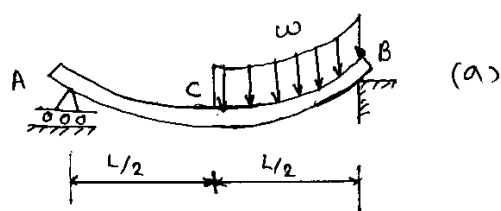


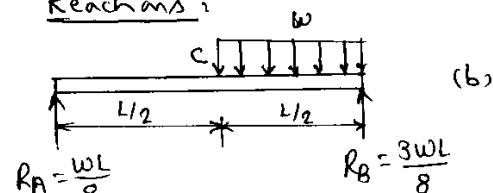
## Solution to H/W and Practice

### Problems for Chapter 8

#### Solution to problem 8.2



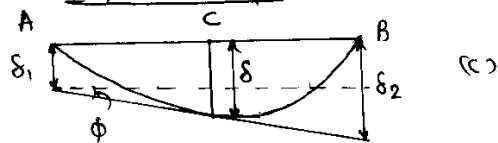
Reactions:



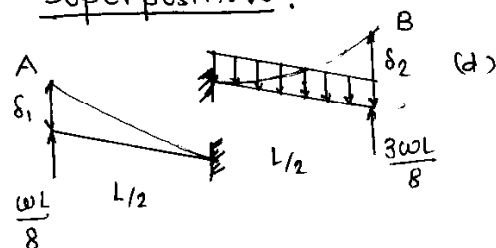
$$R_A = \frac{wL}{8}$$

$$R_B = \frac{3wL}{8}$$

Deflected shape:



Superposition:



- Let  $\delta_1$  and  $\delta_2$  be the distances of the end-points A and B from the tangent at the midpoint C (Fig. (c)). Then, they can be found as the deflections of the 2 cantilever beams shown in Fig. (d):

$$\delta_1 = \frac{wL/8 (L/2)^3}{3EI} = \frac{wL^4}{192EI}$$

$$\begin{aligned} \delta_2 &= \frac{3wL/8 (L/2)^3}{3EI} - \frac{w(L/2)^4}{8EI} \\ &= \frac{wL^4}{64EI} - \frac{wL^4}{128EI} = \frac{wL^4}{128EI} \end{aligned}$$

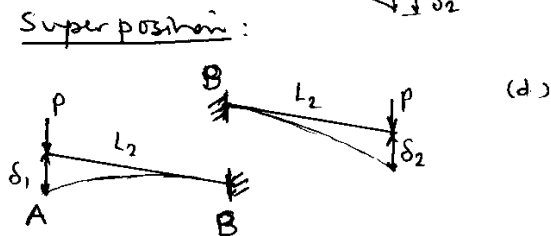
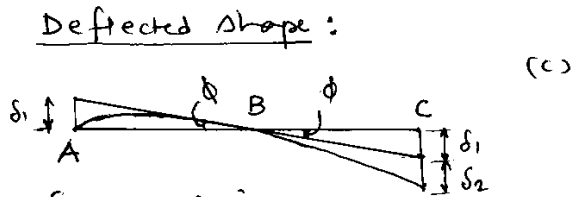
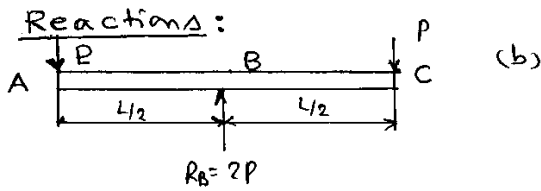
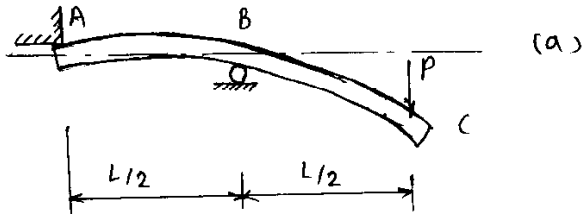
- Then, the slope of the tangent at C is:

$$\begin{aligned} \phi &= \frac{\delta_2 - \delta_1}{L} \\ &= \frac{1}{L} \left[ \frac{wL^4}{128EI} - \frac{wL^4}{192EI} \right] = \frac{wL^3}{384EI} \end{aligned}$$

- Finally, the deflection of midpoint C is:

$$\begin{aligned} \text{Central deflection: } \delta &= \delta_1 + \phi \frac{L}{2} \\ &= \frac{wL^4}{192EI} + \frac{wL^3}{384EI} \cdot \frac{L}{2} \\ &= \frac{5wL^4}{768EI} \end{aligned}$$

# Solution to problem 8.3



• Let  $\delta_1$  be the distance of point A from the tangent at point B. Further, let  $\delta_2$  be the distance of the deflected position of point C from the tangent at point B. (Fig. (c)). Then,  $\delta_1$  and  $\delta_2$  can be found as the deflections of the 2 cantilevers shown in Fig. (d).

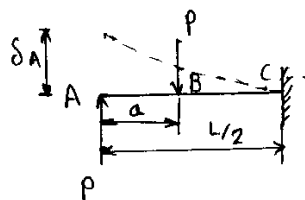
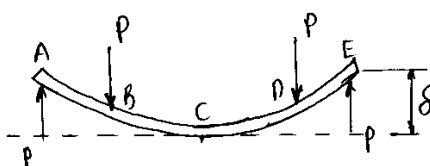
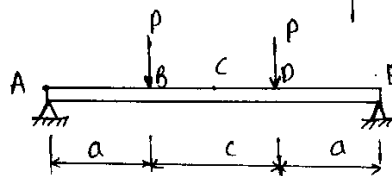
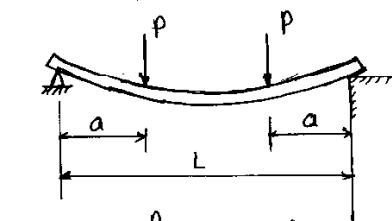
• Actual deflection at C =  $\delta_1 + \delta_2$

$$= 2\delta_2$$

$$= \frac{2P(L/2)^3}{3EI}$$

$$= \frac{PL^3}{12EI}$$

# Solution to problem 8.4



To find: central deflection.

- note:
- due to symmetry reactions at A and E are P each.
  - The slope at C is zero.
  - Therefore, the deflected shape is symmetric about point C.

\* consider ~~either~~ the beam ABC.

- Since the slope at C is zero, the point C can be considered fixed.

