1.1

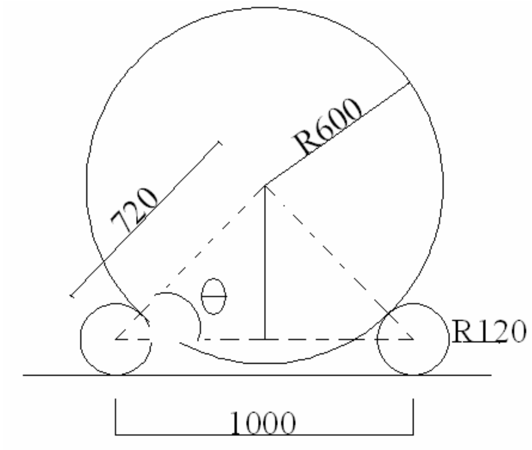


Fig 1.1 System Geometry

See Fig 1.1:
$\text{Cos}\theta = (1000/2)/(720);$
$\rightarrow \theta = 46.02°$

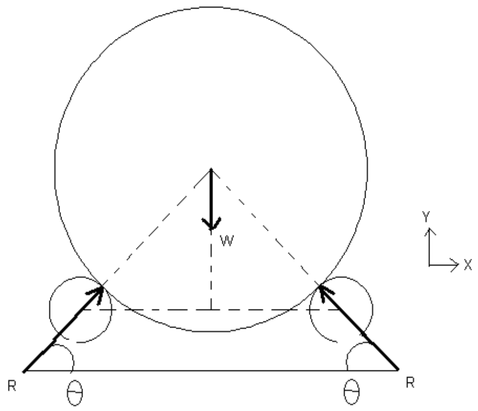


Fig .1.2 FBD of Drum

$\sum F_y = 0$ (Fig 1.2)
$\rightarrow (R\ \text{Sin}\theta) + (R\ \text{Sin}\theta) - W = 0$
$\rightarrow R = 1111.8$ Kg $= 10.91$ KN [Ans.]

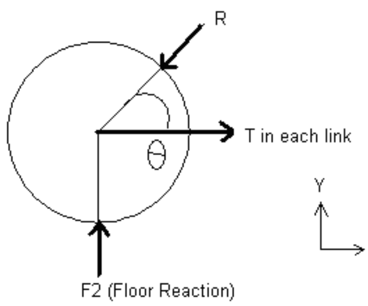
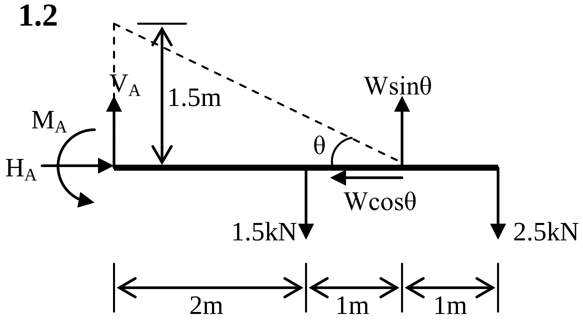


Fig.1.3 FBD of Left Roller

$\sum F_x = 0$ (Fig 1.3)
$\rightarrow 2T - R\ \text{Cos}\theta = 0$
$\rightarrow T = 3.79$ KN [Ans]

1.2



$\theta = \tan^{-1}(1.5/3) = \tan^{-1}0.5$
$\sin\theta = 1/\sqrt{5}$ $\cos\theta = 2/\sqrt{5}$

(a) $W = 6.5kN$

$$\sum F_X = 0 \Rightarrow H_A = W\cos\theta = 6.5\times\frac{2}{\sqrt{5}} = 5.82kN$$

$$\sum F_Y = 0 \Rightarrow W\sin\theta + V_A = 1.5 + 2.5 \Rightarrow V_A = 4.0 - 6.5\times\frac{1}{\sqrt{5}} = 1.09kN$$

$$\sum M_{AZ} = 0 \Rightarrow W\sin\theta\times 3 + M_A - 1.5\times 2 - 2.5\times 4 = 0$$

$$\Rightarrow M_A = 13 - 6.5\times\frac{3}{\sqrt{5}} = 4.28kNm$$

(b) M_A has a maximum magnitude of 2.5kNm

$$\sum M_{AZ} = 0 \Rightarrow W\sin\theta\times 3 + M_A - 1.5\times 2 - 2.5\times 4 = 0$$

$$\Rightarrow 3W\sin\theta = 13 - M_A$$

M_A can be +ve (CCW) or –ve (CW)

