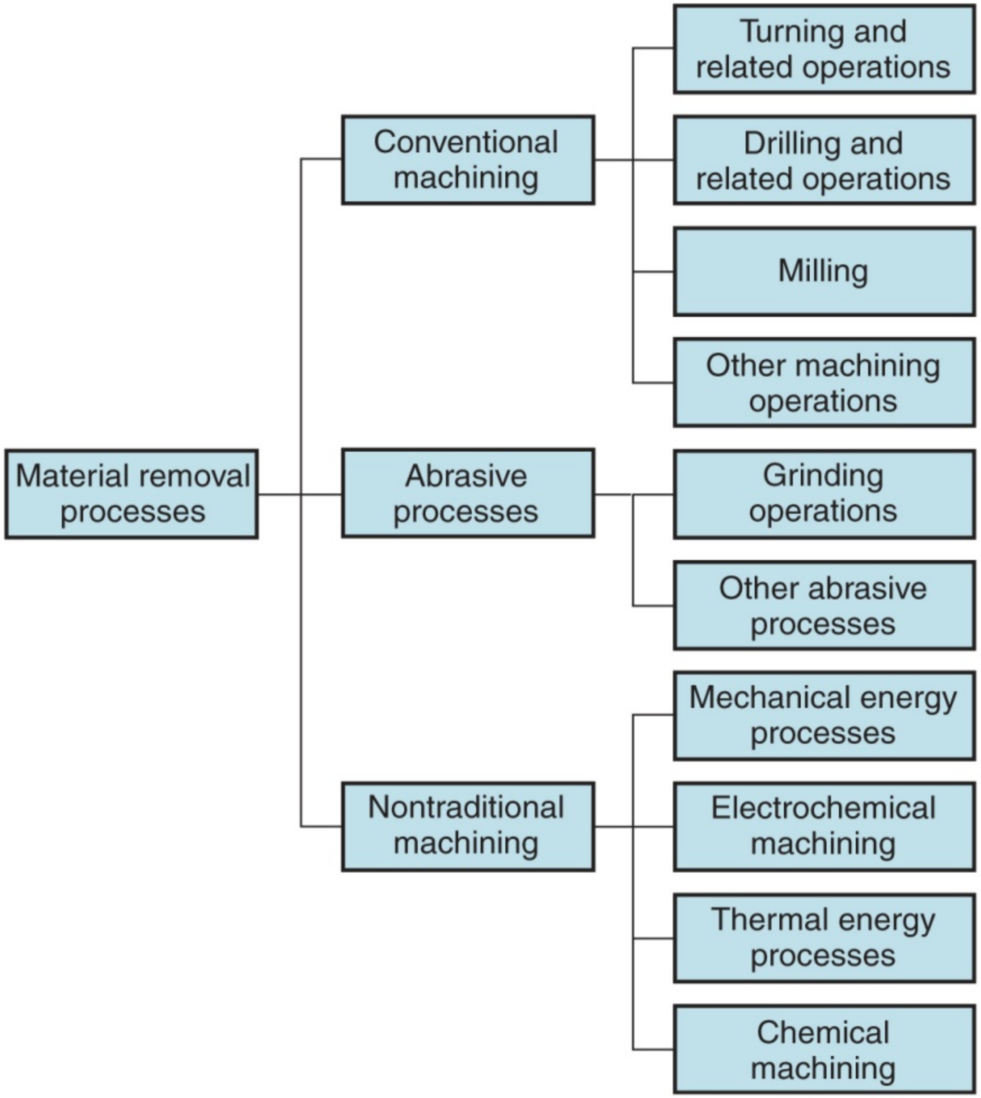
Material Removal Processes

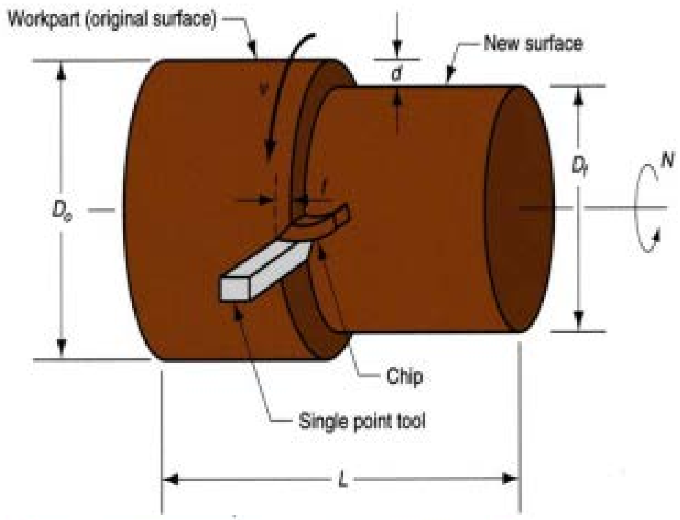
- The family tree

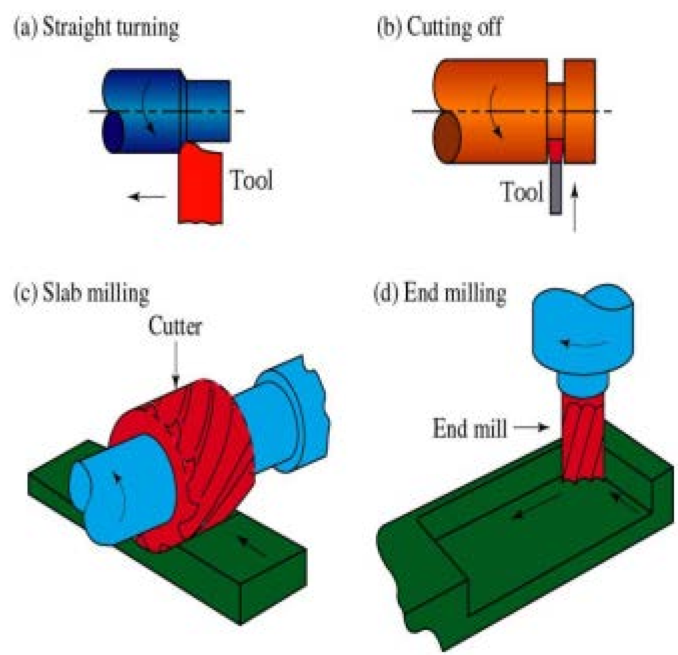


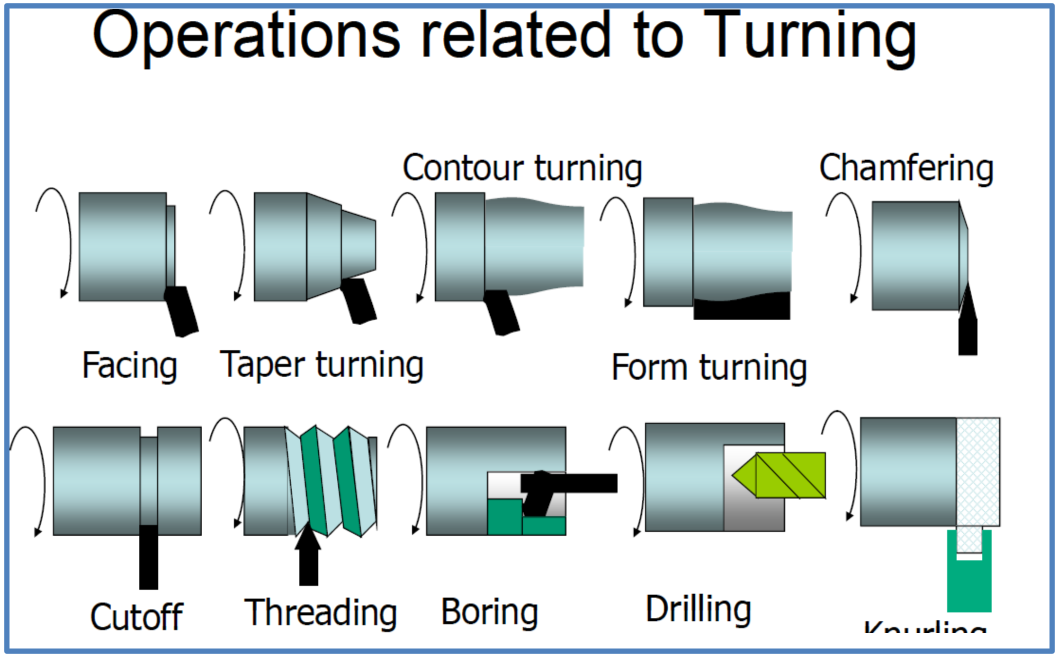
Machining

In MACHINING, the shape, size, finish and accuracy are obtained by removing the excess material from the workpiece surface.

Various surfaces are obtained as an interaction between a workpiece and a cutting tool with the help of a contrivance known as MACHINE TOOL.







Operations related to Turning
Contour turning
Chamfering
Facing
Taper turning
Form turning
Cutoff
Threading
Boring
Drilling

Why Machining is Important

- Variety of work materials can be machined
  - Most frequently used to cut metals
- Variety of part shapes and special geometric features possible:
  - Screw threads
  - Accurate round holes
  - Very straight edges and surfaces
- Good dimensional accuracy and surface finish

Disadvantages with Machining

- Wasteful of material
  - Chips generated in machining are wasted material
    - At least in the unit operation
- Time consuming
  - A machining operation generally takes longer to shape a given part than alternative shaping processes

Machining in the Manufacturing Sequence

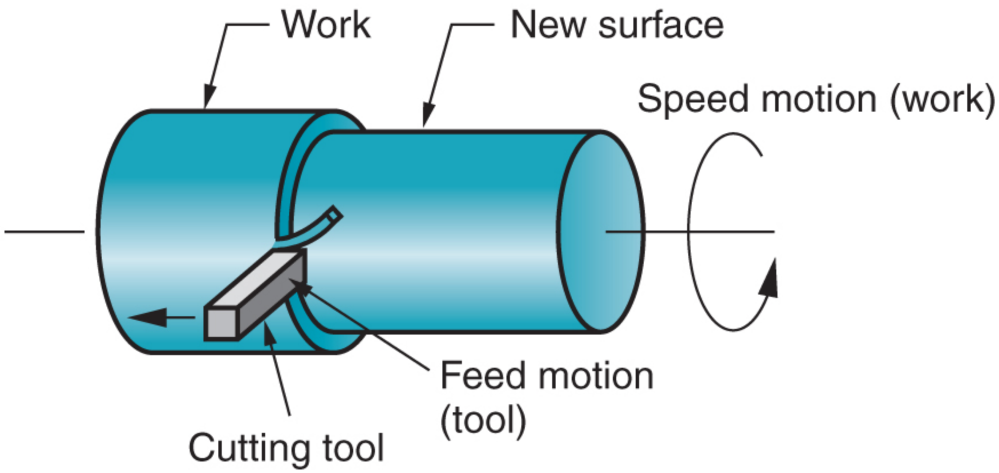
- Generally performed after other manufacturing processes, such as casting, forging, and bar drawing
  - Other processes create the general shape of the starting workpart
  - Machining provides the final shape, dimensions, finish, and special geometric details that other processes cannot create

Machining Operations

- Most important machining operations:
  - Turning
  - Drilling
  - Milling
- Other machining operations:
  - Shaping and planing
  - Broaching
  - Sawing

Turning

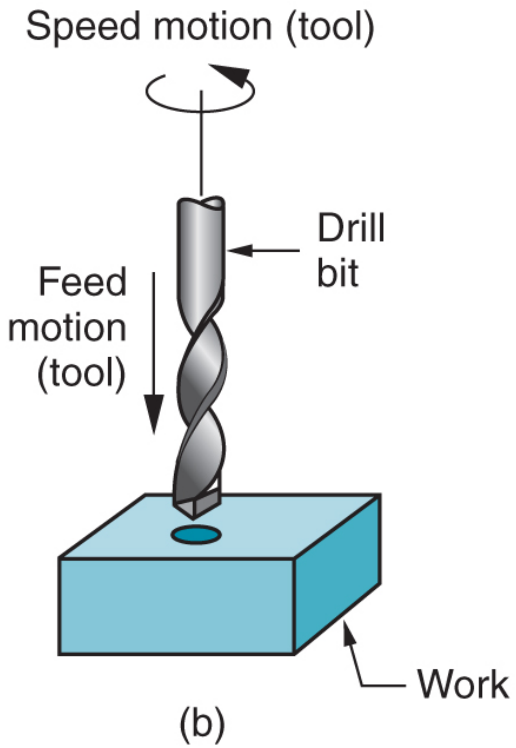
- Single point cutting tool removes material from a rotating workpiece to form a cylindrical shape



(a)

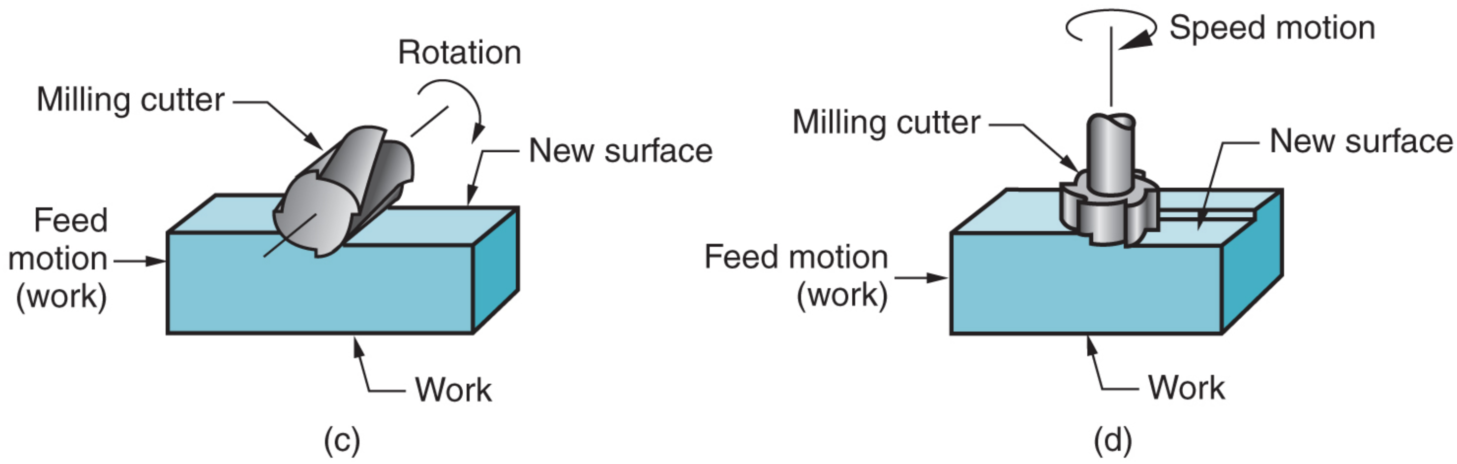
Drilling

- Used to create a round hole, usually by means of a rotating tool (drill bit) with two cutting edges