• Using superposition, deflection at A is:

$$(\delta_A)_{\uparrow P} = \frac{P(L/2)^3}{3EI} = \frac{PL^3}{24EI} \text{ upward.}$$

$$(\delta_A)_{\downarrow P} = \frac{P(L/2 - a)^2 \left[ 3\frac{L}{2} - (L/2 - a) \right]}{6EI} \text{ downward}$$

$$= \frac{P(L^3 - 3L^2a + 4a^3)}{24EI}$$

$$\text{Net upward displacement} = \frac{PL^3}{24EI} - \frac{P(L^3 - 3L^2a + 4a^3)}{24EI}$$

$$= \frac{Pa}{24EI} (3L^2 - 4a^2).$$

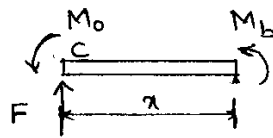
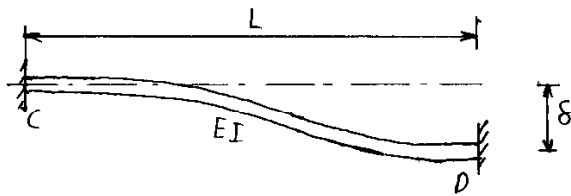
\* Note:  $\delta$  (central deflection of original beam)

$$= \delta_A$$

$$= \frac{Pa(3L^2 - 4a^2)}{24EI}$$

④

### Solution to problem 8.8



• Statistically indeterminate problem

• Since, all the beams deform in the same manner, consider one of the beams, labelled 1.

• Reactions:

Let the reaction at D consist of a force F and moment Mb.

• Bending Moment:

$$EI \frac{d^2v}{dx^2} = M_b = F \cdot x - M_o.$$

• Integrating,  $EI \frac{dv}{dx} = \frac{Fx^2}{2} - M_o x + C_1$

$$EI v = \frac{Fx^3}{6} - \frac{M_o x^2}{2} + C_1 x + C_2.$$

• Boundary conditions: at  $x=0$ ,  $v=0$   $\frac{dv}{dx} = 0$ .

at  $x=L$ ,  $v=\delta$   $\frac{dv}{dx} = 0$ . (Compatibility Conditions)

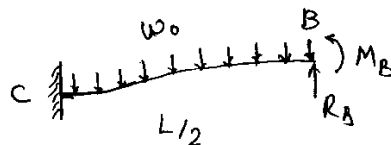
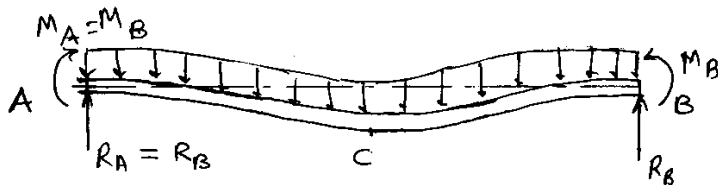
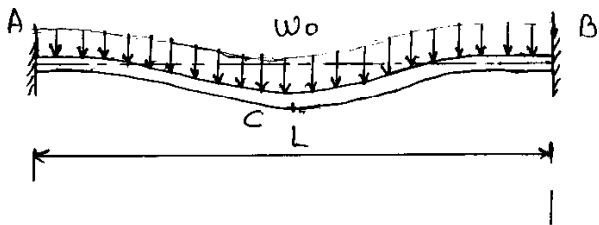
These B.C.s give  $C_1 = C_2 = 0$ .  $F = \frac{12EI\delta}{L^3}$ ,  $M_o = \frac{6EI\delta}{L^2}$ .

• Max. BM:

$$M_b = \frac{12EI\delta}{L^3} x - \frac{6EI\delta}{L^2}$$

$$\therefore |M_b|_{\max} \text{ at } x=0, x=L = \frac{6EI\delta}{L^2}.$$

# Solution to problem 8.11



\* Superposition:

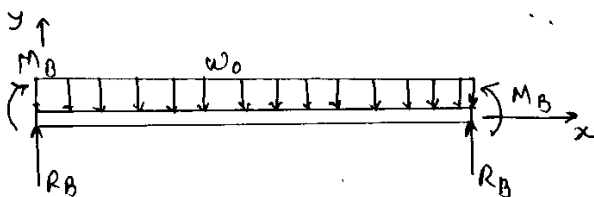
$$\phi_B = \frac{w_0(L/2)^3}{6EI} - \frac{R_B(L/2)^2}{2EI} - \frac{M_B(L/2)}{EI}$$

$$= \frac{w_0 L^3}{48EI} - \frac{R_B L^2}{8EI} - \frac{M_B L}{2EI}$$

$$= \frac{L}{48EI} [w_0 L^2 - 6R_B L - 24M_B]$$

\* Compatibility:  $\phi_B = 0$

$$\therefore w_0 L^2 - 6R_B L - 24M_B = 0 \quad \text{--- ①}$$



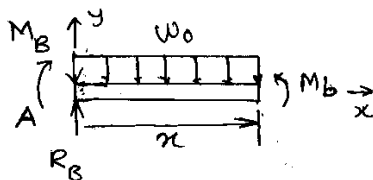
\* Equilibrium of the whole beam

$$R_B = \frac{w_0 L}{2} \quad \text{--- ②}$$

$$\text{① and ②} \Rightarrow w_0 L^2 - 6\left(\frac{w_0 L}{2}\right)L - 24M_B = 0$$

$$\Rightarrow M_B = -\frac{w_0 L^2}{12}$$

\* To find maximum Bending moment:

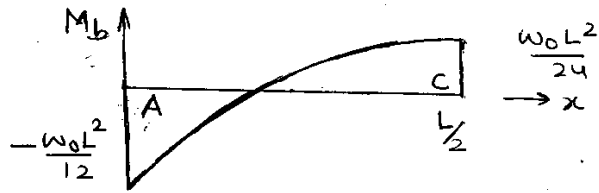


Bending moment at x:

$$M_b = R_B \cdot x + M_B - w_0 \frac{x^2}{2}$$

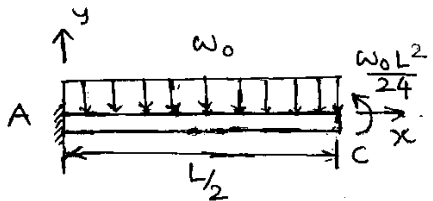
$$= \frac{w_0 L}{2} x - \frac{w_0 L^2}{12} - w_0 \frac{x^2}{2}$$

(problem 8.11 contd.)

B.M.D.

From the BM Diagram,

$$|(M_b)_{\max}| = |M_b|_{x=0} = \frac{w_0 L^2}{12}$$

\* To find maximum deflection.

