Some extensions of the CAPM for individual assets

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Capital Asset Pricing Model (CAPM)

Definition (Wikipedia)

A model used to determine a theoretically appropriate required rate of return of an asset, to make decisions about adding assets to a well-diversified portfolio.

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f) \text{ where:}$$

$$\begin{aligned}
E(R_i) &: \text{Expected return of capital asset} \\
R_f &: \text{Risk-free rate of return} \\
\beta_i &: \text{Beta (measure of sensitivity) of the investment} \\
E(R_m) &: \text{Expected return of the market} \\
E(R_m) - R_f &: \text{Market risk premium}
\end{aligned}$$

Limitations of CAPM

- Beta (β_i) is not the only relevant systematic risk and hence is not sufficient to explain average returns
- CAPM doesn't hold unconditionally.

The following attempts were made to mitigate the limitations:

- Multi-factor models were introduced to add more proxies for systematic risks, i.e. factors other than market portfolio were introduced.
- Single-factor (market portfolio) multi moment models were introduced. They were theoretically more appealing than multi-factor single moment models.

This paper introduces 4-moment model (mean, variance, skewness, kurtosis) which give rise to missing systematic risks from the empirical CAPM.

4-moment CAPM

Idea

The four moment CAPM model has been derived by employing both expected utility optimisation and mean–variance–skewness–kurtosis frontier optimisation.

The variance, skewness and kurtosis of a portfolio p are given as:

$$\text{Variance: } \sigma^2(R_p) = E[R_p - E(R_p)]^2$$

$$\text{Skewness: } s^3(R_p) = E[R_p - E(R_p)]^3$$

$$\text{Kurtosis: } k^4(R_p) = E[R_p - E(R_p)]^4$$

where R_p is the return of portfolio

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4-moment CAPM continued

The coskewness and cokurtosis (counterparts of covariance) between asset i and portfolio p are given as:

$$\text{Coskewness: } Cos(R_i, R_p) = E\{[R_i - E(R_i)][R_p - E(R_p)]^2\}$$

$$\text{Cokurtosis: } Cok(R_i, R_p) = E\{[R_i - E(R_i)][R_p - E(R_p)]^3\}$$

Assets with positive coskewness are considered less risky while those with positive cokurtosis are considered more risky.

A mean–variance–skewness–kurtosis optimisation implies the following 4-moment CAPM:

$$\begin{aligned} E(R_i) - R_f &= \lambda_\beta \frac{E(r_i.r_m)}{E(r_m^2)} + \lambda_s \frac{E(r_i.r_m^2)}{E(r_m^3)} + \lambda_k \frac{E(r_i.r_m^3)}{E(r_m^4)} \\ &= \lambda_\beta \beta_i + \lambda_s \gamma_i + \lambda_k \delta_i \end{aligned}$$

where $r \equiv R - E(R)$

4-moment CAPM continued

- The coefficients λ_β, λ_s and λ_k are interpreted as risk premia.
- A positive λ_β is expected as investors require higher returns for higher systematic beta risk.
- If market portfolio returns (R_p) have negative skewness, investors should prefer asssets with lower coskewness.
- If market portfolio returns (R_p) have positive skewness, investors prefer assets with higher coskewness and therefore a negative coefficient for λ_s is expected as investors are willing to forego some returns for positive skewness.
- A positive λ_k is expected as investors require higher compensation for assets with a greater probability of extreme outcomes.

CAPM extensions in literature

Here is a brief timeline.

- 1972 : Arditti and Levy show that nonincreasing absolute risk aversion implies investor preference for positive skewness.
- 1976 : Kraus and Litzenberger derive a 3-moment CAPM by adding coskewness risk to the standard CAPM and apply it for portfolios of stocks double-sorted on beta and systematic coskewness.
- 1980 : Scott and Horvath shows that risk-averse investors with decreasing marginal utility have a positive preference for mean and skewness and a negative preference for variance and kurtosis.
- 1980 : Friend and Westerfield find results that partly contradict the findings of Kraus and Litzenberger, with a significant non-zero intercept and a time-varying coefficient for coskewness.

