0}{\sigma_0}\right)$$
$$= 28.15°$$

3.4 Maximum shear stress = radius = 500 MPa

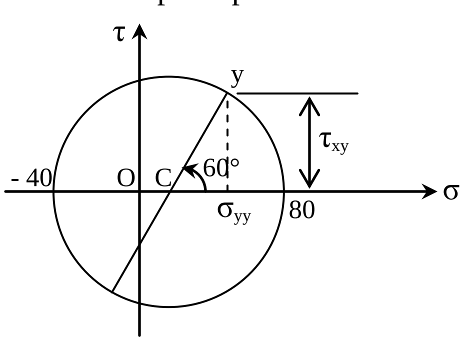


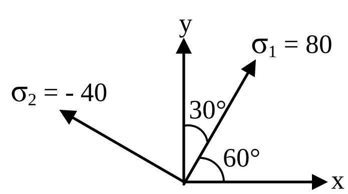
OR



$\sigma_{yy} = \pm\ 500\ \cos60° = \pm\ 250$ MPa
$\tau_{xy} = \pm\ 500\ \sin60° = \pm\ 433$ MPa

3.5 Since the two given planes are perpendicular and there are no shear stresses on these planes, the given stresses are the principal stresses.

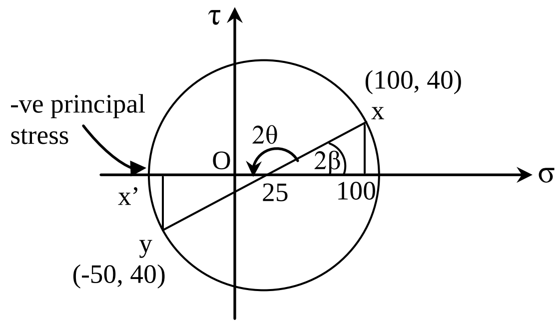




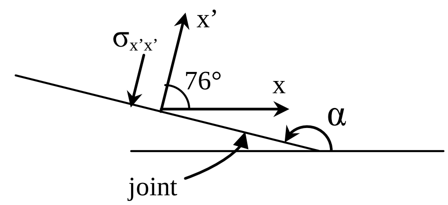
$\sigma_{yy} = 20 + 30 = 50$ MPa
$\tau_{yy} = 60\ (\sqrt{3}/2) = 30\sqrt{3}$ MPa

Maximum Shear Stress = 60 MPa

3.6 Since the joint cannot take any shear or tension, the inclination α, (2α in Mohr's circle) will be such that the principal stress perpendicular to the joint is negative.



$2\beta = \tan^{-1}(40/75) = 28°$
$2\theta = 180° - 28° = 152°$
$\theta = 76°$



Obviously, $\alpha = 90 + 76 = 166°$
$\sigma_{yy} = -50$ MPa

3.7 The centre of Mohr's circle C should be on σ-axis such that the angle between Cn_1 and Cn_2 should be 240^0 in CW sense.
Draw the Mohr's circle.

$$OD = 2\sigma_0$$
$$Dn_1 = \sqrt{3}\,\sigma_0$$
$$\tan 60^0 = \sqrt{3} = \frac{Dn_1}{CD}$$

or,
$$CD = \frac{\sqrt{3}\sigma_0}{\sqrt{3}}$$

$\therefore$
$$OC = OD + CD = 3\sigma_0$$

Also,
$$Cn_1 = [\sigma_0^2 + (\sqrt{3}\sigma_0)^2]^{\frac{1}{2}}$$
$$= 2\sigma_0$$

$\therefore$ Principal stresses are,

$$\sigma_1 = OC + \text{radius}(Cn_1)$$
$$= 3\sigma_0 + 2\sigma_0 = 5\sigma_0 \quad \text{[Ans]}$$
$$\sigma_2 = 3\sigma_0 - 2\sigma_0 = \sigma_0$$