Polymer Mechanical Properties

Smart Materials Structures and Systems Laboratory
IIT Kanpur

Contents

- ✓ Stress-Strain relationship
- ✓ Parameter models
- ✓ Stress-Strain behavior
- ✓ Fracture & Fatigue
- ✓ Factors affecting mechanical behavior

Stress – Strain Relationship

- The force-deformation relationship in a polymer is governed by the loading rate.
- The Stress(σ)–Strain(ε) relationship, in the most general case for the polymers is,

$$[\boldsymbol{a_o} + \boldsymbol{a_1}\left(\frac{\partial}{\partial t}\right) + \boldsymbol{a_2}\left(\frac{\partial^2}{\partial t^2}\right) + \dots + \boldsymbol{a_n}\left(\frac{\partial^n}{\partial t^n}\right)]\boldsymbol{\sigma} = [\boldsymbol{b_o} + \boldsymbol{b_1}\left(\frac{\partial}{\partial t}\right) + \boldsymbol{b_2}\left(\frac{\partial^2}{\partial t^2}\right) + \dots + \boldsymbol{b_m}\left(\frac{\partial^m}{\partial t^m}\right)]\boldsymbol{\varepsilon}$$

Or

$$\boxed{a_0\sigma + \sum_{i=1}^{n} a_i \frac{d^i\sigma}{dt^i} = b_0\varepsilon + \sum_{j=1}^{m} b_j \frac{d^j\varepsilon}{dt^j}}$$

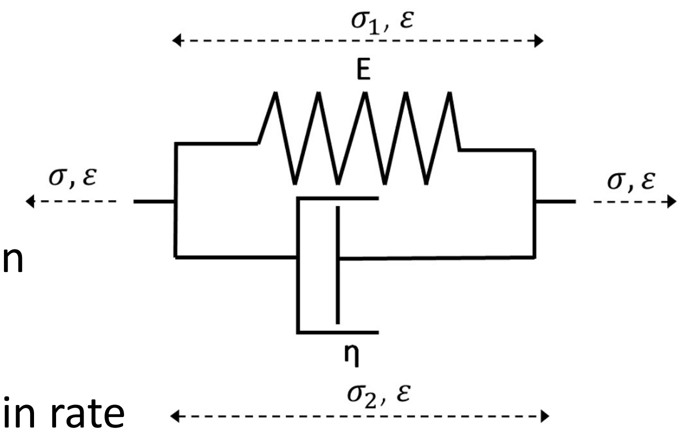
Generalized Hooke's Law

- If all the **coefficients** $a_o, a_1 .. a_n$ and $b_o, b_1 .. b_m$ are **constant** – **Linear Viscoelastic Material**

For metals, $a_1 \dots a_n = 0$
$b_1 \dots b_m = 0$
Then, $a_0\sigma = b_0\varepsilon$
Thus, $\sigma = (b_0/a_0)\ \varepsilon = E\varepsilon$

Kelvin-Voight (K-V) Mechanical Model

- **Parallel combination** of a linear spring of **stiffness k** and a viscous dashpot of **damping coefficient η**



Linear Spring (elastic) : Stress is proportional to strain

$$\sigma_1 = E\varepsilon_1$$

Linear Viscous dashpot : Stress is proportional to strain rate

$$\sigma_2 = \eta \frac{d\varepsilon_2}{dt}$$

$$\eta = viscosity$$

For parallel combination

$$\varepsilon = \varepsilon_1 = \varepsilon_2 \qquad \sigma = \sigma_1 + \sigma_2$$

$$\boxed{\sigma = E\varepsilon + \eta \frac{d\varepsilon}{dt}}$$

Or

$$\boxed{a_0\sigma + a_1 \frac{d\sigma}{dt} = b_0\varepsilon + b_1 \frac{d\varepsilon}{dt}}$$

$$\boxed{a_0\sigma + \sum_{i=1}^{n} a_i \frac{d^i\sigma}{dt^i} = b_0\varepsilon + \sum_{j=1}^{m} b_j \frac{d^j\varepsilon}{dt^j}}$$

