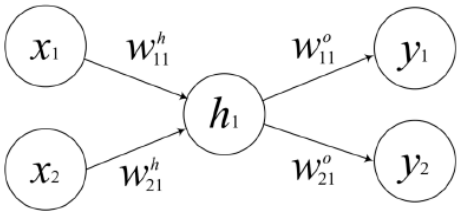
9.3) Problem Solving manually on Paper

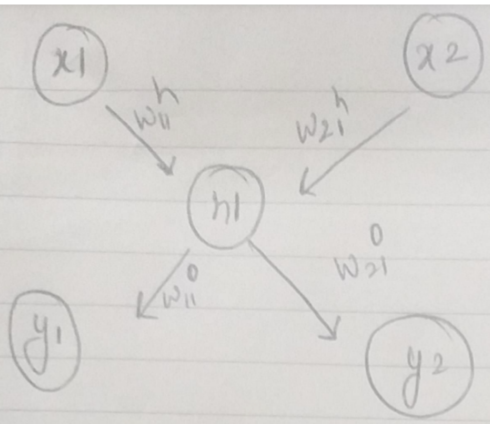
Assume the following neural network structure and the initial values indicated in the following table. Your task is to manually calculate the following:

1. Using the feed forward process calculate the output at y_1 and y_2.

2. Assuming the correct output should have been $y_1 = 1$ and $y_{2=0}$, calculate the error for both output nodes and the hidden node. Then calculate the new weights for each of the four arcs connecting the nodes. Assume no bias nodes.



Variable	Initial Value	Description
x_1	0.59	First input node
x_2	0.1	Second input node
w^h_{11}	0.6	Weight from first input to hidden node
w^h_{21}	0.1	Weight from second input to hidden node
w^o_{11}	0.9	Weight from hidden node to first output node
w^o_{21}	0.3	Weight from hidden node to second output node
k	1	Sigmoid k value
η	0.3	Learning rate



weights	value	connection
w_{11}^h	0.6	$x_1 \rightarrow h_1$
w_{21}^h	0.1	$x_2 \rightarrow h_1$
w_{11}^0	0.9	$h_1 \rightarrow y_1$
w_{21}^0	0.3	$h_1 \rightarrow y_2$

$\left.\begin{array}{l} x_1 \rightarrow h_1 \\ x_2 \rightarrow h_1 \end{array}\right\} \rightarrow h_1$

$h_1 \rightarrow y_1 \} \rightarrow y_1$

$h_1 \rightarrow y_2 \} \rightarrow y_2$

A = Sigmoid Activation function.

$$h1 = A\left(w_{11}^h \cdot x_1 + w_{21}^h \cdot x_2\right)$$
$$= A\left(0.6 \cdot (0.59) + 0.1 \cdot (0.1)\right)$$

$$h1 = A(0.365)$$
$$h1 = 0.59.$$

$$y1 = w_{11}^0 \times h_1 = 0.9 \times 0.59 = 0.531$$

$$y_2 = w_{21}^0 \times h_1 = 0.3 \times 0.59 = 0.177$$

$$E_{Total} = \sum \frac{1}{2} (\text{target} - \text{output})^2$$

$$= \frac{1}{2} (T_1 - \text{out } y_1)^2 + \frac{1}{2} (T_2 - \text{out } y_2)^2$$

$$= \frac{1}{2} (-0.531 + 1)^2 + \frac{1}{2} (0 - 0.177)^2$$

$$= 0.1099 + 0.015$$

$$\boxed{E_{Total} = 0.125}$$

$E_1 = 0.1099$

$E_2 = 0.015.$

Backward pass

At w_{11}^0. Error $= \frac{\partial E_{Total}}{\partial w_5}$

$$\frac{\partial E_{total}}{\partial w_{11}^0} = \frac{\partial E_{total}}{\partial \text{out } y_1} \times \frac{\partial \text{ out } y_1}{\partial y_1} \times \frac{\partial y_1}{d w_{11}^0}$$

$$\frac{\partial E_{Total}}{\partial \text{ out } y_1} = -(T_1 - \text{out } y_1)$$

$$= -0.469$$

$$\text{out } y_1 = \frac{1}{1 + e^{-y_1}}$$

$$\frac{d y_1}{\partial w_{11}^0} = \text{out } h_1 = 0.59$$

$$\frac{\partial \text{ out } y_1}{\partial y_1} = \text{out } y_1 (1 - \text{out } y_1)$$

$$= .531 \, (1 - .531)$$

$$= 0.249$$

$$\frac{\partial E_{total}}{\partial w_{11}^{0}} = 0.249 \times 0.59 \times (-0.469)$$