Accordingly, $W = \dfrac{13 \mp M_A}{3\sin\theta}$

$$W_{\min} = \frac{13-2.5}{3\times {}^{1}\!/\!{}_{\sqrt{5}}} = 7.83kN$$

$$W_{\max} = \frac{13+2.5}{3\times {}^{1}\!/\!{}_{\sqrt{5}}} = 11.55\ KN$$

1.3

$$\left.\begin{aligned}
\vec{R}_A &= \left(R_x\hat{i} + R_y\hat{j} + R_z\hat{k}\right)N \\
\vec{F}_{EF} &= F_{EF}\hat{\lambda}_{EF} = F_{EF}\left(-1.5\hat{i} + 0.75\hat{j} + 0.5\hat{k}\right)\frac{1}{1.75} \\
F_{BG} &= F_{BG}\hat{\lambda}_{BG} = F_{BG}\left(-2\hat{i} + \hat{j} - 2\hat{k}\right)\frac{1}{3}
\end{aligned}\right\}[A]$$

$$\sum \vec{F} = \vec{0} \Rightarrow \vec{R}_A + \vec{F}_{EF} + \vec{F}_{BG} - 1350\hat{j} = \vec{0}$$

$$\left.\begin{aligned}
\hat{i} &\to R_x - \frac{1.5}{1.75}F_{EF} - \frac{2}{3}F_{BG} = 0 \\
\hat{j} &\to R_y - \frac{0.75}{1.75}F_{EF} + \frac{1}{3}F_{BG} - 1350 = 0 \\
\hat{k} &\to R_z + \frac{0.5}{1.75}F_{EF} - \frac{2}{3}F_{BG} = 0
\end{aligned}\right\}[B]$$

$$\sum M_A = 0:$$

$$1.5\hat{i}\times\vec{F}_{EF} + 2\hat{i}\times\vec{F}_{BG} - \hat{i}\times 1350\hat{j} = \vec{0}$$

Use expressions of $\vec{F}_{EF}$ and $\vec{F}_{BG}$ from [A] and then collect coefficients of $\hat{j}$ and $\hat{k}$:

$$\hat{j} \rightarrow F_{EF} = 3.11 F_{BG}$$

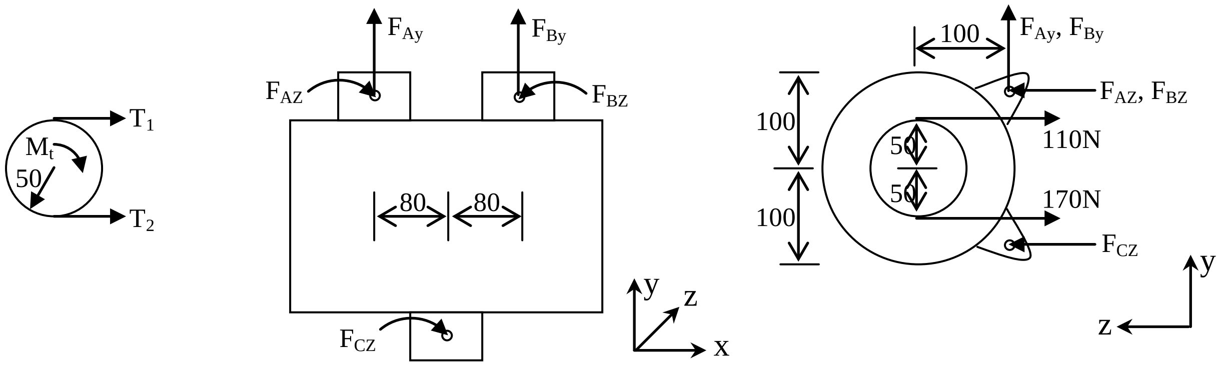
$$\hat{k} \rightarrow F_{BG} \times 3.11 \times \frac{1.5 \times 0.75}{1.75} + \frac{2}{3} F_{BG} = 1350$$