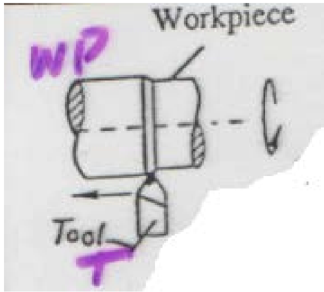
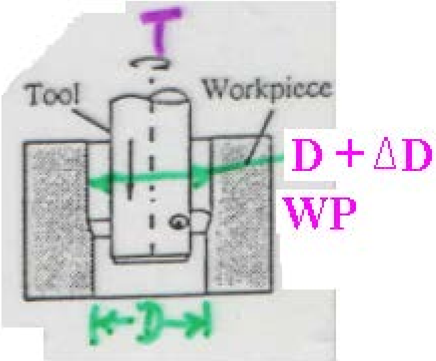
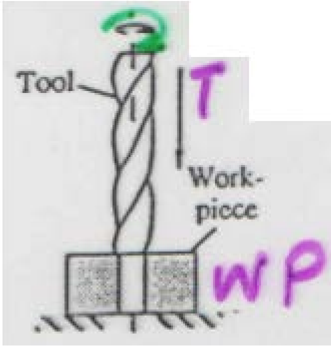


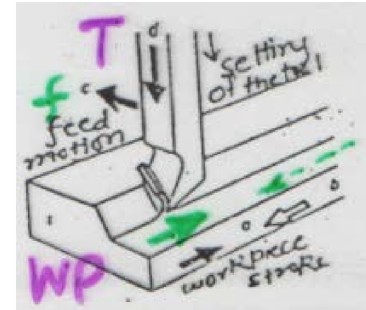
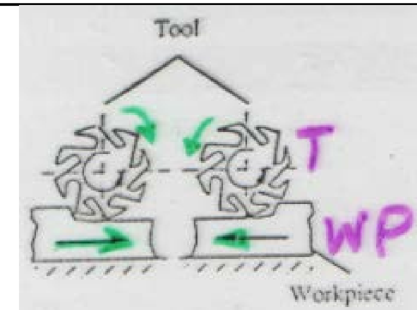
Milling

- Rotating multiple-cutting-edge tool is moved across work to cut a plane or straight surface
- Two forms: (c) peripheral milling and (d) face milling



NATURE OF RELATIVE MOTION BETWEEN THE TOOL AND WORKPIECE

OPERATION	MOTION OF JOB	MOTION OF CUTTING TOOL	FIGURE OF OPEARTION
TURNING	ROTARY	TRANSLATORY (FORWARD)	
BORING	ROTATION	TRANSLATION (FORWARD)	
DRILLING	FIXED (NO MOTION)	ROTATION AS WELL AS TRANSLATORY FEED	

PLANING	TRANSLATORY	INTERMITTENT TRANSLATION	
MILLING	TRANSLATORY	ROTATION	
GRINDING	ROTARY / TRANSLATORY	ROTARY	

WHAT IS THE BASIC DIFFERENCE BETWEEN ?

TURNING
BORING
PLANING
SINGLE POINT

AND

DRILLING
MILLING
GRINDING
MULTI POINTS

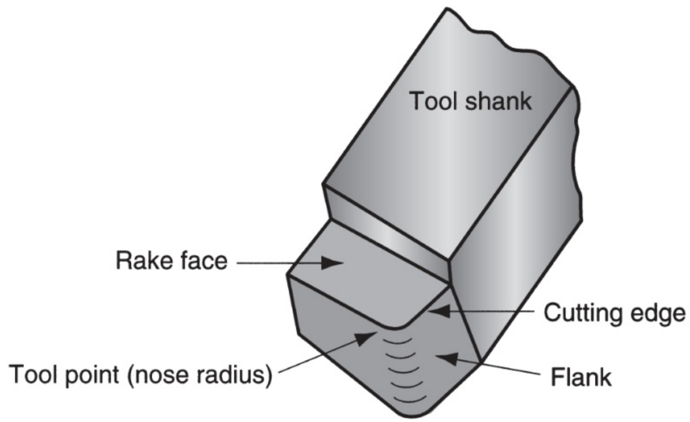
- SINGLE VS MULTI POINTS
- CONTINUOUS AND INTERMITTENT

Cutting Tool Classification

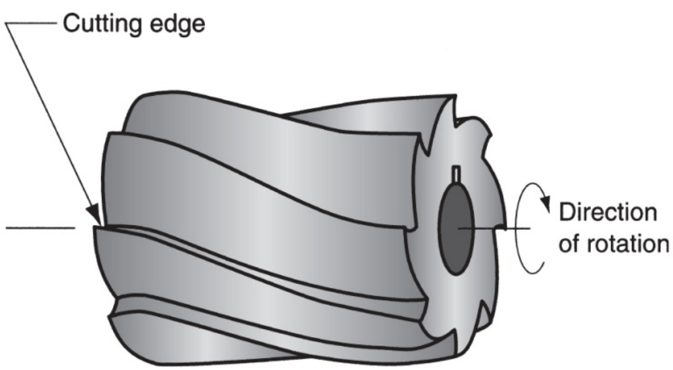
1. Single-Point Tools
   – One dominant cutting edge
   – Point is usually rounded to form a nose radius
   – Turning uses single point tools
2. Multiple Cutting Edge Tools
   – More than one cutting edge
   – Motion relative to work achieved by rotating
   – Drilling and milling use rotating multiple cutting edge tools

Cutting Tools

- (a) Single-point tool showing rake face, flank, and tool point; and (b) a helical milling cutter, representative of tools with multiple cutting edges



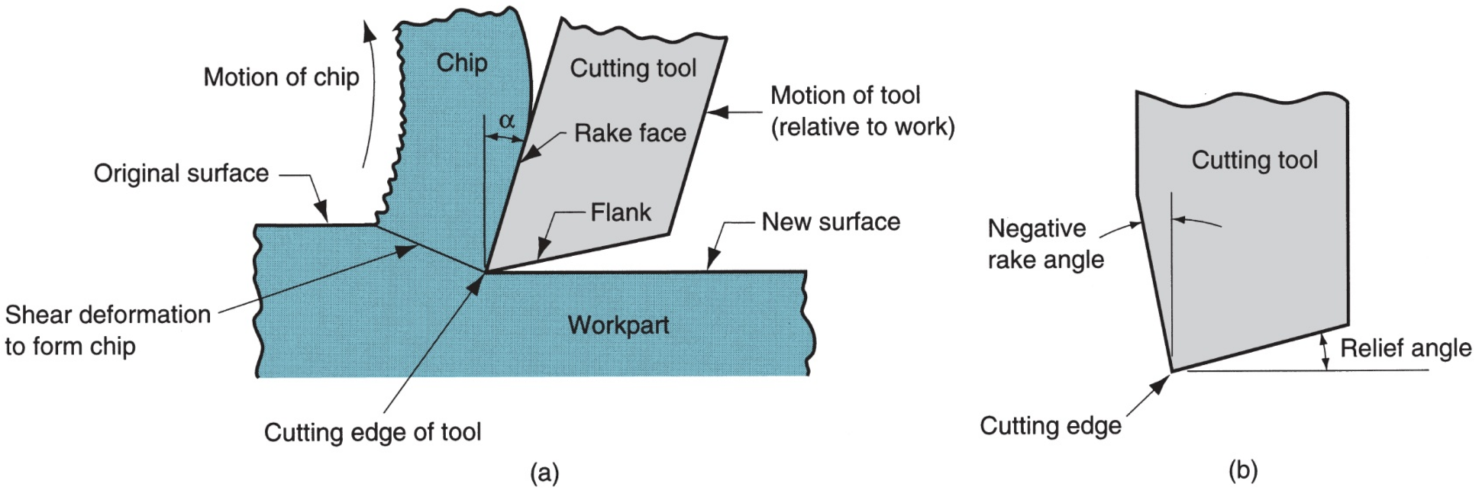
(a)

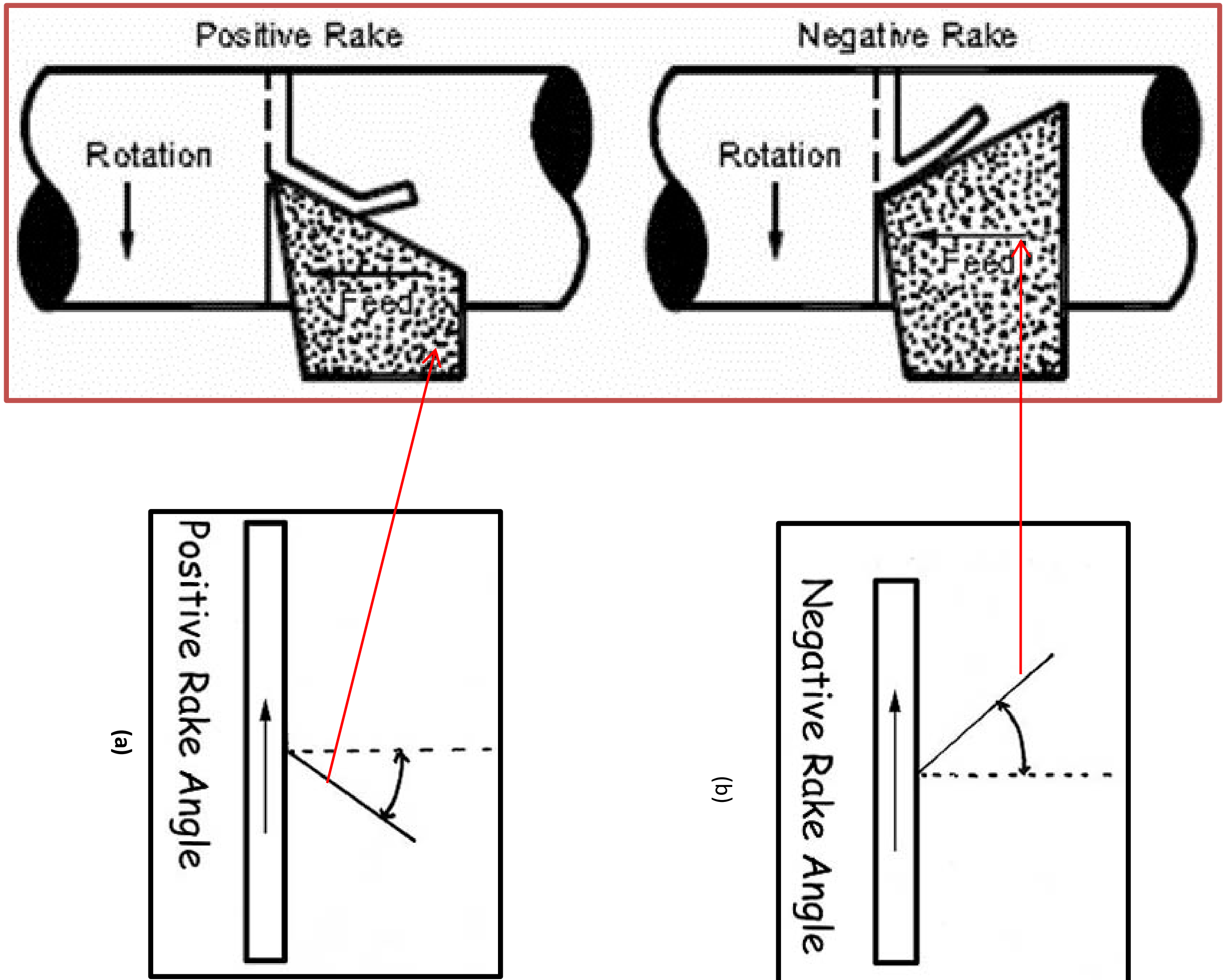


(b)

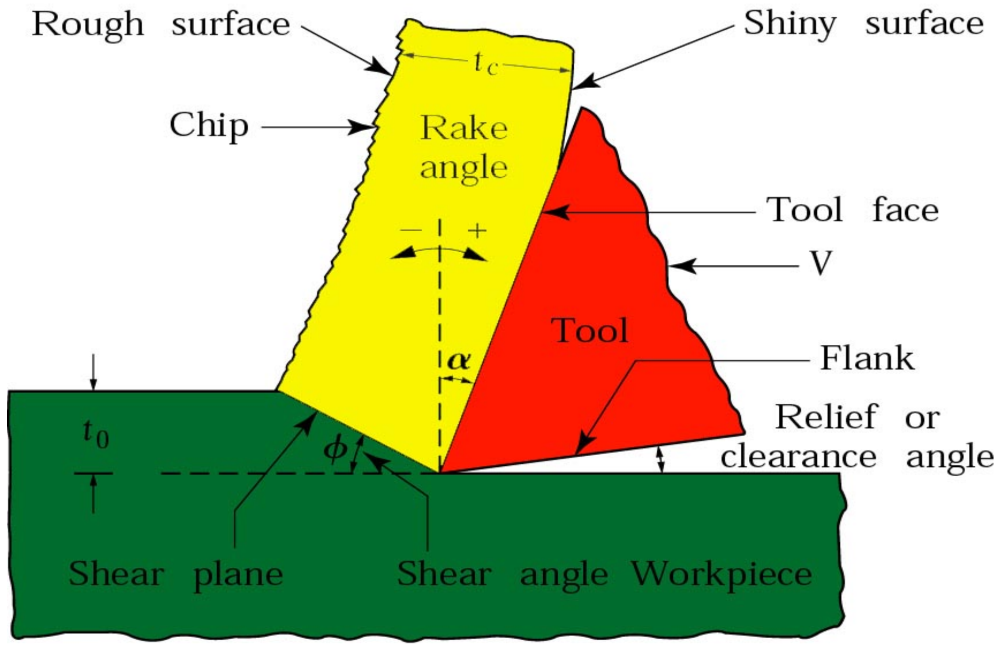
Cutting action

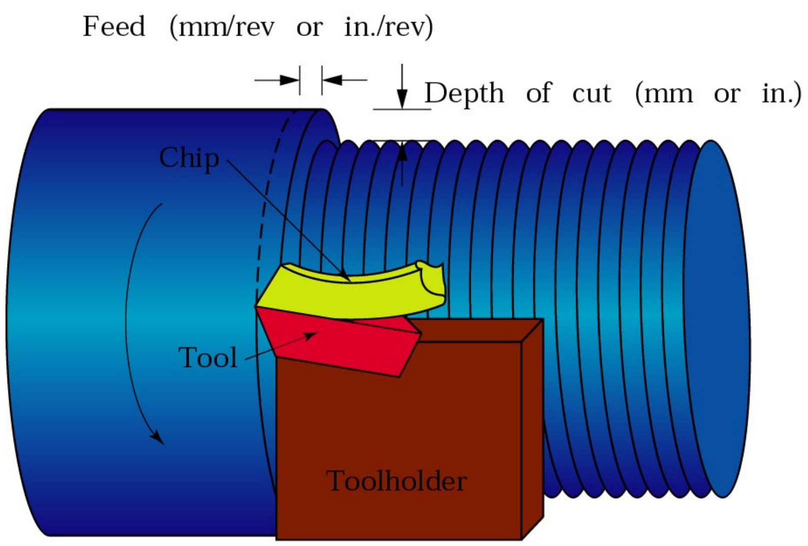
- Cutting action involves shear deformation of work material to form a chip, and as chip is removed, new surface is exposed: (a) positive and (b) negative rake tools