Shear force  
= 0 at C  
due to  
symmetry

Superposition:

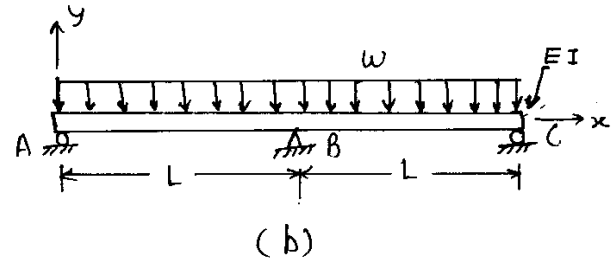
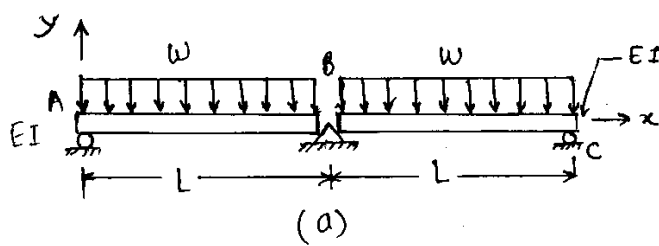
$$\delta_{\max} = \delta|_C = \delta|_C \text{ due to } w_0$$

$$+ \delta|_C \text{ due to } M_0 = \frac{w_0 L^2}{24}$$

$$= \frac{w_0 (L/2)^4}{8EI} - \frac{(w_0 L^2 / 24) L}{2EI}$$

$$= \frac{w_0 L^4}{384EI} \quad (\text{downward})$$

# Solution to problem 8.16

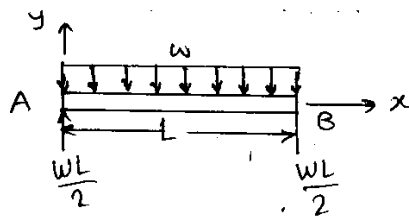


Statically indeterminate problem

To compare: i) Reactions at central support  
ii) Max  $M_b$ .  
iii) Max. deflection

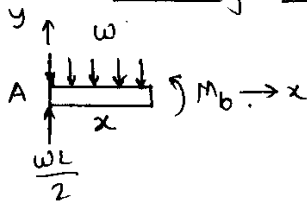
} in the two cases.

a) AB and BC are 2 different beams. Consider beam AB  
(i) Reactions :



$$\therefore \text{Total reaction at centre} = 2 \frac{wL}{2} = wL.$$

(ii) Bending moment :

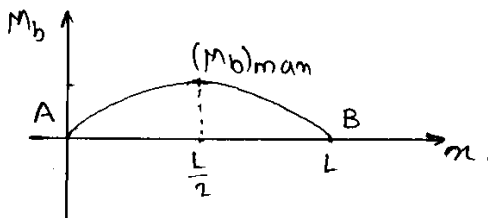


$$M_b = \frac{wL}{2}x - \frac{wx^2}{2}.$$

$$\frac{dM_b}{dx} = 0 \Rightarrow \frac{wL}{2} - \frac{1}{2}w \cdot 2x = 0.$$

$$\Rightarrow x = \frac{L}{2}.$$

$$\begin{aligned} \therefore (M_b)_{\max} &= M_b|_{x=L/2} \\ &= \frac{wL}{2} \left(\frac{L}{2}\right) - \frac{w}{2} \left(\frac{L}{2}\right)^2 \\ &= \frac{wL^2}{8}. \end{aligned}$$



Bending Moment Diagram is same for beam BC.

$$\therefore (M_b)_{\max} = M_b|_{x=L/2} = \frac{wL^2}{8} \text{ for BC}$$

(problem 8.16 contd.)

(iii) Deflection:

$$EI \frac{d^2v}{dn} = M_b = \frac{wL}{2}n - \frac{1}{2}wn^2$$

$$EI \frac{dv}{dn} = \frac{wL}{2} \frac{n^2}{2} - \frac{1}{2}w \frac{n^3}{3} + C_1$$

$$EI v = \frac{wL}{4} \frac{n^3}{3} - \frac{1}{6}wn^4 + C_1n + C_2$$

$$v=0 \text{ at } n=0 \Rightarrow C_2=0$$

$$v=0 \text{ at } n=L \Rightarrow \frac{wL^4}{12} - \frac{1}{6} \frac{wL^4}{4} + C_1L = 0$$

$$\Rightarrow C_1 = -\frac{wL^3}{24}$$

$$\therefore v = \frac{w}{EI} \left[ \frac{Ln^3}{12} - \frac{n^4}{24} - \frac{L^3n}{24} \right]$$

$$= \frac{w}{24EI} [2Ln^3 - n^4 - L^3n]$$

$$\frac{dv}{dn} = 0 \Rightarrow \frac{w}{24EI} [6Ln^2 - 4n^3 - L^3] = 0$$

$$\Rightarrow n = 1.366L, -0.366L, 0.5L$$

In the above solution  $n=0.5$  is only ~~ex~~ feasible solution.

$$\therefore \underline{v_{max}} = v|_{n=\frac{L}{2}} = \frac{w}{24EI} \left[ 2L \left( \frac{L^3}{8} \right) - \frac{L^4}{16} - wL^3 \left( \frac{L}{2} \right) \right]$$

$$= \frac{wL^4}{24EI} \left[ \frac{1}{4} - \frac{1}{16} - \frac{1}{2} \right]$$

$$= -\frac{5wL^4}{384EI}$$

Deflection expression is same for beam BC.

$$\therefore \underline{v_{max}} = v|_{x=\frac{3L}{2}} = -\frac{5wL^4}{384EI} \text{ for BC}$$



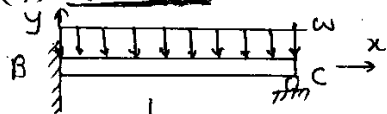
(problem 8.16 contd.)

(b) continuous beam.

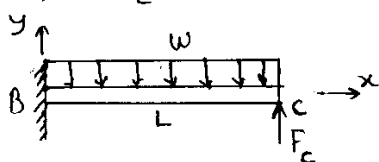
Slope and deflection at B <sup>are</sup> zero.

consider either the part AB or BC. It can be considered as a cantilever beam.