$$F_{BG} = 505N$$

$$F_{EF} = 1570N$$

From [B], R_x = 1681N, R_y = 509N, R_z = -112N

1.4



M_t = torque supplied to pulley by motor

$$\frac{T_1 + T_2}{2} = T_m = 140$$

$$T_2 \times 50 - T_1 \times 50 = 3 \times 1000$$

$$\Rightarrow T_1 = 110N \qquad T_2 = 170N$$

Equation of motor:

$$\sum F_X = 0 \rightarrow identically\ satisfied$$

$$\sum F_Y = 0: \quad F_{AY} - F_{BY} - 100 = 0$$

$$\sum F_Y = 0: \quad F_{AZ} + F_{BZ} + F_{CZ} - 280 = 0$$

$$\sum M_{AX} = 0: \quad 100 \times 10 + 110 \times 50 + 170 \times 50 = F_{CZ} \times 200 \Rightarrow F_{CZ} = 205N$$

$$\sum M_{AX} = 0: \quad - F_{BZ} \times 160 + 110 \times 240 + 170 \times 240 - F_{CZ} \times 80 = 0$$

$$\sum M_{AX} = 0: \quad -100 \times 80 + F_{BY} \times 160 = 0 \Rightarrow F_{BY} = 50N$$

From other equations,

$$F_{BZ} = 317.5N \quad F_{AY} = 50N \quad F_{AZ} = -242.5N$$

$$\Rightarrow \vec{F}_A = (50\hat{j} - 242.5\hat{k})N \quad \vec{F}_B = (50\hat{j} + 317.5\hat{k})N$$

1.5

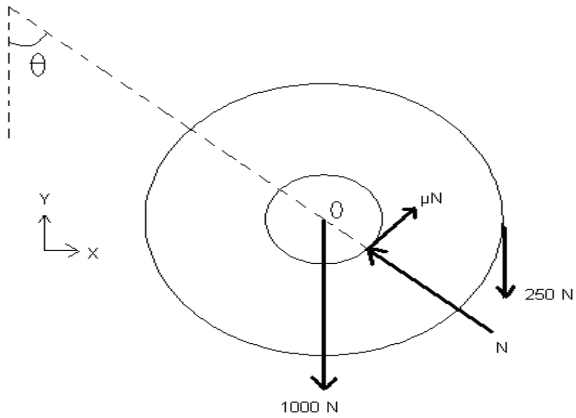


Fig 1.5.1 FBD of wheel

N= normal reaction, μ= coefficient of friction

$\sum$Mo = 0 (Fig.1.5.1)

→ (μN) (300) – (250) (800) = 0

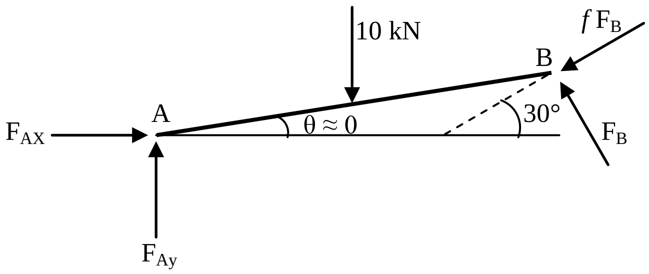
→ μN = 2000/3 …………….(1)

$\sum F_y = 0$ & $\sum Fx = 0$
N Cosθ + μN Sinθ = 1250 …………….(2)
- N Sinθ + μN Cosθ = 0 …………….(3)

Solving (1),(2) & (3):
$\mu_{min} = 0.63$
$\theta = \text{Sin}^{-1}(8/15) = 32.2°$

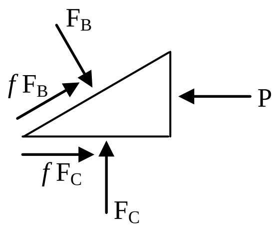
1.6 Assume a general friction coefficient f.
When the wedge is moving, tangential force at contact = f x normal force