Positive Rake
Negative Rake
Rotation
Feed
Rotation
Feed
(a)
Positive Rake Angle
(b)
Negative Rake Angle

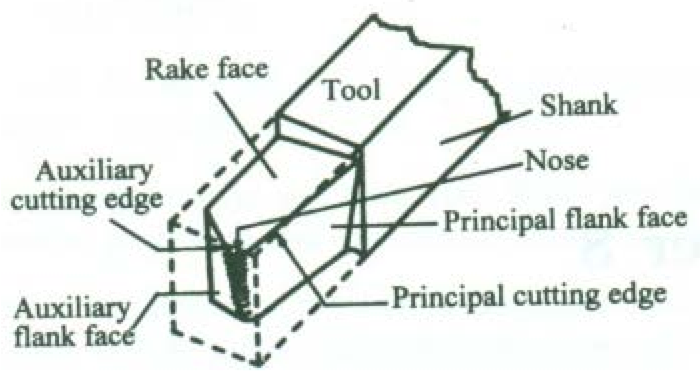




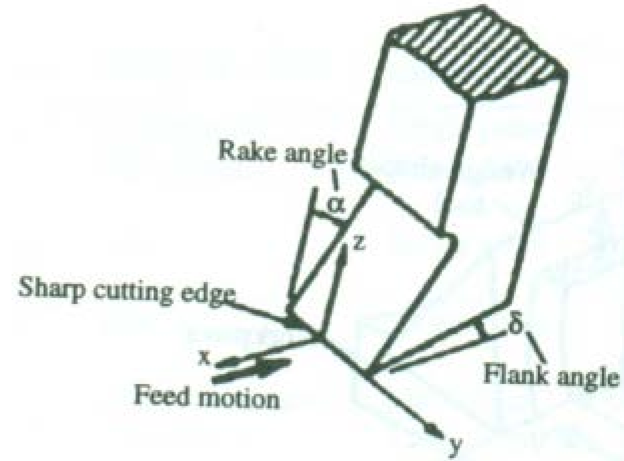
Two-dimensional cutting process, also called orthogonal cutting. Note that the tool shape and its angles, depth of cut, t_o, and the cutting speed, V, are all independent variables.

Dependent Variables (Responses): Forces, Temperature (workpiece, tool, chip), tool wear, etc.

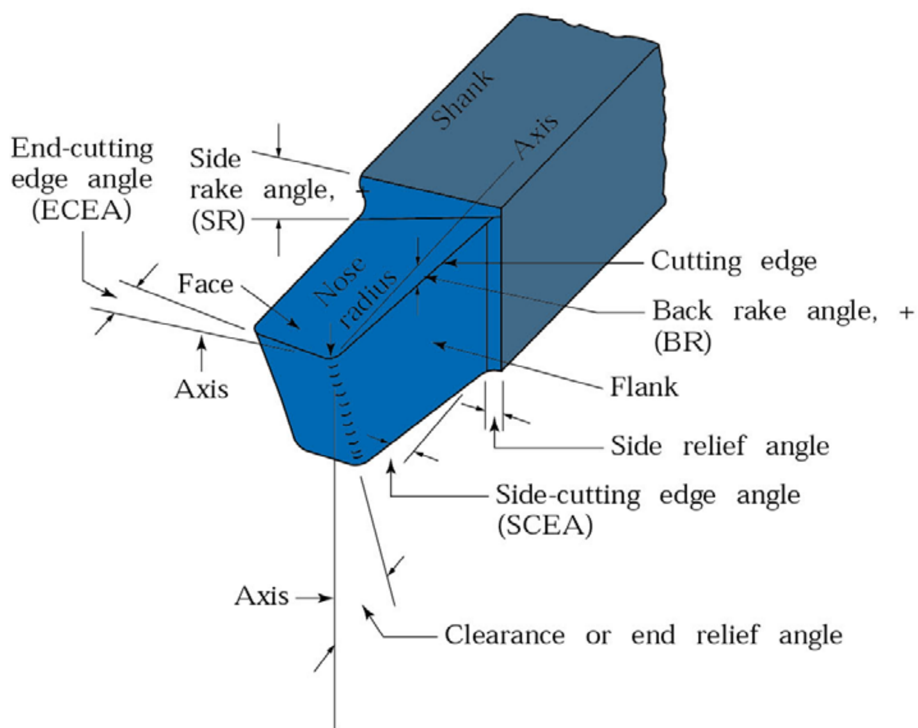
Cutting Tools and Types of Machining



A Typical Lathe Tool



Wedge-Shaped tool



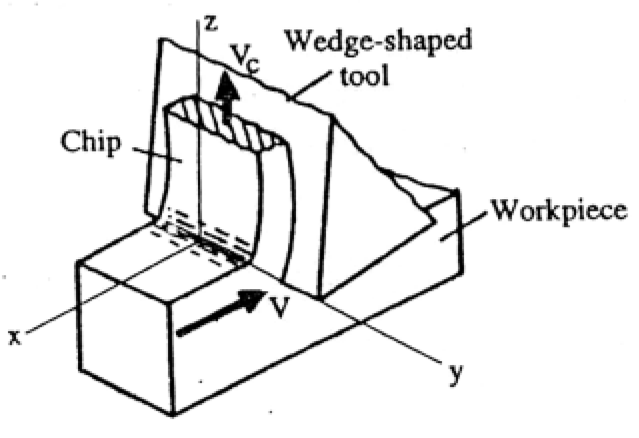
o Orthogonal Cutting (2-D Cutting):

Cutting edge is (1) straight,
(2)parallel to the original plane surface on the work piece and
(3)perpendicular to the direction of cutting. For example:
Operations: Lathe cut-off operation, Straight milling, etc.

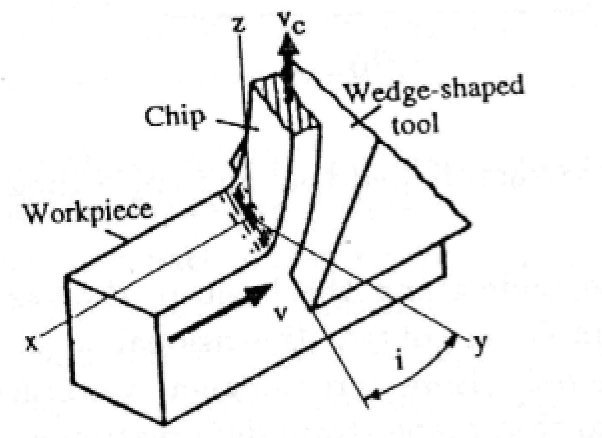
o Oblique Cutting (3-D Cutting):

Cutting edge of the tool is inclined to the line normal to the cutting direction.

In actual machining, Turning, Milling etc., cutting operations are oblique cutting(3-D)

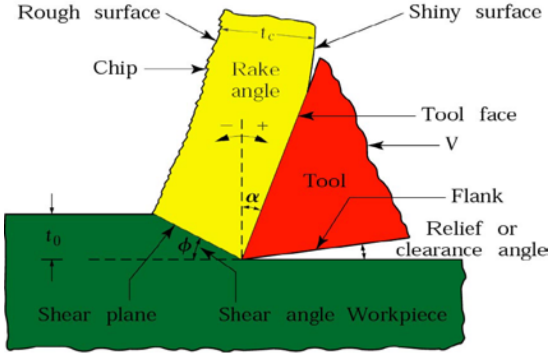


Orthogonal Cutting



Oblique Cutting

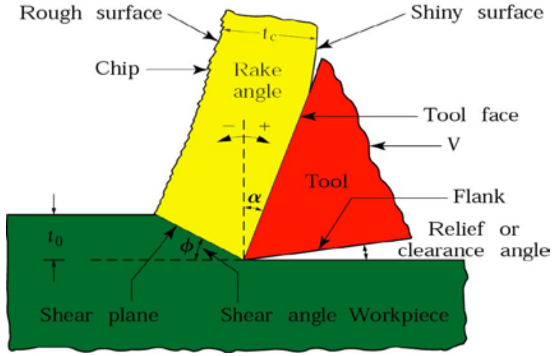
Tool Angles



Rake Angle (α)

- Influence cutting forces, power and surface finish
- Large α
  - lowers forces and improves surface finish
  - In general, power consumption ↓ by ~ 1% for 1 degree change in α
  - Has adverse effect on tool strength because less metal is available to support the tool
  - Greatly reduced capacity to conduct heat away from the cutting edge
  - Increases tool forces, but keeps the tool in compression and provides added support to the cutting edge

- 0 or negative rake angles employed on carbide, ceramic and similar "hard" tools
- Particularly important in making intermittent cuts and in absorbing impact during initial tool - workpiece contact
- Rake angles: 5 – 15 degrees for HSS; Lower for harder materials



Flank Angle

- Minimizes rubbing of flank faces with the machined surface
- Higher values of flank angle will reduce rubbing but also weaken the tool
- Flank angles have no influence on cutting forces and power. So angles large enough to avoid rubbing is generally chosen
- Angle: 5 – 12 degrees for HSS; higher for softer and lower for brittle material

Cutting Edge Angle

- Provided to clear the cutting edge from the machined surface
- To Reduce tool chatter
- Affects tool life as well as surface finish

Tool Parameter

Nose radius

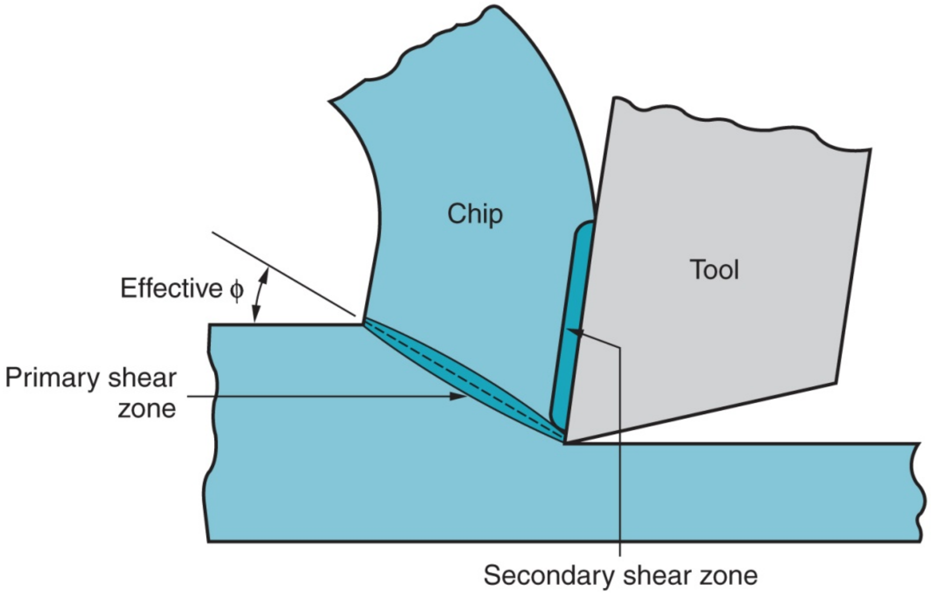
- ➢ Improves tool life and surface finish

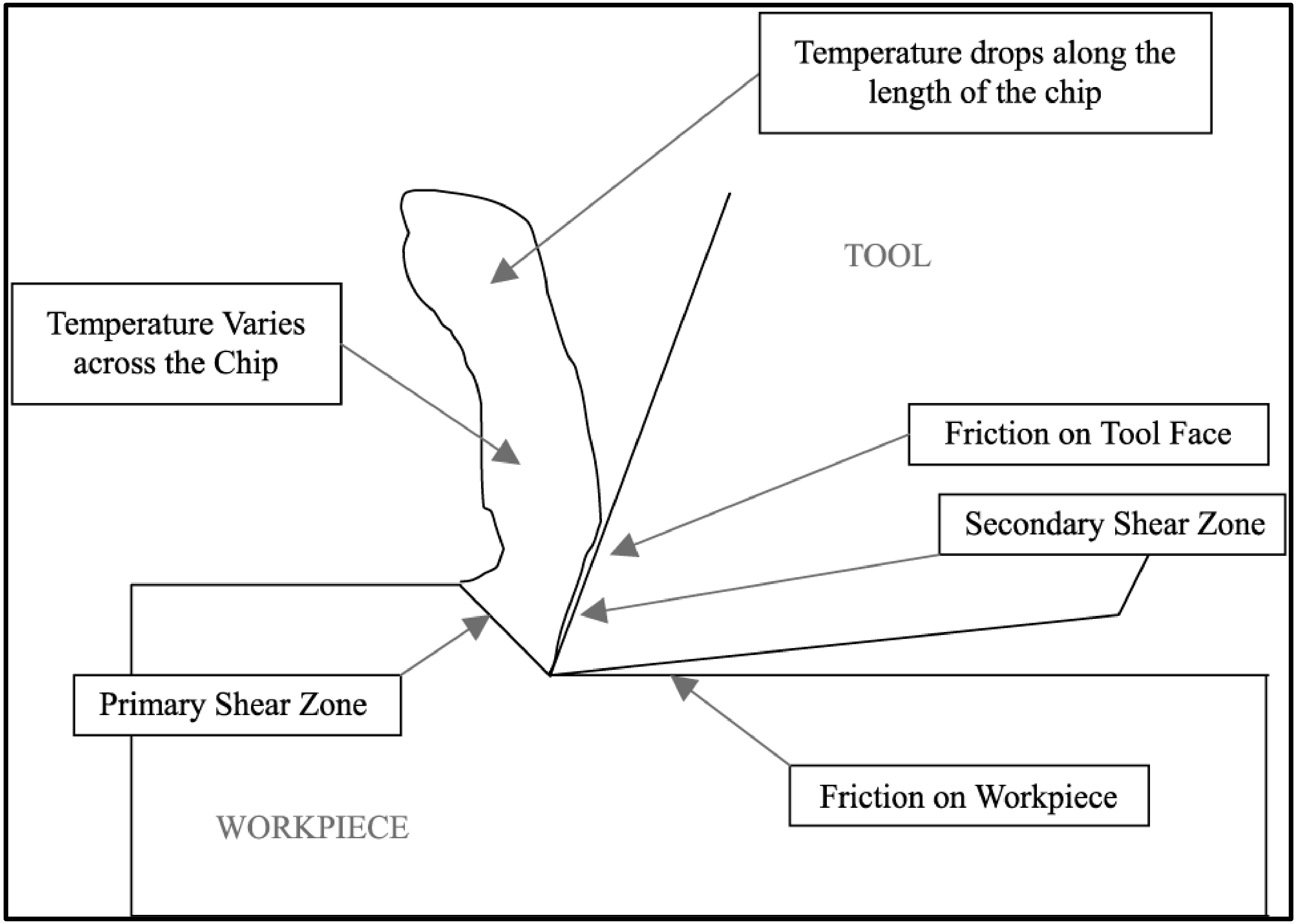
- Large nose radius
  - Increases cutting forces and power
  - Causes chatter (self-excited vibration)