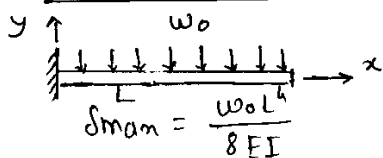
(i) Reaction:



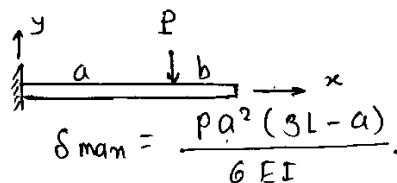
Replace the support at C by a force  $\Rightarrow$



\* Superposition:



$$\delta_{\text{max}} = \frac{w_0 L^4}{8EI}$$



$$\delta_{\text{max}} = \frac{Pa^2(3L-a)}{6EI}$$

i) deflection due to  $w$ :

$$w_0 \rightarrow w.$$

$$\delta_w = \frac{wL^4}{8EI} \quad (\text{downward})$$

ii) deflection due to  $F_C$

$$a \rightarrow L, P \rightarrow -F_C$$

$$\delta_F = \frac{FL^3}{3EI} \quad (\text{upward}).$$

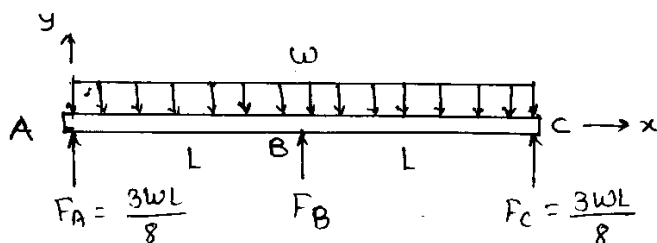
iii) Net downward deflection:

$$\delta_c = \frac{wL^4}{8EI} - \frac{F_c L^3}{3EI}.$$

\* compatibility:

$$\delta_c = 0 \Rightarrow \frac{wL}{8} - \frac{F_c}{3} = 0 \Rightarrow F_c = \frac{3}{8} wL.$$

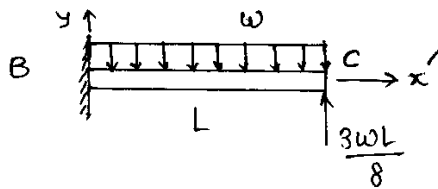
Reaction at the centre. Consider the whole beam



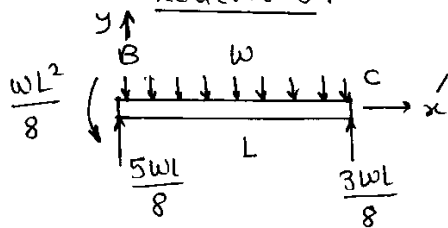
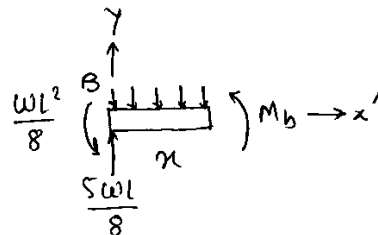
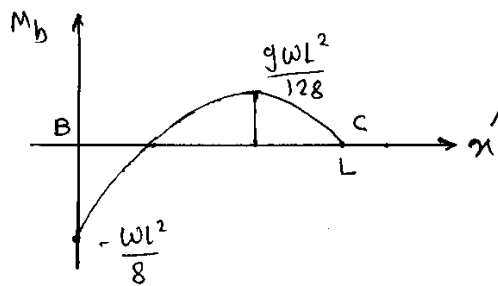
By symmetry,  $F_A = \frac{3}{8} wL$

$$\begin{aligned} \therefore F_B &= 2wL - 2\left(\frac{3}{8} wL\right) \\ &= \frac{5wL}{4} \end{aligned}$$

(problem 8.16 contd.)

(ii) Bending Moment: Again consider part BC

(x is measured from point A)

\* Reactions:\* Bending Moment:

$$M_b = -\frac{wL^2}{8} + \frac{5wL}{8}x' - \frac{1}{2}wx'^2$$

$$\frac{dM_b}{dx'} = 0 \Rightarrow x' = \frac{5L}{8}$$

$$M_b|_{x'=\frac{5L}{8}} = -\frac{wL^2}{8} + \frac{5wL}{8}\left(\frac{5L}{8}\right) - \frac{1}{2}w\left(\frac{5L}{8}\right)^2$$

$$= \frac{9}{128}wL^2$$

But this value is less than  $|M_b|_{x'=0} = \frac{wL^2}{8}$ .

$$\therefore |(M_b)_{\max}| = \frac{wL^2}{8}$$

This occurs at point B

(problem 8.16 contd.)

(11)

(iii) Deflection:

$$EI \frac{d^2 v}{dx'^2} = -\frac{wl^2}{8} + \frac{5wl}{8} x' - \frac{1}{2} w x'^2.$$

$$\therefore EI \frac{dv}{dx'} = -\frac{wl^2}{8} x' + \frac{5wl}{8} \frac{x'^2}{2} - \frac{1}{2} w \frac{x'^3}{3} + C_1$$

$$EI v = -\frac{wl^2}{8} \frac{x'^2}{2} + \frac{5wl}{16} \frac{x'^3}{3} - \frac{1}{6} w \frac{x'^4}{4} + C_1 x' + C_2.$$

$$\frac{dv}{dx'} = 0 \text{ at } x' = 0 \Rightarrow C_1 = 0.$$

$$v = 0 \text{ at } x' = 0 \Rightarrow C_2 = 0.$$

$$\therefore v = \frac{w}{48EI} [-3l^2 x'^2 + 5L x'^3 - 2x'^4].$$

$$\frac{dv}{dx'} = 0 \Rightarrow \frac{wx'}{48} [-6l^2 + 15Lx' - 8x'^2] = 0.$$

$$\Rightarrow x' = 0, \quad 8x'^2 - 15Lx' + 6l^2 = 0.$$

$$\Rightarrow x' = 0, 0.58L, 1.3L.$$

$x' = 0.58L$  is only possible solution.

$$\underline{v_{\max}} = v|_{x'=0.58L} = \frac{w}{48EI} [-3l^2 (0.58L)^2 + 5L (0.58L)^3 - 2(0.58L)^4]$$

$$= \frac{wL^4}{48EI} [-1.0092 + 0.976 - 0.226].$$

$$= -\frac{0.259 wL^4}{48EI}.$$

$$= -\frac{2.07 wL^4}{384EI}.$$

(This is at distance,  $1.58L$  from A since  $x'$  is measured from B.)