Hence, for this model

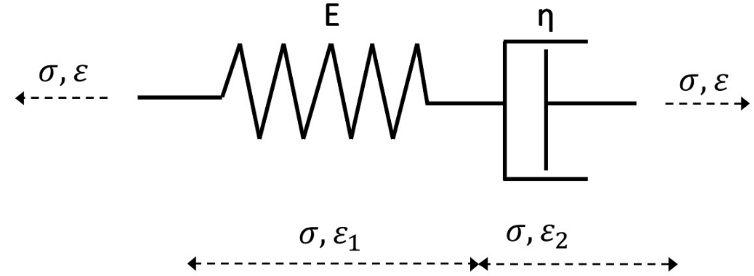
$a_0 = 1$, $a_1 = 0$,

$b_0 = E$, $b_1 = \eta$

Maxwell Mechanical Model

In this model, the **spring and dashpot** are connected in **series**. In this case,

$$\sigma_1 = E\varepsilon_1 \qquad \sigma_2 = \eta\frac{d\varepsilon_2}{dt}$$



$$\varepsilon = \varepsilon_1 + \varepsilon_2 \qquad \text{..........(1)}$$

$$\sigma = \sigma_1 = \sigma_2 \qquad \text{..........(2)}$$

Taking derivative of strain w.r.t time (eq.1), we get

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon_1}{dt} + \frac{d\varepsilon_2}{dt} \qquad \text{..........(3)}$$

Since, $\frac{d\sigma_1}{dt} = E\frac{d\varepsilon_1}{dt}$

On Substituting the values in right hand side, we get

$$\boxed{\frac{d\varepsilon}{dt} = \frac{1}{E}\frac{d\sigma}{dt} + \frac{\sigma}{\eta}}$$

$$\boxed{a_0\sigma + \sum_{i=1}^{n} a_i\frac{d^i\sigma}{dt^i} = b_0\varepsilon + \sum_{j=1}^{m} b_j\frac{d^j\varepsilon}{dt^j}}$$

Hence, for this model
$a_0 = 1/\eta$, $a_1 = 1/E$,
$b_0 = 0$, $b_1 = 1$

Parameter Models

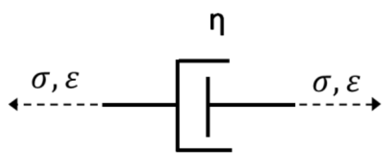
1 PARAMETER

Linear Elastic Spring

$$\boldsymbol{\sigma = E\varepsilon}$$

- Perfectly elastic behaviour

Linear Viscous dashpot

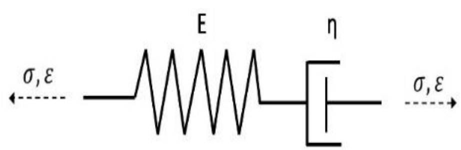


$$\boldsymbol{\sigma = \eta\dot{\varepsilon}}$$

- Perfectly viscous behaviour

2 PARAMETER

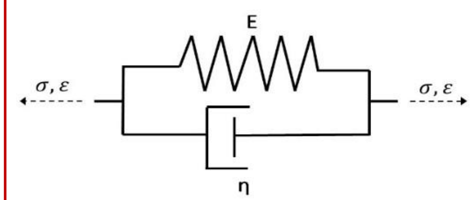
Maxwell model



$$\boldsymbol{\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\dot{\varepsilon}}$$

- Predicts fluid-like behavior.
- Do not describe recovery.

Kelvin − Voight model

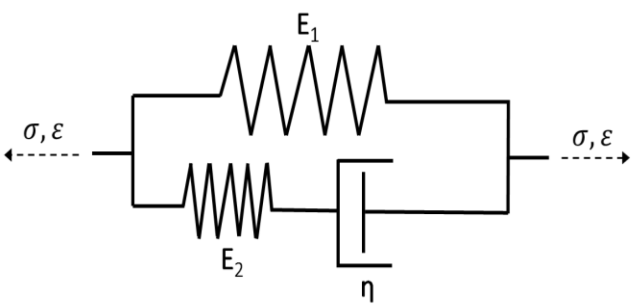


$$\boldsymbol{\sigma = E\varepsilon + \eta\dot{\varepsilon}}$$

- Predicts solid-like behavior.
- Do not describe stress relaxation.