- ➢ Recommended value: 1 – 3 mm

Chip Formation

- More realistic view of chip formation, showing shear zone rather than shear plane
- Also shown is the secondary shear zone resulting from tool-chip friction





Schematic of chip formation

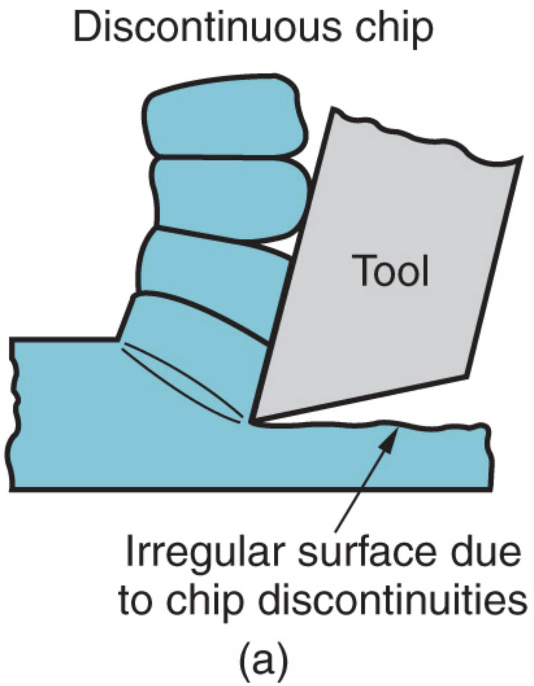
Four Basic Types of Chip in Machining

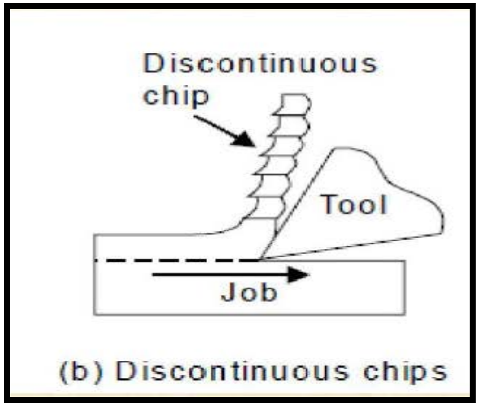
1. Discontinuous chip
2. Continuous chip
3. Continuous chip with Built-up Edge (BUE)
4. Serrated chip

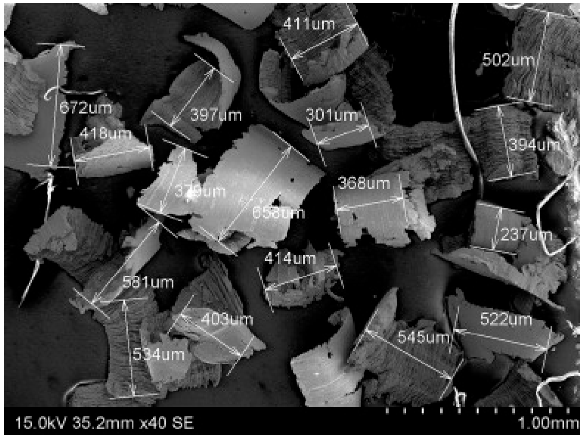
Discontinuous Chip

Conditions

- Brittle work materials
- Low cutting speeds
- Large feed and depth of cut
- Small rake angle
- High tool-chip friction







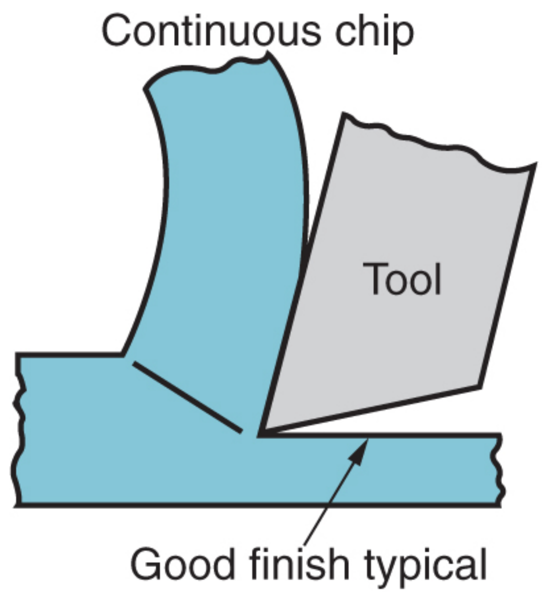
Chip in the form of discontinuous segments:

- ➢ **Easy disposal**
- ➢ **Good surface finish**

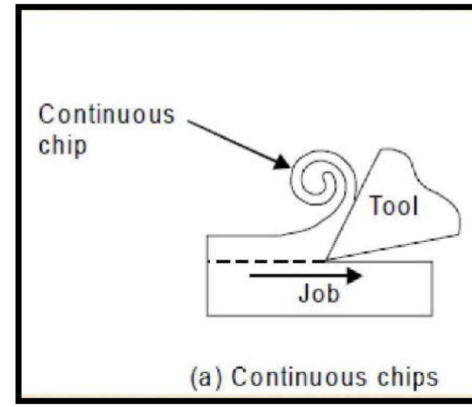
Continuous Chip

Conditions

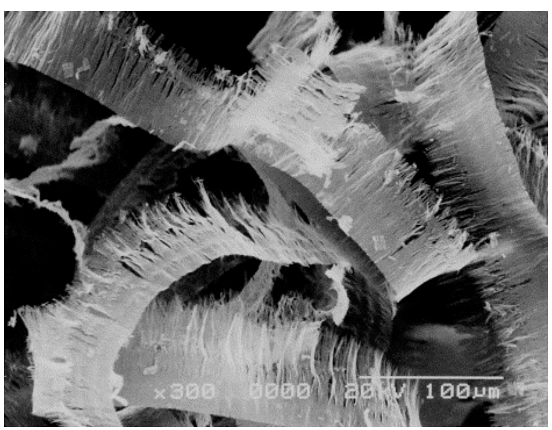
- Ductile work materials
- High cutting speeds
- Small feeds and depths
- Large rake angle (+ve)
- Sharp cutting edge
- Low tool-chip friction



(b)



(a) Continuous chips



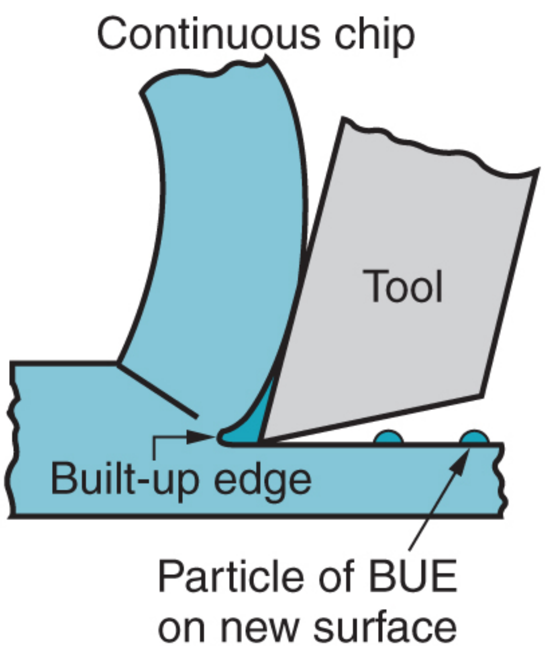
Continuous chip Results in:

- **Good surface finish**
- **High tool life**
- **Low power consumptions**

Continuous with BUE

Conditions

- Ductile materials
- Low-to-medium cutting speeds
- Large feed
- Small rake angle
- Tool-chip friction causes portions of chip to adhere to rake face
- BUE forms, then breaks off, cyclically



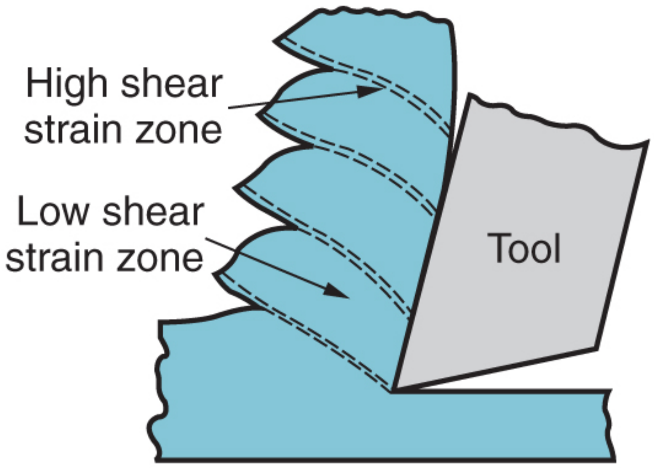
(c)

Built up Edge:

- **High friction between Tool & chip**
- **Particles of chip adhere to the rake face of the tool near cutting edge**
- **Some part of BUE get adhered to the machined surface hence poor surface finish**

Serrated Chip

- Semi-continuous - saw-tooth appearance
- Cyclical chip forms with alternating high shear strain then low shear strain
- Associated with difficult-to-machine metals at high cutting speeds



(d)

Types of Chips

(a) Continuous chip with narrow, straight primary shear zone

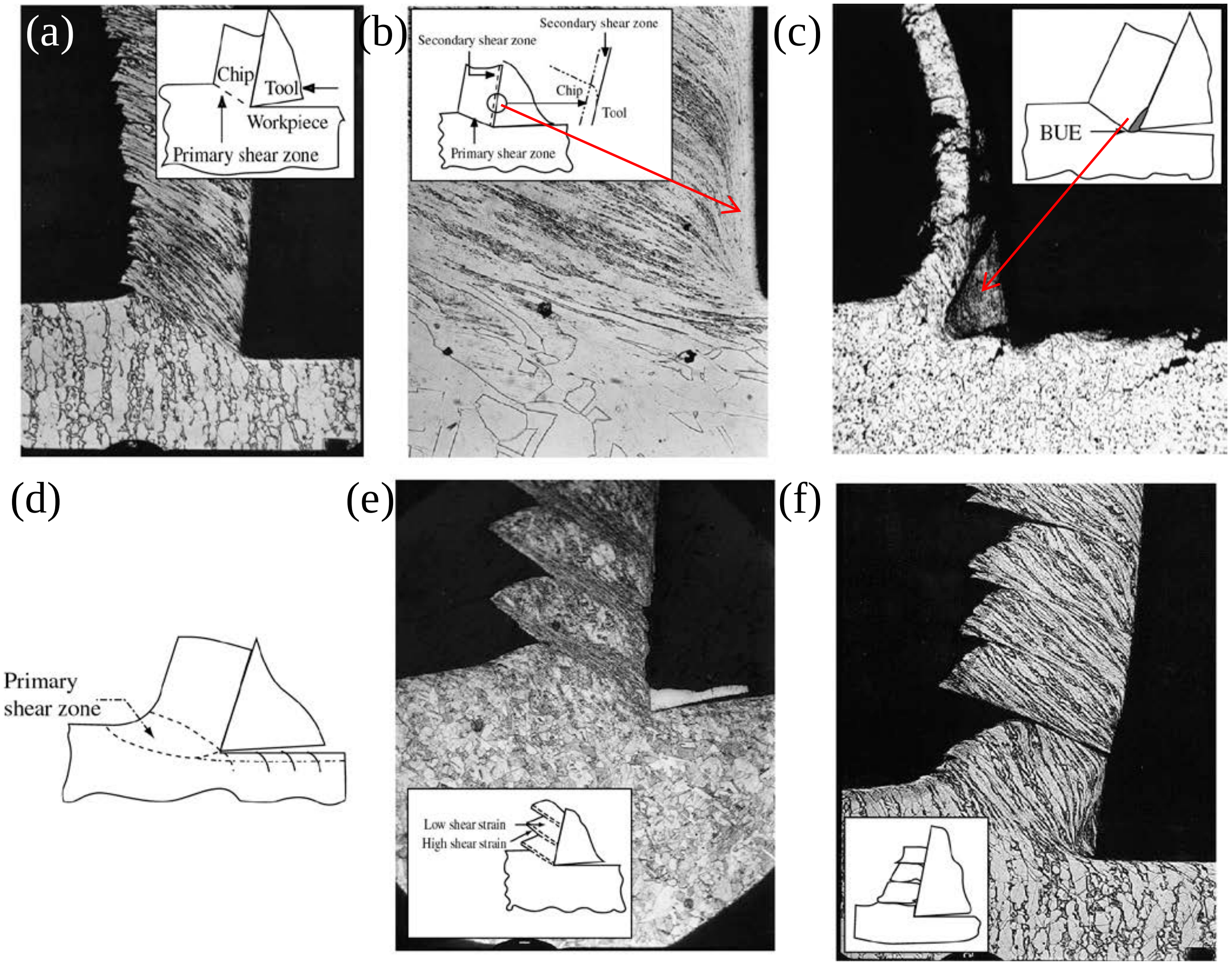
(b) Secondary shear zone at the chip-tool interface

(c) Continuous chip with built-up edge

(d) Continuous chip with large primary shear zone

(e) Segmented (Serrated) or nonhomogeneous chip and

(f) Discontinuous chip



Source: After M. C. Shaw, P. K. Wright, and S. Kalpakjian

Chip- Breaking

- **The chip breaker breaks the produced chips into small pieces.**
- **The work hardening of the chip makes the job of the chip breakers easy.**
- **When a strict chip control is desired, some sort of chip breaker has to be employed.**