In part AB,  $v_{\max}$  occurs at a distance  $0.58L$  from B

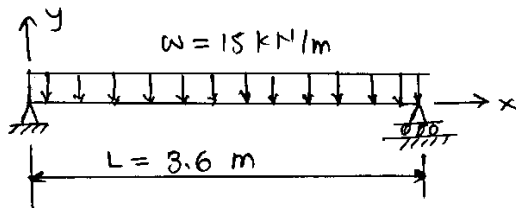
$$\therefore \underline{v_{\max}} = -\frac{2.07 wL^4}{384EI} \text{ at } 0.42L \text{ from A for AB}$$

(problem 8.16 contd.)

Comparison:

|   | <u>Case (a)</u>             | <u>Case (b)</u>          |
|---|-----------------------------|--------------------------|
| i) Central Reaction $\rightarrow$   | $wL$                        | $\frac{5wL}{4}$          |
| ii) $M_{\max}$ $M_b$ $\left\{ \begin{array}{l} \rightarrow \end{array} \right.$     | $\frac{wL^2}{8}$            | $\frac{wL^2}{8}$         |
| location $\left\{ \begin{array}{l} \rightarrow \end{array} \right.$                 | $\frac{L}{2}, \frac{3L}{2}$ | at B (L)                 |
| iii) $M_{\max}$ $\theta$ $\left\{ \begin{array}{l} \rightarrow \end{array} \right.$ | $\frac{5wL^4}{384EI}$       | $\frac{2.07wL^4}{384EI}$ |
| (downward) $\left\{ \begin{array}{l} \rightarrow \end{array} \right.$               |                             |                          |
| location $\left\{ \begin{array}{l} \rightarrow \end{array} \right.$                 | $\frac{L}{2}, \frac{3L}{2}$ | 0.42L, 1.58L.            |

Solution to problem 8.28 :



Given :

•  $E = 7 \text{ GN/m}^2,$

•  $\frac{\delta_{\max}}{L} = \frac{1}{360}.$

To find :  $I_{\min}$

From the results of problem 8.16 (a),

$$\delta_{\max} = \frac{5 w L^4}{384 E I} \Rightarrow \frac{\delta_{\max}}{L} = \frac{5 w L^3}{384 E I}$$

$$\frac{5 w L^3}{384 E I} = \frac{1}{360}$$

$$\Rightarrow \frac{5 \times 15 \times 10^3 \times 3.6^3}{384 \times 7 \times 10^9 I} = \frac{1}{360}$$

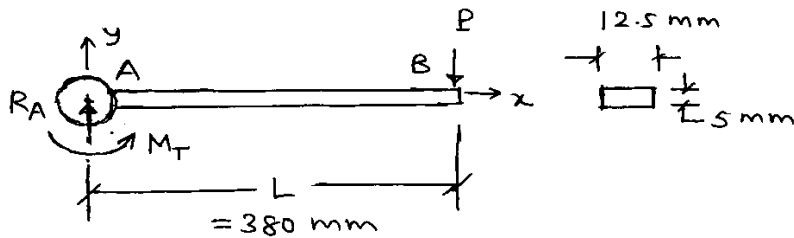
$$\Rightarrow I = \frac{360 \times 5 \times 15 \times 10^3 \times 3.6^3}{384 \times 7 \times 10^9}$$

$$= 4.686 \times 10^{-4} \text{ m}^4.$$

This is the minimum value of  $I$  (i.e.  $I_{\min}$ )

# Solution to problem 8-31

(15)



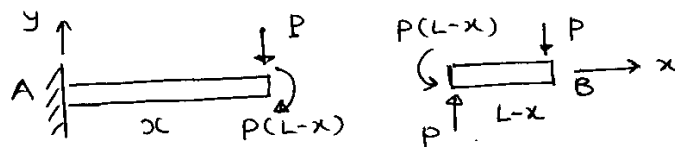
Given:

- $E (\text{Steel}) = 210 \text{ GN/m}^2$
- Scale at  $x = 300 \text{ mm}$

• Equilibrium  $\Rightarrow R_A = P$   
 $M_T = PL \quad \text{or} \quad P = \frac{M_T}{L}$

•  $I = \frac{1}{12} (12.5)(5)^3 = 130.2 \text{ mm}^4$

• Deflection at  $x$ :



\* Superposition:

$$\delta_x = \delta_x \text{ due to } P + \delta_x \text{ due to } P(x-L)$$

$$= \frac{Px^3}{3EI} + \frac{[P(L-x)]x^2}{2EI}$$

$$= \frac{Px^2}{6EI} [2x + 3(L-x)]$$

$$= \frac{Px^2(3L-x)}{6EI}$$

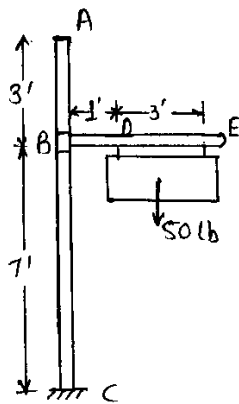
$$= \frac{M_T x^2(3L-x)}{6LEI} \quad \left( \text{Since } P = \frac{M_T}{L} \right)$$

\* At the scale,

$$\delta \Big|_{x=300 \text{ mm}} = \frac{M_T (300)^2 [3 \times 380 - 300]}{6(380) \times 210 \times 10^3 \times 130.2}$$

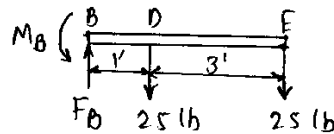
$$= 1.21 \times 10^{-3} M_T$$

# Solution to problem 8.22



## Equilibrium :

Free body diagram of BE



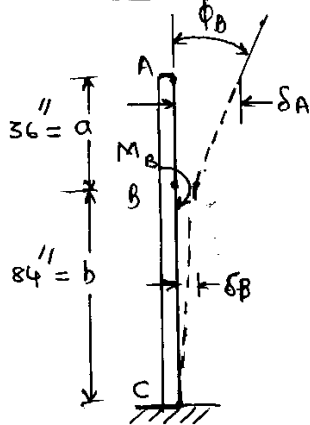
Reactions at B :

$$* F_B = 50 \text{ lb}$$

$$* M_B = 1500 \text{ lb}\cdot\text{in}$$

Note that the effect of the axial force  $F_B$  ~~contribute~~ on the bending moment (in the signpost) is negligible.