CAPM extensions in literature continued

- 1989 : Lim tests the three-moment CAPM using a generalised method of moments (GMM) approach and shows that investors prefer coskewness when market returns are positively skewed, and dislike coskewness when market returns are negatively skewed.
- 1991 : Tan applies a 3-moment CAPM on a sample of mutual funds and finds results that do not support the three-moment CAPM.
- 1997 : Fang and Lai test the 4-moment CAPM on portfolios triplesorted on β, coskewness and cokurtosis ,where the factor loadings are estimated using time series regressions of the cubic market model and concluded that skewness, rather than kurtosis, plays the most important role.
- 1999 : Hwang and Satchell estimate an unconditional four-model CAPM for emerging market and concludes that higher moments can add explanatory power to model return for emerging market.

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CAPM extensions in literature continued

- 2002 : Dittmar estimates a conditional 4-moment CAPM using a stochastic discount factor approach with 2 model as equity market index and human labor wealth and noticing a reduction in pricing error significantly.
- 2010 : Doan, Lin, and Zurbruegg analyse a higher-moment CAPM for US and Australian stocks and find that higher moments explain a portion and sensitivity of returns not explained by the Fama and French factors.
- 2012 : Kostakis, Muhammad, and Siganos test the higher-moment CAPM for UK stocks and find coskewness and cokurtosis have additional explanatory power to covariance risk, size, book-to-market and momentum factors.

With all these extensions in mind the test on past trends follows.

Past trends (NYSE, AMEX and NASDAQ)

December (Year)	1940	1950	1960	1970	1980	1990	2000	2010
Number of stocks	852	986	1159	2242	4733	7507	9681	7820
Mean excess return	0.54%	2.00%	0.39%	1.55%	1.63%	0.50%	0.62%	1.84%
t-statistic	7.17	50.02	7.03	36.0	40.72	15.34	16.11	42.85
Standard deviation	0.022	0.013	0.019	0.020	0.028	0.028	0.038	0.038
Skewness	1.14	0.28	7.10	0.70	1.18	0.47	1.34	0.71
Kurtosis (excess)	9.46	0.65	146.79	1.06	3.04	20.09	9.65	18.77

Table: Summary statistics for the cross-section of stock returns

Observations

- Two distinct volatility regimes (pre and post 1980)
- The normality of the distribution of returns, that is the joint hypothesis that skewness and excess kurtosis are equal to zero, is rejected.

Testing the models on trends

CAPM

- Intercept is zero $\alpha = 0$
- Risk premium is positive $\lambda_\beta > 0$
- Risk premium is equal to the average historical market excess return $\lambda_\beta = E(R_m) - R_f$

In 4-moment CAPM, $E(r_m^3)$ may be zero. So, we modify:

$$
\begin{aligned}
E[R_i - R_f] &= \lambda_* \frac{E(r_i r_m)}{E(r_m^2)} + \lambda_s E(r_i r_m^2) + \lambda_k \frac{E(r_i r_m^3)}{E(r_m^4)} \\
&= \lambda_* \beta_i + \lambda_s s_i + \lambda_k k_i \\
E[R_m - R_f] &= \lambda_* + \lambda_s s_m + \lambda_k \\
\therefore E[R_i - R_f] &= \lambda_\beta \beta_i + \lambda_s (s_i - s_m \beta_i) + \lambda_k (k_i - \beta_i)
\end{aligned}
$$

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Testing the models on trends continued

4-moment CAPM

- Price of beta is positive $\lambda_\beta > 0$ and $\lambda_\beta = E(R_m) - R_f$
- The premium for (excess) coskewness is negative $\lambda_s < 0$.
- The premium for (excess) cokurtosis is positive $\lambda_k > 0$.

- Authors criticize using portfolios to estimate market premium. Reason is the spread in β's being too small, leads to large errors.
- Limitations of static models is overcomed by this two-step method.
- Expected excess return by short-window (24 month) time series regression of monthly individual asset excess returns over market excess return.
- Average excess return of individual stocks regressed in cross-section over conditional co-moments to estimate risk premia.

