Ex. 1: $\vec{V} = \hat{i}u + \hat{j}v = \hat{i}(0.5 + 0.8x) + \hat{j}(1.5 - 0.8y)$ in m/s

$u = 0.5 + 0.8x$
$v = 1.5 - 0.8y$

Find stagnation pt. and velocity vector at various pts in the domain; Check if the flow is steady / unsteady.

At stagnation pt, $u = v = 0 \Rightarrow 0.5 + 0.8x = 0$
$\Rightarrow x = -0.625$ m

$1.5 - 0.8y = 0$
$\Rightarrow y = 1.875$ m

There is a stagnation pt, the co-ordinate is $(-0.625\text{m}, 1.875\text{m})$

Velocity vector has been found out by calculating resultant vector at any x & y and angle α with x-direction using $\tan^{-1}\left(\frac{v}{u}\right)$. For example at pt $x = 2$m, $y = 2$m, $u = 0.5 + 0.8(2) = \underline{2.1 \text{ m/s}}$, $v = 1.5 - 0.8(2) = -0.1$ m/s

Resultant vel $= (u^2 + v^2)^{1/2} = \left[(2.1)^2 + (-0.1)^2\right]^{1/2}$

$= 2.102$ m/s, Angle $\alpha = \tan^{-1}\left(-\frac{0.1}{2.1}\right) = -2.7°$

since $u \neq f(t)$ & $v \neq f(t) \Rightarrow$ The flow is steady.

Ex. 2: Find the acceleration using velocity at Ex-1. at (2m, 2m)

$\frac{\partial u}{\partial t} = 0$, $\frac{\partial v}{\partial t} = 0$
$\frac{\partial u}{\partial x} = 0.8$, $\frac{\partial v}{\partial x} = 0$
$\frac{\partial u}{\partial y} = 0$, $\frac{\partial v}{\partial y} = -0.8$

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = 0 + (0.5 + 0.8x)(0.8)$$
$$= 0.4 + 0.64x$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = 0 + 0 + (1.5 - 0.8y)(-0.8)$$
$$= -1.2 + 0.64y$$

Accl^n at $x = 2$m, $y = 2$m, $\vec{a} = a_x\hat{i} + a_y\hat{j} = (1.68\hat{i} + 0.08\hat{j})$ m/s^2

Ex. 3: Draw streamline after finding equation of streamline using velocity given in Ex. 1.

Equ^n of streamline $\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{(0.5 + 0.8x)} = \frac{dy}{(1.5 - 0.8y)}$

Integrating the equ^n (using separation of variable)

$$y = \frac{c}{0.8(0.5 + 0.8x)} + 1.875$$

Ex. 4: $\vec{V} = \hat{i}u + \hat{j}v = (0.5 + 0.8x)\hat{i} + (1.5 + 2.5\sin(\omega t) - 0.8y)\hat{j}$

$\omega = 2\pi$ rad/s

When $t = 2$ sec. $\vec{V} = (0.5 + 0.8x)\hat{i} + (1.5 - 0.8y)\hat{j}$

Equ^n ~~[illegible]~~ is identical to Ex 1. Therefore, the streamlines look identical to Ex 3. for $t = 2$ sec.

However, at other time bet^n 0 – 2 secs. (more than 0 but less than 2 sec), the flow field is unsteady.

Therefore, the pathlines and streaklines differ as shown in Fig. 8 (ppt slides).

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\,\vec{i} + (1.5 - 0.8y)\,\vec{j}$$