(a) Equilibrium of AB:



$$\sum M_{AZ} = 0$$
$$-10\times 1 + F_B \cos 30° \times 2 - fF_B\cos 60^0 \text{ x } 2 = 0$$
$$F_B = 5 \big/ (0.866 - 0.5f)$$

Equilibrium of wedge:



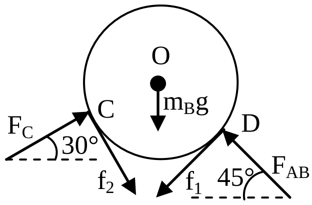
$$\sum F_y = 0: \quad F_C - F_B \cos 30° + fF_B \sin 30° = 0$$
$$\sum F_X = 0: \quad -P + fF_C + (f \cos 30° + \sin 30°)F_B = 0$$

Solution leads to:
$$F_C = 5kN \quad P = 5\left[f + \frac{0.866f + 0.5}{0.866 - 0.5f} \right]$$
For $f = 0.3 \quad P = 6.81kN$

(b) If f is very small, the wedge will slip out when the force P is removed. The borderline occurs when the wedge is just prevented from slipping out by the friction forces. We go through the previous analysis, replacing *f* by *–f* everywhere, because the tendency we are investigating now is the slipping in the opposite direction.
Thus:
$$P = 5\left[-f + \frac{-0.866f + 0.5}{0.866 + 0.5f} \right]$$
$$i.e.\ P = 0 \Rightarrow \frac{-0.866f + 0.5}{0.866 + 0.5f} - f = 0 \Rightarrow f = 0.27(borderline)$$

1.7

For the paper towel to start moving vertically downward, two possibilities exist:
(i) Rolls A and B rotate together. This is possible when the contact at C slips and the contact at D does not slip
(ii) Roll B does not rotate. Slip occurs at D only and the paper goes down. The frictional forces developed at C are such that no slip occurs at C and the roll B is stationary.



FBD of roll B

[B]

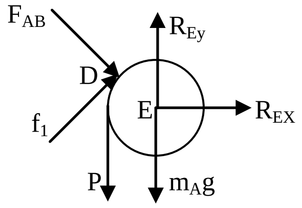
$$\sum F_X = 0 \Rightarrow$$

$$F_C \cos 30° - F_{AB} \cos 45° + f_2 \cos 60° - f_1 \sin 45° = 0 \qquad (i)$$

$$\sum F_Y = 0 \Rightarrow$$

$$F_C \sin 30° + F_{AB} \sin 45° - f_2 \sin 60° - f_1 \sin 45° - m_B g = 0 \qquad (ii)$$

$$\sum M_{OZ} = 0 \Rightarrow f_1 = f_2 \qquad (iii)$$



FBD of roll A

[A]

$$\sum M_{EZ} = 0 \Rightarrow f_1 = P \qquad (iv)$$

$\sum F_X = 0, \sum F_Y = 0$ help to get R_{EY}, R_{EX} but they are not required.

Using *(iii)* in *(i)*,

$$\frac{F_C}{f_2}\cos 30° + (\cos 60° - \sin 45°) = \frac{F_{AB}}{f_1}\cos 45°$$

$$\frac{F_C}{f_2}\frac{\sqrt{3}}{2} - 0.207 = \frac{F_{AB}}{f_1}\frac{1}{\sqrt{2}} \qquad (v)$$

Moreover, given that $\dfrac{f_2}{F_C} \le 0.2$; $\dfrac{f_1}{F_{AB}} \le 0.5$ $\qquad (vi)$

Case I: Slip at C

then $\dfrac{f_2}{F_C} = 0.2$

from *(v)* we get $\dfrac{f_1}{F_{AB}} = 0.171 \qquad (vii)$

Case II: Slip at D

then $\dfrac{f_1}{F_{AB}} = 0.5$

from *(v)* we get $\dfrac{f_2}{F_C} = 0.534$ which is not possible in view of *(vi)*

Therefore *(vi)* is satisfied only when slip occurs at C.
Using *(iii)*, *(vii)* and *(ii)*, we can find f_1 = 4.85N
From *(iv)* $P = f_1$ = 4.85N