3 PARAMETER MODELS

Standard Linear Solid

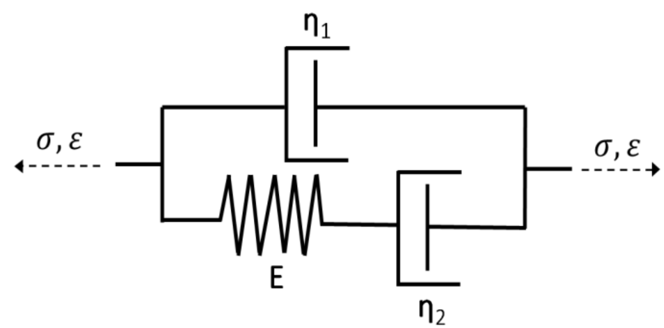


$$\boldsymbol{\sigma + \frac{\eta}{E_2}\dot{\sigma} = E_1\varepsilon + \frac{\eta(E_1+E_2)}{E_2}\dot{\varepsilon}}$$

$a_o = 1$, $a_1 = \frac{\eta}{E_2}$

$b_o = E_1$, $b_1 = \frac{\eta(E_1+E_2)}{E_2}$

Standard Linear Fluid



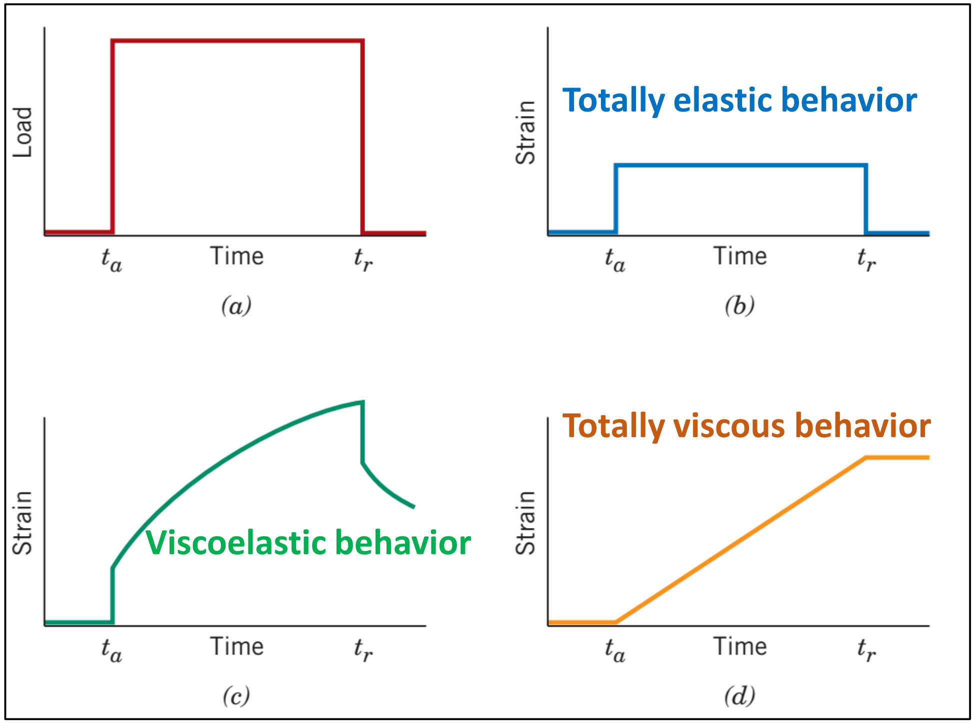
$$\boldsymbol{\sigma + \frac{\eta_2}{E}\dot{\sigma} = (\eta_1+\eta_2)\dot{\varepsilon} + \frac{(\eta_1\eta_2)}{E}\ddot{\varepsilon}}$$

$a_o = 1$, $a_1 = \frac{\eta_2}{E}$

$b_o = 0$, $b_1 = (\eta_1 + \eta_2)$, $b_2 = \frac{(\eta_1\eta_2)}{E}$

Viscoelastic deformation

- An amorphous polymer may behave like a
  - Glassy polymer at low temperatures.
  - A rubbery solid at intermediate temperatures (above T_g).
  - A viscous liquid as the temperature is further raised.
- For **intermediate temperatures** the polymer is a rubbery solid – **Viscoelastic behavior.**