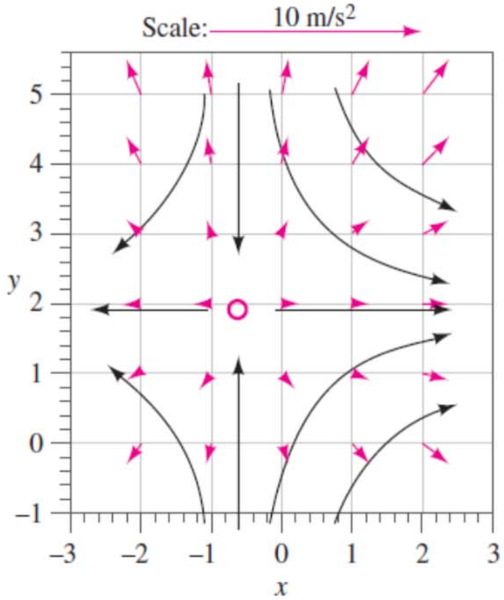


Fig. 1

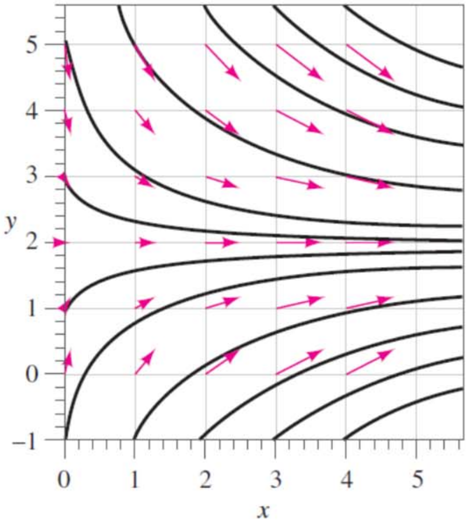


Fig. 2

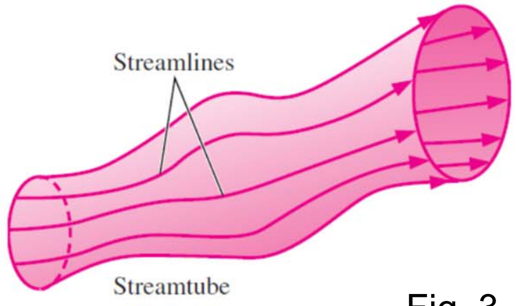


Fig. 3

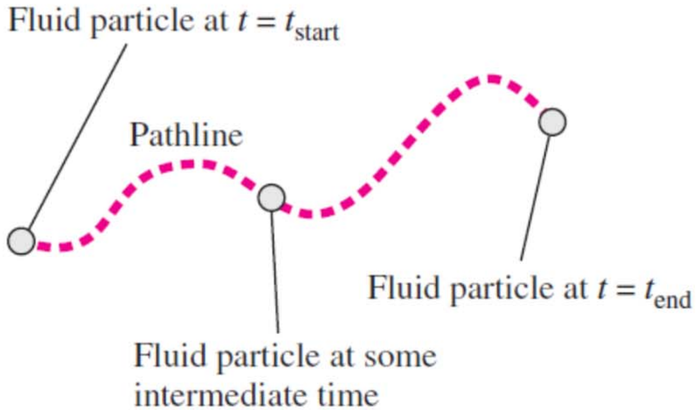
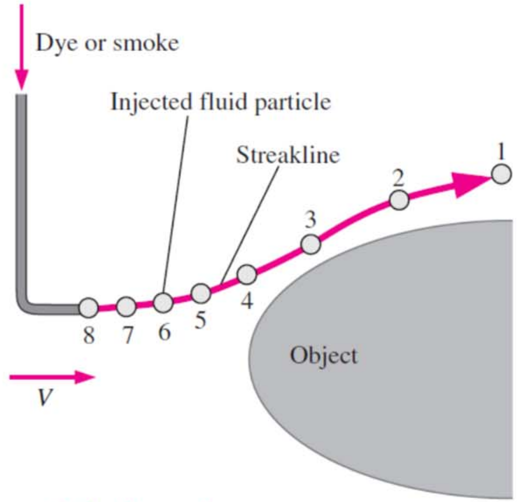


Fig. 4



Integrated tracer particle location:

Tracer particle location at time t: $\vec{x} = \vec{x}_{\text{start}} + \int_{t_{\text{start}}}^{t} \vec{V}\, dt$

$$\vec{x} = \vec{x}_{\text{injection}} + \int_{t_{\text{inject}}}^{t_{\text{present}}} \vec{V}\, dt$$

Fig. 5

Timeline: set of adjacent fluid particles that were marked at the same (earlier) instant of time