## Deflected shape of ABC :



\* Part AB bends as a straight line since  $M_B = 0$  in AB

## Method of superposition.

$$\delta_A = \delta_B + AB \times \phi_B.$$

$$= \delta_B + a \phi_B.$$

$$\delta_B = \frac{M_B b^2}{2EI}, \quad \phi_B = \frac{M_B b}{EI}$$

$$* E = 30 \times 10^6 \text{ psi (steel)}$$

\* Outer diameter of the C/S = 4"

inner diameter of the C/S = 3.5"

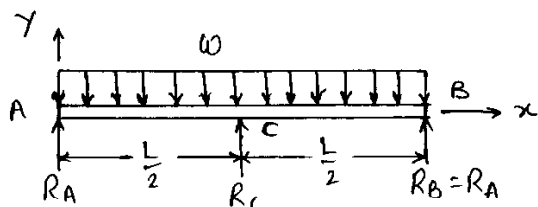
$$\therefore I = \frac{\pi}{64} (4^4 - 3.5^4) = 5.2 \text{ in}^4$$

$$\therefore \delta_A = \frac{M_B b}{EI} (b/2 + a)$$

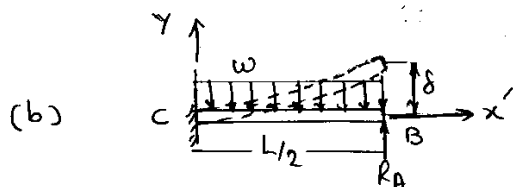
## Numerical Value :

$$\delta_A = \frac{1500 \times 84}{30 \times 10^6 \times 5.2} \left( \frac{84}{2} + 36 \right) = 0.063''$$

# Solution to problem 8.33



(a) FBD of the whole beam



(b)

• Statistically indeterminate problem

• Symmetry  $\Rightarrow$

\* Reactions  $R_B = R_A$

\* slope at  $C = 0$

• consider part CB as a cantilever beam (Fig (b))

$\delta = \delta$  due to  $w$  +  $\delta$  due to  $R_A$ .

$$= -\frac{w(L/2)^4}{8EI} + \frac{R_A(L/2)^3}{3EI} \quad \text{--- (1)}$$

• Compatibility for the original beam:

$R_C = \text{buoyancy force} = \gamma A \delta \quad \text{--- (2)}$

① & ②  $\Rightarrow$

$$\frac{R_C}{\gamma A} = \frac{R_A L^3}{24EI} - \frac{w L^4}{128EI}$$

$$\therefore \frac{48EI R_C}{\gamma A L^3} - 2R_A = -\frac{3wL}{8} \quad \text{--- (3)}$$

• Equilibrium of the original beam (Fig (a))

$$R_C + 2R_A = wL \quad \text{--- (4)}$$

• Solution

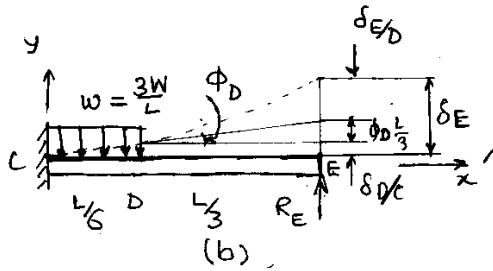
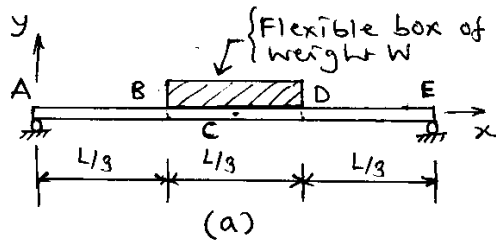
$$\text{③} \& \text{④} \Rightarrow \left(1 + \frac{48EI}{\gamma A L^3}\right) R_C = \frac{5wL}{8}$$

$$\therefore R_C = \frac{5wL}{8} \cdot \frac{1}{\left(1 + \frac{48EI}{\gamma A L^3}\right)} \quad \text{--- (5)}$$

$$\begin{aligned} \text{② and ⑤} \Rightarrow \delta &= \frac{R_C}{\gamma A} = \frac{5wL}{8} \cdot \frac{1}{\gamma A \left(1 + \frac{48EI}{\gamma A L^3}\right)} \\ &= \frac{5wL^4}{384EI \left(1 + \frac{\gamma A L^3}{48EI}\right)} \end{aligned}$$



# Solution to problem 8.35



• Statically indeterminate problem.

• Assume: The force exerted by the block is UDL with intensity  $w = \frac{W}{L/3}$

• Symmetry  $\Rightarrow$

$$* R_A = R_E = W/2$$

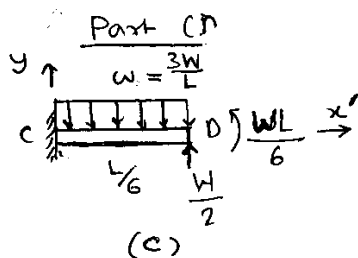
\* Slope at point C is zero.

• Consider CDE as a cantilever beam fixed at C. (Fig. (b))

\*  $\delta_E$  (Deflection at E) can be found by superposition by considering the parts CD and DE as cantilever beams

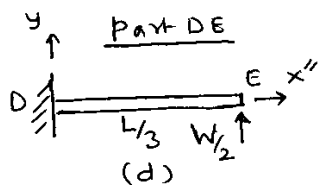
(Figures (c) & (d))

$$* \text{Superposition} \Rightarrow \delta_E = \delta_{D/C} + \phi_D \frac{L}{3} + \delta_{E/D} \quad (1)$$



$$\begin{aligned} \delta_{D/C} &= -\frac{w \left(\frac{L}{6}\right)^4}{8EI} + \frac{W}{2} \frac{\left(\frac{L}{6}\right)^3}{3EI} + \frac{WL}{6} \frac{\left(\frac{L}{6}\right)^2}{2EI} \\ &= -\frac{29 WL^3}{10368 EI} \quad (2) \end{aligned}$$

$$\begin{aligned} \phi_D &= -\frac{w \left(\frac{L}{6}\right)^3}{6EI} + \frac{W}{2} \frac{\left(\frac{L}{6}\right)^2}{2EI} + \frac{WL}{6} \frac{\left(\frac{L}{6}\right)}{EI} \\ &= \frac{42 WL^2}{1296 EI} \quad (3) \end{aligned}$$



$$\delta_{E/D} = \frac{W}{2} \frac{\left(\frac{L}{3}\right)^3}{3EI} = \frac{WL^3}{162 EI} \quad (4)$$

$$(1)-(4) \Rightarrow \delta_E = \frac{WL^3}{EI} \left( \frac{1}{162} + \frac{29}{10368} + \frac{14}{1296} \right) = \frac{205 WL^3}{10368 EI}$$