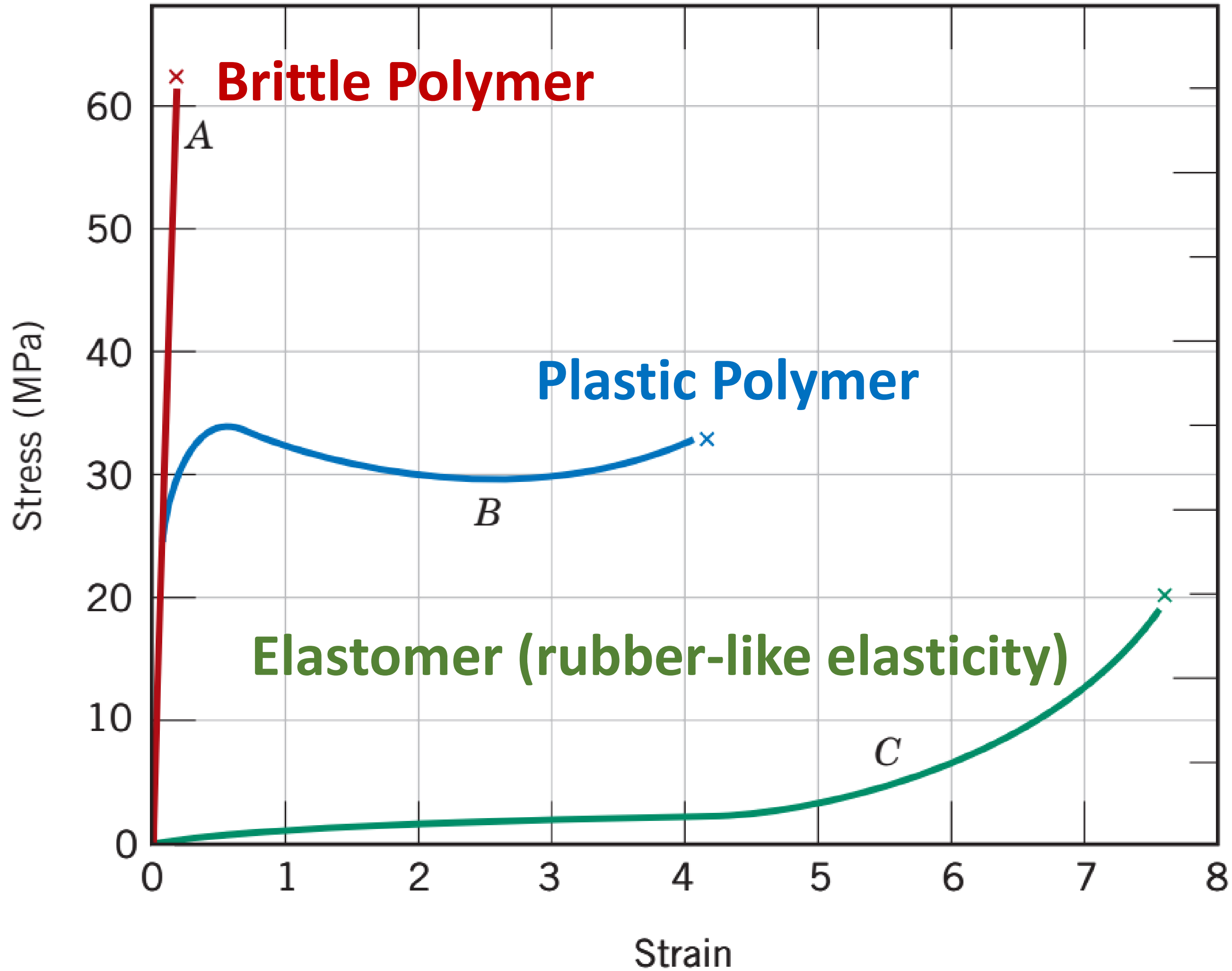


t_a = Load applied time

t_r = Load release time

Reference: W.D Callister, 7 Ed.

Stress – Strain Behavior



Reference: W.D Callister, 7 Ed.

Comparison

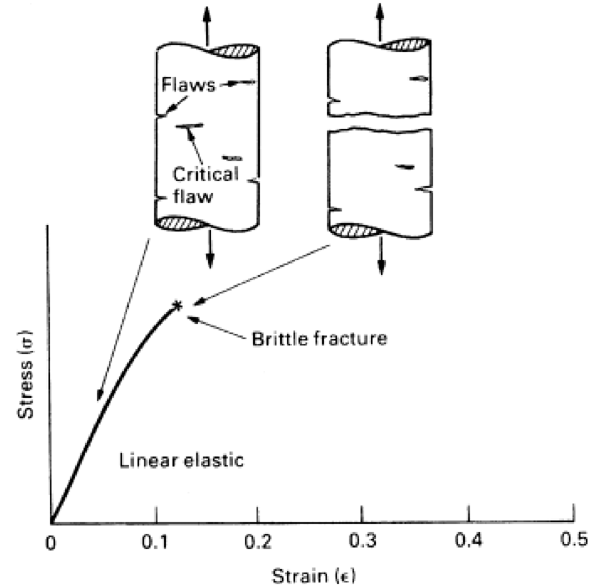
- **Moduli of elasticity**
  - Polymers ≈ 7 MPa - 4 GPa
  - Metals ≈ 50 - 400 GPa