Testing the models on trends continued

Augmented 4-moment CAPM

SMB factor: the return of a portfolio of small-capitalization stocks minus return of portfolio of large-capitalization stocks.
HML factor: the return of a portfolio of high book-to-market stocks minus return of portfolio of low book-to-market stocks.
The new model becomes

$$
\begin{aligned}
R_i - R_f = & \lambda_\beta + \lambda_s(s_i - s_m\beta_i) + \lambda_k(k_i - \beta_i) \\
& + \lambda_s mb \frac{E(r_i smb)}{E(smb^2)} + \lambda_{hml} \frac{E(r_i hml)}{E(hml^2)} + \epsilon_i
\end{aligned}
$$

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Results

Table 2 This table has the CAPM evaluated over 1930-2010

	CAPM	4-Moment CAPM (adjusted)	4-Moment CAPM (unadjusted)	3-Moment CAPM (adjusted)	3-Moment CAPM (unadjusted)
α	0.0007 (3.80)***/[1.74]*	0.0006 (3.33)***/[1.53]	0.0006 (3.33)***/[1.53]	0.0006 (2.91)***/[1.33]	0.0006 (2.91)***/[1.33]
λ_β	0.0067 (22.14)***/[10.32]***	0.0067 (23.88)***/[11.15]***	0.0062 (11.78)***/[5.60]***	0.0068 (23.14)***/[10.79]***	0.0061 (17.10)***/[8.07]***
λ_s		−73.16 (−14.27)***/[−7.00]***	0.0010 (3.41)***/[1.73]*	−70.4126 (−17.08)***/[−8.01]***	0.0008 (5.63)***/[2.81]***
λ_k		−0.0004 (−0.90)/[−0.45]	−0.0004 (−0.90)/[−0.45]		
Market risk premium $(\lambda_\beta + \lambda_s s_m + \lambda_k)$		0.0073 (19.43)***/[9.58]***		0.0076 (25.84)***/[12.20]***	
Market risk premium $(\lambda_\beta + \lambda_s + \lambda_k)$			0.0067 (23.88)***/[11.15]***		0.0068 (23.14)***/[10.79]***

Table 3 Test of the CAPM over the subsample period of 1980–2010

	CAPM	4-Moment CAPM (adjusted)	4-Moment CAPM (unadjusted)	3-Moment CAPM (adjusted)	3-Moment CAPM (unadjusted)
α	0.0008 (4.03)***/[1.92]*	0.0004 (2.77)***/[1.32]	0.0004 (2.77)***/[1.32]	0.0004 (2.48)**/[1.18]	0.0004 (2.48)**/[1.18]
λ_β	0.0059 (18.98)***/[8.94]***	0.0059 (19.10)***/[9.00]***	0.0063 (15.48)***/[7.50]***	0.0059 (19.19)***/[9.03]***	0.0064 (22.01)***/[10.47]***
λ_s		−32.79 (−6.61)***/[−3.15]***	−0.0011 (−3.15)***/[−1.60]	−35.57 (−8.99)***/[−4.38]***	−0.0004 (−3.36)***/[−1.63]
λ_k		0.0007 (1.44)/[0.72]	0.0007 (1.44)/[0.72]		
Market risk premium $(\lambda_\beta + \lambda_s s_m + \lambda_k)$		0.0056 (10.61)***/[5.02]***		0.0055 (14.45)***/[6.82]***	
Market risk premium $(\lambda_\beta + \lambda_s + \lambda_k)$			0.0059 (19.11)***/[9.00]***		0.0059 (19.18)***/[9.03]***

This table has the CAPM evaluated over 1930-2010. The coefficients are reported for the conditional alpha, the conditional beta, the conditional coskewness, the conditional cokurtosis, and the conditional overall risk premium. Conventional t-statistics are reported in parentheses, while HAC t-statistics are given in square brackets. Significant coefficients at the 1%, 5% and 10% levels are indicated with ***, ** and *, respectively

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Results

Table 4 Tests of the adjusted four-moment CAPM augmented with SMB and HML, and the three-factor model of Fama and French using short-window regressions, on individual assets over the period 1930–2010 and 1980–2010.

