Homework-12 Solutions

Q.12-49

Analysis: The definition for the isothermal compressibility is

$$\alpha = -\frac{1}{\boldsymbol{v}}\left(\frac{\partial \boldsymbol{v}}{\partial P}\right)_T$$

The derivative is

$$\left(\frac{\partial P}{\partial \boldsymbol{v}}\right)_T = -\frac{RT}{(\boldsymbol{v}-b)^2} + \frac{a}{T^{1/2}}\frac{2\boldsymbol{v}+b}{\boldsymbol{v}^2(\boldsymbol{v}+b)^2}$$

Substituting,

$$\alpha = -\frac{1}{\boldsymbol{v}}\left(\frac{\partial \boldsymbol{v}}{\partial P}\right)_T = -\frac{1}{\boldsymbol{v}}\left(\frac{1}{-\frac{RT}{(\boldsymbol{v}-b)^2} + \frac{a}{T^{1/2}}\frac{2\boldsymbol{v}+b}{\boldsymbol{v}^2(\boldsymbol{v}+b)^2}}\right) = -\frac{1}{-\frac{RT\boldsymbol{v}}{(\boldsymbol{v}-b)^2} + \frac{a}{T^{1/2}}\frac{2\boldsymbol{v}+b}{(\boldsymbol{v}+b)^2}}$$

Q.12-66

***Analysis*:** From Eq. 12-52 of the text,

$$c_p = \frac{1}{\mu}\left[T\left(\frac{\partial \boldsymbol{v}}{\partial T}\right)_P - \boldsymbol{v}\right]$$

Expanding the partial derivative of $\boldsymbol{v}/T$ produces

$$\left(\frac{\partial \boldsymbol{v}/T}{\partial T}\right)_P = \frac{1}{T}\left(\frac{\partial \boldsymbol{v}}{\partial T}\right)_P - \frac{\boldsymbol{v}}{T^2}$$

When this is multiplied by T^2, the right-hand side becomes the same as the bracketed quantity above. Then,

$$\mu = \frac{T^2}{c_p}\left(\frac{\partial(\boldsymbol{v}/T)}{\partial T}\right)_P$$