- **Tensile strengths**
  - Polymers ≈ 10 - 100 MPa (fracture point)
  - Metals ≈100 - 1000 MPa

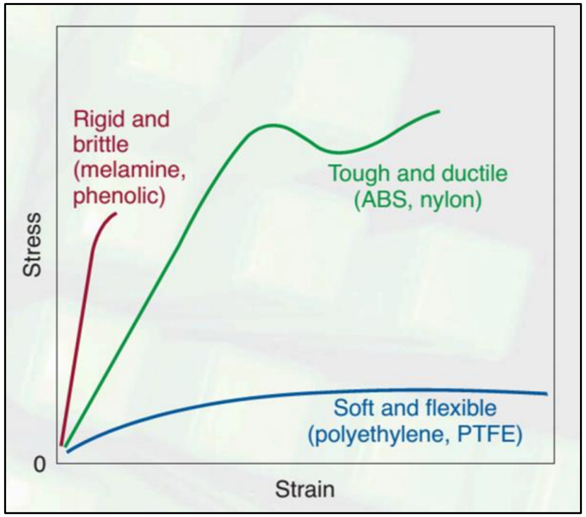
- **Elongation**
  - Polymers - up to 1000 % in some cases
  - Metals - < 10%

Fracture in Polymers

- **Low fracture strength** compared to metals and ceramics.
- Fracture mode in **thermosetting polymers** (highly crosslinked) – **Brittle**
- Fracture mode in **thermoplastic polymers** – Both **brittle & ductile** possible.
- **Crack forms** at region of localized **stress concentration** – scratches, notches, sharp flaws.
- Factors favoring brittle fracture: -
  - ✓ Temperature reduction.
  - ✓ Increase in strain rate.
  - ✓ Sharp notch presence.
  - ✓ Increased specimen thickness.



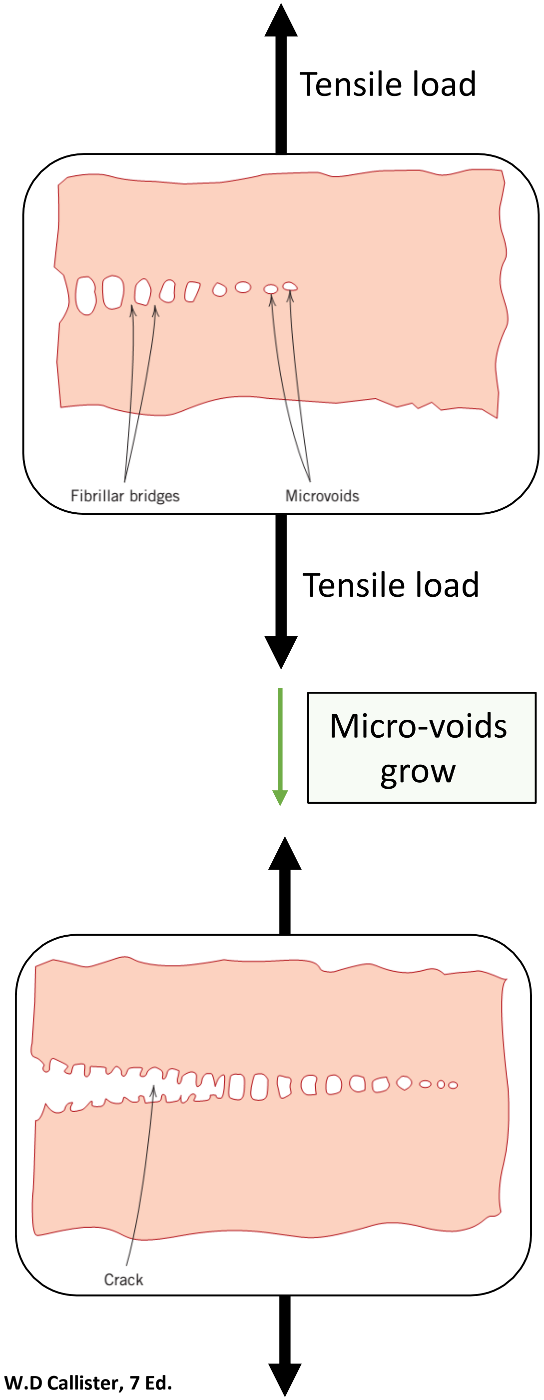
Reference: Engineering Materials 2: Ashby & Jones, 4th Ed.