	(1930–2010)		(1980–2010)	
	Adjusted 4M-CAPM with FF factors	Fama–French 3-factor model	Adjusted 4M-CAPM with FF factors	Fama–French 3-factor model
α	0.0007	0.0009	0.0002	0.0003
	(3.77)***/[1.72]*	(4.65)***/[2.13]**	(1.17)/[0.56]	(1.69)*/[0.81]
λ_β	0.0058	0.0053	0.0043	0.0041
	(22.83)***/[11.00]***	(20.00)***/[9.50]***	(14.78)***/[7.10]***	(15.28)***/[7.35]***
λ_s	−49.4709		−10.32	
	(−11.70)***/[−5.64]***		(−2.08)**/[−1.00]	
λ_k	0.0014		0.0018	
	(3.60)***/[1.86]*		(3.38)***/[1.65]*	
λ_{smb}	0.0016	0.0016	0.0028	0.0026
	(8.29)***/[3.86]***	(8.19)***/[3.78]***	(13.60)***/[6.44]***	(14.73)***/[7.00]***
λ_{hml}	−0.0004	−0.0005	−0.0003	0.0003
	(−2.97)***/[−1.46]	(−3.87)***/[−1.87]*	(−1.55)/[−0.74]	(−1.57)/[−0.75]

Table 5 Test of the CAPM and four-moment CAPM based on 25 ME/BM portfolios.

Models	α	λ_β	λ_s	λ_k	$\lambda_\beta+\lambda_s s_m+\lambda_k$	SMB	HML
Panel A: 1930–2010							
CAPM	0.0075	−0.0001					
	(3.55)***	(0.02)					
Adjusted 4-moment CAPM	0.0068	−0.0000	50.31	−0.0033	−0.0009		
	(3.08)***	(−0.01)	(0.89)	(−0.46)	(−0.14)		
4-moment CAPM + FF	0.0091	−0.0026	16.37	0.0001		0.0013	0.0021
	(4.69)***	(−1.22)	(0.30)	(0.02)		(1.23)	(1.73)*
FF	0.0094	−0.0030				0.0012	0.0038
	(5.35)***	(−1.58)				(1.21)	(3.82)****
Panel B: 1980–2010							
CAPM	0.0107	−0.0042					
	(2.73)***	(−0.91)					
Adjusted 4-moment CAPM	0.0107	−0.0049	−4.44	−0.0055	−0.0063		
	(2.75)***	(−1.16)	(−0.04)	(−0.40)	(−0.61)		
4-moment CAPM + FF	0.0143	−0.0084	34.34	0.0091		−0.0007	0.0003
	(4.36)***	(−2.37)**	(0.34)			(−0.36)	(0.11)
FF	0.0124	−0.0067				0.0000	0.0032
	(3.92)***	(−1.91)*				(0.03)	(1.71)*

Results

- The results are summarised from tables 2 to 4.
- There is a sharp contrast between the conventional t-statistics and HAC t-statistics.
- Test of conditional CAPM on individual assets
    - The corrected statistics are around half or less the value of standard t-statistics and some risk premiums also become insignificant after adjustment.
    - The conventional t-statistic suggests that the intercept is highly significant, thus leading to a rejection of the CAPM, but after taking into account potential heteroscedasticity and autocorrelation, it shows that the intercept is insignificant.
    - Since the market risk premium is positive and significant, the CAPM cannot be rejected.
    - In the subsample from Table 3, intercept is insignificant and beta premium has positive and significant coefficient.

Results

- Test of conditional four-moment CAPM on individual assets.
  - Higher moment CAPM models have insignificant intercepts.
  - As beta is priced and price skewness is significant, it rejects standard CAPM.
  - $K^4(R_p)$ for 4-moment CAPM is approximately 0 for both adjusted and unadjusted models indicating that 3-moment CAPM is more accurate.
  - The results for 3 moment are not much different from 4 moment CAPM as shown in Table 2.
  - From Table 3, $E(R_M)$ suggested by higher moment CAPM is lower than that of simple CAPM.
  - From Table 2, the coskewness premium for the adjusted model is negative but positive in unadjusted model.
  - Such result is due to possible low market skewness which could inflate the scale coskewness.

Results

- Four moment CAPM with SMB and HML factors.
  - Tested for sample 1930-2010 and subsample 1980-2010.
  - Coskewness and cokurtosis do not seem to be able to capture the size effect.
  - Scale of prices change in the subsample period.
- Impact of using portfolios
  - Analysis repeated with 25 ME/MB sorted portfolios.
  - Results in Table 5 estimate Fama-Macbeth average $E(R_P) - r_f$.
  - The portfolios were tested against CAPM model, adjusted four-moment CAPM model, FF augmented model and the simple FF model and all the models were rejected due to high values of intercept.
  - The portfolio results explain the difficulties in the different models of the CAPM.