ME321: Advanced Mechanics of Solids

Assignment 1: Cartesian Tensors.

1. Simpify the following expressions by employing properties of the Kronecker delta and the e-permutation symbol:

   (a) $e_{ijk}\delta_{kn}$ (b) $e_{ijk}\delta_{is}\delta_{jn}$ (c) $e_{ijk}\delta_{is}\delta_{jm}\delta_{kn}$,

   (d) $a_{ij}\delta_{in}$ (e) $\delta_{ij}\delta_{jn}$ (f) $\delta_{ij}\delta_{jn}\delta_{ni}$

   Note: Try not to perform explicit summation.

2. Use the summation convention to evaluate:

   (a) δ_{ii}, (b) $e_{ijk}a_i a_j a_k$, (c) $\delta_{ij}\delta_{ij}$,

   (d) $a_i b_j \delta_{ji} - b_m a_n \delta_{mn}$.

3. Express the following in indical notation.

   (a) $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$, (b) $\boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$,

   (c) $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})$.

4. Show that $e_{ijk} = -e_{jik} = -e_{ikj} = -e_{kji}$.

5. Using the results of Pr.3 show that,

   $\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c})=\boldsymbol{b}(\boldsymbol{a} \cdot \boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a} \cdot \boldsymbol{b})$.

6. Use the $e - \delta$ identity to simplify.

   (a) $e_{ijk}e_{jik}$, (b)$e_{ijk}e_{jki}$.

7. For a symmetric tensor $\boldsymbol{S}$ and a skew-symmetric tensor $\boldsymbol{W}$, show that

$$\boldsymbol{S} : \boldsymbol{W} = 0. \tag{1}$$

8. Using the divergence theorem show that

$$\int_V \frac{\partial u_i}{\partial x_j} dV = \int_{\partial V} u_i n_j dS$$