Reference: Kalpakjian, Schmid - Manufacturing Processes for Engineering Materials, 5th ed.

Fracture mechanism

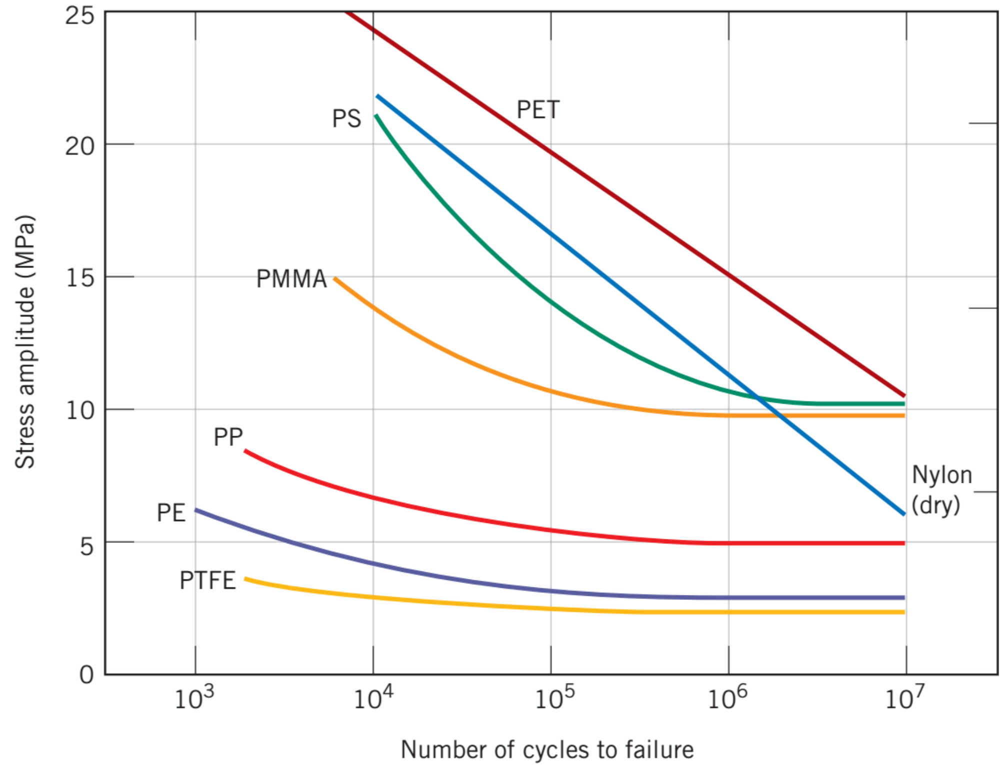
- **Amorphous thermoplastics**
  - ✓ Below T_g – Brittle (low fracture resistance)
  - ✓ Above T_g – Ductile (high fracture resistance)
  - ✓ Plastic yielding prior to fracture
- Fracture phenomenon – **Crazing**
- **Crazes** – Region of localized plastic deformation
  - ✓ **Form** at **highly stressed regions** associated with scratches, flaws, and molecular in-homogeneities.
  - ✓ Leads to the **formation** of small and interconnected **micro-voids**.
  - ✓ Region between micro-voids : **Fibrillar bridges**
  - ✓ Under **load** ,the fibrillar bridges **elongate and break**.
  - ✓ Micro-voids **grow and merge**.
  - ✓ Thus, **crack propagate** perpendicular to the applied tensile stress.



Reference: W.D Callister, 7 Ed.