- The following types of chip breakers are commonly used:

  a) Groove type
  b) Step type
  c) Secondary Rake type
  d) Clamp type

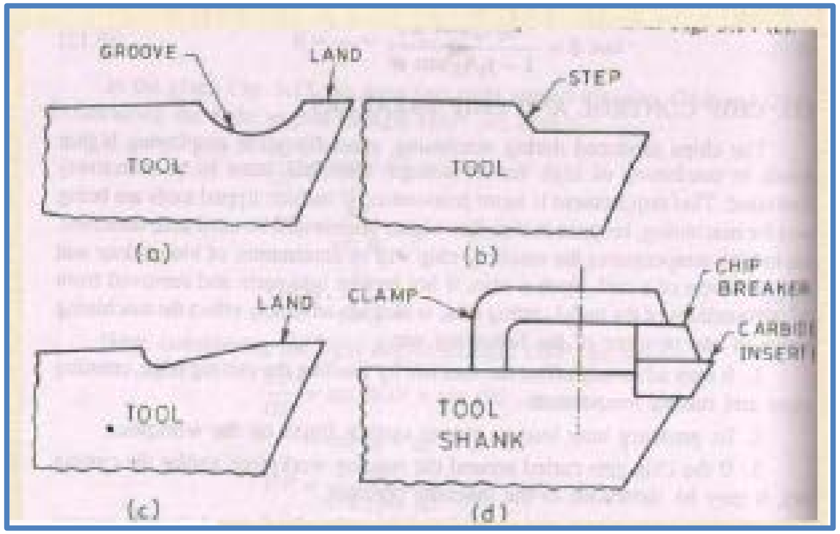
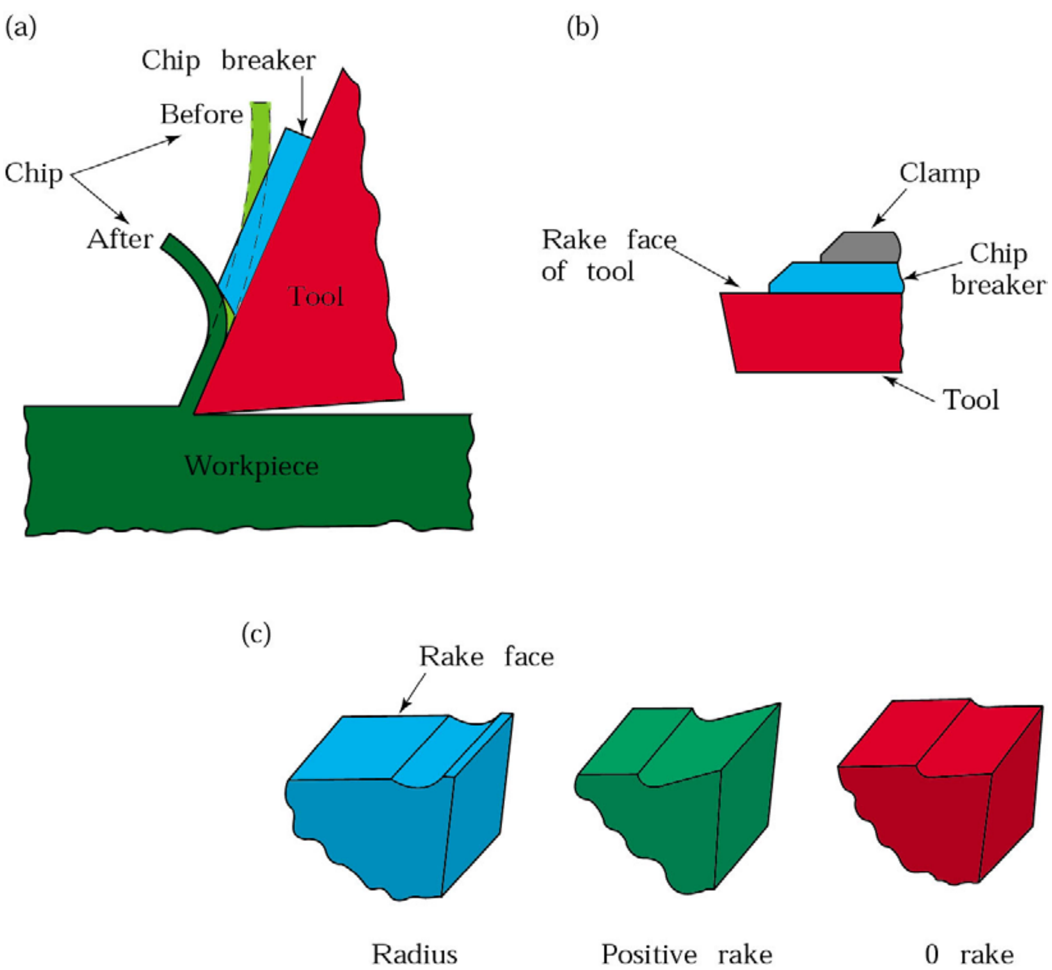


Fig: Schematics of different types of chip breakers



(a) Schematic illustration of the action of a chip breaker. Note that the chip breaker decreases the radius of curvature of the chip. (b) Chip breaker clamped on the rake face of a cutting tool. (c) Groove in a cutting tool acts as a chip breaker.

Cutting Conditions in Machining

- Three dimensions of a machining process
  - Cutting speed v – primary motion [m/min]

    Relates velocity of the cutting tool to the work piece
  - Feed f – secondary motion [mm/rev]

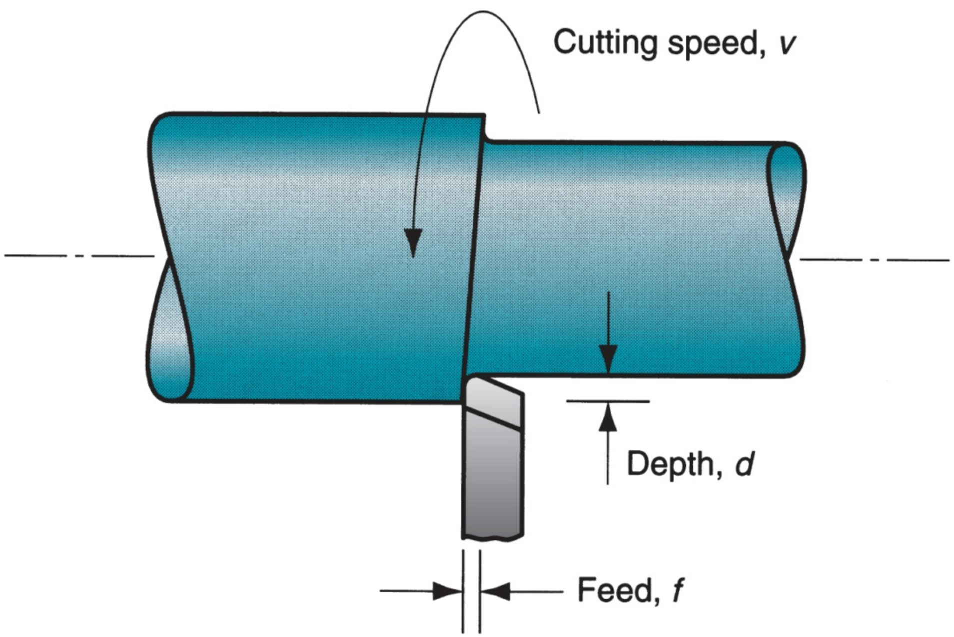
    Movement (advancement) of the tool per revolution of the workpiece
  - Depth of cut d – penetration of tool below original work surface [mm]

- For certain operations (e.g., turning), material removal rate R_{MR} (or MRR) can be computed as

$$R_{MR} = v\,f\,d$$

Cutting Conditions in Turning

- Speed, feed, and depth of cut in a turning operation



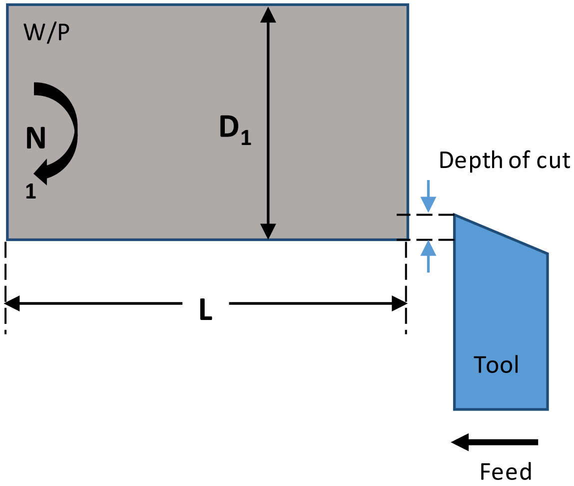
Roughing vs. Finishing Cuts

- In production, several roughing cuts are usually taken on a part, followed by one or two finishing cuts
  - Roughing - removes large amounts of material from starting workpart
    - Some material remains for finish cutting
    - High feeds and depths, low speeds
  - Finishing - completes part geometry
    - Final dimensions, tolerances, and finish
    - Low feeds and depths, high cutting speeds

Problem-1:

A turning operation has to be performed on an aluminum rod of diameter 50 mm and length 300 mm. The Spindle speed of lathe is given to be 500 RPM. The feed and depth of cut are 0.15 mm/rev and 0.3 mm, respectively. Draw a neat sketch of the turning operation described above. Find out the cutting speed in mm/s and the volumetric material removal rate (MRR_v) in mm^3 / s.

Solution:



Turning operation

$$N_1 = 500 RPM, D1 = 50mm$$

$$f_1 = 0.15 mm/rev$$

$$d_1 = 0.3 mm$$

$$Cutting Speed, \ V_c = \omega . R$$

$$V_c = \left[\frac{500 \times 2\pi}{60}\right] \times 25$$

$$V_c = 1308.9 mm/s$$

$$MRR_v = (\pi \times D_1 \times N_1) f_1 \cdot d_1 / 60$$

$$MRR_v = (V_c) f_1 \cdot d_1$$

$$MRR_v = 1308.9 \times 0.15 \times 0.3$$

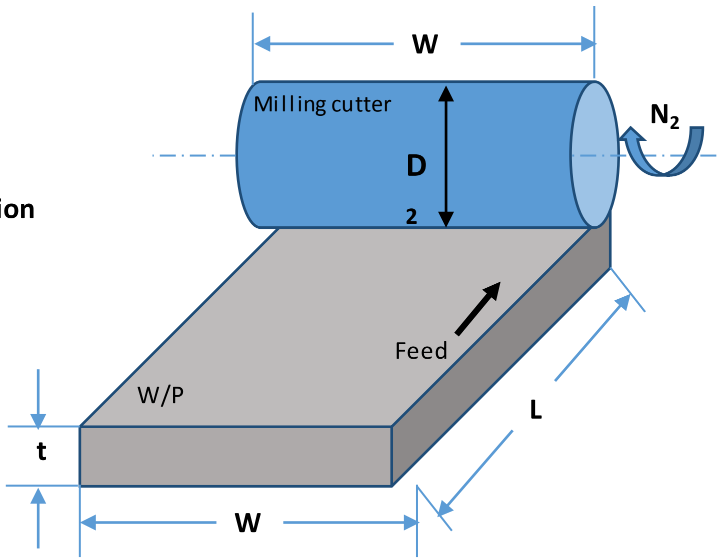
$$MRR_v = 58.905 \ mm^3 / s$$

Problem-2

An aluminum block of length 50 mm and width 70 mm is being milled using a slab milling cutter with 50 mm diameter. The feed of the table is 15 mm/min. The milling cutter rotates at 60 RPM in clockwise direction and width of cut is equal to the width of the workpiece. Draw a neat sketch of the milling operation describing above conditions. The thickness of the workpiece is 20 mm. If depth of cut of 2 mm is used then find out cutting speed and volumetric material removal rate (MRR_v).

Solution:

Milling operation



$$Milling\ Cutter\ Diameter, D_2 = 50mm$$

$$Width\ of\ cut, WOC = 70mm$$

$$Depth\ of\ cut, d_2 = 2mm$$

$$feed, f_2 = 15mm/\min$$

$$Cutting\ Speed,\ V_c = \frac{\pi D_2 N_2}{1000} m/\min$$

$$V_c = \left[\frac{50 \times \pi \times 60}{1000}\right]$$

$$V_c = 9.424m/\min$$

$$MRR_v = WOC \cdot f_2 \cdot d_2$$

$$MRR_v = 70 \times \frac{15}{60} \times 2$$

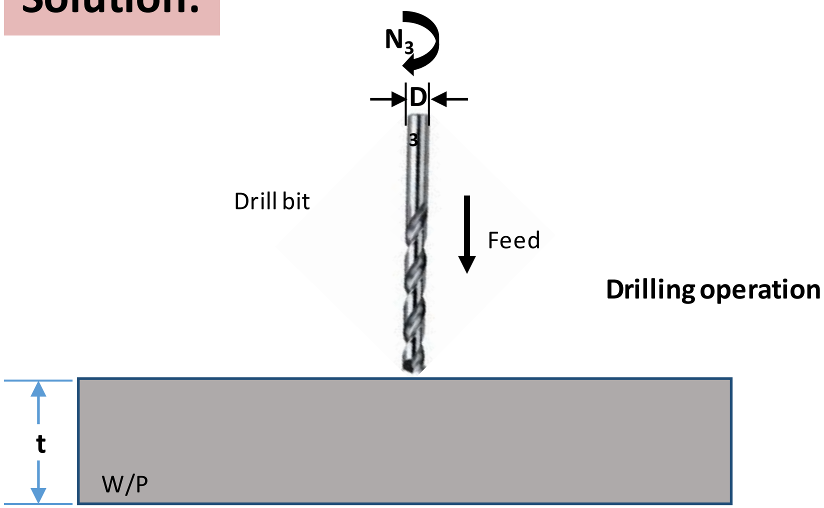
$$MRR_v = 35\ mm^3/s$$

Problem-3

Following the milling operation, a through hole is to be drilled on the same workpiece.

Find out the cutting speed and volumetric material removal rate if the drill of diameter 10 mm is being rotated at the same RPM as in case of milling cutter with feed rate as 0.5 mm/rev.