• This is same as  $\delta_C$  (deflection of the midpoint) of the original beam

$$\delta_C = \frac{205 WL^3}{10368 EI} \quad (a)$$

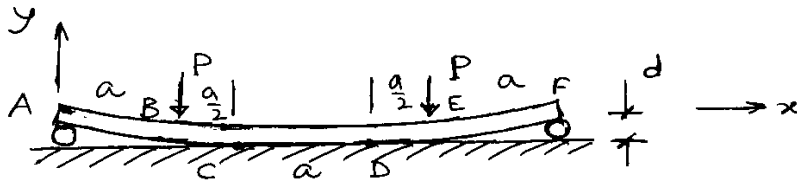
• Deflection  $\delta_C$  by replacing box by a concentrated force  $\frac{W}{2}$  at centre:

$$(\delta_C)_{\text{approx}} = \frac{WL^3}{48EI} \quad (b)$$

$$(a) \& (b) \Rightarrow \frac{(\delta_C)_{\text{approx}}}{(\delta_C)_{\text{exact}}} = \frac{10368}{48 \times 205} = 1.05$$

# Solution to Problem 8-36

(19)



- Determination of the reactions for the part CD:

In part CD,  $\frac{1}{\rho} = 0 \Rightarrow M_b = 0$  for  $\frac{3a}{2} \leq x \leq \frac{5a}{2}$   
 $(M_b = EI/\rho) \Rightarrow \frac{dM_b}{dx} = 0$

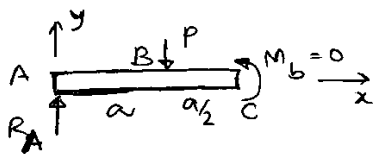
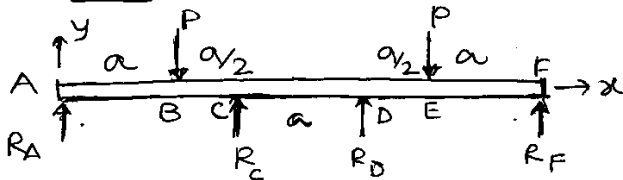
$(V = -\frac{dM_b}{dx}) \Rightarrow V = 0$  for  $\frac{3a}{2} \leq x \leq \frac{5a}{2}$

$\Rightarrow \frac{dV}{dx} = 0$

$(q = -\frac{dV}{dx}) \Rightarrow q = 0$

$\therefore$  No distributed force between C and D. Only the point forces at C and D

- FRD:



\* Symmetry  $\Rightarrow$

$R_A = R_F, R_C = R_D$  — (1)

\* Equilibrium:

$\sum F_y = 0 \Rightarrow 2R_A + 2R_C = P$  — (2)

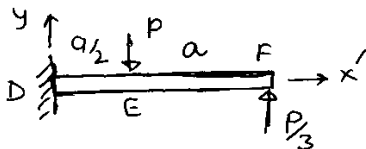
\* Bending Moment at C = 0

$\sum M_C = 0 \Rightarrow \frac{3a}{2} R_A - \frac{a}{2} P = 0$  — (3)

(1) - (3)  $\Rightarrow R_A = R_F = \frac{P}{3}, R_C = R_D = \frac{2P}{3}$

deflection and

- Since the slope at D are zero, consider the part DEF as a cantilever beam fixed at D



By superposition,

$\delta_F = \delta_F \text{ due to } P + \delta_F \text{ due to } P/3$

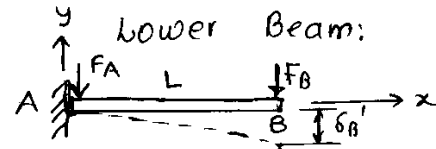
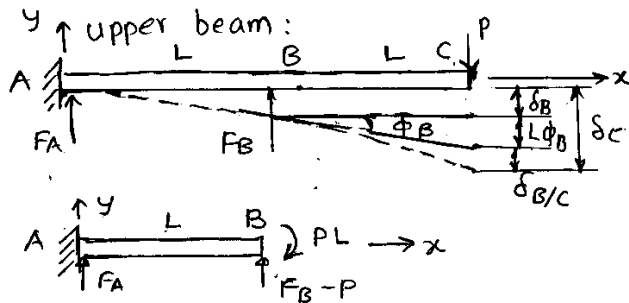
$= -\frac{P(a/2)^2 [3(\frac{3a}{2}) - a/2]}{6EI} + \frac{P/3 (3a/2)^3}{3EI}$

$= \frac{5Pa^3}{24EI}$

•  $\delta_F = d \Rightarrow \frac{5Pa^3}{24EI} = d \Rightarrow P = \frac{24EId}{5a^3}$

### Solution to problem 8.37

- Since the beams touch each other only at the points A ( $x=0$ ) and B ( $x=L$ ), the interactions at A and B and the deflection curves are as follows.



#### \* Deflection:

\* upper beam:

$$\delta_B = -\frac{(F_B - P)L^3}{3EI} + \frac{PL \cdot L^2}{2EI}$$

$$= \frac{L^3}{6EI} (5P - 2F_B)$$

\* lower beam:

$$\delta_B' = \frac{F_B L^3}{3EI} \quad (\text{downward})$$

#### \* Geometric compatibility: $\delta_B = \delta_B'$

$$\therefore \frac{L^3}{6EI} (5P - 2F_B) = \frac{F_B L^3}{3EI}$$

$$\Rightarrow F_B = \frac{5}{4} P.$$

#### \* Deflection at C for upper beam: $\delta_C = \delta_B + L\phi_B + \delta_{B/C}$

$$\therefore \delta_C = \frac{L^3}{6EI} (5P - 2 \cdot \frac{5}{4} P) + L \left[ \frac{PL \cdot L}{EI} - \frac{(\frac{5P}{4} - P)L^2}{2EI} \right] + \frac{PL^3}{3EI}$$

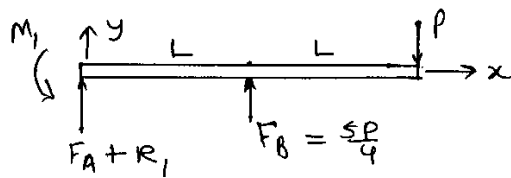
$$\text{or } \delta_C = \frac{13 PL^3}{8EI}$$

#### \* Deflection curves:

#### \* F.B.D.s and Reactions

(problem 8.37 contd.)