Fatigue

- Polymers undergo **fatigue** failure under **cyclic loading**.
- Fatigue behavior more sensitive to loading frequency than for metals.
- **High frequencies** and/or relatively large stresses can cause **localized heating** which **softens** the material leading to **failure.**



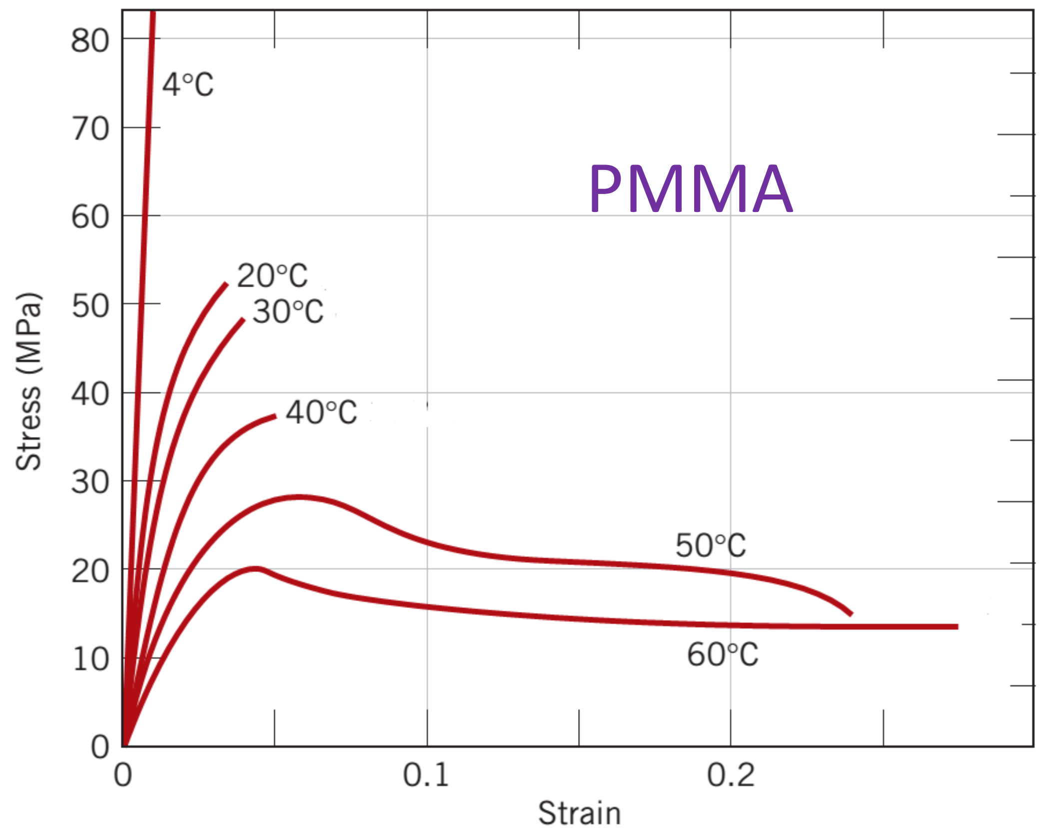
Reference: W.D Callister, 7 Ed.

Factors affecting Mechanical Properties

1. Temperature

An increase in temperature produces:-

- ✓ Decrease in elastic modulus
- ✓ Reduction in tensile strength
- ✓ Enhancement of ductility

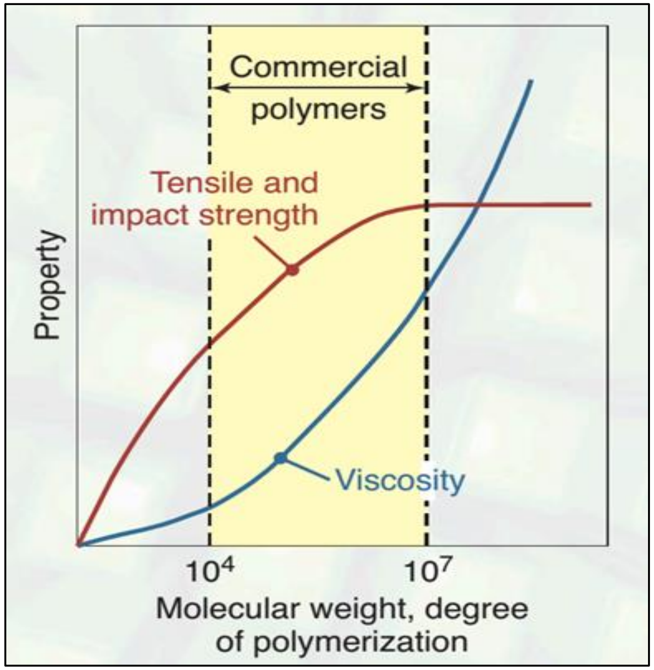


Reference: W.D Callister, 7 Ed.