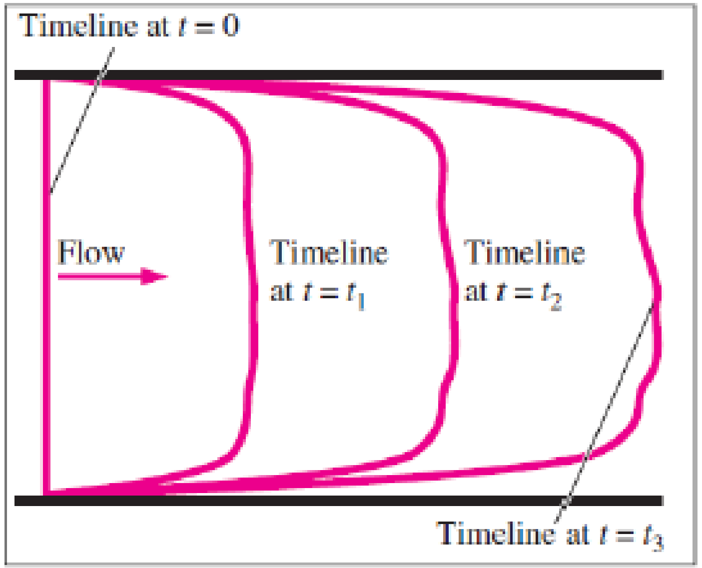


Fig. 6

Flow through a channel

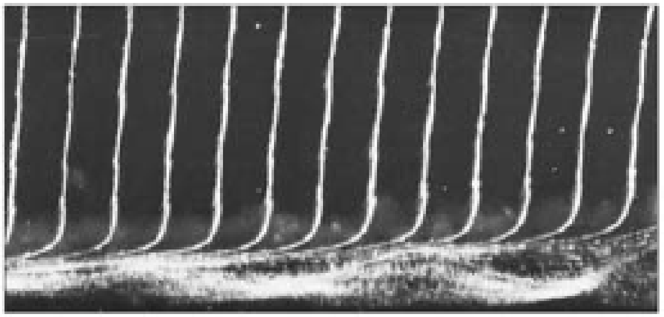
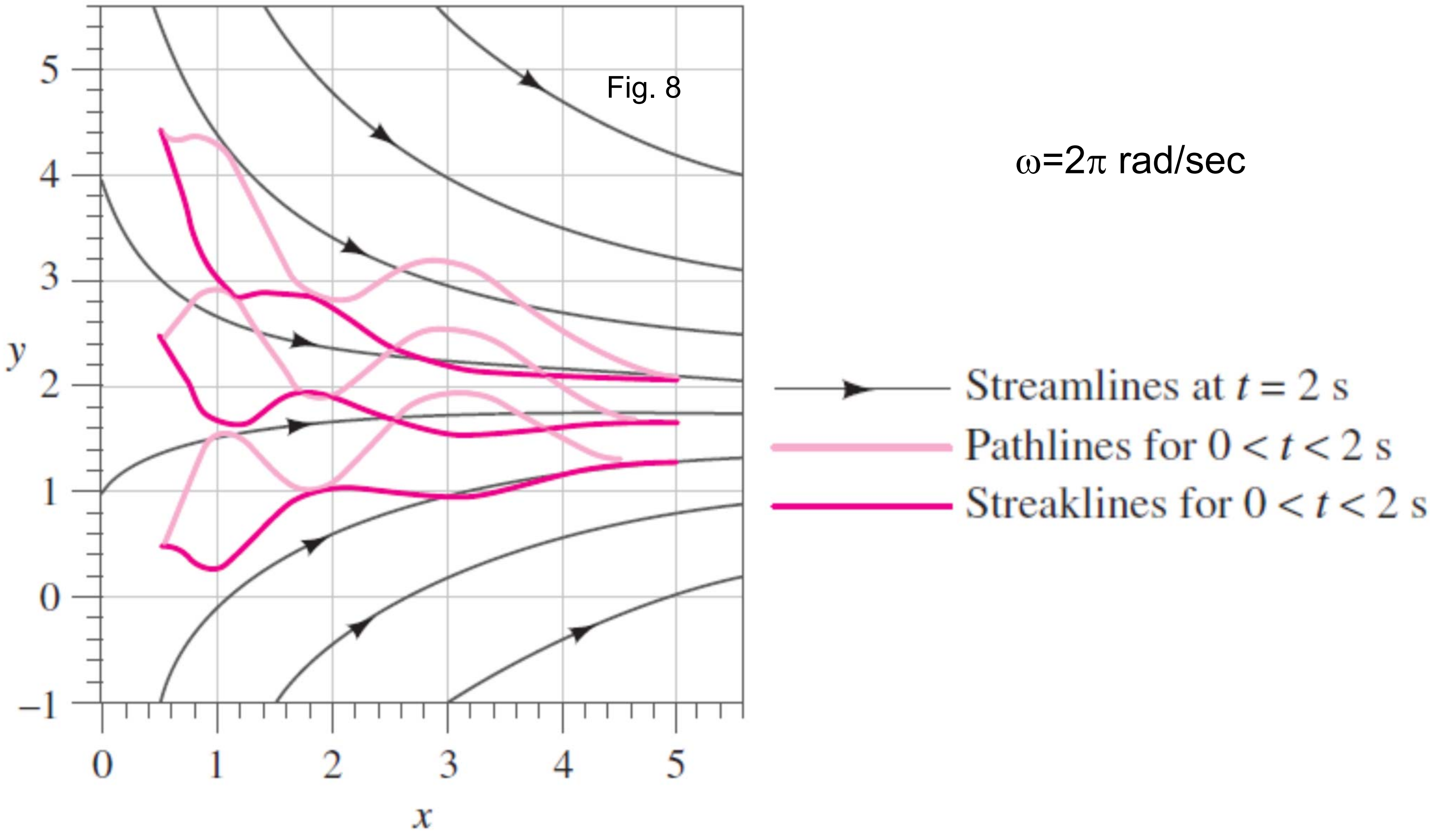