Solution:



$$Diameter\ of\ \ Drill, D_3 = 10mm$$

$$N_3 = 60 RPM$$

$$feed,\ f_3 = 0.5 mm/rev$$

$$Cutting\ \ Speed,\ V_c = \frac{\pi N_3 D_3}{1000} m/\min$$

$$V_c = \left[\frac{\pi \times 60 \times 10}{1000}\right] m/\min$$

$$V_c = 1.884 m/\min = 31.4 mm/s$$

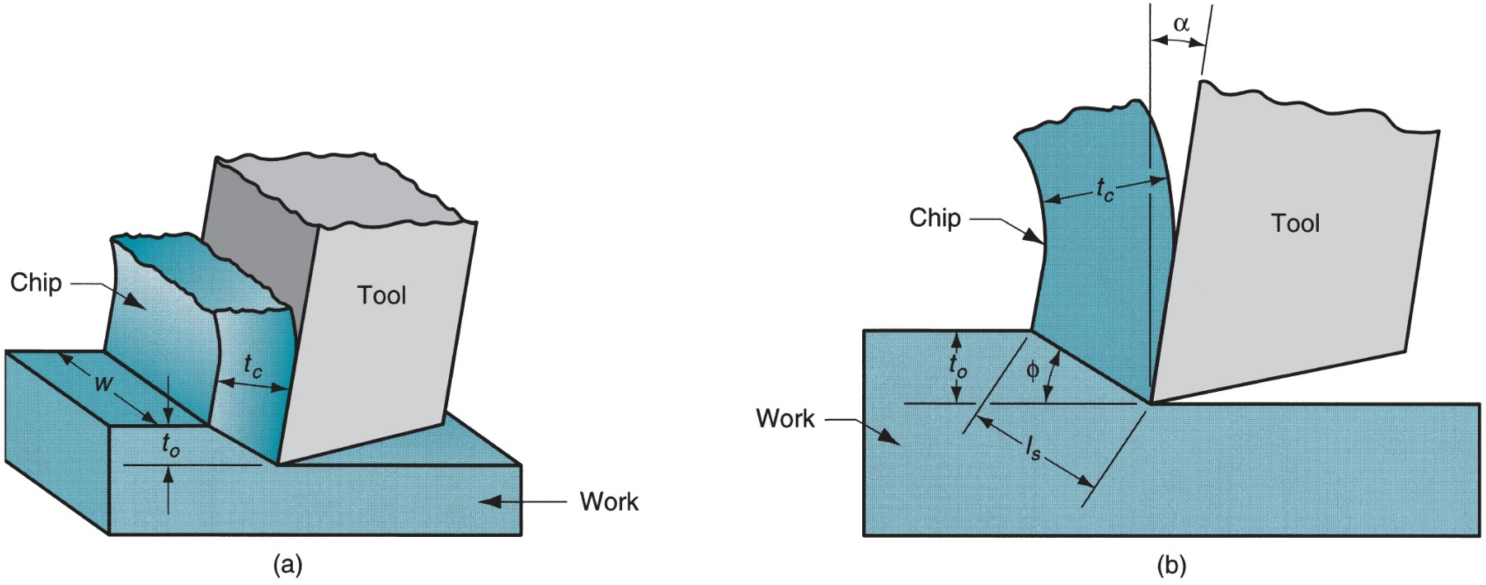
$$MRR_v = \frac{\pi \times D_3^{\,2}}{4} \times f_3 \times N_3$$

$$MRR_v = \frac{\pi \times 10^2}{4} \times 0.5 \times 60$$

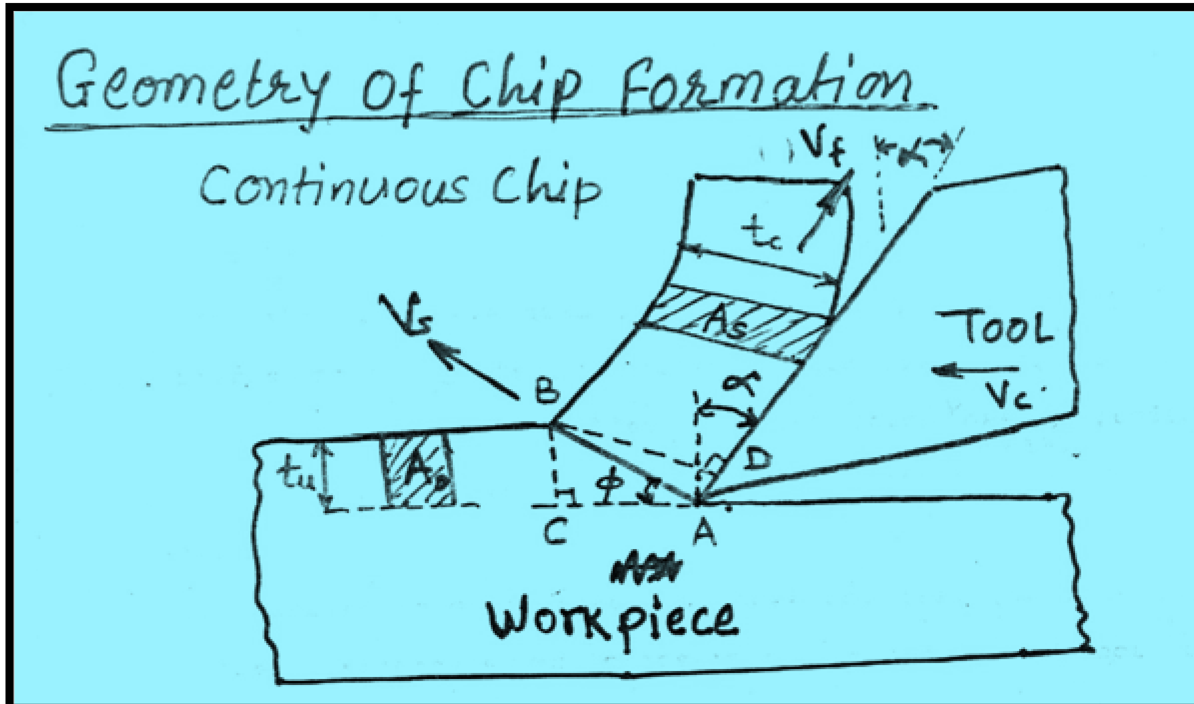
$$MRR_v = 2356.19\ mm^3/\min = 39.27 mm^3/s$$

Orthogonal Cutting Model

- Simplified 2-D model of machining that describes the mechanics of machining fairly accurately



Geometry of chip Formation



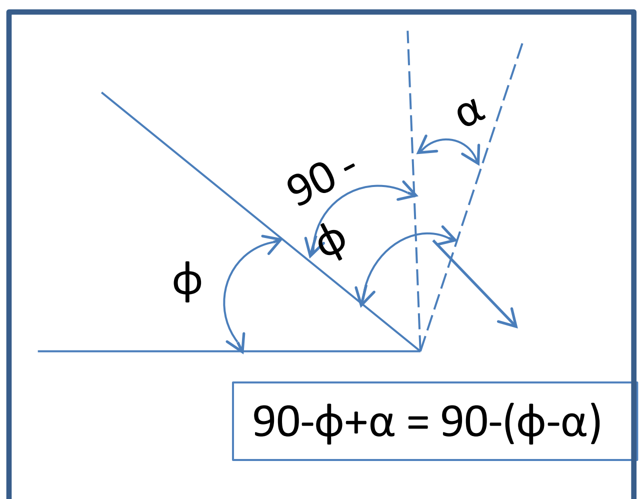
t_c	: *Chip thickness*
t_u	: *Uncut chip thickness*
V_f	: *Chip Sliding Velocity*
V_s	: *Shear Velocity*
V_c	: *Cutting Velocity*
ϕ	: *Shear Angle*

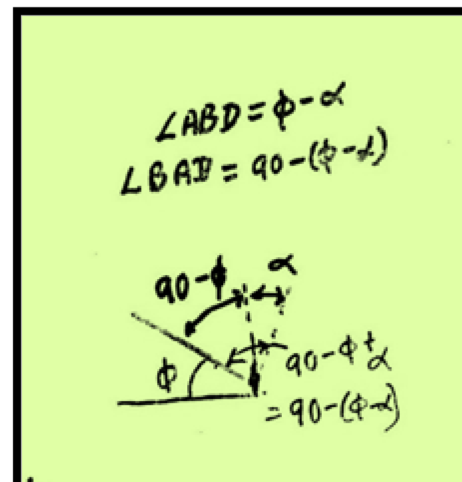
ΔABC & ΔABD

$$AB = \frac{t_u}{sin\phi}$$

$$also,\ AB = \frac{t_c}{\sin(90-(\phi-\alpha))} = \frac{t_c}{\cos(\phi-\alpha)}$$

$$\frac{t_u}{t_c} = \frac{\sin\phi}{\cos(\phi-\alpha)}$$





SHEAR ANGLE AND CHIP THICKNESS RATIO EVALUATION

$$r_c = \frac{t_u}{t_c} : Chip\ thickness\ Ratio / Coefficient$$

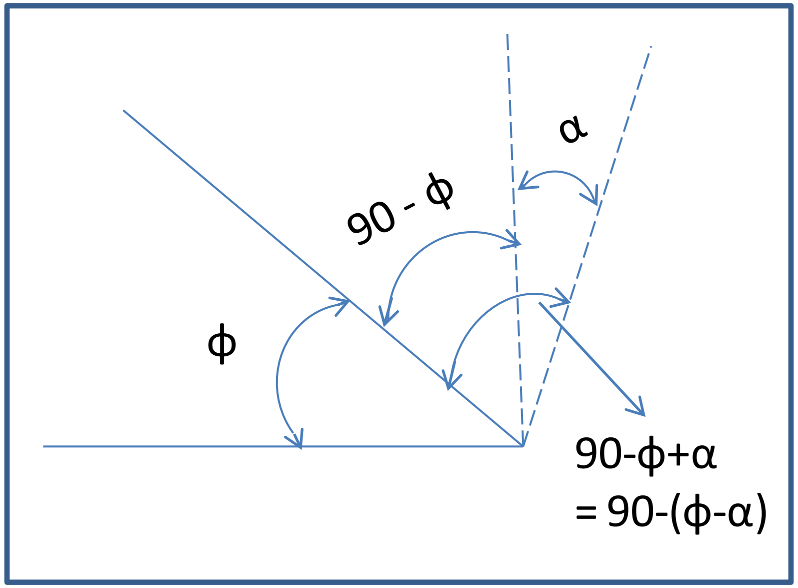
Substitute the value of t_u / t_c from earlier slide and simplify to get:

$$\frac{1}{r_c} = \frac{\cos\phi \cos\alpha + \sin\phi \sin\alpha}{\sin\phi}$$

$$1 = r_c \cot\phi \cos\alpha + r_c \sin\alpha$$

$$r_c \cos\alpha = (1 - r_c \sin\alpha) \tan\phi$$

$$\therefore \boxed{\tan\phi = \left(\frac{r_c \cos\alpha}{1 - r_c \sin\alpha} \right)}$$



How to determine ϕ & r_c ?

t_c should be determined from the chip. t_u (= feed) and α are already known.

To determine **t_c** with micrometer, is difficult and **not so accurate** because of uneven surface. **How?** *(say, f=0.2 mm/rev. An error of even 0.05 mm will cause an error of 25 % in the measurement of t_c)*

Volume Constancy Condition: Volume of Uncut chip = Volume of cut chip

$$L_u t_u b = L_c t_c b$$

$$\therefore L_c t_c = L_u t_u$$

$$or, r_c = \frac{t_u}{t_c} = \frac{L_c}{L_u}$$

L_c = Chip length

L_u = Uncut chip length

b = Chip width

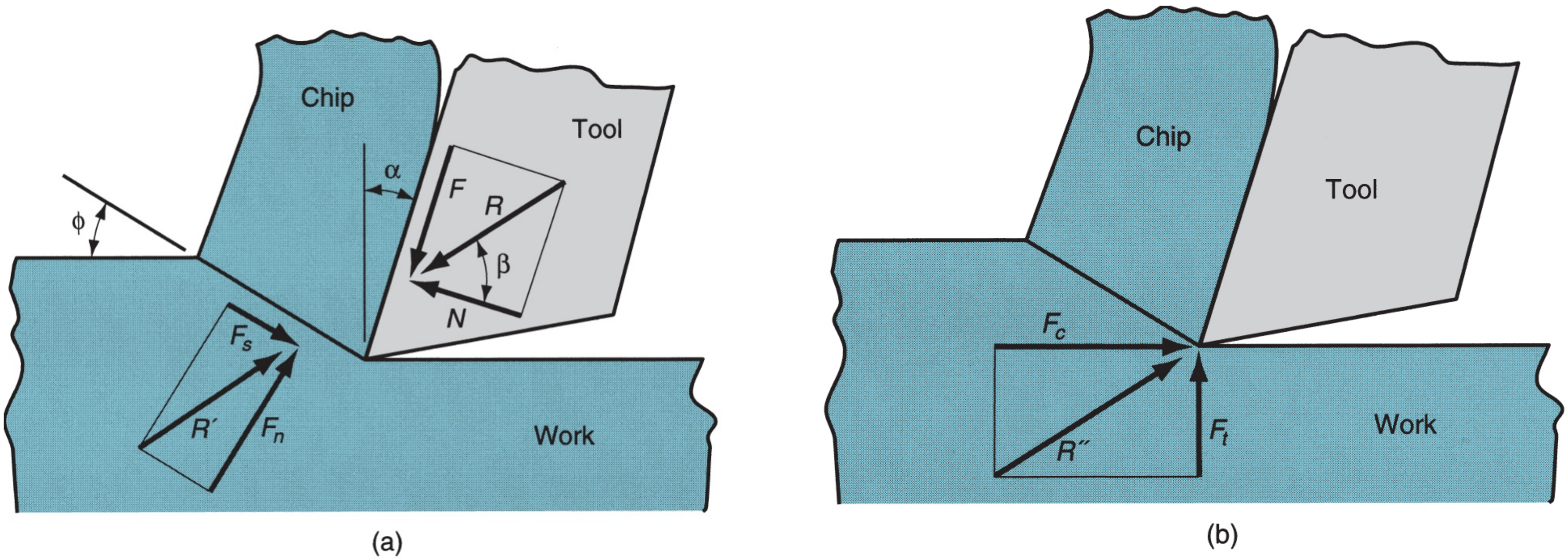
(2-D Cutting)

LENGTH OF THE CHIP MAY BE MANY CENTIMETERS HENCE THE ERROR IN EVALUTION OF r_c WILL BE COMPARATIVELY MUCH LOWER.

$r_c = L_c / L_u$

- Chip thickness after cut is always greater than before, so chip ratio is always less than 1.0
- Why is $t_c > t_o$?