➢ Reducing the strain rate also produces the same effects.

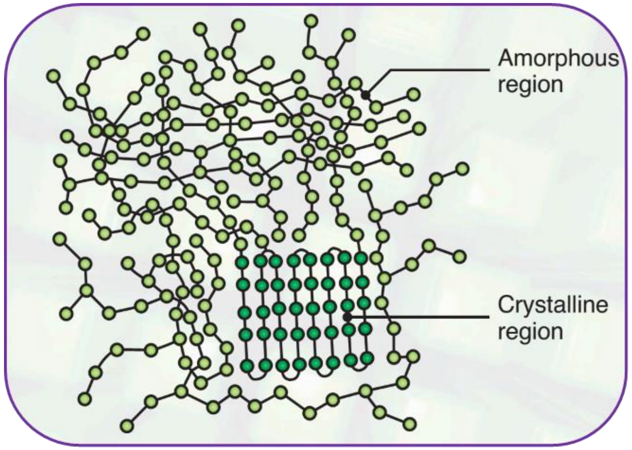
2. Molecular Weight

- For many polymers **tensile & impact strength increases** with **increasing molecular weight** due to increased **chain entanglements**.
- Viscosity also increases.



3. Degree of Crystallinity

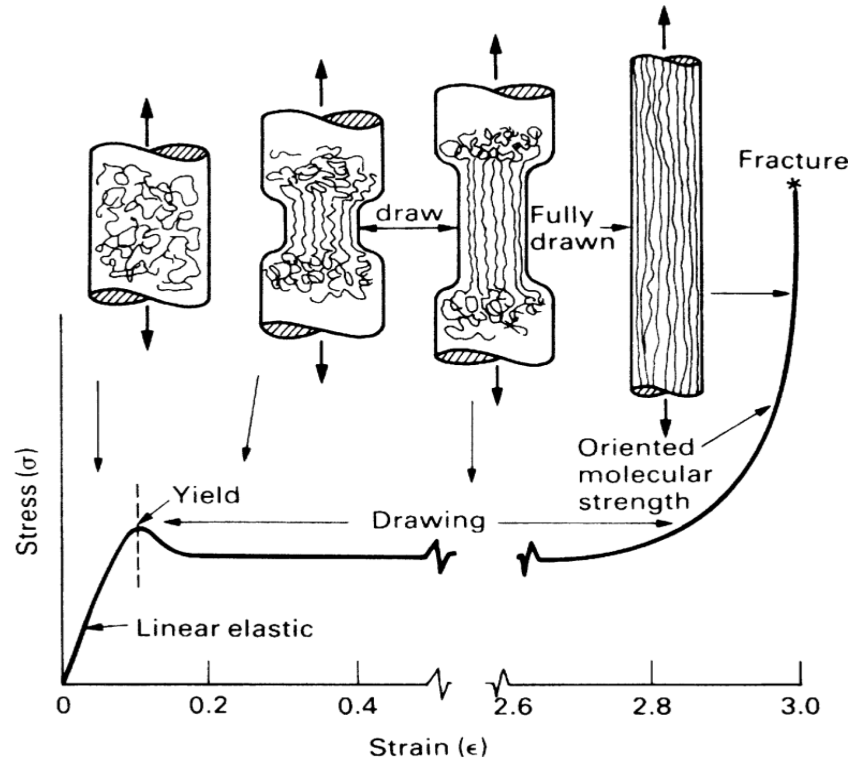
- Higher the crystallinity – higher the close packing.
- Thus**, higher density**, **more strength**, **higher resistance** to both dissolution and softening by heating.



Reference: Kalpakjian, Schmid - Manufacturing Processes for Engineering Materials, 5th ed.

4. Cold Drawing

- Analogous to strain hardening in metals.
- Used in production of fibers and films.
- Molecular chains become **highly oriented**.
- **Properties** of drawn material are **anisotropic**. (perpendicular to the chain alignment direction strength is reduced).



Cold drawing (Linear polymer)

Reference: Engineering Materials 2: Ashby & Jones, 4th Ed.

In the **next lecture**, we will learn about

- ✓ Basics of Composites
- ✓ Classification

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