Fig. 7

Timeline produced by, hydrogen bubble visualization for flow over flat plate

An *unsteady*, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 + 2.5\sin(\omega t) - 0.8y)\vec{j}$$

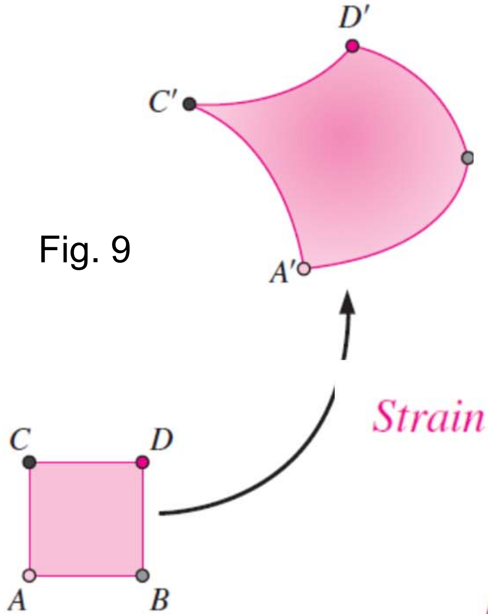


$\omega = 2\pi$ rad/sec

Linear strain rate in Cartesian coordinates:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} \qquad \varepsilon_{yy} = \frac{\partial v}{\partial y} \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z}$$

Fig. 9



Shear strain rate in Cartesian coordinates:

$$\varepsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \varepsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) \qquad \varepsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

Strain rate tensor in Cartesian coordinates:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{pmatrix} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) & \frac{\partial w}{\partial z} \end{pmatrix}$$