Upper beam

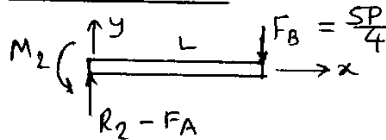


Equilibrium  $\Rightarrow$

$$F_A + R_1 = -P/4$$

$$M_1 = \frac{3PL}{4}$$

Lower beam



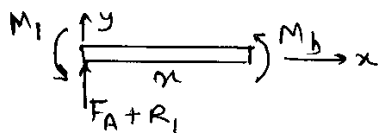
Equilibrium  $\Rightarrow$

$$R_2 - F_A = \frac{5P}{4}$$

$$M_2 = \frac{5PL}{4}$$

\* Bending Moments & Boundary Conditions

Upper Beam



$$M_b = (F_A + R_1)x - M_1$$

$$= -\frac{P}{4}(x + 3L)$$

$$\therefore EI \frac{d^2 v_1}{dx^2} = -\frac{P}{4}(x + 3L)$$

$$EI \frac{dv_1}{dx} = -\frac{P}{4}\left(\frac{x^2}{2} + 3Lx\right) + C_1$$

$$EI v_1 = -\frac{P}{4}\left(\frac{x^3}{6} + \frac{3}{2}Lx^2\right) + C_1 x + C_2$$

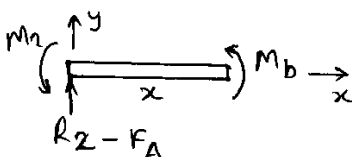
Boundary conditions

$$\text{at } x=0 \quad v_1=0 \Rightarrow C_2=0$$

$$\frac{dv_1}{dx}=0 \Rightarrow C_1=0$$

$$\therefore v_1 = -\frac{P}{4EI}\left(\frac{x^3}{6} + \frac{3}{2}Lx^2\right) \quad \text{--- (1)}$$

Lower beam:



$$M_b = (R_2 - F_A)x - M_2$$

$$= \frac{5P}{4}(x - L)$$

(problem 8.37 contd.)

$$\therefore EI \frac{d^2 \theta_2}{dn^2} = \frac{5P}{4} (n-L)$$

$$\therefore EI \frac{d \theta_2}{dn} = \frac{5P}{4} \left( \frac{n^2}{2} - Ln \right) + C_1$$

$$EI \theta_2 = \frac{5P}{4} \left( \frac{n^3}{6} - \frac{Ln^2}{2} \right) + C_1 n + C_2$$

Boundary conditions:

$$\text{at } n=0 : \theta_2 = 0 \Rightarrow C_2 = 0$$

$$\frac{d \theta_2}{dn} = 0 \Rightarrow C_1 = 0$$

$$\therefore \theta_2 = \frac{5P}{4EI} \left( \frac{n^3}{6} - \frac{Ln^2}{2} \right) \quad \text{--- (2)}$$

\* Comparison

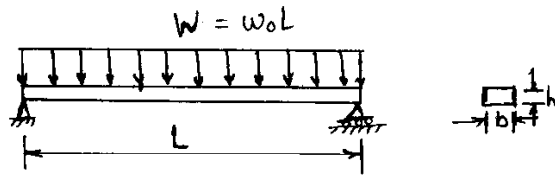
$$(1) \& (2) \Rightarrow \theta_1 - \theta_2 = \frac{P}{4EI} (Lx^2 - x^3) > 0 \quad \text{for } 0 < x < L \quad \left( \begin{array}{l} \theta_1 \text{ is} \\ \text{less} \\ \text{negative} \end{array} \right)$$

$$= 0 \quad \text{for } x = L.$$

This means the beams touch only at B.

Hence the assumption is correct.

# Solution to problem 8.68



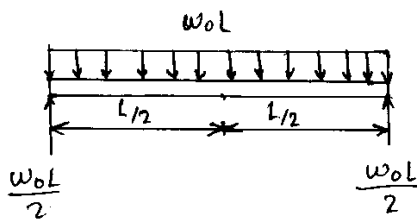
Given:

- beam with minimum beam weight  $W_b$  to be designed.
- $W_b \ll W$
- $L$  and  $h$  fixed;  $\sigma_{\max}$ ,  $E$ , and weight density  $\gamma$  to be varied.

To show: a) strength based design:  $\left(\frac{W}{W_b}\right)_{\max} \Rightarrow \left(\frac{\sigma_{\max}}{\gamma}\right)_{\max}$ .

b) rigidity based design:  $\left(\frac{W}{W_b}\right)_{\max} \Rightarrow \left(\frac{E}{\gamma}\right)_{\max}$ .

- FBD & Reactions



$$\bullet \text{ a) } (M_b)_{\max} = \frac{w_0 L}{2} \left(\frac{L}{2}\right) - \left(\frac{w_0 L}{2}\right) \frac{L}{4} = \frac{w_0 L^2}{8}$$

$$\sigma_{\max} = \frac{(M_b)_{\max} h/2}{\frac{bh^3}{12}} = \frac{\frac{w_0 L^2}{8} \cdot \frac{h}{2}}{bh^3/12} = \frac{3}{4} \frac{WL}{bh^2}$$

$$\therefore W = \frac{4bh^2}{3L} \sigma_{\max}$$

$$W_b = (Lbh)\gamma$$

$$\therefore \frac{W}{W_b} = \frac{4bh^2}{3L} \cdot \frac{1}{Lbh} \cdot \frac{\sigma_{\max}}{\gamma}$$

$$= \left(\frac{4h}{3L^2}\right) \frac{\sigma_{\max}}{\gamma} \quad \left(\frac{4h}{3L^2} \text{ is fixed}\right)$$

$\therefore \max\left(\frac{W}{W_b}\right)$  is achieved by choosing the material with largest  $\frac{\sigma_{\max}}{\gamma}$  ratio.

( $W_b$  neglected while calculating  $\sigma_{\max}$ )

(problem 8.68 contd.)

$$\cdot b) \delta_{max} = \frac{5 w_0 L^4}{384 EI} = \frac{5 W L^3}{384 E \left( \frac{bh^3}{12} \right)} = \frac{5 W L^3}{32 E b h^3} \cdot \left( \begin{array}{l} W_b \text{ neglected} \\ \text{while} \\ \text{calculating} \\ \delta_{max} \end{array} \right)$$

$$\therefore W = \frac{32 E b h^3}{5 L^3} \delta_{max}$$

$$W_b = (L b h) \gamma$$

$$\therefore \frac{W}{W_b} = \frac{32 E b h^3 \delta_{max}}{5 L^3} \cdot \frac{1}{L b h \gamma}$$

$$= \left( \frac{32}{5} \frac{\delta_{max} h^2}{L^4} \right) \frac{E}{\gamma}$$

note that  $\delta_{max}$ ,  $h$ ,  $L$  are fixed.

$\therefore \max \frac{W}{W_b}$  is achieved by choosing the material with the largest  $\frac{E}{\gamma}$  ratio.