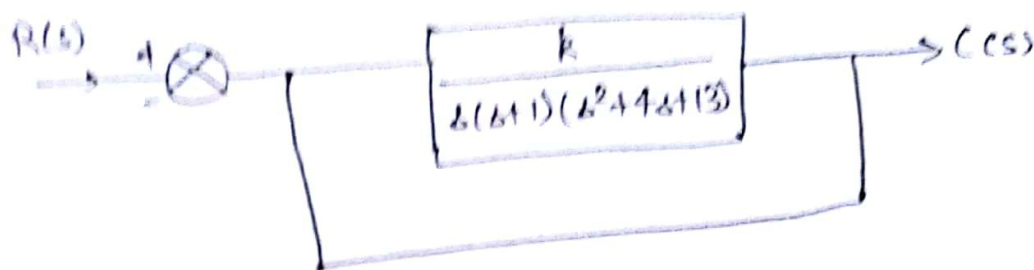Find / Draw the root locus for the following :-

$$G_{e}(s) = \frac{k}{s(s+1)(s^2+4s+13)+k}$$

① Number of branches = # of closed loop poles.

Open loop, Poles are

$$s = 0, -1, \quad \frac{-4 \pm \sqrt{16-52}}{2}$$

$$s = 0, -1, -2 \pm 3i$$

∴ # of branches = 4

② It has to be symmetric about $\sigma$-axis.

③ Real axis intercept.

$$\sigma_a = \frac{\Sigma \text{ finite poles} - \Sigma \text{ finite zeros}}{\# \text{ finite poles} - \# \text{ finite zeros}}$$

$$= \frac{0 + (-1) + (-2+3i) + (-2-3i) - 0}{4 - 0}$$

$$= \frac{-5}{4} = -1.25$$

Angle $\theta_a = \dfrac{(2k+1)\pi}{\# \text{ finite poles} - \# \text{ finite zeros}} = \dfrac{(2k+1)\pi}{4}$   where $k = 0, 1, 2, 3 ..$
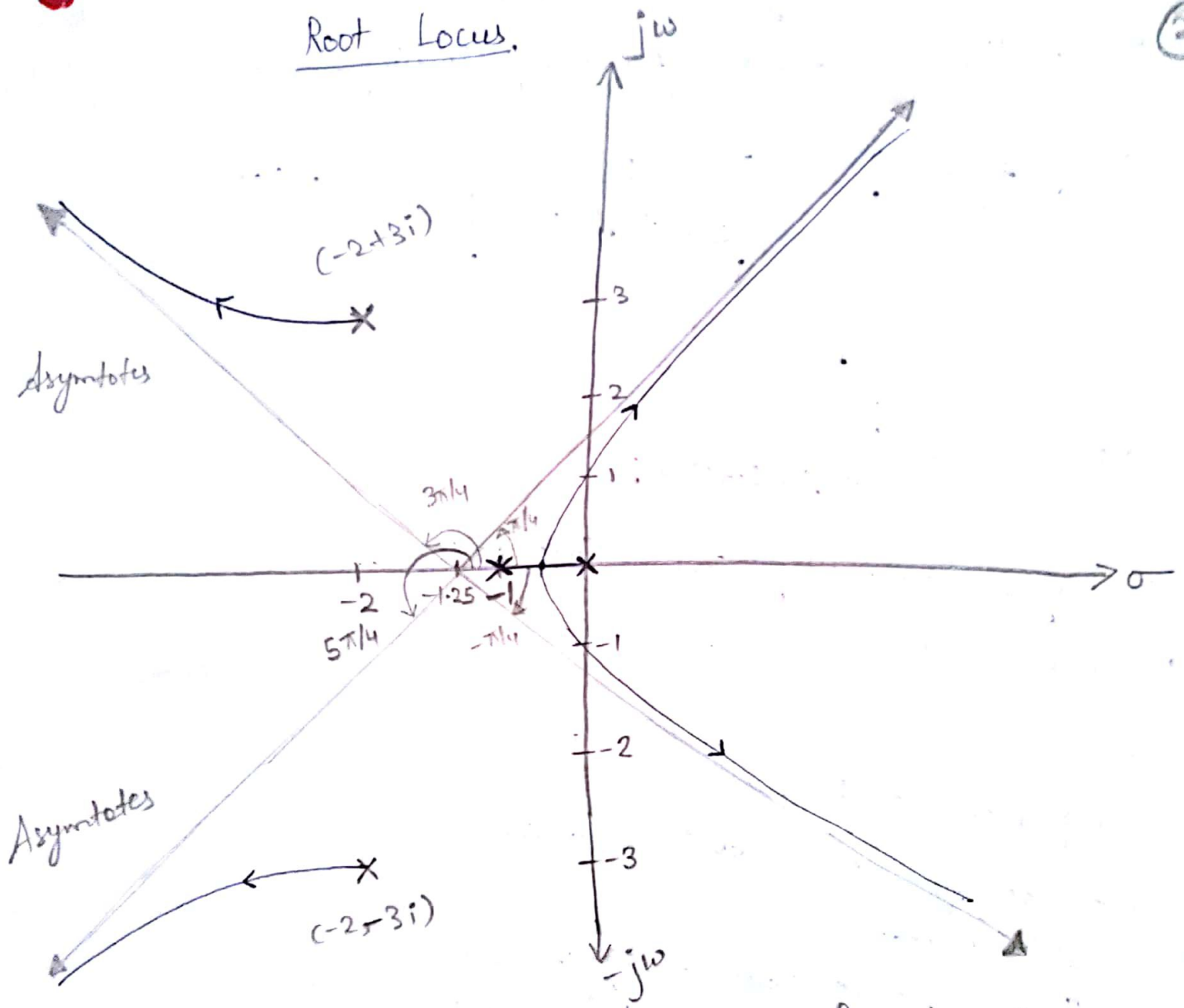
$$= \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

As $s \rightarrow \infty$, func. $\rightarrow 0 \Rightarrow \exists$ a zero at $\infty$

This function also has zeros at $\infty$. ( # = 4 )

# Root Locus.



Asymptotes

$(-2+3i)$

$3\pi/4$

$\pi/4$

$5\pi/4$

$-2$ $-1.25$ $-1$

$-\pi/4$

Asymptotes

$(-2-3i)$

four branches.

Root locus symmetric about $\sigma$-axis.

$j\omega$

$-j\omega$

$\sigma$