A steady, incompressible, two-dimensional velocity field is given by

$$\vec{V} = (u, v) = (0.5 + 0.8x)\vec{i} + (1.5 - 0.8y)\vec{j}$$

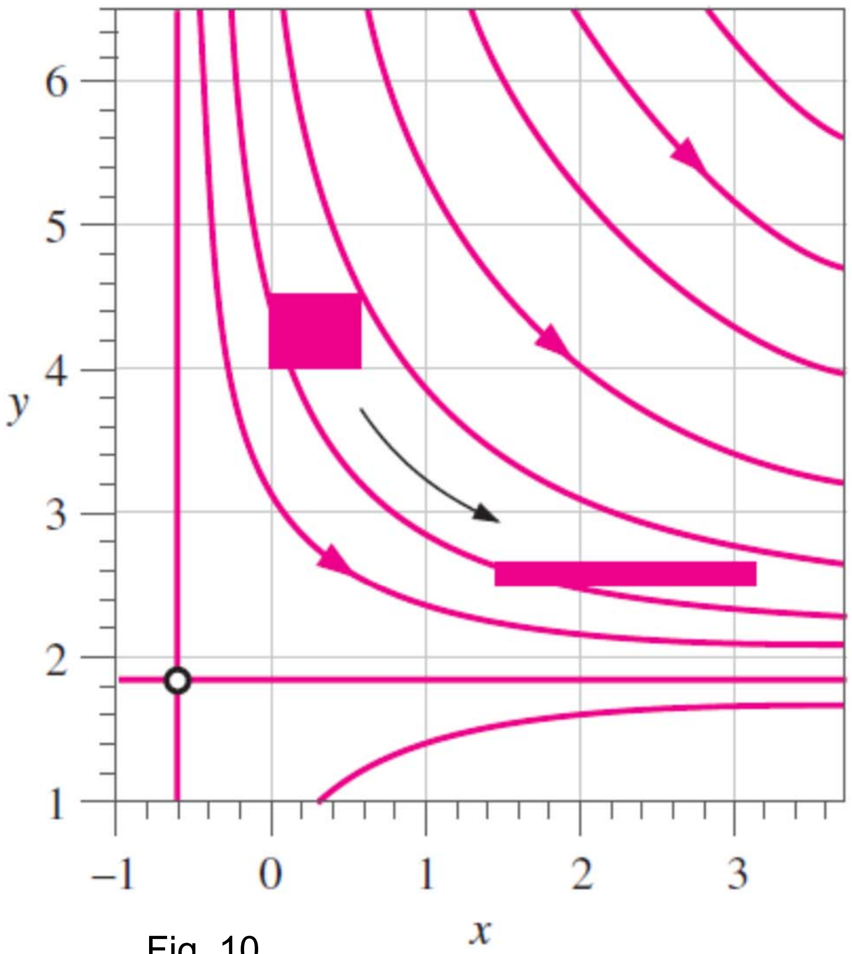


Fig. 10

$$\frac{\partial u}{\partial x} = 0.8 \text{ and } \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = 0 \text{ and } \frac{\partial v}{\partial y} = -0.8$$

The element is subjected to plain strain but no shear strain as

$$\varepsilon_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = 0$$

The element becomes longer in x-direction but shrinks in y-direction because of the sign of the strain rate