$$= -0.069$$

$$\text{new } w_{11}^{0} = w_{11}^{0} - \alpha \frac{\partial E_{Total}}{\partial w_{11}^{0}}$$

$$= 0.9 + 0.3 \times 0.069$$

$$\boxed{w_{11}^{0} = 0.921}$$

$$\boxed{w_{11}^{0} = 0.921}$$

Backward pass at

w_{21}^{0} Error $= \dfrac{\partial \mathcal{E}_{Total}}{\partial w_{21}^{0}}$

$$\frac{\partial \mathcal{E}_{Total}}{\partial w_{21}^{0}} = \frac{\partial \mathcal{E}_{Total}}{\partial out\, y_0} \times \frac{\partial out\, y_0}{\partial y_0} \times \frac{\partial y_0}{\partial w_{21}^{0}}$$

$$\frac{\partial \mathcal{E}_{Total}}{\partial out\, y_0} = -(T_0 - out\, y_0)$$

$$= -(0 - 0.177)$$

$$= 0.177$$

$$out\, y_0 = \frac{1}{1+e^{-y_0}}$$

$$\frac{\partial out\, y_0}{\partial y_0} = out\, y_0(1 - out\, y_0)$$

$$= 0.177(1 - 0.177)$$

$$= 0.146$$

$$\frac{\partial y_0}{\partial w_{21}^{0}} = out\, h_1$$

$$= 0.59$$

$$\frac{\partial \mathcal{E}_{Total}}{\partial out\, y_0} = 0.177 \times 0.146 \times 0.59$$

$$= 0.015$$

$$w_{21}^{0} = w_{21}^{0} - \alpha.(0.015)$$

$$= 0.3 - (0.3 \times 0.015)$$

$$\boxed{w_{21}^{0} = 0.295}$$

Now at hidden layer (w_{11}^{h})

$$\frac{\partial E_{total}}{\partial w_{11}^{h}} = \frac{\partial E_{Total}}{\partial\, out\, h_1} \times \frac{\partial\, out\, h_1}{\partial h_1} \times \frac{\partial h_1}{\partial w_1}$$

$$\frac{\partial E_{total}}{\partial\, out\, h_1} = \frac{\partial E_{\cancel{t}}}{\partial\, \cancel{out}\, y_1} \times \frac{\partial y_1}{\partial\, out\, h_1}$$

$$= \frac{\partial E_1}{\partial\, out\, y_1} \times \frac{\partial\, out\, y_1}{\partial y_1} \times W_{11}^{o}$$

$$= -0.469 \times .249 \times 0.9$$

$$= -0.105$$

$$\frac{\partial\, out\, h_1}{\partial h_1} = out\, h_1\,(1 - out\, h_1)$$

$$= 0.59 \times (1 - 0.59)$$

$$= 0.2419$$

$$\frac{\partial h_1}{\partial w_1} = x_1 = 0.59$$

$$\frac{\partial E_{total}}{\partial w_{11}^{h}} = -0.105 \times 0.2419 \times 0.59$$

$$= -0.0152$$

Updating

$$w_{11}^h = w_{11}^h - \alpha \cdot -0.0152$$

$$= 0.6 - (+0.3) \times (-0.0152)$$

$$\boxed{w_{11}^h = 0.60457}$$

Hidden layer (w_{21}^h)

$$\frac{\partial E_{total}}{\partial w_{21}^h} = \frac{\partial E_{total}}{\partial out\,h_1} \times \frac{\partial out\,h_1}{\partial h_1} \times \frac{\partial h_1}{\partial w_2}$$

$$\frac{\partial E_{total}}{\partial out\,h_1} = \frac{\partial E_2}{\partial y_0} \times \frac{\partial y_0}{\partial out\,h_1}$$

$$= \frac{\partial E_2}{\partial out\,y_0} \times \frac{\partial out\,y_0}{\partial y_0} \times w_{21}^0$$

$$= 0.177 \times 0.146 \times 0.1$$

$$= 0.00258$$

$$\frac{\partial E_{total}}{\partial w_{21}^h} = 0.00258 \times 0.2419 \times 0.1$$

$$= 0.0000625179$$

Updating

$$w_{21}^{h} = w_{21}^{h} - 0.3 \times 0.0000625$$

$$\boxed{w_{21}^{h} = 0.099981}$$

Updated weights

$$w_{11}^{o} = 0.921$$

$$w_{21}^{o} = 0.295$$

$$w_{11}^{h} = 0.60457$$

$$w_{21}^{h} = 0.099981$$