Forces in Machining



F = Frictional force between the tool and chip

N = Normal force

β = Friction angle

F_S = Shear force

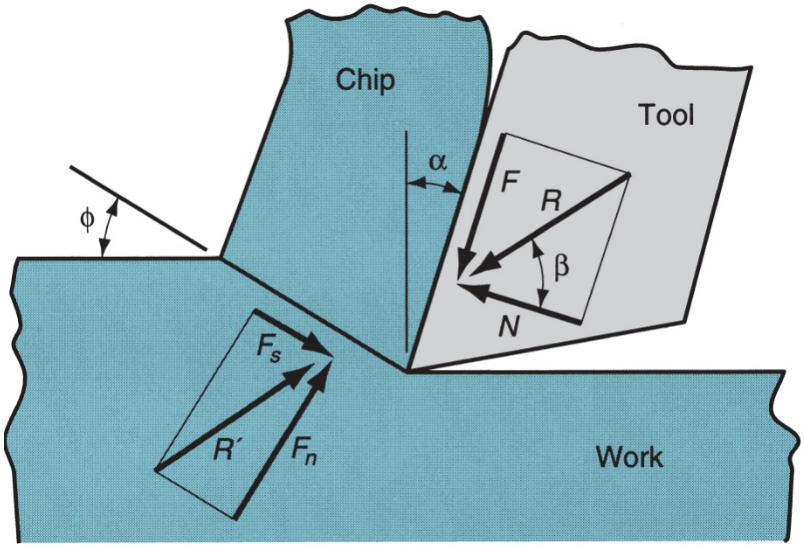
F_n = Normal force to shear

F_C = Cutting force

F_t = Thrust force

Forces Acting on Chip

- (a) Friction force F and Normal force to friction N
- (b) Shear force F_s and Normal force to shear F_n



(a)

Coefficient of Friction

- Coefficient of friction between tool and chip

$$\mu = \frac{F}{N}$$

- Friction angle related to coefficient of friction as

$$\mu = \tan \beta$$

Shear Stress

- Shear stress acting along the shear plane

$$S = \frac{F_s}{A_s}$$

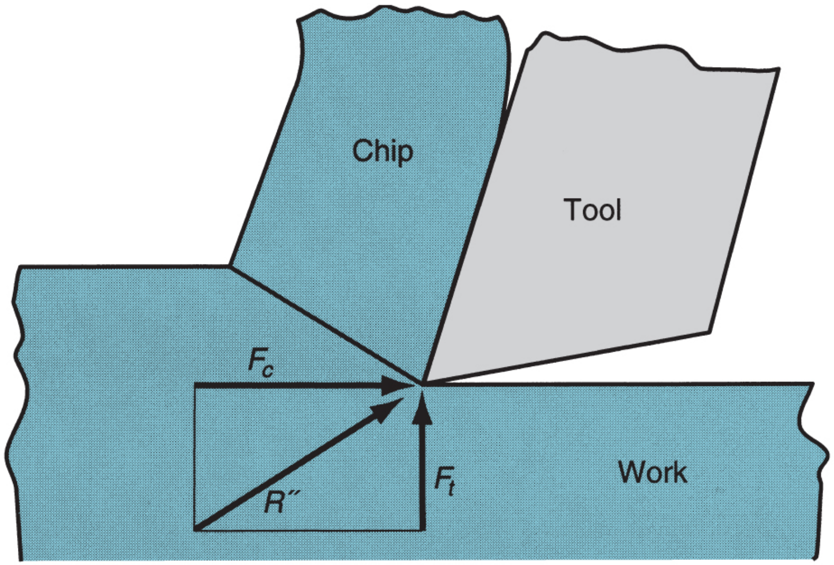
where A_s = area of the shear plane

$$A_s = \frac{t_o w}{\sin\phi}$$

- Shear stress = shear strength of work material during cutting

Cutting Force and Thrust Force

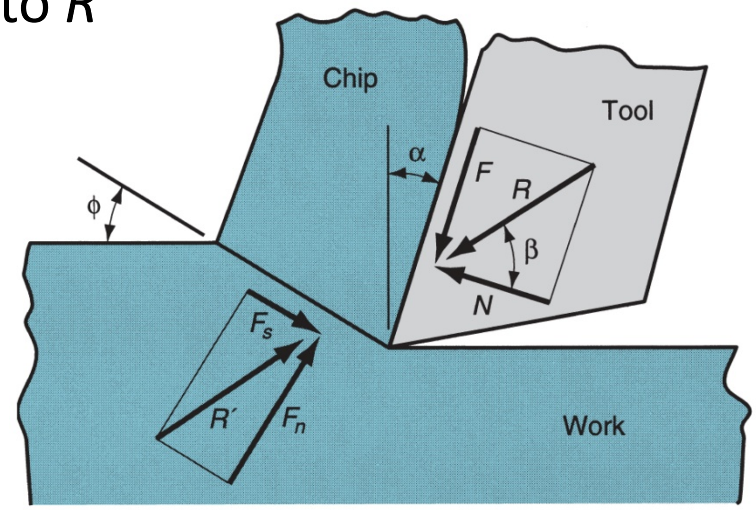
- F, N, F_s, and F_n cannot be directly measured
- Forces acting on the tool that can be measured: Cutting force F_c and Thrust force F_t



(b)

Resultant Forces

- Vector addition of F and N = resultant R
- Vector addition of F_s and F_n = resultant R'
- Forces acting on the chip must be in balance:
  - R' must be equal in magnitude to R
  - R' must be opposite in direction to R
  - R' must be collinear with R



(a)

Approximation of Turning by Orthogonal Cutting

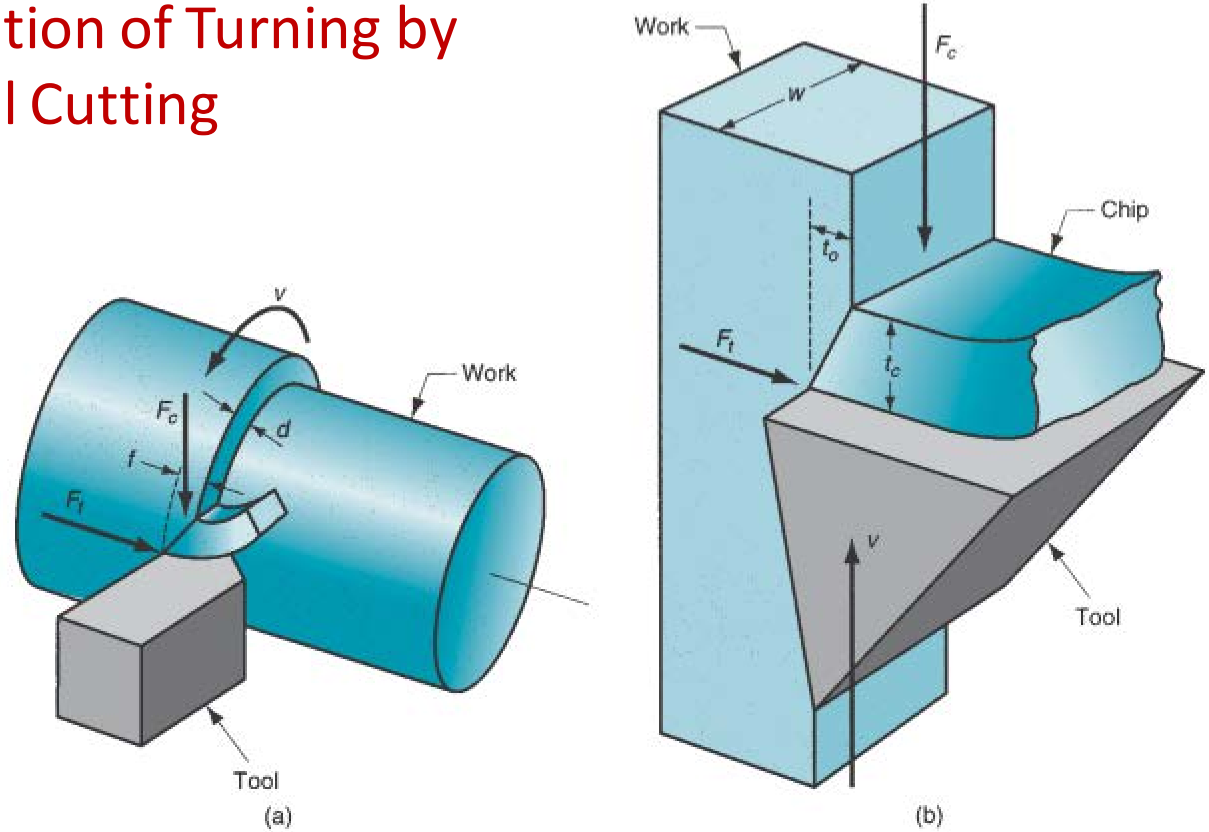


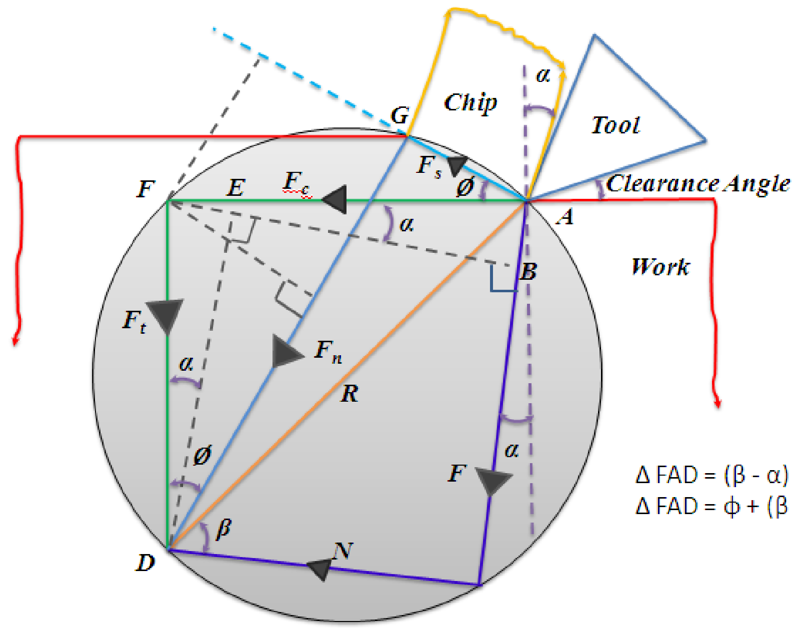
TABLE 21.1 Conversion key: turning operation vs. orthogonal cutting.

Turning Operation	Orthogonal Cutting Model
Feed $f =$	Chip thickness before cut t_o
Depth $d =$	Width of cut w
Cutting speed $v =$	Cutting speed v
Cutting force $F_c =$	Cutting force F_c
Feed force $F_f =$	Thrust force F_t

Force Analysis

Forces in Orthogonal Cutting:

- Friction force, F
- Force normal to Friction force, N
- Cutting Force, F_C
- Thrust force, Ft
- Shear Force, F_S
- Force Normal to shear force, F_n
- Resultant force, R



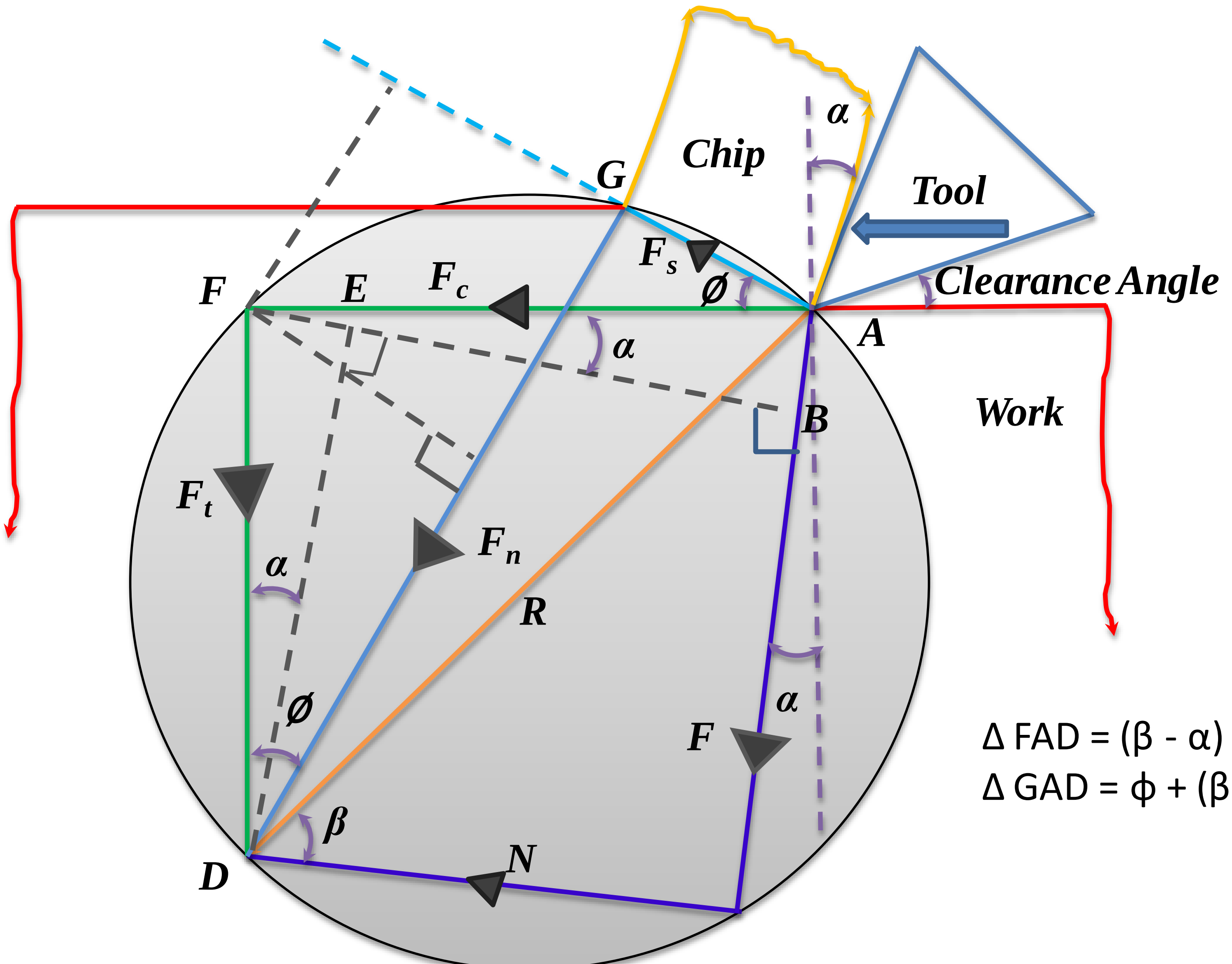
Δ FAD = (β - α)
Δ FAD = ϕ + (β - α)

FREE BODY DIAGRAM

$$R' = \vec{F} + \vec{N}$$

$$R = \vec{F}_S + \vec{F}_N = \vec{F}_c + \vec{F}_t = \vec{R}'$$

FORCE CIRCLE DIAGRAM



Δ FAD = $(\beta - \alpha)$

Δ GAD = $\phi + (\beta - \alpha)$

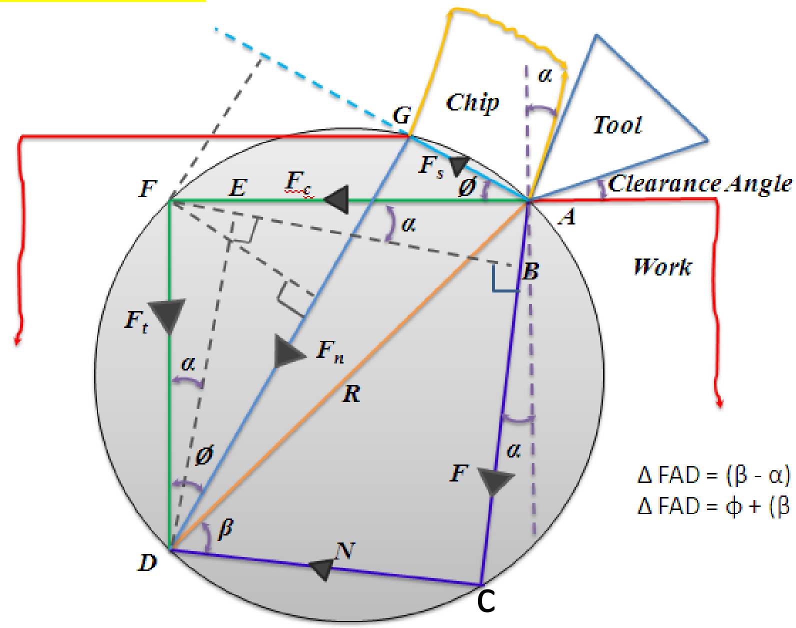
Force Analysis

F=CB+BA

$$F = F_t \cos\alpha + F_c \sin\alpha$$

N=BE-EF

$$N = F_c \cos\alpha - F_t \sin\alpha$$



Δ FAD = (β - α)