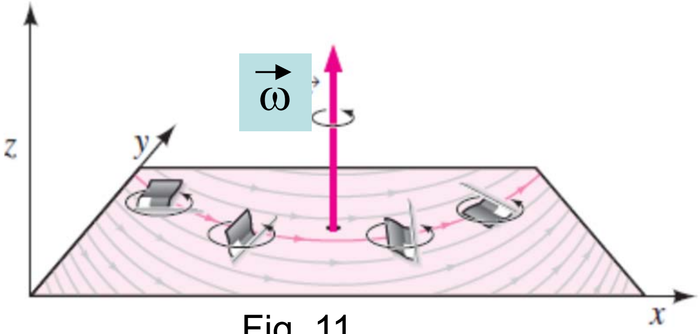


Fig. 11

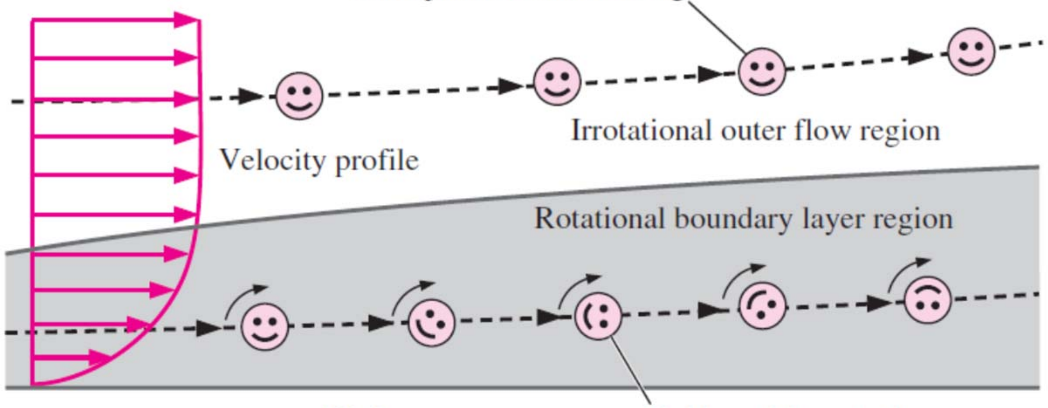


Fig. 12

Analogous to rotational circular flow

Fig. 13

Roundabout

Ferris wheel

Analogous to irrotational circular flow

$$\vec{V} = (u, v) = x^2\vec{i} + (-2xy - 1)\vec{j}$$

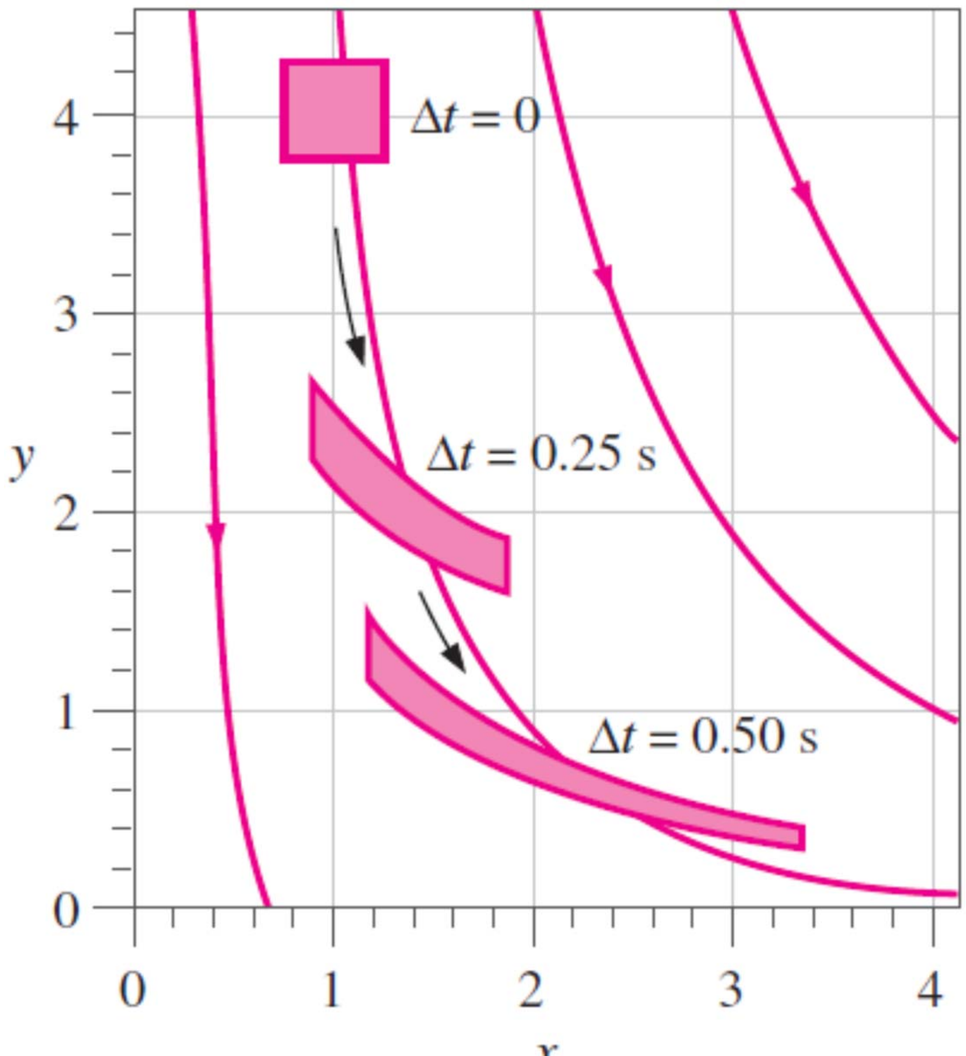


Fig. 15

$$\frac{\partial u}{\partial x} = 2x \text{ and } \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial v}{\partial x} = -2y \text{ and } \frac{\partial v}{\partial y} = -2x$$

The element is subjected to both plain as well as shear strain

$$\varepsilon_{xy} = \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = -2x$$