Δ FAD = ϕ + (β - α)

$$Coefficient\ of\ Friction\ (\mu)$$

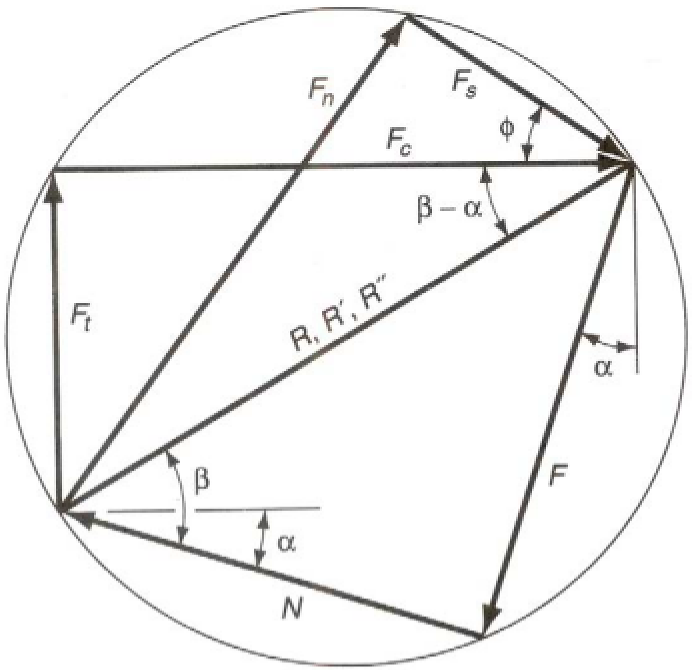
$$\mu = \tan\beta = \frac{F}{N} = \frac{F_t \cos\alpha + F_c \sin\alpha}{F_c \cos\alpha - F_t \sin\alpha}$$

$$\beta = Friction\ Angle$$

DIVIDE R.H.S. BY Cos α

$$\mu = \frac{F_t + F_c \tan\alpha}{F_c - F_t \tan\alpha} \qquad also,\ \beta = \tan^{-1}(\mu)$$

Merchant's Circle Diagram



$$Fs = RCos(\emptyset + \beta - \alpha)$$

$$Fn = RSin(\emptyset + \beta - \alpha)$$

$$Fc = RCos(\beta - \alpha)$$

$$F_T = R\sin(\beta - \alpha)$$

$$F = R\sin\beta$$

$$N = R\cos\beta$$

Expressing through F_c,

$$F_S = F_C\ Cos\ \emptyset - F_T\ Sin\emptyset$$

$$F_n = F_C Sin\emptyset + F_T Cos\emptyset$$

$$F = F_C\ Sin \propto + F_T\ Cos \propto$$

$$N = F_C\ Cos \propto - F_T\ Sin \propto$$

$$R = \frac{F_C}{\cos(\beta - \alpha)} = \frac{F_S}{\cos(\emptyset + \beta - \alpha)}$$

$$F_C = \frac{F_S \cos(\beta - \alpha)}{\cos(\emptyset + \beta - \alpha)}$$

Forces in Metal Cutting

- Equations to relate the forces that cannot be measured to the forces that can be measured:

$$F = F_c \sin\alpha + F_t \cos\alpha$$

$$N = F_c \cos\alpha - F_t \sin\alpha$$

$$F_s = F_c \cos\phi - F_t \sin\phi$$

$$F_n = F_c \sin\phi + F_t \cos\phi$$

- Based on these calculated force, shear stress and coefficient of friction can be determined

Force Analysis

$\boxed{F_S = AH - GH}$

$$F_S = F_c \cos\phi - F_t \sin\phi$$

$\boxed{\mathbf{F_N = AJ + JG}}$

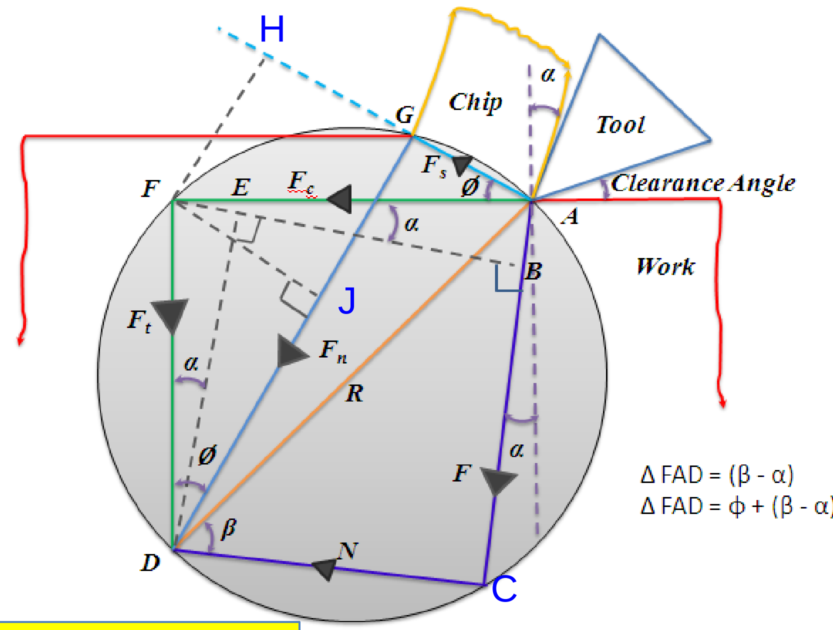
$$F_N = F_t \cos\phi + F_c \sin\phi$$

also,

$$F_C = R\cos(\beta - \alpha)$$

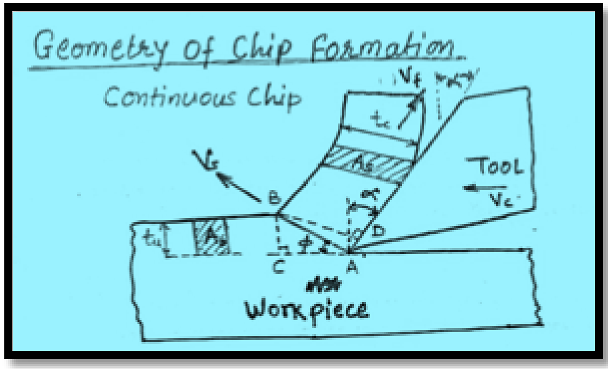
$$F_S = R\cos(\phi + \beta - \alpha)$$

$$\therefore \frac{F_C}{F_S} = \frac{\cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$



Δ FAD = (β - α)
Δ GAD = ϕ + (β - α)

$$Shear Plane Area\ (A_S) = \frac{t_u b}{\sin\phi}\left(= \frac{t_u}{\sin\phi} \times b\right)$$



Force Analysis

Let τ be the strength of work material

$$F_S = A_S \tau = \frac{t_u b}{sin\phi} \tau$$

$$\mathrm{F_C} = \left(\frac{t_u b \tau}{sin\phi} \right) \left(\frac{cos(\beta - \alpha)}{cos(\phi + \beta - \alpha)} \right) \quad and, \quad R = \left(\frac{t_u b \tau}{sin\phi} \right) \times \left(\frac{1}{cos(\phi + \beta - \alpha)} \right)$$

$$F_t = R \sin(\beta - \alpha) = \frac{t_u b \tau}{sin\phi} \times \frac{sin(\beta - \alpha)}{cos(\phi + \beta - \alpha)}$$

$$\boxed{\frac{F_t}{F_c} = \tan(\beta - \alpha)}$$

Force Analysis

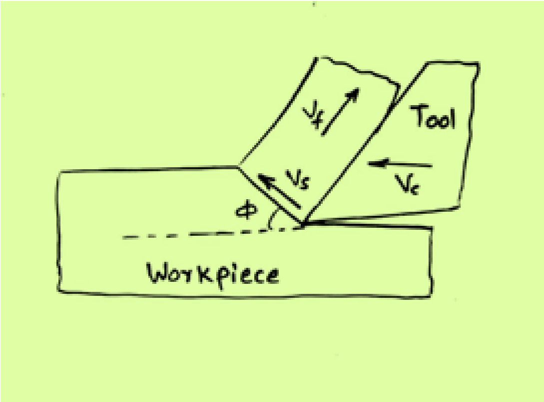
$$Mean\ Shear\ Stress\ (t_{chip}) = \frac{F_S}{A_S}$$
$$(On\ Chip)$$

$$= \frac{(F_c\ cos\phi - F_t\ sin\phi)\sin\phi}{b\ t_u}$$

$$Mean\ Normal\ Stress\ (\sigma_{chip}) = \frac{F_N}{A_S}$$
$$(On\ Chip)$$

$$= \frac{(F_t\ cos\phi + F_c\ sin\phi)\sin\phi}{b\ t_u}$$

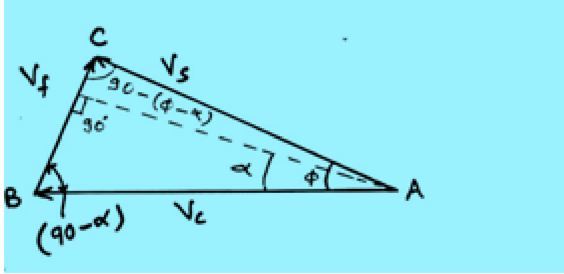
VELOCITY ANALYSIS



V_c : *Cutting velocity of tool relative to workpiece*

V_f : *Chip flow velocity*

V_s : *Shear velocity*



Using sine Rule:

$$\frac{V_c}{\sin(90-(\phi-\alpha))} = \frac{V_f}{\sin\phi} = \frac{V_s}{\sin(90-\alpha)}$$

$$\frac{V_c}{\cos(\phi-\alpha)} = \frac{V_f}{\sin\phi} = \frac{V_s}{\cos\alpha} \quad \textit{and} \quad V_f = \frac{V_c \sin\phi}{\cos(\phi-\alpha)} = V_c \cdot r_c$$

$$\boxed{V_s = \frac{V_c \cos\alpha}{\cos(\phi-\alpha)} \Rightarrow \frac{V_s}{V_c} = \frac{\cos\alpha}{\cos(\phi-\alpha)}}$$

Shear Strain & Strain Rate

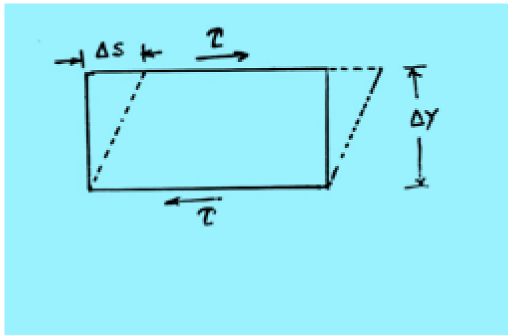
Thin Zone Model: Merchant

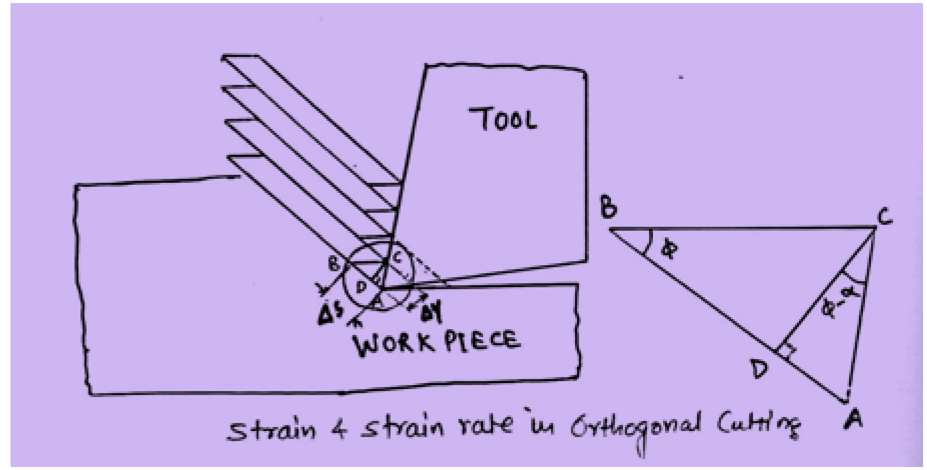
ASSUMPTIONS:-

- Tool tip is sharp, No Rubbing, No Ploughing
- 2-D deformation.
- Stress on shear plane is uniformly distributed.
- Resultant force R on chip applied at shear plane is equal, opposite and collinear to force R' applied to the chip at tool-chip interface.

Expression for Shear Strain

The deformation can be idealized as a process of block slip (or preferred slip planes)





$$ShearStrain(\gamma) = \frac{deformation}{Length}$$

DAC = (90-Φ+α)

$$\gamma = \frac{\Delta s}{\Delta y} = \frac{AB}{CD} = \frac{AD}{CD} + \frac{DB}{CD} \qquad = \tan(\phi - \alpha) + \cot\phi$$

$$\frac{\sin(\phi-\alpha)\sin\phi + \cos\varphi\cos(\phi-\alpha)}{\sin\phi\cos(\phi-\alpha)},$$

$$\boxed{\therefore \gamma = \frac{\cos\alpha}{\sin\phi\cos(\phi-\alpha)}}$$

Expression for Shear Strain rate

In terms of shear velocity (V_s) and chip velocity (V_f), it can be written as

$$\therefore \quad \gamma = \frac{V_s}{V_c \sin\phi} \qquad \left(\text{since} \frac{V_s}{V_c} = \frac{\cos\alpha}{\cos(\phi - \alpha)} \right)$$

$$\textit{Shear strain rate } (\dot{\gamma})$$

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{\left(\dfrac{\Delta s}{\Delta y}\right)}{dt} = \left(\frac{\Delta s}{\Delta y}\right)\frac{1}{\Delta t}$$

$$= \frac{V_s}{\Delta y} = \frac{V_c \cos\alpha}{\cos(\phi - \alpha)\Delta y}$$

$$\textit{where, } \Delta y : \textit{Mean thickness of PSDZ}$$